SPX Option Strategy: Straddle

Table of Contents

1. Objective
2. Aim
3. Risk Involved
4. Market Analysis
5. Strategy Implementation
    - a) Index Calculation
    - b) Units Calculation
    - c) Greeks Calculation
    - d) Selection of Strike
    - e) Necessary Assumptions
6. Appendix

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1. Objective

The primary objective of the SPX Option Strategy: Straddle is to capture the volatility risk premium embedded in S&P 500 options. This strategy involves selling at-the-money straddles, which consist of both call and put options.

Mathematics Behind the Objective

- **Straddle Definition**: A straddle involves buying or selling a call and a put option with the same strike price and expiration date. In this strategy, we focus on selling the straddle.
- **Volatility Risk Premium**: The volatility risk premium is the difference between the implied volatility (the market's forecast of future volatility) and the realized volatility (the actual volatility observed). By selling straddles, the strategy aims to profit from the decay of the options' time value as the expiration date approaches, especially if the underlying asset does not move significantly.

The profit from selling a straddle can be expressed mathematically as:

$$\text{Profit} = \text{Premium Received} - \text{Loss from Price Movement}$$

Where:

- **Premium Received**: The total premium collected from selling both the call and put options.
- **Loss from Price Movement**: This occurs if the underlying asset moves significantly away from the strike price, leading to potential losses on the options sold.

Delta-Hedging

To manage the risk associated with price movements, delta-hedging is employed. Delta (Δ) measures the sensitivity of the option price to changes in the underlying asset price. The delta of a straddle can be calculated as:

$$\Delta_{\text{straddle}} = \Delta_{\text{call}} + \Delta_{\text{put}}$$

Where:

- $\Delta_{\text{call}} = N(d_1)$
- $\Delta_{\text{put}} = N(d_1) - 1$

Here, $N(d_1)$ is the cumulative distribution function of the standard normal distribution, and d_1 is calculated using the Black-Scholes formula.

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2. Aim

The aim of this strategy is to replicate the performance from January 2023 to November 2024 using mocked S&P options data, volatilities, and underlying index levels. The strategy will focus on selling straddles that expire in 3-4 months, allowing for effective management of risk and potential profit from market movements.

Interpretation of the Formula

- **Vega Weight**: This is a measure of how much the price of the options will change with a 1% change in implied volatility. It reflects the sensitivity of the options' prices to changes in volatility. A higher vega weight indicates a greater sensitivity to volatility changes.

- **Index Previous Close**: This is the closing value of the underlying index (e.g., S&P 500) from the previous trading day. It serves as a reference point for calculating the potential exposure of the options position.
- **Vega of Call and Vega of Put**: These represent the vega values for the call and put options, respectively. The sum of these two values gives the total vega exposure of the straddle position. This is important because it indicates how much the overall position will be affected by changes in implied volatility.

Overall Interpretation

The formula calculates the number of units to trade based on the vega exposure of the options position relative to the previous close of the index. By multiplying the vega weight by the index's previous close and dividing by the total vega of the call and put options, we determine how many contracts are needed to achieve a desired level of exposure to volatility.

This approach helps in managing the risk associated with changes in implied volatility, allowing the trader to adjust their position size accordingly. The goal is to maintain a balanced exposure to volatility while capturing the volatility risk premium through the sale of straddles.

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3. Risk Involved

Detailed Risk Factors

- **Market Risk**: The strategy is exposed to significant market risk, particularly if the underlying index experiences large price movements in either direction. If the index moves significantly away from the strike price, the losses can be substantial.
- **Volatility Risk**: Changes in implied volatility can impact the profitability of the straddle. A decrease in volatility may lead to losses if the options are not managed properly. Conversely, an increase in volatility can lead to larger losses.
- **Liquidity Risk**: The ability to enter and exit positions may be affected by market conditions, leading to potential slippage or inability to execute trades at desired prices. This is particularly relevant in volatile markets.
- **Execution Risk**: Delays or errors in executing trades can lead to unintended exposure or losses. This includes the risk of not being able to hedge positions effectively.

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4. Market Analysis

The semiconductor sector, represented by the SOX index, is known for its volatility and growth potential. Key factors influencing this market include:

- **Technological Advancements**: Innovations in semiconductor technology can drive demand and affect stock prices.
- **Economic Indicators**: Macroeconomic factors such as interest rates, inflation, and consumer demand can impact market sentiment and volatility.
- **Global Supply Chain Dynamics**: Disruptions in the supply chain can lead to fluctuations in stock prices, creating opportunities for options trading.

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5. Strategy Implementation

a) Index Calculation

The index level at time (t) is calculated using the following formula:

$$\text{Index}_t = \text{Index Previous Close} + \text{Option PnL} - \text{Delta PnL}$$

- **Option PnL**: The profit and loss from the options position can be calculated as:

$$\text{Option PnL} = \begin{cases} (\text{Underlying Price} - \text{Strike Price}) \times \text{Units} & \text{if long position} \\ (\text{Strike Price} - \text{Underlying Price}) \times \text{Units} & \text{if short position} \end{cases}$$

- **Delta PnL**: The profit and loss from the delta hedge can be calculated as:

$$\text{Delta PnL} = (\text{Underlying Price at Time } t - \text{Underlying Price at Time } t-1) \times \text{Units} \times \Delta$$

b) Units Calculation

The number of units to be traded is determined based on the vega exposure and the previous close of the index:

$$\text{Units} = -100 \times \left(\frac{\text{Vega Weight} \times \text{Index Previous Close}}{\text{Vega of Call} + \text{Vega of Put}} \right)$$

c) Greeks Calculation

The Greeks are calculated using the Black-Scholes model:

- **Delta**: Measures the sensitivity of the option price to changes in the underlying asset price. For a call option, Delta is calculated as:

  $$\Delta_{\text{call}} = N(d_1)$$

  For a put option, Delta is calculated as:

  $$\Delta_{\text{put}} = N(d_1) - 1$$

- **Calculation of (d_1)**: The value of (d_1) is calculated using the following formula:

  $$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}$$

  Where:

  - S = Current stock price (underlying asset price)
  - K = Strike price of the option
  - T = Time to expiration in years
  - σ = Volatility of the underlying asset (annualized standard deviation of returns)
  - r = Risk-free interest rate (annualized)

The components of the formula are as follows:

- $\ln\left(\frac{S}{K}\right)$: This term represents the natural logarithm of the ratio of the current stock price to the strike price. It indicates how far the stock price is from the strike price, with a higher value suggesting that the stock is more likely to be in-the-money.

- $\left(r + \frac{\sigma^2}{2}\right)T$: This term adjusts the expected return of the stock price by incorporating both the risk-free rate and the volatility. The term $\frac{\sigma^2}{2}$ accounts for the effect of volatility on the expected return, scaled by the time to expiration T.
- $\sigma\sqrt{T}$: This term represents the standard deviation of the stock price over the time to expiration. It reflects the uncertainty in the stock price, with greater volatility leading to a larger value, indicating a wider range of potential future prices.

- **Vega**: Measures the sensitivity of the option price to changes in volatility.

$$\text{Vega} = S \cdot N'(d_1) \cdot \sqrt{T}$$

- **Theta**: Measures the sensitivity of the option price to the passage of time (time decay).

$$\Theta = -\frac{S \cdot N'(d_1) \cdot \sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

- **Gamma**: Measures the rate of change of Delta with respect to changes in the underlying price.

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}}$$

Where:

- $N(d_1)$ and $N(d_2)$ are the cumulative distribution functions of the standard normal distribution.
- $N'(d_1)$ is the probability density function of the standard normal distribution.

d) Selection of Strike

The strike price for the straddle is selected based on the closest strike to the underlying asset price at the time of trade execution. The process involves:

1. **Filtering available options** for the desired expiration date.
2. **Calculating the absolute distance** between the strike prices and the underlying asset price.
3. **Selecting the strike price** with the minimum distance:

$$\text{Closest Strike} = \text{argmin}(|\text{Strike} - \text{Underlying Price}|)$$

e) Necessary Assumptions

1. **Inception Date**: The strategy will generate the straddle on January 3, 2024, based on the same price on that date. Going forward, the same methodology will be applied.
2. **Interest Rates**: It is assumed that interest rates are zero throughout the strategy's duration.
3. **Frictionless Transactions**: All transactions in options or the underlying asset are assumed to be frictionless, with no bid/offer spread applied & no transaction cost involved.
4. **Flat Implied Volatility**: Implied volatilities are provided and are flat across strikes and maturities.
5. **Data Integrity**: The strategy assumes that the data used for calculations is accurate and complete.

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6. Appendix

Key	Value
Inception Date	3 Jan 2023
End Date	15 Nov 2024
Inception Level	100
Vega Weight	0.00015873