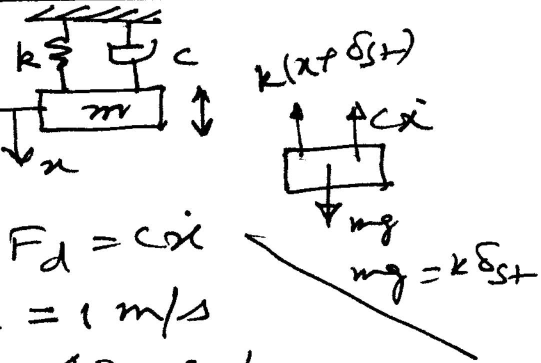
Problem ① The system is:



$k = 1000\,N/m$, $m = 5.5\,kg$

If $F_d$ = damping force, then, $F_d = c\dot{x}$

when $F_d = 40\,N$, $\dot{x} = 1\,m/s$

So, $\underline{\underline{c}} = F_d/\dot{x} = \underline{40\ Ns/m}$

ⓐ The DEOM is: $m\ddot{x} + c\dot{x} + kx = 0$

→ The first step should be computation of $\zeta$. $\zeta = \dfrac{c}{2\sqrt{km}} = 0.2697 < 1$

Hence, the system is underdamped.

So, $x(t) = X_0 e^{(-\zeta\omega_n t)} \sin(\omega_d t + \phi)$

$\underline{\underline{or}}$

$x(t) = e^{(-\zeta\omega_n t)}[A \sin\omega_d t + B\cos\omega_d t]$

Note
→ You may use either form.
→ Be patient with computations, they are ~~a~~ often a bit lengthy.

Here $\omega_n = \sqrt{\dfrac{k}{m}} = 13.484$ rad/s, $\zeta\omega_n = 3.637$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \cancel{3.637}\ 12.984$ rad/s

Then $x = e^{(-3.637t)}(A \sin 12.984t + B\cos 12.984t)$ --- (1)

To evaluate A & B, we use initial conditions

$x(0) = 50\times10^{-3}\,m$, $\dot{x}(0) = 0$

Then, ① gives:- $\underline{\underline{50\times10^{-3} = B\ (\text{in } m)}}$

Now, $\dot{x} = e^{(-3.637t)}(A\times12.984\cos 12.984t - 12.984B\sin 12.984t)$

$- 3.637 e^{(-3.637t)}(A\sin 12.984t + B\cos 12.984t)$ --- (I)

Then $\dot{x}(0)=0$ gives: $0 = 12.984A - 3.637B$

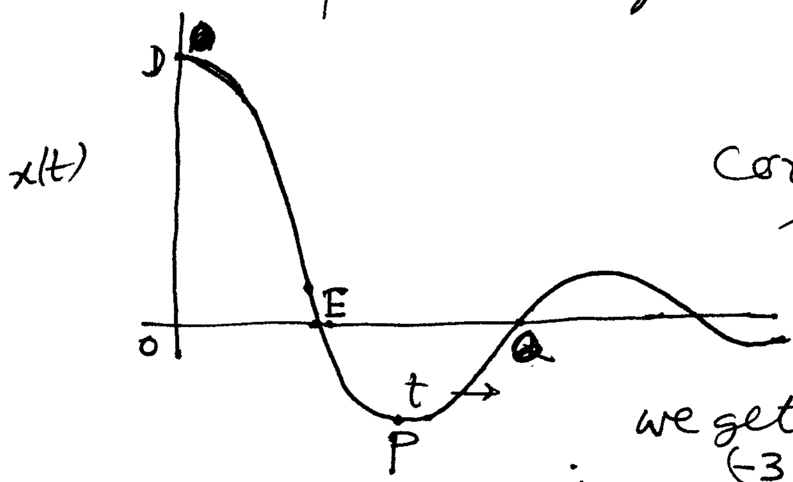
So, $A = 14.0 \times 10^{-3}$ m

Thus, the required expression for displacement is:

$$x(t) = e^{(-3.637t)}\left[14\sin 12.984t + 50\cos 12.984t\right] \text{ mm} \quad \text{(II)}$$

$\boxed{\text{check}}$

Part (b) [A sketch of $x(t)$ versus $t$ may help visualize the situation.




Note that point P corresponds to the highest point.

Substituting values of A & B in (I), we get:

$$\dot{x} = e^{(-3.637t)}\left[-0.700112\sin 12.984t\right] \text{ m/s}$$

At P, $\dot{x}=0 \Rightarrow \sin 12.984t = 0.$

$\Rightarrow$ So $12.984t = 0, \pi, 2\pi$ etc.

Obviously, P corresponds to $t = \frac{\pi}{12.984}$ s $= 0.24195$ s $\simeq 0.242$ s

( $t=0$ corresponds to point A)

Putting this value of $t$ in (II), we get

$x = -20.741 \times 10^{-3}$ m. $= -20.741$ mm.

Hence, required total distance moved $= 50 + 20.741 = 70.741$ mm & time taken is $0.242$ s.

$\boxed{\text{Part c}}$ Here time corresponding to point E is required. At E, $x=0$. From (II), we get $\tan 12.984t = -3.5714$ ~~$12.984t = 105$~~

$\Rightarrow 12.984t = 105.64 \times \pi/180 \Rightarrow t = 0.142$ s.

$\longrightarrow$

**Problem 2** (a) $\delta = \ln\left(\frac{14.4}{1.2}\right)$ or $\ln\left(\frac{1.2}{0.1}\right)$

Hence, $\delta = \underline{2.485}$

(b) $\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$ or $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} = 0.368$

(c) $k_t = \frac{GJ}{l}$, $G = 34.5\times10^9\ N/m^2$, $l = 0.4\ m$,

$J = \frac{\pi d^4}{32} = \frac{\pi \times (9\times10^{-3})^4}{32}\ m^4$

$\therefore k_t = 55.55\ N\text{-}m/rad$, $I_d = 0.6\ kgm^2$

So, $\omega_n = \sqrt{\frac{k_t}{I_d}} = 9.622\ rad/s$

(d) $\omega_d = \omega_n\sqrt{1-\zeta^2} = 8.95\ rad/s$

(e) $T_{damping} = c_t\dot{\theta}$ Hence, damping torque at unit angular velocity is nothing but $c_t$ only.

Now, $\zeta = \frac{c_t}{2I_d\omega_n} \Rightarrow c_t = 4.25\ Nms/rad.$

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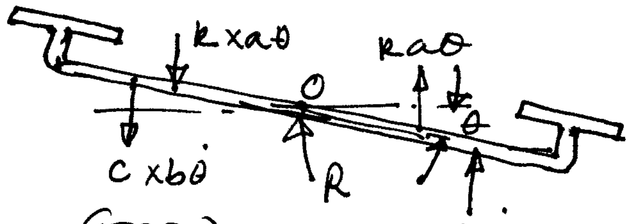
**Problem 3** This problem is a little tricky, be careful.

Let $\theta$ (+ive CW) be the generalized coordinate.



(FBD)

Here $I_O = 0.01\ Kgm^2$
$a = 0.08\ m$, $b = 0.1\ m$,
$k = 5\times10^3\ N/m$, $c = 70\ Ns/m$

$T = \frac{1}{2}I_O\dot{\theta}^2$, $V = 2\times\frac{1}{2}k(a\theta)^2$; $D = 2\times\frac{1}{2}c(b\dot{\theta})^2$

$\Rightarrow$ The DEOM is : $I_O\ddot{\theta} + \cancel{2ka^2}\ 2cb^2\dot{\theta} + 2ka^2\theta = 0$

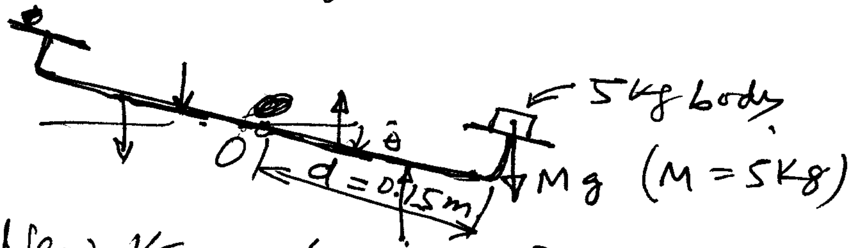
$\therefore \omega_n = \sqrt{\frac{2Ka^2}{I_0}} = 80$ rad/s, $\zeta = \frac{2cb^2}{2I_0\omega_n} = 0.875$

<u>After 5 kg mass is placed:-</u>

We have a new system with 5 kg extra mass on righthand pan. We treat it as a particle & hence, now $I_0'$ = new moment of inertia of this new system about an axis through $O = I_0' = 0.01 + 5 \times (0.15)^2 = 0.1225$ kgm².

→ A new equilibrium position results at $\theta = \theta_0$. <u>Show</u> that (by taking moments about $O$ in the FBD for this <u>new</u> system) $\theta_0 = 0.115$ rad.

→ We may take a new generalized coordinate measured from this $\theta_0$ line. <u>But we can also continue to measure rotation from horizontal position only.</u> You will see that this will introduce an extra constant term in the DEOM & make it nonhomogeneous, as follows:-




New KE $= T' = \frac{1}{2} I_0' \dot{\theta}^2$, $D' = D = 2 \times \frac{1}{2} c (b\dot{\theta})^2$,

$V' = V - Mg \times d \times \theta$ (Note) $= 2 \times \frac{1}{2} k (a\theta)^2 - Mgd\theta$

Then, the DEOM will be: $I_0' \ddot{\theta} + 2cb^2\dot{\theta} + 2ka^2\theta = Mgd$ — (1)

Then, $\zeta' = \zeta_{new} = \dfrac{2Cb^2}{2\sqrt{2ka^2 \times I_0'}} = 0.25 < 1.$

$\omega_n' = \sqrt{\dfrac{2ka^2}{I_0'}} = 22.86$ rad/s.

$\zeta\omega_n' = 5.714$ & $\omega_d' = \omega_n'\sqrt{1-\zeta'^2} = 22.13$ rad/s

Hence, $\theta(t) = e^{(-5.714t)}[A\sin 22.13t + B\cos 22.13t] + \dfrac{Mgd}{K}$

→ Find $\dot{\theta}(t)$

→ Use initial conditions: $\theta(0) = 0$ & $\dot{\theta}(0) = 0$.

Do the Computations in detail.

These should give $A = -0.02969$ & $B = -0.11496$.

Then, $\dot{\theta} = 0$ would give

$22.13t = 0, \pi, 2\pi$ -- etc.

We have to take $22.13t = \pi$

$\Rightarrow t = 0.14196$ s.

Then, $\theta(0.14196) = 0.166$ rad $= 9.51°$

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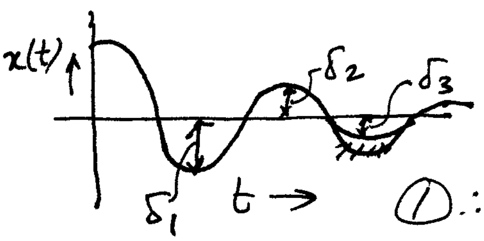
Problem 4 $\quad I_d = 50\times10^{-3}\times(12\times10^{-3})^2 = 72\times10^{-7}$ kgm²

$\tau_d = \dfrac{2\pi}{\omega_d} = 4.2$ s $\Rightarrow \omega_d = 1.496$ rad/s



(Not to scale)

$\dfrac{\delta_1}{\delta_2} = \dfrac{90}{12}, \quad \dfrac{\delta_2}{\delta_3} = \dfrac{90}{12} \Rightarrow \dfrac{\delta_1}{\delta_3} = \left(\dfrac{90}{12}\right)^2$

(1): Logarithmic decrement $= \delta = \ln\left(\dfrac{\delta_1}{\delta_3}\right)$

or, $\delta = 2\ln\left(\dfrac{90}{12}\right) = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0.5399$

(2) $\omega_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}} = 1.7774$ rad/s

(3) $\omega_n = \sqrt{\dfrac{K_t}{I_d}} \Rightarrow K_t = \omega_n^2 I_d = 22.75\times10^{-6}$ Nm/rad →

4 $C_t = 2\zeta I_d \omega_n \quad \left[\because \zeta = \frac{C_t}{2 I_d \omega_n}\right]$

$= 13.82 \times 10^{-6}$ Nm/rad/s

Part (b)

$\theta = e^{-\zeta\omega_n t}(A \sin\omega_d t + B \cos\omega_d t)$

(i) $= e^{(-0.95956t)}[A \sin 1.496t + B \cos 1.496t]$

$\dot\theta = e^{(-0.95956t)}[1.496A \cos 1.496t - 1.496B \sin 1.496t]$

(ii) $-0.95956\, e^{(-0.95956t)}[A \sin 1.496t + B \cos 1.496t]$

(Patience, patience please!)

→ Let $\dot\theta(0) = \omega_0$, the unknown initial angular velocity generated by the angular impulse. (Such a short duration impulse generates a finite angular velocity, but no appreciable angular displacement, do you visualize?)

So, $\theta(0) = 0$.

[ Also, if $T$ is the time reqd to swing through $25^\circ$, then $\dot\theta(T) = 0$ when $\theta(T) = \frac{25 \times \pi}{180}$ rad.

Using $\theta(0)=0$ & $\dot\theta(0) = \omega_0$ in (i) & (ii), you should get $B = 0$, $A = \frac{\omega_0}{1.496}$

Also, $\dot\theta(T) = 0$ & $\theta(T) = \frac{25\pi}{180}$ rad would yield $T = 0.6684$ s, $A = 0.98478$ & $\omega_0 = 1.4732$ rad/s

Finally, the energy supplied by the impulse

goes into making the initial kinetic energy of the rotor of magnitude $\frac{1}{2} I_d \omega_0^2 = 7.813 \times 10^{-6} J$.
Hence, the energy supplied by the impulse $= 7.813\ \mu J$

Problem 5 The bag strikes the platform with velocity $= \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.4}$ m/s
Conservation of momentum gives

$$870\, V = 70 \sqrt{2 \times 9.81 \times 0.4}$$

$\Rightarrow V = 0.2254$ m/s, which is our $\dot{x}(0)$.
Assuming inelastic collision, the bag + platform is the new system & there will be a new mass & there will be a new equilibrium position, note. However, we can measure $x$ from earlier equilibrium position only. This gives the following DEOM:

$$870 \ddot{x} = -100 \times 10^3 x - 500 \dot{x} + 70 \times 9.81$$

$$\Rightarrow \quad \ddot{x} + 0.5747 \dot{x} + 114.94 x = 0.7893 \quad \text{--- (a)}$$

Now note the following interesting point:-
Let the DEOM be $m\ddot{x} + c\dot{x} + kx = F$
Then, $\ddot{x} + \frac{c}{m} \dot{x} + \frac{k}{m} x = F/m$
But $c/m = 2\zeta\omega_n$, $k/m = \omega_n^2$. Hence,

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = F/m \quad \text{--- (b)}$$

Now compare (a) & (b). Then, $2\zeta\omega_n = 0.5747$ &

$\omega_n^2 = 114.94 \Rightarrow \omega_n = 10.721$ rad/s

$2\zeta\omega_n = 0.5747 \Rightarrow \zeta = 0.0268 < 1.$

→ It is not essential to do it this way, but the form ③ is used in many text books etc. & you should be familiar with it!

Then, $\omega_d = 10.717$ rad/s

Next, note that the P.I. of ⓐ is simply $x_p = \frac{0.7893}{114.94} = 6.867 \times 10^{-3}$

Then, the general solution of ⓐ is:-

$$x = 6.867 \times 10^{-3} + e^{(-0.28735t)} [A \sin 10.717t + B \cos 10.717t]$$

Now, $x(0) = 0$, $\dot{x}(0) = 0.2254$ m/s.

These would give $A = 0.02085$.

$B = -0.006867$

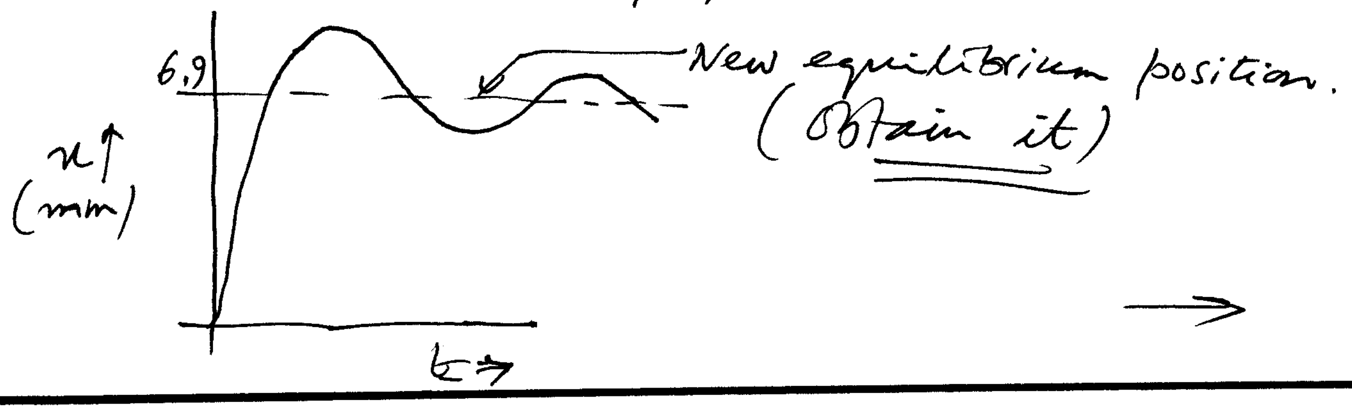
→ At the lowest position, $\dot{x} = 0$

$\Rightarrow \tan 10.717t = -3.3343$

$\Rightarrow t = 0.174$ s.

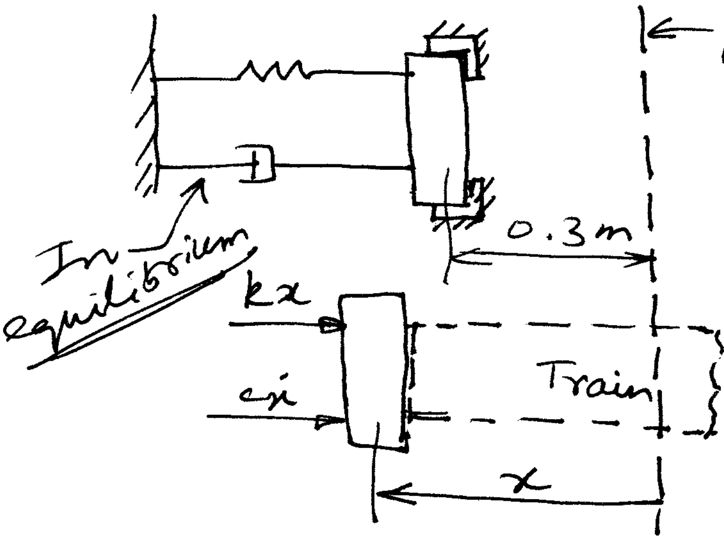
$\boxed{\text{Carry out the details}}$

Then, $x(0.174\text{s}) = 27.7$ mm.




→

Problem 6 Note that the spring initial compression is provided by some pre-compression device like the one shown in the figure below:




← position of buffer with no pre-compression.

If $x$ is measured the way shown, the DEOM becomes

$\ddot{x} + 2.4\dot{x} + 0.6x = 0$, check this!

(You may use a different $x$, of course, measured from static eqbm position of buffer)

Then, $\omega_n = \sqrt{0.6} = 0.7746$ rad/s,

$2\zeta\omega_n = 2.4 \Rightarrow \underline{\zeta = 1.55} > 1$

→ So, this system is overdamped.

So, $x(t) = A e^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + B e^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t}$

etc.

Now Complete the solution

END of Tu-2, Part I