Books

⇒ Introduction to Tribology of Bearings — B.C Majumdar

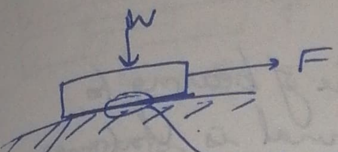
⇒ Theory of Hydrodynamic Lubrication — Pinkus

⇒ The Principles of Lubrication — Cameron

⇒ Fluid Flow Lubrication — Hamrock & Dowson

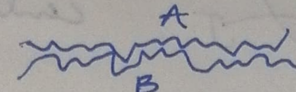
Assignment Book of Online

Lubrication and Rotor Dynamics



(Sliding/Contact) (Dry friction)

μ_s, μ_K → Zoom →



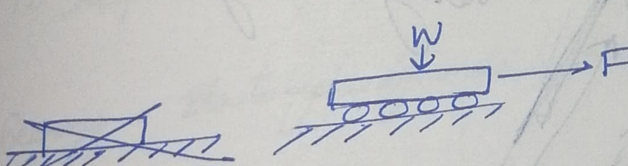
→ F_s, F_k (motion)

result of welding of peaks (Enough pressure and time)

Less or Instanteous welding
∴ Less force F_k needed.

Abrasion → Soft and hard material wearing out
(Material removed from soft material)
(Machining Process) In Abrasion, peaks will not be welded (only penetration)

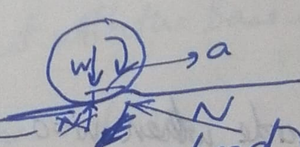
Adhesion → Welding between peaks



Rolling ~~Friction~~
(Friction is due to relative motion on two points)

(No relative friction)

We call it as rolling resistance



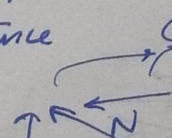
Reaction is not vertical but at some angle

(Microscopic Bump ahead)

a (horizontal distance) is the rolling resistance

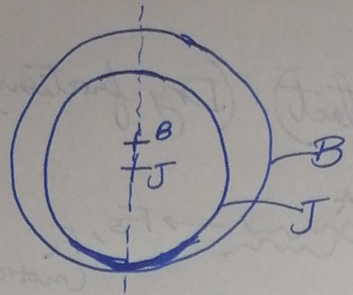
a is a dimensional quantity.

Components



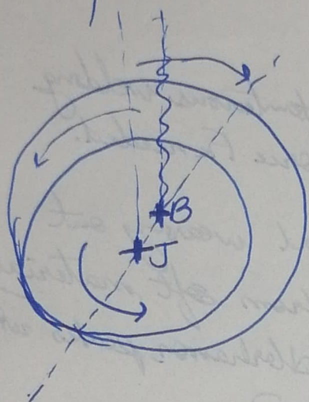
Hydrodynamic bearing:-

Stationary:



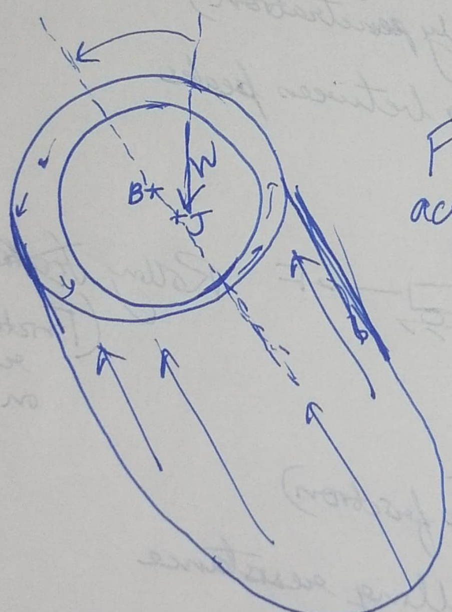
Line from centre of bearing to centre of journal is stationary.

Just started:-



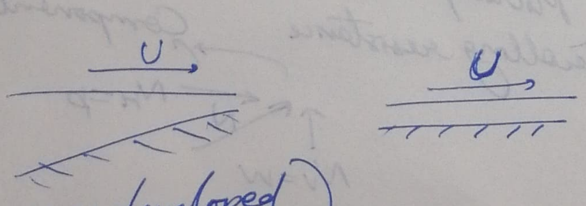
Fluid dragged to narrow zone, pressure increases, ∴ journal pushed away as designated speed is approached

Designated speed



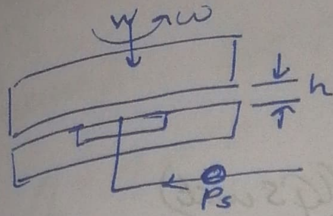
Fluid pressure balances the load W acting on the journal

Centres coincide when $W=0$ in ideal case.

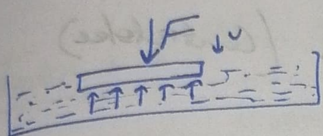


(pressure developed in narrow zone)

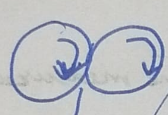
Hydrostatic bearing:-



Squeeze film Bearings:



When two balls come close in a ball bearing (when cage is not present):-

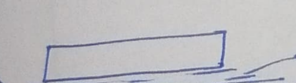


Friction at contact due to relative motion

∴ To keep bearings seperated, cage is necessary

Ball Bearing → Elastohydrodynamic lubrication (EHL)

Solid Lubricants

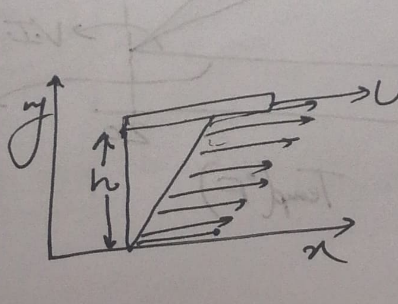
a)  → Low shear strength (Reduce friction)

b) Surface protection → should not react with the base surface

c) Good bonding → should stick to the surface.

Properties of Lubricant

Viscosity



For Newtonian fluids

$$\tau \propto \frac{\partial u}{\partial y}$$

$$\tau = \eta \frac{\partial u}{\partial y}$$

proportionality const = coefficient of dynamic / absolute viscosity

$\tau = \eta \frac{U}{h}$ (Considering the case)

$\eta = \frac{\tau h}{U} = \frac{Ns}{m^2} = Pa\cdot s \rightarrow$ SI unit

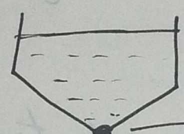
$\frac{dyne \cdot s}{cm^2} \rightarrow$ Poise (CGS unit)

$Cp = \frac{Poise}{100}$ (centipoise)

Kinematic Viscosity $\nu = \frac{\eta}{\rho} = m^2/s$ | $\frac{cm^2}{s}$ = Stoke

$CS = \frac{Stoke}{100}$ (Centistoke)

Saybolt Universal Seconds



Kinematic Viscosity

$\nu = \left(0.22t - \frac{180}{t}\right)$

$\eta = \rho \nu$

Effects of Temperature on Viscosity

Temp↑ Oil↓ Gas↑

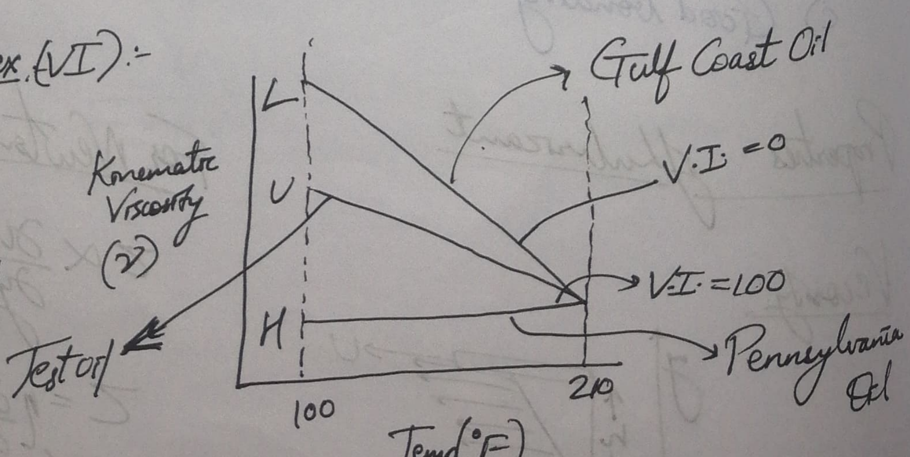
ASTM: $\log_{10} \log_{10} (\nu + 0.8) = n \log_{10} T + C$

$n, C \rightarrow$ constants

Viscosity Index (VI):-

$V.I. = \frac{U-L}{H-L} \times 100$

$L, U, H \rightarrow$ Kinematic Viscosity (Centistoke)



Effect of Pressure on Viscosity

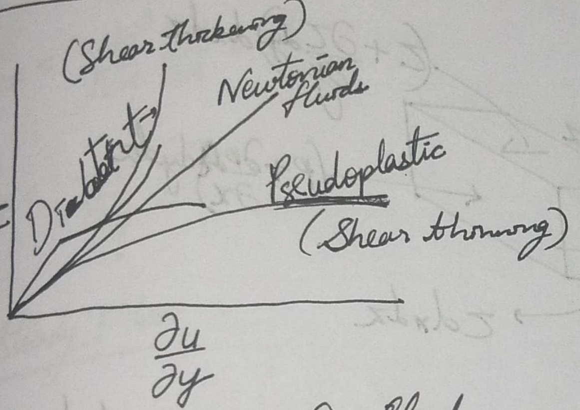
Pressure ↑ Oil ↑ Gas ↑

$$\eta = \eta_0 e^{\alpha p}$$

(Barus relation)

α - pressure viscosity coefficient

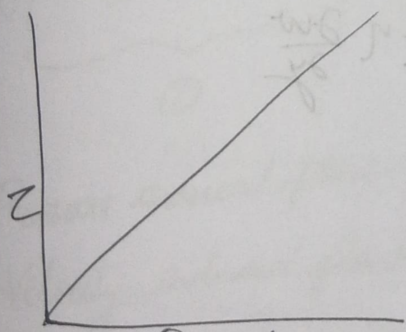
Newtonian and Non-Newtonian Fluids



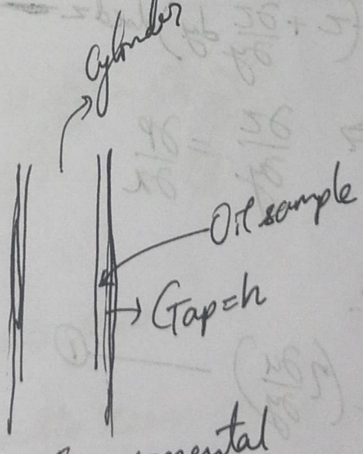
Shear Thickening → Corn Starch

Shear Thinning → Paint

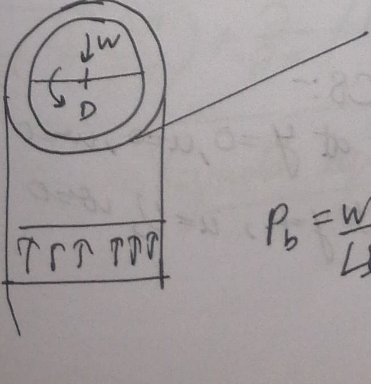
Experiment:-



$$\frac{\partial u}{\partial y} = \frac{U}{h}$$

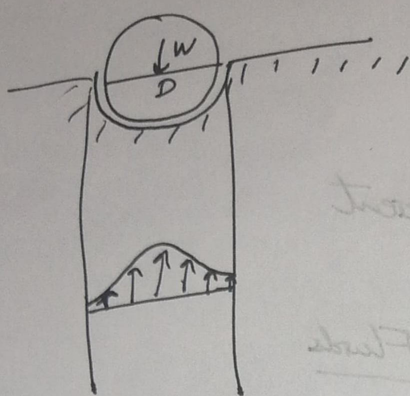


Experimental Setup

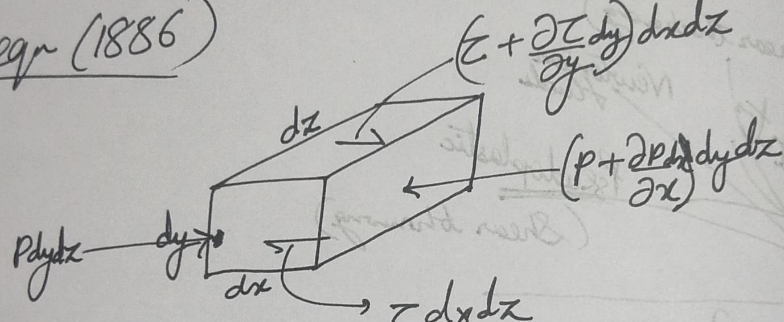


$$P_b = \frac{W}{LD}$$ (Bearing Pressure)

Tower Bearing (1883)



Reynolds eqn (1886)



x-direction

$$p\,dy\,dz + \left(\tau + \frac{\partial \tau}{\partial y} dy\right) dx\,dz - \left(p + \frac{\partial p}{\partial x} dx\right) dy\,dz - \tau\, dx\,dz = 0$$

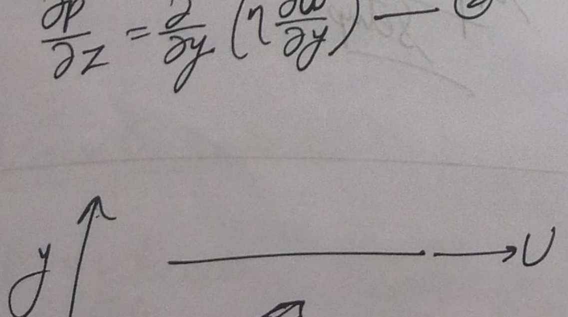
$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$$

$$\tau_{yx} = \eta \frac{\partial u}{\partial y} \qquad \tau_{yz} = \eta \frac{\partial w}{\partial y}$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\eta \frac{\partial u}{\partial y}\right) \quad \text{— (1)}$$

Similarly,

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial y}\left(\eta \frac{\partial w}{\partial y}\right) \quad \text{— (2)}$$



BCS:-

at $y = 0$, $u = 0$, $w = 0$

at $y = h$, $u = U$, $w = 0$

Integrating,

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y(y-h) + \frac{U}{h}y$$

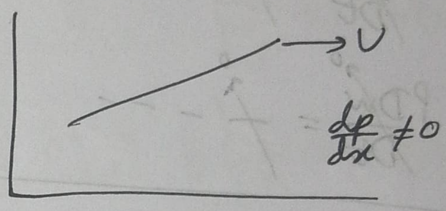
$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y(y-h)$$

Flow in x and z directions per unit width,

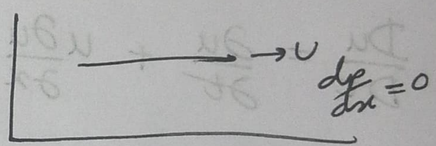
$$q_x = \int_0^h u\,dy \;, \quad q_z = \int_0^h w\,dy$$

$$q_x = \frac{-h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{Uh}{2}$$

$$q_y = \frac{-h^3}{12\eta}\frac{\partial p}{\partial z}$$



For Steady Flow :-



$$\frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} = 0$$

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial z}\right)}_{1} = \underset{2}{6U\frac{\partial h}{\partial x}} + \underset{3}{6h\frac{\partial U}{\partial x}}$$

1 Pressure induced flow - Poiseuille flow

2 Velocity induced flow - Couette term

3 Stretch term

As U is constant w.r.t. x,

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x}}$$

Reynolds eqn

Lubrication Technology

Navier-Stokes eqn

$$\rho \frac{\cancel{Du}^{\,0}}{Dt} = \cancel{\rho X}^{\,0} - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left\{\eta\left[2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right]\right\}$$
$$+ \frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)\right]$$

$$\rho \frac{\cancel{Dv}^{\,0}}{Dt} = \cancel{f}^{\,0} - - -$$

$$\rho \frac{\cancel{Dw}^{\,0}}{Dt} = \cancel{f}^{\,0} - - -$$

Momentum equations

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \rightarrow \text{Continuity equation}$$

Assumptions of thin-film lubrication:-

① Inertia & Body forces are negligible compared with the pressure & viscous terms.

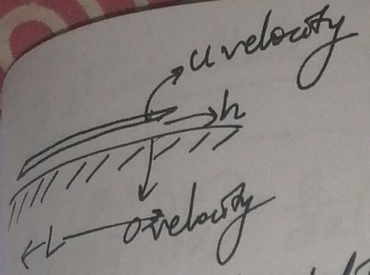
② Film is thin, there is no variation of pressure across the fluid film. $\left(\frac{\partial p}{\partial y} = 0\right)$.

③ No slip across the fluid boundaries

④ No external forces act on the film

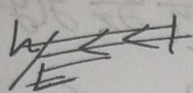
⑤ Flow is viscous and laminar

⑥ Derivatives of u & w w.r.t y are much larger than the other derivatives of velocity components.



$\frac{\partial u}{\partial y}$ & $\frac{\partial w}{\partial y}$ are large

1) Height of fluid film is very small compared to the length of the contact.



$$\underline{h/L < 10^{-3}}$$

Using assumptions,

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\eta \frac{\partial u}{\partial y}\right)$$

$$0 = -\frac{\partial p}{\partial y} \Rightarrow p = f(x, z)$$

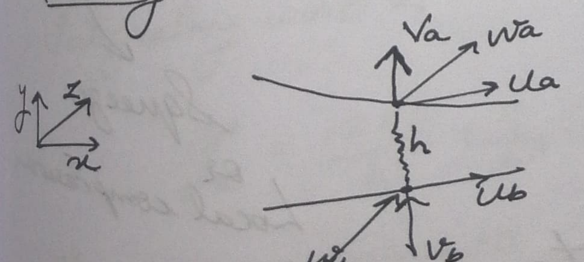
$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial y}\left(\eta \frac{\partial w}{\partial y}\right)$$

We obtain,

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y^2 + C_1 y + C_3$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y^2 + C_2 y + C_4$$

<u>Geometry</u>:-



h varies across <u>x and z</u>.

<u>B.C.S.</u> :-

at $y = 0$, $u = u_b$, $w = w_b$

at $y = h$ = $u = u_a$, $w = w_a$

Using BCs, we obtain,

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y(y-h) + \left(\frac{h-y}{h}\right)u_b + \frac{y}{h}u_a$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y(y-h) + \left(\frac{h-y}{h}\right)w_b + \frac{y}{h}w_a$$

Substitute in continuity eqn, we have,

$$\frac{\partial}{\partial y}(\rho v) = -\frac{1}{2}\left\{\frac{\partial}{\partial x}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial x} y(y-h)\right] + \frac{\partial}{\partial z}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial z} y(y-h)\right]\right\}$$
$$- \frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-y}{h}\right)u_b + \frac{y}{h}u_a\right\}\right] - \frac{\partial}{\partial z}\left[\rho\left\{\left(\frac{h-y}{h}\right)w_b + \frac{y}{h}w_a\right\}\right] - \frac{\partial}{\partial t}$$

Integrating w.r.t. y,

$$\rho(v_a - v_b) = -\frac{1}{2}\left\{\int_0^h \frac{\partial}{\partial x}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial x} y(y-h)\right]dy + \int_0^h \frac{\partial}{\partial z}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial z} y(y-h)\right]dy\right\}$$
$$- \int_0^h \frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-y}{h}\right)u_b + \frac{y}{h}u_a\right\}\right]dy$$
$$- \int_0^h \frac{\partial}{\partial z}\left[\rho\left\{\left(\frac{h-y}{h}\right)w_b + \frac{y}{h}w_a\right\}\right]dy - h\frac{\partial \rho}{\partial t}$$

Applying Leibnitz rule,

$$\int_0^h \frac{\partial}{\partial x} f(x,y,z)\,dz = \frac{\partial}{\partial z}\int_0^h f(x,y,z)\,dz - f(x,y,h)\frac{dh}{\partial x}$$

$$\boxed{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial z}\right) = \left(\frac{U_a+U_b}{2}\right)\frac{\partial(\rho h)}{\partial x} + \left(\frac{W_a+W_b}{2}\right)\frac{\partial}{\partial z}(\rho h) + \frac{\partial(\rho h)}{\partial t}}$$

Generalized Reynolds equation

Squeeze or Local compression

LHS → Poiseuille term

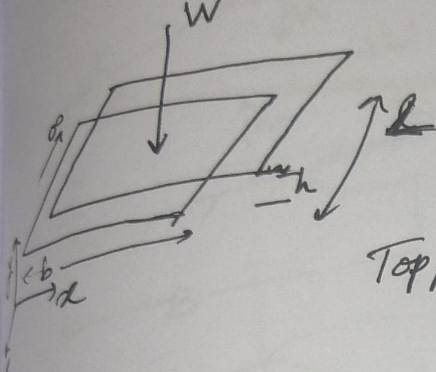
RHS 1st 2 terms → Couette terms

last term → Squeeze or Local Compression

2D steady State ~~Reynolds eqn~~

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial z}\right)=\left(\frac{U_a+U_b}{2}\right)\frac{\partial h}{\partial x}}$$

$\left.\begin{array}{l} U_a = U \\ U_b = 0 \end{array}\right\}$ We get Reynolds eqn.



$h = f(z, x) \rightarrow$ Profile is known (input)

Top plate is moving.

We need to find pressure as a fn of x and z.

If $L \geq 5B$, we take plate to be long in one direction and assume as ∞ in that direction

$\therefore \frac{\partial p}{\partial z} = 0 \Rightarrow P(x)$, also $h(x)$.

Now we have,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) = \left(\frac{U_a+U_b}{2}\right)\frac{\partial h}{\partial x}$$

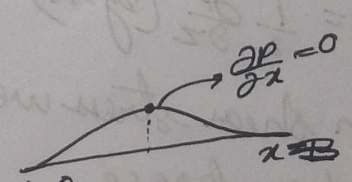
~~$U_a = U, U_b = 0$~~ Let $U = \frac{U_a+U_b}{2}$

Integrating w.r.t. x,

$$\frac{\partial p}{\partial x} = \frac{12\eta U}{h^2} + \frac{\eta}{h^3}C_1$$

At $x=0$, guage pressure $=0$
At $x=B$,

As plate is loaded, pressure profile $\Rightarrow$



Let at $h = h_m$, $\frac{\partial p}{\partial x} = 0$ | $h_m = h(x_m)$

$$\Rightarrow \boxed{\frac{\partial p}{\partial x} = 12\frac{\eta U}{h^3}(h - \underbrace{h_m}_{\text{unknown}})}$$

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6\eta U\frac{\partial h}{\partial x}$$

$p(x,z) = ?$ based on $h(x,z)$

Operating Parameters

Load Carrying Capacity $W = \int p\,dx\,dz$

Flow

$$Q_x = \int_0^h u\,dy, \quad Q_z = \int_0^h w\,dy$$

$$Q_x = -\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{hU}{2}$$

$$Q_z = -\frac{h^3}{12\eta}\frac{\partial p}{\partial z}$$

$u_a = U$

$u_b = 0$

$w_a = w_b = 0$

Shear Stress

$$\tau_x = \eta\frac{\partial u}{\partial y}, \quad \tau_z = \eta\frac{\partial w}{\partial y}$$

Substituting expressions for u and w we have,

$$\tau_x = \frac{1}{2}\frac{\partial p}{\partial x}(2y-h) + \frac{\eta U}{h}$$

$$\tau_z = \frac{1}{2}\frac{\partial p}{\partial z}(2y-h)$$

from shear stress we can calculate the friction force,

Friction force

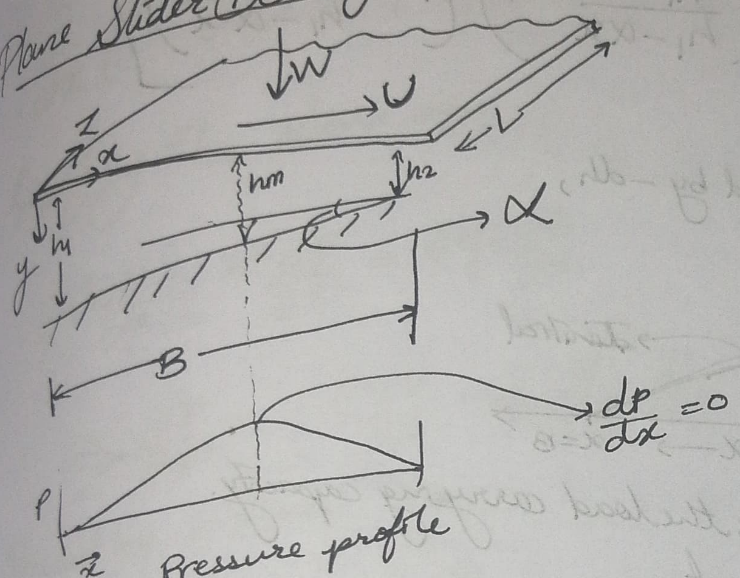
$$\tau_x|_{y=0} = -\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h}$$

$$\tau_x|_{y=h} = \frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h}$$

~~Friction force~~

$$F|_{y=0,h} = \int_0^Z \int_0^x \left[\mp \frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h} \right] dx\,dz$$

Plane Slider Bearing



$L \to \infty$

$L/B >> 1$

We use $U_a = U$, $U_b = 0$

$\therefore$ $U/2 \to$ avg.

$$\frac{\partial p}{\partial x} = \frac{12\eta\,(U/2)}{h^3}(h - h_m) = \frac{6\eta U}{h^3}(h - h_m)$$

$$\tan\alpha = \frac{h_1 - h_2}{B}$$

$$\alpha = \frac{h_1 - h_2}{B}$$

$$h(x) = h_1 - \alpha x$$

$$\frac{dP}{dx} = \frac{6\eta U}{(h_1 - \alpha x)^3}(h_1 - \alpha x - h_m)$$

Integrating w.r.t x, we obtain,

$$P = 6\eta U \int \left[\frac{1}{(h_1 - \alpha x)^2} - \frac{h_m}{(h_1 - \alpha x)^3} \right] dx + C_1$$

We have 2 BCs, $P = 0$, at $x = 0, B$, to find h_m and C_1.

$$h_m = \frac{2 n h_2}{n+1} \qquad \text{where } n = \frac{h_1}{h_2}$$

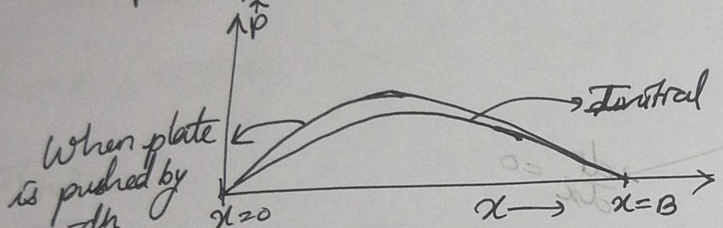
↓

Attitude

$$G = \frac{-6\eta U}{\alpha h_2}\left(\frac{1}{n+1}\right)$$

$$P = \frac{6\eta U B}{h_2^2}\,\frac{1}{(n^2-1)}\left[\left(\frac{h_1}{h_1-\alpha x}-1\right)\left(1-\frac{h_2}{h_1-\alpha x}\right)\right]$$

When ~~pressure~~ plate is pushed by $-dh$,



Area under the curve gives the load carrying capacity.

When $h_2 \to 0$, pressure shoots up.

Load carrying capacity, $W = L\int_0^B p\,dx$

$$\boxed{W = \frac{6\eta U B^2 L}{h_2^2}\,\frac{1}{(n-1)^2}\left[\ln n - \frac{2(n-1)}{n+1}\right]}$$

↓

Inverse solution for Lubrication

__Shear Stress__

$$\tau = \frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta U}{h} \qquad \text{(At moving surface)}$$

Friction force

$$F = L\int_0^B \tau\,dx$$

$$\therefore F = \frac{\eta U B L}{h_2}\,\frac{1}{(n-1)}\left[4\ln n - \frac{6(n-1)}{n+1}\right]$$

$$\mu = \frac{F}{W} = \frac{h_2}{B}\left[\frac{2(n^2-1)\ln n - 3(n-1)^2 \cancel{-3(n-1)^2}}{3(n+1)\ln n - 6(n-1)}\right]$$

Numerical Solution to One-Dimensional problem

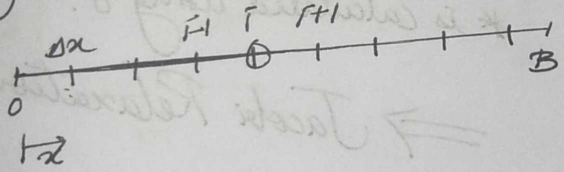
$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 6\eta U \frac{\partial h}{\partial x}$$ (One-Dimensional differential equation)

Simplifying the above eqn,

$$3h^2 \frac{\partial p}{\partial x}\frac{dh}{dx} + h^3 \frac{\partial^2 p}{\partial x^2} = 6\eta U \frac{\partial h}{\partial x}$$

$$\Rightarrow \frac{\partial^2 p}{\partial x^2} = \left(\frac{6\eta U}{h^3} - \frac{3}{h}\frac{\partial p}{\partial x}\right)\frac{\partial h}{\partial x}$$ (Input $\rightarrow h = h_1 - \alpha x$)

Proceed by FDM:-



$$\frac{\partial P_i}{\partial x} = \frac{P_{i+1} - P_{i-1}}{2\Delta x}$$

$$\frac{\partial^2 P_i}{\partial x^2} = \cancel{\frac{P_{i+1} - P_i}{\Delta x}} \quad \frac{\frac{\partial P_{i+1/2}}{\partial x} - \frac{\partial P_{i-1/2}}{\partial x}}{\Delta x}$$

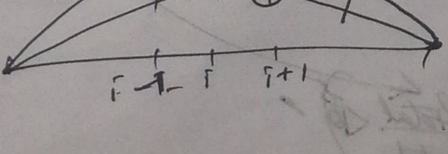
$$= \frac{\frac{P_{i+1} - P_i}{\Delta x} - \frac{P_i - P_{i-1}}{\Delta x}}{\Delta x}$$

$$= \frac{P_{i+1} - 2P_i + P_{i-1}}{\Delta x^2}$$

$$\frac{\partial h}{\partial x} = \frac{h_{i+1} - h_{i-1}}{2\Delta x}$$

Substituting and rearranging, we obtain,

$$P_i = -\frac{\Delta x^2}{2}\left[\frac{6\eta U}{h_i^3} - \frac{3}{h_i}\left(\frac{P_{i+1} - P_{i-1}}{2\Delta x}\right)\right]\left(\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right) + \frac{P_{i+1} + P_{i-1}}{2}$$



* is calculated using O

Gauss-Seidal Relaxation

Faster convergence rate

MATLAB

$x = [0, - - - -];$ n nodes use linspace

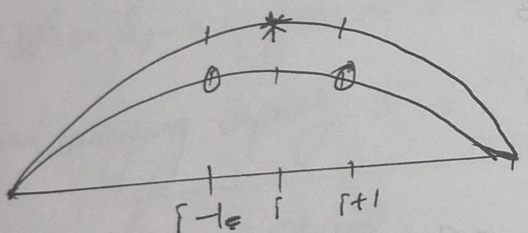
$h = h_1 - \alpha x$

$P = [0, 0 - - - - 0]$ $\left(\begin{matrix} P_1 = 0 \\ P_n = 0 \end{matrix}\right)$

```
while( )
Pold = P
for i=2:n-1
    P(i) = - - -
end
```

$\left.\right\}$ 1 iteration

$$\frac{\Sigma P - \Sigma Pold}{\Sigma Pold} > 10^{-5}$$



* is calculated using o.

$\Rightarrow$ Jacobi Relaxation

$P(i) = - - - - f(Pold)$

Jacobi is useful for unstable solutions

MATLAB

```
x=[0, - - - ];
h=h1-αx
P=[0 0 - - - 0]
while ()
Pold = P
for i=2:n-1
    RHS = f(P)
    E(i)=RHS-P(i)
    P(i)=P(i)+1.25E(i)
end
```

$$\left(\frac{\Sigma P - \Sigma Ptotal}{\Sigma Ptotal} < 10\right)$$

Overrelaxation factor

$\underbrace{1.2 - 1.3}$ we observe stability & faster convergence

(Overrelaxation)

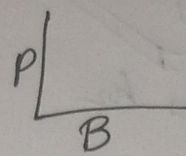
If we use 0.85, we call it underrelaxation

(0.8 — 0.9)

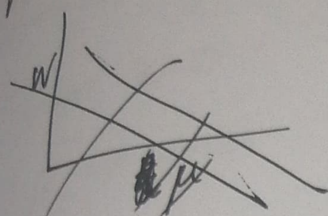
(for stability)

Take some geometry,
h_1, $\alpha \rightarrow$ constants
find pressure profile

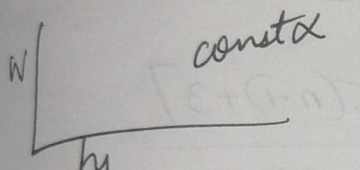
Take n atleast 100.

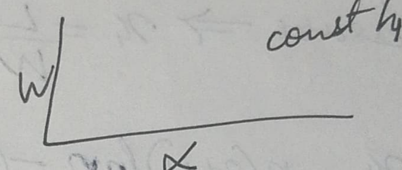


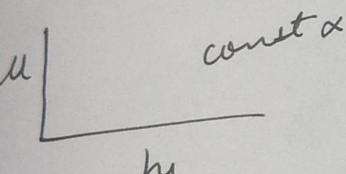
$h_2 \rightarrow$ constant value

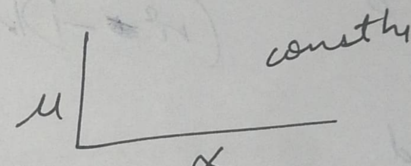


use sum for integration ($\Sigma x \Delta x$)
from p find $\frac{dp}{dx}$
find W
find Friction forces
find μ.

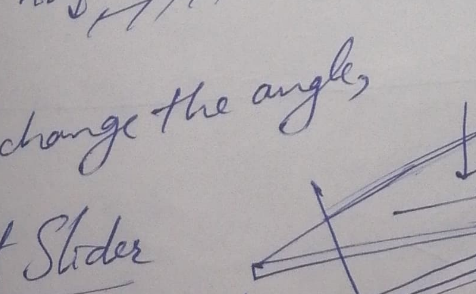








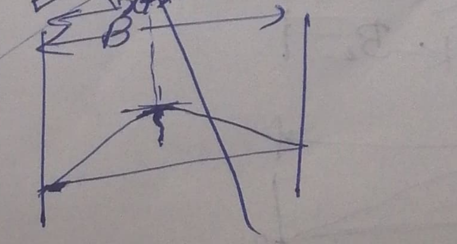
Optimum Performance of Slider Bearing



Angle remains same if W increases.

To change the angle,

Pivot Slider Bearing

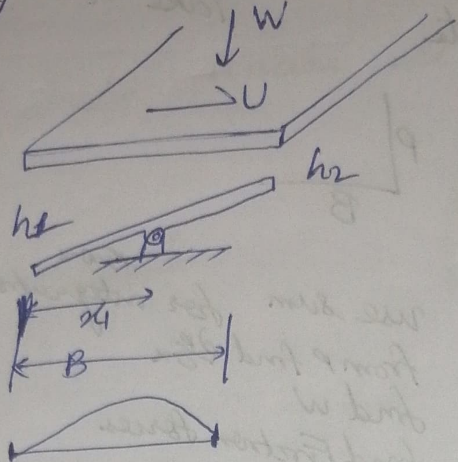


Angle changes

$n = \frac{h_1}{h_2}$

Pivot Slider Bearing

$n = \frac{h_1}{h_2}$



Position of Pivot (x_1)

Centre of Pressure $\Rightarrow x_1 = \frac{L}{W}\int_0^B P\,dx$

$$\frac{x_1}{B} = \frac{n(2+n)\ln n - (n-1)[2.5(n-1)+3]}{(n^2-1)\ln n - 2(n-1)^2}$$

For maximum load,

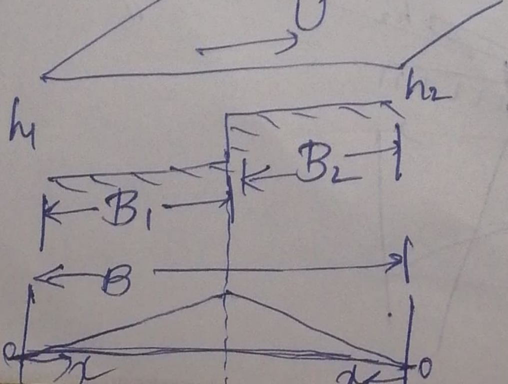
$\frac{dW}{dn} = 0 \Rightarrow n = 2.18$

$W_{max} = 0.1602 \frac{\eta U L B^2}{h_2^2}$

$\frac{x_1}{B} = 0.578$

→ Optimum load will be obtained by the above condition

Rayleigh Step Bearings



$$P = 6\eta U\left[\frac{h - h_m}{h^3}\right]x + C_1$$

Region B_1

$P = 0$, at $x = 0$

$P = P_c$ at $x = B_1$

Condition,

$C_1 = 0, \quad P_c = 6\eta U \dfrac{h_1 - h_m}{h_1^3} B_1$

Region B_2

$P = 0$ at $x = 0$

$P = P_c$ at $x = B_2$

$C_2 = 0, \quad \cancel{P_c = 6\eta U \dfrac{(h_2 - h_m)}{h_2^2}}$

$$P_c = 6\eta U \frac{(h_m - h_2) B_2}{h_2^3}$$

$$\Rightarrow h_m = \frac{h_1 h_2 (B_1 h_2^2 + B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3}$$

Region B_1

$$P_1 = \frac{6\eta U}{h_1^2}\left[1 - \frac{h_2 (B_1 h_2^2 - B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3}\right]x$$

Region B_2

$$P_2 = \frac{6\eta U}{h_2^2}\left[\frac{h_1 (B_1 h_2^2 + B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3} - 1\right]x$$

Load Carrying Capacity :-

$$W = L\int_0^{B_1} P_1\, dx + L\int_0^{B_2} P_2\, dx$$

$$W = \frac{3\eta U L B B_2 (B-B_2)(n-1)}{(B_2 n^3 + B - B_2) h_2^2}$$

$$\frac{dW}{dB_2} = 0 \quad \& \quad \frac{dW}{dn} = 0$$

$$B_2 = \frac{B}{2n^3 - 3n^2 + 1} \qquad n = 1.866$$

$$\frac{B_1}{B_2} = 2.549 \qquad W_{max} = 0.2052 \frac{\eta U L B^2}{h_2^2}$$

$B_1 = B_2 = 1$
$h_1 = 2,\ h_2 = 1$
$\Rightarrow h_m = 1.11$

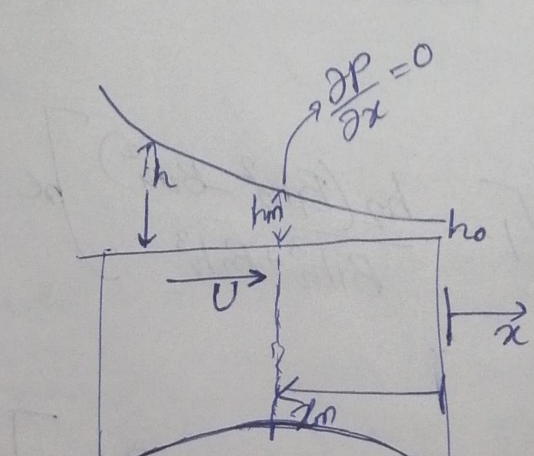
Physically impossible

$\therefore$ P_c is at h_m

Exponential Film

$$h = h_0 e^{-\alpha x}$$

$$h_m = h_0 e^{-\alpha x_m}$$



$$\frac{\partial p}{\partial x} = 6\eta U \left[\frac{h - h_m}{h^3}\right]$$

$$\frac{\partial p}{\partial x} = 6\eta U \left[\frac{h_0 e^{-\alpha x} - h_0 e^{-\alpha x_m}}{h^3}\right]$$

$$p = \frac{6\eta U}{h_o^2}\left[\frac{e^{2\alpha x}}{2\alpha} - \frac{e^{3\alpha x}e^{-\alpha x_m}}{3\alpha}\right] + C_1$$

BCs:-

at $x=0$, $p=0$

at $x=-\infty$, $p=0$

$$\therefore C_1 = 0, \quad e^{-\alpha x_m} = 3/2$$

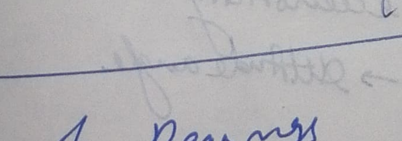
$$p = \frac{3\eta U}{\alpha h_o^2}\left(e^{2\alpha x} - e^{3\alpha x}\right)$$

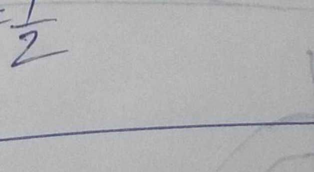
$$W = L\int_{-\infty}^{0} p\,dx = \frac{3\eta U}{h_o^2}\left[\frac{e^{2\alpha x}}{2\alpha} - \frac{e^{3\alpha x}}{3\alpha}\right]_{-\infty}^{0}$$

$$W = \frac{\eta U L}{2h_o^2\alpha^2}$$ (Check with Book)

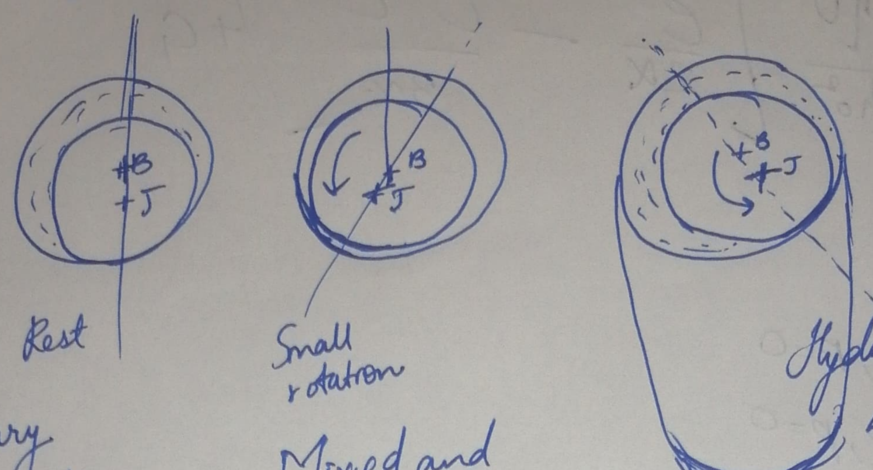
$$\overline{W} = \frac{W h_o^2 \alpha^2}{\eta U L} = \frac{1}{2}$$

Composite Bearings



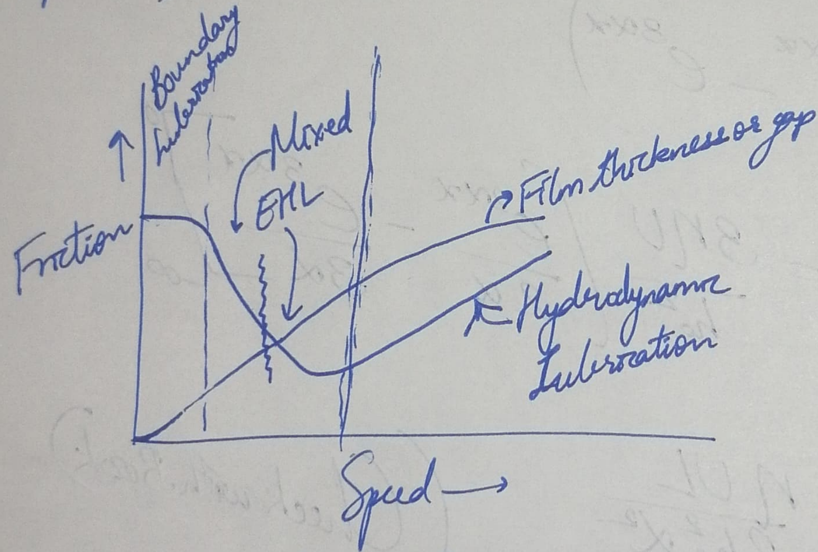


Idealized Journal Bearings



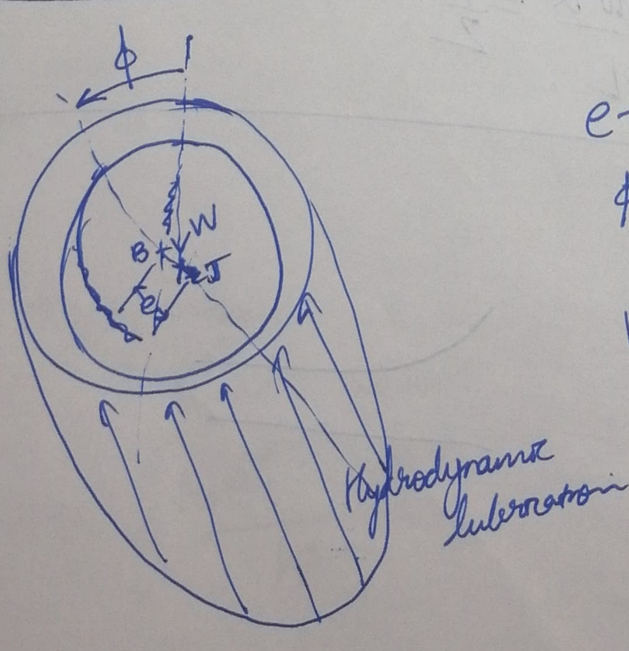
Boundary Lubrication (Surfaces are not separated)

Mixed and Elasto Hydrodynamic Lubrication



(Graph not specific to Journal Bearing)

Stribeck Curve



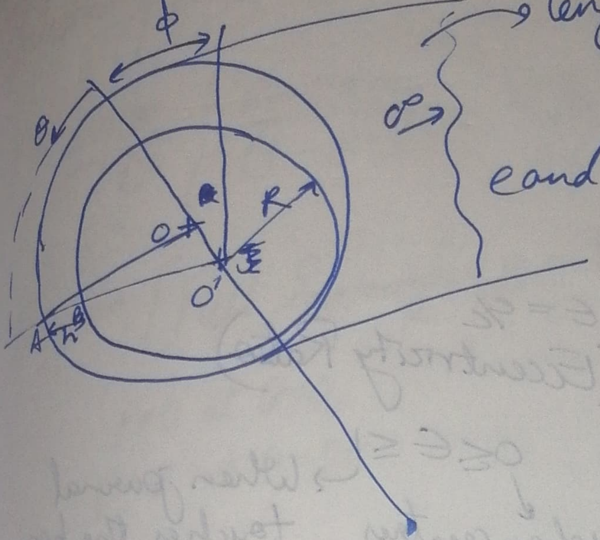
$e \rightarrow$ eccentricity

$\phi \rightarrow$ Attitude angle

When speed increases, but W remains same, pressure profile changes but summation of pressure remains equal to W.

Also, ϕ and e changes.

ϕ, e are dependent on W and rpm.



→ length direction → infinite (z-direction)

e and φ are constant for a particular W & rpm.

Gap AB (h) is a fn of θ only.

Infinitely long Journal Bearings

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) = 6\eta U \frac{\partial h}{\partial x}$$

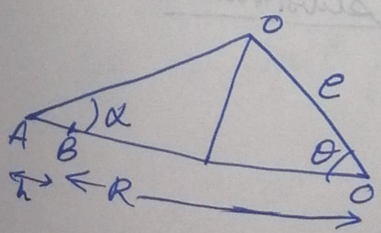
R → radius of Journal

$x = R\theta$
$dx = R\,d\theta$

$h(\theta)$

Output → $p(\theta)$

$$\frac{\partial}{\partial \theta}\left(h^3 \frac{\partial p}{\partial \theta}\right) = 6\eta U R \frac{\partial h}{\partial \theta}$$

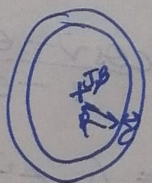


$h = AB$ → film thickness

$AB = AO' - R$

$AO' = AO\cos\alpha + e\cos\theta$

$R_B = R + C$ → Radial Clearance



α is small.
$\therefore \cos\alpha \longrightarrow 1$

$AO' = (R+C)\cos\alpha + e\cos\theta$

$AO' = (R+c) + e\cos\theta$

$h = AB = AO' - R = c + e\cos\theta$

$h = c\left[1 + \frac{e}{c}\cos\theta\right]$

$h = c\left[1 + \epsilon\cos\theta\right]$ where $\epsilon = e/c$
(Eccentricity Ratio)

$0 \leq \epsilon \leq 1$ → when journal touches the bearing surface.
↓
when centres coincide

$$\frac{\partial p}{\partial \theta} = 6\eta UR\left[\frac{h-h_m}{h^3}\right]$$ (Similar to previous case)

$\frac{\partial p}{\partial \theta} = 0$ at $h = h_m$.

using the expression for h,
we obtain,

$$p = \frac{6\eta UR}{c^2}\left[\int\frac{d\theta}{(1+\epsilon\cos\theta)^2} - \frac{h_m}{c}\int\frac{d\theta}{(1+\epsilon\cos\theta)^3}\right] + C_1$$

$$I_2 = \int\frac{d\theta}{(1+\epsilon\cos\theta)^2}, \quad I_3 = \int\frac{d\theta}{(1+\epsilon\cos\theta)^3}$$

To solve the integrations, we use Sommerfeld Substitutions.

$\cos\nu = \frac{\epsilon+\cos\theta}{1+\epsilon\cos\theta}$ (substitute this)

we obtain $\sin\theta = \frac{(1-\epsilon^2)^{1/2}\sin\nu}{1-\epsilon\cos\nu}$

$$\cos\theta = \frac{\cos\nu - \epsilon}{1-\epsilon\cos\nu}$$

and $d\theta = \frac{(1-\epsilon^2)^{1/2}d\nu}{1-\epsilon\cos\nu}$

Substituting, we have,

$$p = \frac{6\eta UR}{C^2}\left[\frac{\gamma - \epsilon \sin\gamma}{(1-\epsilon^2)^{3/2}} - \frac{h_m}{C(1-\epsilon^2)^{5/2}}\left(\gamma - 2\epsilon\sin\gamma + \frac{\epsilon^2\gamma}{2} + \frac{\epsilon^2 \sin 2\gamma}{4}\right)\right] + C_1$$

BC's → [Full Sommerfeld B.Cs]

$p = 0$ at $\theta = 0$,
$p = 0$ at $\theta = 2\pi$

$$p = \frac{6\eta UR\epsilon}{c^2}\,\frac{(2+\epsilon\cos\theta)\sin\theta}{(2+\epsilon^2)(1+\epsilon\cos\theta)^2}$$

Load Carrying Capacity

$$W_r = W\cos\phi = -L\int_0^{2\pi} p\cos\theta R\,d\theta$$

$$W_\phi = W\sin\phi = L\int_0^{2\pi} p\sin\theta\,d\theta \rightarrow \text{Check?}$$

After integration by parts

$$W_r = LR\int_0^{2\pi}\sin\theta\,\frac{dp}{d\theta}\,d\theta$$

$$\frac{\partial p}{\partial\theta} = 6\eta UR\left[\frac{h-h_m}{h^3}\right]$$

$h_m = \dfrac{2c(1-\epsilon^2)}{2+\epsilon^2}$

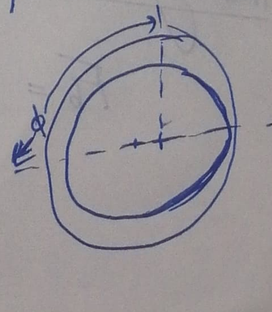
$h = c(1+\epsilon\cos\theta)$

Use Sommerfeld Substitution,

$\rightarrow \phi = 90°$

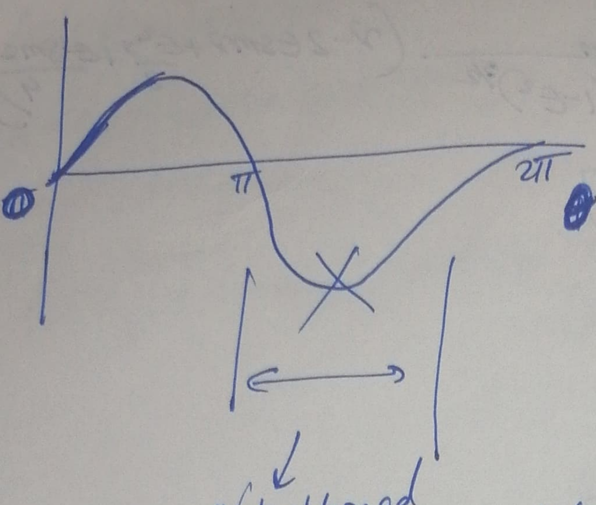
$$W_r = W\cos\phi = 0$$

$$W_\phi = W\sin\phi = \frac{12\eta\pi UL\epsilon(R/c)^2}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$



$$W = \sqrt{W_r^2 + W_\phi^2}$$

$$W = W_\phi = \frac{12\eta UL\epsilon(R/c)^2}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$



→ Pressure profile

→ Full Sommerfeld soln

∴ Wrong

Also $\phi = 90°$, so Journal shifts horizontally ∴ not possible

∴ This is wrong.

Shear stress at the journal surface

$$\tau_j = \frac{h}{2R}\frac{dp}{d\theta} + \eta\frac{U}{h}$$

Friction force

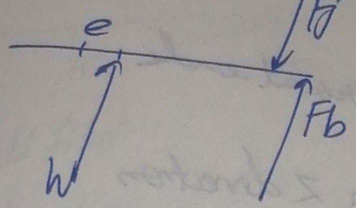
$$F_j = \int_0^{2\pi} \tau_j \, LR\, d\theta$$

$$F_j = \eta U L (R/c) \frac{4\pi(1+2\epsilon^2)}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$

Similarly at the Bearing Surface

$$F_b = \frac{\eta U L R}{c} \frac{4\pi(1-\epsilon^2)^{1/2}}{(2+\epsilon^2)}$$

for this case,



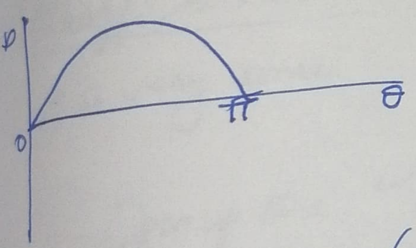
taking c to be small,

$RF_j = RF_b + We$

$\Rightarrow F_j > F_b$

Because of friction force Journal shifts to one side.

As this is not feasible soln, ∴ we neglect the -ve half part.



← Half Sommerfeld Soln

$$P = \frac{6\eta URE}{C^2} \frac{(2+\epsilon\cos\theta)\sin\theta}{(2+\epsilon^2)(1+\cos\theta)^2} \quad , 0<\theta<\pi$$

$P = 0$ for $\pi < \theta < 2\pi$

Integrate By parts of Sommerfeld subs.

$$W_r = W\cos\phi = -LR\int_0^{\pi} P\cos\theta d\theta = 12\eta UL\left(\frac{R}{c}\right)^2 \frac{\epsilon^2}{(2+\epsilon^2)(1-\epsilon^2)}$$

$$W_\phi = W\sin\phi = LR\int_0^{\pi} P\sin\theta d\theta = 6\eta UL\left(\frac{R}{c}\right)^2 \frac{\pi\epsilon}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$

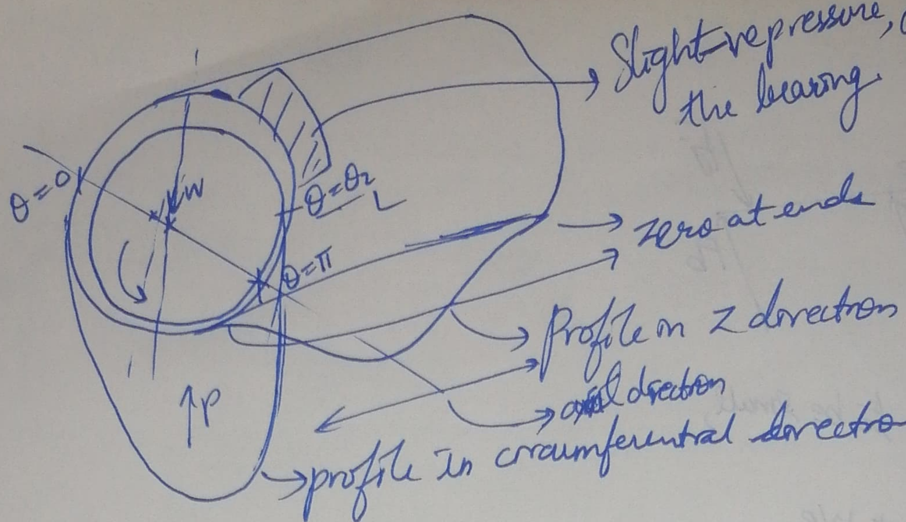
$$W = \sqrt{W_r^2 + W_\phi^2} = 6\eta UL\left(\frac{R}{c}\right)^2 \frac{\epsilon\left[\pi^2 - \epsilon^2(\pi^2-4)\right]^{1/2}}{(2+\epsilon^2)(1-\epsilon^2)}$$

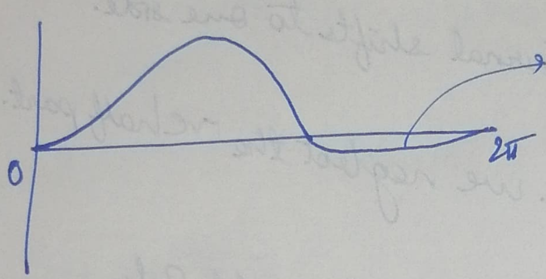
$$\phi = \tan^{-1}\left(\frac{W_\phi}{W_r}\right)$$

$$= \tan^{-1}\left(\frac{\pi}{2\epsilon}\sqrt{1-\epsilon^2}\right)$$

This soln is slightly close to exp. soln.

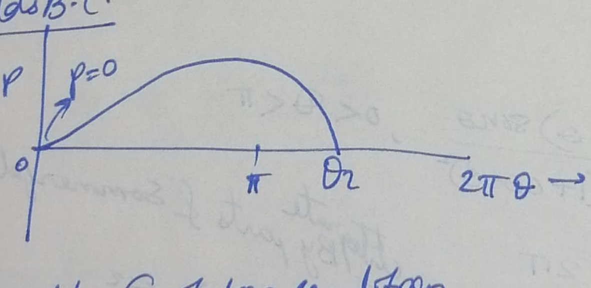
∴ People started using new soln.





People neglected the -ve part,

Reynolds B.C.



Reynolds Cavitation condition

at θ_2, $P=0$, $\frac{\partial P}{\partial \theta}=0$.

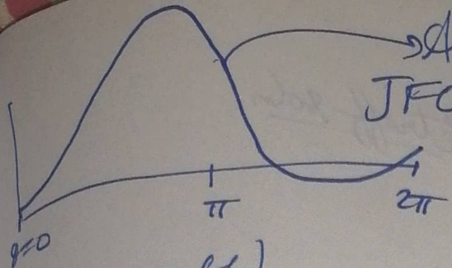
3 BCs,

$P=0$ at $\theta=0$,

$P=0$ at $\theta=\theta_2$,

$\frac{\partial P}{\partial \theta}=0$ at $\theta=\theta_2$.

θ_2 varies with load and RPM.



Actual Pressure Profile

JFO → Cavitation condition

JFO → 3 people

$P_0 = P_{2\pi}$ (Full film)

↳ In reality $P \neq 0$ at $\theta = 0$ as Journal has no boundary

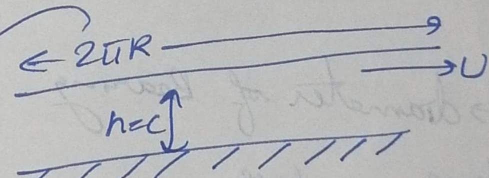
↳ means it is periodic

$\rho_c \rightarrow$ Critical density of fluid (beyond which density is not allowed)
(below)

Petroff Solution (Lightly loaded Journal Bearing)

e is very small

Open up the circumference



Friction = τ*A

$$F = \frac{\eta U A}{h}$$

$U = 2\pi R N$, $A = 2\pi R L$, $h = c$

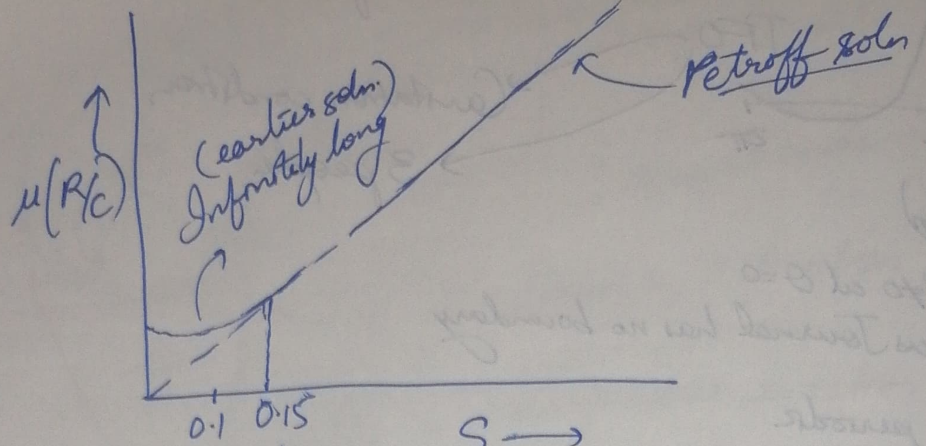
$$\therefore F = \frac{4\pi^2 \eta R^2 N L}{c}$$

$$\mu = \frac{F}{W} = \frac{2\pi^2 \eta N}{p}\left(\frac{R}{c}\right)$$

where $p = \frac{W}{2LR}$ (Bearing pressure)

$$\boxed{\mu\left(\frac{R}{c}\right) = 2\pi^2 S}$$ → Sommerfeld number

$$\boxed{S = \frac{\eta N}{p}\left(\frac{R}{c}\right)^2}$$



beyond 0.15, matches with infinitely long soln.

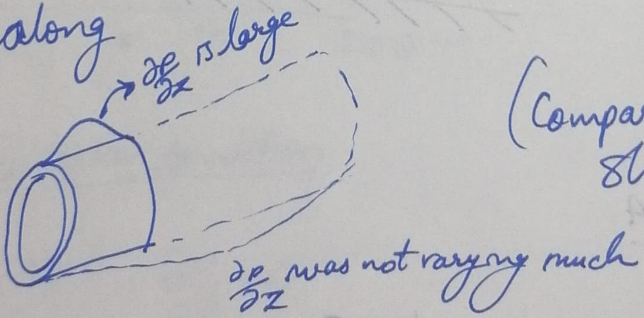
Beyond 0.15, low load & high speed

↳ lightly loaded Journal Bearing

Infinitely short Journal Bearings (Narrow Bearings)

$L < D$ ↳ diameter of Bearing

Variation along



(Comparison between long & short bearing)

$$\frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\eta U \frac{\partial h}{\partial x}$$

$\frac{\partial p}{\partial x}$ term vanishes.

$$\frac{\partial}{\partial z}\left(h^3 \frac{\partial p}{\partial z}\right) = 6\eta \frac{U}{R} \frac{\partial h}{\partial \theta}$$

$h = f_n(\theta)$

$$p = \frac{6\eta U}{R h^3} \frac{\partial h}{\partial \theta} \frac{z^2}{2} + C_1 z + C_2$$

BCs:
$p = 0$ at $z = \pm L/2$

$$p = \frac{3\eta U}{RC^2}\left(\frac{L^2}{4} - z^2\right)\frac{\epsilon \sin\theta}{(1+\epsilon\cos\theta)^3}$$

$$W_r = -2\int_0^{\pi}\int_0^{L/2} p\cos\theta \, R \, d\theta \, dz$$

$$W_\phi = 2\int_0^{\pi}\int_0^{L/2} p\sin\theta \, R \, d\theta \, dz$$

Use Sommerfeld Substitutions

$$W_r = \frac{\eta U L^3}{C^2}\frac{\epsilon^2}{(1-\epsilon^2)^2}$$

$$W_\phi = \frac{\eta U L^3}{4C^2}\frac{\pi\epsilon}{(1-\epsilon^2)^{3/2}}$$

Total Load

$$W = \sqrt{W_r^2 + W_\phi^2} = \frac{\eta U L^3}{4c^2}\frac{\epsilon}{(1-\epsilon^2)^2}\left[\pi^2(1-\epsilon^2) + 16\epsilon^2\right]^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{W_\phi}{W_r}\right) = \tan^{-1}\left[\frac{\pi}{4}\frac{(1-\epsilon^2)^{1/2}}{\epsilon}\right]$$

Shear Stress

$$\tau = \eta\frac{U}{h}$$

Friction Force

$$F = \int_0^{2\pi}\eta\frac{U}{h}LR\,d\theta = \frac{\eta U L R}{C}\frac{2\pi}{(1-\epsilon^2)^{1/2}}$$

$$\mu\left(\frac{R}{C}\right) = \frac{2\pi^2 S}{(1-\epsilon^2)^{1/2}}$$

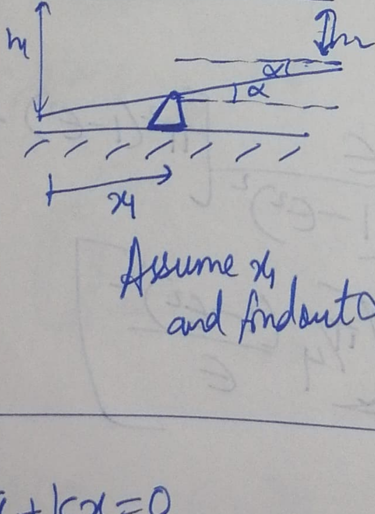
End Flow

$$dQ_z = -\frac{h^3}{12\eta}\frac{\partial p}{\partial z}$$

$$Q_z = -2\int_0^{\pi} \frac{Rh^3}{12\eta}\frac{\partial p}{\partial z}\bigg|_{z=L/2} d\theta$$

Multiplied by 2 for 2 ends

$$\underline{\underline{Q_z = \epsilon ULC}}$$



α is input

find x_1

Assume x_1 and findout α

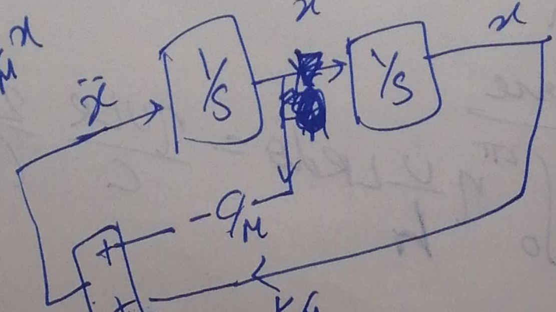
$$M\ddot{x} + C\dot{x} + Kx = 0$$

then change for a Force

$$\ddot{x} + \frac{C}{M}\dot{x} + \frac{K}{M}x = 0$$

$$\ddot{x} = -\frac{C}{M}\dot{x} - \frac{K}{M}x$$

Make a loop

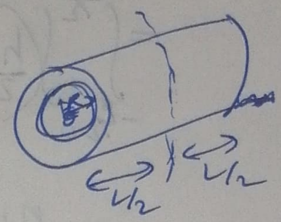


Finite Bearings (Journal)

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial z}\right)=\frac{U_a+U_b}{2}\frac{\partial h}{\partial x}$$

$\theta = x/R$ $\qquad \bar{h} = h/c$

$\bar{z} = \dfrac{z}{L/2}$ $\qquad U = U_a + U_b$



$$\frac{\partial}{R^2\partial\theta}\left(\frac{\bar{h}^3c^3}{6\eta}\frac{\partial p}{\partial\theta}\right)+\frac{4}{L^2}\frac{\partial}{\partial\bar{z}}\left(\frac{\bar{h}^3c^3}{6\eta}\frac{\partial p}{\partial\bar{z}}\right)=\frac{U}{R}\frac{\partial\bar{h}}{\partial\theta}c$$

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=\frac{\partial\bar{h}}{\partial\theta}$$

$\Rightarrow \bar{p} = \dfrac{pc^2}{6\eta UR}$

$\bar{h} =$ [redacted] $+ \epsilon\cos\theta$

Load Carrying Capacity:-

$$W=\int^{L/2}\int_0^{2\pi R} p\,dx\,dz$$

$$=2\int_0^{1}\int_0^{2\pi}\bar{p}\,\frac{6\eta UR}{c^2}\,R\,d\theta\,d\bar{z}\,\frac{L}{2}$$

$$W=\frac{6\eta UR^2L}{c^2}\int_0^1\int_0^{2\pi}\bar{p}\,d\theta\,d\bar{z}$$

$$\bar{W}=\frac{Wc^2}{\eta UR^2L}=6\int_0^1\int_0^{2\pi}\bar{p}\,d\theta\,d\bar{z}$$

[redacted]

Friction

$$F = \int_{-L/2}^{L/2}\int_{0}^{2\pi R} \tau_{x}\, dx\, dz$$

$$= \int_{-L/2}^{L/2}\int_{0}^{2\pi R} \left(\frac{h}{2}\frac{\partial p}{\partial x} + \frac{\eta}{h}U\right) dx\, dz$$

$$= \frac{\eta U R L}{c} \int_{0}^{1}\int_{0}^{2\pi} \left[3\bar{h}\frac{\partial \bar{p}}{\partial \theta} + \frac{1}{\bar{h}}\right] d\theta\, d\bar{z}$$

$$\bar{F} = \frac{Fc}{\eta U R L} = \int_{0}^{1}\int_{0}^{2\pi} \left[3\bar{h}\frac{\partial \bar{p}}{\partial \theta} + \frac{1}{\bar{h}}\right] d\theta\, d\bar{z}$$

$$\mu = \frac{F}{W} = \frac{\bar{F}\, \cancel{\eta U R L}\, c^{2}}{\cancel{c} \times \bar{W}\, \cancel{\eta U R L}}$$

$$\mu = \frac{\bar{F} c}{\bar{W} R}$$

$$\mu\left(\frac{R}{c}\right) = \frac{\bar{F}}{\bar{W}}$$

Friction parameter

End Flow

$$\frac{dQ_z}{dx} = \frac{-h^3}{12\eta}\frac{\partial p}{\partial z}\bigg|_{z=L/2}$$

$$Q_z = \int_{0}^{2\pi R} \frac{h^3}{12\eta}\left(\frac{\partial p}{\partial z}\right)\bigg|_{z=L/2} dx$$

$$= \frac{cUR^2}{L}\int_{0}^{2\pi} -\bar{h}^3 \frac{\partial \bar{p}}{\partial \bar{z}}\bigg|_{\bar{z}=1} d\theta$$

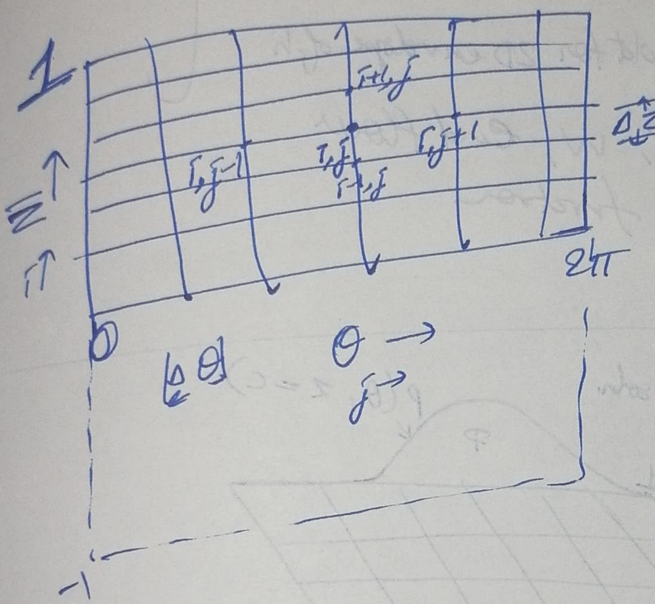
$$\bar{Q}_z = \frac{Q_z L}{cUR^2} = \int_{0}^{2\pi} -\bar{h}^3\frac{\partial \bar{p}}{\partial \bar{z}}\bigg|_{\bar{z}=1} d\theta$$

Numerical soln (FDM)

$$\frac{\partial^2 \bar{p}}{\partial \theta^2} + \left(\frac{D}{L}\right)^2 \frac{\partial^2 \bar{p}}{\partial \bar{z}^2} + \frac{3}{\bar{h}} \frac{\partial \bar{p}}{\partial \theta} \frac{\partial \bar{h}}{\partial \theta} = \frac{1}{\bar{h}^3} \frac{\partial \bar{h}}{\partial \theta}$$

$\frac{D}{L} = ?$ and $\epsilon = ?$ $\longrightarrow$ Input

$\bar{h} = 1 + \epsilon \cos\theta$



$$\left.\frac{\partial p}{\partial \theta}\right|_{i,j} = \frac{P(i, j+1) - P(i, j-1)}{2\Delta\theta}$$

$$\left.\frac{\partial p}{\partial \bar{z}}\right|_{i,j} = \frac{P(i+1, j) - P(i-1, j)}{2\Delta\bar{z}}$$

$$\left.\frac{\partial^2 p}{\partial \theta^2}\right|_{i,j} = \frac{P(i, j+1) + P(i, j-1) - 2P(i, j)}{\Delta\theta^2}$$

$$\left.\frac{\partial^2 p}{\partial \bar{z}^2}\right|_{i,j} = \frac{P(i+1, j) + P(i-1, j) - 2P(i, j)}{\Delta\bar{z}^2}$$

$$\left.\frac{\partial h}{\partial \theta}\right|_{i,j} = \frac{h(i, j+1) - h(i, j-1)}{2\Delta\theta}$$

Substitute and obtain an eqn for $P(i,j)$

$P(i,j) =$

x = linspace (0, 2π, nx+1); x(end) = [ ];

z = linspace (0, 1, nz) → full bearing soln.

$x = [0 \quad 0.5 \quad 1 \quad 2 \quad 2.5$

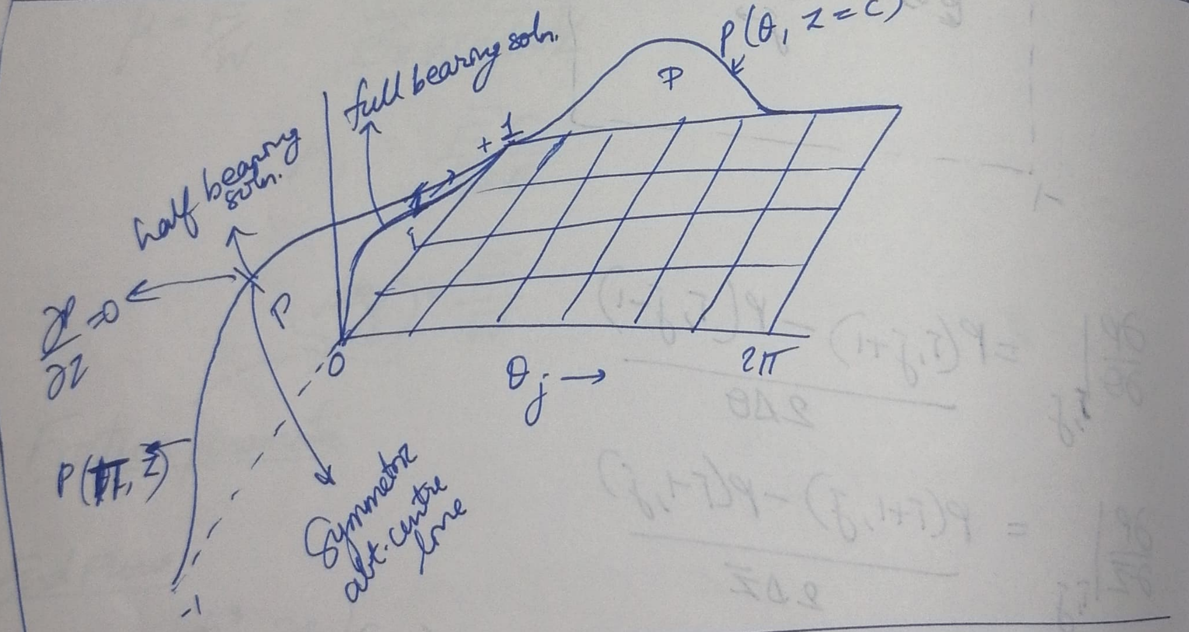
[X, Z] = meshgrid (x, z)

$$X = \begin{bmatrix} 0 & 0.5 & 1 & 2 & 2.5 \\ 0 & 0.5 & 1 & 2 & 2.5 \\ \vdots \\ \vdots \end{bmatrix}_{nz \times nx}$$

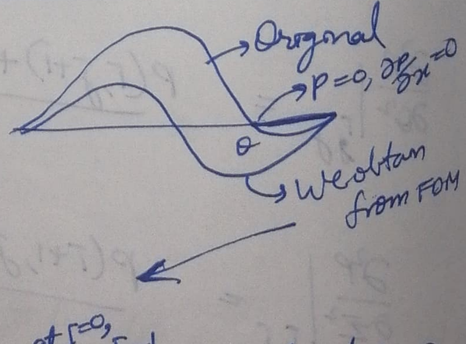
2D ↑ $h = 1 + \epsilon \cos X$

mesh (x, z, h) → plot for 2D envelope of h

use Pold. find P, W, end flow friction




```
h = 1 + ε cos X
mesh (X, Z, h)
p = zeros(size(X))
Pold = P;
while convergence > 10^-5
   for i = 2:nz-1
      for j = 1:nx
         if statement
         (P(i,j) = ______
      end
   end
   convergence = (ΣP - ΣPold) / ΣPold
   Pold = P
end
```



(at j=0, j-1 when → end value at θ=2π p(2π))

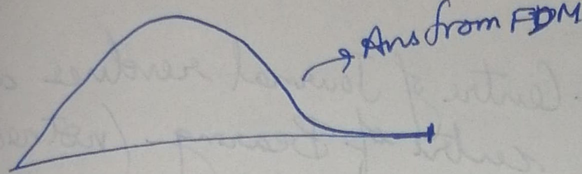
when j=nx → for j+1, p(1)) use if statements

Impose Reynolds ~~condition~~ cavitation $p=0$, $\frac{\partial p}{\partial x}=0$

$P(i,j) = \text{— — — —}$

if $P(i,j) < 0$,

$P(i,j) = 0$

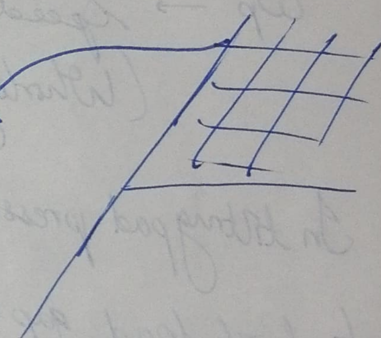


for half bearing soln.

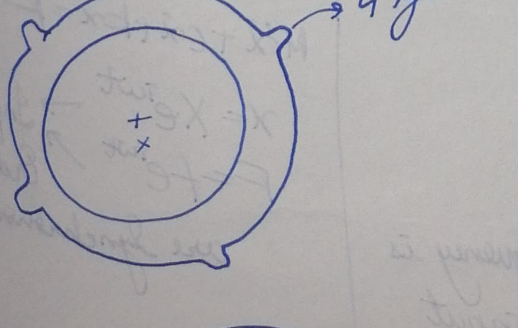
$P(1,j) = P(2,j) \longrightarrow$ BC for half bearing soln.

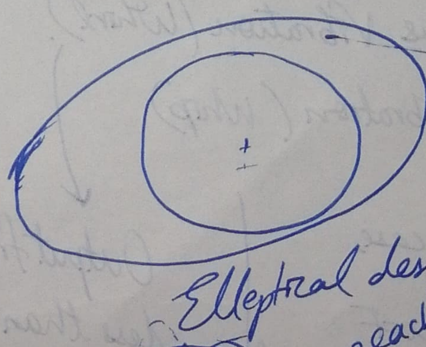
$\frac{\partial p}{\partial z} = 0$

solution :-

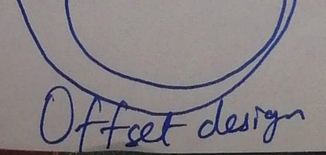


Hydrodynamic Instability (Journal Bearings)

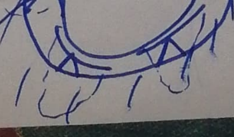




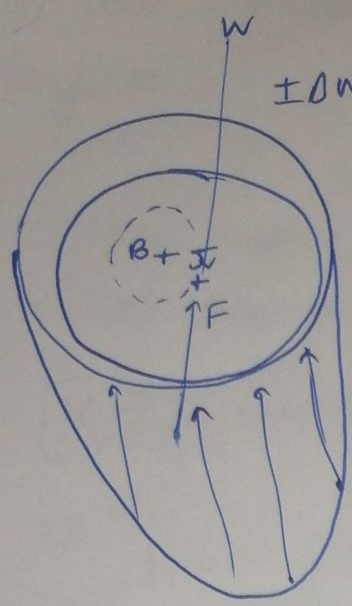
Elliptical design



Offset design



Tilting pad Journal bearing



$\pm \Delta W \rightarrow$ In practical case

$\therefore$ F has to be adjusted ; $\because$ fluid has to be adjusted, which takes time to respond

By that time ΔW changes to ΔW_1 $\therefore$ F is not always equal to W.

$\therefore$ Centre of Journal revolves around centre of Bearing (not necessarily in a circle.)

All of this because of fluid which cannot respond.

$\omega \rightarrow$ Spinning Speed of Journal

$\omega_p \rightarrow$ Speed of Journal centre around Bearing Centre (Whirling)

In tilting pad pressure is adjusted quickly as compared to ~~Normal~~ Journal Bearing.

for high load, gap is small, $\therefore$ small part of fluid gives high pressure, $\therefore$ adjusts quickly corresponding to the load ΔW.

Whirl Ratio $= \frac{\omega_p}{\omega} = \frac{1}{2}$ (Half Whirl mechanism in Journal Bearing)

① Subsynchronous Vibration (Whirl) $\rightarrow$ Output frequency is less than input frequency

② Resonating Vibration (Whip)

In resonating case,

$M\ddot{x} + c\dot{x} + kx = F$

$x = Xe^{i\omega_n t}$ $\rightarrow$ = natural frequency

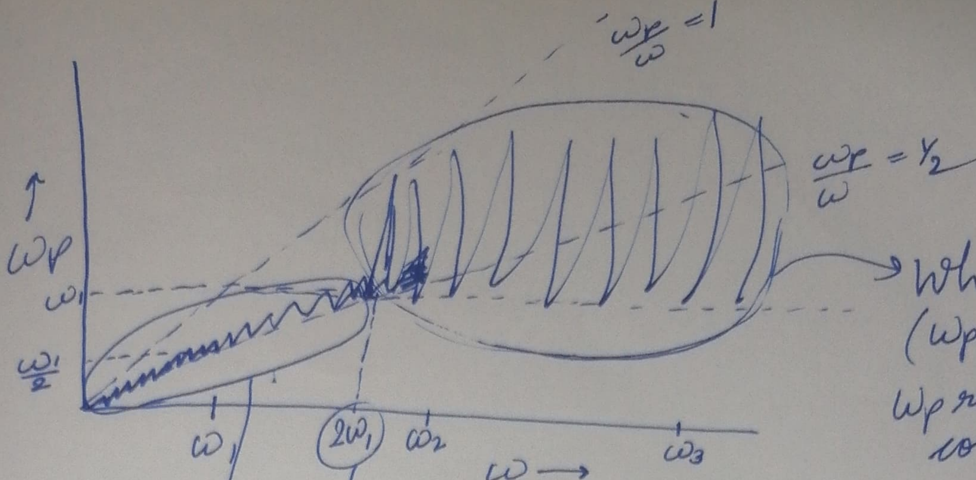
$F = fe^{i\omega t}$ $\rightarrow$ any ω value of output

$M\ddot{x} + c\dot{x} + kx = F$

$x = Xe^{i\omega t}$ $\rightarrow$ Input

$F = fe^{i\omega t}$ $\nearrow$ Output

are Synchronous



Whip ($\omega_p=\omega_1$)

ω_p remains constant

Threshold Speed ω_s.

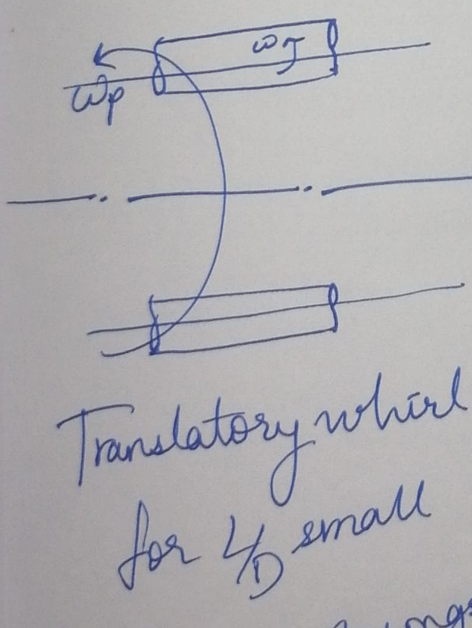
Whirl

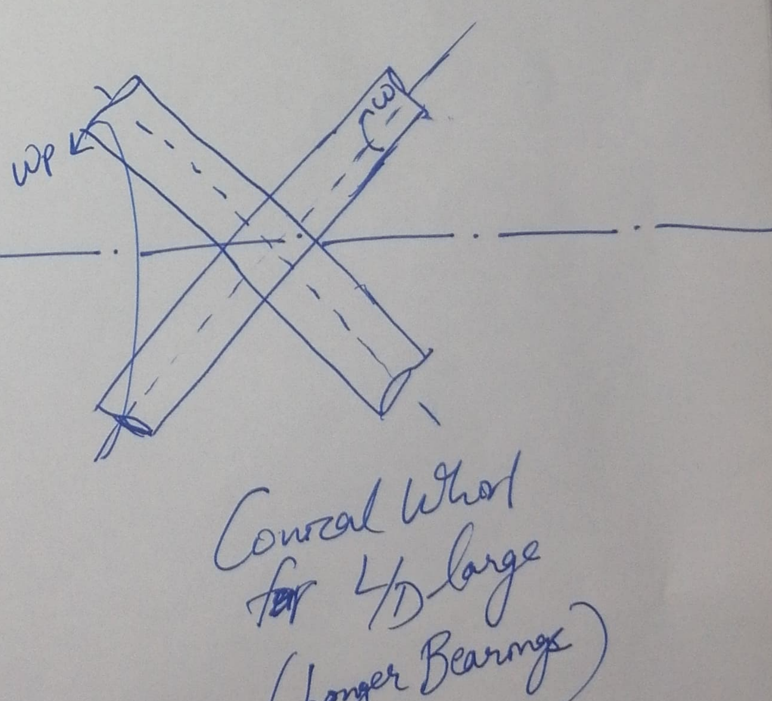
$\frac{\omega_p}{\omega}=\frac{1}{2}$ is only upto $\omega_p=\omega_1$

$\therefore$ Safe zone of working is before $2\omega_1$.

ω_s = Threshold Speed $= 2\omega_1$

Whip doesn't occur in Ball Bearing





Translatory whirl for L/D small (Narrow Bearings)

Conical Whirl for L/D large (Longer Bearings)