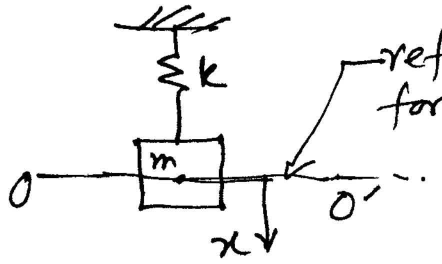
To clarify problem 3, TU/HW Sheet 2:–

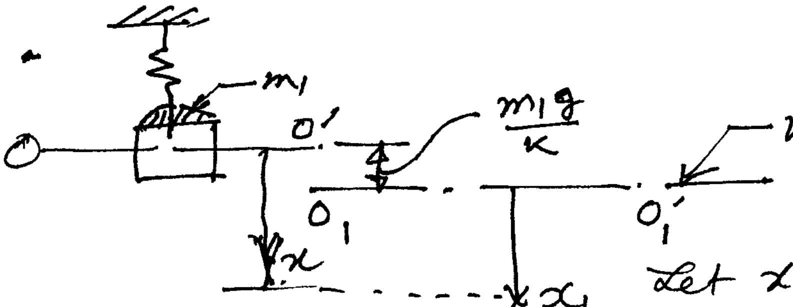
For this, we take a simpler system to illustrate the fact that any reference level can be chosen to measure the generalized coordinate used.

Consider the simple spring-mass system:



reference level (Static eqlbm., $k\delta_{st} = mg$) for measuring x, the generalized coordinate,

Now, let m_1 is added gently, by some means.

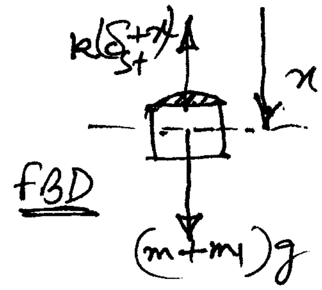


new equilibrium level of ~~(m+m1)~~ $(m+m_1)$

Let x_1 be the generalized coordinate measured from this level.

Then, clearly, $x_1 = x - \frac{m_1 g}{k}$ — (a)

<u>Case 1:–</u> We write DEOM in terms of x:–



<u>FBD</u>

$\Rightarrow (m+m_1)\ddot{x} = (m+m_1)g - k(\delta_{st}+x)$

$\Rightarrow (m+m_1)\ddot{x} + kx = m_1 g$ — (i) $(\because mg - k\delta_{st} = 0)$

$\therefore\ x = A\sin\omega_n t + B\cos\omega_n t + \frac{m_1 g}{k}$ — (ii)

$\boxed{\omega_n = \sqrt{\frac{k}{m+m_1}}}$

Initial conditions:– $x(0)=0$, $\dot{x}(0)=0$

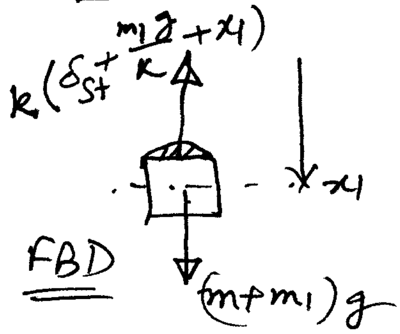
$\Rightarrow 0 = B + \frac{m_1 g}{k}$ or, $B = -\frac{m_1 g}{k}$

$\dot{x} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$

$\therefore\ \dot{x}(0)=0 \Rightarrow A = 0.$

Hence, $x = -\frac{m_1 g}{k}\cos\omega_n t + \frac{m_1 g}{k} = \frac{m_1 g}{k}[1-\cos\omega_n t]$ — (iii) →

Case 2:- We write DEOM in terms of x_1, measured from new equilibrium position:-



FBD

$$\Rightarrow (m+m_1)\ddot{x}_1 = (m+m_1)g - k\left(\delta_{st} + \frac{m_1 g}{k} + x_1\right)$$

$$\Rightarrow (m+m_1)\ddot{x}_1 + kx_1 = 0 \quad \text{--- (I)}$$

$$\Rightarrow x_1 = A' \sin\omega_n t + B' \cos\omega_n t \quad \text{--- (II)}$$

Initial conditions:-

$$\underline{x_1(0) = -\frac{m_1 g}{k}} \;\; \& \;\; \dot{x}_1(0) = 0.$$

note this

from (II), $\quad x_1(0) = B' = -\frac{m_1 g}{k}$

$$\dot{x}_1 = A'\omega_n \cos\omega_n t - B'\omega_n \sin\omega_n t$$

$$\Rightarrow \quad \underline{0 = \dot{x}_1(0) = A'}$$

So, (from (II)) $\quad x_1(t) = -\frac{m_1 g}{k}\cos\omega_n t \quad \text{--- (III)}$

(iii) & (III) are basically the same, since $x_1 = x - \frac{m_1 g}{k}$ (relation (a), pg. 1)

Hence, whether we measure our generalized coordinate from old or new equilibrium position, the final result is the same.

→ The same applies to problem 3, Tu-2. If you want, you may find the new equilibrium position at $\theta = \theta_0$, say, and take a new generalized coordinate θ_1, measured from this new angular position & solve the problem. You should get the same answer both ways. So this & check.

VA TU + HW (I) Contd Page 2

1.1 Determine the natural frequency of small oscillations of the bell crank lever ABC shown in Figure 1.25. The lever is light but has a mass M fixed at C. BC is horizontal when the system is in the equilibrium position.

Answer $f_n = \{1/(2\pi)\}\{(c/d)(k/M)^{0.5}\}$.

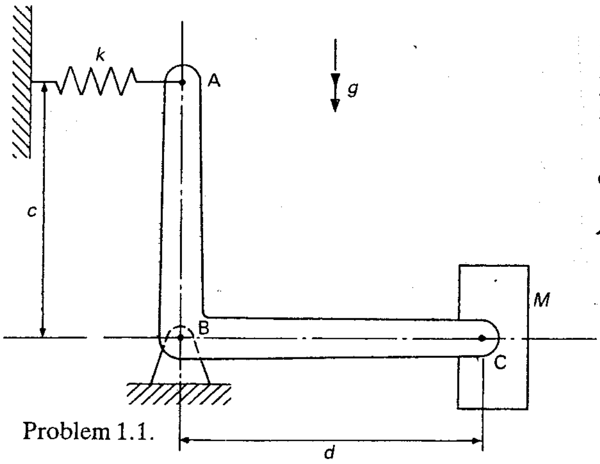


Figure 1.25 Problem 1.1.

1.2 A thin ring of 120 mm radius is placed on a frictionless pivot at O and given a small displacement. Determine the natural frequency of the oscillations. See Figure 1.26.

Answer $\omega_n = 6.39$ rad/s, $f_n = 1.018$ Hz.

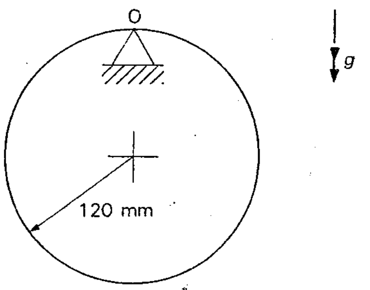


Figure 1.26 Problem 1.2.

1.3 A rotational system is formed by a solid steel shaft fixed at one end and a solid steel disc, as shown in Figure 1.27. In addition a

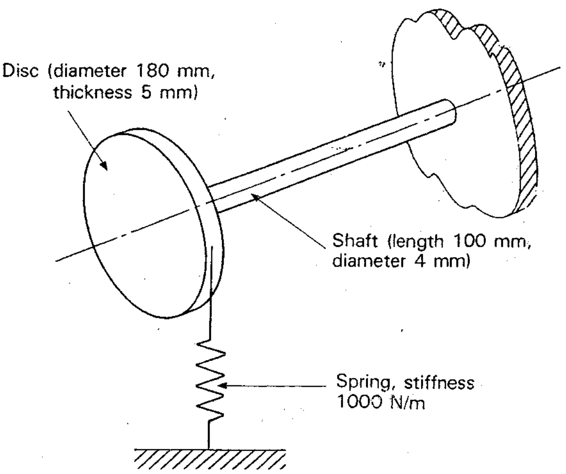


Figure 1.27 Problem 1.3.

inear spring is attached to the periphery of the disc so that its line of ıction is tangential to the disc. (G for steel is 8×10^{10} Pa, density of teel is 7.8×10^3 kg/m^3.)

Determine the natural frequency of small oscillations.

Answer $\omega_n = 83.77$ rad/s ($f_n = 13.33$ Hz) – mass of shaft neglected.

1.4 A drum which is a solid cylinder of mass m and radius r can rotate in frictionless bearings at O as shown in Figure 1.28. A rope passes over the drum and carries a load of mass M. The rope is attached to a fixed support via a spring of stiffness k.

Given that the load is given a small displacement downwards determine the frequency of the resulting vibrations.

Answer $f_n = \{1/(2\pi)\}\{k/(M + m/2)\}^{0.5}$.

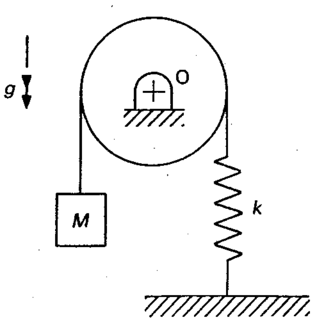


Figure 1.28 Problem 1.4.

1.5 In Figure 1.29 a belt is wrapped round a pulley A (mass 8 kg radius 120 mm and moment of inertia about an axis through the centre of 0.4 kg m^2) and a pulley B (of negligible mass) and is attached to a spring C of stiffness

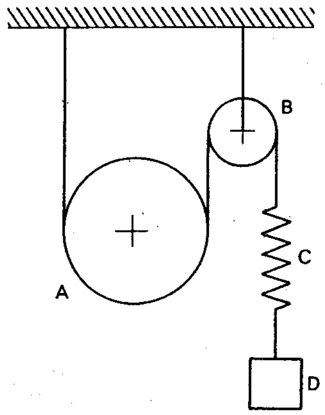


Figure 1.29 Problem 1.5.

1200 N/m. A load D of mass 4 kg is attached to the spring. Given that the system is initially in equilibrium and load D is then given a small displacement, estimate the frequency of vibrations. Ans: $f_n = 3.32$ Hz

Hence $\dot{x}(t) = \frac{dx}{dt} = A\left[-1.5e^{-1.5t}\sin\omega_d t + \omega_d e^{-1.5t}\cos\omega_d t\right]$

So, $\dot{x}(0) = A\omega_d \cancel{\cos\omega_d t}$ or $A = \frac{\dot{x}(0)}{\omega_d} = \frac{20\times10^{-2}}{\cancel{0.989}\ 9.8868}\ m$

$= \cancel{0.202\ m} = 0.0202\ m$

So, $x(t) = 0.0202e^{-1.5t}\sin(0.\cancel{989}\,9.8868\,t)\ m$ ~~Ans~~
is the required response.

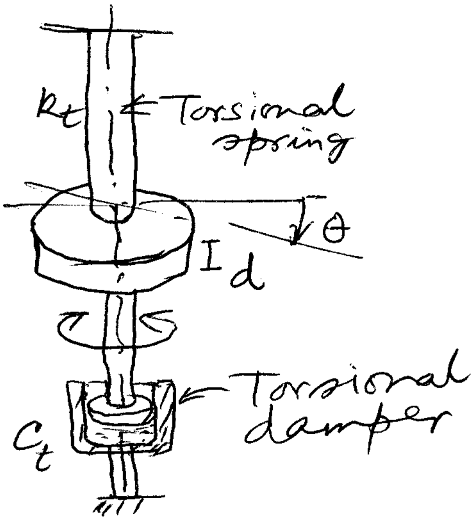
Also, at $t = 0.5$ s,

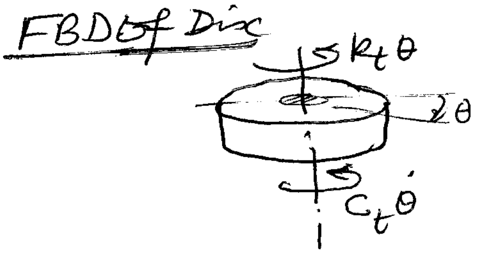
In radian. Be careful

$x(0.5) = 0.0202e^{(-1.5\times0.5)}\sin(9.987\times0.5)\ m$

= whatever comes. $\cancel{= 0.0453\ m}$ = ~~0.00453~~ m Ans.
0.~~009~~.

(8) Free-vibration of a damped torsional system:-





So, $I_d\ddot{\theta} = -c_t\dot{\theta} - k_t\theta$

or, $I_d\ddot{\theta} + c_t\dot{\theta} + k_t\theta = 0$. This is the required DEOM.

Note that SI unit of c_t is $\frac{N\text{-}m}{rad/s}$ or, N-m-s. The constitutive equation for the torsional damper is:

$\tau_d = c_t\dot{\theta}$ where τ_d is the torque in damper.

Hence, c_t is the damping torque per unit relative angular velocity between piston & cylinder like relative velocity between two ends of