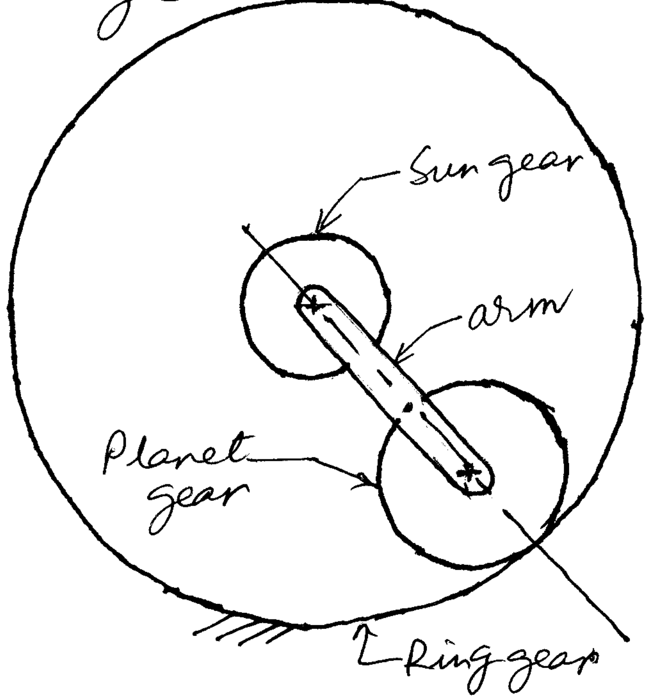
$\frac{\partial T}{\partial \theta} = 0$; $\frac{\partial V}{\partial \theta} = mg(R-r)\sin\theta \cong mg(R-r)\theta$ for small θ. Substitution of these in ① gives the req'd DEOM as:

$$\frac{3}{2}m(R-r)^2\ddot{\theta} + mg(R-r)\theta = 0 \quad \text{--- ④}$$

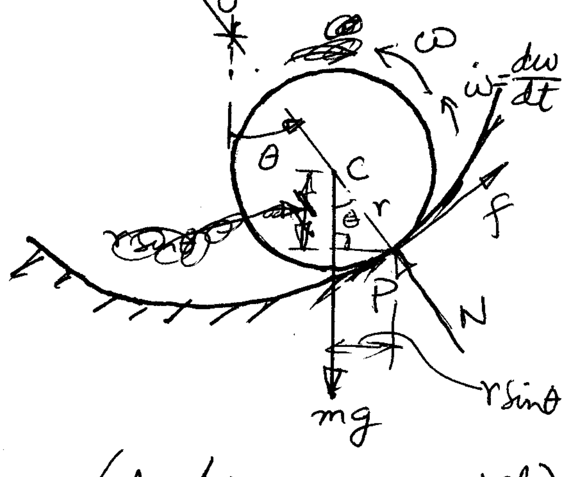
A comparison with $I_d\ddot{\theta} + k_t\theta = 0$ gives

$$\omega_n = \sqrt{\frac{mg(R-r)}{\frac{3m}{2}(R-r)^2}} = \sqrt{\frac{2g}{3(R-r)}},$$ the req'd natural frequency.

→ The question now is — where do we find a set-up such as the one discussed above? You must have studied epicyclic (or, planetary) gear trains. So, if we put an arm and a sun-gear ~~and replace each gear~~ and replace each gear by its pitch cylinder, then the above set-up becomes part of an epicyclic gear train as shown below.



Thus, the above system can be considered to be a subsystem of the gear train shown here and a study of its vibrational characteristics could lead to a study of the same for the whole system shown here!



(Angles exaggerated for clarity)

You could also use the energy method (or, the power balance method) $\frac{d}{dt}(T+V)=0$ to get the same DEOM.

How could you get it using ~~the~~ Newton's method (the force balance method)?

For that, you must draw the FBD as shown in the figure here.

where f is the friction force & N the normal reaction. In order to eliminate the unknown forces f & N, we apply the moment balance method about point P which qualifies ~~for the~~ as the point about which the simple equation $I_P\dot{\omega}$ = Sum of moments of external forces applies, ~~thus, taking~~ since the whole body (cylinder) instantaneously rotate about P.

Thus, ~~taking~~ moments about P, we get

$$I_P\dot{\omega} = -mgr\sin\theta \quad \left(\omega = \left(\frac{R-r}{r}\right)\dot{\theta} \Rightarrow \dot{\omega} = \left(\frac{R-r}{r}\right)\ddot{\theta}\right)$$

or, $$\frac{3}{2}mr^2\left(\frac{R-r}{r}\right)\ddot{\theta} + mgr\theta = 0 \quad (\text{Assuming } \sin\theta \cong \theta)$$

which is basically the same as DEOM (4) derived earlier.