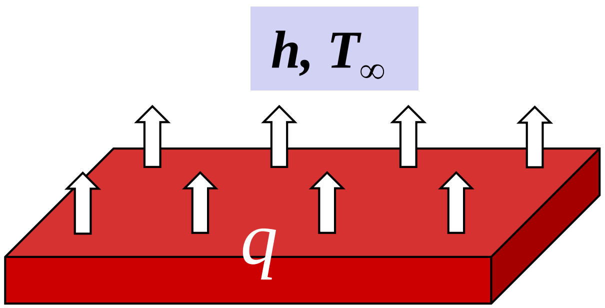
Conduction: Theory of Extended Surfaces

Why extended surface?

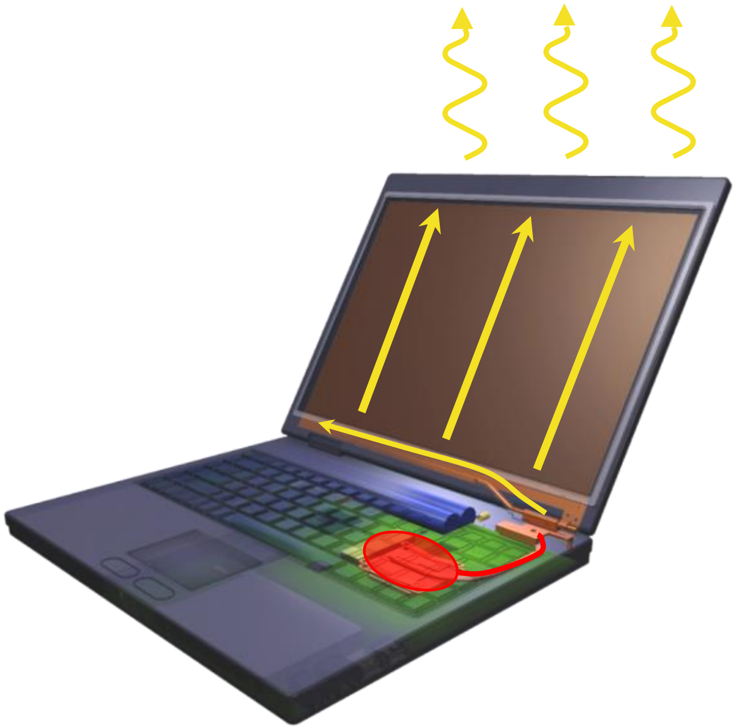


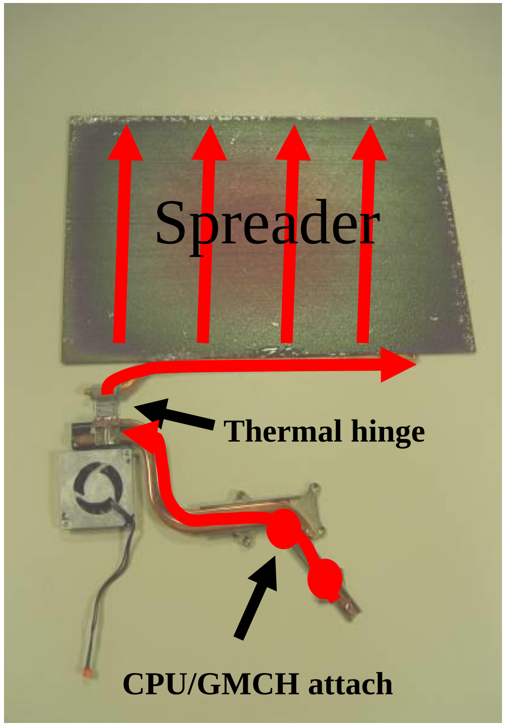
$$q = hA(T_s - T_\infty)$$

Increasing *h*

Increasing *A*

Extended surface: an example



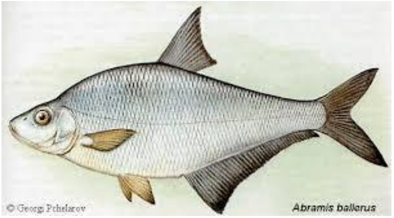


Lid Cooling in laptop computers

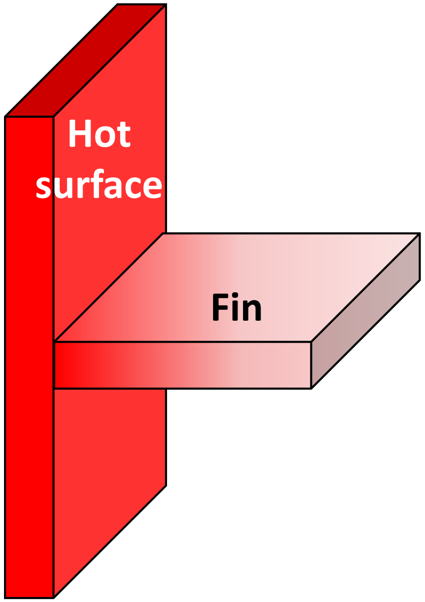
- Laptop display area (lid) used for cooling
- 7-15W of additional cooling achievable

Fins as extended surfaces

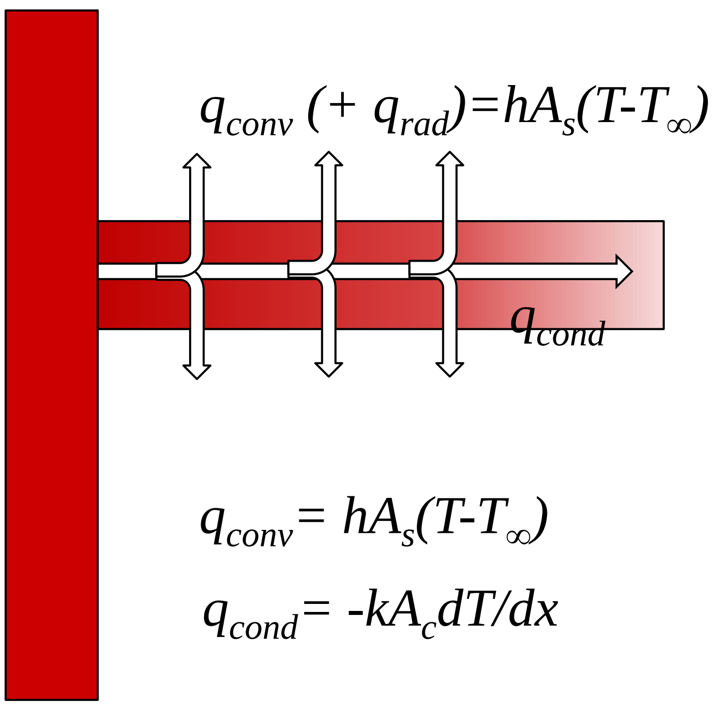
A **fin** is a thin component or appendage attached to a larger body or structure



In the context of heat transfer also, these are components protruding out of a heated (or cold) surface



What happens in a fin?

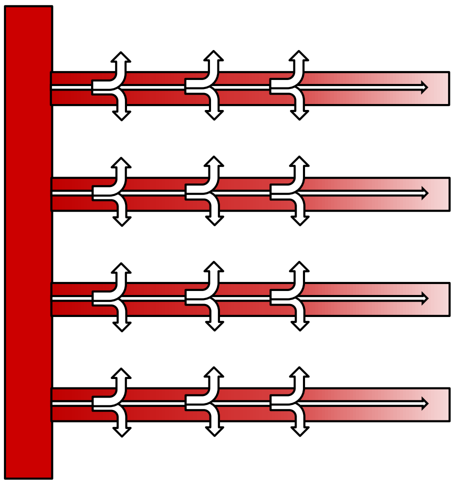


$q_{conv} = hA_s(T-T_\infty)$

$q_{cond} = -kA_c dT/dx$

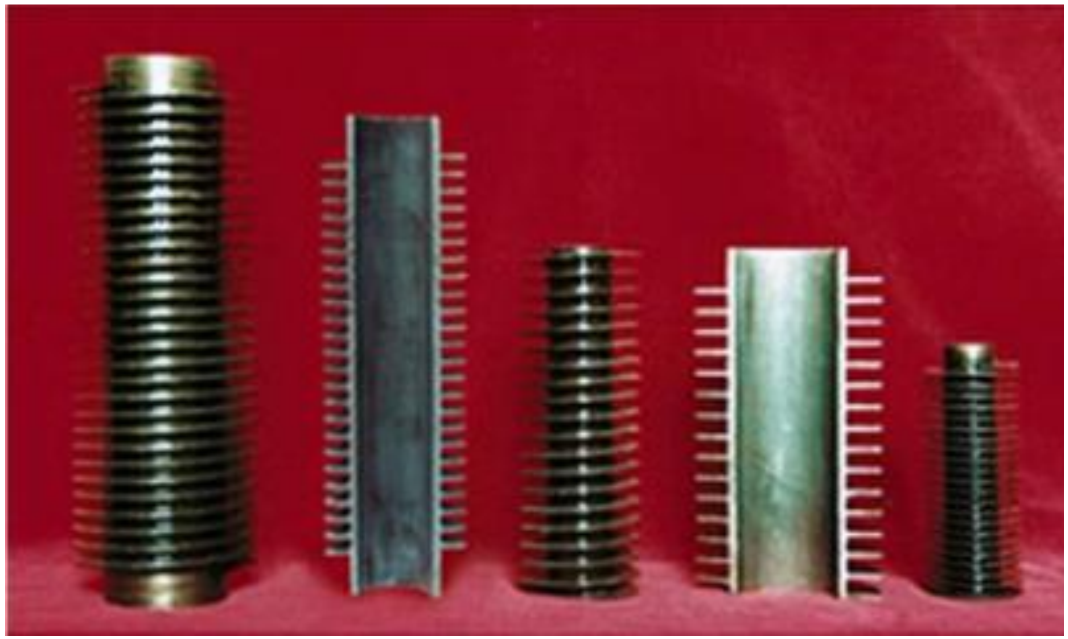
An extended surface (combined conduction-convection system) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.

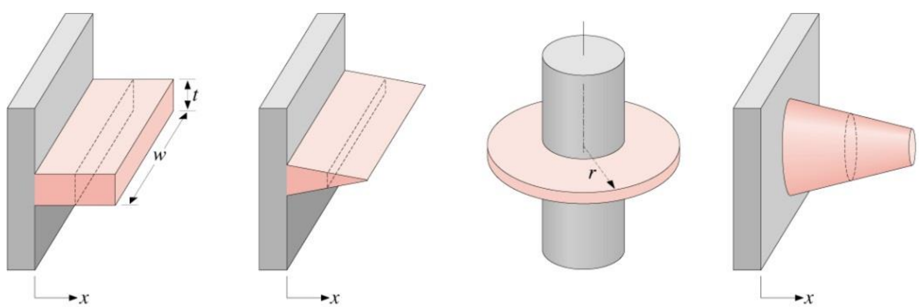
What happens in a fin?



An extended surface (combined conduction-convection system) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.

Examples of Fins





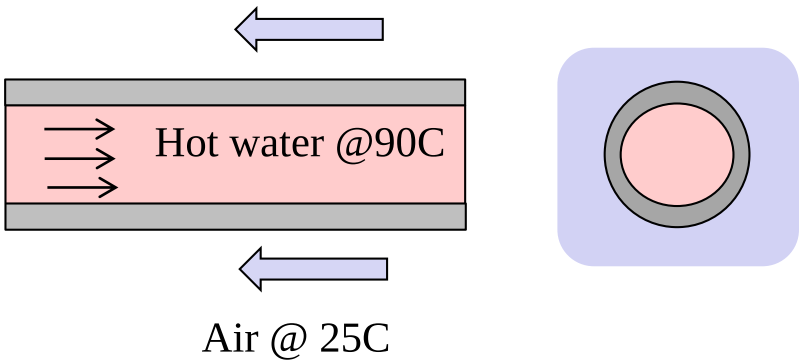
(a) (b) (c) (d)

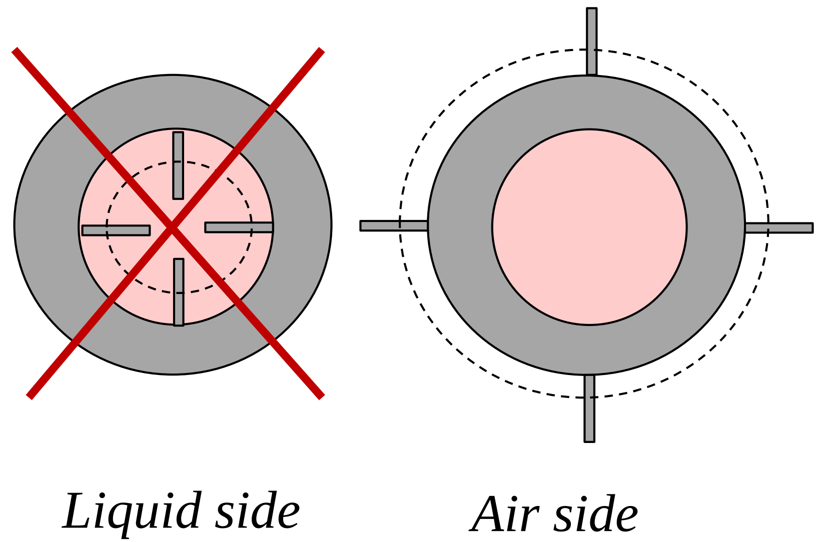
Question time

- Heat is transferred from hot water flowing inside a tube to cooling air flowing over the tube. To enhance heat transfer rate, **which side** should the **fins be installed**?

- Fins are most **beneficial** where ***h* is low**

- Fin *dimensions* and *k* are **critical** design parameters

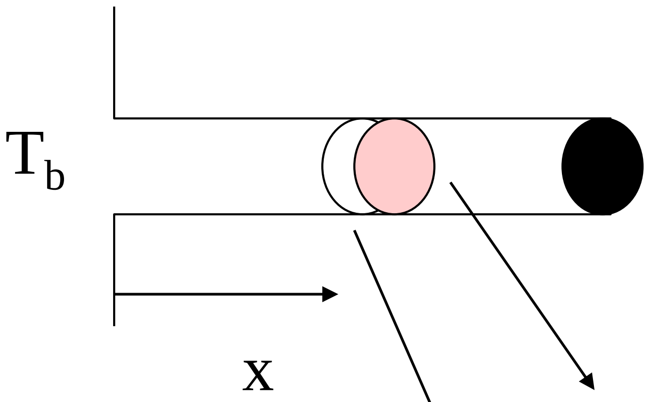




Summary

- Extended surfaces help in enhancing heat dissipation
  - Increases surface area for heat exchange
- Fins: most common embodiment of extended surface
  - Can be of varied shape and forms
- Fin peformance = *f (h, material, size)*

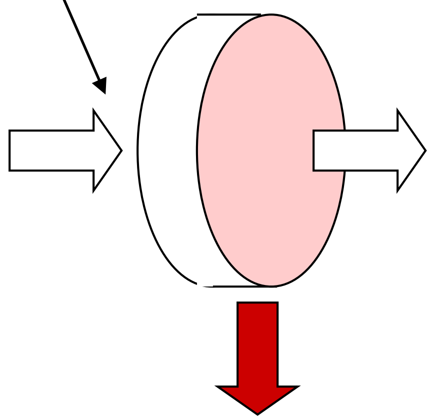
Fin Analysis



P: the fin perimeter

A_c: the fin cross-sectional area

$$q_x = -kA_C \frac{dT}{dx}$$



$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

$dq_{conv} = h(dA_S)(T - T_\infty)$, where dA_S is the surface area of the element

Energy balance: $q_x = q_{x+dx} + dq_{conv} = q_x + \dfrac{dq_x}{dx} dx + hPdx(T - T_\infty)$

Fin Analysis (cont.)

$$\frac{d}{dx}(kA_c\frac{dT}{dx}) - hP(T - T_\infty) = 0 \qquad A_c = A_c(x)$$

$$\theta = T - T_\infty \quad \rightarrow \quad \frac{d}{dx}(A_c\frac{d\theta}{dx}) - \frac{hP}{k}\theta = 0$$

For a constant cross-section A_c

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \quad m^2 = \frac{hP}{kA_c} \quad \longrightarrow \quad \boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx}}$$

Need two boundary conditions

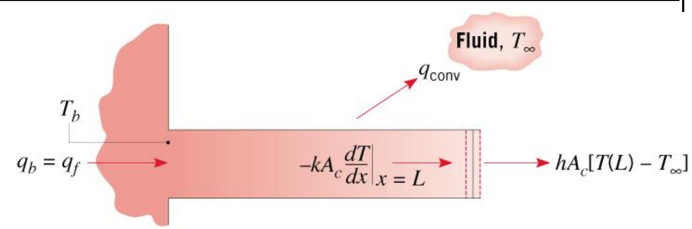
- *Base:* $\theta = \theta_b$ at $x = 0$
- *Tip: 4 scenarios at x = L*

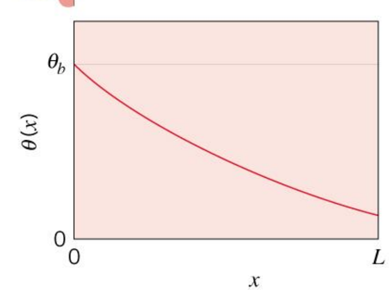
Temperature profiles

Case	Tip Condition	Temp. Distribution	Fin heat transfer
A	Convection heat transfer: $h\theta(L)=-k(d\theta/dx)_{x=L}$	$\dfrac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$	$M\theta_b\dfrac{\sinh mL+(h/mk)\cosh mL}{\cosh mL+(h/mk)\sinh mL}$
B	Adiabatic $(d\theta/dx)_{x=L}=0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M\theta_b \tanh mL$
C	Given temperature: $\theta(L)=\theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh m(L-x)+\sinh m(L-x)}{\sinh mL}$	$M\theta_b\dfrac{(\cosh mL-\theta_L/\theta_b)}{\sinh mL}$
D	Infinitely long fin $\theta(L)=0$	e^{-mx}	$M\ \theta_b$

$$\theta \equiv T-T_\infty, \qquad m^2 \equiv \frac{hP}{kA_C}$$

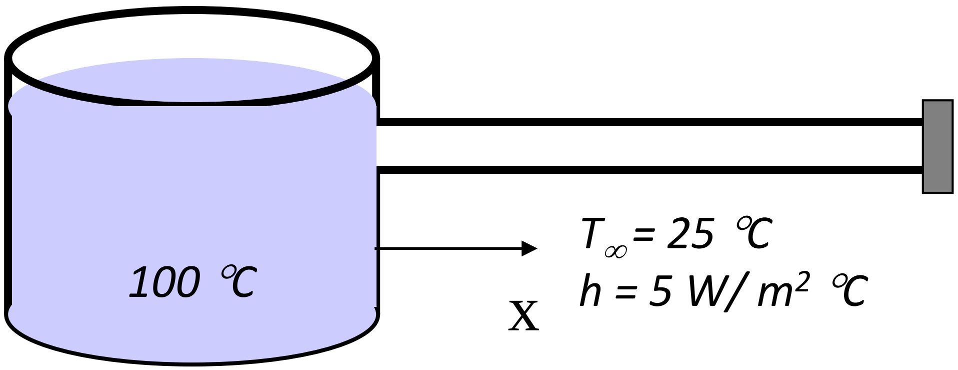
$$\theta_b=\theta(0)=T_b-T_\infty, \qquad M=\sqrt{hPkA_C}\,\theta_b$$





Example Problem

An Aluminum pot is used to boil water as shown below. The handle of the pot is *20-cm long, 3-cm wide*, and *0.5-cm thick*. The pot is exposed to room air at 25°C, and the convection coefficient is *5 W/m²-K*. Assume no heat transfer at the end of the handle. Question: can you touch the handle when the water is boiling? (*k* for Al = 237 W/m-K)

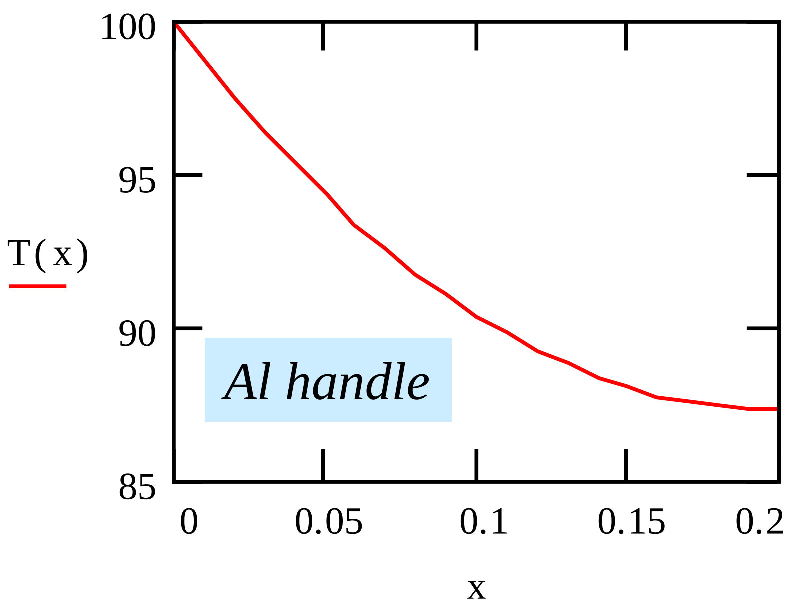


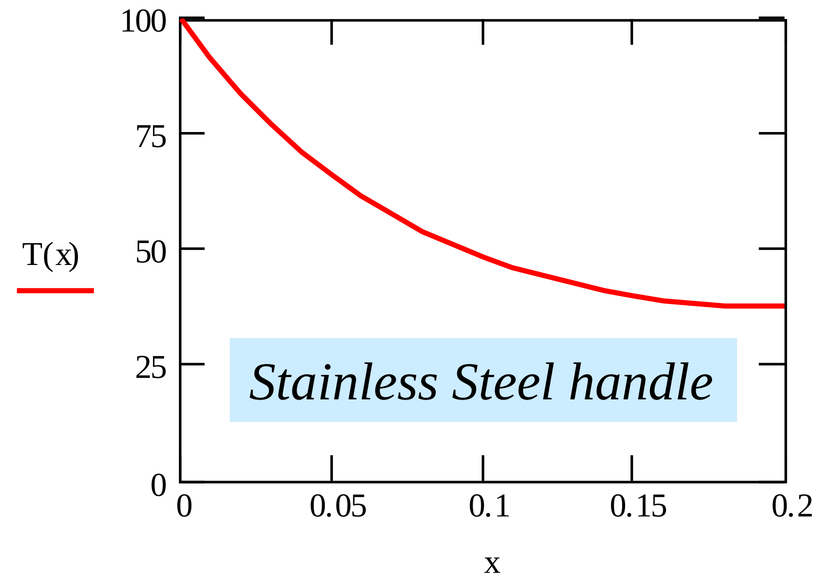
Steps

- Treat handle as fin
- Identify tip condition
  - *Adiabatic*
- Calculate P, m, M
- Get temp. profile
- Calculate T at x=L= 20 cm

Example (contd...)

Temperature distribution along the pot handle





Temperature at the tip = *87.3 ℃* → **Not safe** to touch

Why?

What can you do?

Fin parameters: Effectiveness (ε_f)

Fin effectiveness (ε_f): ***how effective*** is the fin

• *Ratio of heat transferred in presence of fin to in its absence*

$$\boxed{\varepsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b}} \longrightarrow$$

>1 → fin is effective

<1 → should not include fin

$$\varepsilon_f \uparrow \text{ with } \downarrow h, \uparrow k \text{ and} \downarrow A_c / P$$

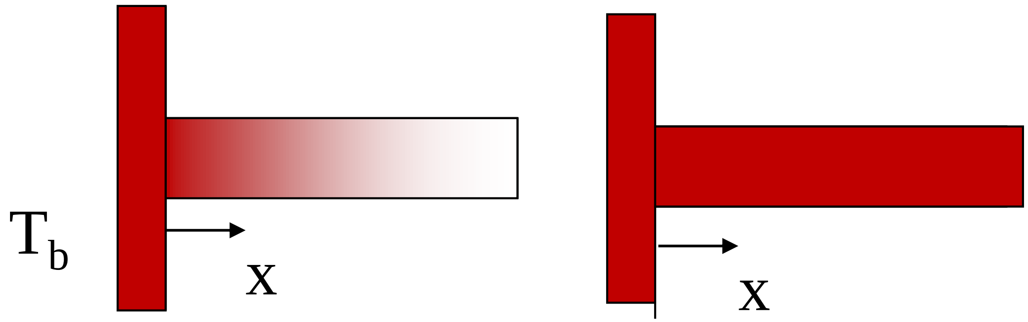
Remember we wanted fins on the air side!!

Fin parameters: Efficiency (η_f)

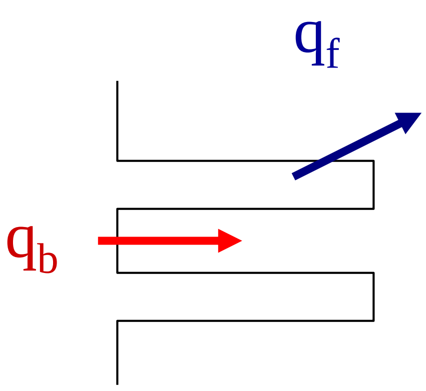
Fin efficiency (η_f): ***how close to ideal scenario*** is the fin

- *Ratio of heat transferred to that if entire fin were at base temp*

$$\boxed{\eta_f \equiv \frac{q_f}{q_{f,\max}} = \frac{q_f}{hA_f\theta_b}}$$



Real situation ***Ideal situation***



For a **fin array** with *N fins*,

A_B: total base area A_b, A_t: base and tip area of fin

A_f: surface area of a single fin (excluding tip)

$$\boxed{q_{tot,array} = h(A_s - NA_b + N\eta_f(A_f + A_t))\theta_b}$$

Fin array in heat sinks

<u>Parallel fins</u>

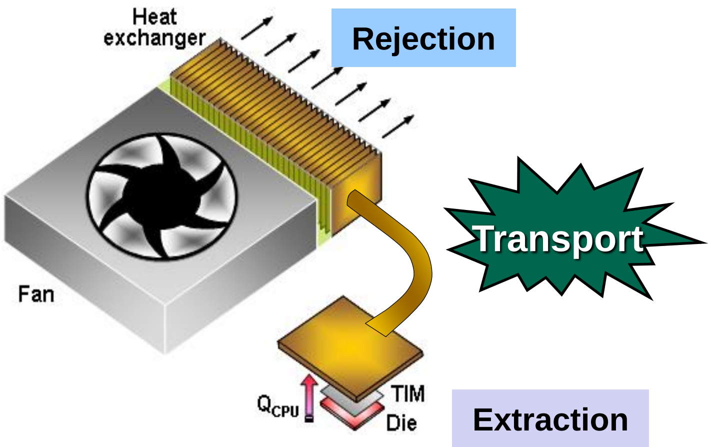
<u>Radial fins</u>

<u>Pin fins</u>

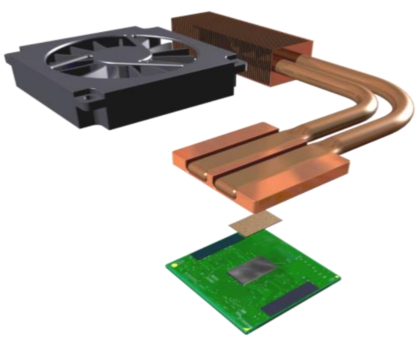
<u>Circular heat sink</u>

<u>Dovetail fins</u>

Fins in computing products



Desktop Computer



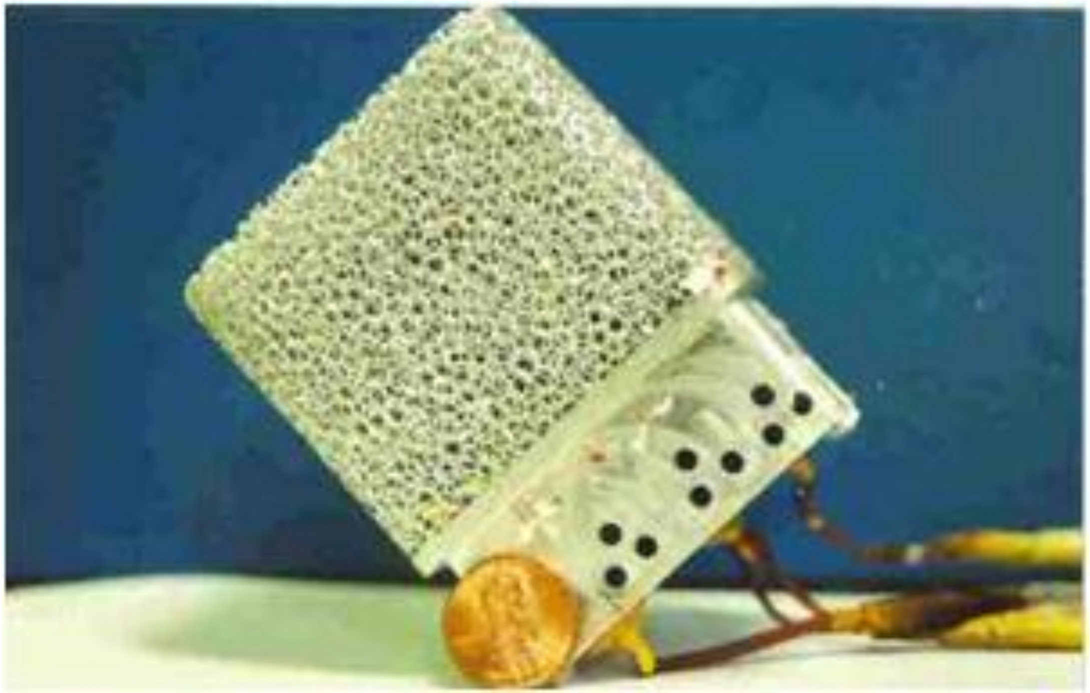
Laptop Computer

Summary

- Solved ODE for 1-D heat conduction to get temp profile
  - 4 different tip conditions

- Fin performance metrics – Effectiveness (ε_f) & Efficiency (η_f)

- Fin arrays used to design heat sinks
  - Commonly used in computing products



Thank you!!