सौरभ यादव
(भारतीय प्रौद्योगिकी संस्थान, खड़गपुर)
⊙ यांत्रिकी अभियांत्रिकी विभाग। ⊙

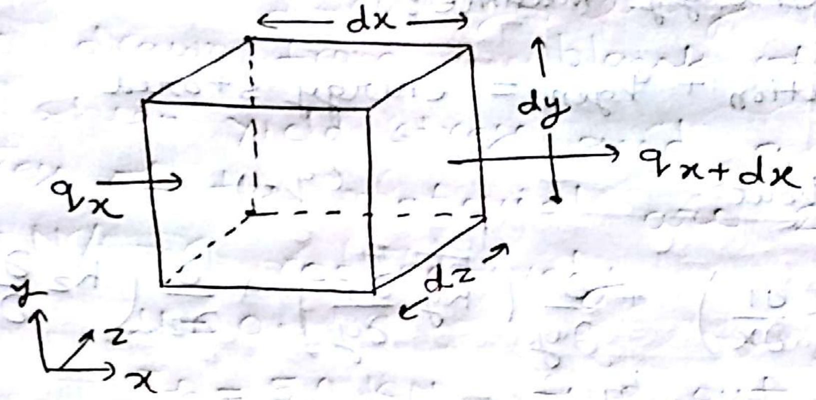
विषय: ऊष्मा संचलन

संवैधानिक चेतावनी:

- ⊙ पाठ्य सामग्री निर्माता परीक्षा मे किसी भी प्रकार की प्राप्तांक हानि के लिए उत्तरदायी नही माना जाएगा।
- ⊙ पाठ्य सामग्री निर्माता किसी प्रकार की मानविक त्रुटि के लिए जिम्मेदार नही होगा।
- ⊙ लोकोक्ति "पहले इस्तेमाल करें ! फिर विश्वास करें !"

21 July 2016

Heat conduction equation:



Fourier's law gives us

$$q_x = -K\cdot(dy\cdot dz)\cdot\frac{\partial T}{\partial x}$$

$$q_{x+dx} = -K(dy\cdot dz)\frac{\partial T}{\partial x} - (dy dz)\cdot\frac{\partial}{\partial x}\left(K\cdot\frac{\partial T}{\partial x}\right)\cdot dx$$

To get q_{x+dx} we use taylor series expansion

$$q_{x+dx} = q_x + \frac{\partial}{\partial x}(q_x)\,dx \ldots\ldots$$ neglecting higher terms

Net heat inflow in x direction

$$q_x - q_{x+dx} = \frac{\partial}{\partial x}\left(K_x\frac{\partial T}{\partial x}\right)dx\cdot dy\cdot dz$$

y-direction

$$q_y - q_{y+dy} = \frac{\partial}{\partial y}\left(K_y\frac{\partial T}{\partial y}\right)dx\,dy\,dz$$

z

$$q_z - q_{z+dz} = \frac{\partial}{\partial z}\left(K_z\frac{\partial T}{\partial z}\right)dx\,dy\,dz$$

Internal energy generatation:

internal energy per unit volume $= q'''$

$$\boxed{q'' = \frac{q}{\text{Area}} \qquad q''' = \frac{q}{\text{Volume}}}$$

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$$q_g = q''' \, dx\,dy\,dz$$

Lets do an energy balance:

$$q_{conduction} + q_{gen} = \text{energy stored}$$

$$\Downarrow \qquad\qquad\qquad\qquad (q_s)$$

$$q_{in} - q_{out}$$

$$\Rightarrow \left[\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) + q'''\right] \cancel{dx\,dy\,dz} = \rho c_p \frac{\partial T}{\partial \tau} \cdot \cancel{(dx\,dy\,dz)}$$

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial T}{\partial z}\right) + q''' = \rho c_p \frac{\partial T}{\partial \tau}$$

Let $K_x = K_y = K_z = K$

Then we get

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + q''' = \rho c_p \frac{\partial T}{\partial \tau}$$

or

$$\boxed{\nabla^2 T + \frac{q'''}{K} = \frac{\rho c_p}{K} \frac{\partial T}{\partial \tau}}$$

Thermal diffusivity $= \dfrac{K}{\rho c_p}$ $(m^2/s) = \alpha$

Revise PDE	Laplace eqn

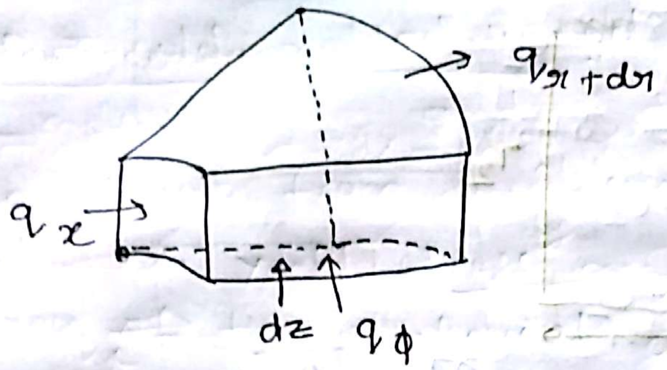
Case ① Is steady state

$$\nabla^2 T + \frac{q'''}{K} = 0$$

case ② No internal energy generation

$$\nabla^2 T = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

Case ① + ② . $\nabla^2 T = 0$



$x = r\cos\phi$

$y = r\sin\phi$

$z = z$

Finally you will get

$$\boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

25/07/16

Anisotropic material:

When K is non-uniform

$$K_{ij} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

Heat cond eq

$$K\nabla^2 T + q''' = \rho c_p \frac{\partial T}{\partial \tau}$$

generic form

$$\boxed{\frac{\partial}{\partial x_i}\left[K_{ij}\frac{\partial T}{\partial x_j}\right] + q''' = \rho c_p \frac{\partial T}{\partial \tau}}$$

orthotropic:

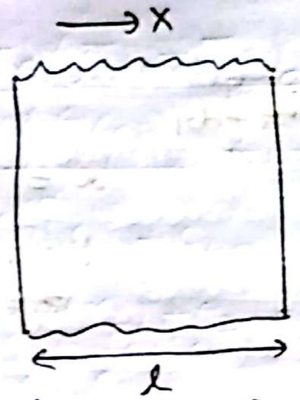
$K_{ij} = 0 \quad \text{for } i \neq j$

Isotropic:

$K_{ij} = K \quad \text{for } i = j$

$\quad\;\; = 0 \quad \text{for } i \neq j$

Plane wall



1. steady state $\Rightarrow \frac{\partial T}{\partial \tau} = 0$
2. No internal energy generation $\Rightarrow q''' = 0$
3. 1-D conduction $\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0$

Therefore the energy equation: becomes:

$$\frac{d^2T}{dx^2} = 0$$

$$\frac{dT}{dx} = C_1$$

$$T = C_1 x + C_2 \quad -(1)$$

Boundary condition

at $x=0$ $\quad T = T_1$

$$C_2 = T_1$$

at $x = l$ $\quad T = T_2$

$$T_2 = C_1 l + T_1$$

$$\left(\frac{T_2 - T_1}{l}\right) = C_1$$

$$\boxed{T = \frac{T_2 - T_1}{l} \cdot x + T_1}$$

$$\Rightarrow \quad q_x = -K \frac{dT}{dx} \cdot A$$

$$\boxed{q_x = -KA\left(\frac{T_2 - T_1}{L}\right)}$$

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$$\boxed{q_x = \frac{T_1 - T_2}{L/KA}}$$ $\frac{L}{KA} = R_{Th}$ thermal resistance

method 1 BC at $x = 0$

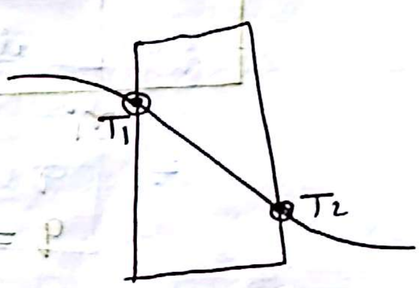
$$h_1(T_{1\infty} - T) = -K\frac{dT}{dx}$$

at $x = L$

$$h_2(T - T_{2\infty}) = -K\frac{dT}{dx}$$

method 2 Resistance Network

$$T_{1\infty} \xrightarrow{\frac{1}{h_1 A}} \xrightarrow[\to q_x]{\frac{l}{KA}} \xrightarrow{\frac{1}{h_2 A}} T_{2\infty}$$



$$q_x = \frac{T_1 - T_2}{\frac{1}{h_1 A} + \frac{L}{KA} + \frac{1}{h_2 A}}$$

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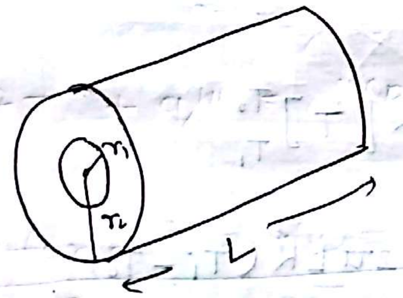
Hollow cylinder:



steady state 1D conduction in radial directio (no energy gen.)

$$K\nabla^2 T = 0$$

$$\Rightarrow \boxed{\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0}$$

B.C $\Rightarrow$ ① $T = T_i$ at $r = r_i$

$= T_0$ at $r = r_0$

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$$T(r) = c_1 \ln(r) + c_2$$

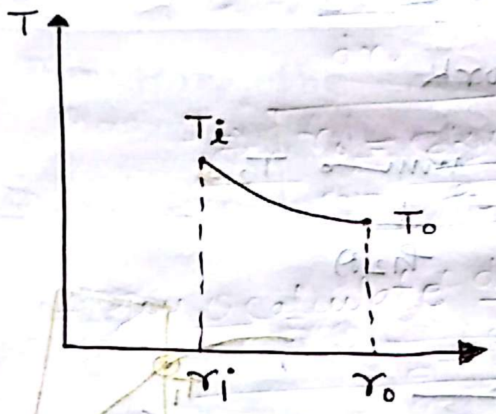
using the boundary conditions

$$T_i = c_1 \ln(r_1) + c_2$$

$$T_o = c_1 \ln(r_2) + c_2$$

$\rightarrow$ get c_1 & c_2

$$\boxed{T(r) = T_i + \frac{T_i - T_o}{\ln(r_i/r_o)} \ln\left(r/r_i\right)}$$



$$q = -KA \frac{dT}{dr}$$

$$q = -2\pi r \cdot L \, k \frac{dT}{dr}$$

$$\frac{q\,dr}{r} = -2\pi L \, k\,dT$$

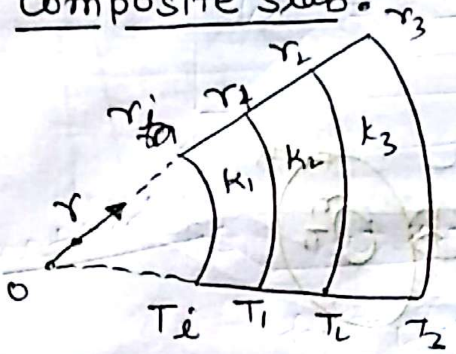
$$q \ln r \Big]_{r_i}^{r_o} = -2\pi L K \left[T\right]_{T_i}^{T_o}$$

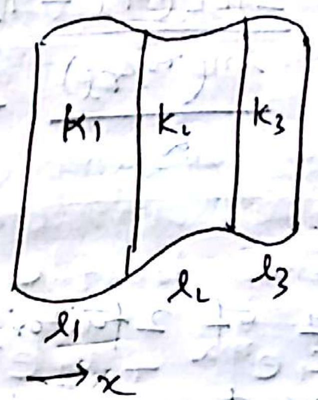
$$q \ln(r_o/r_i) = 2\pi L K (T_i - T_o)$$

$$\boxed{q = \frac{2\pi L K (T_i - T_o)}{\ln(r_o/r_i)}}$$

where $R_{Th} = \dfrac{\ln(r_o/r_i)}{2\pi L K}$

composite slab:





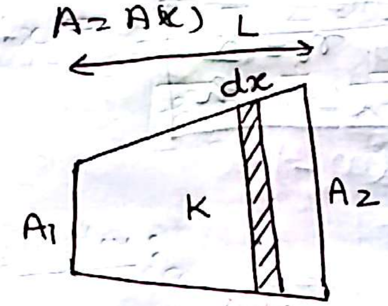
solve by series resistance

cylindrical

$R_1 = \frac{L_1}{K_1 A}$ $R_3 = \frac{L_3}{K_3 A}$

$$\boxed{R_2 = \frac{L_2}{K_2 A_2}}$$

when $A = A(x)$



$$dR_{Th} = \frac{dx}{KA(x)}$$

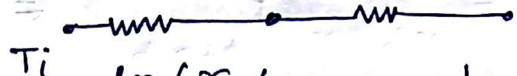
$$\boxed{R_{Th} = \int_0^L \frac{dx}{KA(x)}}$$

Insulated cylinder

area(↑) if K is high enough

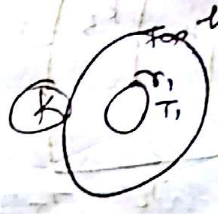
so no profit of insulation

Presence of critical radius



$$\frac{\ln(r_o/r_i)}{2\pi k L} \qquad \frac{1}{h_o(2\pi r_o L)}$$

$$q = \frac{2\pi L(T_i - T_s)}{\frac{\ln(r_o/r_i)}{K} + \frac{1}{r_o h}}$$



interplay of 2 factor

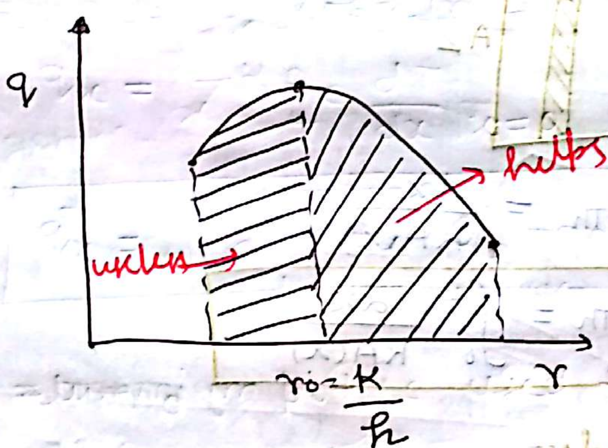
→ reduced conduction
enhanced convection

$$\frac{dq}{dr_o} = 0$$

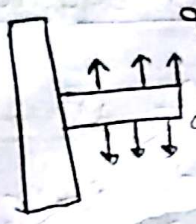
$$\Rightarrow \boxed{r_o\Big|_{crit} = \frac{K}{h}}$$

if you calculate $\frac{d^2q}{dr_o^2}$, $r_o = \frac{K}{h}$ it will be < 0

$$\Rightarrow \boxed{q \text{ is max at } r_o = \frac{K}{h}}$$



Case II $r_i > r_c$ helps



$q_{conv} (+ q_{rad}) = h A_s (T - T_\infty)$

q_{cond}

$$q_{conv} = h A_s (T - T_\infty)$$

$$q_{cond} = -k A_c \frac{dT}{dx}$$

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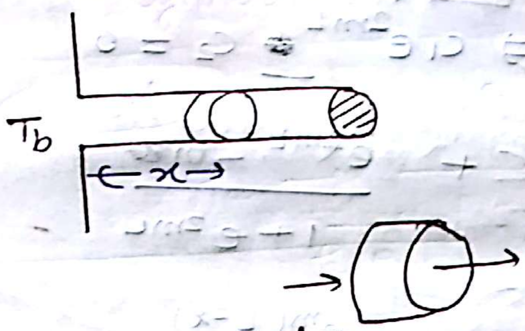
Fin analysis:



$$q_{x+dx} = q_x + \frac{dq_x}{dx} \cdot dx$$

$$q_x = -k A_c \frac{dT}{dx}$$

$\Downarrow$

$$dq_{conv} = h (dA_s)(T - T_\infty)$$ where dA_s is the surface are of the element

energy balance:

$$q_x = q_{x+dx} + q_{conv}$$

$$\boxed{= q_x + \frac{dq_x}{dx} \cdot dx + h (dA_s)(T - T_\infty)}$$

$$\frac{d}{dx}\left(k A_c \frac{dT}{dx}\right) - hP (T - T_\infty) = 0 \qquad A_c = A_c(x)$$

$$\theta = T - T_\infty \rightarrow \frac{d}{dx}\left(A_c \frac{d\theta}{dx}\right) - \frac{hP}{k}\theta = 0$$

for constant cross section A_c

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \ , \quad m^2 = \frac{hP}{kA_c}$$

$$\Rightarrow \boxed{\theta(x) = c_1 e^{mx} + c_2 e^{-mx}}$$

Need two boundary condition

→ Base $\theta = \theta_b$ at $x = 0$

→ Tip: 4 scenarios at $x = L$

Temp profile

$$\theta = c_1 e^{mx} + c_2 e^{-mx}$$

B.C. $\theta(0) = \theta_b \Rightarrow \theta_b = c_1 + c_2$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \Rightarrow 0 = m c_1 e^{mL} - m c_2 e^{-mL}$$

$$\Rightarrow c_1 e^{2mL} - c_2 = 0$$

$$\frac{\theta(x)}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{(2mL - mx)}}{1+e^{2mL}}$$

or $$\frac{\theta}{\theta_b} = \frac{e^{m(x-L)}}{e^{-mL}+e^{mL}} + \frac{e^{m(L-x)}}{e^{-mL}+e^{mL}}$$

$$= \frac{\cosh m(L-x)}{\cosh(mL)}$$

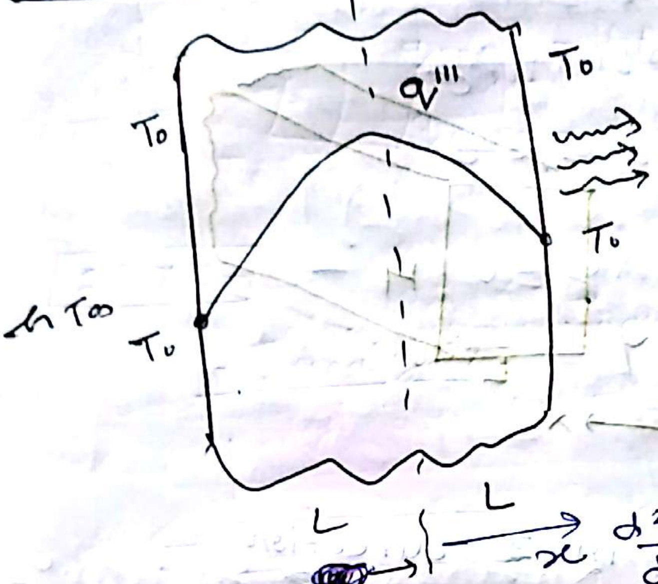
$$q_x = -KA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$\boxed{q_x = \sqrt{hPKA_c}\,\theta_b \tanh(mL)}$$

Ques: An aluminium pot is used to boil water as shown below

01/08/16

~~Internal energy generation~~: Fins



$$\frac{d^2T}{dx^2} + \frac{q'''}{K} = 0$$

steady state

B.C. $T = T_0$ at $x = L$ ($\&\ -L$)

$$\left.\frac{dT}{dx}\right|_{x=0} = 0$$

$$\Rightarrow \frac{dT}{dx} = -\frac{q'''}{K}x + C_1$$

$$T(x) = -\frac{q'''}{2K}x^2 + C_1x + C_2$$

BC 2 ~~Since~~ $\frac{dT}{dx}(x=0) = 0 \Rightarrow C_1 = 0$

BC1 $$C_2 = \frac{q''' L^2}{2K} + T_0$$

$$T(x) = \frac{q'''}{2K}(L^2 - x^2) + T_0$$

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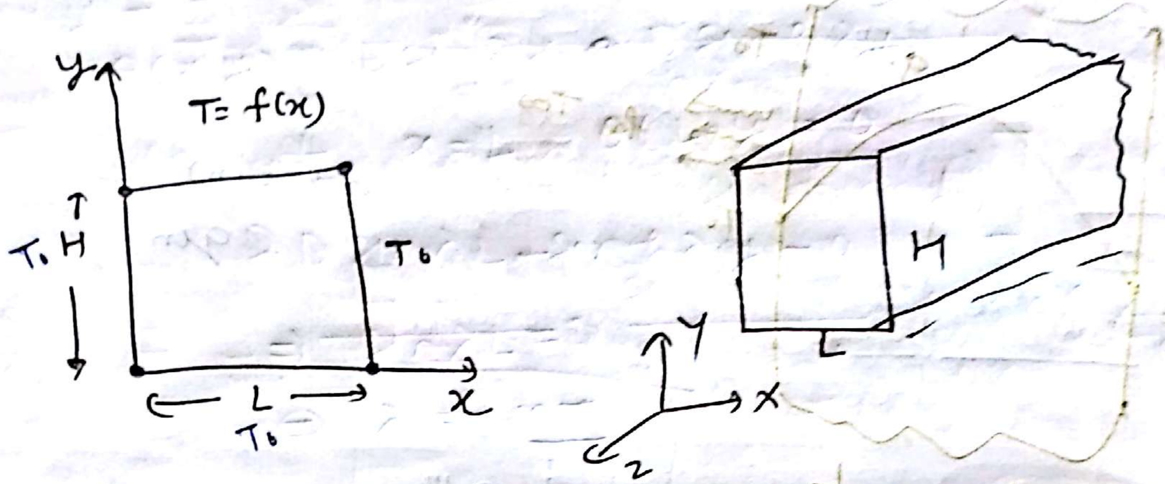
steady state conduction (2D & 3D)



Bar is infinitely long in Z direction

Hence 2D conduction

governing eq.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \text{(homogenous equation)}$$

B.C

$x \Rightarrow$ at $x = 0$, $T = T_0$

$x = L$, $T = T_0$

$y \Rightarrow$ at $y = 0$, $T = T_0$

$y = H$, $T = f(x)$

let $\theta = T - T_0$

$$\Rightarrow \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

& $\theta = 0$ @ $x = 0, L$

$\theta = 0$ @ $y = 0$

$\theta = f(x) - T_0$ @ $y = H$

This is a homogeneous PDE with homogenous BC in x direction

Apply principal of separation of variables

$\Rightarrow \quad \theta(x, y) = X(x) \cdot Y(y)$

$\Rightarrow -\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{Y}\frac{d^2Y}{dy^2} = \pm\lambda^2$

set the value of λ^2 be $-$ve

$$\frac{d^2X}{dx^2} - \lambda^2 X = 0$$

$$\frac{d^2Y}{dy^2} + \lambda^2 Y = 0$$

$X(x) = $

$= A\cosh\lambda x + B\sinh\lambda x$

$Y(y) = $

$= C\cos\lambda y + D\sin\lambda y$

At $\quad x = 0, L \quad , \theta(0) = \theta(L) = 0$

$\Rightarrow \quad A = 0$

$B\sinh\lambda L\,(C\cos\lambda y + D\sin\lambda y) = 0$

This mean $B = 0$, or $\lambda = 0$

$\Rightarrow$ either gives trivial solution

(ii) Let $\lambda^2 = 0$

$\Rightarrow \quad \theta(x, y) = (Ax + B)(Cy + D)$

Applying BC $\quad A = 0$ & $B = 0$

trivial solution

(iii) $\lambda^2 = +$ve

$X(x) = A\cos\lambda x + B\sin\lambda x$

$Y(y) = C\cosh\lambda y + D\sinh\lambda y$

At $\quad x = 0, L$

$\theta(0) = \theta(L) = 0$

(i) $\theta = 0$ @ $y=0$

$A\cos\lambda x + B\sin\lambda x$

$\theta(x,y) = B\sin\lambda x \cdot D\sinh\lambda y$

At $x=L$, $\theta(L,y)=0$

$B.D\sin\lambda L \cdot \sinh\lambda y = 0$

$\Rightarrow \sin\lambda L = 0$

$\Rightarrow \boxed{\lambda = \frac{n\pi}{L}}$

$\Rightarrow \theta(x,y) = \sum_{n=1}^{\infty} K_n \sin\lambda_n x \cdot \sinh\lambda_n y$

where $B.D = K$

(iv) $\theta(x) = f(x) - T_0$ @ $y=H$

$\Rightarrow f(x) - T_0 = \sum_{n=1}^{\infty} K_n(\sin\lambda_n x)(\sinh\lambda_n H)$

---

orthogonal fun

$g_1(x), g_2(x)$

are orthogonal if

$$\int_a^b g_m(x)\, g_n(x) = 0$$

if $f(x)$ can be expressed as

$$f(x) = \sum_{n=1}^{\infty} c_n \cdot g_n(x)$$

$$\int_a^b f(x) g(x)\, dx = c_1 \int_a^b g(x) g_n(x)\, dx$$

$$+ c_2 \int_a^b g_2(x) g_n(x)\, dx$$

$$+ c_m \int_a^b g_m(x) g_n(x)\, dx$$

or: $c_n \int_a^b g_n^2(x)\,dx = \int_a^b f(x)\, g_n(x)\,dx$

become $\int_a^b g_m(x)\, g_n(x)\,dx = 0$

$$\Rightarrow \quad c_n = \frac{\int_a^b f(x)\, g_n(x)\,dx}{\int_a^b g_n^2(x)\,dx}$$

Let us now consider

$g_n(x) = \sin \lambda_n x$

since $\int_0^L \sin(\lambda_m x)\sin(\lambda_n x)\,dx = 0$

if $m \neq n$

Let $c_n = K_n \sinh \lambda_n H$

Therefore we get

$$K_n(\sinh \lambda_n H) = \frac{\int_0^L (f(x) - T_0)(\sin \lambda_n x)\,dx}{\int_0^L \sin^2(\lambda_n x)\,dx}$$

$$\int_0^L \sin^2 \lambda_n x\,dx = \frac{1}{2}\int_0^L (1 - \cos 2\lambda_n x)\,dx$$

$$= \frac{L}{2} - \frac{1}{4\lambda_n} \sin 2\lambda_n L$$

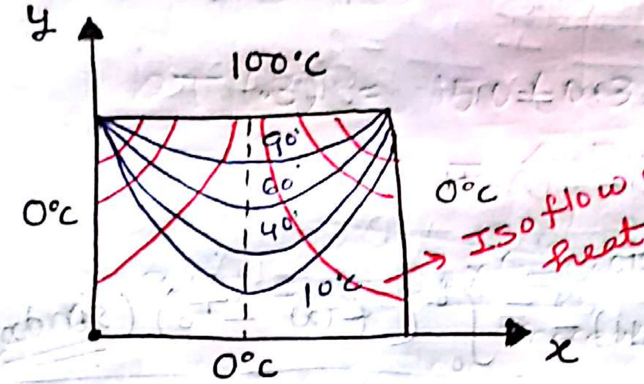
$$\Rightarrow K_n = \frac{2}{L \sinh \lambda_n H} \int_0^L (f(x) - T_0) \cdot \sin \lambda_n x \cdot dx$$

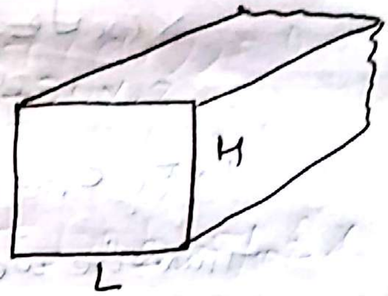
$$\boxed{\theta(x,y) = \frac{2}{L} \sum_{n=1}^{\infty} \left[\frac{\int_0^L (f(x) - T_0)\sin \lambda_n x\,dx}{\sinh \lambda_n H} \cdot (\sin \lambda_n x)(\sinh \lambda_n y) \right]}$$

Example : for $f(x) = T_c$ find $\theta(x, y)$

$$\theta(x,y) = \frac{2}{L}\left[\sum_{n=1}^{\infty}\left(\frac{\int_0^L (T_c - T_0)\sin\lambda_n x\, dx}{\sinh\lambda_n H}\cdot(\sin(\lambda_n x)\cdot\sinh\lambda_n y)\right]\right.$$

$$\frac{\theta}{\theta_c} = 2\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\cdot\frac{\sinh\lambda_n y}{\sinh\lambda_n H}\cdot\sin\lambda_n x$$





$y = H$ isotherm

$y=0$

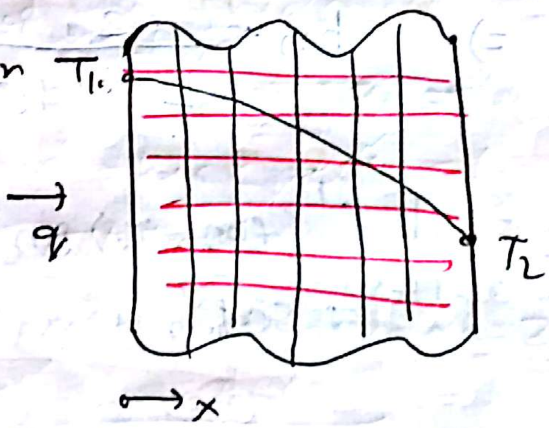
$x=0$

$x = L$

Isotherms ⇒ const, temp lines never intersects

Heat flux lines :

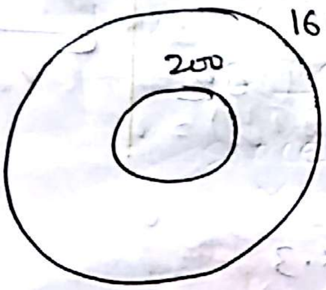
1D - conduction



Qus A steel pipe ~~of~~ of conductivity 50 W/m K has inside and outside temp of 200°C & 160°C find

find the heat flow rate by unit length ~~find the heat flow per unit~~ and heat flux on both inside and outside surface

ID = 25 mm, OD = 40 mm



$K_s = 50$ W/mK

$d_1 = 25$ mm $r_1 = 12.5$ mm

$d_0 = 40$ mm $r_0 = 20$ mm

$T_i = 200°C$

$T_0 = 160°C$

$$T(r) = T_i + \frac{(T_i - T_0)}{\ln(r_i/r_0)} \ln(r/r_i)$$

$$\frac{q}{l} = \frac{40}{\frac{\ln(40/25)}{2\pi \times 50}} = 29 \text{ KW/m}$$

Qus: certain material of K = 0.04 W/mK thickness 30 cm

a particular instant of time the temp distr'n with x x is from left side of bar

$T(x) = 150x^2 - 30x$

x is in [m]

① q at x = 0, 30

② ~~q~~ Is the solid heat up or cooling down

$T(0) = 0$

$$\frac{dT}{dx} = 300x - 30$$

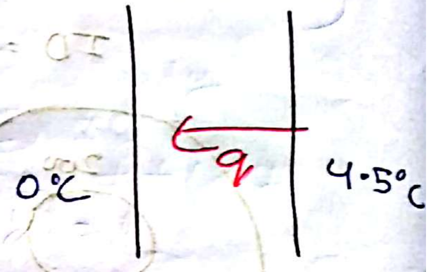
$$q = -KA\frac{dT}{dx}$$

$$= -0.04 \times 1\,(300x - 30)$$

$$= -0.04\,(90 - 30)$$

$$= -0.04 \times 60$$

$$q = -2.4 \text{ W/m}^2$$



$$T(.3) = 150(0.3)^2 - 30 \times .3$$

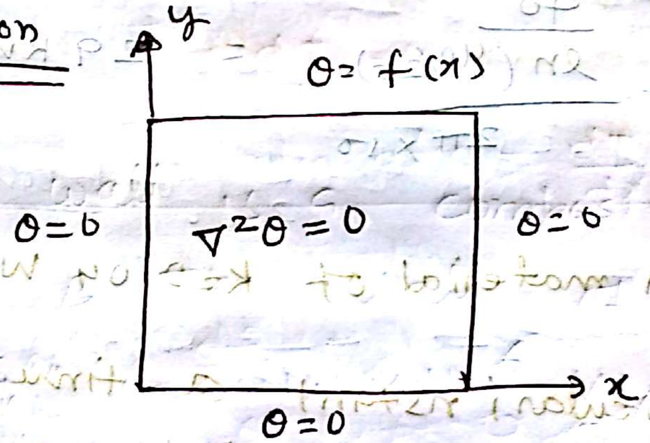
$$= 4.5\,°C$$

$$q(0) = -30 \times (-0.04 \times 1) = 1.2 \text{ W/m}^2$$

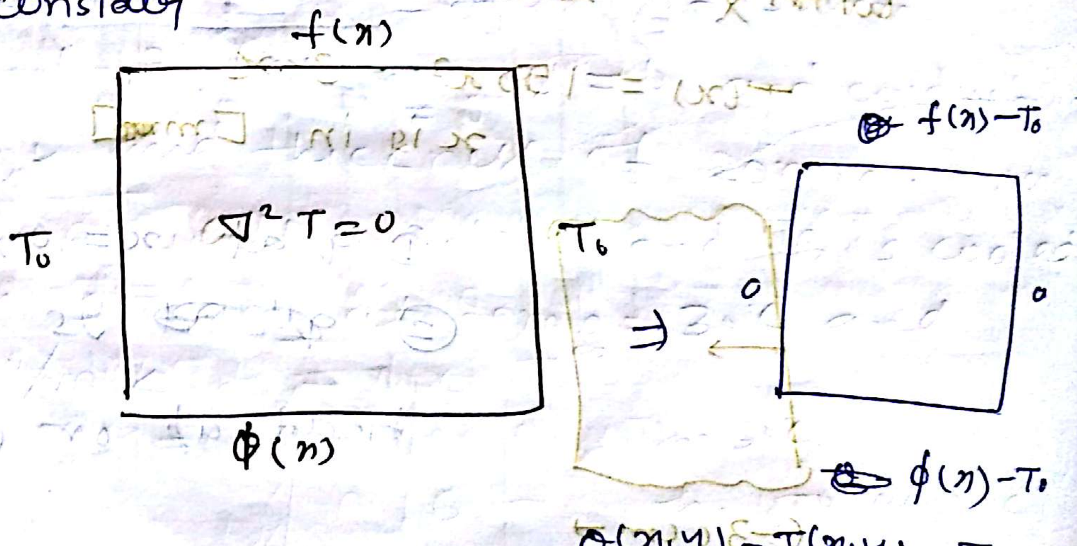
Increasing temperature of block

4 Aug 2016

2D conduction



lets consider

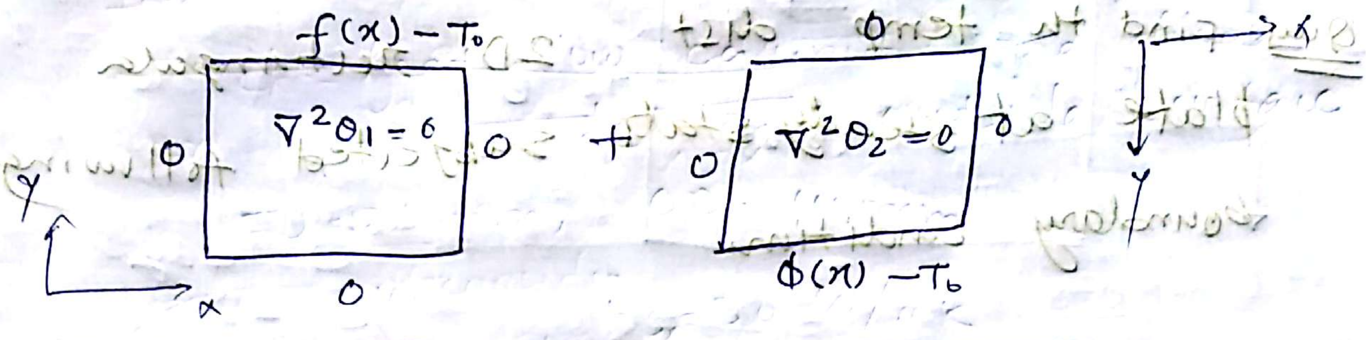


$$\theta(x,y) = T(x,y) - T_0$$

Homogenious PDE

BC

method of ~~superposition~~:

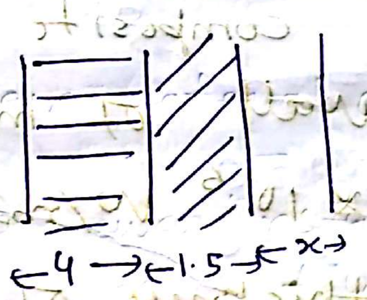


$$\theta(x,y)=\theta_1(x,y)+\theta_2(x,y)$$

Ques An exterior wall of a house may be approx. 4 inch layer of common brick ($K=0.7$ W/mK) followed by 1.5 inch layer of gipsum ($K=0.48$ W/mK).

What thickness of rock wool insulation ($K=0.065$) should be added to reduce heat loss or gain by 80%.

Ans



$\Rightarrow$ $q_{new}=0.2\,q_{old}$

$R_{old}=0.2\,R_{new}$

K is less for wool so that reduce heat loss

$$q=\frac{\Delta T}{R_{Th}}$$

$$\Rightarrow \frac{R_h}{0.2}=5R_{Th}$$

$$R_{Th}=\frac{L}{KA}$$

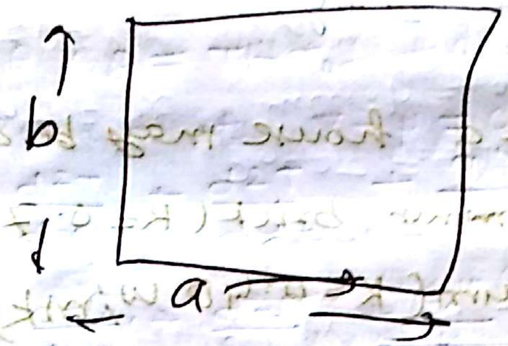
$$\Rightarrow\left(\frac{4}{0.7\times1}+\frac{1.5}{0.48\times1}\right)5=\frac{4}{0.7\times1}+\frac{1.5}{.48}$$

$$44.196-\quad=\frac{x}{0.065}+\frac{x}{.065}$$

$35.3567 \times 0.065 = x$

$x = 2.290$ inch

Ques Find the temp dist 2D rectangular plate at steady state subjected following boundary condition.



$T(0,y) = 0$

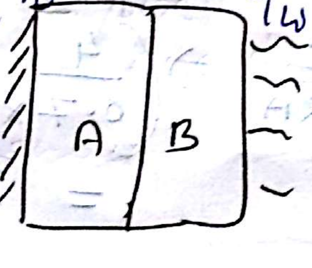
$T(a,y) = 0$

$T(x,b) = 0$

$T(x,b) = Ax$

Sol

Ques A plane wall is a composite of two mater. A & B. The wall of material A has q''' at 1.5×10^6 W/m3, and K = 75 W/mK and the thickness of 50 mm. The material B has no $q''' = 0$ and conductivity of 150 W/mK and thickness of 20mm. inner surface of A well insulated and B is cooled by water stream at $temp_\infty = 30°C$ and h = 1000 W/m2K



$T_\infty = 30°C$

$h = 1000$ W/m²K

① Sketch the temp profile of composite wall at steady state

② determine the temp T_0 and T_∞ at the cooled surface

insulated

heat gen in A per unit area $= 1.5\times10^6 \times 1 \times 0.05$
$= 7.5\times10^4$ W

for T_w

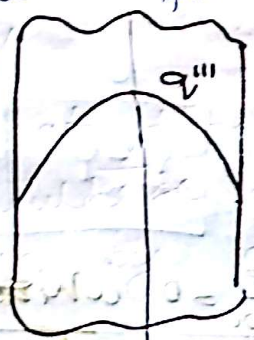
$$h\cdot1\cdot(T_w - T_\infty) = 7.5\times10^4$$

$$T_w - 30 = \frac{7.5\times\cancel{10^4}\times1\cancel{000}}{1\cancel{000}}$$

$$\cancel{T_w - 30 = 75}$$

$$\boxed{T_w = 105°C}$$

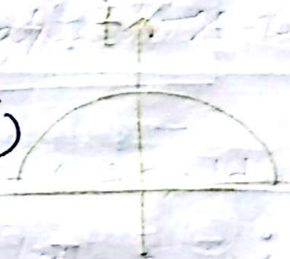
This condition is valid only for identical mirror images



1) $q'' = q'''_A\, t_A$

$= h\,(T_w - T_\infty)$

$\Rightarrow T_w = 105°C$



② $q'' = q'''_A\, t_A = K\,\dfrac{(T_i - T_w)}{t_B}$

$\rightarrow$ get $T_i = 115°C$

3) solve the following problem for A where

$T_1 = T_i$

Temp di $\Rightarrow$ get $T_0 \Rightarrow (T_{max})$

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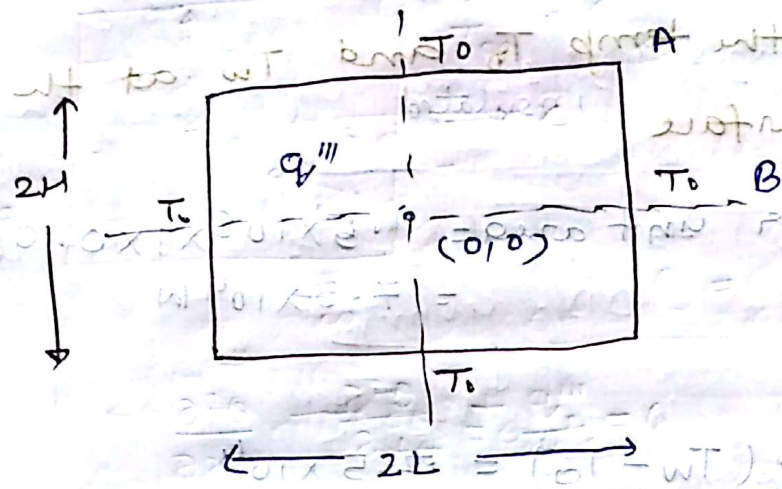
8/8/16 **2D conduction with internal energy generation:**



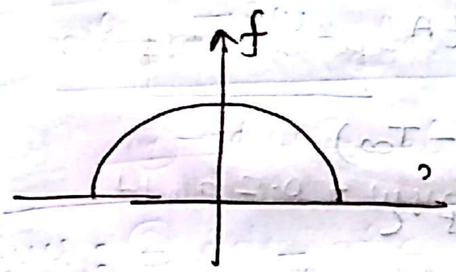
assumptions:
① steady
② 2D
③ $K \neq f(x,y)$

$\theta = T - T_0$

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{q'''}{K} = 0$$

$$\frac{d^2\theta}{dx^2} + \frac{d^2\theta}{dy^2} + \frac{q'''}{K} = 0$$

★ Symmetric problem



$\frac{df}{dx} = 0$

adiabatic ⇒ $\frac{dT}{dx} = 0$ where T = temp

① BC ⇒ $x = 0$: $\frac{d\theta}{dx}(0,y) = 0$

$x = L$: $\theta(L,y) = 0$

② at $y = 0$ ⟶ $\frac{d\theta}{dy}(x,0) = 0$

$y = H$ ⟶ $\theta(x,H) = 0$

since PDE is non-homogeneous, we can not use separation of variables we will define solution as

$$\theta(x,y) = \psi(x,y) + \phi(x) \quad (\text{or } \phi(y))$$

↑ homogenous function

b) include the term $\frac{q'''}{K}$ as part of $\phi(x)$

$$\Rightarrow \frac{d^2\phi}{dx^2} + \frac{q'''}{K} = 0 \quad \Bigg| \quad \frac{d\phi}{dx}(0) = 0, \quad \phi(L) = 0$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \cancel{\left[\frac{d^2\phi}{dx^2} + \frac{q'''}{K}\right]}^{\to 0} = 0$$

$$\Rightarrow \nabla^2\psi = 0 \quad \Bigg| \quad \frac{\partial\psi}{\partial x}(0,y) = \frac{\partial\theta}{\partial x}(0,y) - \frac{\partial\phi}{dx}(0) = 0 - 0 = 0$$

$$\psi(L,y) = \theta(L,y) - \phi(L) = 0$$

$$\frac{\partial\psi}{\partial y}(x,0) = \frac{\partial\theta}{\partial y}(x,0) = 0$$

$$\psi(x,H) = \theta(x,H) - \phi(x) = -\phi(x)$$

$\Rightarrow$ For $\phi(x)$ solve the ODE

$$\Rightarrow \phi(x) = \frac{q'''}{2K}(L^2 - x^2)$$

For $\psi(x,y)$ use separation of variables with x as homogeneous direction

$$\boxed{\psi(x,y) = -\frac{2q'''L^2}{K}\sum_{n=0}^{\infty}\frac{(-1)^n}{(\lambda_n L)^3}\left[\frac{\cosh\lambda_n y}{\cosh\lambda_n H}\right]\cos\lambda_n x}$$

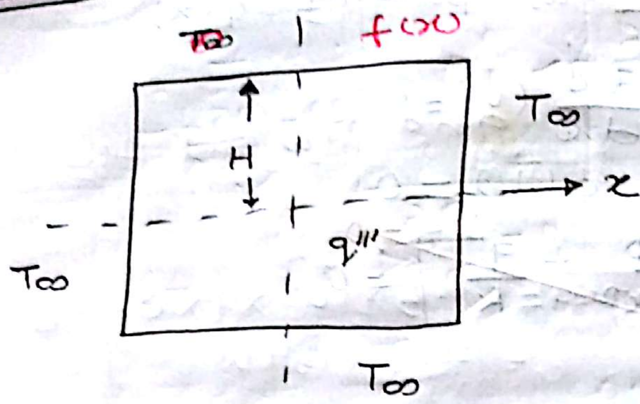
where $\lambda_n L = (2n+1)\pi/2$

final solution

$$\boxed{\theta(x,y) = \psi(x,y) + \phi(x)}$$

40 hours to think about

10/08/16



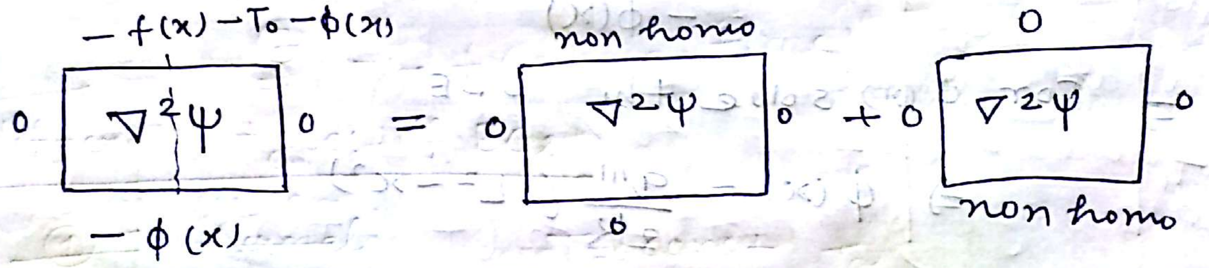
$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{q'''}{K} = 0$$

$$\theta(x,y) = \psi(x,y) + \phi(x)$$

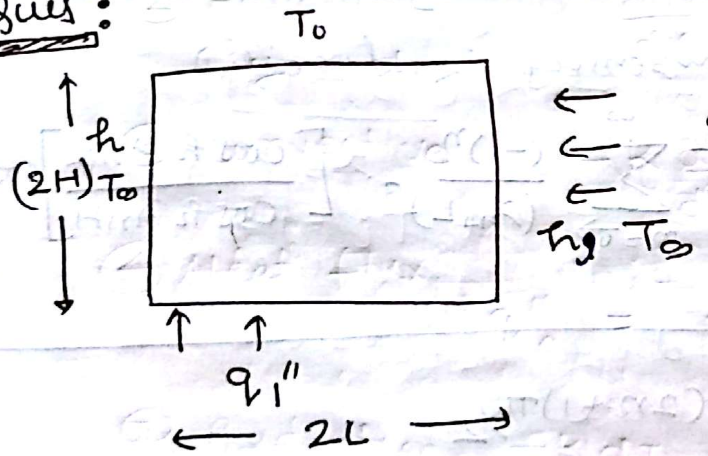
such that $\nabla^2\psi = 0$ → Homogeneous PDE with homogenous x BC
one " y BC

$$\frac{d^2\phi}{dx^2} + \frac{q'''}{K} = 0 \quad \text{ODE}$$

Imp



Ques:

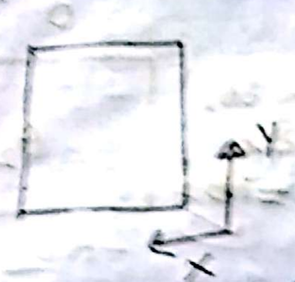


eq ⇒ $\nabla^2 T = 0$

$\theta = T - T_\infty$

⇒ $\nabla^2\theta = 0$

BC:



BC ⇒

At $x = 0$, $\theta \Rightarrow$

$$q_2'' - h\theta = -k\frac{\partial \theta}{\partial x}$$ ⊗

at $x = 2L$, $-k\dfrac{\partial \theta}{\partial x} = h\theta$ ✓

At $y = 0$, $-k\dfrac{\partial \theta}{\partial y} = q_1''$ ⊗

$y = 2H$; $\theta = \theta_0$ ⊗ homo or Not

✓ homogenous

⊗ non-homogen

θ_0

$$-k\frac{\partial \theta}{\partial x} = h\theta \qquad \boxed{\nabla^2\theta = 0} \qquad -k\frac{\partial \theta}{\partial x} = q_2'' - h\theta$$

$$-k\frac{\partial \theta}{\partial y} = q_1''$$

$$\Rightarrow \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

$$\Rightarrow \frac{\partial^2 \theta}{\partial x^2} = -\frac{\partial^2 \theta}{\partial y^2} = m$$

~~$\theta(x,y) = (c_1 \cos \lambda x + c_2 \sin \lambda$~~

~~$\dfrac{\partial^2 \theta}{\partial x^2} = m$~~ ~~$\dfrac{\partial^2 \theta}{\partial y^2} = -m$~~

~~$\dfrac{\partial \theta}{\partial x} = mx + c_1$~~ ~~$\dfrac{\partial \theta}{\partial}$~~

~~$\theta = \dfrac{mx^2}{2} + c_1 x + c_2$~~

$$\Rightarrow \theta(x,y) = (c_1 \cos\lambda x + c_2 \sin\lambda x)\cdot(c_3 e^{\lambda y} + c_4 e^{-\lambda y})$$

1

$$\theta_1 = \theta_0$$

$$= \quad -k\frac{\partial \theta_1}{\partial x} = h\theta_1 \qquad \theta_1 \qquad -k\frac{\partial \theta_1}{\partial x} = h\theta_1 \quad + \quad -k\frac{\partial \theta_2}{\partial x} = h\theta_2 \qquad \theta_2 \qquad -k\frac{\partial \theta_2}{\partial x} =$$

$$\frac{\partial \theta_1}{\partial y} = 0 \qquad\qquad -k\frac{\partial \theta_2}{\partial y} = q_1''$$

$$\theta_3 = 0$$

$$+ \quad -k\frac{\partial \theta_3}{\partial x} = \theta_3 h \qquad \theta_3 \qquad -k\frac{\partial \theta_3}{\partial x} = q_2''$$

$$\frac{\partial \theta_3}{\partial y} = 0$$

for division

① 3 homo BC are in box

②

different types of BCs

① Isothermal $\quad T = \text{const}$

② Adiabatic (symmetry) $\quad \frac{dT}{dx} = 0$

③ convective $\quad -k\frac{dT}{dx} = h(T - T_\infty)$

④ Heat flux

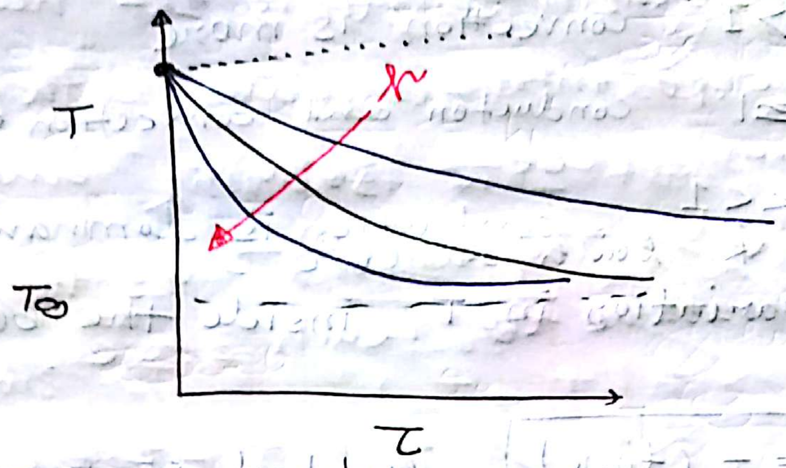
$$q_0'' = -k\frac{dT}{dx}$$

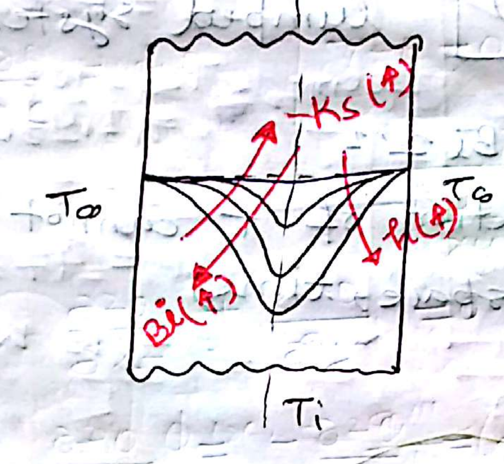
⑤ Radiation $\quad -k\frac{dT}{dx} = \sigma\epsilon(T^4 - T_\infty^4)$

Transient conduction:

Temperature varies with time

Cup of tea left to cool





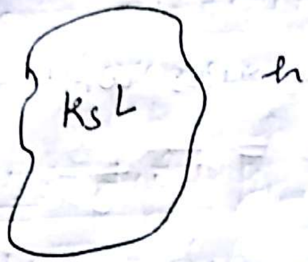
concept of Biot no. (Bi)

when you quench a body in a hot cold fluid There is (a) convection at surface (followed by) (b) conduction inside the body

define: $\boxed{Bi = \dfrac{\text{conduction resistance}}{\text{Convection resistance}}}$

$$Bi = \frac{L/KA}{1/hA} \quad \text{(Relative measure)}$$

$$\boxed{Bi = \frac{hL}{K_{solid}}}$$



If $Bi > 1$ convection is more

$Bi \simeq 1$ conduction and convection compared to

$Bi << 1$ conduction is dominant

(1) if $Bi << 1$ high K low h or low L

spatial variation in T inside the body is negligible

$\Rightarrow$ $\boxed{T = f(\text{time})}$ lumped system

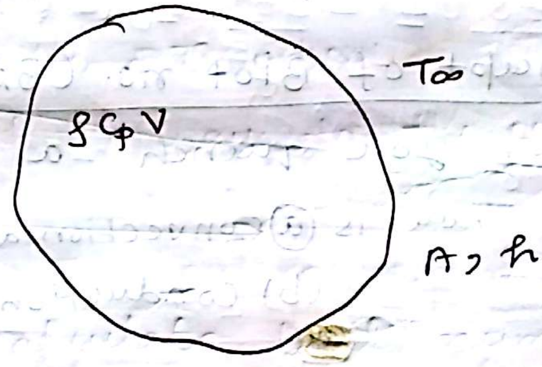
(2) if $Bi > 2$ or $Bi \simeq 1$

$\Rightarrow$ spatial variation of T cannot be neglected

$\Rightarrow$ $T = f(\text{space}, \text{time})$

11/08/2016

lumped system:



valid only if $Bi = \frac{hL}{K} << 1$

Assume a body initially at T_i is suddenly exposed to an ambient at $(T_\infty < T_i)$

Heat loss from the body = heat dissipated to the ambient

$$\Rightarrow \rho V c_p \frac{dT}{d\tau} + hA(T - T_\infty) = 0$$

$$T = T(\tau)$$

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Where $T = T_i$ at $\tau = 0$

$$\Rightarrow \frac{dT}{T - T_\infty} = -\frac{hA}{\rho c_p V} \cdot d\tau$$

$$\Rightarrow \ln(T - T_\infty) = -\frac{hA}{\rho c_p V} \cdot \tau + \ln C$$

put $T = T_i$ at $\tau = 0$

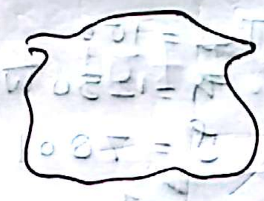
$$\Rightarrow \boxed{\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[\left(-\frac{hA}{\rho c_p V}\right)\tau\right]}$$

Note for L ⇒

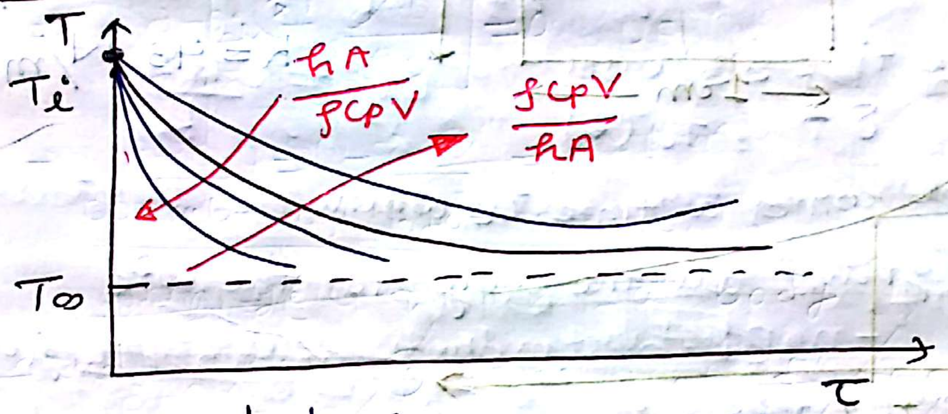
⇒ $L = R$

⇒ $L = D$

2D



$L = \frac{V}{A}$



Let $\frac{\rho c_p V}{hA} = \phi$ ⇒ time constant
e folding time

$$\Rightarrow \frac{T - T_\infty}{T_i - T_\infty} = e^{-\tau/\phi}$$

⊛ Large time constant Lower response

At $\tau = \phi$

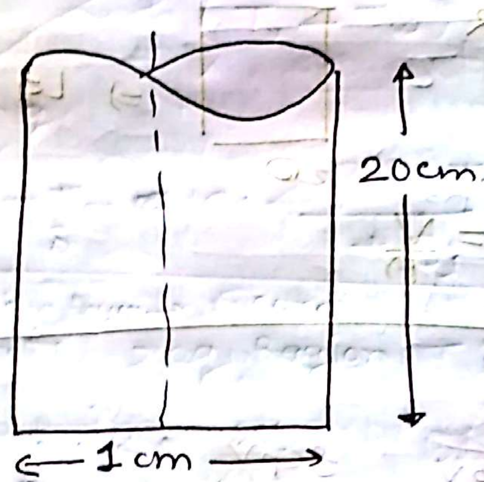
$$T - T_\infty = \frac{1}{e}(T_i - T_\infty)$$

$$= 0.368\,(T_i - T_\infty)$$

temp difference reached 36.8% of inital difference are called time constant.

Ques: during quenching a cylind rod of steel 1cm in dia 20cm in length is 1st heated 750°c and then immersed into a water bath at 100°c the heat transfr coeff 250 W/m²K and $\rho = 7801$ kg/m3 cp = 473 J/kg.K, Thermal cond - 43 W/mK calculate the time to reach 300°C.

Ans:



$T_i = 750°C$

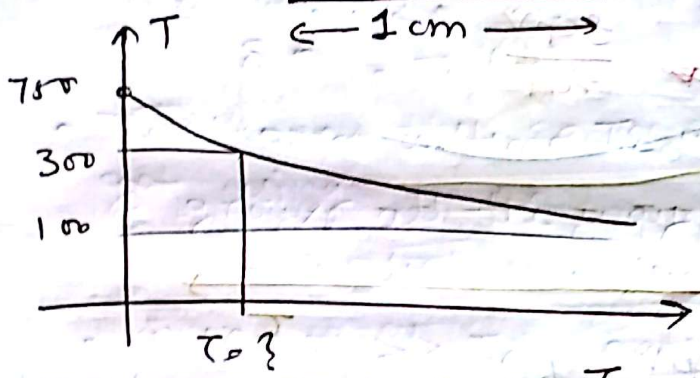
$T_\infty = 100$

$h = 250 \ W/m^2K$

$\rho = 7801 \ Kg/m3$

$cp = 473 \ J/Kg \cdot K$

$K = 43 \ W/mK$



calculate biot number

$$Bi = \frac{hL}{K} = \frac{250 \times 0.005}{43} = 0.029$$

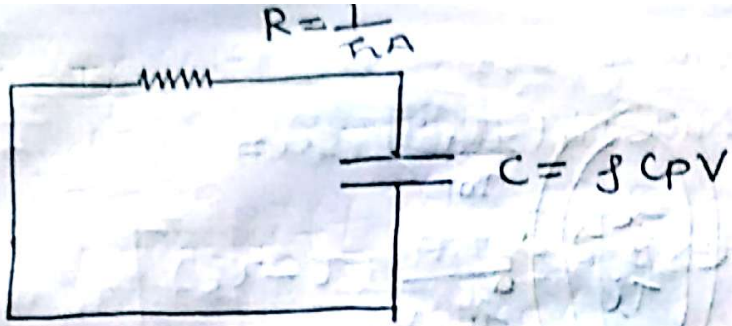
use lumped capacitance method

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA}{\rho c_p V}\right)\tau$$

$$\frac{300-100}{750-100} = 0.3077 = \exp\left(-\right.$$

$$\ln(0.3077) = -0.0271\tau$$

$$\boxed{\tau = 43.492 \text{ second}}$$

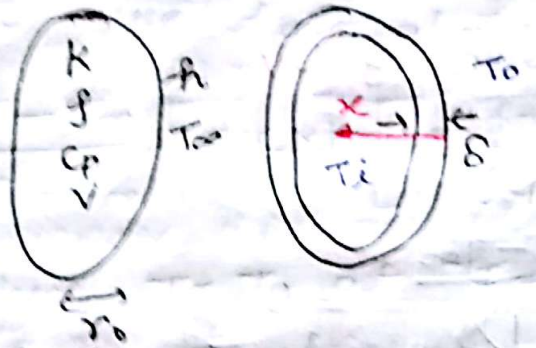


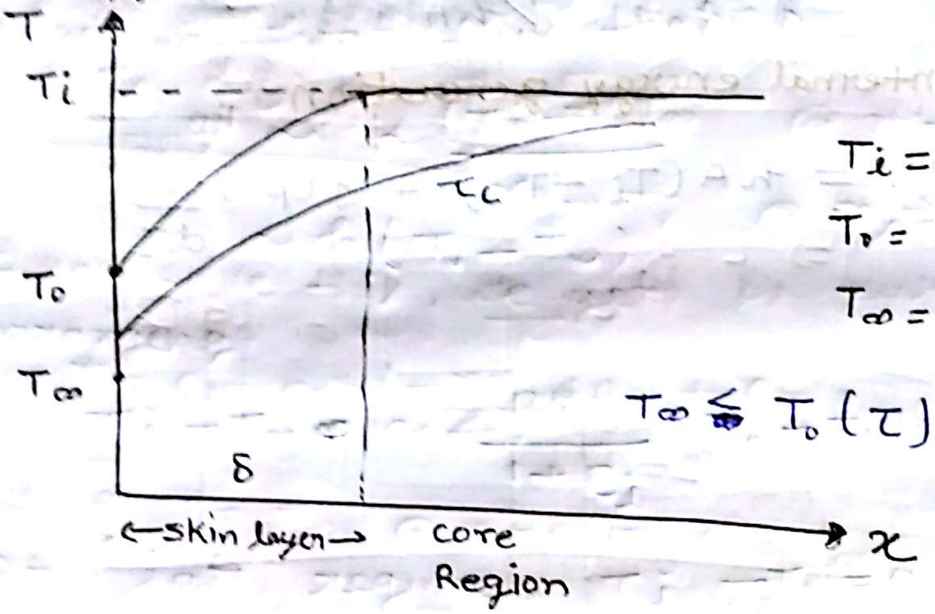
$$\rho C_p V \frac{dT}{d\tau} + \frac{T - T_\infty}{1/hA} = 0$$

with internal energy generation

$$q''' V = hA(T_i - T_\infty) + \rho C_p V \frac{dT}{d\tau}$$

17/08/16





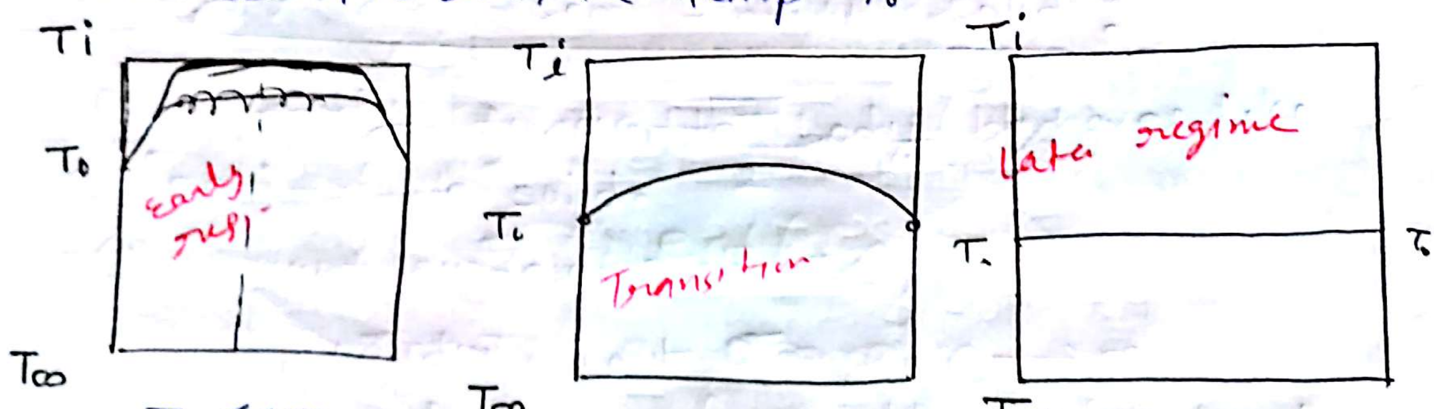
T_i = initial temp

T_0 = surface temp

T_∞ = ambiant temp

$T_\infty \leq T_0(\tau) < T_0$

- As time increases
  $\delta \uparrow$, $T_0 (\downarrow)$
- There comes a time τ_c when
  δ grows all the way to the core
  - no distinct skin and core regions
- when $\tau >> \tau_c$ entire entire body is at the surface temp T_0



$\tau << \tau_c$ $\quad$ $\tau \sim \tau_c$ $\quad$ $\tau >> \tau_c$

$T = T(x, \tau)$ $\quad$ $T = T(x, \tau)$ $\quad$ ⇒ Lumped system $T = T(\tau)$

What is τ_c?

when $\delta << r_0$, we can assume conduction to be 1D

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

Scaling analysis

In skin region, T varies from T_0 to T_i

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} \sim \frac{T_i - T_0}{\delta}$$

$$\left.\frac{\partial T}{\partial x}\right|_{x\sim\delta} \sim 0$$

1st Rule of difference

$$\frac{\partial^2 T}{\partial x^2} \sim \frac{\left.\frac{\partial T}{\partial x}\right|_{x\to\delta} - \left.\frac{\partial T}{\partial x}\right|_{x=0}}{\delta} \sim -\frac{(T_i - T_0)}{\delta^2} \quad (1)$$

In the time scale, RHS of the PDE represents a 'thermal inertia' over τ, surface temp has fallen from T_i to a value "comparable" to T_0.

$$\therefore \frac{\partial T}{\partial \tau} \sim \frac{T_0 - T_i}{\tau - 0} \quad (2)$$

combining the two

$$-\frac{T_i - T_0}{\delta^2} \sim \frac{1}{\alpha}\frac{T_0 - T_i}{\tau}$$

$$\Rightarrow \boxed{\delta \sim \sqrt{\alpha\tau}}$$

At $\tau \to \tau_c$, $\delta \to r_0$

$$\Rightarrow \boxed{\tau_c \sim \frac{r_0^2}{\alpha}}$$

Late Regime: (Lumped system)

we have already seen

$$\frac{T - T_\infty}{T_1 - T_\infty} = \exp\left[-\frac{hA}{\rho c_p V} (\tau - \tau_c) \right]$$

where $T = T_1$ at $\tau = \tau_c$

conduction heat flux then

$$q'' \sim K \frac{(T_i - T_o)}{r_o}$$

$$\sim h (T_o - T_\infty)$$

Rearranging

$$T_i - T_\infty \sim \frac{Bi}{1 + Bi} (T_o - T_i)$$

At 'late' regime, $T_i - T_o = 0$

$\Rightarrow Bi << 1$ $\qquad Bi = \frac{h r_o}{K}$

This is the same condition that we got using

$$Bi \sim \frac{\text{conduction resistance}}{\text{convection resistance}}$$

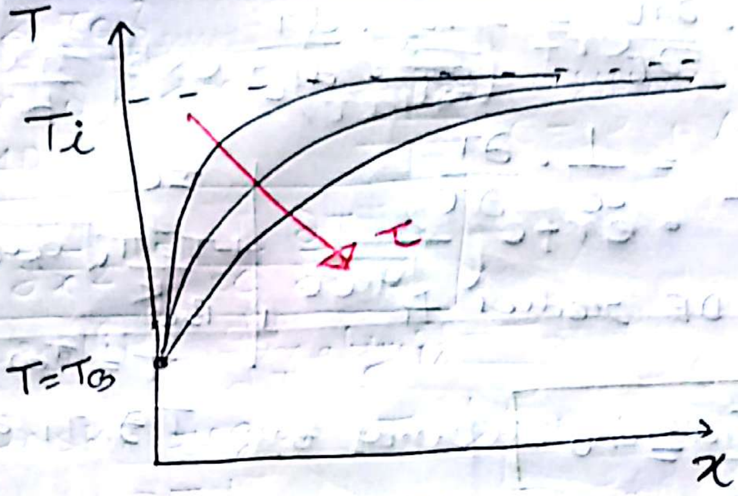
Early regime semi-infinite solid:

At early times, the body behaves like a 'semi-infinite' solid.

(a) constant surface temp (T_∞)

consider $T_o = T_\infty$

- h is so large the surface temp instantly assumes the value of T_∞



equation $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$

Initial cond.

$T = T_i$ at $\tau = 0$

Boundary cond:

$T = T_\infty$ at $x = 0$

$T \to T_i$ as $x \to \infty$

$\Rightarrow$ using our knowledge that

$\delta \sim \sqrt{\alpha\tau}$, we can sketch the family of T-x curves

T-x curves are similar

- start at T_∞ & cond assymptotes at T_i
- has one "knee"
- $\delta \sim \sqrt{\alpha\tau}$ (location of "knee")

Is it possible to plot them on a single curve

define $\eta = \frac{x}{\sqrt{\alpha\tau}}$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial \eta}\cdot\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{\alpha\tau}}\cdot\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left(\frac{\partial T}{\partial x}\right)\frac{d\eta}{dx} = \frac{1}{\alpha\tau}\cdot\frac{d^2 T}{d\eta^2}$$

$$\frac{\partial T}{\partial \tau} = \frac{\partial T}{\partial \eta} \cdot \frac{\partial \eta}{\partial \tau} = \frac{-\eta}{2\tau} \cdot \frac{dT}{d\eta}$$

Eq' $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial \tau}$$

IC

$T = T_i$; at $\tau = 0$

BC

$T = T_\infty$ at $x = 0$

$T \to T_i$ as $x \to \infty$

The governing PDE reduces to

$$\boxed{\frac{d^2 T}{d\eta^2} + \frac{\eta}{2} \cdot \frac{dT}{d\eta} = 0}$$

$T = T_\infty$ @ $\eta = 0$

$T \to T_i$ @ $\eta \to \infty$

If you solve this, we get

$$T = C_1 \int \exp\left(-\frac{\eta^2}{4}\right) d\eta + C_2$$

using $T = T_\infty$ @ $\eta = 0 \Rightarrow C_2 = T_\infty$

$$T - T_\infty = C_1 \int_0^\eta \exp\left(-\left(\frac{\beta}{2}\right)^2\right) d\beta$$

we see the emergence of an error function

$$\boxed{\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-m^2)\, dm}$$

with properties $\mathrm{erf}(0) = 0$ & $\mathrm{erf}(\infty) = 1$

using this in our problem

$$T - T_\infty = 2C_1 \frac{\sqrt{\pi}}{2} \cdot \mathrm{erf}(\eta/2)$$

$$= C_3\, \mathrm{erf}(\eta/2)$$

using $T \to T_i$ as $\eta \to \infty$

$$C_3 = T_i - T_\infty$$

$$\Rightarrow \boxed{\frac{T - T_\infty}{T_i - T_\infty} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha\tau}}\right)}$$

ONLY valid for early regime and $T_0 = T_\infty$
(very high 'h')

$$\frac{d}{dx} \operatorname{erf}(x)\Big|_{x=0} = \frac{2}{\sqrt{\pi}}$$

$$q''(\tau) = -k \frac{\partial T}{\partial x}\Big|_{x=0}$$

$$= -\frac{k(T_i - T_0)}{\sqrt{\pi \alpha \tau}}$$

(b) constant heat flux

B.C changes to $k\frac{dT}{dx}\Big|_{x=0} = q_0''$

Solution is

$$\boxed{T(x,\tau) - T_i = 2\frac{q_0''}{k}\sqrt{\frac{\alpha\tau}{k}} \cdot \exp\left(\frac{-x^2}{4\alpha\tau}\right) - \frac{q_0'' x}{k} \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{4\alpha\tau}}\right)}$$

$\because \operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$

surface temp

$$T_0(\tau) - T_i = 2\frac{q_0''}{k}\sqrt{\frac{\alpha\tau}{\pi}}$$

(c) surface in contact with fluid

Most generic scenario of the three at $x = 0$, $h(T_\infty - T) = -k\frac{\partial T}{\partial x}\Big|_{x=0}$

Solution

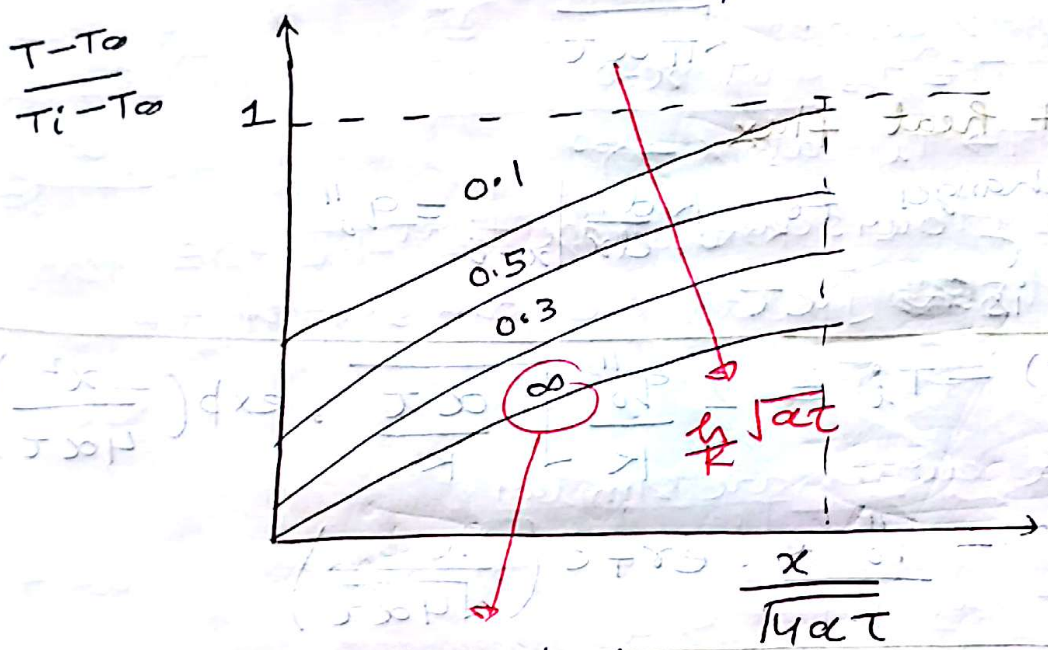
$$\cancel{\frac{T(x,\tau) - T_\infty}{T_i - T_\infty} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha\tau}}\right)}$$

$$\boxed{\frac{T(x,\tau)-T_\infty}{T_i-T_\infty} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha\tau}}\right) + \exp\left(\frac{hx}{K} + \frac{h^2\alpha\tau}{K^2}\right) \cdot \operatorname{erfc}\left(\frac{x}{\sqrt{4\alpha\tau}} + \frac{h}{K}\sqrt{\alpha\tau}\right)}$$

Temp variation depends on

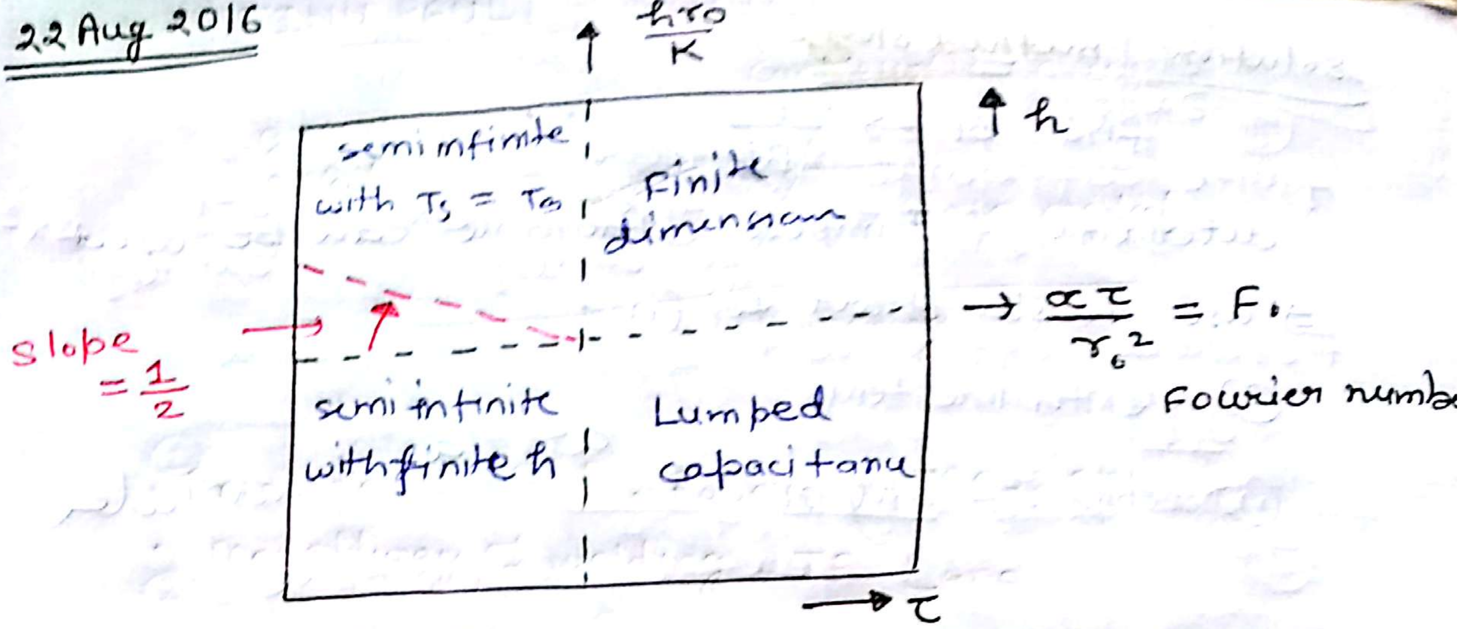
$\frac{x}{\sqrt{4\alpha\tau}}$ & $\frac{h}{K}\sqrt{\alpha\tau}$

$\frac{h\delta}{K} \Rightarrow Bi$ based on δ



corresponds to $T_0 = T_\infty$

22 Aug 2016



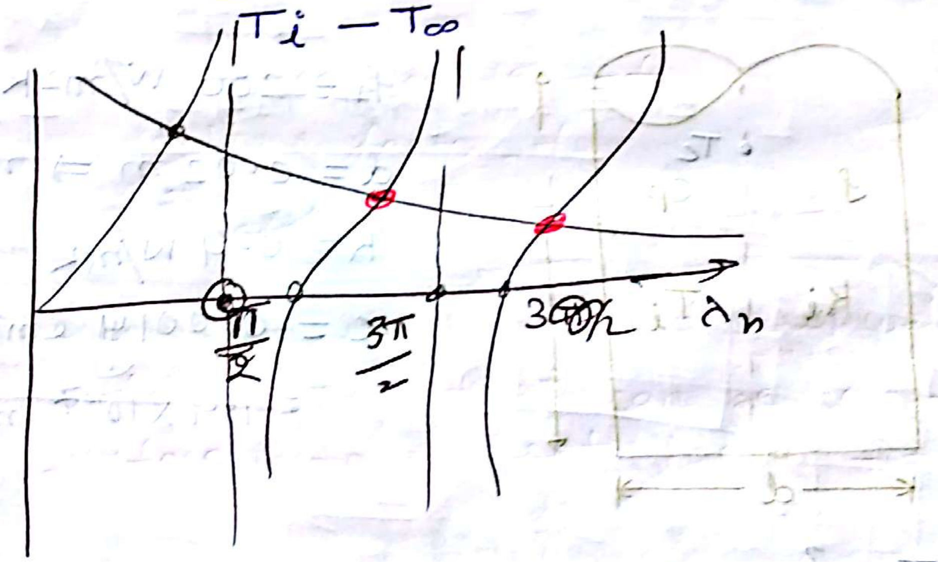
Domain of applicability of Lumped capacitance & semi infinite methods

slide ⇒ Transient condition: spatial effect & the role of analytical solutions

$$\tau = \frac{\alpha t}{L^2}$$

$$X = +\frac{x}{L}$$

$$\theta = \frac{T(x,t) - T_\infty}{T_i - T_\infty}$$



2nd Heisler chart $\frac{\theta_n}{\theta_c}$

Solution methodology:

find Bi → $\frac{1}{Bi}$

determine if lumped capacitance can be used

⇒ use heisler chart to find

(i) Center line temp
(ii) temp at any x

} at a particular time

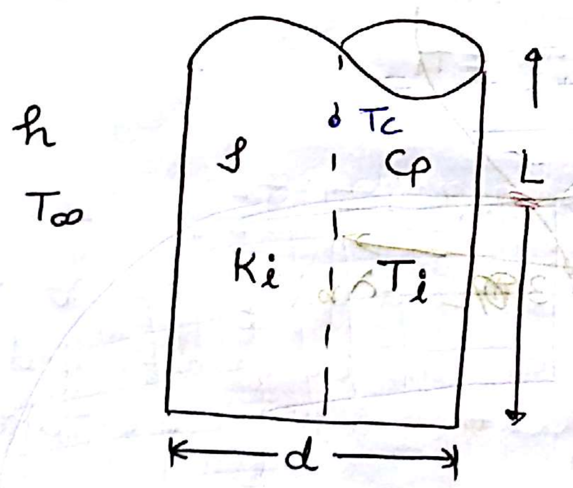
for the particular value of $1/Bi$

Que A hot dog with initial temp of 50°c is dipped in a pool of 95°c water $h = 200 \frac{W}{m^2K}$ (const)

The thermal $K = 0.4\ W/mK$, $\alpha = 0.0014\ cm^2/s$

(A) How long to wait unit hot dog centre at 65°C

(B) estimate surface temp at time calculated for part (a)



$h = 200\ W/m^2K$

$d = 0.02\ m \Rightarrow r_o = 0.01\ m$

$K = 0.4\ W/mK$

$\alpha = 0.0014\ cm^2/s$
$= 1.4 \times 10^{-7}\ m^2/s$

(a) $\tau = ?$

when $T_c = 65\,°C$, $T_i = 20\,°C$

(b) $T_s = ?$ $T_\infty = 95$

at τ in (a)

$$Bi = \frac{h r_0}{K} = 5$$

we cannot use lumped capacitance

(a) $\frac{1}{Bi} = 0.2$

$$\frac{T_c - T_\infty}{T_i - T_\infty} = \frac{65-95}{20-95} = 0.4$$

at the above value $F_0 = \frac{\alpha \tau}{r_0^2}$

$= 0.4 \rightarrow$ from Hiesler

$$\Rightarrow \tau = 0.4 \times \frac{r_0^2}{\alpha} = 2808$$

(b) for surface temp

$\frac{r}{r_0} = 1 \left[\& \frac{1}{Bi} = 0.2 \right]$

form heisler chart

$$\frac{T - T_\infty}{T_c - T_\infty} = 0.18$$

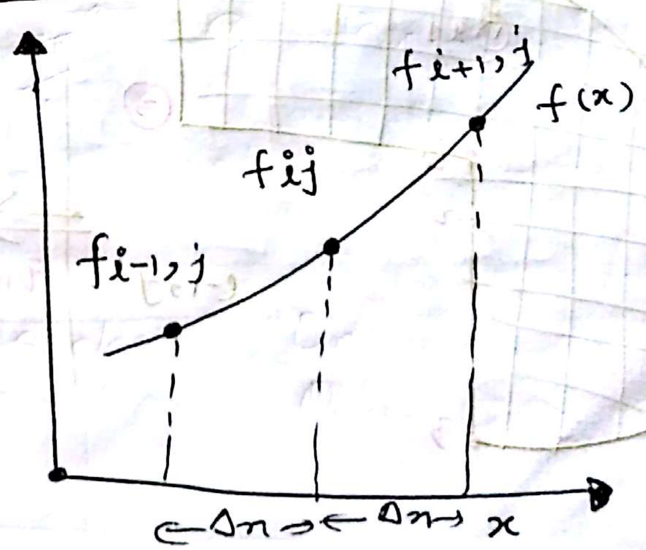
$\Rightarrow T_s = T_\infty + 0.18 \times (65-95)$

$= 95 - 5.4$

24/08/2016

$$\boxed{T_s = 89.6}$$

Numerical Methods:



$$\left.\frac{\partial f}{\partial x}\right|_{ij} =$$

finite difference methods

$$f_{i+1,j} = f_{i,j} + \left.\frac{\partial f}{\partial x}\right|_{i,j} \Delta x + \left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} \Delta x^2 + \ldots$$

reduce

Forward difference →

$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

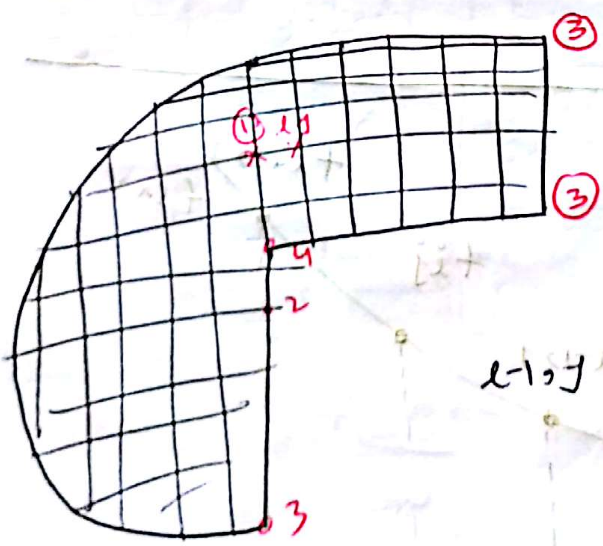
$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i,j} - f_{i-1,j}}{\Delta x}$$ Backward difference

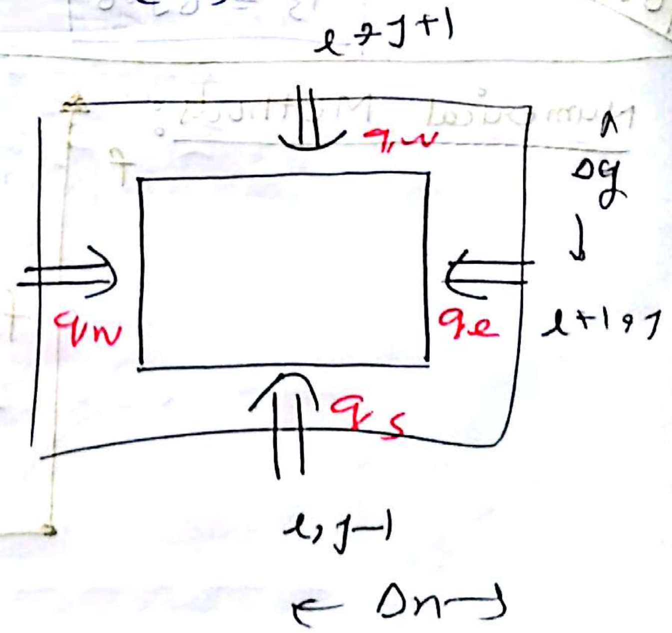
$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}$$

second order derivative

$$\left.\frac{\partial^2 f}{\partial x^2}\right|_{i,j} = \frac{\left.\frac{\partial f}{\partial x}\right|_{i+\frac{1}{2},j} - \left.\frac{\partial f}{\partial x}\right|_{i-\frac{1}{2},j}}{\Delta x}$$

$$= \frac{f_{i+1,j} + f_{i-1,j} - 2f_{ij}}{(\Delta x)^2}$$





- Domain
- Mesh
- Cell
- Node

depth of nodel = w

$q_W = K W \Delta x$

$$q_S = K W \Delta x \frac{(T_{i,j-1} - T_{i,j})}{\Delta y}$$

$$q_E = K W \Delta y \frac{T_{i-1,j} - T_{i,j}}{\Delta x}$$

From energy balance

$$q_N + q_S + q_E + q_W + q''' \Delta x \Delta y\, w = 0$$

$$\Rightarrow T_{i,j} = \frac{1}{4}\left(T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1}\right) + \frac{q'''}{4K}(\Delta x)^2$$

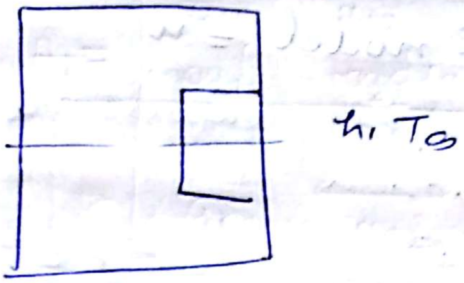
[for $\Delta x = \Delta y$]

① $T_{i-1,j} + T_{i+1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{ij} = 0$

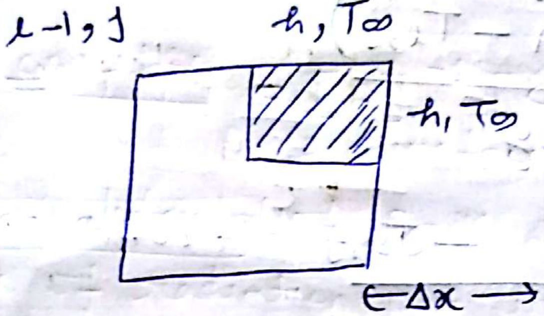
② $2T_{i-1,j} + T_{i,j-1} + T_{i,j+1} + \frac{2h\Delta x}{K} \cdot T_\infty - 2\left(\frac{h\Delta x}{K} + 2\right) T_{i,j} = 0$

③ $T_{i,j-1} + T_{i-1,j} + \frac{2h\Delta x}{K} T_\infty - 2\left(1 + \frac{h\Delta x}{K}\right) \cdot T_{ij} = 0$

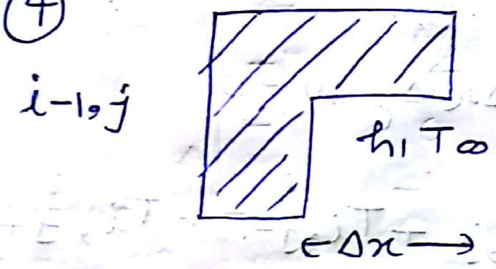
(II)



(III)



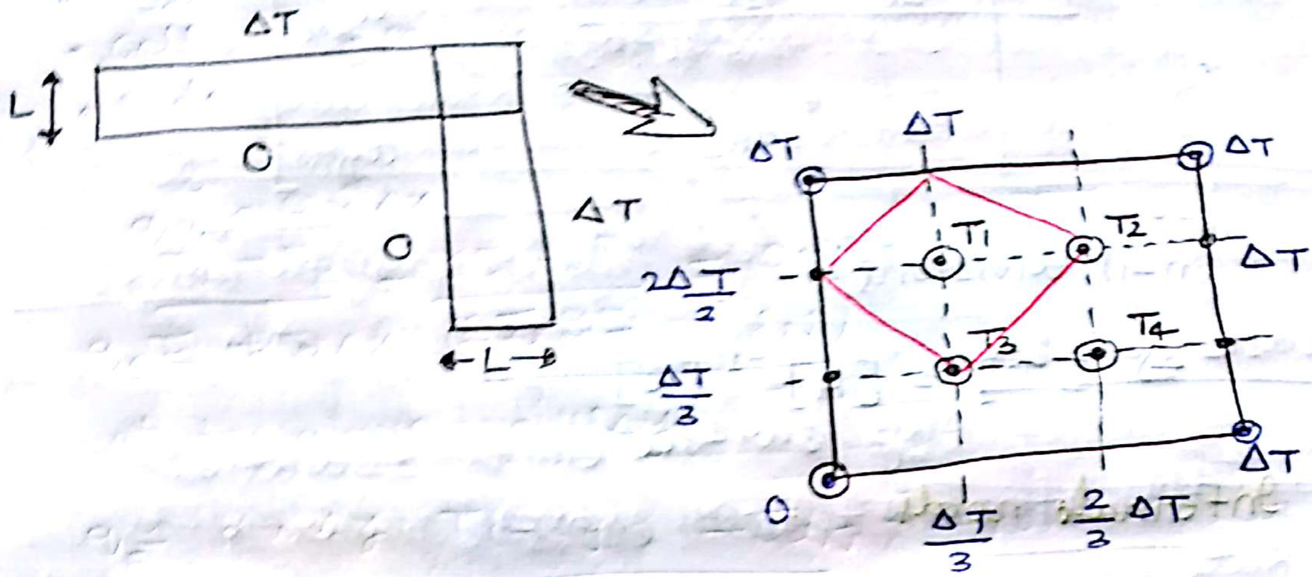
(4)



$$2(T_{i-1,j} + T_{i,j+1}) + T_{i+1,j} + T_{i,j-1} + \frac{2h\Delta x}{K} T_\infty - 2\left(3 + \frac{h\Delta x}{K}\right) T_{i,j} = 0$$

Case study - Prism Edge

25/08/2016



devide 3x3 grid

12 ● external, 4 ● internal nodes

2D domain assume a linear temp variation along the internal edges

Internal nodes:

$$4T_{ij} - (T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1}) = 0$$

use these expression for $\{T_1 \cdot T_4\}$

$$\Rightarrow 4T_1 - T_2 - T_3 - \frac{2}{3}\Delta T - \Delta T = 0$$

$$\Rightarrow 4T_1 - T_2 - T_3 = \frac{5}{3}\Delta T$$

$$-T_1 + 4T_2 - T_4 = 2\Delta T$$

$$-T_1 + 4T_3 - T_4 = \frac{2}{3}\Delta T$$

$$-T_2 - T_3 + 4T_4 = 5/3\,\Delta T$$

This is of the form

$$[A][T] = [C]$$

in general

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & & a_{nn} \end{bmatrix}$$

$(n-1)$ divisions

$$\Rightarrow [T] = [A]^{-1}[C]$$

<u>Internals noods</u>

For our core

$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2 \\ 2/3 \\ 5/3 \end{bmatrix} \Delta T$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 0.2916 & 0.0833 & 0.0833 & 0.0416 \\ 0.8333 & 0.2916 & 0.0416 & 0.8333 \\ 0.0833 & 0.0416 & 0.2916 & 0.8333 \\ 0.0416 & 0.0833 & 0.0833 & 0.2916 \end{bmatrix}$$

Thus gives

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 0.7778 \\ 0.8089 \\ 0.5556 \\ 0.7778 \end{bmatrix} \Delta T$$

gauss siedel Iteration method :

$$T_1 = \frac{1}{4}\left(T_2 + T_3 + \frac{5}{3}\Delta T\right)$$

$$T_2 = \frac{1}{4}\left(T_1 + T_4 + 2\Delta T\right)$$

$$T_3 = \frac{1}{4}\left[T_1 + T_4 + \frac{2}{3}\Delta T\right]$$

$$T_4 = \frac{1}{4}\left[T_2 + T_4 + \frac{5}{3}\Delta T\right]$$

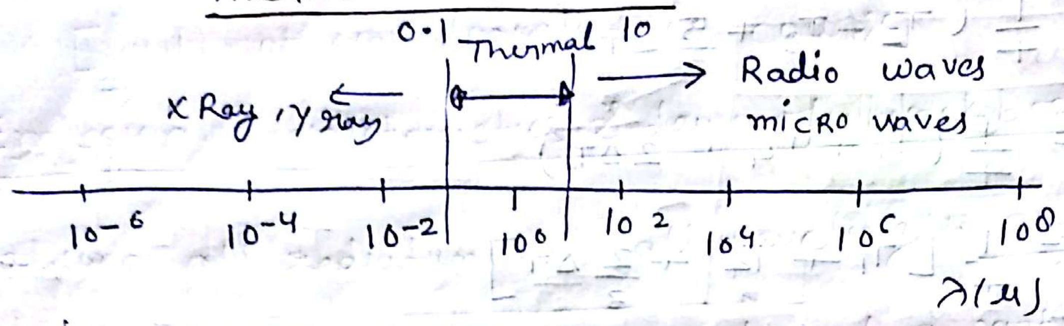
- Assign intelligently guessed initial values
- use calculated values for next iteratn
- continue till change in each node b/w two successive iterations if sufficienlt small

⇒ started with $T_1 = T_2 = T_3 = T_4 = \frac{\Delta T}{2}$

iteratin number	$T_1/\Delta T$	$T_2/\Delta T$	$T_3/\Delta T$	$T_4/\Delta T$
0	1/2	1/2	1/2	1/2
1	0.6458	0.8224	0.4617	0.6458
4	0.7757	0.8879	0.5514	0.7757
5	0.7773	0.8886	0.5545	0.7773

20/08/2016

Thermal Radiation



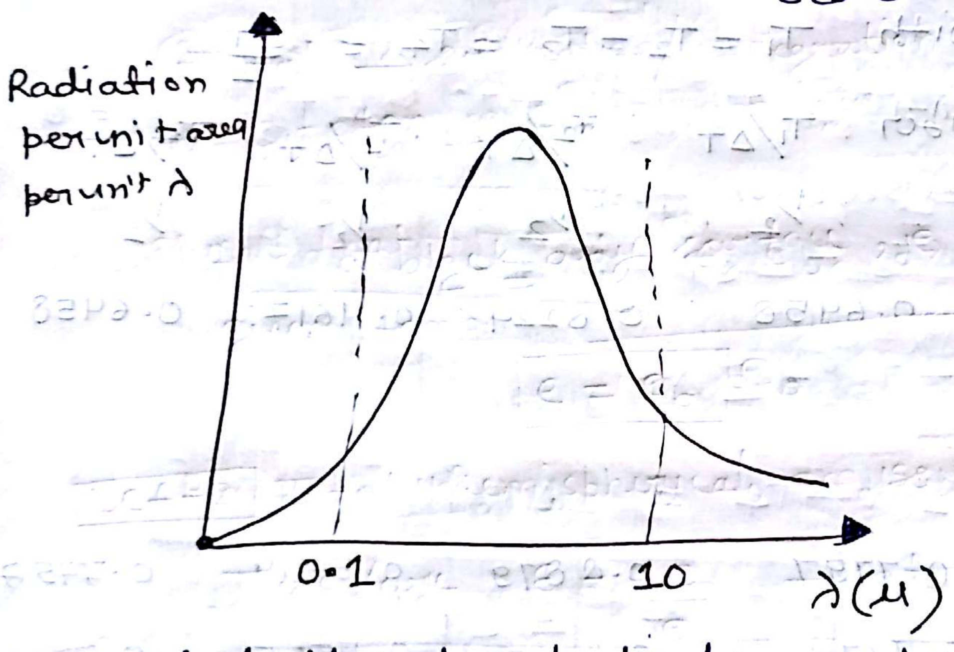
Basic concepts :

Any surface emits radiation in the form of E-M waves

Rate of radiation per unit area is $\propto T^4$

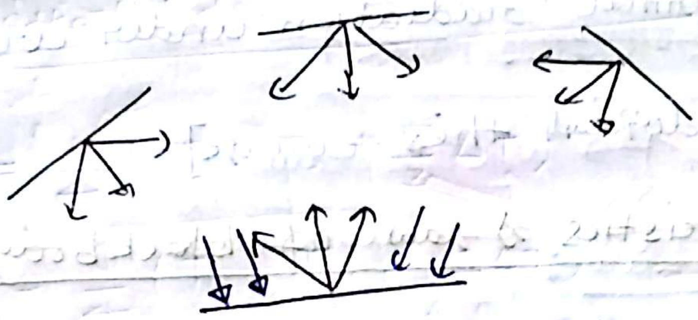
or $q'' \propto T^4$

(W/m^2) ∴ only K use in temp Scale



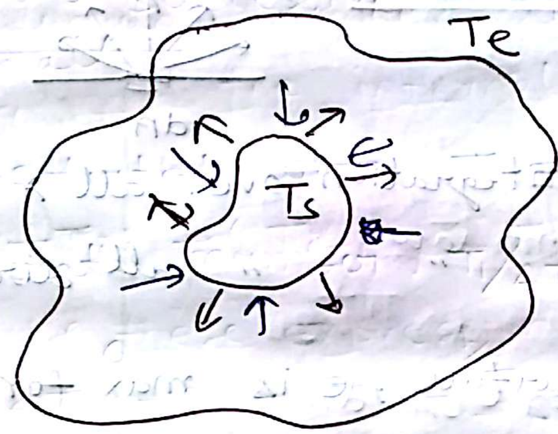
- heat flux at a particular wavelengths
- most of the flux concentrated in 0.1 to 10 μ wave length range

E-M waves do not require any material medium for transmission



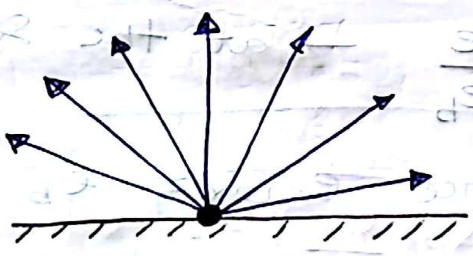
→ radiation emitted by surrounding surface & falling on the target surface

② ⇒ In addition to emitting radiation a body at temp T also absorbs all or part of the thermal radiation emitted by surrounding surface and falling on it.



absorbing more than radiating ⇒ T(↑)

③ ⇒ directional nature of radiation



surface ⇒ hemi sphere

point ⇒ sphere

~~Emission~~ Emission is directional independent

Emission of radiation happen in all directions (encompassed by hemisphere)

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gases also emit thermal radiation under certain conditions

[will not consider in this course]

Emission characteristics & law of black body radiation

absorb all radiation at all wavelength <u>Black body</u>

black surface emits max rad. at a particular temp

$\boxed{\text{def 1}}$

Total hemispherical emissive power

Radiant flux emitted from unit surface of the body.

$e\ [W/m^2]$

Total → summation/Integration over all λ

Hemispherical → " " " all directions

for a black body surface, e is max for a given temp.

$$\boxed{e = e_b}$$

$\boxed{\text{Def-2}}$ Total hemispherical emissivity (ϵ)

Therefore $\epsilon = \dfrac{e}{e_b}$ → at the same temp

$0 < \epsilon < 1$ since $e_{max} = e_b$ for a given temp

31/08/2016

Total hemispherical Emissive power (e)

Total " Emissivity (ϵ)

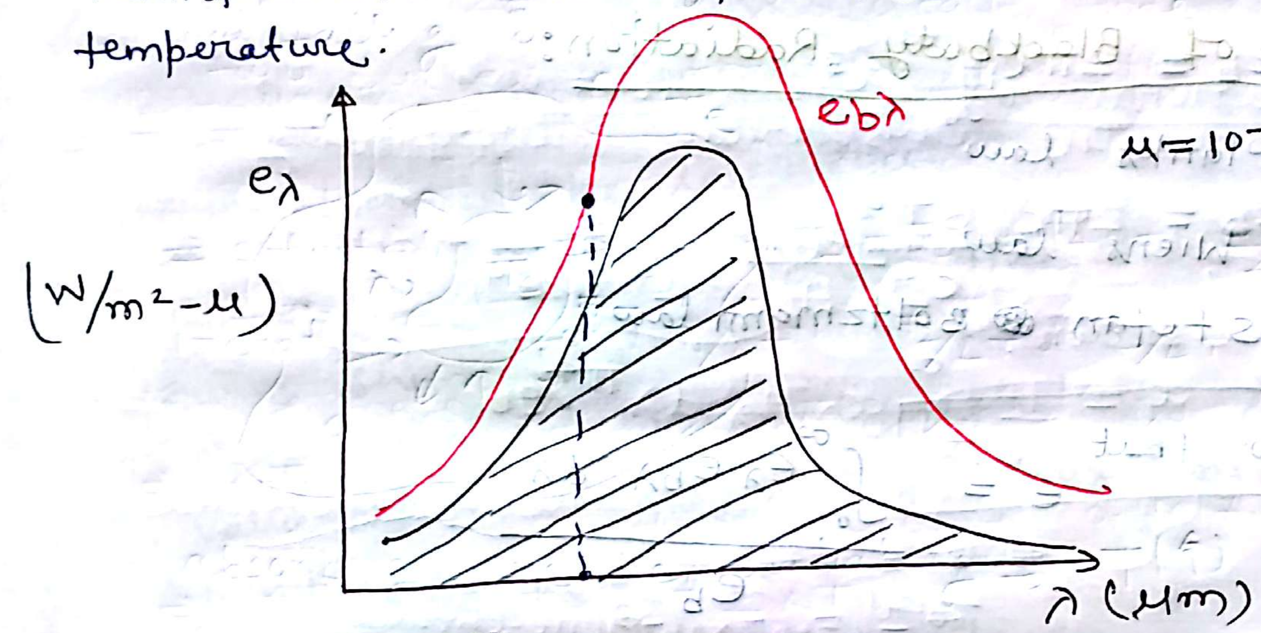
Monochromatic " Emissive power (e_λ)

" " Emissivity (ϵ_λ)

grey surface:

Monochromatic Hemispherical emissive power : (e_λ)

e_λ is that quantity which when integrated across all wavelengths gives the total hemispherical emissive power (e) at the given temperature.



$$e = \int_0^\infty e_\lambda \, d\lambda$$

or $$e_\lambda = \frac{de}{d\lambda}$$

For a black body $e_\lambda = e_{b\lambda}$

Monochromatic hemispherical emissivity (ϵ_λ)

Ratio of e_λ to $e_{b\lambda}$ at the same temp and wavelength.

$$\epsilon_\lambda = \frac{e_\lambda}{e_{b\lambda}} \Big|_{\text{temp} \,\&\, \lambda}$$

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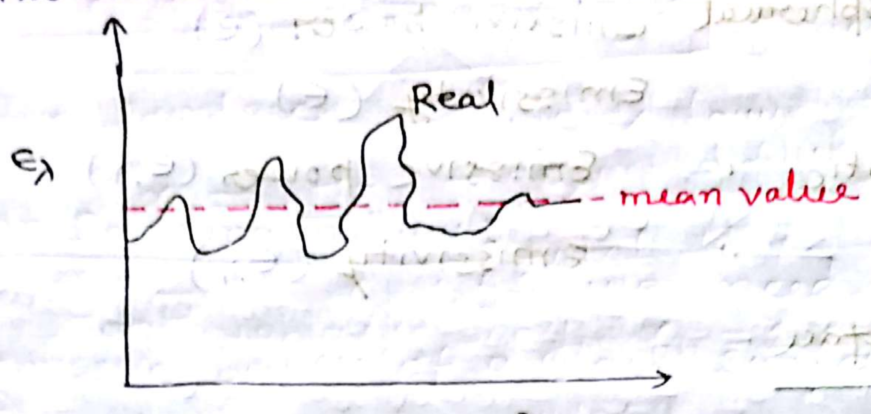
will lie between 0 & 1



grey surface:

grey surface approximation average ϵ_λ over spectrum.

Laws of Blackbody Radiation:

1. Planks law
2. Wiens law
3. Stefan Boltzmamm law

Show that
$$\epsilon = \frac{\int_0^\infty \epsilon_\lambda e_{b\lambda} \, d\lambda}{e_b}$$

Planks law: allow us to calculate $e_{b\lambda}$ at a given temp and λ

$$\boxed{e_{b\lambda} = \frac{2\pi c_1}{\lambda^5 \left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]}}$$

Constants: $c_1 = 0.596 \times 10^{-16}$ W.m2

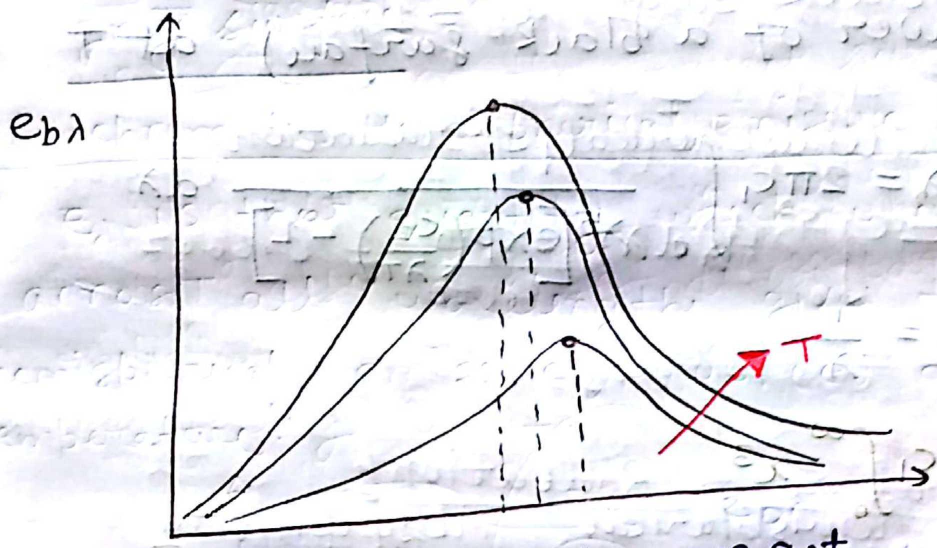
$c_2 = 0.014387$ m.K

come from exp. measurements

→ nature/form of this expression comes from E-M Theory.

for a non black surface

$$\boxed{e_\lambda = \epsilon_\lambda \cdot e_{b\lambda}}$$



If we plot planks law we get

① $e_{b\lambda}$ reaches a peak and then reduces

② curve shifts with T

⇒ radiation is higher at High temp

③ value of λ corresponding to max $e_{b\lambda}$ shifts Right when T(↓)

To locate the peak

$$\frac{de_{b\lambda}}{d\lambda} = 0$$

let $\frac{c_2}{\lambda T} = y$ ---- follow the steps

$$\Rightarrow e^{y}(5-y) = 5$$

using trial and error

$$\frac{c_2}{\lambda_m T} = [y] = 4.965$$

$$\boxed{\lambda_m T = 0.0029 \text{ mK}}$$

↳ wien's law

Stefan Boltzmann Law:

If we need to calculate e_b (emissive power of a black surface) at T

$$e_b = \int_0^\infty e_{b\lambda} \cdot d\lambda = 2\pi c_1 \int_0^\infty \frac{1}{\lambda^5 \left[\exp\left(\frac{c_2}{\lambda T}\right) - 1\right]} d\lambda$$

put $x = \frac{1}{\lambda}$

$$e_b = 2\pi c_1 \int_0^\infty x^3$$

$$e_b = 2\pi c_1 \frac{6T^4}{c_2^4} \cdot \underbrace{\left(1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right)}_{\pi^4/90}$$

$$\boxed{e_b = \sigma T^4}$$

$\Rightarrow \sigma =$ stefan constant

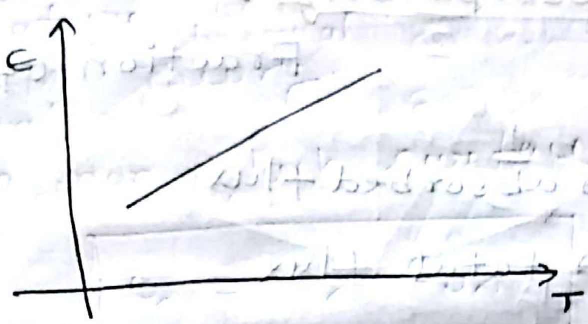
$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

since we are neglecting participation of gases

Typical ϵ value

Metal:

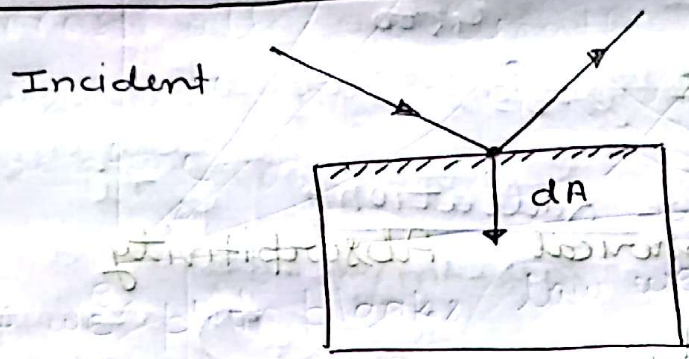
		ϵ
Brass	Polished	0.09
	oxidized	0.6
copper	Polished	0.04
	oxi	0.5 – 0.8
steel	Pol.	0.08 – 0.15
	oxi.	0.8



non metal 0.89 – 0.95

water at room temp 0.95

Irradiation - radiatio incident on surface:

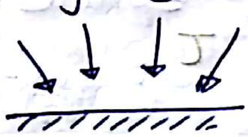


Incident Radiation

- Reflected
- Absorbed
- Transmitted (non opaque body)

① Total Hemispherical Irradiation: Total radiant flux incident on a surface

H [W/m²]



• can be due to radiation emitted from surrounding surface may also have argucated from the surface and got reflected back from another surface

② Total hemispherical Absorptivity: Fraction of H absorbed at the surface.

$$\alpha = \frac{H_a}{H} \quad \begin{matrix} \rightarrow \text{absorbed flux} \\ \rightarrow \text{total flux} \end{matrix}$$

For a black body $\alpha_b = 1$

③ Monochromatic Hemispherical Irradiation.

$$H = \int_0^\infty H_\lambda \, d\lambda$$

④ Monochromatic hemispherical Absorpitivity

$$\alpha_\lambda = \frac{H_{a\lambda}}{H_\lambda}$$

It can be shown:

$$\alpha = \frac{1}{H} \int_0^\infty \alpha_\lambda H_\lambda \, d\lambda$$

	Total	Mono-chromal
Emissivity	ε	ϵ_λ
absorpitivity	α	α_λ
Reflectivity	ρ	ρ_λ
transmissivity	τ	τ_λ

$$\alpha + \rho + \tau = 1$$

$$\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$$

For opaque surfaces $\tau = \tau_\lambda = 0$

$$\Rightarrow \alpha + \rho = 1$$

$$\alpha_\lambda + \rho_\lambda = 1$$

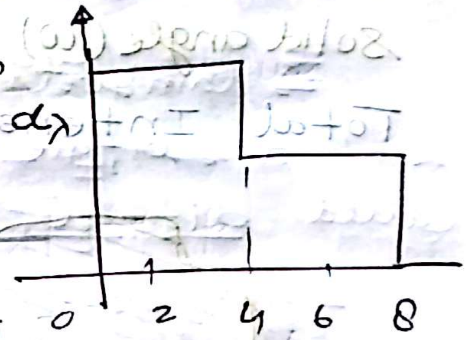
1/09/2016

Ques The variation of the monochromatic irradiation incident on an opaque surface and the monochromatic absorptivity of the surface (α_λ) with wavelength in as follows.

$H_\lambda = 0$, $0 \le \lambda \le 2$ μm

$= 750$ W/m²-μm , $2 < \lambda < 8$ μm

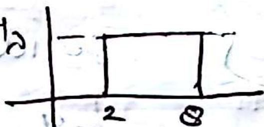
$= 0$ for $\lambda \ge 8$ μm



$\alpha_\lambda = 1$, $0 \le \lambda \le 4$ μm

$= 0.5$ $4 < \lambda < 8$

$= 0$ $\lambda \ge 8$ μm



Calculate the absolute radiant flux, the absorptivity and ~~refle~~ reflectivity of the surface (H, α, ρ).

Ans

$$H = H_\lambda \cdot d\lambda$$

$$H = 750 \times 6 = 4500 \text{ W/m}^2$$

absorbed $\Rightarrow$

$$e = \int e_\lambda \, d\lambda \Rightarrow H = \int \alpha_\lambda H_\lambda \, d\lambda$$

$$= 2 \times 750 \times 1 + 750 \times 0.5 \times 4$$

$$= 1500 + 1500 = 3000 \text{ W/m}^2$$

$$\therefore \alpha = \frac{3000}{4500} = \frac{2}{3}$$

$$\rho = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\alpha = \frac{1}{H}\int_0^\infty \alpha_\lambda H_\lambda \, d\lambda$$

$$\Rightarrow \quad H = \int_0^\infty H_\lambda \, d\lambda = 4500$$

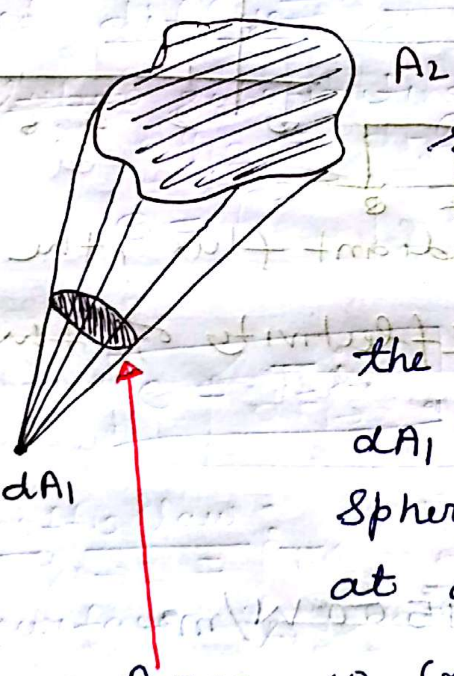
$$\alpha = \frac{1}{H}\int_0^\infty \alpha_\lambda H_\lambda \, d\lambda = \frac{3000}{4500} = 0.6667$$

$$\rho = 1 - \alpha = 0.333$$

directional nature of Radiation:

Solid angle (ω)

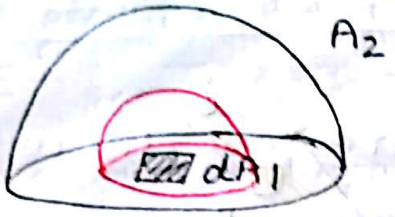
Total Intensity of radiation



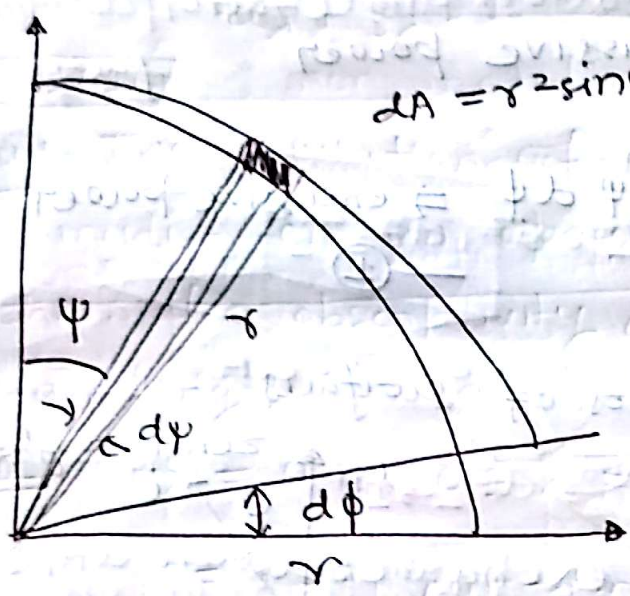
solid angle (ω) subtended by A_2 at dA_1

→ numerically equal to the area cut out from cone joining dA_1 to the perimeter of A_2 by a sphere of unit radius centered at dA_1.

Area = ω (numerically)



$\omega = 2\pi$



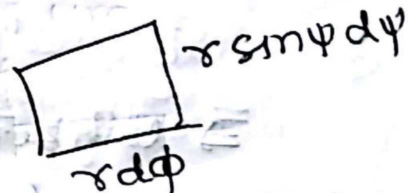
$dA = r^2 \sin\psi \, d\psi \, d\phi$

r = radius

ψ = zenith angle

φ = azimuth angle

solid are subtended by



$$dA = r^2 \sin\psi \, d\psi \, d\phi$$

$$d\omega = \frac{dA}{r^2}$$

$$\boxed{d\omega = \sin\psi \, d\psi \, d\phi}$$

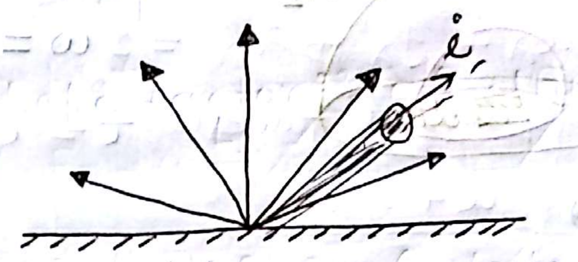
Thus differential solid angle in spherical coordinates is $\sin\psi \, d\psi \, d\phi$

<u>Radiation Intensity</u>: Total Intensity of radiation in a given direction (i) is the radiant flux passing in the specified direction per unit solid angle.

$$i = \frac{de}{dw}$$

or

$$e = \int i\,dw$$



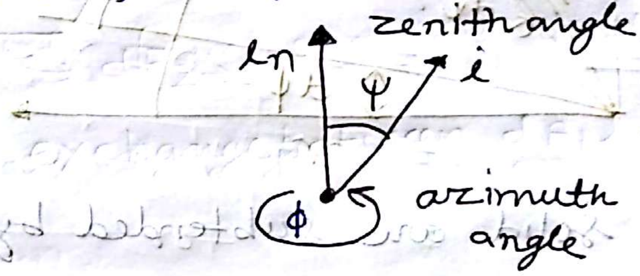
Total hemispherical emissive power

$$e = \int_0^{2\pi}\int_0^{\pi/2} i \sin\psi \, d\psi \, d\phi \Rightarrow \text{emissive power} \quad —— (1)$$

05/09/2016 =

for many/several types of surfaces

$$i = i_n \cos\psi$$



⇒ lamberts law

Such a surface that follows Lamberts law is called diffuse surface.

~~Subs~~ b substituting lamberts law in (1)

$$e = \int_0^{2\pi}\int_0^{\pi/2} i_n \cos\psi \sin\psi \, d\psi \, d\phi$$

$$\boxed{e = \pi i_n} \Rightarrow \text{if the surface diffuse}$$

Kirchoff's law : The monochromatic emissivity of a surface is equal to its monochromatic absorptivity if the outer surface emit in a diffuse manner.

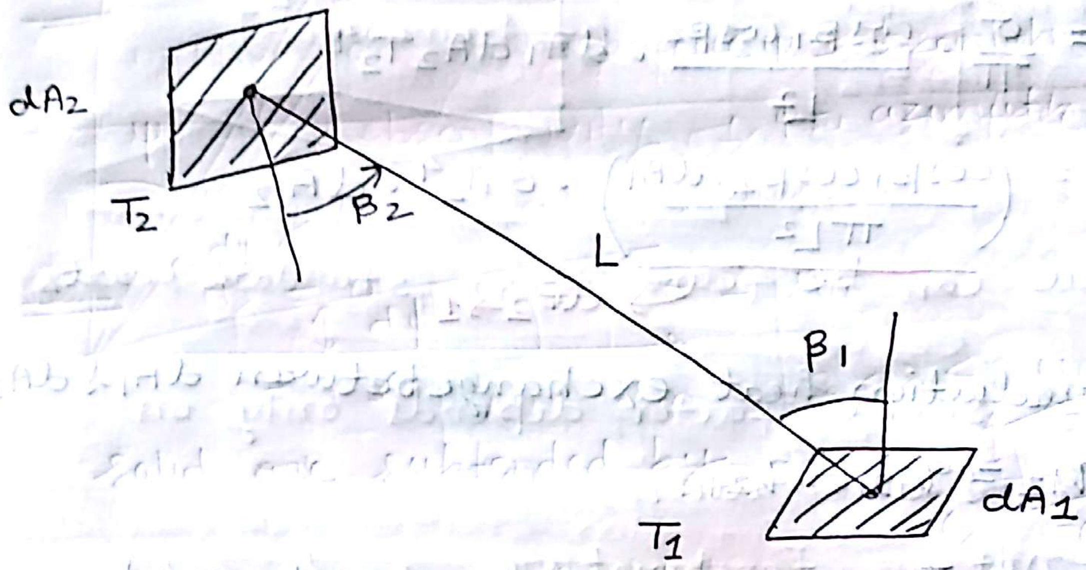
$$\boxed{\epsilon_\lambda = \alpha_\lambda} \rightarrow \text{Imp law relating emission \& absorption characteristics}$$

Therefore for a grey surface

$$\epsilon = \alpha$$

we will normally assume a real surface to be diffuse and grey

Heat transfer by Radiation between two black **surfaces** :



Intensity of radiation (i) emitted by dA_1 in the direction of dA_2 =

$$= i_n \cos\beta_1 = \frac{\sigma T_1^4}{\pi} \cdot \cos\beta_1$$

(using lamberts law)

solid angle substracted by dA_2 at dA_1

$$= \frac{dA_2 \cos\beta_2}{L^2}$$

∴ Rate at which radiation emitted by dA_1 flows towards dA_2

$$q_{12} = i\,d\omega\,dA_1 = \left(\frac{\sigma T_1^4}{\pi} \cdot \cos\beta_1\right)\left(\frac{dA_2 \cos\beta_2}{L^2}\right) dA_1$$

or $$q_{12} = \left(\frac{\cos\beta_1 \cos\beta_2\, dA_2}{\pi L^2}\right) \sigma T_1^4\, dA_1$$

$$q_{12} = dF_{1-2}\ \sigma T_1^4 \cdot dA_1$$

dF_{1-2} is called "shape factor" of dA_1 with respect to dA_2

$\Rightarrow$ other names: view factor / Angle factor / configuration factor

similarly

$$q_{21} = \frac{\sigma}{\pi} \cdot \frac{\cos\beta_1 \cos\beta_2}{L^2} \cdot dA_1\, dA_2\, T_2^4$$

$$= \underbrace{\left(\frac{\cos\beta_1 \cos\beta_2 \cdot dA_1}{\pi L^2}\right)}_{dF_{2\to1}} \cdot \sigma T_2^4 \cdot dA_2$$

The net radiation heat exchange between dA_1 & dA_2

$$\Rightarrow dq_{12} = q_{12} - q_{21}$$

$$= \frac{\sigma}{\pi} \frac{\cos\beta_1 \cos\beta_2}{L^2} \cdot dA_1\, dA_2\, (T_1^4 - T_2^4)$$

or $$dq_{12} = dF_{1\to2}\ \sigma (T_1^4 - T_2^4)\, dA_1$$

$$= dF_{2\to1}\ \sigma (T_1^4 - T_2^4)\, dA_2$$

so that $$\boxed{dF_{1\to2}\, dA_1 = dF_{2\to1}\, dA_2}$$

$$q_{1\to2} = \int_{A_1}\int_{A_2} \frac{\sigma}{\pi} \cdot \frac{\cos\beta_1 \cos\beta_2}{L^2} (T_1^4 - T_2^4)\, dA_1\, dA_2$$

$$= \left[\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} \cdot dA_1\, dA_2\right] \sigma (T_1^4 - T_2^4)$$

defining shape factor of surface 1 w.r.t 2

$$F_{1\to2} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} \cdot dA_2\, dA_1$$

$$\boxed{q_{1\to2} = F_{1\to2}\ \sigma A_1 (T_1^4 - T_2^4)}$$

we can also define a reverse shape factor

$$F_{2\to1} = \frac{1}{A_2}\int_{A_2}\int_{A_1} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} \cdot dA_1\, dA_2$$

It is obvious that

$$\boxed{F_{1\to2} \cdot A_1 = F_{2\to1} \cdot A_2}$$

for a finite surface & small surface dA_2

$$q_{1\to2} = \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} \cdot dA_2 \times \sigma A_1 (T_1^4 - T_2^4)$$

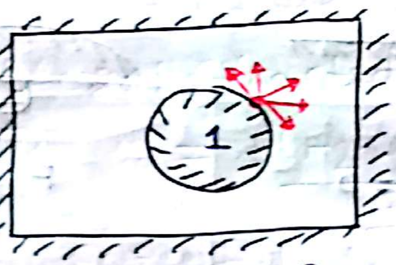
07/09/2016

shape factor: $[0, 1]$

① $F_{1-2} \cdot A_1 = F_{2-1} \cdot A_2$ — reciprocal relation

— very handy especially when one of shape factor is unity

Ex:



$F_{1\to2} = 1$

$F_{2\to1} = ?$

using reciprocal relation

$$F_{2\to1} \cdot A_2 = F_{1\to2} \cdot A_1$$

$$F_{2\to1} = \frac{A_1}{A_2} \cdot (1)$$

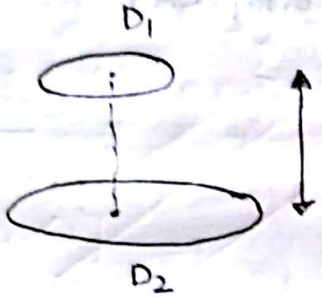
Standard charts are availble for common configuration

(1) Parrallel rectangular plates

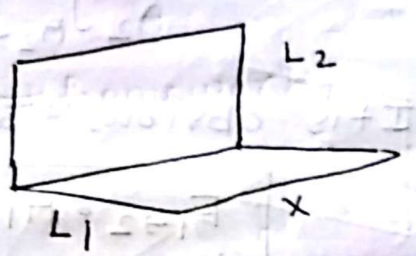
graphs available

for $F_{1\to 2} = f\left(\frac{L_1}{D}, \frac{L_2}{D}\right)$

(ii)



(iii)



value can be found from standard charts

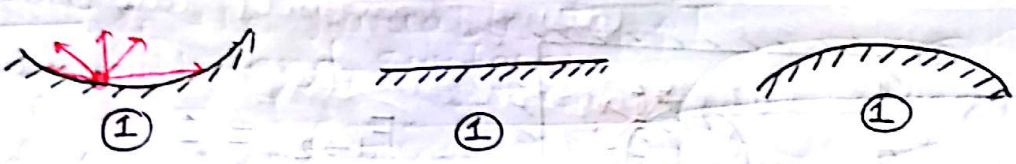
(2). Value of shape factor depends only on geometry & orientation

→ not on temperature

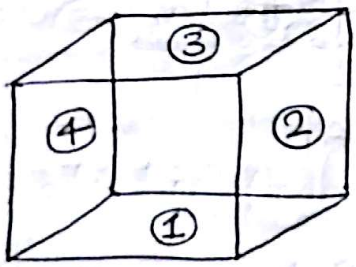
(3) A surface can have a shape factor wrt to itself

= 0 ⇒ flat/convex

> 0 ⇒ for concave surfaces



$F_{1\to 1} > 0$ $F_{1\to 1} = 0$ $F_{1\to 1} = 0$



Treat each face separately

$F_{1\to 1} = F_{2-2} = F_{3-3} = \cdots = 0$

combine surface 1 & 2 and call that surface A

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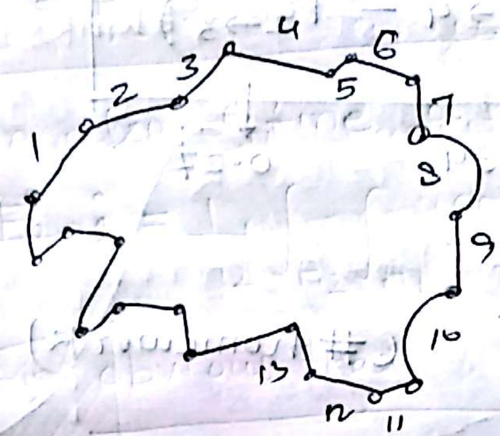
$1+2 = A$

$F_{A\to A} \neq 0$

④



consider an n-sided enclosure

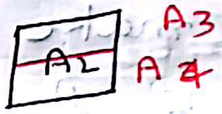
Radiation emitted from any surface is interupted by itself on the other $(n-1)$ surface

$$F_{1-1} + F_{1-2} + \cdots + F_{1-n} = 1$$

$$F_{2-1} + F_{2-2} + \cdots + F_{2-n} = 1$$

$$F_{n-1} + F_{n-2} + \cdots + F_{n-n} = 1$$

⑤ Additive Relation



if $A_3 + A_4 = A_2$

$$\boxed{F_{1\to 2} = F_{1\to 3} + F_{1\to 4}}$$

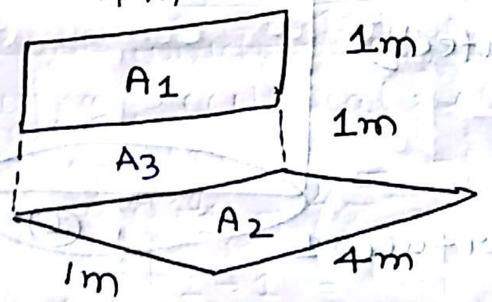
Note, the converse is not true

i.e

$$\boxed{F_{2\to 1} \neq F_{3\to 1} + F_{4-1}}$$

Problem :

①



$F_{1\to 2} = ?$

$F_{2\to 4} - F_{2\to 3}$

let A_3

$A_1 + A_3 = 4$

$F_{2\to 1} = F_{2\to 4} - F_{2\to 3}$

standard standard standard

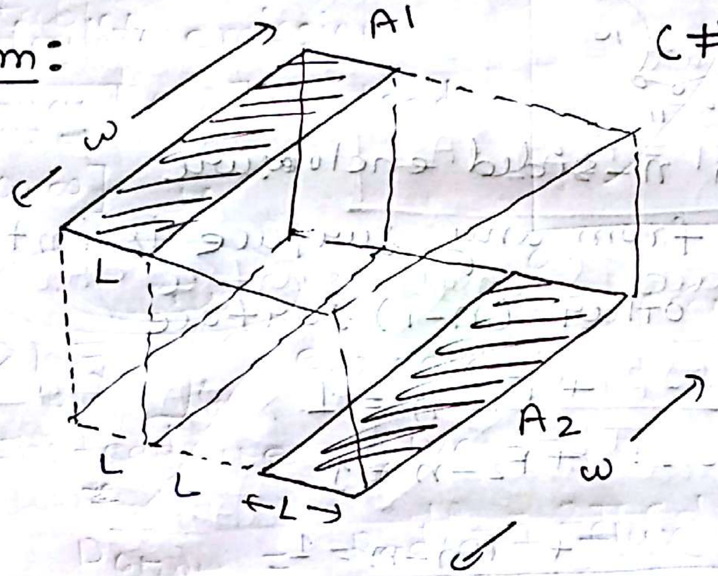
$F_{2\to1} \cdot A_2 = F_{1\to2} \cdot A_1$

$$\boxed{F_{1\to2} = \frac{A_2}{A_1}\left(F_{2\to4} - F_{2\to3}\right)}$$

$\frac{A_2}{A_1} \to 1$, $F_{2\to4} \to 0.34$, $F_{2\to3} \to 0.27$

$F_{1\to2} = 0.07$

Problem: (# homework)



make three strips and solve

why is $A_{2-1} \neq A_{3-1} + A_{4-1}$

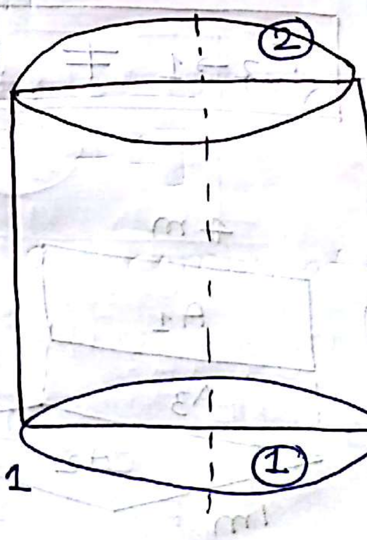
Radiant Heat exchange in an enclosure with black surface:

cylinder with end surfaces

surface ① is at T_1

" ② is at T_2

" ③ is perfectly insulated



Net radiation from surface 1

$q_1 = q_{1\to2} + q_{1\to3}$

$$= A_1 F_{1\to2}(e_{b1} - e_{b2}) + A_1 F_{1\to3}(e_{b1} - e_{b3})$$

similarly

$$q_2 = A_2 F_{2\to1}(e_{b2} - e_{b1}) + A_2 F_{2\to3}(e_{b2} - e_{b3})$$

$$q_3 = A_3 F_{3\to1}(e_{b3} - e_{b1}) + A_3 \cdot F_{3\to2} \cdot (e_{b3} - e_{b2})$$

we know $e_{b1} = \sigma T_1^4$

& $e_{b2} = \sigma T_2^4$

$q_3 = 0 \rightarrow$ perfectly insulated

$\therefore$ we have 3 eq. and

3 unknown $\Rightarrow q_1, q_2$, and e_{b3} (or T_3)

If we solve

(A) $q_1 = -q_2$

$$= \sigma(T_1^4 - T_2^4)\left[A_1 F_{1-2} + \frac{A_1 F_{1-3} F_{3-2}}{F_{3-2} + F_{3-1}}\right]$$

(B) $\sigma T_3^4 [e_{b3}] = \dfrac{F_{3-2}\, e_{b2} + F_{3-1}\, e_{b1}}{F_{3-2} + F_{3-1}}$

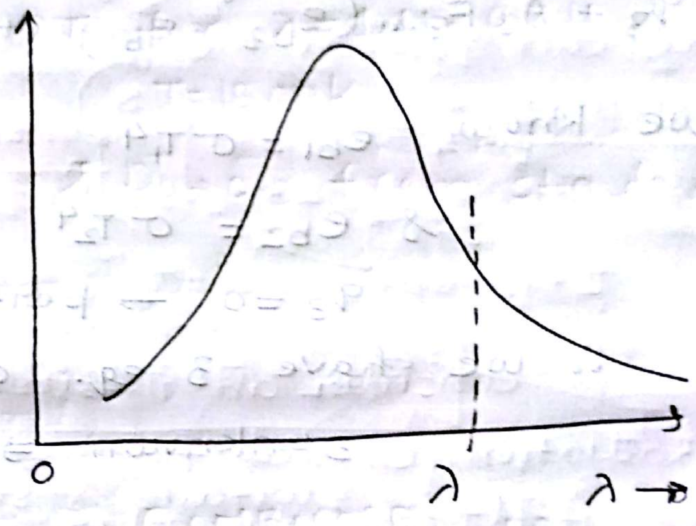
simplify $F_{1-1} = F_{2-2} = 0$

$$q_1 = -q_2 = \sigma A_1 (T_1^4 - T_2^4)\left[\frac{A_2 - A_1 F_{1\to2}^2}{A_1 + A_2 - 2A_1 F_{1-2}}\right]$$

$$T_3 = \left[\frac{A_2 - A_1 F_{1-2} T_2^4 + A_1(1 - F_{1\to2}) T_1^4}{A_1 + A_2 - 2A_1 F_{1-2}}\right]^{1/4}$$

Emission over λ range :

$$e_b = \int_0^\infty e_{b\lambda}\, d\lambda \rightarrow e_{b\lambda} \text{ can be calculated from planks law}$$



let us say we will $e_{b\lambda}$ be calculate emission power over $[0, \lambda]$ instead of $[0, \infty]$

$$e_{b(0-\lambda)} = \int_0^\lambda e_{b\lambda} \cdot d\lambda$$

Instead we give

$$D_{0-\lambda} = \frac{\text{Radiant flux emitted over } 0 \to \lambda}{\text{Radiant flux emitted over } 0 \to \infty}$$

$$= \int_0^\lambda \frac{2\pi C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} \frac{1}{\sigma T^4}\, d\lambda$$

using plank & S.B law

change the variable to λT

$$D_{0\to\lambda} = \int_0^{\lambda T} \frac{2\pi C_1}{\sigma (\lambda T)^5 \left[e^{\frac{C_2}{\lambda T}} - 1\right]}\, d(\lambda T)$$

This helps us to get a combination parameter λT

The

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The numerical integration has been done s to a

λT (μK)	$D_{0-\lambda}$
600	0.000011
800	0.000020
1000	0.000320
⋮	⋮
4000	0.4800
⋮	
1000	0.9142
50,000	0.998923

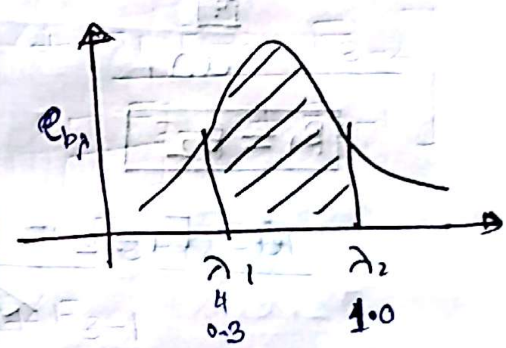
Ques: The e_λ of a surface varries as

$\epsilon_\lambda = 0$ for $\lambda < 0.3\,\mu$

$\epsilon_\lambda = 0.9$ for $0.3 \le \lambda \le 1\,\mu$

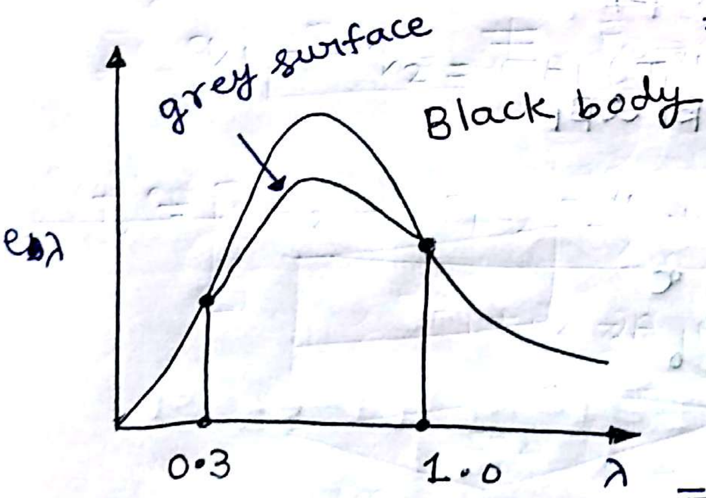
$= 0$ for $\lambda > 1\,\mu$

$\lambda_1 \to \lambda_2 \Rightarrow (0 \to \lambda_2) - (0 \to \lambda_1)$



calculate: e_b at 1500K

$$e_b = \sigma T^4 = 5.67 \times 10^{-8} \times 1500^4$$
$$= 287.044 \text{ kW}$$



$$e_\lambda = \epsilon_\lambda \, e_{b\lambda}$$

$$e = \int_0^\infty e_\lambda \, d\lambda$$

$$= \int_0^{0.3} \epsilon_\lambda e_{b\lambda} \, d\lambda + \int_{0.3}^{1.0} \epsilon_\lambda e_{b\lambda} \, d\lambda$$

$$+ \int_1^\infty \epsilon_\lambda e_{b\lambda} \, d\lambda$$

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or $e = 0.9 \int_{0.3}^{1.0} e_{b\lambda} \, d\lambda$

$= 0.9 \, D_{0.3 \to 1.0} \, \sigma T^4$

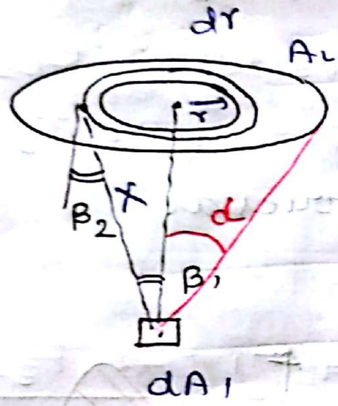
$= 0.9 \left[D_{0-1} - D_{0 \to 0.3} \right] \sigma T^4$

$\Rightarrow$ calculate λT for $\lambda = 1\mu, 0.3\mu$ & $T = 1500K$

$\Rightarrow$ got values of $D_{0\to1}$ & $D_{0\to0.3}$ from standard table

$\boxed{08-09-2016}$

$\boxed{S6}$ tut $\Rightarrow$



$F_{1\to2} = \int \frac{\cos\beta_1 \cos\beta_2}{\pi x^2} \cdot dA_2$

$dA_2 = 2\pi r \, dr$

$\tan\beta = \frac{r}{L}$

$\boxed{\beta_1 = \beta_2}$

$\therefore F_{1\to2} = \cancel{\int_0^{D/2} \frac{2\pi \cos^2\beta_1}{\pi r^2} \cdot \frac{r \, dr}{\tan^2\beta_1}}$

$= \int_0^{\alpha} \frac{2\pi \cos^2\beta_1}{\pi L^2 \sec^2\beta_1} \cdot L\tan\beta_1 \times L\sec^2\beta_1 \, d\beta_1$

$= 2\int_0^{\alpha} \sin\beta_1 \cos\beta_1 \, d\beta_1$

$= -\frac{\cos 2\beta}{2} \Big|_0^{\alpha}$

$= \sin^2\alpha$

$F_{1\to2} = \frac{D^2/4}{L^2 + D^2/4}$

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Ques – 9

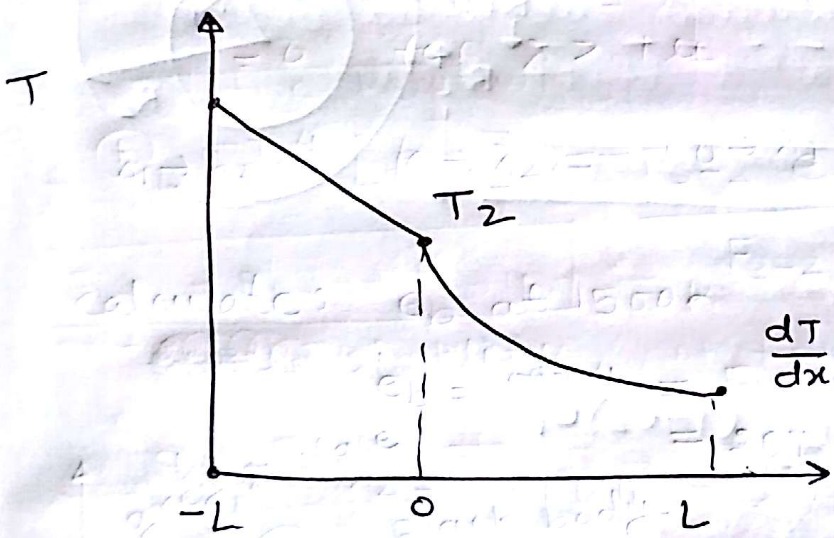
$$\cancel{F_{1\to1}}^{0} + F_{1\to2} + F_{1\to3} = 1$$

$$F_{2\to1} + \cancel{F_{2\to2}}^{0} + F_{2\to3} = 1$$

$$F_{3\to1} + F_{3\to2} + F_{3\to3} = 0$$

$$F_{1\to2} \cdot A_1 = F_{2\to1} \cdot A_2$$

Q–2 (class test



$\frac{dT}{dx} = 0$ at $x = L$

do a heat balance at $x = 0$

$$q_{in} = KA_c \cdot \frac{T_1 - T_2}{L}$$

$$q_{out} = \theta_b \cdot M \tanh(mL)$$

$$M = \sqrt{hPKA_c}$$

$$m = \sqrt{\frac{hP}{KA_c}}$$

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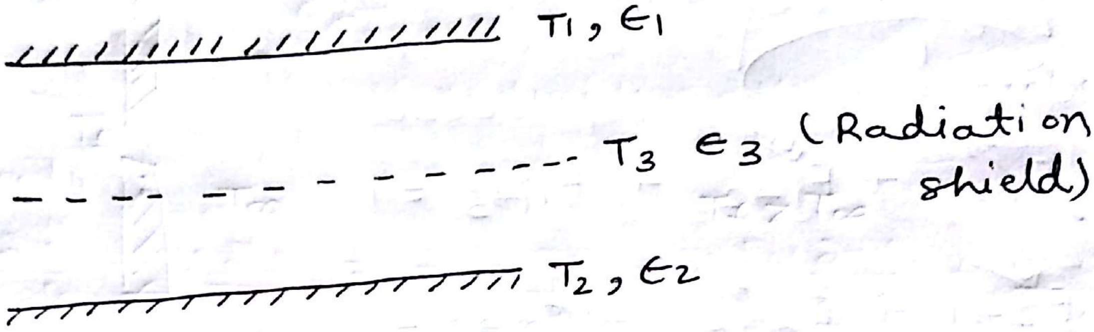
26/09/2016

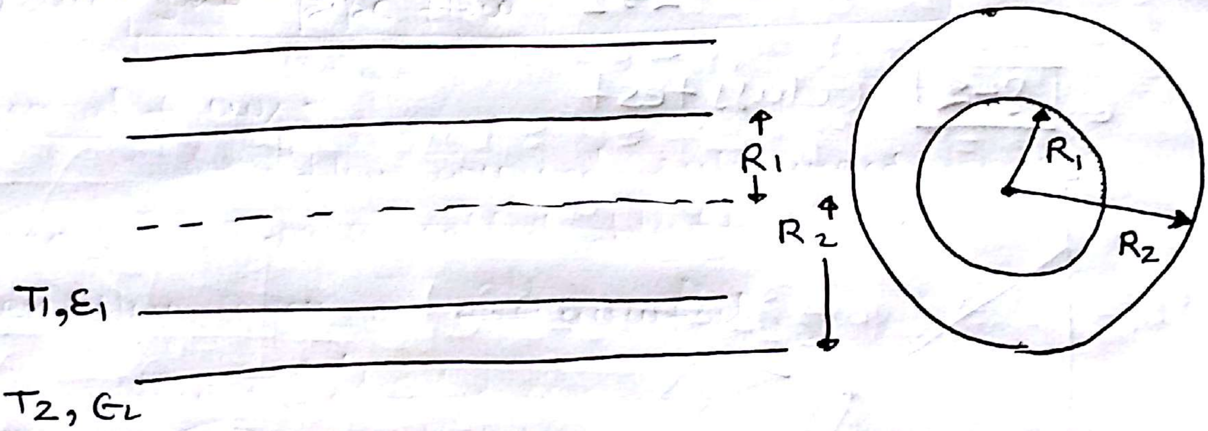
radiation exchange between gray bodies:

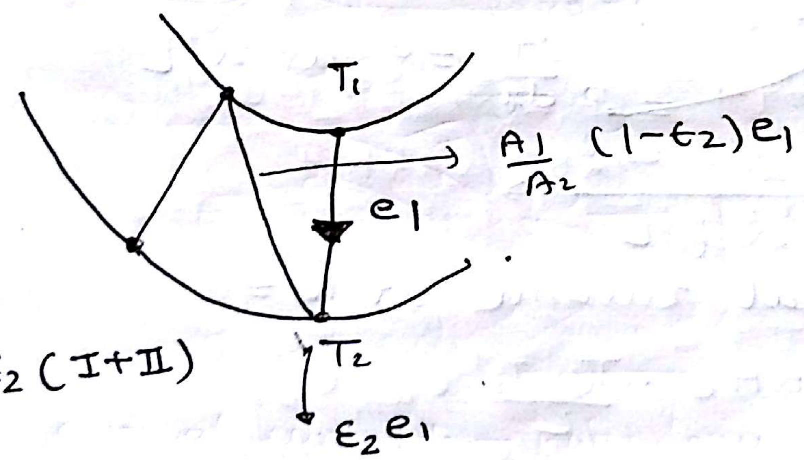
① Infinite surface



$$q''_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

②





$$I = \left(1 - \frac{A_1}{A_2}\right)(1-\epsilon_2) e_1$$

$$II = (1-\epsilon_1)\frac{A_1}{A_2} \cdot (1-\epsilon_2) e_2$$

Rate at which radiation emitted by A_1 & absorbed at A_2

$$= \epsilon_2 e_1 A_1 \left[1 + \left(1-\epsilon_1 \frac{A_1}{A_2}\right)(1-\epsilon_2) + \left(1-\epsilon_1 \frac{A_1}{A_2}\right)^2 (1-\epsilon_2)^2 + \cdots\cdots \right]$$

$$= A_1 e_1 \epsilon_2 \cdot \frac{1}{1 - \left(1-\epsilon_1 \frac{A_1}{A_2}\right)(1-\epsilon_2)}$$

$$= \frac{A_1 e_1 \epsilon_2}{\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}} \quad \text{--- (1)}$$

similarly rotation emitted by A_2 and absorbed at A_1

$$= \frac{A_1 e_1 \epsilon_1}{\epsilon_1 \frac{A_1}{A_2} + \epsilon_2 - \epsilon_1 \epsilon_2 \frac{A_1}{A_2}} \quad \text{--- (2)}$$

Net radiation exchange

$$\boxed{q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}}$$

• यही गोले के लिए होगा।

(a) Valid for concentric spheres:

→ $\frac{A_1}{A_2} = \frac{R_1}{R_2}$ cylinder

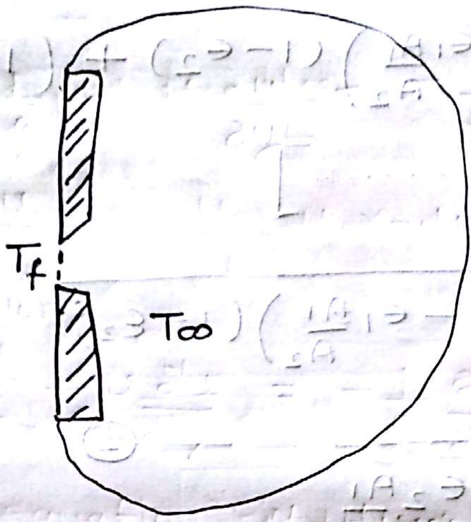
→ sphere $\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^2$

(b) No need to be concentric

(c) Valid for 2 surfaces as long as $F_{1\to 2} = 1$ & $F_{2\to 1} = A_1/A_2$

Problem

glass / Transparent window in a furnace wall



$T_f > T_\infty$

① mass and momentum eq.

② duct and pipe flow

28/09/2016

Convection:

$$\boxed{q_{conv.} = hA(T_W - T_\infty)}$$

$h \Rightarrow$ heat transfer coeff. $(W/m^2 \cdot K)$

At $y=0$

$$-k \left.\frac{\partial T}{\partial y}\right|_{y=0} = h(T_W - T_\infty) \rightarrow \text{condition at wall}$$

$$\Rightarrow \boxed{h = \frac{-k \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_W - T_\infty)}}$$

Review of fluid mechanics:

governing equation of fluid flow:

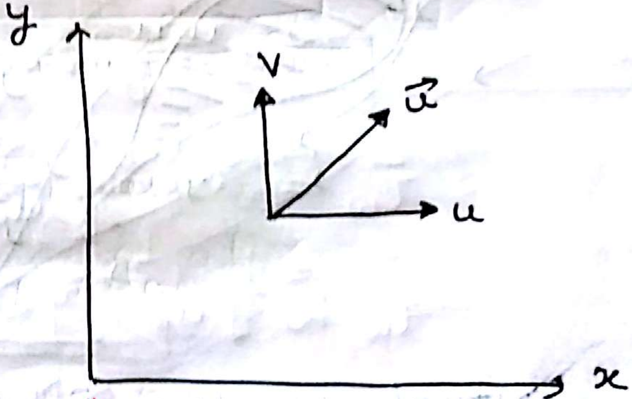
- conservation of mass $\rightarrow$ continuity equation
- conservation of moment $\rightarrow$ 2nd law of motion
- stokes Relation $\rightarrow$ relates stress to rate of strain

$$\tau = \mu \left(\frac{du}{dy}\right)$$

(i)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\nabla \vec{u} = 0$$



(ii)

$$\rho \underbrace{\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]}_{\text{Inertial}} = \underbrace{-\frac{\partial p}{\partial x}}_{\text{pressure}} + \underbrace{\mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]}_{\text{viscous}} \underbrace{+ F_x}_{\text{Body (external)}}$$

At steady state & no external force

$$\frac{\partial u}{\partial t} = 0 \quad , \quad F_x = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

X-direction Navior-stroks eq.

Y-direction:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$

Y-direction N-S eq.

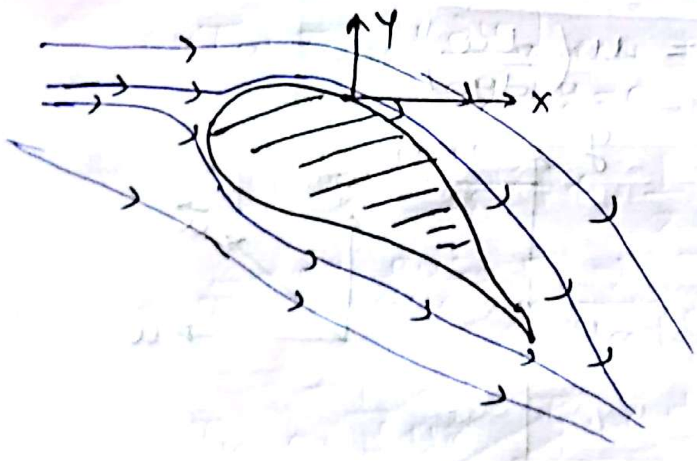
Vector form

$$\boxed{\rho\cdot\bar{u}\cdot\nabla\vec{u} = -\nabla p + \mu\nabla^2\bar{u}}$$

External flows:

- flow around submerged bodies
- also called bluff bodies

ex. flow over an ~~a~~ aerofoil



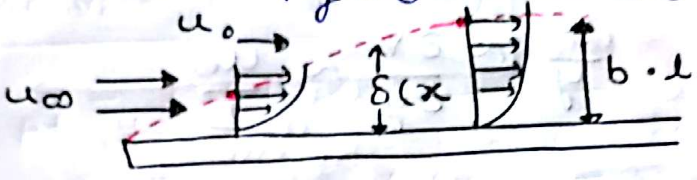
critical concept - Boundary layer

- Small region surrounding the body where the effect of presence of submerged body, ~~positi~~ particularly viscous forces are felt

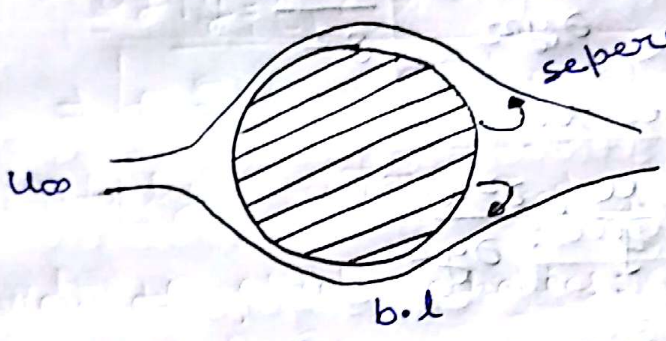
→ beyond b.d is the zone of inviscid flow where flow, pressure etc. are not impacted by stress force

→ at the edge of b.d ($y \simeq \delta$)

$$u|_{y=\delta} = 0.99\,u_\infty$$



• 99% velocity



⇒ velocity must be zero at surface | $u = v = 0$ @ $y = 0$

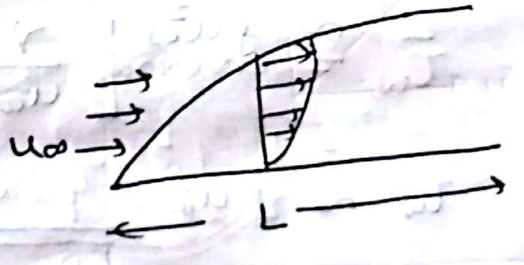
– No slip condition

⇒ B.L characteristics

(a) $\delta << L$ (b) $v << u$ © $\frac{\partial u}{\partial x} << \frac{\partial u}{\partial y}$

Solve N-S equation for flat plate:

continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$



scaling analysis

$$\frac{\partial u}{\partial x} \sim \frac{u_\infty}{L} \checkmark$$

$$\therefore \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} \sim \frac{u_\infty}{L}$$

momentums

$X \rightarrow \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$

$$P + \frac{1}{2}\rho u_\infty^2 = 0$$

$$u\frac{\partial u}{\partial x} \sim u_\infty \frac{u_\infty}{L}$$

$$v\frac{\partial u}{\partial y} \sim \frac{u_\infty}{L} u_\infty \qquad \frac{v}{y} \sim \frac{\partial v}{\partial y} \sim \frac{u_\infty}{L}$$

$$\frac{\partial^2 u}{\partial x^2} \sim \frac{u_\infty}{L^2} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{u_\infty}{\delta^2}$$

N-S equation simplifies to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\cdot\frac{\partial^2 u}{\partial y^2}$$

Y- all terms are insignificant compared to X-direction terms

since $v \sim \delta \qquad \frac{\partial v}{\partial y} \sim \frac{v}{\delta}$

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$\sim \mu \frac{u_\infty}{\delta}$$

$$\sim \rho u_\infty^2 \left[\frac{\nu}{u_\infty L}\right]^{1/2}$$

$Re = \frac{\rho u_\infty L}{\mu}$

$= \frac{u_\infty \cdot L}{\nu}$

$$\tau_w \sim \rho u_\infty^2 \cdot \frac{1}{\sqrt{Re_L}}$$

Locally,

$$\tau_{wx} \sim \rho u_\infty^2 \frac{1}{\sqrt{Re_x}}$$

define C_{fx} = skin friction coefficient

$$= \frac{\tau_{wx}}{\frac{1}{2}\rho u_\infty^2}$$

$$\Rightarrow \boxed{C_{fx} \sim \frac{1}{\sqrt{Re_x}}}$$

lets get back to N-S eq.

$u = \frac{\partial \psi}{\partial y}$ $\quad v = \frac{\partial \psi}{\partial x}$ stream function

let us define $\eta = \frac{y}{x} Re_x^{1/2}$

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^2 \psi}{\partial y^3}$$

$u = \frac{\partial \psi}{\partial y} = 0$ at $y = 0 \rightarrow$ no slip

$\psi = 0$ at $y = 0$ - impermeable wall

$\frac{\partial \psi}{\partial y} \rightarrow u_\infty$ at $y \rightarrow \infty$ - free stream

Now $\psi(x, y) = [u_\infty \nu x]^{1/2} \cdot f(\eta) \Rightarrow f =$ similarity streamline funct

$u = u_0 \frac{\partial f}{\partial \eta}$

N-S eqn.

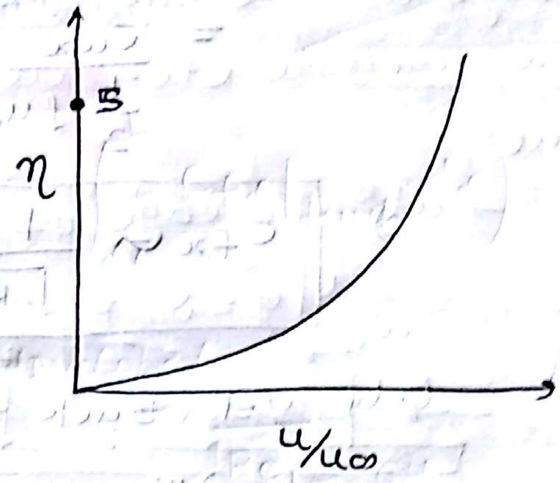
$2f''' + ff'' = 0 \rightarrow$ Blasius equation

$\Rightarrow f' = f = 0$ at $\eta \rightarrow 0$

$f' \rightarrow 1$ as $\eta \rightarrow \infty$

Looking at the solution:

$$\frac{u}{u_\infty} = 0.99 \quad \text{at } \eta \sim 5$$



$$\eta = \frac{y}{x} Re_x^{1/2}$$

$$\Rightarrow \frac{\delta}{x} = 5 / Re_x^{1/2}$$

or $\boxed{\delta(x) = \frac{5x}{\sqrt{Re_x}} \propto x^{1/2}}$

Blassius solution also gives:

$$\left.\frac{\partial^2 f}{\partial \eta^2}\right|_{\eta=c} = 0.332$$

$$\therefore \; C_{fx} = \frac{\mu \left.\frac{\partial u}{\partial y}\right|_{y=0}}{\frac{1}{2}\rho u_\infty^2} = 2 \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0} Re_x^{-1/2}$$

or $\quad C_{fx} = 0.664\, Re_x^{-1/2}$

$$\tau_{wx} = C_{fx} \cdot \frac{1}{2} \rho u_\infty^2 \rightarrow \text{local shear stress}$$

$$\bar{\tau}_w = \text{average shear stress} = \frac{1}{L}\int_0^L \tau_{wx}\, dx$$

$$\bar{c}_f = \frac{\bar{\tau}_w}{\frac{1}{2}\rho u_\infty^2}$$

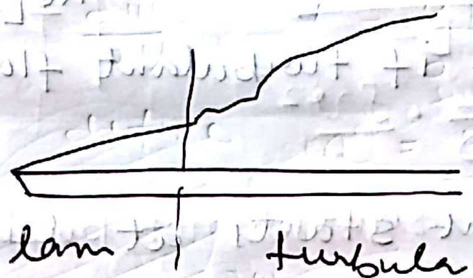
$\boxed{\bar{c}_f = 1.328\, Re_L^{-1/2}}$ average skin friction coefficient

29/09/2016

Turbulent flow: velocity vs time plot varies both magnitude and direction

For flow through a tube turbulance starts right at inlet

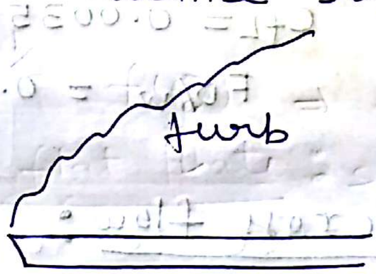
$\Rightarrow$ for flat plate/ext flow it is possible to have laminar flow followed by turbulant flow.



critical Re for flow over a flat plate

$$Re_c = 3\times10^5 = \frac{\rho u_\infty x_c}{\mu}$$

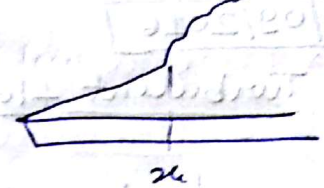
If we make the leading edge ragged and rough we can have turbulance right from start.



Recommended relation for C_{fx} for turbulant flow

$$C_{fw}\Big|_{turb} = 0.0592\,(Re_x)^{-1/2}$$

Laminar followed by turbulant :



$$\bar{C}_{fL} = \frac{1}{L}\left(\int_0^{x_c} C_{fx}\big|_{lam} \cdot dx + \int_{x_c}^{L} C_{fx}\big|_{turb} \cdot dx\right)$$

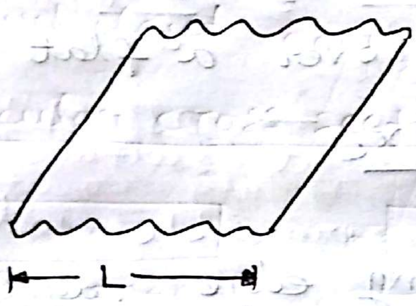
This gives

$$C_{fL} = 0.074\, Re_L^{-0.2} - \frac{C_1}{Re_L}$$

Where $C_1 = \cancel{1500}\ 1050$ if turbulant flow starts at $Re = 3\times10^5$

$C_1 = 0$ if turbulant starts at leading edge

Ques :



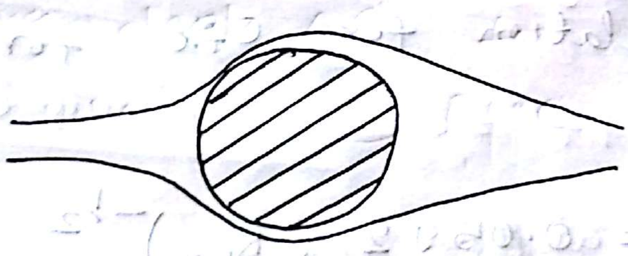
$C_{fL} = ?$

$F_D/w = ?$

$x_L = ?$

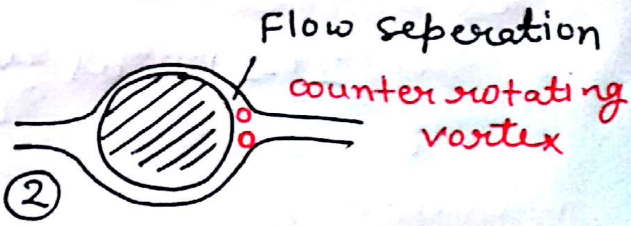
[Ans: $C_{fL} = 0.00357$, $x_c = 0.751$ m
$F_D/w = 0.103$ N/m]

Cylinder in cross flow :

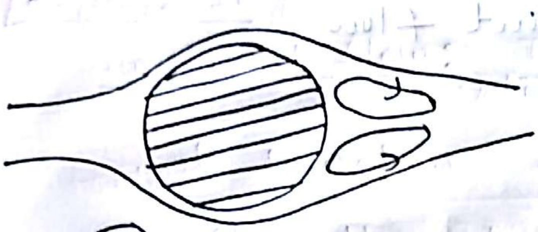


As Re (↑)

① very low Re (creeping flow)



②



③

④ Turbulant flow

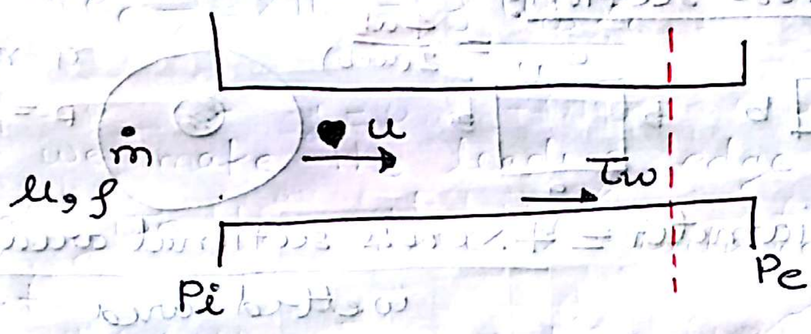
von Karmann vortex street

$$C_D = \frac{F_D / A_P}{\frac{1}{2}\rho u_\infty^2}$$

A_P = Projected area

$C_D = A\,Re_D^n$ | values available in table & charts

Internal flow:



consider flow inside a duct / tube / pipe of some cross section.

$\Delta P = P_i - P_e$ $\quad P_i > P_e$ due to viscous drag

$= f$(geometry, fluid, velocity, L) ← length

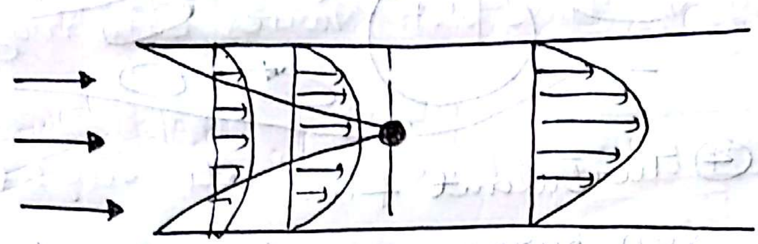
"Friction factor"

$$f = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2} = \frac{-4\left(\frac{dp}{dx}\right)A_c}{P\cdot\rho u_\infty^2}$$

$f \rightarrow$ non dimensional

$f \rightarrow f(Re)$ only for laminar, fully developed

$= f\left(Re, \frac{\epsilon}{D}\right)$ for turbulent flow



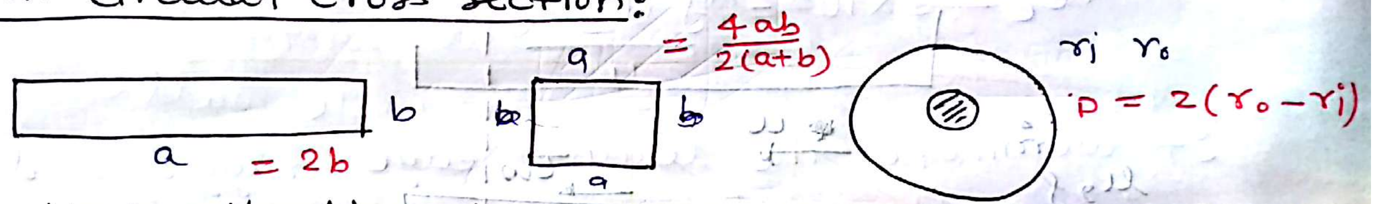
developing   fully developed

Laminar: $\Delta P = \frac{32 \mu u L}{D^2}$ | $f = \frac{64}{Re}$

$= f \cdot \frac{L}{D} \cdot \frac{\rho v^2}{2}$

Turbulent: Analytical solution not possible
- use moody chart or chans corelation
  Turbulent occurs at $Re > 2000$

Non circular cross section:



Hydraulic diameter $= \frac{4 \times \text{cross sectional area of flow}}{\text{wetted area}}$

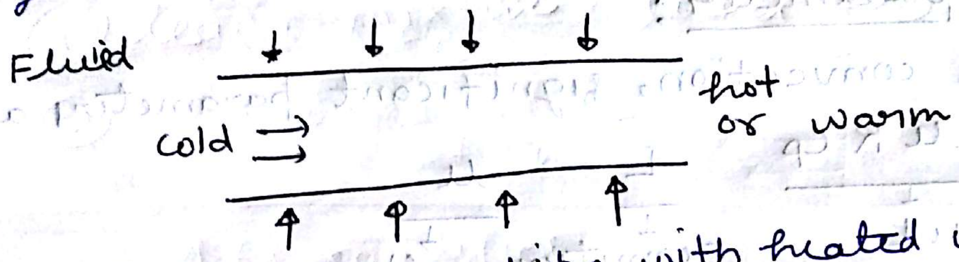
Ques Take a tube 1cm dia, $u = 0.2 m/s$, $L = 3m$

cal. pressure drop, ΔP for water coming in

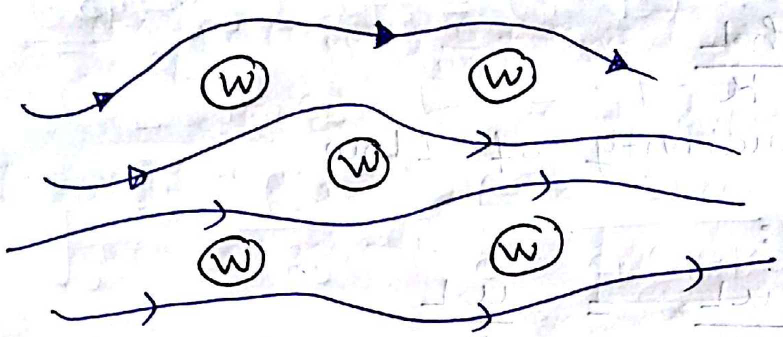
(a) 10°C   (2) 80°C

03/10/16

Forced convection: occurs due to fluid flow caused by an external agent like fan, pump, blower etc.

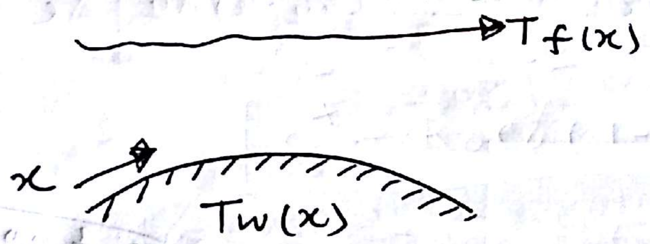


flow Here a pipe with heated walls



flow over a bank of tubes

- we will limit ourselves to single phase
  → No boiling, condensating, freezing



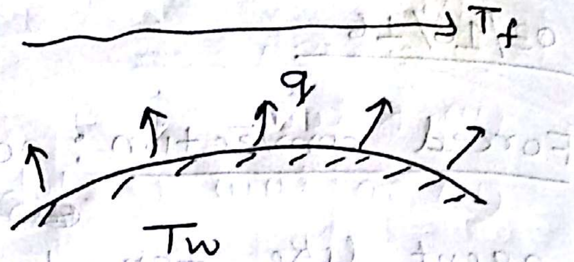
$T_w(x) > T_f(x)$

$$h(x) = \frac{q''(x)}{T_w(x) - T_f(x)}$$

↓

heat transfer coefficient $[W/m^2K]$

$$\bar{h} = \frac{q_{w \to f}}{A[\bar{T}_w - \bar{T}_f]}$$



dimensionless parameter:

- For forced convection, significant parameters are

h $\quad \underbrace{\rho\ \mu\ K\ C_p}_{\text{fluid}} \quad \underbrace{L}_{\text{geometry}} \quad \underbrace{u}_{\text{flow}}$

(a) Nusselt number:

$$Nu = \frac{hL}{K}$$

$K \to$ conductivity of flow

(b) Reynolds number:

$$Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu}$$

(c) Prandtl number:

$$Pr = \frac{\mu C_p}{K} = \frac{\mu/\rho}{K/\rho C_p}$$

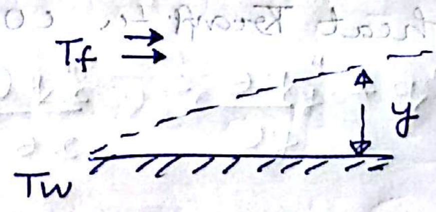
$$\boxed{Pr = \frac{\nu}{\alpha}}$$

Nu #

(a) $Nu = \dfrac{hL}{K} = \dfrac{L/KA}{1/Ah} = \dfrac{R_{Th}/cond}{R_{Th}/conv}$ Biot no

(b) Nu = dimensionless temp gradient

$$= \frac{-\frac{\partial T}{\partial y}\Big|_{y=0^+}}{(T_w - T_f)/L}$$



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$$-KA\frac{\partial T}{\partial y}\Big|_{y=0^+} = hA(T_w - T_f)$$

$$Nu = \frac{hL}{k} = \frac{-k\frac{\partial T}{\partial y}\Big|_{y=0^+}}{(T_w - T_f)} \cdot \frac{L}{k}$$

$$= -\frac{\partial T}{\partial y}\Big|_{y=0^+} \Big/ \left(\frac{T_w - T_f}{L}\right)$$

$\boxed{Pr\ \#}$

- fluid property
- measure of momentum diffusivity over thermal diffusivity
- measure of ease of transfer of shear force Through the fluid to transfer of thermal energy.

ⓐ engine oil – high μ, low k

$Pr \gtrsim 100$

ⓑ most gases $Pr \sim 1$ (low μ, k)

ⓒ liq metals [eg, Hg, liq Na, k] $Pr \lesssim 0.01$

governing equations:

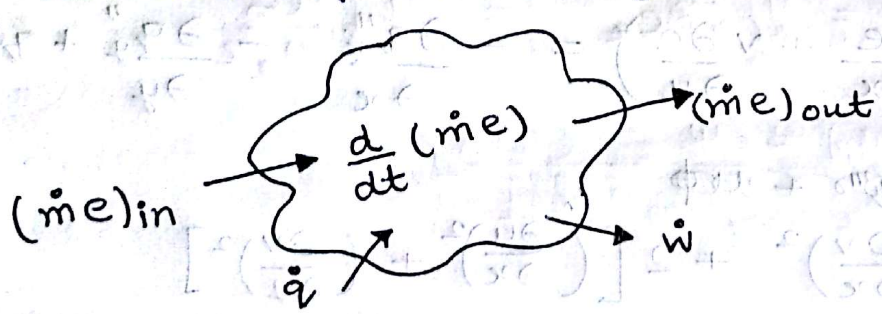
continuity } we know from fluid mechanics
Momentum

Energy → Imp

apart from these Fouriers law is also used

Energy Equation:

Ist law of thermodynamics

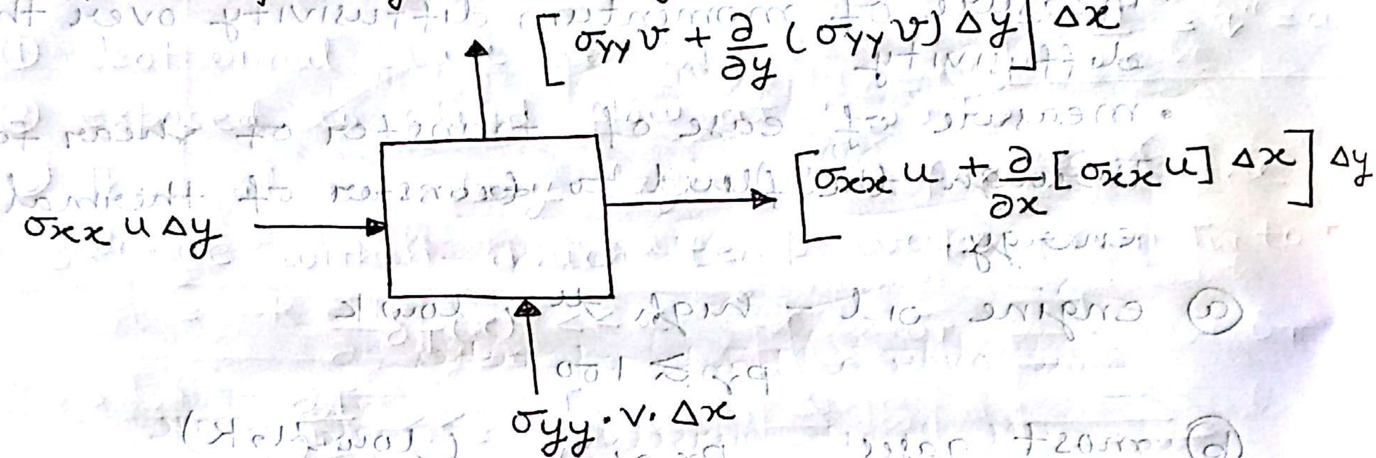


apply Ist law

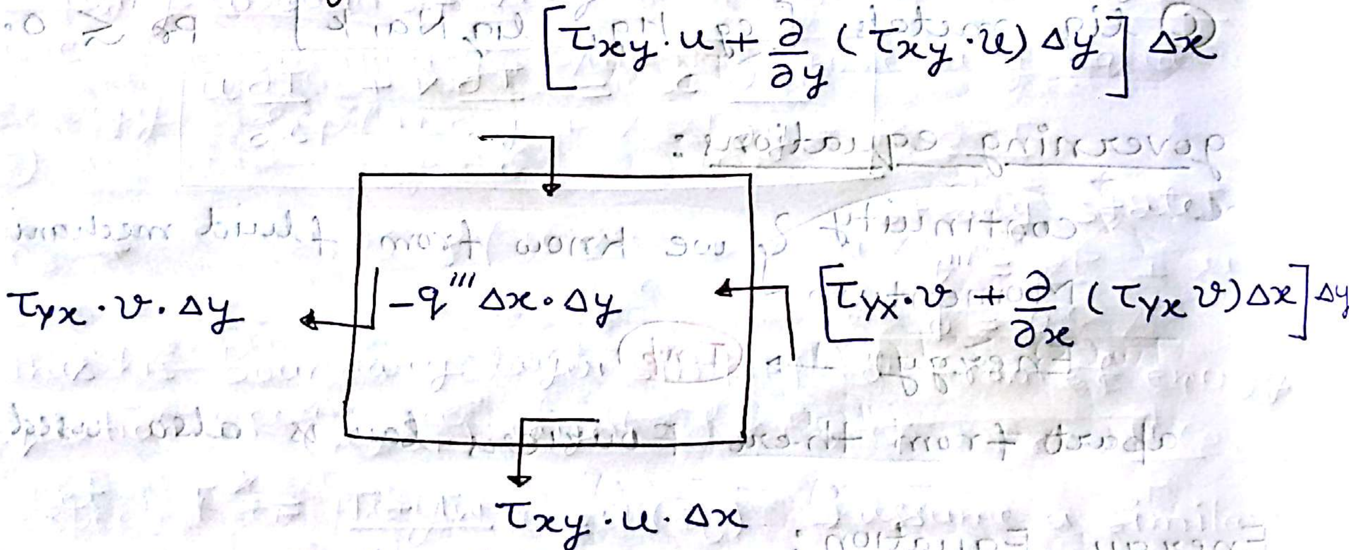
$$\frac{\partial}{\partial t}(\dot{m}e) = (\dot{m}e)_{in} - (\dot{m}e)_{out} + \dot{q} - \dot{W}$$

② Heat transfer by conduction

③ Work transfer by normal forces



④ Work done by shear forces + int energy generation



<u>Energy balance</u>:

$$\rho\left(\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y}\right) = -\frac{\partial q''_x}{\partial x} - \frac{\partial q''_y}{\partial y} + q''' + \mu\phi$$

$$\phi = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$$

where ϕ is the viscous dissipation function for a fluid with constant 'ρ'

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\sigma_{xx} = -P = -2\mu\frac{\partial u}{\partial x}$$

We need to restate energy equation in terms of $T \Rightarrow$

Internal energy $e = i - PV$, i = specific enthalpy

for single phase fluid $i = c_p T$

$$di = c_p dT + (1-\beta T)\frac{1}{\rho}dP$$

β = Volumetric thermal expansion coefficient $= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)$

⊙ For $\rho \sim$ constant, LHS of energy equation

$$\Rightarrow \rho\left[\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y}\right] = \rho c_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right]$$

⊙ contribution due to pressure terms is negligible

RHS becomes $\Rightarrow \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + q''' + \mu\phi$

Final energy equation:

$$\boxed{\rho c_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + q''' + \mu\phi}$$

Note ① for fluid at rest $u = v = \phi = 0$

energy equation reduces to heat diffusion equation

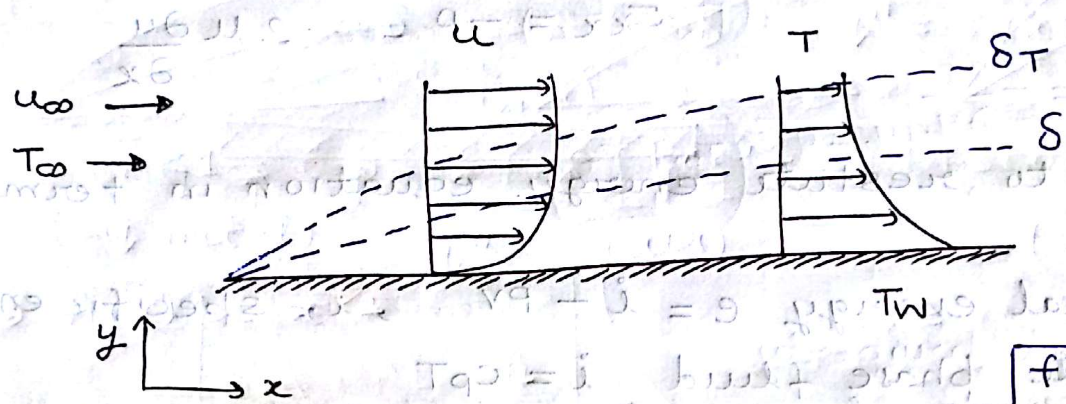
② In most convection problems, viscous dissipation is negligible ($\phi = 0$)

★★★ Thinner boundary layer = Better heat transfer

Plane wall or flat plate:

$T = T_w$ @ $y=0$

$T \to T_\infty$ @ $y \to \infty$



for iso flux

$-k\frac{\partial T}{\partial x} = q'' = const$

① Isothermal plate at T_w

② exposed to fluid flow at u_∞, T_∞

③ Thermal boundary layer (B.L.) forms on the surface within which temp drops from T_w to T_∞

$$\delta_T(x) << x$$

$$\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$$

using order of magnitude analysis, we get

$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}}$$

steady state

$q''' = 0$

$\phi = 0$

This is boundary layer simplified from of energy equation in (x, δ_T) layer

Let, $T^* = \frac{T - T_w}{T_\infty - T_w} = T(\eta) \Rightarrow$ assume a similarity solution

Where $\eta = \frac{y}{x} \cdot Re_x^{1/2} = y\sqrt{\frac{u_\infty}{\nu x}}$

$$f(\eta) = \frac{\psi}{\sqrt{u_\infty \nu \cdot x}}$$

using the above, energy equation reduces to

$$\boxed{\frac{\partial^2 T}{\partial \eta^2} + \frac{Pr}{2} + \frac{dT^*}{d\eta} = 0}$$

$T^*_{(0)} = 0$

$T^*(\infty) = 1$

note dependence of T on (u, v) is captured through 'f'

Numerically solving the above you get

$$\frac{dT^*}{d\eta} = 0.332\, Pr^{1/3} \quad \Big| \text{ for } (Pr \geq 0.6$$

Note $h(x) = \dfrac{q''(x)}{T_W - T_\infty} = \dfrac{-(T_\infty - T_W)}{T_W - T_\infty} \cdot K \left.\dfrac{\partial T}{\partial y}\right|_{y=0}$

$$= K\left(\frac{u_\infty}{\nu x}\right)^{1/2} \left.\frac{dT^*}{d\eta}\right|_{\eta=0}$$

$$= 0.332\, Pr^{1/3} \cdot K \sqrt{\frac{u_\infty}{\nu \cdot x}}$$

or $\boxed{Nu_x = \dfrac{h(x) \cdot x}{K} = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}}$

① $\overline{Nu}_L = \dfrac{1}{L}\displaystyle\int_0^L Nu_x \cdot dx = 0.221\, Re_L^{1/2} \cdot Pr^{1/3}$ ✗

② $\overline{Nu}_L = \dfrac{\bar{h} L}{K}$ $\quad \bar{h} = \dfrac{1}{L}\displaystyle\int_0^L h(x)\, dx$ ✓

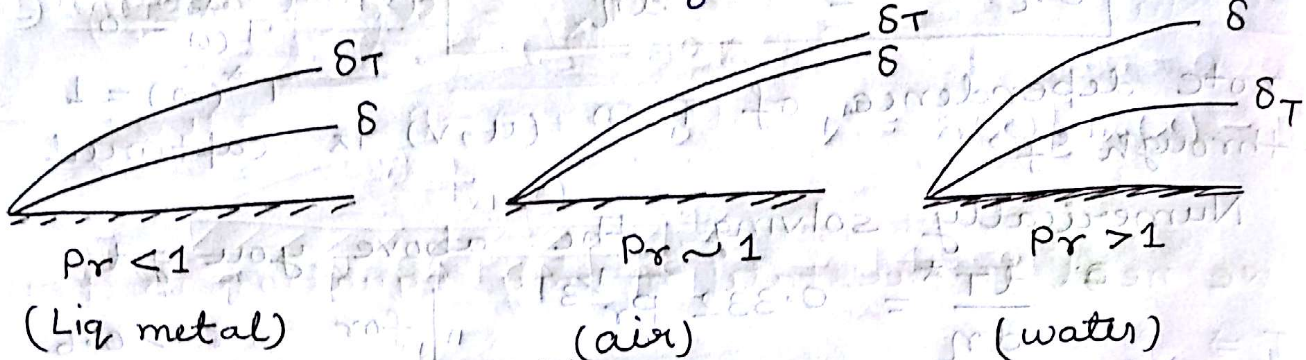
$$= 0.664\, Re_L^{1/2} \cdot Pr^{1/3}$$

⇒ which one is correct

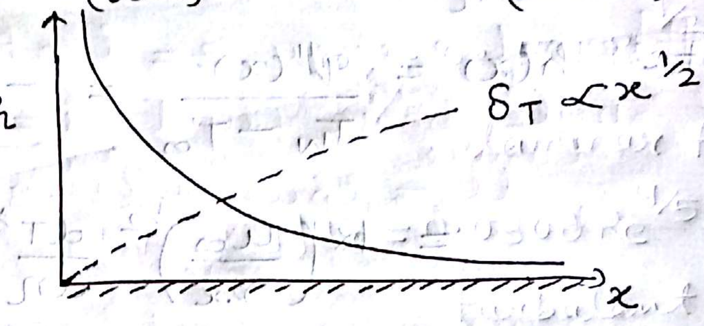
① heat transfer coefficient is a real parameter hence it has to be averaged

② There's nothing as average nusselt number observations:

① It can be shown that $\frac{\delta_T}{\delta} = Pr^{1/3}$

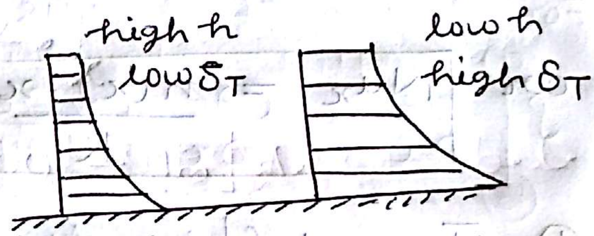


② $\Rightarrow h(x) \propto x^{-1/2}$



③ low δ_T = high h and

$$h = \frac{-K \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_w - T_\infty}$$



<u>Analogy between Heat and momentum transfer</u>

- Note the similarity b/w the governing equations

energy $\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = K\frac{\partial^2 T}{\partial y^2}$

X-momentum: $\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \cancel{-\frac{\partial p}{\partial x}}^{\,0} + \mu\frac{\partial^2 u}{\partial y^2}$

- consequently we expect the solution to be similar due to occurence of μ, ρ, K, c_p, $Pr = \frac{\mu c_p}{K}$ is expected to play a role
- lets look at flow (laminar) cover flat plate

$C_f(x) = 0.664\, Re_x^{-1/2}$ | $Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$

$$\Rightarrow \frac{Nu_x}{Re_x\, Pr^{1/3}} = \frac{C_f(x)}{2}$$

- even for average values $\frac{\overline{Nu}}{Re_L \cdot Pr^{1/3}} = \frac{\bar{C}_f}{2}$
- This relation is called colburn analogy
- assumes such a relation exists for other flows (lam & turb) as long as the geometry is the same for $\overline{Nu}$ and $\bar{C}_f$

For Internal flow:

$$\frac{Nu_D}{Re_D^{1/2}\ Pr^{1/3}} = \frac{f}{2} \qquad D \rightarrow \text{diameter of the tube}$$

Turbulent flow over flat plate

$$C_f(x) = 0.0592 \cdot Re_x^{-0.2}$$

using Colburn analogy

$$Nu_x = \frac{C_f(x)}{2} \cdot (Re_x)(Pr)^{1/3}$$

$$\boxed{Nu_x = 0.0296\ Re_x^{4/5} \cdot Pr^{1/3}}$$

$$\overline{Nu}_L = \frac{5}{4} \times 0.0296\ Re_L^{4/5} \cdot Pr^{1/3}$$

Correlation matches very well with experimental data for $Re_c < Re_x < 10^7$ & $0.7 \le Pr \le 100$

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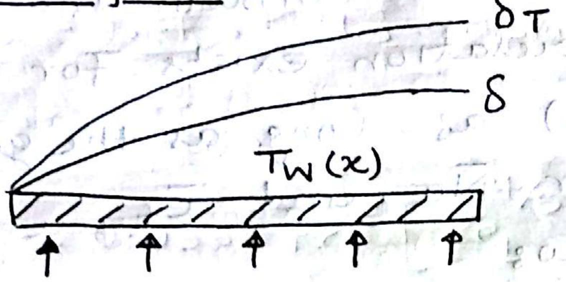
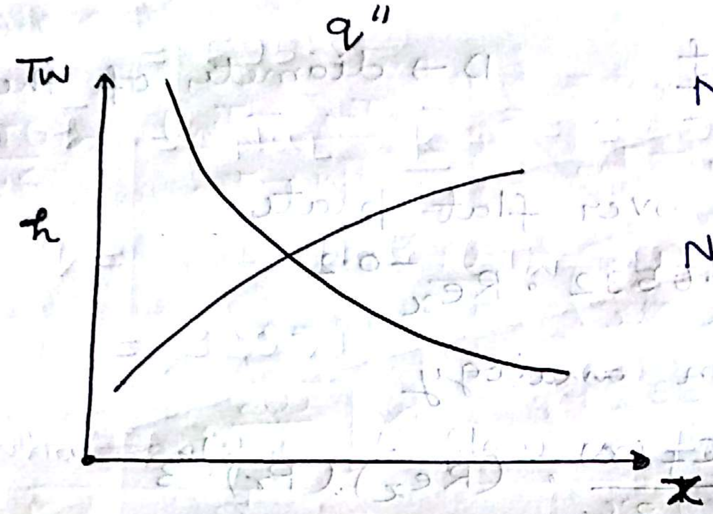
Special cases:

① uniform flux:



$$q'' = h(x)\left[T_W(x) - T_\infty\right]$$



$$Nu_x = 0.453\, Re_x^{1/2}\, Pr^{1/3}$$

laminar flow

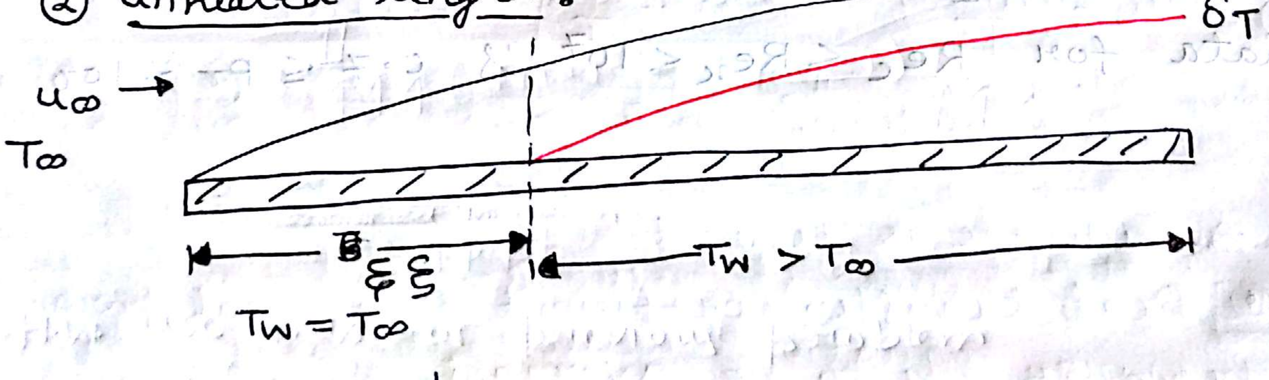
$$Nu_x = 0.0308\, Re^{4/5} \cdot Pr^{1/3}$$

turbulant flow

$$\overline{(T_W - T_\infty)} = \frac{1}{L}\int_0^L \left[T_W(x) - T_\infty\right] dx$$

$$= \frac{q'' L}{K\, \overline{Nu}_L} \qquad \left\{ \overline{Nu}_L = \frac{\bar{h} L}{K} \right\}$$

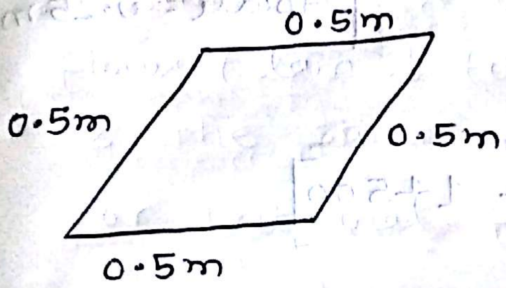
② unheated length:



$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{\left[1 - \left(\frac{\xi}{x}\right)^{3/4}\right]^{1/3}} \quad \text{— laminar}$$

$$= \frac{Nu_x\big|_{\xi=0}}{\left[1 - \left(\xi/x\right)^{9/10}\right]^{1/9}} \quad \text{— turbulance}$$

Ques A square plate of 0.5 m × 0.5 m is exposed to air flow in both surfaces at 15 m/s at 30°C and the wall temp maintained at 50°C.

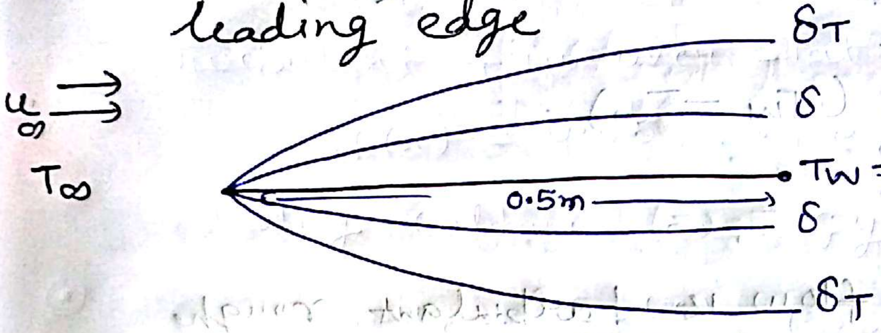


$u_\infty = 15\ m/s$

$T_\infty = 30°C$

$T_W = 50°C$

(a) calculate F_D and q ? assume a sharp leading edge



calculate fluid properties at mean temp

$$T_m = \frac{T_W + T_\infty}{2}$$

$$T_m = 40°C$$

At 40°C, $\rho = 1.128\ kg/m^3$ $\nu = 1.696 \times 10^{-5}\ m^2/sec$

$K_f = 0.027\ W/m\text{-}k$ $P_r = 0.7$

$$Re = \frac{u_\infty \cdot L}{\nu} = 4.422 \times 10^5 > Re_c\ (3 \times 10^5)$$

Lamianar followed by turbulant

⇒ boundary layer will be

$$Nu_L = 0.0366\ Pr^{1/3}\left[(Re)^{4/3} - C_1\right]$$

$C_1 = 14500$ if $Re_c = 3 \times 10^5$

$= 0$ if boundary layer (b.l.) is turbulant from the start itself

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$$\bar{C}_f = 0.074\, Re_L^{-0.2} - \frac{1050}{Re_L} = 3.122\times 10^{-3}$$

$$F_D = 2\cdot\frac{\bar{C}_f}{2}\cdot \rho\, u_\infty^2 \cdot A_{plate} \qquad \Big|\; A_{plate} = 0.25\ m^2$$

2 b.l: $\frac{①}{②}$

$$= 0.198\ N$$

$$\overline{Nu}_L = 0.0366\, Pr^{1/3}\left[Re_L^{4/5} - 14500\right]$$

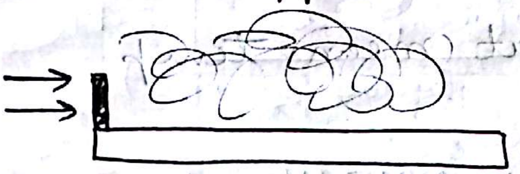
$$= 595.9$$

$$\Rightarrow \bar{h} = \overline{Nu}_L \cdot \frac{k}{L} = 32.9\ W/m^2.K$$

$$\therefore q = 2\bar{h}\, A_{plate}\,(T_w - T_\infty)$$

$$= 329\ W$$

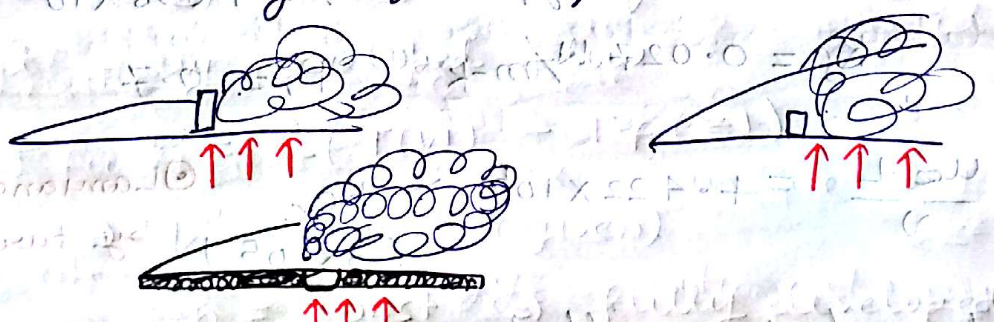
ⓑ What happen if flow is turbulant rough



Tripping of b.l.

Put an obstruction

make leading edge rough and ragged



H.W use this for previous problem

$$\left.\begin{aligned} F_D &= 0.349\ N \\ q &= 589\ W \end{aligned}\right\}$$

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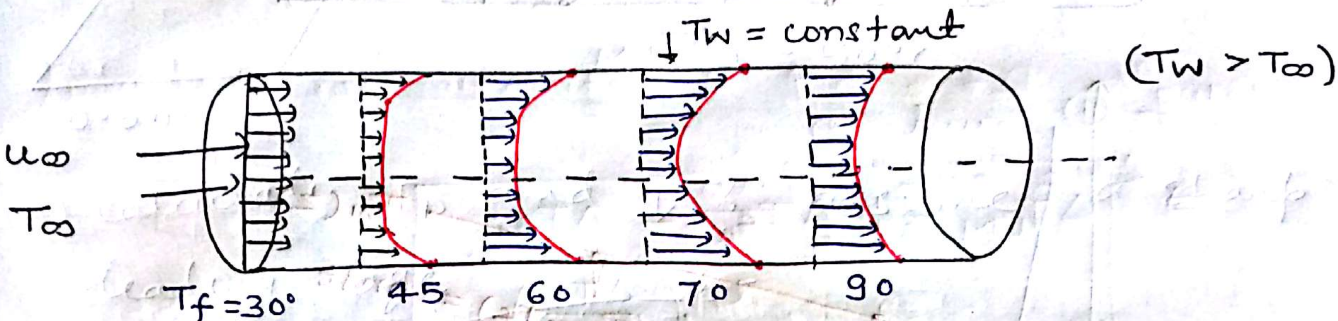
Forced convection (Internal flow):

Flow through a tube

- steady laminar flow
- Two boundary condition
  ① Iso thermal ② Isoflux



if $T_W = 100°C$

① Fluid temp Increases at the wall - at short distance, heat front does not propagate to the core

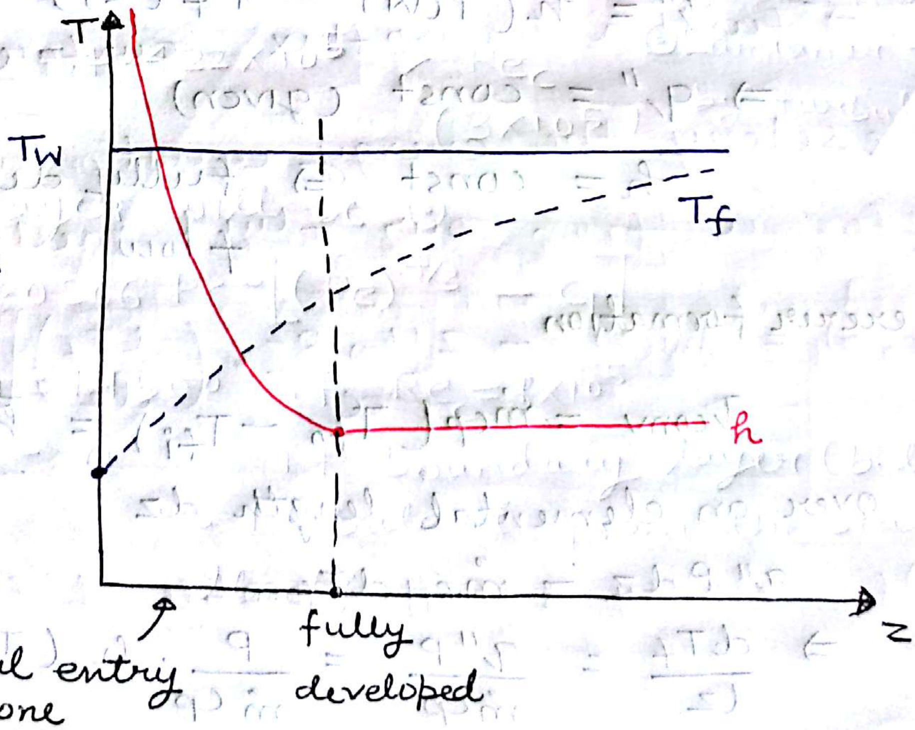
② after a distance the effect of heated wall penetrates the centerline

③ Fully developed.

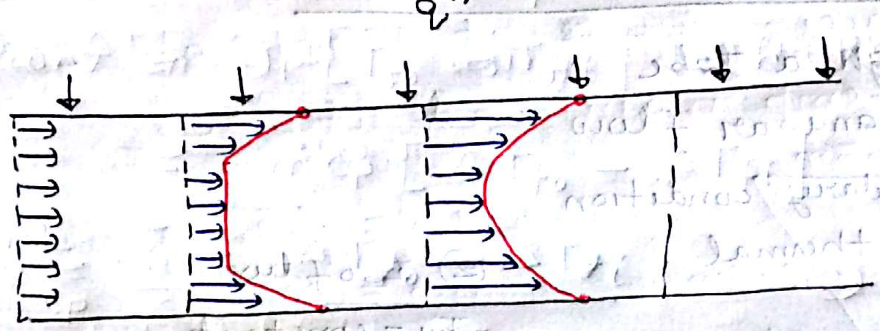
④ further downstream $(T_W - T_f)$ keeps getting reduced.

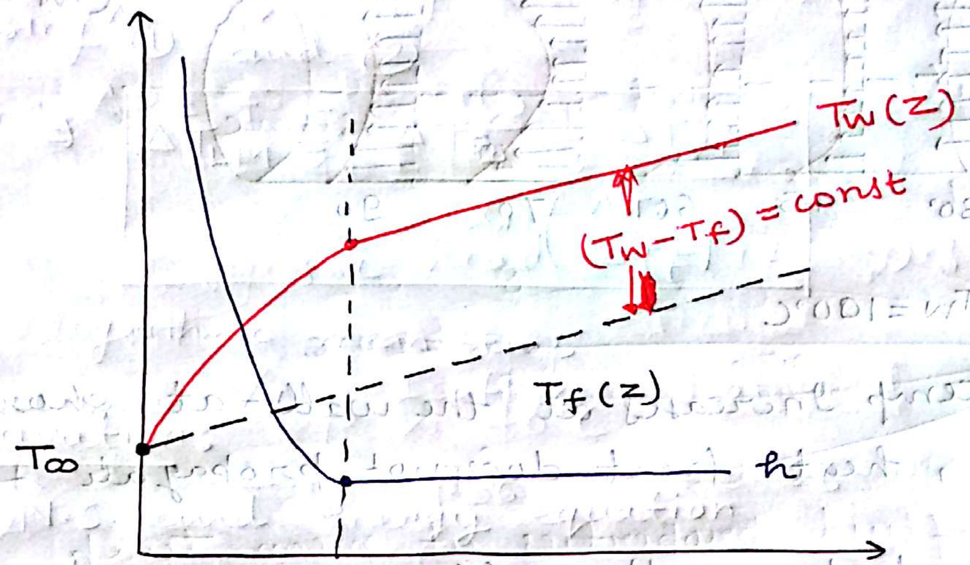
Isoflux is also same as Isothermal

★ h for fully developed constant



uniform flux condition:





at a distance z

Heat input $= q''\cdot P\cdot z = \dot{m} c_p (T(z) - T_\infty)$

$\Rightarrow T(z)$ varies linearly with z

At fully developed flow

$$q'' = h(T(W) - T_f(z))$$

$\Rightarrow q'' =$ const (given)

$h =$ const $\Rightarrow$ fully developed laminar flow

gererir formation

$$q_{conv} = \dot{m} c_p (T_{fo} - T_{fi}) = \bar{h} A (T_w - T_f)_m$$

over an elemental length dz

$$q'' P dz = \dot{m} c_p dT_f$$

$$\Rightarrow \frac{dT_f}{dz} = \frac{q'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} \cdot h (T_w - T_f)$$

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q'', T_f, h, T_w are all functions of z

① Isoflux B.C.

$$q'' = q''_w = \text{const}$$

$$\Rightarrow \frac{dT_f}{dz} = \frac{q''_w \cdot P}{\dot{m} \cdot c_p} \neq f(z)$$

$$\Rightarrow \boxed{T_f(z) = T_{fi} + \frac{q''_w P}{\dot{m} c_p} \cdot z}$$

mean temp of fluid increases linearly along flow length

$$T_w(z) = T_f(z) + \frac{q''_w(z)}{h(z)}$$

② Isothermal B.C.

$$\frac{dT_f}{dz} = \frac{P}{\dot{m} c_p} h (T_w - T_f)$$

using $\Delta T = T_w - T_f$ and solving ODE

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = -\frac{PL}{\dot{m} c_p} \int_0^L \frac{1}{L} \cdot h \, dz$$

$$= -\frac{PL}{\dot{m} c_p} \bar{h} \quad \text{—— ①}$$

$$\Rightarrow \frac{T_w - T_{fe}}{T_w - T_{fi}} = \exp\left[-\frac{PL}{\dot{m} c_p} \cdot \bar{h}\right]$$

$\rightarrow$ exit

along any distance z

$$\frac{T_w - T_f(z)}{T_w - T_{fi}(0)} = \exp\left[-\frac{Pz}{\dot{m} c_p} \cdot \bar{h}_z\right]$$

$\Rightarrow$ $(T_w - T_f)_z$ has an exponential decay along flow length

Total convective heat loss/transfer

$$q_{conv} = \dot{m} c_p [T_{fo} - T_{fi}] = \bar{h}(PL) \Delta T_{LM}$$

$$= -\dot{m} c_p [\Delta T_{fo} - \Delta T_{fi}]$$

$$\Rightarrow -\frac{PL}{\dot{m} c_p} \cdot \bar{h} = + \frac{\Delta T_{o} - \Delta T_{i}}{\Delta T_{LM}} \quad \text{--- (2)}$$

using (1) and (2)

$$\Rightarrow \boxed{\Delta T_{LM} = \frac{\Delta T_{o} - \Delta T_{i}}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}}$$

logrithmic mean temp

Derivation:

N.S. and energy equation

- at any point in the pipe, with b.l approximities, energy equation

$$u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} = \frac{\alpha}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

→ viscous dissipation is neglected

- Inside b.l $v = 0$, $\frac{\partial u}{\partial z} = 0$

⇒ further for isoflux surface $\frac{\partial^2 T}{\partial z^2} = 0$

- solving the energy equation at any expression for u from our fluid mechanics knowledge

$$T(r,z) = T_W(z) - \frac{2 u_\infty R^2}{\alpha} \cdot \left(\frac{dT_f}{dz}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{R}\right)^4 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right] \quad \text{--- (1)}$$

If velocity & temp profile of fluid are used

$$\frac{u(r)}{u_\infty} = 2\left(1 - \left(\frac{r}{R}\right)^2\right]$$

$$T_f(z) = T_w(z) - \frac{11}{48}\left(\frac{u_\infty R^2}{\alpha}\right)\left(\frac{dT_f}{dz}\right)$$

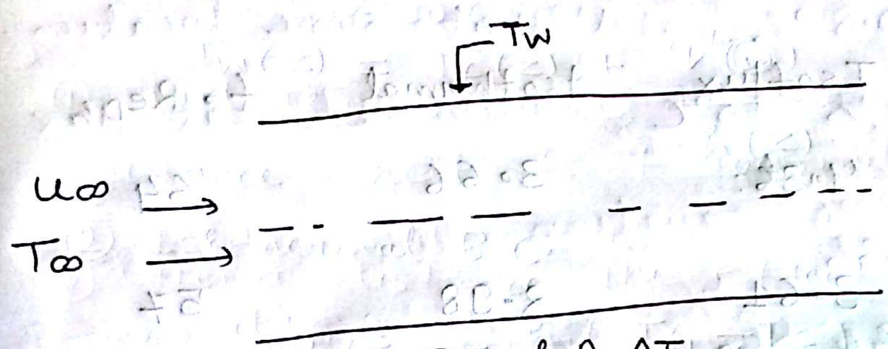
$\boxed{\begin{array}{l} P = \pi D \\ \dot{m} = \rho u \cdot \pi D^2/4 \end{array}}$

$$\Rightarrow T_w(z) = T_f(z) + \frac{11}{48}\,\frac{q''\cdot D}{K}$$

$$\Rightarrow h = \frac{48}{11}\,\frac{K}{D} \Rightarrow \boxed{Nu_D = \frac{hD}{K} = 4.36}$$

$\boxed{20/10/2016}$

Slide



$q = hA_s\Delta T$

Independent of u_∞

Nu = constant at fully developed

$\Rightarrow h =$ constant

$\rightarrow$ independent of u_∞

$$q = \dot{m}c_p(T_{fo} - T_{fi})$$

$\Rightarrow$ If $\dot{m}\uparrow$ $(T_{fo} - T_{fi})\downarrow$ & vice versa

\# $q = hA_s\,\Delta T_{LM}$

$\underset{\text{log mean}}{\uparrow}$

$$\Delta T_m = T_w - \frac{T_{fi} + T_{fo}}{2}$$

$$\Delta T_{Lm} = \frac{(T_w - T_{fi}) - (T_w - T_{fo})}{\ln\left((T_w - T_{fi}/T_w - T_{fo}\right)}$$

$\Delta T_{LM} \Rightarrow$ Increasing with increasing $\dot{m}$ (or) u_∞

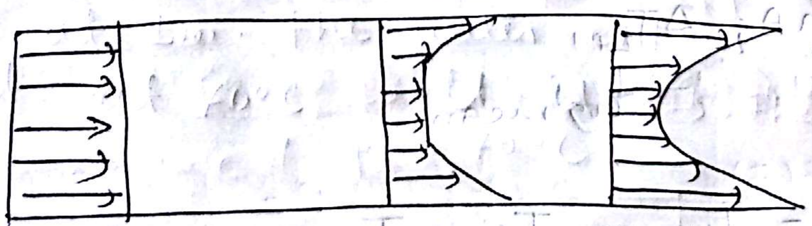
$$Nu = \frac{hD}{K} = const$$

As $D\downarrow$, $h\uparrow \Rightarrow$ motivation for μ channel

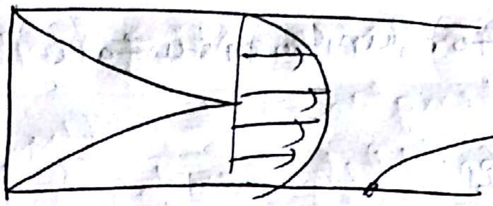
$$\Delta P = f \cdot \frac{L}{D} \cdot \frac{\rho u_\infty^2}{2}$$

ΔP pressure head $\uparrow$ = if, $D\downarrow$

cross section	b/a	Isoflux	Isothermal	$f_F\, Re_{Dh}$
circle	—	4.36	3.66	64
square (a, b)	1	3.61	2.98	57
	2	4.12	3.39	62
rectangle (a, b)	4	5.33	4.44	73
parallel plates (b)	$\rightarrow \infty$	8.33	7.54	96
triangle	—	3.11	2.47	53



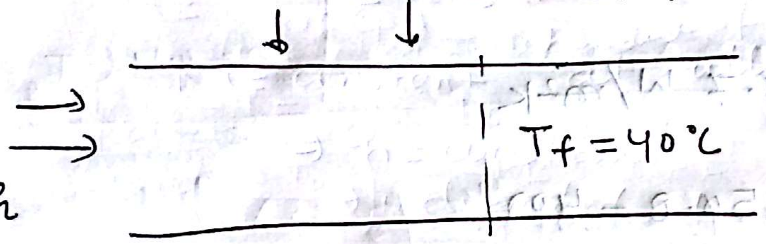
1>



(2)

23/10/16

Ques: consider a smooth tube of ID ~~3cm~~ 1.5 cm water flows at 50 kg/h the tube heated 2000 W/m2 at a location z the mean bulk temp of water is 40°C assume the velocity and temp profile fully developed and find wall temp and Tw.



temp of fully developed fluid given = 40°C

water 40°C ⇒ $K_f = 0.634$ W/m-K

$\rho = 992.$ kg/m3

$\nu = 6.59 \times 10^{-7}$ m²/sec

(i) Laminar vs turbulence

$$u = \frac{50}{\left(3600 \times \pi \frac{(0.015)^2}{4}\right) 992} = \frac{78.63}{992} \text{ m/sec} = 0.0792 \text{ m/sec}$$

$$Re = \frac{78.63 \times 0.015}{6.59 \times 10^{-7}} = 1803$$

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$Nu_D = \frac{hD}{K} = 4.36$ (for uniform surface heat flux)

$$h = \frac{0.634 \times 4.36}{0.015}$$

$$h = 184.28 \ W/m^2K$$

$$q'' = h(\Delta T)$$

$$\Delta T = \frac{2000}{184.28}$$

$$T_W = 40 + 10.85$$

$$= 50.85°C$$

Last one

Ques: wall at 50.8 °C and that some location fluid temp is 40°C determine heat flux and coefficient.

Ans: $Nu_D = 3.66$

$$\frac{hD}{K} = 3.66$$

$$h = 154.7 \ W/m^2K$$

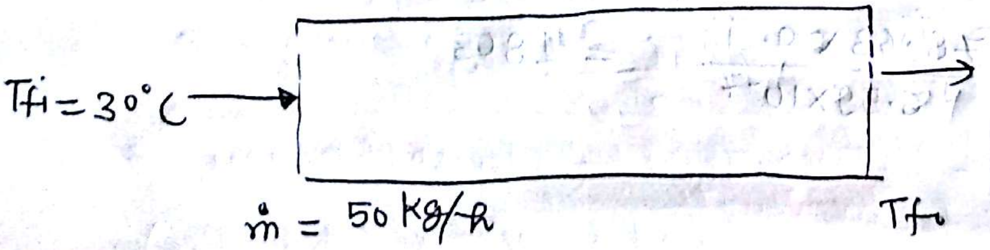
$$q'' = h(50.8 - 40)$$

$$q'' = 1670.72 \ W/m^2$$

$$q''_{isoflux} > q''_{isothermal}$$

Ques: consider the same tube, the length of tube fixed at 1m. the water enters at 30°C and wall at 70°C wall at Isothermal 70°C

$T_a = 70°C$



$\mu_{40} = 631 \times 10^{-6}$ unit

$\mu_{70°C} = 389 \times 10^{-6}$

at 40°C $\quad Pr = 4.31$

$c_p = 4.17$ KJ/kg.K

combined entry length

$$\left[Re_D \, Pr / (L/D)\right]^{1/3} (\mu/\mu_s)^{0.14} > 2$$

$$Nu = 1.86 \, (5.14)$$

$$= 9.56 = \frac{hD}{k}$$

$$h = \frac{9.56 \times .634}{0.015}$$

$$h = 404.069 \frac{W}{m.K}$$

$$Q = h A_s (T_w - T_f)$$

we don't know mean temp

LM

Logrethamic mean

$$\Delta T_{Lm} = \frac{\Delta T_o - \Delta T_i}{\ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$\frac{50}{3600} \times 4170 \times (T_{fo} - 30) = 404.069 \times \pi \times 0.015 \times 1 \times \frac{70 - 40}{\ln\left(\frac{70}{40}\right)}$$

$$T_{fo} = 41.2°C$$

Ques same old pipe with $T_w = 90°C$, $V_i = 1\,m/sec$ what should be the length of the tube to maintain 65°C outlet temp

$59.1°C \Rightarrow T : k_f = 0.656\ W/m\,K$

$\rho = 984.4\ kg/m^3$

$\nu = 4.97 \times 10^{-7}\ m^2/sec$

$C_p = 4.17\ kJ/kg\,K$

$P_r = 3.12$

$Re_D = \frac{uD}{\nu} = 30301$ (turbulent)

dittus Boelter equation

$Nu_D = 0.023\ Re_D^{4/5} \cdot P_r^{n}$ | $n = 0.3\ (T_s < T_m)$
$n = 0.4\ (T_s > T_m)$

$Nu_D = 0.023\,(30301)^{4/5}\,(3.12)^{0.4}$

$\frac{\bar{h}D}{K} = 139.49$

$\boxed{\bar{h} = 6100}$

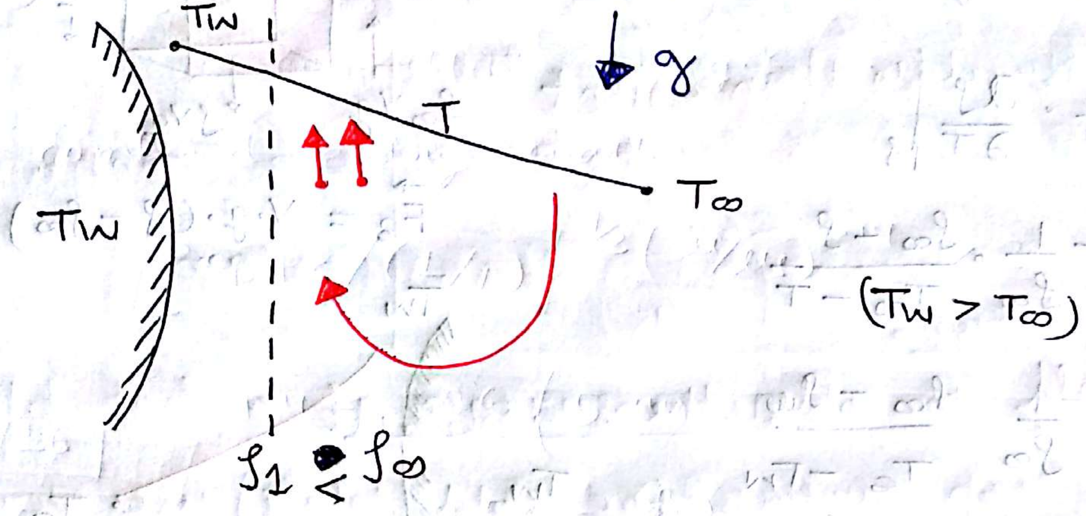
$h \cdot A(\Delta T_{Lm}) = \dot{m}C_p(T_{fo} - T_{fi})$

$61\cancel{00 \times \pi (0.015)^2/4}\,(\Delta T_{lm}) = 984.4 \times \pi \frac{(0.015)^2}{4} \times 4170 \times (65 - 50$

$= 10875.54$

Natural convection:

Buoyancy induced convection // gravity assisted

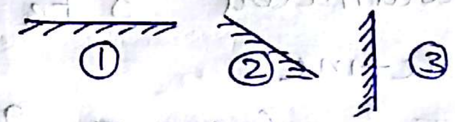


characteristics of N.C. ⇒

① There is no bulk flow and no power consumption

② $\bar{h}$ is very low

③ orientation dependent



④ difficult to control

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Natural convention:

dimensionless numbers:

- Reynolds number (Re) does not play a role

(i) Nusselt Number: $Nu = \frac{hL}{K}$

(ii) Prandtl Number: $Pr = \frac{\mu c_p}{K} = \frac{\nu}{\alpha}$

(iii) grashoff number:

$$Gr = \frac{g \beta \Delta T x^3}{\nu^2}$$

β = thermal expansion coefficient

$$= -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

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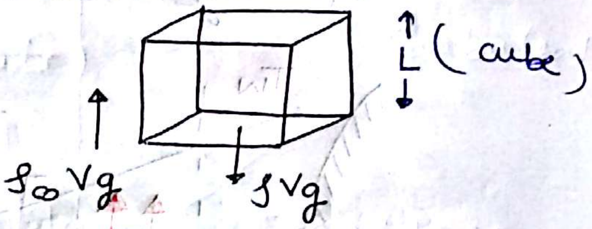
$$F_{Buoyancy} = F_B = -\text{volume} \times g \times (\rho - \rho_\infty)$$

$$F_B \sim g L^3 (\rho_\infty - \rho)$$

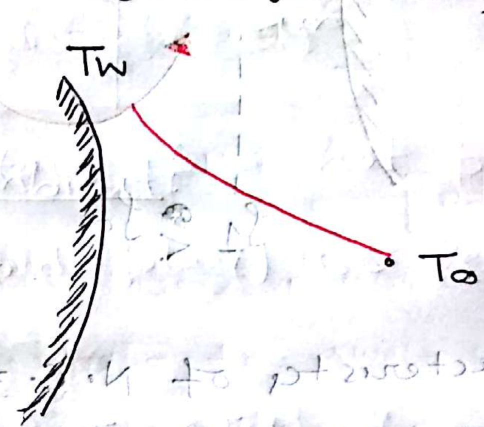


$$\vec{F}_B = V \cdot g \cdot (\rho - \rho_\infty)$$

$$\beta = -\frac{1}{\rho} \cdot \frac{\partial \rho}{\partial T}\Big|_\rho$$

$$\sim -\frac{1}{\rho_\infty} \cdot \frac{\rho_\infty - \rho}{T_\infty - T}$$

$$\sim -\frac{1}{\rho_\infty} \frac{\rho_\infty - \rho_w}{T_\infty - T_w}$$



$$\Rightarrow \rho_\infty - \rho_w \sim \beta \rho_\infty (T_w - T_\infty)$$

$$F_B \sim g \beta \rho_\infty (T_w - T_\infty) L^3$$

If we use Buckingham π theorem, and various parameters of F_B expression (scaling) we can define

$$Gr = \frac{g \beta \Delta T L^3}{\nu^2} \sim \frac{F_{Buoyancy}}{F_{viscous}}$$

(v) Rayleigh #

$$Ra_L = Gr_L Pr = \frac{g \beta \Delta T L^3}{\alpha \nu}$$

Typically

$$Nu_L \sim f(Ra_L, Pr)$$

Mixed convection:

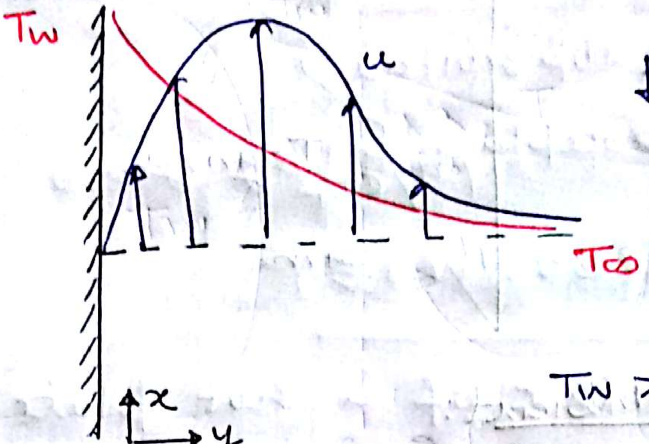
- convection where effects of NC & FC are comparable
- true if $\frac{Gr_L}{Re_L^2} \sim O(1)$

N.C $Gr_L > Re_L^2$

FC $\Rightarrow$ $Gr_L < Re_L^2$

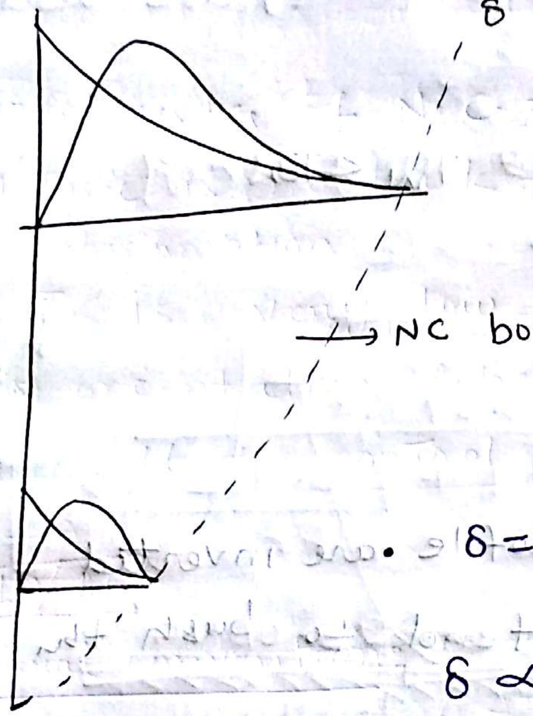
$$Nu^n \sim N_{FC}^n \pm N_{NC}^n$$

$$n = 3$$



$\downarrow g$ (imp factor in N.C diagram)

$T_W > T_\infty$



$$\delta = 5x\left(\frac{Gr_x}{4}\right)^{-1/4}$$

$$\delta \propto x^{1/4}$$

analytical solution gives

$$Nu_x = \left(\frac{Gr_x}{4}\right)^{1/4} \cdot g(Pr)$$

where $g(Pr) = \dfrac{0.75\, Pr^{1/2}}{(0.609 + 1.221 Pr^{1/2} + 1.238\, Pr)^{1/4}}$

$$\bar{h} = \frac{1}{L}\int_0^L h_x\, dx \Rightarrow \overline{Nu}_L = \frac{4}{3} \cdot Nu_L$$

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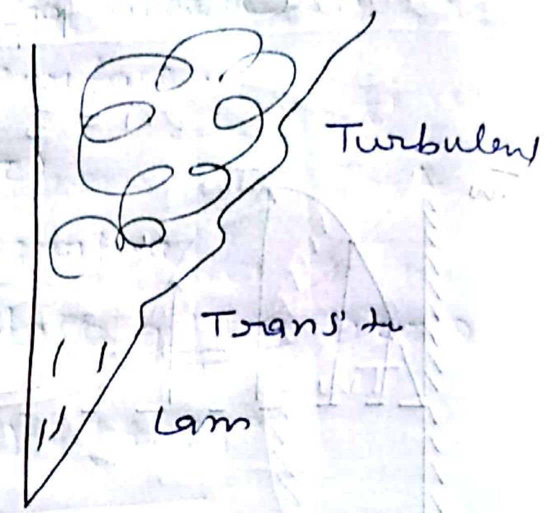
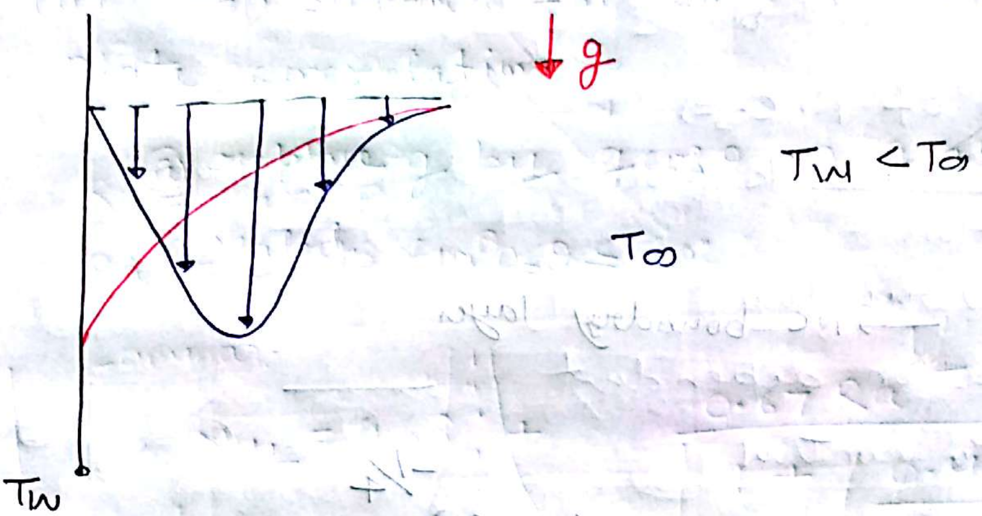
- Transition to turbulence:

$Ra_{x,e} \simeq 10^9$

If wall is colder than ambient



If wall is colder then ambient:



- velocity and temp profile are inverted
  - buoyant force tend to push the heavier fluid down
- B.L develops at the 'top' end (tip) grows downwards.

Correlation:

@ churchill and chu (1975)

@ Laminar

$Ra_L < 10^9$ $\quad \overline{Nu_L} = 0.68 + \dfrac{0.67\, Ra^{1/4}}{\left[1 + \left(\dfrac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$

(b) Turbulent

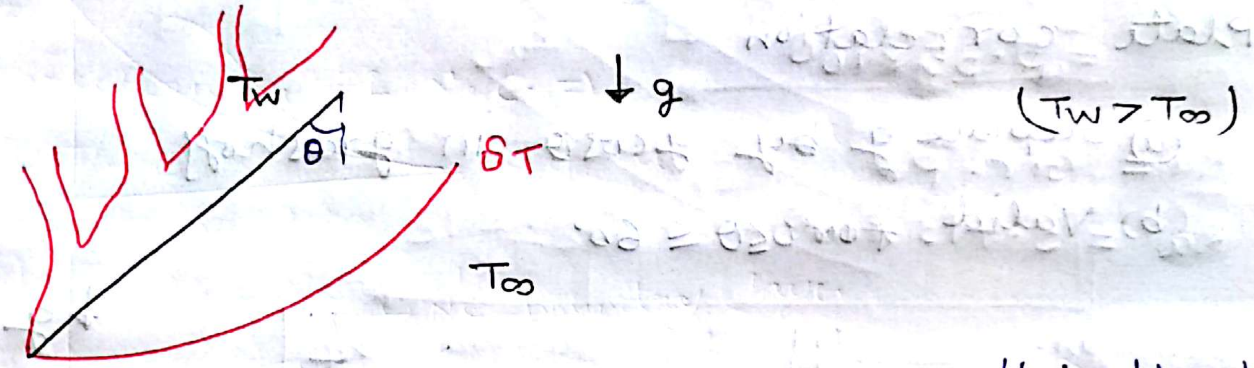
$10^9 < Ra_L < 10^{12}$

$$\overline{Nu_L} = \left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}\right]^2$$

These are average Nu # corellation

25/10/2016

Inclined & Horizontal Plates:

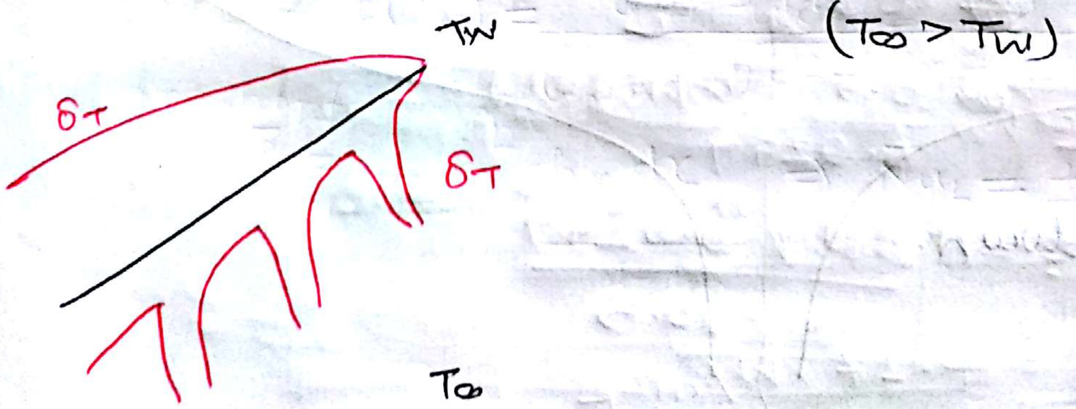


Buoyancy force has a component // to the plate
-remaining component is ⊥ to the plate

→ For bottom surface, flow is hindered → Reduction in h

⇒ on top surface, packets of hot fluid break away from the thermal b.l. and escape vertically upwards

⊙ This interrupts the thermal b.l. and enhances 'h'.



converse is true for $T_w < T_\infty$ where 'h' is enhanced on the bottom surface.

In hot surface (hot plate)

→ No correlation for top surface

→ For lower part, replace g by $g\cos\theta$

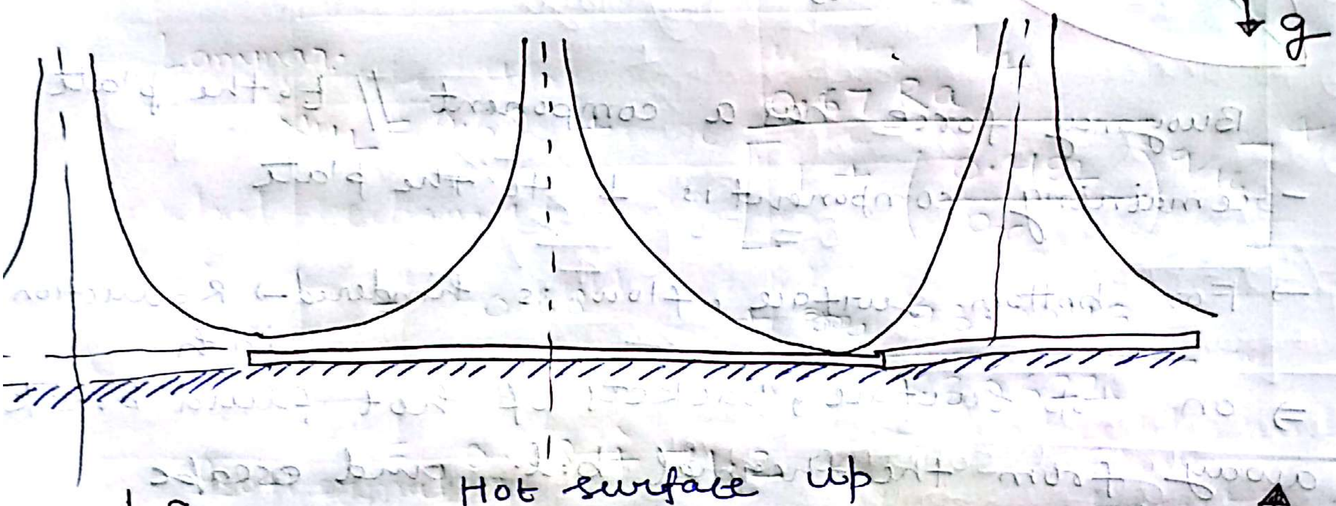
$\theta < 60$

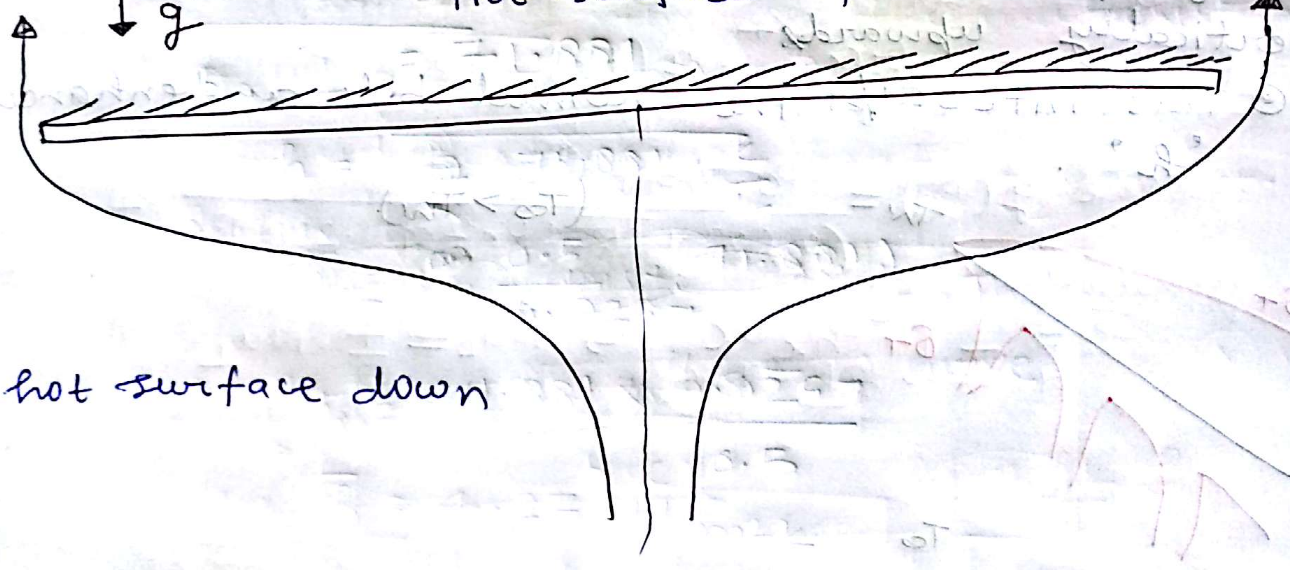
Correlation:

1> For bottom surface of a hot plate (or top surface for cold plate) use vertical plate correlation

(a) replace g by $g\cos\theta$ in grashoff

(b) Valid for $0 \le \theta \le 60°$





hot surface down

$$\left\{\begin{array}{ll} \overline{Nu}_L = 0.54\, Ra_L^{1/4} & (10^4 < Ra_L < 10^7) \\ \overline{Nu}_L = 0.15\, Ra_L^{1/3} & (10^7 < Ra_L < 10^{11}) \end{array}\right\}$$

Hot surface up $\overline{Nu}_L^{*} = 0.54\, Ra_L^{*\,1/4}$

Hot surface down $\overline{Nu}_L^{*} = 0.27\, Ra_L^{*\,1/4}$

↓

finite plate to escape the air

Ques consider a flat plate of dim. (0.5m × 1m) held vertically and maintained at 100°C
The plate is exposed to 40°C and 1atm.
calculate average heat transfer coeff and total heat transfer.

** $Gr_x = \frac{g\beta\Delta T x^3}{\nu^2}$

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p \qquad P = \rho RT$$

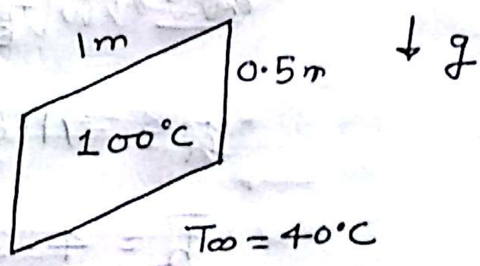
$$\rho = \frac{P}{RT}$$

$$\beta = -\frac{RT}{P}\left(-\frac{P}{RT^2}\right)$$

$$\boxed{\beta = \frac{1}{T}}$$

---

① find fluid properties
② calculate Gr (or Ra or Re)
③ determine appropriate correlation



$T_{mean} = \frac{100+40}{2} = 70°C$

$= 343K$

air at 70°C & 1 atm

$\nu = 2.002 \times 10^{-5}\ m^2/sec$

$Pr = 0.694$

$K = 0.0297\ W/m\text{-}K$

$$Gr = \frac{\beta \Delta T \cdot g\, x^3}{\nu^2} \qquad \beta = \frac{1}{T}$$

$$= \frac{9.81 \times \left(\frac{1}{343}\right) \times (60) \cdot (0.5)^3}{(2.002 \times 10^{-5})^2}$$

$$= 2.140 \times 10^9 \times (0.5)^2$$

$$= 5.35 \times 10^8$$

$Ra = Gr \cdot Pr$

$Ra = 3.713 \times 10^8$

Laminar:

$$Nu_L = 0.68 + \frac{0.67\, Ra^{1/4}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}}$$

$$= 0.68 + \frac{93.0049}{1.30622}$$

$$= 0.68 + 71.2$$

$$= 71.881$$

$$h = (71.881) \cdot \frac{k}{L}$$

$$\bar{h} = 71.881 \times \frac{0.0297}{0.5}$$

$$\bar{h} = 4.27\ W/m^2K$$

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$q = hA(\Delta T)$

$= 4.27 \times 0.5 \times 1 (60)$

$= 128.092\ W$

$q_{rad} = \sigma A (373^4 - 313^4)$ $\epsilon = 1$ blackbody

$= 5.67 \times 10^{-8} \times 0.5 \times 1 (\quad)$

$= 276.67\ W$

$q_{total} = q_{conv} + q_{rad}$

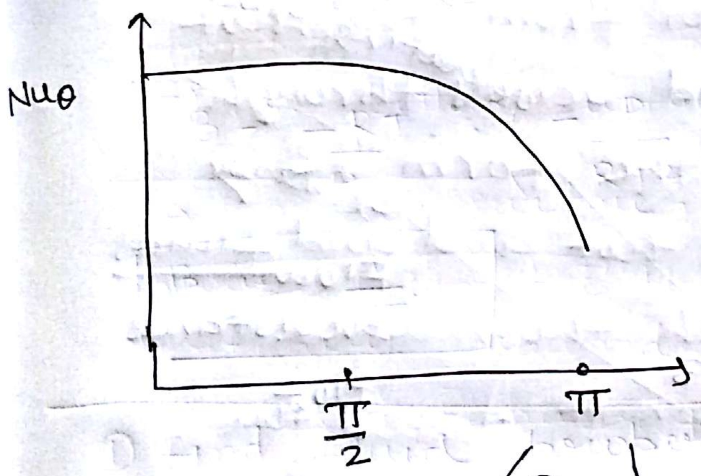
$= 404.76\ W$

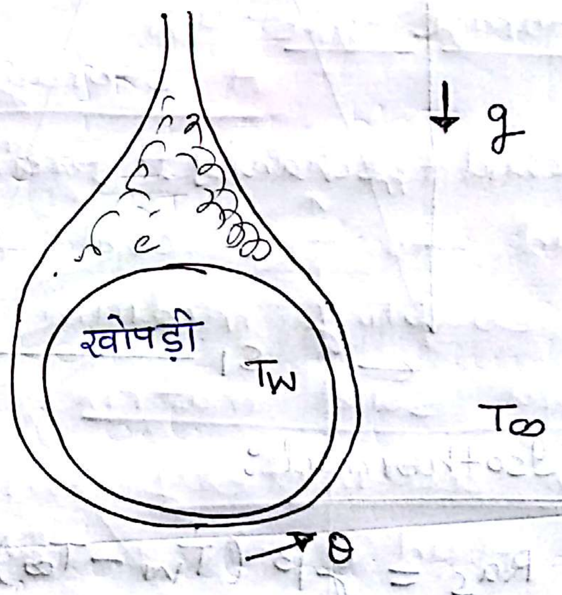
a grey body $\epsilon = 0.5$ then

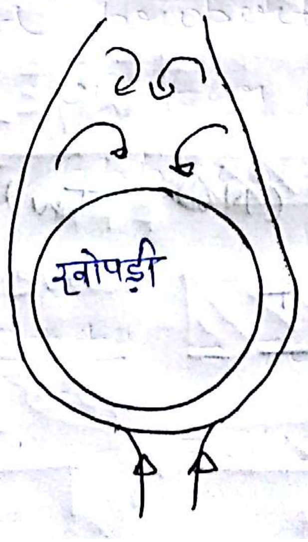
$q_{rad} = 138.34\ W$

radiation and convection in NC is comparable

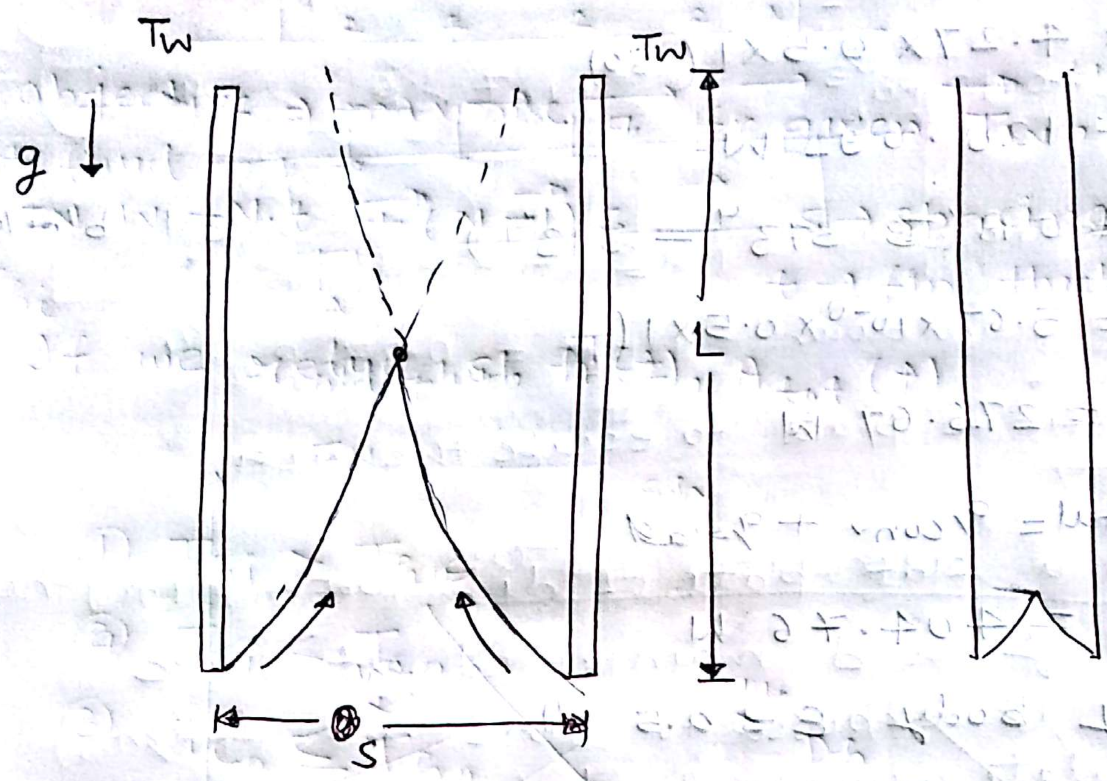
Plume (आभामंडल)

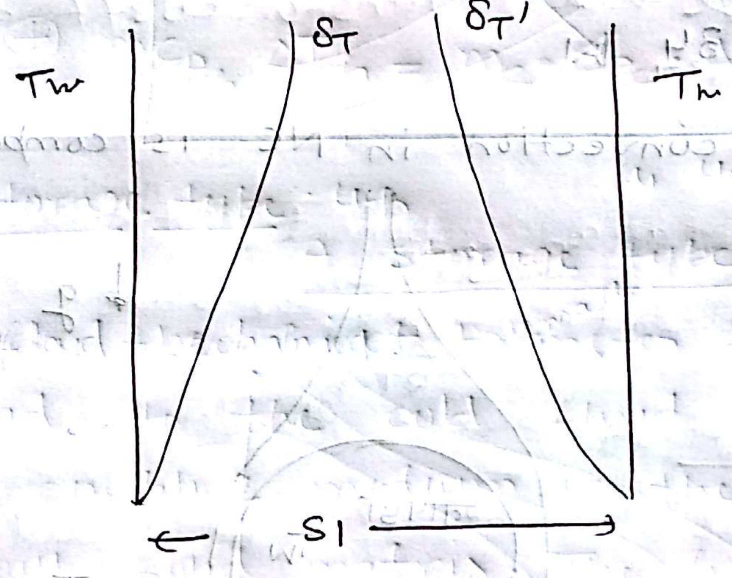






Internal Flow through vertical channels:





Isothermal:

$$Ra_S = \frac{g\beta (T_W - T_\infty) s^3}{\alpha \nu}$$

Isoflux:

$q_w'' =$ const (Not Tw)

$$Ra_S^* = \frac{g \beta q_w'' s^4}{K \alpha \nu}$$

$$Nu_s = \frac{hs}{K} =$$

Isothermal

$$\overline{Nu_s} = \left[\frac{c_1}{\left(Ra_s \frac{S}{L}\right)^2} + \frac{c_2}{\left(Ra_s \cdot \frac{S}{L}\right)^{1/2}}\right]^{-1/2}$$

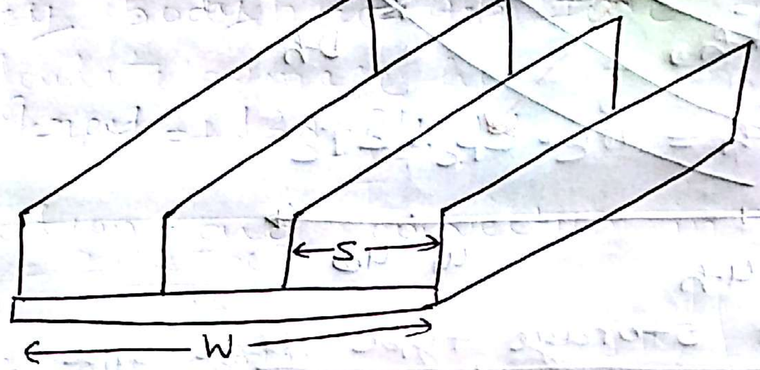
Isoflux:

$$\overline{Nu_{s,L}} = \left[\frac{c_1}{\left(Ra_s^* \frac{S}{L}\right)} + \frac{c_2}{\left(Ra_s^* \frac{S}{L}\right)^{2/5}}\right]^{-\frac{1}{2}}$$

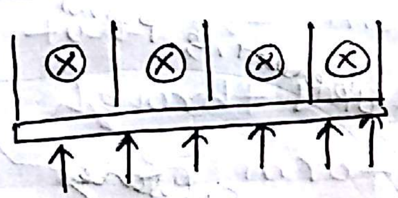
\# $S \uparrow \Rightarrow \overline{Nu_s} (\downarrow\downarrow)$

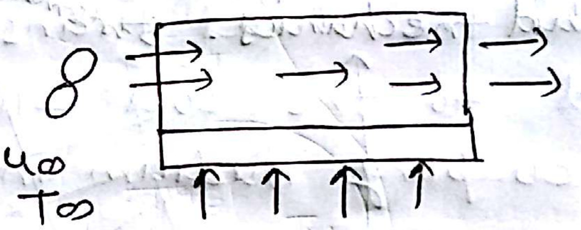
26/10/16

Flow and HT through parrallel channels:



W = constant
s = Variable





Heat Transfer

$$q = \bar{h} A_{tot} (T_w - T_\infty)$$

$$\bar{h} \Rightarrow \overline{Nu}_{D_h} = 0.023\, Re_{D_h}^{4/5} \cdot Pr^{0.4}$$

D_h = hydraulic diameter

$$h \propto D_h^{-1/5}$$

$$Re_{D_h} = \frac{u_\infty \cdot D_h}{\nu}$$

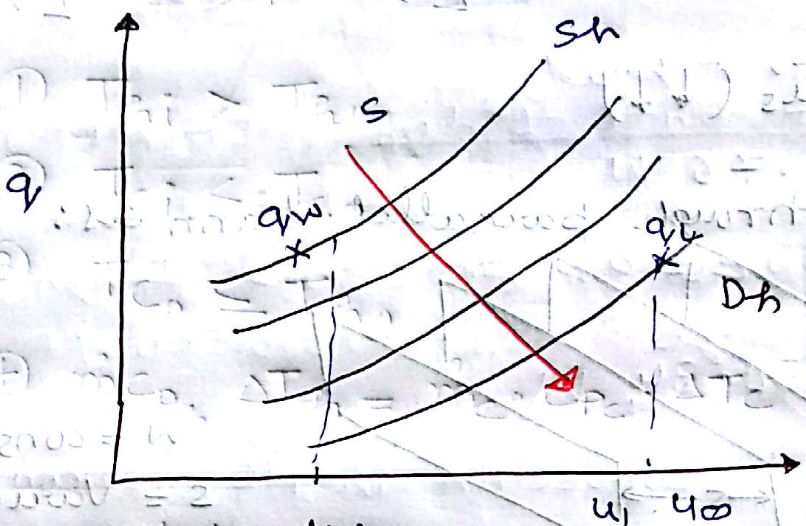
$A_{tot} = A_b + \eta_f \cdot N \cdot A_{fin}$ (assuming thin fins)

objective ⇒ maximise q for given T_w, T_∞

$W = N \cdot t + (N-1) s$ $s \rightarrow$ spacing

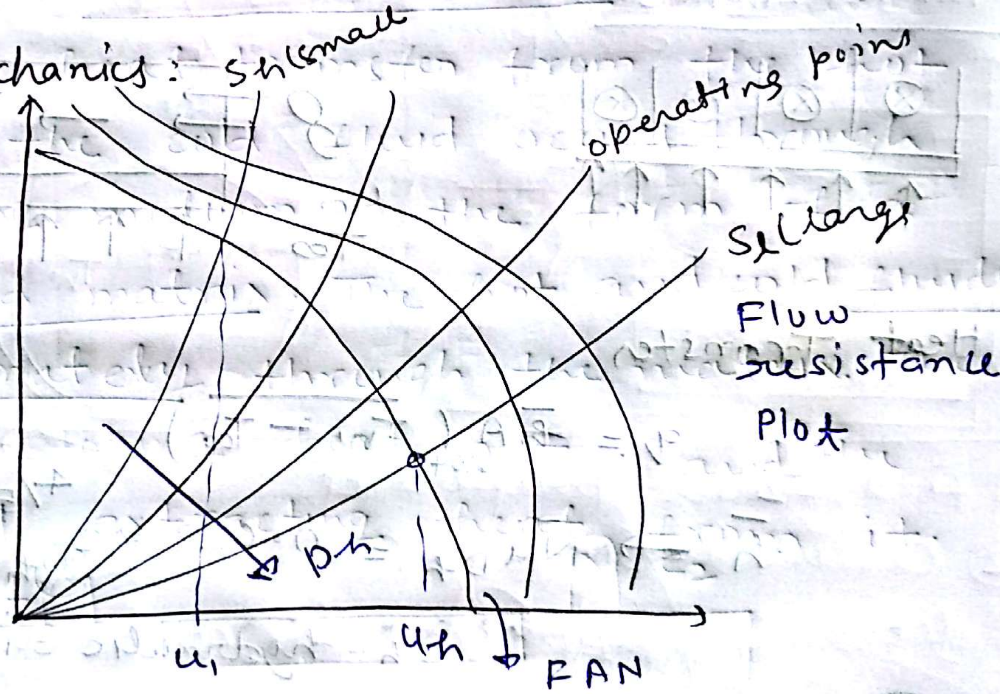
$t \rightarrow$ fin thickness

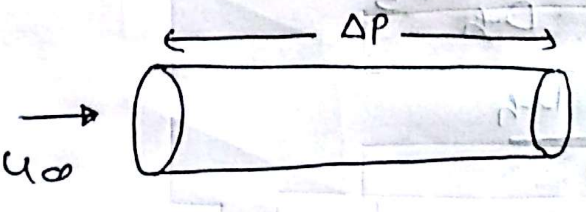
If we reduce s, $h (\uparrow)$, $A_{tot} (\uparrow)$



u_h

fluid mechanics:





f

$$\Delta P = \frac{fL}{D} \cdot \frac{\rho u_\infty^2}{2}$$

Function of a fan = increase pressure head

operating point ⇒ higher flow rate → large spacing

31/10/16

Heat exchanger :

Device where exchange of heat b/w two liquids at diff temp takes place, there is no mixing of fluid.

Heat exchanger (HX) is a device where heat is transferred between two fluids at different temperature without mixing of any fluid.

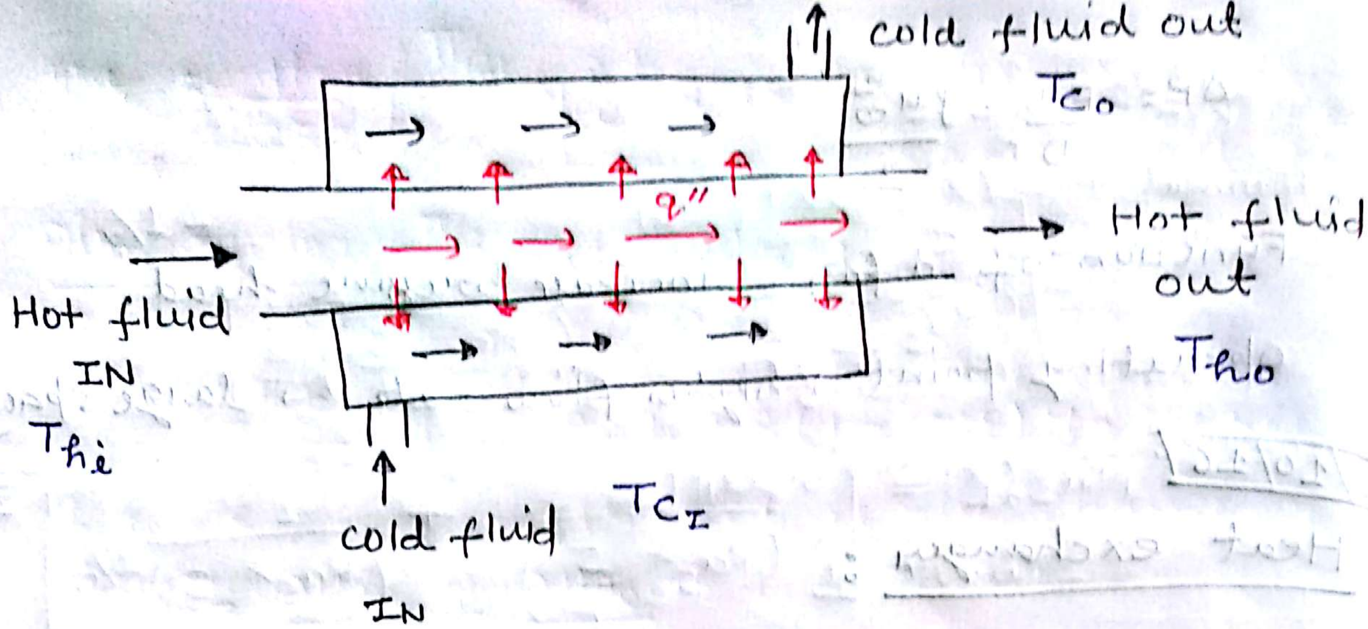
Classifications

(a) Direct transfer type
(b) storage type
(c) Direct contact type

$\left.\begin{array}{l}(a)\\(b)\\(c)\end{array}\right\}$ 3 most broad classification

Direct transfer type :

a HX where hot and cold fluid flows simultaneously through the device and heat is transferred through a wall separating the two fluid.



① $T_{hi} > T_{ho}$

② $T_{ci} < T_{co}$

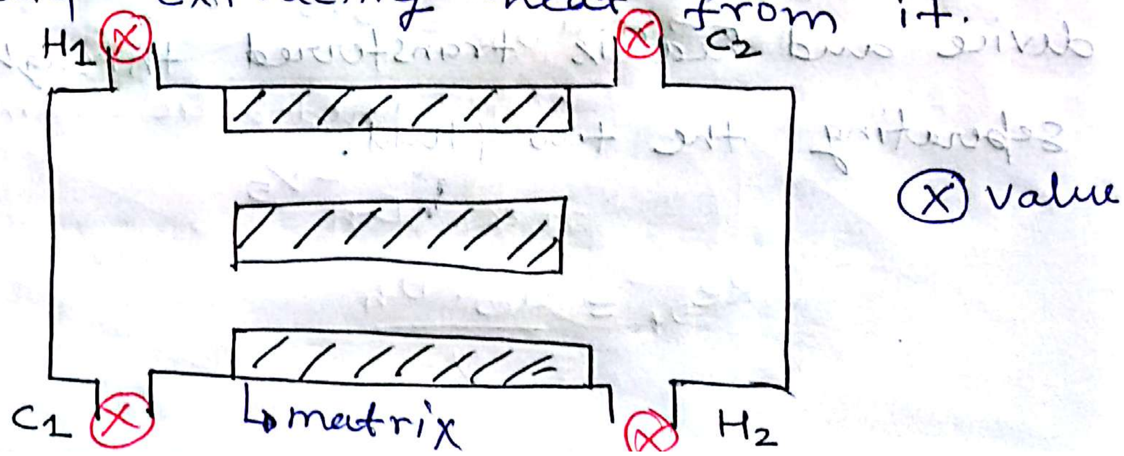
③ $T_{co} \leq T_{ho}$

④ $\dot{m}_h c_{p_h} \Delta T_h = \dot{m}_c \cdot c_{p_c} \cdot \Delta T_c$

---

storage type :

A storage type HX is one in which the heat transfer from the hot fluid to the cold fluid occurs through a coupling medium in the form of a porous solid matrix. The hot and cold fluids flows alternately through the matrix. The hot fluid storing heat in it and the cold fluid extracting heat from it.

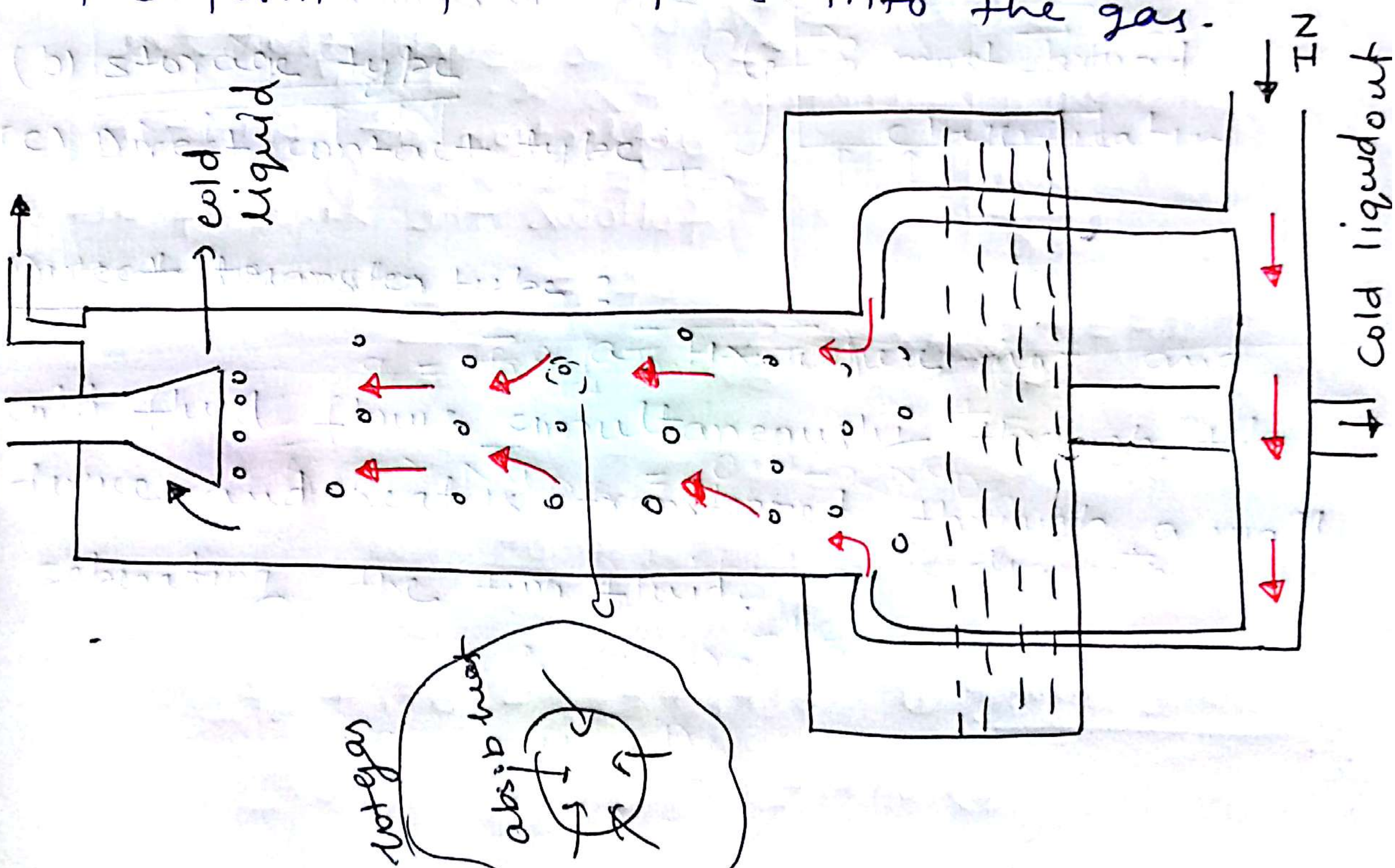


For $0 < \tau < \tau_1$ - hot fluid flows the matrix
matrix absorbs thermal energy (heat) rejected by fluid and itself gets heated (~~[illegible]~~) energy stores

- At τ_1, close H_1 & H_2 & open C_1 & C_2
- For $\tau_1 < \tau < \tau_2$, cold fluid flows through the matrix
  → matrix rejects heat ~~[illegible]~~ which is absorbed by the cold fluid.

Direct contact type heat exchangers :

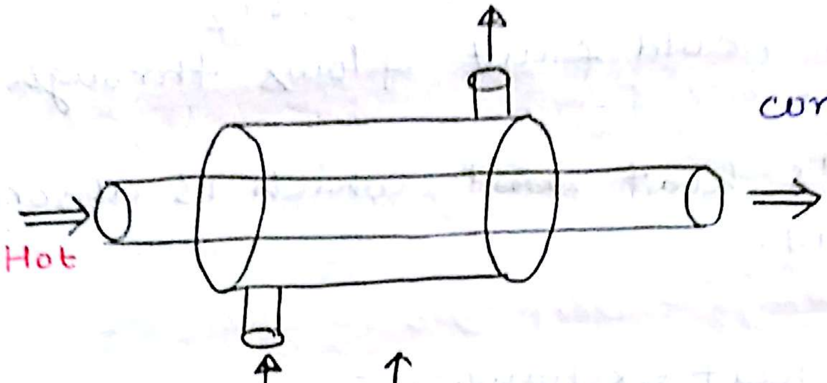
Direct contact type heat exchangers is one where the two fluids are not separated. If heat is to be transferred b/w a gas and a liquid. The gas is either bubbled through the liquid or the liquid is sprayed is the form of droplets into the gas.



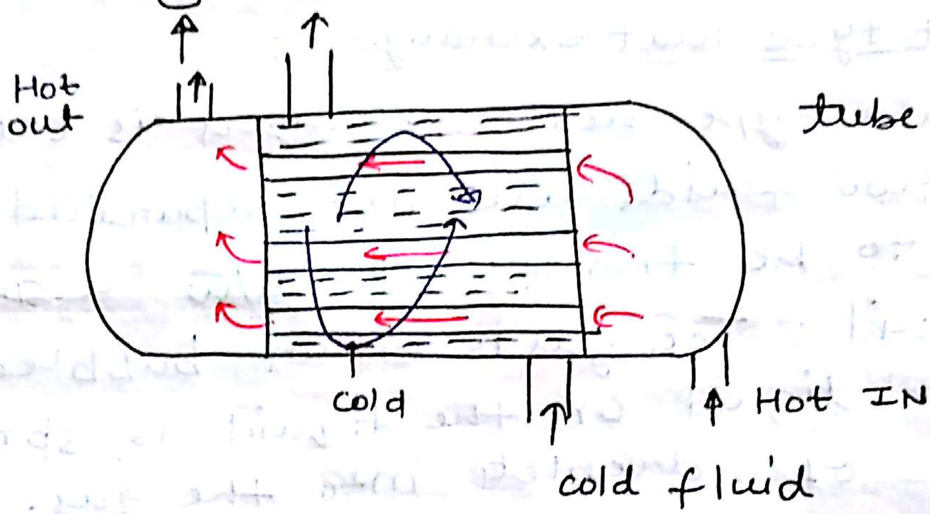
02/11/16

Direct transfer type:

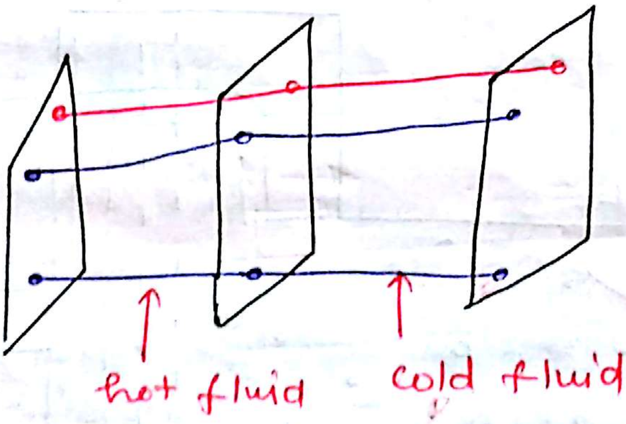
① Design type
- (a) Tubular → tube & shell, concentric tubes
- (b) Parallel Plate
- (c) Extended surface (mixed and unmixed)



concentric tube



tube and shell



② Flow arrangement
- (a) Parallel Flow
- (b) counter flow
- (c) Cross flow

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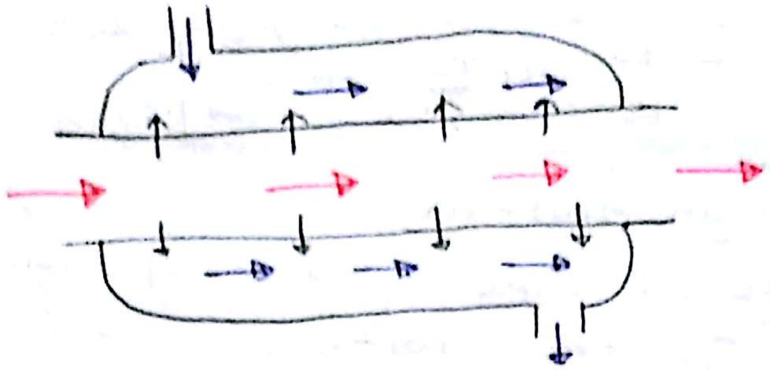
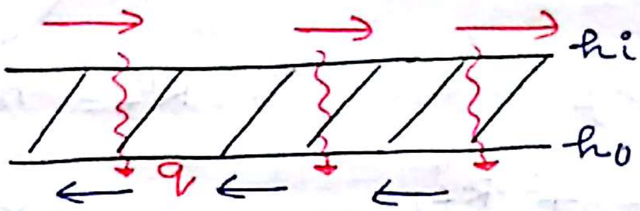


$\longrightarrow \;\; \longrightarrow$ hot $\Rightarrow$ counter flow
$\longleftarrow \;\; \longleftarrow$ cold

overall heat transfer coefficient (U) :

(a) across a plane surface



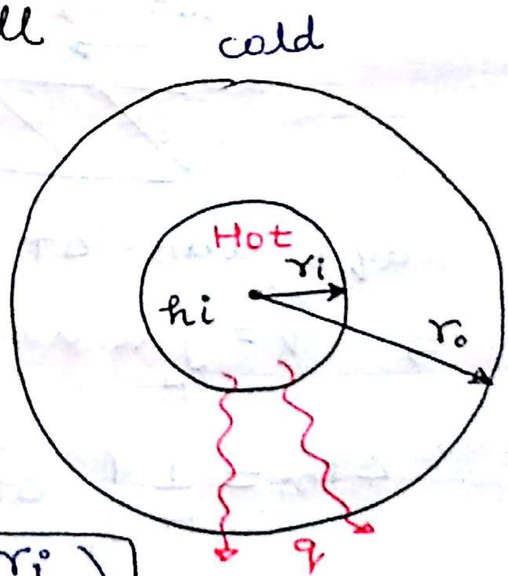
$$\frac{1}{UA} = \frac{1}{h_i A} + \frac{t}{KA} + \frac{1}{h_o A}$$

$$q = UA(T_h - T_c)$$

$U \rightarrow$ overall h.t.c

$$\boxed{\frac{1}{U} = \frac{1}{h_i} + \frac{t}{K} + \frac{1}{h_o}}$$

(b) across a tubular wall



$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi K L} + \frac{1}{h_o A_o}$$

$$A_i = 2\pi r_i L$$

$$A_o = 2\pi r_o L$$

$$\boxed{\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{K}\ln\frac{r_o}{r_i} + \frac{1}{h_o}\left(\frac{r_i}{r_o}\right)}$$

Like wise

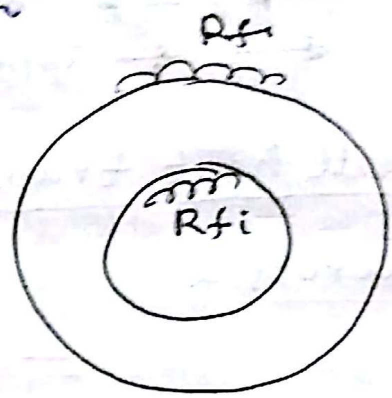
$$\boxed{\frac{1}{U_o} = \frac{r_o}{r_i}\cdot\frac{1}{h_i} + \frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o}}$$

Valid for clean surface.

with passage of time deposite of solid matter happens on the surface

- Introduces additional thermal resistance
- captured through

Fouling Factor (R_f'')



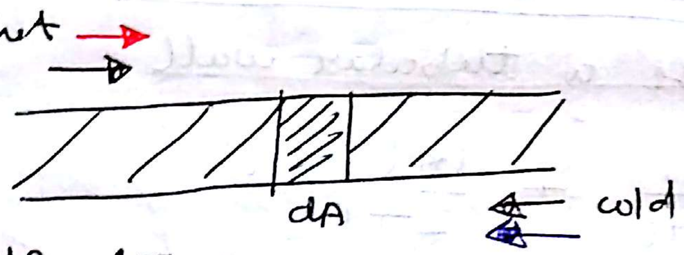
with fouling $\qquad R''_f = m^2\text{-}K/W$

$$\frac{1}{U_iA_i} = \frac{1}{h_iA_i} + \frac{R''_{fi}}{A_i} + \frac{1}{2\pi KL}\ln\left(r_o/r_i\right) + \frac{R''_{fo}}{A_o} + \frac{1}{h_oA_o}$$

$$\boxed{q = UA\,(\Delta T)_m}$$

??

mean temp diff $(\Delta T)_m$:



$$dq = U\,dA\cdot\Delta T$$

$$\Rightarrow \quad q = \int_A U\cdot dA\cdot\Delta T = U\int_A \Delta T\cdot dA$$

$$\Delta T_m = \frac{1}{A}\int_A \Delta T\, dA$$

$$\Rightarrow \quad q = UA\,\Delta T_m$$

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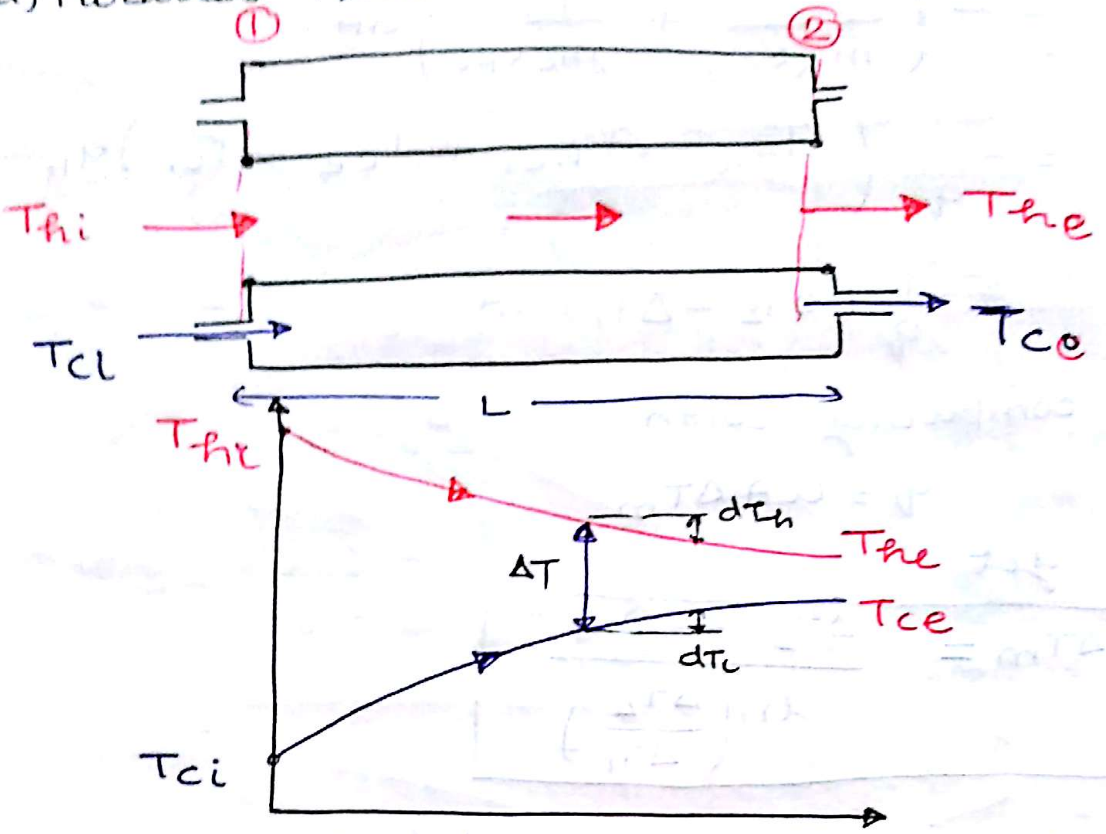
Mean temp difference (ΔT_m)

(a) Parallel Flow



w = width of HX

L = Length of HX

$\Delta T = T_h - T_c$

convection: $dq = u\, dA\, \Delta T$

$= u\, w\, dx\, \Delta T$

energy: $dq = -\dot{m}_h c_{p_h} \cdot dT_h$

$= \dot{m}_c c_{p_c} \cdot dT_c$

$d(\Delta T) = dT_h - dT_c$

$= -dq \left[\frac{1}{m_h c_{p_h}} + \frac{1}{m_c \cdot c_{p_c}} \right]$

or $d(\Delta T) = -u\, w\, \Delta T\, dx \left(\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c \cdot c_{p_c}} \right)$

$$\Rightarrow \int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -(\;---\;)\, u\, w \int_0^L dx$$

$$\ln\frac{\Delta T_2}{\Delta T_1} = -\left(\quad \right) U W L$$

$$= -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}} \right) UA$$

$$= -\frac{1}{q}\left(T_{hi} - T_{he} + T_{ce} - T_{ci} \right) UA$$

$$= \frac{1}{q}\left(\Delta T_2 - \Delta T_1 \right) UA$$

comparing with

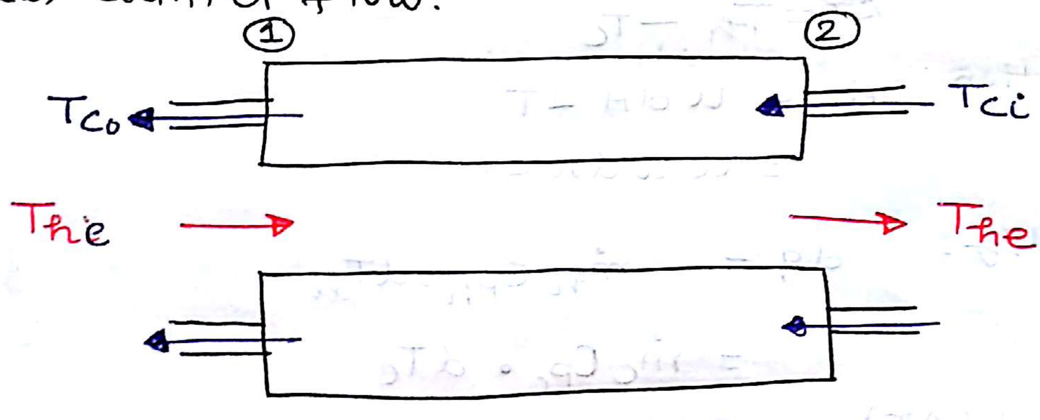
$$q = UA\,\Delta T_m$$

we get

$$\boxed{\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}}$$

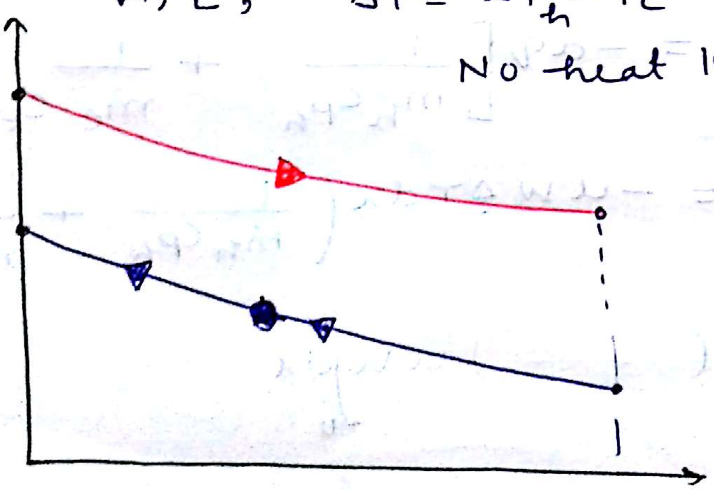
LMTD = Log mean temp difference

(b) counter flow:



W, L, $\Delta T = \Delta T_h - T_c$

No heat loss to surroundings



derive at the home

$$dq = -\dot{m}_h c_{p_h} \cdot dT_h$$
$$= -\dot{m}_c c_{p_c} \cdot dT_c$$

you will end up with

$$\boxed{\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}$$

- for same set of temp diff. in counter and parallel flow LMTD will be different

special case :

$$\dot{m}_h c_{p_h} = \dot{m}_c c_{p_c}$$

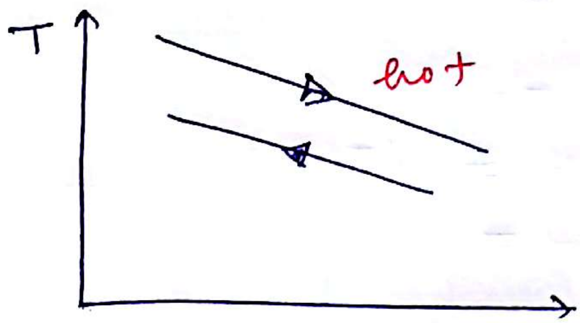
$$\Rightarrow T_{hi} - T_{he} = T_{ce} - T_{ci}$$

$$\Rightarrow T_{hi} - T_{ce} = T_{he} - T_{ci}$$

$$\Rightarrow \Delta T_1 = \Delta T_2$$

$$\Rightarrow LMTD = \lim_{x \to 1} \frac{\Delta T_2 (x-1)}{\ln x} \quad \Big| \quad \frac{\Delta T_1}{\Delta T_2} = x$$

$$= \Delta T_2 \rightarrow \text{L hospital law}$$

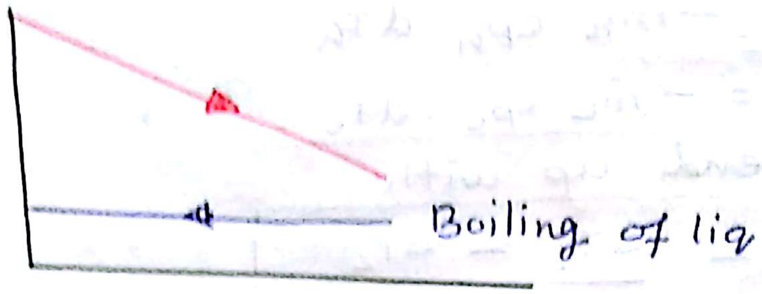


Specific case

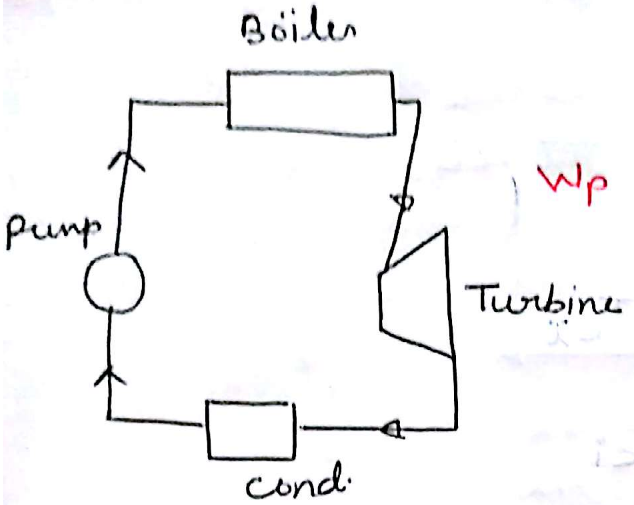
① $T_h =$ constant $\Rightarrow \dot{m}_h c_{p_h} \rightarrow \infty$

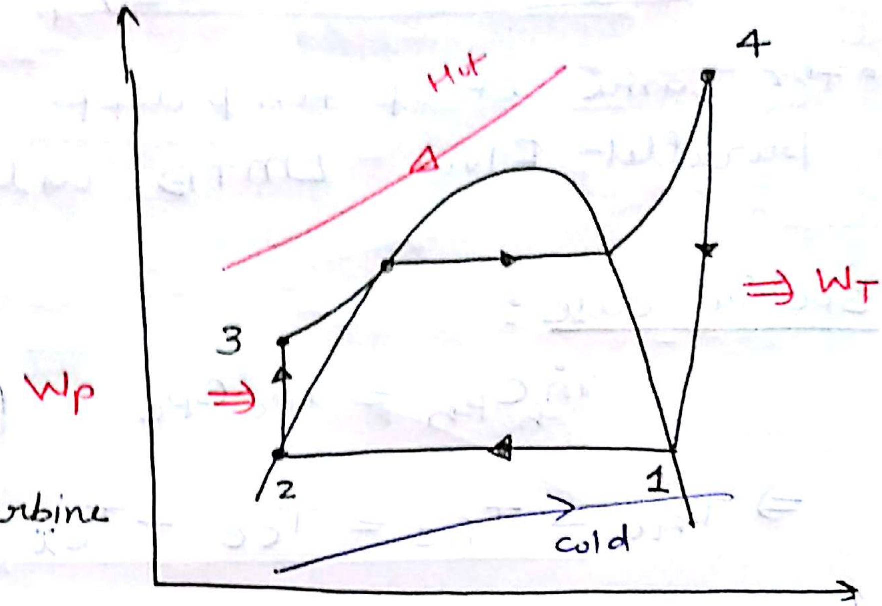
condensation of gas stream

(II) $T_c = \text{const} \Rightarrow \dot{m} c_{p_c} \Rightarrow \infty$



Rankine cycle:





only temp needs to be read from the graph (Not entropy)