Tutorial 9
ME-30602, 2017-18 Spring Semester

April 02, 2018

<u>Problem:</u> Desired parameters for a spur gear set to be designed are given in the table below.

Pressure Angle, ϕ	20^{o}
Gear Ratio, m_G	4
Center to center distance, c	200mm
Pinion Material	2.5% Chrome Steel Grade 2 with Hardness HB 250
Gear Material	2.5% Chrome Steel Grade 2 with Hardness HB 200
Quality Number Q_v	10 (precession gearing)
Mounting	Accurate mounting, at the midspan between two bearings, with low bearing clearance, enclosed.
Power source	Light Shock (see fig 14-17, page 766)
Driven Machine	Heavy Shock (see fig 14-17, page 766)
Desired Reliability on life	0.99
Desired Pinion Life, N	10^8 cycles
Power Rating, H	25kW
Pinion Speed, n	2000rpm
Face width, b	50mm
Ratio of rim thickness to tooth depth	1.5

(a) For the above gear set first decide on the number of pinion teeth and module.

(b) Determine the factors of safety against bending (S_F) and pitting failure (S_H) of the pinion tooth and gear tooth (you should determine four such factors).

(c) Compare S_F with $(S_H)^2$ and determine which failure mode is likely to occur and which component (gear or pinion) is likely to fail first.

(d) Calculate the power rating at which the set will fail.

<u>Notes:</u>

1. Assume the strength against pitting resistance S_c=1350 MPa (for both gear and pinion).
2. For surface factor use $K_s = 1$.
3. Obtain the mesh alignment factor C_{ma} for the calculation of load distribution factor K_H from Figure 14-10 (be careful about the units used for face width in the figure).
4. Obtain elasticity coefficient Z_E from table 14-8.
5. Obtain bending-strength geometry factor Y_J from figure 14-6 and pitting-resistance geometry factor Z_I using Eq. (14-23).

Given: Gear ratio $m_G = \frac{N_G}{N_P} = 4$

Pressure angle $\phi = 20°$

(a) To avoid interference:

$$N_P \geq \frac{2k}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (2m_G+1)\sin^2\phi}\right)$$

$$= 15.46$$

$$\Rightarrow \boxed{N_P = 16, \quad N_G = 4 \times 16 = 64} \text{ (Less than 101)}$$

Also check: Ng_max = 101 (for Np=16, pressure angle =20 degrees).

Module:

Center to center distance:

$$C = \frac{(N_P + N_G)}{2} m = 200 \text{ mm}$$

$$\Rightarrow \boxed{m = 5 \text{ mm}}$$

Diameters:

$$\boxed{\begin{aligned} d_P &= N_P \cdot m = 80 \text{ mm} \\ d_G &= N_G \cdot m = 320 \text{ mm} \end{aligned}}$$

(b) Power rating $H = 25$ kW, pinion speed $n = 2000$ rpm

$\Rightarrow$ Transmitted load: $W_t = \frac{60000 H}{\pi d n}$

$$= \frac{60{,}000 \times 25}{\pi \times 80 \times 2000} = 2.984 \text{ kN}$$

$$\Rightarrow \boxed{W_t = 2.984 \text{ kN}}$$, Pitch line vel. $V = 8.378$ m/s

Factors for bending stress and strength calculation

Over load factor: $K_o = 2.0$ (L.S. Power, H.H. driven) (Fig 14-17)

Size factor: $K_s = 1$

Dynamic factor: (Eq 14-27 & 28)

$$K_v = \left(\frac{A + 200\sqrt{V}}{A}\right)^B, \quad A = 50 + 56(1-B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$ ← quality no.

$$\Rightarrow \boxed{\begin{array}{l} Q_v = 10 \\ B = 0.3969 \\ A = 83.776 \end{array}}$$

$$\Rightarrow \boxed{K_v = 1.171}$$

Load distribution factor: $b = 50\,mm$

Accurate mounting: $K_H = 1.3$ (table)

Rim thickness factor: Rim thickness to tooth height ratio $\doteq 1.5 > 1.2$

$\Rightarrow K_B = 1$ (Fig 14-16)

Geometry factor:

$(Y_J)_P = 0.27$ (pinion – 16 teeth)

$(Y_J)_G = 0.41$ (gear – 64 teeth)

(Fig 14-6)

Stress cycle factor:

~~$Y_N = 0.9$~~

$H_B = 250$, Desired pinion life $= 10^8$ cycles $= N_p$

~~$(Y_N)_G$~~ $H_B = 200$, $N_G = \frac{10^8}{4} = 2.5 \times 10^7$ cycles

$\Rightarrow Y_N = 1.3558 \times N^{-0.0178}$

$$\Rightarrow \boxed{\begin{array}{ll} (Y_N)_P = 0.977 & (\text{pinion}, N = 10^8) \\ (Y_N)_G = 1.009 & (\text{gear}, N = 2.5\times10^7 \text{ cycles}) \end{array}}$$ (Fig. 14-14)

Temperature factor: $\boxed{Y_\theta = 1}$

Reliability factor: $Y_Z = 1$ (Reliability $= 0.99$, Table 14-10)

Bending strength:

$$S_t = 0.7255\, H_B + 153.63$$

Nitrided, 2.5% Chrome Steel, Grade 2, HB 250 – (P), 200 – (G)

$$(S_t)_P = 335\ \text{MPa} \quad (\text{pinion } HB = 250)$$
$$(S_t)_G = 298.73\ \text{MPa} \quad (\text{Gear } HB = 200)$$

(Fig – 14 – 4)

Bending failure:

(1) Pinion:

$$\sigma = \frac{W_t}{bm} K_o K_v K_s \frac{K_H K_B}{Y_J}$$

$$= \frac{2984}{50 \times 5} \times 2 \times 1.171 \times 1 \times \frac{1.3 \times 1}{0.27}$$

$$(\sigma)_P = 134.6\ \text{MPa}$$

Allowable stress: $\sigma_{all} = \dfrac{S_t \cdot Y_N}{S_F\, Y_\theta\, Y_Z}$

$$(\sigma_{all})_P = \frac{335 \times 0.977}{S_F} = \frac{327.3}{S_F}$$

Factor of safety: $\boxed{(S_F)_P = \dfrac{327.3}{134.6} = 2.43}$

2. Gear:

$$(\sigma)_G = \frac{W_t}{bm} \times 2 \times 1.171 \times \frac{1.3 \times 1}{0.41} = 88.6\ \text{MPa}$$

$$(\sigma_{all})_G = \frac{298.73 \times 1.009}{S_F} = \frac{299}{S_F}$$

$\Rightarrow$ Factor of Safety: $\boxed{(S_F)_G = 3.373}$

Factors for pitting resistance and strength:

Elasticity coefficient : $Z_E = 191 \sqrt{MPa}$ (table 14-8)

Surface condition factor: $Z_R = 1$

Geometry factor : $Z_I = \frac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \frac{m_G}{m_G+1}$ (Eq 14-23)

$\phi_t = \phi = 20°$, $m_N = 1$ (spur gear)

$m_G = 4$

$$\Rightarrow \boxed{Z_I = 0.1285}$$

Stress cycle factor: $Z_N = 1.4488\, N^{-0.023}$ (Fig-14-15)

$$\Rightarrow \begin{aligned} (Z_N)_P &= 0.948 \quad \text{(pinion)} \\ (Z_N)_G &= 0.979 \quad \text{(gear)} \end{aligned}$$

Hardness ratio factor :

$$1.7 \geq \frac{H_{BP}}{H_{BG}} = \frac{250}{200} = 1.25 \geq 1.2$$

$$\Rightarrow \begin{aligned} (Z_W)_P &= 1 \quad \text{(pinion)} \\ (Z_W)_G &= 1.009 \quad \text{(gear)} \end{aligned} \qquad \text{(fig-14-12)}$$

Contact strength:

$S_c = 1350$ MPa for both gear & pinion

Pitting failure :

Pinion

$$(\sigma_c)_P = Z_E \cdot \left\{ \frac{W_t}{d_P b} \cdot K_o K_v K_s \cdot \frac{K_H Z_R}{Z_I} \right\}$$

$$= 191 \left\{ \frac{2984}{80 \times 50} \times 2 \times 1.171 \times 1 \times \frac{1.3 \times 1}{0.1285} \right\}^{1/2}$$

$$= 803. \text{ MPa}$$

Allowable contact Stress:

$$(\sigma_{c,all})_P = \frac{S_c}{S_H} \cdot \frac{Z_N Z_W}{Y_\theta Y_Z} = \frac{1350 \times 0.948 \times 1}{S_H}$$

$$= \frac{1279.8}{S_H} \text{ MPa}$$

Factor of safety

$$\Rightarrow (S_H)_p = \frac{1279.8}{803} = 1.594$$

Gear :

$$(\sigma_c)_G = 803\ MPa$$

Allowable contact stress: $(\sigma_{c,all})_G = \frac{S_c \times 0.979 \times 1.009}{S_H}$

$$= \frac{1333.54}{S_H}\ MPa$$

$\Rightarrow$ Factor of safety:

$$(S_H)_G = \frac{1333.54}{803} = 1.66$$

(c) Likely failure mode:

Pinion : $(S_F)_p = 2.43$, $(S_H)_p^2 = (1.594)^2 = 2.54$

Gear : $(S_F)_G = 3.373$, $(S_H)_G^2 = (1.66)^2 = 2.756$

$\Rightarrow$ Most likely threat to failure is from bending of pinion tooth.

(d) Power to failure

$(\sigma_{all})_p = (\sigma)_p$ for pinion choose $(S_F)_p = 1$

$$\Rightarrow \sigma_p = 327.3\ MPa$$

$$\Rightarrow W_t = \frac{(\sigma)_p \times b \times m \times Y_J}{K_o K_v K_s K_H K_B} = \frac{327 \times 80 \times 5 \times 0.27}{2 \times 1.171 \times 1.3 \times 1}$$

$= 7.25\ kN$ at failure

Also : $W_T = \left(\frac{\sigma_{all}}{\sigma}\right)_p \times W_{t\,old} = \frac{327.3}{134.6} \times 2.984\ kN$

$= 7.25\ kN$

$\Rightarrow H = 60 \cdot 7$ KN at failure

$\Rightarrow$ This is 143 % overload.

Solution to Tutorial 9 Problem Discussed in Class
April 02, 2018

In this version, the size factor Ks and load distribution factors are calculated from Formulae

Given: Gear ratio $m_G = \frac{N_G}{N_P} = 4$.

Pressure angle $\phi = 20°$.

(a) ⇒ To avoid interference: No. of teet in pinion :- N_P

$$N_P \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= 15.44$$

($K = 1$ full depth teeth)

⇒ Choose $\boxed{N_P = 16}$ ⇒ $\boxed{N_G = M_G N_P = 64}$

Module:

Center to Center distance:

$$C = \left(\frac{N_P + N_G}{2}\right) \cdot m = 200 \text{ mm}$$

$$\Rightarrow \boxed{m = 5\text{mm}}$$

(b) • Diameters: $\boxed{\begin{aligned} d_P &= 80\text{mm} \\ d_G &= 320\text{mm}\end{aligned}}$

Power rating: $H = 25$ kW

⇒ Transmitted load: $W_t = \frac{60000 H}{\pi d n} = \frac{60000 \times 25}{\pi \times 80 \times 2000}$ kN

$$\Rightarrow \boxed{W_t = 2.984 \text{ kN} = 2984 \text{ N}}$$

Pitch line velocity $\boxed{V = \frac{\pi d n}{60} \times 10^{-3} \text{ m/s} = 8.378 \text{ m/s}.}$

Factors for Bending stress and Strength Calculation

Overload Factor $\boxed{K_o = 2.0}$ (for Light shock power source and Heavy shock driven machine)

Size Factor: $K_s = 1.192 \left(bm\sqrt{Y}\right)^{0.0535}$

facewidth $b = 50$mm, $m = 5$mm,

$\boxed{\begin{aligned} Y_P &= 0.296 \text{ (pinion)} \\ Y_G &= 0.425 \text{ (gear)}\end{aligned}}$

(Table 14-2 and linear interpolation)

$$\Rightarrow \boxed{\begin{aligned} (K_s)_P &= 1.0964 \text{ (pinion)} \\ (K_s)_G &= 1.1071 \text{ (gear)}\end{aligned}}$$

Dynamic Factor

Equation 14-27

$$K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B \quad : \quad A = 50+56(1-B), \quad B = 0.25(12-Q_v)^{2/3}$$

Q_v ← Quality no.

$\Rightarrow$ ~~A~~ $B = 0.3969$

$A = 83.776$

$\Rightarrow \boxed{K_v = 1.171}$

Load distribution factor

~~K_H~~

$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$. (Eq. 14-30)

$C_{mc} = 1$ (uncrowned teeth)

$C_{pf} = \frac{b}{10d} - 0.0375 + 0.0125\frac{b}{25.4}$ (Eq. 14-32)

$$\Rightarrow \boxed{\begin{array}{l}(C_{pf})_P = 0.0496 \text{ (pinion)} \\ (C_{pf})_G = 0.0371 \text{ (gear)}\end{array}}$$

$\frac{b}{10d} = 0.0156 \le 0.05$

$\Rightarrow \frac{b}{10d} = 0.05$ is chosen

$S_1/S = 0$ (see Fig 14-10)

$\Rightarrow \boxed{C_{pm} = 1}$

$C_e = 1$, $C_{ma} \cong 0.1$ (curve-3 Fig 14-11, b= 50mm= 1.97 in)

$\Rightarrow (K_H)_P = 1 + 1(0.0496 * 1 + 0.1)$

$$\Rightarrow \boxed{\begin{array}{l}(K_H)_P = 1.1496 \text{ (pinion)} \\ (K_H)_G = 1.1371 \text{ (gear)}\end{array}}$$

Rim thickness-to-tooth height ratio. $= 1.5$

$\Rightarrow \boxed{K_B = 1}$ (Figure 14-16)

Geometry Factor:

$$\boxed{\begin{array}{l}(Y_J)_P = 0.27 \text{ (pinion)} \\ (Y_J)_G \simeq 0.41 \text{ (gear)}\end{array}}$$

(Fig 14-6)

Stress Cycle factor: $Y_N = 1.3558 \times N^{-0.0178}$ (Fig 14-14)

$$\Rightarrow \boxed{\begin{array}{l}(Y_N)_P = 0.977 \text{ (pinion, } N = 10^8 \text{ cycles)} \\ (Y_N)_G = 1.009 \text{ (gear, } N = \frac{10^8}{4} \text{ cycles)}\end{array}}$$

Temperature factor: $\boxed{Y_\theta = 1}$

Reliability factor: $\boxed{Y_Z = 1}$ (Reliability = 0.99) (Table 14-10)

Bending strength:

$S_t = 0.7255\, H_B + 153.63$ (Nitriding steel, 2.5% Chrome, Grade 2) Figure 14-4

$$\boxed{\begin{aligned}(S_t)_P &= 335\,\text{MPa} \quad (\text{Pinion},\ HB = 250)\\ (S_t)_G &= 298.73\,\text{MPa} \quad (\text{Gear},\ HB = 200)\end{aligned}}$$

Bending failure

Pinion: $\sigma = \dfrac{W_t}{bm} K_o K_v K_s \dfrac{K_H K_B}{Y_J}$

$= \dfrac{2984}{50\times5} \times 2 \times 1.171 \times 1.0964 \times \dfrac{1.1496\times1}{0.27}$

$\Rightarrow \boxed{(\sigma)_P = 130.5\,\text{MPa}}$

Allowable stress: $\sigma_{all} = \dfrac{S_t}{S_F} \cdot \dfrac{Y_N}{Y_\theta Y_Z}$

$\Rightarrow (\sigma_{all})_{pinion} = \dfrac{335 \times 0.977}{S_F \times 1 \times 1} = \dfrac{327.3}{(S_F)_P}$ MPa.

$\Rightarrow$ Factor of Safety: $\boxed{(S_F)_P = 2.508}$

Gear: $(\sigma)_G = \dfrac{2984}{50\times5} \times 2\times1.171\times1.1071\times \dfrac{1.1371\times1}{0.41}$

$\Rightarrow \boxed{(\sigma)_G = 85.83\,\text{MPa}}$

Allowable stress

$(\sigma_{all})_{gear} = \dfrac{298.73}{S_F} \times \dfrac{1.001}{1\times1} = \dfrac{299}{S_F}$ MPa

Factor of Safety: $\boxed{(S_F)_G = 3.484}$

$\Rightarrow \boxed{(S_F)_P = 2.508,\ (S_F)_G = 3.484}$

B Factors for pitting-resistance & strength :

Elasticity coefficient : $Z_E = 191 \sqrt{MPa}$ (Table 14-8)

Surface Condition Factor : $Z_R = 1$

Geometry factor : $Z_I = \frac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \frac{m_G}{m_G+1}$ (Equation 14-23)

$\phi_t = \phi = 20^\circ$, $m_N = 1$, (spur gear)
$m_G = 4$

$\Rightarrow \boxed{Z_I = 0.1285}$

Stress cycle factor: $Z_N = 1.4488 N^{-0.023}$ (Fig 14-15)

$$\Rightarrow \boxed{\begin{array}{l}(Z_N)_P = 0.948 \text{ (pinion)} \\ (Z_N)_G = 0.979 \text{ (gear)}\end{array}}$$

Hardness Ratio factor:

$$1.7 \geq \frac{H_{BP}}{H_{BG}} = \frac{250}{200} = 1.25 \geq 1.2$$

$$\Rightarrow A' = \left(8.98\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29\right) \times 10^{-3} \quad \text{(Eq. 14-36)}$$
$$= 2.935 \times 10^{-3}$$

$$\Rightarrow (Z_W)_G = 1.0 + A'(m_G - 1) = 1.009. \quad \text{(Eq 14-36)}$$

$$\Rightarrow \boxed{\begin{array}{l}(Z_W)_P = 1. \text{ (pinion)} \\ (Z_W)_G = 1.009 \text{ (gear)}\end{array}}$$

Contact Strength: $\boxed{S_c = 1350 \text{ MPa}}$ for both pinion and gear.

Pitting Failure:

Pinion: $(\sigma_c)_P = Z_E \left\{ W_t K_o K_v K_s \cdot \frac{K_H}{d_P \cdot b} \frac{Z_R}{Z_I} \right\}^{1/2}$

$$= 191 \left\{ 2984 \times 2 \times 1.171 \times 1.0964 \times \frac{1.1496}{80 \times 50} \times \frac{1}{0.1285} \right\}^{1/2}$$

$$\Rightarrow \boxed{(\sigma_c)_P = 790.68 \text{ MPa}}$$

Allowable contact stress: $\sigma_{c,all} = \frac{S_c}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$

$$\Rightarrow \boxed{(\sigma_{c,all})_P = \frac{1350 \times}{S_H} \frac{0.948 \times 1}{1 \times 1} = \frac{1279.8}{S_H} \text{ MPa}}$$

Factor of Safety : $\boxed{(S_H)_P = 1.619}$

Gear :

$$(\sigma_c)_G = 191 \left\{ 2984 \times \frac{2 \times 1.171 \times 1.1071 \times 1.1371}{50 \times 80} \times \frac{1}{0.1285} \right\}$$

$$\boxed{(\sigma_c)_G = 790.2 \text{ MPa}}$$

Allowable contact stress

$$\boxed{(\sigma_{c,all})_G = \frac{1350 \times 0.979 \times 1.009}{S_H \times 1 \times 1} = \frac{1333.59}{S_H} \text{ MPa}}$$

Factor of Safety : $\boxed{(S_H)_G = 1.688}$

$$\Rightarrow \boxed{(S_H)_P = 1.619, \quad (S_H)_G = 1.688}$$

(c) Likely Failure Mode :

Pinion : $\boxed{(S_F)_P = 2.508, \quad (S_H)_P^2 = 2.621}$

Gear : $\boxed{(S_F)_G = 3.484, \quad (S_H)_G^2 = 2.849}$

$\Rightarrow$ Most threat to failure is from bending of pinion tooth.

(d) Power to failure :

$$(\sigma_{all})_P = (\sigma)_P \quad \text{for pinion with } (S_F)_P = 1.$$

$$\Rightarrow \sigma_P = 327.3 \text{ MPa}.$$

$$\Rightarrow \boxed{W_t = 7.484 \text{ kN}} \text{ at failure}$$

$$\Rightarrow \boxed{H = 62.7 \text{ kW}} \text{ power rating at failure}$$

This is 151% over load