22/07/2019

A brief recap:

Hamilton's Eq^{ns}

$$H(p, q, t)$$

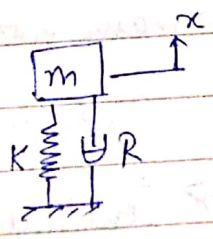
p = generalized momentum
q = generalized charge/displacement

$$\frac{d}{dt}\underbrace{\begin{Bmatrix} p \\ q \end{Bmatrix}}_{X} = A\underbrace{\begin{Bmatrix} p \\ q \end{Bmatrix}}_{X} + Bu$$ ← State Space Equations

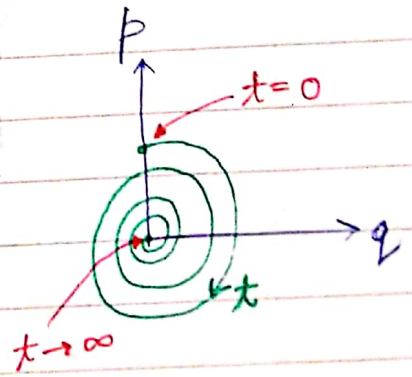
where,
A = state matrix
X = state vector
B = input participation matrix
U = input / forcing vector

$\dot{X} = AX + BU$

Eigen values of A, $|dI - A| = 0$ are natural frequencies.



$p = m\dot{x}$
$q = x$



$\dot{p}$ → generalized effort
$\dot{q}$ → generalized flow

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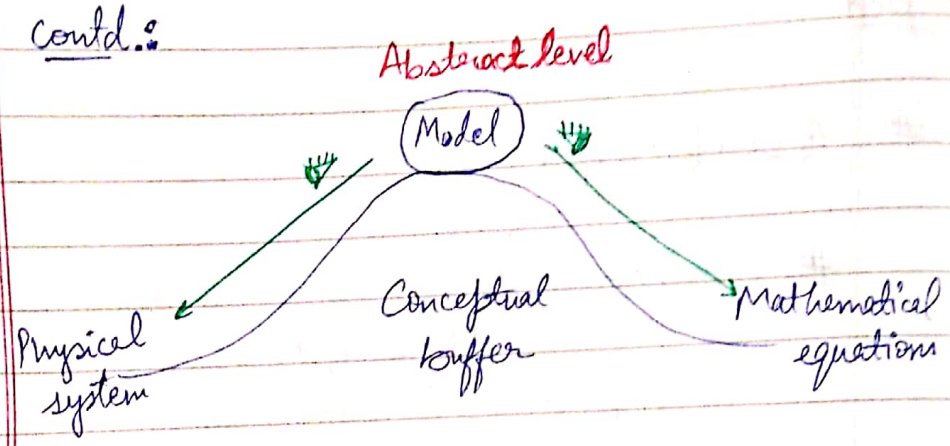
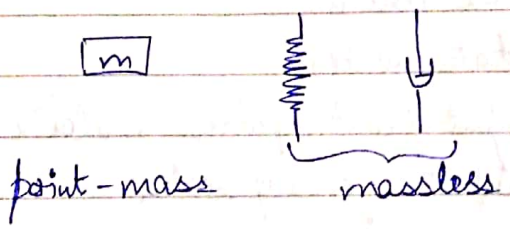
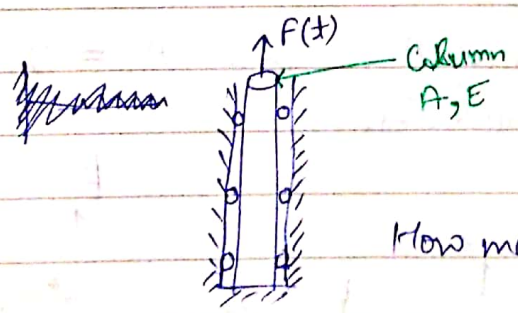
Contd.:



Abstract ~~level~~ model reveals the structure of the system as well as the form of mathematical eq^ns.

Simplification

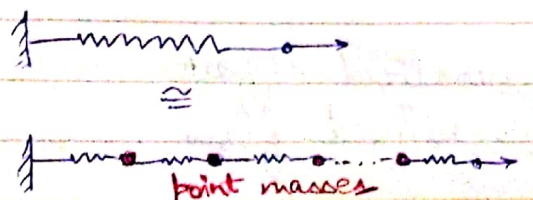




How many natural frequencies?

Infinite

$\Rightarrow$ Continuous/ distributed parameter system.



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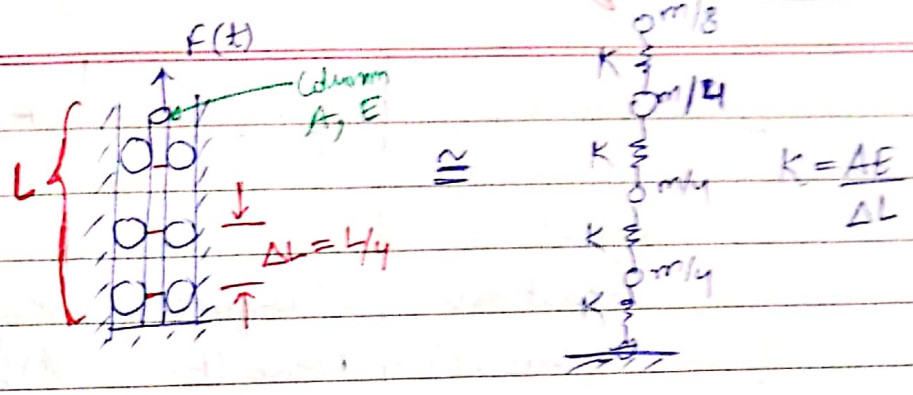


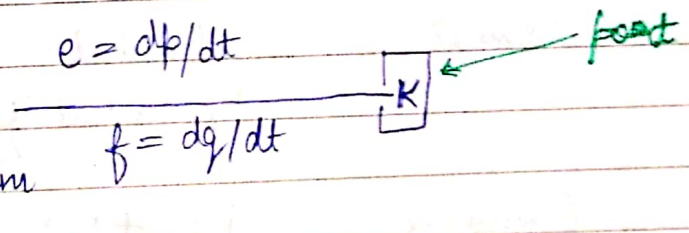
⇓

1st & 2nd ω_n will be closer to the actual.

Any engg. dynamic system with some or other type of impact must have a factor of safety of minimum 2.

Port-Hamiltonian System:

Some kind of door



domain wise def^n of e & f:

$e = \frac{dp}{dt}$ → generalized effort

$f = \frac{dq}{dt}$ → generalized flow

$e \cdot f$ = power ← free from domain

Dot product

Domain	Effort (e)	Flow (f)
Mechanical		
Rotation	Torque (T)	Angular vel. (ω)
Translation	Force (F)	Velocity (v)
Electrical	Potential difference (Δv)	Current (i)
Hydraulic	Pressure difference (ΔP)	Volume flow rate ($\dot{V}$)
Thermal	Pressure	Volume change rate ($\dot{V}$)
($dU = pdV - Tds + hdm$)	Temperature	Entropy change rate
	Specific Enthalpy	Mass flow rate
Chemical	Chemical potential (μ)	Mole flow/change rate ($\dot{n}$)
Economic	Unit price	Order flow rate

Power variables = (e, f)

Energy variables = State variables
$= (\int e\,dt, \int f\,dt)$

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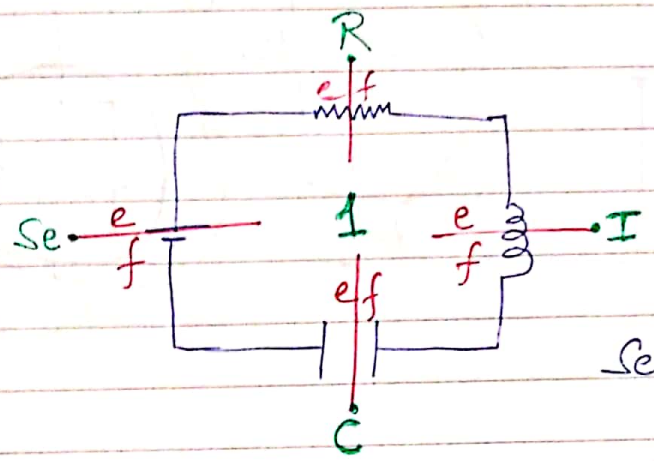
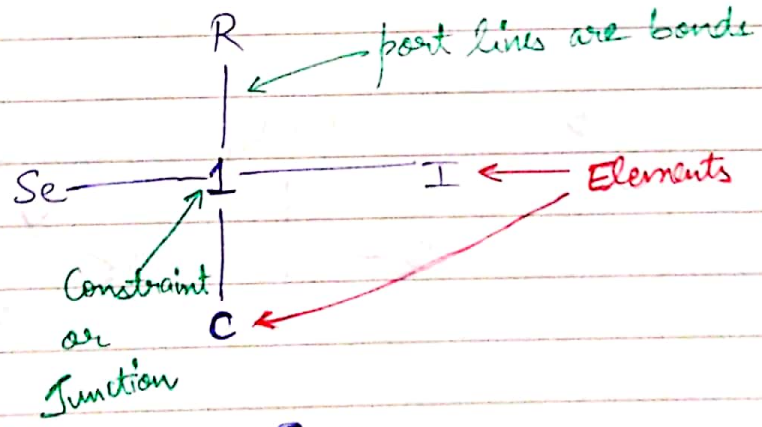
Except temperature, all efforts have momentum. Bcoz if we have temp. momentum possible, we can have a system having oscillations but it will violate the laws of Thermodynamics.



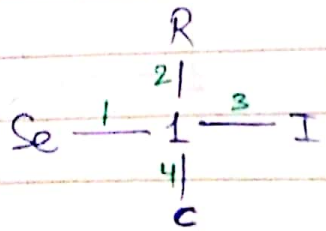
Se = Source for effort

1 ≡ Common flow (current i)



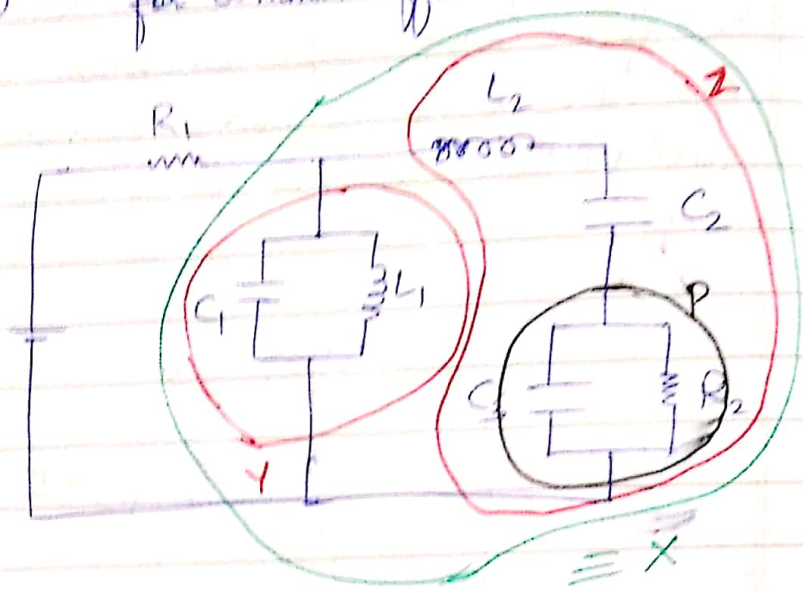
Bond Graph

LISP →



((Se 1)(R 2)(C 4)(I 3)(1 1 2 3 4))

0 → for common effort



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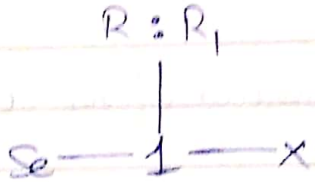

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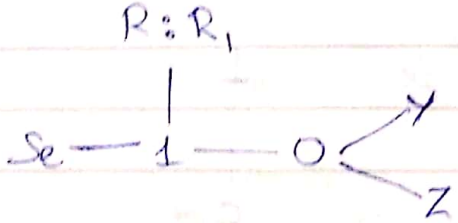

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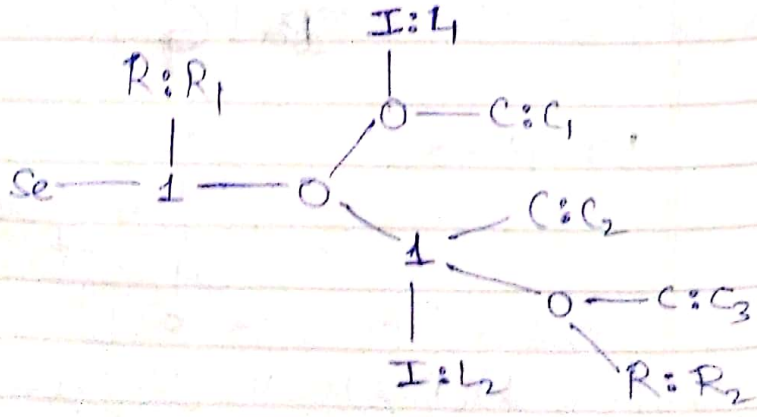
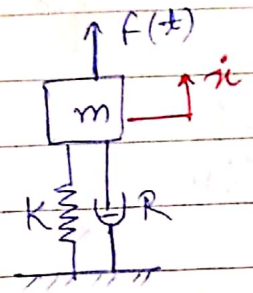
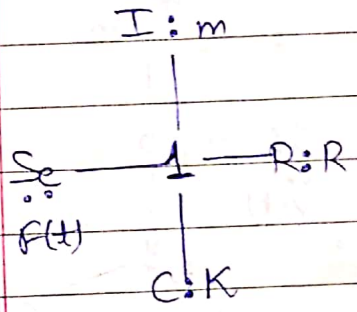


23/07/2019

(i)



v(t) will be represented by Sf (source of flow)
F(t) by Se (source of effort)
m by I
K by C
R by R

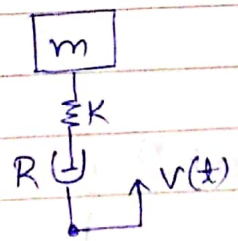


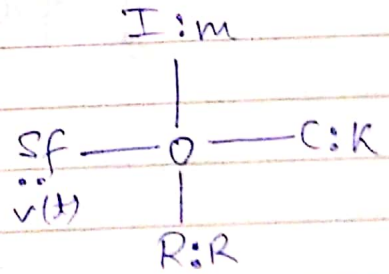
Constraints:
Same flow 1
Same effort 0

Flow = velocity for m
= rate of deformation for C & R

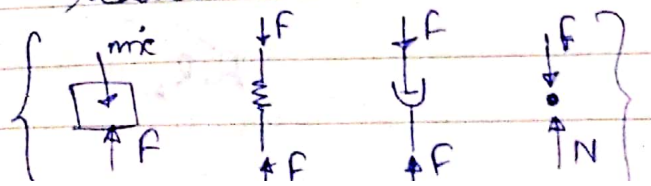
velocity of m, F(t) = $\dot{x}$
rate of deform^n of C, R = $\dot{x}$
⇒ Same flow, 1 junction

(ii)





force in mass, spring, damper & ground reaction are same.



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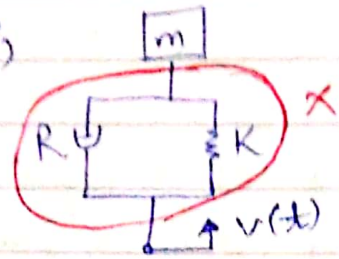
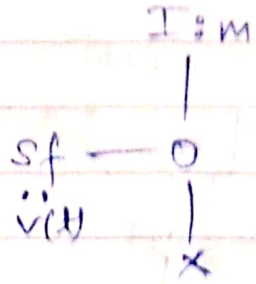
(iii)





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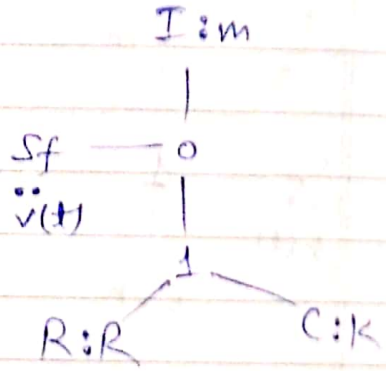

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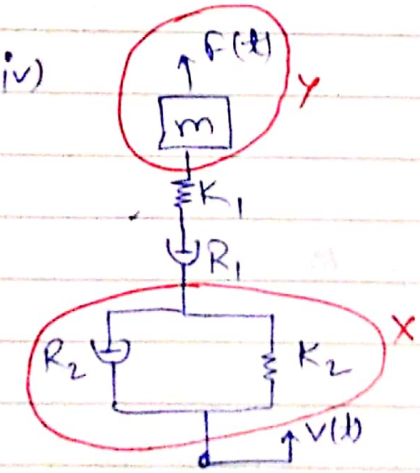

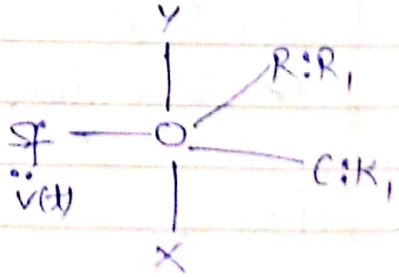

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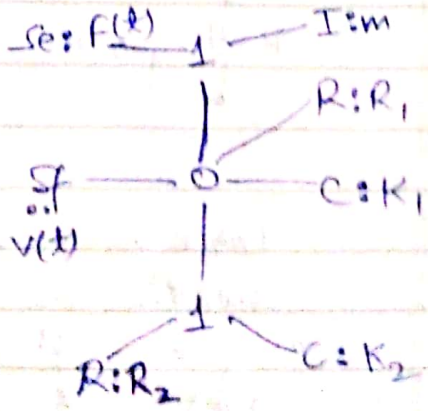

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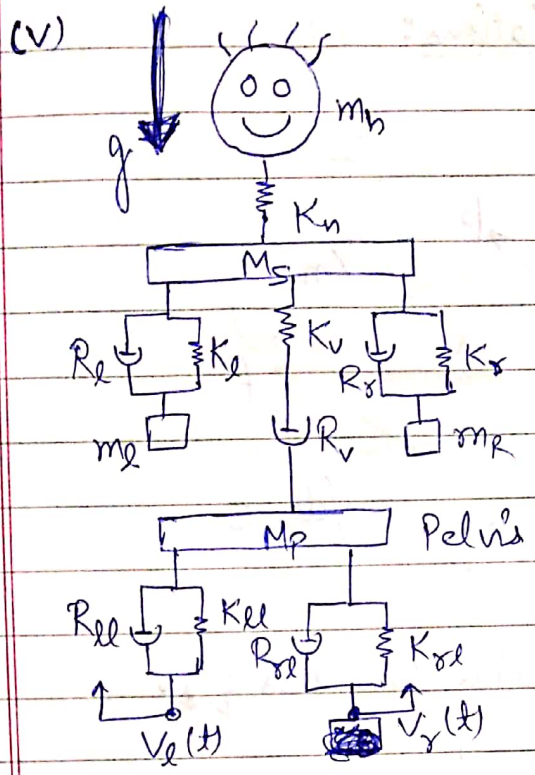
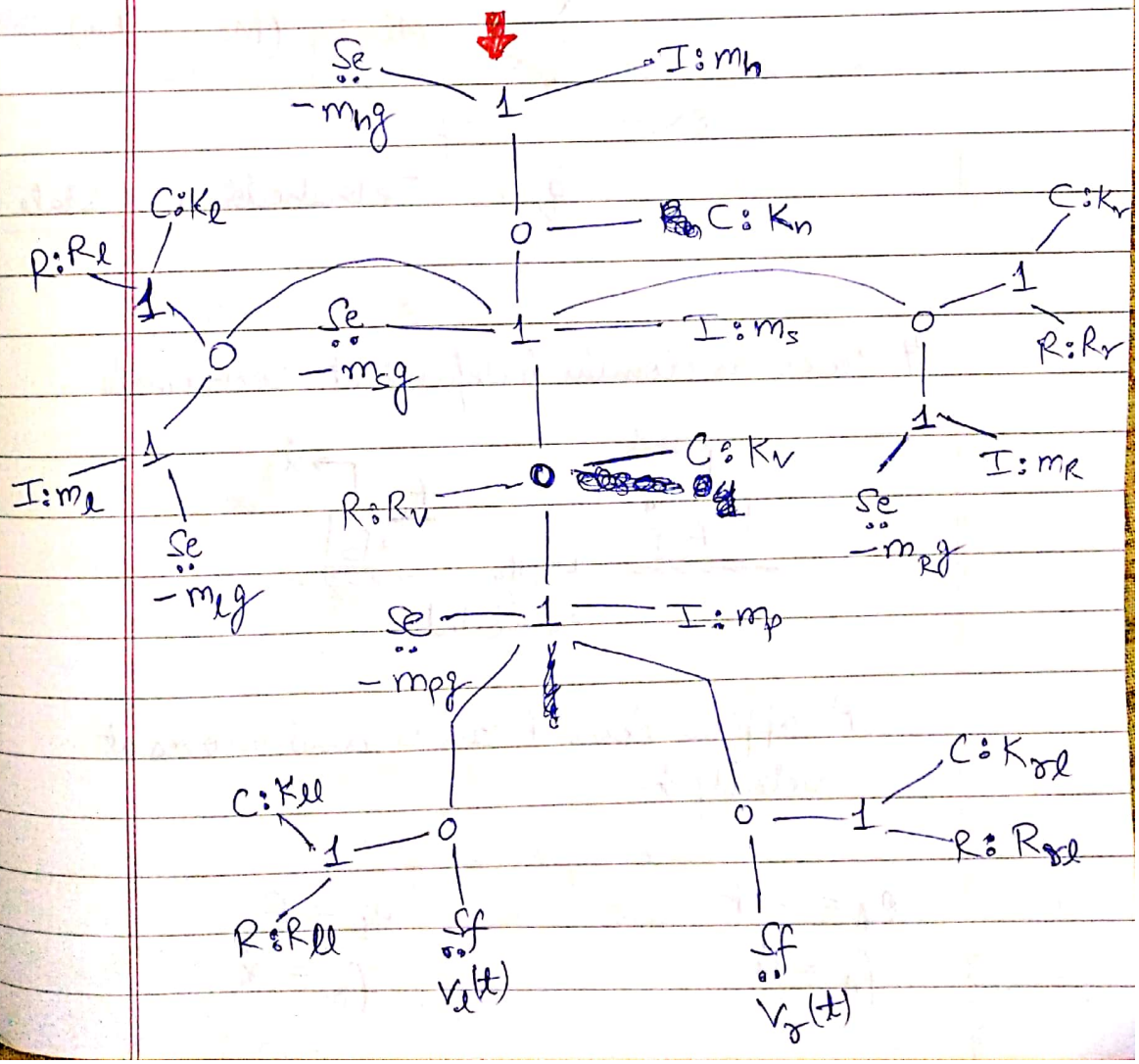
(v)





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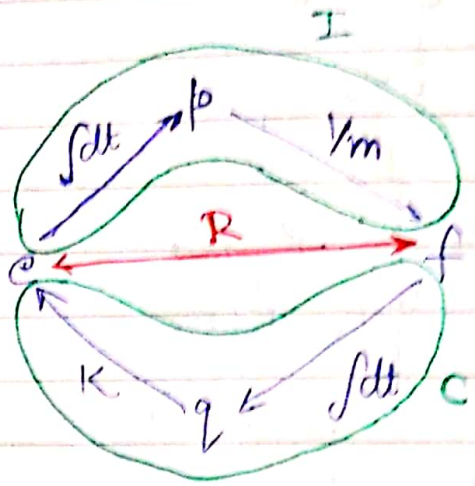
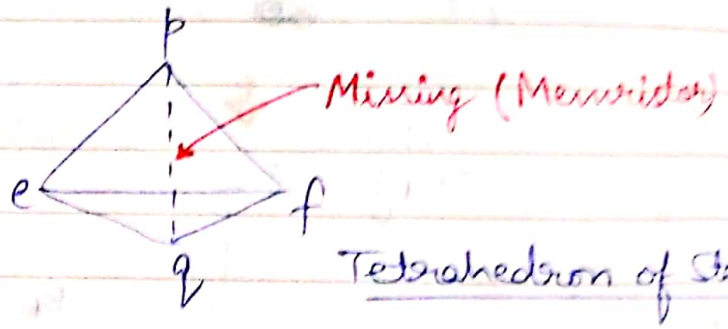
Constitutive Relations:

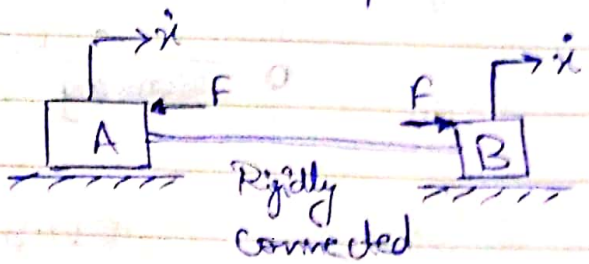


But no connection b/w p & q.



Tetrahedron of State

Power as domain independent coordinate



A applies force F on B and moves at velocity $\dot{x}$.

$e_A = -F$ $\qquad$ $e_B = F$

$f_A = \dot{x}$ $\qquad$ $f_B = \dot{x}$

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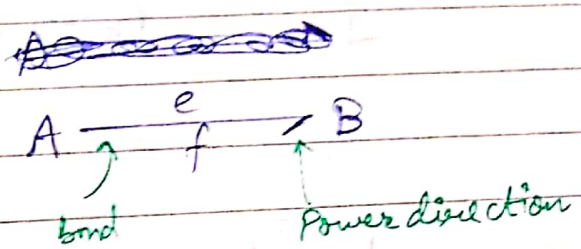
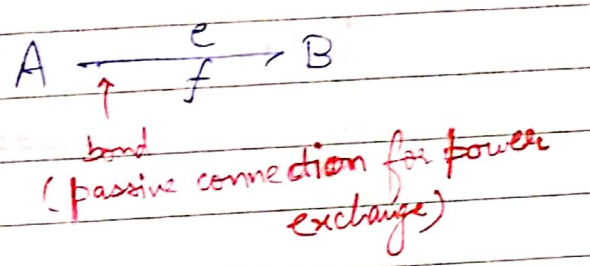
$e_A \cdot f_A = -f\dot{x}$ $\qquad$ $e_B \cdot f_B = f\dot{x}$

$f > 0, \dot{x} > 0$

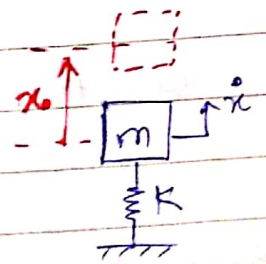


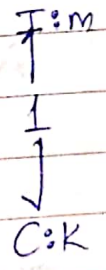


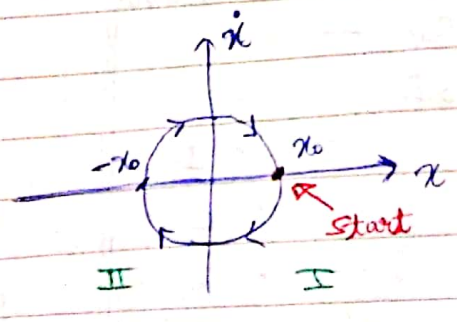
⇒ Power flows from A to B if $e \cdot f > 0$

$e > 0, f > 0$ or $e < 0, f < 0$ ⇒ Power from A to B

$e < 0, f > 0$ or $e > 0, f < 0$ ⇒ Power from B to A







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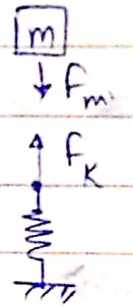
(I)



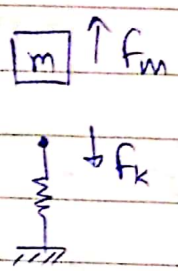
$f_m < 0, \dot{x} < 0$

~~e.f<0~~ $e.f > 0$

$f_K > 0, \dot{x} < 0$

$e.f < 0$

(II)



$f_m > 0, \dot{x} < 0$

$e.f < 0$

~~$f_K > 0$~~ $f_K < 0, \dot{x} < 0$

$e.f > 0$

Power flow as per indicated direction only if $e.f > 0$.

A ⟶ B

A ⟵ B

A ⟶ B

A ⟵ B

A ⟷ B or A ⟶ B

Default power directions →

Se ⟶

Sf ⟶

⟶ I

⟶ C

⟶ R

It might be reverse

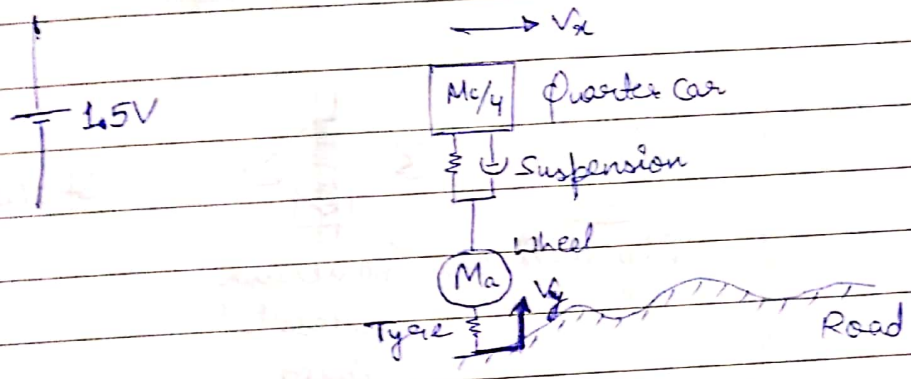
1 — 0

arbitrary direction or as for Kinematics

24/02/2021

Source:

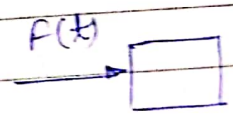
Source supplies one factor of power (e or f) independent of the other factor.



$$\frac{dy}{dt} = \left(\frac{dy}{dx}\right) \times \left(\frac{dx}{dt}\right)$$

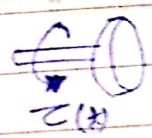
$$= \underbrace{\phi(x,y)}_{\text{Road profile}} \times V_x$$

Force

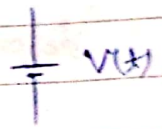


$Se \longrightarrow$ $F(t)$

Torque

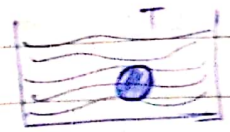


$Se \longrightarrow$ $\tau(t)$



$Se \longrightarrow$ $V(t)$

Hot bath



$Se \longrightarrow$ $T(t)$

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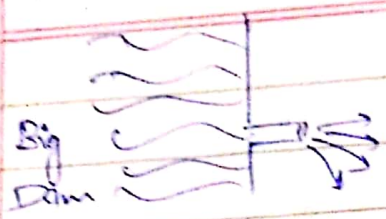
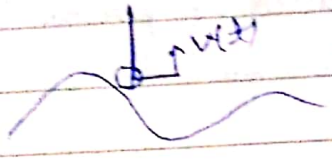
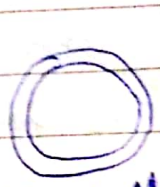


Se : P(t) →

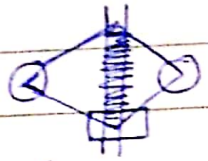


Base Excitation

Sf : v(t) →



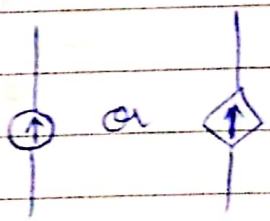
Flywheel or



Governor Controlled speed

Sf : ω(t) →

Positive displacement pump Sf : Q(t) →



or Current source Sf : i(t) →

One-port elements (I, C & R):

I - element $\xrightarrow[f]{e}$ I : ??

default power dir

$\phi_I(f, \int e\,dt) = 0$ → may be non-linear or linear

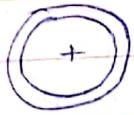
Linear case,

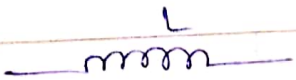
$$f = \frac{1}{m}\int_{-\infty}^{t} e\,d\tau = \frac{1}{m}\left[\int_{0}^{t} e(\tau)\,d\tau + p(0)\right]$$

Initial momentum @ $t=0$

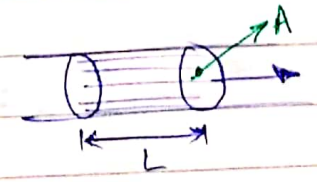
$$e = m\frac{df(t)}{dt}$$

Mass $\boxed{m}$ $\longrightarrow$ I : m

J disc  $\longrightarrow$ I : J



$\longrightarrow$ I : L

Fluid flow 

$\longrightarrow$ $I : \frac{\rho A}{L}$

C - element

$\xrightarrow[f]{e}$ C : ??

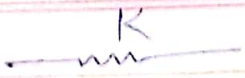
$$\phi_c(e, \int f\,dt) = 0$$

Linear case,

$$e = K\int_{-\infty}^{t} f\,d\tau = K\left[\int_{0}^{t} f\,d\tau + q(0)\right]$$

Initial displacement / charge

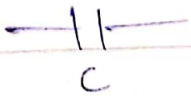
classmate
Date
Page



$\longrightarrow C : K$

$\longrightarrow C : K_t$

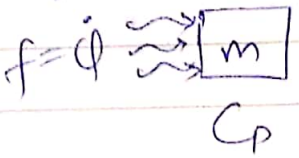
Torsional spring



$\longrightarrow C : \frac{1}{C}$

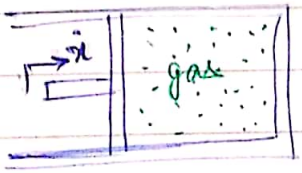
Accumulator (non-linear capacitor) $\longrightarrow C : ??$

Hydraulic capacity



$T = \frac{1}{mC_p} \int \dot{\phi}\, dt \longrightarrow C : \frac{1}{mC_p}$

Thermal Capacity

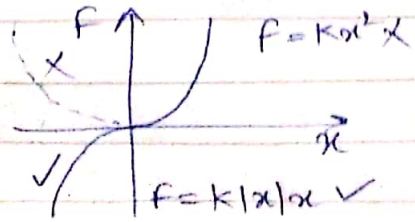


$\longrightarrow C : ??$ (gas field)

$\longrightarrow C : R_l$ (Reluctance)

Magnetized material

$$MMF = R_l \int \dot{\phi}\, dt$$



$\longrightarrow C : ??$

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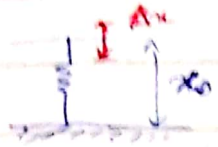
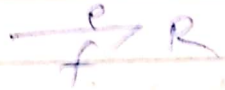


$$\left.\frac{\partial f}{\partial x}\right|_{x=x_0} = 2Kx_0$$

Oscillating spring about x_0

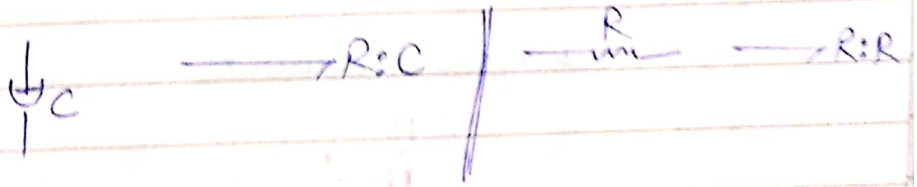
$\longrightarrow C : 2Kx_0$

R-element

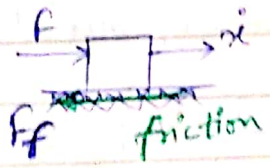


$$\phi_R(e, f) = 0$$

Linear case, $e - Rf = 0$

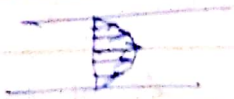


Torsional resistance



$\longrightarrow R_f$

$$f_f = \begin{cases} f & \text{if } \dot{x} = 0 \text{ \& } f < \mu N \\ \mu N & \text{if } f > \mu N, \ \dot{x} = 0 \\ \mu_K N \operatorname{sign}(\dot{x}) & \text{for } \dot{x} \neq 0 \end{cases}$$



viscous-friction

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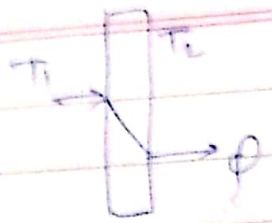


$$\varphi = \lambda (T_1 - T_2)$$

Thermal resistance

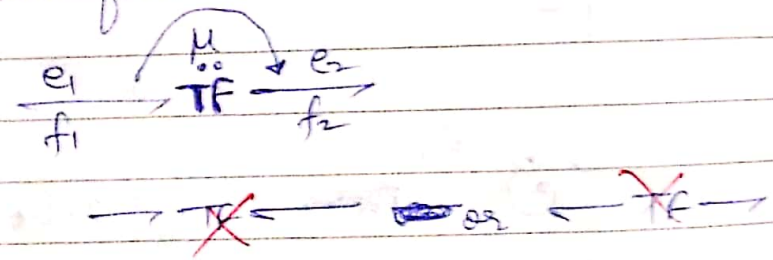
λ → overall heat transfer coefficient

Two-port elements:

Transducers:

Change of energy domain.

Transformer element



μ is called transformer modulus

↷ is transformer orientation

$$f_2 = \mu f_1 \quad \text{——— (i)}$$
$$e_1 = \mu e_2 \quad \text{——— (ii)}$$

Main
Corollary

So that $\mu e_1 f_1 = \mu e_2 f_2$

or $e_1 f_1 = e_2 f_2$

⇒ Power/energy is conserved

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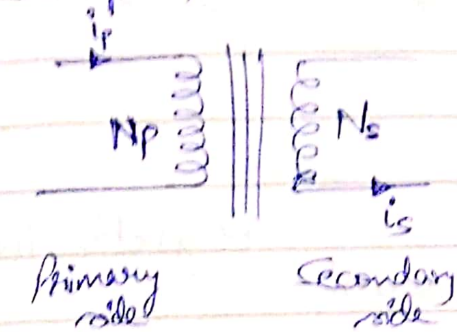
examples,

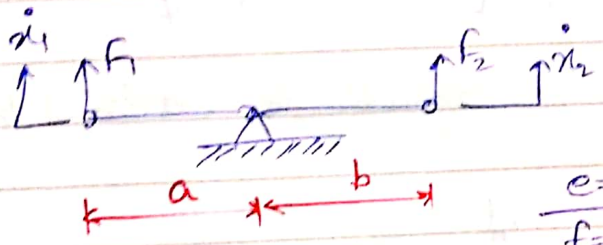


$$\frac{e = V_p}{f = i_p} \xrightarrow{N_p/N_s} \overset{\cdot\cdot}{TF} \frac{e = V_s}{f = i_s}$$

$$i_s = \frac{N_p}{N_s} \cdot i_p$$

Hidden magnetic domain

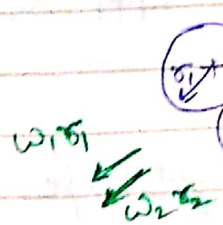
$$V_p = \frac{N_p}{N_s} \cdot V_s$$



$$\frac{e = F_1}{f = \dot{x}_1} \xrightarrow{-b/a} \overset{\cdot\cdot}{TF} \frac{e = F_2}{f = \dot{x}_2}$$

via Rotation domain

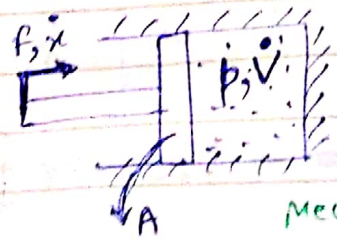
$$\dot{x}_2 = -\frac{b}{a}\dot{x}_1$$



Gear pairs

$$\frac{e = T_1}{f = \omega_1} \xrightarrow{r_1/r_2} \overset{\cdot\cdot}{TF} \frac{e = T_2}{f = \omega_2}$$

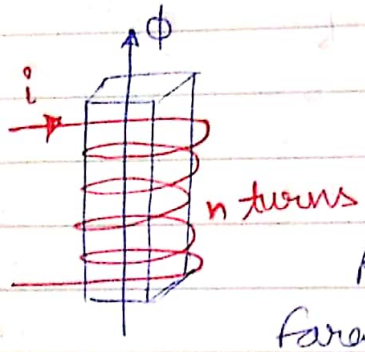
Rack & Pinion



$$\dot{V} = A\dot{x}, \quad F = Ap$$

Mech. translation to Hydraulic

$$\xrightarrow[f=\dot{x}]{e_\phi = f} \overset{n}{\underset{TF}{:}} \xrightarrow[f=\dot{v}]{e = p}$$



magnetic induction

Ampere's law, $MMF = ni$

Faraday's law, $e_b = -n\dot{\phi}$

(induced emf)

~~Am~~

$$\xrightarrow[f=i]{e=e_b} \overset{n}{\underset{TF}{:}} \xrightarrow[f=\dot{\phi}]{e=MMF}$$

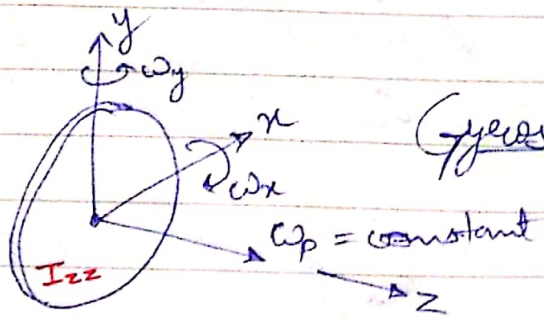
~~magnetic to~~ electrical to magnetic

$$\xrightarrow[f_1]{e_1} \overset{\mu}{\underset{GY}{:}} \xrightarrow[f_2]{e_2}$$

$$\boxed{\begin{aligned} e_2 &= \mu f_1 \\ e_1 &= \mu f_2 \end{aligned}}$$

Gyrator

$$e_1 f_1 = e_2 f_2$$



Gyroscope

$$\vec{T} = I_{zz}(\omega_x \hat{i} + \omega_y \hat{j}) \times \omega_p \hat{k}$$

$$T_x \hat{i} + T_y \hat{j} = -I_{zz}\omega_x\omega_p \hat{j} + I_{zz}\omega_y\omega_p \hat{i}$$

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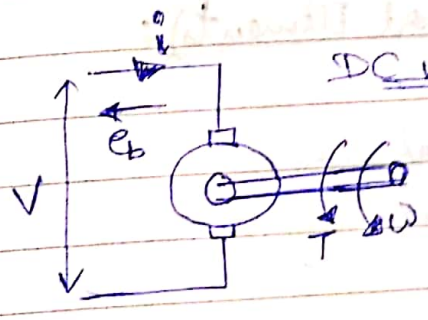
$$e = T_x \qquad I_{zz}\omega_p \qquad e = T_y$$
$$f = \omega_x \qquad GY \qquad f = \omega_y$$

$$T_x \omega_x + T_y \omega_y = 0$$



DC motor

μ_m = motor characteristic constant (torque const / speed const)

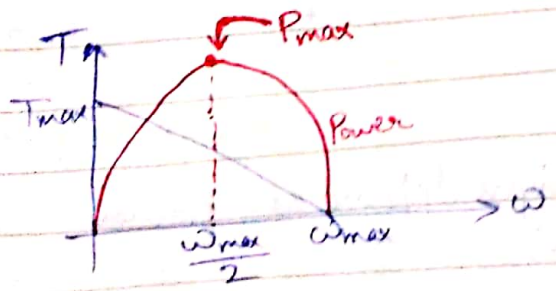
$$T = \mu_m i$$
$$e_b = -\mu_m \omega \quad \text{(Back emf)}$$

$$i = 0 \text{ if } V + e_b = 0$$

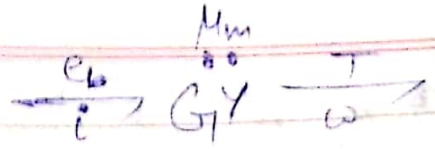
$$\omega_{max} = \frac{V}{\mu_m}$$

$$i = i_{max} \text{ if } e_b = 0 \Rightarrow \omega = 0$$

$$T_{max} = \left(\frac{V}{R_a}\right)\mu_m \quad ; \quad R_a = \text{armature resistance}$$



$$P = T\omega$$

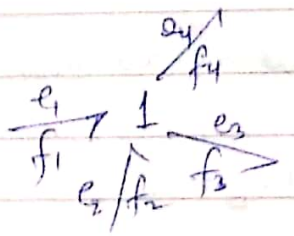


30/07/2011

Junctions (Multi-port Elements):

1 junc^n for equal flow
0 junc^n for equal effort

1-junc^n



8 power variables
4e & 4f

$f_1 = f_2 = f_3 = f_4$ (strong law of 1-junc^n)

Constraint neither creates, stores or dissipates energy.

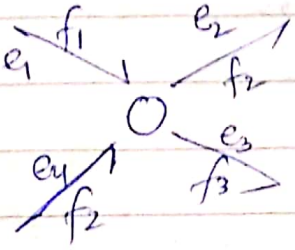
$$e_1 f_1 + e_2 f_2 - e_3 f_3 - e_4 f_4 = 0$$

$$\Rightarrow \underbrace{e_1 + e_2 - e_3 - e_4 = 0}_{\text{effort balance equation}}$$

(Corollary or weak law of 1-junc^n)

+ & − signs depend on power direction.

0-junc^n



$e_1 = e_2 = e_3 = e_4$ (strong law of 0-junc^n)

$$e_1 f_1 - e_2 f_2 - e_3 f_3 + e_4 f_4 = 0$$

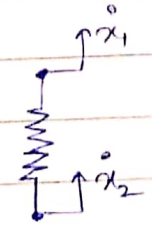
$$\Rightarrow \underbrace{f_1 - f_2 - f_3 + f_4 = 0}_{\text{flow balance equation}}$$

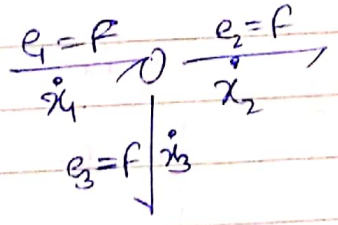
(corollary on weak law of 0-junc^n)

+ & − signs depend on power direction.

Kinematic Relations:

$$\dot{x}_3 = \dot{x}_1 - \dot{x}_2$$

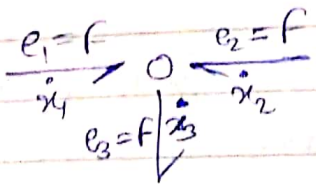




$$F\dot{x}_1 - F\dot{x}_2 - F\dot{x}_3 = 0$$

$$\Rightarrow \dot{x}_3 = \dot{x}_1 - \dot{x}_2$$

$$\dot{x}_3 = \dot{x}_1 + \dot{x}_2$$



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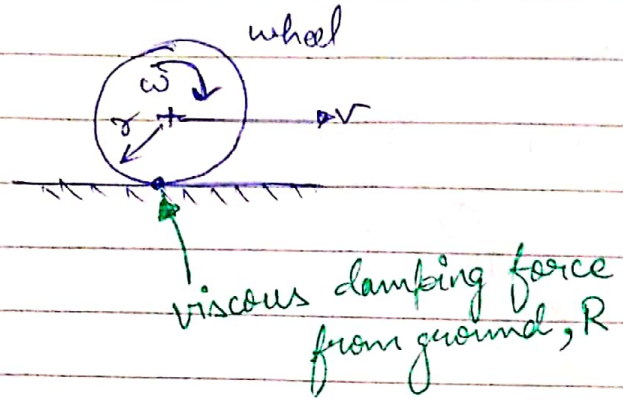
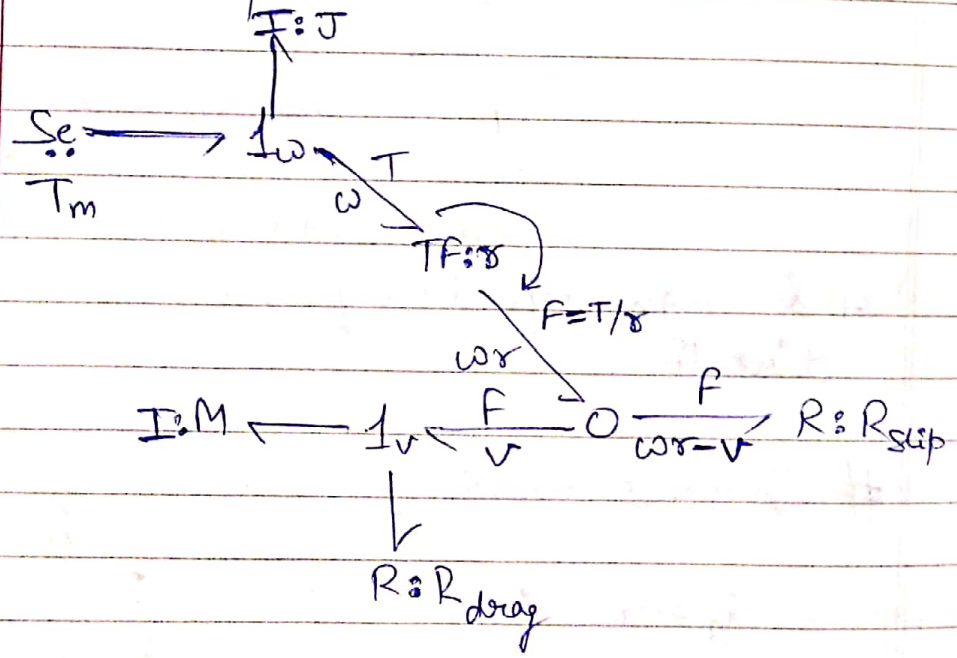
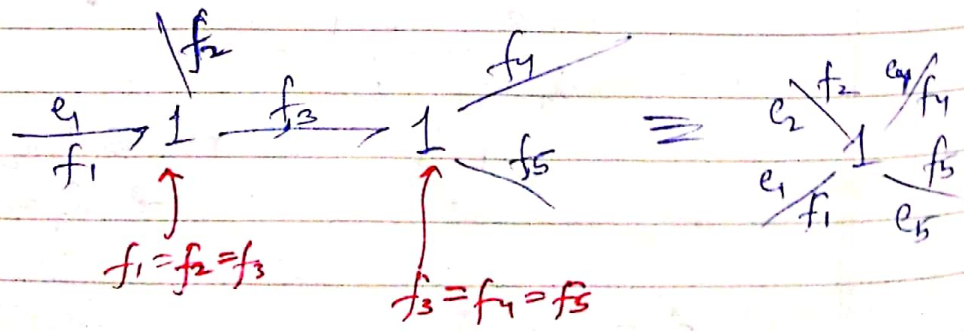


$F = R \cdot Slip$ (Traction force)

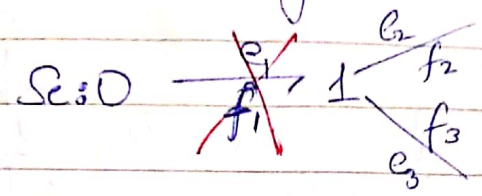
$Slip = \omega r - v$



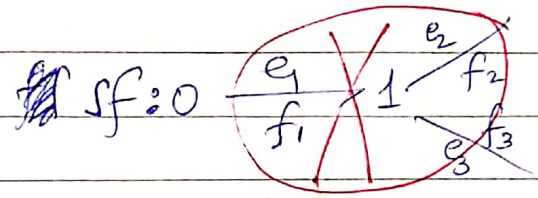


Two adjacent 1-junc^{ns} (or 0-junc^{ns}) can be merged together.

Powerless bonds & junc^{ns} can be removed.

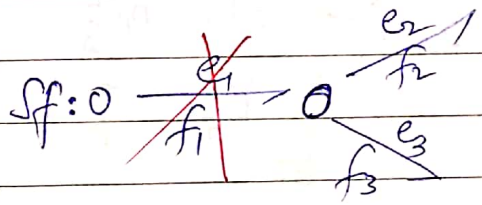


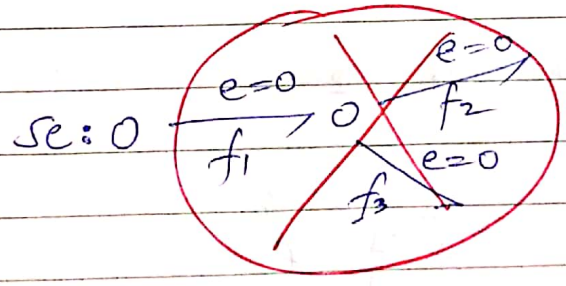
$e_1 = 0 \Rightarrow e_1 f_1 = 0$

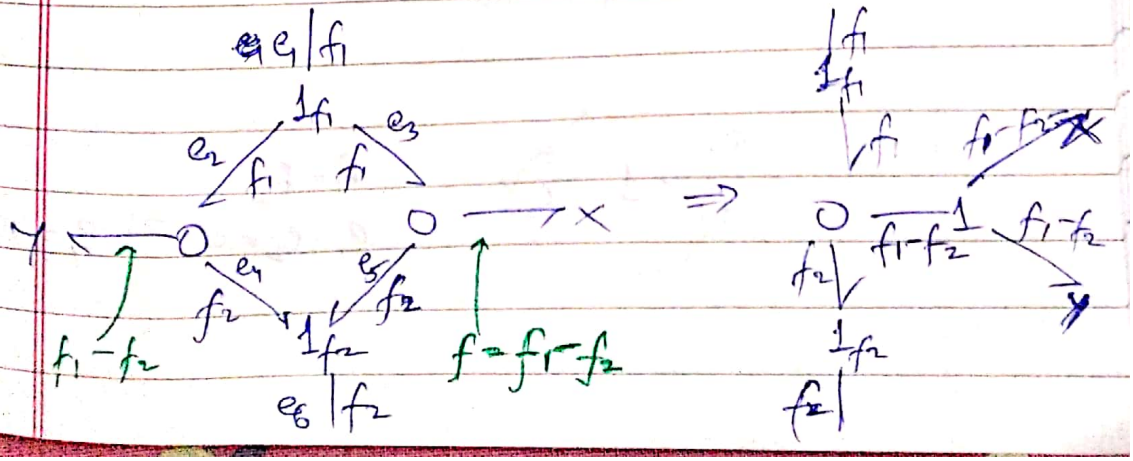


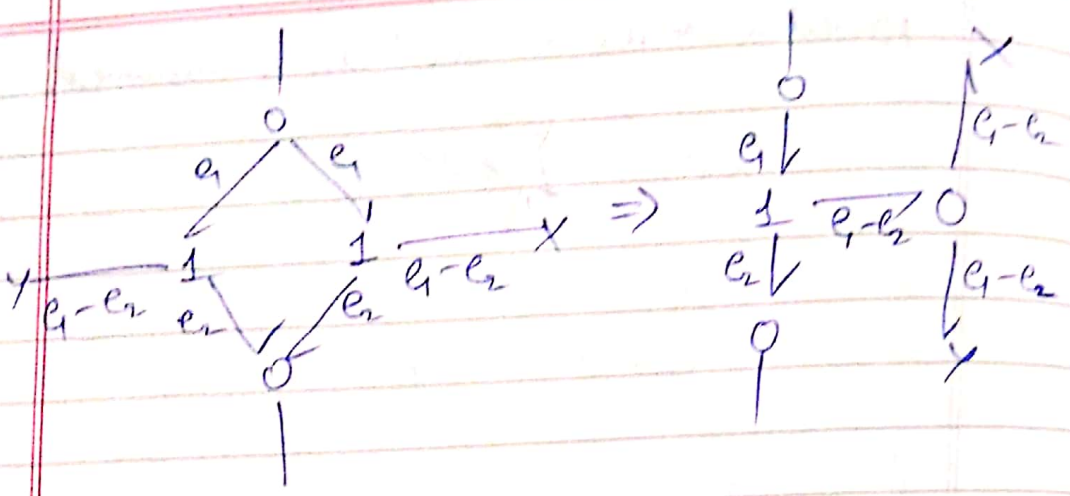
$f_1 = 0 \Rightarrow f_2 = f_3 = 0$

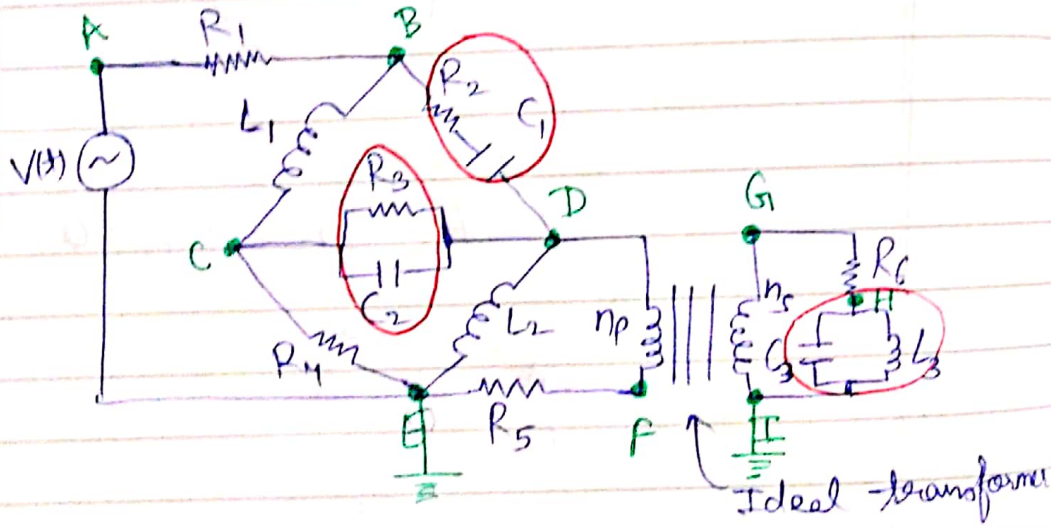
$\Rightarrow e_1 f_1 = e_2 f_2 = e_3 f_3 = 0$



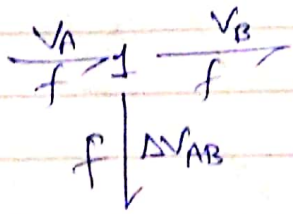


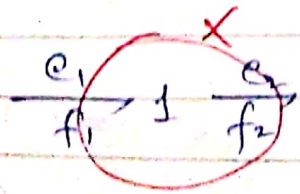






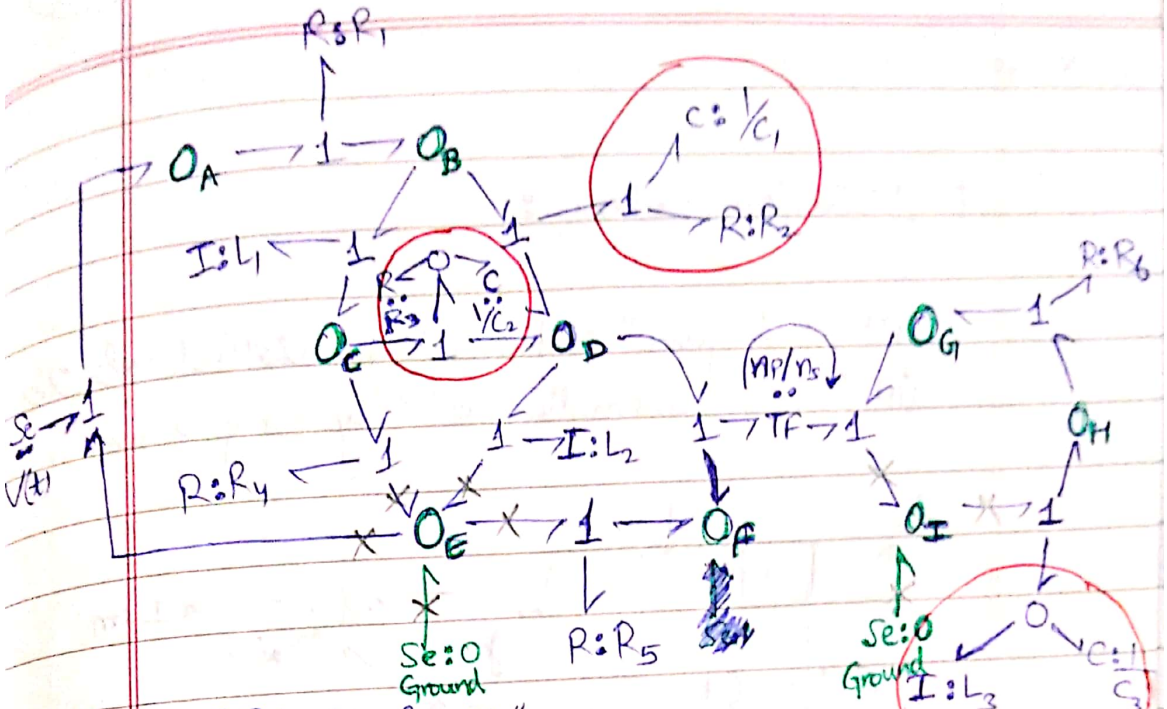
$$\Delta V_{AB} = V_A - V_B$$





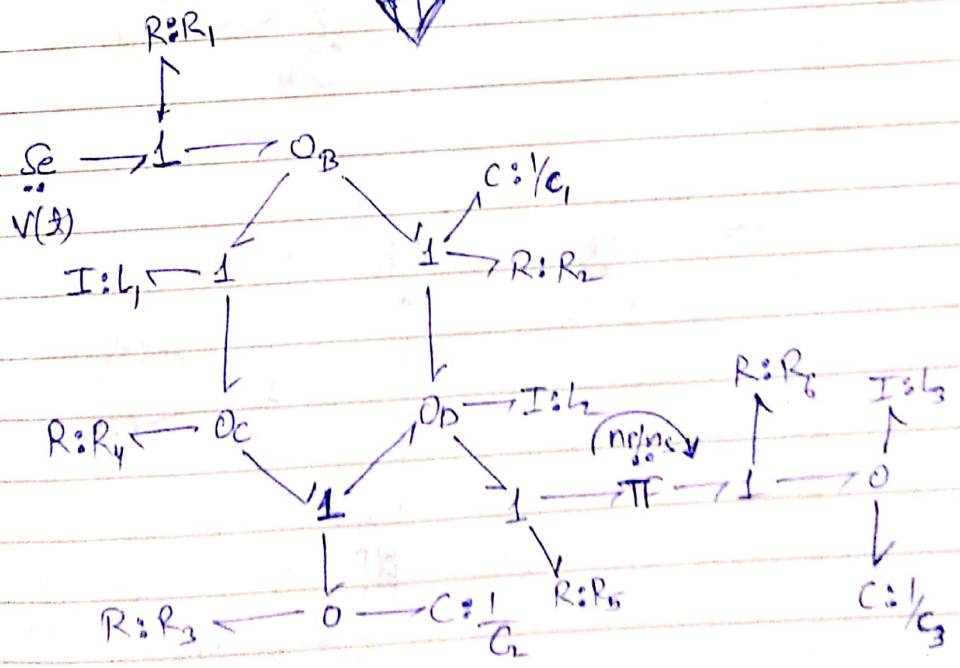
$f_1 = f_2$

$e_1 - e_2 = 0 \Rightarrow e_1 = e_2$



"Floating Circuit"
(Circuit without a ground)

Simplified



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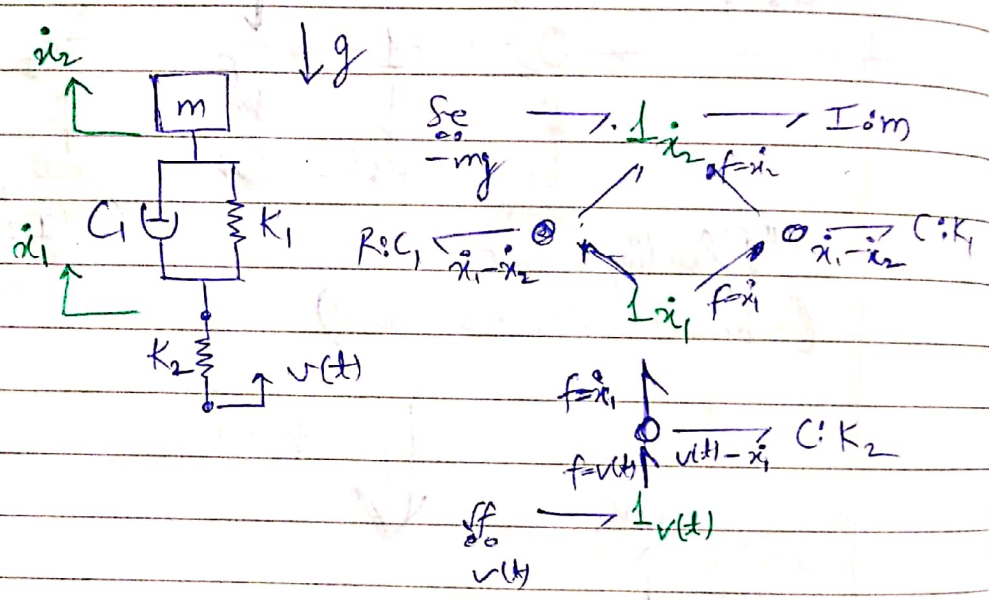
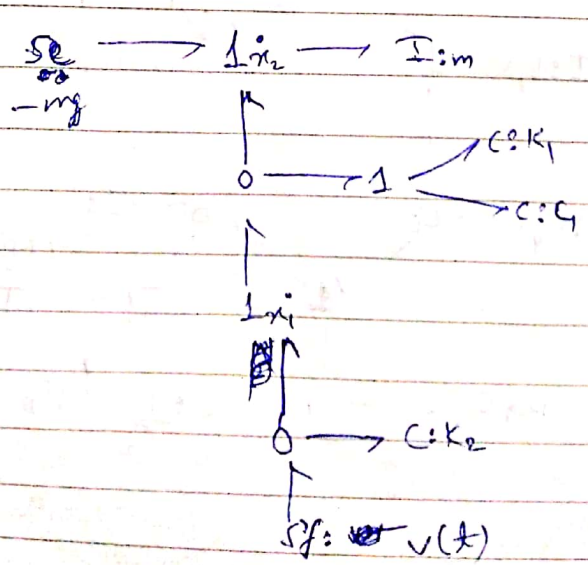
05/08/2019

Mechanical Systems :

(i) Add 1 junc^ns representing distinct velocities.
(ii) Model Kinematic rel^ns by using 0 junc^ns.



⇒



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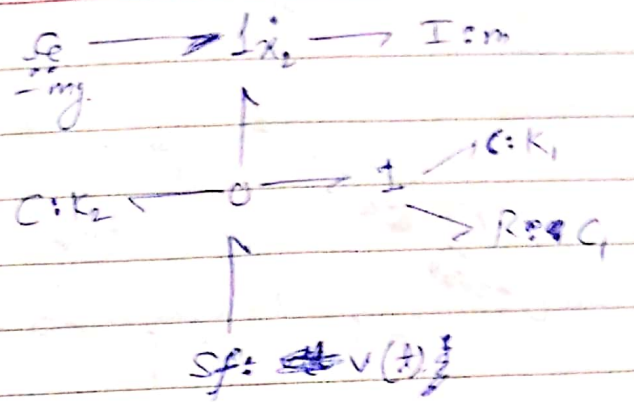

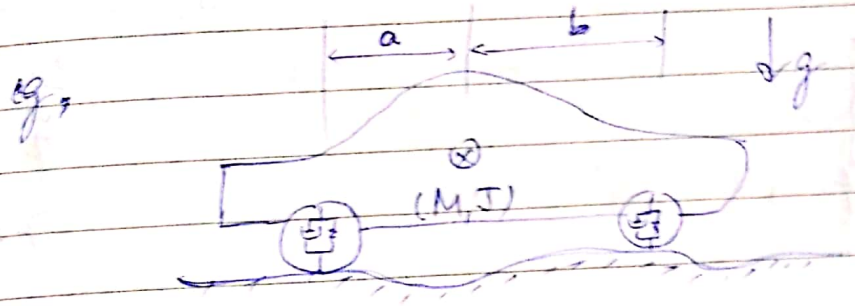

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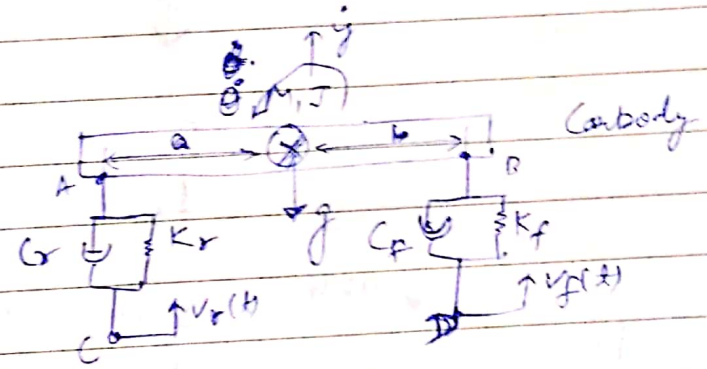
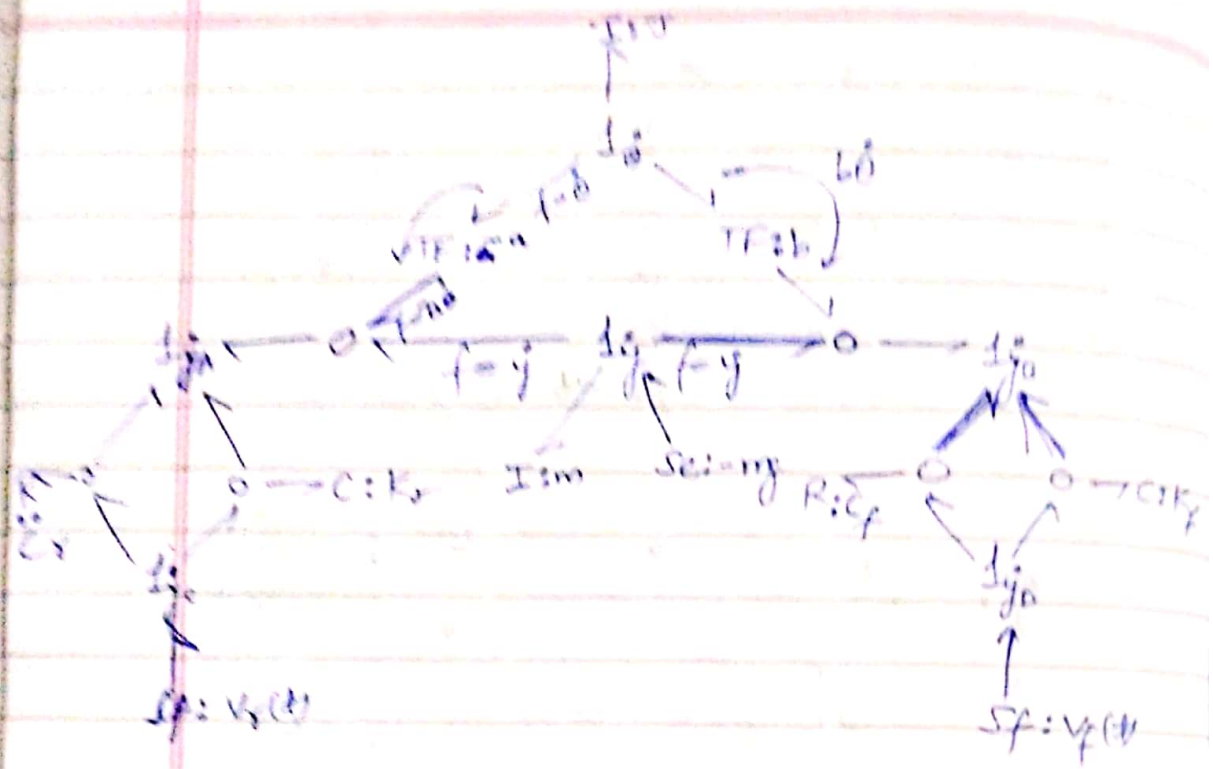


$$V_r(t) = V_f\left(t - \frac{a+b}{V_x}\right)$$

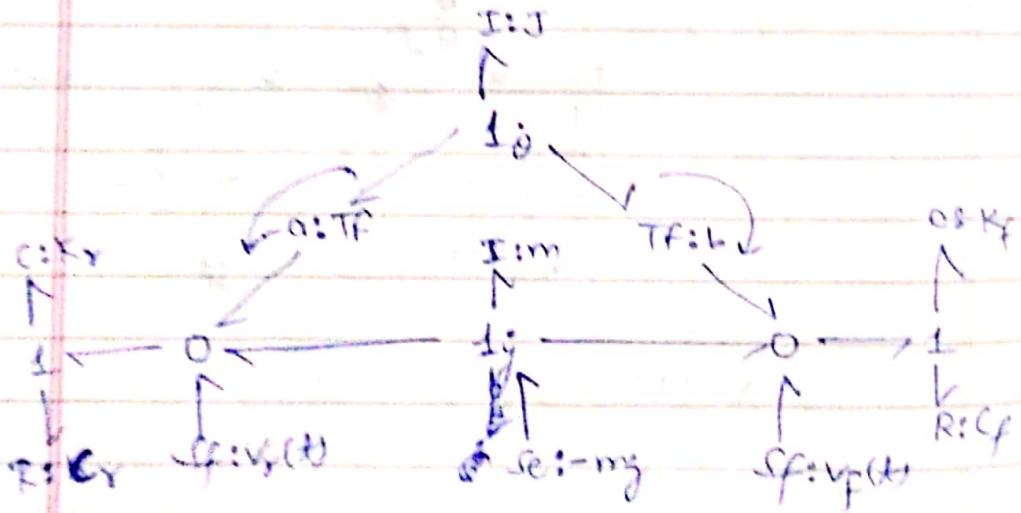
wheelbase a+b

$\dot{y}$ is heave velocity
$\dot{\theta}$ is pitch angular velocity

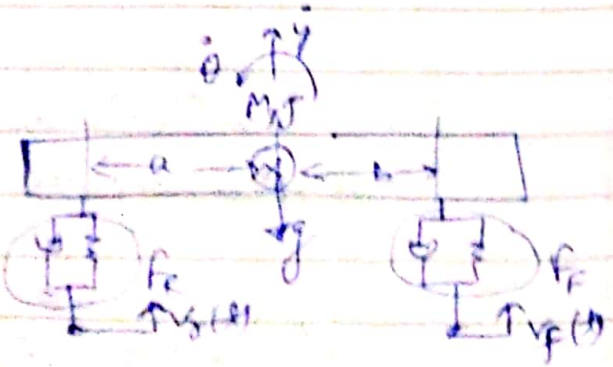
$$\dot{y}_A = \dot{y} - a\dot{\theta}$$
$$\dot{y}_B = \dot{y} + b\dot{\theta}$$



⇓

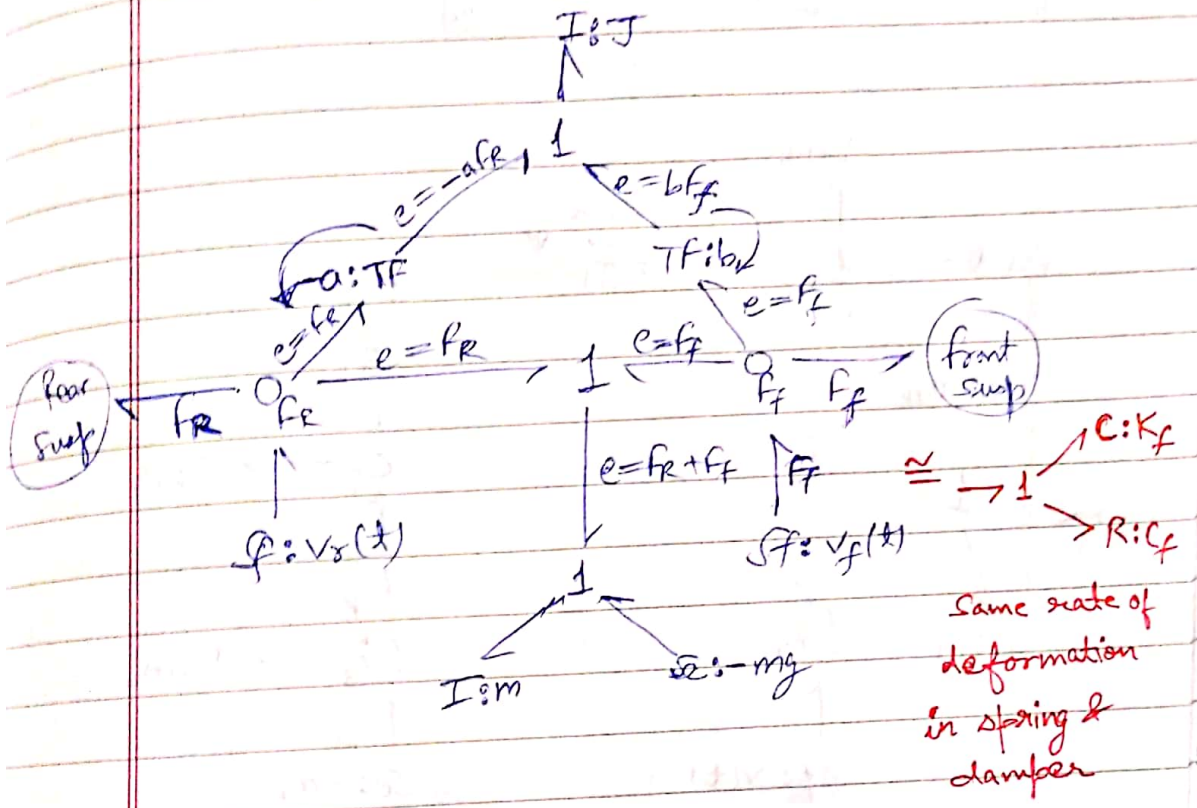


eg.



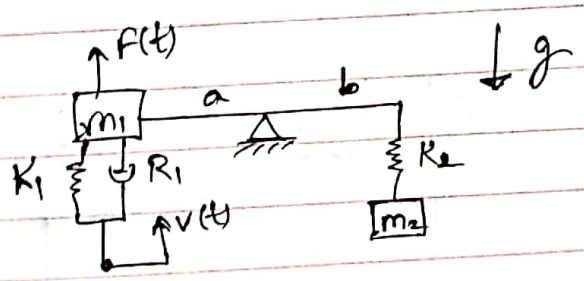
F_r & F_f are the suspension forces

$$F_G = F_R + F_F$$
$$M_G = F_F\, b - F_R\, a$$
} Equivalent force system at G



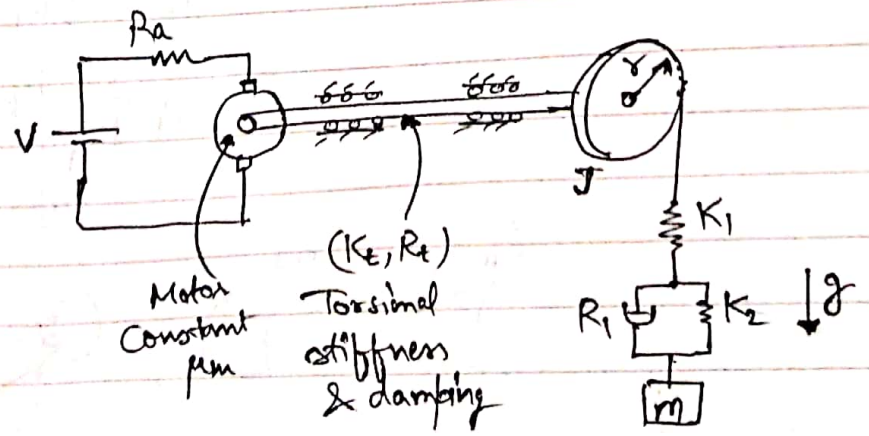
Same rate of deformation in spring & damper

Prob. ①



MODEL THE FOLLOWING SYSTEMS.

②



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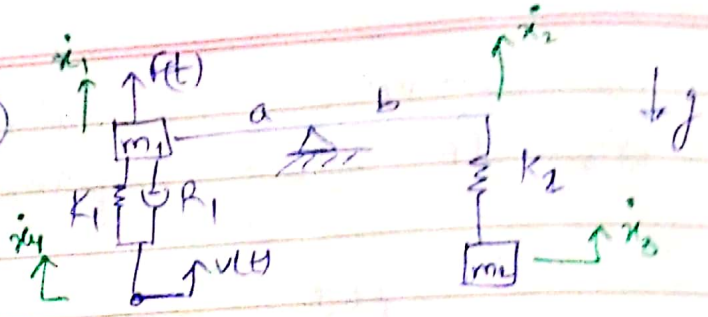
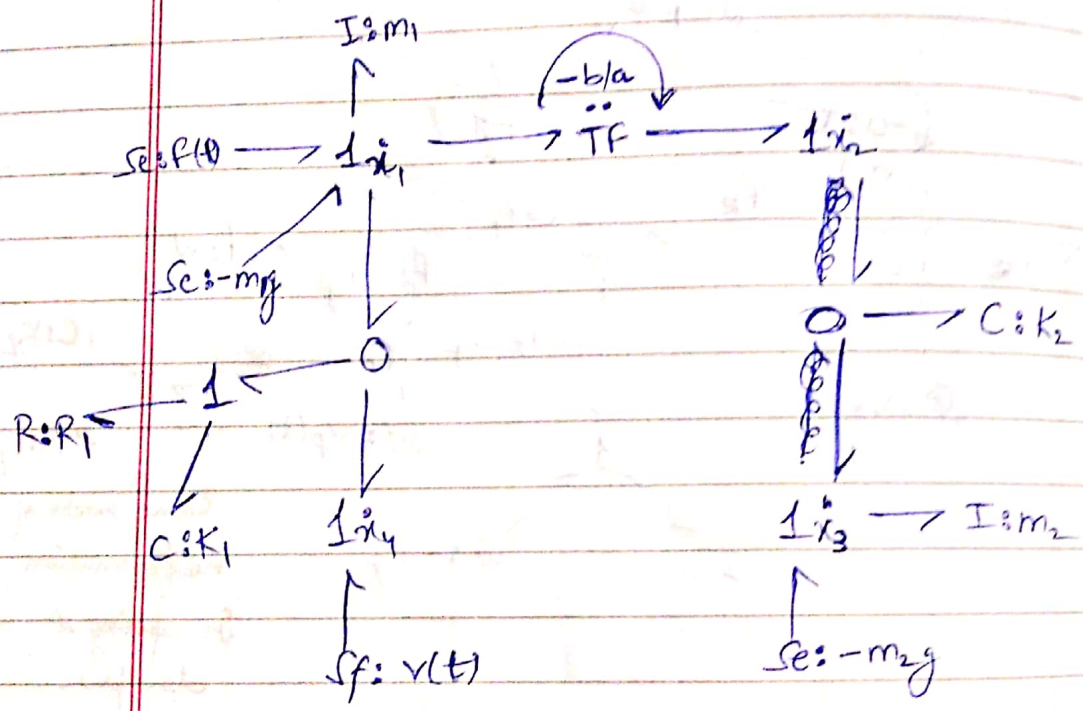
Sol^n ①





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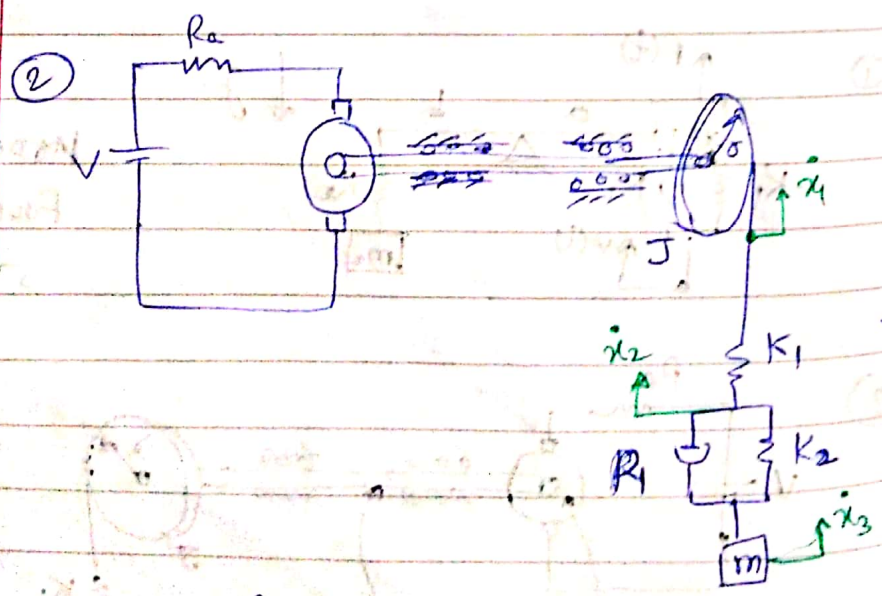

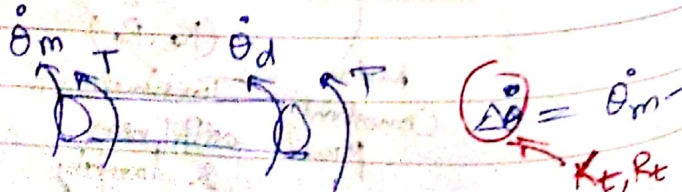

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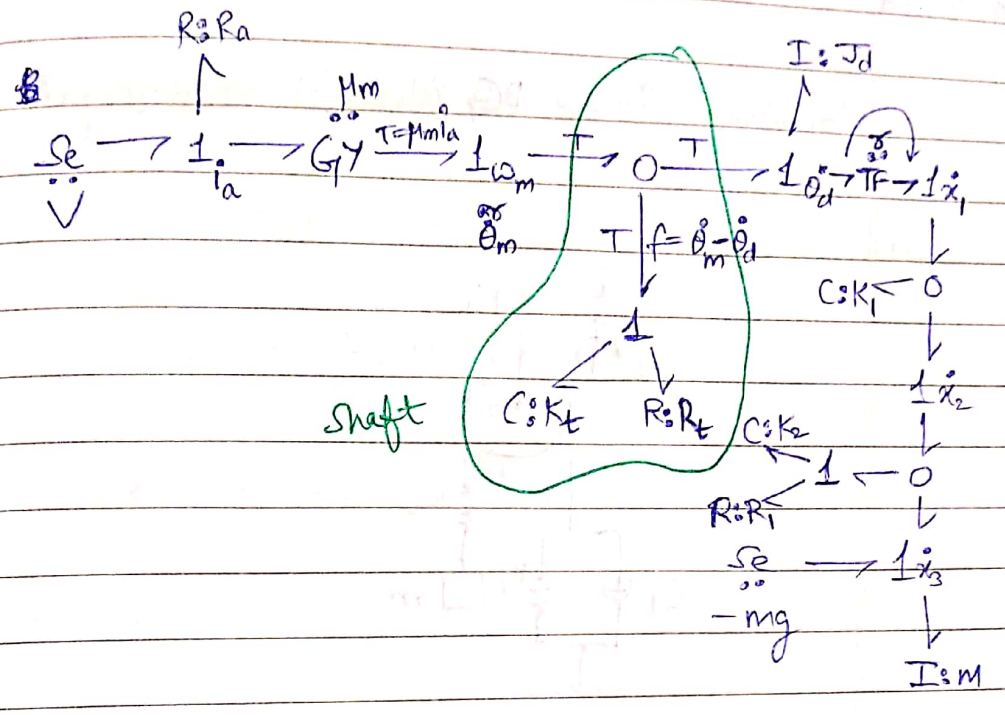
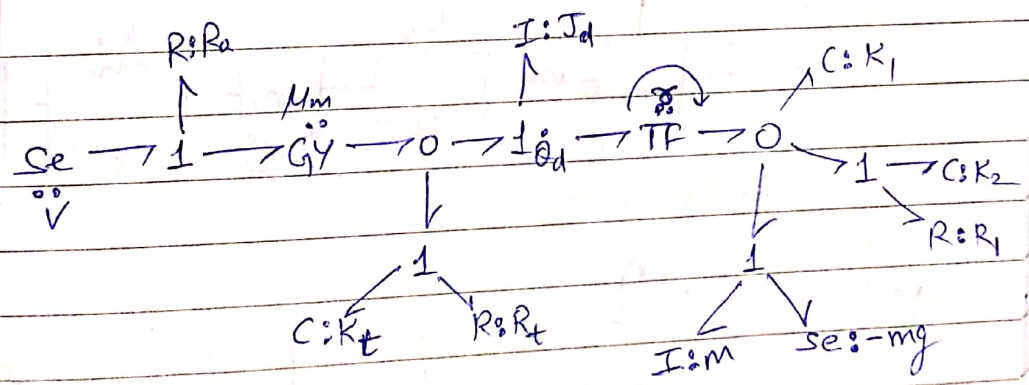

R:Ra
μm
Se
V
1
i_a
GY
$T=\mu m i_a$
1
ω_m
θ_m
T
0
T
f = $\dot{\theta}_m - \dot{\theta}_d$
1
C:Kt
R:Rt
shaft
I:Jd
1
θ_d
TF
1
$\dot{x}_1$
C:K1
0
1
$\dot{x}_2$
C:K2
1
0
R:R1
Se
-mg
1
$\dot{x}_3$
I:M

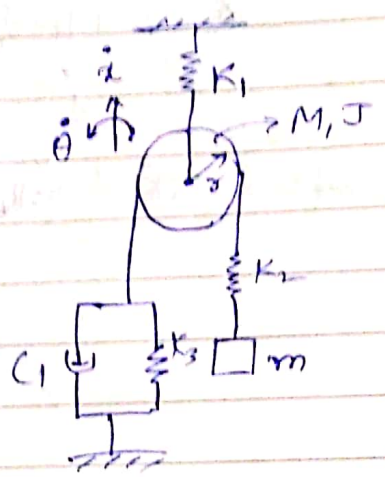
⇓


R:Ra
μm
I:Jd
C:K1
Se
V
1
GY
0
1
$\dot{\theta}_d$
TF
0
1
C:K2
R:R1
1
C:Kt
R:Rt
1
I:M
Se:-mg

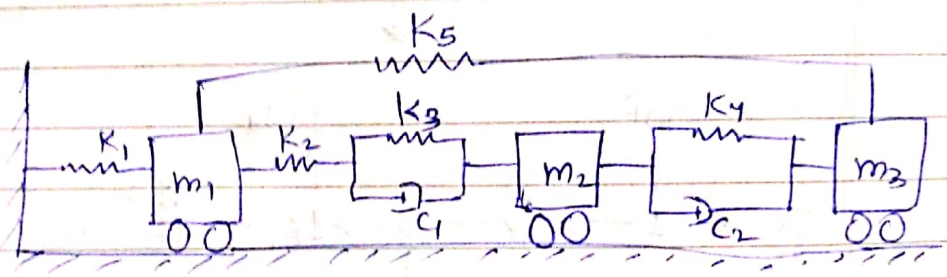
6/08/2021

Problems (From PGT Model)

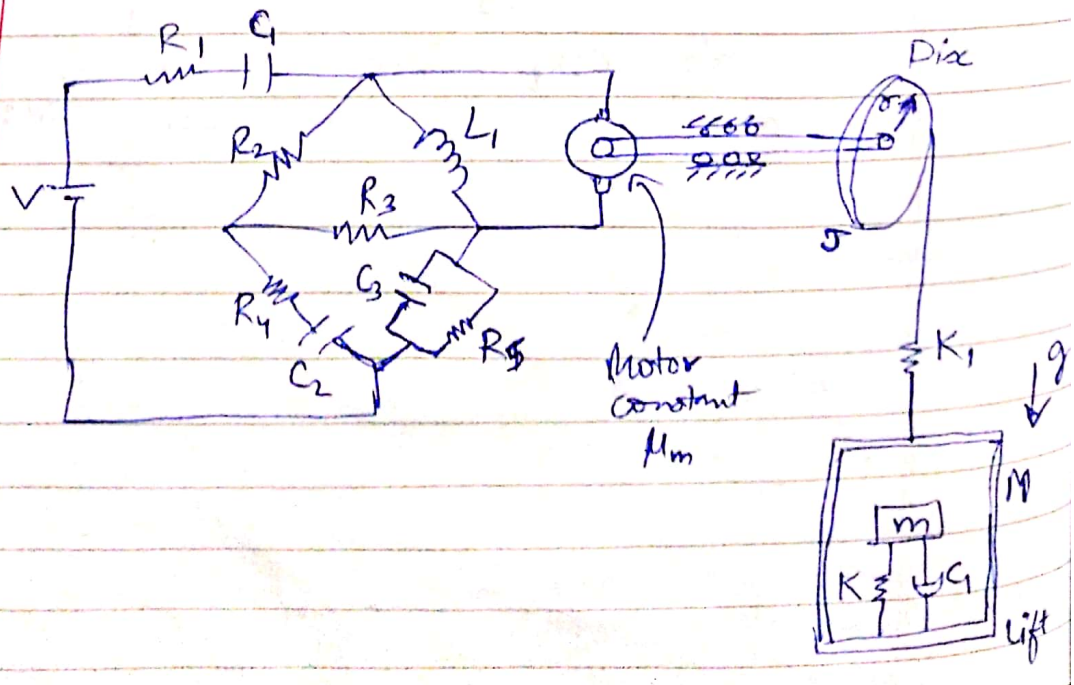
(i)



(ii)



(iii)



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13/06/2021

Information Exchange: → causality

Derive eq^n from bond graphs

cause $\xrightarrow[\text{to}]{\text{leading}}$ effect

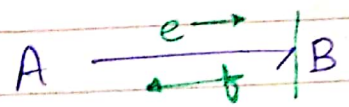
antecedent $\xrightarrow[\text{to}]{\text{leading}}$ consequence

$$\begin{cases} F = ma \\ v = \frac{1}{m}\int F\,dt \\ \text{Proper causal relation} \end{cases}$$

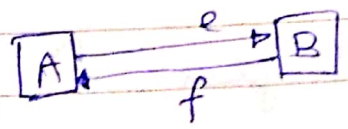
$$\underset{\text{effect}}{a} = \underbrace{b + c}_{\text{causes}} \leftarrow \text{causal (one variable on LHS)}$$

$$\left.\begin{array}{l} a - b - c = 0 \\ a - b = c \end{array}\right\} \text{not causal or acausal}$$

$$a = b + c - \sin(a) \quad \times$$



causal stroke



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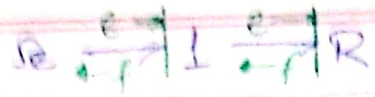
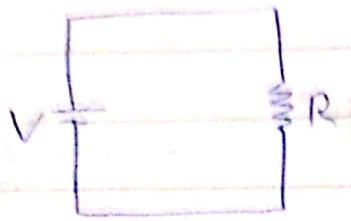
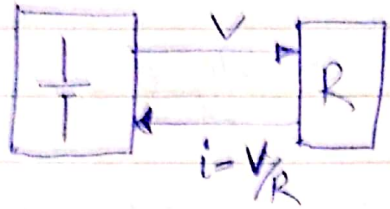
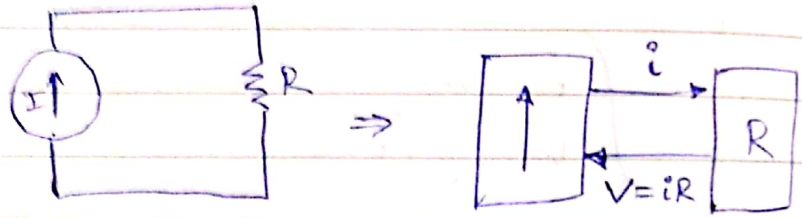
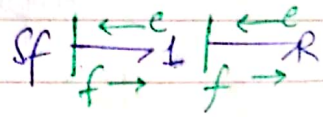






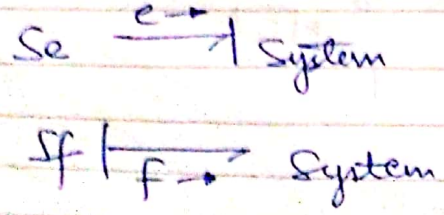




Power dir^n & causality are <u>unrelated</u>.

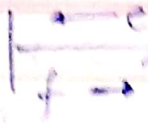
{ Stroked end receives effort information.
{ Unstroked end receives flow information.

<u>Causality in Elements:</u>

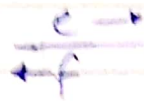


} fixed Causalities

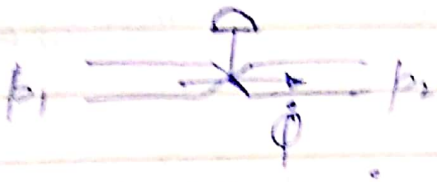
classmate
Date
Page



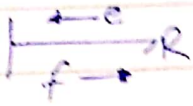
$e = Rf$



$f = e/R$



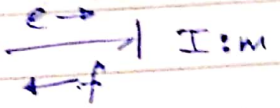
$e = \Delta P, \; f = \dot{\phi}$



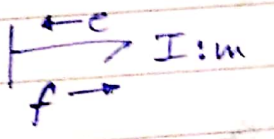
$\Delta P = \frac{\dot{\phi}^2}{C_d^2 A^2} \times 2\rho \Rightarrow e = f_R(f)$

$\dot{\phi} = C_d A \sqrt{\frac{\Delta P}{2\rho}}$

$\Rightarrow f = f_R(e)$



$\Rightarrow f = \frac{1}{m}\int e\,dt$ ✓ {INTEGRAL CAUSALITY}



$\Rightarrow e = m\frac{df}{dt}$ ✗ {DERIVATIVE OR DIFFERENTIAL CAUSALITY}

$\Rightarrow$ Integral causality is preferred.

$$f = \frac{1}{m}\int_{-\infty}^{t} e\,dt = \frac{1}{m}\left[\int_{0}^{t} e(\tau)\,d\tau + p(0)\right]$$

initial cond^ns can always be introduced

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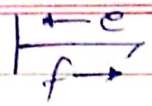
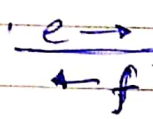


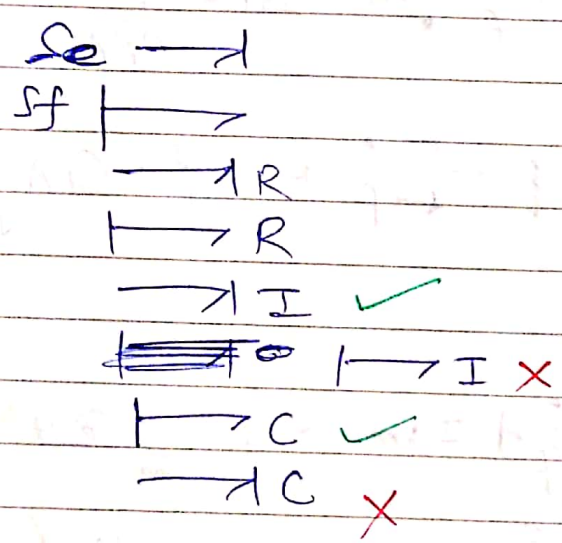
$\Rightarrow e = K\int f\,dt$

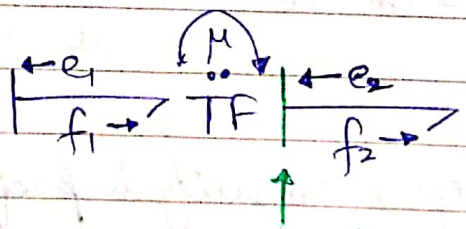
$= K\left[\int_0^t f(\tau)\,d\tau + q(0)\right]$

INTEGRAL CAUSALITY



$\Rightarrow f = \frac{1}{K}\frac{de(t)}{dt}$ ✗ DERIVATIVE CAUSALITY



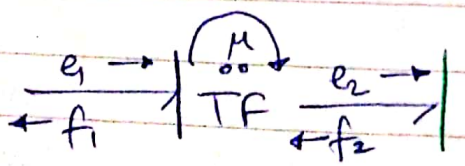


determined automatically by dir^n of first stroke

f_1 is known

$f_2 = \mu f_1$ } f_1 & e_2 are input

$e_1 = \mu e_2$ } f_2 & e_1 are output



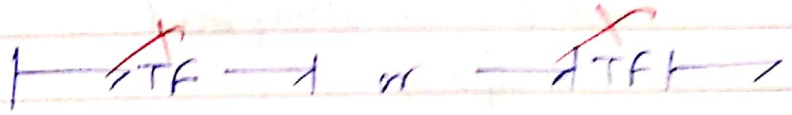
e_1 is known

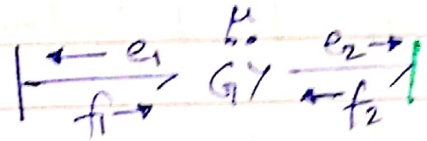
$$e_2 = \frac{1}{\mu} e_1$$
$$f_1 = \frac{1}{\mu} f_2$$

e_1 & f_2 are inputs

f_1 & e_2 are outputs

One causal stroke near 1 & one away from TF.

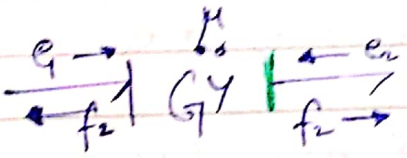




f_1 is known

$e_2 = \mu f_1$

$e_1 = \mu f_2$

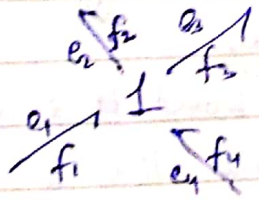


e_1 is known

$f_2 = e_1/\mu$

$f_1 = e_2/\mu$

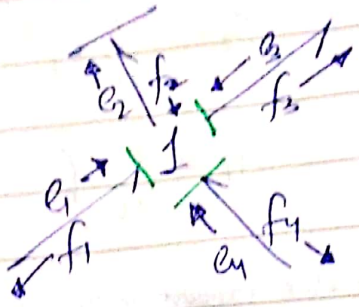
Junctions



$$f_1 = f_2 = f_3 = f_4$$

f_4 is known

The bond with known flow is called strong bond (master) others are weak (slave) bonds.



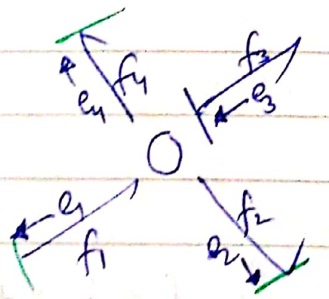
say,
bond #2 is strong bond

Then
$$\left.\begin{aligned} f_1 &= f_2 \\ f_3 &= f_2 \\ f_4 &= f_2 \end{aligned}\right\} \text{strong law of 1-junction}$$

$\Rightarrow$ f_1, f_3 & f_4 are outputs

Weak law of 1-junction,
$$e_2 = e_1 - e_3 + e_4$$

$\left\{\begin{array}{l}\text{LHS \& RHS terms depend on causality.} \\ \text{+ \& − signs depend on power direction.}\end{array}\right.$



Strong (master) bonds decide effort at the junc^n.
say,
bond # 3 is strong bond.
Bond #s 1, 2 & 4 are weak (slave) bonds.

$$\left.\begin{aligned} e_1 &= e_3 \\ e_2 &= e_3 \\ e_4 &= e_3 \end{aligned}\right\} e_1, e_2 \text{ \& } e_4 \text{ are outputs}$$

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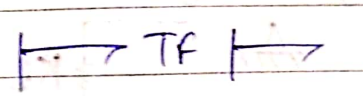
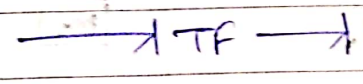
weak law,

$$f_3 = f_1 - f_2 - f_4$$

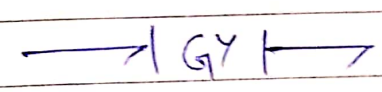
NOTE: e_3 came on RHS so f_3 came on LHS in 0 junc^n example.
f_2 came on RHS so e_2 came on LHS in 1 junc^n example.

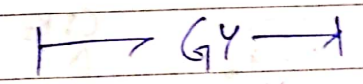
NOTE:



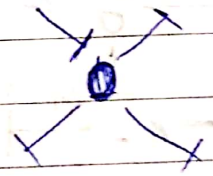


one near, one away

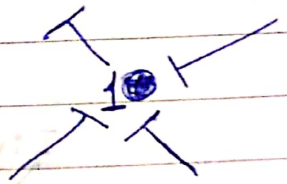




both near or both away

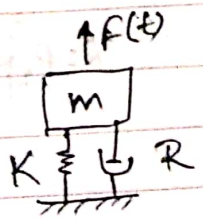


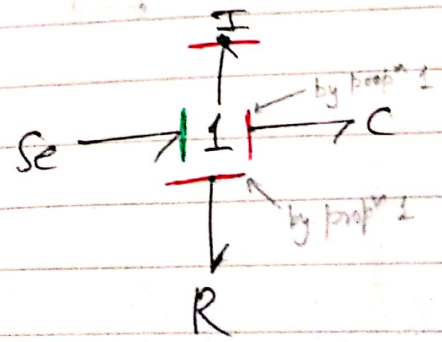
one near, others away



one away, others near

Prob. ①





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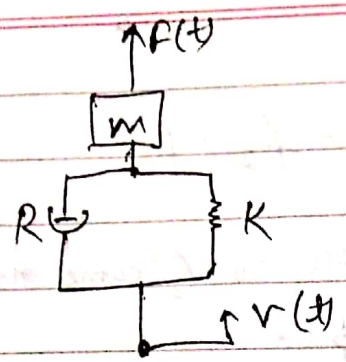
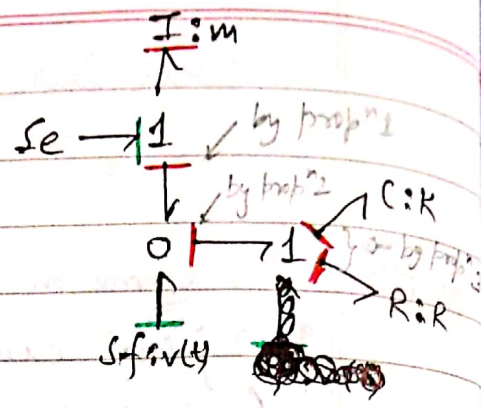
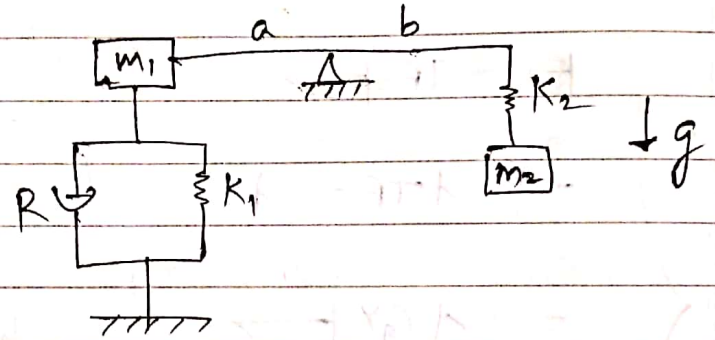
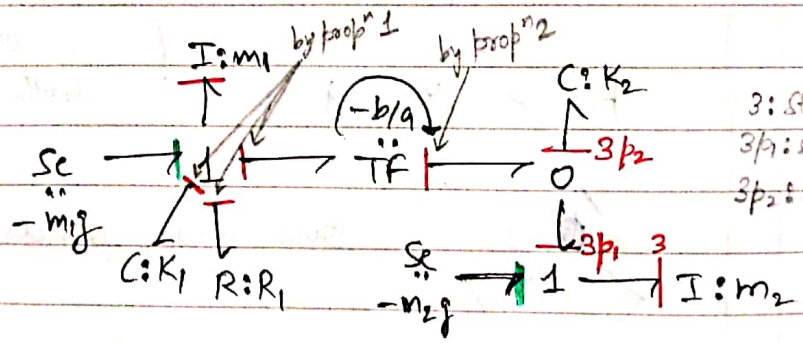
(2)





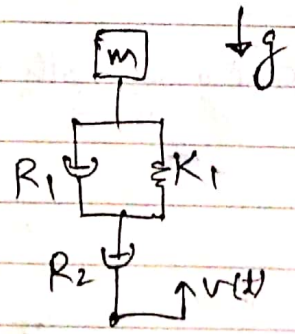
(3)

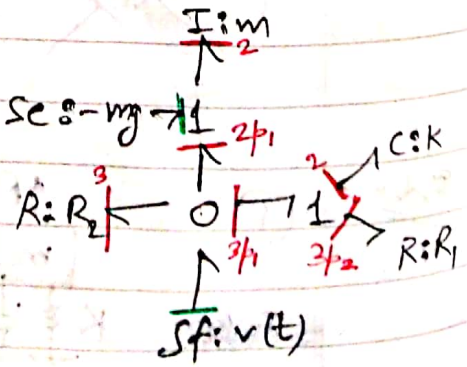




3: Step 3
3p1: Step 3 prop^n 1
3p2: Step 3 prop^n 2

(4)





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Sequential Causality Assignment Procedure (SCAP Algo)

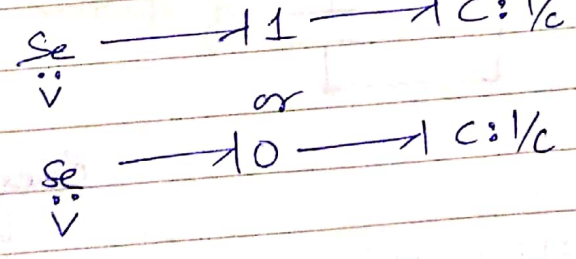
(i) Assign fixed causalities to sources, one-by-one & propogate. (based on last note)

(ii) Assign preferred integral causality to I & C elements one-by-one & propogate ($\dashv I$ & $\vdash C$).
(First prefer I in mechanical system & First prefer C in electrical system.)

(iii) Use arbitrarily causality in R-element one-by-one & propogate.

19/08/2019

Problem with Differential or Derivative Causality :



$\vdash\!\!\rightarrow C:K \qquad e = K\int f\,dt$

$\dashv\!\!\rightarrow C:K \qquad f = \frac{1}{K}\frac{de}{dt}$ ✗

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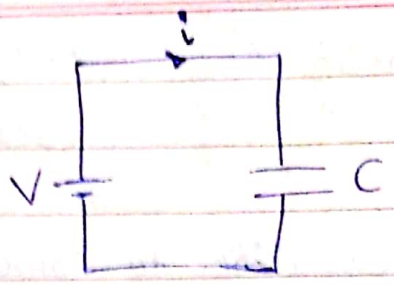


$$E_C = \frac{\varphi^2}{2C}$$

$$E_V = \int V i\,dt$$

$$= V\int i\,dt$$

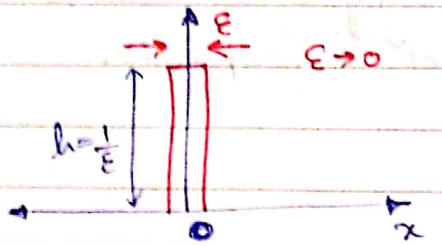
$$= V\varphi = \frac{\varphi^2}{C}$$

$$\therefore E_C = \frac{E_V}{2}$$

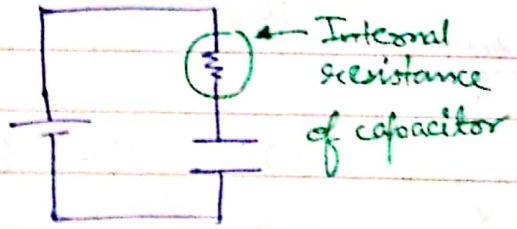
But where is another half of the energy?

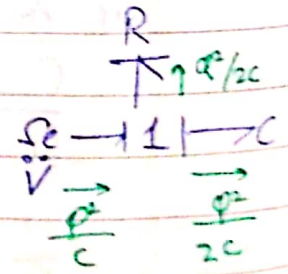
$\Downarrow$

energetically inconsistent



$$\int_{-\infty}^{\infty} h\,dx = 1$$





Energetically consistent

Differential causality $\longrightarrow$ Loss of time constant or information exchange at 0 time.

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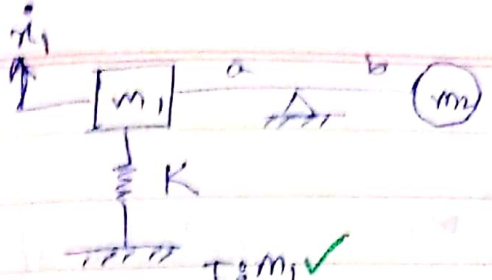
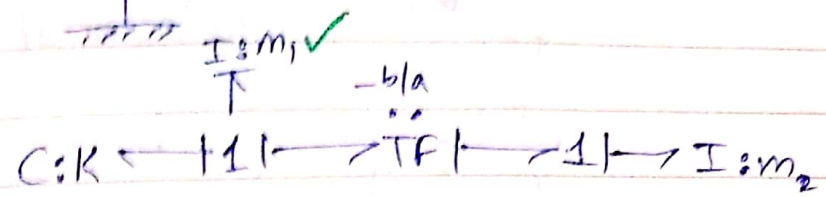




Differential causality

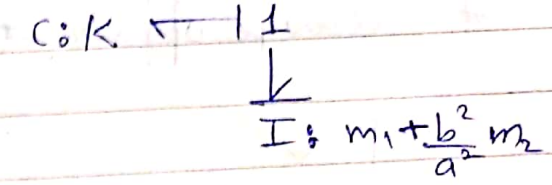
D.C. is ~~too~~ this system has ~~two~~ one dof but we have assumed 2 dof.

∴ Either transfer m_2 to m_1 or assume rod to be flexible.

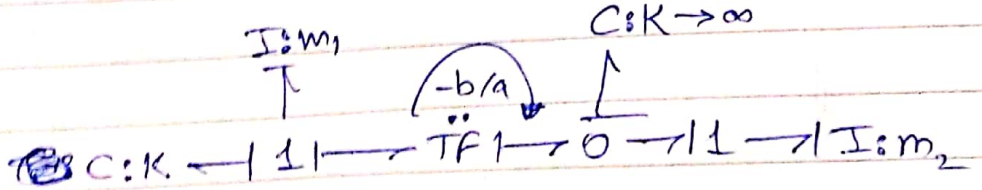
$$\frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \left[\left(-\frac{b}{a}\right)\dot{x}_1\right]^2$$

$$= \frac{1}{2}\underbrace{\left[m_1 + \left(\frac{b^2}{a^2}\right) m_2\right]}_{\text{equivalent mass}} \dot{x}_1^2$$

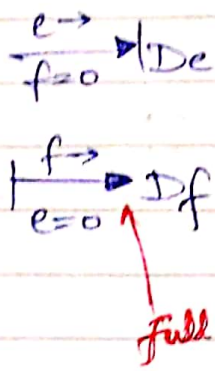
$\Rightarrow$



Or



Sensors:



(Detector of effort)

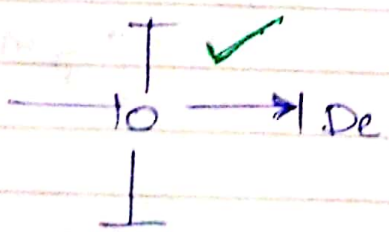
(Detector of flow)

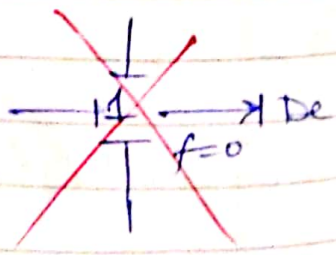
→ power connection

→ Signal/no power connection

For De, f=0 and for Df, e=0 so that power = 0 which implies that the sensors never draw ~~or provide~~ power from ~~or to~~ the system. Their source of power is external.

De & Df causalities are fixed.
De always appear at 0-junc^n.
Df always appear at 1-junc^n.





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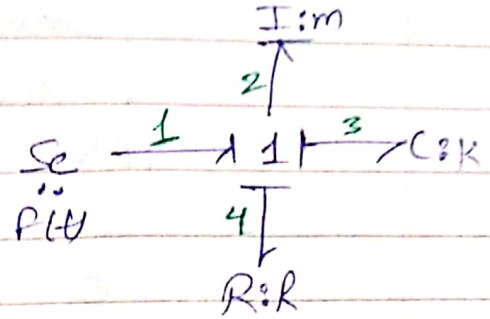
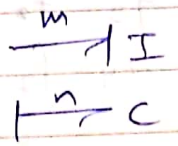
State-Space Eqns :



Augment the model
⇒ numbering of bonds
⇒ assign causality
⇒ arrange state, input & output vectors

All I and C in integral causality lead to state ~~vect~~ variables.



$p_m = \int e_m dt$

$q_n = \int f_n dt$

For the given system model,

State vector, $X = \begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix}$, $\dot{X} = \begin{Bmatrix} e_2 \\ f_3 \end{Bmatrix}$

Input vector, $U = \{Se_1\}$

Output vector, $Y = \phi$

$$\left.\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX + DU \end{aligned}\right\} \text{form of equation}$$

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Step 1: Write all constitutive element eqns (sources, parameters & states as known)

(AS PER CAUSALITY)

4 eqns
$$\begin{cases} e_1 = se_1 \\ f_2 = p_2/m_2 \\ e_3 = K_3 q_3 \\ e_4 = R_4 f_4 \end{cases}$$

Step 2: Write ~~constant~~ constraint (1, 0, TF, GY) eqns as per causality.

4 eqns
$$\begin{cases} \text{Strong law,} & f_1 = f_2 \\ & f_3 = f_2 \\ & f_4 = f_2 \\ \text{Weak law,} & e_2 = e_1 - e_3 - e_4 \end{cases}$$

Total 8 eqns, unknowns = 8 ($e_1, f_1, e_2, f_2, \ldots, e_4, f_4$)

Step 3: Triangular ordering of equations (Pick the eqns whose RHS is known & tick them after writing down)

$e_1 = se_1$

$f_2 = p_2/m_2$

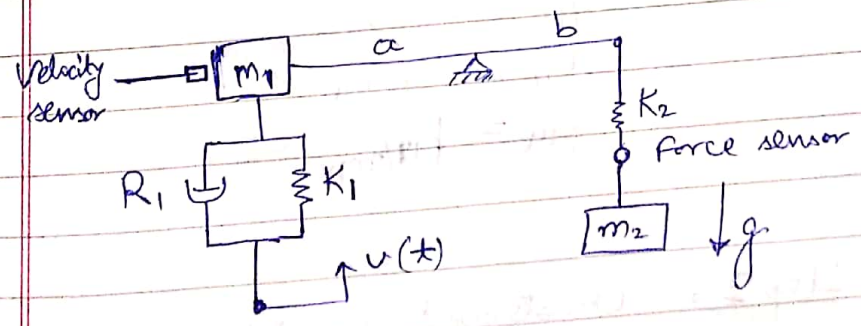
$e_3 = K_3 q_3$

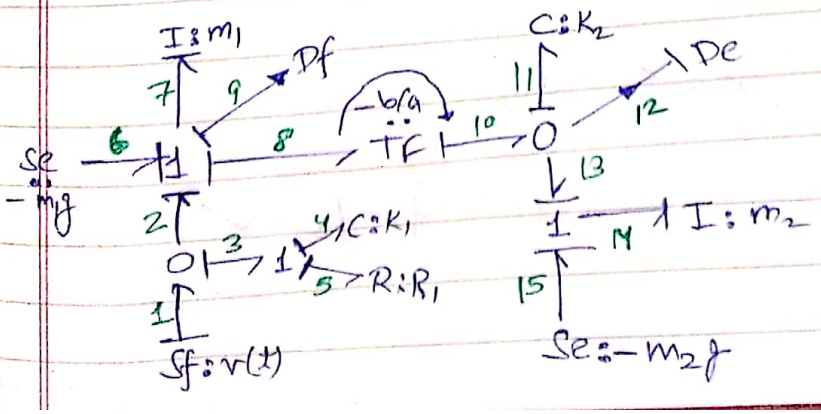
$$f_1 = p_2/m_2$$
$$f_3 = p_2/m_2$$
$$f_4 = p_2/m_2$$

$$e_4 = R_4 p_2/m_2$$
$$e_2 = Se_1 - K_3 q_3 - R_4 p_2/m_2$$

$$\dot{X} = AX + BU$$

$$\Rightarrow \frac{d}{dt}\begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} e_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} -R_4/m_2 & -K_3 \\ 1/m_2 & 0 \end{bmatrix}\begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\{Se_1\}$$





$$X = \begin{Bmatrix} p_7 \\ p_{14} \\ q_4 \\ q_{11} \end{Bmatrix}, \quad \dot{X} = \begin{Bmatrix} e_4 \\ e_{14} \\ f_4 \\ f_{11} \end{Bmatrix}, \quad U = \begin{Bmatrix} Sf_1 \\ Se_6 \\ Se_{15} \end{Bmatrix}$$

$$Y = \begin{Bmatrix} f_9 \\ e_{12} \end{Bmatrix}$$

Step 1: Element equations

10 eqns

$$\begin{cases} f_1 = Sf_1 \\ e_6 = Se_6 \\ e_{15} = Se_{15} \\ e_4 = K_4 q_4 \\ e_5 = R_5 f_5 \\ f_7 = p_7/m_7 \\ e_9 = 0 \\ f_{12} = 0 \\ e_{11} = K_{11} q_{11} \\ f_{14} = p_{14}/m_{14} \end{cases}$$

sensors

Step 2: Constraint equations

$0_{1,2,3} \Rightarrow$

$$e_7 = e_3$$
$$e_1 = e_4$$
$$f_3 = f_1 - f_2$$

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$1_{3,4,5} \Rightarrow$ $f_4 = f_3$

$f_5 = f_3$

$e_3 = e_4 + e_5$

$1_{2,6,7,8,9} \Rightarrow$ $f_6 = f_7$

$f_2 = f_7$

$f_9 = f_7$

$f_8 = f_7$

$e_7 = e_6 + e_2 - e_8 - e_9$

$0_{10,11,12,13} \Rightarrow$ $e_{10} = e_{11}$

$e_{12} = e_{11}$

$e_{13} = e_{11}$

$f_{11} = f_{10} - f_{12} - f_{13}$

$1_{13,14,15} \Rightarrow$ $f_{13} = f_{14}$

$f_{15} = f_{14}$

$e_{14} = e_{15} + e_{13}$

$TF_{8,10} \Rightarrow$ $f_{10} = -\frac{b}{a} f_8$

$e_8 = -\frac{b}{a} e_{10}$

20 eqn.

Step 3: Sorting

$f_1 = sf_1$

$e_6 = se_6$

$e_{15} = se_{15}$

$e_4 = K_4 q_4$

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$f_7 = p_7/m_7$

$e_9 = 0$

$f_{12} = 0$

$e_{11} = K_{11} q_{11}$

$f_{14} = p_{14}/m_{14}$

$f_6 = p_7/m_7$

$f_2 = p_7/m_7$

$f_9 = p_7/m_7$ sensor equations

$f_8 = p_7/m_7$

$e_{10} = K_{11} q_{11}$

$e_{12} = K_{11} q_{11}$

$e_{13} = K_{11} q_{11}$

$f_{13} = p_{14}/m_{14}$

$f_{15} = p_{14}/m_{14}$

$f_{10} = -\frac{b}{a} \frac{p_7}{m_7}$

$e_8 = -\frac{b}{a} K_{11} q_{11}$

$f_3 = Sf_1 - p_7/m_7$

$f_4 = Sf_1 - p_7/m_7$ $\frac{dq_4}{dt}$

$f_5 = Sf_1 - p_7/m_7$

$f_{11} = -\frac{b}{a} \frac{p_7}{m_7} - 0 - p_{14}/m_{14}$ $\frac{dq_{11}}{dt}$

$e_{14} = Se_{15} + K_{11} q_{11}$ $\frac{dp_{14}}{dt}$

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$$e_5 = R_5\left(Sf_1 - \frac{p_7}{m_7}\right)$$

$$e_3 = K_4 q_4 + R_5\left(Sf_1 - \frac{p_7}{m_7}\right)$$

$$e_2 = K_4 q_4 + R_5\left(Sf_1 - \frac{p_7}{m_7}\right)$$

$$e_1 = K_4 q_4 + R_5\left(Sf_1 - \frac{p_7}{m_7}\right)$$

$$\cancel{e_7 = K_4 q_4 + R_5\left(Sf_1 - \frac{p_7}{m_7}\right)}$$

$$\frac{dp_7}{dt} = e_7 = Se_6 + K_4 q_4 + R_5\left(Sf_1 - \frac{p_7}{m_7}\right) + \frac{b}{a} \cdot K_{11} q_{11} - 0$$

30 eqns all with known RHS.

$$\frac{d}{dt}\begin{Bmatrix} p_7 \\ p_{14} \\ q_4 \\ q_{11} \end{Bmatrix} = \begin{bmatrix} -\frac{R_5}{m_7} & 0 & K_4 & \frac{b}{a}K_{11} \\ 0 & 0 & 0 & K_{11} \\ -\frac{1}{m_7} & 0 & 0 & 0 \\ -\frac{b}{am_7} & -\frac{1}{m_{14}} & 0 & 0 \end{bmatrix} \begin{Bmatrix} p_7 \\ p_{14} \\ q_4 \\ q_{11} \end{Bmatrix}$$

$$+$$

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$$\begin{bmatrix} R_5 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} Sf_1 \\ Se_6 \\ Se_5 \end{Bmatrix}$$

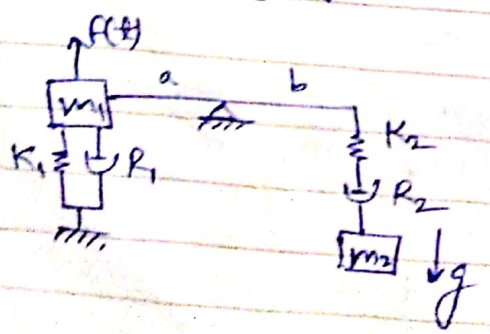
$$Y = \begin{Bmatrix} f_9 \\ e_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{m_7} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{11} \end{bmatrix} \begin{Bmatrix} p_7 \\ p_4 \\ q_4 \\ q_{11} \end{Bmatrix}$$

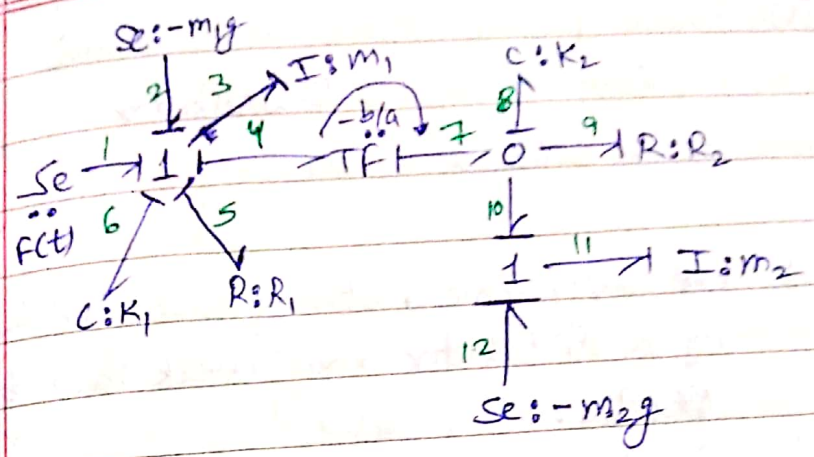
$+ DU$ not present in this case

20/08/2019

Block Diagram Model

Matlab Simulink



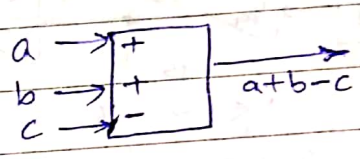


Prepare the model for eqn derivation.

NOTE :

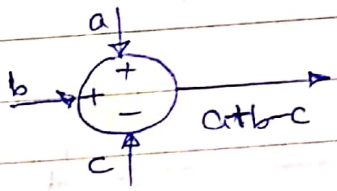
(i)

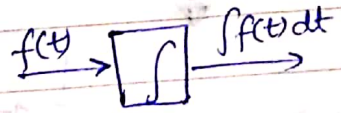
gain block

(ii) 

Sum blocks

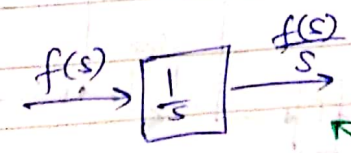
or



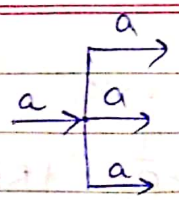
(iii) 

Integrator

or



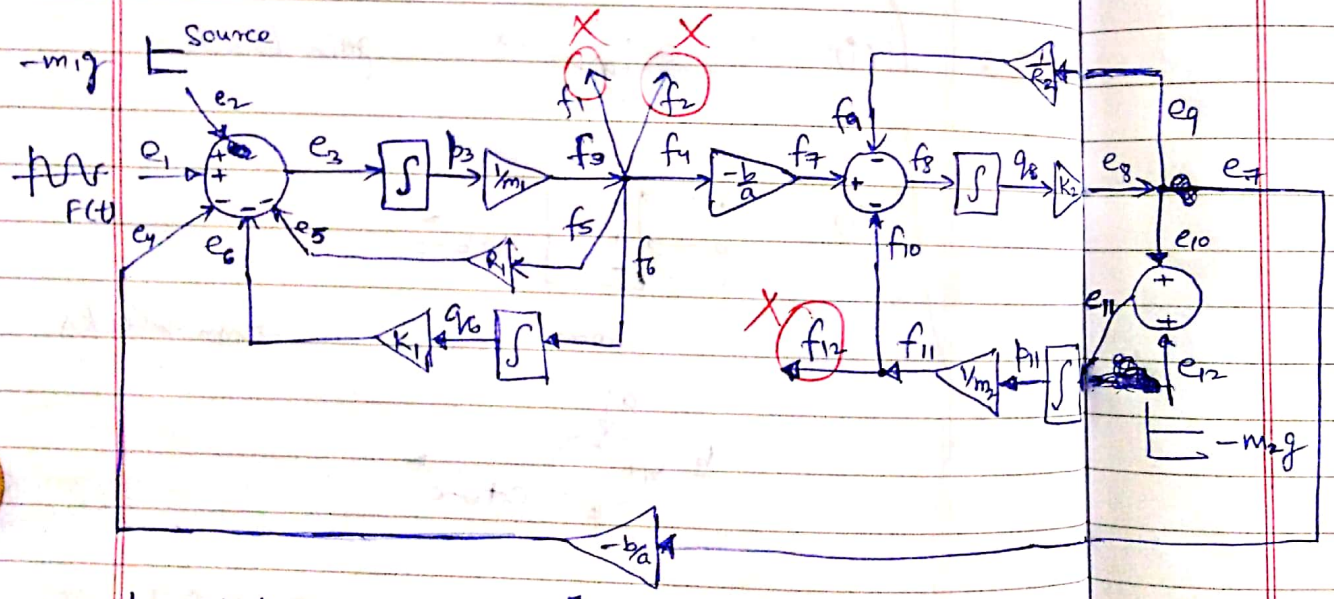
Laplace transform of integration

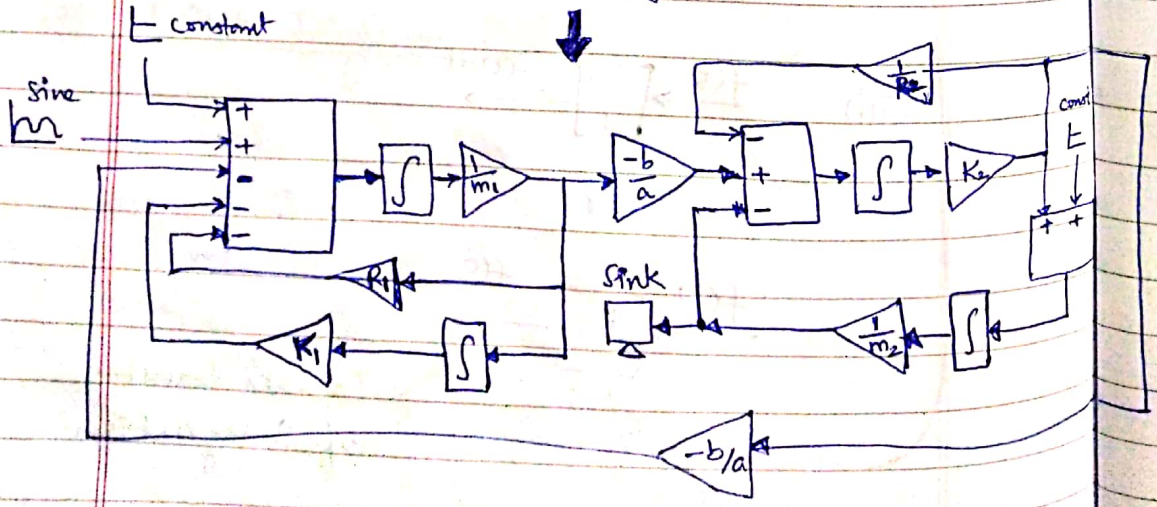
(iv) 

Distributor

(v) For every jmc^n, strong law will be shown by a distributor and weak law by sum blocks.

(vi) For transformer & gyrator there will use be gain blocks.





Sol^n:

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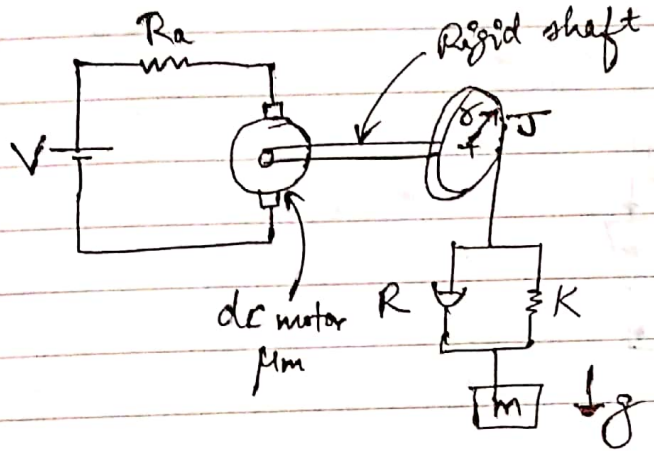
Prob. ~~Draw~~ Draw block diagram model and simulate in Simulink for 10 sec. duration. Output velocity of mass. Use bond no.s as shown.

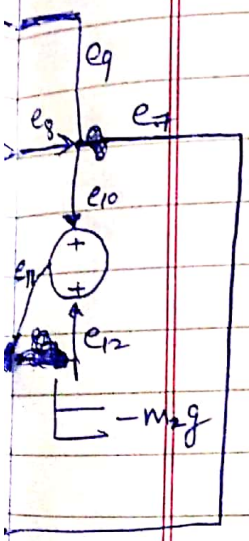
Data :

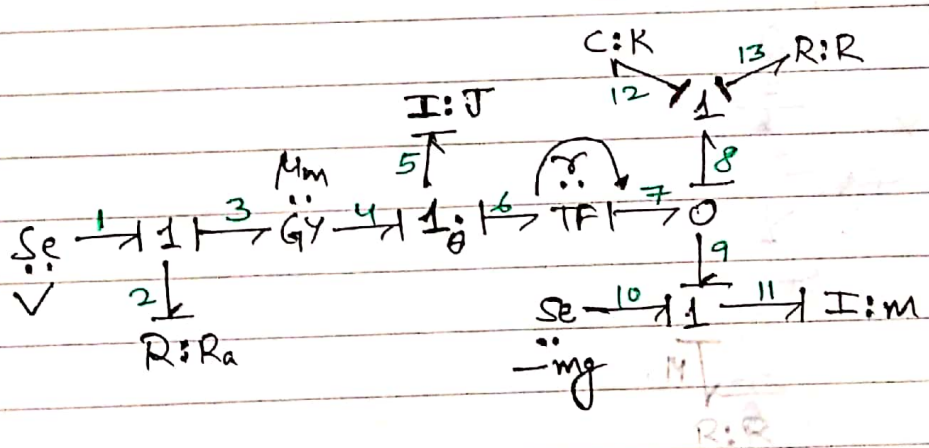
$V = 50\,V$, $R_a = 10\,\Omega$, $\mu_m = 0.5$ Nm/A

$J = 0.1$ kg m^2, $r = 0.1$ m, $m = 1$ kg

$K = 1000$ N/m, $R = 10$ Ns/m.

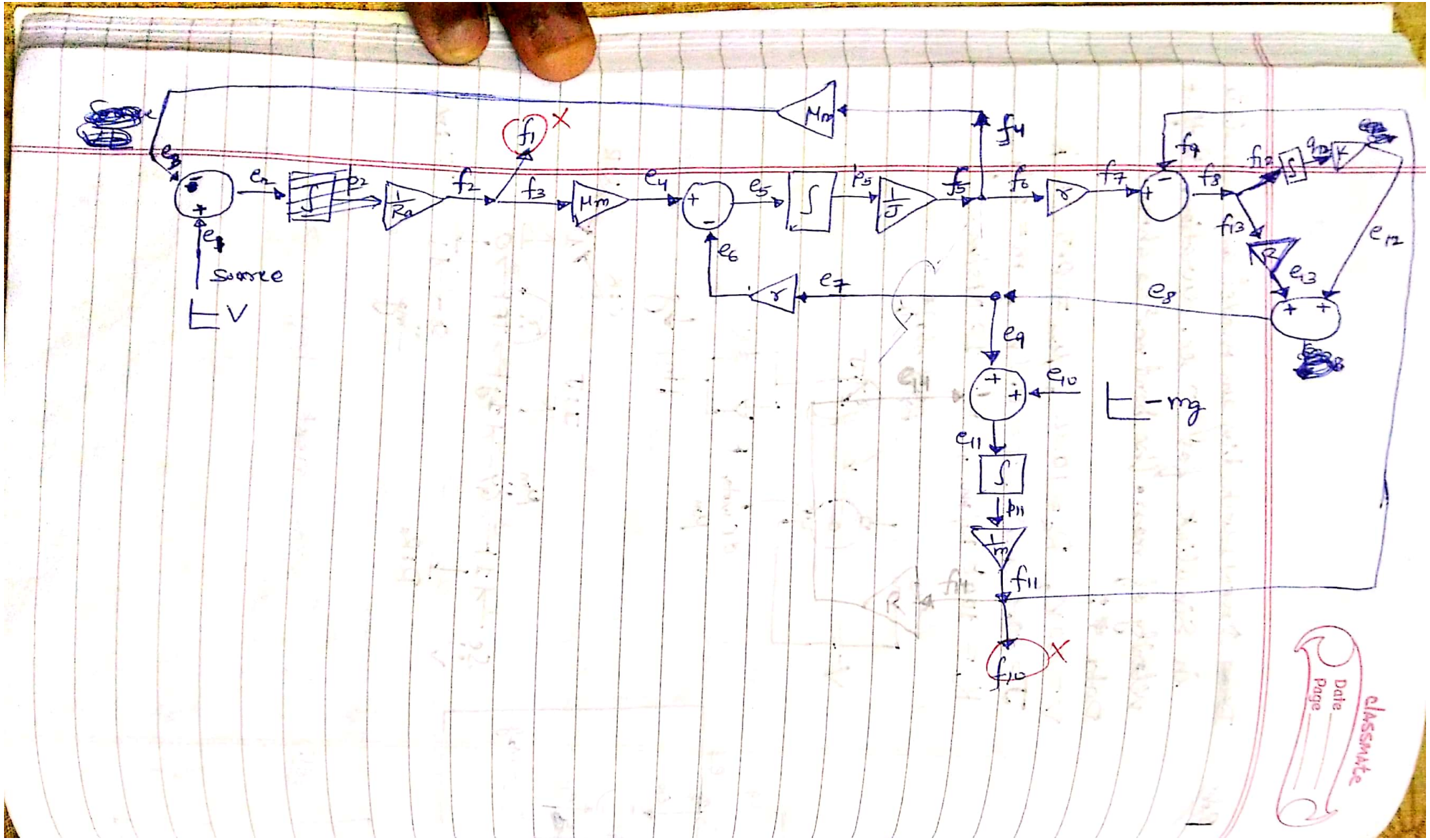






Sol^n:

on next page →



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page #1

Name Roll no.

Block diagram copied from simulink

Signature

page #2 (back to back)

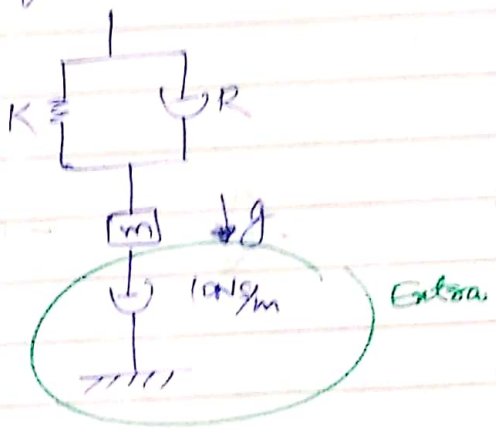
Name Roll no.

Signature

Submit printout (1 page back to back) with signature

No photocopy is allowed.

Extra mark for including drag on mass.



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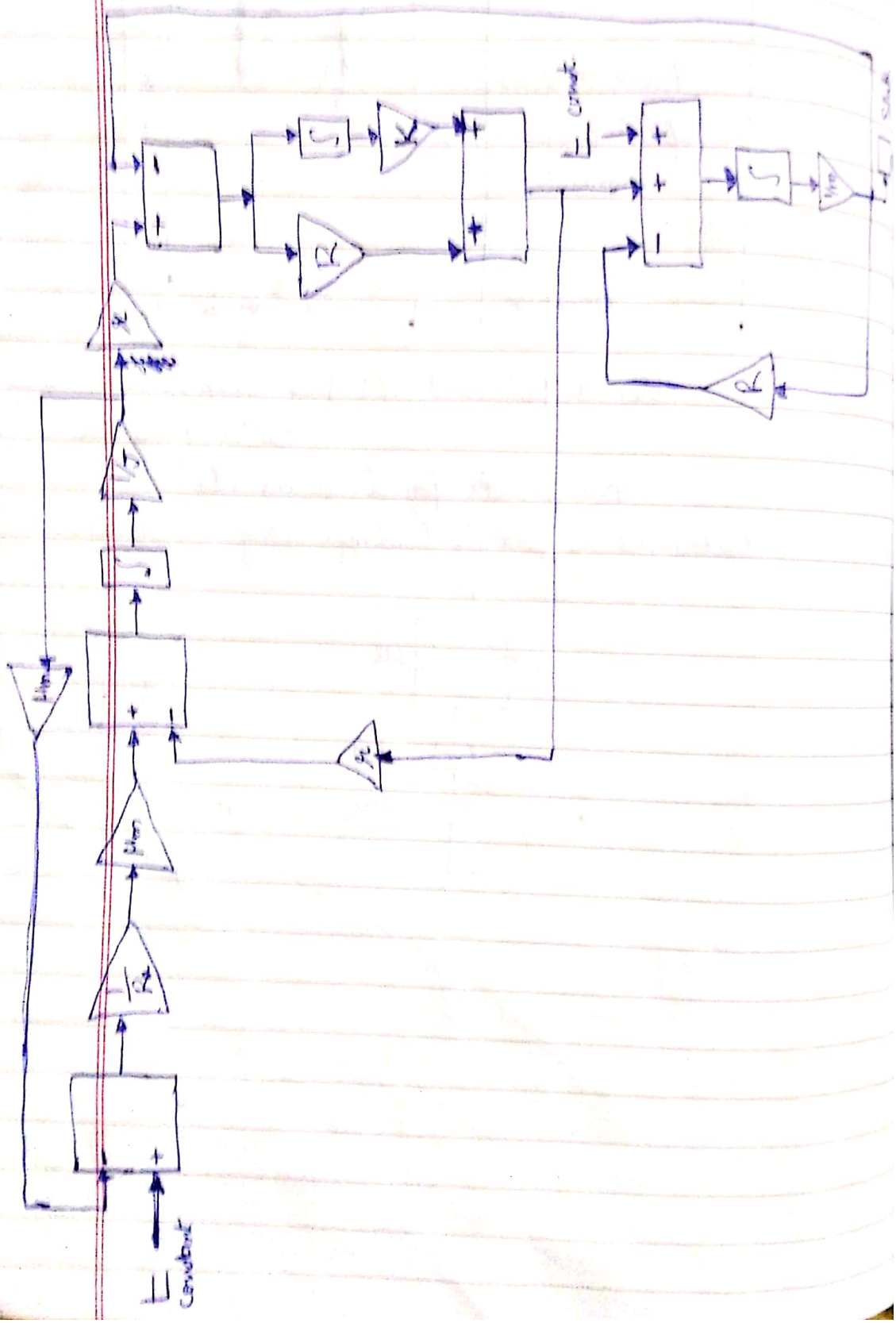
Block diagram model with drag on mass.



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26/08/2019

Laplace Transform :

Analyze the system in frequency domain

with $s = \sigma + j\omega$

freq.

Earlier we were analyzing in time domain.

$$f(s) = \mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\left[\int f(t)\, dt\right] = \frac{1}{s} f(s)$$

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sf(s) - f'(0^-)$$

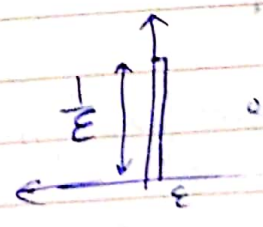
$$\mathcal{L}\left[\frac{d^2 f(t)}{dt}\right] = s^2 f(s) - sf'(0^-) - f''(0^-)$$

All initial cond^ns are zero (for freq. domain).

$f'(0), f''(0), f'''(0), \ldots = 0$

$\mathcal{L}[\delta(t)] = 1$

unit impulse

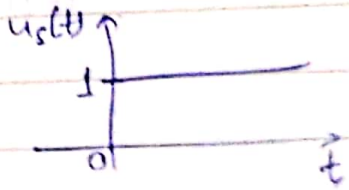


$$\int_0^\infty \delta(t)\, dt = \int_0^\infty \frac{1}{\varepsilon} e^{-st}\, dt \quad (e^{-st} \simeq 1)$$

$$= 1$$

$\mathcal{L}[u_s(t)] = 1/s$

unit step



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$$u_r(t) = \int \delta(t)\, dt$$

$$\mathcal{L}[u_r(t)] = \mathcal{L}\left[\int \delta(t)\, dt\right] = \frac{1}{s}\mathcal{L}[\delta(t)] = \frac{1}{s}$$

$$\mathcal{L}[e^{-at}] = \frac{1}{s+a}$$

$$\mathcal{L}[\sin\omega t] \text{ or } \mathcal{L}[\cos\omega t]$$

$$\downarrow$$

$$\mathcal{L}[e^{i\omega t}] = \frac{1}{s-i\omega t} = \underbrace{\frac{s}{s^2+\omega^2}}_{\cos} + i\underbrace{\frac{\omega}{s^2+\omega^2}}_{\sin}$$

Final Value Theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to 0} s.f(s)$$

Transfer Function

$$\xrightarrow[\text{input}]{u(t)} \boxed{\text{System}} \xrightarrow[\text{output}]{y(t)}$$

$$\text{Transfer func}^n = \frac{\text{L.T. of output}}{\text{L.T. of input}}$$

with all initial cond^{ns} being zero.

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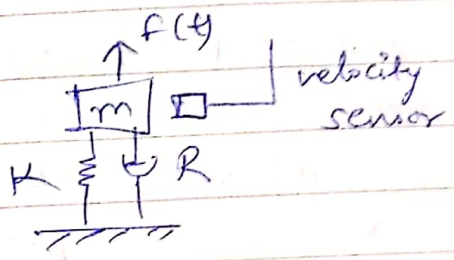
$\cancel{G(s) = \mathcal{L}[y]}$

$$G(s) = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} = \frac{y(s)}{u(s)}$$



$$m\ddot{x}(t) + R\dot{x}(t) + Kx(t) = f(t)$$

$$\Rightarrow \mathcal{L}[m\ddot{x}(t) + R\dot{x}(t) + Kx(t)] = \mathcal{L}[f(t)]$$

$$\Rightarrow ms^2x(s) + Rsx(s) + Kx(s) = f(s)$$

$$\Rightarrow (ms^2 + Rs + K).x(s) = f(s)$$

$$\Rightarrow \frac{x(s)}{f(s)} = \frac{1}{ms^2 + Rs + K}$$

$$\mathcal{L}[\dot{x}(t)] = sx(s)$$

So, $$G(s) = \frac{\mathcal{L}[y(t)]}{\mathcal{L}[u(t)]} = \frac{\mathcal{L}[\dot{x}(t)]}{\mathcal{L}[f(t)]}$$

$$= \frac{sx(s)}{f(s)}$$

$$= \frac{s}{ms^2 + Rs + K}$$

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State-Space form

$$\dot{X}(t) = AX(t) + Bu(t)$$

$$Y(t) = CX(t) + \cancel{Du(t)}$$

usually $D = 0$

$$sX(s) = AX(s) + Bu(s)$$

~~or SI - A~~

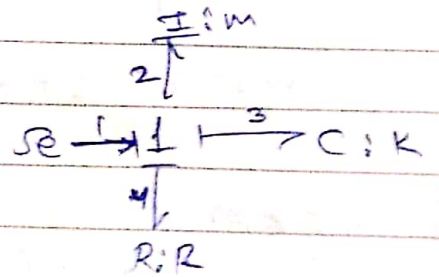
or, $(sI - A)X(s) = Bu(s)$

or, $X(s) = (sI - A)^{-1}Bu(s)$

$$Y(s) = CX(s)$$

$$\Rightarrow Y(s) = C(sI - A)^{-1}Bu(s)$$

$$\Rightarrow \boxed{\frac{Y(s)}{u(s)} = C(sI - A)^{-1}B}$$



$$\frac{d}{dt}\begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix} = \overbrace{\begin{bmatrix} -R/m & -K \\ 1/m & 0 \end{bmatrix}}^{A}\begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix} + \overbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}^{B} f_1$$

$$Y = \frac{dq_3}{dt} = \underbrace{\begin{bmatrix} \frac{1}{m} & 0 \end{bmatrix}}_{C}\begin{Bmatrix} p_2 \\ q_3 \end{Bmatrix}$$

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$$\frac{Y(s)}{u(s)} = \begin{bmatrix} \frac{1}{m} & 0 \end{bmatrix} \begin{bmatrix} s+\frac{R}{m} & K \\ -\frac{1}{m} & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{\frac{1}{m} \cdot s}{\left(s^2 + \frac{Rs}{m} + \frac{K}{m}\right)} \longleftarrow |sI - A|$$

Roots of denominator of $G(s)$, i.e. roots of $|sI - A| = 0$ are <u>eigen values</u>.

<u>Time Response:</u>

$$\frac{y(s)}{u(s)} = G(s) \Rightarrow y(s) = u(s) \cdot G(s)$$

$$\therefore y(t) = \mathcal{L}^{-1}\left[u(s) \cdot G(s)\right]$$

Let $f(t) = A\cos\omega t \Rightarrow u(s) = \frac{As}{s^2+\omega^2}$

$$y(s) = \left(\frac{As}{s^2+\omega^2}\right) \times \left(\frac{s}{ms^2+Rs+K}\right)$$

$$= \underbrace{\frac{Bs+C}{s^2+\omega^2} + \frac{Ds+E}{ms^2+Rs+K}}_{\text{Partial fraction}}$$

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Stability:

A system is stable if all its eigen values have negative real part.

Roots of $|sI-A|$ must have negative real part in root.

$$G(s) = \frac{N(s)}{D(s)} = \frac{\text{Numerator polynomial}}{\text{Denominator polynomial}}$$

Roots of $N(s)=0$ are called zeroes of the system.

Roots of $D(s)=0$ are called poles of the system (eigen values).

$$G(s) = \frac{\prod_{i=1}^{m}(s-z_i)}{\prod_{j=1}^{n}(s-p_j)}$$

$z_1, z_2, \ldots, z_m$ are zeroes.

$p_1, p_2, \ldots, p_n$ are poles.

$n > m$ always!! → Physically realizable system

$$|G(s)| = \frac{|N(s)|}{|D(s)|} = \frac{|(s-z_1)|\,|(s-z_2)|\cdots}{|(s-p_1)|\,|(s-p_2)|\cdots}$$

Magnitude

$$\angle G(s) = (\angle s-z_1 + \angle s-z_2 + \angle s-z_3 + \ldots) - (\angle s-p_1 + \angle s-p_2 + \angle s-p_3 + \ldots)$$

Argument

For example, if $s = j\omega$,

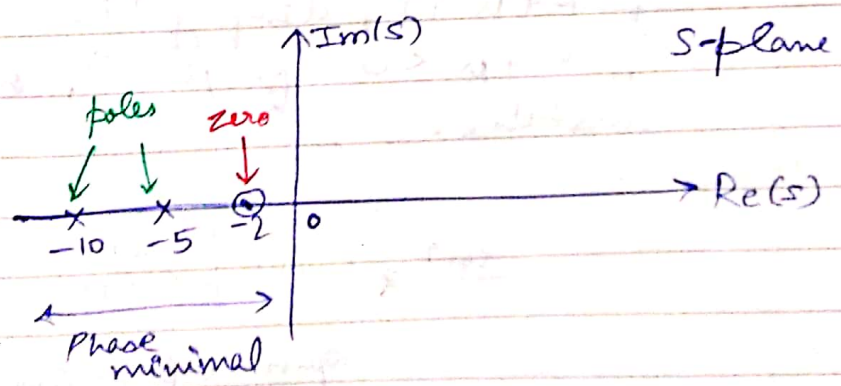
$$\angle s-5 = \angle j\omega-5 = \tan^{-1}\left(\frac{\omega}{-5}\right)$$

Phase Minimal System

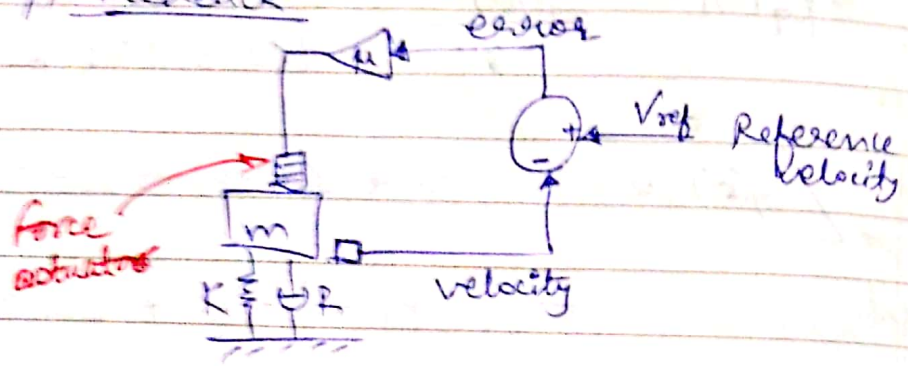
All its poles and zeroes are in left half of complex plane (s-plane).

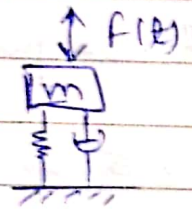
For example, $G(s) = \dfrac{s+2}{(s+5)(s+10)}$

$$z_1 = -2, \quad p_1 = -5, \quad p_2 = -10$$



Feedback





$$m\ddot{x} + R\dot{x} + Kx = F(t)$$

Eq^{ns} of open-loop system

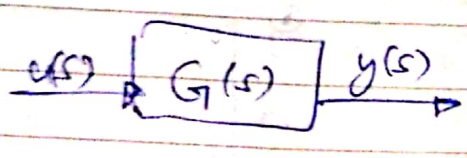
$$F(t) = \mu(V_{ref} - \dot{x}) \quad , \text{ let } V_{ref} = 0$$

$$\therefore m\ddot{x} + R\dot{x} + Kx = -\mu\dot{x}$$

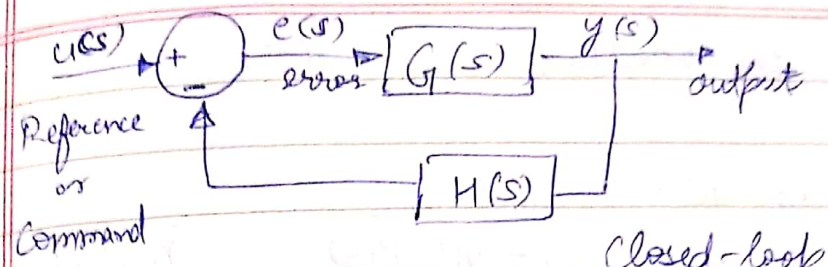
$$\Rightarrow m\ddot{x} + (R+\mu)\dot{x} + Kx = 0$$

Eq^{n} of closed-loop system

If $R+\mu \uparrow\uparrow$, damping $\uparrow\uparrow$.
If $R+\mu < 0$ (for $\mu < 0$), system becomes unstable.



open-loop system



Closed-loop system

H(s) is controller transfer func^n.

$$y(s) = e(s)\,G(s)$$

$$e(s) = u(s) - H(s)\,y(s)$$

$$= u(s) - e(s)\,G(s)\,H(s)$$

$$\Rightarrow \left[1 + G(s)\,H(s)\right] e(s) = u(s)$$

$$\Rightarrow \quad e(s) = \frac{u(s)}{1 + G(s)\,H(s)}$$

$$y(s) = e(s)\,G(s) = \frac{u(s)\,G(s)}{1 + G(s)\,H(s)}$$

$$\therefore \quad G_c(s) = \frac{y(s)}{u(s)} \Rightarrow \boxed{G_c(s) = \frac{G(s)}{1 + G(s)\,H(s)}}$$

closed-loop transfer func^n

Loop Rule

if $H(s) = 0$, $G_c(s) = G(s)$.

= open-loop transfer func^n

For example, $G(s) = \dfrac{s}{ms^2 + Rs + K}$

input force, output vel.

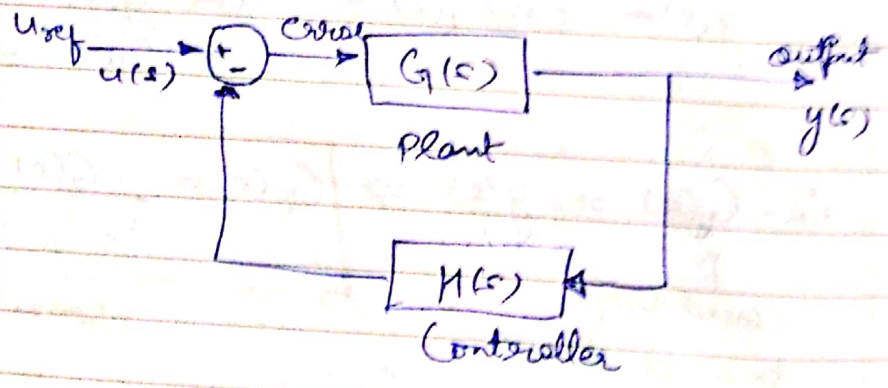
~~Ex~~ Feedback = μ = $H(s)$

$$\therefore G_c(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{s}{ms^2+\beta s+k}}{1+\frac{\mu s}{ms^2+\beta s+k}}$$

$$= \frac{s}{ms^2+(\beta+\mu)s+k}$$

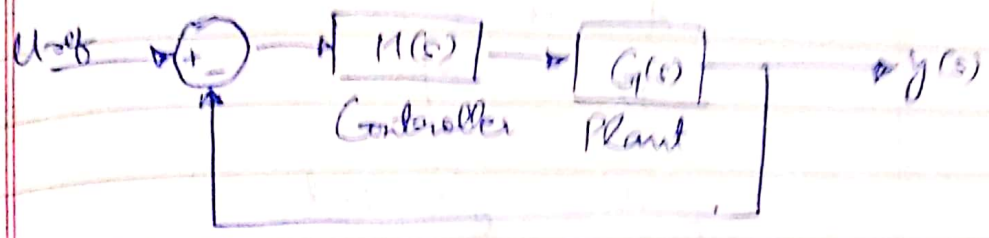
Only the ~~outp~~ input changes from a open-loop system to closed-loop system. The output remains the same.

31/08/20A



$$G_c(s) = \frac{y(s)}{u(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Controller in the feedback pt



Controller in the forward path

$$G_c(s) = \frac{G(s)H(s)}{1+G(s)H(s)}$$

Denominator is same for both.
$\Rightarrow$ Stability analysis for both are same.

Let $H(s) = K$ (constant), let $G(s) = \frac{N(s)}{D(s)}$

$\therefore G_c(s) = \frac{N(s)}{D(s)+KN(s)}$ or $G_c(s) = \frac{KN(s)}{D(s)+KN(s)}$

As $K \to 0$, $G_c(s) = G(s)$
As $K \to \infty$, $G_c(s) = \frac{1}{K} (\to 0)$

As $K \to 0$, $G_c(s) = 0$
As $K \to \infty$, $G_c(s) = 1$

For closed-loop system $\phi(s) = D(s) + KN(s)$ is called characteristic eqn of the system.

For stability, all roots of $\phi(s) = 0$ must have negative real part.

$$\phi(s) = (s-p_1)(s-p_2)\cdots\cdots$$
$$G_c(s) = \frac{N(s)}{\phi(s)}$$

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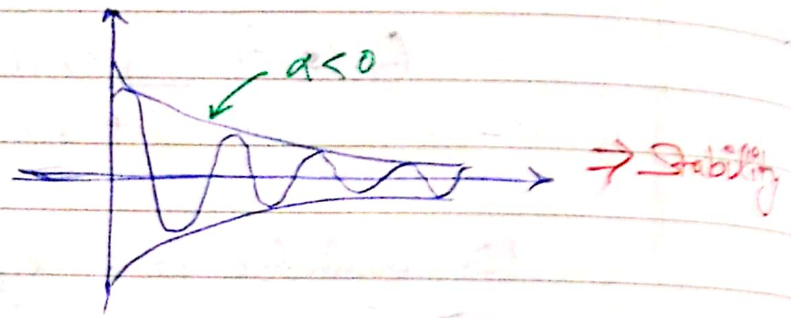
$$= \frac{A}{s-p_1} + \frac{B}{s-p_2} + \dots$$

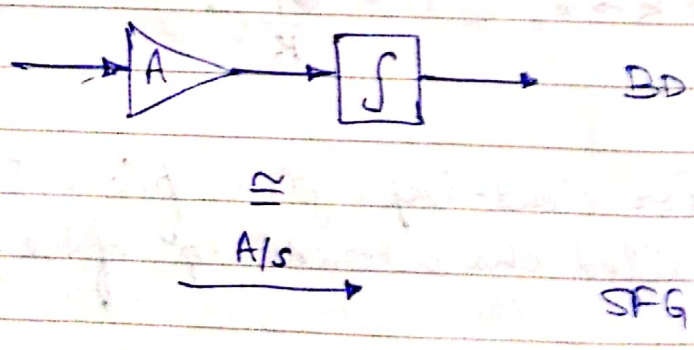
$$e^{p_1 t} = e^{(\alpha_1 + i\beta_1)t}$$
$$= e^{\alpha_1 t} \cdot e^{i\beta_1 t}$$



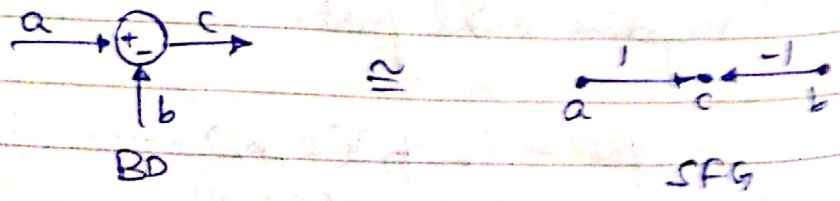
Signal flow Graph (SFG) :

Similar to block diagram but SFG exists only for linear time invariant (LTI) systems.

(i)



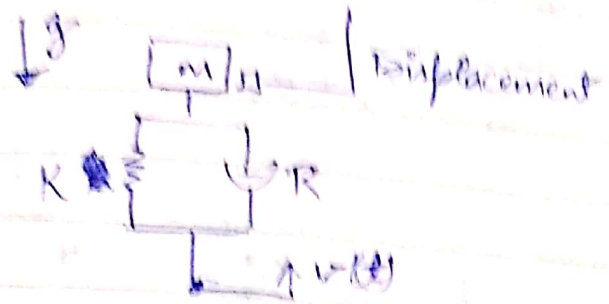
(ii)



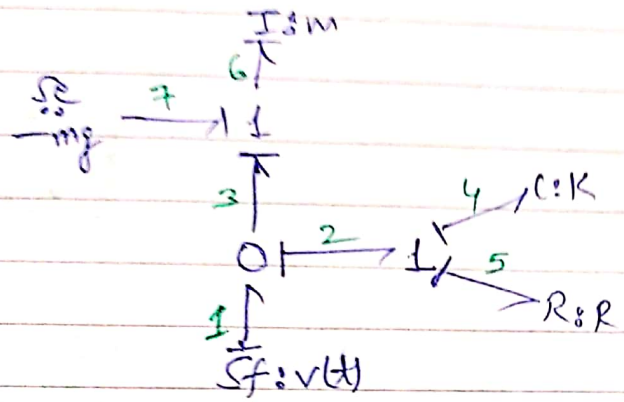
Variables a & f are bonds

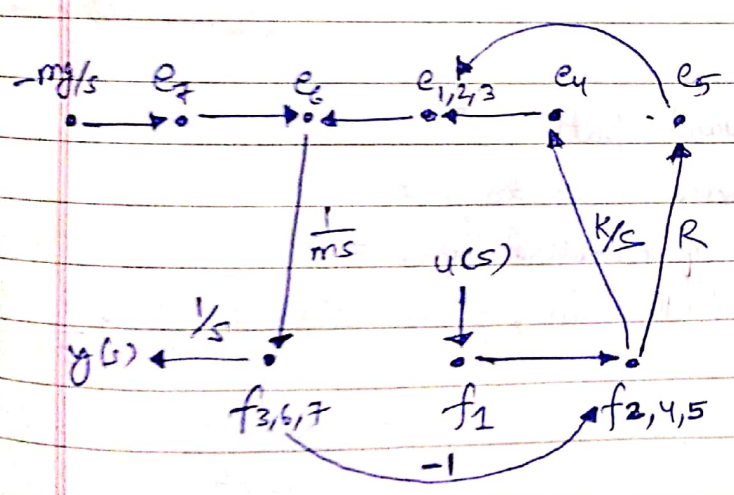
Variables a & f are nodes

Example



Derive transformation transfer func^n b/w ground excitation to the displacement of the block.





$$f_6 = \frac{1}{m_6}\int e_6\,dt \Rightarrow f_6(s) = \frac{1}{ms}\,e_6(s)$$

$$e_4 = K\int f_4\,dt \Rightarrow e_4(s) = \frac{K}{s}\,f_4(s)$$

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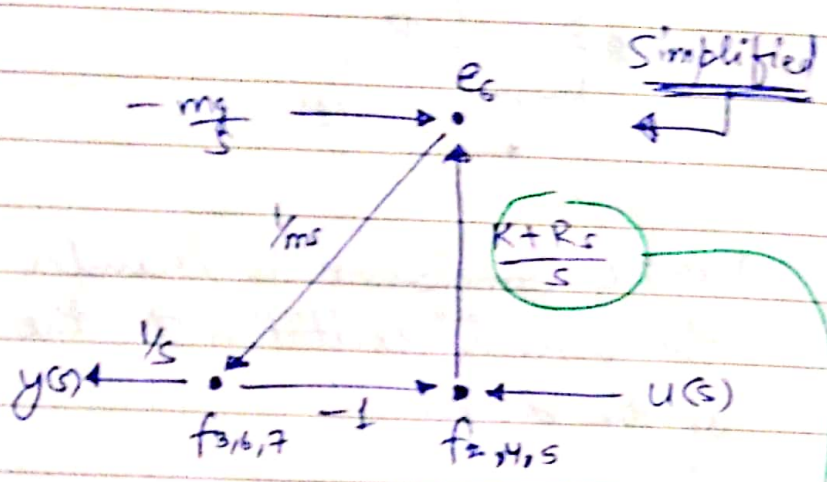
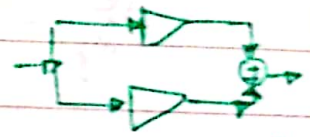
$e_6 = e_2 + e_3$

$f_2 = f_1 - f_3$

$e_2 = e_4 + e_5$





Two gains in parallel & in same direction

(Can only be done if gains are parallel & in same direction)

Forward path gain

from $u(s)$ to $y(s)$

No. of forward paths $= 1$.

Path gain, $p_1 = \frac{K+Rs}{s} \cdot \frac{1}{ms} \cdot \frac{1}{s}$

$= \frac{K+Rs}{ms^3}$

Number of loops $= 1$

Loop gains, $L_1 = \frac{K+Rs}{s} \cdot \frac{1}{ms} \cdot (-1)$

$= -\frac{(K+Rs)}{ms^2}$

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Mason's Gain Rule :

$$G(s) = \frac{\sum p_i \Delta_i}{\Delta}$$

where p_i is the ith forward path gain
Δ is the graph determinant

$$\Delta = 1 - \sum L_j + \sum L_j L_K - \sum L_j L_K L_m + .$$

j & K non-touching

j, K & m non touching

& $\Delta_i = \Delta$ after removing p_i from graph.

From the example above,

$$\Delta = 1 - L_1 = \frac{ms^2 + K + Rs}{ms^2}$$

$$\Delta_1 = 1$$

$$\therefore G(s) = \frac{p_1 \Delta_1}{\Delta}$$

$$= \frac{(K+Rs)/ms^3}{(ms^2+K+Rs)/ms^2}$$

$$\Rightarrow G(s) = \frac{K+Rs}{s(ms^2+K+Rs)}$$

Base vel. to block disp.

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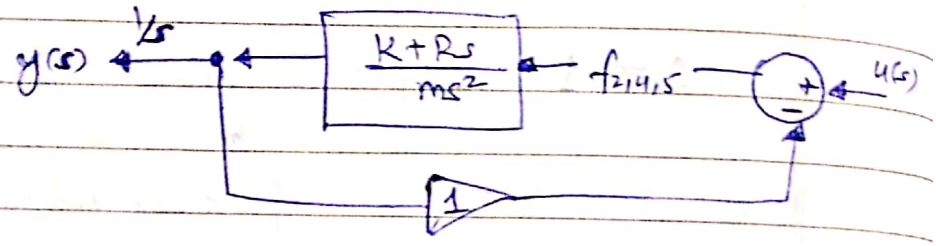
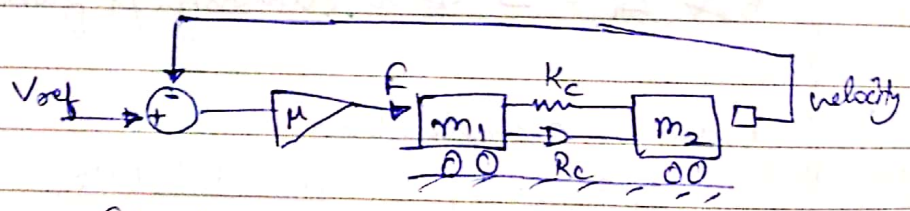
$$G(s) = \frac{K+Rs}{ms^2+Rs+K}$$

Base vel. to block vel.

Alter:

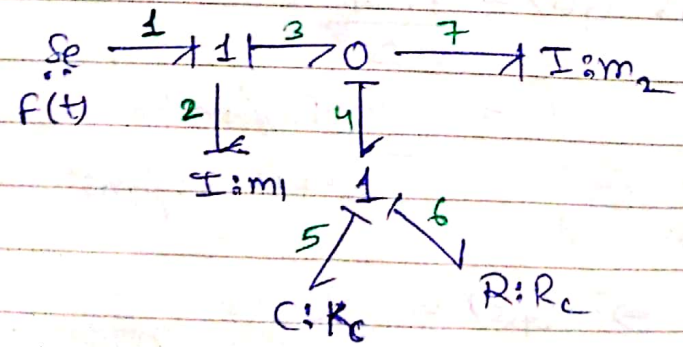


Example:



Closed loop control of train of vehicles

Consider open-loop model front,



NOTE: For a stable system, all loop gains will be negative.

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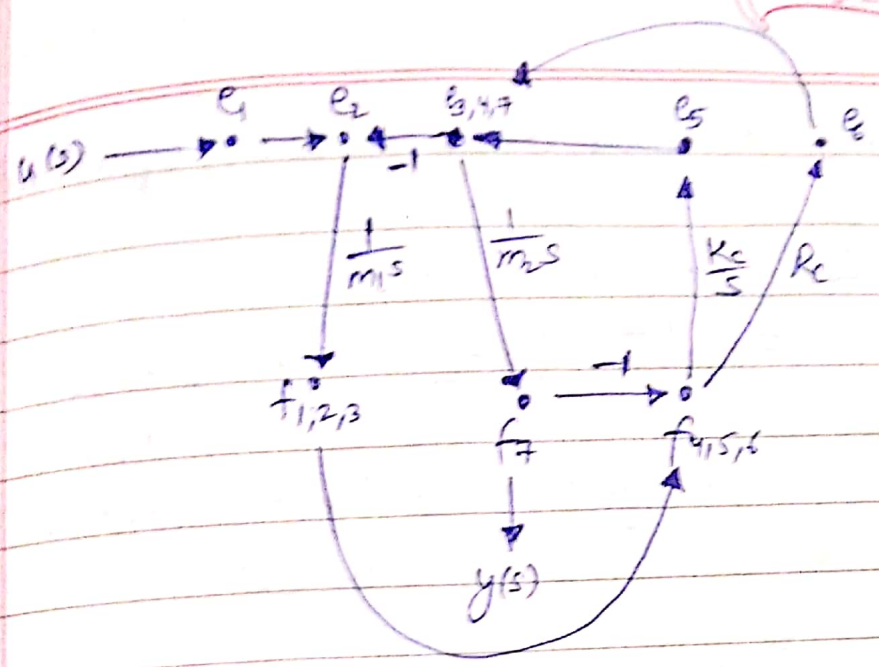

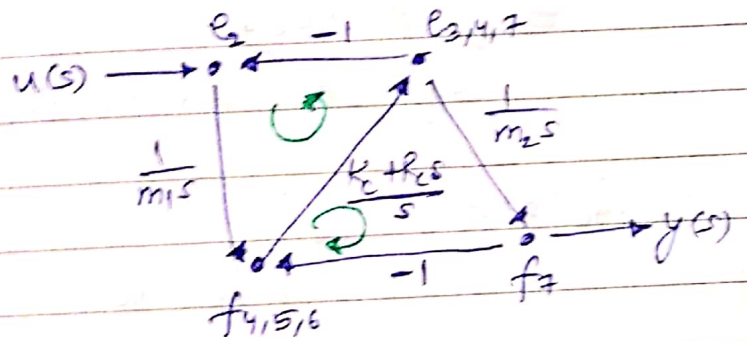

Forward paths = 1, forward path gain

$$P_1 = \frac{1}{m_1 s} \cdot \frac{K_c + R_c s}{s} \cdot \frac{1}{m_2 s}$$

$$= \frac{K_c + R_c s}{m_1 m_2 s^3}$$

$$\Delta_1 = 1$$

No. of loops = 2.

Loop gains : $L_1 = \frac{K_c + R_c s}{s} \cdot (-1) \cdot \frac{1}{m_1 s}$

$$= -\frac{(K_c + R_c s)}{m_1 s^2}$$

$$L_2 = \frac{(K_c + R_c s)}{s} \cdot \frac{1}{m_2 s} \cdot (-1)$$

$$= -(K_c + R_c s)/m_2 s^2$$

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Non-touching loop pairs L_j, L_k do not exist.

$$\Delta = 1 - (L_1 + L_2) + \dots$$

$$= 1 + \frac{K_c + R_c s}{m_1 s^2} + \frac{K_c + R_c s}{m_2 s^2}$$

$$= \frac{m_1 m_2 s^2 + (m_1 + m_2)(K_c + R_c s)}{m_1 m_2 s^2}$$

$$G_1(s) = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{K_c + R_c s}{s\left[m_1 m_2 s^2 + (m_1 + m_2) R_c s + (m_1 + m_2) K_c\right]}$$

$$G_c(s) = \frac{G_1(s)\mu}{1 + G_1(s)\mu}$$

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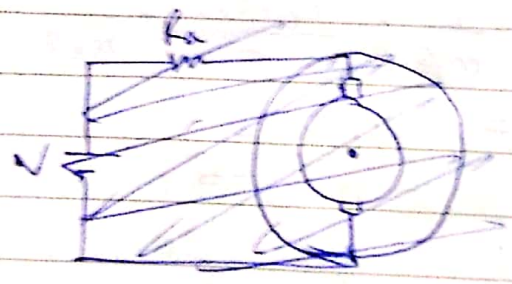
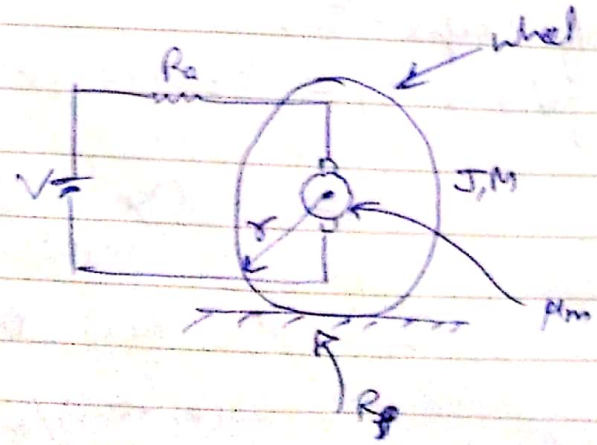
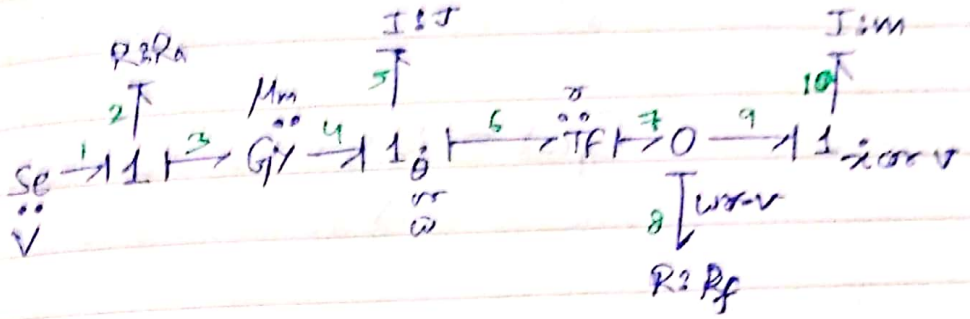
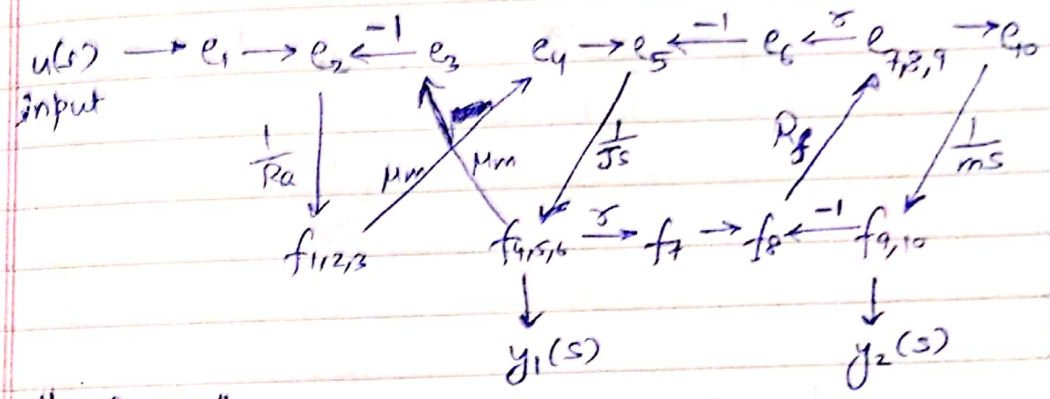




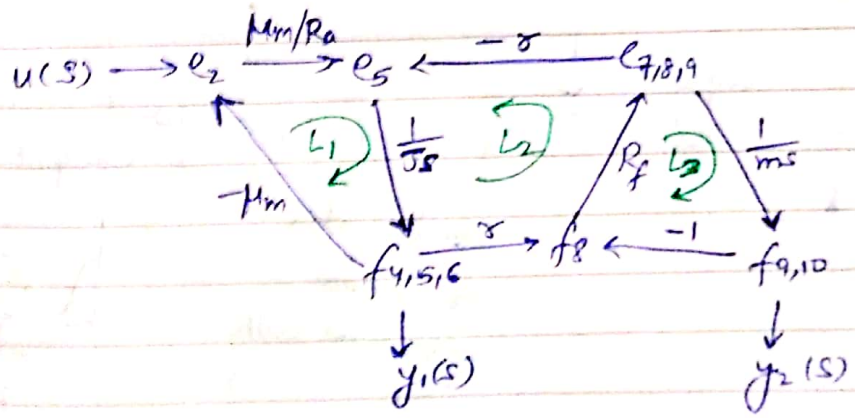
Derive transfer funⁿ b/w V & linear velocity
V & angular velocity.





"3 loops"

⇓ simplifying



$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_3$$

$$L_1 = \frac{-\mu_m^2}{R_a J s} \; ; \; L_2 = \frac{-\gamma^2 R_f}{J s} \; ; \; L_3 = \frac{-R_f}{m s}$$

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$\frac{y_1(s)}{u(s)} = G_1(s)$. No. of forward paths = 1

$P_1 = \frac{\mu_m}{R_a J s}$, $\Delta_1 = 1 - L_a = \frac{ms + R_f}{ms}$

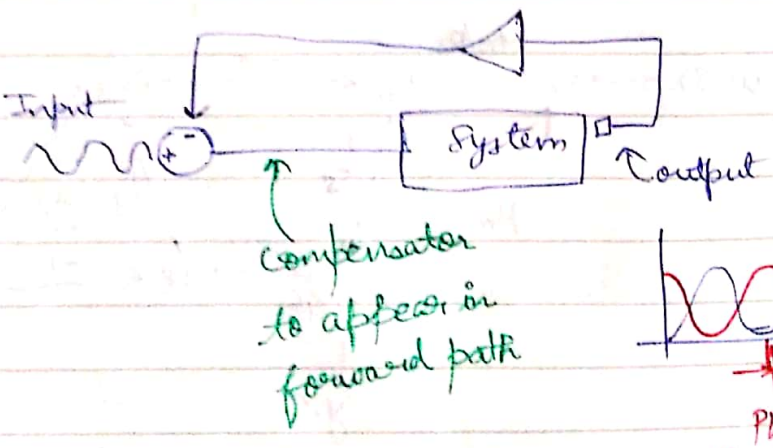
$\Rightarrow G_1(s) = \frac{P_1 \Delta_1}{\Delta}$

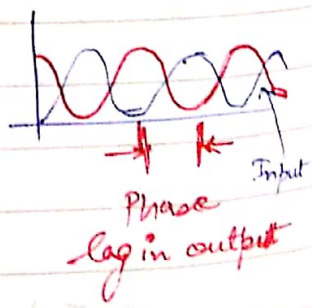
$\frac{y_2(s)}{u(s)} = G_{p2}(s)$. No. of forward paths = 1.

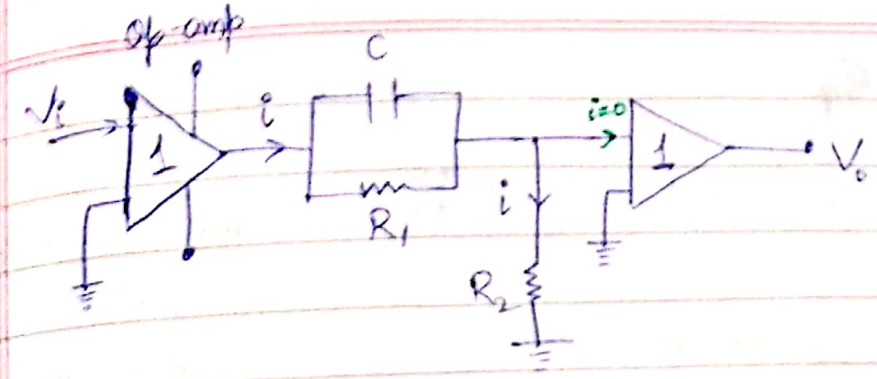
$P_1 = \frac{\mu_m R_f}{R_a m J s^2}$, $\Delta_1 = 1$

$\Rightarrow G_{p2}(s) = \frac{P_1 \Delta_1}{\Delta}$

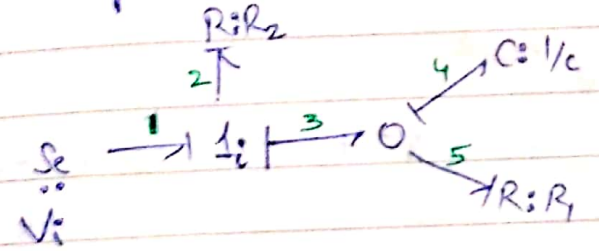
High-pass Filter or Lead Compensator

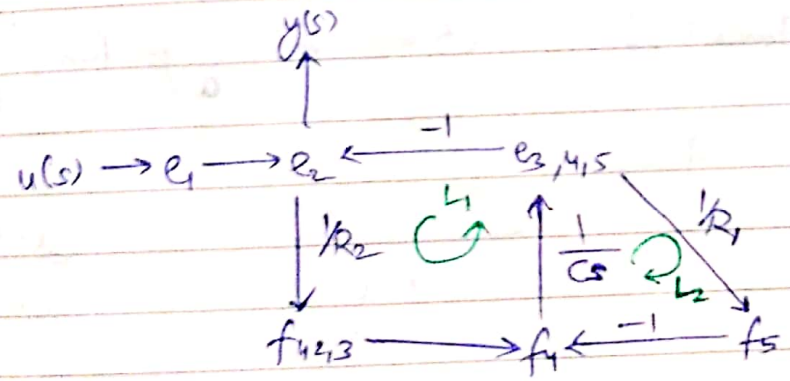






Op-amps act as buffers, i.e. isolate the controller.
Input current to op-amps $\cong 0$.





Forward path $= 1$, $P_1 = 1$, $\Delta_1 = 1 - L_2$

$$L_1 = \frac{-1}{R_2 C s}, \quad L_2 = -\frac{1}{R_1 C s}, \quad \Delta = 1 - L_1 - L_2$$

$$G_f(s) = \frac{P_1 \Delta_1}{\Delta} = \frac{s + \frac{1}{R_1 C}}{s + \left(\frac{1}{R_1 C} + \frac{1}{R_2 C}\right)}$$

$$\Rightarrow G_f(s) = \frac{s+a}{s+b}, \quad b > a$$

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$$\angle G_f(s=j\omega) = \angle \frac{\cancel{j\omega a}}{j\omega+b} = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$$

> 0 for $b > a$ & all $\omega \neq 0$

we study sinusoidal waves for filter & ~~use~~ complex no. generates sinusoidal signal

Plant

$$\angle G_f(s)\, G(s) = \angle G_f(s) + \angle G(s)$$ $\Rightarrow$ Filter adds phase to $G(s)$

$\phi_{max} = ??$ $\qquad \phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$

$$\tan\phi = \frac{\frac{\omega}{a} - \frac{\omega}{b}}{1 + \frac{\omega^2}{ab}} = \frac{(a-b)\omega}{\omega^2+ab}$$

$$\frac{d\tan\phi}{d\omega} = 0 \Rightarrow (\omega^2+ab)(a+b) - (a+b)2\omega = 0$$

$$\Rightarrow ab - \omega^2 = 0$$

$$\Rightarrow \omega = \sqrt{ab}$$ (Break frequency)

$$\boxed{\phi_{max} = \tan^{-1}\sqrt{\frac{b}{a}} - \tan^{-1}\sqrt{\frac{a}{b}}}$$

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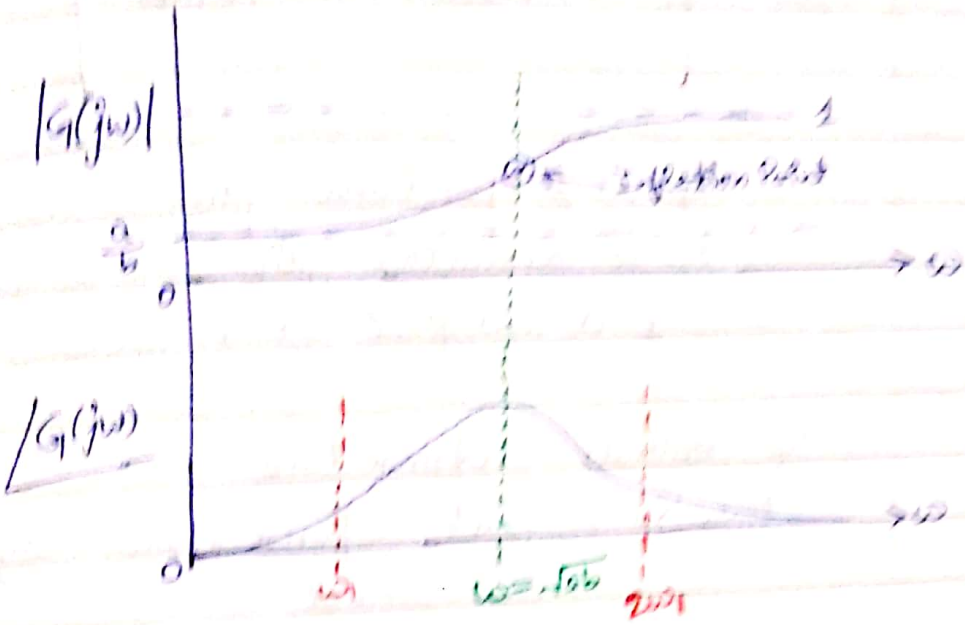
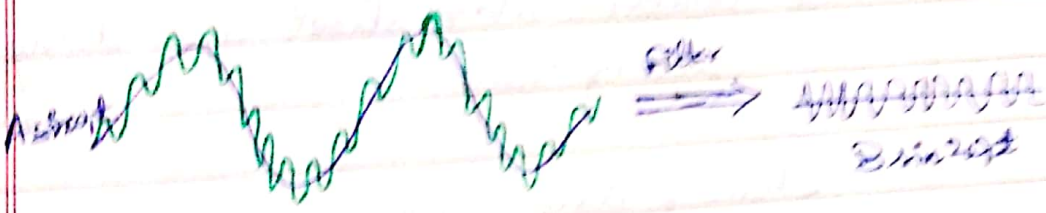
Frequency Domain Response





PID Control

Proportional - Integral - Derivative Control

Error = $e(t) = u(t) - y(t)$

Then PID output

$$y_{PID}(t) = K_P e(t) + K_I \int e(t)\,dt + K_D \frac{de(t)}{dt}$$

PID Controller Transfer func^n = $\frac{Y_{PID}(s)}{E(s)}$

$$H(s) = K_P + K_D s + \frac{K_I}{s}$$

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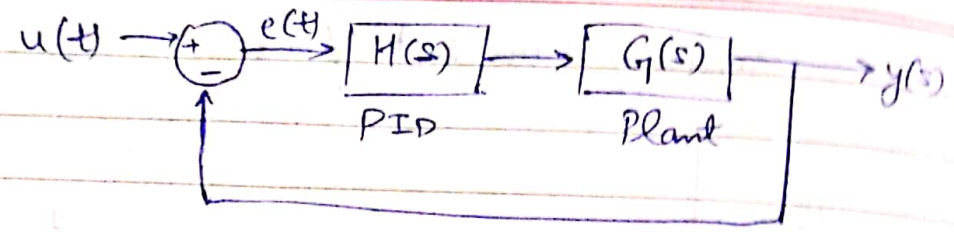


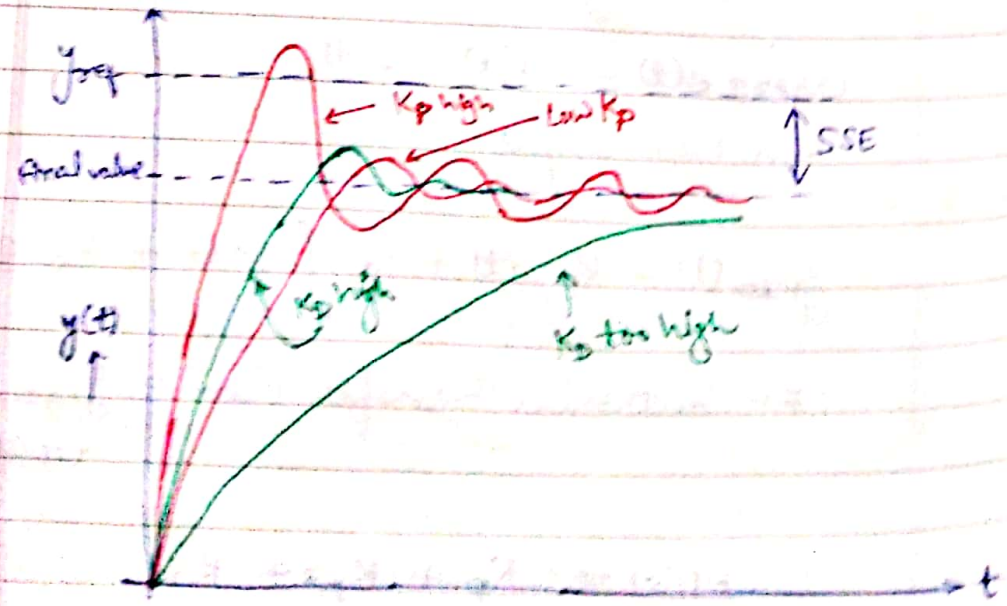
K_P is proportional gain
K_D is derivative gain
K_I is integral gain.

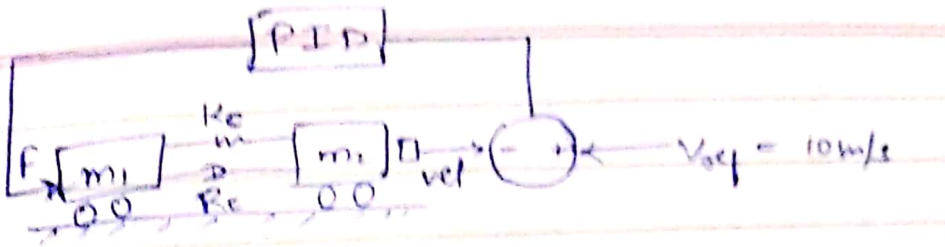
K_P controls response time
High K_P ⇒ quick response, but large over-shoot

K_D controls over-shoot and decay rate / settling time

Steady State Error (SSE) $= y_{ref} - \lim_{t \to \infty} y(t)$

K_I controls SSE.





$m_1 = m_2 = 1\,kg$, $K_c = 100\ N/m$, $R_c = 1\ Ns/m$

Open-loop, $G(s) = \dfrac{K_c + R_c s}{s\left[m_1 m_2 s^2 + R_c(m_1+m_2)s + (m_1+m_2)K_c\right]}$

$$= \frac{100 + s}{s\left[s^2 + 2s + 200\right]}$$

$$\frac{y(s)}{u(s)} = G_c(s) = \frac{G(s)\,H(s)}{1 + G(s)H(s)}$$

$$y(s) = G_c(s)\,u(s), \quad u(s) = \frac{10}{s}$$

Let us assume only proportional control ($K_I = 0$, $K_D = 0$, $H(s) = K_p$)

$$y(s) = \frac{\dfrac{s+100}{s[s^2+2s+200]} \times K_p}{1 + \dfrac{s+100}{s[s^2+2s+200]} \times K_p} \times \frac{10}{s}$$

$$= \frac{(s+100)K_p}{s(s^2+2s+200) + (s+100)K_p} \times \frac{10}{s}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot y(s) = 10 \times \frac{100 K_p}{100 K_p}$$

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03/09/2019

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_0}$$

$, n > m$

$$G_c(s) = \frac{G(s)H(s)}{1 + G(s)H(s)}$$

$$= \frac{K_p (a_m s^m + \dots + a_0)}{(b_n s^n + \dots + b_0) + K_p (a_m s^m + \dots + a_0)}$$

$$= \frac{K_p N(s)}{D(s) + K_p N(s)}$$

for proportional control, $K_D = K_I = 0$

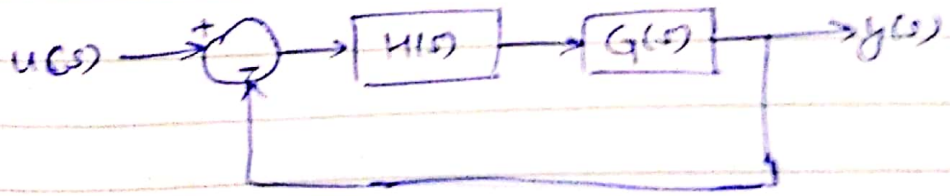
$$u(s) = \frac{C}{s}, \quad y(s) = u(s)\, G_c(s)$$

$$\lim_{t \to 0} y(t) = \lim_{s \to 0} s\, y(s) = \lim_{s \to 0} C \cdot G_c(s)$$

$$= \frac{K_p a_0}{K_p a_0 + b_0} \times C$$

$$SSE = u(t) - \lim_{t \to \infty} y(t) = 0$$

If $b_0 = 0$, output $= C$ (no steady-state error).
Otherwise, use full PID control for $SSE = 0$



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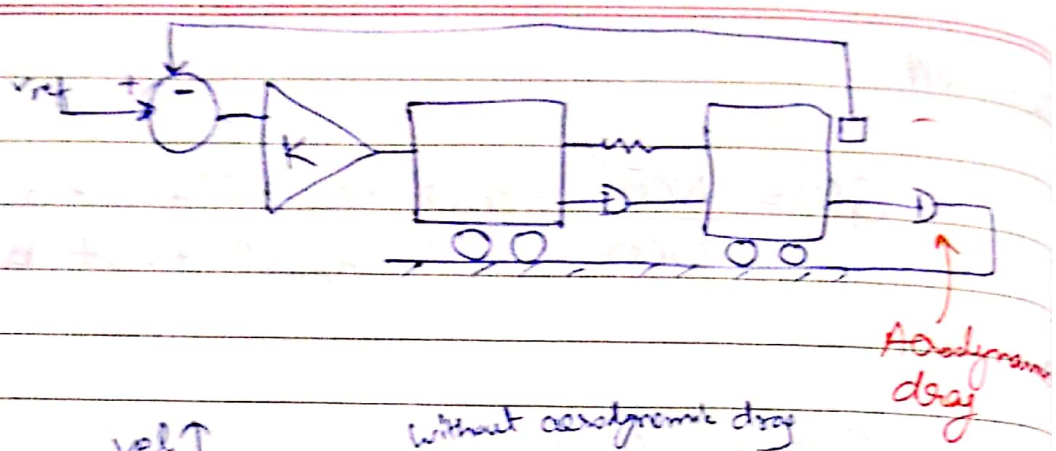

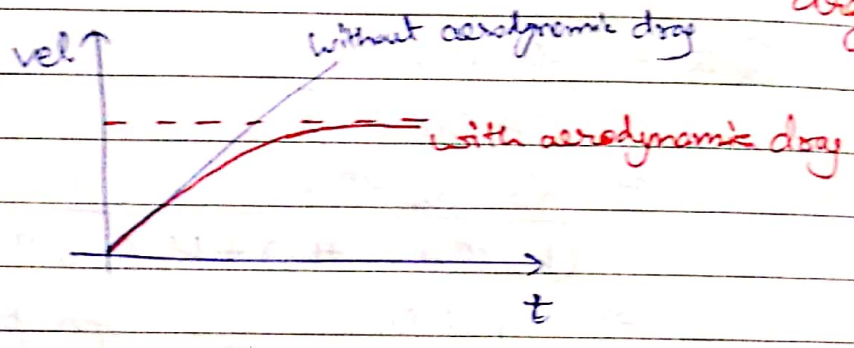

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