

# Assignment - 7

PDE (MA20103)

Course teacher

Koeli Ghoshal

23.10.2017

1. A homogeneous string is stretched and its ends are at  $x=0$  and  $x=l$ . Motion is started by displacing the string into the form  $f(x) = u_0 \sin \frac{\pi x}{l}$  from which it is released at time  $t=0$ . Find the displacement of the string at any point  $x$  and time  $t$  by using D'Alembert's sol<sup>n</sup>.

[Hint: Start from  $u(x,t) = \frac{1}{2} [f(x+ct) + f(x-ct)]$  and then apply  $u = f(x) = u_0 \sin \frac{\pi x}{l}$ .

Answer:  $u(x,t) = u_0 \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$  ]

2. Solve the one dimensional wave eqn.

$$u_{tt} = c^2 u_{xx} \quad 0 \leq x \leq \pi, \quad t \geq 0$$

subject to  $u=0$  when  $x=0$  and  $x=\pi$

$$u_t = 0 \quad " \quad t=0 \quad \text{and} \quad u(x,0) = x \quad 0 < x < \pi.$$

[Ans:  $u(x,t) = \sum_{n=1}^{\infty} b_n \sin nx \cos(nct)$  where  $b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nxdx$   
 $= \frac{2}{n} (-1)^{n+1}$  ]

3. Solve the BVP described by

$$\text{PDE: } u_{tt} - c^2 u_{xx} = 0 \quad 0 \leq x \leq l, \quad t \geq 0$$

$$\text{BCs: } u(0,t) = u(l,t) = 0 \quad t \geq 0$$

$$\text{ICs: } u(x,0) = 10 \sin \frac{\pi x}{l} \quad 0 \leq x \leq l$$

$$u_t(x,0) = 0$$

[Ans:  $u(x,t) = 10 \cos \frac{\pi ct}{l} \sin \frac{\pi x}{l}$  ]

\*\*\* The End \*\*\*