Natural Convection: Correlations and slides

- Pertinent Dimensionless Parameters

  ➢ Grashof Number:

$$Gr_L = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu^2} \sim \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ coefficient of thermal expansion

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Perfect Gas: $\beta = 1/T\left(\text{K}\right)$

  ➢ Rayleigh Number:

$$Ra_L = Gr_L \Pr = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu\alpha}$$

- Mixed Convection
  - A condition for which forced and free convection effects are comparable.
  - Exists if $\dfrac{Gr_L}{Re_L^2} \sim O(1)$
  - Heat Transfer Correlations for Mixed Convection:

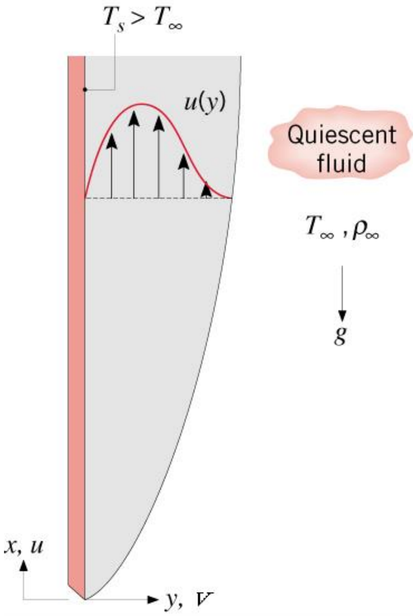
$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

$+ \rightarrow$ assisting and transverse flows
$- \rightarrow$ opposing flows

$$n \approx 3$$

Vertical Plates

- Free Convection Boundary Layer Development on a Heated Plate:



  - ➢ Ascending flow with the maximum velocity occurring in the boundary layer and zero velocity at both the surface and outer edge.

  - ➢ How do conditions differ for a cooled plate $\left(T_s < T_\infty\right)$?

- Form of the *x*-Momentum Equation for Laminar Flow

$$\underbrace{u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}}_{\substack{\text{Net Momentum Fluxes}\\ \text{(Inertia Forces)}}}=\underbrace{g\beta\left(T-T_\infty\right)}_{\text{Buoyancy Force}}+\underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}}$$

  - Temperature dependence requires that solution for $u\ (x,y)$ be obtained concurrently with solution of the boundary layer energy equation for $T\ (x,y)$.

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

    – The equations and solutions are said to be <u>coupled</u>.

Derivation – Similarity Solution

out of scope for this class

- Nusselt Numbers $\left(Nu_x \text{ and } \overline{Nu}_L\right)$:

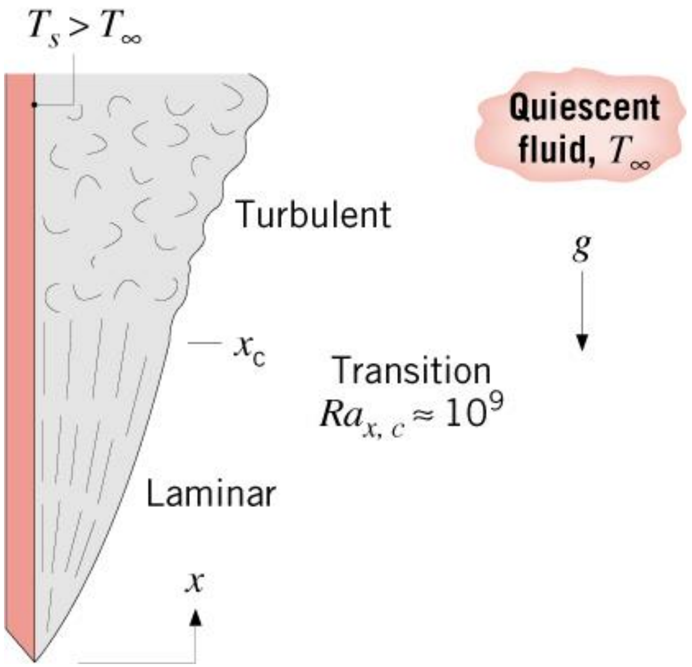
$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \left.\frac{dT^*}{d\eta}\right|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g(\mathrm{Pr})$$

$$g(\mathrm{Pr}) = \frac{0.75\,\mathrm{Pr}^{1/2}}{\left(0.609 + 1.221\,\mathrm{Pr}^{1/2} + 1.238\,\mathrm{Pr}\right)^{1/4}} \qquad (0 < \mathrm{Pr} < \infty)$$

$$\bar{h} = \frac{1}{L}\int_o^L h\,dx \rightarrow \overline{Nu}_L = \frac{4}{3} Nu_L$$

- Transition to Turbulence
  - Amplification of disturbances depends on relative magnitudes of buoyancy and viscous forces.
  - Transition occurs at a critical Rayleigh Number.

$$Ra_{x,c} = Gr_{x,c}\,\mathrm{Pr} = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha} \approx 10^9$$



- Empirical Heat Transfer Correlations
  - Laminar Flow $\left(Ra_L < 10^9\right)$:

$$\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}$$

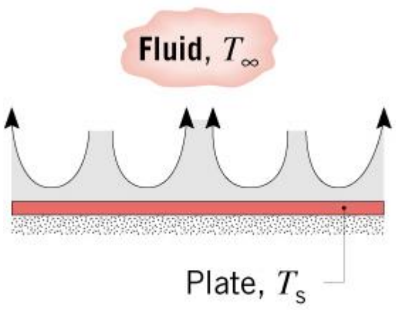
  - Turbulent Flow (*note this is average over entire length – no need to average over laminar and turbulent lengths separately)*

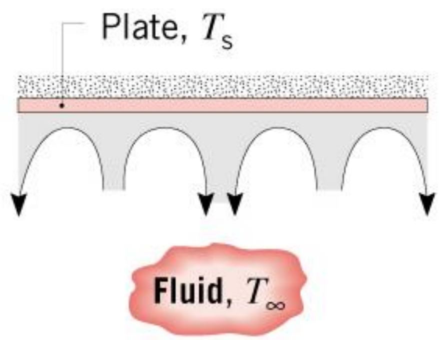
$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1+\left(0.492/\Pr\right)^{9/16}\right]^{4/9}}\right\}^2$$

  - *Note that average Nu varies as* $Ra_L^{1/3}$

Horizontal Plates

- Buoyancy force is normal, instead of parallel, to the plate.
- Flow and heat transfer depend on whether the plate is heated or cooled and whether it is facing upward or downward.
- Heated Surface Facing Upward or Cooled Surface Facing Downward





$T_s > T_\infty$ $\quad$ $T_s < T_\infty$

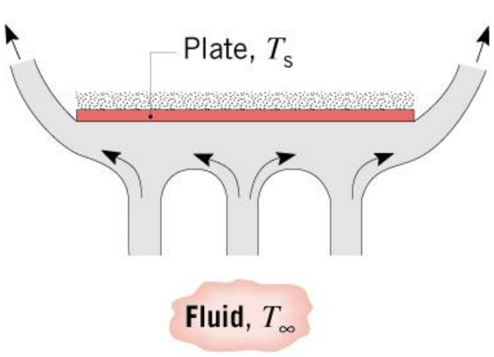
$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \left(10^4 < Ra_L < 10^7\right)$$

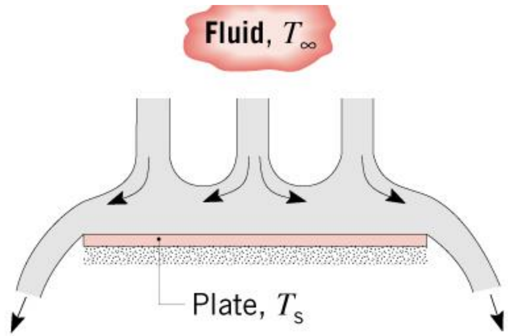
$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \left(10^7 < Ra_L < 10^{11}\right)$$

$$\boxed{L = A/P}$$

How does $\bar{h}$ depend on L when $\overline{Nu}_L \propto Ra_L^{1/3}$?

- Heated Surface Facing Downward or Cooled Surface Facing Upward





$$T_s > T_\infty$$

$$T_s < T_\infty$$

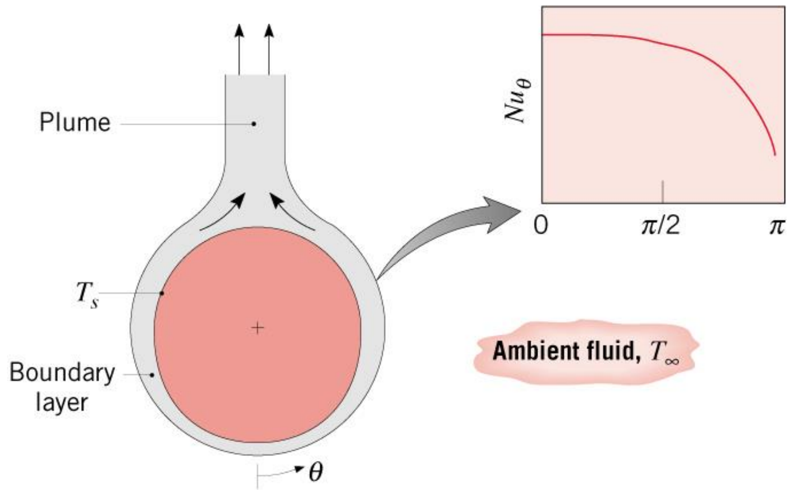
$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \qquad \left(10^5 < Ra_L < 10^{10}\right)$$

Inclined Plates

- Refer to class notes

The Long Horizontal Cylinder

- Boundary Layer Development and Variation of the Local Nusselt Number for a Heated Cylinder:



- The Average Nusselt Number:

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[1+\left(0.559/\Pr\right)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D < 10^{12}$$

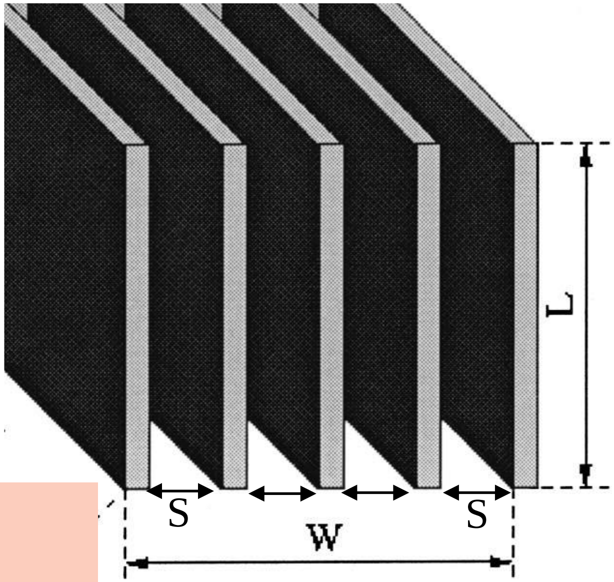
Spheres

- The Average Nusselt Number:

$$\overline{Nu}_D = 2 + \frac{0.589\ Ra_D^{1/4}}{\left[1+\left(0.469/\Pr\right)^{9/16}\right]^{4/9}}$$

Natural Convection through vertical channels

- Limits – large spacing and narrow spacing
- Concept of optimal spacing for maximum heat transfer from an array of vertical plates under forced convection
- Correlations of Bar Cohen and Rohsenow for different boundary conditions



Isothermal plates:

$$\overline{Nu}_S = \left[\frac{C_1}{(Ra_S\,S/L)^2} + \frac{C_2}{(Ra_S\,S/L)^{1/2}}\right]^{-1/2}$$

Isoflux plates:

$$Nu_{S,L} = \left[\frac{C_1}{Ra_S^*\,S/L} + \frac{C_2}{(Ra_S^*\,S/L)^{2/5}}\right]^{-1/2}$$

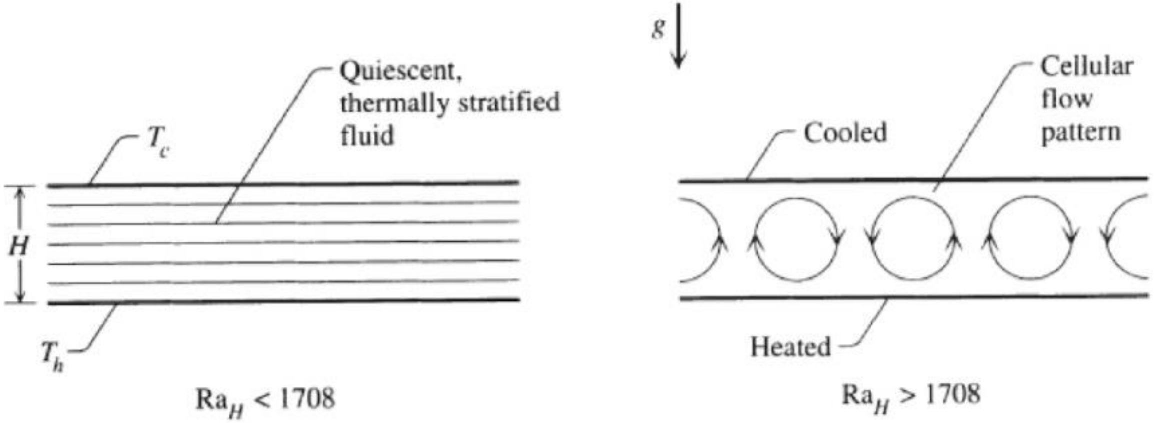
$$Ra_S^* = \frac{g\beta q_s'' S^4}{k\alpha\nu}$$

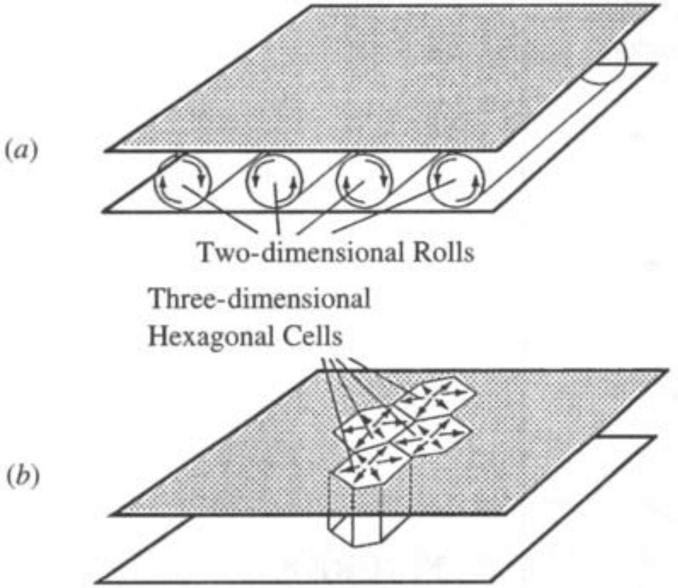
$$Nu_{S,L} = \left(\frac{q_s''}{T_{s,L} - T_\infty}\right)\frac{S}{k}$$

Surface Condition	C_1	C_2	S_{opt}	S_{max}/S_{opt}
Symmetric isothermal plates ($T_{s,1} = T_{s,2}$)	576	2.87	$2.71(Ra_S/S^3L)^{-1/4}$	1.71
Symmetric isoflux plates ($q''_{s,1} = q''_{s,2}$)	48	2.51	$2.12(Ra_S^*/S^4L)^{-1/5}$	4.77
Isothermal/adiabatic plates ($T_{s,1}$, $q''_{s,2} = 0$)	144	2.87	$2.15(Ra_S/S^3L)^{-1/4}$	1.71
Isoflux/adiabatic plates ($q''_{s,1} = q''_{s,2} = 0$)	24	2.51	$1.69(Ra_S^*/S^4L)^{-1/5}$	4.77

Heat Transfer in Enclosures

- **Heated from below**





- Onset of convection at *$Ra_H > 1708$*
- Counter-rotating 2-D rolls
  - called Benard Convection or Benard Cells
- At still higher RaH, the 2-D rolls break up into 3-D cells that appear hexagonal when viewed from above

[Video]