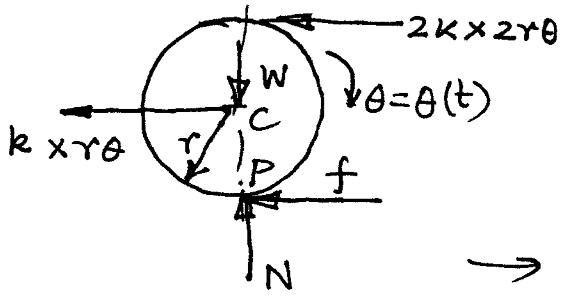
Problem ① See fig. 1. Take rotation angle θ as the generalized coordinate. Let θ is +ive CW. Then for small θ, the top spring is compressed by $\approx 2r\theta$ & the central spring is extended by $\approx r\theta$ at time t. There will be friction force, weight W & normal reaction N as shown in the FBD.



→ So, if you are using the moment balance method (Do this), take moments about P, the point of contact of the central transverse section of the ~~cylinder~~, disk as shown in the FBD.

So, $I_P \ddot{\theta}$ = etc. Note that by taking moments about P, we get rid of the unknowns f & N which are not required ~~by~~ ~~?~~ at this moment.

$I_P = I_C + mr^2$, by the parallel-axes theorem; $I_C = \frac{1}{2} m r^2$.

→ Obtain the DEOM by Lagrange's method also. Here there is instantaneous rotation about the line of contact & hence, $T = \frac{1}{2} I_P \dot{\theta}^2$ ($\omega = \dot{\theta}$ here). You could also obtain T from another point of view. Consider motion of the centre of mass C as well as rotation about the disk axis through C. Then, $T = T_{translation} + T_{rotation}$

$V = \frac{1}{2}\times 2K \times (2r\theta)^2 + \frac{1}{2}\times K \times (r\theta)^2$ etc.

$$= \frac{1}{2} m v_c^2 + \frac{1}{2} I_C \omega^2$$
$$= \frac{1}{2} m (r\dot{\theta})^2 + \frac{1}{2}\cdot\frac{1}{2} m r^2 \dot{\theta}^2$$
$$= \frac{1}{2}\left(1+\frac{1}{2}\right) m r^2 \dot{\theta}^2 = \frac{1}{2}\cdot\left(\frac{3}{2} m r^2\right)\dot{\theta}^2$$
$$= \frac{1}{2}\cdot I_p \dot{\theta}^2 \text{ only}$$

Here Lagrange's eqn. is:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = 0$$

Now complete the problem solution.

×

Problem (2) Obtain the DEOM using Lagrange's equation as well by the moment balance method.

$m_d = \rho_{steel} \times V_{disc} = 0.9924\ kg$

I_d = disc MI about its own axis $= \frac{1}{2} m_d r^2$

$= 4.019 \times 10^{-3}\ kg\ m^2$

> Check all numerical Computations. Mistakes, if detected, should be corrected by yourselves.

K_t = Torsional stiffness of the shaft $= \frac{GJ}{l}$ (Formula already explained before).

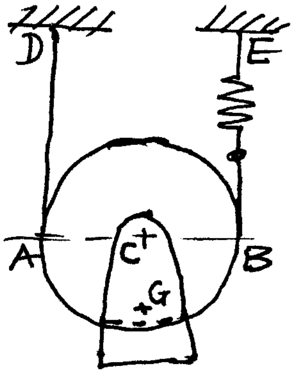
$J = \frac{\pi d^4}{32}$; $d = 12\times 10^{-3}\ m$; $l = 0.2\ m$; $G = 8\times 10^{10}\ Pa$

So, $k_t = 814.301$ N-m/rad $\quad \omega_n = \sqrt{\frac{k_t}{I_d}}$ = etc.

→

Problem 3] This deceptively simple looking problem requires special attention. You have to make certain assumptions. The spring part is flexible but the rest of the rope/belt is inextensible.

So, could we just pull the pulley-load assembly down & release to get free oscillations? Think a little bit. Doesn't portion DA remain as it is during the oscillations? You actually have to turn the pulley a little about A & release it to get the small oscillations you seek. Of course, there are other ways to do it such as an angular impulse (this generates an initial angular velocity) &/or an initial angular displacement etc.



→ So, one important point to realize is that the disc will be rotating about point A, as if it was 'hinged' at A. (A is the point of tangency of rope portion DA with the cylinder)

→ Also, the load won't be rigidly connected to the pulley (Afterall, why should we do it that way, the load should be easily removable, isn't it?)

So, we assume the load is <u>pinned</u> at C to the pulley so that it can turn freely about the pin.

→ But this assumption immediately poses another question. Isn't our system then having <u>two</u> degrees of freedom? The pulley executing rotational oscillations about A & the load doing what a compound pendulum does?

→ So, to resolve matters for now, we shall assume the load does vertical translatory motion only, at least approximately.

→ By now, you might have given up! (Wondering why we are making a mess of this problem when we are brilliant students after cracking the JEEs and GATEs etc? We <u>can</u> solve this problem without all this, can't we?

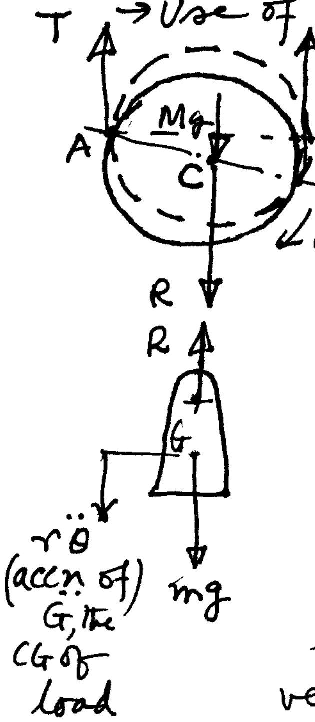
The answer probably is - may be.

But you probably can't deny the possibilities this simple looking problem presents. So, the suggestion is - gather a bit of courage & complete the solution!)

→

→ Use of Lagrange's equation :–

→ Use of moment balance etc.



M = Mass of pulley

m = Mass of load

$\boxed{\text{The FBDs}}$

Note that velocity of c is $r\dot\theta$, approximately vertical. Also, the load has the same velocity & it is translating up & down with acceleration $r\ddot\theta$

So, translation of load gives:

$$m r\ddot\theta = mg - R \quad (1)$$

Rotation of pulley about A gives:–

$$I_A\ddot\theta = (Mg+R)r - k(\delta_{st}+2r\theta)\times 2r \quad (2)$$

& using (1) & (2) together and simplifying, you should get

$$[I_c + (M+m)r^2]\ddot\theta + 4kr^2\theta = 0,$$ the required DEOM.

$$T = \underbrace{\tfrac{1}{2}m(r\dot\theta)^2}_{\text{KE of load, translating}} + \underbrace{\tfrac{1}{2}I_A\dot\theta^2}_{\text{KE of pulley, rotating about A}}$$

$$I_A = I_c + Mr^2$$

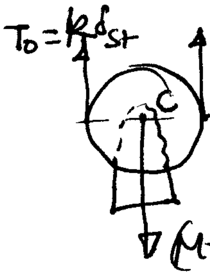
So, $T = \frac{1}{2}[I_c + (M+m)r^2]\dot\theta^2$

$$V = \frac{1}{2}k(2r\theta+\delta_{st})^2 - \frac{1}{2}k\delta_{st}^2 - (M+m)g\times r\theta$$

$$= 2kr^2\theta^2 \cancel{+ (M+m)g\, r\theta}$$

(∵ At static equilibrium,

$$2\times k\delta_{st} = (M+m)g \quad \left[\begin{array}{l}\text{Vertical}\\ \text{force}\\ \text{balance}\end{array}\right.$$



($T_0 = k\delta_{st}$, by $\Sigma M_c = 0$))

Important :– Thus, here also, we could simply overlook δ_{st} etc. & straightaway take

$$V = \frac{1}{2}k(2r\theta)^2.$$

So, $\frac{d}{dt}\left(\frac{\partial T}{\partial\dot\theta}\right) - \frac{\partial T}{\partial\theta} + \frac{\partial V}{\partial\theta} = 0$

gives:

$$[I_c + (M+m)r^2]\ddot\theta + 4kr^2\theta = 0$$

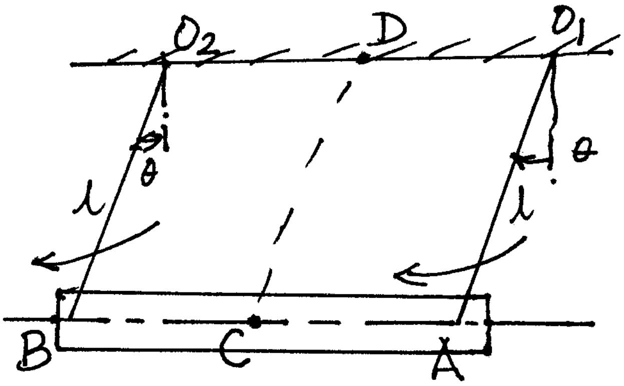
& consequently,

$\omega_n = \sqrt{\text{etc.}} = 6.325$ rad/s; $f_n = \frac{\omega_n}{2\pi}$ Hz

→ So, finally, all *you* need to do to solve such a problem is given on page 5 alone, once you understand what you're doing!

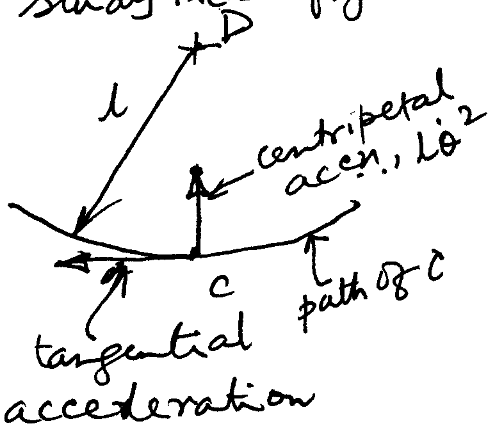
→ Practice ~~this~~ problem, an important one. You even might find a shorter way to do it!

Problem 4

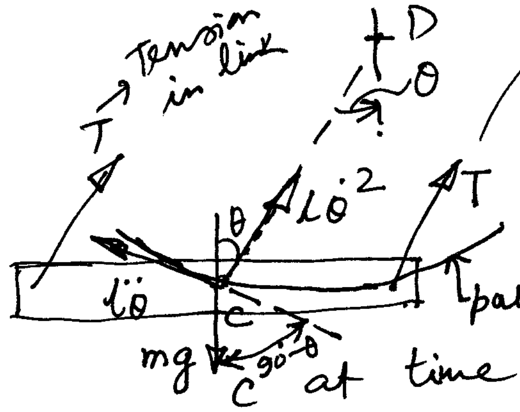


Do you see this is basically an inverted 4-bar parallelogram mechanism?

If you do, then you know ~~that~~ the centre of mass C of the bar executes a circular motion about D, midway between O_1 & O_2? And so, study these figs.



This fig. is for understanding, not essential here



C at time t

(Draw this figure properly)

[T is $\perp r$ to $l\ddot{\theta}$. Hence T doesn't arise

Apply $ml\ddot{\theta} = -mg\cos(90-\theta)$

& $\sin\theta \cong \theta$ to get

$\boxed{ml\ddot{\theta} + mg\theta = 0}$ & so, $\omega_n = \sqrt{\frac{mg}{ml}} = \sqrt{g/l}$

2nd part (A good one) We have already written the DEOM in tangential direction.

Now write the dynamical equation in the radial direction at c at time t.

Then, $$ml\dot{\theta}^2 = T+T - mg\cos\theta$$

$$\text{or, } ml\dot{\theta}^2 = 2T - mg\cos\theta. \quad \text{—(1)}$$

This DE gives T.

We take $\cos\theta = 1 - \frac{\theta^2}{2!}$ (So, don't linearize $\cos\theta$, note

since 2nd order terms are to be taken into account, as per the problem statement, note. Then,

From (1), $$T = \frac{1}{2}ml\dot{\theta}^2 + \frac{mg}{2}\left(1 - \frac{\theta^2}{2}\right) \quad \text{—— (2)}$$

Now, from $ml\ddot{\theta} + mg\theta = 0$, we have

$$\theta = A\sin\omega_n t + B\cos\omega_n t$$

$$\therefore \theta(0) = \theta_0 = B$$

$$\dot{\theta} = A\omega_n\cos\omega_n t - B\omega_n\sin\omega_n t$$

$$\therefore \dot{\theta}(0) = 0 \Rightarrow 0 = A$$

Hence, $\theta = \theta_0\cos\omega_n t$

& $\dot{\theta} = -\theta_0\omega_n\sin\omega_n t$

Put these in (1) & write the expression for T.

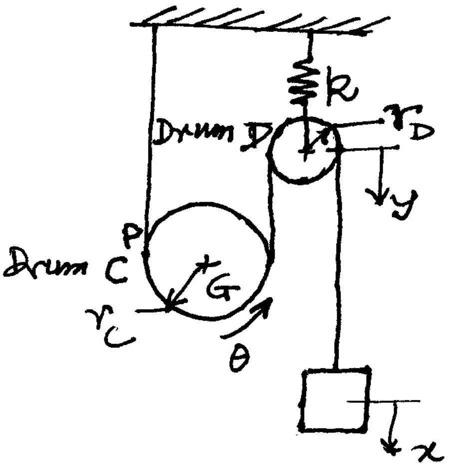
~~Put these~~

→ Case (a) In the vertical position, $\theta = 0$ & get T from the expression you've written.

→ Case (b) when $\theta = \theta_0$, $\cos\omega_n t = 1$ etc. & get T.

→ Complete the solution. The answer is yours to find. →

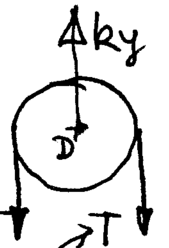
Problem (5) :- (We won't consider static forces & gravity. You may check if it is OK to do so!)



(If mass of drum D is considered, this is a 2-DOF system, note.
→ We shall go into a detailed discussion of the term DOF soon.)

→ The drum C rotates about P, as before.

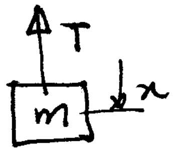
→ $x = 2y + 2r_c\theta$ (kinematical condition) --(1)

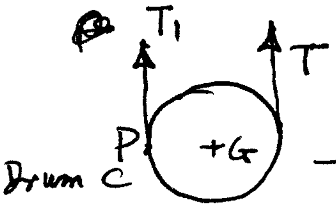


These are equal since drum is inertialess. Otherwise, an infinite angular acceleration would result!

$\Rightarrow \quad 2T = ky$ --(2)

$m\ddot{x} = -T$ --(3)





← T_1 & T are different. Why?

Taking moments about G, we get,

(4) --- $I_p\ddot{\theta} = 2r_cT$ [c is not the cm of drum, G is]

$\left[I_P = I_c + m_c r_c^2 = m_c k_G^2 + m_c r_c^2 = 1.096\ \text{kg m}^2\right]$

→ Eliminate T, y, θ from (1), (2), (3) & (4) to get

(!) $\frac{4m}{k}\ddddot{x} + \left[1 + \frac{4mr_c^2}{I_c}\right]\ddot{x} = 0 \Rightarrow \frac{4m}{k}\ddot{x} + \left[1 + \frac{4mr_c^2}{I_c}\right]x = 0$ (?) →

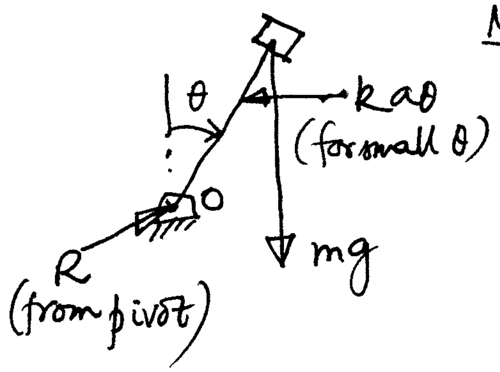
$$\Rightarrow \omega_n = \sqrt{\frac{\left(1+\frac{4mr_c^2}{I_c}\right)}{4m/k}} = 18.174 \text{ rad/s}$$

Check

$$\Rightarrow f_n = \frac{\omega_n}{2\pi} = 2.892 \text{ Hz}$$

Question:– Could you eliminate a different set of variables (say, T, θ & x) & get a 2nd order DE instead of a 4th order?

Problem (6):~ A simple one! But an important point to note is that the weight of the block doesn't support any static force in the spring & hence, its moment about the pivot matters.



Moment-balance method:–

$$I_o \ddot{\theta} = Mg(a+b)\sin\theta - ka\theta \times a$$

$$\Rightarrow M(a+b)^2\ddot{\theta} + [ka^2 - Mg(a+b)]\theta = 0$$

(Linearizing, $\sin\theta \cong \theta$)

$$\Rightarrow \omega_n = \sqrt{\frac{ka^2 - Mg(a+b)}{M(a+b)^2}}$$

= The given answer.

→ TRY THE OTHER PROBLEMS.
WE SHALL DISCUSS THEM LATER.

(END OF Tu-1, Discussions-I)