# Assignment 2

## Transform Calculus (MA20101)

### Section 4

To be submitted on or before 11th September 2017 (Monday)

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Q1) Sketch the function

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

extended periodically with period $2\pi$.
Find $\mathcal{L}\{f(t)\}$.

Q2) Find $\mathcal{L}\{t^{-1/2}\}$.

Q3) Verify the initial value theorem for the two functions
(a) $2 + \cos t$ & (b) $(4+t)^2$

Q4) Prove that $\mathcal{L}\{Ei(t)\} = \mathcal{L}\left[\int_t^\infty \frac{e^{-u}}{u}\,du\right]$

$$= \frac{\ln(s+1)}{s}.$$

** Q5) Show that $\mathcal{L}\{\ln(t)\} = \dfrac{\Gamma'(1) - \ln s}{s}$

$$= -\frac{\gamma + \ln s}{s}$$

where $\gamma = 0.5772156\ldots$
is Euler's constant.

Q6) Prove that

$$\mathcal{L}\{\operatorname{erf}\sqrt{t}\} = \mathcal{L}\left[\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-u^2}\,du\right]$$

$$= \frac{1}{s\sqrt{s+1}}.$$

Q7) Solve the integral eqn

$$\int_0^t y(u)\,y(t-u)\,du = 16\sin 4t.$$

Q8) Evaluate each of the following by using the Convolution Theorem:

(i) $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$, (ii) $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}$.

Q9) Use the Laplace transform to solve the system of O.D. eqns

$$x_1'' + 10x_1 - 4x_2 = 0$$

$$-4x_1 + x_2'' + 4x_2 = 0.$$

subject to $x_1(0)=0$, $x_1'(0)=1$, $x_2(0)=0$, $x_2'(0)=-1$.

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