Indian Institute of Technology, Kharagpur
Mid-Autumn Semester Examination, 2017
Mechanical Engineering Department
Subject: Applied Elasticity (ME 60401 / ME40605)

17MF63R51

Full Marks : 30 Time : 2 Hrs

Answer all questions. Symbols are self explanatory. Adopt reasonable assumption if it is required.

1. Two small straight lines **OP** and **OQ** in an undeformed solid (Ω) have been mapped as **O'P'** and **O'Q'** , respectively in the same solid when it is deformed (Ω'). The lines **OP** and **OQ** are parallel to the vectors $2\hat{i}+2\hat{j}+\hat{k}$ and $3\hat{i}-6\hat{j}$, respectively. Determine the change in angle between the lines after deformation if the state of strain at the point **O** is given by

$$[\mathbf{E}] = \begin{bmatrix} 0.5 & 0.3 & 0 \\ 0.3 & 0.4 & -0.1 \\ 0 & -0.1 & 0.2 \end{bmatrix}. \qquad (5)$$

2. (a) How the second Piola Kirchoff stress tensor is defined? Show that the components of a traction vector at a point on a plane ($\hat{\mathbf{n}}$) can be expressed as $\mathbf{t}_i(\hat{\mathbf{n}}) = \sigma_{ij}\mathbf{n}_j$. (1+4)

(b) Prove one of the governing equilibrium equations given by $\sigma_{ij,j} + \mathbf{f}_i = 0$. (3)

(c) Show that in a formulation the term $\sigma_{ij}\mathbf{u}_{i,j}$ can be expressed as $\sigma_{ij} \in_{ij}$. (2)

3. Show that the constitutive relation of a Green elastic solid can be given by $\sigma_{ij} = \dfrac{\partial \mathbf{U}}{\partial \in_{ij}}$ where **U** is the strain energy density function. (5)

4. The elastic constant matrix of an orthotropic solid is given by

$$[\mathbf{C}] = \begin{bmatrix} C_{1111} & C_{1122} & C_{1133} & 0 & 0 & 0 \\ C_{1122} & C_{2222} & C_{2233} & 0 & 0 & 0 \\ C_{1133} & C_{2233} & C_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{1212} \end{bmatrix}$$

Show that the number of independent elastic constants can be further reduced by 4 if an axis of symmetry exists in the solid. (5)

5. A simply supported prismatic straight beam of rectangular transverse cross section is transversely loaded with a sinusoidally distributed force on its top surface. The beam is made of homogeneous isotropic material. Show that the exact solutions for the displacement field in the beam can be given by the following expressions:

$$\mathbf{u}(\mathbf{x},\mathbf{z}) = \{(\mathbf{U}_1 + \mathbf{z}\mathbf{U}_2)\mathbf{e}^{\lambda z} + (\mathbf{U}_3 + \mathbf{z}\mathbf{U}_4)\mathbf{e}^{-\lambda z}\}\cos\lambda\mathbf{x}$$

$$\mathbf{w}(\mathbf{x},\mathbf{z}) = \{(\mathbf{W}_1 + \mathbf{z}\mathbf{W}_2)\mathbf{e}^{\lambda z} + (\mathbf{W}_3 + \mathbf{z}\mathbf{W}_4)\mathbf{e}^{-\lambda z}\}\sin\lambda\mathbf{x}$$

where λ is a characteristic parameter, $\mathbf{U}_i$ and $\mathbf{W}_i$ ($\mathbf{i}$=1, 2, 3, 4) are unknown constants. (5)