

Bending $\rightarrow \sigma_b \rightarrow K_f \cdot \sigma_b$ fatigue SIF

Axial $\rightarrow \sigma_{\text{axial}} \rightarrow K_f \cdot \sigma_{\text{axial}}$

Torsion/ shear: $\tau \rightarrow K_{fs} \cdot \tau$

fatigue SIF

in shear

Calculate von Mises stress

$$\sigma' = \sqrt{(K_f \sigma_{\text{bending}} \pm K_f \cdot \sigma_{\text{axial}})^2 + 3(K_{fs} \cdot \tau)^2}$$

factor safety coefficient

takes into account
 $K_c = 0.59$ implicitly

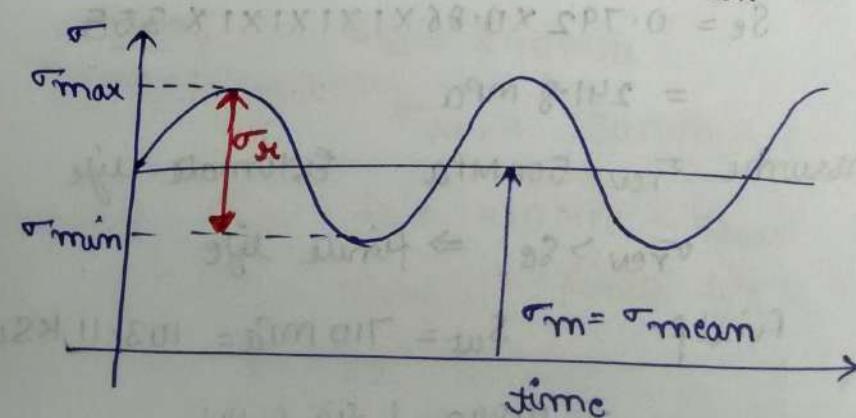
Fatigue under fluctuating load

- Load is not completely reversible.

$$\sigma_{\text{mean}} = \frac{\sigma_{\max} + \sigma_{\min}}{2} \neq 0$$

- S-N diagram works only when

$$\sigma_{\text{mean}} = 0$$



σ_{\max} := maximum stress

σ_{\min} := minimum stress

$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$:= amplitude complement

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} := \text{midrange component}$$

$$\sigma_a = (\sigma_{max} - \sigma_{min}) = \text{range of stress}$$

Notch sensitivity:

Practise : Before yielding,

$$\sigma_m = \sigma_{mo} \cdot k_f \quad \xrightarrow{\text{Nominal values}}$$

$$\sigma_a = \sigma_{ao} \cdot k_f \quad \xrightarrow{\text{same factor } k_f \text{ to amplitude and midrange component}}$$

After yielding,

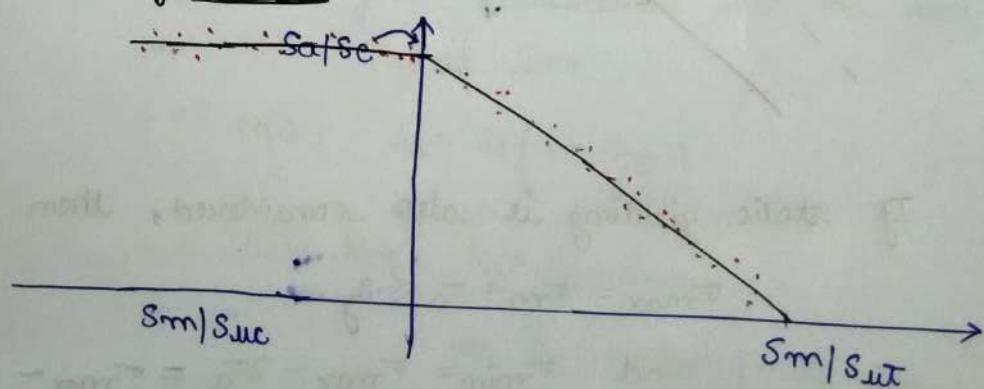
$$\sigma_a = k_f \cdot \sigma_{ao}$$

$$\sigma_m = \sigma_{mo} \quad (\text{No } k_f \text{ on } \sigma_{mo})$$

Modified Goodman Criterion

Observation: S_a , S_m are limiting values of amplitude and midrange components of stress.

If $N > 10^6$:



M-G eqn.

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad \text{for } \sigma_m > 0$$

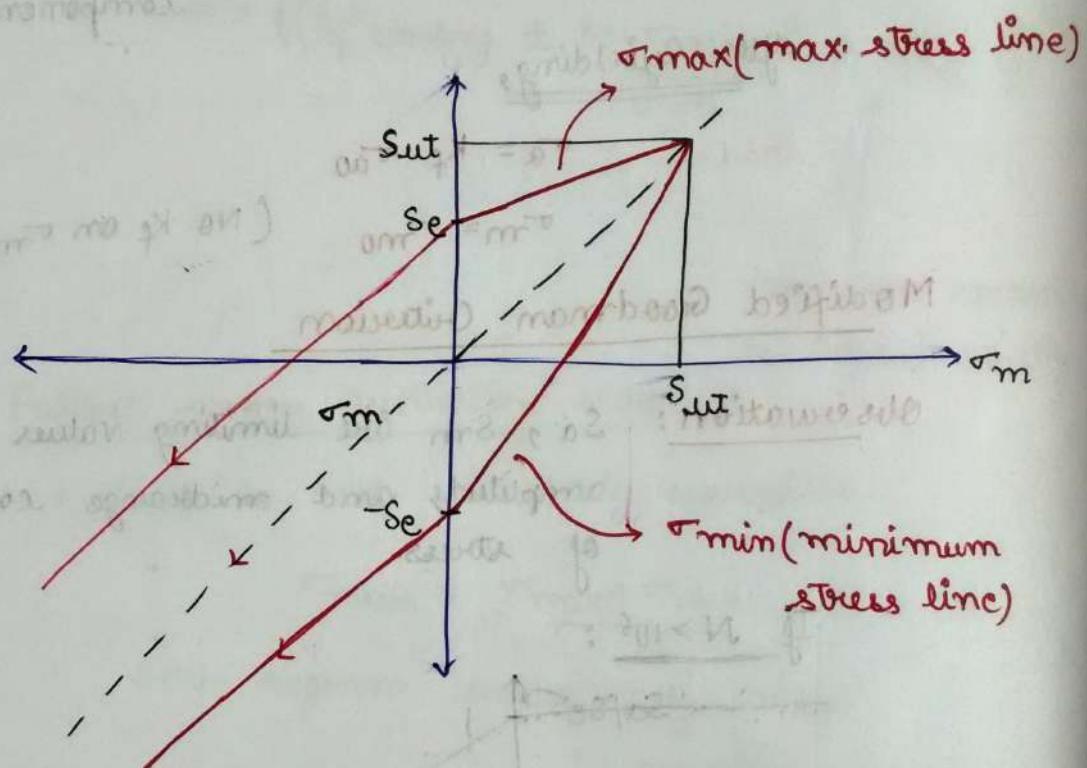
$$\text{or } \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1 \quad - " -$$

and $\frac{\sigma_a}{S_e} = 1$ or $\frac{\sigma_a}{S_c} = 1$ for $\sigma_m < 0$.

$$\sigma_a = S_e \left(1 - \frac{\sigma_m}{S_{UT}} \right) \quad \text{for } \sigma_m > 0$$

or $\sigma_{max} = \sigma_m + \sigma_a = S_e + \sigma_m \left(1 - \frac{S_e}{S_{UT}} \right)$

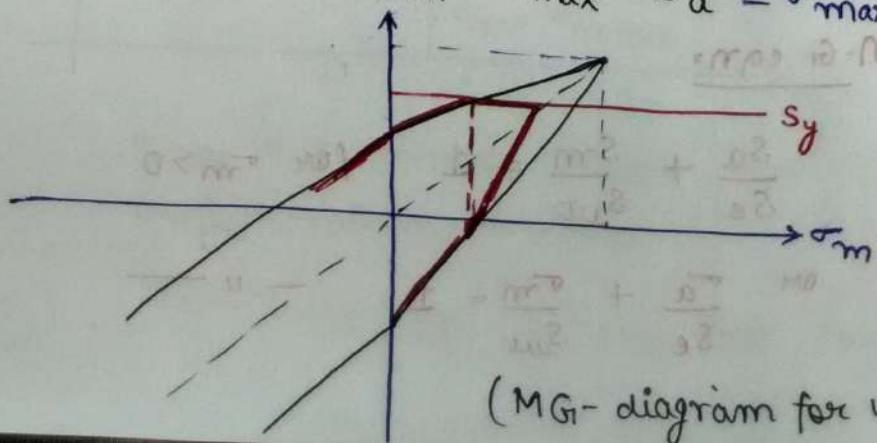
$$\sigma_{min} = \sigma_m - \sigma_a = -S_e + \sigma_m \left(1 + \frac{S_e}{S_{UT}} \right)$$



If static yielding is also considered, then

$$\sigma_{max} = \sigma_m + \sigma_a \leq S_y$$

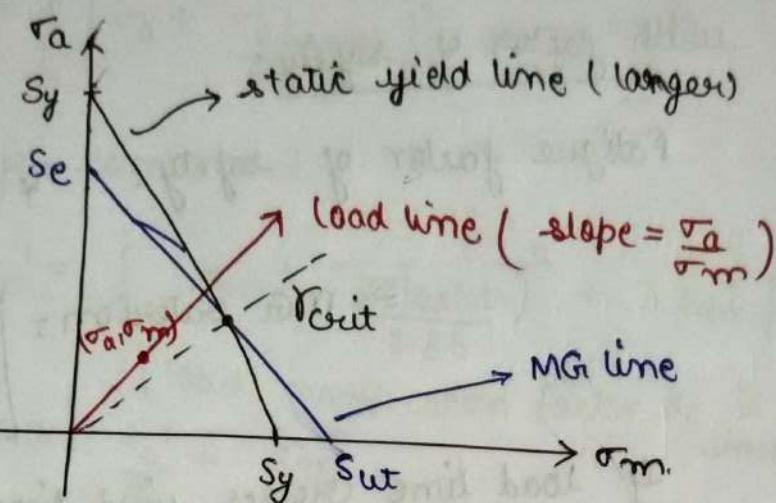
and $\sigma_{min} = \sigma_{max} - 2\sigma_a = \sigma_{max} - 2(S_y - \sigma_m)$



(MG-diagram for infinite life)

For finite life: replace S_e by S_f .

MG diagram combined with static yield (for $\sigma_m > 0$)



Load line slope:

$$r = \frac{r_a}{r_m}$$

If $r > r_{out} \Rightarrow$ fatigue failure

$r < r_{out} \Rightarrow$ static yielding

Load line:

$$r = \frac{r_a}{r_m} = \frac{S_a}{S_m}$$

for r_{out} : find intersection b/w MG line and yield line

$$MG: S_a = S_e \left(1 - \frac{S_m}{S_{ut}}\right)$$

$$Yield line: S_a = S_y - S_m.$$

$$\Rightarrow S_e \left(1 - \frac{S_m}{S_{ut}}\right) = S_y - S_m$$

$$\Rightarrow S_e - S_y = S_m \left(\frac{S_y - S_e}{S_{ut}} - 1\right)$$

$$\Rightarrow S_m = \frac{S_y - S_e}{S_{ut} - S_e} \times S_{ut}$$

$$\Rightarrow S_a = \left(\frac{S_{ut} - S_y}{S_{ut} - S_e}\right) S_e$$

$$\gamma_{uit} = \frac{s_a}{s_m} = \frac{s_{uit} - s_y}{s_y - s_e} \times \frac{s_e}{s_{uit}}$$

With factor of safety:

Fatigue factor of safety: $\eta_f = \frac{\sigma_a}{s_e} = \frac{\sigma_m}{s_m}$

\Rightarrow MG criterion:

$$\frac{\sigma_a}{s_e} + \frac{\sigma_m}{s_m} = \frac{1}{\eta_f}$$

If load line crosses yield line first:

$$\sigma_{max} = \frac{s_y}{n}$$

$$\Rightarrow \frac{\sigma_a}{s_y} + \frac{\sigma_m}{s_y} = \frac{1}{n}$$

For pure torsional loading:

Use MG criterion with

$$\begin{aligned} s_{sy} &= 0.577 s_y \\ s_{su} &= 0.67 s_{ut} \\ \text{or } K_c &= 0.59 \end{aligned}$$

combined loading:

Bending: $\sigma_a = K_f \cdot \sigma_{ao}$ fatigue SCF

$$\sigma_m = K_f \cdot \sigma_{mo}$$

Axial: $(\sigma_a)_{axial} = K_f \cdot (\sigma_{ao})_{axial}$

$$(\sigma_m)_{axial} = K_f \cdot (\sigma_{mo})_{axial}$$

Torsion/shear:

$$\tau_a = K_{fs} \cdot \tau_{ao}$$

$$\tau_m = K_{fs} \cdot \tau_{mo}$$

Von Mises Stress:

Alternating component

$$\sigma_a' = \left\{ \left(\frac{\sigma_a + \tau_{axial}}{0.85} \right)^2 + 3\tau_a^2 \right\}^{1/2}$$

Midrange component

$$\tau_m' = \left\{ \left(\frac{\tau_m + \tau_{m\text{axial}}}{0.85} \right)^2 + 3\tau_m^2 \right\}^{1/2}$$

(load modification factor K_c is incorporated)

$$MG: \frac{\sigma_a'}{S_e} + \frac{\tau_m'}{S_{ut}} = \frac{1}{n_f}$$

⇒ Choose load factor $K_c = 1$ for calculating S_e

Example → loading

$$\begin{aligned} \sigma_{\text{Bending}} &= 200 \text{ to } 260 \text{ MPa} \\ \tau &= -120 \text{ to } 120 \text{ MPa} \\ \tau_{\text{Axial}} &= -100 \text{ MPa} \end{aligned} \quad \left. \begin{array}{l} \text{After correcting} \\ \text{for notch} \\ \text{sensitivity} \end{array} \right\}$$

Material: $S_{ut} = 1200 \text{ MPa}$ $S_y = 900 \text{ MPa}$

$$S_e = 400 \text{ MPa} \quad (\text{fully corrected})$$

Calculate n_f for infinite life

Solution →

	Alternating (MPa)	Mid Range (MPa)
Bending	30	230
Tension	120	0
Axial	0	-100

Von Mises

$$\sigma_a' = \left\{ \left(30 + \frac{0}{0.85} \right)^2 + 3 \times 120^2 \right\}^{1/2} = 210 \text{ MPa}$$

$$\sigma_m' = \left\{ \left(230 - \frac{100}{0.85} \right)^2 + 3 \times 0^2 \right\}^{1/2} = 112.35 \text{ MPa}$$

Load line : $R = \frac{\sigma_a'}{\sigma_m'} = \frac{210}{112.35} = 1.866$

$$r_{exit} = \frac{S_{UT} - S_e}{S_{UT} - S_c} \times \frac{S_c}{S_{UT}}$$

$$= \frac{1200 - 900}{900 - 400} \times \frac{400}{1200} = 0.2$$

since, $R > r_{exit} \Rightarrow$ fatigue failure

$$\Rightarrow \frac{1}{n_f} = \frac{\sigma_a'}{S_c} + \frac{\sigma_m'}{S_{UT}}$$

$$\Rightarrow n_f = \left(\frac{210}{400} + \frac{112.35}{1200} \right)^{-1}$$

$$\Rightarrow n_f = 1.62$$

Cumulative fatigue damage

Fully reversible

stress level : σ_1 for n_1 cycles $\xrightarrow{\text{cycle of operation}}$

N_1 be the life at stress σ_1 .

After n_1 cycles at σ_1 , apply stress σ_2 for n_2 cycles

N_2 be the life at stress σ_2 .

Example \Rightarrow At σ_1 , $N_1 = 10^4$ cycles

$n_1 = 3 \times 10^3$ cycles

life remaining at σ_1 : $N_1 - n_1$

$= 7 \times 10^3$ cycles

Now let $\sigma_2 = S_c$, $N_2 = 10^6$ cycles

What is the remaining life n_2 ?

Palmgreen - Miner rule:

Damage gets accumulated at every cycle of operation

- Damage variable D .

$$\text{stress level : } D_1 = \frac{n_1}{N_1}, \quad \text{stress level : } D_2 = \frac{n_2}{N_2}$$

$$\text{stress level } \sigma_i \quad D_i = \frac{n_i}{N_i}$$

Total / cumulative damage

$$D = D_1 + D_2 + \dots + D_i$$

For failure it is fully damaged
 $\Rightarrow D = 1$.

\Rightarrow For failure

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots + \frac{n_i}{N_i} = 1$$

Eg. $n_1 = 3 \times 10^3 \quad N_1 = 10^4$

$n_2 = ? \quad N_2 = 10^6$

$$\Rightarrow 1 = \frac{n_1}{N_1} + \frac{n_2}{N_2}$$

$$\Rightarrow 1 = \frac{3}{10} + \frac{n_2}{10^6} \Rightarrow n_2 = 7 \times 10^5 \text{ cycles}$$

(Not $10^6 - 3000$ cycles)

Eg \rightarrow for a machine part

$$S_{UT} = 595 \text{ MPa}, \quad f = 0.86$$

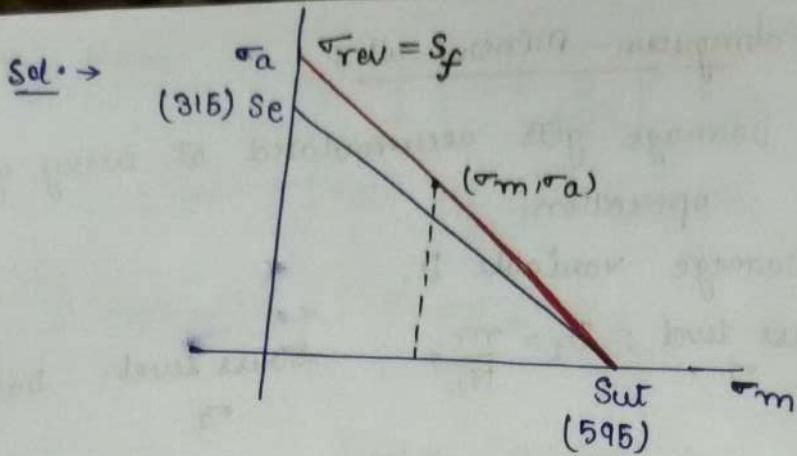
Fully corrected endurance strength

$$S_e = 315 \text{ MPa.}$$

Loading: $\sigma_a = 245 \text{ MPa}, \quad \sigma_m = 210 \text{ MPa. for}$

$$12 \times 10^3 \text{ cycles.}$$

Find remaining life at $\sigma_a = S_e, \sigma_m = 0$.



Load point is beyond MG line

⇒ Finite life

→ Draw a straight line joining $(S_{ut}, 0)$ & (σ_m, σ_a)

⇒ Intersection with y-axis gives $\sigma_{rev} = S_f$.

$$\Rightarrow \frac{\sigma_{rev}}{S_{ut}} = \frac{\sigma_a}{S_{ut} - \sigma_m} \Rightarrow S_f = \sigma_{rev} = \frac{\sigma_a}{S_{ut}} \times \frac{1}{(S_{ut} - \sigma_m)}$$

Eg → $S_f = 378.6 \text{ MPa} > 315 \text{ MPa} = S_e$

⇒ Finite life.

$$f \times S_{ut} = 0.86 \times 595 = 511.7 \text{ MPa} > S_f$$

⇒ High cycle fatigue

⇒ $N_1 = \text{life at } \sigma_{rev} = S_f$ (completely reversible)

S-N diagram

$$N_1 = \left(\frac{\sigma_{rev}}{a} \right)^{1/b} = \left(\frac{S_f}{a} \right)^{1/b}$$

$$a = \left(f \cdot S_{ut} \right)^2 / S_e \quad b = -\frac{1}{3} \log_{10} \left(f \cdot S_{ut} \right) / S_e$$

$$\Rightarrow a = 831.23 \quad b = -0.07$$

$$\Rightarrow N_1 = 75 \times 10^3 \text{ cycles}$$

$$(\sigma_a, \sigma_m) = (245, 210) \equiv \sigma_i = S_f = 378.6 \text{ MPa}$$

$$n_1 = 12 \times 10^3$$

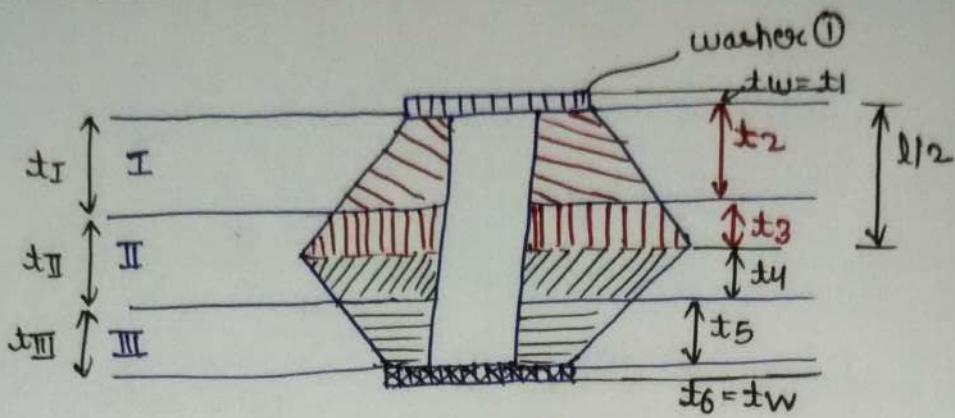
$$N_1 = 75 \times 10^3$$

$$\text{Miner: } \frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \frac{n_2}{N_2} = 1 - \frac{12 \times 10^3}{75 \times 10^3}$$

$$\Rightarrow n_2 = 8.4 \times 10^5 \text{ cycles.}$$

(remaining life)

Pressure cone



Member 1 (Washer)

$$D = 1.5 d$$

$$t_1 = t_w$$

$$E_1 = E_w.$$

Member 2:

$D_2 = \text{calculate} = \text{Top dia of seg. 2}$

$$t_2 = t_1 = D + 2t_1 * \tan\alpha$$

$$E_2 = E_I$$

Member 3:

$D_3 = \text{top dia of seg 3}$

$= \text{bottom dia of seg 2}$

$$= D_2 + 2t_2 \tan\alpha$$

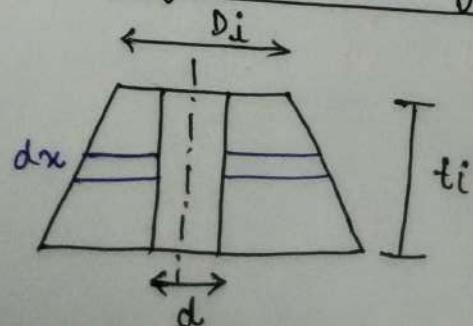
$$E_3 = E_{II}$$

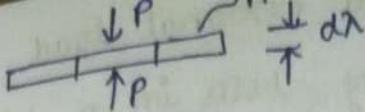
$$t_3 = \frac{l}{2} - (t_1 + t_2)$$

$$= \frac{l}{2} - (t_w + t_1)$$

Similarly for members 4, 5, 6.

For any member segment-i



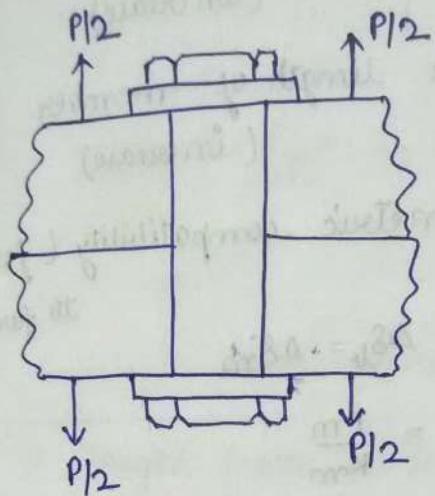


$$d\delta = \frac{P}{AE} \times dx$$

Total elongation

$$\delta = \int_0^{x_i} \frac{P}{AE} dx$$

$$= \frac{P}{E} \int_0^{x_i} \frac{dx}{A}$$



F_i = initial pre load per bolt

K_b = bolt stiffness

K_m = member stiffness

- Upon application of external load P

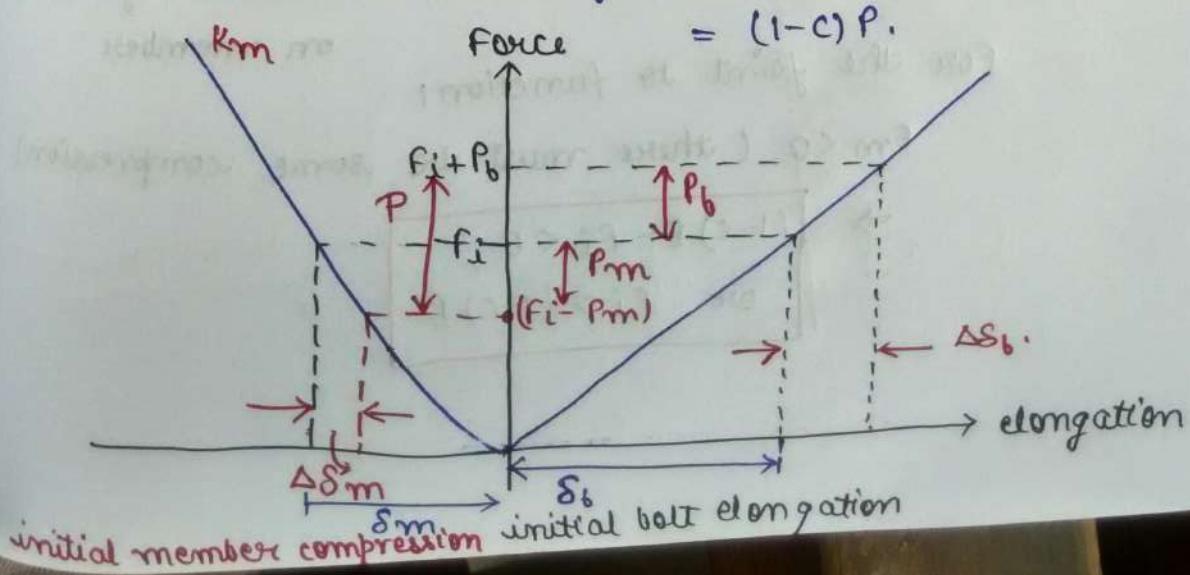
- bolt tension increases
- member compression decreases.

- Bolts and members \rightarrow two equivalent springs in parallel.

- $P_b =$ portion of P taken by bolt = $C P$

- $P_m =$ portion of P taken by member

$$= (1-C) P.$$



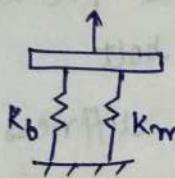
P_{total} = total external load

N = number of bolts in a joint

$p = \frac{P_{\text{total}}}{N}$ = external load per bolt
(+ pressure cone)

upon application of P

- $\Delta \delta_b \rightarrow$ change in length of bolt
(increase)
- $\Delta \delta_m \rightarrow$ change in length of member
(increase)



for geometric compatibility (for joint to function)

$$\Delta \delta_b = \Delta \delta_m$$

$$\Rightarrow \frac{P_b}{K_b} = \frac{P_m}{K_m}$$

$$\text{and } P_b + P_m = P$$

$$\Rightarrow P_b = \frac{K_b}{K_m + K_b} \times P, \quad P_m = \frac{K_m}{K_b + K_m} \times P$$

$$C = \frac{K_b}{K_m + K_b} := \text{stiffness constant of the joint}$$

$$1-C = \frac{K_m}{K_b + K_m} := \text{stiffness constant of the member}$$

$$\Rightarrow F_b = f_i + CP \rightarrow \text{resultant force on bolt}$$

$$F_m = (1-C)P - f_i \rightarrow \text{resultant force on member}$$

For the joint to function:

$F_m < 0$ (there must be some compression)

$$\Rightarrow (1-C)P - f_i < 0$$

or $f_i > (1-C)P$

How to apply preload on bolt?

- Related to bolt tightening torque
- Torque $T = K f_i d$

\downarrow
torque factor

(Table 8-15)

Recommended pre load:

$$f_i = \begin{cases} 0.75 F_p & \text{for non-permanent joint} \\ 0.9 F_p & \text{for permanent joint} \end{cases}$$

Proof load $\leftarrow f_p = A_t \times S_p \rightarrow$ proof strength
(proportional limit)

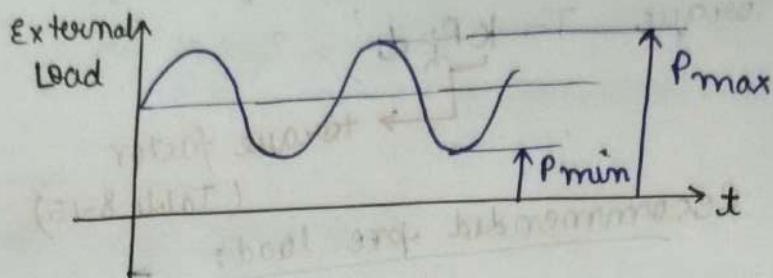
table 8.9 - 8.11

for steel bolts

$\rightarrow S_p = 0.75 S_y$ for other materials

→ Taught from slides - Bolt Analysis

Fatigue loading on bolt:



Load on bolt:

$$F_{b\max} = cP_{\max} + f_i$$

$$F_{b\min} = cP_{\min} + f_i$$

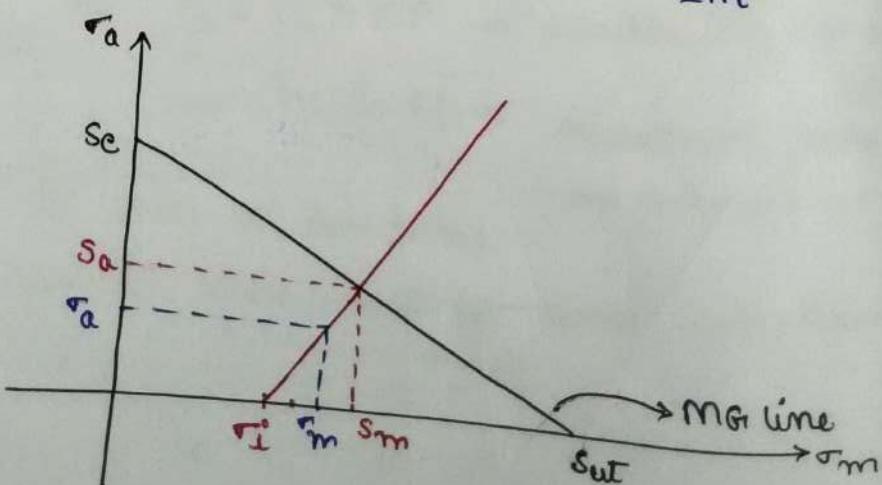
Stresses on bolt:

Alternating: $\sigma_a = \frac{F_{b\max} - F_{b\min}}{2A_t} = \frac{c(P_{\max} - P_{\min})}{2A_t}$

Mid Range: $\sigma_m = \frac{F_{b\max} + F_{b\min}}{2A_t} = \frac{f_i}{A_t} + \frac{c(P_{\max} + P_{\min})}{2A_t}$

When $P_{\min}=0$: $\sigma_a = \frac{cP_{\max}}{2A_t}$

$$\sigma_m = \sigma_i + \frac{cP_{\max}}{2A_t} = \sigma_i + \sigma_a$$



Fatigue factor of safety

$$\eta_f = \frac{S_a}{\sigma_a} = \frac{S_m - \sigma_i}{\sigma_m - \sigma_i} \neq \frac{S_m}{\sigma_m}$$

→ increases factor of safety

- Bolt preload improves factor of safety.
- Endurance strength: $S_e = K_a K_b K_c K_d K_e K_f S_e'$
or

$$S_e = 0.85 S_e'$$

K_f → fatigue SCF

K_f ← accounts for notch sensitivity and surface finish

↓
Table 8-16

Eg.

Pressure vessel: internal pressure varies from 0 to P_{max} .

External load on bolt: 0 to P ($P_{min}=0$, $P_{max}=P$)

with preload: $\sigma_a = \frac{c P}{2 A_t}$, $\sigma_m = \sigma_a + \sigma_i$

Factor of safety: $\eta_f = \frac{S_a}{\sigma_a} = \frac{S_m - \sigma_i}{\sigma_m - \sigma_i}$

$$= \frac{S_m - \sigma_i}{\sigma_a}$$

→ Refer to dig. on left side.

M.G. Line

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Also, from load line,

$$S_m = \sigma_i + S_a$$

$$\Rightarrow \frac{S_a}{S_e} + \frac{\sigma_i + S_a}{S_{ut}} = 1$$

$$\Rightarrow S_a \left(\frac{1}{S_e} + \frac{1}{S_{ut}} \right) = 1 - \frac{\sigma_i}{S_{ut}} \Rightarrow S_a = S_e \left(\frac{S_{ut} - \sigma_i}{S_{ut} + S_e} \right)$$

$$\Rightarrow n = \frac{S_p}{\sigma_m + \sigma_a} = \frac{S_p A_t}{(f_i + cP)}$$

Let $c = 0.2$, Grade 9-8 bolt M10x1.5

$$S_p = 650 \text{ MPa}$$

$$S_{ut} = 900 \text{ MPa}, A_t = 58 \text{ mm}^2$$

$$S_e = 140 \text{ MPa}$$

Load: $P = 0 \text{ to } 20 \text{ kN}$

$$\sigma_a = \frac{cP_{max}}{2A_t} = \frac{0.2 \times 20 \times 10^3}{2 \times 58} = 34.5 \text{ MPa}$$

$$\sigma_m = \sigma_i + \sigma_a = 0.9 S_p + \sigma_a$$

$$\sigma_i = 0.9 S_p \text{ (permanent joint)}$$

$$\Rightarrow \sigma_m = 619.5 \text{ MPa}$$

$$\Rightarrow S_m = \frac{S_{ut} \times (S_e + \sigma_i)}{(S_{ut} + S_e)}$$

$$S_a = S_m - \sigma_i = 42.4 \text{ MPa} \quad (S_a > \sigma_a)$$

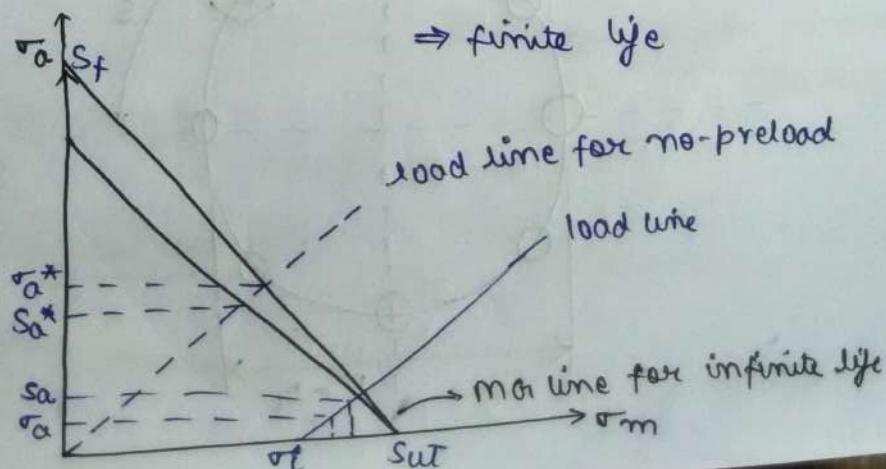
\Rightarrow Infinite life

when preload is zero,

$$c=1, \sigma_i=0.$$

$$\sigma_m^* = \sigma_a^* = \frac{cP_{max}}{2A_t} = \frac{20 \times 10^3}{2 \times 58} = 172 \text{ MPa}$$

$$S_a = \frac{S_e S_{ut}}{S_{ut} + S_e} = 121.2 \text{ MPa} < \sigma_a^*$$



$\sigma_a > \tau_a \Rightarrow$ infinite life

$\sigma_a^* < \tau_a^* \Rightarrow$ finite life

Eg. Pressure vessel: CI grade 30

Cyl Head: Steel

No of bolts: N

Gasket effective dia: D

Cylinder pressure: P_g

$$A = 20 \text{ mm}$$

$$C = 100 \text{ mm}$$

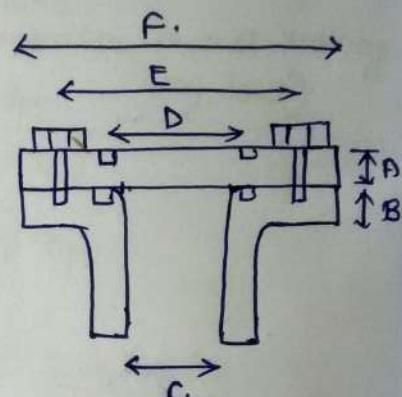
$$B = 20 \text{ mm}$$

$$D = 150 \text{ mm}$$

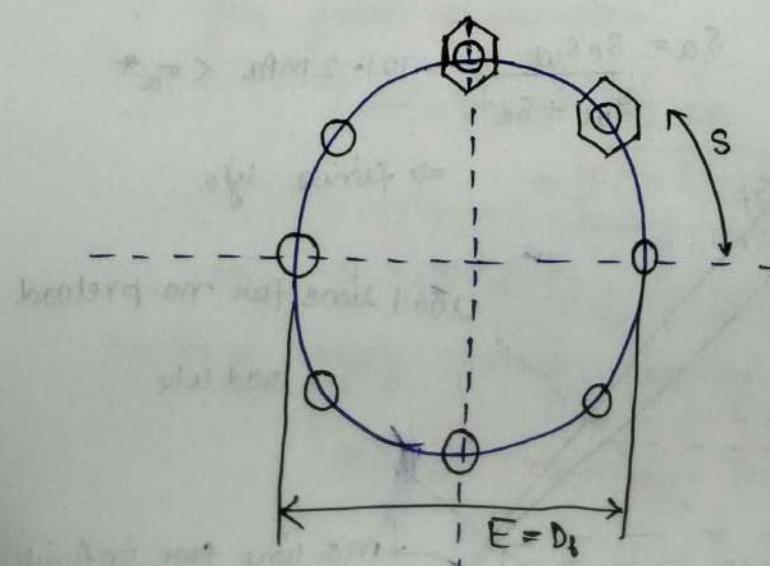
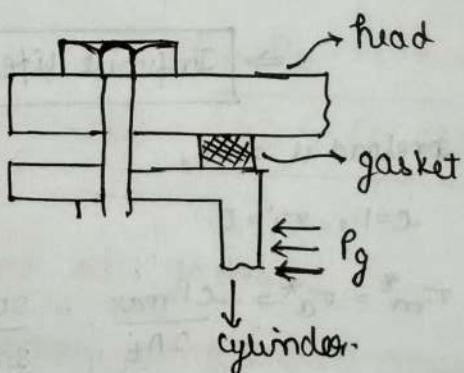
$$E = 200 \text{ mm}$$

$$F = 300 \text{ mm}$$

$$N = 10, P_g = 6 \text{ MPa.}$$



Gasketed Joint



$D_b = E =$ bolt circle dia.

Bolt spacing

$$S = \frac{\pi D_b}{N} = \frac{\pi E}{N}$$

$$3d < \frac{\pi D_b}{N} < 6d$$

d = dia. of bolt

for wrench
clearance
(tightening)

for uniform gasket
pressure.

* Joint separation $f_m < 0$

at separation, $f_m = 0$

$$\Rightarrow (1-C) P_0 - f_i = 0$$

$$\Rightarrow P_0 = \frac{f_i}{1-C} \leftarrow \text{load at separation}$$

$\frac{F_A}{F_B}$

Factor of safety against joint separation

$$n_o = \frac{P_0}{P} = \frac{f_i}{(1-C)P}$$

Factor of safety against overloading

$$n_L = \frac{s_p A_t - f_i}{c P}$$

Bolt length:

grip length: $l = A + B = 40\text{ mm}$

Nut height: $H = 10.8\text{ mm}$ for M12 bolts
(Table A31)

\Rightarrow Bolt length: $L > l + H = 50.8\text{ mm}$

Standard value = 60mm
(Table A17)

Threaded length

$$L_t = 2d + 6 = 30\text{ mm}$$

Unthreaded length

$$L_d = l_d = L - l_t = 30\text{ mm}$$

Threaded length in grip
 $l_f = l - l_d = 10\text{ mm}$.

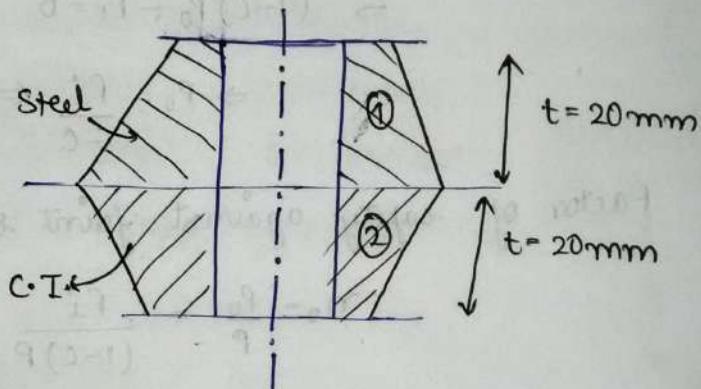
Bolt areas: $A_d = \frac{\pi d^2}{4} \approx 113.1 \text{ mm}^2$

$$A_t = 84.3 \text{ mm}^2$$

Bolt stiffness: $K_b = \frac{A_d A_t E}{A_d l_d + A_t l_d}$

$$= 534 \text{ MN/m}$$

Member stiffness:



Pressure-Cone

Member 1 →

$$K_1 = \frac{\pi E_1 d \ tan \alpha}{\ln \left[\frac{2t \ tan \alpha + D - d}{2t \ tan \alpha + D + d} \cdot \left(\frac{D+d}{D-d} \right) \right]}$$

$$d = 12 \text{ mm}$$

$$D = 1.5d = 18 \text{ mm}$$

$$\alpha = 30^\circ$$

$$t = 20 \text{ mm}$$

$$E_1 = 205 \text{ GPa (Steel)}$$

$$\Rightarrow K_1 = 4426.95 \text{ MN/m.}$$

Member 2 →

Only E_2 is different. $E_2 = 100 \text{ GPa}$

$$K_2 = K_1 \frac{E_2}{E_1} = 4426 \times \frac{100}{205} = 2159.48 \text{ MN/m.}$$

$$\Rightarrow K_m = \left(\frac{1}{K_1} + \frac{1}{K_2} \right)^{-1} = 1451.45 \text{ MN/m.}$$

stiffness constant:

$$c = \frac{k_b}{k_b + k_m} = \frac{534}{534 + 1451.45} = 0.268$$

Bolt material: ISO 9.8 grade

$$S_p = 650 \text{ MPa}, S_{ut} = 900 \text{ MPa}$$

$$S_y = 720 \text{ MPa} \quad (\text{Table 8-11})$$

Preload:

$$f_i = 0.75 S_p A_t \quad (\text{non permanent}) \\ = 41.1 \text{ kN}$$

Total external load:

$$P_{\text{total}} = p_g \times \frac{\pi c^2}{4} \\ = \frac{6 \times 10^6 \times \pi \times (100)^2 \times 10^{-6}}{4} = 47.12 \text{ kN}$$

$$P = \text{External load per bolt} = \frac{P_{\text{total}}}{N} = 4.71 \text{ kN,}$$

factor of safety

$$\text{Yielding: } n_p = \frac{S_p A_t}{C_p + f_i} = 1.14$$

$$\text{Joint separation: } n_0 = \frac{f_i}{P(1-c)} = 13.67$$

$$\text{Load factor: } n_L = \frac{S_p A_t - f_i}{C_p} = 6.67$$

→ Repeat the calculation when pressure varies from

$$0 \text{ to } p_g = 6 \text{ MPa.}$$

$$\Rightarrow \frac{f_A''}{\sigma_A} = \frac{f_B''}{\sigma_B} = \frac{f_C''}{\sigma_C} = \frac{f_D''}{\sigma_D} \rightarrow ②$$

Calculate $f_A'', f_B'', f_C'', f_D''$ using ① & ②

If bolts have different dia.

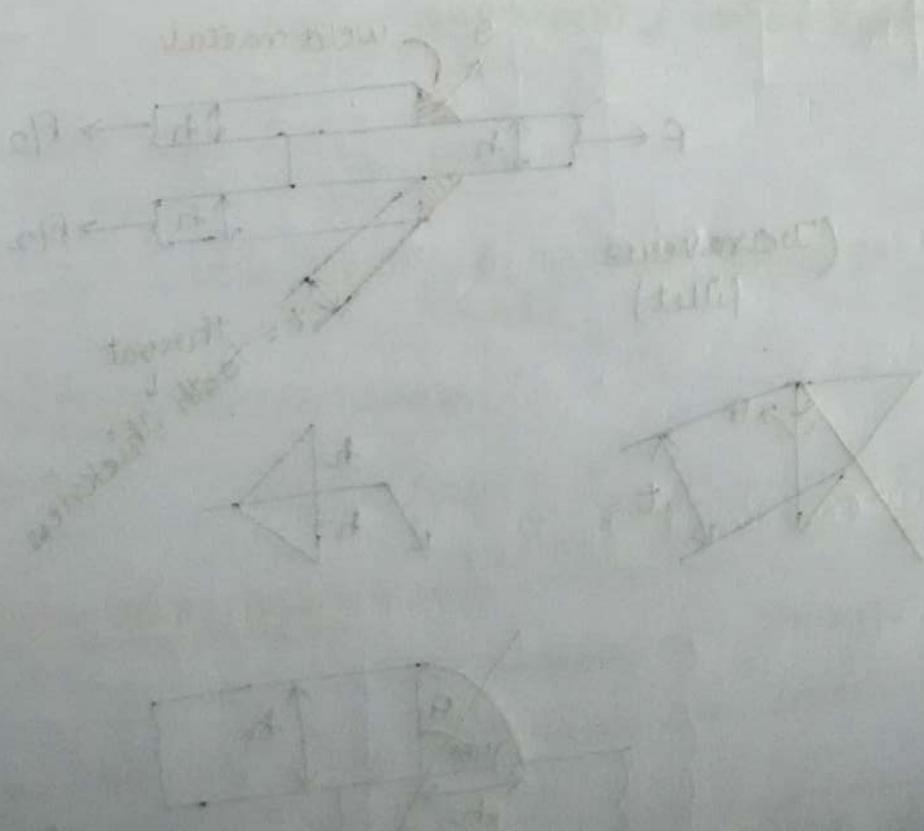
$$\tau_A'' = \frac{M \sigma_A}{J} \Rightarrow f_A'' = \tau_A'' A_A = \left(\frac{\pi M}{4 J} \right) \sigma_A d_A^2$$

$$\Rightarrow \frac{f_A}{\sigma_A d_A^2} = \frac{f_B}{\sigma_B d_B^2} = \frac{f_C}{\sigma_C d_C^2} = \frac{f_D}{\sigma_D d_D^2} \rightarrow ②$$

Primary shear: $f_A' = f_B' = f_C' = f_D' = \frac{V}{4}$ (if same dia)

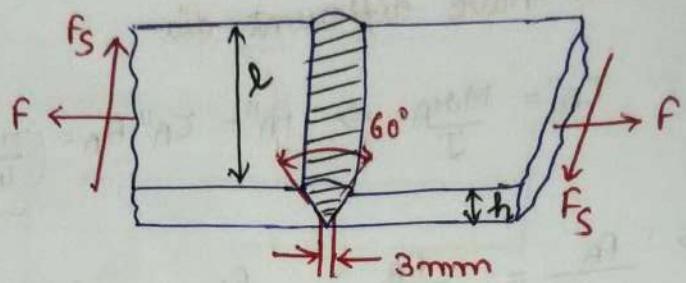
f_A is resultant of $f_A' \& f_A''$
 f_B — " — $f_B' \& f_B''$
 f_C — " — $f_C' \& f_C''$
 f_D — " — $f_D' \& f_D''$.

} Design bolt based
on resultant
shear.

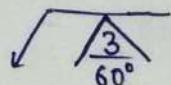


Welded Joints (Butt and Fillet Joints)

Butt-joint :-



Symbol:

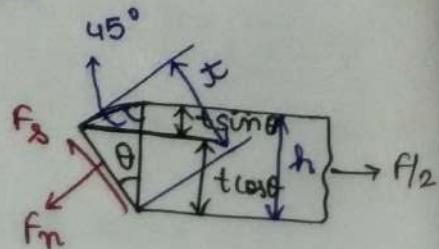
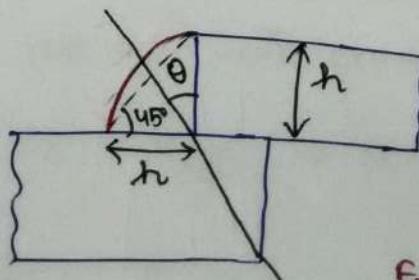
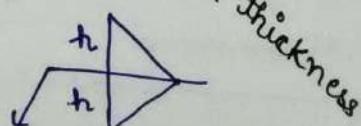
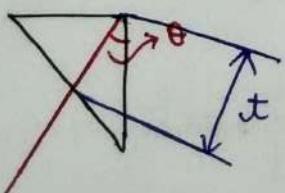
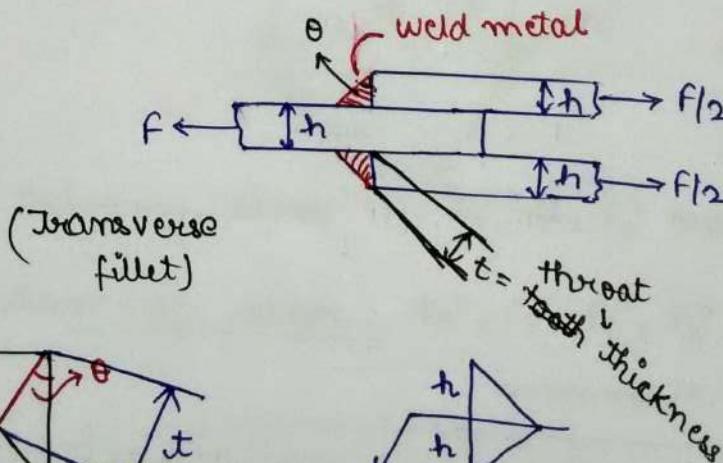


can be loaded in tension or shear

$$\text{Avg. normal stress: } \sigma = \frac{F}{th}$$

$$\text{Avg. shear stress: } \tau = \frac{F_S}{th}$$

Fillet Joint :-



$$F_n = \frac{F}{2} \cos \theta$$

$$F_s = \frac{F}{2} \sin \theta$$

Throat thickness

$$h = t \cos \theta + t \sin \theta$$

$$\Rightarrow t = \frac{h}{(\cos \theta + \sin \theta)}$$

Throat area: $A_{\text{throat}} = t \cdot l = \frac{hl}{(\cos \theta + \sin \theta)}$

Shear stress: $\tau = \frac{F_s}{A_{\text{throat}}} = \frac{F}{2hl} \sin \theta (\sin \theta + \cos \theta)$

Normal stress: $\sigma = \frac{F_n}{A_{\text{throat}}} = \frac{F}{2hl} \cos \theta (\sin \theta + \cos \theta)$

Von Mises stress: $\sigma' = \sqrt{\sigma^2 + 3\tau^2}$

$$= \frac{F}{2hl} (\sin \theta + \cos \theta) \sqrt{\cos^2 \theta + 3 \sin^2 \theta}$$

Maximum von Mises stress:

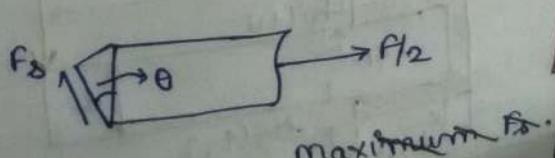
$$\sigma' = 2.016 \left(\frac{F}{2hl} \right) \text{ at } \theta = 62.5^\circ$$

Maximum shear stress:

$$\tau_{\max} = 1.207 \left(\frac{F}{2hl} \right) \text{ at } \theta = 67.5^\circ$$

Conservative Design:

Ignores normal stress at throat.
All the load is taken by shear.



$$F_s \sin \theta = \frac{F}{2}$$

maximum F_s .

$$F_s = \frac{1}{2} \tan 45^\circ$$

Minimum throat length

$$t_{\min} = \frac{h}{\sqrt{2}} \text{ at } \theta = 45^\circ$$

$$f_s|_{at t_{\min}} = \frac{f}{(2L)} \cdot \frac{1}{\sin 45^\circ} = \frac{1.414 f}{2L}$$

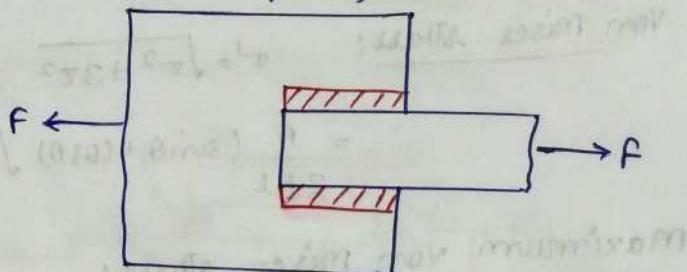
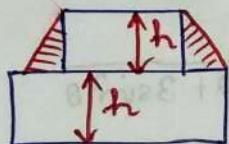
$$\tau|_{\theta=45^\circ} = \left(\frac{f}{2L}\right) \times 1.414 \rightarrow \textcircled{*}$$

$\tau_{\max} = \tau|_{\theta=45^\circ}$ is slightly larger than

$$\tau = 1.207 \left(\frac{f}{2L}\right) \text{ at } \theta = 67.5^\circ$$

Parallel fillet:

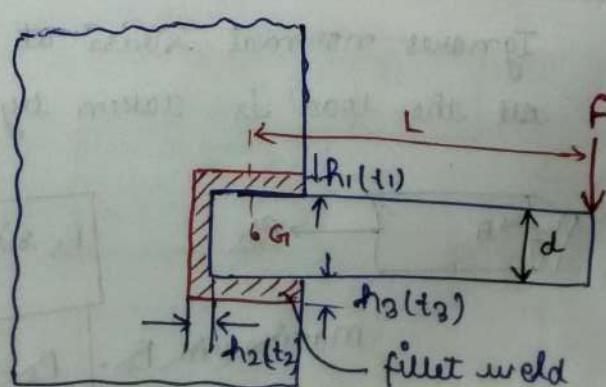
(top view)



• minimum throat area: $A_{throat} = \frac{2hL}{\sqrt{2}}$

$$\bullet \tau_{\max} = \frac{f}{\frac{1}{h} \sqrt{2}} = 1.414 \left(\frac{f}{2L}\right)$$

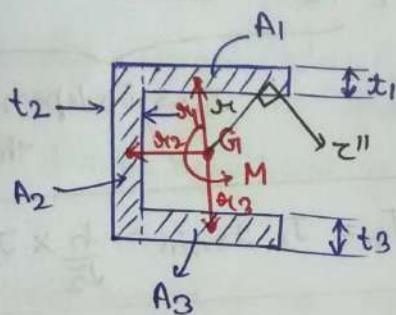
Combined loading



$$t_i = \text{throat thickness} = \frac{h_i}{\sqrt{2}}$$

$$\text{Primary shear: } \tau' = \frac{f}{A_{\text{throat}}}$$

Secondary shear:



G_1 : centroid of weld group

$$M = FL$$

$$\text{Throat area: } A_1 = b t_1, \quad A_2 = d t_2, \quad A_3 = b t_3$$

r_1, r_2, r_3 : distance from G to centroids

of A_1, A_2, A_3 respectively

Weld group is under torsion due to M

$$\Rightarrow \tau = \frac{M r}{J} \quad J \rightarrow \text{polar M.I. of weld group}$$

$$J_{1G_1} = \frac{b t_1}{12} (b^2 + t_1^2) + A_1 r_1^2$$

$$J_{2G_1} = \frac{d t_2}{12} (d^2 + t_2^2) + A_2 r_2^2$$

$$J_{3G_1} = \frac{b t_3}{12} (b^2 + t_3^2) + A_3 r_3^2$$

$$J_{G_1} = J_{1G_1} + J_{2G_1} + J_{3G_1}$$

$$= \frac{1}{12} \left[(b^3 + b t_1^2) t_1 + (d^3 + d t_2^2) t_2 + \left(\frac{b^3}{12} + b t_3^2 \right) t_3 \right]$$

$+ \frac{1}{12} (b t_1^3 + d t_2^3 + b t_3^3)$

Ignore

$$\Rightarrow J_G = \left(\frac{b^3}{12} + b r_1^2 \right) t_1 + \left(\frac{d^3}{12} + d r_2^2 \right) t_2 + \left(\frac{b^3}{12} + b r_3^2 \right) t_3$$

$$\text{If } t_1 = t_2 = t_3 = t$$

$$\Rightarrow J_G = \left[\frac{2b^3}{12} + \frac{d^3}{12} + b(r_1^2 + r_3^2) + d r_2^2 \right] t$$

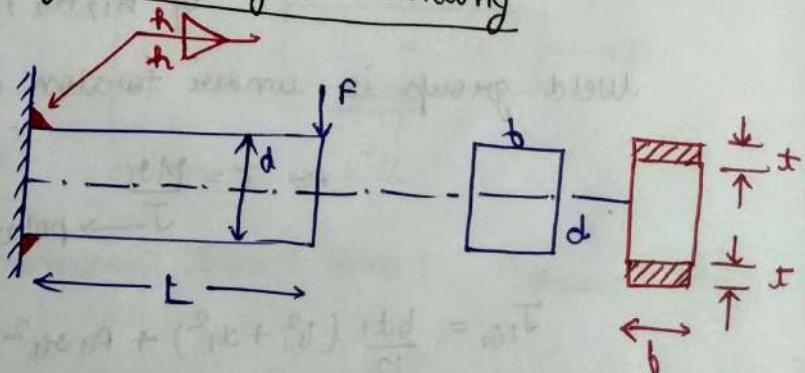
$J_u \rightarrow$ independent of weld thickness

$$\Rightarrow J_{Gr} = J_u t_{min} = \frac{t}{\sqrt{2}} \times J_u$$

$$\Rightarrow \tau'' = \frac{M c}{J_{Gr}} \Rightarrow \tau''_{max} = \frac{M c_{max}}{J_{Gr}}$$

Calculate resultant of τ' and τ'' for design

secondary shear for bending



$$\text{Bending moment} = M = F L$$

Area moment of inertia:

$$I = I_u \cdot t = \frac{t}{\sqrt{2}} I_u$$

unit m.i. of weld group about centroid

$$\tau'' = \frac{M c}{I}$$

For J_u and I_u see table 9-1 & 9-2.

steps for combined loading

1. compute centroid of weld group
2. ——" Bending moment about G
3. ——" $J_u(9-1)$ or $I_u(9-2)$
4. ——" find τ''_{\max}
5. Primary shear $\tau' = \frac{F}{A_{\text{flange}}}$
find resultant of τ' and τ'' .

Note:

- ① When base metal is a cold drawn steel, the cold drawn properties are replaced by hot rolled properties near weld group.
- ② Always check for base metal failure
- ③ Shear loading - shear stress on base metal is less than $0.4S_y \rightarrow$ (of base metal)

P-9-17 →

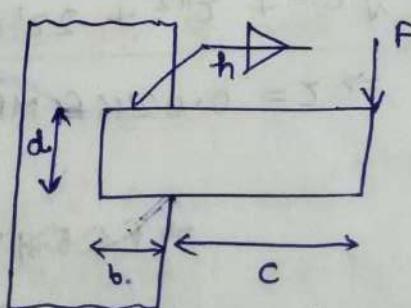
$$b = 50 \text{ mm}$$

$$d = 50 \text{ mm}$$

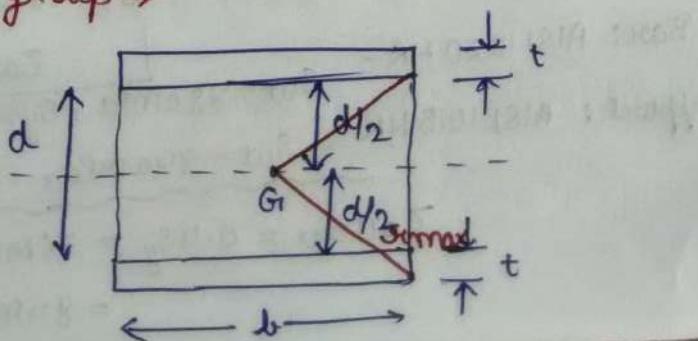
$$c = 150 \text{ mm}$$

$$h = 5 \text{ mm}$$

$$\tau_{\text{allow}} = 140 \text{ MPa.}$$



Sol. → Weld group →



$$A_{throat} = 26 \times t \\ = (0.707 \cdot h) 26$$

$$= 1.414 \cdot hb$$

$$J_u = \frac{\pi}{6} (3d^2 + b^2) = 83.33 \times 10^3 \text{ mm}^3$$

$$J_G = (0.707 \cdot h) J_u$$

$$L = c + \frac{b}{2} = 175 \text{ mm}$$

$$M = F \cdot L$$

Primary shear: $\tau' = \frac{F}{A} = \frac{F}{353} \text{ MPa}$ (F in N)

Secondary shear: $\tau'' = \frac{M \cdot r_{max}}{J_G}$

$$r_{max} = 37.9 \text{ mm}$$

$$\Rightarrow \tau'' = \frac{F \times 175 \times 37.9}{(0.707 \cdot h) J_u} \text{ MPa}$$

$$= 0.0225 F \text{ MPa}$$

Angle between τ' and τ'' is $\sim 45^\circ$.

Resultant

$$\tau = \sqrt{\tau'^2 + \tau''^2 + 2\tau' \tau'' \cos 45^\circ}$$

$$\Rightarrow \tau = 0.0246 F \text{ MPa} \leq 140 \text{ MPa} \text{ (allowable shear)}$$

$$\Rightarrow F \leq 5.69 \text{ KN}$$

$$\Rightarrow F \leq 5.69 \text{ KN}$$

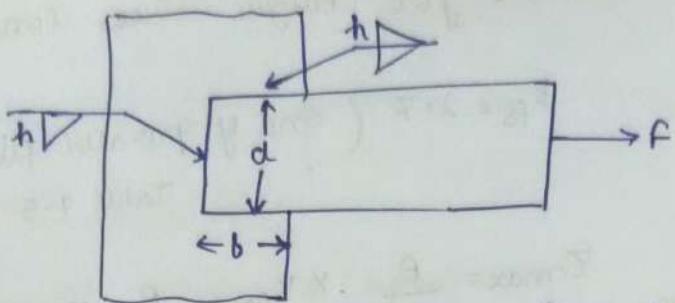
⑥ Electrode: E7010 - $S_{ut} = 482 \text{ MPa}$ $S_y = 393 \text{ MPa}$ [Table 9-3]

Bar: AISI 1020 HR - $S_{ut} = 380 \text{ MPa}$, $S_y = 210 \text{ MPa}$ } Table A20

Support: AISI 1015 HR - $S_{ut} = 340 \text{ MPa}$, $S_y = 190 \text{ MPa}$ } Table A20

$$\tau_{allow} = 0.4 S_y = 76 \text{ MPa} \text{ for support} \\ = 84 \text{ MPa for bar}$$

9-25)



Base and support: AISI 1010 HR $\rightarrow S_{ut} = 320 \text{ MPa}$
 $S_y = 180 \text{ MPa}$

Electrode: E 6010 $\rightarrow S_{ut} = 427 \text{ MPa}$

alternating load F

$b=d=50 \text{ mm}$, $h=5 \text{ mm}$.

$$\text{Throat area, } A_{\text{throat}} = (2b+d) \cdot t = 0.707 \cdot h(b+d)$$

$$= 530.25 \text{ mm}^2.$$

Endurance strength of a member

$$S_e = K_a K_b K_c K_d K_e K_f \underbrace{S_e'}_{0.5 S_{ut}}$$

$$K_a = \alpha S_{ut}^b = 0.7 \times (320)^{-0.718} = 0.917 \quad (\text{Hot Rolled})$$

$$K_b = 1.$$

$$K_c = 0.59$$

$$K_d = K_e = K_f = 1.$$

$$S_e = 0.917 \times 0.59 \times 160 = 83.63 \text{ MPa}$$

Endurance strength of weld material

$$S_e = K_a K_b K_c K_d K_e K_f S_e'$$

$$K_a = \alpha S_{ut}^b = 0.7 \times (427)^{-0.718} = 0.745$$

$$K_b = 1 \quad K_c = 0.59 \quad K_d = K_e = K_f = 1.$$

$$S_e = 0.745 \times 0.59 \times \left(\frac{427}{2}\right) = 93.84 \text{ MPa.}$$

\Rightarrow Member is weaker.

Use factor of safety = 1.

Fatigue SCF?

Table 9-5 for Fatigue-Stress Concentration Factor

$$K_{fs} = 2.7 \quad (\text{end of parallel fillet})$$

Table 9-5

$$\tau_{max} = \frac{F}{A_{throat}} \times K_{fs} = \frac{F}{530.3} \times 2.7$$

$$\tau_{max} \leq \tau_{allow} = 83.8 \text{ MPa}$$

$$\Rightarrow \frac{F}{530.3} \times 2.7 \leq 83.8 \text{ MPa}$$

$$F_{throat} = I(b + dC) = 530.3 \times 16.4 = 83.8 \text{ MPa} \Rightarrow F \leq 16.4 \text{ kN}$$

$$I = 3.08 \text{ cm}^4$$

Minimum to fit correctly. dimension

$$3.08 \times 3.0 \times 6.0 \times 2.0 \times 0.8 = 53$$

$$F_{throat} = 3.08 \times 16.4 = 50 \text{ kN}$$

$$F_{throat} = 16.4 \text{ kN}$$

$$1 - 1 = 34 - 34$$

$$3.08 \times 3.0 \times 6.0 \times 1.0 = 53$$

Minimum to fit correctly. dimension

Gears

→ Introduction from slides.

Design of spur gear

Angular velocity: ω (r.p.m)

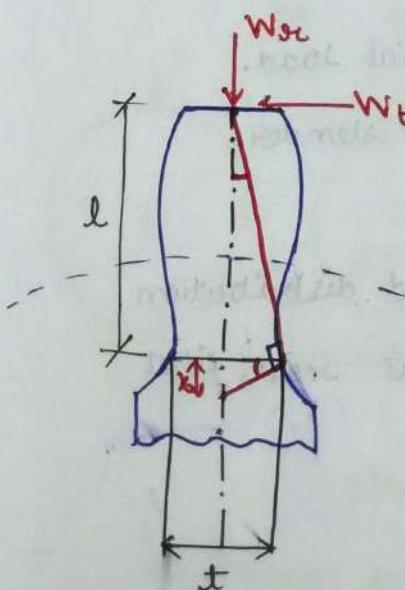
$$\begin{aligned} \text{Pitch line velocity: } V &= \frac{\omega d}{2} = \frac{2\pi n}{60} \times \frac{d}{2} \\ &\quad (\text{m/s}) \\ &= \frac{\pi d n}{60000} \quad (\text{if } d \text{ is in mm}) \end{aligned}$$

Transmitted load: $W_t = \text{Power}/\text{Velocity}$

$$= \frac{60000 H}{\pi d n} \text{ KN}$$

(if H is in kW)

Bending stress:



a := point of max. bending stress (at root fillet)

t := tooth thickness at a .

l := tooth depth

b := width of tooth

$$\tau_{\text{bending}} = \frac{M c}{I} = \frac{M t}{2I}, \quad I = \frac{b t^3}{12}, \quad M = W_t l$$

$$\Rightarrow \tau_{\text{bending}} = \frac{W_t l \times t}{2 \frac{b t^3}{12}} = \underline{\underline{\frac{6 W_t \left(\frac{l}{t^2} \right)}{b}}} = \frac{3 W_t}{2 b t}$$

Lewis bending eqn.:

$$\frac{x}{(t/2)} = \frac{(t/2)}{l} \Rightarrow \frac{x^2}{l} = 4x.$$

$$\Rightarrow \sigma_{\text{bending}} = \frac{3W_t}{2bx}$$

$$\sigma_{\text{bending}} = \frac{3W_t}{2bx} \cdot \frac{p}{p} = \frac{W_t}{bp} \cdot \left(\frac{3p}{2x}\right)$$

$$\boxed{\sigma_{\text{bend.}} = \frac{W_t}{bp} \cdot y}$$

$$y := \frac{3p}{2x} = \text{Lewis form factor}$$

$p \rightarrow$ pitch (circular pitch)

\rightarrow depends on tooth profile

Note: Not considered

- ① Compression due to radial load.
- ② Beam assumed is not slender.
- ③ Dynamic effects.
- ④ Non-uniformity of load distribution
- ⑤ Stress concentration at root fillet
- ⑥ Rim can be elastic

$$\sigma = \frac{W}{bp} \cdot y$$

with all factors

$$\Rightarrow \sigma = K_O K_V K_S \left(\frac{W}{bm_t} \right) \cdot \frac{1}{y_J} \cdot \frac{K_H K_B}{\left(\frac{P}{\pi} \right)} \cdot \text{load distribution factor}$$

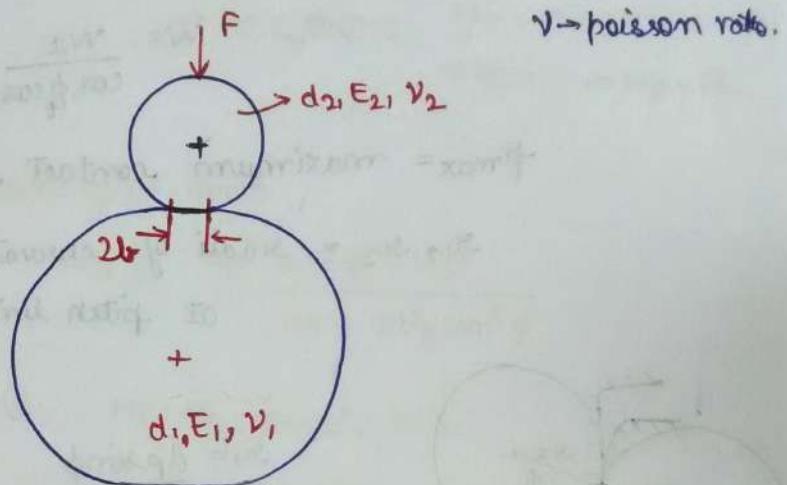
size factor rim thickness factor
 ↓ ↑
 Dynamic factor ↓

m_t - transverse module

$m_t = m$ for spur gear

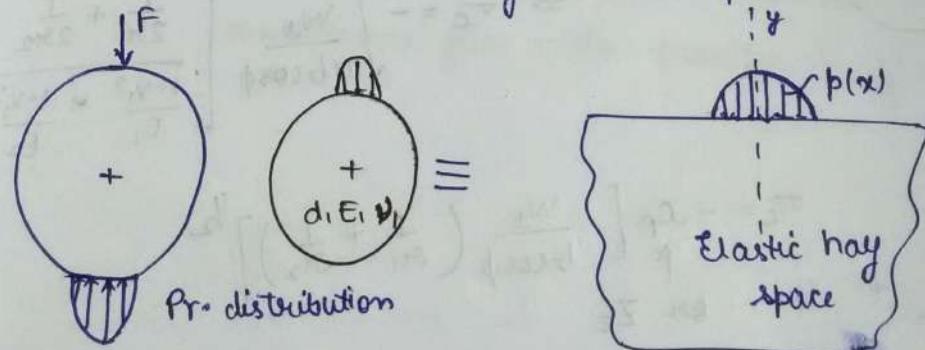
→ See the related slide also.

Hertzian Contact betw two cylinders



ν → poisson ratio.

$2b$: width of contact patch



When $r_1, r_2 \gg b$

Maximum pressure:

$$p_{\max} = \frac{2F}{\pi b l} = \sqrt{\frac{2F}{\pi l}} \left\{ \frac{\frac{1}{r_1} + \frac{1}{r_2}}{\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}} \right\}^{1/2}$$

$$p(x) = p_{\max} \left(1 - \frac{x^2}{b^2} \right)$$

$l \rightarrow$ length of cylinder

$$b = \sqrt{\left\{ \frac{2f}{\pi l} \left[\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{d_1 + d_2} \right] \right\}}$$

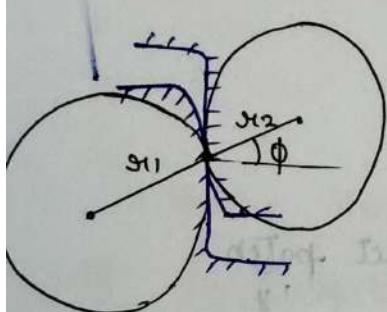
For gear tooth contact

Normal load on tooth: $W = \frac{W_t}{\cos\phi}$ (spur gear)

$W = \frac{W_t}{\cos\phi_t \cos\psi}$ (helical gear)

τ_{max} = maximum contact stress = σ_c

r_1, r_2 = radii of curvatures of tooth profile at pitch line.



$$r_1 = \frac{dp \sin\phi}{2} \quad r_2 = \frac{dp \sin\phi}{2}$$

$$\Rightarrow \sigma_c = - \sqrt{\frac{W_t}{\pi b \cos\phi}} \left[\frac{\frac{1}{2r_1} + \frac{1}{2r_2}}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}} \right]^{\frac{1}{2}}$$

$$\sigma_c = - c_p \left[\frac{W_t}{b \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{\frac{1}{2}}$$

$$c_p = \text{Elastic constant} = \left[\frac{1}{2\pi \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)} \right]^{\frac{1}{2}}$$

Problem 13-10)

$$N_p \geq \frac{2K}{(1+2m_G) \sin^2\phi} \left[m_G + \sqrt{m_G^2 + (1+2m_G)^2 \sin^2\phi} \right]$$

a) Take $m_G = 1$, $\phi = 20^\circ$, $K = 1$ [full depth tooth]

$$N_p = 13, 14, \dots$$

(b) $m_G = 2.5$

$$N_p \geq 14.63 \Rightarrow N_p = 15.$$

(c) smallest pinion that will run with rack

$$m_G = \infty$$

$$\text{as } m_G = \frac{dG_1}{dp} = \frac{N_{G_1}}{N_p}$$

$$\Rightarrow N_p \geq 18.0744 \quad N_p \geq 17.097$$

$$\Rightarrow N_p = \Rightarrow N_p = 18.$$

Maximum gear tooth

$$N_{G_1} \leq \frac{N_p^2 \sin^2 \phi - 4k^2}{4k - 2N_p \sin^2 \phi}$$

for (b), $N_p = 15, \phi = 20^\circ, k = 1.$

$$N_{G_1} \leq 45.489 \Rightarrow N_{G_1 \max} = 45$$

$$N_{\text{pinion}} = 15.$$

maximum gear ratio possible is $\frac{45}{15} = 3.$

Tutorial-9 Problem - Gear design

@ Pinion and Gear teeth number

Interference condition

$$N_p \geq \frac{2K}{(1+2m_g) \sin^2 \phi} [m_g + \sqrt{m_g^2 + (1+2m_g)^2 \tan^2 \phi}]$$

$K=1$ (full depth)

$$m_g = \frac{N_g}{N_p} = 4.$$

$$\phi = 20^\circ$$

$$\Rightarrow N_p \geq 15.44$$

Choose $N_p = 16$

$$\Rightarrow N_g = 64.$$

for $N_p = 16, \phi = 20^\circ$

$$N_g \leq 101.07$$

module :

$$d_p = m N_p$$

$$d_g = m N_g$$

center-to-centre distance :

$$c = \frac{d_p + d_g}{2} = 200 \text{ mm}$$

$$\Rightarrow \frac{m(N_p + N_g)}{2} = 200$$

$$\Rightarrow m = 5 \text{ mm}$$

$$\Rightarrow d_p = 80 \text{ mm.}$$

$$d_g = 320 \text{ mm.}$$

⑥ Power rating $H = 25 \text{ kW}$

Pinion speed $n = 2000 \text{ rpm}$

Transmitted load:

$$W_t = \frac{60000 H}{\pi d p N} \text{ KN}$$

$$\Rightarrow W_t = \frac{60,000 \times 25}{\pi \times 80 \times 2000} = 2.984 \text{ KN}$$

$$W_t = 2.984 \text{ KN}$$

Pitch line velocity:

$$v = \frac{\pi d p n}{60,000} \text{ m/s.} = 8.378 \text{ m/s.}$$

Factors for AGMA bending stress eqn.

Overload factor: $K_o = 2.0$ (fig 14-17)

→ Power source light shock
Driven heavy shock

size
surface factor: $K_s = 1.$

Dynamic factor: (Eq 14-27, 14-28)

$$K_v = \left(A + \frac{\sqrt{200} v}{A} \right)^B$$

$$A = 50 + 56(1-B)$$

$$B = 0.25 (12 - Q_v)^{2/3} = 0.25 (12 - 10)^{2/3}$$

as $Q_v = 10.$

$$\Rightarrow B = 0.3969 \quad \left. \begin{matrix} \\ A = 83.776 \end{matrix} \right\} \Rightarrow K_v = 1.171$$

Load distribution factor

$$K_H = 1.03 \text{ (from table)} \downarrow \text{slides}$$

Bending strength:

$$S_t = 0.7255 H_B + 153.63 \text{ (MPa)} \quad (\text{fig 14-4})$$

Material: Nitrided, 2.5% chrome steel

$$H_B = 250 \text{ (P)}$$

$$H_B = 200 \text{ (G)}$$

$$(S_t)_P = 335 \text{ MPa} \quad (\text{Pinion})$$

$$(S_t)_G = 298.73 \text{ MPa} \quad (\text{gear})$$

allowable stress:

$$\tau_{\text{all}} = \frac{S_t \cdot Y_N}{S_f \cdot Y_Q Y_Z}$$

$$\text{Pinion } \tau_{\text{all}}|_P = \frac{335 \times 0.977}{S_f} = \frac{327.3}{S_f}$$

$$\text{Gear } \tau_{\text{all}}|_G = \frac{298.73 \times 1.009}{S_f} = \frac{299}{S_f}$$

Factor of safety

$$\text{Pinion } S_F_P = \frac{\tau_{\text{all}}}{\sigma_P} = \frac{327.3}{139.6} = 2.373$$

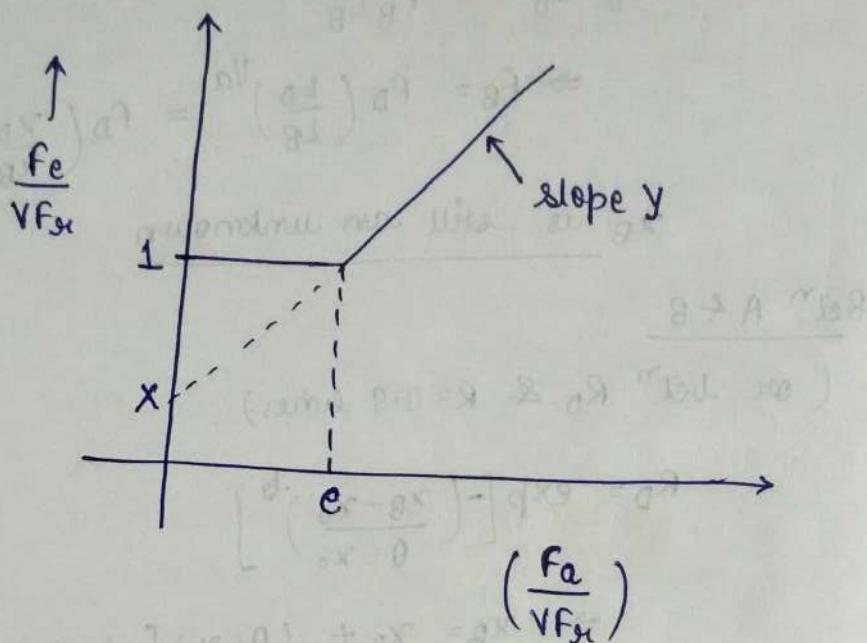
$$\text{Gear } S_F_G = \frac{\tau_{\text{all}}}{\sigma_G} = \frac{299}{88.6} = 3.373$$

Combined Radial + Thrust Load

F_a = Thrust load (along shaft axis)

F_r = Radial load

Find equivalent radial load $\rightarrow F_e$



$$\left(\frac{F_a}{V F_{r \perp}} \right)$$

$$F_e = V F_{r \perp} \cdot 1 \text{ for } \frac{F_a}{V F_{r \perp}} \leq e$$

$$= x + y \left(\frac{F_a}{V F_{r \perp}} \right) \text{ for } \frac{F_a}{V F_{r \perp}} \geq e$$

$$V = \begin{cases} 1 & \text{if inner race rotates} \\ 1.2 & \text{if outer race rotates} \end{cases}$$

"e" depends on static load rating C_0

C_0 := basic static load rating

- A load that produces a total permanent deformation of 10^{-4} times that of rolling element size.

→ For e, x, y see Table II-1.

→ For C_{10}, C_0 see table II-2, II-3.

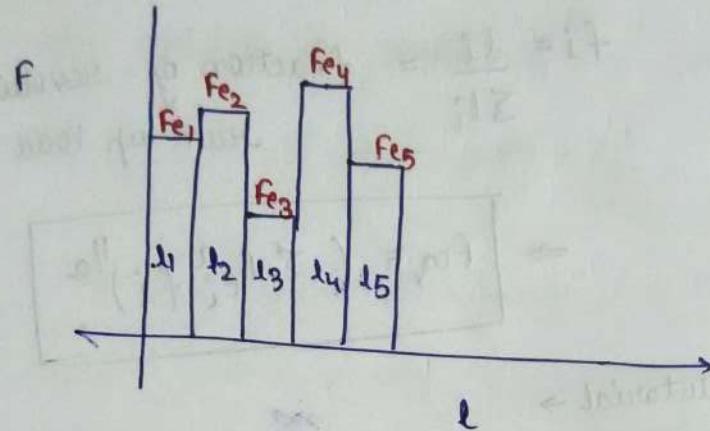
→ Load application factor α_f (Table II-5)

Variable Loading

$f_{e_1} \leftarrow (F_{e_1}, f_{x_1})$ load for $-l_1$ cycles

$f_{e_2} \leftarrow (F_{e_2}, f_{x_2})$ load for $-l_2$ cycles

⋮



$$f_L^{\alpha} = \text{constant}$$

↳ life to failure

$$\Rightarrow f_{e_1} L_1^{\alpha} = f_{e_2} L_2^{\alpha} = \dots \quad (\text{since bearing is same})$$

L_i is life at load f_{e_i}

$$l_1 < L_1, \dots, l_i < L_i$$

$$\Rightarrow f_{e_i}^{\alpha} L_i = \text{constant}$$

Linear damage hypothesis:

$$\Rightarrow \text{Damage: } D_1 = f_{e_1}^{\alpha} l_1$$

$$D_2 = f_{e_2}^{\alpha} l_2, \dots$$

Total Damage

$$D = D_1 + D_2 + \dots = \sum_{i=1}^N f_{eq_i}^{\alpha} l_i$$

Let bearing run for $(l_1 + l_2 + \dots + l_N)$ cycles.

at load F_{eq} and cause same damage

$$\Rightarrow L_{eq} = (l_1 + l_2 + \dots + l_N)$$

$$\Rightarrow f_{eq}^{\alpha} L_{eq} = \sum f_{eq}^{\alpha} l_i = D.$$

$$f_{eq}^a = \frac{\sum f_{ei}^a l_i}{\sum l_i} = \sum f_{ei}^a \left(\frac{l_i}{\sum l_i} \right)$$

$$f_i = \frac{l_i}{\sum l_i} = \frac{\text{fraction of revolution}}{\text{sum up load}}$$

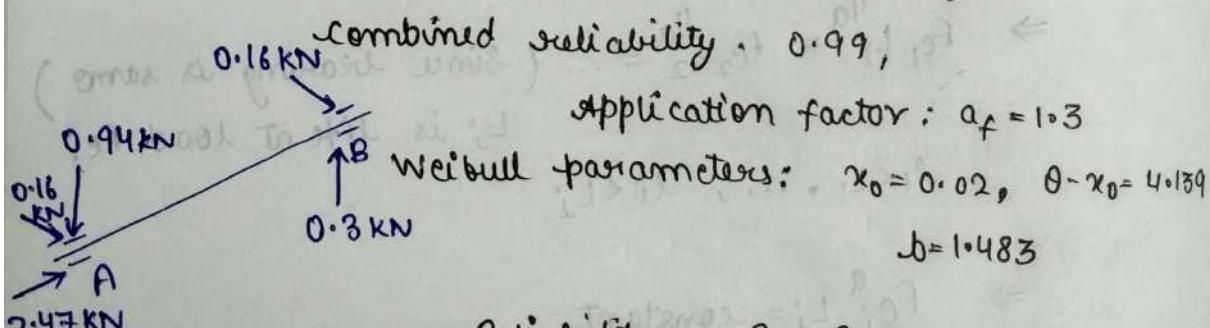
$f_i = \frac{l_i}{\sum l_i}$ = fraction of revolution
sum up load

$$\Rightarrow f_{eq}^a = \left(\sum f_{ei}^a f_i \right)^{1/a}$$

Q → from tutorial →

Given: At A: Angular contact bearing }
Takes all the axial thrust } 02-Series

At B: straight roller bearing



$$\text{Reliability : } R = R_A R_B \geq 0.99$$

combined

$$\text{Either } R_A = R_B = \sqrt{0.99}$$

In fig. loads are in lbf or choose: $R_A = 0.99$
convert to KN. $R_B = 1$

Selection of Bearing at A:

$$f_{eq} = \sqrt{(0.16^2 + 0.94^2)} = 0.954 \text{ kN}$$

$$f_A = 2.47 \text{ kN}$$

Find equivalent radial load: F_e

Inner race loading: $V=1$

$$\frac{f_a}{\sqrt{f_{y2}}} = \frac{2.47}{0.954} = 2.59$$

Need to know: e from table H-1.

→ Choose a starting value of $\frac{f_a}{C_0}$ at somewhere in the middle of table H-1.

$$\text{Let } \frac{f_a}{C_0} = 0.07 \Rightarrow e = 0.27 < \frac{f_a}{\sqrt{f_{y2}}}$$

$$X = X_2 = 0.56, \quad Y = y_2 = 1.63$$

$$\Rightarrow f_e = \sqrt{f_{y2}} \left(X + Y \cdot \frac{f_a}{\sqrt{f_{y2}}} \right) = 4.57 \text{ kN}$$

$$C_0 = \frac{f_a}{0.07} = 35.3 \text{ kN}$$

$$C_{10} = \alpha_f \cdot f_D \cdot \left\{ \frac{x_D}{x_0 + (\theta - x_0) \left\{ \ln \left(\frac{x_D}{R_D} \right) \right\}^{1/6}} \right\}^{1/\alpha}$$

$\alpha = 3$ for ball bearing

$$C_{10} = 96.93 \text{ kN}$$

$$L_D = 25 \times 10^3 \times 600 \times 60 = 900 \times 10^6 \text{ cycles}$$

$$x_D = \frac{L_D}{L_{10}} = \frac{900 \times 10^6}{10^6} = 900.$$

Now select a bearing from table H-2.

Select: 02-series bearing with bore dia 90-mm

$$\Rightarrow C_{10} = 106 \text{ kN} > 96.93 \text{ kN}$$

$$C_0 = 73.5 \text{ kN} > 35.3 \text{ kN}$$

$$\text{Check: } \frac{f_a}{C_0} = \frac{2.47}{73.5} = 0.034$$

$$\Rightarrow e = 0.23, \quad (\text{table H-1}) \& \text{ linear interpolation}$$

$$X_2 = 0.56, \quad Y_2 = 1.93$$

$$F_e = V F_{d2} \left(0.56 + 1.93 \times \frac{f_d}{V F_{d2}} \right)$$

$$= 5.3 \text{ kN} = f_d.$$

$$\Rightarrow C_{10} = 110.2 \text{ kN} > 106 \text{ kN}$$

Not satisfied

Now, select bearing with bore dia 95 mm.

$$\frac{f_d}{C_0} = \frac{2.47}{85} = 0.029$$

$$\Rightarrow c = 0.221, x_2 = 0.56$$

$$V F_{d2} = \left(\frac{0.221 \cdot 0.56 + 0.56}{0.221} \right) \Rightarrow f_d = 5.425 \text{ kN}$$

$$\Rightarrow C_{10} = 112.8 \text{ kN} < 121 \text{ kN}$$

Satisfied

Bearing at A: 02-series, 95 mm bore dia Angular contact

Bearing at B: $R_D = 1$, straight roller

$$F_{d2} = f_d = \sqrt{(0.16^2 + 0.3^2)} = 0.34 \text{ kN}$$

$$C_{10} = \alpha_f f_d \left(\frac{x_D}{0.02} \right)^{1/\alpha}$$

$$C_{10} = 11 \text{ kN}$$

($\alpha = 10/3$ for
roller bearing)

See bearing from table 11-3.

Choose bore dia = 25 mm (02-series)

$$\Rightarrow C_{10} = 16.8 \text{ kN}, C_0 = 8.8 \text{ kN}$$

or choose same bore dia as bearing at A

i.e. 95 mm $\Rightarrow C_{10} = 165 \text{ kN}, C_0 = 112 \text{ kN}$