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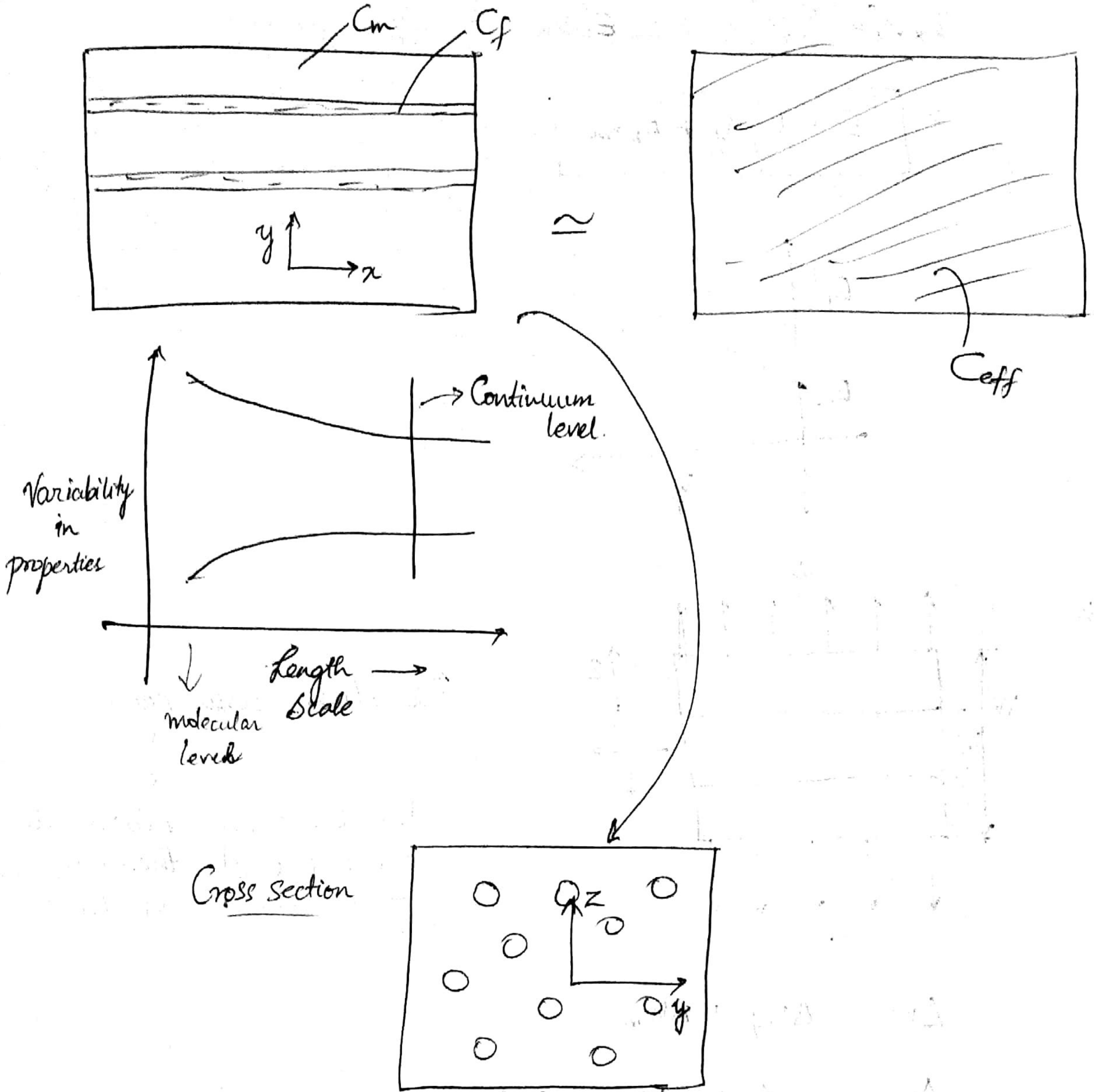
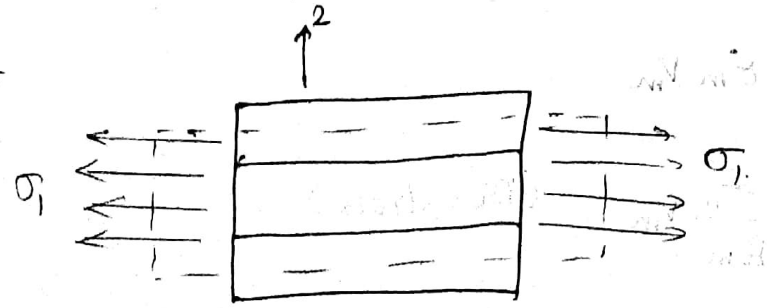


→ Assume ~~fiber~~ & matrix ~~are~~ is isotropic.

*



Strain in matrix & fiber are the same

* σ gets distributed so that the strain remains the same.

$$\sigma_1 A = \sigma_f A_f + \sigma_m A_m$$

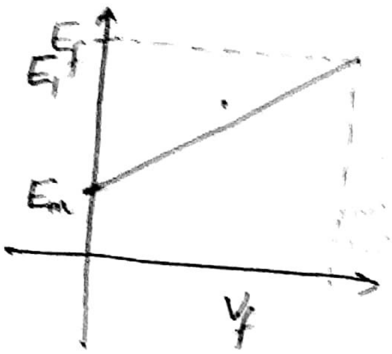
$$\sigma_1 = \sigma_f \frac{A_f}{A} + \sigma_m \frac{A_m}{A}$$

$$= \sigma_f V_f + \sigma_m V_m$$

V_f, V_m → vol. fractions

$$E_1 \varepsilon_1 = E_f \varepsilon_f V_f + E_m \varepsilon_m V_m \quad \text{(Iso-strain)}$$

$$\Rightarrow \boxed{E_1 = E_f V_f + E_m V_m}$$



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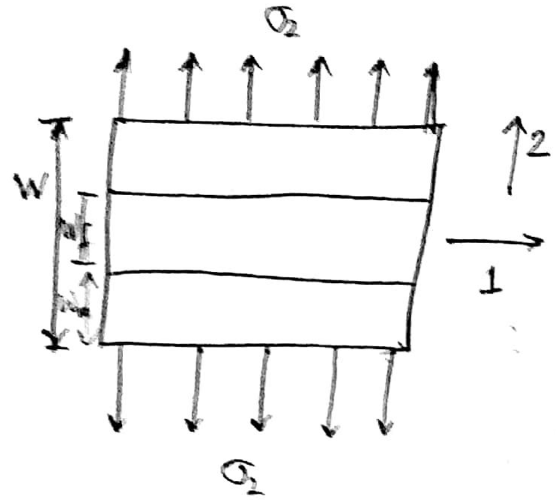
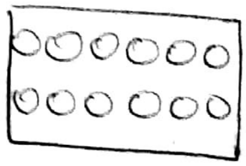


Iso-stress assumption.



Fibers align themselves in st. lines.

$$\Delta W = \Delta W_f + \Delta W_m$$

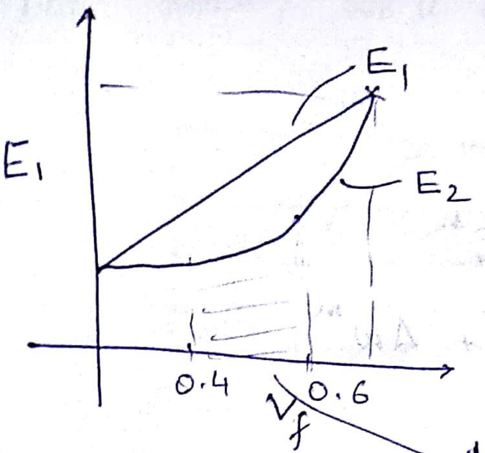
$$\frac{\Delta W}{W} = \frac{\Delta W_f}{W_f} \cdot \frac{W_f}{W} + \frac{\Delta W_m}{W_m} \cdot \frac{W_m}{W}$$

$$\varepsilon_c = \varepsilon_f V_f + \varepsilon_m V_m$$

$$\frac{\cancel{\sigma_c}}{E_c} = \frac{\cancel{\sigma_f}}{E_f} V_f + \frac{\cancel{\sigma_m}}{E_m} V_m \quad \text{(Iso-stress)}$$

$$\therefore \frac{1}{E_2^{composite}} = \frac{V_f}{E_2^f} + \frac{V_m}{E_2^m}$$ [inverse rule of mixtu

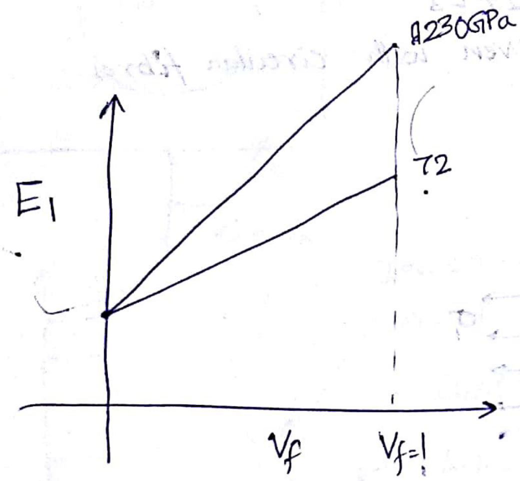
↘ Same for E_3

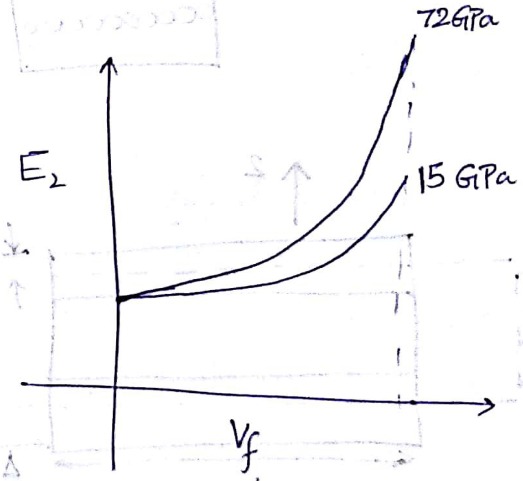


Most composites are in this range.

$E_2^{Glass} = 72\text{ GPa}$

$E_2^{carbon} = 15\text{ GPa}$



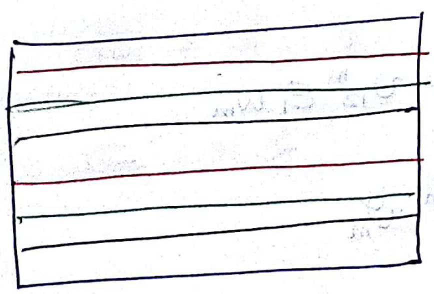


* Glass - epoxy

$E_2 \rightarrow 20\text{ GPa}$ desired $\rightarrow 23\text{ GPa}$

(a) $V_f \uparrow$
(b) Replace Glass with carbon.
(c) " matrix with matrix

→ Rules of mixtures apply to composites with more constituents also.



$$E_1 = V_f^{gl} E_{gl} + V_f^{c} E_c + V_f^{m} E_m$$

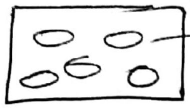
Poisson's ratio

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$$

Axial - isostrain $\varepsilon_1^c = \varepsilon_1^f = \varepsilon_1^m$

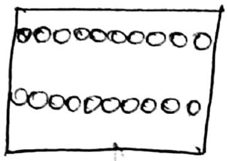
Trans - isostress $\Delta w_c^c = \Delta w^f + \Delta w^m$

* Ellipsoid fibers don't have $E_2 = E_3$



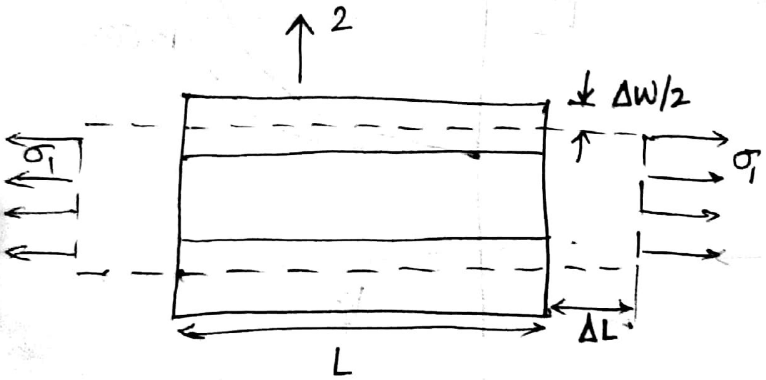
Ex: Natural fibers.

or,



$E_2 \neq E_3$
even with circular fibres.

*



$$\Delta w = \Delta w_f + \Delta w_m$$

$$\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} \quad \Big| \quad \varepsilon_1^c = \varepsilon_1^f = \varepsilon_1^m$$

$$\frac{\Delta w}{w} = \varepsilon_2$$

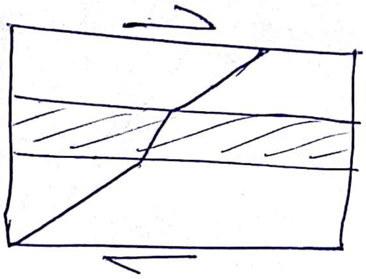
$$\Delta w = w\,\varepsilon_2$$

$$= \nu_{12}\,\varepsilon_1\,w$$

$$-\nu_{12}\,\varepsilon_1\,w = -\nu_{12}^f\,\varepsilon_1\,w_f - \nu_{12}^m\,\varepsilon_1\,w_m$$

$$+\ \nu_{12} = \nu_{12}^f\,\vartheta_f + \nu_{12}^m\,\vartheta_m$$

For shear, we'll again get inverse rule.



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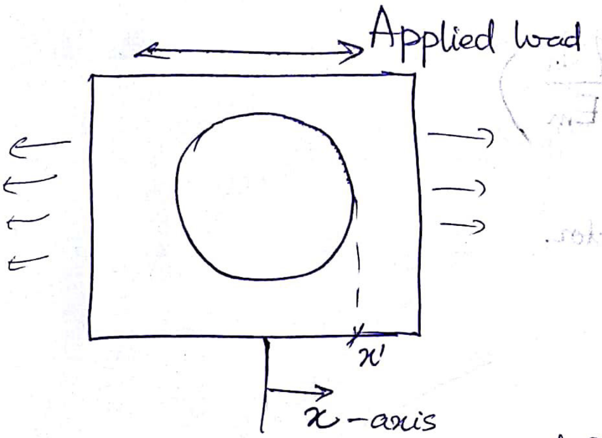
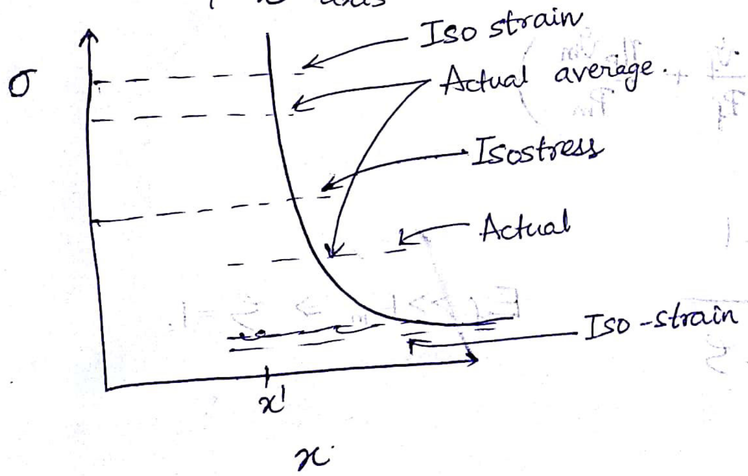




Assumptions:-

1) Fiber is stiffer than matrix $\Rightarrow \sigma_f >> \sigma_m$

$\Rightarrow$ Stress in fiber >>> Stress in matrix.

2) Avg. Stress in matrix = f(Avg. stress in fibre)

$\sigma_m = f(\sigma_f)$

$\sigma_m^{2} = \eta\, \sigma_f^{2}$ → superscript is not square (transverse direction)

In transverse direction, we have,

$$\frac{\sigma_c}{E_c} = \frac{\vartheta_f \sigma_f}{E_f} + \frac{\vartheta_m \sigma_m}{E_m}$$

$$\frac{\sigma_c}{E_c} = \frac{\vartheta_f \sigma_f}{E_f} + \frac{\vartheta_m \eta \sigma_f}{E_m}$$

$$\left.\begin{aligned} \sigma_c &= \vartheta_f \sigma_f + \vartheta_m \sigma_m \\ \sigma_c &= \vartheta_f \sigma_f + \vartheta_m \eta \sigma_f \end{aligned}\right\}$$

$$\frac{1}{E_c} = \frac{1}{\vartheta_f + \eta \vartheta_m} \left(\frac{\vartheta_f}{E_f} + \frac{\eta \vartheta_m}{E_m} \right)$$

$\eta \rightarrow$ stress correction factor.

$$\eta = f\left(\frac{E_f}{E_m}, \vartheta_f\right)$$

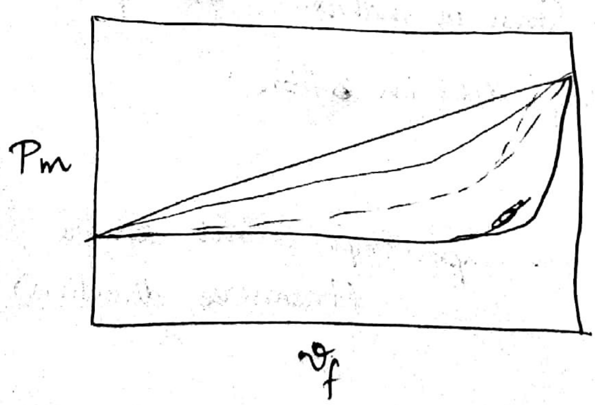
$$\frac{1}{P} = \frac{1}{\vartheta_f + \eta_p \vartheta_m} \left(\frac{\vartheta_f}{P_f} + \frac{\eta_p \vartheta_m}{P_m} \right)$$

$$\eta = \frac{\frac{P_f}{P_m} - 1}{\frac{P_f}{P_m} + \varsigma}$$

$E_f >> E_m \Rightarrow \varsigma = 1.$

$\varsigma = 0$ Isostress

$\varsigma = \infty$ Isostrain



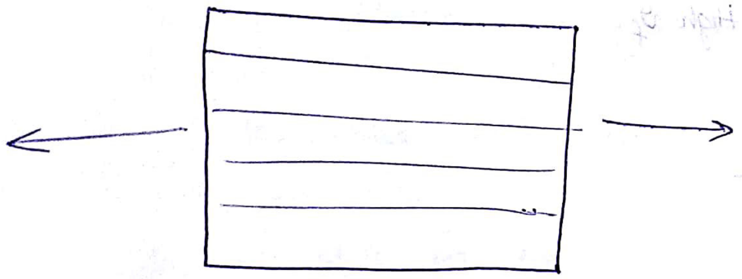
Chamis formula

$$\frac{1}{E_c} = \left(\frac{\vartheta_f}{E_f} + \frac{\vartheta_m}{E_m} \right)$$

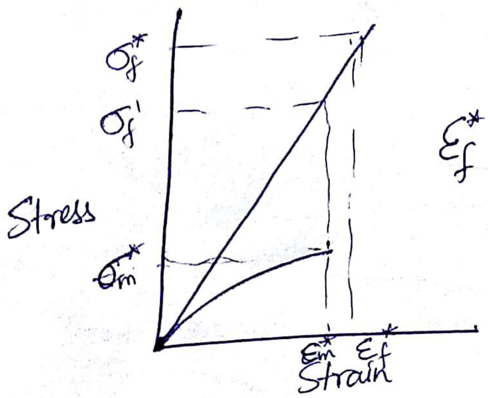
$$\left.\begin{aligned} \vartheta_m &= 1 - \vartheta_f \\ \vartheta_f &\rightarrow \sqrt{\vartheta_f} \end{aligned}\right\} \text{Correction for rule of mixtures.}$$

Same for shear modulus.

Strength is a local property.

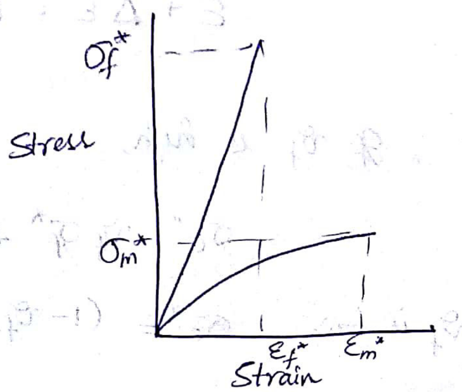


Case - 1



$\varepsilon_f^* > \varepsilon_m^*$

Case - 2



Assumptions :-

1) All fibres have same strength
2) All fibres carry equal load.
3) Both the fibre & matrix follow linear till failure.

→ In case - 1,

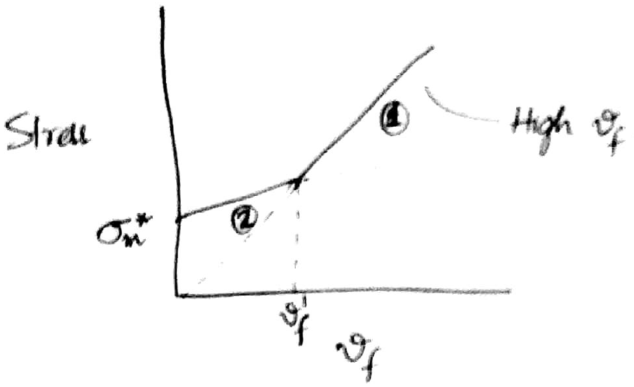
$$\varepsilon + \Delta\varepsilon = \varepsilon_m^*$$

If ϑ_f is high,

$$\sigma_c^* = \vartheta_f \sigma_f^* \quad \ldots (1)$$

ϑ_f is low,

$$\sigma_c^* = \vartheta_m \sigma_m^* + \vartheta_f(\sigma_f') \quad \ldots (2)$$



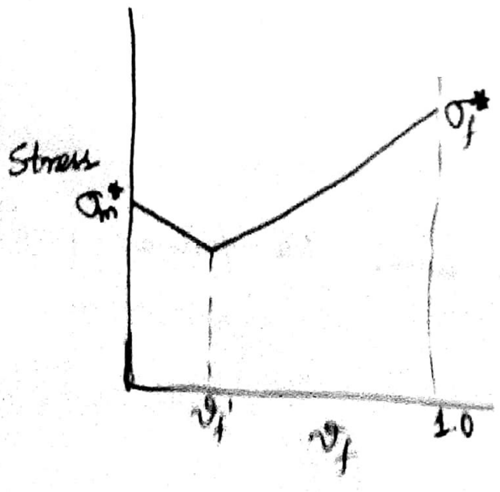
→ In case - 2,

$$\varepsilon + \Delta\varepsilon = \varepsilon_f^*$$

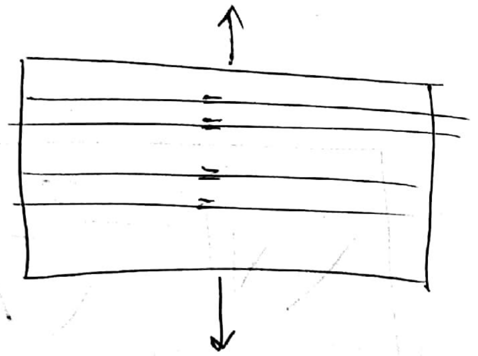
If ϑ_f is high,

$$\sigma_c'' = \vartheta_f \sigma_f^* + (1-\vartheta_f)\sigma_m'$$

ϑ_f is low $\quad \sigma_c'' = (1-\vartheta_f)\sigma_m^*$



HW Calculate ϑ_f' for glass & carbon.
[it will be very low].



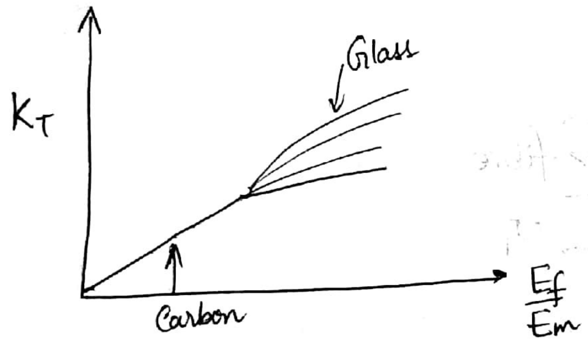
$$K_T = \frac{\sigma_{local}}{\sigma_{composite} \rightarrow avg.}$$

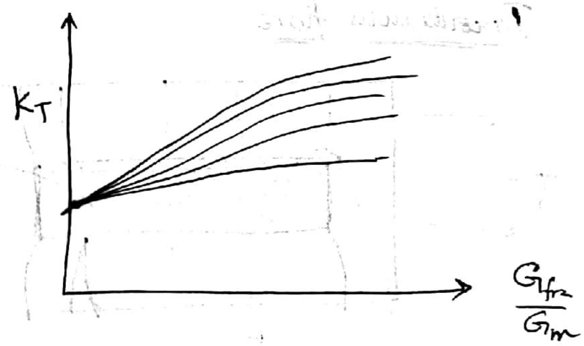
If $\sigma_{local} = \sigma_m^*$
composite will fail.

$$\sigma_{comp} = \frac{\sigma_m^*}{K_t}.$$

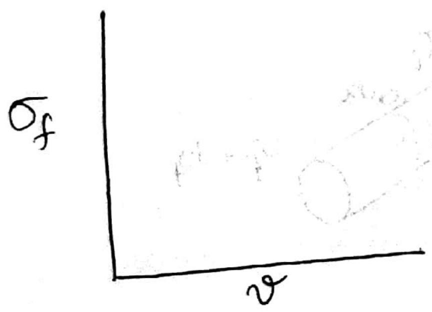
K_t also will depend on $\frac{E_f}{E_m}$, ϑ_f.

<u>Skudra's plots for K_T</u>

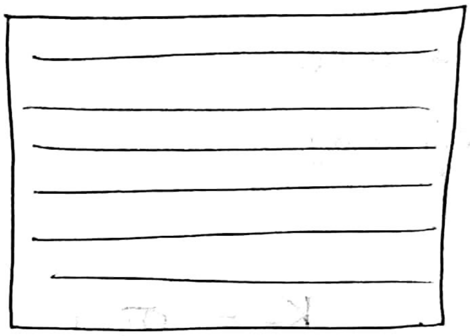




* <u>Term project</u>



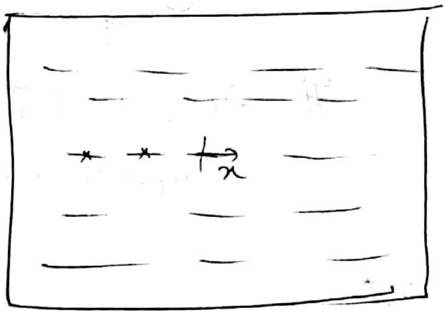
Continuous Composites ←

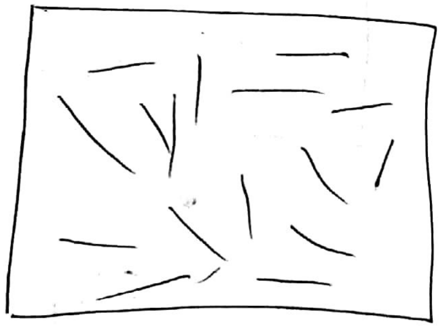


→ $\varepsilon_f = \varepsilon_m = \varepsilon_1 = \text{const.}$

Discontinuous Composites

$\varepsilon_f \neq \varepsilon_m$ →



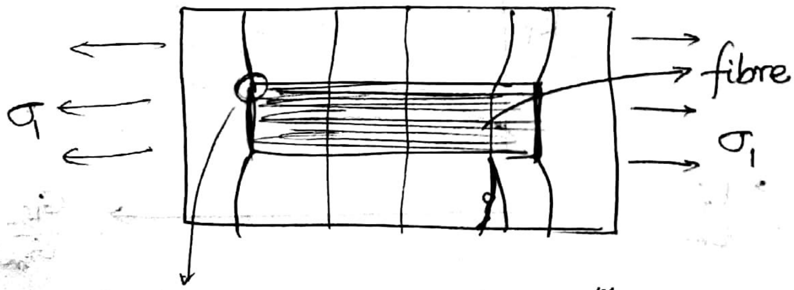


Aligned

NOT Aligned
(RANDOM)

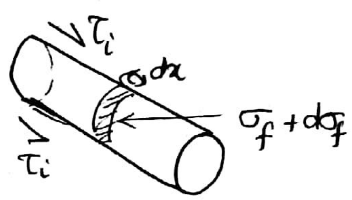
$E_{eff.} = f(E_f, E_m, \vartheta_f, f_{length}, f_{orientation})$

Discontinuous fibre



At the tip shear stress is max^m.

Simple force eqbm



$$\pi r^2 \, d\sigma_f = -2\pi r \tau_i \, dx$$

$$\frac{d\sigma_f}{dx} = -\frac{2\tau_i}{r}$$

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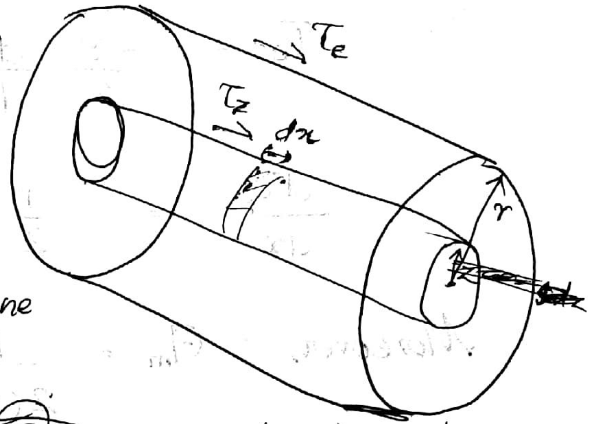
Add^l. assumption

* Shear force is constant @ concentric cylinders.

$$2\pi z \cdot \tau_z \, dx = 2\pi r \, \tau_e \, dx$$



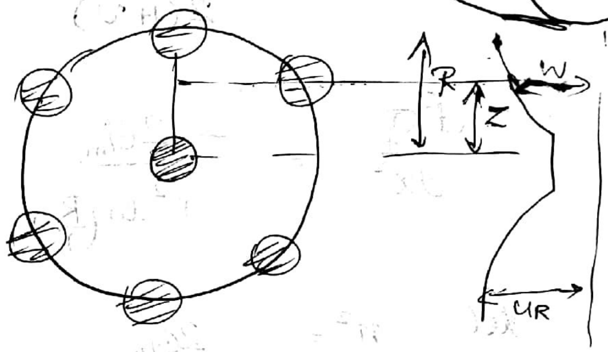
$$\tau_z = \frac{r\tau_e}{z}$$

Shear stress $\propto \frac{1}{z}$

Let us consider strain in x-z plane

$$\gamma = \frac{dw}{dz} = \frac{\tau}{G_m}$$

$$= \frac{r\tau_e}{z G_m}$$



Integrating,

$$\int_{u_f}^{u_R} dW = \int_{r}^{R} \frac{r\tau_e}{z G_m} dz$$

$$u_R - u_f = \frac{r\tau_e}{G_m} \ln\left(\frac{R}{r}\right)$$

$$\tau_e = \frac{G_m \, (u_R - u_f)}{r \cdot \ln\left(\frac{R}{r}\right)}$$

Diff. w.r.t x,

$$\frac{d\tau_e}{dx} = \frac{G_m}{r\ln\left(\frac{R}{r}\right)} \left[\frac{du_R}{dx} - \frac{du_f}{dx}\right]$$

$$\frac{du_f}{dx} = \varepsilon_f = \frac{\sigma_f}{E_f}$$

$$\frac{du_R}{dx} = \varepsilon_m \text{ (say)}$$ [strain at a point in the matrix]

We are making simplification that strain in matrix is same everywhere.

$$\frac{d\sigma}{dx} = -\frac{2\tau_e}{r}$$

$$\tau_e = -\frac{r}{2}\frac{d\sigma}{dx}$$

$$\frac{d\tau_e}{dx} = -\frac{r}{2}\frac{d^2\sigma_f}{dx^2}$$

Moreover, $G_m = \dfrac{E_m}{2(1+\nu)}$

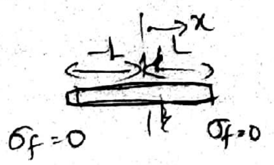
$$\therefore \frac{d^2\sigma_f}{dx^2} = -\frac{2G_m}{r^2\ln\left(\frac{R}{r}\right)}\left[\varepsilon_1 - \frac{\sigma_f}{E_f}\right]$$

Let $n^2 = \dfrac{2G_m}{E_f \ln\left(\frac{R}{r}\right)}$

$$\frac{d^2\sigma_f}{dx^2} = \frac{n^2}{r^2}\left(\sigma_f - E_f\varepsilon_1\right)$$

General sol^n is

$$\sigma_f = E_f\varepsilon_1 + B\sinh\left(\frac{nx}{r}\right) + D\cosh\left(\frac{nx}{r}\right)$$



$\rightarrow$ even ~~odd~~ fn.

$\therefore \quad B = 0. \quad$ (sinh is odd fn)

$$D = \frac{-E_f\varepsilon_1}{\cosh\left(\frac{nL}{r}\right)}$$

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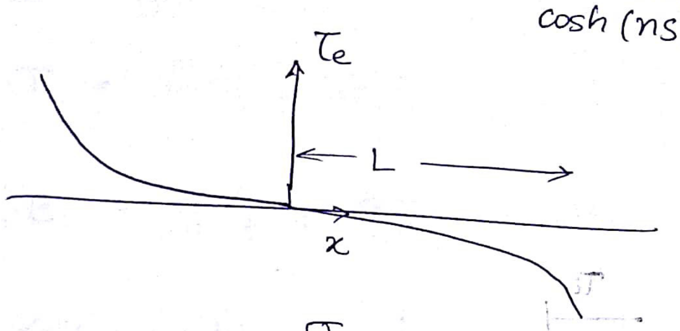
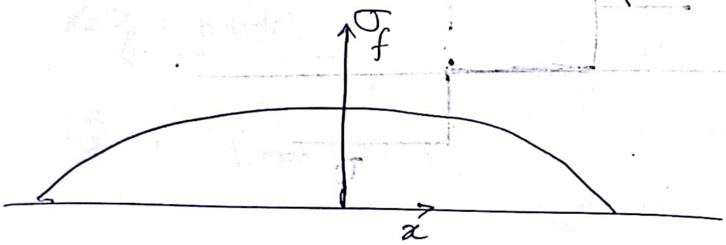
Important Results

$$\sigma_f = E_f \cdot \varepsilon_1 \left[1 - \frac{\cosh\left(\frac{nx}{r}\right)}{\cosh(ns)} \right] \nearrow \sigma_f \propto E_f$$

$$\tau_e = \frac{1}{2} n E_f \cdot \varepsilon_1 \frac{\sinh\left(\frac{nx}{r}\right)}{\cosh(ns)} \quad ; \quad s = \frac{L}{r}$$





Maximum σ_f is at center

$$\sigma_{f,max} = E_f \varepsilon_1 \left(1 - \text{sech}(ns)\right)$$

$$\tau_{e,max} = \frac{1}{2} n E_f \varepsilon_1 \tanh(ns)$$
(at tip)

$$\sigma_{f,avg} = \frac{1}{L} \int_0^L \sigma_f \, dx$$

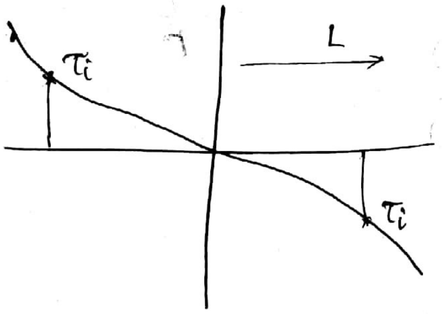
(Avg. stress across the fiber) $= E_f \cdot \varepsilon_1 \left[1 - \frac{\tanh(ns)}{ns} \right]$

This derivation is called Cox formula.

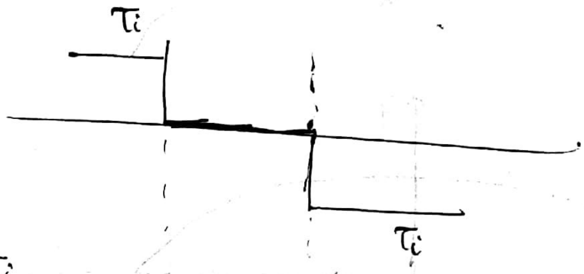
→ Cox & Kelly are called shear lag models.

Let τ_i be critical shear stress (at the interface of fibre & m when it will be de-bonded

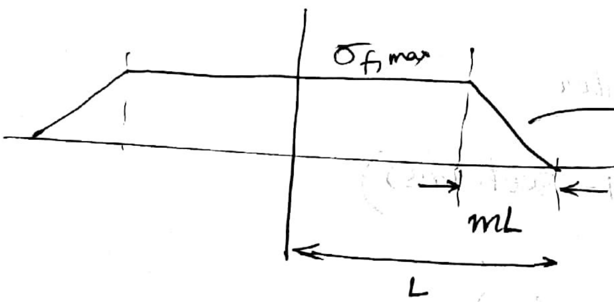
Kelly's model



Simplification →



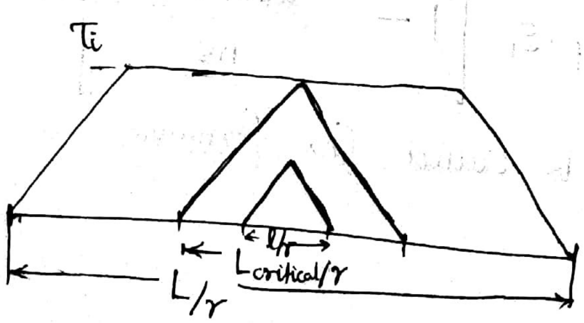
$$\frac{d\sigma_f}{dx} = \frac{-2\tau_i}{r}$$



$$\sigma_f = \frac{2\tau_i}{r}(L-x)$$

$$\sigma_{f,max} = 2\tau_i\, m \left(\frac{L}{r}\right)$$

Critical aspect ratio is the smallest fiber length at which stress in fiber can reach the ultimate fiber strength for the given interfacial stress τ_i



$$m = \frac{\sigma_{f,max}}{2\tau_{i,s}}$$

$m = 1$ in ~~a~~ critical case.

$$\Rightarrow \boxed{S_c = \frac{\sigma_{f,max}}{2\tau_i}} \rightarrow \text{Critical aspect ratio}$$

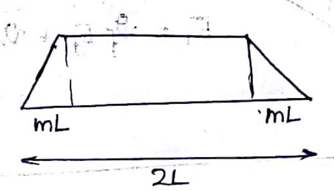
Average fibre stiffness

$$\sigma_1 = \vartheta_f \overline{\sigma_f} + \vartheta_m \overline{\sigma_m}$$

$$E_1^c \cdot \varepsilon_1 = \vartheta_f \overline{\sigma_f} + \vartheta_m \overline{\sigma_m}$$

(i) Kelly's model

$$\overline{\sigma_f} = \sigma_{f,max} \times \left(1 - \frac{m}{2}\right)$$



$$E_1^c \cdot \varepsilon_1 = \vartheta_f E_f \left(1 - \frac{m}{2}\right) \varepsilon_1 + \vartheta_m E_m \varepsilon_1$$

$$m = \frac{E_f \varepsilon_1}{2\tau_{i,s}}$$

Substituting m & rearranging,

$$E_1^c \cdot \varepsilon_1 = (\vartheta_f E_f + \vartheta_m E_m) \varepsilon_1 - \frac{\vartheta_f E_f^2 \varepsilon_1}{2\tau_{is}} \cdot \varepsilon_1$$

$$\boxed{E_1^c = \vartheta_f E_f + \vartheta_m E_m - \frac{\vartheta_f E_f^2}{2\tau_{is}} \varepsilon_1}$$

Physically unacceptable (since youngs modulus is dependent on strain)

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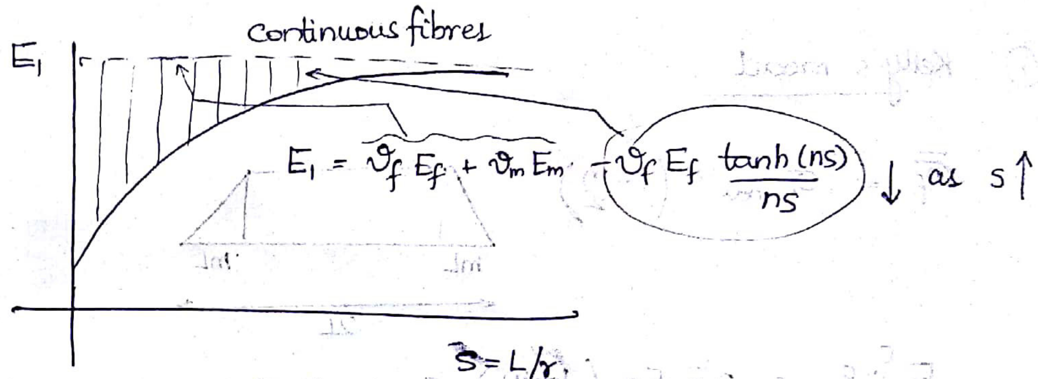
Cox's model

$$\sigma_1 = \vartheta_f \overline{\sigma_f} + \vartheta_m \overline{\sigma_m}$$

$$\overline{\sigma_f} = E_f \varepsilon_1 \left[1 - \frac{\tanh(ns)}{ns} \right]$$

$$\Rightarrow \quad E_1^c \cdot \varepsilon_1 = E_f \varepsilon_1 \vartheta_f \left[1 - \frac{\tanh(ns)}{ns} \right] + \vartheta_m E_m \varepsilon_1$$

$$E_1^c = \vartheta_f E_f + \vartheta_m E_m - \vartheta_f E_f \frac{\tanh(ns)}{ns}$$



Strength — Kelly's model

$$\sigma_1 = \vartheta_f E_f \left(1 - \frac{m}{2}\right) \varepsilon_1 + \vartheta_m E_m \varepsilon_1$$

$$= \sigma_f \vartheta_f \left(1 - \frac{m}{2}\right) + \frac{\vartheta_m E_m \cdot \sigma_{f,max}}{E_f}$$

$$= \sigma_{f,max} \left[\vartheta_f \left(1 - \frac{m}{2}\right) + \vartheta_m \frac{E_m}{E_f} \right]$$

$$\sigma_{f,max} \geq \sigma_f^{UTS}$$

$$\boxed{\sigma_1^c = \sigma_f^{UTS} \left[\vartheta_f \left(1 - \frac{m}{2}\right) + \vartheta_m \frac{E_m}{E_f} \right]}$$

Case - 1 :-

$$s \gg s_c$$

$$m = \frac{s_c}{s}$$

$$\sigma_1^c = \sigma_f^{UTS} \left[\vartheta_f \left(1 - \frac{s_c}{2s}\right) + \vartheta_m \frac{E_m}{E_f} \right]$$

$\therefore$ If $s \gg s_c$ $\quad \sigma_1^c \uparrow$

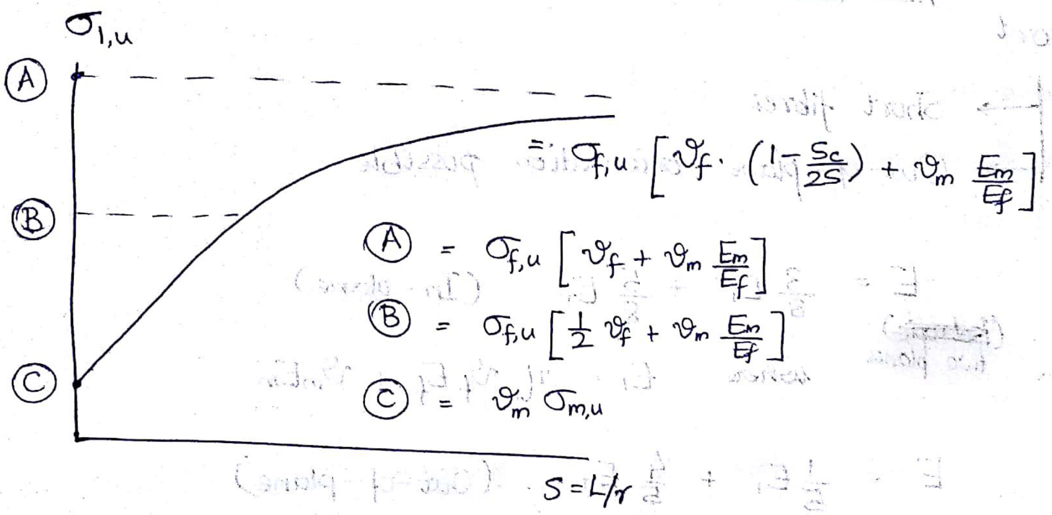
Case – 2 :- If $s = s_c$

$$\sigma_1^c = \sigma_f^{UTS} \left(\frac{\vartheta_f}{2} + \vartheta_m \frac{E_m}{E_f} \right)$$

reducing contribution of ϑ_f by $1/2$.

Case – 3 :- $s \ll s_c$

Fiber contribution becomes lower & lower.



In reality, all fibres are not of same length & not oriented in same direction. Then, a correction factor is introduced.

$$E = \eta_L E_f \vartheta_f + E_m \vartheta_m \; - \dots$$

$\eta_L \rightarrow$ length correction factor

If all are in certain direction,

$$E = \eta_o \eta_L E_f \vartheta_f + E_m \vartheta_m \; - \dots$$

$\eta_o \rightarrow$ orientation correction factor (all oriented in some other direction

$\eta_L \rightarrow$ Length correction factor

Random mat (Like shown in first class)

- → Long fibres
- → in-plane orientation.

Empirical formula } $E = \frac{3}{8}E_1 + \frac{5}{8}E_T$

E_1, E_T UD continuous composites

E_1 → Axial, E_T → Transverse

Short

- → Short fibres
- → Out-of-plane orientation possible

$E = \frac{3}{8}E_1 + \frac{5}{8}E_T$ (In-plane)

Same in (~~random~~) two planes where $E_1 = \eta_1 \vartheta_f E_f + \vartheta_m E_m$

$E = \frac{1}{5}E_1 + \frac{4}{5}E_T$ (Out-of-plane)

$E_1 = \eta_1 \vartheta_f E_f + \ldots$

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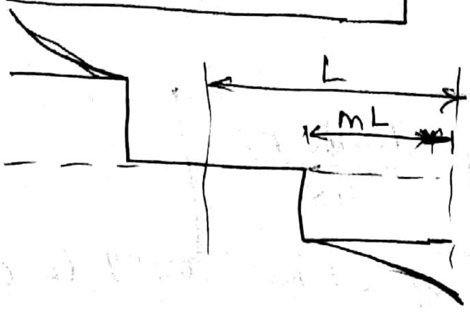
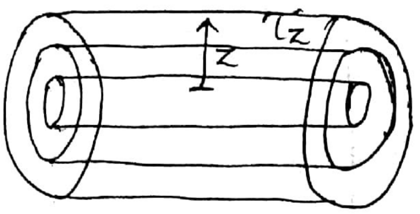
Important formulae

① $$\boxed{\frac{d\sigma_f}{dx} = -\frac{2\tau_i}{r}}$$



Derived from traction continuity conditions. This formula gives relation b/w shear & normal stress

② $$\boxed{\tau_z = \frac{r \cdot \tau_e}{z}}$$



If shear stress at distance z from the fiber surface is known, then ~~interphase~~ interface stress can be estimated.

Interface - Mathematical contact surface b/w fiber & matrix

Interphase - actual material region of the fiber matrix connection point.

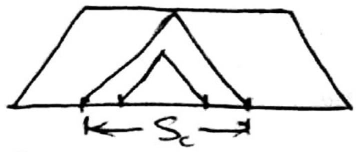
③
$$\sigma_f = E_f \varepsilon_1 \left[1 - \frac{\cosh\left(\frac{nx}{r}\right)}{\cosh(ns)} \right]$$

$$\tau_e = \frac{1}{2} n E_f \varepsilon_1 \cdot \frac{\sinh\left(\frac{nx}{r}\right)}{\cosh(ns)}$$

Stress profile along the length of fiber

Cox Model

Kelly's Model



$$\boxed{S_c = \frac{\sigma_{f,max}}{2\tau_i}}$$

Critical aspect ratio of fiber

$$\boxed{E_1^c = \vartheta_f E_f + \vartheta_m E_m - \vartheta_f E_f \frac{\tanh ns}{ns}}$$

Average stiffness of SFRC acc^ng to Cox

$$\sigma_1 = \sigma_f^{max}\left[\vartheta_f\left(1-\frac{m}{2}\right) + \vartheta_m \frac{E_m}{E_f}\right]$$

$$m = \frac{S_c}{S}$$

$$\sigma_f^{max} = 2\tau_i \cdot m \cdot S$$

Strength of SFRC acc^ng to Kelly
↓
(Short Fiber Reinforced Composites)

Problem

* During production of a flexible glass fiber-polyester hollow tub (diameter 20mm, wall thickness 10% of diameter, average fiber length 5mm, fibers oriented along the tube axis), something went wrong, and important fiber fracture during the manufacturing has occured. The glass fibers of the tube wall have now an average length of only 1mm.

redundant information (not needed)

- Calculate Stiffness of tube in tension
- " strength "

Data :- Matrix → Stiffness : 2GPa, strength : 100MPa
$\nu = 0.35$

Fiber → Stiffness : 70GPa, strength : ~~100~~ 2400MPa ; $\vartheta_f = 60\%$

- Interface strength : 30 MPa

$$n^2 = \frac{2G_m}{E_f \cdot \ln(R/r)} \qquad \text{use } R/r = 20.$$

- Typical glass fiber dia. = 45 μm
- Assume other values.

Sol^n:-

$$E_1^c = \vartheta_f E_f + \vartheta_m E_m - \vartheta_f E_f \frac{\tanh(ns)}{ns}$$

— known

○ unknown

$$n = \sqrt{\frac{2G_m}{E_f \ln(R/r)}} = 0.084$$

$$G_m = \frac{E_m}{2(1+\nu)} = 0.741$$

★★ Aspect Ratio $= l/r = \frac{2l}{d} = \frac{\text{Overall length}}{\text{Diameter}}$

$$s = \frac{1}{0.045} \quad \Bigg| \quad s = \frac{5}{0.045}$$

$$E_1^c = 42 + 0.8 - 42 \times \frac{\tanh(0.084 \times 22.2)}{(0.084 \times 22.2)}$$

$$= 42 + 0.8 - 42 \times 0.1$$

$$= 38.6 \text{ GPa}$$

Similarly calculate for other s. value.

Strength

$$\sigma_1 = \sigma_f^{max} \left[\vartheta_f \left(1 - \frac{m}{2}\right) + \vartheta_m \cdot \frac{E_m}{E_f} \right]$$

$$m = \frac{S_c}{S} \quad \Bigg| \quad S_c = \frac{\sigma_{f\,max}}{2\tau_i} \rightarrow \text{max. stress that fiber can take}$$

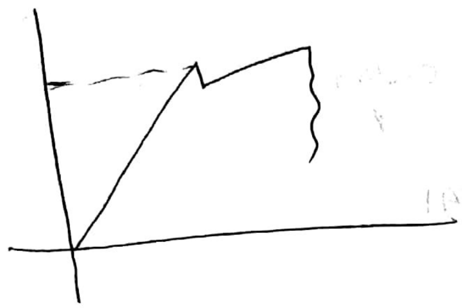
$$\sigma_{f\,max} = 2\tau_i m s. \rightarrow \text{max. stress that fiber will take.}$$

Calculating, we get

$$\sigma_1 \approx 248.67 \text{ MPa}$$

t_1 E_1 (layer 1)
t_2 E_2 (layer 2)
t_3 E_3 (layer 3)

$$E_1^c = \frac{t_1 E_1 + t_2 E_2 + t_3 E_3}{t_1 + t_2 + t_3}$$



Stress strain curve

Three steps in composite design

Micromechanics   Mesomechanics

1.) Generalized Hooke's law

$$\underset{2}{\sigma_{ij}} = \underset{4}{C_{ijkl}}\ \underset{2}{\varepsilon_{kl}} \qquad i,j,k,l = 1,2,3$$

$\sigma_{ij} = \sigma_{ji}$, $\varepsilon_{kl} = \varepsilon_{lk}$

$3^4 = 81$ $\downarrow$ reduced to 36

$$\therefore \quad \sigma_{ij} = [C_{ij}]\ \varepsilon_j \qquad i,j = 1,2,\ldots,6$$

Also, $C_{ij} = C_{ji}$

Derive

$$W = \frac{1}{2} C_{ij} \varepsilon_i \varepsilon_j \quad \Big| \quad \frac{\partial W}{\partial \varepsilon_i} = \sigma_i$$

Now 36 reduces to 21. $\longrightarrow$ even for anisotropic materials.

$\sigma_1 = \sigma_{11}$

$\sigma_2 = \sigma_{22}$

$\sigma_3 = \sigma_{33}$

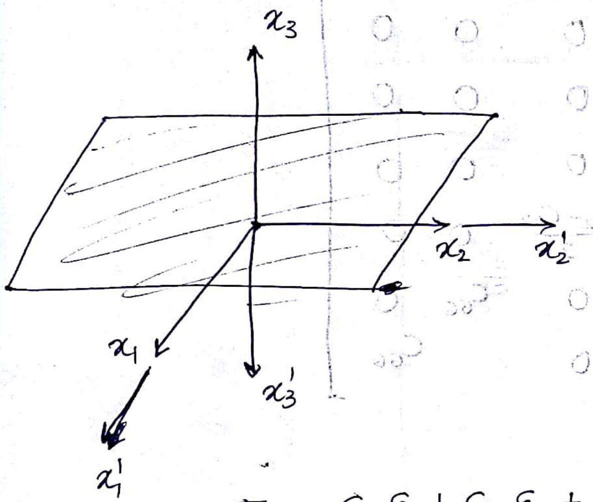
$\sigma_4 = \sigma_{23}$

$\sigma_5 = \sigma_{13}$

$\sigma_6 = \sigma_{12}$

For anisotropic / ~~mono~~ triclinic material

Monoclinic



$\sigma_1 = \sigma_1'$ $\qquad \varepsilon_1 = \varepsilon_1'$

$\sigma_2 = \sigma_2'$

$\sigma_3 = \sigma_3'$ $\qquad \vdots$

$\sigma_4 = -\sigma_4'$

$\sigma_5 = \sigma_5'$ $\qquad \varepsilon_5 = -\varepsilon_5'$

$\sigma_6 = -\sigma_6'$ $\qquad \varepsilon_6 = \varepsilon_6$

$$\sigma_1 = C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + \dots + C_{16}\varepsilon_6$$

$$\sigma_1' = C_{11}'\varepsilon_1' + C_{12}'\varepsilon_2' + \dots + C_{16}'\varepsilon_6'$$

$$C_{ij} = C_{ij}'$$

Then $C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix} \rightarrow$ 13 constants

Orthotropic

$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \rightarrow$ 9 constants

$$G = \frac{E}{2(1+\nu)}$$ → valid only if material is isotropic.

(only in 23 direction for transversely isotropic material) ★★

Transversely isotropic

5 constants

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C_{22}-C_{23}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{66} & 0 \\ 0 & 0 & 0 & 0 & & C_{66} \end{bmatrix}$$

For isotropic materials,

two independent constants, C_{11} & C_{12} are required.

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

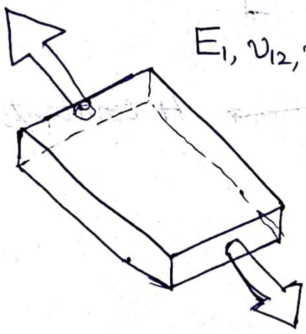
or $$\sigma_i = [C_{ij}]\,\varepsilon_j$$

$$\varepsilon_j = [S_{ji}]\,\sigma_i$$
$$= [C_{ij}]^{-1}$$

$$\varepsilon_1 = \frac{\sigma_1}{E} - \nu_{21}\frac{\sigma_2}{E} - \nu_{31}\frac{\sigma_3}{E}$$

$$S_{11} = \frac{1}{E_1} \; ; \; S_{12} = \frac{-\nu_{21}}{E} \; ; \; S_{13} = \frac{-\nu_{31}}{E} \; ; \; S_{66} = \frac{1}{G_{23}}$$

To calculate the engg. constants for an orthotropic material, 6 tests (3 normal & 3 shear tests are to be performed).



E_1, ν_{12}, ν_{13}

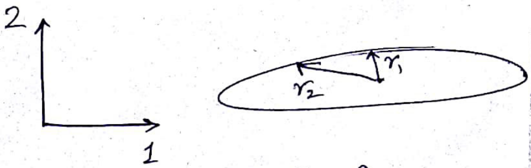
$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_1} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{Bmatrix}$$

$$-\frac{\nu_{21}}{E_2} = -\frac{\nu_{12}}{E_1}$$

Usually $\rightarrow$ $\nu_{21} \neq \nu_{12}$

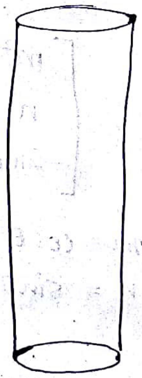
$\nu_{31} \neq \nu_{13}$

Plane stress & plane strain



$\left.\begin{aligned} \sigma_3 &= 0 \\ \sigma_4 &= 0 \\ \sigma_5 &= 0 \end{aligned}\right\}$ plane stress

$t << r_1, r_2$

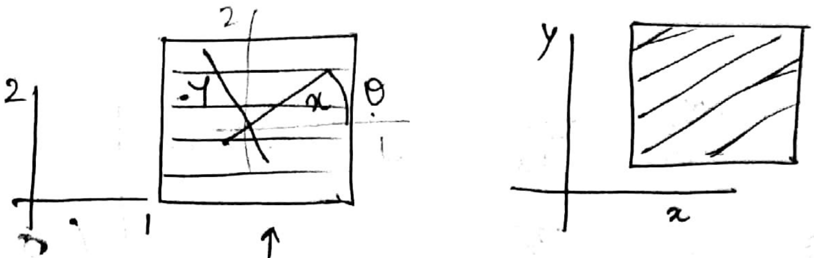


$\left.\begin{aligned} \varepsilon_3 &= 0 \\ \varepsilon_4 &= 0 \\ \varepsilon_5 &= 0 \end{aligned}\right\}$ plane strain

$t >> r_1, r_2$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix}$$

→ 4 constants



Known

Unknown

We want to calculate in xy direction

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T_\sigma] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \end{bmatrix} = [T_\varepsilon] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix}$$

$$[T_\sigma] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix}$$

$$[T_\varepsilon] = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix}$$

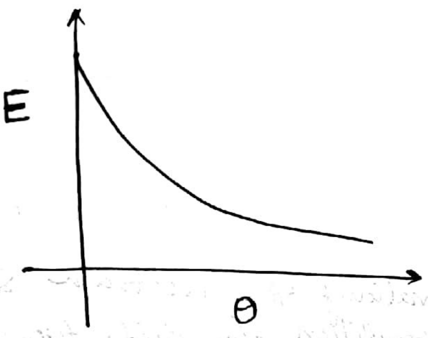
$m = \cos\theta$

$n = \sin\theta$

$$[\bar{S}] = [T_\varepsilon]^{-1}[S][T_\sigma]$$

x, y direction — 1, 2 direction stiffness

$$E_x = f(m, n, E_1, \nu_{12})$$

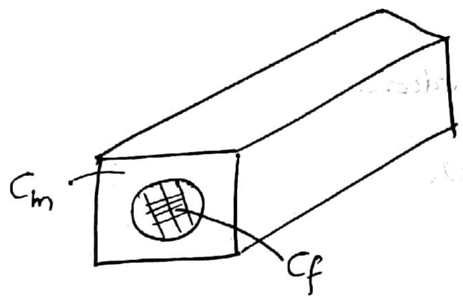


$$\{\varepsilon^{12}\} = [C^{12}]\{\sigma^{12}\} \quad \text{--- } ③$$

$$\{\varepsilon^{xy}\} = [C^{xy}]\{\sigma^{xy}\}$$

Abaqus → server abqs...

No UNDO



Parts

Section → for assigning elastic consts.

Module → Step → type of calculation to solve

Section Assignment → Tie ⇒ Glued.

Stiffer material → master

Display Group → Hiding

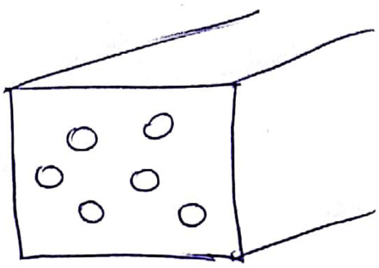
Master surface and slave surface.

We can see yellow surface indicating tying.

Module → Load (Load/ Displacement)

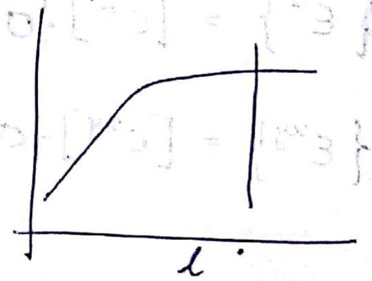
Load -ve ⇒ Tension.
+ve ⇒ Inward (Compression)

Term project



Check validity of isostrain & isostress condition for diff. types of loading.

Test atleast for two diff. fiber volume fractions.



janga1997.github.io/abaqus

→ Qualitative Analysis - used for evaluation.

.inp file → you can change the inputs

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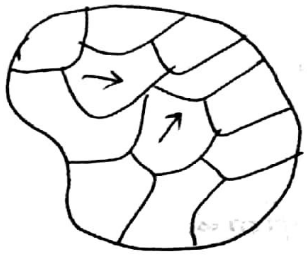
$$\eta_{i,ij} = \frac{\varepsilon_i}{\nu_{ij}}$$



$$\eta_{ij,i} = \frac{\nu_{ij}}{\varepsilon_i}$$

↓

Coefficient of mutual dependence

= 0 in fiber direction

* **Properties of E-glass**

	E-glass	Epoxy
	$E_1 = 87.0$ GPa	→ 3 GPa
	$\nu = 0.3$	0.3
	$G = 33.5$ GPa	1.2 GPa

$\vartheta_f = 0.4$

UD Composites.

① Find the maximum theoretical values of $E_1, E_2, G_{12} \longrightarrow 4.16$ GPa

② How much extra fiber will have to added if off-axis ply is not used to get same shear modulus (Max.)

Sol:-

$$E_1 = E_f \vartheta_f + E_m \vartheta_m$$

$$= 87(0.4) + 3(0.6)$$

$$= 36.6 \text{ GPa} \longrightarrow \text{Max.}$$

$$1/E_2 = \vartheta_f/E_f + \vartheta_m/E_m$$

$$= \frac{0.4}{87} + \frac{0.6}{3}$$

$$\Rightarrow E_2 = 4.89 \text{ GPa}$$

$$1/G_{12} = \frac{0.4}{33.5} + \frac{0.6}{1.2} \overset{\Rightarrow G_{12}}{\approx} 1.95 \text{ GPa}.$$

After rotation to 45°.

$$\frac{1}{G_{xy}} = 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)^2 \left\{\frac{1}{E_1} + \frac{1}{E_2} + \frac{2\nu_{12}}{E_1}\right\} + (0)\frac{1}{G_{12}} \Rightarrow G \approx 4.03 \text{GPa}$$

$$\frac{1}{4.16} = \frac{v_f}{33.5} + \frac{1-v_f}{1.2}$$

$$\underline{\underline{v_f = 0.74}}$$

* Randomly oriented fibres

$$E = \frac{3}{8}E_1 + \frac{5}{8}E_t$$

$$= \eta \frac{3}{8}E_1 + \frac{5}{8}E_t \qquad (2D)$$

$$= \eta \frac{1}{5}E_1 + \frac{4}{5}E_t \qquad (3D)$$

Midsem $\rightarrow$ 2 Numerical + 1 Logical Question.