(8) <u>Critical speeds</u> of rotating shafts & <u>Whirling</u> of shafts

At certain angular speeds a rotating shaft carrying one or more rotors can exhibit excessive lateral deflection and vibration. Such an angular speed is called a <u>critical speed</u> or <u>critical whirling speed</u>. The shaft deflection can sometimes become so excessive that permanent deformation & structural damage takes place. For instance, the rotor blades of a turbine may contact the stator blades. Also, large bearing reactions ~~can~~ occur & can result in bearing failure & structural damage to the bearing supports. This phenomenon is seen to occur even for very accurately balanced rotors. A machine should never be operated at a critical speed. We now study why this happens and how a rotor-shaft system can be designed to avoid the critical speeds as the operating speed.

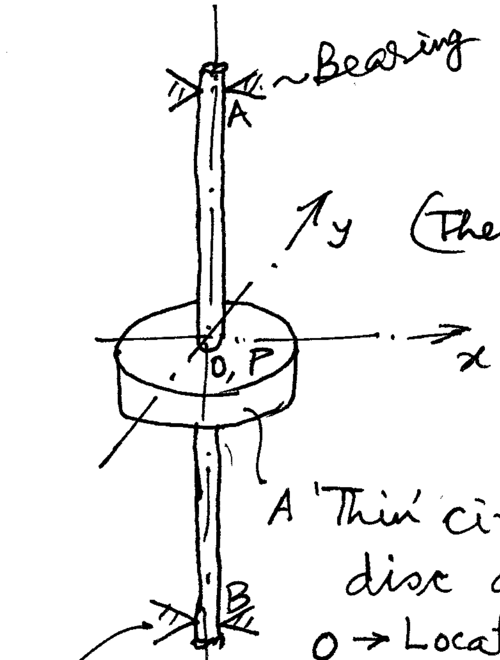
→ A shaft can contain multiple rotors such as a compressor, a turbine and an electric generator. Such shafts will have multiple critical speeds. However, most such set-ups almost never operate at or beyond the <u>second</u> critical speed. Hence, the lowest critical speed turns out to be the most important one.

→ for the sake of simplicity in understanding the phenomenon, many simplifying assumptions are made such as:

- The shaft is of negligible inertia and acts as a linear spring to resist deflection in a lateral direction with constant 'k'
- The bearings are rigid, that is, we neglect the flexibility of the bearings. (What happens if bearing flexibility is taken into account shall be briefly discussed later)
- The rotor (a single rotor only!) is rigid & connected at the mid-span between the bearings at the ends. It has an unbalance (a rotating unbalance).

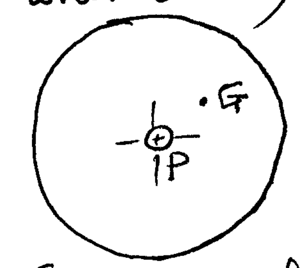
We consider the system in fig. 1 (page 3) which corresponds to the static equilibrium condition. The centre of gravity, G of the rotor, considered as a circular disc, is away from its geometric centre P by an amount 'e' ('e' for eccentricity). Mass of rotor = m. This whole mass causes unbalance.

- A damping force is assumed which is proportional to the speed of the geometric centre P. Such a damping force may arise due to air or fluid (liquid) friction. The rotor may rotate in a liquid such as water as in a water turbine, note.



Ox, Oy → inertial axes. (These <u>do not</u> rotate with disc)

Figures not to scale



A 'Thin' circular disc as rotor

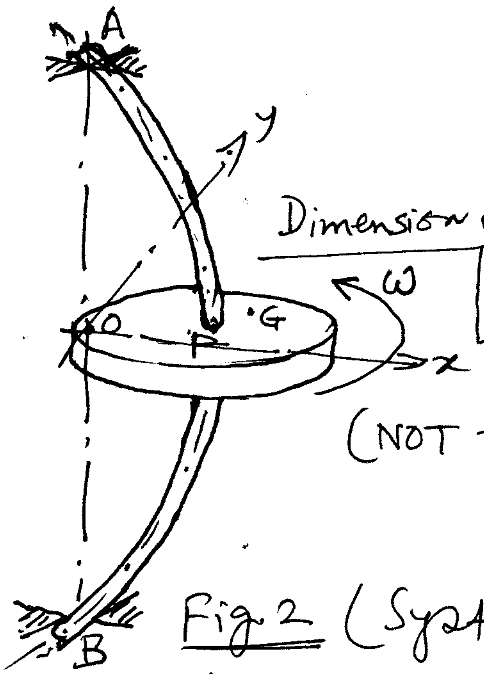
G → Centre of gravity of disc
<u>PG = e</u> (very much exaggerated)

O → Location of disc geometric centre P at static eqbm. O lies on the axis of rotation AB. (So, O & P coincide at static eqbm.)

<u>Fig. 1</u>

{System in static equilibrium}

[G is different from P due to many reasons such as non-uniform material density, asymmetry of nuts, bolts etc., elongation of bolt holes etc.



Dimension OP highly exaggerated. While disc rotates, P is not on x-axis, in general, note

(NOT TO SCALE)

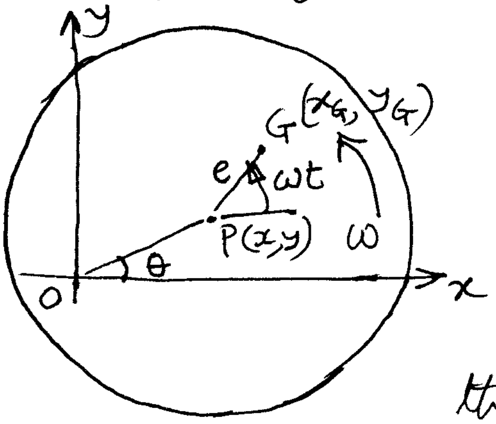
<u>Fig. 2</u> (System while disc rotates)

Note that due to unbalance (since geometric centre P & centre of gravity G of the disc don't coincide), the shaft takes a bow shape and <u>'whirl'</u>. <u>Whirling</u> is defined as the rotation of the plane formed by the curved axis of the bent shaft and the line of centers of the bearings, AB.

→ Although the above definition of <u>whirling</u> appears to be pretty simple, visualizing the phenomenon is far from simple. The (rigid) disc is assumed to rotate at an angular velocity ω, i.e., the (imaginary) line segment PG rotates at a constant $\measuredangle r$ speed ω, whereas whirling speed is the speed of rotation of the imaginary line segment OP & these two speeds may differ! We have so called <u>'synchronous' whirling</u> if whirl speed $= \omega$. Otherwise, we have 'asynchronous' whirl. To make matters further complicated, whirling may take place in the same or opposite direction as ~~that~~ of the disc rotation!

→ A detailed study of this is made in books on <u>'Rotor Dynamics'</u> (Such as, Rotor Dynamics by J. S. Rao; Dynamics of Rotating Systems by Genta) and is beyond the scope of our syllabus. However, you may note in passing that this phenomenon results from various causes such as mass unbalance, gyroscopic forces, fluid friction in <u>Journal Bearings</u>, hysteresis effect in shaft material, flexibility of bearings etc. <u>We assume synchronous whirl.</u>

A top view of the rotor (disc) at time t is shown below clarifying these further. (The rotor size is increased for clarity)



Let at $t=0$, segment PG was parallel to the x-axis.

Then, in time t, it rotates through ~~an~~ angle 'ωt' CCW where ω is the angular velocity of the rotor.

At this instant, segment OP makes an angle θ with the x-axis. Note that $\dot{\theta} = \frac{d\theta}{dt}$ is the whirl speed and for synchronous whirl, $\dot{\theta} = \omega$.

Let (x, y) ~~be~~ the location of the geometric centre at time t.

From above fig., we see that

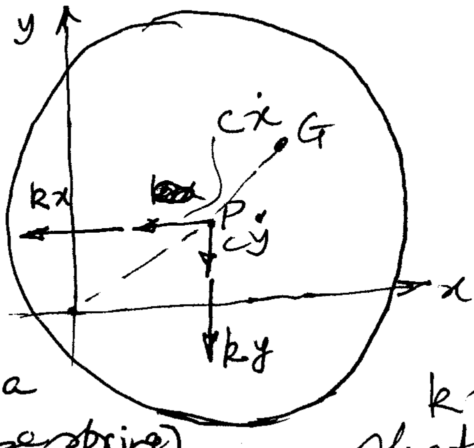
x_G = x-coordinate of CG = $x + e\cos\omega t$

& y_G = y- " " " = $y + e\sin\omega t$ (i)

Hence, $\ddot{x}_G$ = x-component of acceleration of G = $\ddot{x} - e\omega^2\cos\omega t$

& $\ddot{y}_G$ = y - " " " " " = $\ddot{y} - e\omega^2\sin\omega t$ (ii)

~~Ext. forces~~ External force components acting on rotor in the plane of motion are shown below:



[The shaft acts like a transverse spring]

$c\dot{x}$ & $c\dot{y}$ are damping forces due to air/liquid friction where this damping force is assumed ~~to be~~ proportional to the velocity of the geometric centre P of rotor.

kx & ky are due to the shaft deflection at P. →

We now apply Newton's 2nd law to the CG 'G' of the rotor:

$$m\ddot{x}_G = \Sigma \text{ Ext forces in } x\text{-direction}$$
$$\& \quad m\ddot{y}_G = \Sigma \text{ " " " } y\text{-direction}$$
} m = mass of disc/rotor

These lead to: (using (i) & (ii) on page 5)

$$\begin{cases} m\ddot{x} - me\omega^2 \cos\omega t = -kx - c\dot{x} \\ m\ddot{y} - me\omega^2 \sin\omega t = -ky - c\dot{y} \end{cases}$$

OR

$$m\ddot{x} + c\dot{x} + kx = me\omega^2 \cos\omega t \quad \text{--- (iii)}$$
$$m\ddot{y} + c\dot{y} + ky = me\omega^2 \sin\omega t \quad \text{--- (iv)}$$

Do you remember we came across DEOM similar to (iii) & (iv) when we discussed rotating unbalance for a single DOF system?

Imp. → The only difference is that now m is the mass of the whole rotor and not the small, unbalanced mass in the case of rotating unbalance topic!

→ We know from our previous studies that the forced response of $m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega_f t$ is given by: $x = x(t) = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \sin(\omega_f t - \psi)$

& for $m\ddot{y} + c\dot{y} + ky = F_0 \cos\omega_f t$, it is given by: $y = y(t) = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \cos(\omega_f t - \psi)$.

$\left(r = \frac{\omega_f}{\omega_n}, \zeta = \frac{c}{2\sqrt{km}} \text{ etc.}\right)$

Here $F_0 \equiv me\omega^2$, $\omega_f \equiv \omega$,

$$\frac{F_0}{k} = \frac{me\omega^2}{k} = \frac{e\omega^2}{k/m} = \frac{e\omega^2}{\omega_n^2} = er^2 \text{ where}$$

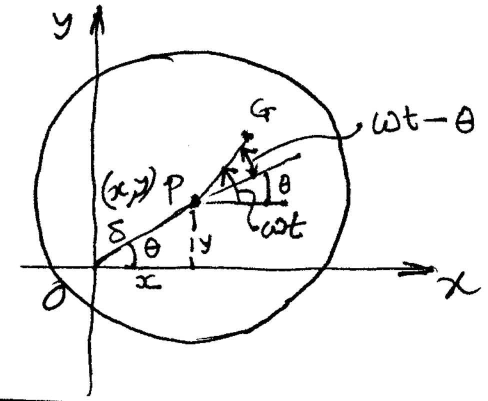
$\sqrt{\frac{k}{m}} = \omega_n$ = natural frequency of the rotor-shaft system for transverse vibration.

Hence, the forced responses corr. to (iii) & (iv) are:

$$x = \frac{er^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\cos(\omega t-\psi) \quad \text{--- (V)} \quad \& \quad y = \frac{er^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\sin(\omega t-\psi) \quad \text{--- (VI)}.$$

$= OP\cos\theta$ $\quad \boxed{\therefore\ \theta = \omega t - \psi} \quad$ $= OP\sin\theta$

The phase lag ψ is shown in the fig. below.



Also, $\psi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$, as you already know. $\boxed{\psi = \text{Constant for given } \omega}$

Let $OP = \delta$, which is the distance of the disc center from axis of rotation.

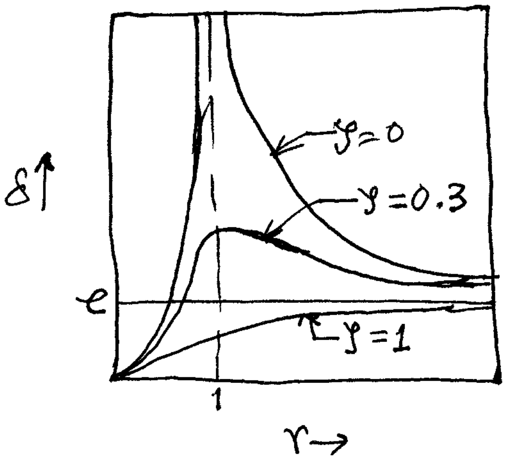
$\omega t - \theta = \psi$
$\Rightarrow \omega = \dot{\theta}$, as it should be for synchronous whirl

$\boxed{\omega, \psi \text{ are Constants}}$

Then, using (V) & (VI) & above figure, we have

$$\delta = \sqrt{x^2+y^2} = \frac{er^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \quad \text{(Remember)}$$

The plots of δ vs r for various ζ values look similar to the ones obtained for studying rotating unbalance:



From these plots, it can be seen that for small ζ, δ can be quite large for values of r close to unity.

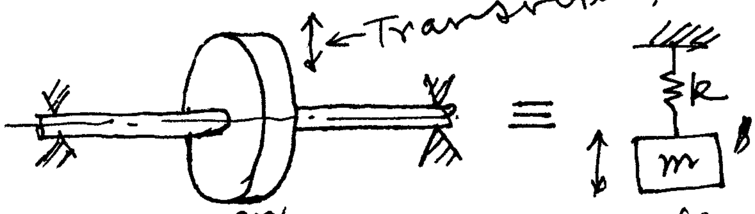
<u>Home Work</u>:- From a textbook, study the figures of disc configuration corresponding to $\psi < \pi/2$, $\psi = \pi/2$ & $\psi > \pi/2$ after studying the ψ vs r plots. (You have actually already studied these ψ vs r plots before.)

$\longrightarrow$ Let us <u>sum up</u>: $\rightarrow$

→ <u>Whirling</u> of a rotating shaft occurs when an unbalance exists.

→ For a flexible shaft of negligible mass carrying a rigid rotor at its mid-span, violent vibrations may ~~occur~~ occur at a certain rotational speed called a <u>critical speed</u>. If damping is negligible, this critical speed is nothing but the undamped natural frequency of the system for <u>transverse</u> vibrations:

Transverse, small vibration



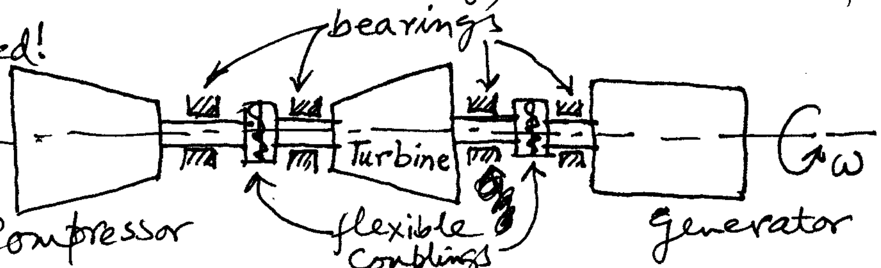
(No rotation)

$\omega_n = \sqrt{\frac{k}{m}}$

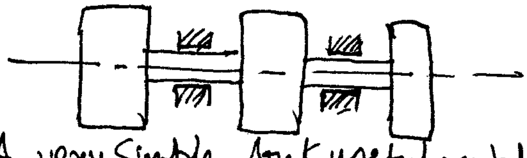
k = stiffness of shaft in a transverse direction.

→ Note that our ~~sy~~ system model above is too simple to represent many practical systems where there shall be multiple rotors (each of which can be modeled by a disc, approximately). An example:-

[Still over-simplified! Many connections, reduction gears etc. not shown!



$\equiv$

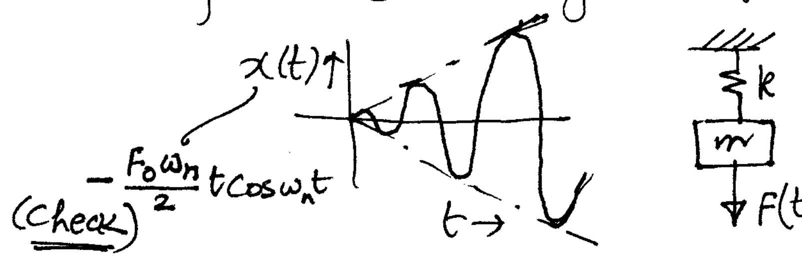


(A very simple but useful model)

Hence, multiple discs (rotors) will cause the system to have multiple natural frequencies for transverse vibration, i.e., $\omega_1, \omega_2, \omega_3$ etc.

→ Each of these will now be a critical speed.

→ for most machines, $\omega < \omega_2$. Hence, ω_1 or, the lowest natural frequency is the most important one. ω should never be equal to or close to ω_1. Hence, if a machine operates at an ω such that $\omega_1 < \omega < \omega_2$, special precautions must be exercised when operations begin. The amplitude of shaft vibration at a critical speed reaches a dangerous level only if time is allowed for the amplitude to build up. (Remember what happened at resonance for the single DOF undamped system?



$\omega_n = \sqrt{\frac{k}{m}}$ (Will be done soon))

Therefore, the m/c must be <u>accelerated</u> (i.e. quickly pass ~~through~~) through the critical speed so that vibration level is acceptable. Examples are :- some centrifuges and some high-speed turbines operate at a speed well above the critical speed and must be brought up to the operating speed by passing quickly through their critical speed.

→ for a non-circular shaft (which is not very common) the shaft stiffness will be different (say, k_x & k_y) in x- & y-directions. In this case, point P (the centre of the rotor) describes an ellipse as the shaft whirls. [Meirovitch – Fundamentals of vibrations, 1st edition, § 3.4, page 126].

→ When damping is present, ~~considerable~~ the max^m^ amplitude of vibration occurs at some $r<1$ (you already know about it) but usually, the critical speed is still said to occur at an undamped natural frequency, i.e. at $r=1$.

→ At speeds near the critical speed, the shaft deflections are large and the forces on the bearings will be large too. In addition, these forces will be changing directions constantly & may give rise to bearing fatigue failure.