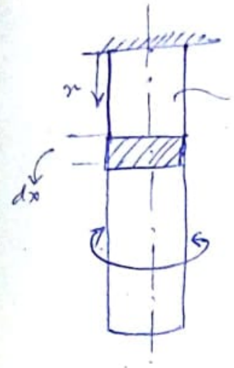
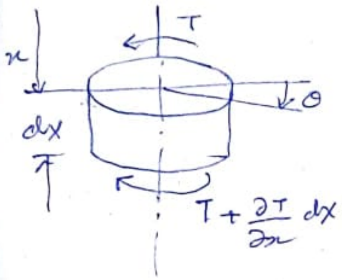
§ Torsional free vibrations of a shaft



$G, J, l, \theta(x,t)$
$\downarrow$
angular displacement of a section at (x,t)



$T(x,t) =$ Torque at (x,t)

$$\underset{\text{element}}{I} \frac{\partial^2\theta}{\partial t^2} = \frac{\partial T}{\partial x} dx$$

$I_{\text{element}} = I\,dx$

I = MI of the shaft per unit length about its own axis

$$I \frac{\partial^2\theta}{\partial x^2} = \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(GJ\frac{\partial\theta}{\partial x}\right) \rightarrow \text{the req.d DEOM}$$

$$\frac{T(x,t)}{J} = G\frac{\partial\theta}{\partial x}(x,t)$$

$$T = GJ\frac{\partial\theta}{\partial x}$$

For a clamped free shaft,

B.Cs are: $\theta(0,t) = 0$

& $GJ\frac{\partial\theta}{\partial x}\Big|_{x=l} = 0$

For a uniform shaft,

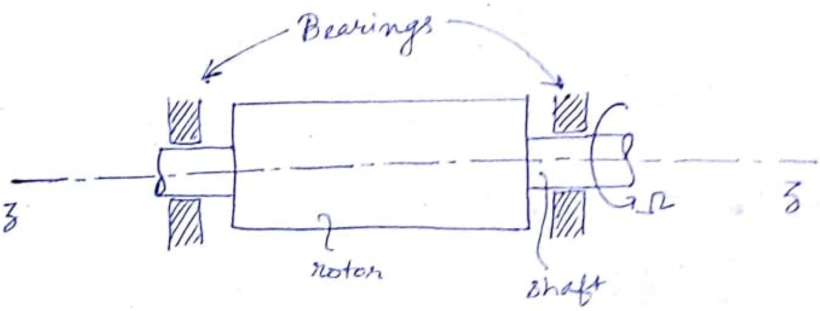
$$I\frac{\partial^2\theta}{\partial t^2} = GJ\frac{\partial^2\theta}{\partial x^2} \rightarrow \text{1-D wave eq}^n$$

$$\frac{\partial^2\theta}{\partial t^2} = c^2\frac{\partial^2\theta}{\partial x^2}$$

$$c = \sqrt{\frac{GJ}{I}}$$

= velocity of shear waves.

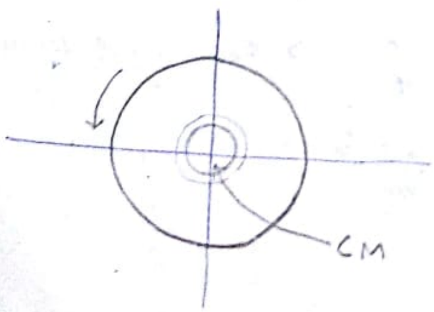
<u>11.4.18</u>

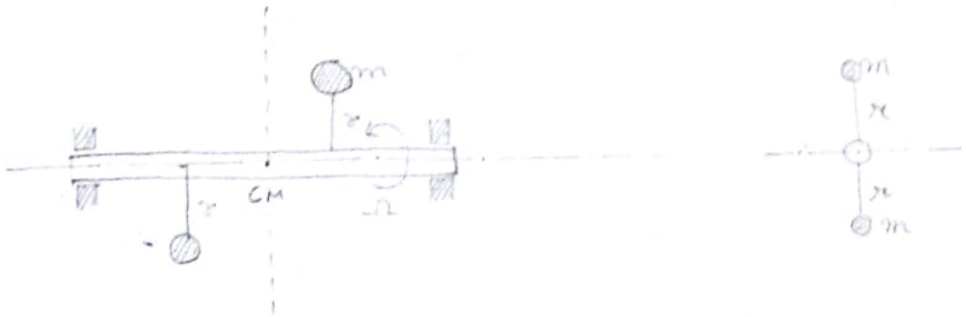


Q. When do we say the rotor-bearing shaft is in <u>dynamic balance?</u>

Static balance ⇒ The CM must lie on axis of rotation

<u>Dynamic balance:</u>





We have to continuously monitor the vibration.

Necessary and Sufficient conditions for the Dynamic Balance

~~Conditions of dynamics balance:~~

of a roller-bearing system are as follows:-

(1) The CG/CM must lie on the axis of rotation at all times

(2) The axis of rotation must be a principal axis at any point on the axis of rotation.

zz-axis must be a principal axis at any arbitrary point A on it.

This means we must have $I_{xz} = 0$ and $I_{yz} = 0$ at all times.

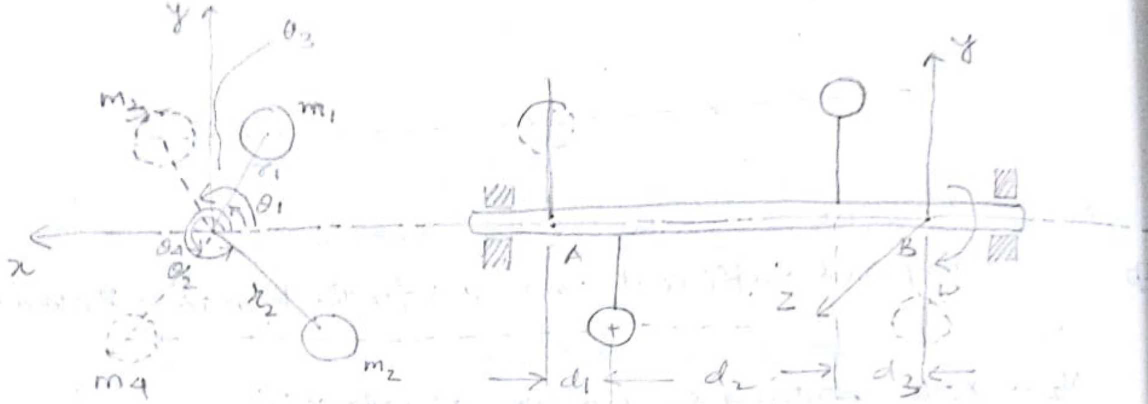
The benefit of this scheme is that if we prove it for one point on the axis, it is automatically proven for other points on the axis.

$$M_x = -I_{xz}\dot{\omega}_z + I_{yz}\omega_z^2$$

$$M_y = -I_{yz}\dot{\omega}_z - I_{xz}\omega_z^2$$

$$M_z = I_{zz}\dot{\omega}_z$$

Example



Note: Sometime chunks of mass are welded and sometimes, mass are drilled out to balance out an unbalancing. Note that the correction weights with generally be on two planes to achieve a balanced system.

Known: m_1, r_1, θ_1

m_2, r_2, θ_2

unknowns: m_3, r_3, θ_3

m_4, r_4, θ_4

$\boxed{r_3, r_4} \rightarrow$ specify because this has to be constrained.

$$M x_c = \int_M x\, dm = 0$$

$$M y_c = \int_M y\, dm = 0$$

$$I_{xz} = 0 \Rightarrow \int_M xz\, dm = 0$$

$$I_{yz} = 0 \Rightarrow \int_M yz\, dm = 0$$

$$\therefore M x_c = m_1 r_1(-\cos\theta_1) + m_3 r_3 (\cos(180^\circ - \theta_3)) + m_4 r_4 \cos(\theta_4 - 180^\circ) + m_2 r_2 (\cos(2\pi - \theta_2)) = 0$$

Or, $$-m_1 r_1 \cos\theta_1 - m_3 r_3 \cos\theta_3 - m_4 r_4 \cos\theta_4 - m_2 r_2 \cos\theta_2 = 0$$

Or, $$m_1 r_1 \cos\theta_1 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4 + m_2 r_2 \cos\theta_2 = 0$$

Or, $$\sum_i m_i r_i \cos\theta_i = 0 \quad \text{— (1)}$$

Similarly,

$$y_c = 0 \Rightarrow \sum_i m_i r_i \sin\theta_i = 0 \quad \text{— (2)}$$

$$I_{xz} = \int xz\, dm = 0$$

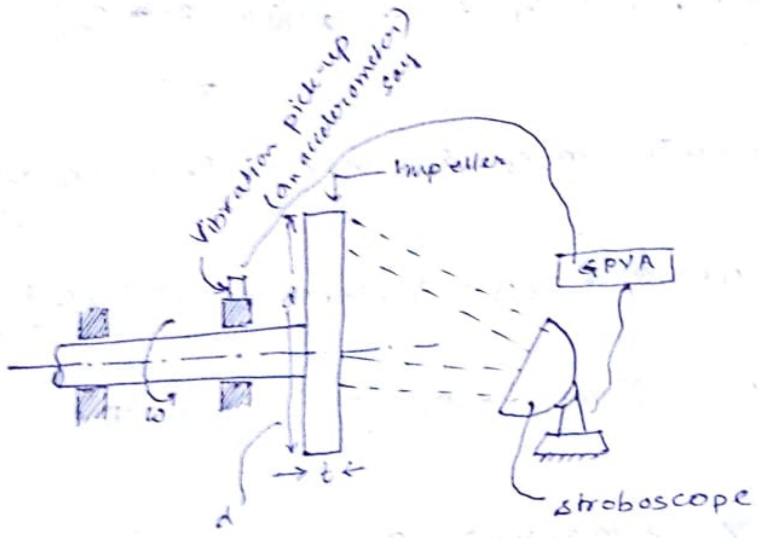
$$x_1 z_1 m_1 + x_2 z_2 m_2 + x_3 z_3 m_3 + x_4 z_4 m_4 = 0$$

$$\Rightarrow \sum_{i=1}^{4} m_i r_i z_i \cos\theta_i = 0 \quad \text{— (3)}$$

Similarly,

$$\sum_{i=1}^{4} m_i r_i z_i \sin\theta_i = 0 \quad \text{— (4)}$$

Field Balancing of Rotors.



Aim: Detect (measure) the unbalance and take corrective measures.

If $\frac{d}{t} > 5$, we shall call the impellor a thin one for which single-plane balancing will do.

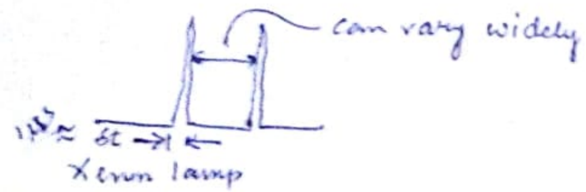
GPVA → General purpose Vibration Analyser.

It is an electronic equipment which can integrate the signal from the accelerometer (which measures acceleration at the bearing) & producing displacement)

The stroboscope flashes at the instant when the bearing experiences the maximum vertical displacement

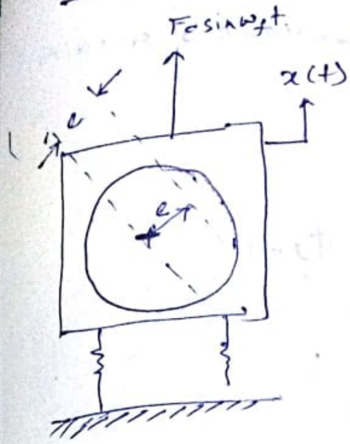
Whirling of shafts

The impeller has developed unacceptable vibration occuring at frequency ω. This is most likely due to the presence of an unbalance.



1 flash/s
20000 flash/s

Unbalance



$$F_0 = m e \omega_f^2$$

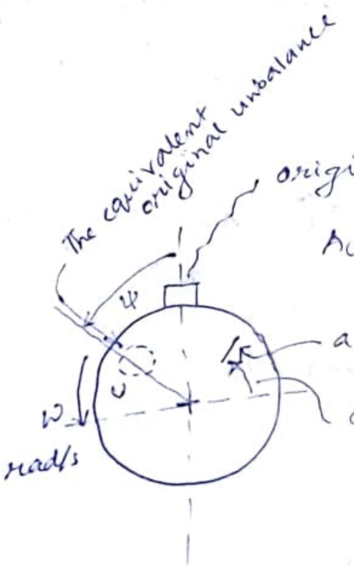
$$x(t) = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \sin(\omega_f t - \psi)$$

$$\psi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

$$x(t) = \frac{\frac{m e \omega_f^2}{k}}{\sqrt{\cdots}}$$

$$= \frac{\frac{m e r^2}{M}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \sin(\omega_f t - \psi)$$

$me = U =$ unbalance.



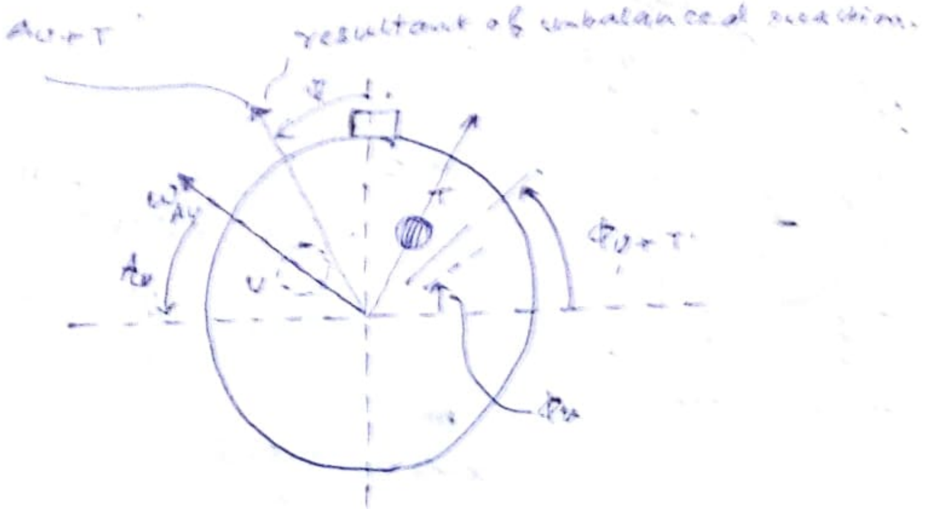
A_u = Amplitude of vertical vibration at bearing with original unbalanced U.

The m/c is stopped. We put a trial known unbalance 'T' and then run the m/c again.

$\omega_n \rightarrow$ remains practically same after T is attached.

$\because$ T ~ (50-20) gm in comparison to 1 ton blade (turbine)

ψ remains unchanged.



A_U, Φ_U

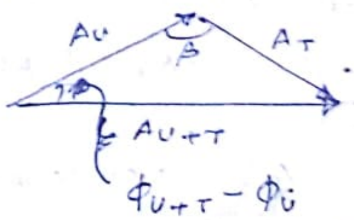
A_{U+T}, Φ_{U+T}

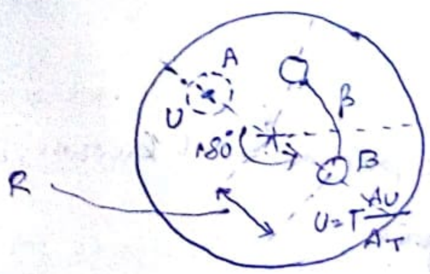
$$\frac{A_U}{A_T} = \frac{U}{T}$$

So, $U = T \dfrac{A_U}{A_T}$

$$A_T = \left[A_U^2 + A_{U+T}^2 - 2 A_U A_{U+T} \cos(\Phi_{U+T} - \Phi_U) \right]^{1/2}$$

$$\bar{A}_{U+T} = \bar{A}_U + \bar{A}_T$$





So, mass to be attached (welded) at $B = \dfrac{U}{R}$

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