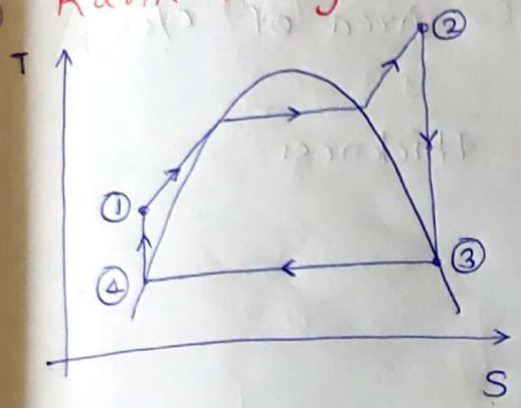
Introduction

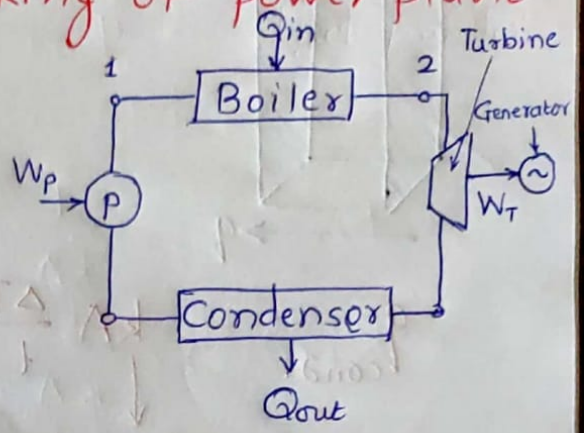
18/07/2018

① Conduction, convection, radiation.

② IInd Law of Thermodynamics.

③ Rankine Cycle : Working of power plant.





1 → 2 - Constant Pressure

2 → 3 - Turbine (Work Output)

3 → 4 - Condenser (const. Pressure)

4 → 1 - Pump (compressor)

$[W_T >> W_P]^*$

{ Turbomachinery - Pump/Turbine }

④ Heat Transfer - Extraction of heat/energy due to temperature diff. within a media or b/w diff. medium.

① Mechanism of heat transfer

② Rate of heat transfer.

⑤ Syllabus -

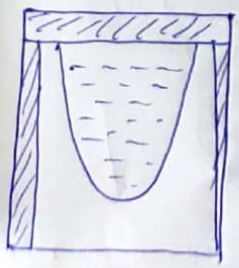
(1) Intro - Modes, multi-mode heat transfer

(2) Conduct^n, convection, Radiat^n

(3) Boiling & Condensat^n

(4) Heat Exchangers.

⑥ Thermoflask :



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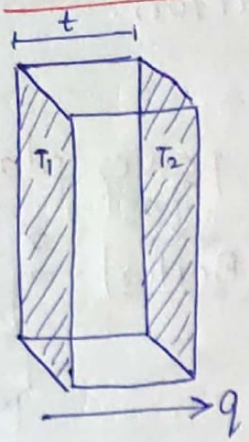
⑦ Conduction :



* Vibration of molecules
* $q \propto (T_1 - T_2)$

$\propto$ Area of c/s

$\propto \dfrac{1}{\text{thickness}}$

$$q_{cond.} = kA\frac{\Delta T}{t}$$

$k \rightarrow$ Thermal conductivity

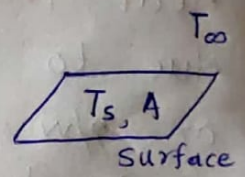
$q \rightarrow W$

$k \rightarrow \dfrac{W}{m.K}$

for $t \rightarrow 0$: $q = -kA\dfrac{dT}{dx}$

⑧ Convection :

- Heat transfer due to fluid motion.
- $q_{conv.} = hA(T_s - T_\infty)$



$h \rightarrow$ heat transfer coefficient $\left(\dfrac{W}{m^2.K}\right)$

- Density driven bulk flow of fluid.

{ Hot air (low dens.) ↳ Natural Convection.

- Forced Convection : Blow air / Switch fan ON.

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Radiation:

Transmission of Electromagnetic Waves.

Always b/w 2 surfaces.

Every surface at some temperature radiates energy!

* $q_{rad} \propto (T_{abs})^4$ ⇒ {Always convert to Kelvin!}

$\propto A$

$\propto n^2$ → {n = refractive index of medium} [typically $n = 1$]

- $q_{rad} = \sigma A T_1^4$ → {For black surface @ temp. T_1}

σ ↓ Stefan Boltzmann Constant $= 5.67 \times 10^{-8}\ W/m^2K^4$

- For grey surface @ temp T_1 : $q_{rad} = \sigma \varepsilon A T_1^4$

ε ↓ emmisivity

$\therefore q_{rad_{1-2}} = \sigma \varepsilon A (T_1^4 - T_2^4)$

↳ Very specific example of gray surface exchanging radiation with an infinite surface @ temp. T_2 around it.

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① Conduction → $q_{cond.} = kA\frac{\Delta T}{L}$

② Convection → $q_{conv} = hA\Delta T$

③ Radiation → $q_{rad} = \sigma \epsilon A (T_1^4 - T_2^4)$

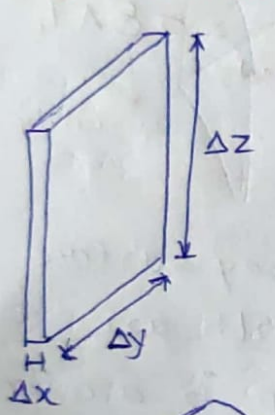
④ Thermal Resistance : $R_{th} = \frac{\text{Temp. diff.}}{\text{heat transf. rate}}$ $\left[\frac{K}{W}\right]$

- $R_{th\,(cond.)} = \frac{L}{kA}$
- $R_{th\,(conv.)} = \frac{1}{hA}$

⑤ Steady State:

- Temp. gradient w.r.t 'L' is zero.
- Heat transfer balances each other.

⑥ 1-Dimensional Conduction:-

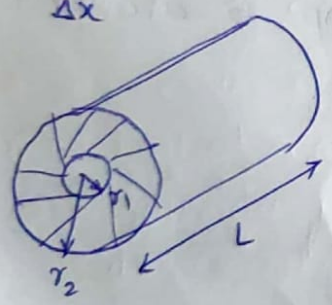


(1) Heat conduction is prominent in a single direction.

(2) $\Delta x << \Delta y, \Delta z$

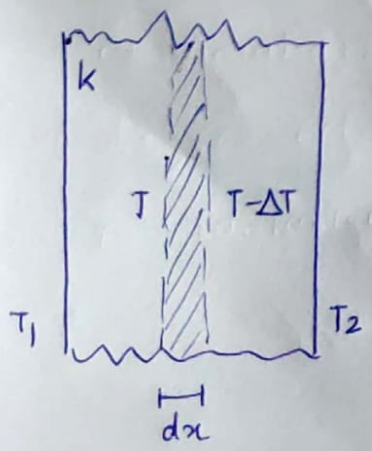
{Assumption is acceptable}

(3) $r_1, r_2 << L$



① $q = -kA\frac{dT}{dx}$

② Heat flux $= -k\frac{dT}{dx} = q''$



③ Generally

$q_i'' = -k_i\frac{dT}{di}$ $\quad i = x, y, z$

[Examples: graphite, graphene]

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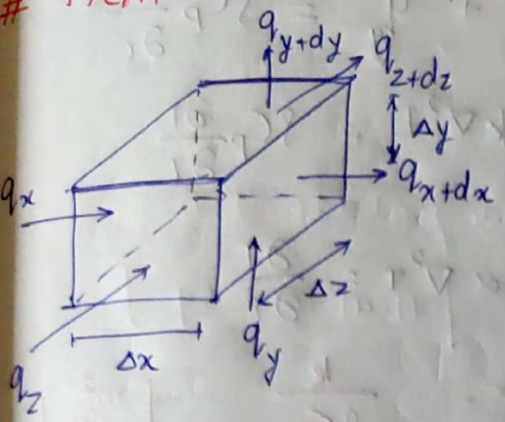
HEAT CONDUCTION EQUATION :—



$$q_x = -k(\Delta y \cdot \Delta z)\frac{\partial T}{\partial x}$$

$$q_{x+dx} = q_x + \left(\frac{\partial q_x}{\partial x}\right)\Delta x + \dots$$

$$\therefore \quad q_x - q_{x+dx} = -\frac{\partial q_x}{\partial x}\Delta x = \frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right]\Delta x \Delta y \Delta z \quad \text{— (1A)}$$

$$q_y - q_{y+dy} = -\frac{\partial q_y}{\partial y}\Delta y = \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right]\Delta x \Delta y \Delta z \quad \text{— (1B)}$$

$$q_z - q_{z+dz} = \frac{-\partial q_z}{\partial z}\Delta z = \frac{\partial}{\partial z}\left[k\frac{\partial T}{\partial z}\right]\Delta x \Delta y \Delta z \quad \text{— (1C)}$$

* Assume internal heat generation:

q''' $[W/m^3]$ ↓ inside cuboid

Heat generated...

$$q_{gen} = q''' \Delta x \Delta y \Delta z$$

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① Energy Balance:

Heat (in) + Heat (gen) = Heat (out) + Energy stored.

$$\therefore \quad (q_{in} - q_{out}) + q_{gen} = q_{stored}.$$

$$\text{or} \left[\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q'''\right]\Delta x \Delta y \Delta z = \rho C_p \frac{\partial T}{\partial \tau} \cdot \Delta x \Delta y \Delta z$$

$$\therefore \quad \frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q''' = \rho C_p \frac{\partial T}{\partial \tau}$$

Heat conduction eq^n :

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* Homogenous Eq^{n}: $k\nabla^2 T + q_g''' = \rho C_p \frac{\partial T}{\partial \tau}$

* No internal energy gen: $k\nabla^2 T = \rho C_p \frac{\partial T}{\partial \tau}$
  ($q''_{gen} = 0$)

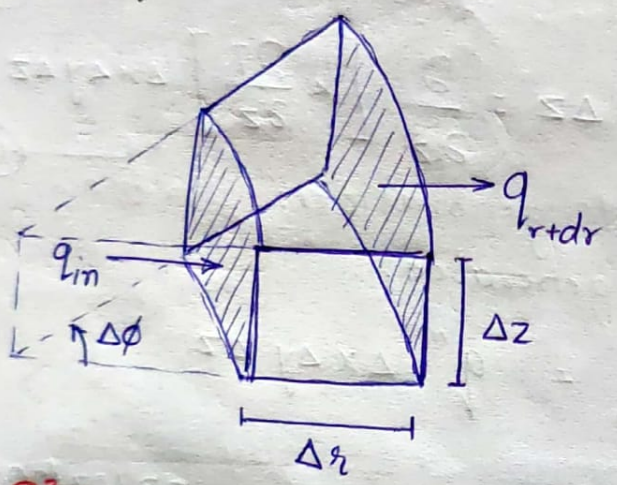
  i.e $\alpha \nabla^2 T = \frac{\partial T}{\partial \tau}$

Thermal diffusivity... ← $\alpha = \frac{k}{\rho C_p}$ $[m^2/s]$

* Steady State: $k\nabla^2 T + q_g''' = 0$

[Diffusion function]

* Steady State + no internal energy gen: $k\nabla^2 T = 0$



$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial T^2}{\partial z^2} + \frac{q'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

* In order to solve heat conduction eq^{n}:

① Initial condition

② Boundary condition:

a) Const. temp.

b) Const. heat flux: $\{ q_x'' = k\frac{\partial T}{\partial x} = \text{const.} \}$

c) Adiabatic: $\{ q_x'' = 0 \text{ or } \frac{\partial T}{\partial x} = 0 \}$

d) convective: $\{ q_x'' = -k\frac{\partial T}{\partial x} = h(T - T_\infty) \}$

e) radiative: $\{ q_x'' = -k\frac{\partial T}{\partial x} = \sigma \epsilon (T^4 - T_\infty^4) \}$

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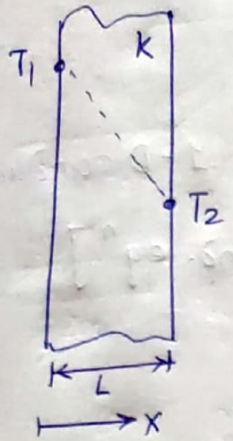
[A] Plane Wall :-



$$k \frac{\partial^2 T}{\partial x^2} = 0$$

$$T(x) = c_1 x + c_2$$

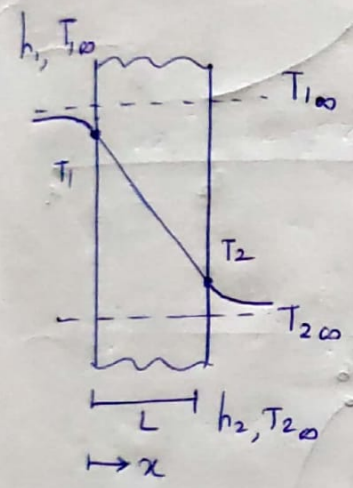
$$[BC \Rightarrow \quad T(0) = T_1, \quad T(L) = T_2]$$

$$\therefore \boxed{T(x) = T_1 - \left(\frac{T_1 - T_2}{L}\right) x}$$

Heat conducted : $q = -kA \frac{\partial T}{\partial x}$

$$= kA \frac{T_1 - T_2}{L}$$

$$= \frac{T_1 - T_2}{(L/kA)} = \frac{\Delta T}{R_{th}}$$



Convective Boundary Cond.

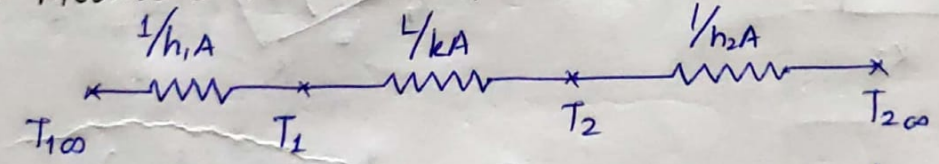
$$\frac{d^2 T}{dx^2} = 0$$

B.C: ① $-k \frac{dT}{dx} = h_1 (T - T_{1\infty}) \quad \forall x = 0$

② $-k \frac{dT}{dx} = h_2 (T - T_{2\infty}) \quad \forall x = L$

Method 1 : Solve ODE

Method 2 : Use Resistance Network.



$$T_{1\infty} - T_{2\infty} = q\left(\frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}\right) \quad \text{— ①}$$

$$T_1 - T_{1\infty} = q\left(\frac{1}{h_1 A}\right) \quad \text{— ②}$$

$$T_2 = T_{2\infty} + \frac{q}{h_2 A} \quad \text{— ③}$$

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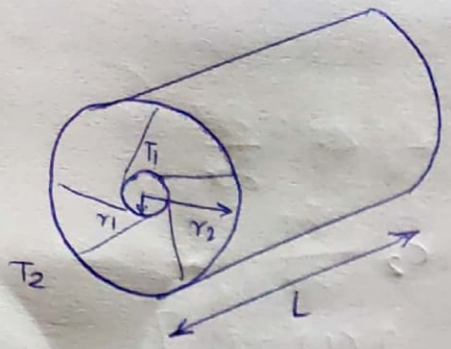


$T = T_1$ @ $r = r_1$

$T = T_2$ @ $r = r_2$

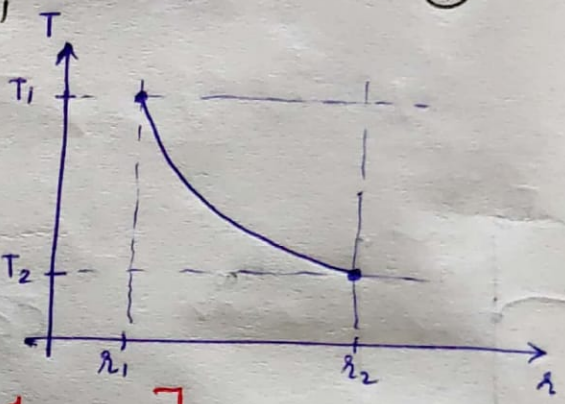
$L >> r_1, r_2$

For steady state 1-D conductio

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$ [Heat cond. eq^n]

$$T(r) = C_1 \ln(r) + C_2$$

$$T(r) = T_1 - \frac{T_1 - T_2}{\ln(r_2/r_1)} \ln\frac{r}{r_1} \quad \text{——— (1)}$$

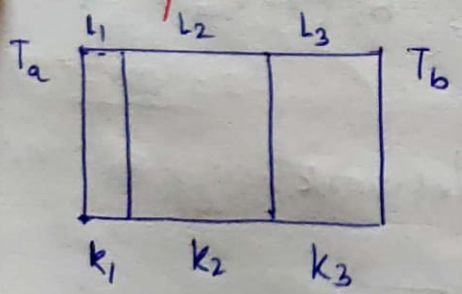


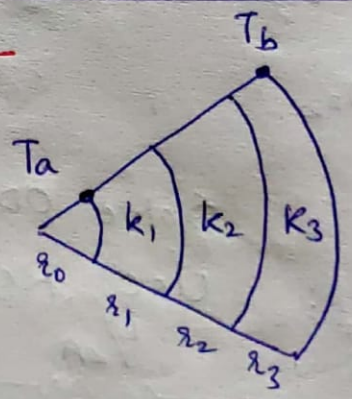
$$q = -kA \left.\frac{dT}{dr}\right|_{r=r_1}$$

$$= \frac{2\pi k L}{\ln(r_2/r_1)} (T_1 - T_2)$$

$$R_{th} = \frac{\Delta T}{q} = \ln\left(\frac{r_2}{r_1}\right) \left[\frac{1}{2\pi k L}\right]$$

Composite wall / pipe :—





$$R_i = \frac{L_i}{k_i A}$$

$$R_i = \frac{1}{2\pi k_i L} \ln\left(\frac{r_i}{r_{i-1}}\right)$$

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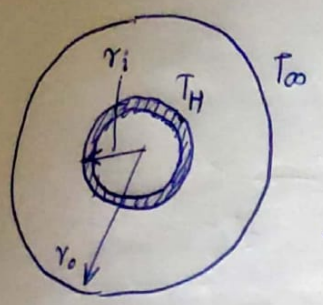
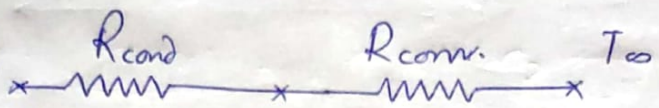
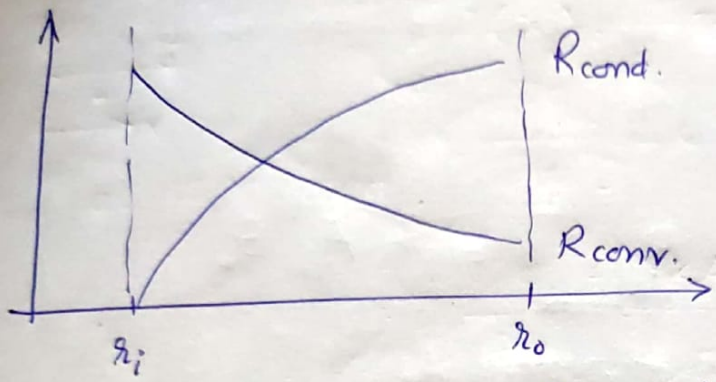
CRITICAL RADIUS OF INSULATION {Hollow Cylinder}



{Neglect ΔT across c/s}



$$\frac{1}{2\pi k_{in} L} \ln\left(\frac{r_o}{r_i}\right) \qquad \frac{1}{h(2\pi r_o L)}$$



It is possible that for given r_o, r_i

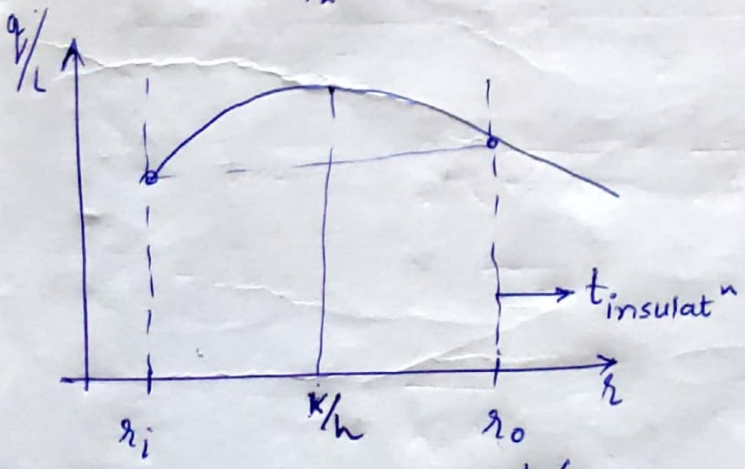
$$\frac{1}{2\pi k_{in} L} \ln\frac{r_o}{r_i} + \frac{1}{2\pi r_o L h} < \frac{1}{2\pi r_i L h}$$

Heat flow per unit lenght:

$$\frac{q}{L} = \frac{T_H - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi k} + \frac{1}{2\pi r_o h}}$$

$$\frac{d(q/L)}{dr_o} = 0 \quad \Big\} \quad \frac{1}{k r_o} - \frac{1}{h r_o^2} = 0 \quad \Big\} \quad r_o = \frac{k}{h}$$

Also, $\left.\frac{d^2(q/L)}{dr^2}\right|_{k/h} < 0$



Scenario 1 → $r_i < k/h$

Scenario 2 : $r_i > k/h$

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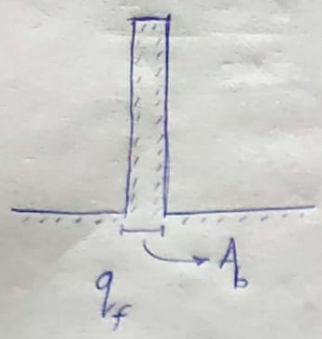
Fin parameter : Effectiveness



$$\epsilon_f = \frac{q_f}{h A_b \theta_b}$$

→ $>1 \Rightarrow$ effective fin

→ $<1 \Rightarrow$ shouldn't include fin

{ $h \downarrow$ $\epsilon_f \uparrow$: Remember? we wanted fin on the air side. }

Efficiency :—

① Ideal situation : $q_{f|max} = h A_{fin} \theta_b$

② Reality : $q_f = \int_0^L h\, \theta(x) P(x) \cdot dx$

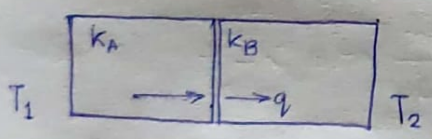
$$\therefore \eta = \frac{q_f}{q_{f|max}} = \frac{q_f}{h A_{fin} \theta_b}$$

③ Array of fins :

$$q_{tot} = h\left[A_s - N A_b + N \eta_f (A_f + A_t) \right] \theta_b$$

[For efficiency, h should be low! Not to care about total heat dissipited.]

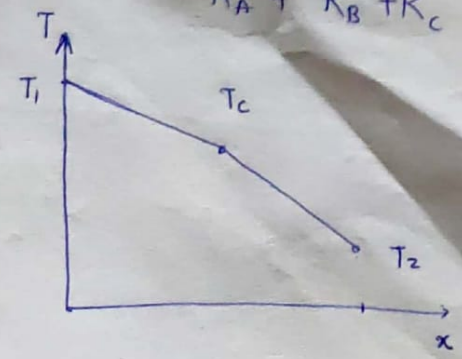
Contact Resistance (R_c) :—



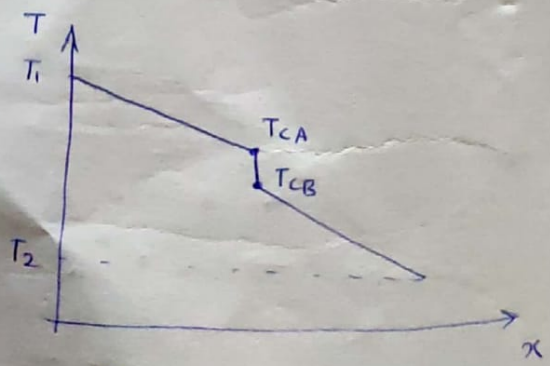
$$R_A = \frac{L_A}{k_A A_A}, \quad R_B = \frac{L_B}{k_B A_B}$$

$$\therefore q = \frac{(T_1 - T_2)}{R_A + R_B + R_c}$$

$$R_c = \frac{T_{CA} - T_{CB}}{q} \quad \left[{}^\circ C / W \right]$$



Ideal situation...



Real situation

- Sometimes, it is given as R_c'' $\left[\frac{{}^\circ C - cm^2}{W} \right]$

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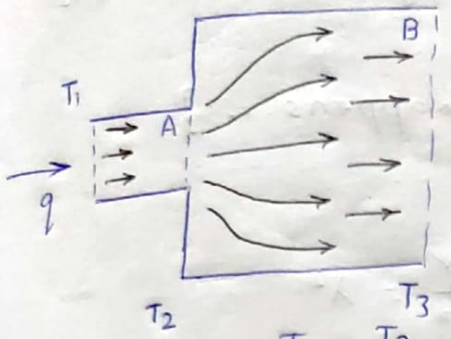
- Increase q across contact resistance, we use thermal paste : TIM : thermal interface material...

Spreading Resistance (R_s) :



$$R_A = \frac{L_A}{k_A A_A}, \quad R_B = \frac{L_B}{k_B A_B}$$

R_s = Spreading Resistance.

$$q = \frac{T_1 - T_3}{R_A + R_s + R_B}$$

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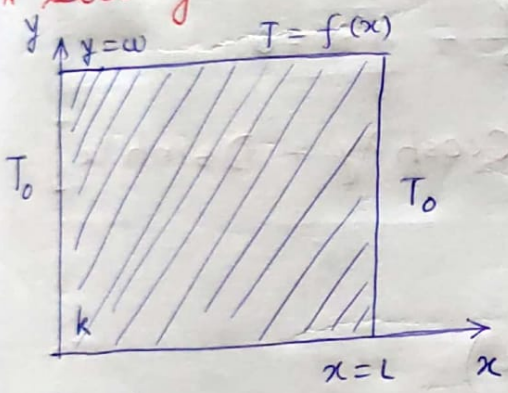
* Steady state conduction : 2D/3D problems :-



- Bar in ∞ long in z direct^n.

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0}$$

⇓ Boundary conditions :-

- B.C's : $T = T_0$ @ $x = 0, L$

  $T = T_0$ @ $y = 0$, & $T = f(x)$ @ $y = \omega$

[Sol^n for P.D.E : for homogenous eq^n...

: Atleast one B.C is homogenous ...]

Let $\theta = T - T_0$: governing P.D.E becomes :

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

B.C : $\theta = 0$ @ $x = 0, L$

$\theta = 0$ @ $y = 0$

$= f(x) - T_0$ @ $y = \omega$

Using seperation of variables :

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$\theta(x,y) = X(x) \cdot Y(y)$

$$\Rightarrow \boxed{\frac{-1}{X}\frac{d^2X}{dx^2} = \frac{1}{Y}\frac{d^2Y}{dy^2} = \pm\lambda^2}$$

[λ^2 is chosen in a way that we get a sol^n in the direction of homogeinity]

[H.W. Try $-\lambda^2$ to see what happens...]

- Taking $+\lambda^2$

$$\frac{-1}{X}\frac{d^2X}{dx^2} = \lambda^2 \quad ; \quad \frac{1}{Y}\frac{d^2Y}{dy^2} = \lambda^2$$

$$\frac{d^2X}{dx^2} = -X\lambda^2 \quad ; \quad \frac{d^2Y}{dy^2} = Y\lambda^2$$

$$\frac{d^2X}{dx^2} + \lambda^2 X = 0 \quad ; \quad \frac{d^2Y}{dy^2} - \lambda^2 Y = 0$$

$$\theta(x,y) = [A\cos\lambda x + B\sin\lambda y][C\cosh\lambda y + D\sinh\lambda x]$$

At, $y=0, \theta=0 \longrightarrow C=0$

$x=0, \theta=0 \longrightarrow A=0$

$$\theta(x,y) = (B\sin\lambda y)(D\sinh\lambda x)$$

At $x=L, \theta=0 \longrightarrow \sin(\lambda L) = 0$

$$\lambda L = n\pi$$

$$\therefore \theta(x,y) = \sum_{n=0}^{\infty} K_n \sin\left(\frac{n\pi}{L}x\right) \cdot \sinh\left(\frac{n\pi}{L}y\right)$$

At, $y=\omega, \theta = f(x) - T_0$

$$\therefore f(x) - T_0 = \sum_{n=0}^{\infty} K_n \sin\left(\frac{n\pi x}{L}\right)\sinh\left(\frac{n\pi\omega}{L}\right)$$

Property of orthogonal equat^n

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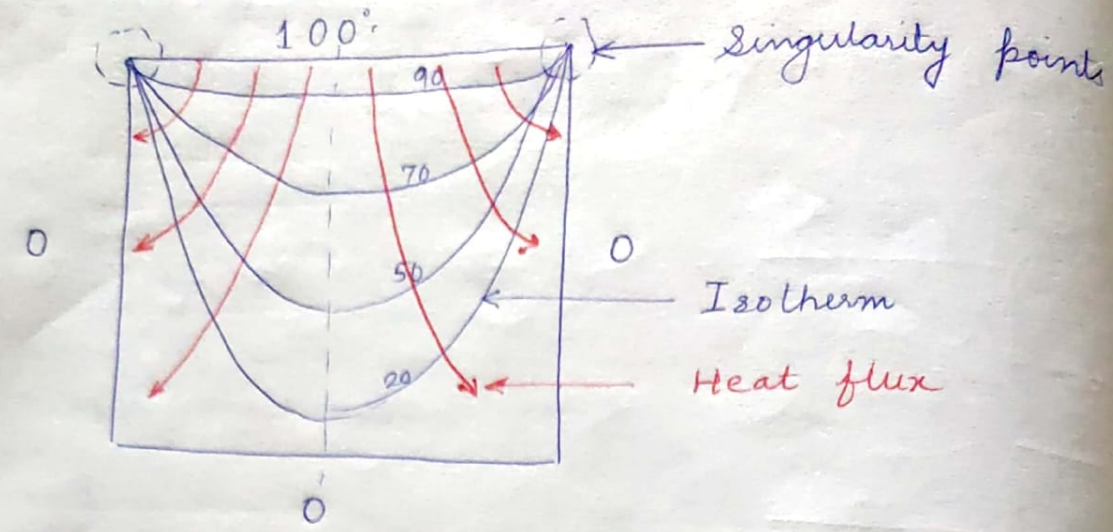
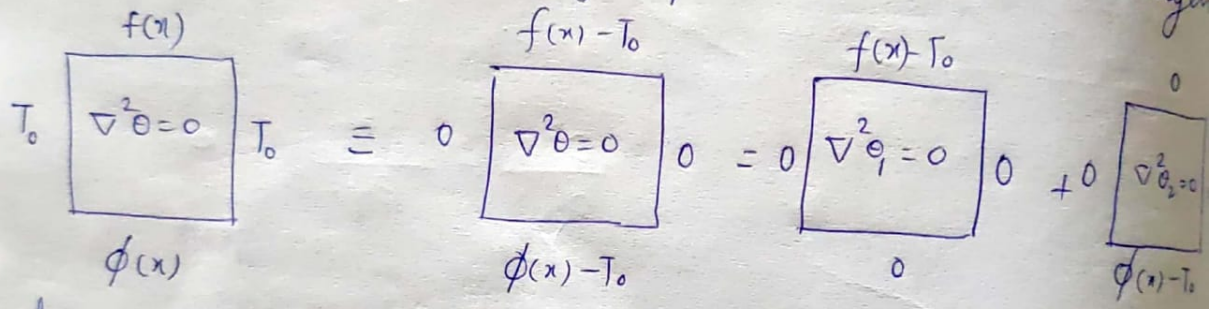
① Consider a case : $f(x) = T_h$



Method of Superposition :-

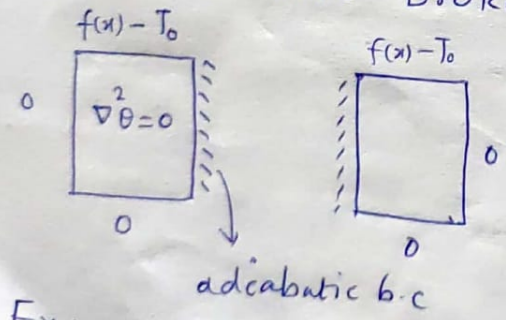
(1) Applied when boundary cond. can't be made homogenous in a direction.

(2) Or, when governing equation is non-homogen

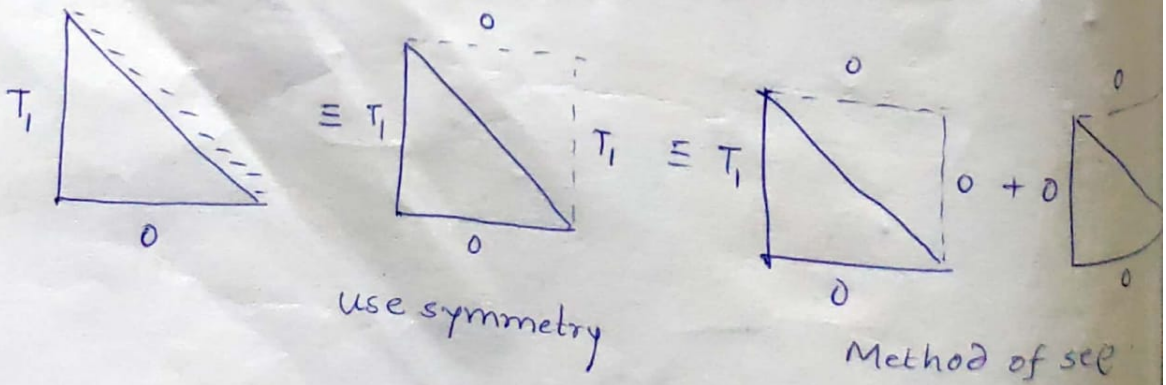


where, $\theta = \theta_1 + \theta_2$

(3) We could have broken the top problem as:



(4) Ex:



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06/08/18

) Evaluation of K_n :

{Orthogonal funct^n: Set of fn $g_1(x), g_2(x) \dots g_m(x) \dots g_n(x)$

$$\int_a^b g_m(x) \cdot g_n(x) = 0 \quad \text{if } m \neq n \quad \}$$

→ Let us consider $g_n(x) = \sin(\lambda_n x)$

$$\therefore \int_0^L \sin(\lambda_m x) \cdot \sin(\lambda_n x) = 0 \quad \text{if } m \neq n$$

$$\lambda_n = \frac{n\pi}{L}$$

$$\theta(x, w) = f(x) - T_0$$

$$\Rightarrow \sum_{n=1}^{\infty} (K_n \sinh \lambda_n w) \sin \lambda_n x = f(x) - T_0$$

$$\Rightarrow \sum_{n=1}^{\infty} C_n \sin \lambda_n x = f(x) - T_0 \quad \{\text{where } C_n = K_n \sinh(\lambda_n w)\}$$

$\therefore$ Using condition of orthogonality.

$$C_n = K_n \sinh(\lambda_n w) = \frac{\int_0^L (f(x) - T_0) \sin(\lambda_n x) \, dx}{\underbrace{\int_0^L \sin^2(\lambda_n x) \, dx}_{L/2}}$$

$$\therefore C_n = \frac{\int_0^L \{ \quad \} \cdot dx}{L/2}$$

$$\therefore K_n = \frac{2}{L \sinh(\lambda_n w)} \int_0^L (f(x) - T_0) \sin(\lambda_n x) \cdot dx$$

$$\boxed{\therefore \theta(x,y) = \frac{2}{L} \sum_{n=1}^{\infty} \left[\frac{\int_0^L (f(x) - T_0) \sin(\lambda_n x) \cdot dx}{\sinh(\lambda_n w)} \right] \times \sin \lambda_n x \times \sinh \lambda_n y}$$

* Exercise: Assume $f(x) = T_c$ $(T_c \neq T_0)$

Get a sol^n

$$\rightarrow \theta(x,y) = \frac{2}{L} \sum_{n=1}^{\infty} \left[\frac{(T_c - T_0) \int_0^L \sin \lambda_n x \cdot dx}{\sinh(\lambda_n w)} \right] \times \sin(\lambda_n x) \times \sinh(\lambda_n y)$$

$$\Rightarrow \frac{T(x,y) - T_0}{T_c - T_0} = 2 \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n\pi} \frac{\sinh \lambda_n y}{\sinh(\lambda_n w)} \cdot \sin(\lambda_n x)$$

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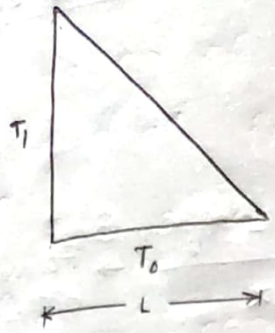
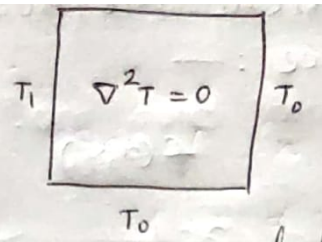
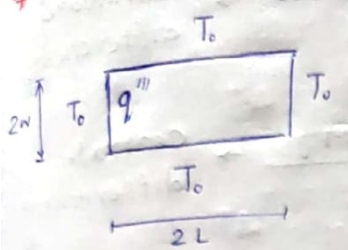
Ex





(Method of superposition)

2-D conduction + heat generation :—



$$\boxed{\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{q'''}{k} = 0}$$

B.C: $\frac{\partial\theta}{\partial x}(0,y) = 0$, $\frac{\partial\theta}{\partial y}(x,0) = 0$

$\theta(L,y) = 0$; $\theta(x,w) = 0$

{ Since PDE is not homogenous, we can't use seperation of variable technique }

Assume, $\theta(x,y) = \psi(x,y) + \phi(x)$

$$\Rightarrow \underbrace{\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right)}_{\text{homogenous P.D.E}} + \underbrace{\left(\frac{\partial^2\phi}{\partial x^2} + \frac{q'''}{k}\right)}_{\text{O.D.E}} = 0 \quad \text{——} \quad (1)$$

A] $\frac{\partial^2\phi}{\partial x^2} + \frac{q'''}{k} = 0$ | $\boxed{BC}$ $\frac{d\phi}{dx}(0) = 0$, $\phi(L) = 0$

B] $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$ | $\boxed{BC}$ $\downarrow$ Use seperation of variables.

ⓐ $\left[\frac{\partial\psi}{\partial x}(0,y) = \frac{\partial\theta}{\partial x}(0,y) - \frac{d\phi}{dx}(0,y) = 0\right]$

ⓑ $\left[\psi(L,y) = \theta(L,y) - \phi(L) = 0\right]$

ⓒ $\left[\frac{\partial\psi}{\partial y}(x,0) = \frac{\partial\theta}{\partial y}(x,0) = 0\right]$

ⓓ $\left[\psi(x,w) = \theta(x,w) - \phi(x) = -\phi(x)\right]$

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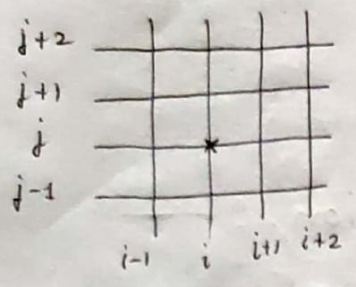
NUMERICAL METHODS ...

08/08/2018

1. Finite Difference :—

$f(x,y)$



$$\left.\frac{\partial f}{\partial x}\right|_{i,j} = \frac{f_{i+1,j} - f_{ij}}{\Delta x} \rightarrow \text{Forward differenc}$$

$$= \frac{f_{ij} - f_{i-1,j}}{\Delta x} \rightarrow \text{Backward differenci}$$

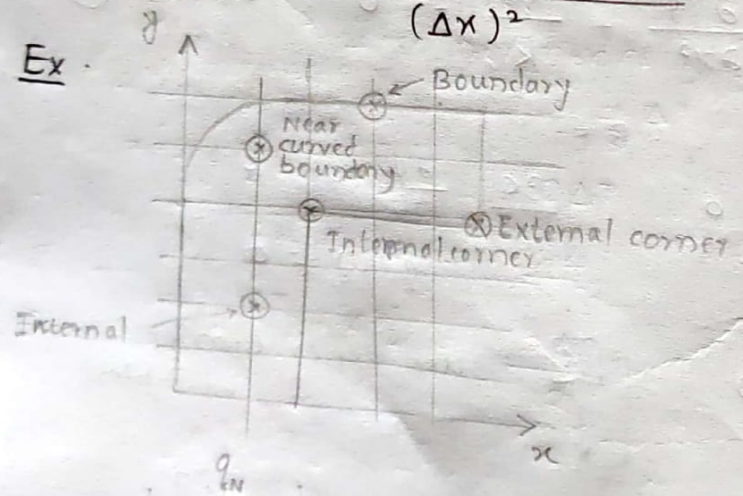
$$= \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} \rightarrow \text{central differenci}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\left.\frac{\partial f}{\partial x}\right|_{i+1/2,j} - \left.\frac{\partial f}{\partial x}\right|_{i-1/2,j}}{\Delta x}$$

$$= \frac{(f_{i+1,j} - f_{i,j}) - (f_{ij} - f_{i-1,j})}{(\Delta x)^2}$$

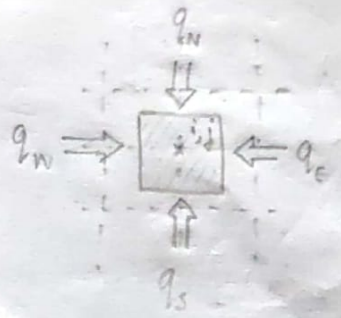
$$= \frac{f_{i+1,j} - 2f_{ij} - f_{i-1,j}}{(\Delta x)^2}$$

Ex.



Nodes

Elements



Internal node...

$$q_N = k \cdot \Delta x \cdot w \cdot \frac{T_{i,j+1} - T_{ij}}{\Delta y}$$

$$q_S = k \cdot \Delta x \cdot w \cdot \frac{T_{i,j-1} - T_{ij}}{\Delta y}$$

$$q_E = k \cdot \Delta y \cdot w \cdot \frac{T_{i+1,j} - T_{ij}}{\Delta x}$$

$$q_W = k \cdot \Delta y \cdot w \cdot \frac{T_{i-1,j} - T_{ij}}{\Delta x}$$

$$q_{gen} = q''' \cdot w \cdot \Delta x \cdot \Delta y$$

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$$\therefore\ q_N + q_S + q_W + q_E + q_{gen} = 0$$

$$\Rightarrow k \cdot w \left[\frac{\Delta x}{\Delta y}\left(T_{i,j+1} - 2T_{i,j} + T_{i,j-1}\right) + \frac{\Delta y}{\Delta x}\left(T_{i+1,j} - 2T_{ij} + T_{i-1,j}\right)\right] + q''' \cdot w \cdot \Delta x \Delta y = 0$$

$$\Rightarrow 2\left(\frac{\Delta x}{\Delta y} + \frac{\Delta y}{\Delta x}\right) T_{ij} \simeq \frac{\Delta x}{\Delta y}\left(T_{i,j-1} + T_{i,j+1}\right) + \frac{\Delta y}{\Delta x}\left(T_{i+1,j} - T_{i-1,j}\right) + \frac{q'''}{k}\Delta x \cdot \Delta y$$

Case 1 : If $\Delta x = \Delta y$ [square grid]

$$4T_{i,j} = T_{i,j-1} + T_{i,j+1} + T_{i+1,j} + T_{i-1,j} + \frac{q'''}{k}(\Delta x)^2$$

$\Downarrow$ Internal Node

$$q_{EW} = k \cdot \frac{\Delta y}{2} \cdot w \cdot \frac{T_{i-1,j} - T_{i,j}}{\Delta x}$$

$$q_S = k \cdot \frac{\Delta x}{2} \cdot w \cdot \frac{T_{i,j-1} - T_{i,j}}{\Delta y}$$

$$q_{conv,N} = h \frac{\Delta x}{2} w \left(T_\infty - T_{i,j}\right)$$

$$q_{conv,E} = h \frac{\Delta y}{2} w \left(T_\infty - T_{i,j}\right)$$

Using energy balance, $\Delta x = \Delta y$, $q_{gen}''' = 0$

$$T_{i,j-1} + T_{i-1,j} + 2h\frac{\Delta x}{k} T_\infty - 2\left(1 + \frac{h\Delta x}{k}\right) T_{ij} = 0$$

$\Downarrow$ External corner

$$2T_{i-1,j} + T_{i,j-1} + T_{i,j+1} + 2\frac{h\Delta x}{k} T_\infty - 2\left(\frac{h\Delta x}{k} + 2\right) T_{ij} = 0$$

$\Downarrow$ surface (boundary)

$$2\left(T_{i-1,j} + T_{i,j+1}\right) + T_{i+1,j} + T_{i,j-1} + 2\frac{h\Delta x}{k} T_\infty - 2\left(3 + \frac{h\Delta x}{k}\right) T_{ij} = 0$$

$\Downarrow$ Internal corner

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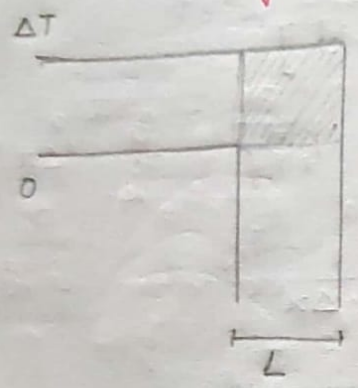
Special conditions:

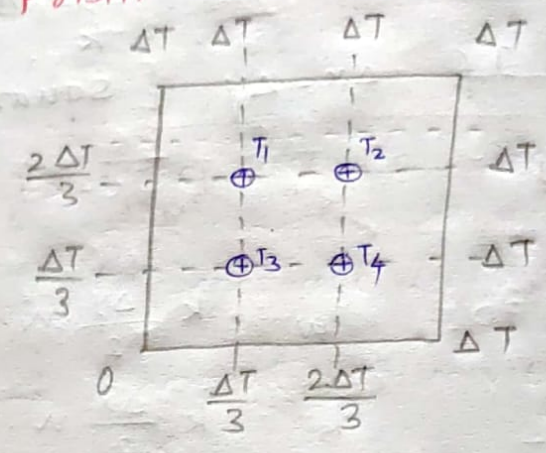
① $\frac{h\Delta x}{k}$ = Grid Biot no.

$= 0 \longrightarrow$ Adiabatic Boundary

$= \infty \longrightarrow T_{i,j} = T_\infty$ (instant cooling)

* Case Study : Edge Prism





16 NODES → 4 internal
→ 8 surface
→ 4 corner nodes

$$\therefore \quad 4T_1 - T_2 - T_3 = \Delta T + \frac{2}{3}\Delta T = \frac{5}{3}\Delta T \quad \text{—①}$$

$$-T_1 + 4T_2 - T_4 = 2\Delta T \quad \text{—②}$$

$$-T_1 + 4T_3 - T_4 = \frac{2\Delta T}{3} \quad \text{—③}$$

$$-T_2 - T_3 + 4T_4 = \frac{5\Delta T}{3} \quad \text{—④}$$

$$\Rightarrow \quad [A][T] = [C]$$

$$\therefore \quad [T] = [A]^{-1}[C]$$

So
$$\begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 2 \\ 2/3 \\ 5/3 \end{bmatrix} \Delta T$$

$$\Rightarrow \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 0.7778 \\ 0.8889 \\ 0.5556 \\ 0.7778 \end{bmatrix} \Delta T \quad \text{——— ANS.}$$

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Gauss - Siedel Method :—

$$T_1 = \frac{1}{4}\left[T_2 + T_3 + \frac{5}{3}\Delta T \right]$$

$$T_2 = \frac{1}{4}\left[T_1 + T_4 + 2\Delta T \right]$$

$$T_3 = \frac{1}{4}\left[T_1 + T_4 + \frac{2}{3}\Delta T \right]$$

$$T_4 = \frac{1}{4}\left[T_2 + T_3 + \frac{5}{3}\Delta T \right]$$

- Use intelligent guess as initial values for $T_{1,2,3,4}$ say ΔT.
- Use these values for next iteration
- Stop when values for successive iteration are very close.

Iteration	$T_1/\Delta T$	$T_2/\Delta T$	$T_3/\Delta T$	$T_4/\Delta T$
0	0.5	0.5	0.5	0.5
1	0.6458	0.8229	0.4167	0.6458
2	0.7448	0.8724	0.4896	0.7448
3	0.7695	0.8848	0.5391	0.7695
4	0.7757	0.8879	0.5541	0.7757
5	0.7773	0.8886	0.5545	0.7773

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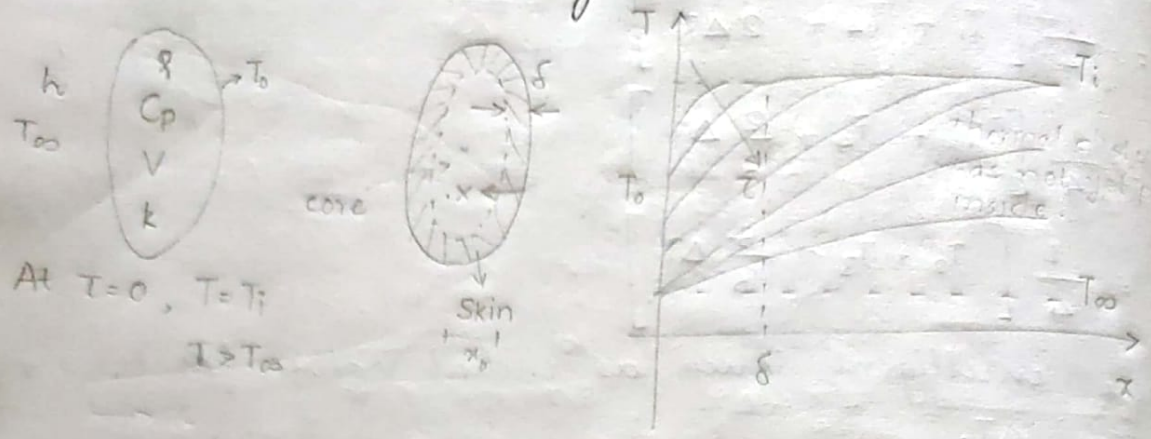
04/08/2018

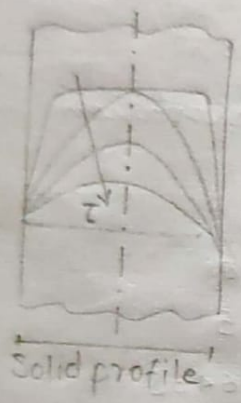
TRANSIENT CONDUCTION ...

Case: Immersion Cooling of a solid.



At $\tau = 0$, $T = T_i$

$T_i > T_\infty$



$\tau_c \rightarrow$ time taken for the thermal disturbance at the surface to penetrate upto the core.

$\alpha \rightarrow$ thermal diffusivity

$$= \frac{k}{\rho C_p} \qquad \tau_c \propto (\alpha)$$

* **Estimate τ_c (using scaling analysis) :—**

- 1-D Transient Conduction –

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

$$\therefore \frac{\partial^2 T}{\partial x^2} = \frac{\left.\frac{\partial T}{\partial x}\right|_{x=\delta} - \left.\frac{\partial T}{\partial x}\right|_{x=0}}{\delta}$$

$$= \frac{0 + \frac{T_0 - T_i}{\delta}}{\delta} \Rightarrow \boxed{\frac{\partial^2 T}{\partial x^2} \sim \frac{T_0 - T_i}{\delta^2}}$$

$$\frac{\partial T}{\partial \tau} \approx \frac{T_i - T_0}{0 - \tau}$$

The temp. within skin reduces from T_i to a value "comparable to" T_0 in time τ.

$\therefore$ We can write,

$$\frac{T_0 - T_i}{\delta^2} \sim \frac{1}{\alpha}\frac{T_i - T_0}{\tau}$$

$$\therefore \delta \propto \sqrt{\alpha \tau}$$

$\frac{x^2}{\alpha t} \rightarrow$ Fourier number

At $\tau = \tau_c$, $x = x_0 \longrightarrow \tau_c = \frac{x_0^2}{\alpha}$

$[\alpha \rightarrow \frac{m^2}{s}]$

remember!!

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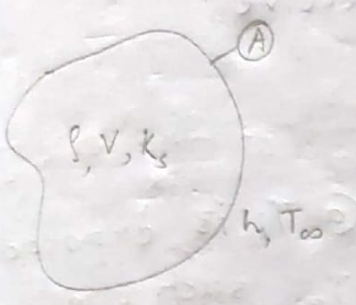
LUMPED SYSTEM	DISTRIBUTED SYSTEM
* $T = F(\tau)$	$T = F(x, y, z, \tau)$

1.

* Biot no. $\rightarrow (Bi) \rightarrow \dfrac{hL}{k_{solid}}$

$$= \frac{L/k_s A}{1/hA} = \frac{R_{cond.}}{R_{conv.}}$$

16/8/18



Body initially at T_i is immersed in a hot fluid at T_∞.

Assume : $\left\{ B_i = \dfrac{hL}{k_s} < 0.1 \right\}$ lumped sys.

Using energy balance,

$$hA(T_\infty - T_i) = \rho V c_p \frac{dT}{d\tau}$$

$T = T_i$ @ $\tau = 0$

$$hA(T_\infty - T_i)\, d\tau = \rho V c_p\, dT$$

$$\therefore hA\, d\tau = \rho V c_p \frac{dT}{T_\infty - T_i}$$

$$\therefore \frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{ \frac{-hA}{\rho V c_p} \tau \right\} = e^{-\tau/\phi}$$

* $\phi = \dfrac{\rho V c_p}{hA} \longrightarrow$ time constant

[Larger the time constant, slower is the response to temp. rise / fall]

[Time constant (ϕ), the temp. rise is 36.4% of the theoritical max. value]

* $B_i = \dfrac{hL}{k_s} \Rightarrow$ 'L' → charachteristic lenght scale.

'r' → sphere / cyl. radius.

't' → thickness for plane wall.

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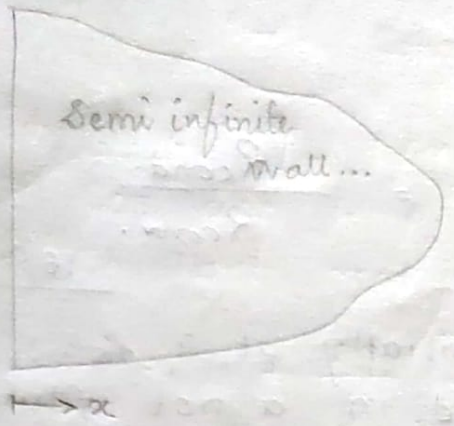
* DISTRIBUTED SYSTEMS :—

1. Semi- Infinite WALL :



B.C :

① Uniform temp.

② Uniform q''

③ Convective

- **Case 1** : Semi infinite wall @ T_i is exposed to an ambient of T_∞ such that $T(0,\tau) = T_\infty$

Equation :

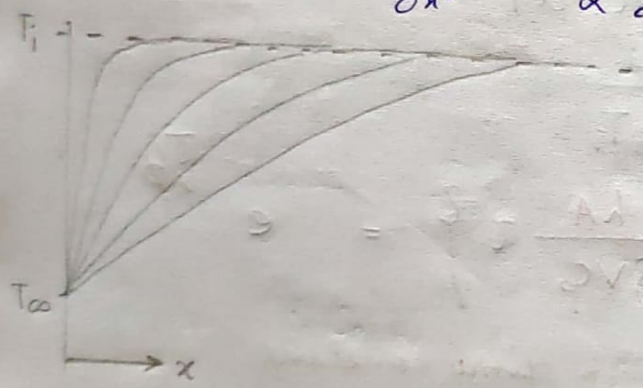
$$k \frac{\partial^2 T}{\partial x^2} = \rho C_p \frac{\partial T}{\partial \tau}$$

or

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

I.C - $T(x,0) = T_i$

B.C : $T(0,\tau) = T_\infty$



* Using our knowledge, $\delta \sim \sqrt{\alpha \tau}$
  - One can sketch a family of curve for $T(x,\tau)$
  - Curves become shallower as the thermal disturbance @ $x=0$ diffuses through the wall

* Define $\eta = \dfrac{x}{\sqrt{\alpha \tau}}$

$$\frac{dT}{dx} = \frac{dT}{d\eta} \times \frac{d\eta}{dx} = \frac{1}{\sqrt{\alpha \tau}} \times \frac{dT}{d\eta}$$

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$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{1}{\alpha \tau}\frac{d^2 T}{d\eta^2}$$

$$\frac{\partial T}{\partial \tau} = \frac{\partial T}{\partial \eta}\cdot\frac{\partial \eta}{\partial \tau} = \frac{x\sqrt{\tau}}{2\sqrt{\alpha}}\frac{dT}{d\eta}$$

$$= \frac{-x}{2\sqrt{\alpha}\,\tau^{3/2}}\cdot\frac{dT}{d\eta}$$

$$= \frac{\eta}{-2\tau}\cdot\frac{dT}{d\eta}$$

$\therefore$ PDE : $\frac{1}{\alpha\tau}\cdot\frac{d^2T}{d\eta^2} = \frac{-\eta}{2\alpha\tau}\frac{dT}{d\eta}$

$$\therefore \boxed{\frac{d^2T}{d\eta^2} + \frac{\eta}{2}\frac{dT}{d\eta} = 0}$$

BC: $\eta = 0$, $T = T_\infty$

$\eta \to \infty$, $T = T_i$

* Sol^n :

$$\boxed{\frac{T(x,\tau) - T_\infty}{T_i - T_\infty} = \underbrace{\mathrm{erf}}_{\text{error function}}\left(\frac{x}{\sqrt{4\alpha\tau}}\right)}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$$

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$q''(\tau) = -k_s \left.\frac{\partial T}{\partial x}\right|_{x=0}$

$\rightarrow \quad q''(\tau) = -k_s \dfrac{T_i - T_\infty}{\sqrt{\pi \alpha \tau}}$

- Objective : $T = T(x, \tau)$
  - (a) Constant Temp.
  - (b) Constant heat flux.

$$T(x,\tau) = T_i + \frac{2q''}{k}\sqrt{\frac{\alpha\tau}{\pi}}\exp\left\{\frac{-x^2}{4\alpha\tau}\right\} - \frac{q''x}{k}\operatorname{erfc}\left(\frac{x}{4\alpha\tau}\right)$$

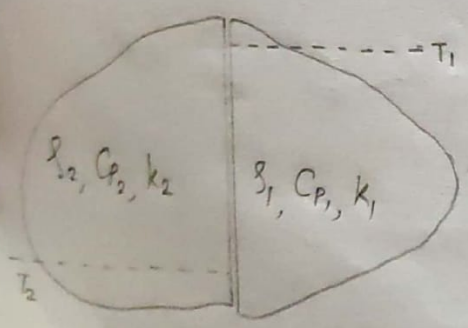
(c) Convective boundary condition (h, T_∞)

$$\frac{T(x,\tau)-T_\infty}{T_i - T_\infty} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha\tau}}\right) + \exp\left\{\frac{hx}{k} + \frac{h^2\alpha\tau}{k^2}\right\}\operatorname{erfc}\left[\frac{x}{\sqrt{4\alpha\tau}} + \frac{h}{k}\sqrt{\alpha\tau}\right]$$

[H.W : For (c) plot :

① $\dfrac{T - T_\infty}{T_i - T_\infty}$ v/s $\dfrac{x}{\sqrt{4\alpha\tau}}$ (family of curves)

② $\dfrac{T(0,\tau) - T_\infty}{T_i - T_\infty}$ vs $\ln\left(\dfrac{h}{k}\sqrt{\alpha\tau}\right)$] (single curve)



* Find the contact temperature

$T_1 > T_c > T_2$ (intuition)

Find $T(x,\tau)$ for both and equate q'' at $x = 0$

$$\Rightarrow k_1 \frac{T_1 - T_c}{\sqrt{\pi\alpha_1\tau}} = k_2 \frac{T_c - T_2}{\sqrt{\pi\alpha_2\tau}}$$

? Contact temperature is independent of time.

$$\frac{k_1}{\sqrt{\pi\alpha_1\tau}}T_1 + \frac{k_2}{\sqrt{\pi\alpha_2\tau}}T_2 = \left[\frac{k_1}{\sqrt{\pi\alpha_1\tau}} + \frac{k_2}{\sqrt{\pi\alpha_2\tau}}\right]T_c$$

$$T_c = \frac{k_1\sqrt{\pi\alpha_2\tau}\,(T_1) + k_2\sqrt{\pi\alpha_1\tau}\,(T_2)}{k_1\sqrt{\pi\alpha_2\tau} + k_2\sqrt{\pi\alpha_1\tau}}$$

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Q.1 consider 1.6 cm thick plate initially at 600°c plunged in water bath @ 15°c.

$h = 10^4 \, W/m^2\text{-}K$

$k = 40 \, W/m\text{-}k$

$\alpha = 0.1 \, cm^2/s$

Determine :

(a) $\tau_1 = ?$ when $T_c = 1000°c$

(b) correspond. T at 0.2 cm away from surf.

(c) Q/Q_{max} at $\tau = \tau_1$

$\rightarrow Bi = \frac{hL}{k} = 2$

(a) When $T_c = 100°c$: $\frac{T_c - T_\infty}{T_i - T_\infty} = 0.145$

Use heisler chart : $1/B_i = 0.5$

$\downarrow$

$F_0 = \frac{\alpha \tau}{L^2} = 1.8$

$\therefore \tau_1 = 11.5 sec$

(b) $\frac{x}{L} = \frac{0.8 - 0.2}{0.8} = 0.75$ [Temp. dist. Heisler chart]

$\left.\frac{T - T_\infty}{T_c - T_\infty}\right|_{\substack{B_i = 2 \\ F_0 = 1.8}} = 0.7$

$\Rightarrow T = 74.5°c$

(c) $Bi = 2 \rightarrow \frac{\alpha \tau_1 h^2}{k^2} = 7.2 = B_i^2 F_0$.

$Q/Q_{max} = 0.85$

Q.2. An isothermal icicle @ $T_i = 0°c$ is placed inside a freezer @ $-5°c$. $h = 10 \, W/m^2\text{-}K$. Icicle can be modelled as cyl. (dia = 2cm). Calculate

(a) T_c @ $\tau = 10 \, \cancel{sec}$ min. [$k_{ice} = 2.25 \, W/m\text{-}K$]

[$\alpha = 0.012 \, cm^2/s$]

$\rightarrow Bi = \frac{hR_0}{k} = 0.044$ (Use lumped sys.)

$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{\frac{-hA}{\rho V C_p}\tau\right\}$ $A \rightarrow 2\pi r l$, $V \rightarrow \pi r^2 l$

$\alpha = \frac{k}{\rho C_p}$

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Q.3. The shape in which a potato is cut is responsible of how fast each peice cooks. Study this effect by considering 3 diff. sh each 5 gm : 1) A sphere

2) Cylinder ($l = 6cm$)

3) Thin disk (dia = 4 cm)

$T_i = 30°c$ @ $\tau = 0$ placed in $T = 100°c$.

$h = 2 \times 10^4 \ W/m^2\text{-}K$

$k = 0.6 \ W/m\text{-}K$

$\rho = 0.9 \ gm/cc$; $\alpha = 0.002 \ cm^2/s$

Calculate τ when avg. temp = 65°c

→ Thin disk : $\dfrac{T_{avg} - T_\infty}{T_i - T_\infty} = \dfrac{Q}{Q_{max}}$

Now, use grober chart to find $\dfrac{h^2 \alpha \tau}{k^2}$

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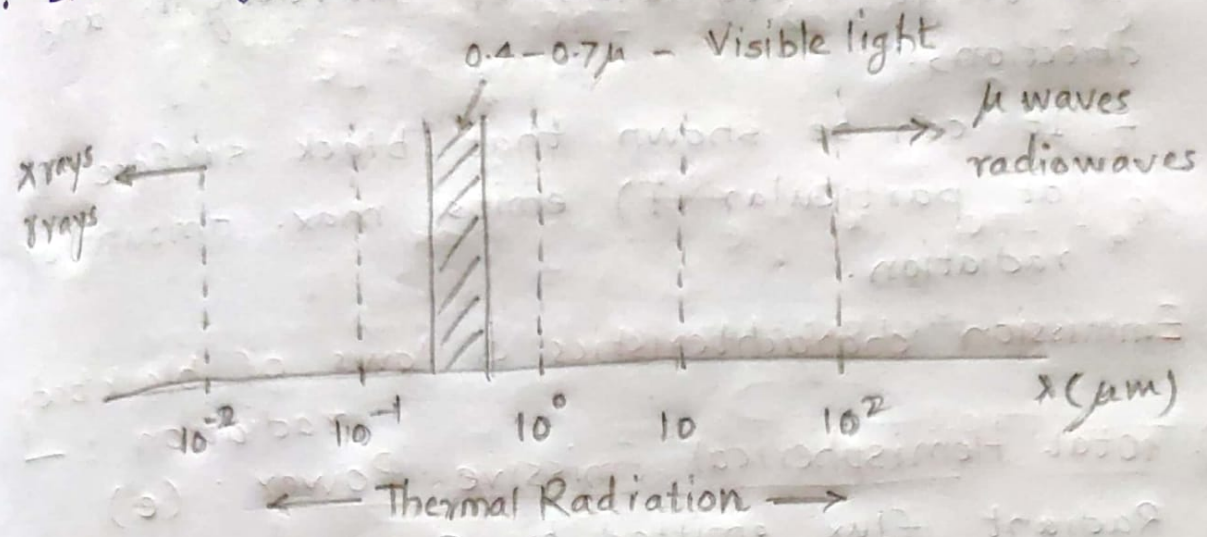
THERMAL RADIATION

Monday
Aug. 27. 2018

- Electromagnetic Radiation - waves @ certain λ.

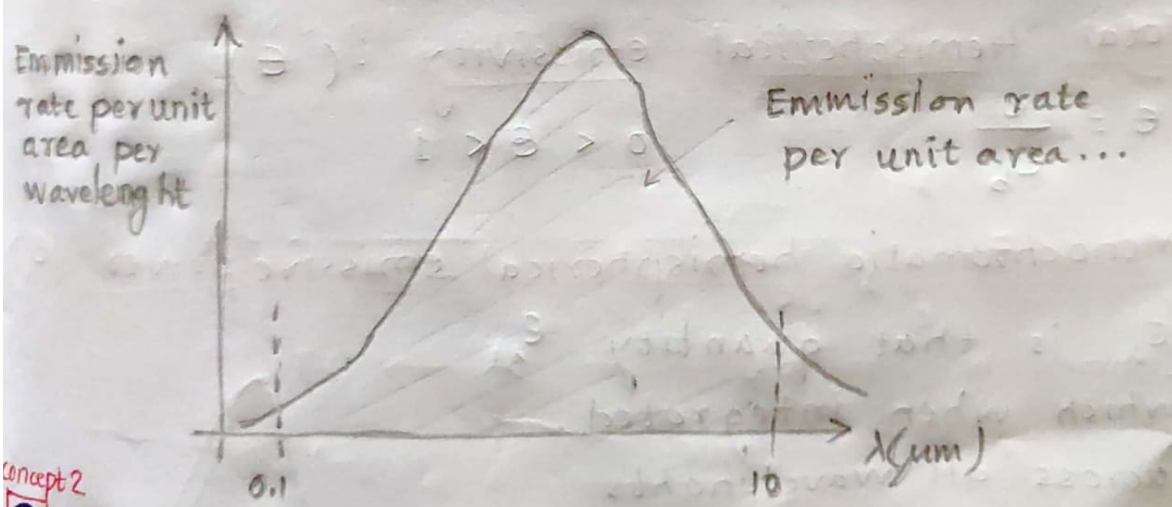


concept 1.

① Any surface @ $T > 0K$ emits radiation : $q \propto T^4$

② 0.1 - 10μ is the range where thermal radiation is most interesting.

③ Visible light falls within thermal radiation spectrum.



concept 2

④ A surface can also absorb radiation emmitted by other surfaces and falling on it.

concept 3.

⑤ A surface emits radiation in all directions encompassed by a hemisphere around it.

concept 4 - Radiation is direction dependent.

⑥ Gases also emit radiation.

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BLACK SURFACE :

→ Absorbs all the thermal radiations that fall on it, regardless of wavelenghts and directions.

- It can be shown that black surface (at particular T) emits max. amount of radiation.

Emmission charachteristics & laws of black body radiation :—

① Total Hemispherical Emissive Power : (e)

- Radiant flux emitted by a surface at a particular T in all directions across all wavelenghts.

$e \rightarrow [W/m^2]$

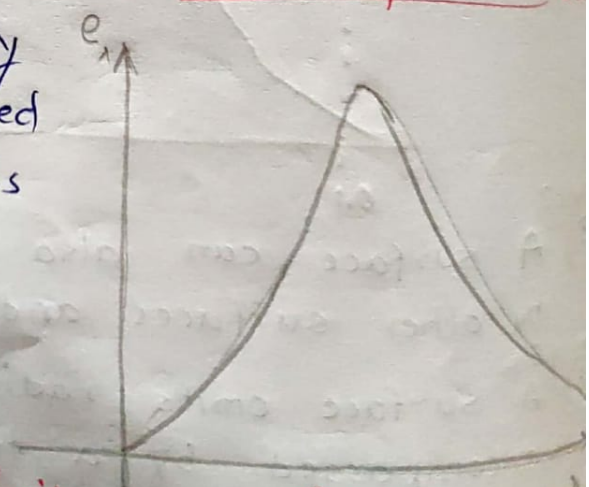
$e_b \rightarrow$ for a black surface.

② Total Hemispherical emissivity : (ϵ)

$$\epsilon = \frac{e}{e_b} \qquad 0 < \epsilon < 1$$

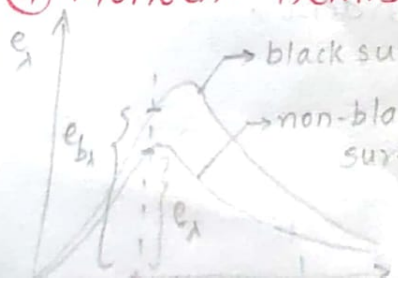
③ Monochromatic hemispherical emissive power : (e_λ)

e_λ is that quantity which when integrated across all wavelenghts gives e.



$$\boxed{\int_0^\infty e_\lambda \cdot d_\lambda = e}$$

④ Monoch. hemisph. emissivity :— (ϵ_λ)



$$\epsilon_\lambda = \frac{e_\lambda}{e_{b\lambda}}$$

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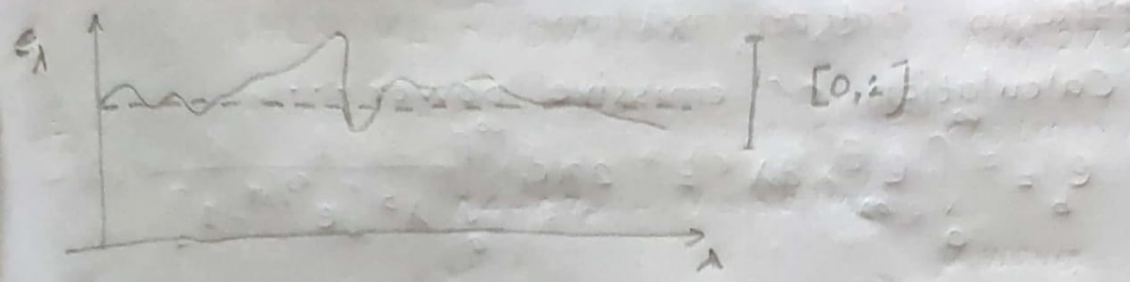


⑤ Gray Surface :

A surface whose emissivity is constant for particular wavelenght.

[sometimes, monochromatic is replaced by spectral]

Planck / Stefan-Boltzmann / Wein displ.

Wednesday
Aug. 28. 2018

Laws of Blackbody Radiation :-

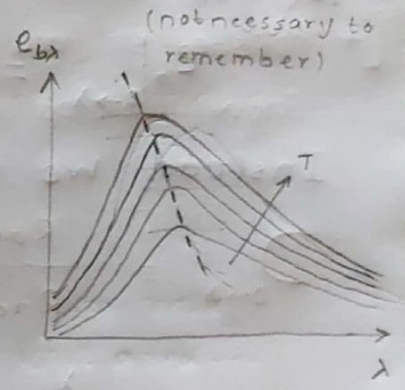
(1) Planck's Law:

- Helps calculate $e_{b\lambda}$ for a given T & λ.

$$e_{b\lambda} = \frac{2\pi C_1}{\lambda^5 \left[e^{C_2/\lambda T} - 1\right]}$$

where $C_1 = 0.596 \times 10^{-16}$ W-m²

$C_2 = 0.014387 \times 10^{0}$ m-K



For a non-black surface:

$$\boxed{e_\lambda = \varepsilon_\lambda e_{b\lambda}}$$

* Plot observation :-

① Shapes are similar for different T.

② Curve shifts upwards if T↑

③ Curve shifts towards the left if T↑

* Locate the peak :-

$$\frac{d(e_{b\lambda})}{d\lambda} = 0 \qquad \text{put } y = \frac{C_2}{\lambda T}$$

$\vdots$

$\Rightarrow e^y(5-y) = 5$

$\Rightarrow C_2/\lambda_m T = 4.965$

$\Rightarrow \boxed{\lambda_m T = 0.0029 \text{ m-K}} \rightarrow$ Wien's Law — (2)

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(3) Stefan- Boltzmann Law:

Calculation of emissive power ...

$$e_b = \int_0^\infty e_{b\lambda} \cdot d\lambda = 2\pi C_1 \int_0^\infty \frac{1}{\lambda^5 \left[e^{C_2/\lambda T} - 1\right]} d\lambda$$

$$e_b = 2\pi C_1 \int_0^\infty x^3 \left\{ e^{\frac{C_2 x}{T}} - 1 \right\}^{-1} \cdot dx \qquad \text{where } x = 1/\lambda$$

Expand the exponential term and integrate termwise

$$\therefore e_b = 2\pi C_1 \frac{6T^4}{C_2^4} \left[\underbrace{1 + \frac{1}{2^4} + \frac{1}{3^4} \cdots}_{\pi^4/90} \right]$$

$$\therefore \boxed{e_b = \sigma T^4}$$

where $\sigma = 5.67 \times 10^{-8}$ W/m²-K⁴

* To get emissive power over a range of λ:

$$e_b\Big|_{0\to\lambda} = \int_0^\lambda e_{b\lambda} \cdot d\lambda$$

Instead we define:

$$D_{0\to\lambda} = \frac{\text{Radiant flux over } (0 \to \lambda)}{\text{Radiant flux over } (0 \to \infty)}$$

$$= \int_0^\lambda \frac{2\pi C_1}{\lambda^5 \left[e^{C_2/\lambda T} - 1\right]} \times \frac{1}{\sigma T^4} \times d\lambda$$

Change a variable from $\lambda \longrightarrow \lambda T$

$$= \int_0^{\lambda T} \frac{2\pi C_1}{(\lambda T)^5 \left[e^{C_2/\lambda T} - 1\right] \sigma} \times d(\lambda T)$$

λT (μm-K)	$D_{0\to\lambda}$
600	0.00000
800	0.0002
1000	0.0032
⋮	⋮
5000	0.99895

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Q.1. Calculate max. value of monochromatic emissive power in the radiation emitted from the surface of the sun. $T_{sun} = 5779K$

① Assume sun surf. is black. Find $e_\lambda|_{max} = ?$

→ • Wiens law to get λ_m

• Substitute in plan

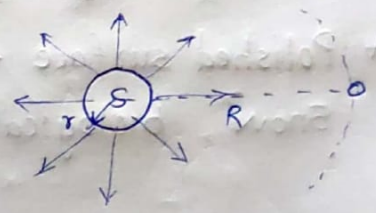
Ans: 8.303×10^7 W/m²-μm

② Calculate emissive power of sun's surface.

→ $e_b = \sigma T^4$

③ What's the value of radiation when it reaches earth surface? [Mean distance $= 1.496 \times 10^8$ km]
[$R_{sun} = 6.95 \times 10^5$ km]

→ $$e_{b\lambda}|_{earth} = e_{b\lambda}|_{sun} \left(\frac{r_{sun}}{R}\right)^2$$



$$= 1792 \text{ W/m}^2\text{-}\mu m$$

$$e_b|_{earth} = e_b|_{sun} \left(\frac{r_{sun}}{r_{earth-sun}}\right)^2 = \underbrace{1365 \text{ W/m}^2}_{\text{SOLAR CONSTANT...}}$$

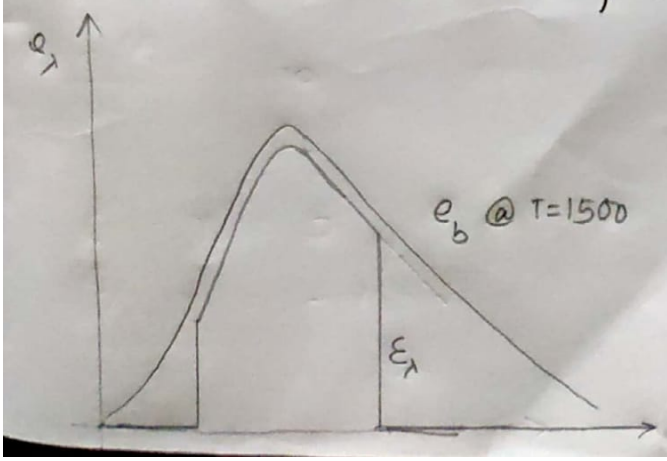
Q.2. Monochromatic emissivity (e_λ) of surface varies over wavelenght as:

$\epsilon_\lambda = 0 \qquad \lambda < 0.3\mu m$

$= 0.9 \qquad 0.3 \le \lambda \le 1\mu m$

$= 0 \qquad \lambda > 1\mu m$

Calculate emissive power @ 1500 K



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$$\rightarrow \quad e_\lambda = \int_0^\infty \epsilon_\lambda \, e_{b\lambda} \, d\lambda$$

$$= \int_0^{0.3} \cancel{\epsilon_\lambda}^{\,0} e_{b\lambda} \, d\lambda + \int_{0.3}^{1} \epsilon_\lambda \cdot e_{b\lambda} \, d\lambda + \int_1^\infty \cancel{\epsilon_\lambda}^{\,0} \cdot e_{b\lambda} \, d\lambda$$

$$= 0.9 \int_{0.3}^{1} e_{b\lambda} \, d\lambda$$

$$= 0.9 \left[\int_0^1 e_{b\lambda} \, d\lambda - \int_0^{0.3} e_{b\lambda} \, d\lambda \right]$$

$$= 0.9 \left[D_{0\to1} - D_{0\to0.3} \right] \sigma T^4$$

$$e_\lambda = 3553 \quad W/m^2$$

$$\epsilon = \frac{e}{e_b} = 0.12 \qquad \text{(gray surface approx)}$$

[* Polished surface → Low emissivity
Snow, biological bodies → high emissivity]

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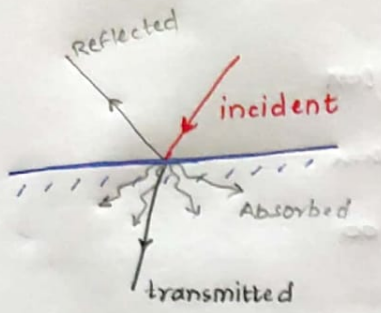
IRRADIATION :— {Radiation incident on a surface}



1) Total Hemispherical Irradiation :— (H)

- Radiant Heat flux incident on the surface [W/m^2]

2) Total Hemispherical Absorptivity :— (α)

- Fraction of (H) absorbed by the surface.

$$\alpha = \frac{H_a}{H} \quad \Big| \quad \alpha_{black} = 1.$$

3) Monochr. Hemisph. Irradiation :- (H_λ) [$W/m^2\text{-}\mu m$]

4) Monochr. Absorptivity :— (α_λ)

Similar to Absorptivity (α), we define :

- Reflectivity (ρ)
- Transmittivity (τ)

It is obvious :

$$\boxed{\begin{aligned} \alpha + \rho + \tau &= 1 \\ \alpha_\lambda + \rho_\lambda + \tau_\lambda &= 1 \end{aligned}}$$

* Properties :

	Total	Monochromatic
1) Emissivity	ε	ε_λ
2) Absorptivity	α	α_λ
3) Reflectivity	ρ	ρ_λ
4)		

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Q. Variation of monochr. irradiation: incident on an opaque surf. ($\tau = 0$):

$H_\lambda = 0$ @ $0 \le \lambda \le 2\,\mu m$

$= 750$ @ $2 < \lambda < 8\,\mu m$

$= 0$ @ $8 < \lambda < \infty$

$\alpha_\lambda = 1$ @ $0 \le \lambda \le 4\,\mu m$

$= 0.5$ @ $4 \le \lambda < 8\,\mu m$

$= 0$ @ $\lambda \ge 8\,\mu m$

Calculate H_a, α, ρ

$\rightarrow H = \int_0^\infty H_\lambda \, d\lambda \longrightarrow 4500\ W/m^2$

$H_a = \int_0^\infty \alpha_\lambda H_\lambda \cdot d\lambda \longrightarrow 3000\ W/m^2$

$\alpha = H_a/H = ? \quad 2/3$

$\rho = 1 - \alpha = ? \quad 1/3$

$H = \int_{02}^{08} 750 \cdot d\lambda = 750 \times 6 = 4500\ W/m^2$

$H_a =$

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Convection

Convection – heat transfer in presence of bulk movement of fluid

to analyze convection, we need to understand fluid flow.
– velocity, Pressure info.

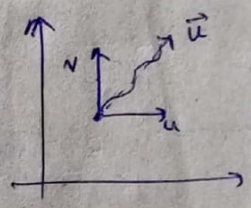
Governing Equation :

- Continuity Equation – Conservation of Mass
- Conservation of Momentum – essentially Newton's 2nd law in an open system.
- Stokes ~~Equation~~ Relation – relates stress to strain (in a Newtonian Fluid)

Combining Newton's Second law and Stokes Relation, we get Navier-Stokes Equation.

i) Continuity Equation – $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\nabla \cdot \vec{u} = 0$$



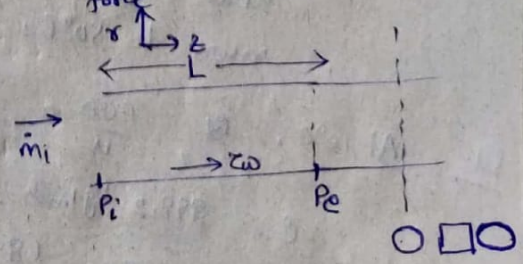
ii) Momentum Equ. (N-S Eqn)

$$x: \quad \rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + F_x$$

$$y: \quad \rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + F_y$$

$$\underbrace{\cancel{\rho\frac{d\vec{u}}{dt}}^{\,0 \text{ for steady state}} + \rho u \cdot \nabla u}_{\text{Net out flow of momentum}} = \underbrace{-\nabla p}_{\text{Force due to pressure variation.}} + \underbrace{\mu \nabla^2 u}_{\text{viscous force}} + \underbrace{F}_{\text{Body Force.}}$$

Internal flow



- Consider flow through a duct. $P_i > P_e$ (due to viscous forces)

$$\Delta P = P_i - P_e$$
$$= f(\text{geometry, velocity, fluid properties, } L)$$

$\tau_w \rightarrow$ shear stress at the wall.

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$\tau_w \rightarrow \mu \frac{\partial u}{\partial r}$

ΔP related to τ_w?

Friction factor $(f) = \frac{\tau_w}{(\frac{1}{2}\rho v^2)}$

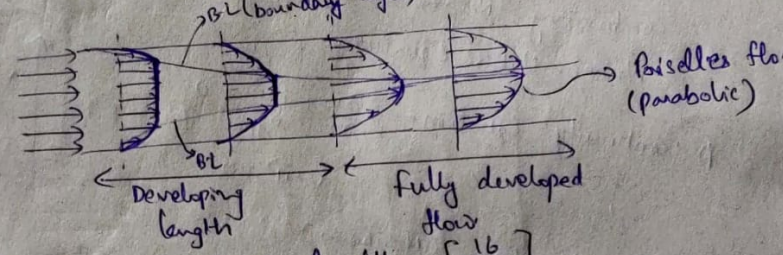
For a pipe, $f = \frac{-4\left(\frac{dP}{dz}\right) A_c}{P\left(\frac{\rho v^2}{2}\right)}$

$P \rightarrow$ perimeter
$A_c \rightarrow$ cross-section of flow

- f is dimensionless
- for laminar, fully developed flow, f is a function of Re only.
- for turbulent, fully developed flow, $f = f(Re, \varepsilon/D)$ roughness parameter

laminar → Momentum doesn't Mix in adjacent layer

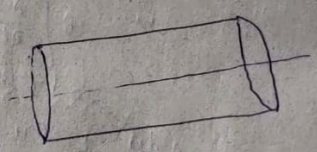
fully developed → τ_w, $\frac{dP}{dz}$ remain constant, (B.L from adjacent surface meet)



For laminar, F.D flow, $f = \frac{64}{Re}$ $\left[\frac{16}{Re}\right]$

Darcy wiesbach — Fanning

Q. Water @ 10°C, $L = 3m$, $v = 0.2 m/s$, $D = 1cm$, Calculate ΔP.



What happens to ΔP, when Temp rises to 80°C.

(i. critical Re = 2000 laminar to turbulent)

Sol. At 10°C, $Re = \frac{\rho v D}{\mu}$

$\rho = 999.2 \, kg/m^3$, $\mu = 1.31 \, kg/m\text{-}s$

$Re = \frac{(999.2)(0.2)(10^{-2})}{1.31} = 1.52$

As $Re < 2300$, flow is laminar

At 80°c,

$\rho = 971\ kgm^{-3}$, $\mu = 3.47\ kg/ms$

$$Re = \frac{(971)(3.47)(10^{-2})}{3.47} = 9.71$$

both cases are in laminar.

At 10°c, $f_1 = \frac{64}{Re} = \frac{64}{1.52}$

using $f = \frac{-4\left(\frac{dp}{dz}\right) A_c}{P\left(\frac{\rho v^2}{2}\right)} = \frac{64}{1.52}$

$$\frac{2(-\Delta P)\pi D^2}{\pi D L(\rho v^2)} = \frac{64}{1.52}$$

$$(-\Delta P) = \frac{64}{2 \times 1.52} \times \frac{\rho v^2 L}{D}$$

$$(-\Delta P) = \frac{32}{1.52} \times \frac{(999.2)(0.2)^2(3)}{10^{-2}} = 2.52 \times 10^5\ Pa$$

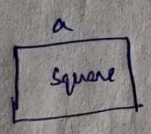
At 80°c, $(-\Delta P) = \frac{32}{9.71} \times \frac{(999.2)(0.2)^2(3)}{10^{-2}} = 0.39 \times 10^5\ Pa$

$\nu = \mu/\rho$.

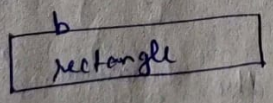
for Circular Cross-section, $Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu}$

for Non-Circular, $D_h = \frac{4A}{P}$ (hydraulic Diameter)

for



$D_h = \frac{4 \cdot a^2}{4a} = a$



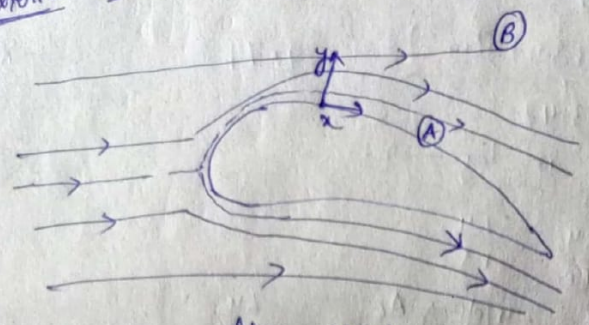
$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$.

$D_h = \frac{4 \cdot \pi(r_o^2 - r_i^2)}{2\pi(r_o + r_i)} = 2(r_o - r_i)$

for rectangle, if $b >> a$, $D_h = \frac{2a}{\left(\frac{a}{b} + 1\right)} = 2a$

(Eg. Microchannels)

External Flow :-

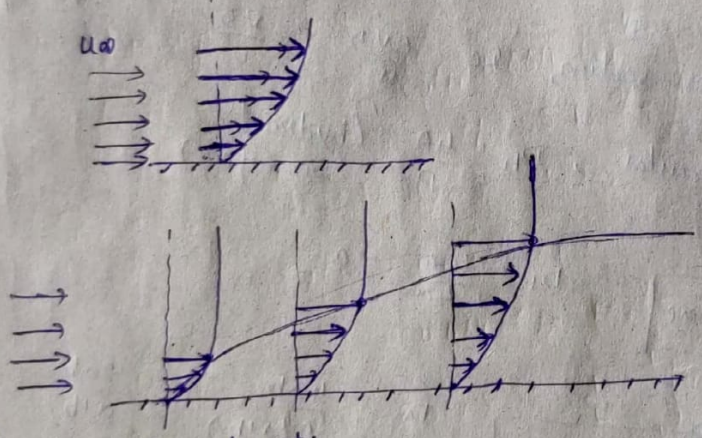


(A) → viscous flow region
presence of bluff body is felt.
No-slip condition at the wall.

(B) → Inviscid region
flow is not impacted by the presence of body.

Critical Concept :-

Boundary Layer – Small region surrounding the body where velocity increases from zero to 99% of the free stream value (u_∞).



imp for viva.

BL characteristic

i) Thin Layer compared to the dimension of the body
$\delta << L$, $\delta(x) << x$

ii) Normal component of velocity is negligible $v << u$.

iii) Gradient along flow is neglible $\frac{\partial u}{\partial x} << \frac{\partial u}{\partial y}$

N-S Equation:

X-momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$

$O(x) \sim L$ $\quad O(u) \sim L$, $\quad O(L)$

$O(y) \sim \delta$ $\quad O(v) \sim \delta$ $\quad O(L)$

$O(1)$ $\quad O(\frac{1}{\delta^2})$

$O(\frac{1}{\delta})$

So, we can neglect $\partial^2 u/\partial x^2$ term,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\cdot\frac{\partial^2 u}{\partial y^2}$$

y-Momentum,

Since v is negligible at every point inside b.l

$$\frac{\partial p}{\partial y} = 0$$

$$\Rightarrow p = f(x)$$

For a flat plate,

$$p + \frac{1}{2}\rho u_\infty^2 = \text{constant}$$

$$\Rightarrow \frac{\partial p}{\partial x} = 0 \rightarrow \text{flat plate (close to boundary layer)}$$

N-S reduces to,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

– it can be shown that,

$$\delta/L \sim \frac{1}{Re_L^{1/2}} \Rightarrow \delta \sim \left(\frac{\nu L}{u_\infty}\right)^{1/2}$$

$\Rightarrow \delta$ vs. x is parabolic profile.

There is never a fully developed flow in External flow.

Stress at the wall,

$$\tau_w(x) = \mu\left.\frac{\partial u}{\partial y}\right|_x$$

shear stress decreases along the boundary layer, it is highest at the initial point.

Non-Dimensionlising x-momentum Equation :-

$x' = x/L$, $y' = y/L$, $u' = u/u_\infty$, $v' = v/v_\infty$, $\delta' = \delta/L$

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$$

$$O(1) = O(1)$$

$$\Rightarrow v' \sim O(\delta').$$

$$\underbrace{u'\frac{\partial u'}{\partial x'}}_{O(1)\,O(1)} + \underbrace{v'\frac{\partial u'}{\partial y'}}_{O(\delta')\,O\left(\frac{1}{\delta'}\right)} = \underbrace{\frac{\mu}{\rho u_\infty L}\frac{\partial^2 u'}{\partial y'^2}}_{O\left(\frac{1}{\delta'^2}\right)}$$

$$\delta'^2 = \frac{\delta^2}{L^2}$$

$$\frac{\mu}{\rho u_\infty L} = \delta'^2$$

$$\frac{\delta}{L} = \frac{1}{\sqrt{Re}}$$

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$$\tau_{wx} = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$= \mu \cdot \left(\frac{u_\infty}{L}\right) \frac{\partial u^*}{\partial y^*}$$

$$\sim \mu \cdot \left(\frac{u_\infty}{L}\right) \frac{1}{\delta'}$$

$$\sim \mu \frac{u_\infty}{\delta}$$

$$\sim \mu u_\infty \left(\frac{u_\infty}{\nu L}\right)^{1/2}$$

$$\sim \rho u_\infty^2 Re^{-1/2}$$

$$\delta \sim \left(\frac{\nu L}{u_\infty}\right)^{1/2}$$

$$\left(\frac{\rho u_\infty L}{\rho \mu L}\right)^{1/2}$$

$$\rho \mu \frac{u_\infty^2}{\rho u_\infty} Re^{1/2}$$

$$\frac{\rho u_\infty^2}{Re^{1/2}}$$

$$\frac{\frac{1}{kA}}{\frac{1}{hA}} = \frac{hl}{k}$$

Define friction factor

$$C_{fx} = \frac{\tau_{wx}}{\left(\frac{1}{2}\rho u_\infty^2\right)}$$

$$\sim \frac{1}{\sqrt{Re_x}}$$

Exact Solution of N-S Equation

→ Ludwig Equation and Blassius

Exact solution yields, $\boxed{C_{fx} = 0.664\, Re_x^{-1/2} \quad \delta/x = 5\, Re_x^{-1/2}}$ → laminar

So, we can calculate shear stress,

$$\tau_w(x) = \frac{1}{2}\rho u_\infty^2 C_{fx}$$

$$= 0.664\, \rho u_\infty^2 Re_x^{-1/2}$$

$$\frac{\rho v L}{\mu}$$

average stress $\bar{\tau}_w(x) = \frac{1}{L}\int_0^L 0.664\, \rho u_\infty^2 \left(\frac{\mu}{\rho u_\infty x}\right)^{1/2}$

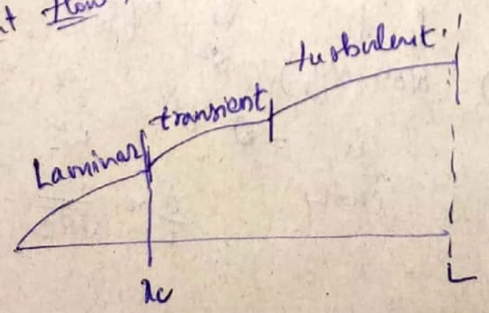
$$x^{-1/2} \quad 2x^{1/2}$$

$$= \frac{0.664\, \rho u_\infty^2}{L}\left(\frac{\mu}{\rho u_\infty}\right)^{1/2} 2\sqrt{L}$$

$$= 2 \times 0.664\, \rho u_\infty^2 Re_L^{-1/2}$$

$$\bar{\tau}_w(x) = \cancel{1.328}\; 2\, \tau_w(L)$$

Now, for Turbulent flow:



For flow through a pipe, turbulence occurs at $Re = \frac{\rho u_\infty D}{\mu} = 2000$ (for all liquids water, oil, ~ ~)

flow is laminar or turbulent throughout.

- For a flate plate flow is laminar followed by turbulent.

  - transition $Re_c = \frac{\rho u_\infty x_c}{\mu}$

    $\sim 3\times10^5$

In turb. flow, analytical solution becomes difficult.

- recommended relation

$$\boxed{C_{fx}\big|_{turb} = 0.0592\, Re_x^{-0.2}} \rightarrow \text{turbulent}$$

Calculation of $\bar{C}_f$.

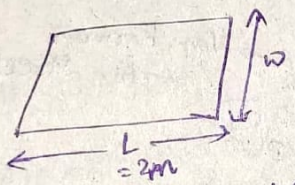
- Is it laminar or turbulent?

  If turb, calculate x_c using $Re_c = 3\times10^5$

$$\bar{C}_f = \frac{1}{\left(\frac{1}{2}\rho u_\infty^2\right)}\frac{1}{L}\left[\int_0^{x_c} \tau_{wx}\, dx\Big|_{lami} + \int_{x_c}^{L} \tau_{wx}\, dx\Big|_{turb}\right]$$

Ex:- $u_\infty = 4\,m/s$

Air @ 20°C



$\bar{C}_{fL} = ?$

$x_c = ?$

$F_D/w =$

At 20°C, 1 atm, $\mu = 1.81\times10^{-5}$ Pa-s

$\rho = 1.81$ kg/m³ (∵ transient area can be neglected)

$$Re = \frac{\rho u_\infty x_c}{\mu} = 3\times10^5$$

$$\frac{\cancel{1.81}\times 4\times x_c}{\cancel{1.81}\times\cancel{10^{-5}}} = 3\times\cancel{10^5}$$

$$x_c = 0.75\,m$$

$$\frac{F_D}{w} = \frac{\bar{\tau}_w (Lw)}{w} = \frac{1}{2}\rho u_\infty^2 \bar{C}_{fL} L$$

$$= \int_0^{x_c} \tau_{wx}\, dx\Big)_{lamin} + \int_{x_c}^{L} \tau_{wx}\, dx\Big)_{turb}$$

$$= \frac{1}{2}\rho u_\infty^2 \left[0.664 \int_0^{x_c} Re_x^{-1/2}\, dx + 0.0592 \int_{x_c}^{L} Re_x^{-0.2}\, dx\right]$$

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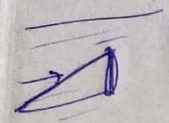
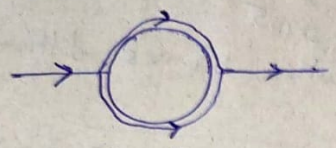
$$\frac{F_D}{w} = \rho u_\infty^2 \left[0.664 \int_0^{0.75} \left(\frac{\rho u_\infty x}{\mu}\right)^{-1/2} dx + 0.0592 \int_{0.75}^{2} \left(\frac{\rho u_\infty x}{\mu}\right)^{-0.5} dx \right] \frac{\rho u_\infty x}{\mu}$$

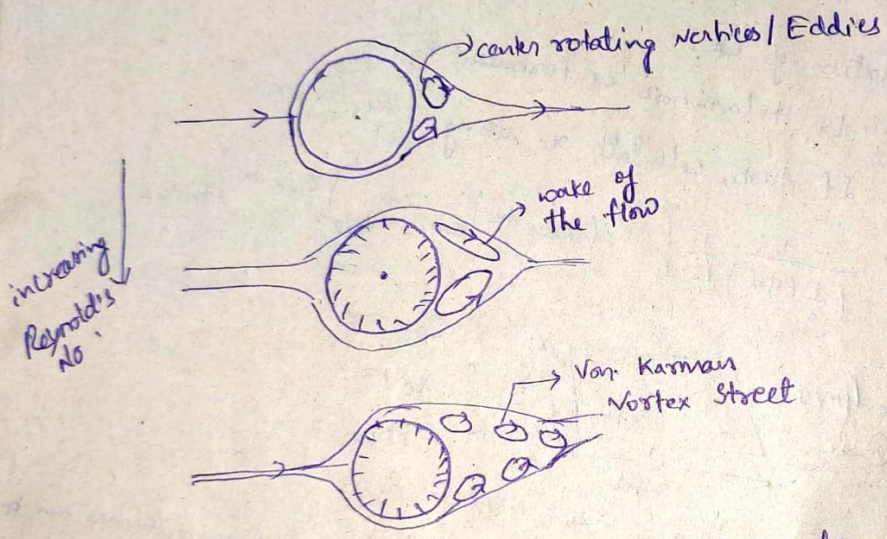


Solving we get 0.103 N/m

Cylinder in Cross Flow :-



low Re,
Creeping / Stokes flow
Inertial force << Viscous force



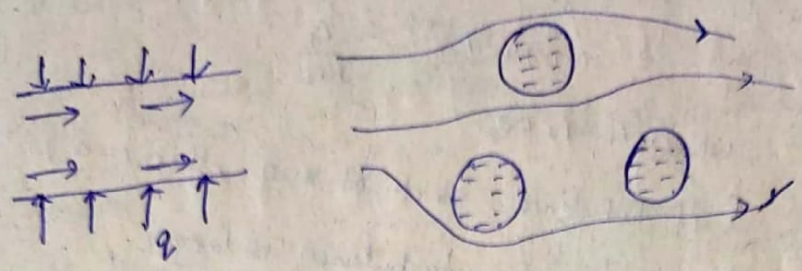
$$\text{Drag Coefficient} = \frac{F_D/A_p}{\left(\frac{1}{2}\rho u_\infty^2\right)}$$

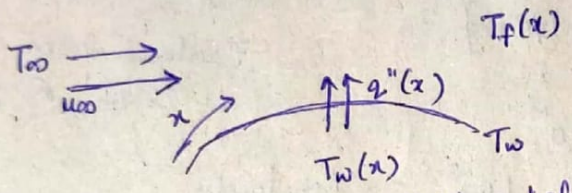
$F_D \rightarrow$ drag force
$A_p \rightarrow$ projected area

Typically $C_D = f(Re_\infty)$

Forced Convection :-

- Occurs in presence of fluid flow caused by an external device like fan, pump, blower etc.





Goal: establish a relation between $q''(x)$, $T_w(x)$ & $T_f(x)$.

$$h = \frac{q''(x)}{T_w(x) - T_f(x)}$$

$$\bar{h} = \frac{q''(x)}{T_w - T_f}$$

Dimensionless Parameter :-

h depends on $\mu, \rho, K_f, C_p, L, u_\infty$

i) Reynolds No (Re) $= \frac{\rho u_\infty L}{\mu}$.

ii) Nusselt No (Nu) $= \frac{hL}{K_f}$ (∴ here K_f is of fluid) (In Biot No K is of solid)

iii) Prandtl No (Pr) $= \frac{\mu C_p}{K_f}$

$$Re = \frac{\text{inertial force}}{\text{viscous force}}$$

$$Nu = \frac{hL}{K_f}$$

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Correlation for turb. drag coeff.

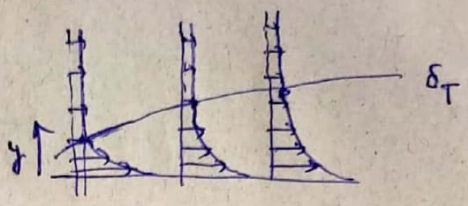
$$\bar{C}_{fL} = \frac{1}{L}\left[\int_0^x C_{fx|lam}\,dx + \int_x^L C_{fx|tur}\,dx\right]$$

$$= 0.074\, Re_L^{-0.2} - \frac{C_1}{Re_L}$$

where $C_1 = 1050$

(if turbulence start at $x=0$, then $c=0$)

∴ shear stress for turbulence is larger.



At any location x,

$$q''(x) = h(x)\left[T_w(x) - T_\infty\right]$$

$$= -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0^+}$$

T_∞ reduces from $T=T_w$ @ $y=0$ to $T=T_\infty$ @ $y=\delta$

$$\text{Nusselt \#} = \frac{hL}{k} = \frac{L/kA}{1/hA} = \frac{R_{th}|_{conduc}}{R_{th}|_{convec}}$$

- if we consider a slab of fluid area A and thickness L above hot surface, then Nu# represent ratio of conduction to convection resistance.

ideally $R_{th}|_{con} \sim \frac{\delta_T}{k_f A}$

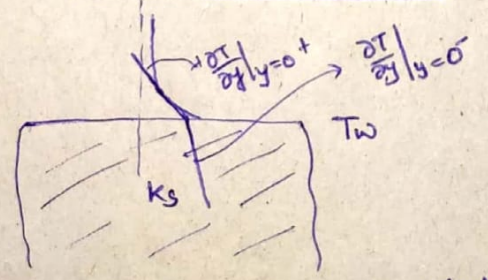
but one may argue that $\delta_T \sim L$

ii) $$Nu = \frac{\left.\frac{\partial T}{\partial y}\right|_{y=0^+}}{(T_w - T_\infty)/L} = \frac{h(T_w - T_\infty)}{k_f\cdot\left(\frac{T_w - T_\infty}{L}\right)}$$

- Non-dimensional temp gradient at the wall.

high Nu ⇒ high convection effects.

$$q''(x) = \underbrace{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0^+} = -k_s \left.\frac{\partial T}{\partial y}\right|_{y=0^-}}$$



if $k_s > k_f$, temp gradient in solid is lesser compared to fluid.

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Prandtl

- Fluid Property
- Measure of momentum diffusivity over thermal diffusivity

$$Pr = \frac{\mu/\rho}{k_f/\rho c_p} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k_f}$$

- Measure of ease of momentum (shear forces) to transfer of thermal energy through fluid.
  - viscous fluids or low thermal conductivity fluids have high Pr.
  - high thermal conduc. fluids have low Pr.

for gases, typically $Pr \sim 1$

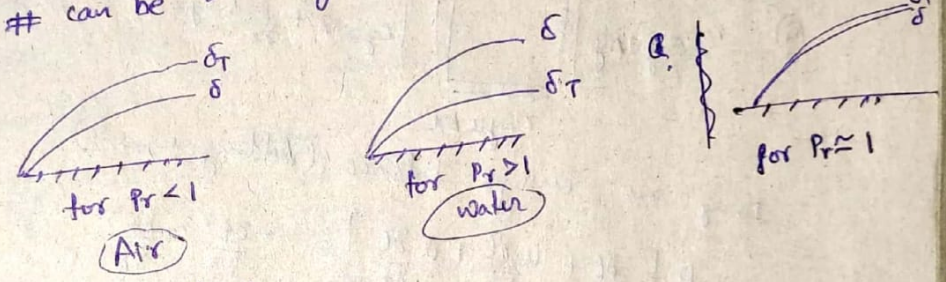
oil, $Pr \geq 100$ (high μ)

Hg, $Pr \leq 0.01$ (high k_f)

Air at room tem $Pr = 0.7$

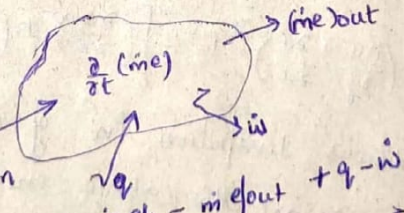
Water at room temp. $Pr \sim 7$

- Pr # can be a strong fun of temp.



Governing Equations:-

- Continuity } → done in fluid flow
- Momentum }
- Energy → 1st law & fourier's Law



e → int. energy
q → heat
w → work
i → enthalpy

$$\frac{\partial}{\partial t}(\dot{m}e) = \underbrace{\dot{m}e_{in} - \dot{m}e_{out}}_{\text{Net Energy In.}} + q - \dot{w}$$

↓ accumulation of energy in CV

→ 1st Law

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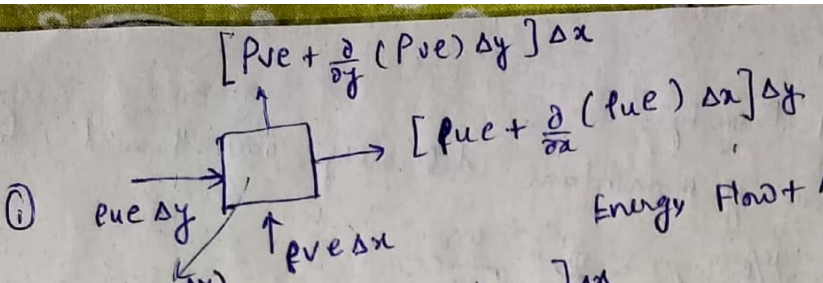
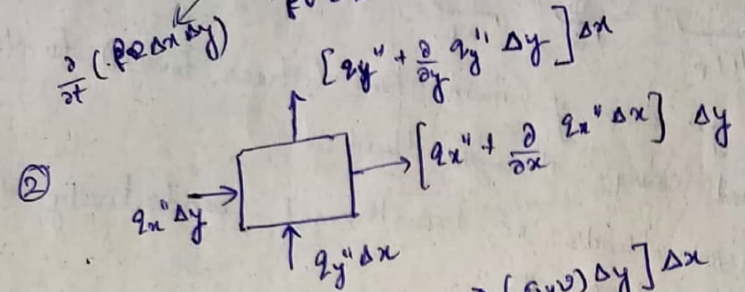
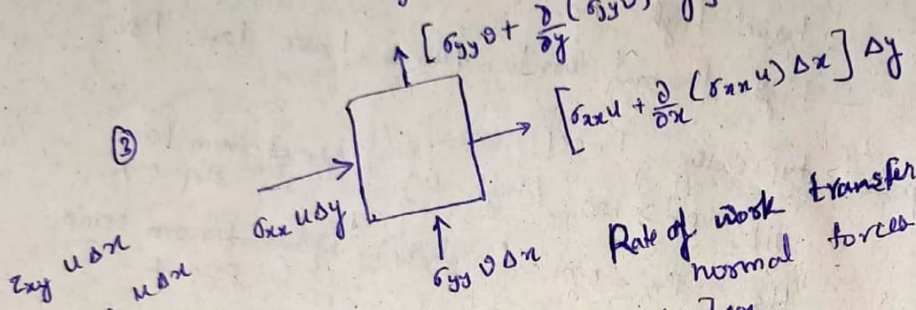
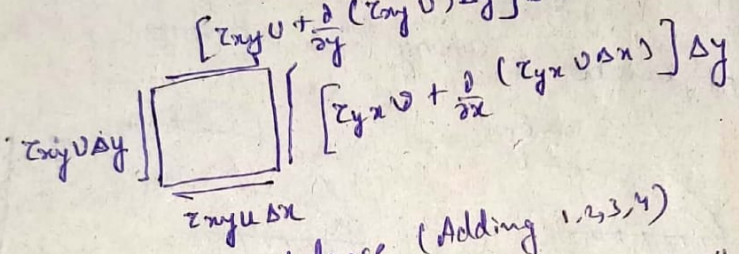


Energy Flow + Accumulation.





Rate of work transfer due to normal forces



Doing an energy balance (Adding 1,2,3,4)

$$\rho\left[\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y}\right] = -\frac{\partial q''}{\partial x} - \frac{\partial q''}{\partial y} + q'' + \mu\phi$$

$$\tau_{xy} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]$$

$$\sigma_{xx} = p - 2\mu\frac{\partial u}{\partial x} + \frac{2}{3}\mu\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]$$

$$\phi = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$$

— viscous dissipation fn of a fluid with constant ρ.

We need to recast the above energy in terms of temp T.

Internal Energy, $e = i - p\theta$

If e = const, L.H.S

$$\rho\left[\frac{\partial e}{dt} + u\frac{\partial e}{\partial x} + v\frac{\partial e}{\partial y}\right] = \rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

(neglecting pressure force terms and ρ_v constant)

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R.H.S becomes,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + q''' + \mu\phi$$

$\therefore$ Energy eq. becomes [unit ρ, μ]

$$\boxed{\rho C_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + q''' + \mu\phi}$$

For fluid at rest, $u = v = \phi = 0$

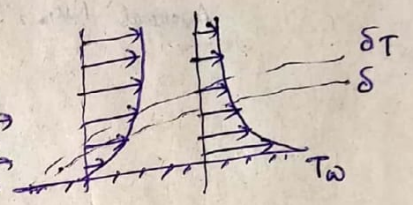
– Equ. reduces to heat diffusion Equation.

In most problem $q''' = 0$ & viscous dissipation is negligible

If you further include steady state & const k.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

<u>Plane Wall</u> :-



i) momentum b.l δ, thermal b.l δ_T

ii) For thermal b.l

$$\frac{\partial T}{\partial y} \sim \frac{T_w - T_\infty}{\delta_T}$$

B.L assumption $\xrightarrow{u_\infty}$ $\xrightarrow{T_\infty}$

(a) $\delta_T << L$ or $\delta_T(x) << x$

b) $\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$

Based on same order of mag. analysis :

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

This is the simplified form of energy equ. in the (x, δ_T) layer

. We know that $q'' = -k\left.\frac{\partial T}{\partial y}\right|_{y=0^+} = h(x)[T_w - T_\infty]$

$$\sim k\frac{\Delta T}{\delta_T}$$

Let $\theta = T^* = \frac{T_w - T}{T_w - T_\infty} = \theta(\eta) \rightarrow$ assuming a similarity solution.

Recall, $\eta = \frac{y}{x}\sqrt{Re_x} = y\sqrt{\frac{u_0}{\nu x}}$

$$f(\eta) = \frac{\psi}{\sqrt{u_0 \nu x}}$$

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$$\frac{d^2\theta}{d\eta^2} + \frac{Pr}{2\alpha} f \frac{d\theta}{d\eta} = 0 \qquad \Big| \quad \theta(\delta_T) = 1$$

Numerical Soln. gives,

$$\boxed{\frac{d\theta}{d\eta} = 0.332\, Pr^{1/3}}$$

$$\frac{d\theta}{dy} = \frac{dT}{dy} \cdot \frac{1}{T_w - T_\infty}$$

$$h_x = \frac{q''(x)}{T_w - T_\infty} = -\frac{T_\infty - T_w}{T_w - T_\infty} \cdot k \frac{\partial\theta}{\partial y}\Big|_{y=0}$$

$$= 0.332\, Pr^{1/3}\, k \sqrt{\frac{u_\infty}{\nu x}}$$

$$\boxed{Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}}$$ → remember this for exam

It is valid for laminar flow over flat plate.

Average Nu_x, $\Rightarrow 0.332 Pr^{1/3} \left(\frac{\rho v}{\mu}\right)^{1/2} \frac{1}{L} \int_0^L x^{1/2}\, dx$

$\frac{\rho v x}{\mu}$

$$\Rightarrow 0.332\, Pr^{1/3} \left(\frac{\rho v}{\mu}\right)^{1/2} \frac{1}{L} \left(x^{3/2}\right)_0^L \cdot \frac{2}{3}$$

$$\Rightarrow \frac{2}{3} \times 0.332\, Pr^{1/3}\, Re_L^{1/2}$$

$$\Rightarrow 0.22\, Re_L^{1/2}\, Pr^{1/3} = \frac{2}{3} Nu_{x=L}$$

This is wrong,

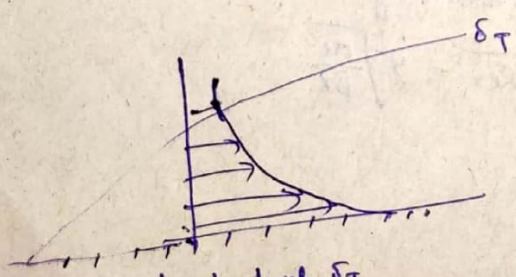
To find $\overline{Nu}_L$, you need to find $\bar{h}_L$

$$\bar{h}_L = \frac{1}{L} \cdot \int_0^L h_x\, dx$$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

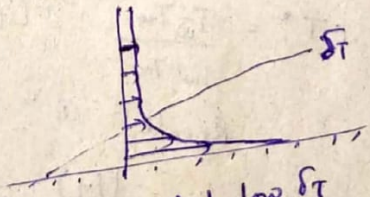
$$\overline{Nu}_L = 2\, Nu_{x=L}$$

It can be shown that $\cancel{\frac{\delta_T}{\delta} \simeq Pr^{1/3}}$ $\frac{\delta}{\delta_T} \simeq Pr^{1/3}$



low h, high δ_T

low $\frac{\partial T}{\partial y}\Big|_{y=0^+}$



high h, low δ_T

high $\frac{\partial T}{\partial y}\Big|_{y=0^+}$

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Using these, the energy eq. becomes

$$\frac{d^2\theta}{d\eta^2} + \frac{Pr}{2\alpha} f \frac{d\theta}{d\eta} = 0 \qquad \Big| \quad \theta(0) = 0, \quad \theta(\delta_T) = 1$$

Numerical soln. gives,

$$\boxed{\frac{d\theta}{d\eta} = 0.332\, Pr^{1/3}}$$

$$\frac{d\theta}{dy} = \frac{dT}{dy} \cdot \frac{1}{T_w - T_\infty}$$

$$h_x = \frac{q''(x)}{T_w - T_\infty} = -\frac{T_\infty - T_w}{T_w - T_\infty} \cdot k \frac{\partial\theta}{\partial y}\Big|_{y=0}$$

$$= 0.332\, Pr^{1/3}\, k \sqrt{\frac{u_\infty}{\nu x}}$$

$$\boxed{Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}}$$ → remember this for exam

It is valid for laminar flow over flat plate.

Average Nu_x, $\Rightarrow 0.332 Pr^{1/3} \left(\frac{\rho v}{\mu}\right)^{1/2} \frac{1}{L} \int_0^L x^{1/2}\, dx$

$\frac{\rho v x}{\mu}$

$$\Rightarrow 0.332\, Pr^{1/3} \left(\frac{\rho v}{\mu}\right)^{1/2} \frac{1}{L} \left(x^{3/2}\right)_0^L \cdot \frac{2}{3}$$

$$\Rightarrow \frac{2}{3} \times 0.332\, Pr^{1/3}\, Re_L^{1/2}$$

$$\Rightarrow 0.22\, Re_L^{1/2}\, Pr^{1/3} = \frac{2}{3} Nu_{x=L}$$

This is wrong,

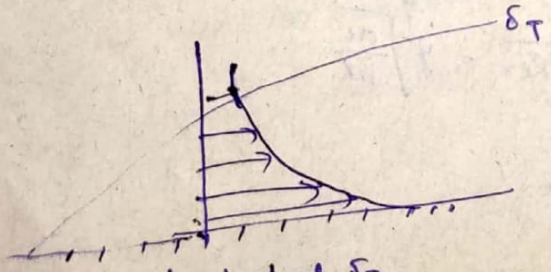
To find $\overline{Nu}_L$, you need to find $\bar{h}_L$

$$\bar{h}_L = \frac{1}{L} \cdot \int_0^L h_x\, dx$$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

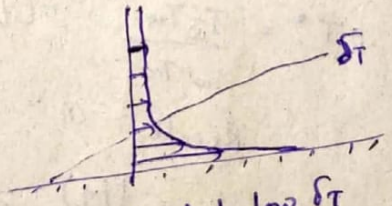
$$\overline{Nu}_L = 2\, Nu_{x=L}$$

It can be shown that $\cancel{\frac{\delta_T}{\delta} \simeq Pr^{1/3}}$ $\frac{\delta}{\delta_T} \simeq Pr^{1/3}$



low h, high δ_T

low $\frac{\partial T}{\partial y}\Big|_{y=0^+}$



high h, low δ_T

high $\frac{\partial T}{\partial y}\Big|_{y=0^+}$

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Reynold's Analogy :-

Momentum equ :- $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$

Energy equ :- $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$

. note the term by term correspondence,

Assume $Pr \simeq 1 \Rightarrow \nu \simeq \alpha$

$\Rightarrow u = T,$

$\Rightarrow \cancel{\frac{\partial u}{\partial y}\Big|_{y=0^+}} \simeq \frac{\partial T}{\partial y}\Big|_{y=0^+}$

$\frac{\tau_w}{\mu} \simeq -h\frac{(T-T_\infty)}{k}$

$C_f \cdot \frac{1}{2}\rho u_\infty^2 = \frac{\mu}{k} h (T_\infty - T)$

$C_f/2 = \frac{\mu}{k \rho u_\infty^2} h (T_\infty - T)$

In order to make boundary condition same, we have to non-dimensionalise the equation

$u' = \frac{u}{u_\infty}, \; v' = \frac{v}{u_\infty}, \; \theta' = \frac{T - T_w}{T_\infty - T_w}$

$u'\frac{\partial u'}{\partial x} + v'\frac{\partial u'}{\partial y} = \frac{\nu}{u_\infty}\frac{\partial^2 u'}{\partial y^2} \rightarrow u' = 0$ at $y = 0$, $u' = 1$ at $y = \delta$

$u'\frac{\partial \theta'}{\partial x} + v'\frac{\partial \theta'}{\partial y} = \frac{\alpha}{u_\infty}\frac{\partial^2 \theta'}{\partial y^2} \rightarrow \theta' = 0$ at $y = 0$, $\theta' = 1$ at $y = \delta$

Since g.d.e & b.c's are identical :

(governing differential equation) $u' = \theta'$

$\frac{\partial u'}{\partial y} = \frac{\partial \theta'}{\partial y}$

$\frac{1}{u_\infty} \cdot \frac{\partial u}{\partial y}\Big|_{y=0^+} = \frac{1}{T_\infty - T_w} \cdot \frac{\partial T}{\partial y}\Big|_{y=0^+}$

$\frac{1}{u_\infty} \cdot \frac{\tau_w}{\mu} = \frac{1}{T_\infty - T_w} \cdot \frac{-q''}{k}$

$\tau_w = \frac{1}{2} C_f \rho u_\infty^2 = \frac{\mu u_\infty}{T_\infty - T_w} \cdot \frac{-h(T_w - T_\infty)}{k}$

$\frac{C_f}{2} = \frac{\mu u_\infty h}{k} \cdot \frac{1}{\rho u_\infty^2} = \frac{hL}{k} \cdot \frac{\mu}{\rho u_\infty L}$

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$$\frac{C_f}{2} = Nu_L \cdot \frac{1}{Re_L}$$

$$\boxed{\frac{C_f}{2} = \frac{Nu_L}{Re_L \cdot Pr}} \qquad [\because Pr = 1]$$

↳ Reynold's Analogy

$$St = \frac{Nu}{Re \cdot Pr}$$

↓

Stanton #

$$Pe = Re \cdot Pr$$

↓

Peclet #

Colburn extended this study and gave:

$$\boxed{\frac{C_f}{2} = \frac{Nu}{Re\, Pr^{1/3}}}$$ → Reynolds Colburn Analogy.

↓

holds good for variety of internal flow.

Ex: Turbulent flow over flat plate.

$$C_f(x) = 0.0592\, Re_x^{-0.2}$$

$$Nu_x = \frac{C_f(x)}{2} \cdot Re\, Pr^{1/3}$$

$$\boxed{Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3}}$$ → for turbulent

Q. The cooling of fruits and veget at refreger air at $t = 4°C$, $P = 1atm$, $v = 0.3\ m/s$. This is modelled as convection, radiation and evaporation of air. The heat transfer coefficient for combined of these in the velocity range of 0.1–0.3 m/s is determined exp. and found as $h = 5K + \frac{Re^{1/3}}{D}$ where

$\frac{\rho v D}{\mu}$

D → characteristic length

K → thermal conductivity

Calculate the initial rate of heat transfer whose area is equivalent to As ~ 5mm dia sphere and initial temp $(T_i) = 20°C$.

K of fruit = 0.8 W/m-K

b) the temp. gradient at the surface of the fruit and Nusset No. corresponding to that.

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A. Step 1 : Find h → use fluid properties

$$q = hA_s(T_s - T_\infty)$$

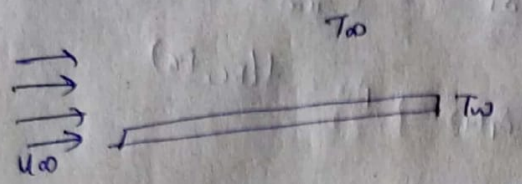
$T_s \to 20$, $T_\infty \to 4$

n → unit normal

Step 2 : $q'' = -k_f \left.\frac{\partial T}{\partial n}\right|_{n=0^+} = -k_s \left.\frac{\partial T}{\partial n}\right|_{0^-}$

Step 3 : Nusset $Nu = \frac{hx}{k_f} = \frac{hD}{k_f} \propto Re^{1/3}$

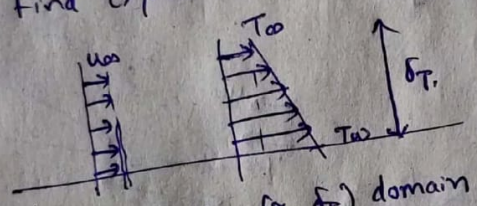
Q.



Given :→ Velocity is uniform u_∞ everywhere (∵ means that u_∞ is everywhere and momentum boundary layer is neglible compared to thermal boundary layer ⇒ $Pr << 1$)

→ Temp vary linearly from the surface to free stream

Find expression for Nu.



$$\left.\frac{\partial T}{\partial y}\right|_{y=0^+} = \frac{T_w - T_\infty}{\delta_T}$$

$u = u_\infty$ in the (x, δ_T) domain

$$\frac{T - T_\infty}{T_w - T_\infty} = \left(1 - \frac{y}{\delta_T}\right)$$

Energy Equ.

$$u\frac{\partial T}{\partial x} + \cancel{v}\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$v = 0$

integrate w.r.t y, from $(0, \delta_T)$

$$u \cdot \int_0^{\delta_T} \frac{\partial T}{\partial x} \cdot dy = \alpha \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}^{y=\delta_T} = \alpha(T_w - T_\infty)\left(\frac{-1}{\delta_T}\right)$$

$$\int_0^{\delta_T} u_\infty \cdot \frac{\partial}{\partial x}\left(\frac{T - T_\infty}{T_w - T_\infty}\right)^{\times (T_w - T_\infty)} dy = \alpha(T_w - T_\infty)\left(\frac{-1}{\delta_T}\right)$$

$$\frac{d}{dx}\int_0^{\delta_T} u_\infty \cdot \left(1 - \frac{y}{\delta_T}\right)\cancel{(T_w - T_\infty)}\, dy = \alpha\cancel{(T_w - T_\infty)}\left(\frac{-1}{\delta_T}\right)$$

$$\frac{d}{dx}\int_0^{\delta_T} u_\infty\left(1 - \frac{y}{\delta_T}\right) dy = \frac{-\alpha}{\delta_T}$$

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$$\frac{d}{dx}\left[\left(\frac{y-\frac{y^2}{2\delta_T}}{\delta_T}\right)_0^{\delta_T}\right] = \frac{-\alpha}{\delta_T u_\infty} \qquad \delta_T - \frac{\delta_T^2}{2}$$

$$\frac{d\delta_T}{dx} = \left(\frac{-2\alpha}{u_\infty}\right)\frac{1}{\delta_T}$$

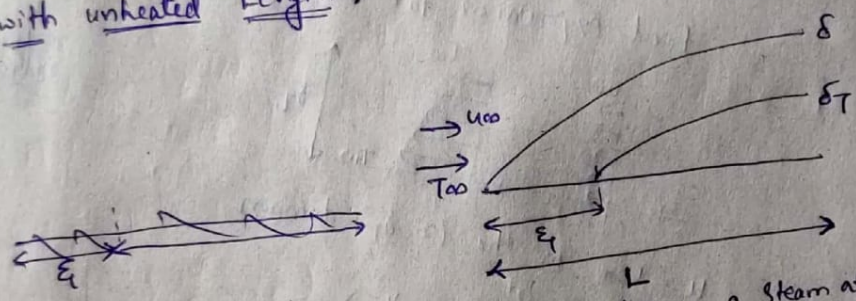
$$\delta_T \frac{d\delta_T}{dx} = -\frac{2\alpha}{u_\infty}$$

$$\frac{\delta_T^2}{2} = \left(\frac{-2\alpha}{u_\infty}\right)x$$

$\left.\frac{\partial T}{\partial y}\right|_{y=0}$ → replace $\delta_T(x)$ ⇒ $Nu = f(Re, Pr)$

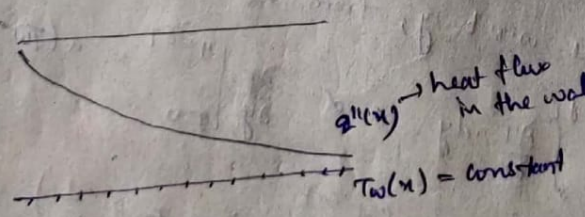
Q. ~~A~~ flat plate of length of 1m, constant T at 315 K is placed vert

Plate with unheated Length :-

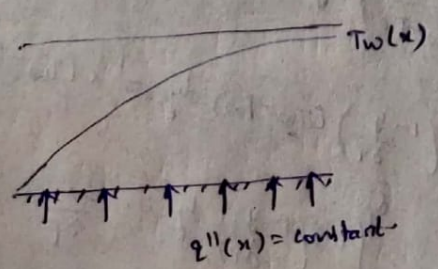


(∵ constant Temp can be maintained by introducing a steam at 100°C " " " " " heater)
(∵ " flux " " " " " "

Isothermal:



Isoflux:



$q''(x) = h_x (T_w(x) - T_\infty)$

$x\uparrow,\ h_x\downarrow$

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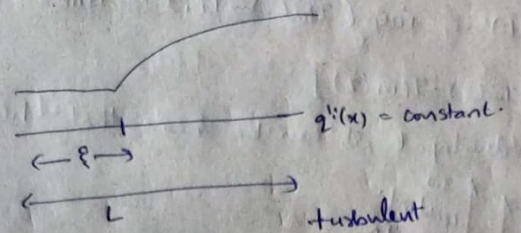
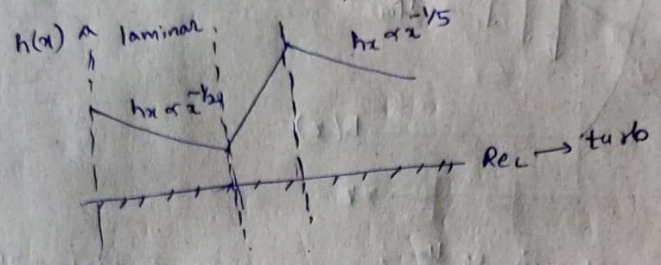
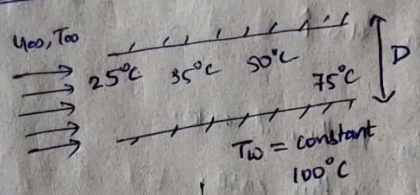
Unheated length:

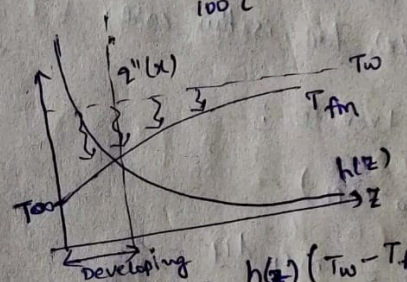




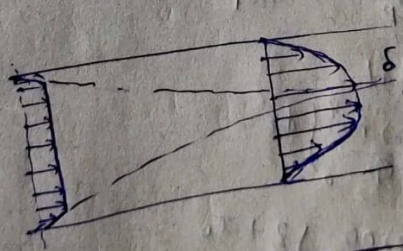
Internal Flow

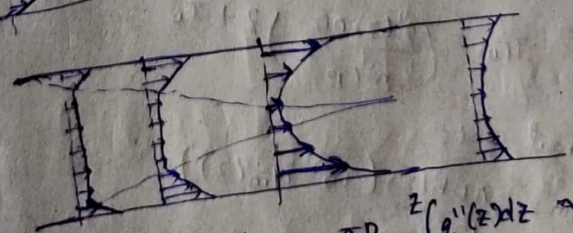


Const. Temp :-



Heat flux $q''(z) = h(z)\left(T_w - T_{fm}(z)\right)$ → mean fluid temp from $y = -\delta$ to δ :





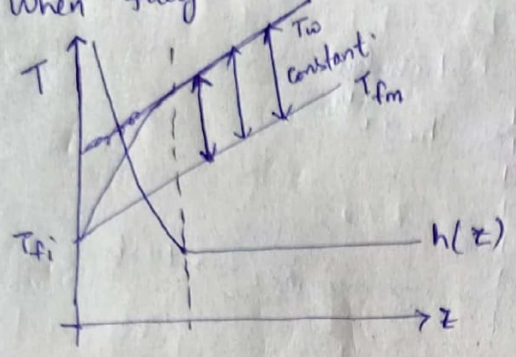
$$q(z) = \int_0^z q''(z)\,\pi D\,dz = \pi D \int_0^z q''(z)\,dz = \dot{m}C_p = \cancel{\dot{m}C_p(T_{ote})} - \dot{m}C_p\left(T_{fm}(z) - T_{fi}\right)$$

for constant heat flux:

$$q(z) = q''(z\pi D) = \dot{m}C_p\left(T_{fm}(z) - T_{fi}\right) \Rightarrow T_{fm}(z) \text{ is linear fn of } z$$

$$q'' = h(z)\left[T_w(z) - T_{fm}(z)\right]$$

When thermal fully developed, it can be shown that $h(z) = \text{constant}$



<u>Influence of $\dot{m}$ or u_∞:</u>

Rate of increase of T_{fm} reduces with increase in $\dot{m}$ (or u_∞)

$$\frac{dT_{fm}}{dz} \downarrow \text{ as } u_\infty \uparrow$$

- Results in
  a) higher $q''(z)$ for isothermal case.
  b) lower $T_w(z)$ for isoflux case.

<u>Total convective heat transfer:</u>

$$q_{conv} = \dot{m}C_p\left(T_{fo} - T_{fi}\right)$$

$$q''(z)\pi D\,dz = \dot{m}C_p\,dT_f$$

$$\frac{dT_f}{dz} = \frac{q''(z)\pi D}{\dot{m}C_p} = \frac{\pi D}{\dot{m}C_p}h_z\left(T_w(z) - T_{fm}(z)\right)$$

- for $q''(z)$ to be constant

$$\int_{T_o}^{T_{fm}} dT_f = \frac{q''(z)\pi D}{\dot{m}C_p}\int_0^z dz$$

$$T_{fm} = \left(\frac{q''(z)\pi D}{\dot{m}C_p}\right)z + T_o$$

- for isothermal case:

$$\frac{dT_{fm}}{dz} = \frac{\pi D h(z)}{\dot{m}C_p}\left(T_w - T_{fm}(z)\right)$$

Let $\Delta T = T_w - T_{fm}$

$$d(\Delta T) = -d(T_{fm})$$

$$\frac{d(\Delta T)}{dz} = \frac{-\pi D h(z)}{\dot{m}C_p}\Delta T$$

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for constant heat flux:

$$q(z) = q''(z\pi D) = \dot{m}C_p\left(T_{fm}(z) - T_{fi}\right) \Rightarrow T_{fm}(z) \text{ is linear fn of } z$$

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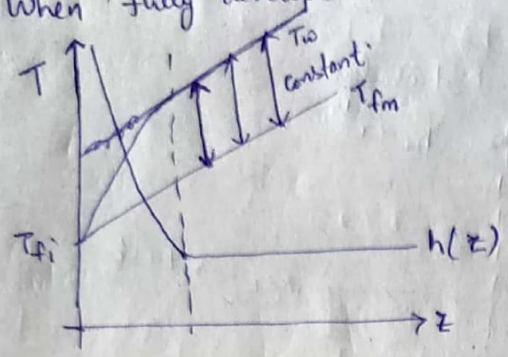

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$$\int_{T_0}^{T_{fm}} dT_f = \frac{q''(z)\pi D}{\dot{m}C_p}\int_0^z dz$$

$$T_{fm} = \left(\frac{q''(z)\pi D}{\dot{m}C_p}\right)z + T_0$$

- for isothermal case:

$$\frac{dT_{fm}}{dz} = \frac{\pi D h(z)}{\dot{m}C_p}\left(T_w - T_{fm}(z)\right)$$

Let $\Delta T = T_w - T_{fm}$

$$d(\Delta T) = -d(T_{fm})$$

$$\frac{d(\Delta T)}{dz} = \frac{-\pi D h(z)}{\dot{m}C_p}\Delta T$$

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$$\int_{\Delta T_i}^{\Delta T_o} \frac{d(\Delta T)}{\Delta T} = \frac{-\pi D}{\dot{m} C_p} \int_0^L h(z)\, dz$$

$$\ln\left(\frac{\Delta T_o}{\Delta T_i}\right) = \frac{-\pi D L}{\dot{m} C_p} \cdot \left[\frac{1}{L}\int_0^L h(z)\, dz\right] \qquad (\because P = \pi D)$$

$$= \frac{-PL}{\dot{m} C_p} \bar{h}$$

$$\frac{T_w - T_{fo}}{T_w - T_{fi}} = \exp\left(\frac{-PL\bar{h}}{\dot{m} C_p}\right)$$

$$q_{conv} = -\dot{m} C_p (\Delta T_o - \Delta T_i) = \bar{h}(PL)\Delta T_m$$

$$\Delta T_o - \Delta T_i = -\frac{\bar{h} PL}{\dot{m} C_p} \Delta T_m$$

$$\Delta T_m = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o) - \ln(\Delta T_i)}$$

ΔT_m → LMTD

Logarithmic Mean Temp Difference

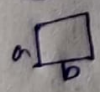
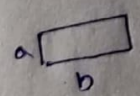
$q_{conv} = \bar{h} A_s \Delta T_{LMTD}$

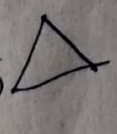
valid when both T_w & T_{fm} are varying.

$Nu_D = \frac{hD}{k} = 4.36$

for Non-circular, D can be hydraulic diameter $\left(\frac{4A}{P}\right)$

$\frac{4 \cdot \pi r^2}{2\pi r} = 2r$

Laminar Cross-section	b/a	Isoflux	Isothermal	fRe_{D_h}
	—	4.36	3.66	64 $\left(f = \frac{64}{Re}\right)$
	1	3.61	2.98	57
	2	4.12	3.39	62
	4	5.33	2.97	62
	8	6.49	2.54	62
	16	7.54	2.36	62
	∞	8.33	2.47	96
	—	3.11	2.47	53

(equilateral triangle) 

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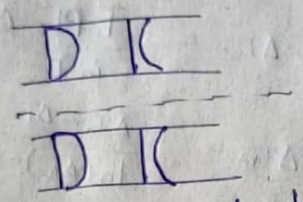
- from here, we conclude the Nusset No. depends on ~~gtm~~ cross-section area, geometry because for differ- cross-section, boundary layer formed will be different.

- for turbulent flow, Nusset No. doesn't depend on cross-section area

$$\frac{Re_D Pr}{x/D} = \text{Graetz \#}$$
$$= Gz$$

Concentric



There will be 2 boundary layer, ⇒ 2 different Nusset No.

Nu_D derivation - internal flow

Isoflux

At any point in the tube (with b.l theory)

we can write,

$$u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

Inside b.l, for F.D flows

$$v = 0, \quad \frac{\partial u}{\partial z} = 0$$

for isoflux surface,

$$\frac{\partial T_{fm}}{\partial z} = \frac{\partial T_w}{\partial z} = \frac{\partial T}{\partial z} = \text{constant}$$

$$\Rightarrow \frac{\partial^2 T}{\partial z^2} = 0$$

Velocity profile

$$u/u_\infty = 2\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{2u_\infty}{\alpha}\left(\frac{dT}{dz}\right)\left(1-\left(\frac{r}{R}\right)^2\right)$$

Integrating twice wrt 'r' in $(0, R)$

$$T(r,z) = \frac{2u_\infty}{\alpha}\left(\frac{dT}{dz}\right)\left[\frac{r^2}{4} - \frac{r^4}{16R^2}\right] + C_1 \ln r + C_2$$

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B.C's :

At $r=0$, $T(r,z)$ is finite $\Rightarrow C_1 = 0$

At $r=R$, $T(R,z) = T_w(z)$

$$T(r,z) = T_w(z) - \frac{2u_\infty R^2}{\alpha}\left(\frac{dT}{dz}\right)\left[\frac{3}{16} + \frac{1}{16}\left(\frac{r}{R}\right)^4 - \frac{1}{4}\left(\frac{r}{R}\right)^2\right]$$

. We get $T_{fm}(z)$ by using $\frac{1}{R}\int_0^r T(r,z)\,dr$

$$T_{fm}(z) = T_w(z) - \frac{11}{48}\left(\frac{u_\infty R}{\alpha}\right)^2 \frac{dT}{dz}$$

$$h(z) = \frac{q''}{[T_w(z) - T_{fm}(z)]}$$

$$\frac{dT_f}{dz} = \frac{dT}{dz} = \frac{q''P}{\dot{m}C_p} = \frac{q_w''\cdot(\pi D)}{\rho \pi R^2 u_\infty C_p}$$

Use the above and $\alpha = k/\rho C_p$

$$T_w(z) - T_{fm}(z) = \frac{11}{48}\frac{q''D}{k} \qquad Nu_D = \frac{hD}{k} = \frac{48}{11} = 4.36$$

$$h(z) = \frac{48k}{11D}$$

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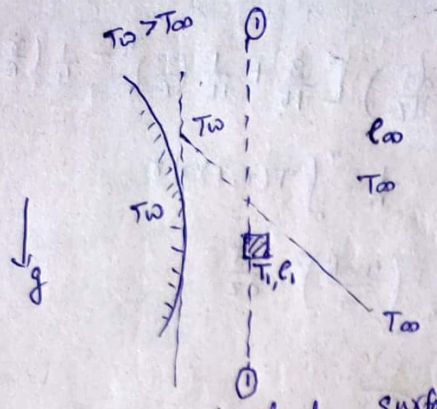

- Consider a heated surface at T_w in a quiescent ambient at T_∞
- We consider temp to reduce from T_w to T_∞ as we move away from the surface.
- At a plane 1–1, we have

$$T_\infty < T_1 < T_w$$

$$\rho_1 < \rho_\infty$$

As a result a net buoncy force acts in the vertically upward directions on the air at 1–1.

- This initiates Natural Convection a.k.a
  - Free convection
  - Buoncy Induced Flow
- Since flow is induced by temperature Gradient, the flow/temp fields are coupled.
  - unlike F.C where we had momentum equ independent of T.

<u>Non-Dimensional Numbers</u> :-

- $Nu = \frac{hL}{k_f}$ where $L \rightarrow$ length scale
- $Pr = \frac{\mu C_p}{k_f}$
- Grashof Number (Gr) $= \frac{g\beta(\Delta T)L^3}{\nu^2}$, where $\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial t}\right)_P$

$$\sim \frac{F_{Buoyancy}}{F_{viscous}}$$

Reynold's Number can't be considered because there is no u_∞ in Natural Convection.

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Scaling:

$F_{buoncy} = Vol \cdot g \cdot (\rho_\infty - \rho)$

$\sim L^3 g (\rho_\infty - \rho)$

$\beta =$ Thermal exp. coeff

$= -\frac{1}{\rho_\infty}\left(\frac{\partial \rho}{\partial T}\right)_p \sim \frac{1}{\rho_\infty}\frac{\rho_\infty - \rho}{T - T_\infty}$ ~~$\Delta L = L\alpha\Delta T$~~

$\rho_\infty - \rho \sim \rho\beta\Delta T \qquad (\Delta T = T - T_\infty)$

So $F_{Buo} = L^3 g \rho \beta \Delta T$

from Buckinghum-PI Theorem

Gr comes as $g\beta\Delta T L^3 \mu^{-2}\rho^2 = \frac{g\beta\Delta T L^3}{\nu^2}$

- Rayleigh $\#(Ra) = Gr \cdot Pr \qquad \left[\because Pr = \frac{\nu}{\alpha}\right]$

$= \frac{g\beta\Delta T L^3}{\alpha\nu}$

$Ra_x = \frac{g\beta\Delta T x^3}{\alpha\nu}$

Typically $Nu = f(Ra, Pr)$

Mixed Convection:

- Buoyancy and Inertial forces are comparable.

$\frac{Gr_L}{Re_L^2} \sim 1 \rightarrow$ Mixed

$\gg 1 \rightarrow$ Natural

$\ll 1 \rightarrow$ Forced.

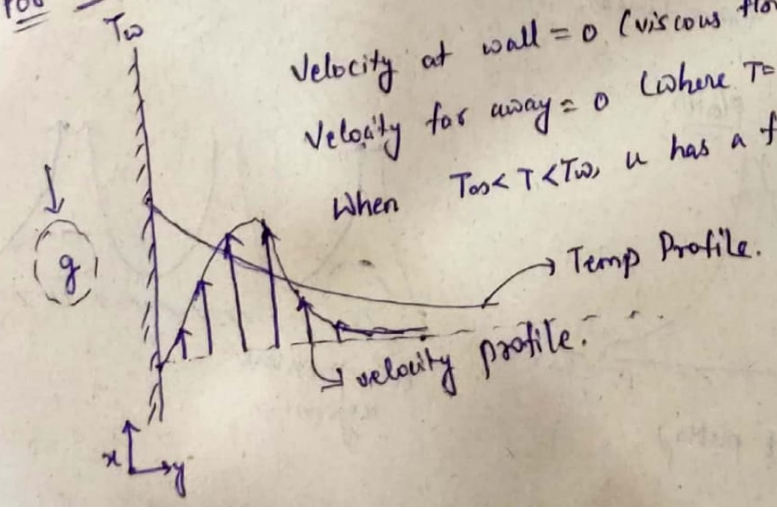
For Mixed Convec.

$Nu^n_{mixed} = Nu^n_{FC} \pm Nu^n_{NC}$

where $n \sim 3$, $\hookrightarrow$ + if gravity assists

− if gravity opposes

For Flat Plate



Velocity at wall = 0 (viscous flow no-slip)

Velocity far away = 0 (where $T = T_\infty$)

When $T_\infty < T < T_w$, u has a finite value.

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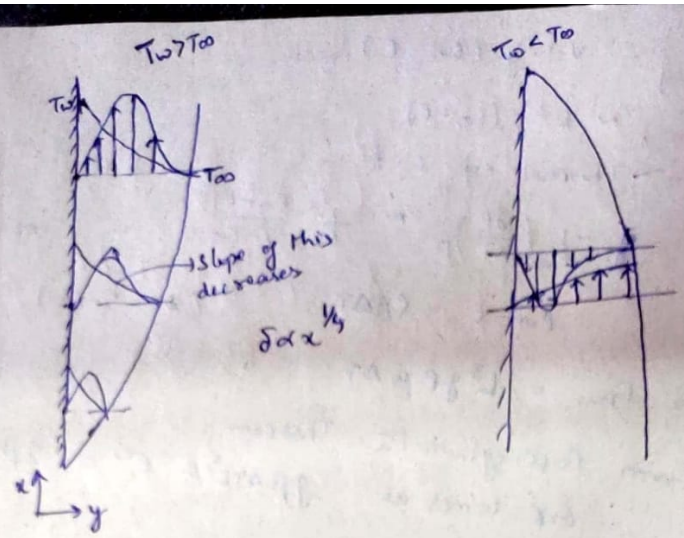


Equations:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + g + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

In the quiescent region, $u = v = 0$

$$\frac{\partial P}{\partial x} = -\rho_\infty g$$

Assuming no body force in y-direction,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\frac{\Delta\rho}{\rho} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\Delta\rho = \rho_\infty - \rho = \beta\rho_\infty(T_\infty - T) \qquad \beta = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\frac{\beta\rho_\infty}{\rho}(T_\infty - T) + \nu\frac{\partial^2 u}{\partial y^2} \quad \text{—(1)}$$

For an ideal gas, $\beta = \frac{1}{\rho}\frac{P}{RT^2} = \frac{1}{T}$

Energy Equation: $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$ —(2)

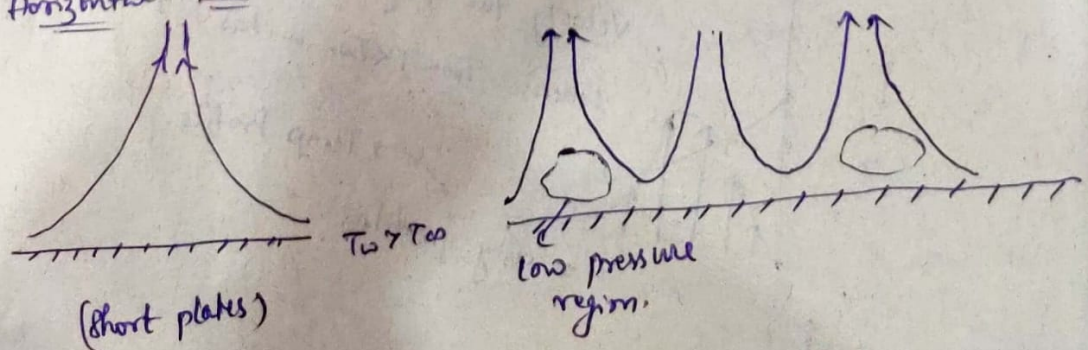
Equation (1) & (2) are solved numerically.

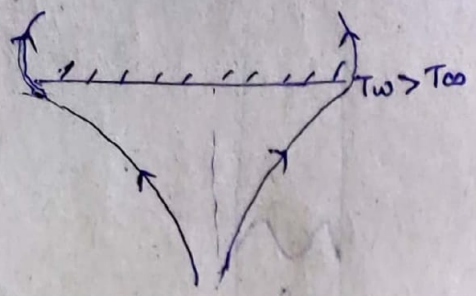
$$Nu_x \propto Gr_x^{1/4}$$

$$\Rightarrow \delta \propto x^{1/4}$$

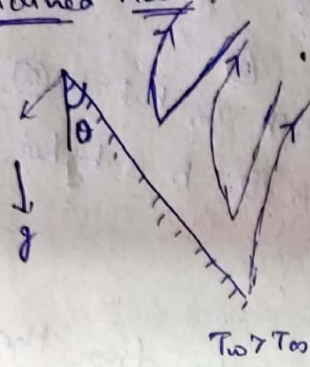
Transition from Linear to turbulent takes place at $Ra_x = 10^9$

For Horizontal Plates



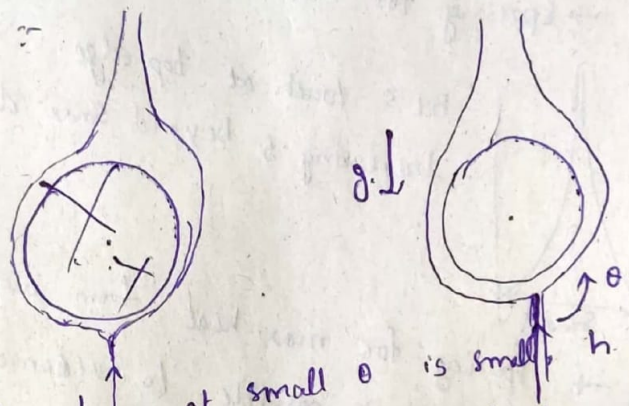


for Inclined Plates:

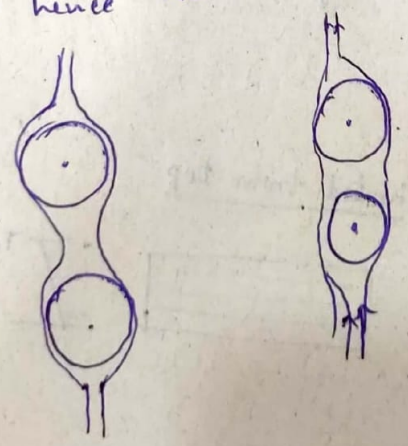


- $F_{Buoyancy}$ will have components normal to the surface and along the surface.
- For small value of $\theta \leq 30°$, use vertical plate co-relation with g replaced by $g\cos\theta$.

for Cylinder :-



As Boundary Layer at small θ is small, h will be large and hence Nu no. decreases with θ.



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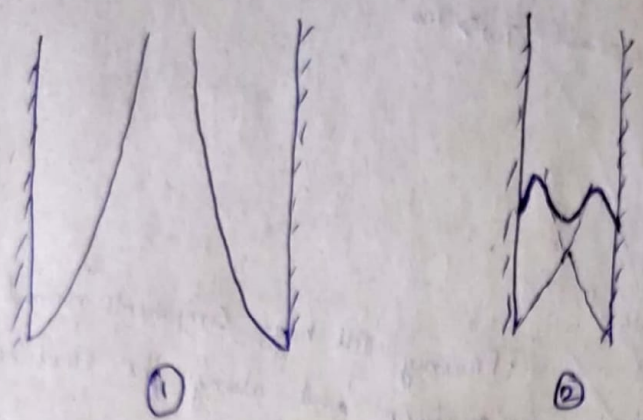
for verticle channels :-

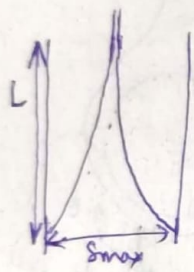


In ①, ~~T~~ heat exchange with T_∞, but in ② heat exchange with higher temp. So in ① h is higher

hence $Nu_{①} > Nu_{②}$

As $S/L \downarrow$, $\overline{Nu} \downarrow$

S_{max} → spacing for max. heat transfer from the plate.



B.L's touch at top edge.

Increasing S beyond S_{max} doesn't increase $\overline{Nu}$.

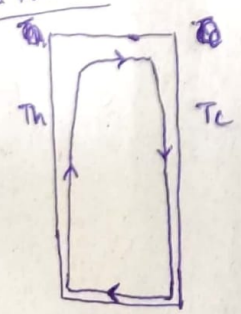
S_{opt} → spacing for max. heat from away of plates.

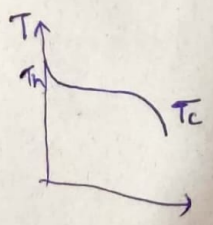
- if $S < S_{max}$, it is possible to accomodate higher no. of heated surfaces ~~by~~ thereby increasing area for heat transfer

$$q = \overline{h} A (T_w - T_\infty)$$

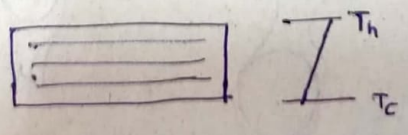
$A \downarrow 2 \times N \times A_w$

heated from sides





heated from top:



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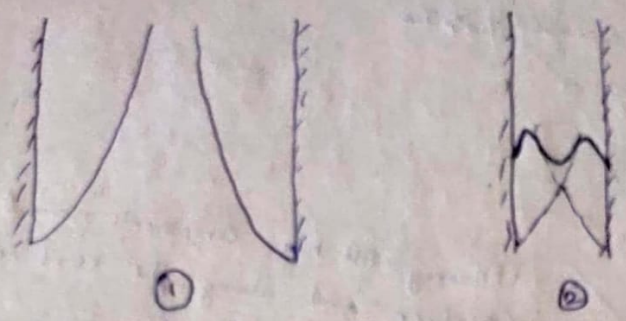
for vertical channels :-



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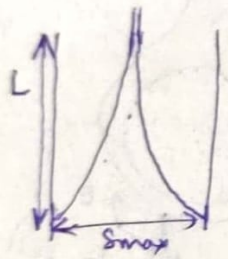

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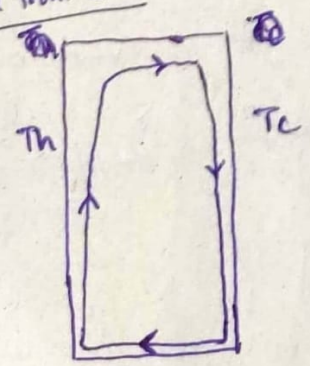
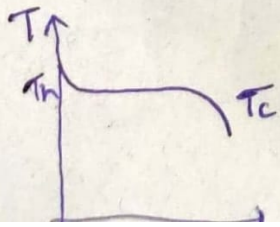
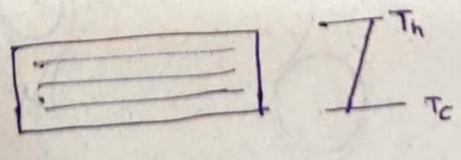




heated from top:



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