Wagner tension field

* 21 constants for an anisotropic material

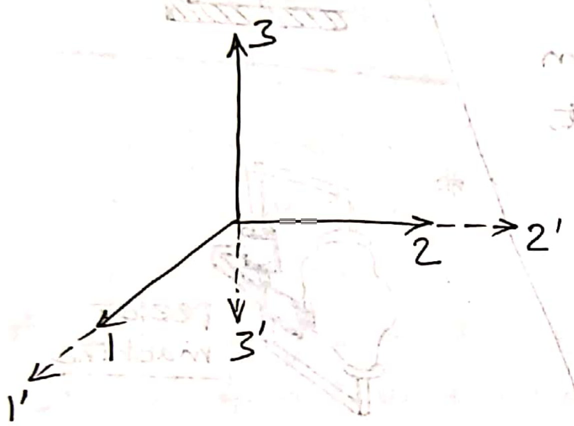
* $\sigma_i = C_{ij}\,\varepsilon_j$

$\varepsilon_i = S_{ij}\,\sigma_j$

($C \equiv$ stiffness, $S \equiv$ compliance)

$$\boxed{S = \frac{1}{C}}$$

*



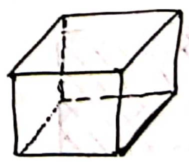
2 coordinate systems; 3 & 3' opposite to each other

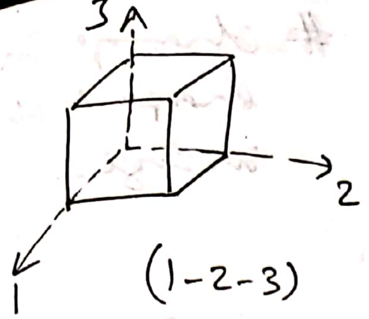
If stiffness of the material is same in both coordinate systems $\Rightarrow$ Monoclinic material (13 constants needed)

* $\sigma_1 = \sigma_{11}$, $\sigma_2 = \sigma_{22}$, $\sigma_3 = \sigma_{33}$
$\sigma_4 = \sigma_{23}$, $\sigma_5 = \sigma_{13}$, $\sigma_6 = \sigma_{12}$ } Voigt notation

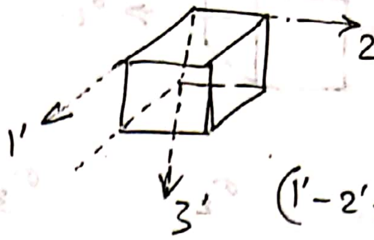
* $\sigma_1 = \sigma_1'$
$\sigma_2 = \sigma_2'$
$\sigma_3 = \sigma_3'$
$\sigma_4 = -\sigma_4'$
$\sigma_5 = -\sigma_5'$
$\sigma_6 = \sigma_6'$

$\varepsilon_1 = \varepsilon_1'$
$\varepsilon_2 = \varepsilon_2'$
$\varepsilon_3 = \varepsilon_3'$
$\varepsilon_4 = -\varepsilon_4'$
$\varepsilon_5 = -\varepsilon_5'$
$\varepsilon_6 = \varepsilon_6'$





(1-2-3)

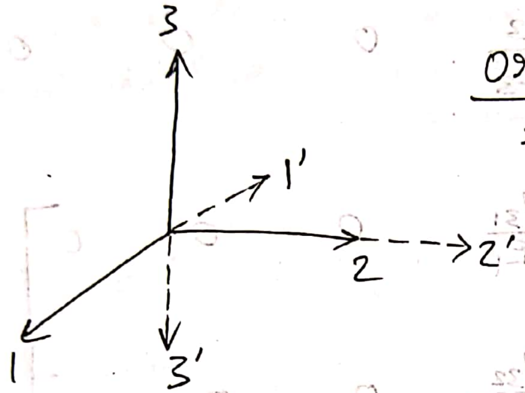


(1'-2'-3')

* $$\sigma_i = \sum_{j=1}^{6} C_{ij}\,\varepsilon_j$$

$$\sigma_i' = \sum_{j=1}^{6} C_{ij}'\,\varepsilon_j'$$

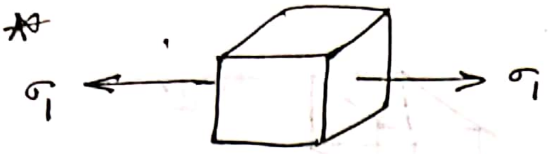
*



<u>Orthotropic material</u>

⇒ S & C same in both coord. axes systems. (9 constants)

$$\begin{array}{l|l}
\sigma_1 = +\sigma_1' & \varepsilon_1 = \varepsilon_1' \\
\sigma_2 = +\sigma_2' & \varepsilon_2 = \varepsilon_2' \\
\sigma_3 = +\sigma_3' & \varepsilon_3 = \varepsilon_3' \\
\sigma_4 = -\sigma_4' & \varepsilon_4 = -\varepsilon_4' \\
\sigma_5 = -\sigma_5' & \varepsilon_5 = -\varepsilon_5' \\
\sigma_6 = -\sigma_6' & \varepsilon_6 = -\varepsilon_6'
\end{array}$$

* 

change in shape & chan[ge] in size.....

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
ε_1	$\frac{\sigma_1}{E_1}$	$\frac{-\nu_{21}}{E_1}\sigma_2$	$\frac{-\nu_{31}}{E_1}\sigma_3$	0	0	0
ε_2	$\frac{-\nu_{12}}{E_2}\sigma_1$	$\frac{\sigma_2}{E_2}$	$\frac{-\nu_{32}}{E_2}\sigma_3$	0	0	0
ε_3	$\frac{-\nu_{13}}{E_3}\sigma_1$	$\frac{-\nu_{23}}{E_3}\sigma_2$	$\frac{\sigma_3}{E_3}$	0	0	0

$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{Bmatrix}
=
\begin{bmatrix}
\frac{1}{E_1} & \frac{-\nu_{21}}{E_1} & \frac{-\nu_{31}}{E_1} & 0 & 0 & 0 \\
\frac{-\nu_{12}}{E_2} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_2} & 0 & 0 & 0 \\
\frac{-\nu_{13}}{E_3} & \frac{-\nu_{23}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2G_4} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2G_5} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2G_6}
\end{bmatrix}
\cdot
\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}
$$

$\downarrow$ 6×1 $\qquad$ $\downarrow$ 6×6 $\qquad$ $\downarrow$ 6×1

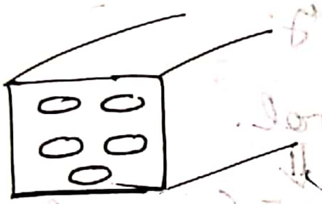
* Transversely isotropic (5 constants needed)

→ valid if exhibited in any 1 coord. axis system (not necessary in ALL coord. axis systems) → material coord. axis system.

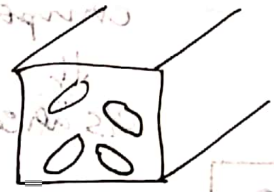
* Isotropic material (2 constants needed) E & ν

* Composites with spherical inclusions randomly distributed — heterogenous & isotropic (2-phase)

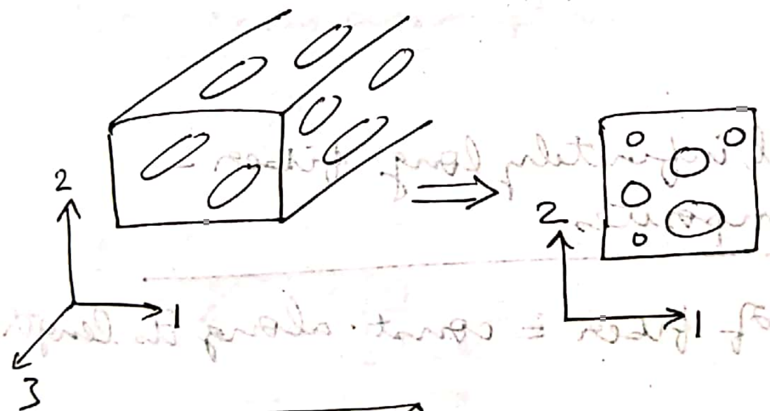
*



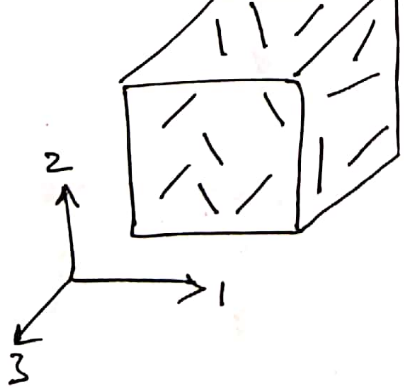
orthotropic



transversely isotropic

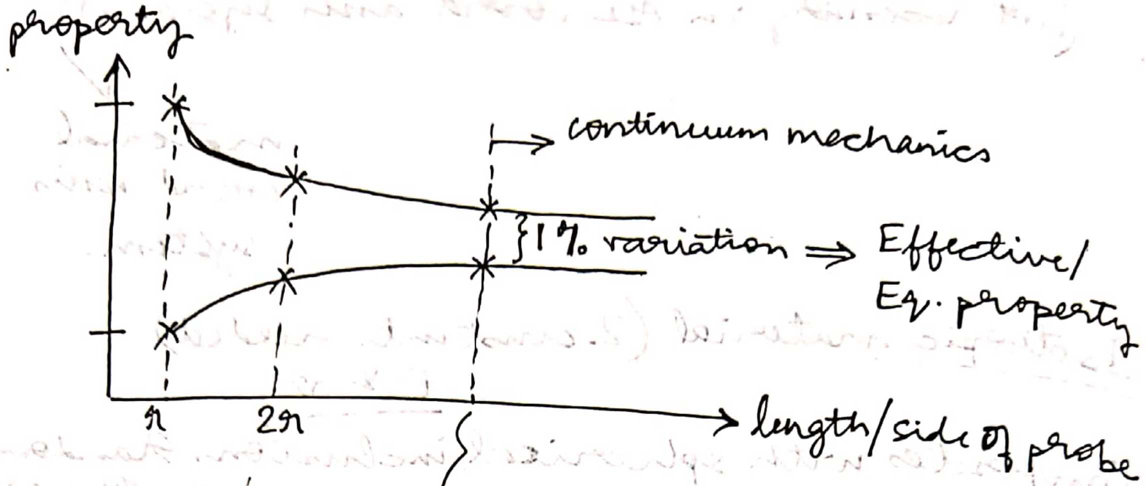


transversely isotropic



transversely isotropic in 2-3 plane. $\Rightarrow G_{23} = \dfrac{E_2}{2(1+\nu_{12})}$

* Average / Effective / Equivalent properties of composites



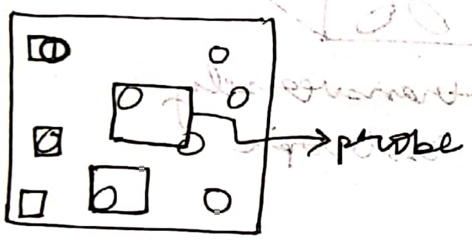
min. length needed to represent composite property.

⇓

c/s area × depth = vol.

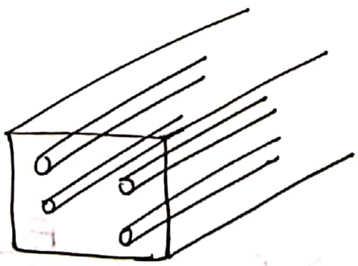
⇓

RVE (min. vol. needed)

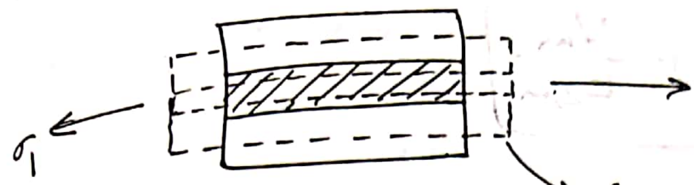


* Uni-directional, infinitely long fiber-reinforced composites

Assumption : c/s of fiber = const. along its length



1) Iso - strain assumption



σ_1 (uniformly across the c/s)

elongations same in fiber & matrix.

$$\varepsilon_{1,c} = \varepsilon_{1,f} = \varepsilon_{1,m}$$

$$P = P_f + P_m$$

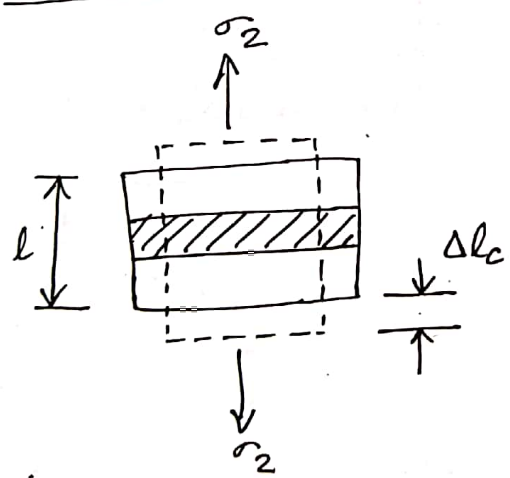
$$\Rightarrow \sigma_{1,c} A = \sigma_{1,f} A_f + \sigma_{1,m} A_m$$

$$\Rightarrow \cancel{\varepsilon_{1,c}} E_{1,c} A = \cancel{\varepsilon_{1,f}} E_{1,f} A_f + \cancel{\varepsilon_{1,m}} E_{1,m} A_m$$

$$\Rightarrow E_{1,c} = E_{1,f} \frac{A_f}{A} + E_{1,m} \frac{A_m}{A}$$

$$\Rightarrow \boxed{E_{1,c} = E_{1,f} v_f + E_{1,m} v_m}$$

2) Iso - stress assumption



$$\sigma_{2,c} = \sigma_{2,f} = \sigma_{2,m}$$

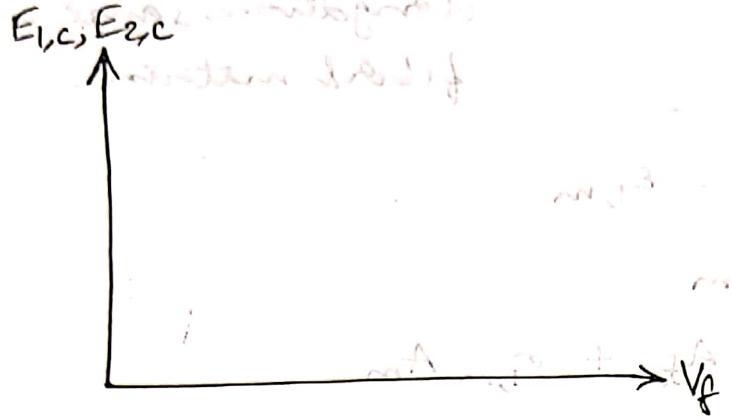
$$\frac{\Delta l_c}{l} = \frac{\Delta l_f}{l} + \frac{\Delta l_m}{l}$$

$$\Rightarrow \frac{\Delta l_c}{l} = \frac{\Delta l_f}{l_f} \times \frac{l_f}{l} + \frac{\Delta l_m}{l_m} \times \frac{l_m}{l}$$

$$\Rightarrow \varepsilon_{2,c} = \varepsilon_{2,f} V_f + \varepsilon_{2,m} V_m$$

$$\Rightarrow \frac{\cancel{\sigma_{2,c}}}{E_{2,c}} = \frac{\cancel{\sigma_{2,f}}}{E_{2,f}} V_f + \frac{\cancel{\sigma_{2,m}}}{E_{2,m}} V_m$$

$$\Rightarrow \boxed{\frac{1}{E_{2,c}} = \frac{V_f}{E_{2,f}} + \frac{V_m}{E_{2,m}}}$$



Q. A composite lamina is made of CF + epoxy. The axial and transverse mod. of CF is 230 & 15 resp. while that of epoxy is 3 GPa. Find the axial & transverse stiffness of composite lamina if $V_f = 0.5$.

$$V_f = V_m = 0.5$$

$$E_{1,f} = 230$$

$$E_{2,f} = 15$$

$$E_m = 3$$

$$E_{1,c} = 230 \times 0.5 + 3 \times 0.5 = 116.5 \text{ GPa}$$

$$E_{2,c} = \frac{1}{\frac{0.5}{15} + \frac{0.5}{3}} = 5 \text{ GPa}$$

Case I : epoxy doped with nanotubes $\Rightarrow E_m = 5\,GPa$

Case II : glass fibers instead of CF $\Rightarrow E_2 = 72\,GPa$

Case III : half CF + half GF .

Case I :

$$E_{1,c} = 230 \times 0.5 + 5 \times 0.5 = 117.5$$

$$E_{2,c} = \frac{1}{\frac{0.5}{15} + \frac{0.5}{5}} = 7.5$$

Case II :

$$E_{1,c} = 230 \times 0.5 + 3 \times 0.5 = 116.5$$

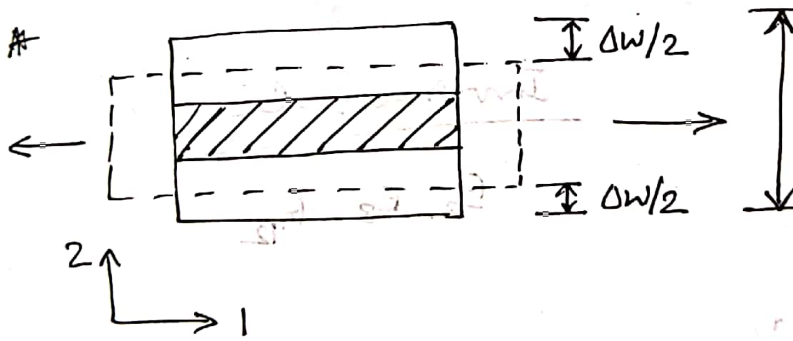
$$E_{2,c} = \frac{1}{\frac{0.5}{72} + \frac{0.5}{3}} = 5.76$$

Case III :

$$E_{1,c} = 230 \times 0.25 +$$

$$E_{2,c} = \frac{1}{\frac{0.25}{15} + \frac{0.25}{3} + \frac{0.25}{72}} = 5.35$$

*



(Effective Poisson's ratio)

$$\nu_{12} = \frac{-\varepsilon_2}{\varepsilon_1}$$

$$\Delta w = \Delta w_f + \Delta w_m$$

$$\varepsilon_{1,f} = \varepsilon_{1,c} = \varepsilon_{1,m} \quad \text{(iso-strain)}$$

$$\varepsilon_2 = \frac{\Delta w}{w} \Rightarrow \Delta w = \varepsilon_2 w = -w \nu_{12} \varepsilon_1$$

$$\varepsilon_{2,f} = \frac{\Delta w_f}{w_f} \Rightarrow \Delta w_f = -w_f \nu_{12,f} \varepsilon_{1,f}$$

$$\varepsilon_{2,m} = \frac{\Delta w_m}{w_m} \Rightarrow \Delta w_m = -w_m \nu_{12,m} \varepsilon_{1,m}$$

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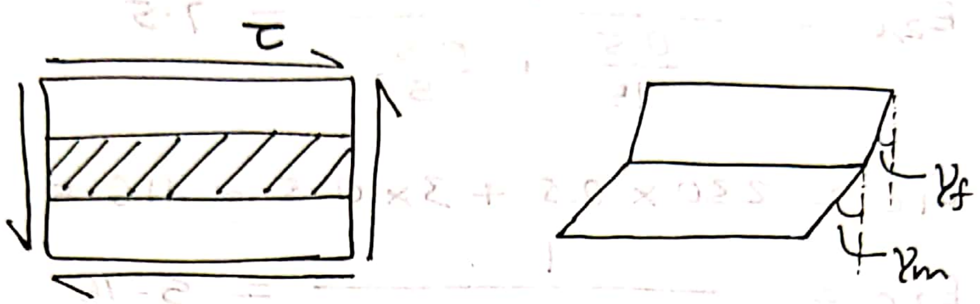
$$\Rightarrow -w\nu_{12}\varepsilon_1 = -w_f\nu_{12,f}\varepsilon_{1,f} - w_m\nu_{12,m}\varepsilon_{1,m}$$

$$\Rightarrow \boxed{\nu_{12,c} = V_f\nu_{12,f} + V_m\nu_{12,m}}.$$

* Shear



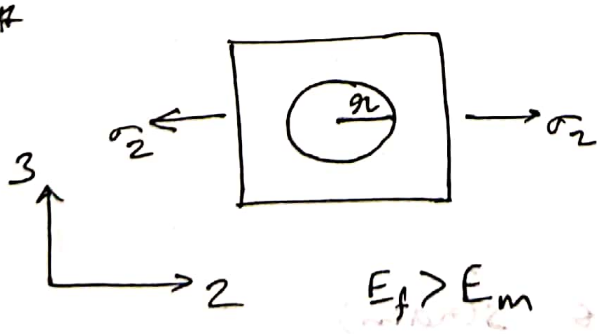
$$\gamma w = \gamma_m w_m + \gamma_f w_f \text{ (iso - stress)}$$

$$\Rightarrow \frac{\tau}{G} w = \frac{\tau_m}{G_m} w_m + \frac{\tau_f}{G_f} w_f$$

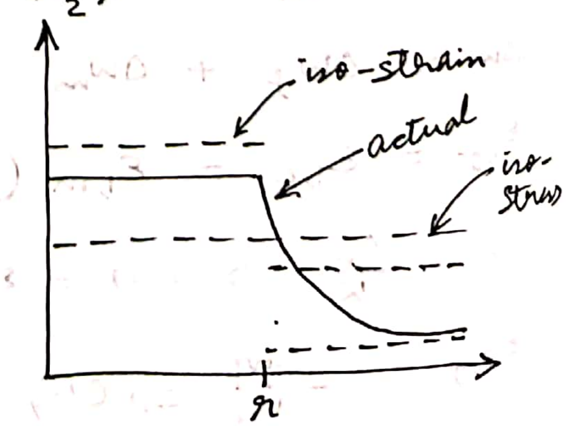
$$\Rightarrow \boxed{\frac{1}{G} = \frac{V_m}{G_m} + \frac{V_f}{G_f}} \quad (\tau = \tau_m = \tau_f)$$

* Rule	Inverse rule
E_1, nu_{12}	E_2, E_3, G_{12}

*



$E_f > E_m$



* Stress-partitioning

$$\sigma_{f,2} \neq \sigma_{m,2} = \eta_2 \sigma_{f,2}$$ (η_2 = stress-partitioning factor)

$$\Rightarrow \frac{\sigma_{c,2}}{E_{c,2}} = \frac{V_f \sigma_{f,2}}{E_{f,2}} + \frac{V_m \eta_2 \sigma_{f,2}}{E_m} \quad (\sigma_{m,2} = \eta_2 \sigma_{f,2})$$

$$\Rightarrow \frac{\sigma_{c,2}}{E_{c,2}} = \left(\frac{V_f}{E_{f,2}} + \frac{V_m \eta_2}{E_m}\right) \sigma_{f,2}$$

$$\Rightarrow \frac{1}{E_{c,2}} = \frac{1}{V_f + \eta_2 V_m} \left(\frac{V_f}{E_{f,2}} + \frac{V_m \eta_2}{E_m}\right)$$

$$\sigma_{c,2} = \sigma_{f,2}(V_f + \eta_2 V_m)$$

$$\sigma_{c,2} = \sigma_{f,2} V_f + V_m \sigma_{m,2}$$

(How?)

$$\frac{1}{P_c} = \frac{1}{V_f + \eta_p V_m} \left(\frac{V_f}{P_f} + \frac{V_m \eta_p}{P_m}\right)$$

$P_f >> P_m \Rightarrow \eta = f(\nu_m)$ $\quad \eta_k = \frac{1}{2(1-\nu_m)}$

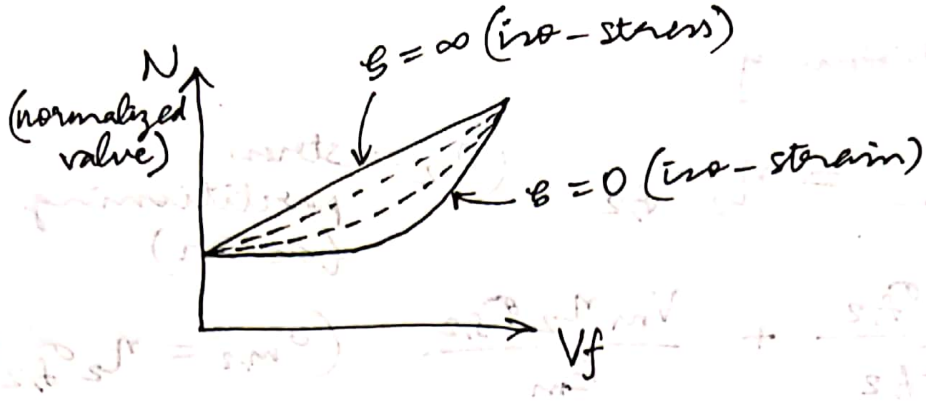
$\eta_2 = \frac{1}{2}$, $\eta_{23} = 0.6$

Bulk modulus

* Halpin-Tsai (unified formula)

$$\frac{P_c}{P_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}$$

$$\Rightarrow \eta = \frac{\frac{P_f}{P_m} - 1}{\frac{P_f}{P_m} + \xi} \quad (\text{for } P_c = P_f)$$

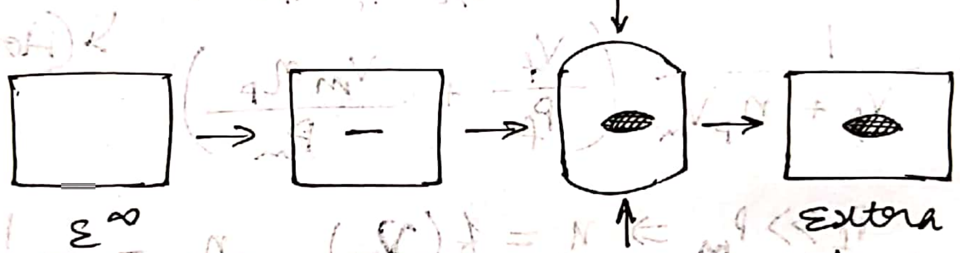


* Chamis rule

(Only) In inverse law of mixtures (only for glass-fiber composites)

Replace $V_f \to \sqrt{V_f}$, etc

* Eigen strain



$$\underbrace{\varepsilon^*}_{\text{eigen strain (extra strain)}} = \underbrace{[E]}_{\text{Eshelby tensor}} \varepsilon^\infty$$

eigen strain (extra strain)

Eshelby tensor $\equiv f(\nu)$ only

(by Eshelby, 1957)

for one ellipsoid void $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2$:

Image strain – strain on an inclusion due to other inclusions

Mean field assumption – image strain is equally distributed among all inclusions.

Mori – Tanaka (1979)
(eff. prop. of heterogenous materials)

Calculations are size independent (fiber-radius)

$\frac{E_f}{E_m}$ is considered, not E_f & E_m separately.

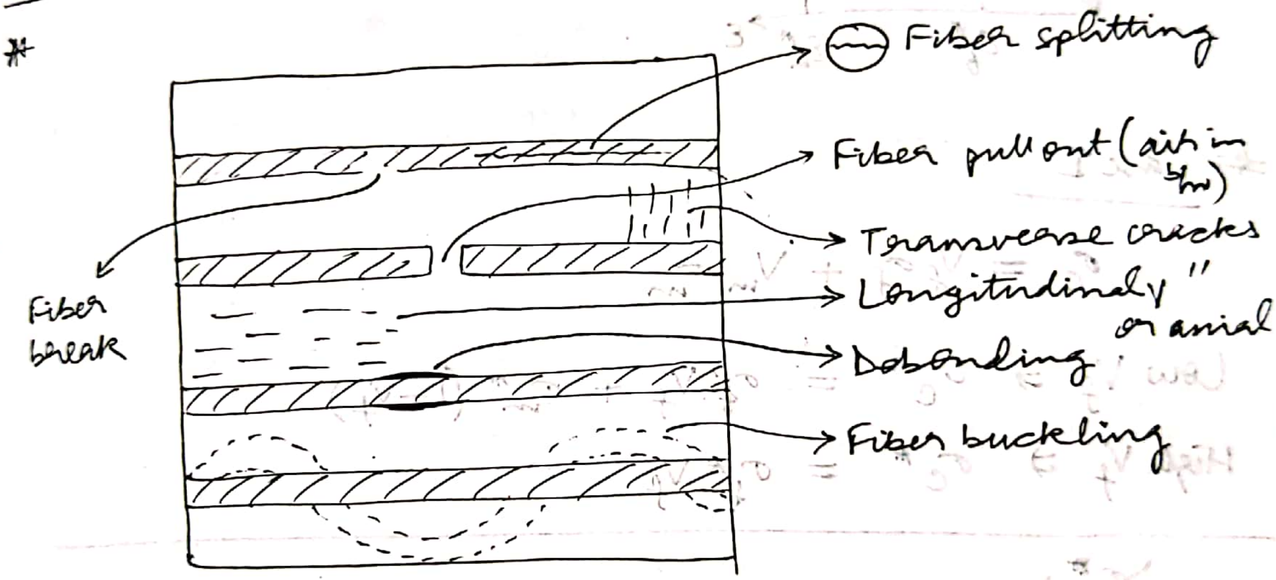
 → same properties if fibers are same. (location-independent)

But

 (diff. cross-section)

× Tutorial notes (see pics) ×

*



* Strength - localised property (no transformⁿ. rules)
  Stiffness - average " (tensor ⇒ transformⁿ. laws apply)

* Stiffness VS Damage models

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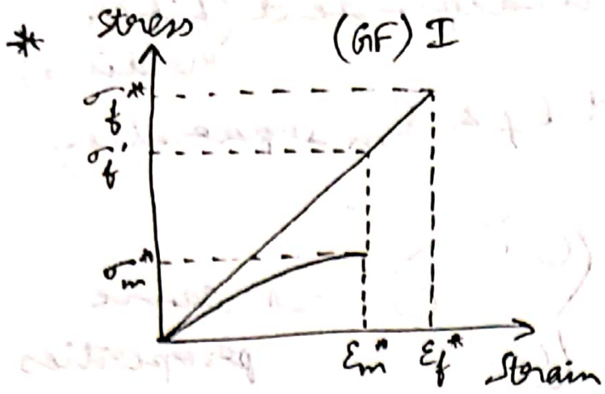
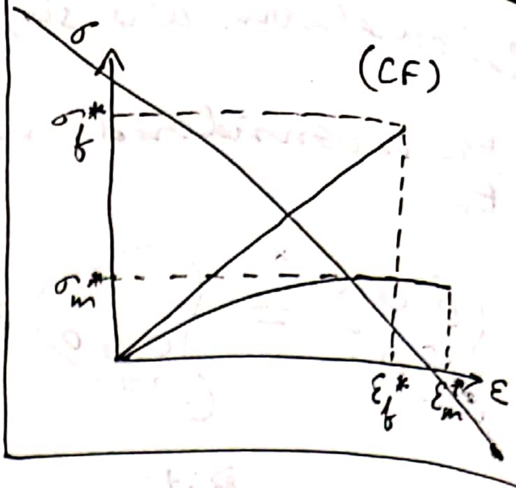
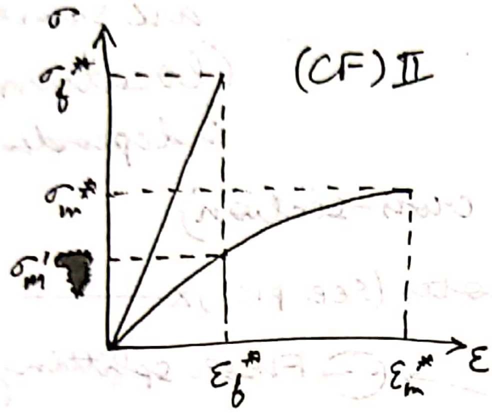
*





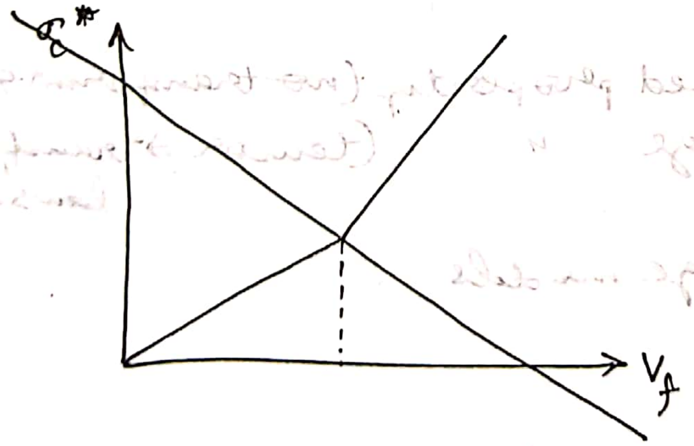


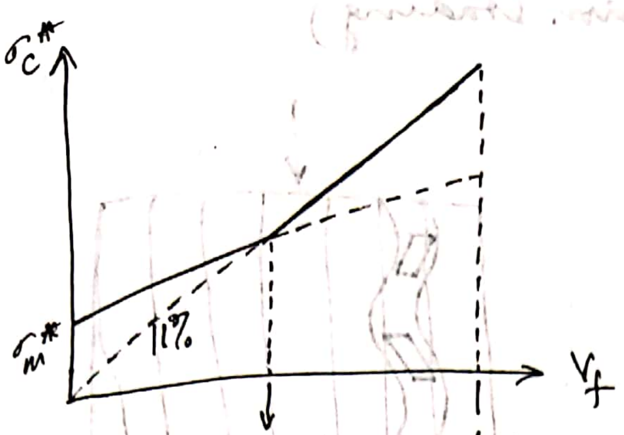
Case I

$$\sigma_c = V_f \sigma_f + V_m \sigma_m$$

Low $V_f \Rightarrow \sigma_c^* = \sigma_f' V_f + \sigma_m^* (1 - V_f)$

High $V_f \Rightarrow \sigma_c^* = \sigma_f^* V_f$





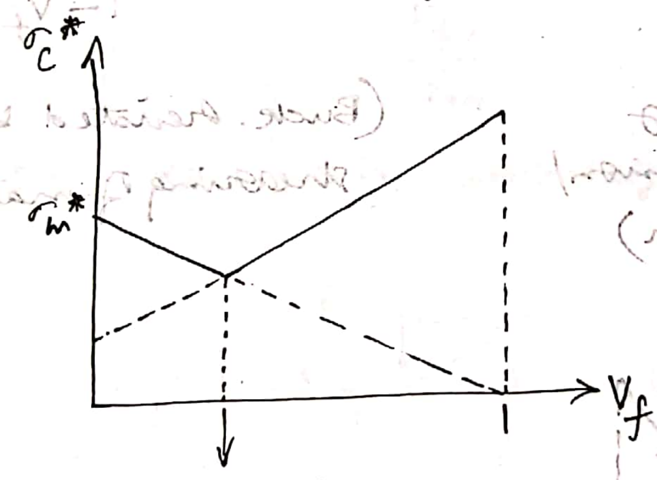
min. V_f reqd. to make effective composites (case I)

Case II

$$\sigma_c = V_f \sigma_f + V_m \sigma_m$$

Low $V_f \Rightarrow \sigma_c^* = \sigma_m^* (1 - V_f)$

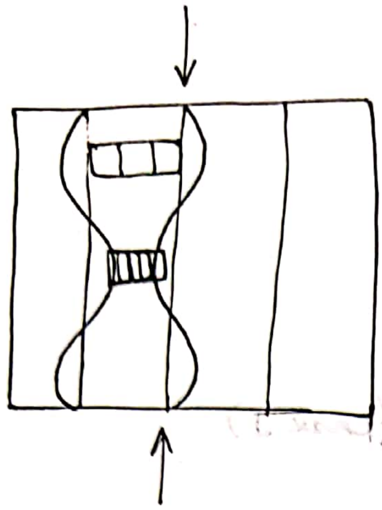
High $V_f \Rightarrow \sigma_c^* = \sigma_f^* V_f + \sigma_m' (1 - V_f)$



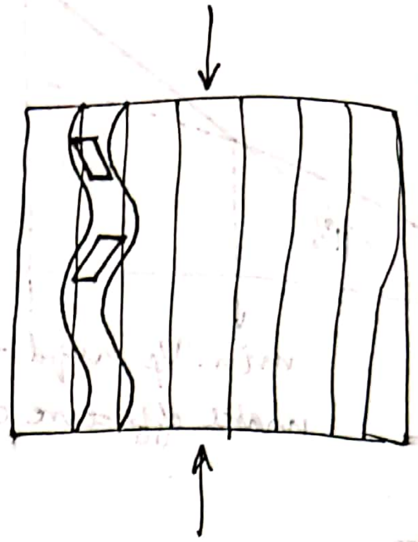
min. V_f reqd. to make effective composites (case II)

PTO

* Buckling (Compression loading)



Low V_f
(Out of plane buckling)



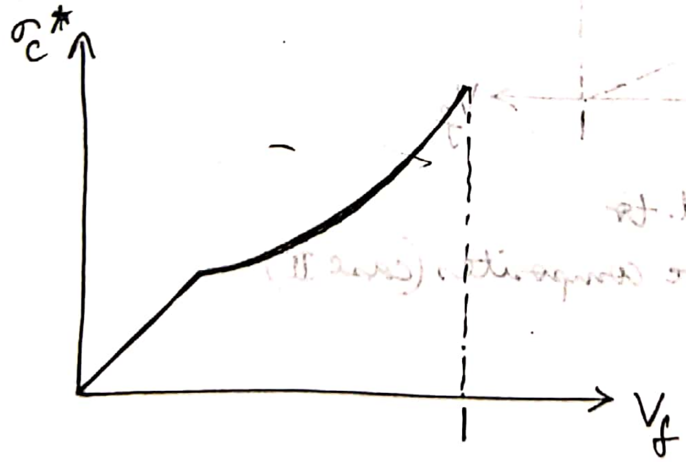
High V_f
(In plane buckling)

$$\sigma_c^* = \frac{2V_f^{3/2}\sqrt{E_m E_f}}{\sqrt{3(1-V_f)}}$$

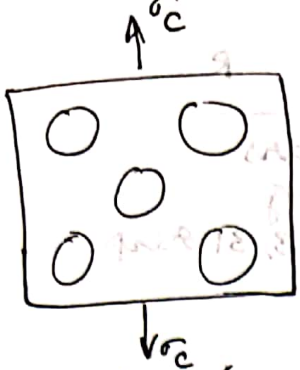
(Buck. resisted by transverse extension/comp. of matrix)

$$\sigma_c^* = \frac{G_m}{1-V_f}$$

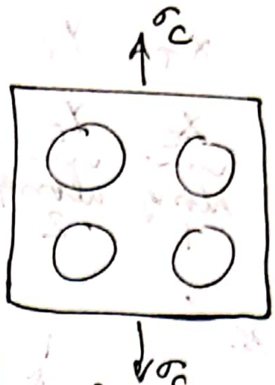
(Buck. resisted by shearing of matrix)



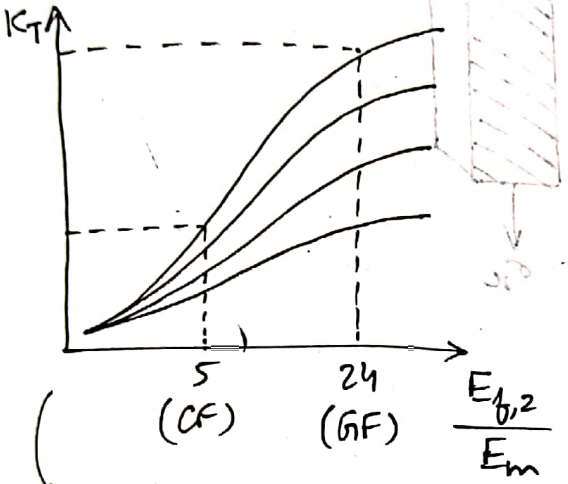
* Stress concn. K_T



Hex (max. 92%)



Square (max. 72%)

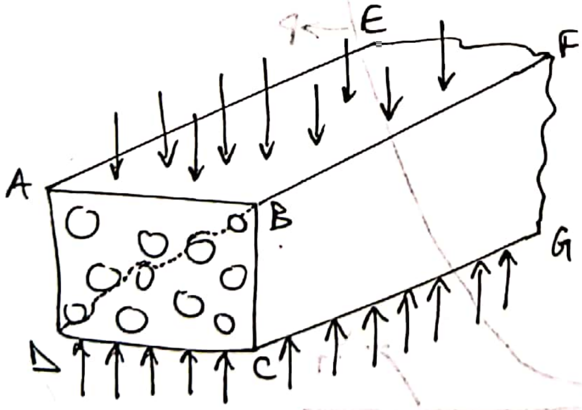


$$\sigma_m^* = K_T \sigma_c$$

$\Rightarrow \sigma_{c,GF} < \sigma_{c,CF}$ for same V_f.

By SKUDRA (for hex packing)

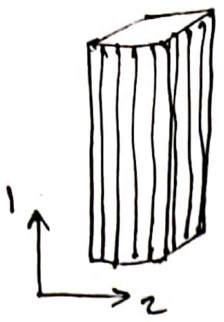
*



Crack along DBFH easier than crack along EFCD.

PTO

*



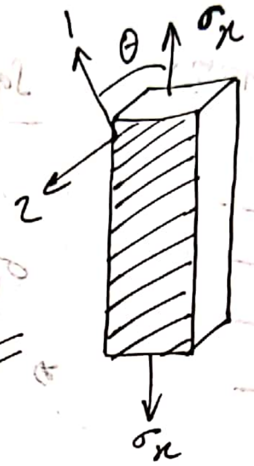
$$\underbrace{\begin{matrix} X_T \\ X_C \end{matrix}}_{\text{along } 1} \quad \underbrace{\begin{matrix} Y_T \\ Y_C \end{matrix}}_{\text{along } 2} \quad \underbrace{P \quad Q \quad R}_{\text{shears along}}$$

12, 23, 31 resp.

$X_C < \sigma_1 < X_T$

$Y_C < \sigma_2 < Y_T$

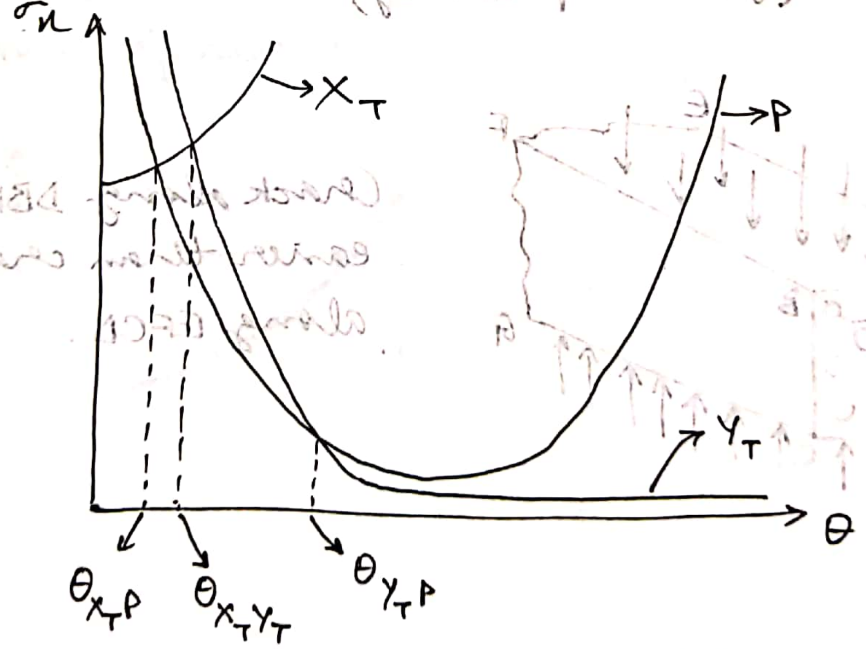
$|\sigma_{12}| < P$



$\sigma_1 = \sigma_x \cos^2\theta$

$\sigma_2 = \sigma_x \sin^2\theta$

$\sigma_{12} = |-\sigma_x \sin\theta\cos\theta|$



$(\theta_{X_TP} < \theta_{X_TY_T} < \theta_{Y_TP})$

Q. $X_T = 1250$ MPa

$Y_T = 53.4$ MPa

$P = 99.3$ MPa

(i) $\sigma_x < \dfrac{X_T}{\cos^2\theta}$

(ii) $\sigma_x < \dfrac{Y_T}{\sin^2\theta}$

(iii) $\sigma_x < \dfrac{P}{\sin\theta\cos\theta}$

(i) $\dfrac{X_T}{\cos^2\theta} = \dfrac{Y_T}{\sin^2\theta}$

$\Rightarrow \theta = \tan^{-1}\left(\sqrt{\dfrac{Y_T}{X_T}}\right)$

$= 10.86^\circ$

(ii) $\dfrac{Y_T}{\sin^2\theta} = \dfrac{P}{\cancel{\sin\theta}\cos\theta}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{Y_T}{P}\right)$

$= 28.26^\circ$

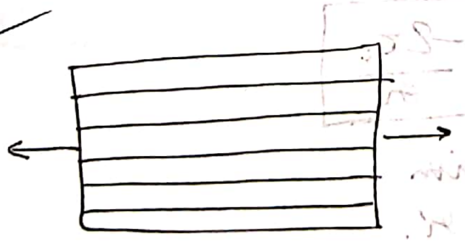
(iii) $\dfrac{X_T}{\cos^2\theta} = \dfrac{P}{\sin\theta\cancel{\cos\theta}}$

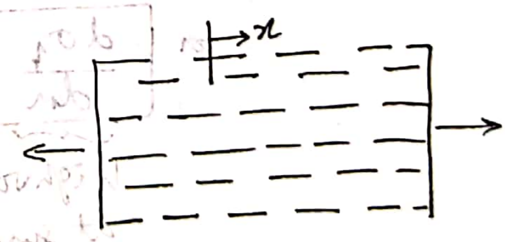
$\Rightarrow \theta = \tan^{-1}\left(\dfrac{P}{X_T}\right)$

$= 3.9^\circ$

Fiber alignment in b/w 0 & 3.9° is better than absolute 0° ⇒ strength increases
(For uniaxial loadings)
For pure shear, 45° alignment optimal.

Q.





$\varepsilon_f = \varepsilon_m = \varepsilon_c$

$\varepsilon_f = E_f\sigma_f$

$E_c = f(E_f, E_m, V_f)$

$\varepsilon_f = \varepsilon_m = \varepsilon_c$? (x)

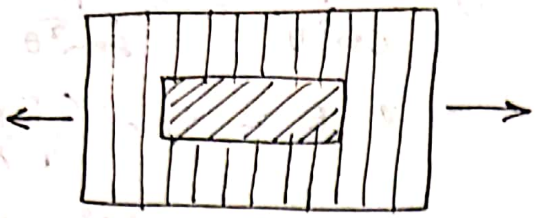
$\varepsilon_f = E_f\sigma_f$? (x)

$\varepsilon_f \neq \varepsilon_m \neq \varepsilon_c$

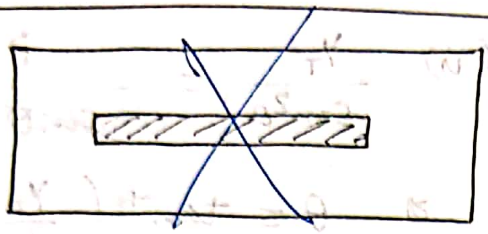
$\sigma_f = f(x)$

$E_c = f(E_f, E_m, AR, V_f)$

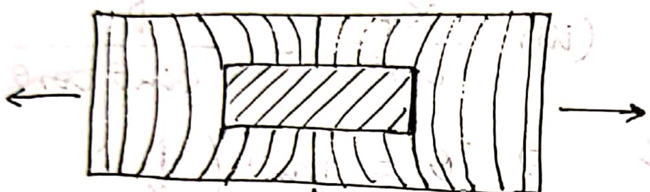
Magnified view :



$E_f >> E_m$



Cox model (Book by GIBSON)

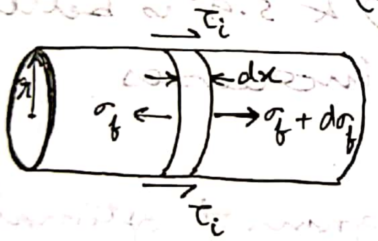


Shear lag models (Cox, Kelly)

(stress transfer from matrix to fiber is only shear. σ_f at vertical i/f $= 0$)

$(x=0)$ max. $\sigma_f \Rightarrow \frac{d\sigma_x}{dx} = 0$

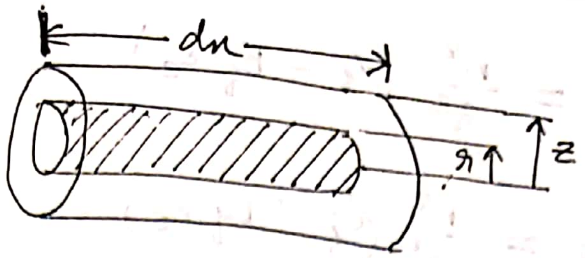
$(\because \tau_i = 0)$



$\Sigma F_x = 0 \Rightarrow (d\sigma_f)\pi r^2 = -2\pi r \tau_i dx$

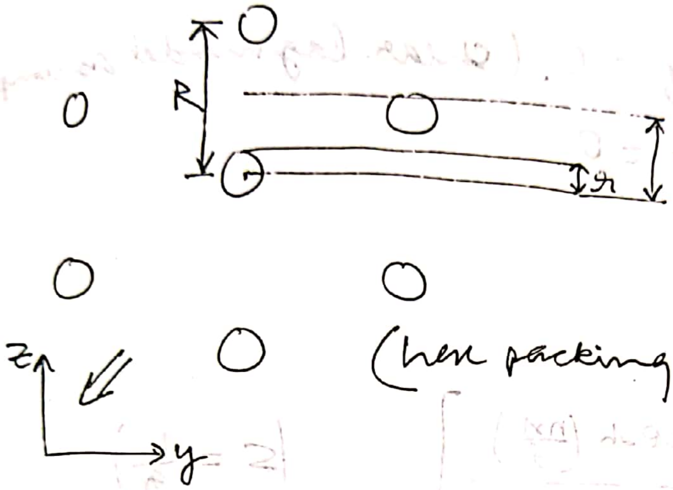
$$\text{or} \quad \boxed{\frac{d\sigma_f}{dx} = \frac{-2\tau_i}{r}}$$

high variation at smaller 'r'.

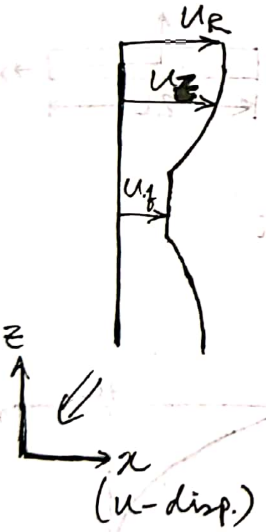


$$2\pi z \tau_z \, dx = 2\pi r \tau_i \, dx$$

$$\Rightarrow \boxed{\tau_z = \frac{r\tau_i}{z}}$$



(hex packing)



$$\gamma = \frac{du}{dz} = \frac{\tau}{G_m}$$

$$\Rightarrow \int_{u_f}^{u_R} du = \int_{r}^{R} \frac{\tau_i r}{z} \cdot \frac{1}{G_m} \, dz$$

$$\Rightarrow \tau_i = \frac{G_m (u_R - u_f)}{r \ln\left(\frac{R}{r}\right)}$$

$$\Rightarrow \frac{\partial \tau_i}{\partial x} = \frac{G_m \left(\frac{\partial u_R}{dx} - \frac{\partial u_f}{dx}\right)}{r \ln\left(\frac{R}{r}\right)}$$

$$\Rightarrow -\frac{r}{2} \cdot \frac{\partial^2 \sigma_f}{\partial x^2} = \frac{G_m (\varepsilon_c - \sigma_f / E_f)}{r \ln\left(\frac{R}{r}\right)}$$

Let $n^2 = \dfrac{2 G_m}{E_f \ln\left(\frac{R}{r}\right)}$ → subst.

$$\Rightarrow \frac{\partial^2 \sigma_f}{\partial x^2} = \frac{n^2}{r^2}(\sigma_f - E_f \varepsilon_c)$$

$$\Rightarrow \frac{d^2 \sigma_f}{dx^2} = \frac{n^2}{r^2}(\sigma_f - E_f \varepsilon_c)$$

$$\Rightarrow \sigma_f = E_f \varepsilon_c + B \sinh\left(\frac{nx}{r}\right) + D \cosh\left(\frac{nx}{r}\right)$$

<u>BCs:</u>

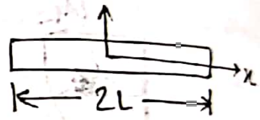
(i) At $x = \pm l$, $\sigma_f = 0$ (shear lag model assump)

(ii) At $x = 0$, $\frac{d\sigma_f}{dx} = 0$

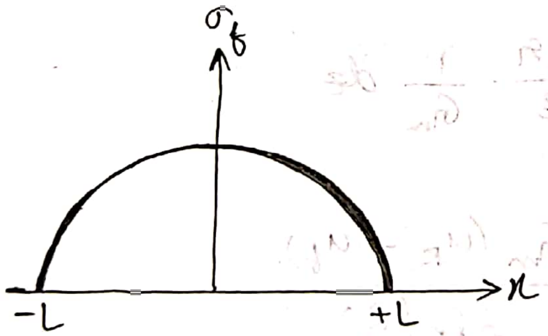
$\Rightarrow B =$

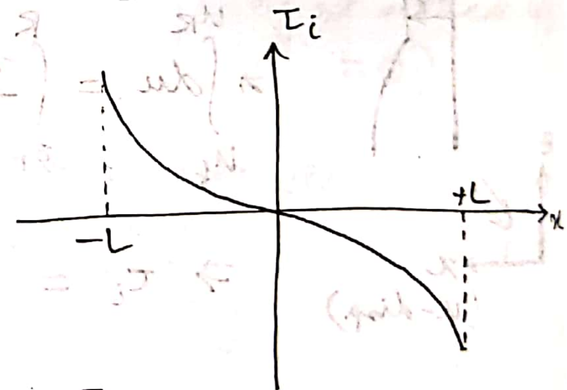
$D =$

$$\Rightarrow \sigma_f = E_f \varepsilon_1 \left[1 - \frac{\cosh\left(\frac{nx}{r}\right)}{\cosh(ns)}\right] \qquad \left(s = \frac{L}{r}\right)$$



$$\Rightarrow \tau_i = \frac{1}{2} n E_f \varepsilon_1 \left[\frac{\sinh\left(\frac{nx}{r}\right)}{\cosh(ns)}\right]$$





$$\sigma_f^{max.} = E_f \varepsilon_1 \left[1 - \frac{1}{\cosh(ns)}\right]$$

$$\tau_i^{max.} = \frac{1}{2} n E_f \varepsilon_1 [\tanh(ns)]$$

$$\sigma_f^{avg.} \neq 0, \quad \tau_i^{avg.} = 0.$$

$$\sigma_f^{avg} = \frac{\int_{-L}^{+L} \sigma_f \, dx}{\int_{-L}^{+L} dx}$$

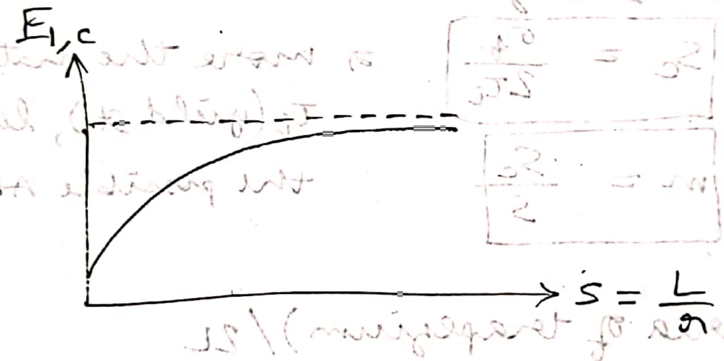
$$\Rightarrow \bar{\sigma}_f = E_f \varepsilon_1 \left\{ 1 - \left[\frac{\tanh(ns)}{ns} \right] \right\}$$

$$\bar{\sigma}_m = E_m \varepsilon_1$$

$$\bar{\sigma}_c = V_f \bar{\sigma}_f + V_m \bar{\sigma}_m$$

$$\Rightarrow E_c \varepsilon_1 = V_f \bar{\sigma}_f + V_m \bar{\sigma}_m$$

$$\Rightarrow E_{1,c} = V_f E_f + V_m E_m - V_f E_f \left(\frac{\tanh(ns)}{ns} \right)$$

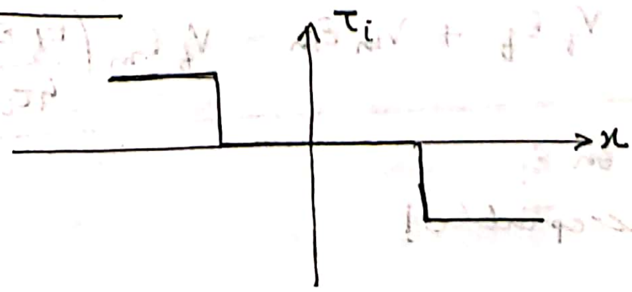


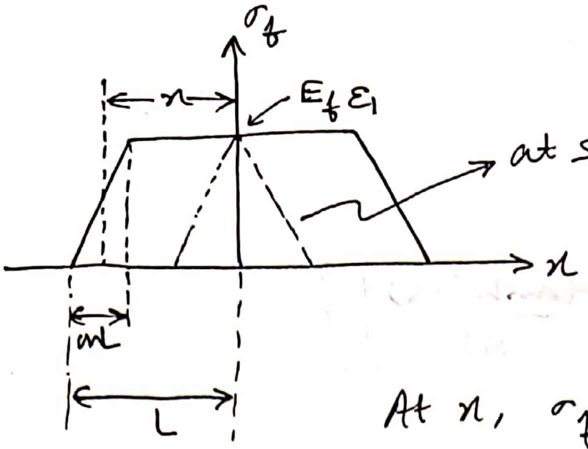
$$\Rightarrow E_{1,c} = \eta_l E_f + V_m E_m \qquad \left[\eta_l = 1 - \frac{\tanh(ns)}{ns} \right]$$

(length η factor)

Min. fiber length reqd. for a meaningful composite = ? (AR)

* Kelly model





At x, $\sigma_f = \frac{2\tau_i}{r}(L-x)$

$= 2\tau_i m s \quad \left(s = \frac{L}{r}\right)$

$\Rightarrow \boxed{m = \frac{\sigma_f}{2\tau_i s} = \frac{E_f \varepsilon_1}{2\tau_i s}}$

At $m=1$, $s = \frac{\sigma_f}{2\tau_i}$ = critical AR for max. theoretical stress at the fiber center. Min. fiber length for max. strength.

$\boxed{S_c = \frac{\sigma_f}{2\tau_i}}$ $\Rightarrow$ more the matrix τ_i (yield st.), lesser the possible AR.

$\Rightarrow \boxed{m = \frac{S_c}{s}}$

Avg. σ_f = (area of trapezium)/2L

$= \frac{1}{2} \times \sigma_f \times (2L + 2L - 2mL) \times \frac{1}{2L}$

$\Rightarrow \bar{\sigma}_f = \sigma_f\left(1 - \frac{m}{2}\right)$

& $\bar{\sigma}_m = E_m \varepsilon_1$

$\Rightarrow E_{1,c} \cancel{\varepsilon_1} = V_f E_f \cancel{\varepsilon_1}\left(1 - \frac{m}{2}\right) + V_m E_m \cancel{\varepsilon_1}$

$\Rightarrow \boxed{E_{1,c} = V_f E_f + V_m E_m - V_f E_m\left(\frac{E_f \varepsilon_1}{4\tau_i s}\right)}$

depends on ε_1

$\therefore$ not acceptable!

Avg. stress in the composite when the fiber fails = ?

$$\overline{\sigma_c} = V_f \sigma_f \left(1 - \frac{m}{2}\right) + V_m \frac{E_m}{E_f} \sigma_f$$

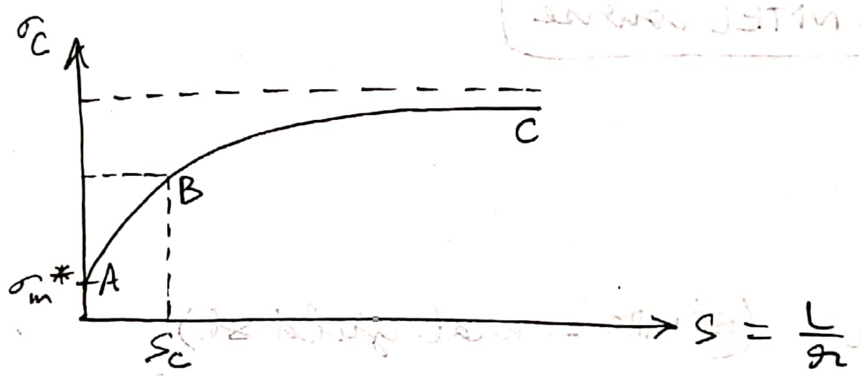
$$\Rightarrow \boxed{\overline{\sigma_c} = \sigma_f \left[V_f \left(1 - \frac{S_c}{2S}\right) + V_m \frac{E_m}{E_f}\right]}$$

$S \to 0 \Rightarrow$ matrix fails ($\sigma_c = \sigma_m^*$)

$S \to \infty \Rightarrow$ same as unidirectional comp. ($\sigma_c = \sigma_f^*$)

$S = S_c \Rightarrow \overline{\sigma_c} = \sigma_f \left(\frac{1}{2} V_f + V_m \frac{E_m}{E_f}\right)$

$\Rightarrow$ fiber takes only 1/2 of its max. possible load limit.



If $\tau_i \uparrow$, AR $\downarrow$ $\Rightarrow$ pt. B shifts left

$\Rightarrow$ cheaper to make such short fiber composites.

If cost is not imp. $\Rightarrow$ UD comp. preferred.

Q. Polyester GF hollow tube,

OD = 20 mm, avg. fiber length = 5mm,

wall thickness = 10% of OD.

Fiber fracture during mfg. process.

Now, GF in outer half have avg. length = 1mm.

Calc strength and stiffness of the tube.

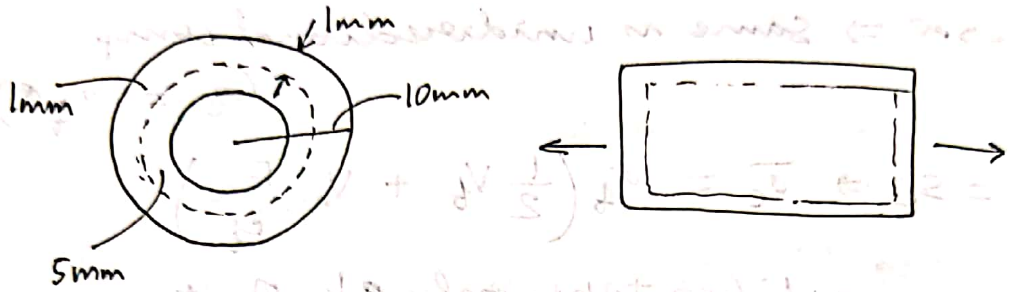
$E_m = 2\ GPa$, $\nu_m = 0.37$

$E_f = 70\ GPa$, $\nu_f = 0.22$

$V_f = 0.60$

$\tau_i = 30\ MPa$, $\frac{R}{r} = \underbrace{16}_{\text{for 1mm}}, \underbrace{20}_{\text{for 5mm}}$

$d_f = 50\ \mu m$



* $\boxed{\text{MOHITE} \rightarrow \text{NPTEL course}}$

Q.1

$V_f = 0.5$

$50 = K_T \sigma_c$ (50 MPa = mat. yield st.)

From graph $K_T = 1.25$ for CF

$\Rightarrow \sigma_c = \frac{50}{1.25} = 40\ MPa$

$K_T \approx 1.75$ for GF

$\Rightarrow \sigma_c = \frac{50}{1.75} = 28.56\ MPa$

As $\frac{E_f}{E_m} \uparrow$, $K_T \uparrow$ and K_T starts depending on V_f.

Q:2

$$\sigma_m^* = \sigma_f^* V_f + \sigma_m' (1-V_f)$$ → V_f so that $\sigma_c^* = \sigma_m^*$

$$= (\sigma_f^* - \sigma_m') V_f + \sigma_m'$$

$$\Rightarrow V_f = \frac{\sigma_m^* - \sigma_m'}{\sigma_f^* - \sigma_m'}$$

$$= \frac{75 - 50}{500 - 50}$$

$$= \frac{\cancel{25}\ 5}{\cancel{450}\ 90}$$

$$\Rightarrow V_f = 0.056$$

$$\sigma_m^* (1-V_f) = \sigma_f^* V_f + \sigma_m' (1-V_f)$$ → V_f for lowest σ_c^*.

$$\sigma_m^* - \sigma_m^* V_f = (\sigma_f^* - \sigma_m') V_f + \sigma_m'$$

$$\Rightarrow V_f = \frac{\sigma_m^* - \sigma_m'}{\sigma_f^* - \sigma_m' + \sigma_m^*}$$

$$= \frac{50 - 25}{500 - 50 + 50}$$

$$= \frac{\cancel{25}}{\cancel{500}\ 20}$$

$$\Rightarrow V_f = 0.05$$

Q:3

$$\varepsilon_1^C < \frac{\sigma_1 - \nu_{12}\sigma_2}{E_1} < \varepsilon_1^T$$

$$\varepsilon_2^C < \frac{\sigma_2 - \nu_{21}\sigma_1}{E_2} < \varepsilon_2^T$$

$$\left|\frac{\tau_{12}}{G_{12}}\right| < |\gamma_{12}|$$

$$\Rightarrow \frac{\sigma_x \cos^2\theta - \nu_{12}\sigma_x \sin^2\theta}{E_1} < \varepsilon_1^T$$

$$\frac{\sigma_x s^2\theta - \nu_{21}\sigma_x c^2\theta}{E_2} < \varepsilon_2^T$$

$$\left|\frac{\sigma_x s\theta c\theta}{G_{12}}\right| < |\gamma_{12}|$$

- ε_1^T & γ_{12}

$$\Rightarrow \frac{\varepsilon_1^T E_1}{c^2\theta - \nu_{12} s^2\theta} = \frac{\gamma_{12} G_{12}}{s\theta\, c\theta}$$

$$\Rightarrow \frac{X_T}{c^2\theta - \nu_{12} s^2\theta} = \frac{P}{s\theta c\theta}$$

$$\Rightarrow \frac{1450}{c^2\theta - \quad s^2\theta} = \frac{99.3}{s\theta\, c\theta} \Rightarrow \theta =$$

ε_1^T & ε_2^T

$$\Rightarrow \frac{X_T}{c^2\theta - \nu_{12} s^2\theta} = \frac{Y_T}{s^2\theta - \nu_{21} c^2\theta}$$

$$\Rightarrow \frac{1450}{c^2\theta - \quad s^2\theta} = \frac{53.4}{s^2\theta - \quad c^2\theta} \Rightarrow \theta =$$

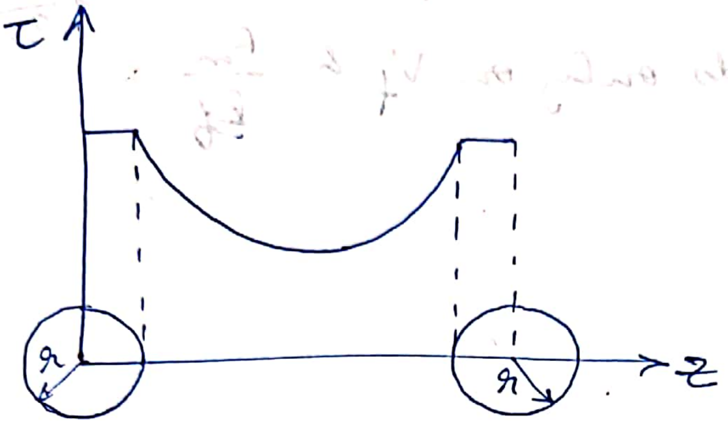
ε_2^T & γ_{12}

$$\Rightarrow \frac{Y_T}{s^2\theta - \nu_{21} c^2\theta} = \frac{P}{s\theta\, c\theta} \Rightarrow \theta =$$

* Composite Express — Google PS.

Q:1

Use $\tau_z = \dfrac{r\tau_i}{z}$ $\quad z = 60 + 25 = 85\,\mu m$



Q:2 Use $S_c = \dfrac{\sigma_f}{2\tau_i}$ (Critical AR)

Then, Cox model:

$$E_{1,c} = \eta_1 E_f + V_m E_m$$

where, $\eta_1 = 1 - \dfrac{\tanh(ns)}{ns}$

& $n = \sqrt{\dfrac{2G_m}{E_f \ln\left(\frac{R}{r}\right)}}$

$\left[\begin{array}{l} E_{CF} = 8.26\ \text{GPa} \\ E_{GF} = 27.7\ \text{GPa} \end{array}\right]$

$\left[\begin{array}{l} \eta_{CF} = \cancel{0.55}\ 0.56 \\ \eta_{GF} = \cancel{0.452}\ 0.40 \end{array}\right]$

$[n = 0.1]$

$\left(\frac{R}{r}\right)_{CF} = 15$ (small AR)

$\left(\frac{R}{r}\right)_{GF} = 20$ (large AR)

Alter Q:

Min. VF reqd. to achieve a particular E_c.

=> Higher the fiber strength, lower the reqd. VF.

$\left[\begin{array}{l} S_{CF} = 15.38 \\ S_{GF} = 38.46 \end{array}\right]$

Q: 3

$$\bar{\sigma}_c = \sigma_f \left(\frac{1}{2} V_f + V_m \frac{E_m}{E_f} \right) \rightarrow \text{finite AR}$$ $\Leftarrow$ from Kelly mo

$$\bar{\sigma}_c = \sigma_f \left(V_f + V_m \frac{E_m}{E_f} \right) \rightarrow \infty \text{ AR } (S \rightarrow$$

$\rightarrow \frac{S_c}{2S} \rightarrow$

$\rightarrow$ Ratio depends only on V_f & $\frac{E_m}{E_f}$.