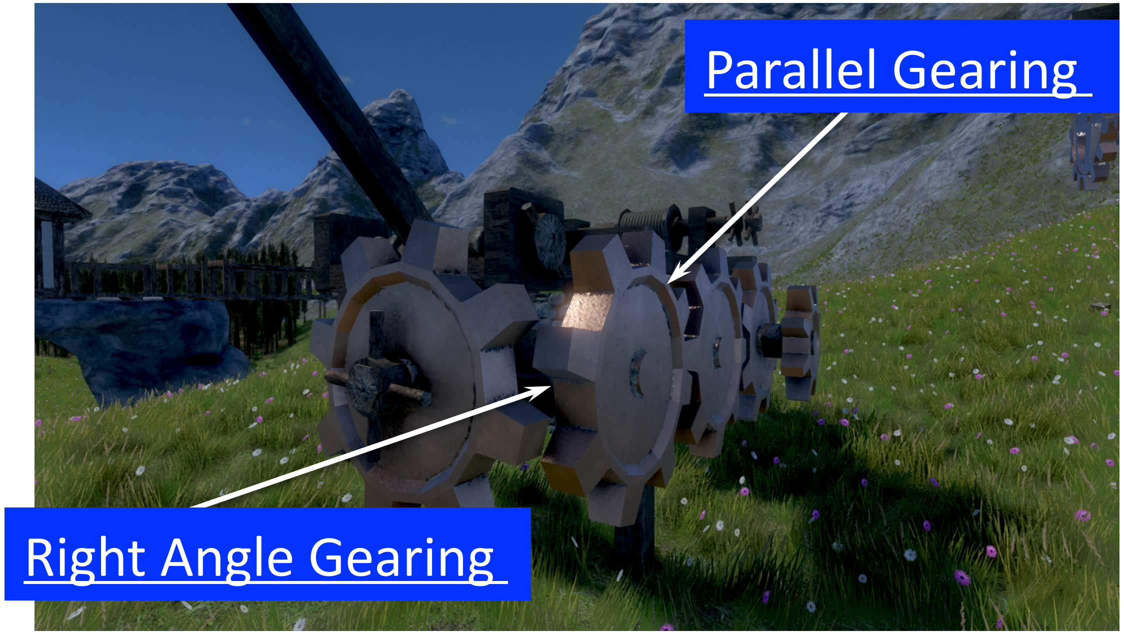
Gears

https://steamcommunity.com/

Gears

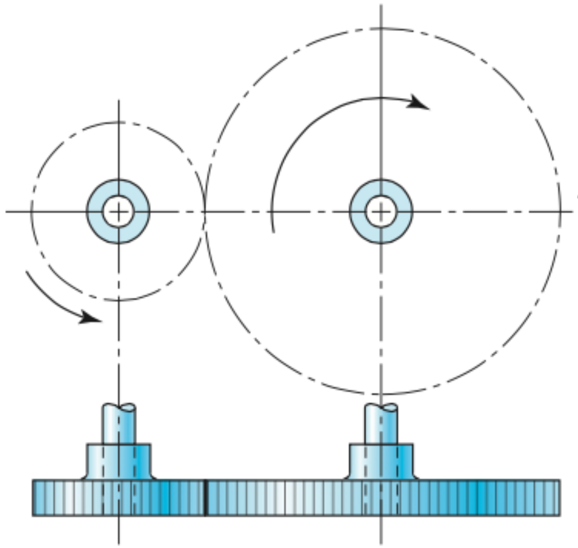


https://steamcommunity.com/

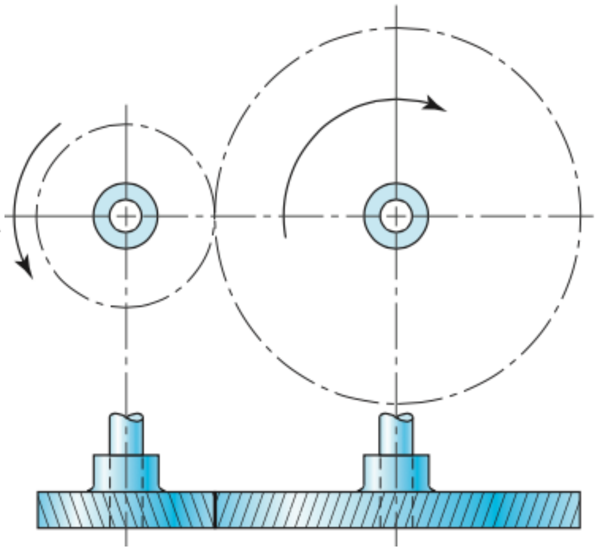
www.indiawaterportal.org/

www.indiamart.com

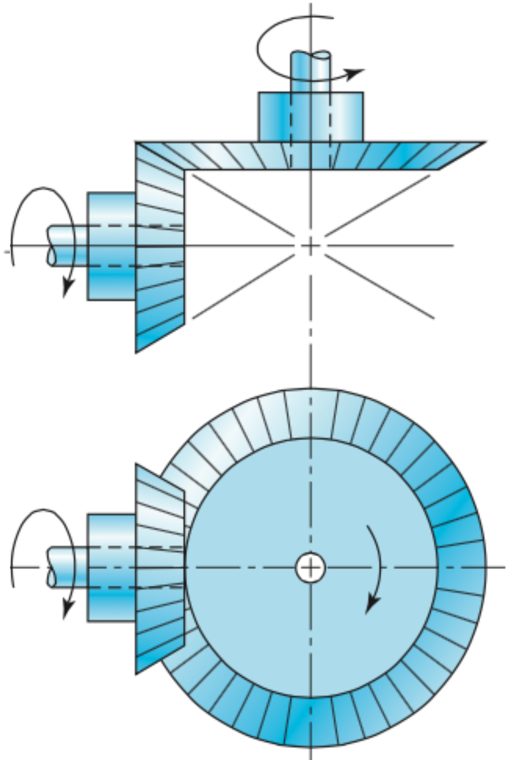
Types of Gears



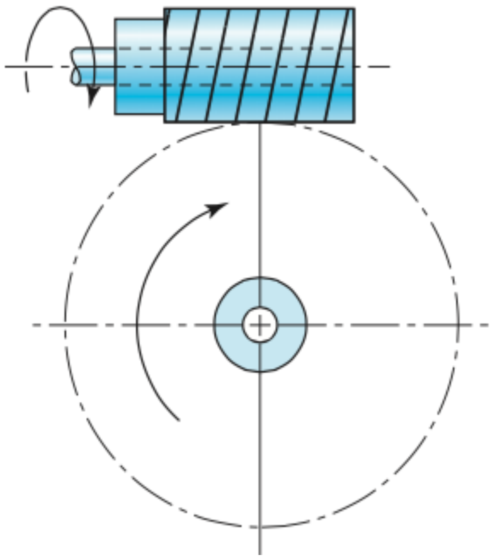
Spur Gear:
Transmits motion between parallel



Helical Gear:
Transmits motion between parallel and non-parallel shafts. Less noisy

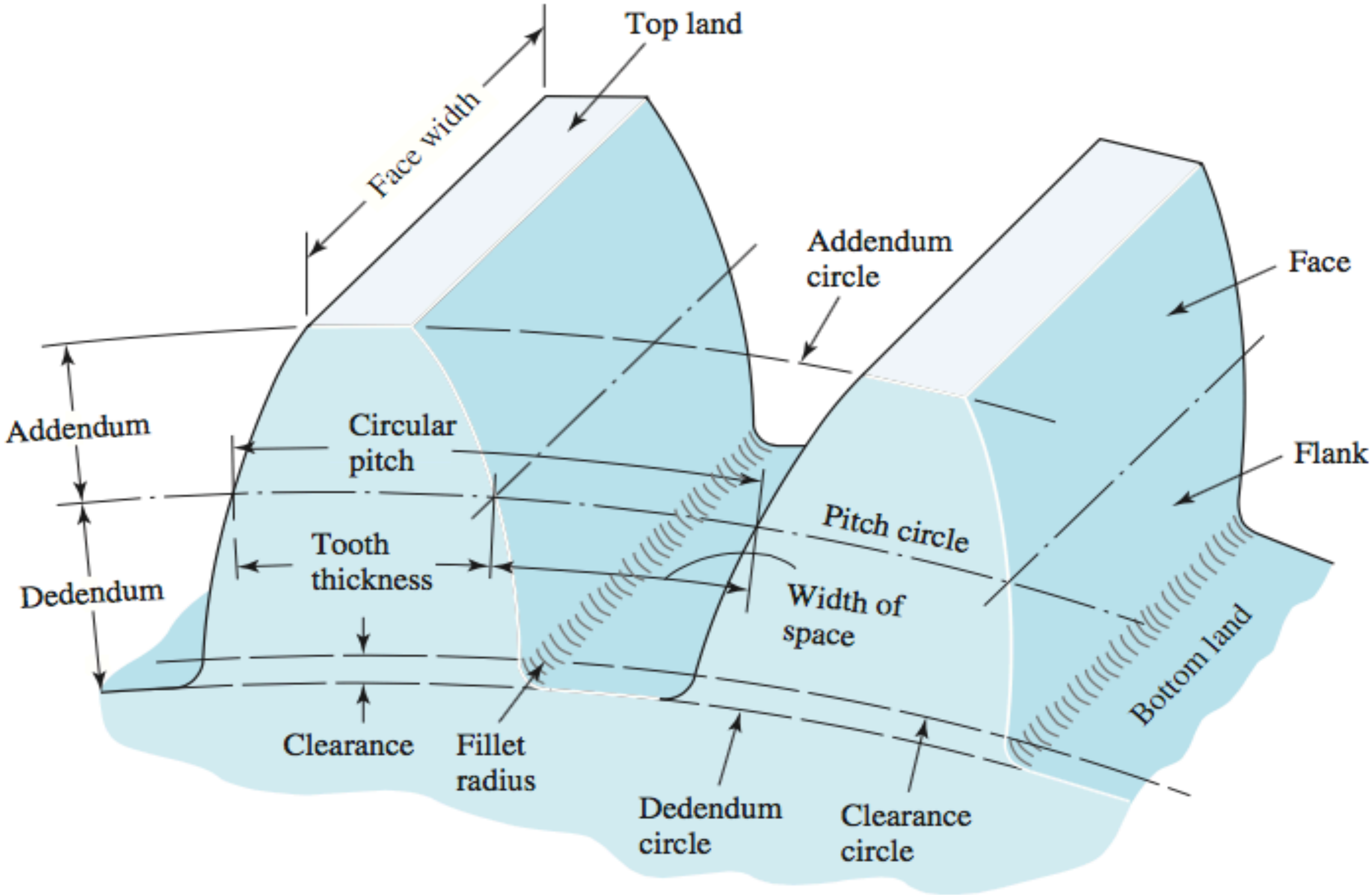


Bevel Gear:
Transmits motion between intersecting shafts



Worm and Worm Gear:
Used for large speed ratio

Nomenclature – Spur Gears



Important Quantities

Pitch Circle: Theoretical Circle.

Pitch circles of a pair of mating gears are parallel to each other

Smaller Mating Gear: Pinion

Larger Mating Gear: Gear

Circular Pitch (p): Distance between a point on one tooth to the corresponding point on an adjacent tooth.

$p =$ (tooth thickness + width of space)

Module: $$m = \frac{\text{pitch circle dia}}{\text{no. of teeth}} = \frac{d}{N} \quad \Rightarrow \quad p = \frac{\pi d}{N} = \pi m$$

Diametral Pitch: $$P = \frac{N}{d} = \frac{1}{m}$$

Addendum: $a =$ radial distance between top land and pitch circle

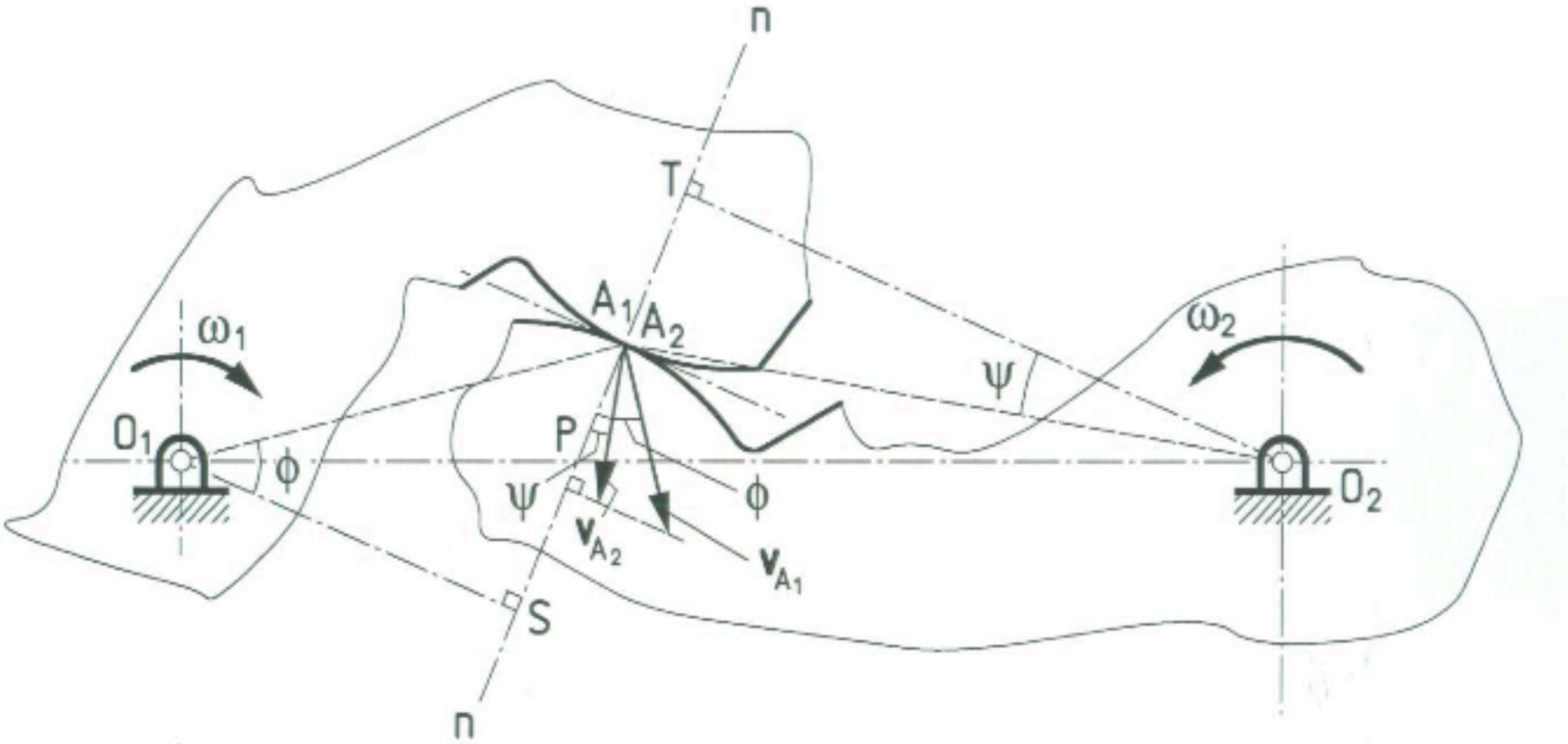
Dedendum: $b =$ radial distance between bottom land and pitch circle

Clearance Circle: circle tangent to the addeddum circle of mating gear

Clearance: $c = b - a$

Back lash: difference between width of tooth space and thickness of the engaging tooth on mating gear on pitch circle

Gears – Angular Velocity Ratio



From the velocities of point A (on bodies 1 and 2)

$$v_{A1}\cos\phi = v_{A2}\cos\psi$$

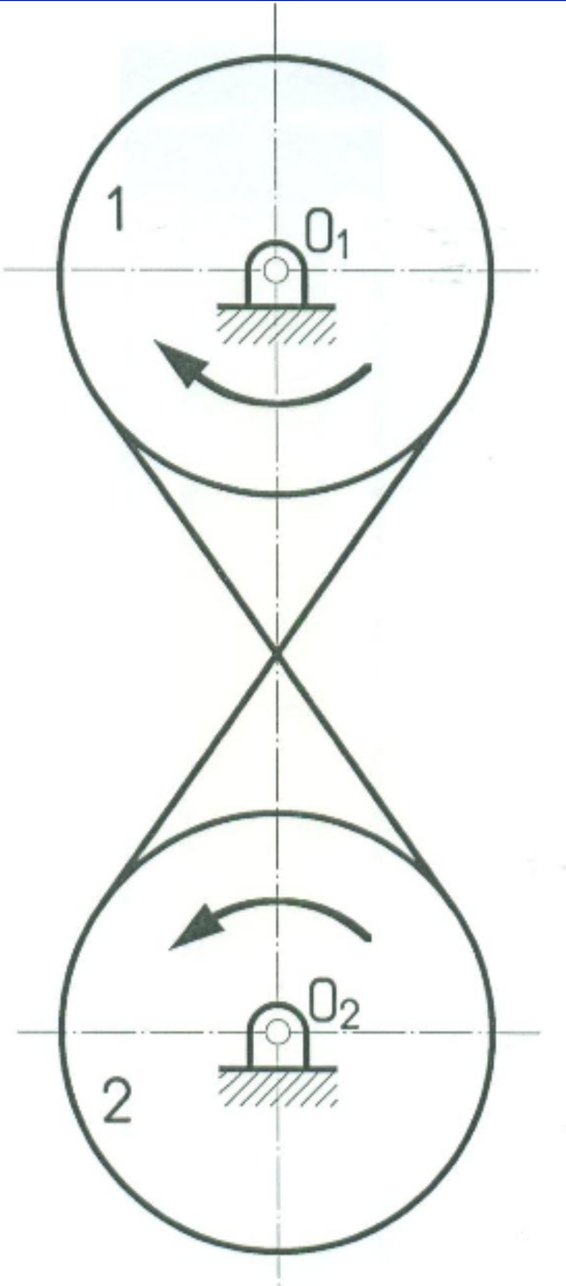
$$\omega_1 . O_1A_1\cos\phi = \omega_2 . O_2A_2\cos\psi$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2T}{O_1S} = \frac{O_2P}{O_1P}$$

(From the similarity of triangles ΔO_1SP and ΔO_2TP)

Source: *Kinematic analysis and synthesis of mechanisms* by Mallik, Ghosh and Dittrich

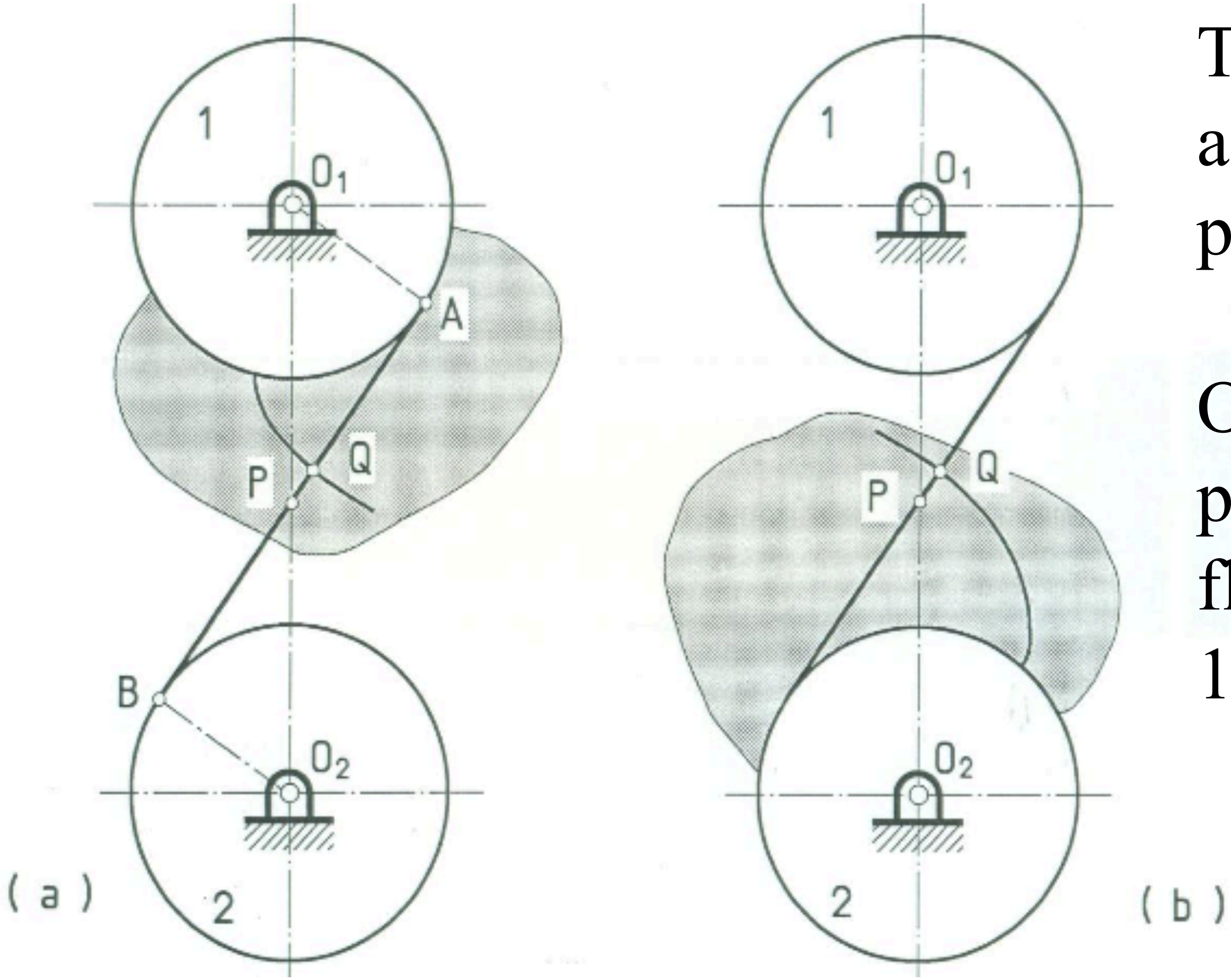
Constant Angular Velocity Ratio



Can be achieved using a belt pulley system:
Angular velocity ratio is same as the inverse ratio of the diameters

Source: *Kinematic analysis and synthesis of mechanisms* by Mallik, Ghosh and Dittrich

Involute Profile

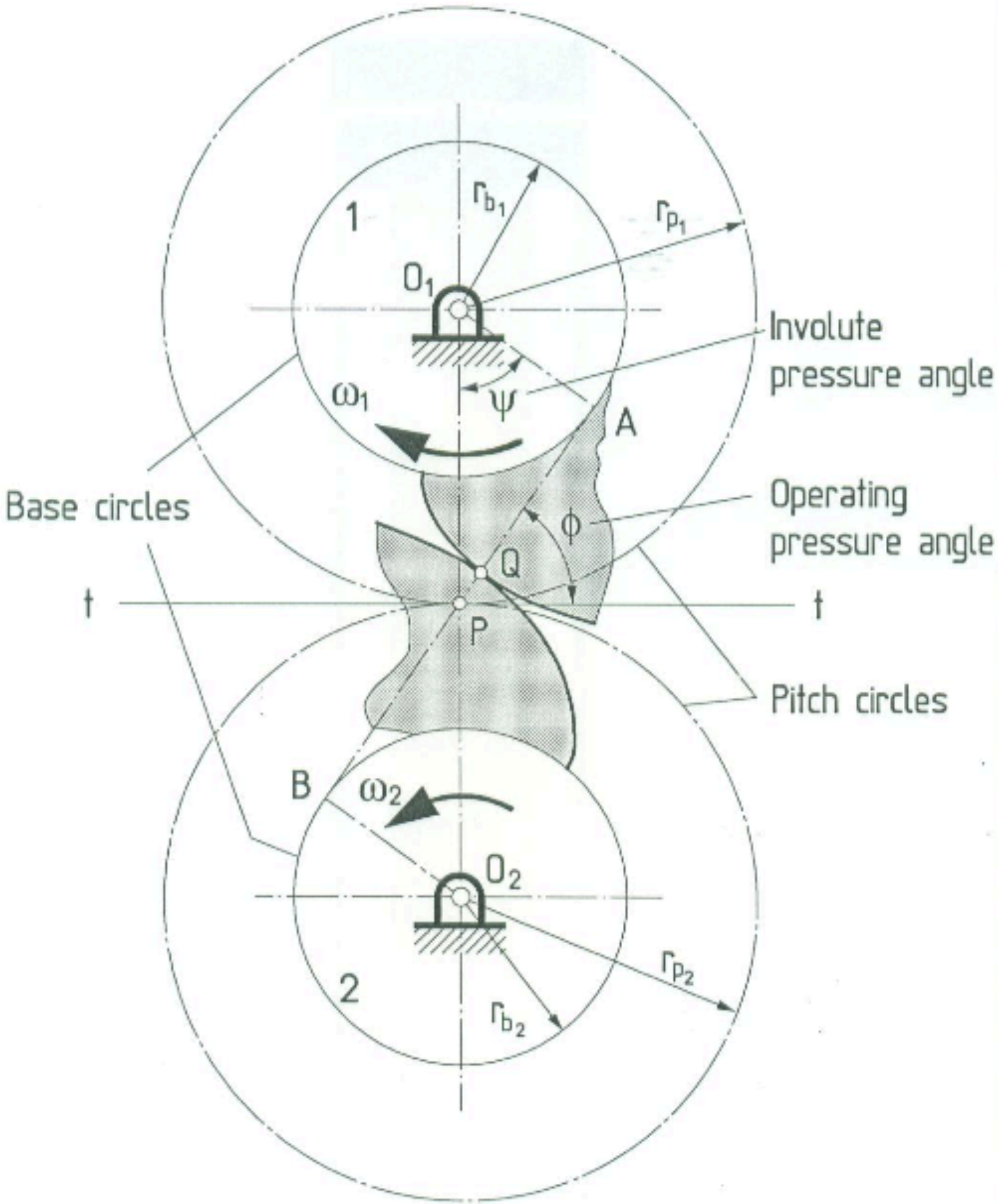


The curves on the shaded area are the involute profiles.

Obtained by tracing the point Q on the shaded flanges attached to pulleys 1 and 2.

Source: *Kinematic analysis and synthesis of mechanisms* by Mallik, Ghosh and Dittrich

Involute Profile

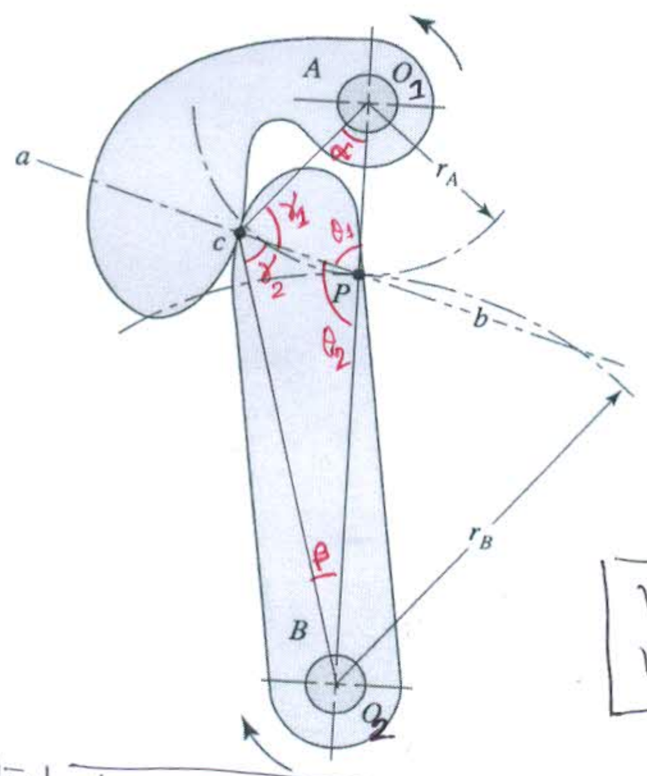


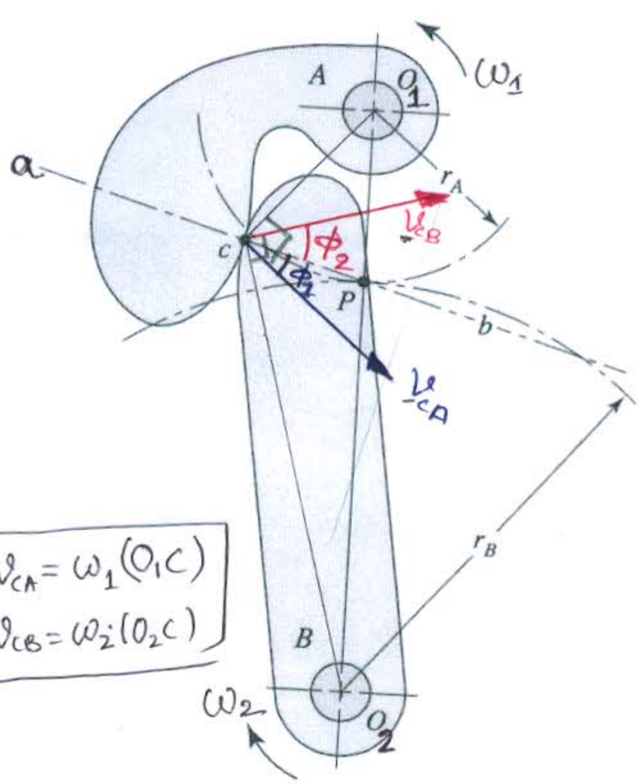
Path of contact is always the line *AB*.

Thus the situation is same as the crossed belt and pulley system

Constant angular velocity ratio is maintained.

Source: *Kinematic analysis and synthesis of mechanisms* by Mallik, Ghosh and Dittrich





$$\boxed{\begin{aligned} v_{CA} &= \omega_1 (O_1 C) \\ v_{CB} &= \omega_2 (O_2 C) \end{aligned}}$$

Note:

$$\boxed{\begin{aligned} \gamma_1 &= \left(\frac{\pi}{2} - \phi_1\right) \\ \gamma_2 &= \frac{\pi}{2} - \phi_2 \\ \theta_2 &= \pi - \theta_1 = 180^\circ - \theta_1 \end{aligned}}$$

- C is contact point.
- ab is common normal to contacting surfaces.
- Condition:

Component of v_{CA} and v_{CB} normal to surfaces at c (along ab) must be same.

$$\Rightarrow v_{CA} \cos\phi_1 = v_{CB} \cos\phi_2$$

$$\Rightarrow \omega_1 (O_1 C \cos\phi_1) = \omega_2 (O_2 C \cos\phi_2)$$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 C \cos\phi_2}{O_1 C \cos\phi_1}}$$

From $\Delta O_1 C P$:

$$\frac{O_1 C}{\sin\theta_1} = \frac{O_1 P}{\sin\gamma_1} = \frac{O_1 P}{\sin(\frac{\pi}{2} - \phi_1)} = \frac{O_1 P}{\cos\phi_1} \quad (1)$$

From $\Delta O_2 C P$:

$$\frac{O_2 C}{\sin\theta_2} = \frac{O_2 P}{\sin\gamma_2} \Rightarrow \frac{O_2 C}{\sin(\pi - \theta_1)} = \frac{O_2 P}{\sin(\frac{\pi}{2} - \phi_2)}$$

$$\Rightarrow \frac{O_2 C}{\sin\theta_1} = \frac{O_2 P}{\cos\phi_2} \quad (2)$$

Dividing (1) by (2):

$$\frac{O_1 C}{O_2 C} = \frac{O_1 P \cos\phi_2}{O_2 P \cos\phi_1}$$

$$\Rightarrow \boxed{\frac{O_1 C \cos\phi_1}{O_2 C \cos\phi_2} = \frac{O_1 P}{O_2 P}} \quad (3)$$

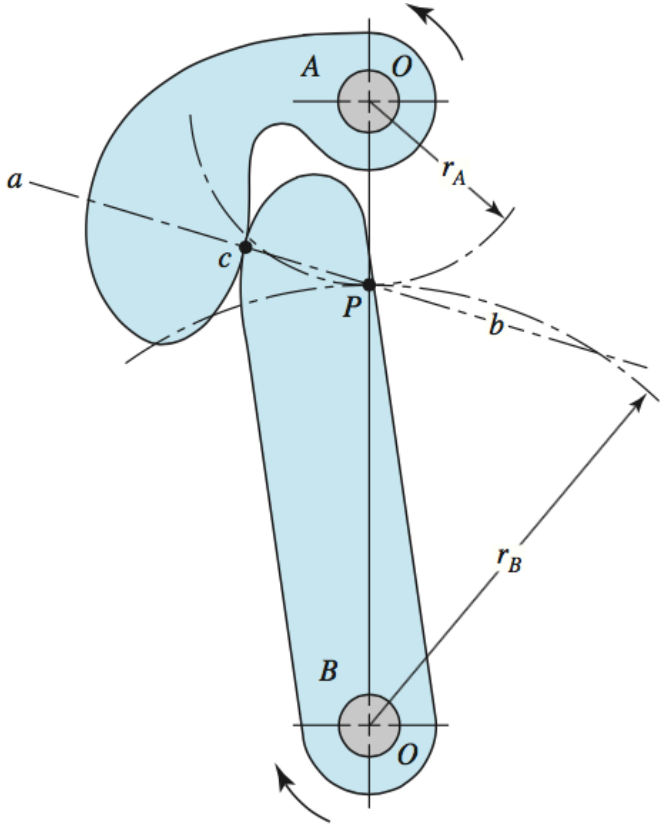
From (3):

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}} = \frac{r_B}{r_A}$$

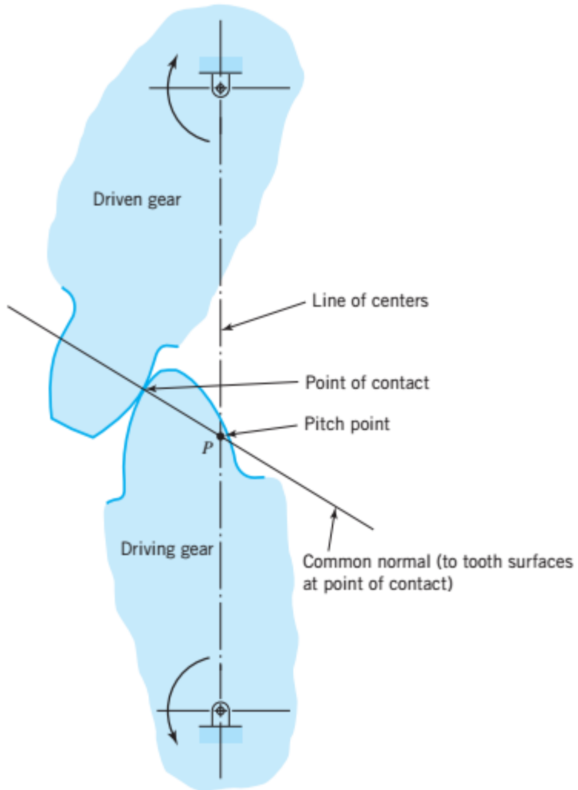
$\Rightarrow$ Ratio of angular velocities is equal to the radii of circles passing through P and with centers at O_2 & O_1, respectively.

Now, during motion: If contact is always along ab, then P is fixed. $\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \text{Constant}}$ and $\frac{\omega_1}{\omega_2} = \frac{O_2 P}{O_1 P}$ $\Rightarrow$ This is the conjugate action used in gears.

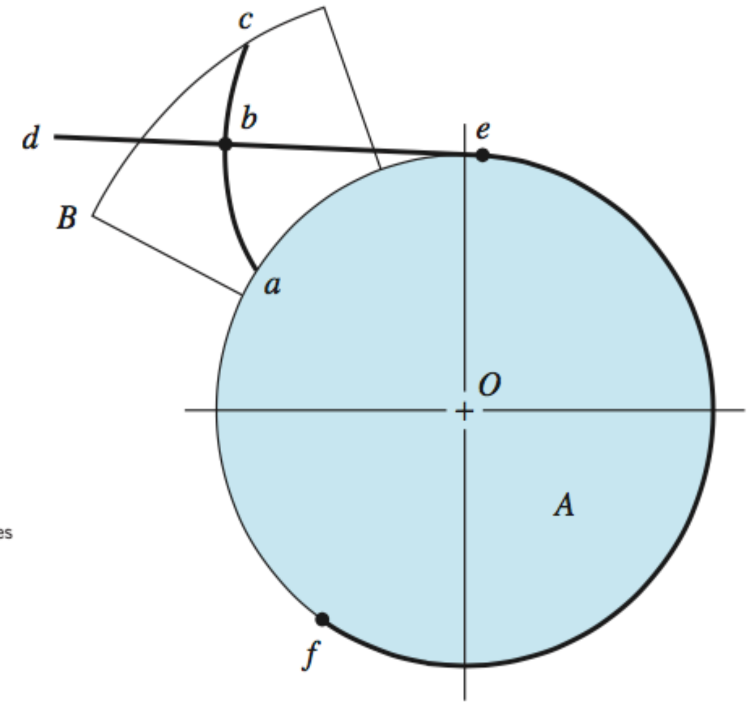
Conjugate Action – Law of Gearing



Conjugate Action



Conjugate Action in Gears

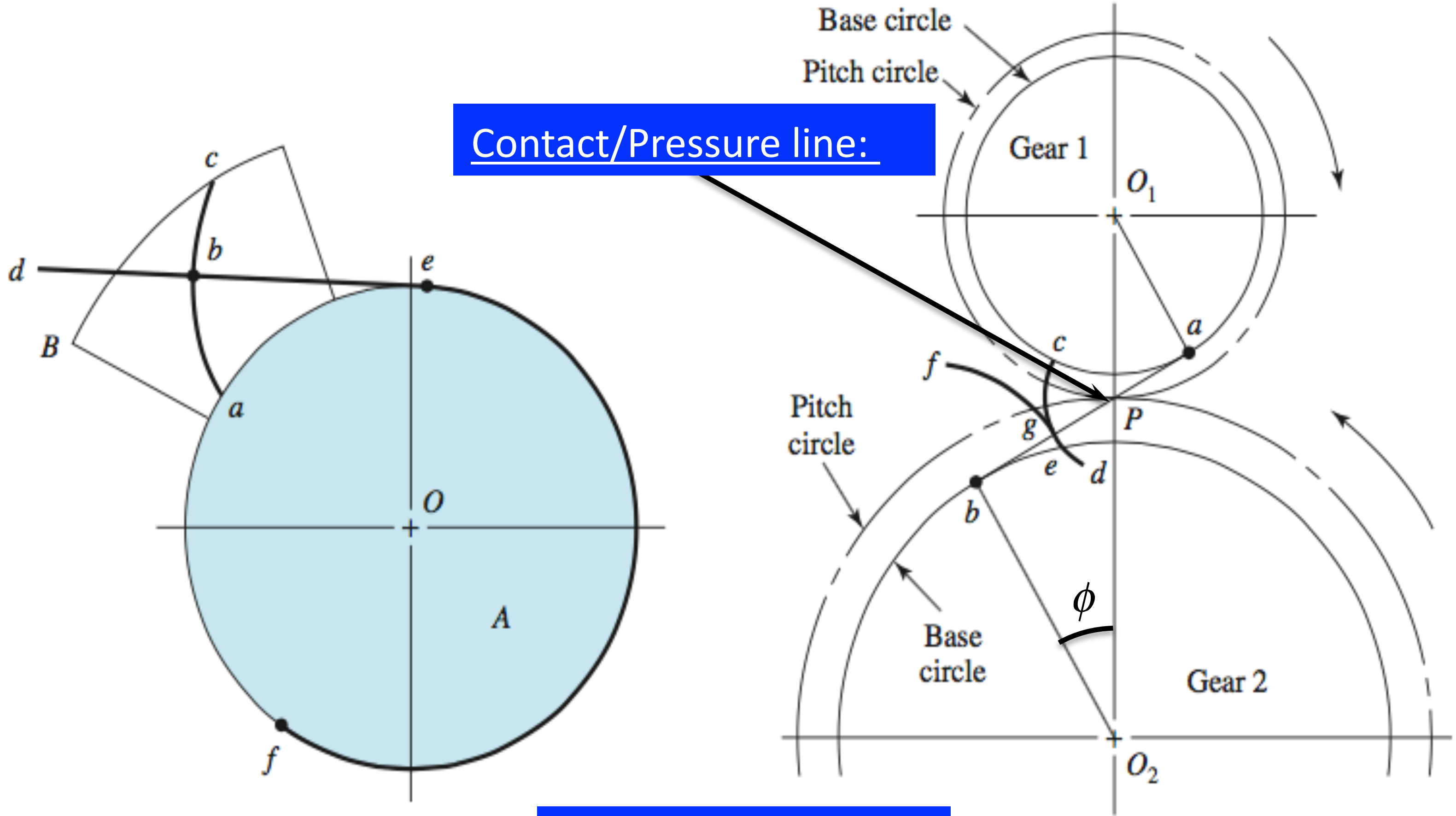


Involute Profile

Conjugate Action: Maintains constant speed ratio during motion transfer
Provides jerk-free and smooth motion transmission

Involute Profile: One way to get conjugate action
Predominantly used for gear tooth profile

Pressure Angle



Involute Profile

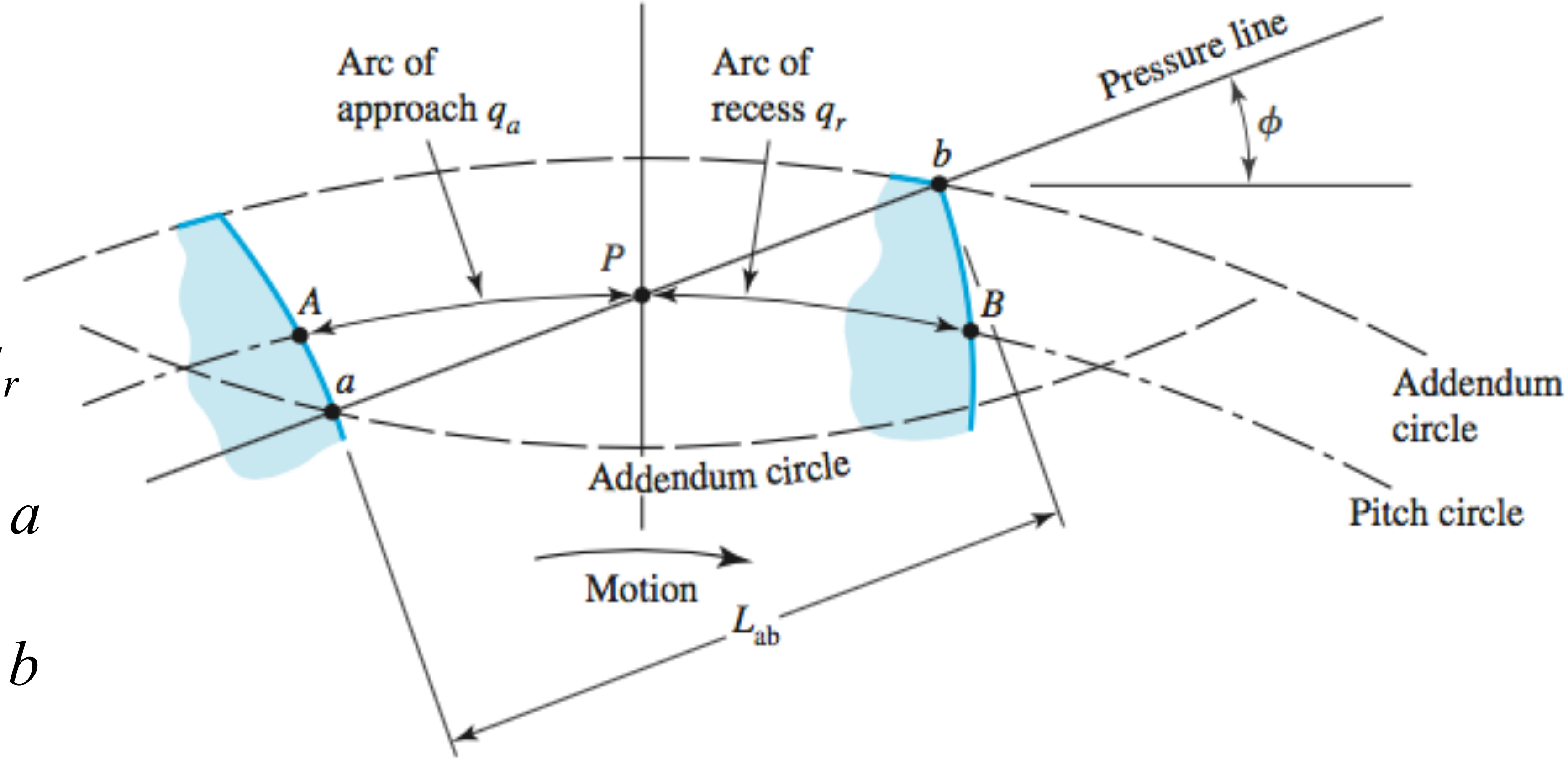
***ab*:** **Contact Line or Pressure Line** (locus of contact point between mating teeth)
Conact line always normal to tooth profile. Contact force acts along this line

***ϕ*:** Pressure Angle

Gear 2: $r = O_2P =$ pitch circle radius; $r_b = O_2b =$ base circle radius

$r_b = r\cos\phi$; base pitch $p_b = p\cos\phi$

Arc of action and Contact Ratio



- Arc of action:
  $$q_t = q_a + q_r$$
- First contact: a
- Final contact: b

$q_t = p$ (circular pitch) $\Rightarrow$ At any instant one one pair of teeth in action

$q_t > p$ $\Rightarrow$ A new pair gets into contact before already engaged pair reaches final contact

Contact Ratio: $$m_c = \frac{q_t}{p} = \frac{L_{ab}}{p\cos\phi} = \frac{L_{ab}}{p_b}$$

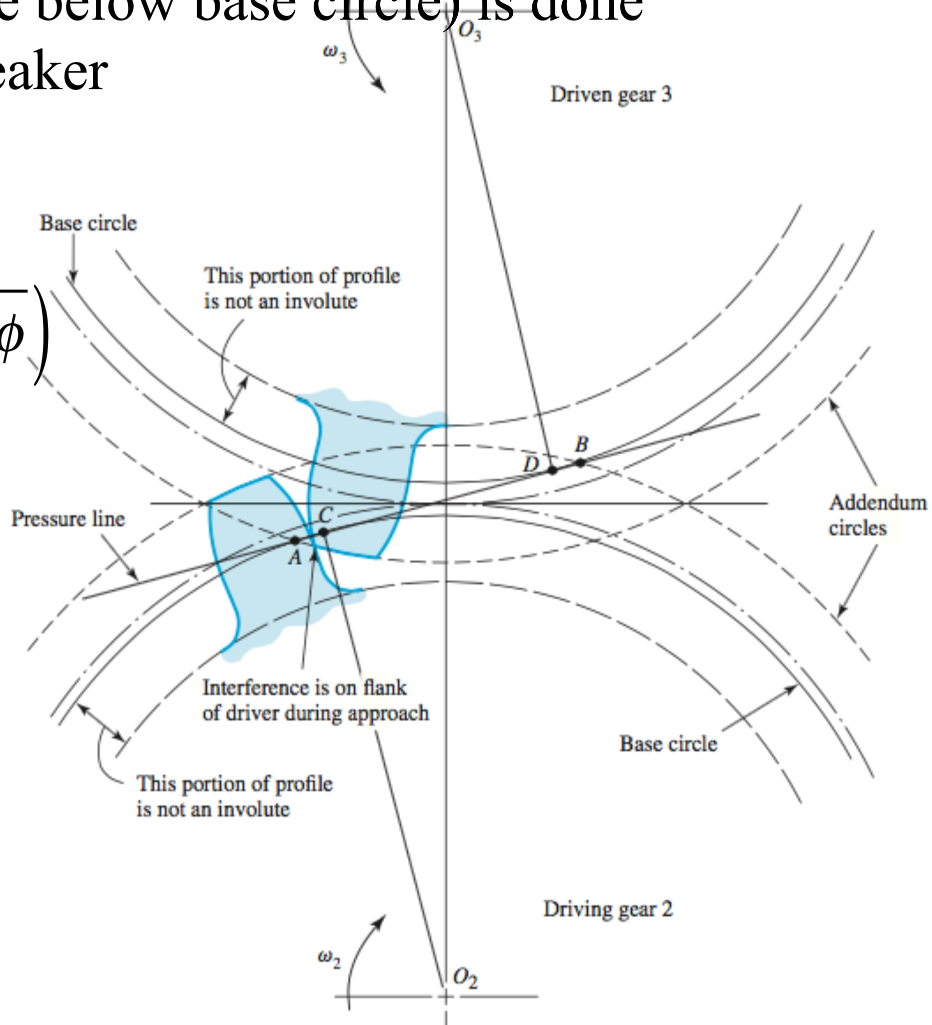
Recommended: $m_c \geq 1.2$ (to avoid mounting inaccuracies)

Interference

Interference:

- Happens due to contact between non-conjugate portion of teeth
- Example: contact occurring in the clearance region
- Causes removal of flank

To avoid interference: undercutting (involute profile below base circle) is done
undercutting makes tooth weaker



To avoid interference without under cutting:

$$N_P \geq \frac{2k}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right)$$

$N_P :=$ number of teeth on pinion

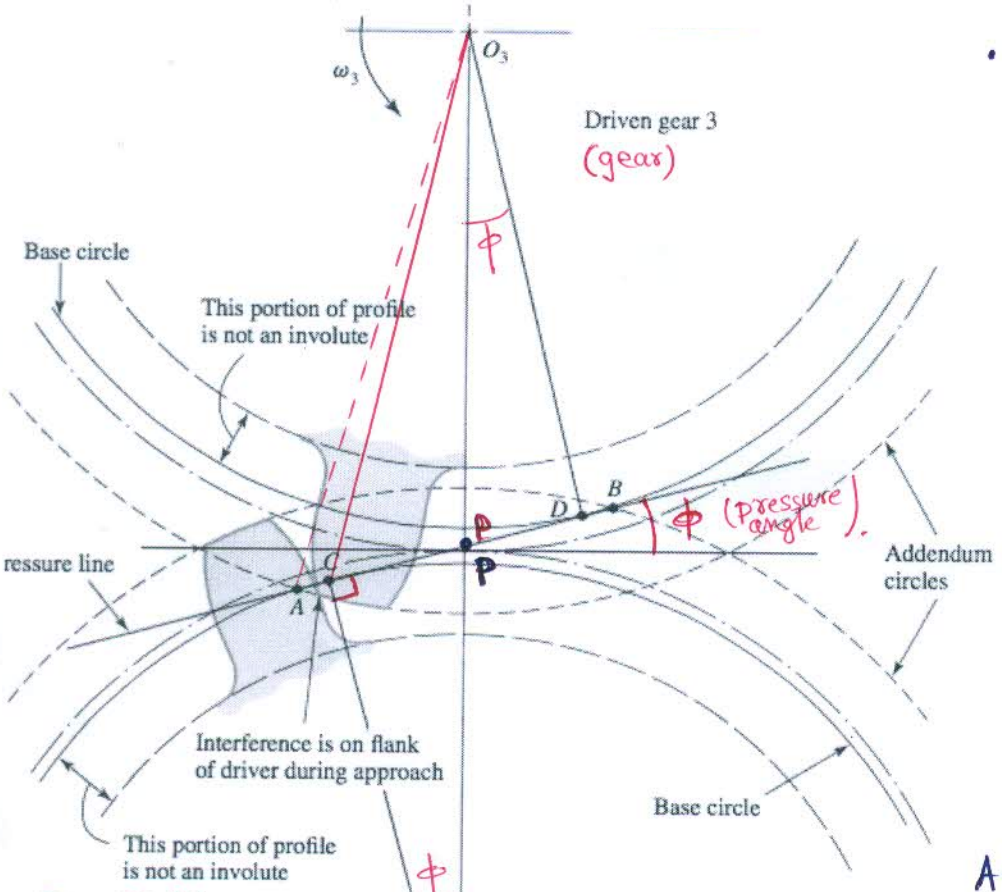
$N_G :=$ number of teeth on gear

$$\mathrm{m}_G = \frac{N_G}{N_P} := \text{ gear ratio}$$

$$k = \begin{cases} 1 & \text{for full depth teeth} \\ 0.8 & \text{for stub teeth (shorter hight)} \end{cases}$$

See Table 13-1 to 13-4 for tooth systems, values of pressure angle, addendum etc.

INTERFERENCE



From ΔO_2PC

$$\boxed{PC = O_2P \sin\phi}$$

- CD — common tangent to base circles
- C on base circle of pinion (gear 2)
- D on base circle of gear (gear 3).
- Conjugate action is between C and D (or inside CD)
- $\underline{AB}$: line of action or contact.
- A := first contact
  B := final contact.

As AB is outside CD interference occurs.

To avoid interference AB must be inside. CD. (Limiting case $A \to C$, $B \to D$)

$\phi :=$ pressure angle

Gear ratio $m_G = \frac{N_3}{N_2} = \frac{O_3P}{O_2P} = \frac{N_G}{N_P}$

$$O_3C^2 = O_3P^2 + PC^2 - 2(O_3P)(PC)\cos\left(\frac{\pi}{2}+\phi\right) = O_3P^2 + PC^2 + 2(O_3P)(PC)\sin\phi$$

(From ΔO_3CP)

$$\Rightarrow O_3C^2 = O_3P^2 + (O_2P)^2\sin^2\phi + 2(O_3P)(O_2P)\sin^2\phi = O_3P^2\left[1 + \left(\frac{1}{m_G^2} + \frac{2}{m_G}\right)\sin^2\phi\right]$$

$$= O_3P^2\left[1 + \left\{\left(\frac{O_2P}{O_3P}\right)^2 + 2\left(\frac{O_2P}{O_3P}\right)\right\}\sin^2\phi\right]$$

$$\Rightarrow O_3C = \left(\frac{O_3P}{m_G}\right)\cdot\sqrt{m_G^2 + (1+2m_G)\sin^2\phi}$$

$\underline{\text{Addendum:}}$ $a = O_3A - O_3P = O_3C - O_3P$ (in limiting case)

To avoid interference: $a \le (O_3C - O_3P)$.

$$\Rightarrow a \le \frac{O_3P}{m_G}\cdot\left(\sqrt{m_G^2 + (1+2m_G)\sin^2\phi} - m_G\right)$$

Let $a = k\cdot m$ ($m :=$ module, $k = 1$ for full depth, 0.8 for stub teeth)

$$\underline{O_3P} = m\cdot\frac{N_3}{2} = m\frac{N_G}{2}.$$

$$\Rightarrow k\cancel{m} \le \frac{\cancel{m}}{2}\cdot\underbrace{\left(\frac{N_G}{m_G}\right)}_{=N_P}\left[\sqrt{m_G^2 + (1+2m_G)\sin^2\phi} - m_G\right]$$

$$\Rightarrow N_P \ge \frac{2k}{\left(\sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right) - m_G} = \frac{2k}{(1+2m_G)\sin^2\phi}\left[m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right]$$

Gear Mesh Design – Spur and Helical

Useful quantities:

Diameters (pinion and dear): d_P and d_G;

Number of teeth (pinion and gear): N_P and N_G

$$\text{Module: } m = \frac{\text{pitch circle diameter}}{\text{Number of teeth}} = \frac{d_P}{N_P} = \frac{d_G}{N_G};$$

$$\text{Circular pitch: } p = \frac{\pi d_P}{N_p} = \frac{\pi d_G}{N_G} = \pi m$$

$$\text{Diametral pitch: P} = \frac{N_P}{d_p} = \frac{N_G}{d_G} = \frac{1}{m}$$

Addendum: a;

Deddendum: d

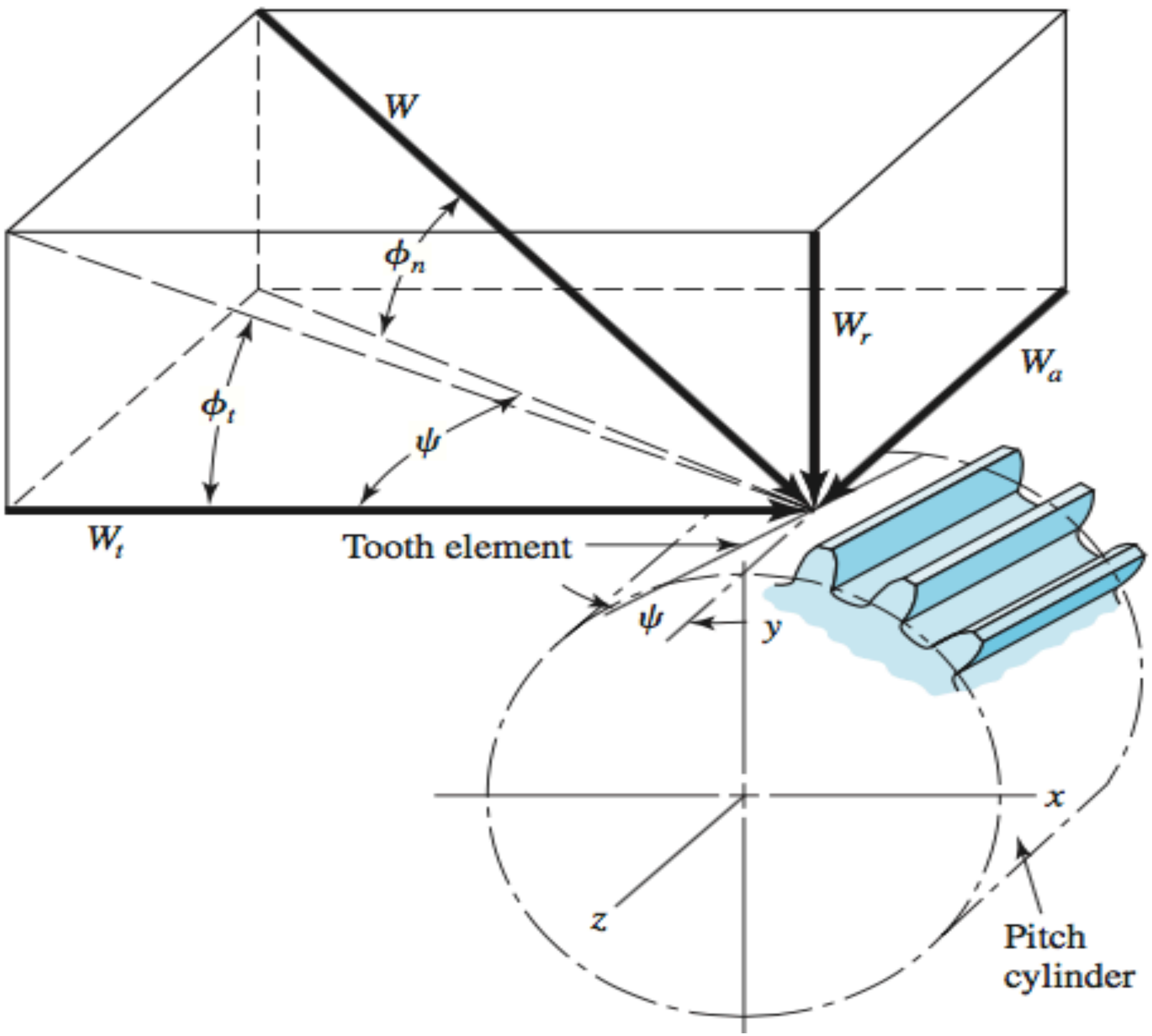
Gear Force Analysis

Transmitted Load: $W_t = \dfrac{60000H}{\pi dn}$ (in kN); Pitch line velocity: $V = \dfrac{\pi dn}{60000}$ (in m/sec)

$H \coloneqq$ Transmitted power in kW

$d \coloneqq$ gear/pinion diameter in mm

$n \coloneqq$ speed in rpm



Helical gear:

Normal tooth force : $W = \dfrac{W_t}{\cos\phi_t \cos\psi}$;

Radial tooth force : $W_r = W_t \tan\phi$;

Axial tooth force : $W_a = W_t \tan\psi$

Spur gear (set helix angle $\psi = 0$):

Normal tooth force : $W = \dfrac{W_t}{\cos\phi}$;

Radial tooth force : $W_r = W_t \tan\phi$

AGMA Equations for Bending (SI Units)

AGMA Stress Equation for Bending:

$$\sigma = K_o K_v K_s \frac{W_t}{b m_t} \frac{K_H K_B}{Y_J}$$

$W_t :=$ Transimtted load (tangential) in N; $b :=$ face width in mm;

$m_t :=$ Transverse metric module in mm (for spur gear $m_t = m$)

$K_o :=$ Overload factor (Figure 14-17 or 14-18)

$K_v :=$ Dynamic factor (Figure 14-9 or Eq. 14-27 and 14-28)

$K_s :=$ Size factor (=1)

$K_B :=$ Rim thickness factor (Eq. 14-40 or Figure 14-16)

$Y_J :=$ Geometry factor for bending resistance (For spur gear Figure 14-6)

(For helical gear Figures 14-7 and 14-8)

$K_H :=$ Load distribution factor (See table below)

Condition of support	**Face width**			
	Up to 50 mm	Up to 150 mm	Up to 225 mm	Up to 400 mm
Accurate mounting, low bearing clearances, minimum elastic deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mounting, less accurate gears, contact across full face	1.6	1.7	1.8	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0			

AGMA Equations for Bending (SI Units)

AGMA Strength Equation for Bending:

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$

$S_t :=$ Allowable bending stress (in N/mm^2) for 0.99 reliability and 10^7 cycles

(Tables 14-3, 14-4; Figures 14-2, 14-3, 14-4)

$Y_N :=$ Stress-cycle factor for bending stress (Figure 14-14)

$Y_\theta :=$ Temperature factor (=1 for less than 100°C)

$Y_Z :=$ Reliability factor (Table 14-10)

$S_F :=$ AGMA factor of safety (calculate in the end as $S_F = \frac{\sigma_{all}}{\sigma}$);

AGMA Equations for Pitting (SI Units)

AGMA Stress Equation for Contact (pitting):

$$\sigma_c = Z_E \sqrt{K_o K_v K_s \frac{W_t}{bd_p} \frac{K_H Z_R}{Z_I}}$$

W_t := Transimtted load (tangential) in N;

$d_p = mN_P$ (Pinion diameter in mm)

Z_E := Elactic coefficient in $\sqrt{\text{N/mm}^2}$; (Table 14-8 or Eq. 14-13)

Z_R := Surface condition factor (Take equal to unity)

Z_I := Geometry factor for pitting resistance (Eq. 14-23 along with 14-12, 14-24, 14-25)

AGMA Equations for Pitting (SI Units)

AGMA Strength Equation for Pitting:

$$\sigma_{c,all} = \frac{S_C}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z}$$

$S_c \coloneqq$ Allowable contact stress (in N/mm^2) for 0.99 reliability and 10^7 cycles

(Tables 14-6, 14-7; Figure 14-5)

$Z_N \coloneqq$ Stress-cycle factor for contact stress (Figure 14-15)

$Z_W \coloneqq$ Hardness ratio factor for pitting resistance

(For pinion $Z_w = 1$; For gear - Figures 14-12, 14-13)

$S_H \coloneqq$ AGMA factor of safety for pitting (calculate in the end as $S_H = \dfrac{\sigma_{c,all}}{\sigma_c}$)

Note: Compare (S_F) and $(S_H)^2$ to decide whether bending or wear is the threat to gear function

Gear Mesh Design – Spur and Helical

Criteria:

Load, speed, reliability, life (N), overload (K_o), design factor (n_d)

***Apriori* Decisions:**

Pressure angle (ϕ), Helix angle (ψ)

Addendum (a), Deddendum (d), root fillet radius (r_F)

Gear Ratio (m_G), Number of teeth (N_p, N_G)

Quality Number (Q_v)

Design Decisions:

Module (m), Face width (b)

Pinion Material, core hardness, case hardness

Gear Material core hardness, case hardness

Design Steps

1. Choose a module:

2. Check for Pinion bending and wear (pitting)
   i. Choose material and a core hardness
   ii. Calculate face width to satisfy safety factor and standardize (ensure $3\pi m \leq b \leq 5\pi m$)
   iii. Compute AGMA factors of safety S_F and S_H
   iv. If not satisfactory modify module and repeat until the design is satisfactory

3. Check for Gear bending and wear (pitting)
   i. Choose material and a core hardness
   ii. Compute AGMA factors of safety S_F and S_H
   iii. If not satisfactory modify module and repeat until the design is satisfactory