Assignment - 2
(P D E)

Course Teacher
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Q1. Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials.

[Ans: $P(x) = \frac{8}{35} P_4(x) + \frac{4}{5} P_3(x) + \frac{40}{21} P_2(x) + \frac{1}{5} P_1(x) - \frac{224}{105} P_0(x)$]

Q2. Show that (i) $P_n(1) = 1$ (ii) $P_n(0) = 0$ for n odd

(iii) $P_n(0) = \dfrac{(-1)^{n/2}\, n!}{2^n \{(n/2)\}^2}$, for n even.

[Hints: (i) Start from generating f^n., put $x=1$ and equate coeff. of h^n. (ii) In the expression for $P_n(x)$, put $n = 2m+1$ and then put $x=0$ (iii) Start from generating f^n, put $x=0$, equate coeff. of h^{2m} on both sides and then put $n=2m$ i.e. $m = \frac{n}{2}$]

Q3. Prove that $\int_{-1}^{+1} (1-x^2) P_m' P_n' \, dx = 0$, m, n are distinct +ve integer.

[Hints: Do by parts integration with P_n as 2nd f^n. and then make use of the fact that P_m is the sol^n. of Legendre eqn.]

Q4. Show that (i) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (ii) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

[Hint: In the expression of $J_n(x)$ and $J_{-n}(x)$, put $n = -\frac{1}{2}$ & $\frac{1}{2}$.]

Q5. Consider the recurrence relations

$$[x^m J_m(x)]' = x^m J_{m-1}(x); \quad [x^{-m} J_m(x)]' = -x^{-m} J_{m+1}(x)$$

If we use these relations, we can show that

$$[J_m^2(x)]' = \frac{x}{A}\left[J_{m-1}^2(x) - J_B^2(x)\right]$$

Determine A and B in terms of m.

[Ans: $A = 2m$, $B = m+1$]

**** The End ****