Department of Mathematics
IIT Kharagpur
MA20103 Partial Differential Equations

Autumn 2015 - Mid-Examination, Time: 2 hrs.; Max. Marks: 30, Number of students: 460

Note: Answer all the questions. Please follow the notations: z: dependent variable; x, y: independent variables; $z_x = p; z_y = q$. Please answer all parts of a specific question at one place. No queries will be entertained during the examination. There are 2 pages in this question paper.

1. (A). Show that $x = 0$ is a regular singular point of the differential equation

$$2x^2y'' - xy' + (1+x)y = 0$$

and hence find the general solution by using Frobenius method.

[5 marks]

(B). Express the polynomial $f(x) = 5x^4 + 3x^3 - 5x^2 + 2x - 3$ in terms of Legendre polynomials, $P_n(x)$. Compute $P_0(x), P_1(x)$ etc. using the generating function

$$\left(1 - 2xh + h^2\right)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x), |h| < 1, |x| \leq 1.$$

[3 marks]

(C). Consider the recurrence relations

$$\left[x^m J_m(x)\right]' = x^m J_{m-1}(x); \quad \left[x^{-m} J_m(x)\right]' = -x^{-m} J_{m+1}(x).$$

If we use these recurrence relations, we can show that

$$\left[J_m^2(x)\right]' = \frac{x}{A}\left[J_{m-1}^2(x) - J_B^2(x)\right].$$

Determine A and B in terms of m by showing details.

[2 marks]

2. (A). Classify (linear, semi-linear, quasi-linear, non-linear) the following first order PDEs with suitable justification. (i). $\sin y z_x - e^x z_y = e^y z$; (ii). $x^2 z_x + (y-x)z_y = y \sin z$.

[2 marks]

(B). Form the first order PDE by eliminating arbitrary constants a and b from $z = (x^2+a)(y^2+b)$.

[1 mark]

(C). Find the general solution of

$$2yz_x + zz_y = 2yz^2$$

using Lagrange's method.

[3 marks]

(D). Find the integral surface of

$$x(y^2+z)z_x - y(x^2+z)z_y = (x^2-y^2)z$$

which passes through $x+y=0, z=1$.

[4 marks]

3. (A). Given the equations

$$z = px + qy \text{ and } 2xy(p^2+q^2) = z(yp+xq)$$

which are compatible, find two common solutions of these. You need not show that these are compatible.

[4 marks]

(B). Find a complete integral of

$$2(z+px+qy) = yp^2$$

[3 marks]

(C). Write the PDE

$$p - q^2 = 3x^2 - y$$

in one of the standard types and hence find a complete integral.

[3 marks]

Best of Luck