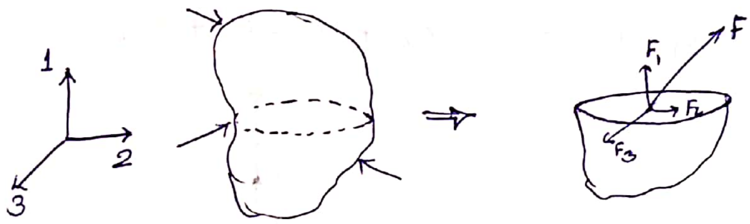
$\underline{\sigma_{ij}} = C_{ijkl}\,\epsilon_{\underline{kl}}$

order of tensor $\longrightarrow$ No. of directions to define the quantity physically.

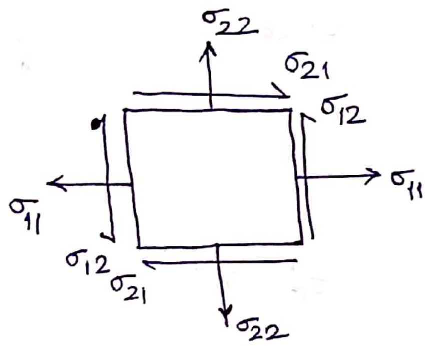


$\lim\limits_{\Delta A \to 0} \dfrac{\Delta F_1}{\Delta A} = \sigma_{11}$

$= \sigma_{12}$

$= \sigma_{13}.$

σ_{11} — first 1: Normal to plane; second 1: $\longrightarrow$ Direction of force.



	σ_{ij}	$=$ C_{ijkl}	ϵ_{kl}
Originally,	9	81	9
Due to Symmetry	6	36.	6

~~Due t~~ $[\sigma_i]_{6\times1} = [C_{ij}]_{6\times6}\,\epsilon_{j\,6\times1}$

$\downarrow$ Symmetric

Due to energy consideratⁿ	6	21	6

$\sigma_1 = \sigma_{11}$

$\sigma_2 = \sigma_{22}$

$\sigma_3 = \sigma_{33}$

$\sigma_4 = \sigma_{23}$

$\sigma_5 = \sigma_{13}$

$\sigma_6 = \sigma_{12}$

For completely defining anisotropic material, we need 21 constants.

Stiffness

$$\sigma_i = [C_{ij}]\, \epsilon_j$$

$$\epsilon_i = [S_{ij}]\, \sigma_j$$

Compliance

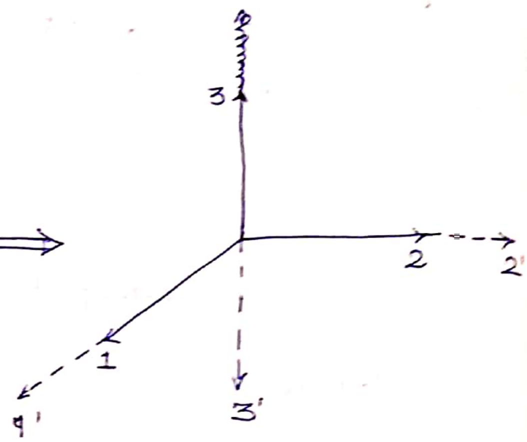
$$\sigma_1 = C_{11}\epsilon_1 + C_{12}\epsilon_2 + C_{13}\epsilon_3 + C_{14}\epsilon_4 + C_{15}\epsilon_5 + C_{16}\epsilon_6$$

$\epsilon_1 = \epsilon_1'$	$\sigma_1 = \sigma_1'$
$\epsilon_2 = \epsilon_2'$	$\sigma_2 = \sigma_2'$
$\epsilon_3 = \epsilon_3'$	$\sigma_3 = \sigma_3'$
$\epsilon_4 = -\epsilon_4'$	$\sigma_4 = -\sigma_4'$
$\epsilon_5 = -\epsilon_5'$	$\sigma_5 = -\sigma_5'$
$\epsilon_6 = \epsilon_6'$	$\sigma_6 = \sigma_6'$

why?

← Mono-clinic ⟹

Only 3 & 3' are in opposite direction.



$\cancel{\sigma_1 = C_{11}\epsilon_1}$ $\cancel{\sigma_1' = C_{11}'}$

$$\sigma_1 = \sum_{j=1}^{6} C_{1j}\,\epsilon_j \qquad \sigma_1' = \sum_{j=1}^{6} C_{1j}\,\epsilon_j'$$

$$\cancel{C_{11}\epsilon_1} + \cancel{C_{12}\epsilon_2} + \cancel{C_{13}\epsilon_3} + C_{14}\epsilon_4 + C_{15}\epsilon_5 + \cancel{C_{16}\epsilon_6} = \cancel{C_{11}\epsilon_1'} + \cancel{C_{12}\epsilon_2'} + \cancel{C_{13}\epsilon_3'} + C_{14}\epsilon_4' + C_{15}\epsilon_5' + \cancel{C_{16}\epsilon_6'}$$

$$\epsilon_4' \to -\epsilon_4 \qquad \epsilon_5' \to -\epsilon_5$$

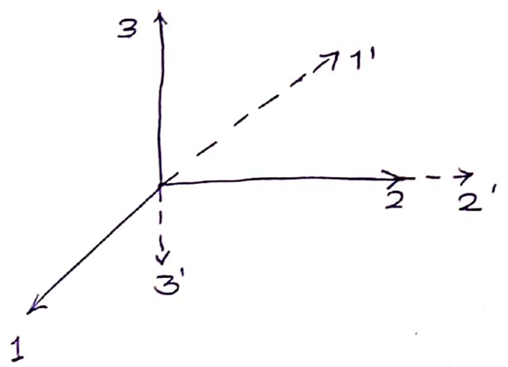
$\cancel{2C_{14}\epsilon_4 + 2C_{15}\epsilon_5 = 0}$.

$C_{14} = 0$, $C_{15} = 0$ {to maintain equality b/w σ_1 & σ_1'}

Total constants need to completely define monoclinic

$$\begin{bmatrix} \overset{1}{C_{11}} & \overset{2}{C_{12}} & \overset{3}{C_{13}} & - & - & \overset{7}{C_{16}} \\ C_{21} & \overset{4}{C_{22}} & \overset{5}{C_{23}} & - & - & \overset{8}{C_{26}} \\ C_{31} & C_{32} & \overset{6}{C_{33}} & - & - & \overset{9}{C_{36}} \\ \epsilon & & & \overset{10}{C_{44}} & \overset{11}{C_{45}} & - \\ & & & C_{54} & \overset{12}{C_{55}} & - \\ & & & - & - & \overset{13}{C_{66}} \end{bmatrix}$$

Orthotropics :- ~~A~~ If any two axes are in different ~~syste coordinate~~ ~~diff~~ directions. (Let's say 1 & 2)



$\sigma_1 = \sigma_1'$

$\sigma_2 = \sigma_2'$

$\sigma_3 = \sigma_3'$

$\sigma_4 = \cancel{\sigma} - \sigma_4'$

$\sigma_5 = -\sigma_5'$

$\sigma_6 = -\sigma_6'$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & - & - & - \\ C_{21} & C_{22} & C_{23} & - & - & - \\ C_{31} & C_{32} & C_{33} & - & - & - \\ - & - & - & C_{44} & - & - \\ - & - & - & - & C_{55} & - \\ - & - & - & - & - & C_{66} \end{bmatrix}$$

→ whatever 2 axes we take symmetry, 3 axis will automatically be symmetric.

	σ_1	σ_2	σ_3
1	$\frac{\sigma_1}{E_1}$	$-\frac{\nu_{21}}{E_1}\sigma_2$	$-\frac{\nu_{31}}{E_1}\sigma_3$
2	$-\frac{\nu_{12}}{E_2}\sigma_1$	$\frac{\sigma_2}{E_2}$	$-\frac{\nu_{32}}{E_2}\sigma_3$
3	$-\frac{\nu_{13}}{E_3}\sigma_1$	$-\frac{\nu_{23}}{E_3}\sigma_2$	$+\frac{\sigma_3}{E_3}$

$$[\epsilon] = [S]_{6\times6}[\sigma]$$

$$[\epsilon] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_1} & -\frac{\nu_{31}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_2} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_3} & -\frac{\nu_{23}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{12} \end{bmatrix} [\sigma]$$

Scanned by CamScanner

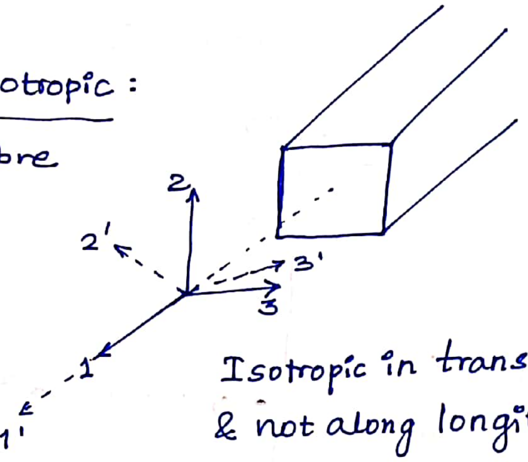
$$\nu_{12} = \frac{\nu_{21} \cdot E_2}{E_1}$$

9 Independent constants :

E_1	~~A_2~~ ν_{12}	G_{12}
E_2	ν_{23}	G_{23}
E_3	ν_{13}	G_{13}

Transversely isotropic :

Eg. Carbon fibre



$C_{12} = C_{13}$

$C_{22} = C_{33}$

$C_{55} = C_{66}$

Isotropic in transverse plane (2-3)
& not along longitudinal axis (1)

$$\begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}$$

$$C_{44} = \frac{C_{22} - C_{23}}{2}$$

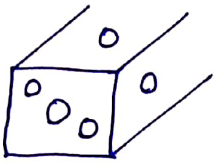
→ Transversely isotropic mat. will remain tran. isotropic in any ~~only~~ one particular chosen axis system.
& that axis system is mat. coordinate axis system

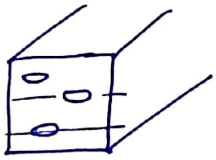
→ So, in total $\underline{\underline{5}}$ constants need to be known to define the structure

C_{11}, C_{12}, C_{22}, C_{23}, C_{55}.

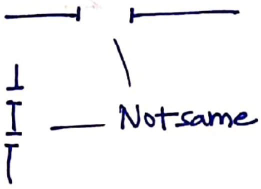
Isotropic

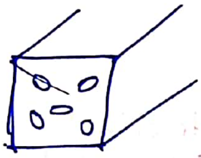
Composites with spherical inclusions are isotropic.



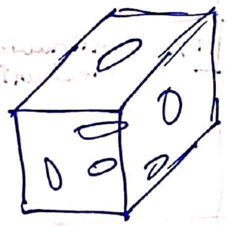


ellipsoids parallel – orthotropic.

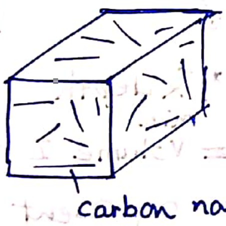




Transversely isotropic.



Trasversely isotropic

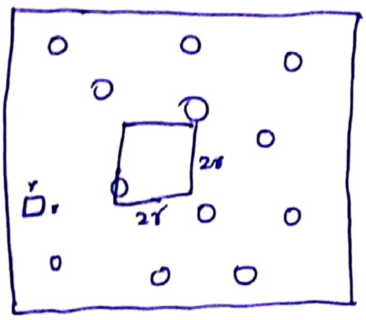


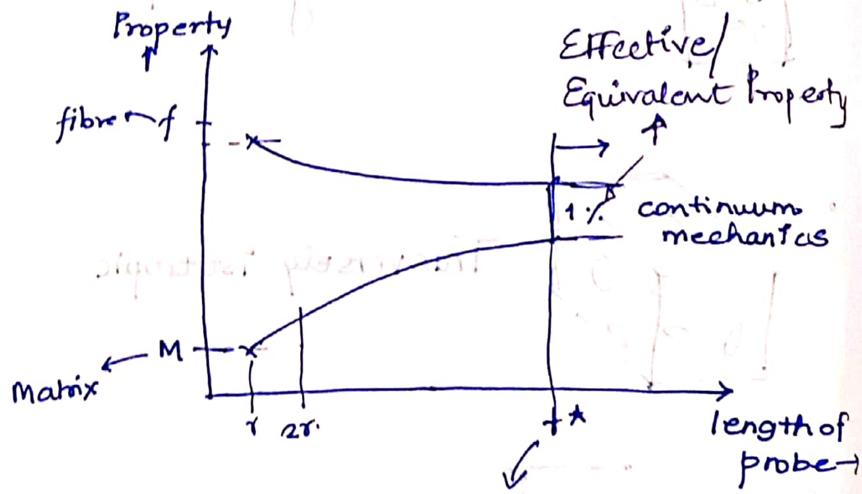
Transversely isotropic

Type of mat.	No. of constants.
Anisotropic	21
Or Monoclinic	13
Orthotropic	9
Transversely Isotropic	5
Isotropic	2

Composites

Consider a transverse plane of composite mat. (matrix + fibre) & checking it by square probe of length 'r',



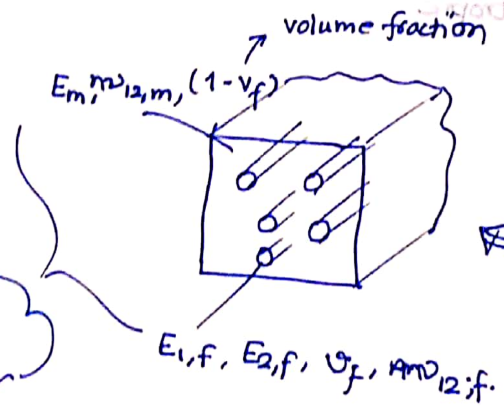


$(r^*)^2_{min} \times$ depth

= Volume I need to represent composite mat.

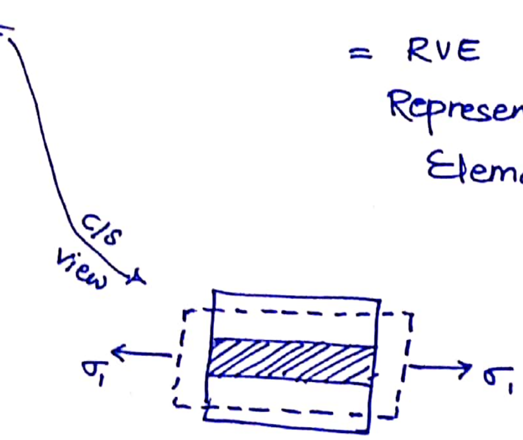
= RVE

Representative Volume Element.



$E_{1,c}$, $E_{2,c}$ = ?

Problem Statement



Iso strain . Assumption.

$\rightarrow \epsilon_{1,c} = \epsilon_{1,f} = \epsilon_{1,m}$.

$\rightarrow P = P_f + P_m$.

$\rightarrow \sigma_{1,c} A = \sigma_{1,f} A_f + \sigma_{1,m} A_m$

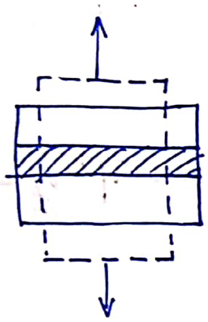
$\cancel{\epsilon_{1,c}} \cdot E_{1,c} A = \cancel{\epsilon_{1,f}} \cdot E_{1,f} A_f + \cancel{\epsilon_{1,m}} \cdot E_{1,m} \cdot A_m$. {∵ Isostrain Condition}

$$E_{1,c} = E_{1,f} \cdot \frac{A_f}{A} + E_{1,m} \frac{A_m}{A}$$

$$= E_{1,f} V_f + E_{1,m} (1 - V_f) \rightarrow \boxed{E_{1,c} = E_{1,f} V_f + E_{1,m} V_m}$$

$$\boxed{E_{1,c} = (E_{1,f} - E_{1,m}) V_f + E_{1,m}}$$ Rule of Mix.

② To find $E_{2,c}$;

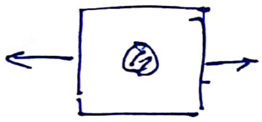


Isostress. Assumption.

$\rightarrow \sigma_{2,c} = \sigma_{2,f} = \sigma_{2,m}$

$\rightarrow \Delta l_c = \Delta l_f + \Delta l_m$.

$$\rightarrow \frac{\Delta l_c}{l} = \frac{\Delta l_f}{l} + \frac{\Delta l_m}{l}$$



$$= \frac{\Delta l_f}{l_f} \times \frac{l_f}{l} + \frac{\Delta l_m}{l_m} \times \frac{l_m}{l}$$

$$\epsilon_{2,c} = \epsilon_{2,f} \cdot V_f + \epsilon_{2,m} V_m.$$

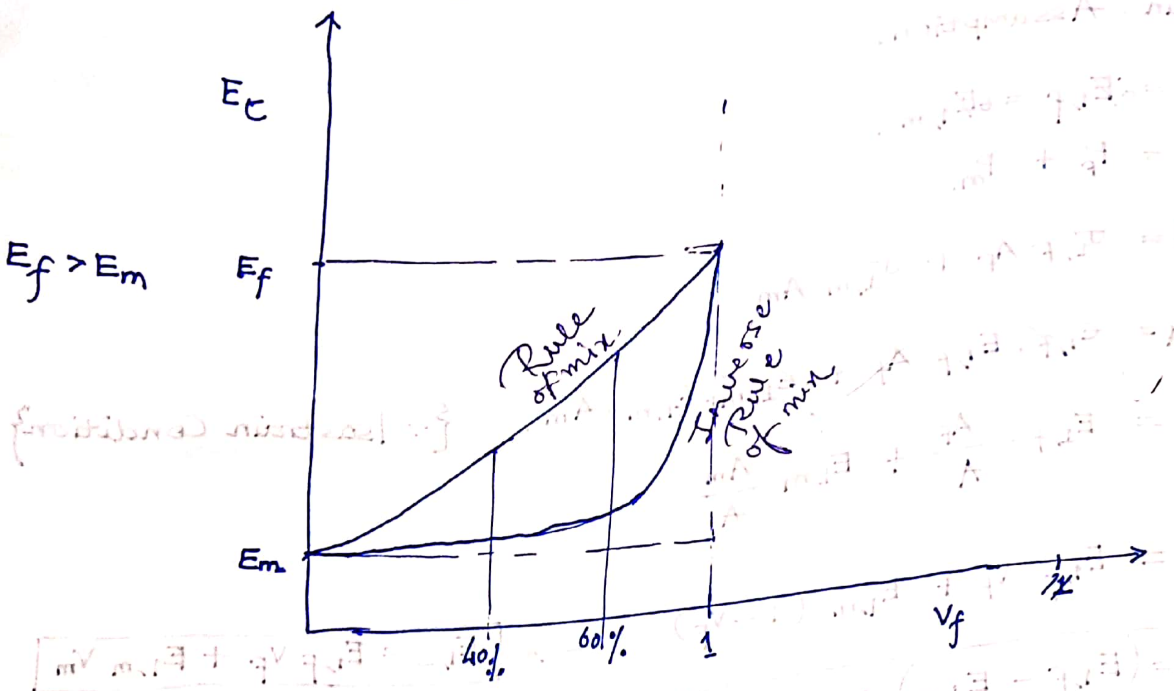
$$\frac{\cancel{\sigma_{2,c}}}{E_{2,c}} = \frac{\cancel{\sigma_{2,f}}}{E_{2,f}} V_f + \frac{\cancel{\sigma_{2,m}}}{E_{2,m}} V_m.$$

$$\boxed{\frac{1}{E_{2,c}} = \frac{V_f}{E_{2,f}} + \frac{V_m}{E_{2,m}}}$$

Inverse Rule of Mixtures.

$$= V_f \left[\frac{1}{E_{2,f}} - \frac{1}{E_{2,m}} \right] + \frac{1}{E_{2,m}}$$

Scanned by CamScanner



Assuming isotropic, $E_{1,m} = E_{2,m}$ & E

Q. A composite lamina is made of carbon fibre & epoxy. The axial & transverse mod. of C fibre is 230 & 15 GPa while epoxy has modulus of 3 GPa. Calculate axial & transverse stiffness of lamina if $V_f = 0.5$.

→ $E_{1,f} = 230$ GPa

$E_{2,f} = 15$ GPa

$E_m = 3$ GPa

$V_f = 0.5$

$$E_{1,c} = (E_{1,f} - E_m) V_f + E_m$$

$$= (230 - 3) \times 0.5 + 3$$

$$= 113.5 + 3$$

$$E_{1,c} = 116.5 \text{ GPa}$$

$$1/E_{2,c} = V_f \left[\frac{1}{E_{2,f}} - \frac{1}{E_m} \right] + \frac{1}{E_m}$$

$$= 0.5 \left[\frac{1}{15} - \frac{1}{3} \right] + \frac{1}{3}$$

$$1/E_{2,c} = -\frac{2}{15} + \frac{5}{15} = \frac{1}{5}$$

$$E_{2,c} = 5 \text{ GPa}$$

To increase $E_{2,c}$,

① Epoxy has doped with carbon fibres $E_m = 5$ GPa

② Carbon fibres are replaced with glass fibres $E_f = 72$ GPa

③ Half of the carbon fibres are replaced with glass fibres.

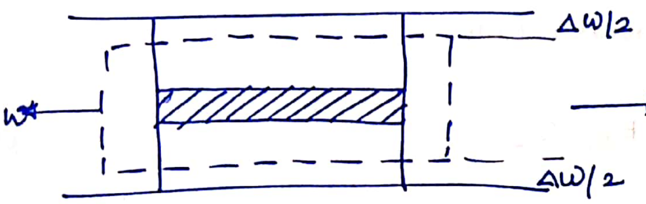
→ ① $\cancel{0.5\left[\frac{1}{15} - \frac{1}{5}\right] + \frac{1}{5}}$ $\quad 0.5\left[\frac{1}{15} + \frac{3}{15}\right] = \frac{2}{15}$

$E_{2,c} = 7.5$ GPa

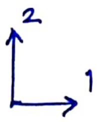
② $1/E_{2,c} = 0.5\left[\frac{1}{72} + \frac{1}{3}\right] = \frac{25}{144} \longrightarrow E_{2,c} = 5.76$ GPa

③ $1/E_{2,c} = 0.25 \times \frac{1}{15} + 0.25 \times \frac{1}{72} + 0.5 \times \frac{1}{3}$

$=$



$\longrightarrow \quad \nu_{12} = -\frac{\epsilon_2}{\epsilon_1}$



$\Delta w = \Delta w_f + \Delta w_m$

$\epsilon_{1,f} = \epsilon_{1,c} = \epsilon_{1,m}$

$\epsilon_2 = \frac{\Delta w}{w}$

$\Delta w = \epsilon_2 \cdot w$

$\epsilon_{2,f} = \frac{\Delta w_f}{w_f}$

$\Delta w = -\nu_{12} \cdot \epsilon_1 \cdot w$

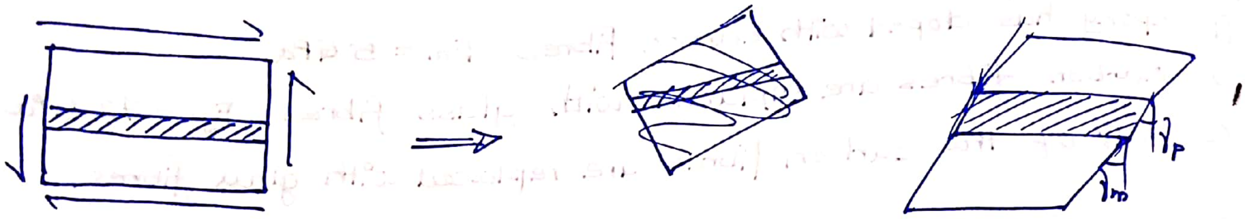
$\epsilon_{2,m} = \frac{\Delta w_m}{w_m}$

$\Delta w_f = -\nu_{12,f} \epsilon_{1,f} \cdot w_f$

$\Delta w_m = -\nu_{12,m} \epsilon_{1,m} w_m$

$-\nu_{12} \cancel{\epsilon_1} w = -\nu_{12,f} \cancel{\epsilon_{1,f}} w_f + -\nu_{12,m} \cdot \cancel{\epsilon_{1,m}} w_m$

$\boxed{\nu_{12,c} = \nu_{12,f} V_f + \nu_{12,m} V_m}$



Isostress Assumption

$$\tau_{1,c} = \tau_{1,f} = \tau_{1,m}$$

$$\gamma_{c} \cdot w = \gamma_m \cdot w_m + \gamma_f \cdot w_f$$

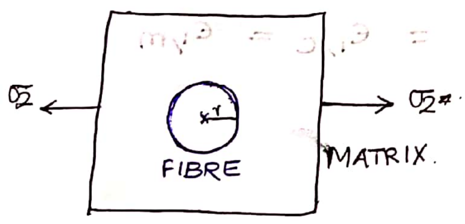
$$\frac{\cancel{\tau}}{G_{12}} w = \frac{\cancel{\tau_{1,m}}}{G_{12,m}} w_m + \frac{\cancel{\tau}}{G_{12,f}} w_f$$

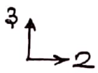
$$\boxed{\frac{1}{G_{12,c}} = \frac{1}{G_{12,m}} V_m + \frac{1}{G_{12,f}} V_f}$$

13:01.2020.

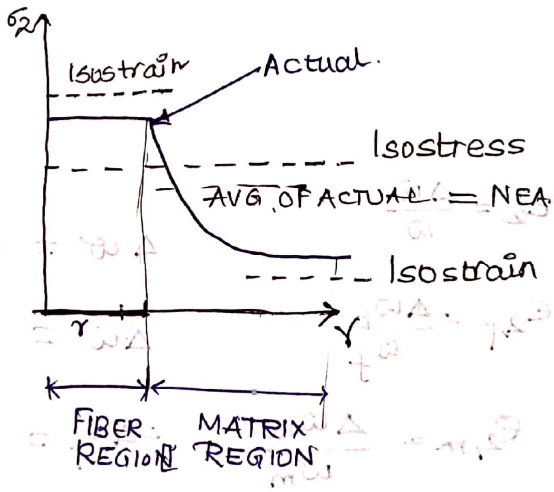
Rule: E_1, ν_{12}

Inverse Rule: E_2, E_3, G_{12}





$E_F > E_M$



∴ ISOSTRESS ASSUMPTION IS NEARLY TRUE.

#1 Stress Partitioning.

$$\sigma_{f,2} \neq \sigma_{m,2} = \eta_2 \, \sigma_{f,2}$$

η_2 → Stress partitioning factor

Scanned by CamScanner

$$\frac{\sigma_{c,2}}{E_{c,2}} = \frac{v_f \sigma_{f,2}}{E_{f,2}} + \frac{v_m \eta_2 \sigma_{f,2}}{\cancel{E_{f,2} E_{m,2}} E_m}$$

$$\frac{\sigma_{c,2}}{E_{c,2}} = \frac{v_f \sigma_{f,2}}{E_{f,2}} + \frac{v_m \eta_2 \sigma_{f,2}}{E_m} \quad \text{—— (1)}$$

$$\sigma_{c,2} = v_f \cdot \sigma_{f,2} + v_m \sigma_m \longrightarrow$$ Sum of respective stresses as per volume fractions.

$$\sigma_{c,2} = (v_f + \eta_2 v_m)\, \sigma_{f,2} \quad \text{—— (2)}$$

From (1) & (2), $$\frac{(v_f + \eta_2 v_m)}{E_{c,2}} = \left[\frac{v_f}{E_{f,2}} + \frac{v_m \eta_2}{E_m}\right]$$

$$\boxed{\frac{1}{E_{c,2}} = \frac{1}{v_f + \eta_2 v_m}\left[\frac{v_f}{E_{f,2}} + \frac{v_m \eta_2}{E_m}\right]}$$

? How to find η_2 beforehand.

$$\frac{1}{P_c} = \frac{1}{v_f + \eta_p v_m}\left[\frac{v_f}{P_f} + \frac{v_m \eta_p}{P_m}\right]$$

SEMI EMPIRICAL METHOD.

$P_f >> P_m \longrightarrow \eta = f(v_m)$

$\eta_k = \frac{1}{2(1 - v_m)}$ [for Bulk Modulus]

$\eta_2 = \frac{1}{2}$ [for E_2]

$\eta_{23} = 0.6$ [for G_{23}]

AVG. STRESS IN PHASE I

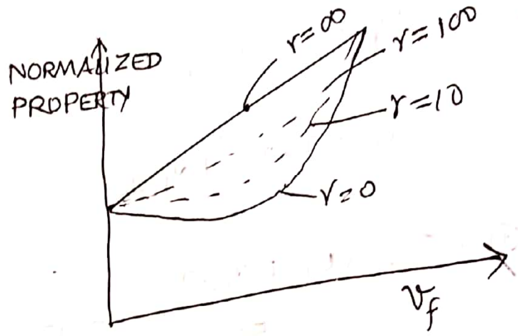
" " " " II

#2 Halpin – Tsai Correction

To get a unique formula, instead of 3 for isostress, isostrain & stress partitioning conditions.

$$\frac{P_c}{P_m} = \frac{1 + \xi \eta v_f}{1 - \eta v_f}$$

$$\eta = \frac{\frac{P_c}{P_m} - 1}{\frac{P_f}{P_m} + \xi}$$



#3 Charmis Correction

— Empirical form

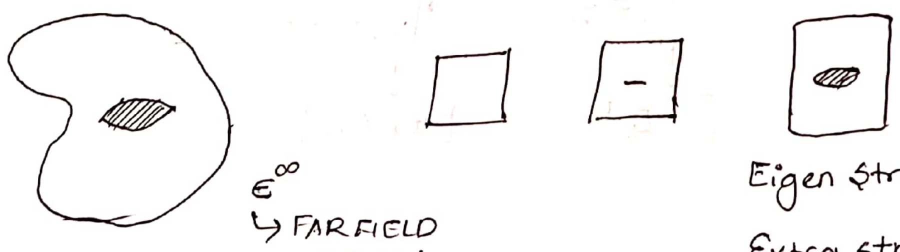
— It does not work for carbon fibres.

– Rule of mix. remain same but for inverse rule of mix,

Replace, $v_m = 1 - v_f$

$v_f \Rightarrow \sqrt{v_f}$

EIGEN STRAIN.



ϵ^∞

↳ FARFIELD STRAIN.

Eigen strain

Extra strain due to inclusion one

Eshelby (1957)

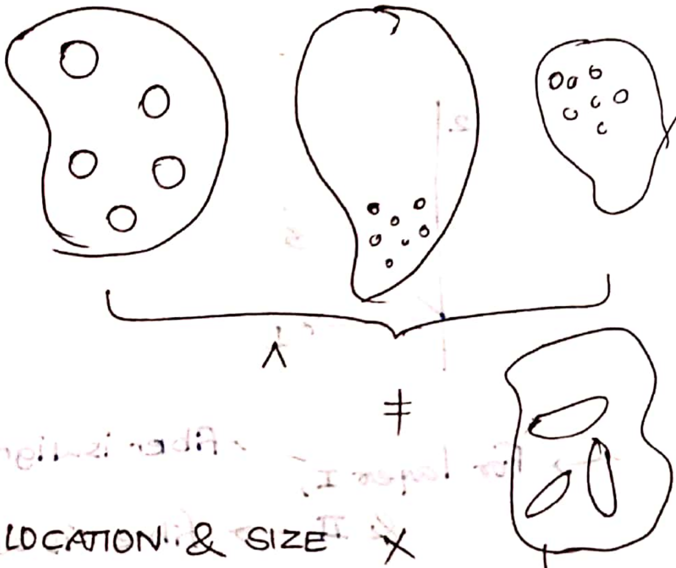
$$\epsilon^* = [E]\,\epsilon^\infty \longrightarrow f(\nu)$$

$\langle \epsilon_i \rangle \longrightarrow$ Average strain in inclusion

Mori & Tanaka (1979)

All inclusions have same amt of image ~~stress~~ strain – Mean field assump

strain on inclusion due to other inclusions



① Calculations are size independent.

② don't depend individually on E_1 & E_2.

③ $[E] = f(v)$

NOT DEPEND → FIBER LOCATION & SIZE ✗

→ FIBER ONLY f^n of a, b, c

DEPEND → FIBER VOL. FRAC ✓ $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$

Not Ellipsoid

As long as ellipsoid shape is observed, these

14/01/2020

1. Hand Layup.
2. Spray Layup.

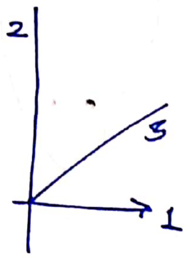
Multiscale mechanics

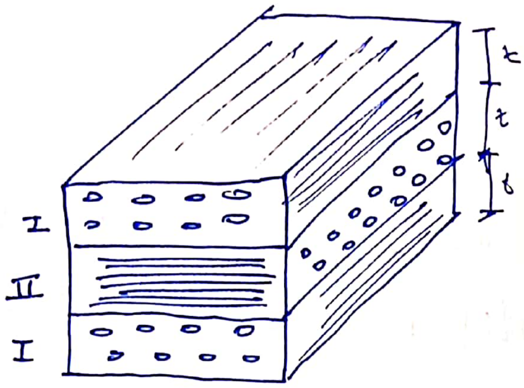
$v_f = 0.5$

$E_{Gf} = 72$ GPa

$E_{epoxy} = 3$ GPa

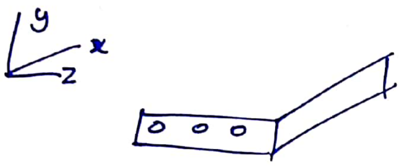
$E_1^{eff}, E_2^{eff}, E_3^{eff}$





→ For layer I, → fiber is aligned to 3 dir.
& II, → fibre is aligned to 1 dir.

① For layer I, Rule of mix



$E_x = 37.5$ GPa

$E_y = E_z = 5.76$ GPa. ← Inv. rule of mix.

② Make it homogenous layers of above prop.
with Layer I, $E_1 = E_2 = 5.76$ & $E_3 = 37.5$
& Layer II, $E_3 = E_2 = 5.76$ & $E_1 = 37.5$

③ ~~Then with equal fractions, apply rule~~ o
Then if load is applied in 1 & 3 apply isostrain condⁿ
& in 2, apply isostress condⁿ.

Effective axial strength does not depend on order (arrangement) in which laminae is kept

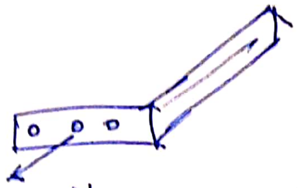
Q. A unidirectional cont. fibre is made of comp carbon fibre-epoxy both with equal vol. fraction, the composite is sub. to load in the fibre dir having magnitude 100N. Find out how much load is taken by the fibre.

→

$$P_f = \sigma_f \times A_f$$

$$P_m = \sigma_m A_m$$

$$P_c = P_f + P_m$$



100 N.

$$\frac{\sigma_f}{E_f} = \frac{\sigma_c}{E_c^{eq}} = \frac{\sigma_m}{E_m}$$

$E_c = E_f \times V_f + E_m \times V_m$

$= (72 + 3) \times 0.5$

$E_c = 37.5 \text{ GPa}$

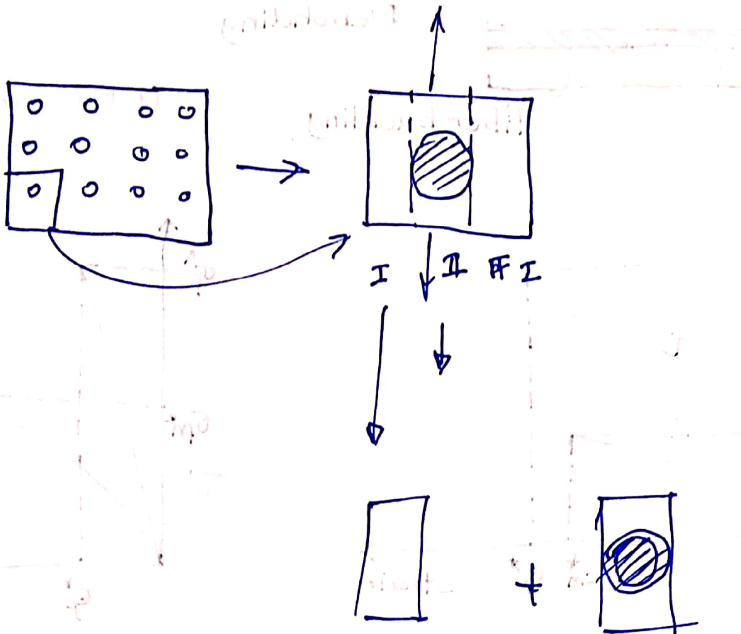
$$\cancel{\sigma_c}\ \frac{P_c \times A_c}{E_c^{eq}} = \frac{\cancel{\sigma}\cdot P_f \times A_f}{E_f}$$

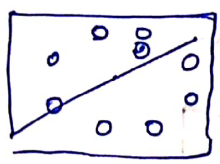
$$\frac{P_c}{37.5} = \frac{P_f \times 0.5}{72}$$

$$P_c = \frac{37.5}{\cancel{36}\ 144} P_f$$

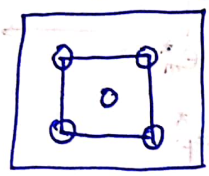
★ Multisection rule of mix.

Square packing

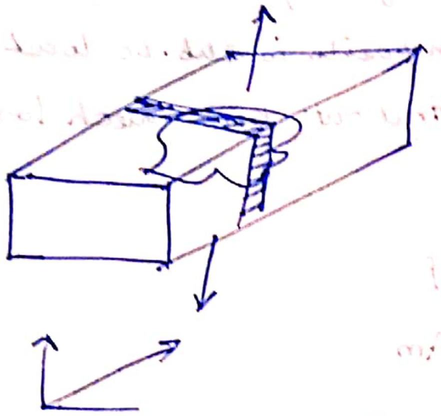


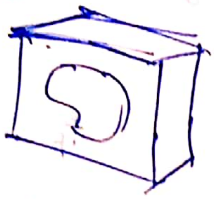


For hex packing,

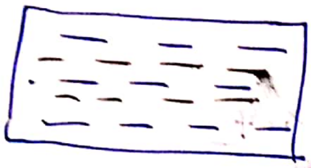


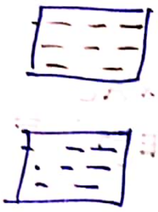
Isostrain For I + Isostress





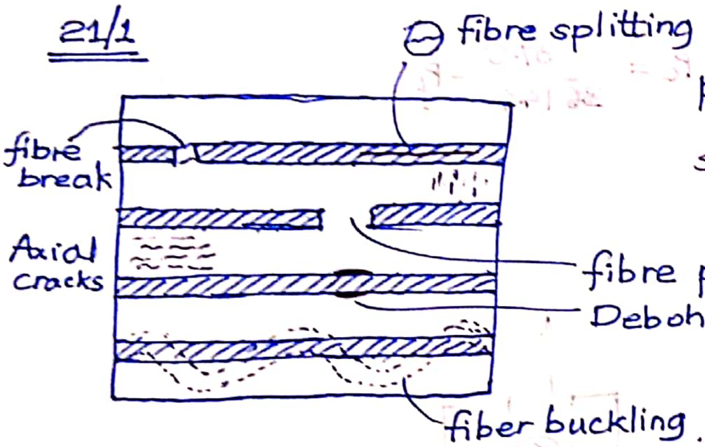
Isostrain.





Isostrain.

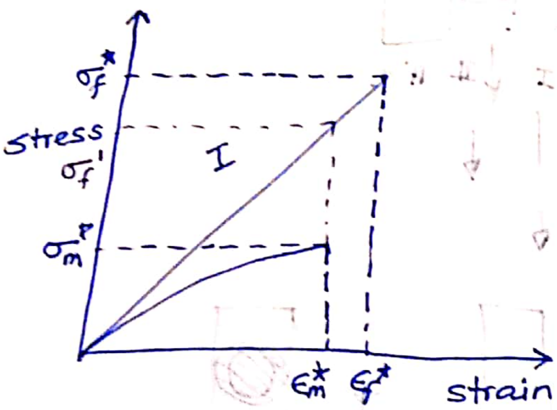
21/1

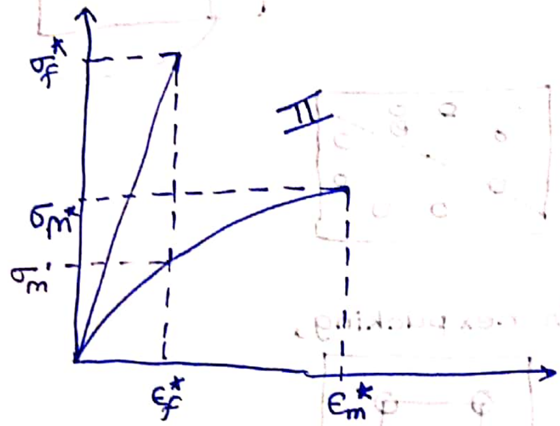


progressive damage.

strength v/s stiffness.

(1) Micromechanics of damage





$\epsilon_m^* < \epsilon_f^*$

$\epsilon_m^* > \epsilon_f^*$

f - fibre
m - matrix

CASE I

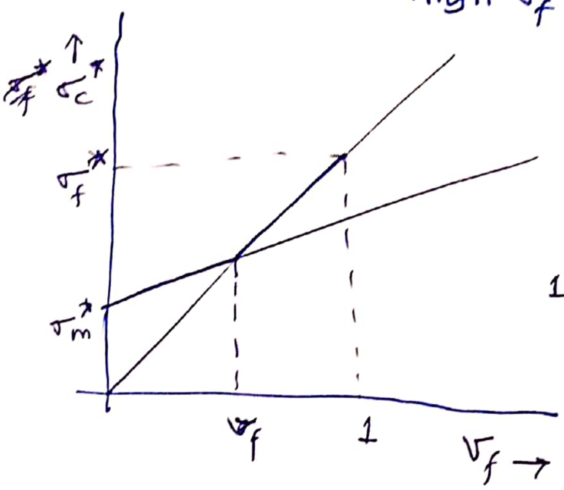
CASE II

Ⅰ

$\sigma_c = v_f \sigma_f + v_m \sigma_m$

low $v_f \rightarrow \sigma_c^* = \sigma_f' v_f + \sigma_m^* (1-v_f)$

High $v_f \rightarrow \sigma_c^* = \sigma_f^* v_f$.

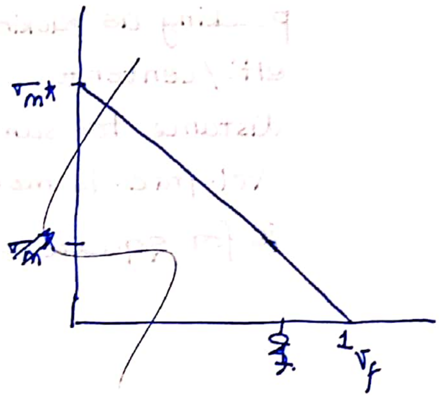


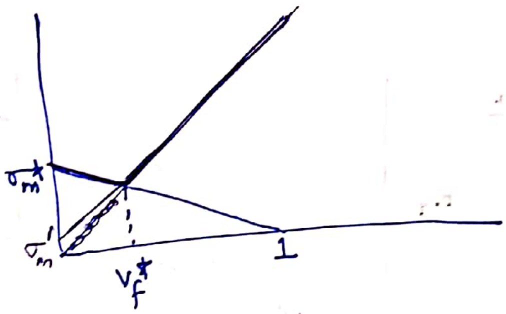
17 % Glass fibre-epoxy $\rightarrow$ Minimum v_f

Ⅱ $\sigma_c = v_f \sigma_f + v_m \sigma_m$

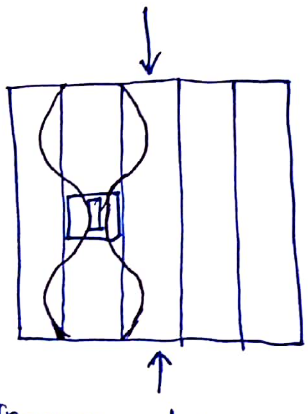
low $v_f \rightarrow \sigma_c^* = \sigma_m^* (1-v_f)$

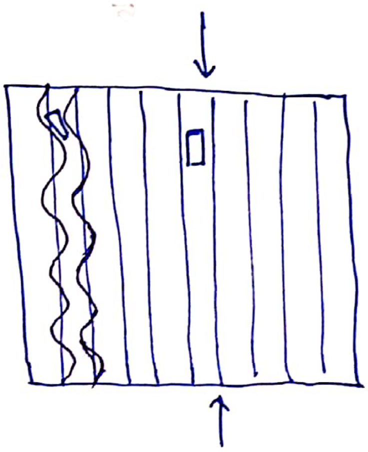
High $v_f \rightarrow \sigma_c^* = \sigma_f^* v_f + \sigma_m' (1-v_f)$





Buckling

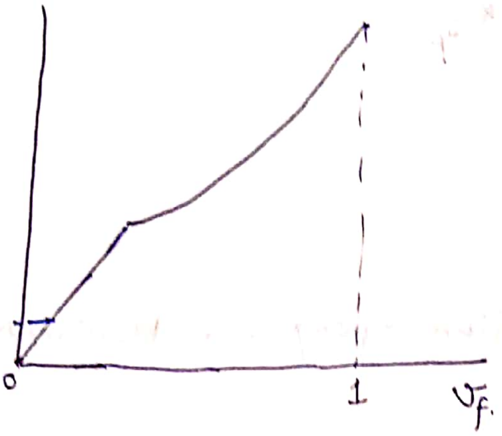




Buckling $\rightarrow$ Transverse behaviour of mat.
Resisted by

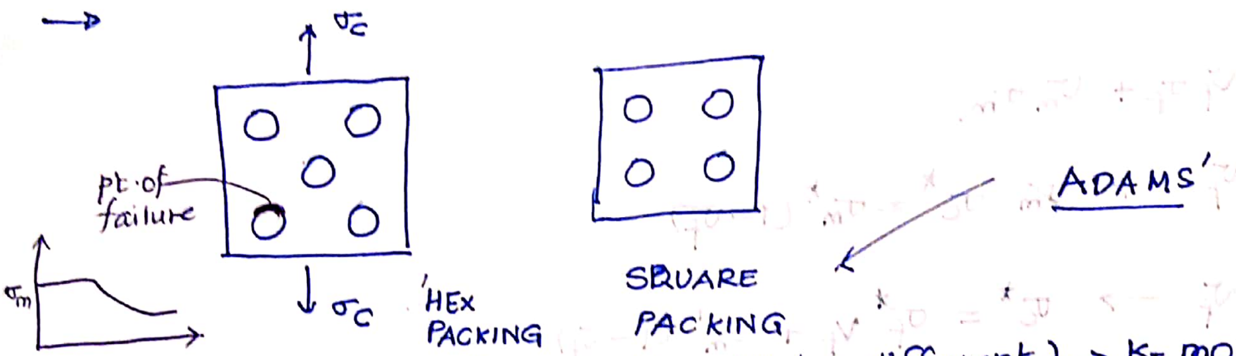
Shear behaviour of mat.

$$\sigma_c^* = \frac{2 v_f^{3/2}}{\sqrt{3(1-v_f)}} \sqrt{E_m E_f} \qquad \sigma_c = \frac{\sigma_m}{1-v_f}$$



Transverse (Tensile)

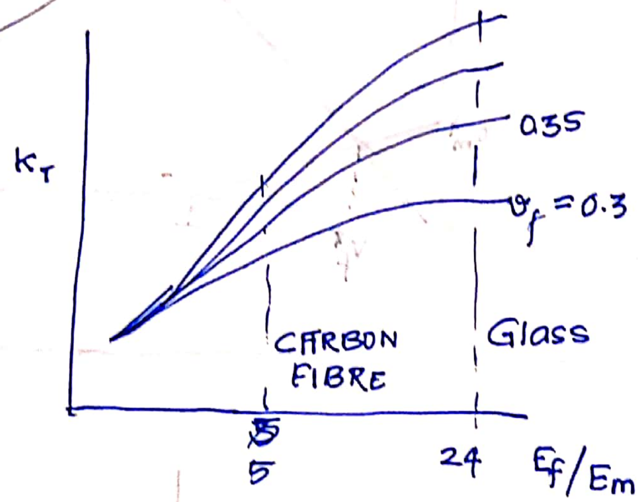
→



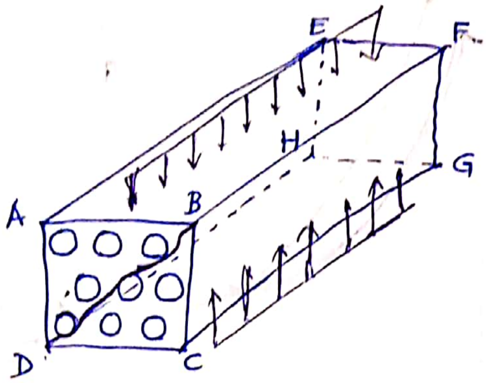
SQUARE PACKING
(K_T will be different) → K_T more in square packing as packing eff. / center to center distance for same vol. frac. is more is for square packing

$\sigma_m^* = K_T \sigma_c$

Skudra



Transverse (Compressive)



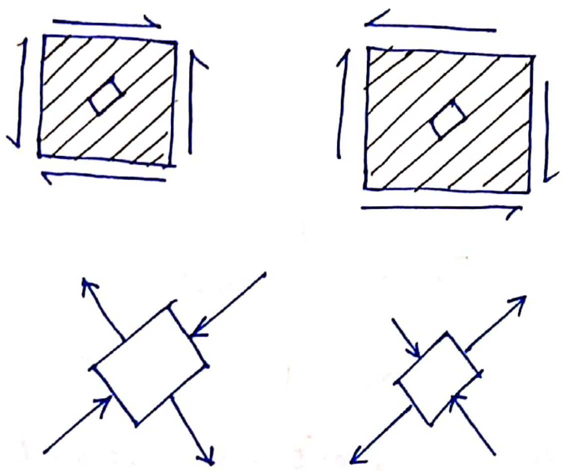
EFCD ✗ {longer, High energy path as it goes across fibre)

DBFH ✓

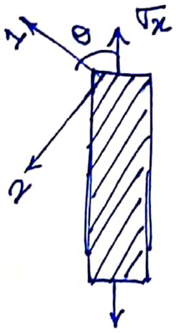
Mode of failure

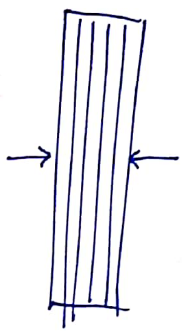
(Crack propagation through matrix avoiding fibre strand).

*



NOT SAME : as axially & transverse, both have opposite loadings





Tensile strength.

↓

X_T $\cancel{X_C}$ Y_T

X_C Y_C

P Q R

'Shear' strength in 3 direc

$$\sigma_1 = \sigma_x \cos^2\theta$$

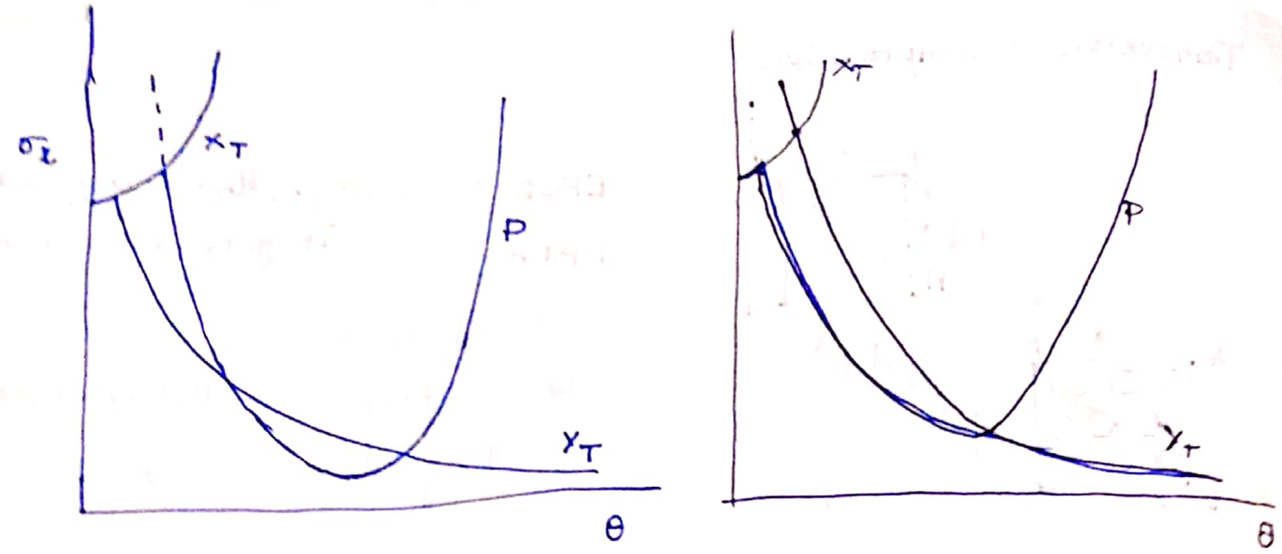
$$\sigma_2 = \sigma_x \sin^2\theta$$

$$\sigma_{12} = P\ |-\sigma_x \sin\theta \cos\theta|$$

MPST : $X_C < \sigma_1 < X_T$

$Y_C < \sigma_2 < Y_T$

$|\sigma_{12}| < P$



- X_T for CF is 1450 MPa.

  Y_T ~~for~~ 53.4 MPa

  P = 99.3 MPa

  find out angles at intersection.

→

$$\sigma_x < \frac{X_T}{\cos^2\theta}, \quad \sigma_x < \frac{Y_T}{\sin^2\theta}, \quad \sigma_x < \frac{P}{\sin\theta\cos\theta}$$

~~$X_T = P$~~

$\cancel{\sigma_x}\cos^2\theta =$

$X_T = Y_T$

$\cancel{\sigma_x}\cos^2\theta = \cancel{\sigma_x}\sin^2\theta$

$2\cos^2\theta - 1 = 0$

$\cos 2\theta = 0$

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

$X_T = P$

$\cancel{\sigma_x}\cos^2\theta = \cancel{\sigma_x}\cos\theta\sin\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta$

$\theta = \frac{\pi}{4}$

$Y_T = P$

$\sigma_x\sin^2\theta = \sigma_x\cos\theta\sin\theta$

$\theta = \frac{\pi}{4}$

$$\sigma_x < \frac{1450}{\cos^2\theta}, \quad \sigma_x < \frac{53.4}{\sin^2\theta}, \quad \sigma_x < \frac{99.3}{\sin\theta\cos\theta}$$

$X_T = Y_T$

$\frac{1450}{\cos^2\theta} = \frac{53.4}{\sin^2\theta}$

~~$\tan^2\theta$~~.

$\theta \rightarrow 10.86°$

$X_T = P$

$\theta \rightarrow 3.92°$

$Y_T = P$

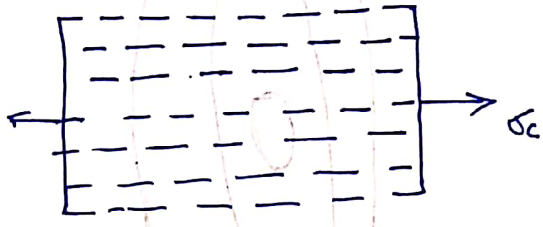
↓

$\theta \Rightarrow 28.27°$

for pure shear, put your fibre at 45° orientation

If we can control fibre orientation in b/w 0 to 3.9°, it would actually make comp. stronger as Strength $\propto \frac{1}{\cos^2\theta}$

27/01



Isostrain (bcoz area of c/s ~~of~~ fibre const.)

$\epsilon_f = \epsilon_m = \epsilon_c$

$\cancel{\epsilon_f}\ \sigma_f = E_f \epsilon_f$

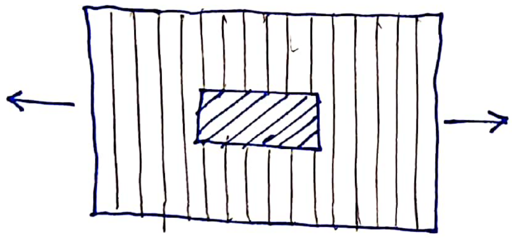
$E_c = f(\epsilon_f, \epsilon_m, \nu_f)$

$\epsilon_f \neq \epsilon_m \neq \epsilon_c$

$\sigma_f = f(x)$

$E_c = f(\epsilon_m, \epsilon_f, AR, V_f)$

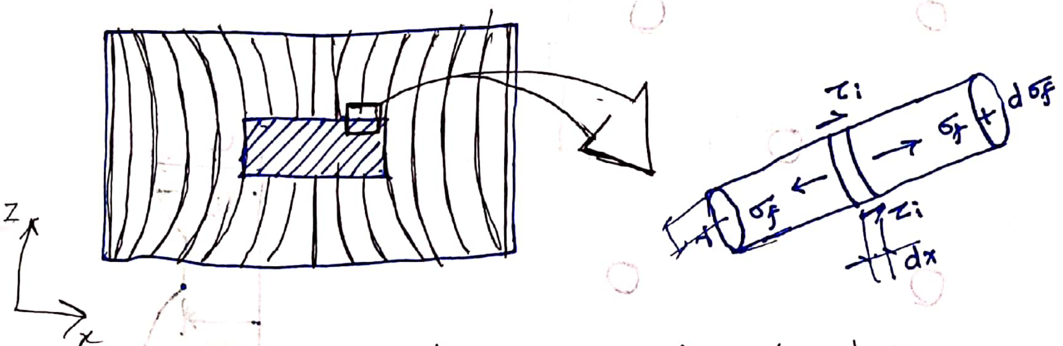
AR: Aspect ratio



$E_f >> E_m$.

SHEAR LAG MODELS (developed to study papers)

COX, KELLY.



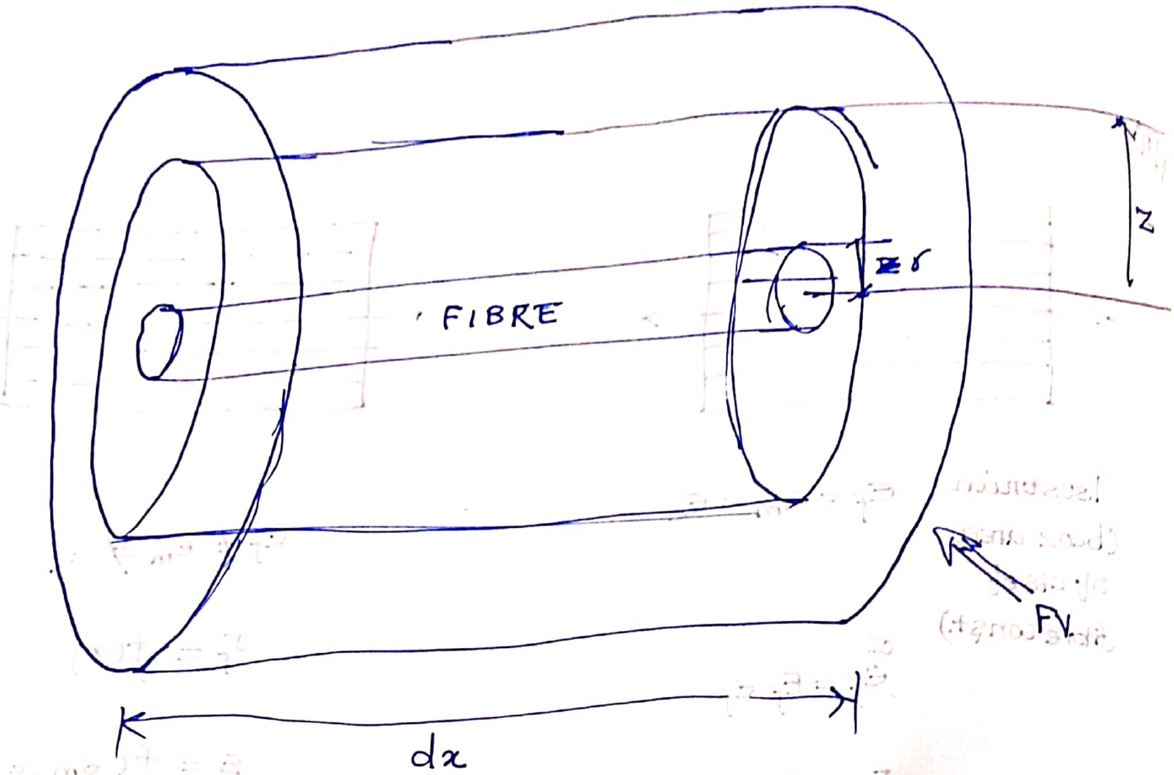
No stress transfer ~~across~~ through edges

Stress transfer through shear along ~~[illegible]~~ periphery around fibre assuming traction continuity surrounding it

$$\cancel{\pi r^2}\, d\sigma_f = -2\cancel{\pi r}\,\tau_i\, dx$$

$$\frac{d\sigma_f}{dx} = -\frac{2\tau_i}{r} \qquad \text{—— (1)}$$

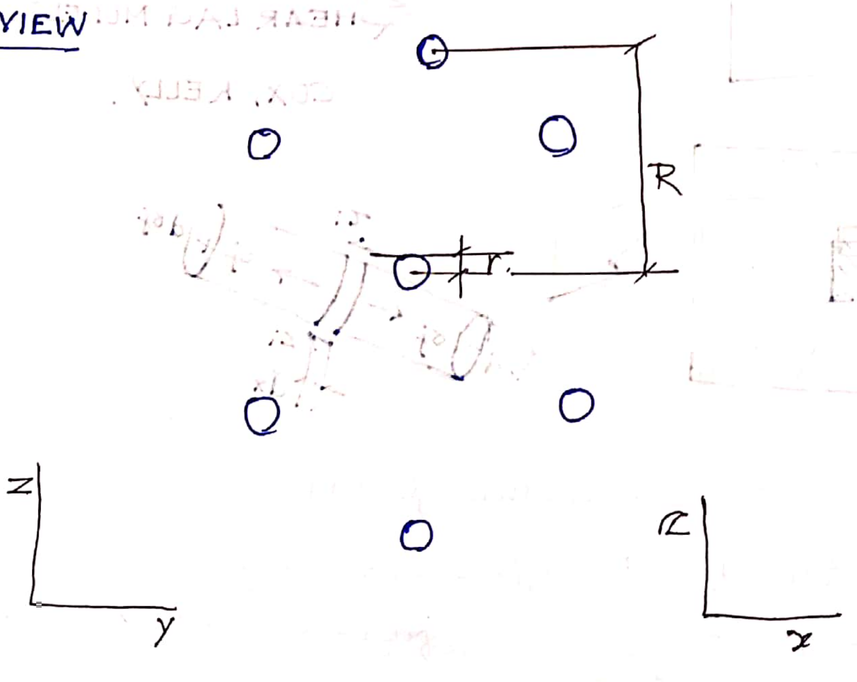
{At center, r=0, local maxima}

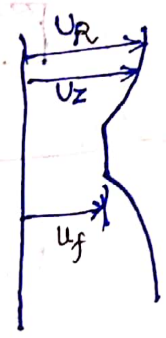


$$\cancel{2\pi} z\, \tau_z \cancel{dx} = \cancel{2\pi} r\, \cancel{dx} \cdot \tau_i$$

$$\tau_z = \frac{r}{z}\tau_i$$

SIDE VIEW





$$\gamma = \frac{\partial u}{\partial z} = \frac{\tau}{G_m}$$

$$\int_{U_f}^{U_R} du = \int_r^R \frac{r\tau_i}{G_m}\cdot\frac{1}{z}\,dz$$

$$\tau_i = \frac{G_m (U_R - U_f)}{r \ln(R/r)}$$

$$\frac{\partial \tau_i}{\partial x} = \frac{G_m\left(\frac{\partial U_R}{\partial x} - \frac{\partial U_f}{\partial x}\right)}{r\ln(R/r)} \qquad \frac{\partial U_R}{\partial x} = \epsilon_m = \epsilon_c .$$

$$= \frac{G_m\left(\epsilon_c - \frac{\sigma_f}{E_f}\right)}{r\ln(R/r)}$$

From (1),
$$-\frac{r}{2}\frac{\partial^2 \sigma_f}{\partial x^2} = \frac{G_m\left(\epsilon_c - \frac{\sigma_f}{E_f}\right)}{\cancel{r}\,r\ln(R/r)} \longrightarrow \text{Let } n^2 = \frac{2G_m}{E_f \ln(R/r)}$$

$$\cancel{r^2 \ln(R/r)} = \cancel{n^2 G}$$

$$\frac{d^2\sigma_f}{dx^2} = \cancel{\sigma_f}\,\frac{n^2}{r^2}(\sigma_f - E_f\epsilon_c)$$

Assuming σ_f is constant ~~Got~~ across c/s

$$\left(D^2 - \frac{n^2}{r^2}\right)\sigma_f = -\frac{n^2}{r^2}E_f\epsilon_c$$

$$D = \pm\frac{n}{r}$$

$$\text{C.F} \Rightarrow \sigma_f = C_1 e^{nx/r} + C_2 e^{-nx/r}$$

$$\text{P.I} \Rightarrow E_f\epsilon_c$$

$$\sigma_f = E_f\epsilon_c + B\sinh\left(\frac{nx}{r}\right) + D\cosh\left(\frac{nx}{r}\right)$$

B.C.s. At $x = L$ & $-L$, $\sigma_f = 0$

at $x=0$, $\frac{d\sigma_f}{dx} = 0 \rightarrow B\,\overset{1}{\cancel{\cosh\left(\frac{nx}{r}\right)}}\cdot\frac{n}{r} + D\,\overset{0}{\cancel{\sinh\left(\frac{nx}{r}\right)}}\frac{n}{r} = 0$

Scanned by CamScanner

$\underline{B = 0}$

$$\sigma_f = E_f \epsilon_c + D \cosh\left(\frac{nx}{r}\right)$$

$$E_f \epsilon_c + D \cosh\left(\frac{nL}{r}\right) = 0$$

$\epsilon_1 \rightarrow$ Applied strain.

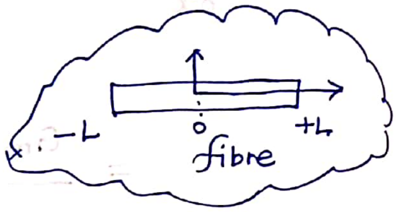
$$D = -\frac{E_f \epsilon_c}{\cosh\left(\frac{nL}{r}\right)}$$

$$\boxed{\sigma_f = E_f \epsilon_1 \left(1 - \frac{\cosh\left(\frac{nx}{r}\right)}{\cosh(ns)}\right)}$$

Aspect ratio.

distance from center

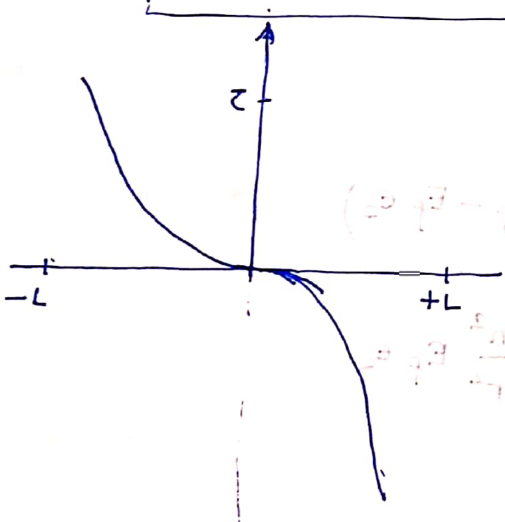
$$s = \frac{L}{r}$$

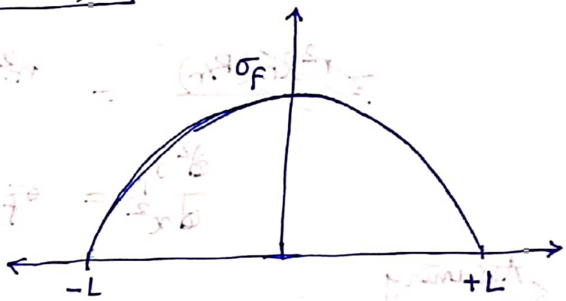


$$\because \frac{d\sigma_f}{dx} = -\frac{2\tau_i}{r}$$

$$\tau_i = -\frac{\cancel{r}}{2} \times \left\{ E_f \epsilon_1 \ominus \frac{\sinh\left(\frac{nx}{r}\right) \cdot \frac{n}{\cancel{r}}}{\cosh(ns)} \right\}$$

$$\boxed{\tau_i = \cancel{\frac{r}{2} \times \frac{n}{r}}\ \frac{1}{2} n E_f \epsilon_1 \frac{\sinh\left(\frac{nx}{r}\right)}{\cosh(ns)}}$$

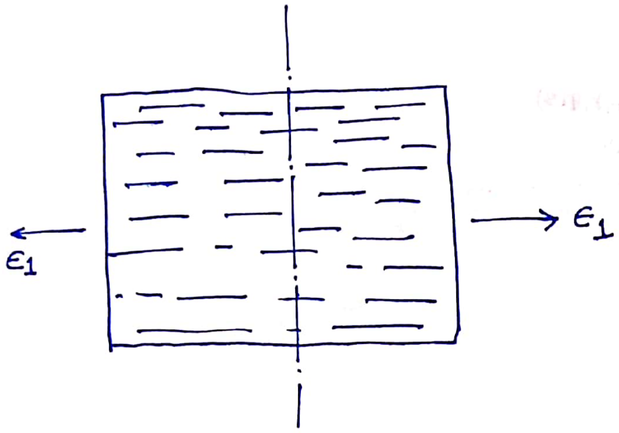


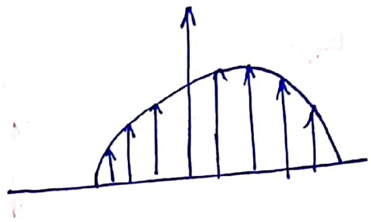


$$\sigma_f^{max} = E_f \epsilon_1 \left(1 - \frac{1}{\cosh(ns)}\right)$$

$$\tau_i^{max} = \frac{1}{2} n E_f \epsilon_1 \tanh(ns)$$

→ How to calculate effective prop.
→ At what load; my composite fails } My goals.





Average stress
= Avg. stress of c/s.

$$\overline{\sigma_m} = E_m \epsilon_1$$

$$\overline{\sigma_f} = \frac{1}{2L}\int_{-L}^{L} \sigma_f \, dx = \frac{1}{2L}\int_{-L}^{L} E_f \epsilon_1 \left(1 - \frac{\cosh\left(\frac{nx}{r}\right)}{\cosh(ns)}\right) dx$$

$$= \frac{1}{2L} E_f \epsilon_1 \left(2L - \left.\frac{\sinh(nx/r) \cdot r}{\cosh(ns) \cdot n}\right|_{-L}^{L}\right)$$

$$= \frac{1}{2L} E_f \epsilon_1 \left(2L - 2\frac{\sinh(ns)}{\cosh(ns)} \cdot \frac{r}{n}\right)$$

$$\overline{\sigma_f} = E_f \epsilon_1 \left(1 - \frac{\tanh(ns)}{ns}\right)$$

$$\cancel{E_{1,c} = (v_f E_f + v_m}$$

$$\sigma_c = v_f \overline{\sigma_f} + v_m \overline{\sigma_m}$$

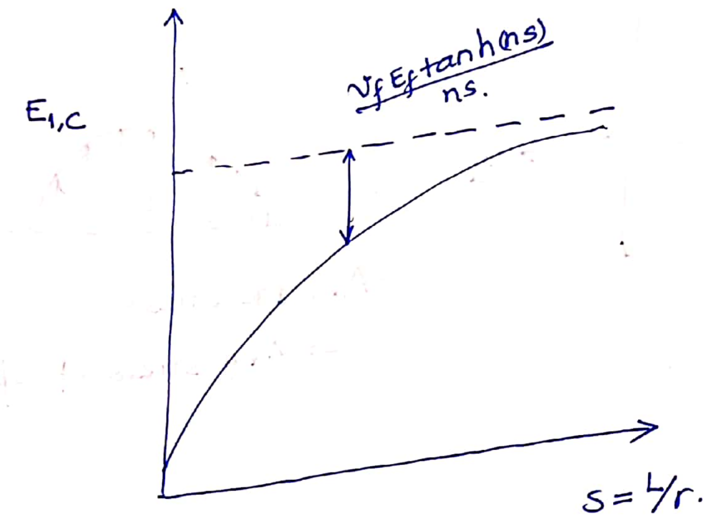
$$E_c \cdot \cancel{\epsilon_1} = v_m \{\cancel{\epsilon_1} E_m\} + v_f \left\{E_f \cancel{\epsilon_1} \left(1 - \frac{\tanh(ns)}{ns}\right)\right\}$$

$$E_c = E_{1,c} = \{v_m E_m + v_f E_f\} \underbrace{- v_f E_f \frac{\tanh(ns)}{ns}}_{\text{penalty for having discont. fibres}}$$

$$\boxed{E_{1,c} = \underbrace{\eta_1}_{\text{Length efficiency factor.}} v_f E_f + v_m E_m}$$

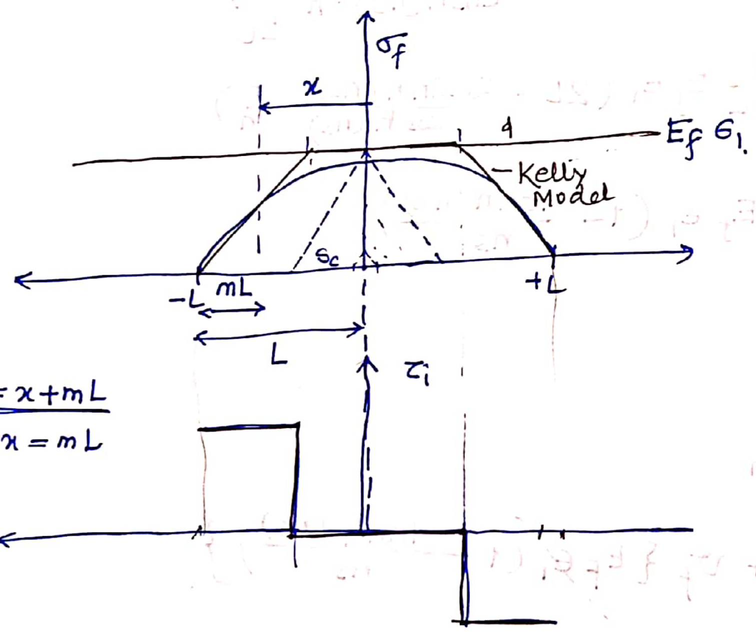
This is Cox model.

Disadv. :— ① what is min. length of fibre that I should have to make
② Max. load ~~can~~ I can apply before it can fail.



Kelly Model (Aligned - discont. fibre)

To overcome earlier disadv.



$L = x + mL$

$L - x = mL$

$$\sigma_f = \frac{2\tau_i}{r}(L-x)$$

$$= 2\tau_i\left(s - \frac{x}{r}\right)$$

$$= 2\tau_i s\left(1 - \frac{x}{L}\right)$$

$$\sigma_f = \frac{2\tau_i L}{r} m$$

$$\sigma_f = 2\tau_i m s$$

$$m = \frac{\sigma_f}{2\tau_i s} \quad \text{—(1)}$$

To get critical aspect ratio, $m = 1$

$$S_c = \frac{\sigma_f}{2\tau_i} \quad \text{—(2)}$$

$\tau_i \rightarrow$ interface strength of matrix.

$\sigma_f \rightarrow$ strength of fibre

In general, $m = \frac{S_c}{S}$

$$\overline{\sigma_f} = \sigma_f\left(1-\frac{m}{2}\right)$$

$$\overline{\sigma_m} = E_f\,\epsilon_1$$

$$\overline{\sigma_c} = v_f\,\overline{\sigma_f} + v_m\,\overline{\sigma_m}$$

$$= v_f\left(\sigma_f\left(1-\frac{m}{2}\right)\right) + \cancel{v}\; v_m E_m \epsilon_1$$

$$E_f\cancel{\epsilon_1} = \cancel{v_f\sigma_f} + v_f\left[E_f\cancel{\epsilon_1}\left(1-\frac{m}{2}\right)\right] + v_m E_m \cancel{\epsilon_1}$$

$$E_c = (E_f v_f + E_m v_m) - E_f v_f \frac{m}{2}$$

$$\Rightarrow E_f v_f \cdot \frac{\sigma E_f \epsilon_1}{4\tau_i S}$$ → This shows that avg. prop. is dep. on applied strain. which is wrong

$$E_{1,c} = \left[E_f v_f + E_m v_m\right] - E_f v_f \frac{E_f\,\epsilon_1}{4\tau_i\, s}$$

$$\overline{\sigma_c} = v_f\,\sigma_f\left(1-\frac{m}{2}\right) + v_m E_m \epsilon_1$$

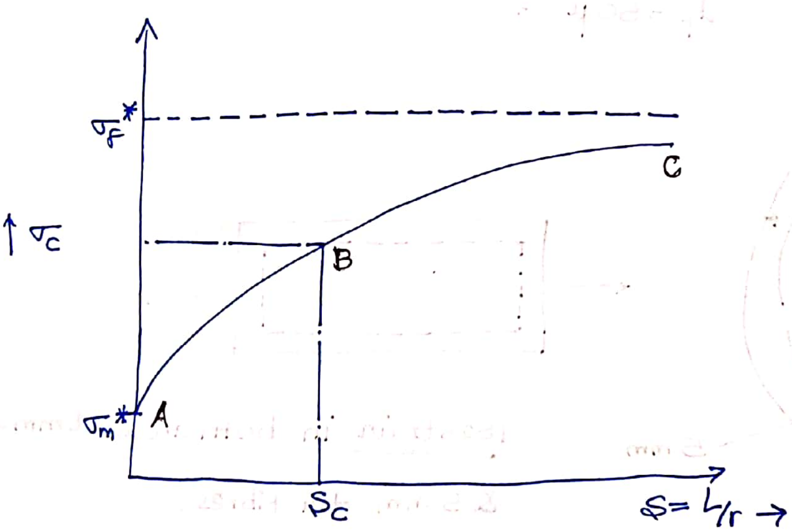
$$= v_f\,\sigma_f\left(1-\frac{m}{2}\right) + v_m E_m \frac{\sigma_f}{E_f}$$

$$= \sigma_f\left[v_f\left(1-\frac{m}{2}\right) + v_m \frac{E_m}{E_f}\right]$$

$$\overline{\sigma_c} = \sigma_f\left[v_f\left(1-\frac{S_c}{2S}\right) + v_m \frac{E_m}{E_f}\right] \quad \text{if } S > S_c$$

If $S_c = S$,

$$\overline{\sigma_c} = \sigma_f \cdot \frac{v_f}{2} + \sigma_f\, v_m \frac{E_m}{E_f}$$



If $S <<< S_c$,

$$\overline{\sigma_{1,c}} = \sigma_m^* \quad \{\text{Strength of Matrix}\}$$

~~If $S >>> S_c$~~

$$\frac{1}{2}\times \sigma_f\left(\frac{2x}{2L}\cancel{2x}\right)(\cancel{2L})\times\frac{1}{\cancel{2L}}(2x+2L) \quad (2L-2mL+2L) = \sigma_f\,(2L-mL)$$

If interfacial strength is higher, then we can shift line towards left. { ↑ i.e. less length is needed to gain the composite} (s_c^*)

	Strength	Stiffness
cont	✗	✓
Discont	✓	✗

Discont →
→ cheaper
→ complex shapes

$$E_{1,c} = \eta_1 \eta_0 E_f v_f + E_m v_m$$

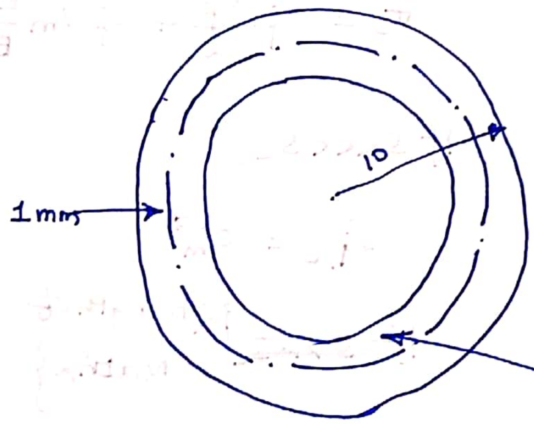
Q → During prod^n of ~~large~~ glass fibre polyester hollow tube (Outer $D_o = 20$ mm, wall thk - 10% dia) - Avg - fibre length → 5 mm. Fibre is oriented along tube axis.
Something went wrong & fibre fracture occured.
The glass fibre on outer ~~fibre~~ half of tube have length of only 1 mm.
Calculate strength & stiffness of wall in tension.

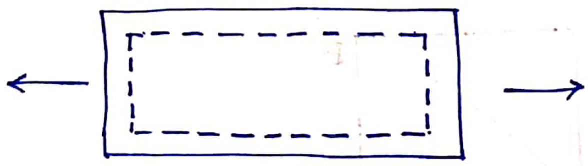
→

$E_m = 2$ GPa, 0.37 $\qquad \tau_i = 30$ MPa

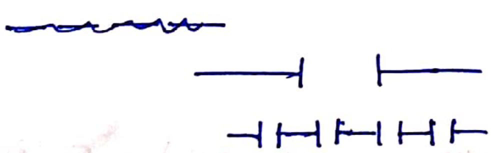
$E_f = 70$ GPa, 0.22 $\qquad R/r = 16, 20$

$v_f = 0.6$ $\qquad d_f = 50\mu$





Isostrain in both areas 1mm & 5 mm dia fibres.



Note: Even if they are {1 mm & 5 mm} fibres together mixed within each other, then, also isostrain can be applied & stiffness & strength can be weighted averaged.

Q30/1 A certain fibre is reinf. comp. has $v_f = 0.5$, calculate strength to first failure under transverse load, if (1) carbon (2) Glass

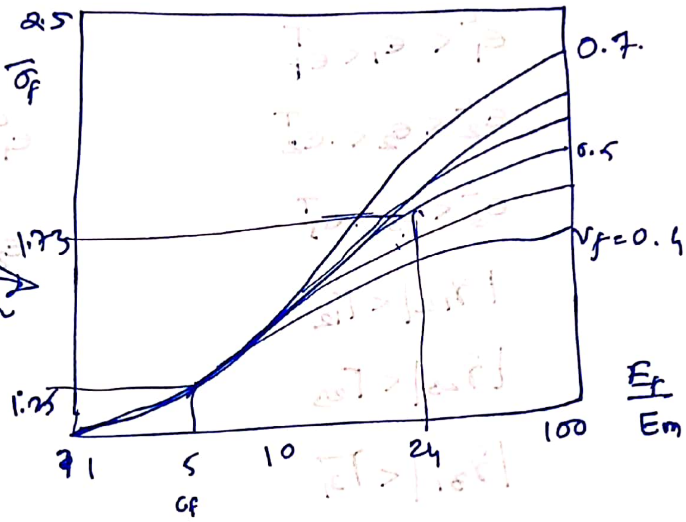
Given →

Yield strength = 50 MPa

SKUDRA (For hex packing)

$\Rightarrow 50 = \underline{k_T} \sigma_c$ → from graph

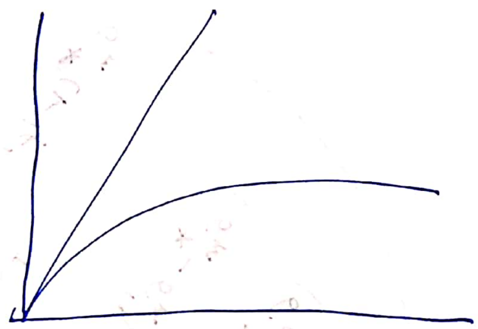
$\Rightarrow k_T = $

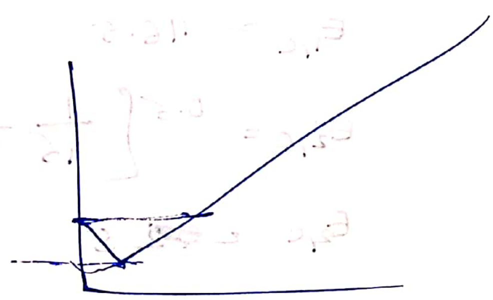


$\sigma_c = \frac{50}{1.25} = 40$ MPa

$= \frac{50}{1.73} =$ ______

2 Unidirectional cont fibre. What is v_f → so that $\sigma_m = \sigma_c$





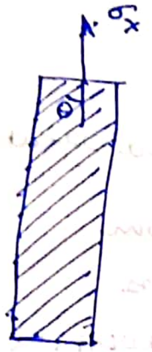
$\sigma_m^* \leftarrow \sigma_c = \sigma_f v_f + \sigma_m'(1 - v_f)$

~~$\sigma_m^* = \sigma_f^* v_f + \sigma_m' - \sigma_m' v_f$~~

$\boxed{v_f = \frac{\sigma_m^* - \sigma_m'}{\sigma_f^* - \sigma_m'}}$

$$V_f = \cancel{\frac{325}{25}} = \cancel{7}$$

(3)



$$\sigma_1 = \frac{\sigma_x}{\cos^2\theta}$$

$$\sigma_2 = \frac{\sigma_x}{\sin^2\theta}$$

$$\sigma_{12} = \frac{\sigma_x}{\sin\theta\cos\theta}$$

$$\epsilon_1^C < \epsilon_1 < \epsilon_1^T$$

$$\epsilon_2^C < \epsilon_2 < \epsilon_2^T$$

$$\epsilon_3^C < \epsilon_3 < \epsilon_3^T$$

$$|\gamma_{12}| < \Gamma_{12}$$

$$|\gamma_{23}| < \Gamma_{23}$$

$$|\gamma_{31}| < \Gamma_{31}$$

$$\epsilon_1^C < \frac{\sigma_1 - \nu_{12}\sigma_2}{E_1} < \epsilon_1^T$$

$$\epsilon_2^C < \frac{\sigma_2 - \nu_{21}\sigma_1}{E_2} < \epsilon_2^T$$

$$\left|\frac{\tau_{12}}{G_{12}}\right| < |\Gamma_{12}|$$

(1) Calc. E_1, E_2, ν_{12}, ν_{21}, G_{12}

(2) Assume carbon fibre, $\nu_f = 50\%$.

$E_1, e_2, m \rightarrow 3$

$\nu_{12_f} = 0.25$

$\nu_{12_E} = 0.37$

$G_{12} = 10 \quad 1$
CF E

$E_1 = 230$

$E_2 = 15$

$$E_{1,c} = E_{1,f} \times \nu_f + E_{1,m} \times \nu_m$$

$$= (230 + 3) \times 0.6$$

$$E_{1,c} = 116.5$$

$$E_{2,c} = 0.5\left[\frac{1}{15} + \frac{1}{3}\right]$$

$$E_{2,c} = \cancel{0.2}\ 5$$

$$\frac{1}{G_{12}} = 0.5[1 + 0.1]$$

$$G_{12} = 1.82$$

$$\nu_{21,c} = \frac{0.25 + 0.37}{2} = 0.31$$

Q.2

$\sigma_m^* (1 - \nu_f) = \sigma_f^* \nu_f$

$+ \sigma_m^* (1 - \nu_f)$

$(\sigma_m^* - \sigma_m')(1 - \nu_f) = \sigma_f^* \nu_f$

$(\sigma_m^* - \sigma_m' + \sigma_f^*)\nu_f$

$= \sigma_m^* - \sigma_m'$

$\cancel{\nu_{21}} =$ $\nu_{21} = \frac{E_1}{E_2}\nu_{12}$

ϑ_{21}/E_1 $\nu_{12} = \frac{E_2}{E_1}\nu_{\cancel{12}21}$

$\frac{\vartheta_{12}}{E_2}$

$= \cancel{\frac{116.5}{5}\times} \frac{5}{116.5} \times \cancel{0.35}\ 0.31$

$\nu_{12,c} = \cancel{0.011}\ 0.0133$

$X_T = 1450,\quad Y_T = 53.4,\quad P = 99.3.$

$\sigma_x < \frac{1450}{\cos^2\theta}$

$$\frac{\sigma_1 - \nu_{12}\sigma_2}{E_1} = \frac{\sigma_2 - \nu_{21}\sigma_1}{E_2}$$

$$\sigma_1 (1+\nu_{21}) = \frac{E_1}{E_2}(1+\nu_{12})\sigma_2$$

$$\frac{\sigma_1}{\sigma_2} = \frac{23.3 \times 1.0133}{1.31}.$$

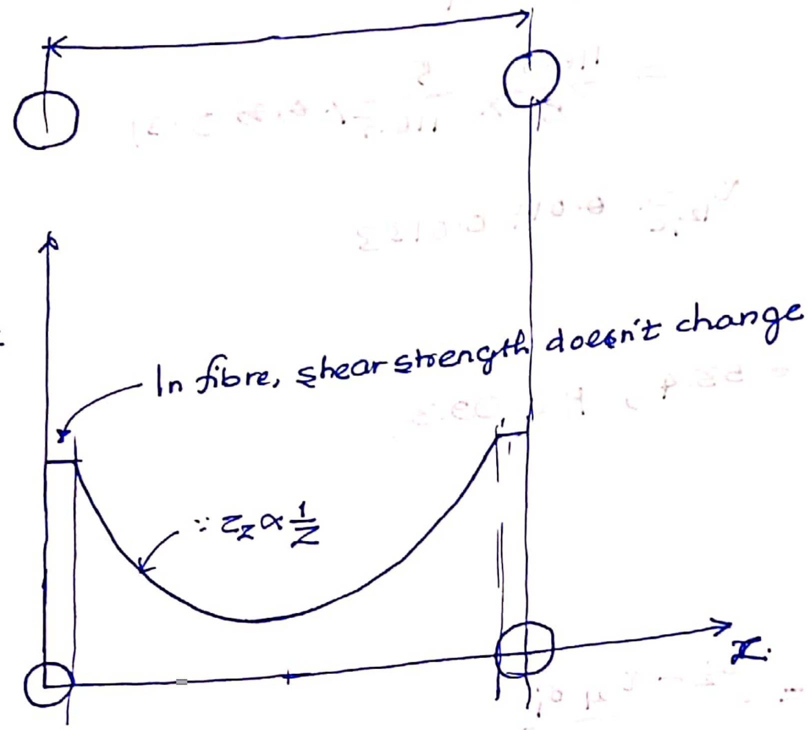
$\tan^2\theta = 17.781$

$\theta = 1.3379$

03.02

1. $\tau_z = \frac{r\tau_i}{z} \longrightarrow = \frac{\cancel{25}^{5} \times 35}{\cancel{8560}_{17}} = 10.294.$

2



Q.2. $\sigma_{f_1} = 5000$ MPa L_c^{*}, E_c.

$\sigma_{f_2} = 2000$ MPa

$\tau_i = 65$ MPa

① $S_{c_1} = \frac{\sigma_f}{2\tau_i} = \frac{5000}{2 \times 65}$ $S_{c_2} = \frac{2000}{2 \times 65}$

$(\approx \sigma_y)$

$S_{c_1} \frac{L}{r} = 38.4615$ $S_{c_2} = 15.3846$

CF GF

$L_c =$

$E_{1,c} = E_{1,f} V_f + E_{1,m} v_m - \cancel{E_f V_f \frac{E_f E_1}{4 G_1 s}}$ $E_f V_f \frac{\tanh(ns)}{ns}$

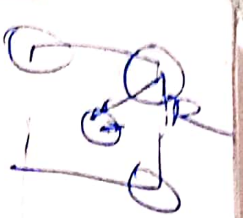
$= \cancel{230 \times 0.2 + 3 \times 0.8 - 230 \times 0.2 \times}$

$=$ $n^2 = \frac{2 G_m}{E_f \ln(R/r)} = \frac{2 \times 1.095}{\cancel{230}\,72 \times \ln(15)}$

G. $n_1^2 = \cancel{0.0593}\ 0.106$

C. $n_2 = 0.0564$

Scanned by CamScanner



$$E_{1,c} = 48.4 - 230 \times 0.2 \times \frac{\tanh(\cancel{0.0593}\,0.106 \times \cancel{15}\,15.3846)}{0.106 \times 15.3846}$$

$$E_{1,c} = 22.275 \text{ GPa}$$

$$\eta = 1 - \frac{\tanh(ns)}{ns}$$

$\cancel{E_{2,c} =}$

	CF	Glass
E	230	72
~~R/r~~ σ_f	~~15~~ 5000	2000
R/r	~~15~~ 20	~~20~~ 15
S_c	38.4615	15.3846
v_f	0.2	0.2
n	0.0564	0.106
η	~~0.4492~~ 0.551	0.432

$E =$	27.746 GPa	8.62 GPa

$$\sigma_\infty = \sigma_f v_f + \sigma_f v_m \frac{E_m}{E_f}$$

$$\sigma_{sc} = \sigma_f v_m \frac{E_m}{E_f}$$

$$\frac{\sigma_\infty}{\sigma_{sc}} = 1 + \frac{\cancel{\sigma_f} v_f}{\cancel{\sigma_f} v_m \frac{E_m}{E_f}} \qquad \sigma_f \left[v_f \left(1 - \frac{S_c}{2S}\right) + v_m \frac{E_m}{E_f} \right]$$

04/02 Micromechanics & Mesomechanics.

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cancel{\epsilon_3} \\ \cancel{\epsilon_4} \\ \cancel{\epsilon_5} \\ \epsilon_6 \end{Bmatrix} = \begin{bmatrix} 1/E_1 & & & & & \\ -\nu_{12}/E_1 & 1/E_2 & & & & \\ & & 1/E_3 & & & \\ & & & 1/G_{23} & & \\ & & & & 1/G_{13} & \\ & & & & & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \cancel{\sigma_3} \to 0 \\ \cancel{\sigma_4} \to 0 \\ \cancel{\sigma_5} \to 0 \\ \sigma_6 \end{Bmatrix}$$

Just Neglected → No rel^n with plane stress assumpt^n

— Plane stress assumption : {Only true for orthotropic mat.}

$\sigma_3, \sigma_4, \sigma_5 = 0$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ e_6 \end{Bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_1 & 0 \\ -\nu/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon \cancel{\sigma_1} \\ \epsilon \cancel{\sigma_2} \\ \epsilon \cancel{\sigma_6} \end{Bmatrix}$$

If $D = 1 - \nu_{12}^2 \dfrac{E_2}{E_1}$

$Q_{11} = E_1/D$

$Q_{12} = \nu_{12} E_2/D$

$Q_{22} = E_2/D$

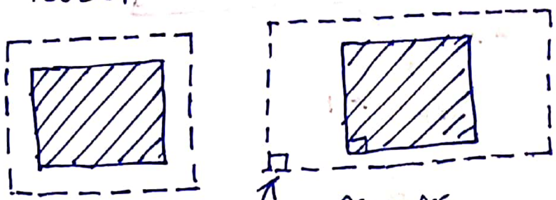
$Q_{66} = G_{12}$

Q → How my constitutive eq^n change based on factors like temp.; humidity, mag. field, $\boxed{\text{for orthotropic mat.}}$ etc.....?

$$\epsilon_1^T = \alpha_1 \Delta T \quad \text{Unit: } \mu c/mm\text{-}^\circ C$$

TEMP.
$$\left\{ \begin{aligned} \epsilon_1^T &= \alpha_1 \Delta T \\ \epsilon_2^T &= \alpha_2 \Delta T \\ \epsilon_{12}^T &= 0 \end{aligned} \right.$$

Isotropic Orthotropic

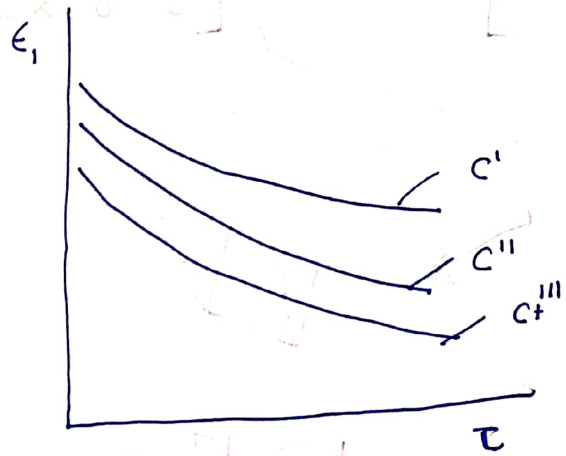


concentration coefficient.

HUMIDITY
$$\left\{ \begin{aligned} \epsilon_1^T &= \beta_1 \Delta C \\ \epsilon_2^T &= \beta_2 \Delta C \\ \epsilon_{12} &= 0 \end{aligned} \right.$$

$$\{\epsilon\} = [S]\{\sigma\} + \{\epsilon^T\} + \{\epsilon^H\} + \dots\dots$$

$$\{\sigma\} = [Q]\left[\{\epsilon\} - \{\epsilon^T\} - \{\epsilon^H\} - \dots\dots\right]$$



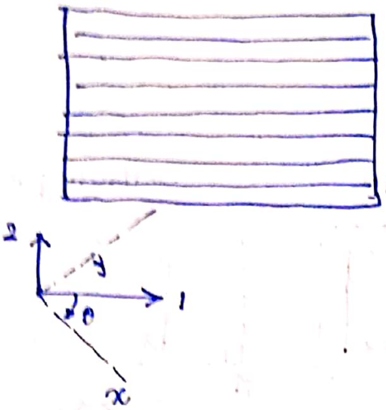
$$\{\sigma\} = [Q]\{\epsilon\} - \cancel{[Q]\{\epsilon^T\}} - \cancel{[Q]\{\epsilon^H\}} \dots$$

bcoz [Q] is a mat. prop. which depends on factors like temp., density, ... etc.

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} + \Delta T \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix} + \Delta C \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix} + \dots$$

Stress-strain relⁿ for orthodropic ply of arbitrary orientation

Goal is to



$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{bmatrix} = [T_\sigma] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 2\epsilon_{12} \end{bmatrix} = [\epsilon_\sigma] \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{bmatrix}$$

$[\sigma] = [Q]\{\epsilon\}$

σ_{12}: $\{\sigma\}^{12} = [Q]^{12}\{\epsilon\}^{12}$ $\quad [T_\sigma^{-1}]$

$\{\sigma\}^{xy} = [Q]^{xy}\{\epsilon\}^{xy}$

$[\bar{Q}] = [T_\sigma]^{-1}[Q][T_\epsilon]$

$[\bar{S}] = [T_\epsilon]^{-1}[QS][T_\sigma]$

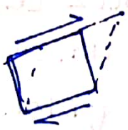
$$\& \quad [T_\sigma] = \begin{array}{ccc} \cos\theta & \sin\theta & \\ \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \end{array}$$

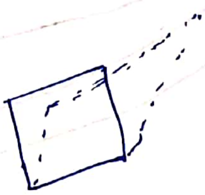
$$\therefore [\bar{Q}] = \begin{bmatrix} \underbrace{\times \quad \times}_{\text{Need not be zero}} & \left.\begin{matrix} 0 \\ \cancel{0} \end{matrix}\right\} \text{Need Not be zero} \\ \times \quad \times & \\ & \times \end{bmatrix}$$

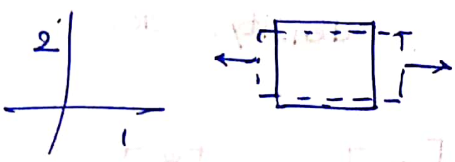
$$[Q] = \begin{bmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{bmatrix} \rightarrow \text{what if} \neq 0 \Rightarrow \text{COUPLING}$$

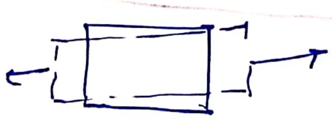
becoz

if "0"









sh:

Shape & size change if "θ" ≠ 0.

HW

$$\alpha_x = m^2\alpha_1 + n^2\alpha_2$$

$$\alpha_y = n^2\alpha_1 + m^2\alpha_2$$

$$\alpha_{xy} = 2(\alpha_1 - \alpha_2)mn.$$

Shear coupling coefficient.

$$\eta_{x,xy} = \frac{\epsilon_x}{2\epsilon_{xy}} \qquad \eta_{y,xy} = \frac{\epsilon_y}{2\epsilon_{xy}}$$

$$\eta_{xy,x} = \frac{2\epsilon_{xy}}{\epsilon_x} \qquad \eta_{xy,y} = \frac{2\epsilon_{xy}}{\epsilon_y}$$

Read as effect on xy dir when load applied @x dir.

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} = \begin{bmatrix} 1/E_x & -\nu_{xy}/E_x & \eta_{xy,x}/E_x \\ -\frac{\nu_{xy}}{E_y} & 1/E_y & \eta_{xy,y}/E_y \\ \frac{\eta_{x,xy}}{G_{xy}} & \frac{\eta_{y,xy}}{G_{xy}} & 1/G_{xy} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}$$

SYMMETRIC.

Q. A unidirectional glass fiber composite has fibre $V_f = 40\%$. (cont.)

Calculate diff mat. prop. for 0° & 90° ply.

GF. $\frac{3000, 0.37}{MPa.}$ $\qquad \mu_{GF} = 0.22.$ $\qquad G = \frac{E}{2(1+\mu)}$

~~ϵ_x~~

~~ϵ_y~~

I. $E_c = \overset{72}{E_f}\,\overset{0.4}{V_f} + \overset{3}{E_m}\,\overset{0.6}{V_m} = 30.6$ GPa.

$$1/E_{2,c} = \frac{V_f}{E_f} + \frac{V_m}{E_m} \Rightarrow E_2 = 4.865 \text{ GPa}$$

$$G_{12,f} = \frac{\cancel{30.6}\ 72}{2(1+0.22)} = \_\_\_\_\_\_$$

$$G_{12,m} = \frac{4.3}{2(1+0.37)} = \_\_\_\_\_\_$$

10/2

How to increase the transverce modulus

$\rightarrow$ change fiber

$\rightarrow$ increase matrix modulus

$\rightarrow V_f \uparrow$

$\rightarrow$ change orientation

① $E_1 = 230 \times 0.5 + 3 \times 0.5 = 116.5$ GPa

$E_2 = 5$ GPa.

$\frac{1}{G_{12}} = \frac{V_f}{G_{12f}} + \frac{V_m}{G_{12m}}$ - ~~0.5~~ [1.9875 GPa

$\nu_{12} = 0.31.$

$m = 0.9962, \quad n = 0.0871$

$\frac{1}{E_x} = 8.454 \times 10^{-3} + 3.7595 \times 10^{-3}$ *

$E_x = 81.7995$ GPa $\longrightarrow$ DROP $\rightarrow \frac{116.5 - 81.79}{116.5} =$

~~$\nu_{xy} =$~~

$E_y = 0.19698 + \longrightarrow E_y = 5.1. \rightarrow$ DROP $\rightarrow \frac{5.1-5}{5} = 2\%$

Tensile Strength

$\sigma_x < \frac{X_T}{\cos^2\theta} \longrightarrow \sigma_x = \frac{1450}{\cos^2 5} = 1461.098$ MPa

$\sigma_x < \frac{Y_T}{\sin^2\theta} \longrightarrow$ ~~7029.9 MPa~~ 53.81 MPa,

$\sigma_x < \frac{S}{-\sin\theta\cos\theta} = 1140.236$ MPa

Min.

Mode of failure $\rightarrow$ ~~shear~~

$\left(\frac{1}{15} + \frac{3}{13}\right) \times 0.5$

$15/6 \times 1/2$