Indian Institute of Technology, Kharagpur

Mid-Autumn Semester Examination, 2015

Mechanical Engineering Department

Subject: Mechanics of Solids Subject No. : ME 31013

Full Marks : 30 Time : 2 Hrs

Adopt appropriate assumption if it is necessary and the stresses mentioned here refer to the Cauchy stresses.

1. (a) Why principal stresses are real? Prove that the principal planes for the distinct principal stresses are orthogonal to each other. (2+2)

(b) Figure 1 shows the normal and shear stresses at a point (**O**) of a deformed solid body based on the two planes **AB** and **CD**. The solid does not have body couples and is in equilibrium. Assuming the state of stress at the point as plane stress, compute the shear stress τ at the point as shown in the figure. (3)

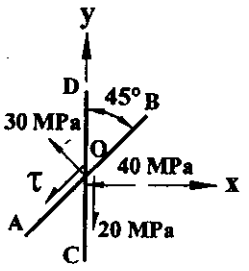


Fig. 1

(c) At a point of a homogeneous isotropic elastic solid body, the principal stresses (σ_1, σ_2 and σ_3) and the normal to the principal planes ($\mathbf{n}_1$ and $\mathbf{n}_2$) for σ_1 and σ_2 are given by

$$\sigma_1 = 4 \text{ MPa}, \ \sigma_2 = 2 \text{ MPa}, \ \sigma_3 = 1 \text{ MPa}, \ \mathbf{n}_1 = (\hat{\mathbf{j}} - \hat{\mathbf{k}})/\sqrt{2} \text{ and } \mathbf{n}_2 = (\hat{\mathbf{j}} + \hat{\mathbf{k}})/\sqrt{2}.$$

The unit vectors along the reference coordinate axes (**x**, **y** and **z**) are $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, respectively. Determine the normal and shear stresses at the same point based on the plane: $2\mathbf{x} + 3\mathbf{y} + \sqrt{3} = \mathbf{0}$ passing through the point. Also, compute the octahedral stresses and the maximum shear stress at the point. (2+2+1)

2. (a) State the difference between the Euler Bernoulli beam theory and the Timoshenko beam theory. Using Timoshenko beam theory, prove that the bending stress in case of symmetric bending of beam made of homogeneous isotropic solid is given by $\sigma_x = -\mathbf{M}_z\mathbf{y}/\mathbf{I}_z$. Prove that the **y**- and **z**- axes refer to the principle centroidal axes. (1+3+2)

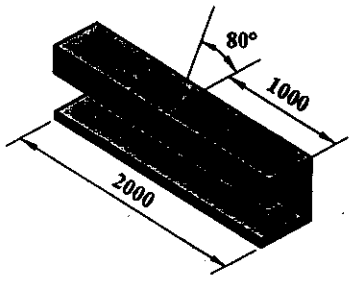
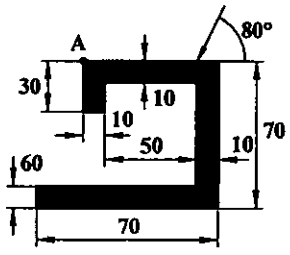


Fig. 2



Enlarged Section at Mid-Span

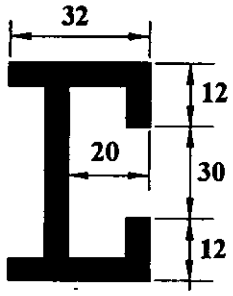


Fig. 3

(b) A straight prismatic beam made of homogeneous isotropic solid is loaded as shown in Fig. 2. The line of action of the load passes through the centroid of the cross-section at the mid-span of the beam. Find the neutral axis in the cross-section and compute the principal stress at point **A**. All dimensions in Fig. 2 are in mm. The ends of the beam are fixed. (1+4)

3. (a) Why the formula $\tau = \mathbf{VQ}/(\mathbf{I}_z\mathbf{t})$ is used for computing the transverse shear stress in a beam instead of using the stress-strain relation? (2)

(b) Show and prove the shear flow diagram for the cross section of a prismatic beam as shown in Fig. 3. Also, find the shear center of the cross-section. The thickness of all walls of the beam is 4 mm. All dimensions shown in Fig. 3 are in mm. (5)