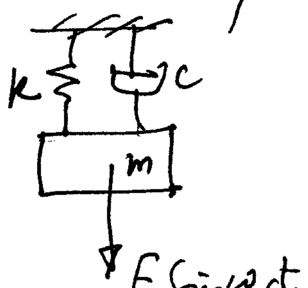
TU/HW-3 → Discussions

Prob. 1 This is a straightforward problem.

$\omega_f = 2\pi \times 3$ rad/s.



$k = 30\times10^3$ N/m, $m = 100$ kg,

$c = 1000$ N-s/m, $F_0 = 80$ N

$$\therefore \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{30\times10^3}{100}} = 17.32 \text{ rad/s}$$

$$r = \frac{\omega_f}{\omega_n} = \frac{2\pi\times3}{17.32} = 1.088$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{1000}{2\times100\times17.32} = 0.2887$$

$$\therefore \text{Reqd. amplitude} = \frac{F_0/k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

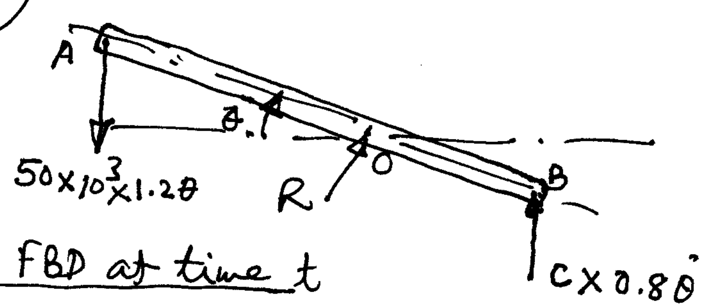
$$= \frac{80/(30\times10^3)}{\sqrt{(1-1.088)^2+(2\times0.2887\times1.088)^2}}$$

$$= 4.2\times10^{-3}\text{ m} = \underline{4.2 \text{ mm}}$$

Also, $\psi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{2\times0.2887\times1.088}{1-(1.088)^2}\right)$

$$= \underline{106.3^\circ}$$

Prob 2



Relevant FBD at time t

Let θ be the generalized coordinate, +ive CW. θ is small & hence, spring deflection $\simeq OA\sin\theta \simeq 1.2\theta$

Similarly, vel. at B is $\simeq 0.8\dot\theta$. Note that there is an unknown reaction force at O. Using the moment balance method, we get

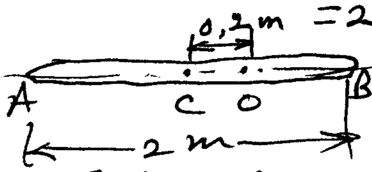
$$I_O\ddot{\theta} = -50\times10^3\times1.2\theta\times1.2 - c\times0.8\dot{\theta}\times0.8$$

(check)

$$I_O = \underbrace{\frac{1}{12}\times80\times2^2}_{I_C} + 80\times(0.2)^2 = 29.867\ \text{kg m}^2$$

or, $\ddot{\theta} + 0.02143c\dot{\theta} + 2410.7\theta = 0$ — (1)

which is the reqd. DEOM for free vibrations.



Note that

$$I\ddot{\theta} + c_t\dot{\theta} + k_t\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{c_t}{I}\dot{\theta} + \frac{k_t}{I}\theta = 0$$

$$\Rightarrow \ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$$

Since, $\zeta = \frac{c_t}{2I\omega_n}$

& $\omega_n = \sqrt{\frac{k_t}{I}}$

Part b comparing (1) with the standard form

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0,$$

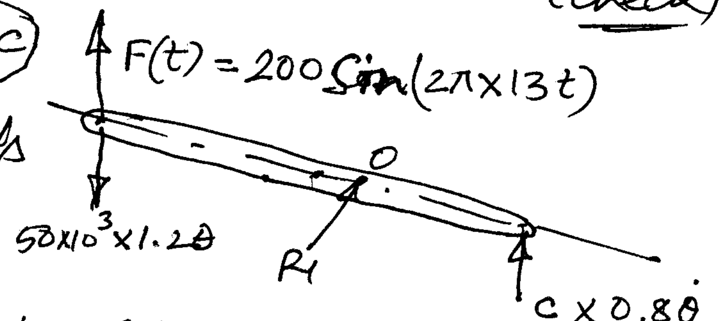
We see that $\omega_n = \sqrt{2410.7} = 49.1$ rad/s

& $2\zeta\omega_n = 0.02143c$

$\zeta = 0.5 \Rightarrow$ $c = 2291$ N-s/m

(check)

Part c



$$\omega_f = 2\pi f_f = 2\pi\times13\ \text{rad/s}$$

After F(t) is applied as shown, the DEOM shall be :-

$$\ddot{\theta} + 49.1\dot{\theta} + 2410.7\theta = 8.0356\,\text{Sin}\,81.68t$$

(check)

Compare with $I\ddot{\theta} + c_t\dot{\theta} + k_t\theta = T_0'\,\text{Sin}\,\omega_f t$.

The amplitude of forced angular oscillations is

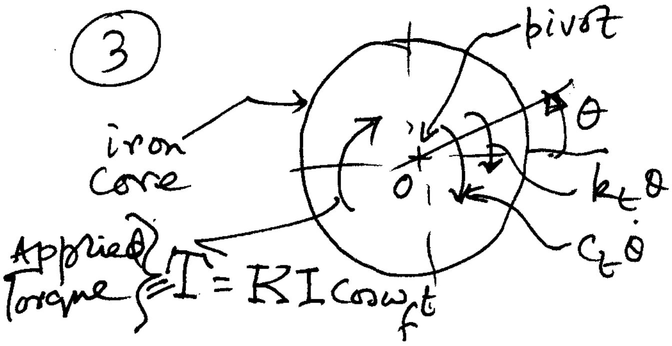
$$\Theta_0 = \frac{T_0/k_t}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\ ;\ r = \frac{\omega_f}{\omega_n}$$

$$= 0.00137\ \text{rad}$$

$$= 0.079^\circ\ \text{(check)}$$

→

③



$\zeta = 1$ (given).

The DEOM is :- (obtain it)

$$I_0\ddot{\theta} + c_t\dot{\theta} + k_t\theta = KI\cos\omega_f t \quad \text{--(1)}$$

$I_0 \rightarrow$ Moment of inertia
$I \rightarrow$ current amplitude

If you don't like this nomenclature, use J_0 instead of I_0

when $\omega_f = 0$ (DC),

$$\Theta_0 = \theta_{ss} = \frac{KI}{k_t} \quad \text{--(2)}$$

So, $\Theta_0 \propto I$

when $\omega_f = 6\times 2\pi$ rad/s, $r = \frac{\omega_f}{\omega_n} = \frac{12\pi}{4\times 2\pi} = 1.5$

Let us denote by Θ_6, the amplitude of θ when ω_f corresponds to 6 Hz.

Then, $\theta_{ss} = \theta(t) = \Theta_6 \cos(\omega_f t - \psi)$

where
$$\Theta_6 = \frac{KI/k_t}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = (KI/k_t)\times 0.308 \quad \text{--(3)}$$

from ② & ③, thus,
$$\frac{\Theta_6}{\Theta_0} = 0.308$$

Hence, if Θ_0 corresponds to 1 amp,
Θ_6 " " 0.308 amp
& the pointer would indicate(?) a current of 0.31 amp approx.

This problem may raise questions. You may leave it out for the moment

$\rightarrow$ This example shows that a DC ammeter is very unsatisfactory for measuring alternating current.

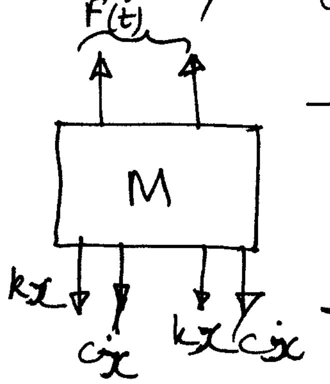
(4) (Note that we used different notations for the theory part. What is given as r is our 'e')

→ With the way θ is given, note that the vertical shaking force is

$2\times m e\omega^2 \cos\omega t$. $m = 1.15$ kg, $e = 0.05$ m

note

So, $f(t) = 5044.5\cos 209.44t$ N (check)



Relevant FBD

→ The DEOM is $\left(\omega_f = \frac{2000}{60}\times 2\pi \text{ rad/s}\right)$

$$M\ddot{x}+2c\dot{x}+2kx = 5044.5\cos 209.44t$$

→ Compare with

$$m\ddot{x}+c\dot{x}+kx = F_0\cos\omega_f t$$

whose SS amplitude is

$$\frac{F_0/k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

So, for this rotating unbalance problem, we didn't use the special formula developed for rotating unbalance case

Read about the phasor method (notes sent already)

Given :- SS amplitude $= 6\times10^{-3}$ m

$$\psi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = 30^\circ$$

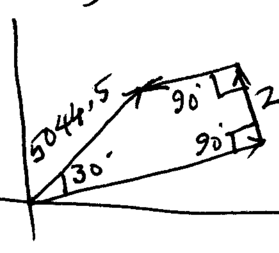
→ Looks like there are 3 unknowns, viz., m, c & k & we have only two relations.

→ Try to solve the problem analytically.

→ Try the phasor method: $x_{ss} = x = 6\times10^{-3}\cos\left(209.44t-\frac{\pi}{6}\right)$ (Given)

⇒ $\dot{x}$ = etc.

⇒ $2c\dot{x}$ = etc.



$2c\times 209.44\times 6\times10^{-3}$

Hence,

$5044.5\cos 30^\circ = 2c\times 209.44\times 6\times10^{-3}$

$\Rightarrow c = 1004$ Ns/m Ans.

(contd) →

(4) 2nd part (Part b) :-

$\omega_f = 209.44$ rad/s, time period $T = \frac{2\pi}{\omega_f}$.

Instantaneous power = $F(t)\dot{x}$ & this varies with time. So, reqd. mean power output $= \frac{1}{T}\int_0^T F(t)\dot{x}\,dt$, where

$x(t) = 6\times10^{-3}\cos(209.44t - \pi/6)$

→ Now perform the integration etc. & get the final answer. [Ans:- 1585 W] (Check)

(5) Case 1:- $\frac{\omega_f}{\omega_n} = 2$. Assume 1 stroke is equivalent to one cycle of motion, we assume.

So, $f_f = \frac{240}{60}$ cycles/s & $\omega_f = 2\pi f_f = 25.13$ rad/s

$m = 2000$ kg.

$F(t) = 12\times100\times10^{-3}\times(25.13)^2 \sin\omega_f t$ N

$= 757.82 \sin\omega_f t$ N, $\varphi = 0$,

$K_{eq} = 4K$ (Let each spring has stiffness k)

Now, the DEOM is: $M\ddot{x} + K_{eq}x = 757.82 \sin\omega_f t$

Case (i) $r = \frac{\omega_f}{\omega_n} = 2 \Rightarrow \omega_n = \frac{\omega_f}{2} = 12.565$ rad/s

(a) Static deflection $= \frac{Mg}{K_{eq}} = \frac{g}{\sqrt{\frac{K_{eq}}{M}}} = \frac{g}{\omega_n^2} = \frac{9.81}{(12.565)^2}$ m

$= 0.0621$ m $= 62.1$ mm Ans

(b) [Note that Transmission Ratio here means 'Transmissibility']

→

$$TR = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \left|\frac{1}{1-r^2}\right| = \left|\frac{1}{1-2^2}\right| = 0.3333$$

(c) $\omega_n = \sqrt{\frac{k_{eq}}{M}} \Rightarrow k_{eq} = M\omega_n^2 = 315758.45 \text{ N/m}$
$= 315.76$ kN/m.

Hence, $k = \frac{k_{eq}}{4} = 78939.61$ N/m $= 78.9$ kN/m.

(d) Reqd. amplitude $= \left|\frac{F_0/k_{eq}}{1-r^2}\right| = 0.8 \times 10^{-3}$ m
$= 0.8$ mm.

→ Do the 2nd part. Answers:– (a) 248 mm
(b) 0.067
(c) 19.7 kN/m
(d) 0.64 mm

Check

→ With the addition of 6000 kg Concrete platform, $M = 8000$ kg. ω_f & r are unchanged. Hence $\omega_n = 12.565$ rad/s as before.

(a) Static deflection $= \frac{g}{\omega_n^2}$, remains same (62.1 mm)

(b) $Tr = \left|\frac{1}{1-r^2}\right|$ → same

(c) $k_{eq} = M_{new}\omega_n^2$ & hence, k_{eq} & so, k will change. $k_{eq} = 8000 \times (12.565)^2$

$\Rightarrow k = \frac{k_{eq}}{4} = 315758.45$ N/m
$\simeq 315.8$ kN/m

(d) Amplitude $= 0.2$ mm

Check

→

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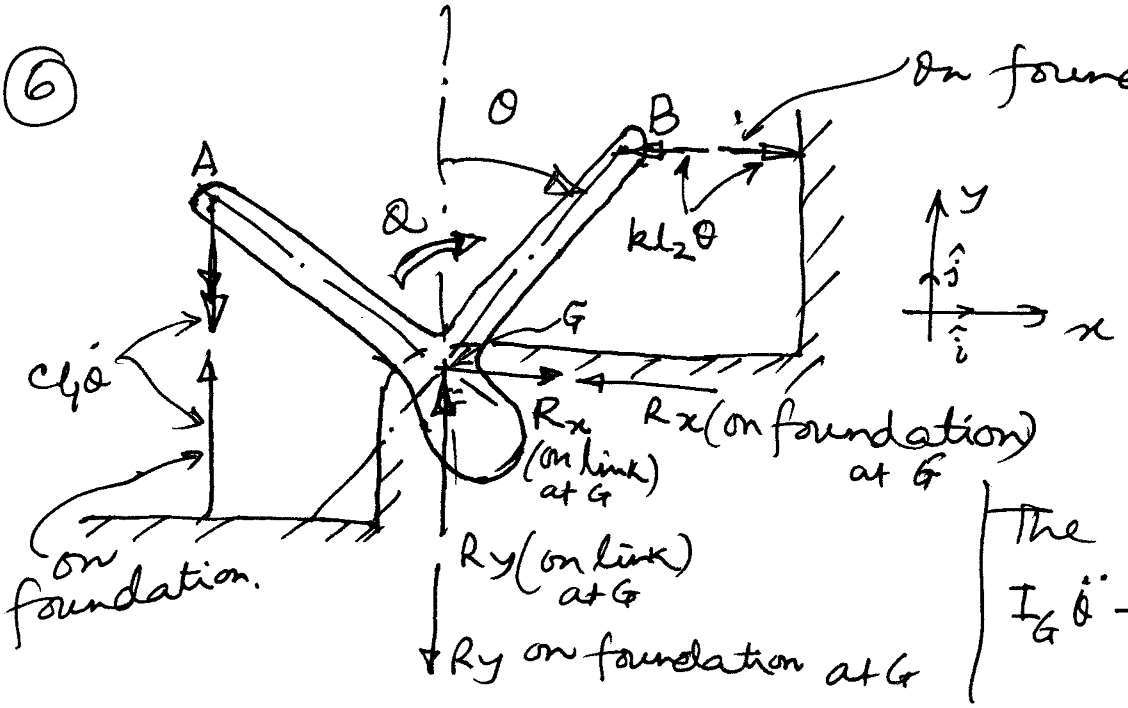


The DEOM is:

$$I_G\ddot{\theta} + cl_1^2\dot{\theta} + kl_2^2\theta = Q$$

Since $\bar{a}_G$ = acceleration of CG 'G' $= 0$,

net force on link in x-direction $= 0$

" " " " " y — " $= 0$.

$$\Rightarrow \quad R_x\hat{i} - kl_2\theta\hat{i} = 0 \quad \text{—(1)}$$

$$\& \quad R_y\hat{j} - cl_1\dot{\theta}\hat{j} = 0 \quad \text{—(2)}$$

Now, net force on foundation due to the vibrating link $= -R_x\hat{i} + kl_2\theta\hat{i} - R_y\hat{j} + cl_1\dot{\theta}\hat{j} = 0$, by virtue of (1) & (2).

(proved)

→ The net couple transmitted to the foundation $= kl_2\theta \times l_2 + cl_1\dot{\theta} \times l_1 = kl_2^2\theta + cl_1^2\dot{\theta}$ —(3)

DEOM:

$$I_G\ddot{\theta} + cl_1^2\dot{\theta} + Kl_2^2\theta = Q_0\sin\omega t \quad \text{—(4)}$$

$$\theta_{ss} = \theta(t) = \Theta_0\sin(\omega t - \psi)$$

$$\Rightarrow \dot{\theta} = \Theta_0\omega\cos(\omega t - \psi)$$

∴ From (3), couple transmitted $= Kl_2^2\Theta_0\sin(\omega t - \psi) + cl_1^2\omega\Theta_0\cos(\omega t - \psi)$

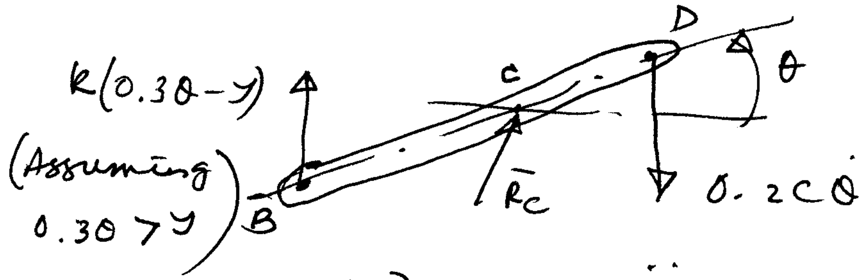
$$= \sqrt{(Kl_2^2)^2 + (cl_1^2\omega)^2}\,\Theta_0\sin(\omega t - \psi + \beta) \text{ etc. (Do it)}$$

$\therefore$ Amplitude of couple transmitted

$$= \frac{\left(\sqrt{k^2 l_2^4 + c^2 l_1^4 \omega^2}\right) \cdot Q_0/(k l_2^2)}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad \left[\because \Theta_0 = \text{etc}\right]$$

= Given expression (Check)

(7) (Base Excitation problem)



$$\Rightarrow \quad 0.25\ddot{\theta} + (0.2)^2 c\dot{\theta} + (0.3)^2 k\theta = 0.3ky \quad (1)$$

$K = 700$ N/m, $c = 60$ N-s/m

$y = 0.01 \sin 10t$ m

$$\Rightarrow \quad 0.25\ddot{\theta} + 2.4\dot{\theta} + 63\theta = 2.1 \sin 10t$$

Let $\theta_{ss} = \theta(t) = \Theta_0 \sin(10t - \psi)$

$\omega_n = \sqrt{\dfrac{63}{0.25}}$
$\simeq 15.874$ rad/s

$r = \dfrac{10}{15.874}$
$= 0.63$

$\zeta = \dfrac{2.4}{2 \times 0.25 \times 15.874}$
$= 0.302$

So, $\Theta_0 = \dfrac{2.1/63}{\sqrt{[1-(0.63)^2]^2 + (2 \times 0.302 \times 0.63)^2}}$ rad

$= 0.0467$ rad

$= 2.678°$ Ans

Check