Casting

7/10/18

C_g must lie close to 0.

$V_{imp-limit}$ → is the limit of erosion taking place

V_{imp} → actual velocity at which erosion takes place

n_{imp} → unit vector towards the velocity

n_f → unit vector normal to surface.

Fettling → cutting the ingate from casting ~~safely~~ safely.

$$C_{gf} = 1 - \frac{t_{gi}}{t_{ci}}$$

t_{gi} ← gate thickness

t_{ci} ← cast thickness

C_{gf} must lie closer to 1.

More fluidity is a required characteristic.

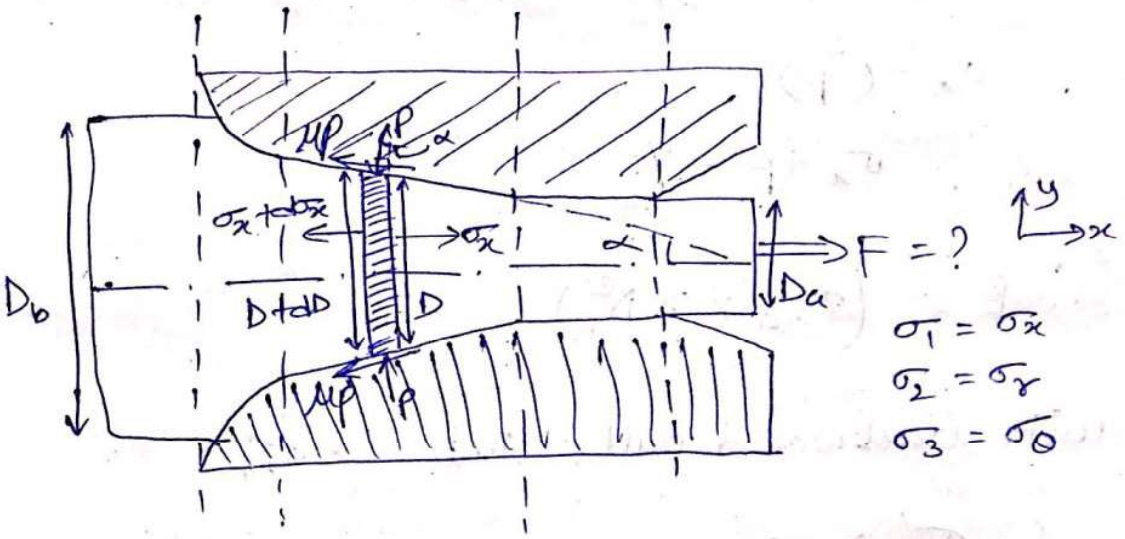
Dendrites reduce fluidity, plus ~~reduces~~ induce a lot of shrinkage cavity.

Fluidity is high for pure metals & at eutectic points.

Forming

8th Oct 18

Drawing
- → Wire
- → Rod
- → Bar
- → Tube ← hollow



$\sigma_1 = \sigma_x$

$\sigma_2 = \sigma_r$

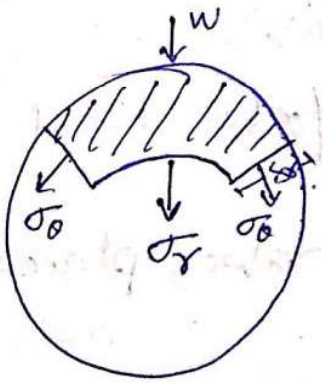
$\sigma_3 = \sigma_\theta$

$$\sum F_x = 0$$

$$\Rightarrow (\sigma_x + d\sigma_x)\frac{\pi}{4}(D+dD)^2 - \sigma_x \frac{\pi}{4} D^2$$

$$+ P\left(\frac{dx}{\cos\alpha}\pi D\right)\sin\alpha$$

$$+ \mu P\left(\frac{dx}{\cos\alpha}\pi D\right)\cos\alpha = 0$$



$$-(P\cos\alpha - \mu P\sin\alpha)$$

$\sigma_r (r\,d\theta\,dx)$ $(\sigma_\theta \delta_t dx)$

$$\sum F_R = 0$$

$$\sigma_r = \sigma_\theta$$

∵ Circumference ∝ radius

$$\epsilon_\theta = \epsilon_r$$

$$\Rightarrow \sigma_r = \sigma_\theta$$

σ_1	σ_2	σ_3
↓	↓	↓
σ_x	$\sigma_{\theta r} = \sigma_\theta$	

Uni-axial loading superimposed with hydrostatic strain

$$\sigma_1 - \sigma_3 = 2K = Y$$

$$\sigma_x - (-p) = Y$$

$$\sigma_x + p = Y$$

$$\therefore F = \left(\sigma_{xa} \times \frac{\pi}{4} D_a^2\right)$$

with backward pull, $\sigma_{xa} \longrightarrow \sigma'_{xa}$

ϕ $\sigma'_{xa} > \sigma_{xa}$

Now, $\sigma'_{xa} + p' = Y$

$$\Rightarrow p' = Y - \sigma'_{xa}$$

Now, $\boxed{p' < p}$ ⇒ Die has to exert less pressure, hence the life of die increases.

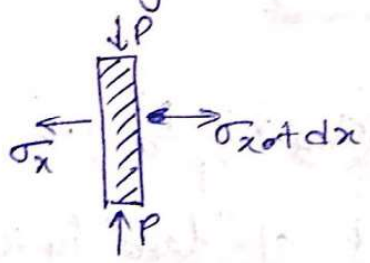
σ_{xa} should not exceed Y else local deformation will take place.

In drawing, deformation takes place only in conical part.

Drawing ratio $= \frac{A_b - A_a}{A_b}$

Front end diameter is kept slightly lower than the required diameter (as the outgoing material will recover elastically).

Considering an element in region ③



$$\Sigma F_x = 0$$

$$D\,d\sigma_x + 2\left[\sigma_x + p(1+\mu\cot\alpha)\right] dD = 0 \quad -①$$

$$\Sigma F_R = 0$$

$$-(p\cos\alpha - \mu p\sin\alpha)(r+\delta r)\,d\theta\,dx - \sigma_{xr}(r\,d\theta\,dx)$$

$$-2\,\sigma_\theta \sin\left(\frac{d\theta}{2}\right)(\delta r\,dx) = 0 \quad -②$$

$\sigma_r \approx -p$

$\sigma_1 = \sigma_x$

$\sigma_2 = \sigma_r$

$\sigma_3 = \sigma_\theta$

$\sigma_x + p = 2k = Y$

From ①

$$D\,d\sigma_x - 2\left[B\sigma_x - Y(1+B)\right] dD = 0$$

$$[B = \mu\cot\alpha]$$

After integration

$$B\sigma_x - Y(1+B) = C D^{2B}$$

Boundary Cond^n: at $D = D_b$, $\sigma_x = \sigma_{x_b} = 0$

$$\Rightarrow C = -\frac{Y(1+B)}{D_b^{2B}}$$

Drawing Stress

$$\sigma_{xa} \Rightarrow \frac{\sigma_{xa}}{Y} = \left(\frac{1+B}{B}\right)\left[1 - \left(\frac{D_a}{D_b}\right)^{2B}\right]$$

$$\frac{\sigma_{xa}}{Y} = \left(\frac{1+B}{B}\right)\left[1 - (1-r)^{B}\right] \qquad r = \frac{D_b^2 - D_a^2}{D_b^2}$$

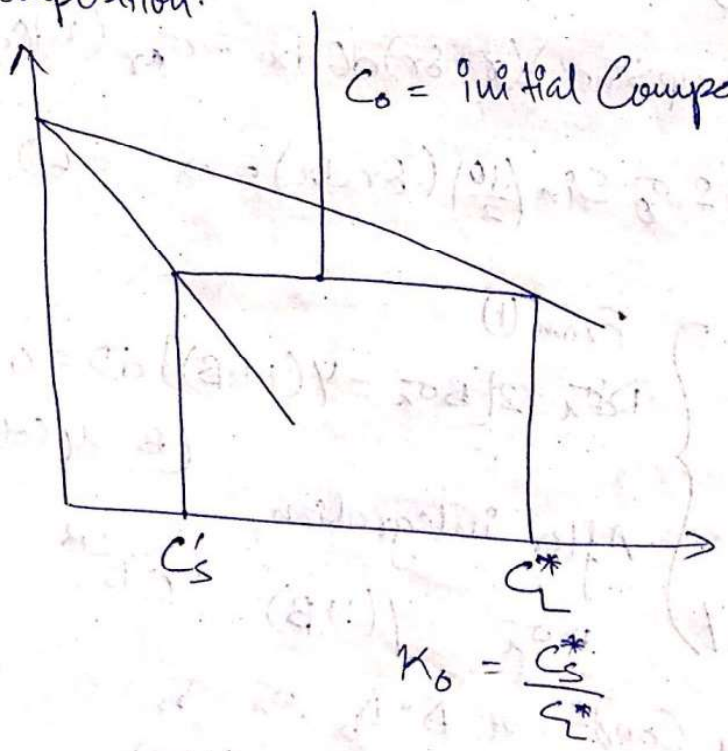
Drawing Power $= \left(\sigma_{xa} \times \frac{\pi}{4} D_a^2\right) \cdot v$

Here, consider the effect of "land" in the die.

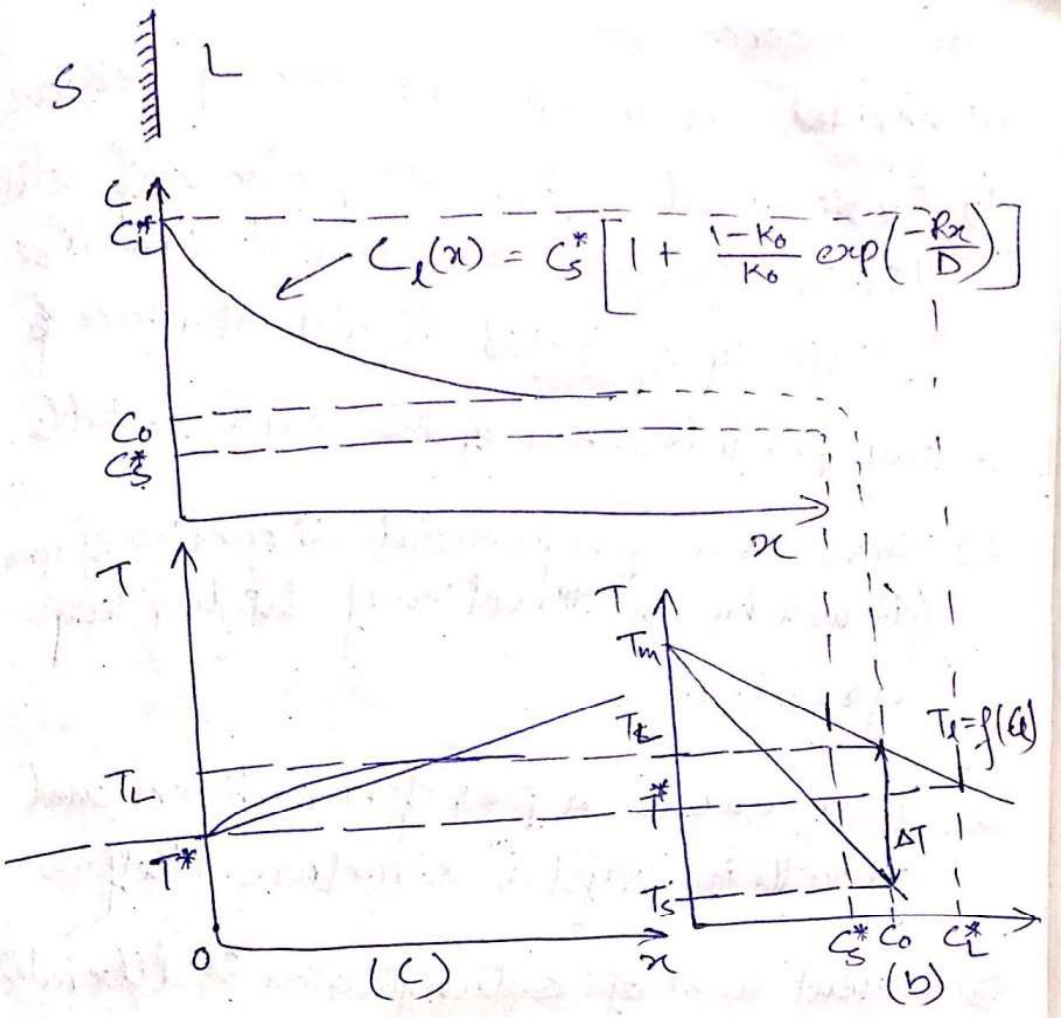
22nd Oct 18

Constitutional Undercooling in an Alloy,

↑ due to change in composition.



$$K_o = \frac{C_s^*}{C_L^*}$$



Due to partioning movement of solid-liquid interface (growth rate R, diffusivity D) a composition profile is developed ahead of solid-liquid interface as shown

S L

$x = 0,\ C = \frac{C_S^*}{K_0}$ $\quad D\frac{d^2C}{dx^2} + R\frac{dC}{dx} = 0$

$x = \infty,\ C = C_0,$

Due to this compositional change, depending on phase diagram (b), freezing temp profile is shown in (c).

It can be seen, that ahead of solid-liquid interface, there is a region that is

more undercooled.

Dendrites grow due to following reasons.

1) Constitutional undercooling ahead of solid-liquid interface, which will create instability of solid-liquid interface & at any perturberation ~~of the~~ can be stable

2) There is a preferential direction of growth. (It will be in direction of higher temp. gradient)

There exists a preferred directional growth in crystal structure itself.

3) Latent heat of solidification is liberated ahead of S-L interface, hence the growth is not planar hence treelike structure spread from interface.

$$G < \left.\frac{\partial T_l(x)}{\partial x}\right|_{x=0}$$

$$T_L = T_m - mC$$

$$T_i = T_m - m\frac{C_s}{K_0}$$

$$T_L = T_i + m\frac{C_s}{K_0} - mC$$

$$\therefore T_l(x) = T_i + mC_s\left(\frac{1-K_0}{K_0}\right)\left[1 - \exp\left(\frac{-Rx}{D}\right)\right]$$

$$\Rightarrow G < mC_s\left(\frac{1-K_0}{K_0}\right)\frac{R}{D}$$

$$\Rightarrow \frac{G}{R} < \frac{mC_s}{D}\left(\frac{1-K_0}{K_0}\right) \Rightarrow \text{Constitutional Cooling}$$

23rd Oct 18

Forming

Tube → Seamless tube can be drawn through Drawing Process or Extrusion Process.

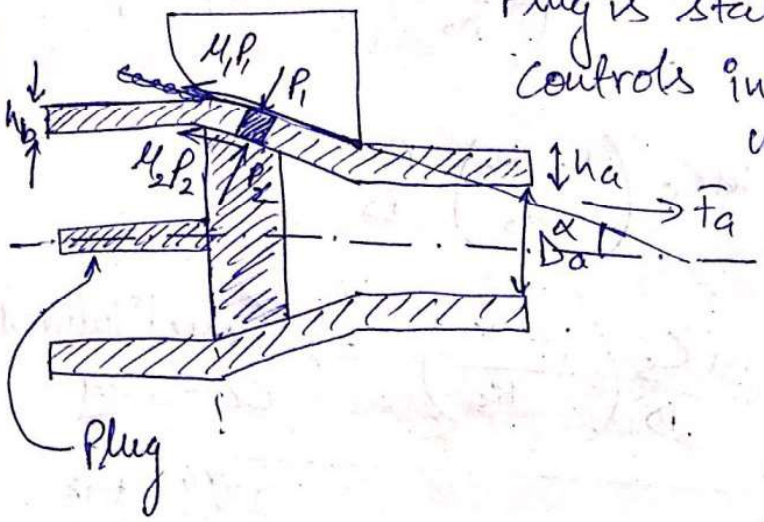
By using drawing, we can perform
1. change of internal dia.
2. change of wall thickness
3. change of external dia.

→ When only external dia is concerned, the process by which we can only control change of external dia is called Tube Sinking.

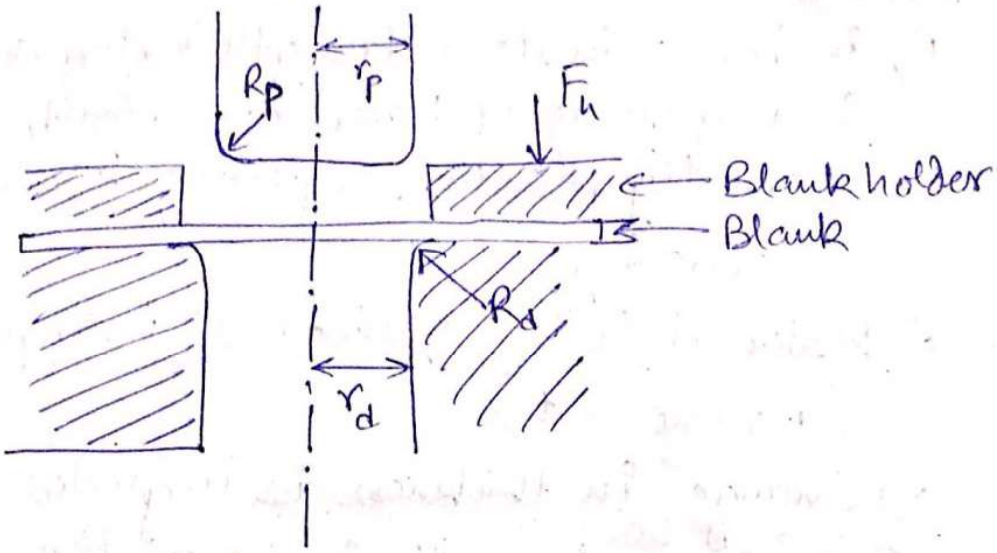
→ For the control of other two also
↳ Tube drawing with mandreal

- Fixed (Plug)
- Moving (Mandreal)

Plug is stationary & controls internal dia & wall thickness.



Deep drawing analysis:

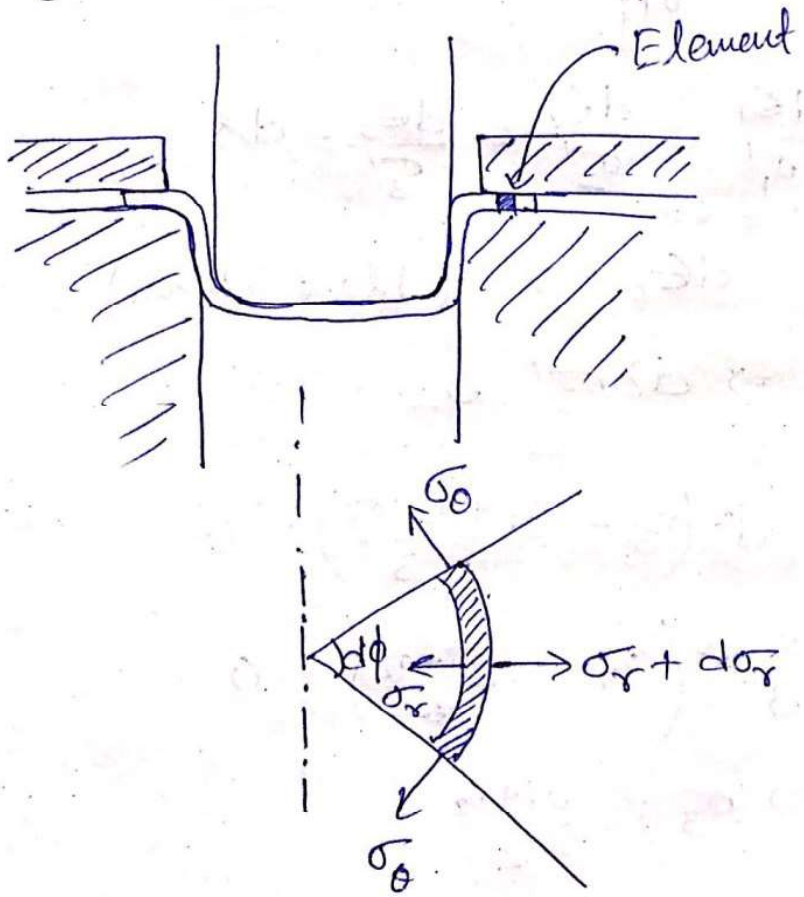


r_p = Punch radius

R_p = Punch corner radius

R_d = Die corner radius

r_d = Die radius.



Objective is to evaluate the force required for deep drawing (it is a sheet forming process)

*Assumptions

1) increase in flow strength during deformation is negligible. (Material is rigid, perfectly plastic, neglecting strain hardening)
2) Material is homogeneous & isotropic (Rvalue = 1)
3) Change in thickness is neglected
4) It is a plane strain deformation.

Levy Mises eqn ← plane strain ka P

$$\frac{d\varepsilon_{ij}}{\sigma'_{ij}} = d\lambda$$

$$\frac{d\epsilon_1}{\sigma_1'} = \frac{d\epsilon_2}{\sigma_2'} = \frac{d\epsilon_3}{\sigma_3'} = d\lambda$$

$$d\epsilon_3 = 0 \quad \text{(Plane strain)}$$

$$\Rightarrow d\lambda \sigma_3' = 0$$

$$\Rightarrow d\lambda \left(\sigma_3 - \frac{\sigma_1+\sigma_2+\sigma_3}{3}\right) = 0$$

$$\Rightarrow \frac{2}{3} d\lambda \left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right) = 0$$

$$\Rightarrow \sigma_3 = \frac{\sigma_1+\sigma_2}{2}$$

Now Von Mises yield criteria

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=2Y^2$$

$$\Rightarrow(\sigma_1-\sigma_2)^2+\left(\sigma_2-\frac{\sigma_1+\sigma_2}{2}\right)^2+\left(\frac{\sigma_1+\sigma_2}{2}-\sigma_1\right)^2=2Y^2$$

$$\Rightarrow(\sigma_1-\sigma_2)^2+\frac{1}{4}(\sigma_2-\sigma_1)^2\times 2=2Y^2$$

$$\Rightarrow \frac{3}{2}(\sigma_1-\sigma_2)^2=2Y^2$$

$$(\sigma_1-\sigma_2)^2=\frac{4}{3}Y^2$$

$$\Rightarrow \sigma_1-\sigma_2=\frac{2}{\sqrt{3}}Y=\sigma_0'$$

4) Blank Holding force is assumed to act just at the rim.

5) Friction between die wall & cup wall is ~~absent~~ not considered

6) Slab analysis

$$(\sigma_r+d\sigma_r)(r+dr)\,d\phi\,t-\sigma_r\cdot r\,d\phi\,t-2\sigma_\theta\,dr\,t\sin\frac{d\phi}{2}=0$$

$$\Rightarrow(\sigma_r+d\sigma_r)(r+dr)-\sigma_r\cdot r-\sigma_\theta\cdot dr=0$$

$$\Rightarrow \sigma_r r+d\sigma_r\cdot r+\sigma_r\,dr-\sigma_r r-\sigma_\theta\cdot dr=0$$

$$\Rightarrow d\sigma_r\cdot r+(\sigma_r-\sigma_\theta)dr=0$$

$$(\sigma_r-\sigma_\theta)=\sigma_0'$$

$$d\sigma_r \cdot r + \sigma_0' dr = 0$$

$$\frac{d\sigma_r}{\sigma_0'} = -\frac{dr}{r}$$

$$\frac{\sigma_r}{\sigma_0'} = -\ln r + C$$

at $r = r_0$, $\sigma_r = \frac{2\mu F_h}{2\pi r_0 t} = \frac{\mu F_h}{\pi r_0 t}$

$$\Rightarrow \frac{\mu F_h}{\sigma_0' \pi r_0 t} = -\ln r_0 + C$$

$$\Rightarrow C = \ln r_0 + \frac{\mu F_h}{\sigma_0' \pi r_0 t}$$

$$\boxed{\frac{\sigma_r}{\sigma_0'} = \ln\left(\frac{r_0}{r}\right) + \frac{\mu F_h}{\sigma_0' \pi r_0 t}}$$

$$\Rightarrow \sigma_r\Big|_{r=r_p} = \sigma_0' \ln\left(\frac{r_0}{r_p}\right) + \frac{\mu F_h}{\pi r_0 t}$$

Stress required to draw the cup at die wall

$$\sigma_w = \sigma_r e^{\mu\pi/2}$$

Force required to draw the material

$$= \sigma_w \times 2\pi r_p t$$

$$\Rightarrow \boxed{F = \left(\sigma_0' \ln\left(\frac{r_0}{r_p}\right) + \frac{\mu F_h}{\pi r_0 t}\right) e^{\mu\pi/2} \times 2\pi r_p t}$$

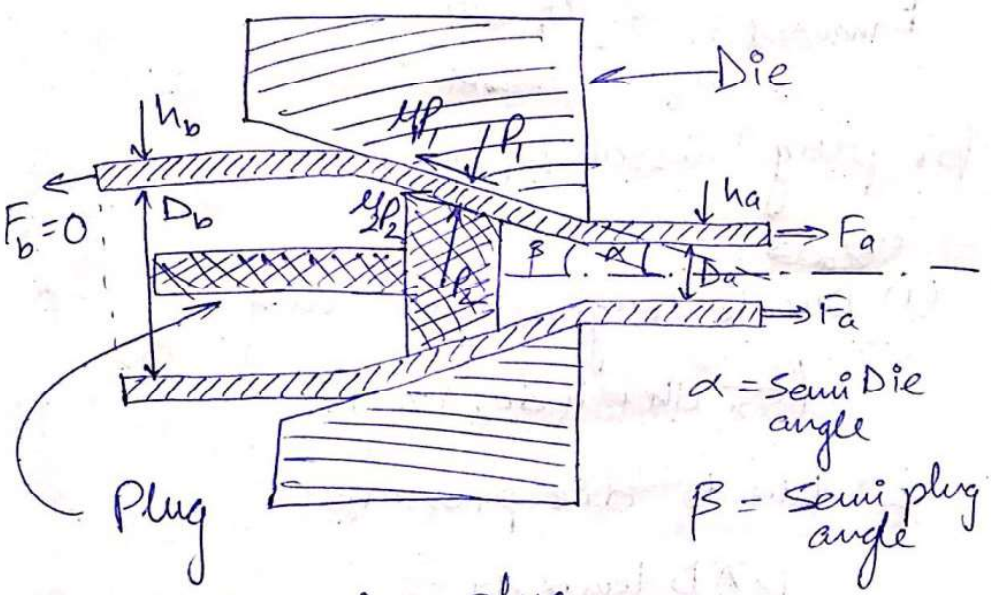
Forming -

6th Nov 18

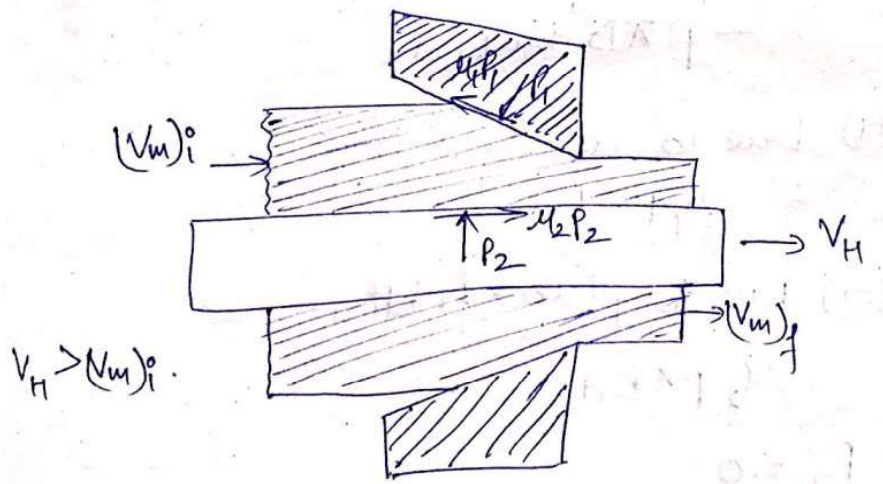
Extrusion & Forging

Only to control outer dia → Tube Sinking.

Outer dia + Inner dia + Wall thickness } → by using plug / by using Mandrel



By using plug.



if $\beta = 0$ (close pass drawing), $h << D$

Plane ~~Stress~~ Strain

D → diameter of the element considered

$B = \mu \cot\alpha \longrightarrow$ Bar Drawing

$$B_{plug} = \left(\frac{\mu_1 + \mu_2}{\tan\alpha - \tan\beta} \right)$$

$$B_{mandrel} = \frac{\mu_1 - \mu_2}{\tan\alpha}$$

for plug assuming $p_1 = p_2$

(1) Due to longitudinal Strain

$$(\sigma_x \, dh + h \, d\sigma_x) \pi D$$

(2) Due to die pressure

$$p \pi D \tan\alpha \, dx$$

(3) Due to plug pressure

$$-p \pi D \tan\beta \, dx$$

(4) Due to die-friction

$$\mu_1 p \pi D dx$$

(5) Due to plug-friction

$$\mu_2 p \pi D dx$$

$$\Sigma F_x = 0$$

$$\Rightarrow h\,d\sigma_x + \left[\sigma_x + p(1+B_{plug})\right]dh = 0$$

$\beta = 0 \Rightarrow$ Plane strain deformation

$$\sigma_x + p = S = 2K$$

$$\int \frac{d\sigma_x}{B_{plug}\,\sigma_x + S(1+B_{plug})} = \int \frac{dh}{h}$$

$$B_{plug}\,\sigma_x + S(1+B_{plug}) = C\,h^{B_{plug}}$$

Boundary Condition $\Rightarrow$ No backpull

at $h = h_b$, $\sigma_x = \sigma_{xb} = 0$

$$\Rightarrow \boxed{\left(\frac{\sigma_x}{S}\right) = \left(\frac{1+B_{plug}}{B_{plug}}\right)\left[1 - \left(\frac{h}{h_b}\right)^{B_{plug}}\right]}$$

for drawing

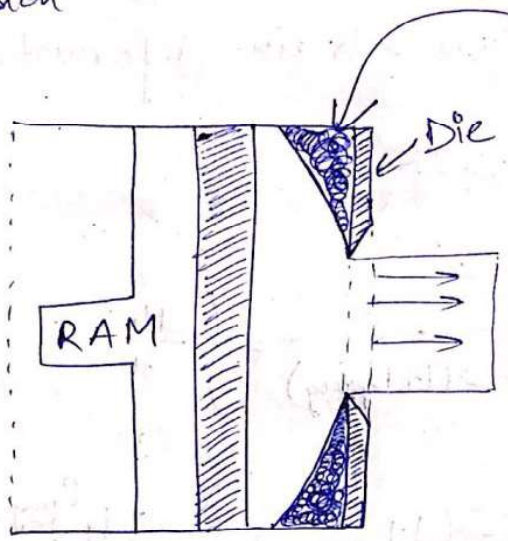
$$\left(\frac{\sigma_{xa}}{S}\right) = \left(\frac{1+B_{plug}}{B_{plug}}\right)\left[1 - \left(\frac{h_a}{h_b}\right)^{B_{plug}}\right]$$

* Extrusion

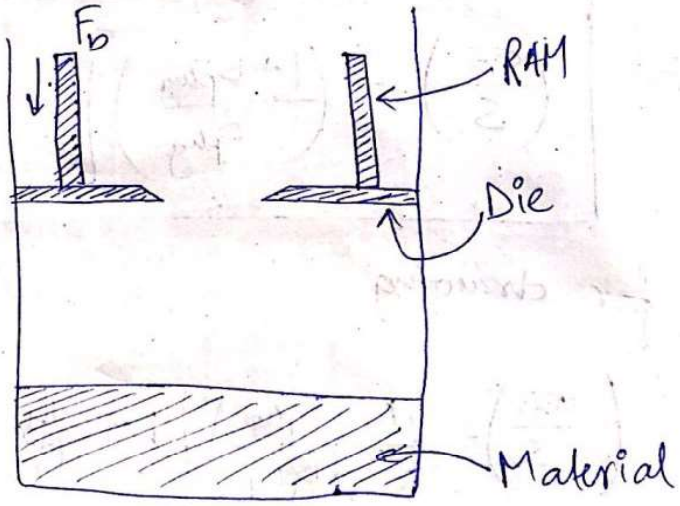
$\sigma_{xa} = 0$

$\because F_a = 0$

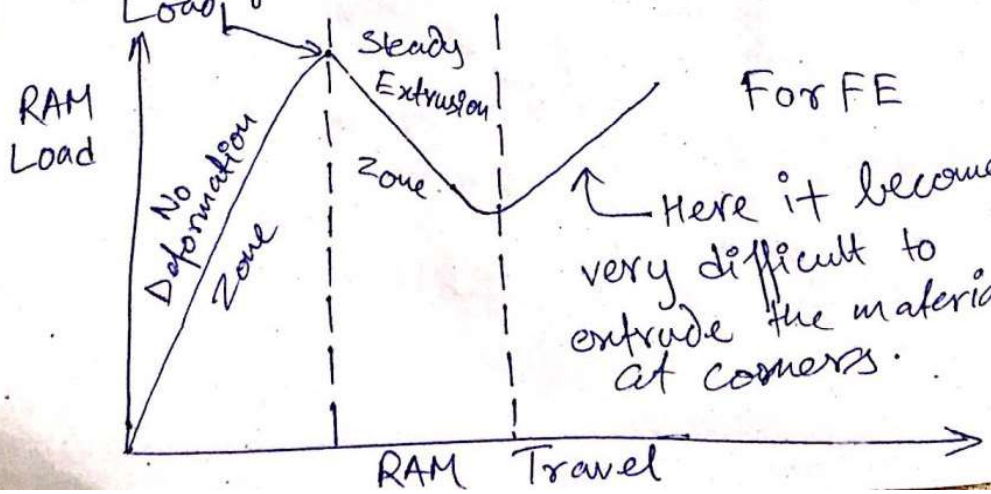
$\sigma_{xb} = -ve$



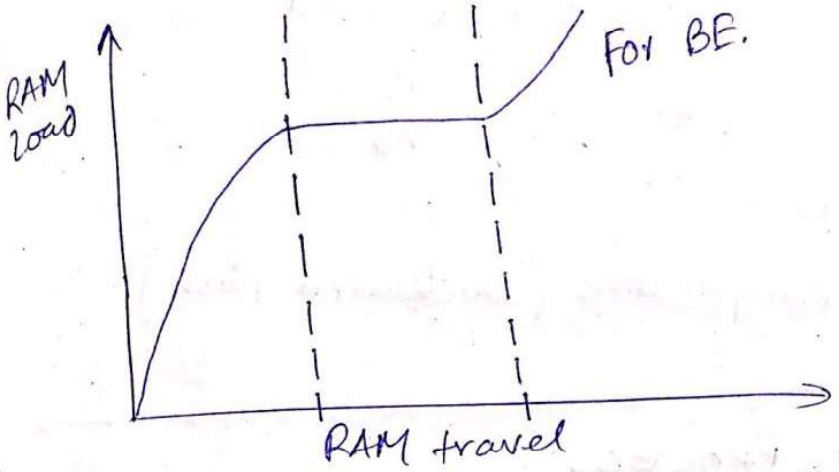
Direct Extrusion or Forward Extrusion.



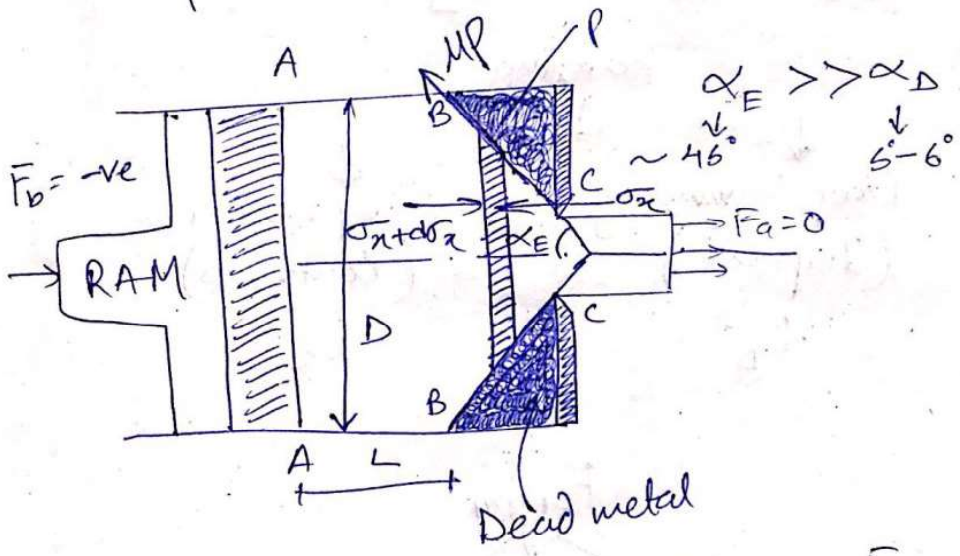
Backward extrusion or Indirect Extrusion



For FE, RAM Load is at peak when material completely fills the internal contour of container.

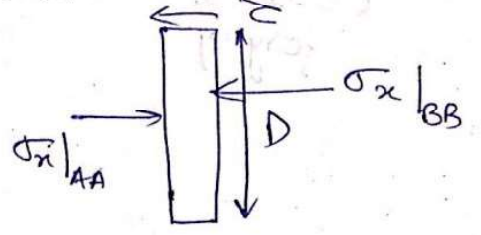


BE is better than FE. It only has infrastructure constraint



from AA to BB

$$\sigma_x|_{AA} = \frac{F_a}{A_x}$$



$$\Rightarrow \sum F_x = 0$$

$$\Rightarrow \sigma_x|_{AA} \times \frac{\pi D^2}{4} = \left(\sigma_x|_{BB} \times \frac{\pi}{4} D^2\right) + \pi D L \times \tau \quad (K)$$

Bar drawing with Backward Pull.

$$\left(\frac{\sigma_{xa}}{Y}\right) = \left(\frac{1+B}{B}\right)\left[1 - \left(\frac{D_a}{D_b}\right)^{2B}\right]$$

$$+ \text{Backward Pull}$$

$$+ \frac{F_b}{A_a}$$

$\sigma_{xa} = 0$

σ_{xb} = Compressive (Backward Push)

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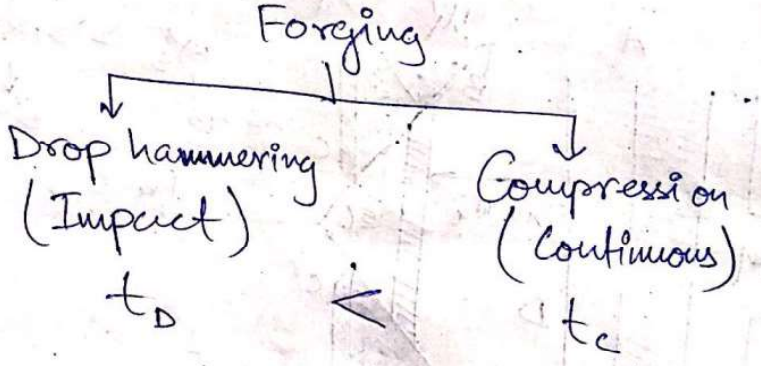
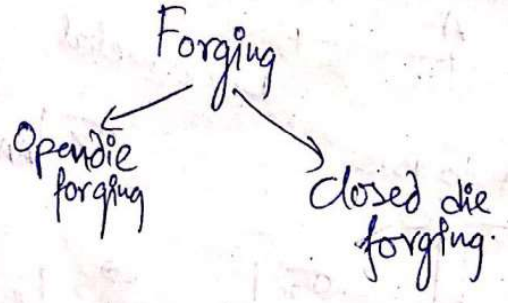
*FORGING PROCESS

Material is given shape by hammering

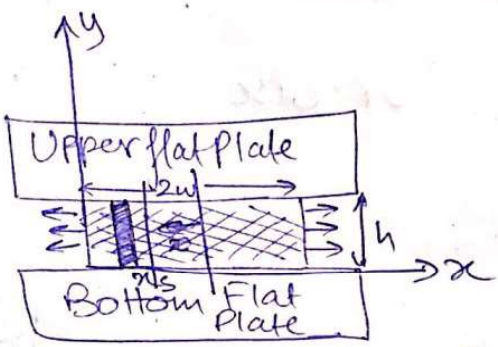
It is a Hot Working Process.





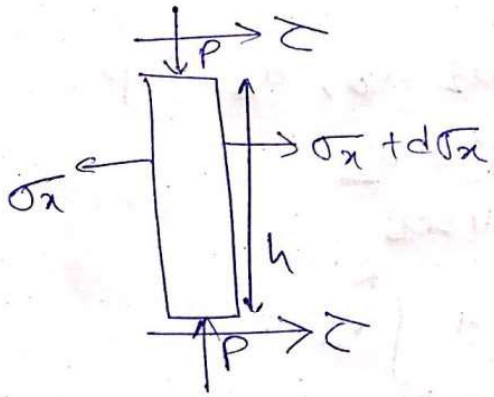
Open die forging

No restriction on radius of material.



Strip forging or Disc forging (nearly same except geometry)

for strip ($L >> 2w >> h$)



$(\sigma_x + p = 2K)$

$\Sigma F_x = 0$

$\tau = \mu p$ ~~K (close to free surface)~~

$x_s < x < w$

$= \mu p$ ~~(near to centre,~~ $0 < x < x_s$)

so for $0 < x < x_s$, $\rightarrow P_1$

& for $x_s < x < w$, $\rightarrow P_2$

at $x = x_s$, $\tau = \mu p_s = k$

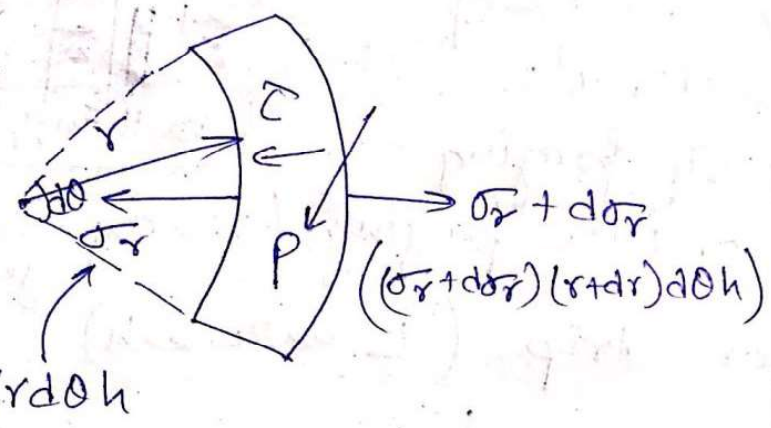
B.C., when $x = 0$, $\sigma_x = 0$

$x = 2w$, $\sigma_x = 0$

$$F = \left[\int_0^{x_s} P_1 + \int_{x_s}^{w} P_2 \right] \times 2$$

F: Force exerted by the Plates on material

For disc (top view)



$\sigma_1 = \sigma_r, \quad \sigma_2 = \sigma_\theta, \quad \sigma_3 = -p$

assume $\sigma_1 = \sigma_2$

Use Von Mises

$$F = \int_0^{r_s} P_1 + \int_{r_s}^{R} P_2$$

$0 < r < r_s$, $\tau = k$ ← Sticking Zone.

$r_s \le r < R$, $\tau = \mu p$ ← Sliding Zone

at $r = R$, $\sigma_r = 0$