Assignment – I

1. Find the Laplace transform of

a) $f(t) = \sin 2t \sin 3t$, b) $f(t) = \sin^3 2t$

c) $f(t) = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$, d) $f(t) = \sin\sqrt{t}$

2. Express the following functions in terms of the Heaviside unit-step function and find their L.T.s

a) $f(t) = \begin{cases} 1, & 0 \le t \le 2 \\ t^2, & t \geq 2 \end{cases}$

b) $f(t) = \begin{cases} 0, & 0 \le t < 3 \\ e^t, & 3 \le t < 4 \\ 0, & t \geq 4 \end{cases}$

c) $f(t) = \begin{cases} \sin t, & 0 \le t < \pi/4 \\ \sin t + \cos\left(t - \frac{\pi}{4}\right), & t \geq \pi/4 \end{cases}$

3. Find L.T. of the periodic function in $(0, \infty)$

$$f(t) = \sin \omega t, \quad 0 < t \le \pi/\omega$$
$$= 0, \quad \frac{\pi}{\omega} \le t < 2\pi/\omega$$

4. Find L.T. of

a) $t \sinh at$, b) $t \sin 3t \cos 2t$

c) $\int_0^t \frac{1}{u}(\cos au - \cos bu)\,du$, c) $\int_0^t \frac{e^u \sin u}{u}\,du$

5. Find

a) $L[t^2 u(t-1) + 8(t-1)]$, c) $J_0(at)$

b) $L[t^3 \delta(t-4)]$, d) $e^{2t} \operatorname{erf}(\sqrt{t})$

e) $L[f(t)]$, $f(t) = [t]$, which is the greatest integer $\le t$.

Assignment - II

1. Find L^{-1} of the following function:

a) $\frac{s}{s^4+s^2+1}$, b) $\log \frac{s^2+1}{s(s+1)}$, c) $\log\left(\frac{s+1}{s-1}\right)$,

d) $\frac{s e^{-s/2} + \pi e^{-s}}{s^2+\pi^2}$, e) $\frac{e^{-s}}{(s+1)^3}$.

f) $\cot^{-1}\frac{s}{2}$, g) $\tan^{-1}\frac{2}{s^2}$, h) $\frac{1}{1+\sqrt{1+s}}$

i) $\frac{1}{\sqrt{s}(s-1)}$, j) $\log\frac{s+2}{s+3}$

k) $\frac{1}{s}\left[\tan^{-1}\frac{a}{s} - \frac{as}{a^2+s^2}\right]$, l) $\frac{1}{\sqrt{s}-\sqrt{a}}$

2. Evaluate the integrals by L.T

a) $\int_0^\infty e^{-tx^2}\,dx$; b) $\int_0^\infty e^{-2x}\frac{\sin 3x - \sin 4x}{x}\,dx$;

Definite integrals.

c) $\int_0^\infty e^{-4t}\cos 2t\,dt$; d) $\int_0^\infty \frac{\sin^2 \theta}{\theta^2}$

e) $f(t) = \int_0^\infty \frac{\cos t\theta}{1+\theta^2}\,d\theta$

f) $\int_0^\infty e^{-2t}\frac{\sinh t \sin t}{t}\,dt$.

3. Solve the following integral equations by L.T.

a) $y(t) = \sin 2t + \int_0^t y(\tau)\sin 2(t-\tau)\,d\tau$

b) $\frac{dy}{dt} + 4y + 5\int_0^t y\,dt = e^{-t}$, $y(0)=0$.

c) $f(t) = 1-\sinh t + \int_0^t (1+\tau)\,y(t-\tau)\,d\tau$.

d) $\int_0^t f(\tau)\,J_0(2(t-\tau))\,d\tau = \sin 2t$.

Assignment – III

1. Q Use Laplace transformation technique to solve the following IVPs

a) $\ddot{y}(t) + 2\dot{y}(t) + 2y(t) = 5\,u(t-2\pi)\sin t,$

$y(0) = 1,\ \dot{y}(0) = 0$

b) $\ddot{y}(t) + 4y(t) = 10\cdot\delta(t-3),\ y(0) = \dot{y}(0) = 0$

c) $\ddot{y}(t) + y(t) = f(t),\ y(0) = \dot{y}(0) = 0$

where $f(t) = n+1$ for $n\pi \le t \le (n+1)\pi,$

$n = 1, 2, \ldots$

d) $t\ddot{y}(t) + \dot{y}(t) + t\,y(t) = 0,\ y(0) = 2,\ \dot{y}(0) = 0$

e) $\ddot{y} + t\dot{y} - y = 0,\ y(0) = 0,\ \dot{y}(0) = 1.$

2. Solve the following BVP by L.T.

a) $y'' + 9y = \cos 2t,\quad y(0) = 1,\ y(\pi/2) = -1.$

b) $y'' + 4y = 0,\quad y(0) = 0,\ y(\pi/4) = -1$

3. Solve the IVP

$y'' + ty' - y = 0,\quad y(0) = 0,\ y'(0) = 1.$

4. $\frac{dx}{dt} - y = e^t$ & $\frac{dy}{dt} + x = \sin t$

$x(0) = 1,\ y(0) = 0.$

5. $3x' + y' = 1 - 2x$

$x' + 4y' + 3y = 0,\quad y(0) = x(0) = 0$

6. Find $y(t)$ when

$y(t) = 1 - \sinh t + \int_0^t (1+\tau)\,y(t-\tau)\,d\tau$

7. Show that $\int_0^t J_0(\tau)\,J_0(t-\tau)\,d\tau = \sin t.$