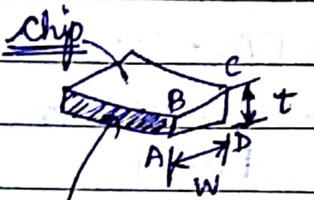
Date: 28/2/18

Module - 4

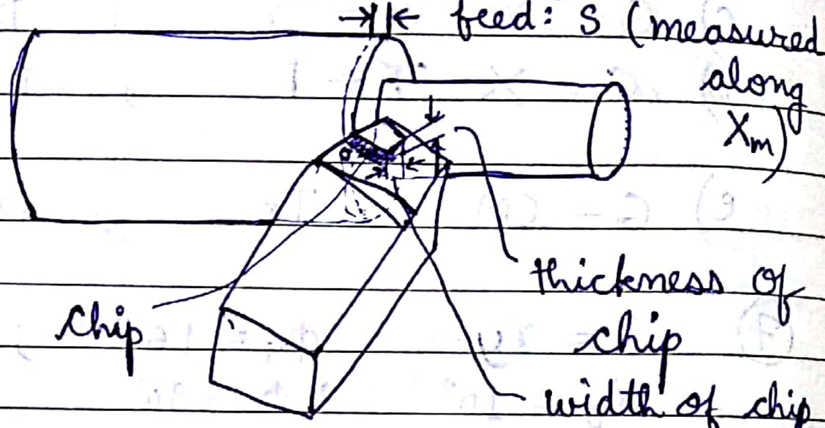
Chip Formation in Machining

→ **Geometry in Chip Formation :-**

Straight Turning



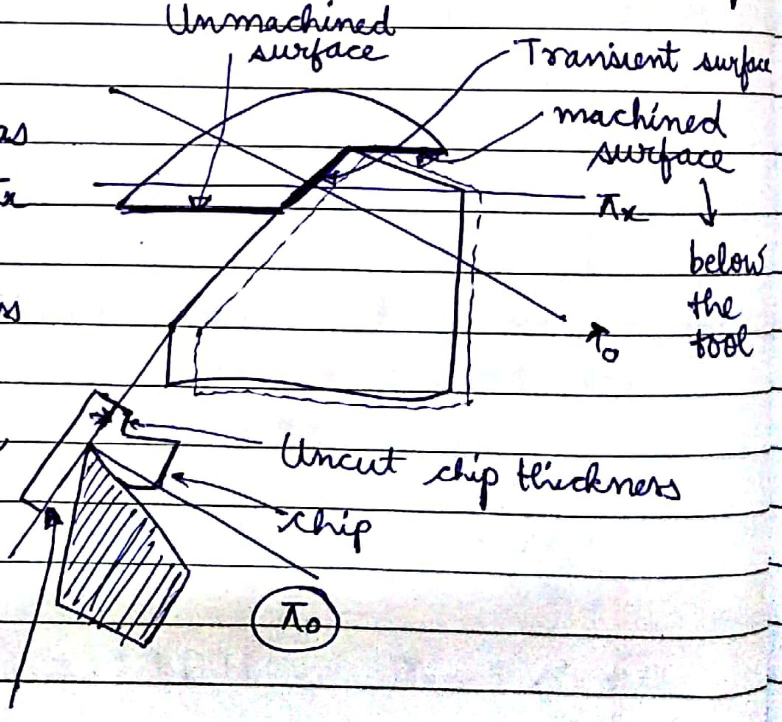
This plane is parallel to orthogonal plane.



→ Uncut chip thickness = feed (which has been depicted in π_r plane)

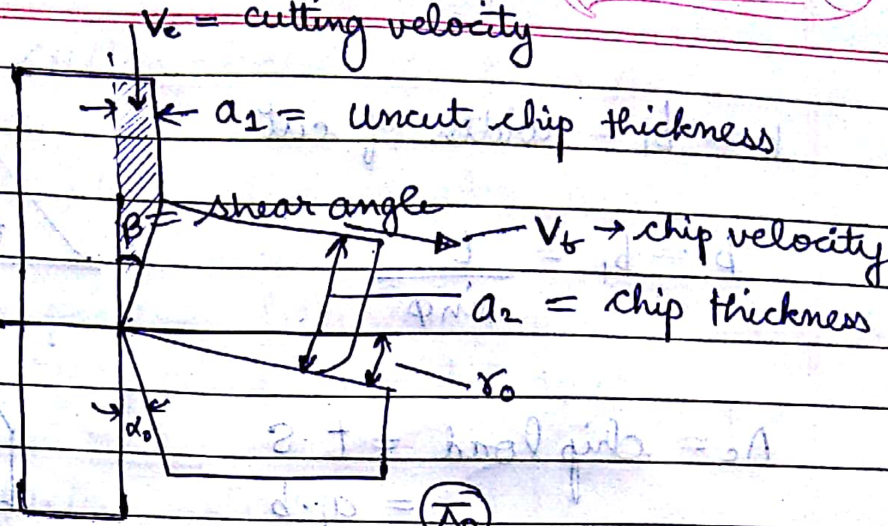
→ Uncut chip thickness is depicted in π_o

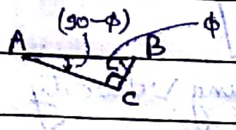
→ Way they are drawn, they may not appear to be related to 1 another.



→ Machined surface is actually a part of transient surface that has been machined which is located below the tool (in π_o plane)

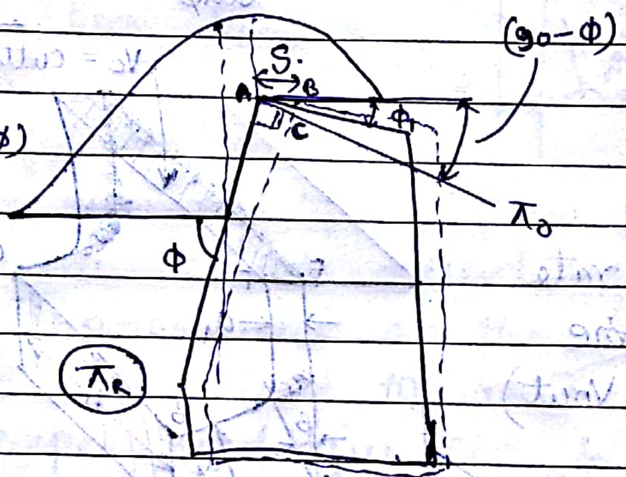
→ Upon machining, we get chip which are thicker. Uncut chip thickness goes down with V_c and chip leaves with V_f.





AC = AB cos(90−φ)

AC = AB sinφ



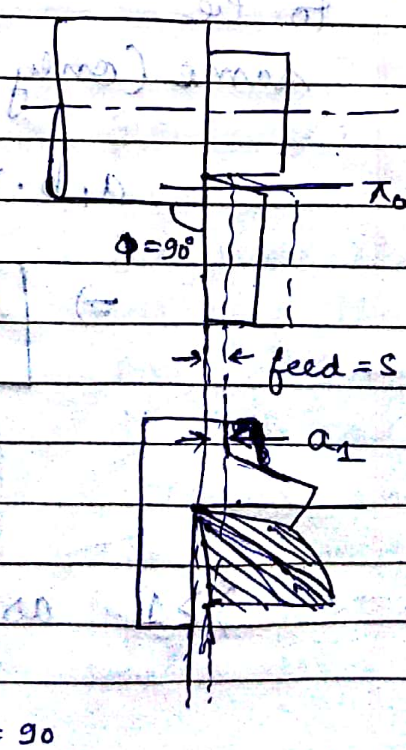
AB = feed = S

AC = a_1 = uncut chip thickness

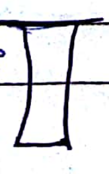
$$AC = AB \cdot \sin\phi = S \cdot \sin\phi \Rightarrow \boxed{a_1 = S \cdot \sin\phi}$$

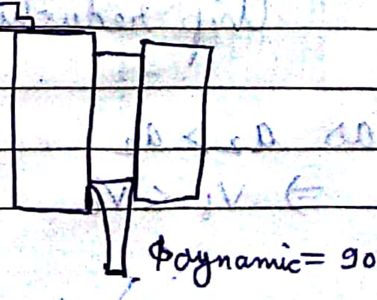
If $\phi = 90$, $a_1 = S$

In this type of turning, shoulder is generated



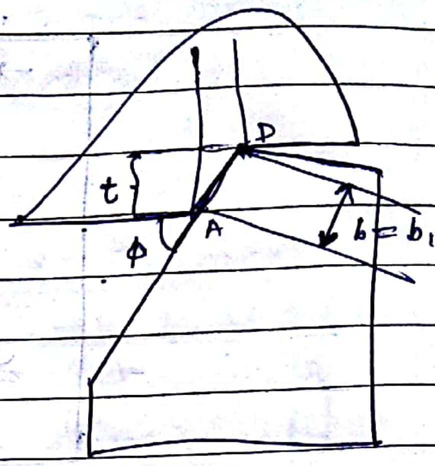
If we take $\phi = \phi$ static
$a_1 = 0$ which isn't possible, so ϕ dynamic is to be taken.





$b = b_1$ = width of cut

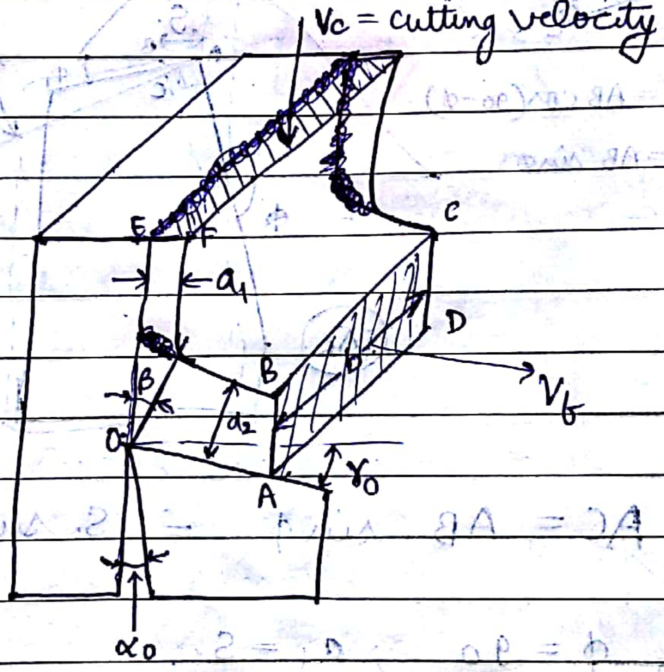
$$b = b_1 = \frac{t}{\sin\phi}$$



A_c = chip load $= t \cdot S$

$= a_1 \cdot b$

$$\left[\text{as } a_1 b = S \sin\phi \cdot \frac{t}{\sin\phi} = S \cdot t \right]$$



→ Volume flow rate is same ($V_{in} = V_{out}$)

→ Width of cut (b) assumed to be same (only in micro machining : $b_1 \neq b_2$)

$$\therefore a_1 b \cdot V_c = a_2 b \cdot V_f \quad \text{(Const. Volume flow rate)}$$

$$\Rightarrow \boxed{\frac{a_2}{a_1} = \frac{V_c}{V_f} = \xi}$$

Chip reduction coefficient (used in ISO)

$\xi > 1$ as $a_2 > a_1$

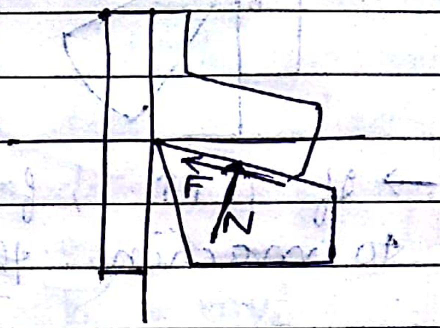
$\Rightarrow V_f < V_c$

r = cutting ratio = $\frac{1}{\xi} = \frac{a_1}{a_2}$

→ Some places, this is also used

→ Friction is present between Rake surface and workpiece surface (towards tool tip).

→ Friction factor calculated from Moody's Chart



Due to friction, V_f is decreasing (similar to fluid flowing over surface), due to $\Delta p = F$, due to change in momentum vector's orientation, Δp = the force transmitted on the surface which here could be related to as a force that will be experienced by C/T & a component of that is the Normal Force).

Contact Pressure b/w chip & tool is 500–600 MPa. Temp is also high → Friction's effect is quite significant.

→ Why $\xi > 1$

$\xi = \frac{a_2}{a_1} > 1$

ξ is more than 1 as chip thickens on machining. Chip thickens as there is hinderance to its movement by the friction ~~factor~~ force, F.

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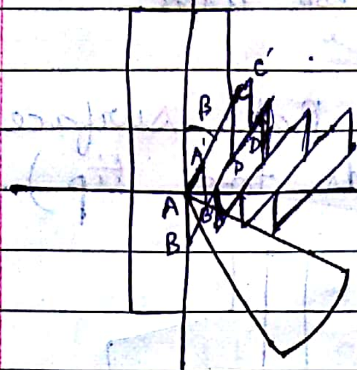
Piispanen Model of chip formation :-
↳ "post card model"

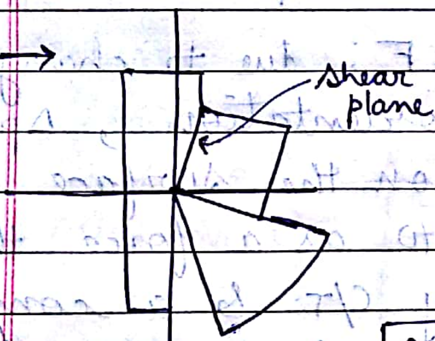


The card ABDC "slides up" or "shears" to its new location A'B'D'C'.

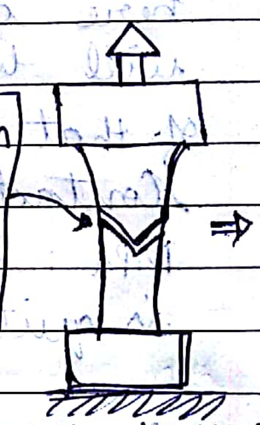
$\boxed{\beta \rightarrow \text{Shear Angle}}$

{ is an easy parameter to decide machinability.

→ If ξ↑↑↑ , friction force is more ⇒ less easy to machine the surface ⇒ less machinable

→ 

Mechanism of chip formation in ductile w/m → shearing



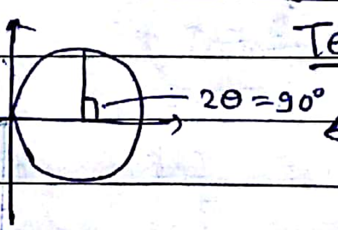
DUCTILE Material
Necking occurs at 45°

Cup & Cone formation for failure in ductile material
Necking occurs.

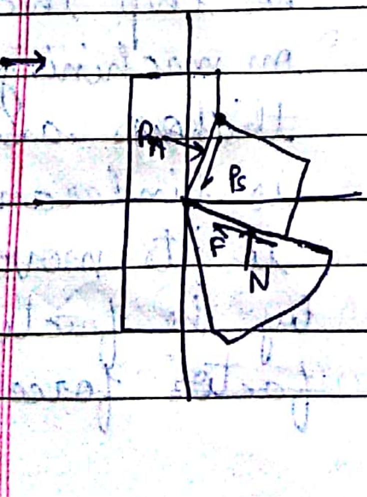
⇒

Here also chip formation takes place.

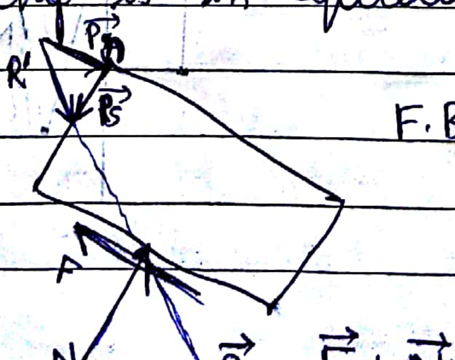
Unconstrained Tensile Test



Mohr Circle

→ 

This Chip is in equilibrium



F.B.D. of chip

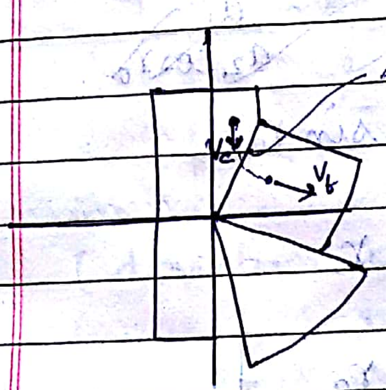
$\vec{R} = \vec{F} + \vec{N}$

$\vec{P_s}$ = shear force which is responsible for introducing shear failure in the W/M.

→ This is directed towards the tool tip as the card is slided up, ∴ hinderance is downward and ductile material fails due to shear.

(chip has no mass, inertia of chip not considered)

→ Material has some planes which on encountering with tool slides upwards due to shear (Analogy with the cards sliding up).

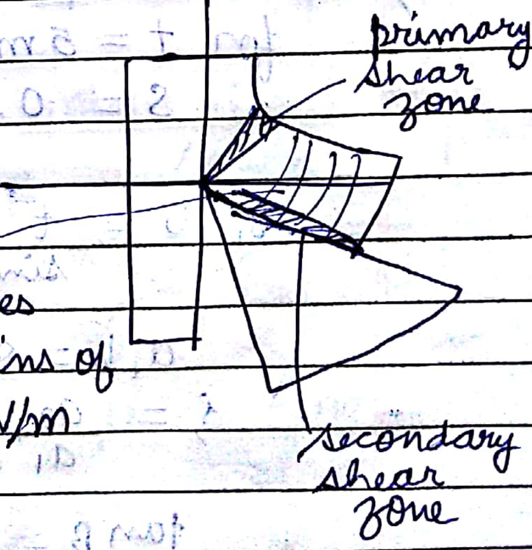


$\vec{p_i} = m\vec{V_c}$

$\vec{p_f} = m\vec{V_f}$

change of momentum is taking place at shear plane ∴ thickness of shear plane is 0, ∴ no time to change

(F→∞) but this is not possible. In actual m/c operation, shear plane has some finite thickness (shear zone).



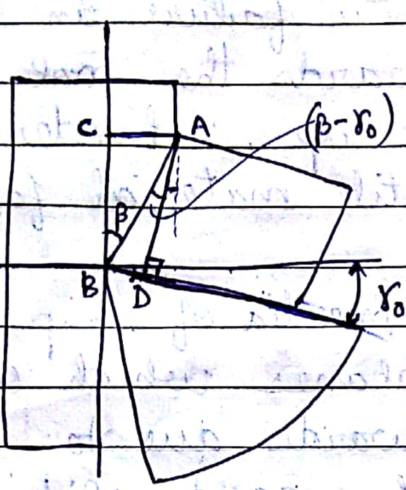
→ As you m/c surface, material gets deformed.

→ In secondary shear zone, $\vec{F}$ is responsible for introducing shear (similar to $\vec{P_s}$ shear in primary shear zone).

lay lines of grains of the W/m

→ Lay lines deform & transform into secondary shear zone as W/m is machined.

⇒ model γ_0, β and ξ



$$\xi = \frac{a_2}{a_1} = \frac{AD}{AC}$$

$$\xi = \frac{AD/AB}{AC/AB}$$

$$\boxed{\xi = \frac{\cos(\beta-\gamma_0)}{\sin\beta}}$$

$$\angle BAD = (\beta-\gamma_0)$$

$$\cancel{\tan\beta = \frac{a_1}{BC}} = \cancel{\frac{a_1}{AD\cos\gamma_0}} = \cancel{\frac{a_1}{a_2\cos\gamma_0}}$$

$$\xi = \frac{\cos\beta\cos\gamma_0 + \sin\beta\sin\gamma_0}{\sin\beta}$$

$$\xi = \frac{\cos\gamma_0}{\tan\beta} + \sin\gamma_0$$

$$\Rightarrow \boxed{\tan\beta = \frac{\cos\gamma_0}{\xi - \sin\gamma_0}}$$

Q4) Estimate β

for $t = 5\,mm$, $b_1 = 5\,mm$ $\quad \gamma_0 = -10^\circ$

$s = 0.2\,mm/rev$ $\quad a_2 = 0.5\,mm$

$$b_1 = 5 = \frac{t}{\sin\phi} = \frac{5}{\sin\phi} \qquad \sin\phi = 1$$
$$\phi = 90^\circ$$

$$a_1 b = s \times t \qquad a_1 = \frac{0.2\times 5}{5} \qquad \underline{a_1 = 0.2}$$

$$\xi = \frac{a_2}{a_1} = 2.5$$

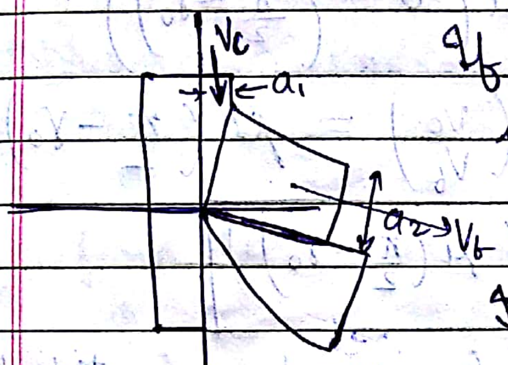
$$\tan\beta = \frac{\cos(-10^\circ)}{2.5-\sin(-10^\circ)} \approx \boxed{20.22^\circ = \beta}$$

→ Mass under the tool is under compression (not uniaxial compression) → 2 constraints

are there → friction & sliding constraint
∴ Shear doesn't occur at exactly 45°
(Also evident in above Qu where $\beta = \cancel{40} \; 20°$)

→ Effect of μ and γ_0 on ξ

(Kronenberg's model) $\xi = \frac{a_2}{a_1}$

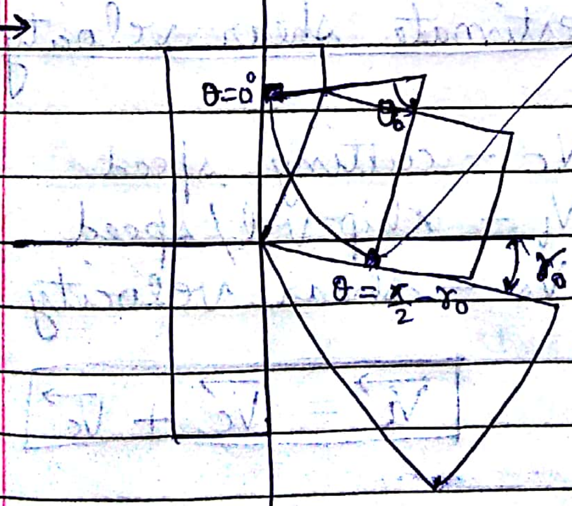


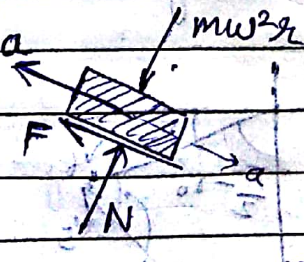
If $\mu \uparrow\uparrow$, friction force $\uparrow\uparrow$
chip ~~becomes~~ thickens,
hence $\xi \uparrow\uparrow$

If γ_0 becomes more −ve ;
change in momentum (direction change) would be much large, ∴ more change in (mv),
∴ $N \uparrow\uparrow$, ∴ $F \uparrow\uparrow$, ∴ $\xi \uparrow\uparrow$.

→ Kronenberg's model is good for qualitative level, don't use for quantitative problems

→ • Particle rotates about some point





$$\frac{F}{N} = \mu = \frac{-ma}{m\omega^2 r}$$

$$\mu = \frac{-dV/dt}{V\,d\theta/dt}$$

$F = \mu N$

$F + ma = 0$

$F = -ma$

$N = m\omega^2 r$

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$$\mu = -\frac{dV}{V d\theta}$$

$$= \mu \int_0^{\frac{\pi}{2}-\gamma_0} d\theta = \int_{V_c}^{V_f} \frac{dV}{V}$$

$$\cdot\ \mu\ \theta \Big|_0^{\frac{\pi}{2}-\gamma_0} = \ln V \Big|_{V_c}^{V_f}$$

$$\ln \frac{V_c}{V_f} = -\mu\left(0 - \left(\frac{\pi}{2} - \gamma_0\right)\right)$$

$$\ln(\xi) = \ln\left(\frac{V_c}{V_f}\right) = \mu\left(\frac{\pi}{2} - \gamma_0\right)$$

$$\therefore \boxed{\xi = \exp\left(\mu\left(\frac{\pi}{2} - \gamma_0\right)\right)}$$

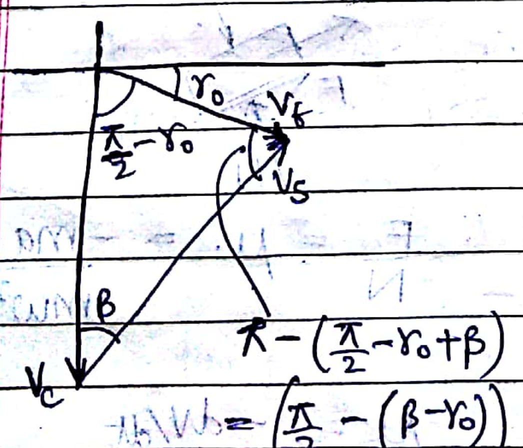
This model should not be used quantitatively

eg.) If $\mu = 0.5$ $\gamma_0 = 10^\circ$

$$\xi = \exp\left(0.5\left(\frac{\pi}{2} - \frac{10 \times \pi}{180}\right)\right)$$

$\boxed{\xi = 2.01}$ (Angle to be converted in radians)

→ Velocity Triangle to estimate shear velocity



V_c – cutting speed
V_f – chip vel/speed
V_s – shear velocity

$$\boxed{\vec{V_f} = \vec{V_c} + \vec{V_s}}$$

Thus,

$$\frac{V_c}{\sin\left(\frac{\pi}{2} - (\beta - \gamma_0)\right)} = \frac{V_f}{\sin\beta} = \frac{V_s}{\sin\left(\frac{\pi}{2} - \gamma_0\right)}$$

$$\frac{V_c}{\cos(\beta-\gamma_0)} = \frac{V_f}{\sin\beta} = \frac{V_s}{\cos\gamma_0}$$

same expressⁿ

$$\boxed{\zeta = \frac{V_c}{V_f} = \frac{\cos(\beta-\gamma_0)}{\sin\beta}}$$

and $\boxed{V_s = V_c \frac{\cos\gamma_0}{\cos(\beta-\gamma_0)}}$

Tool wear is affected by abrasion & Temperature & is decided by energy evolved at chip-tool interface

⇒ Numerical Problem

Q4) $\gamma_x = \gamma_0$ $\quad V_c \perp V_f$ $\quad V_c = 2\ m/s$

$s = 0.2\ mm/rev$ $\quad \beta = 25^\circ$

find a_1, a_2, ζ, V_f, V_s

Solⁿ $\phi = 90^\circ$ $\quad a_1 = s\sin\phi = s$

$a_1 = 0.2\ mm$ $\qquad \zeta = \cot 25$

$\zeta = 2.14$

$\frac{\pi}{2} - \gamma_0 = \frac{\pi}{2}$ $\quad \gamma_0 = 0$

$a_2 = a_1\zeta = 0.428\ mm$

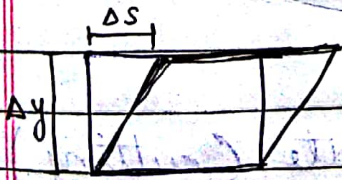
$\frac{V_c}{V_f} = \zeta$ $\quad V_f = \frac{V_c}{\zeta} = 0.934\ m/s.$

$V_s = \frac{V_c \cos 0}{\cos 25} = 2.206\ m/s.$

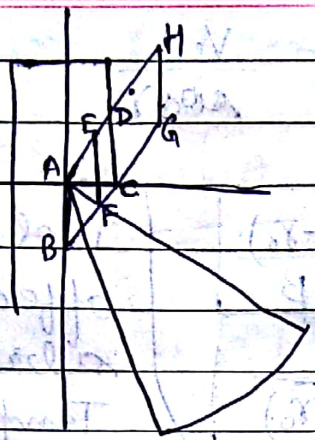
→ We estimate V_s to estimate the amount of energy that is evolved at the shear plane.

Cutting Strain and Strain Rate :-

mechanism of chip formation – shear



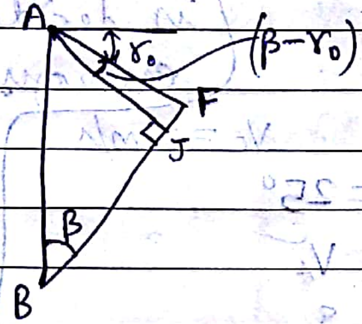
shear strain $= \frac{\Delta s}{\Delta y}$



The segment ABCD sheared along the shear plane to its near location EFGH

ϵ_c = cutting strain (shear)

$$\epsilon_c = \frac{BF}{\text{thickness of the segment}} = \frac{BF}{AJ}$$



$$\epsilon_c = \frac{BJ + JF}{AJ}$$

$$= \frac{BJ}{AJ} + \frac{JF}{AJ}$$

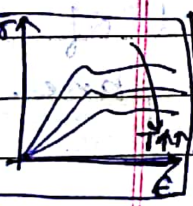
$$\epsilon_c = \cot\beta + \tan\angle JAF$$

$$\boxed{\epsilon_c = \cot\beta + \tan(\beta-\gamma_0)}$$

For previous Ques

$\epsilon_c = 2.61 \leftarrow$ Very large

→ In machining the amount of strain is very large (Min value (when $\gamma_0 = 0°$) is 2). During m/cing, lot of heat evolved at primary ~~m~~ shear zone, ∴ shear plane's ~~strength ↑↑ and it ∴ fails large value~~ ductility ↑↑ ($\sigma-\epsilon$ graph goes down at higher temp.) ∴ it won't be breaking easily (Analogy → Ice cream).



↳ Strain Rate

$\dot{\epsilon}_c$ = strain rate (cutting)

$$\dot{\epsilon}_c = \frac{\text{strain}}{\text{Time}} = \frac{BF/AJ}{BF/V_s} = \frac{V_s}{AJ} = \left(\frac{V_s}{\text{thickness of segment}}\right)$$

Qu.) Estimate $\dot{\epsilon}_c$ if segment thickness is 2μm

$$\dot{\epsilon}_c = \frac{2.2}{2\times10^{-6}} = 1.1\times10^{6} \ /s$$

→ During tensile testing → $\dot{\epsilon}_c = 0.5/500$

→ In m/c ing → large strain, large strain rate.

→ Sheet metal forming → large strain & small strain rate.

→ Grinding → large strain rate.

(*) Effect of process parameters on chip reduction coefficient :-

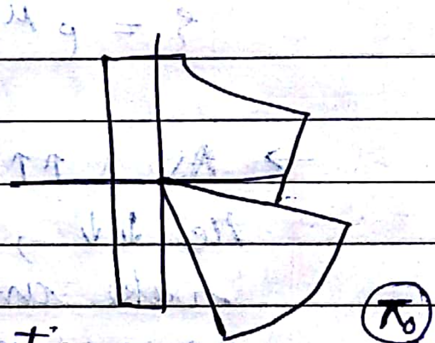
Process parameters

V_c

S

$t \longrightarrow b_1 \ (\pi_c)$

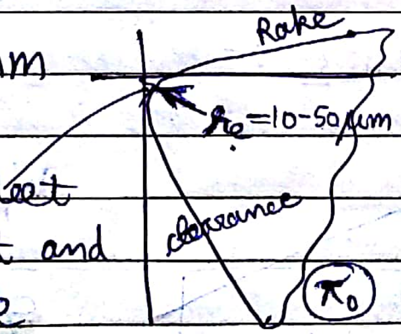
$\downarrow$

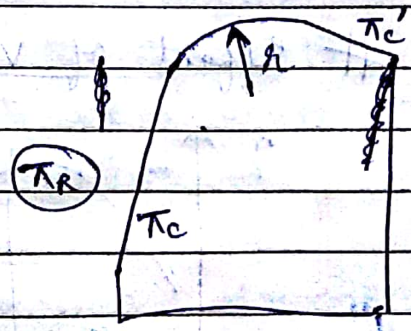


doc, b_1 doesn't affect in chip formation and all other thickness, shear angle unless it is increased by 10 times.

$r = 0.4 - 2$ mm

Does not meet at a point and some edge rounding is there.





Edge rounding not there in natural diamond (Edge Rounding diff. from nose radius)

(b/w rake & clearance on π_0) (b/w π_c & π_c' on π_R)

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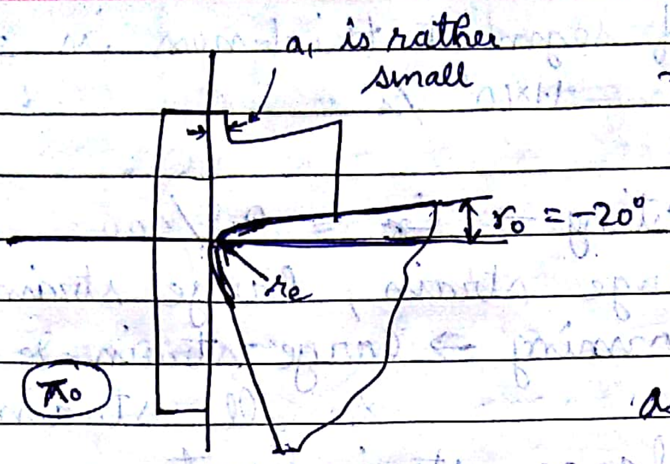


Due to presence of edge radius, dynamics/ effective value of γ_0 gets changed.

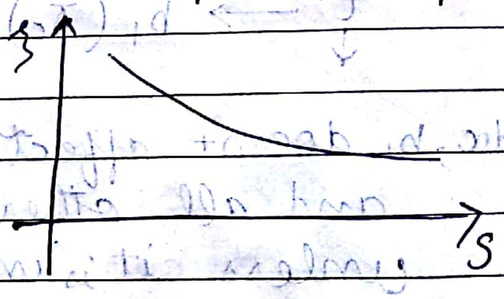
as $S \downarrow a_1 \downarrow$ $\gamma_0 \rightarrow$ becomes more $-ve$

(doesn't necessarily becomes $-ve$ but absolute value reduces mathematically).

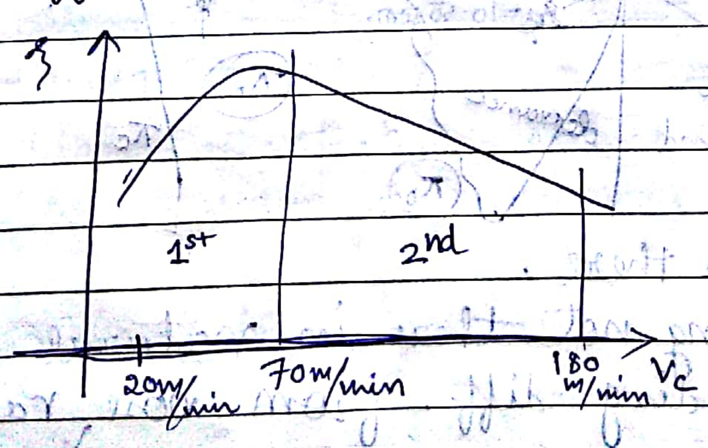
$$\xi = e^{\mu(\frac{\pi}{2} - \gamma_0)}$$

as γ_0 becomes more negative $\xi \uparrow$

→ As $s \uparrow\uparrow$, $a_1 \uparrow\uparrow$ less tool tip participates $r_e \downarrow\downarrow$, effective rake angle becomes more $+ve$, $\therefore \xi \downarrow\downarrow$



Effect of V_c in ξ :-



W/p → low carbon steel

C/T →

As $V_c \uparrow\uparrow \rightarrow \mu \downarrow\downarrow$, $\xi \downarrow\downarrow$ $\left[\because \xi = e^{\mu(\pi/2 - \gamma_0)}\right]$

As $V_c \uparrow\uparrow$ → shrinkage of shear zone

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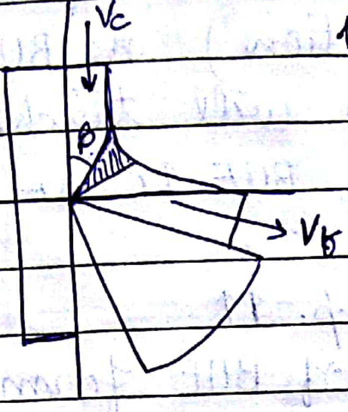
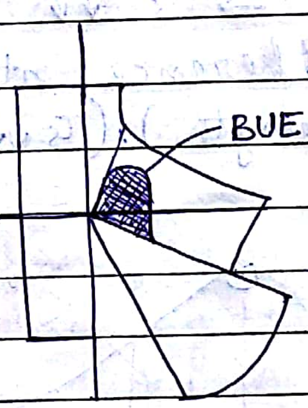


Whatever heat is generated, it flows with the chip.
Shrinkage occurs at the top. As, $V_c \uparrow$, shrinkage of shear zone takes place
$\beta \uparrow\uparrow \longrightarrow \xi \downarrow\downarrow$

→ Built up edge (BUE) :-

Because of prevelant temperature and pressure → welding takes place



→ In the 1st phase of ξ v/s V_c graph :- W/m has been welded at the tool tip, when V_c is low but gradually increasing. Welding takes place due to high temperature at the chip tool interface and contact pressure at the chip-tool interface.
As $V_c \uparrow\uparrow$, energy expended $\uparrow\uparrow$, Temp. $\uparrow\uparrow$, chances $V_c \uparrow$ size of welding $\uparrow\uparrow$, BUE $\uparrow\uparrow$.
→ size of BUE $\downarrow$ as $V_c \uparrow$ in the 2nd phase

2 processes :- ① Adherence
② Removal of BUE by flowing chip

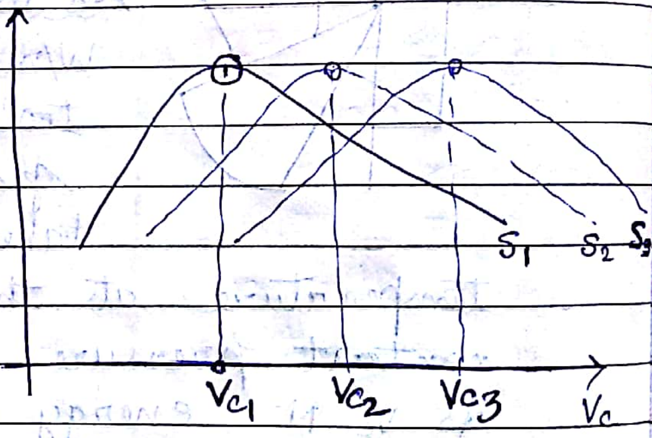
Size of the BUE becomes stable after a point.
→ BUE is gently removed by the chip and at the same time material is added by the chip.
→ There are high chances of adherance as the temp. $\uparrow\uparrow$ ⇒ so material deforms easily, to break that interface it will require less energy.

→ Because of BUE → extra hinderance to chip movement (also by friction). As BUE ↑↑ more hinderance → chip will thicken. In the first phase → size of BUE ↑↑ so ξ ↑↑.

→ 2nd Phase → As V_c ↑↑ , Temp. ↑↑
→ More chances of BUE formation
→ Also removal of BUE is more as Temp. is high so gradual reduction.
→ As V_c ↑↑ , μ is decreasing → ξ ↓↓
→ Issue of removal of adherence at high temperature (loss of strength). (τ_s ↓↓ as Temp. ↑)

→ V_c ↑
S ↑
t ↑ $\Big\}$ Energy ↑

∴ MRR = $V_c \cdot S \cdot t$
MRR is changing in all 3 cases.

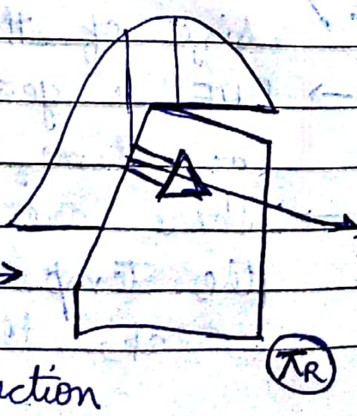


Temp. at which transition will take place will also depend on material.
To achieve the same transition temperature, if we increase V_c, feed Requirement will be minimum and vice versa. ($S_3 < S_2 < S_1$)

3 Types of Machining

① Orthogonal ⟶ Chip velocity is
② Pure Orthogonal | contained in
③ Oblique | π_0 ⟶

↳ Deviation of chip flow direction from π_0



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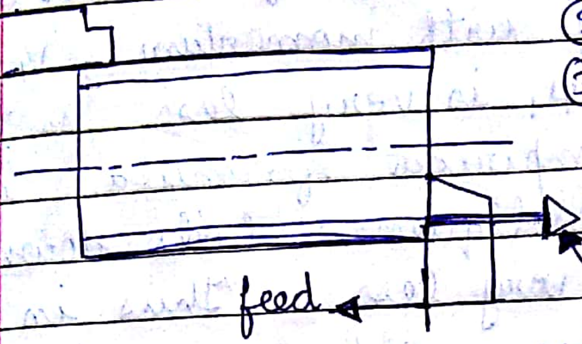
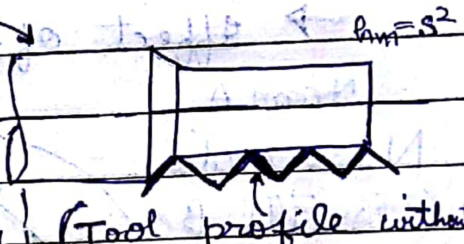
② Pure Orthogonal machining



① $\phi = 90^\circ$
② $\pi_o = \pi_x$
③ Tool tip doesn't participate in machining
④ $\lambda = 0^\circ$

chip flow direction would match with π_o / π_x

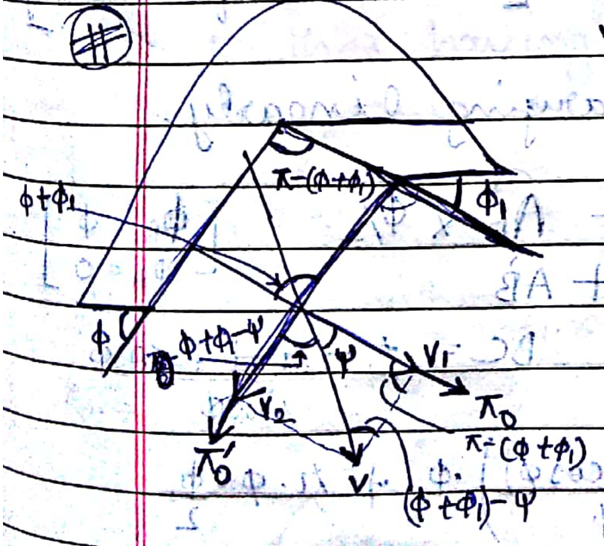
Reasons for chip deviation :-
① non zero λ
② Nose Radius
③ Restricted cutting Effect



When we are m/c ing, the MCE is doing the primary cutting and the ACE is removing the material from W/p though the amount is small → affects surface roughness.

(Tool profile without nose radius) due to geometry & Kinematic reason, some kind of roughness in job.

#



$V_2 \rightarrow$ velocity of aux. chip
$V_1 \rightarrow$ velocity of main chip
When these 2 chips interact, there is a gradual deviation

$$\frac{V_1}{\sin(\phi+\phi_1-\psi)} = \frac{V_2}{\sin\psi}$$

or
$$\frac{V_1}{V_2} = \frac{\sin(\phi+\phi_1-\psi)}{\sin\psi}$$

$$\boxed{\frac{V_1}{V_2} = \frac{t/\sin\phi}{s/2}}$$

(Ermin + Kronenberg)

$$\boxed{\tan\psi = \frac{\sin(\phi+\phi_1)}{\frac{2t}{s\sin\phi} + \cos(\phi+\phi_1)}}$$

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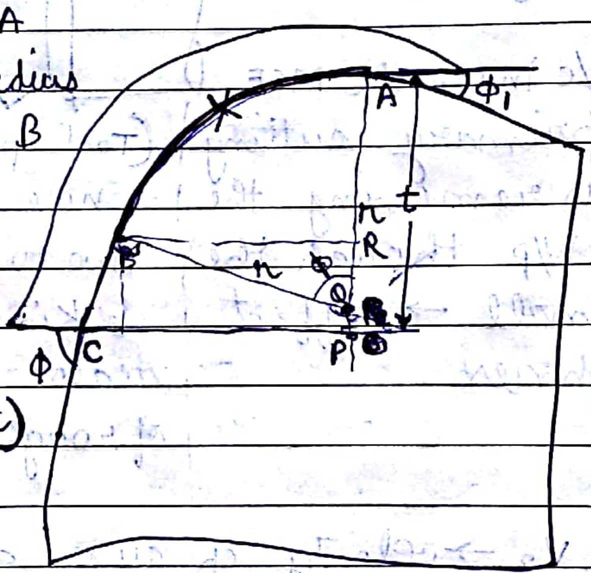
Logic → As V_2 is coming from the A.C.E., so that gate is less open, thus flow is less and the feed is ∴ S/2. The width of gate analogy with momentum, $V_1 \propto (t/\sin\phi_1)$, $V_2 \propto (S/\sin\phi_1)$; but ϕ_1 is very less, so it should be S but from empirical formula, it is S/2.

If $(\phi + \phi_1) = 90^\circ$ ⇒ Obliqueness boils down to $\left[\frac{1}{\left(\frac{2t}{S\sin\phi}\right)}\right]$ which is very less. Thus in normal machining, obliqueness is not so much which is explained provided there is no tool radius.

→ Effect of Nose Radius :-

From A
Nose radius is till B
After B it is linear



PCEA ↓↓ as one progresses towards the nose radius from C to A.
We talk about the portion where nose radius ends. (at B)
[Q → centre of curvature]

AP → t (doc)
AQ → r

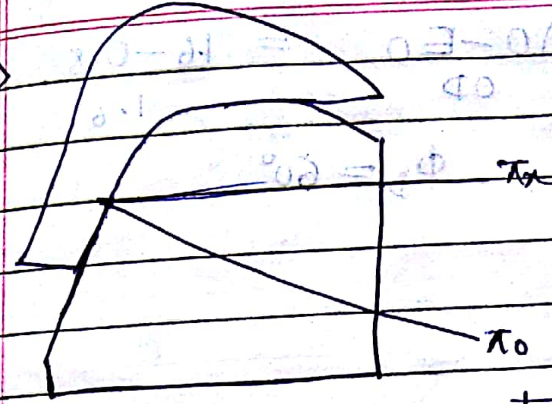
Assumⁿ → Along AB, PCEA is varying linearly.

$$\phi_{av} = \frac{BC \times \phi + \widehat{AB} \times \phi/2}{BC + \widehat{AB}} \qquad \begin{bmatrix} \phi_B = \phi' \\ \phi_A = 0 \end{bmatrix}$$

$$BC = \frac{AP - AQ + RQ}{\sin\phi} \qquad BC = \frac{t - r + r\cos\phi}{\sin\phi}$$

$$\therefore \phi_{av} = \frac{\left(\frac{t - r(1-\cos\phi)}{\sin\phi}\right)\cdot\phi + r\cdot\phi\cdot\frac{\phi}{2}}{\left(\frac{t - r(1-\cos\phi)}{\sin\phi}\right) + r\cdot\phi}$$

Qu.)



$t = 5\,mm$

$r = 0.8\,mm$

$s = 0.2\,mm/rev$

$\phi = 75^\circ$

$\phi_1 = 15^\circ$

Find ϕ_{avg}. Then use $\tan\psi$, where $\phi = \phi_{avg}$.

$$\phi_{avg} = \frac{\left(\frac{5 - 0.8(1-\cos 75^\circ)}{\sin 75} \times [redacted] + \frac{0.8 \times 75 \pi}{360}\right) 75^\circ}{\left[\frac{5-0.8(1-\cos 75^\circ)}{\sin 75} + \left(\frac{0.8 \times 75\pi}{180}\right)\right]}$$

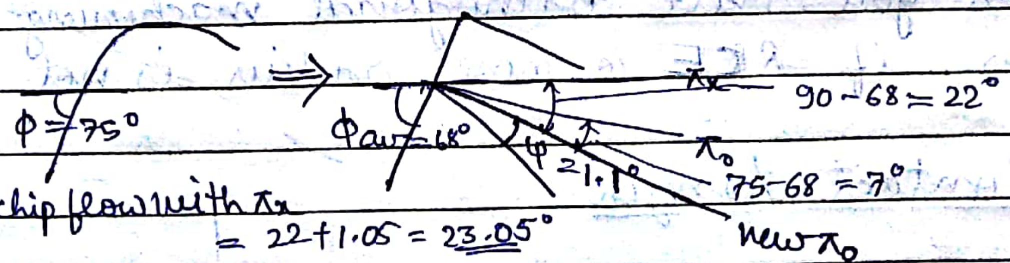
$\underline{\underline{\phi_{avg} = 68^\circ}}$

$$\tan\psi = \left[\frac{\sin(\phi+\phi_1)}{\frac{2t}{s\sin\phi} + \cos(\phi+\phi_1)}\right]$$

$$\tan\psi = \frac{\sin(68+15)}{\frac{2\times 5}{0.2\times(\sin 68)} + \cos(68+15)}$$

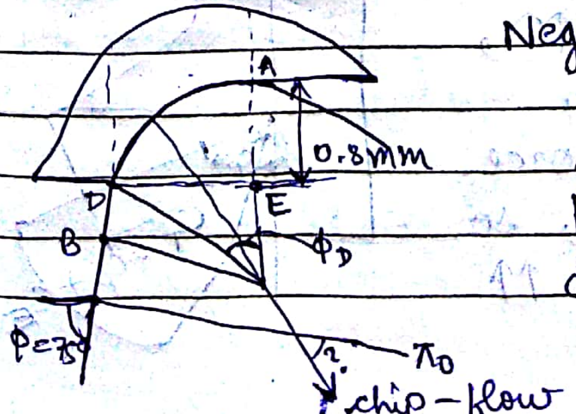
$\psi = 1.052^\circ \approx 1.1^\circ$ ← This value of ψ is from π_o which $\phi = 68^\circ$.

We have changed the C/T having nose radius ($\phi = 75^\circ$) to that having no nose radius ($\phi = 68^\circ$)



Angle of chip flow with π_x $= 22 + 1.05 = 23.05^\circ$

Qu.)



Neglect Restricted cutting effect

$r = 1.6\,mm$

Here C does not exist as doc ends before nose radius

Find ϕ_{avg}.

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$$\cos\phi_D = \frac{OE}{OD} = \frac{AO - EO}{OD} = \frac{1.6 - 0.8}{1.6}$$

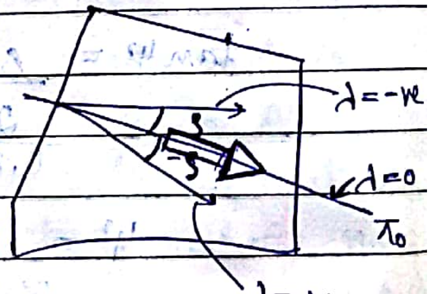
$$\phi_D = \cos^{-1}\left(\frac{1}{2}\right) \quad \phi_D = 60^\circ$$

$$\phi_{avg} =$$

→ Since pt. C is not here & the doc is ending at a pt. in the nose radius, so for finding ϕ_{avg}, we need to find ϕ_A & ϕ_D and take the mean.
$\phi_A = 0$ (since pt. A is [illegible] on π_y).
Here the chip flow is along the new π_o.

Effect of λ :-

$\rho = |\lambda|$ stabler scale
(For small λ)

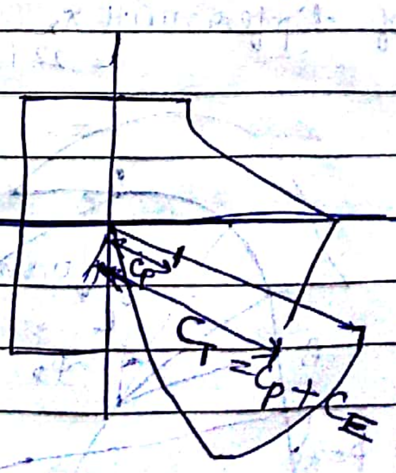


Qu.) Can you have orthogonal machining with λ "-ve"?
Ans.) Yes, if we have nose radius so both the effects are cancelled.

Qu.) Can you have orthogonal machining with $\lambda \to +ve$
Ans.) Yes, if REE & nose radius is not there.

Contact Length

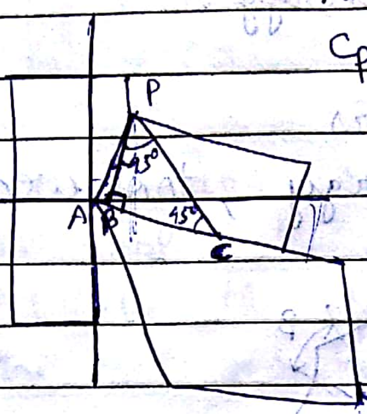
As contact length ↑↑, area over which ~~further~~ friction force is taking place, so hinderance will ↑↑.



In the elastic region, contact length is less
In the plastic region, contact length is more

Abuladze's

At high speed m/c ing, contact length would be plastic entirely



$C_p = AB + BC$

If the contact length is less, contact area friction is less, If we have more area, more chance of welding, so if contact forces are less, less welding.

[Point C is achieved by drawing a line at 45° from P with PB where PB is ⊥].

$$C_p = AB \tan(\beta - \gamma_0) + a_2$$
$$= a_2 \tan(\beta - \gamma_0) + a_2$$
$$C_p = a_2 (1 + \tan(\beta - \gamma_0))$$

Area $= b \times C_T \rightarrow C_p$

$$\sigma_N = \frac{F}{\text{Area}}$$

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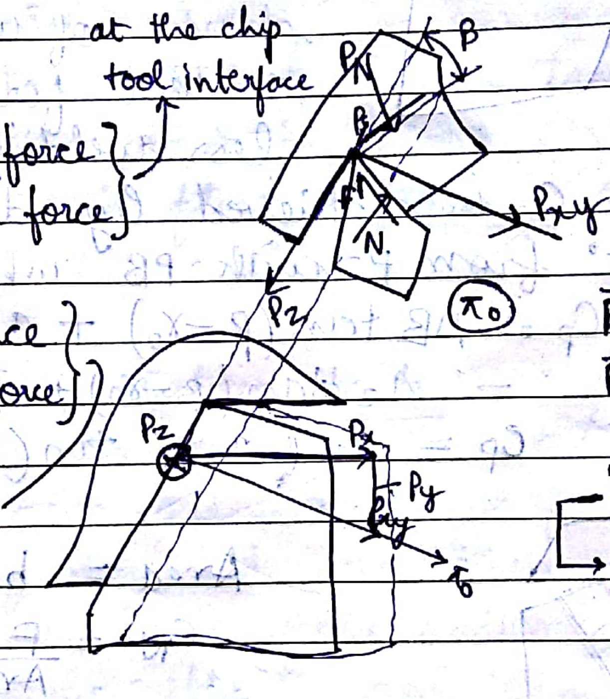
Module - 5

Mechanics of Machining

deals with :
→ machining forces and energy.
→ identification of forces & energy.
→ prediction / estimation
→ effect of process parameters
→ effect of forces on energy, tool wear, accuracy
→ measurement

F = friction force
N = Normal force
} at the chip tool interface

P_S = Shear force
P_N = Normal force
} at the primary shearing plane



$\vec{F} + \vec{N} = \vec{R}$
$\vec{P_S} + \vec{P_N} = \vec{R'}$

$|\vec{R}| = |\vec{R'}|$ → Resultant cutting force

Depending on machine coordinates, $\vec{P_S}$ & $\vec{P_N}$ would change but $\vec{P_{xy}}$ & $\vec{P_Z}$ wouldn't.

$$\vec{R} = \vec{P_{xy}} + \vec{P_Z} = (\vec{P_x} + \vec{P_y}) + \vec{P_Z}$$

} does not depend on rake angle and ξ, thus easy to measure

(whereas P_S & P_N changes as rake angle & ξ changes)

$\vec{P_{xy}}$ will contain in orthogonal m/cing as long as chip floats in orthogonal plane, thus valid only for orthogonal machining.

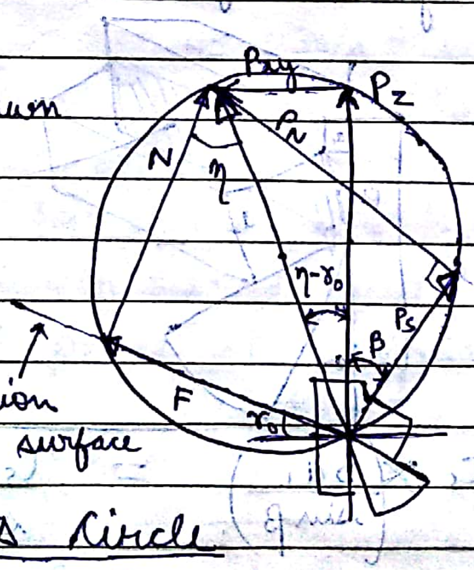
P_z – main cutting force
P_x – feed force
P_y – (radial) thrust force (backforce)
P_{xy} – (resultant) thrust force

→ P_x & P_y & P_z are the forces that are applied on the tool. P_z going in the plane of paper. While the force $\vec{F}$ & $\vec{N}$ are applied by the tool on the ~~uncut~~ cut segment of the w/p. i.e. chip.

→ $\vec{F} \perp \vec{N}$; $\vec{P_S} \perp \vec{P_N}$; $\vec{P_{xy}} \perp \vec{P_z}$ All sum up to give $\vec{R}$ or $\vec{R'}$.

→ $\vec{P_S}$ & $\vec{P_N}$ are applied by the uncut W/P.

These are drawn in reverse direction.



This depicts forces applied by the tool on W/P. A Circle passes through the 3 vertices of Force Triangle.

Extension of rake surface

Merchant's Circle Diagram

capture interrelationship b/w different forces in machining.

$\vec{F}$ & $\vec{N}$ } → are the ~~principal~~ physical forces
($\vec{P_S}$ & $\vec{P_N}$ } → responsible for chip formation

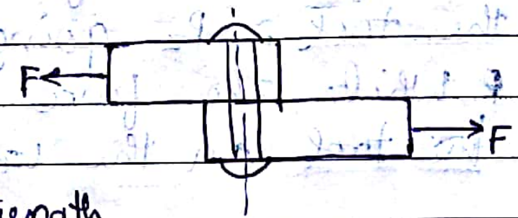
P_{xy} & P_z are components of resultant cutting force.

$P_x = P_{xy} \sin\phi$
$P_y = P_{xy} \cos\phi$

→ Estimation of forces P_z & P_{xy}.

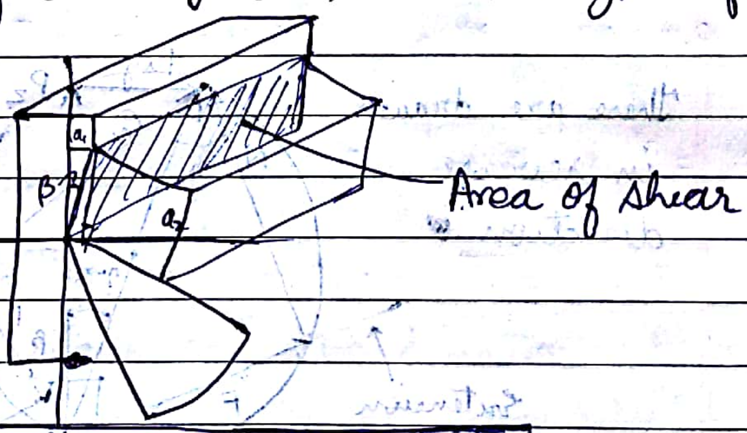
P_s = shear force ← Amount of force required to produce shear in primary shear plane.

$\tau_{shear} = \frac{F_{shear}}{A}$



P_s = W/m shear strength × shear Area
= τ_s × shear Area

τ_s → dynamic yield shear strength of W/m.



$$\therefore \boxed{P_s = \tau_s \times \left(\frac{b a_1}{\sin\beta}\right) = \frac{\tau_s t_s}{\sin\beta}}$$

↳ Here β is func^n of V_c; so V_c is also embedded in the above expression.

→ $R \cdot \cos(\eta + \beta - \gamma_0) = P_s$ (From Merchant circle)
$R \cdot \cos(\eta - \gamma_0) = P_z$

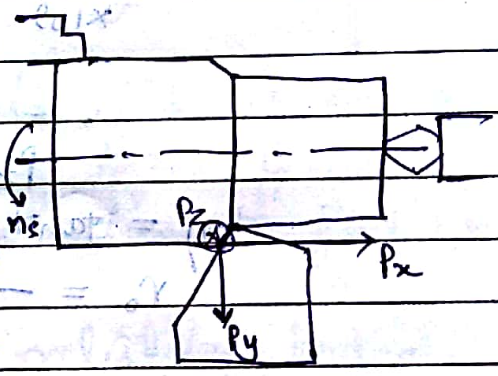
$$\boxed{\frac{P_z}{P_s} = \frac{\cos(\eta - \gamma_o)}{\cos(\eta + \beta - \gamma_o)}}$$

$$\therefore \; P_z = \frac{\tau_s t s}{\sin\beta} \times \frac{\cos(\eta - \gamma_o)}{\cos(\eta + \beta - \gamma_o)}$$

$\tau_s \rightarrow$ gets affected by s, V_c etc. ($\because$ it is dynamic & not static).

→ When you m/c LCS with ..... $\eta \approx 37^\circ$; so for computing P_z, η will be given.

■ Power $= \dot{E}_c = P_z \cdot V_c + P_x S_m$



V_c: 120 m/min
2 m/s

$S_m \longrightarrow$ $s = 0.1$ mm/rev
$n_s = 200$ rpm
$S_m = 20$ mm/min $= \frac{1}{3}$ mm/s

Ratio of V_c to S_m is quite large ($\sim 10^4$)

$\therefore$ For all practical purposes:

$$\boxed{\dot{E}_c = P_z \cdot V_c}$$

→ How much Energy is required to remove unit volume of material → U_c = Specific Cutting Energy

$$\therefore \; U_c = \frac{\text{energy}}{\text{material removed}} = \frac{\dot{\text{energy}}}{\text{material removal rate}}$$

$$U_c = \frac{\dot{E}_c}{MRR}$$

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$$\therefore \quad U_c = \frac{\dot{E}_c}{MRR} = \left(\frac{P_z \cdot V_c}{V_c \cdot s \cdot t}\right) - (J/mm^3) \text{ or } (GJ/m^3)$$

$$U_c = 0.5 - 2 \; J/mm^3$$

Qu. $\gamma_o = -10^\circ$

$s = 0.25$ mm/rev $\qquad \frac{F}{N} = 0.7$

$\phi = 75^\circ$

$t = 4$ mm $\qquad U_c = 2.5 \; J/mm^3$ or GJ/m^3

$V_c = 120$ m/min $\qquad$ estimate $P_z, P_{xy}, P_x, P_y, P_s, \tau_s$

$a_2 = 0.4$ mm

$$\underset{\times 10^9}{2.5} = \frac{P_z}{0.25 \times 10^{-3} \times 4 \times 10^{-3}}$$

$$P_z = 2.5 \times 10^3 = 2.5 kN$$

$\eta = \tan^{-1}(0.7) = 34.99^\circ$

$\gamma_o = -10^\circ$

$\cancel{\zeta = \exp\left(\mu\left(\frac{\pi}{2} + \frac{10 \times \pi}{180}\right)\right)}$ $\qquad \zeta = \frac{a_2}{s \sin\phi}$

$\cancel{\zeta = 3.393 = 3.4}$ $\qquad \boxed{\zeta = 1.656}$

$$\tan\beta = \left(\frac{\cos\gamma_o}{\zeta - \sin\gamma_o}\right)$$

$\beta = 28.291^\circ$

$P_{xy} = P_z \times \tan(\eta - \gamma_o) = 2.5 \times \tan(34.99 + 10)$

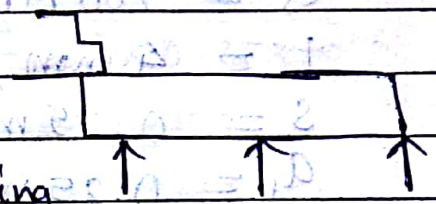
$P_{xy} = 2.499$ kN $\qquad P_s = 1.0169$ kN

$P_s = \frac{\tau_s t s}{\sin\beta} \Rightarrow \tau_s = 481.7$ MPa

$P_{xy} = 2.4198$ kN $\qquad P_y = 0.647$ kN

→ P_z affects how much power you need for machining (results in torsion).

→ P_x was assymetric buckling load (L/D ratio becomes imp.)
→ P_y will introduce bending Moment. It has to be controlled otherwise issues of tolerances due to some less dimensions being machined due to bending (maximum deformation in centre) would be there.



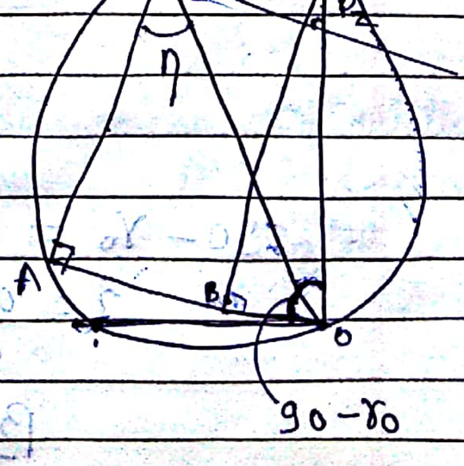
→ During m/c-ing w/p becomes stronger than usual, ∴ we use τ_s ← dynamic shear strength (material in use)

→ With the given instruments, it is easy to measure P_x, P_y, P_z. After finding this, we use Merchant circle to find F, N.

→ P_x, P_y, P_z, are machinability index.
U_c is complex " "
μ is also " " (not direct)

→ $\mu = F/N$

$F = P_z \sin \gamma_o + P_{xy} \cos \gamma_o$
$N = -P_{xy} \sin \gamma_o + P_z \cos \gamma_o$



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Qu) $\vec{F}, \vec{N}, \vec{P_{xy}}, \vec{P_z}$ → forms a rectangle
Angle b/w $\vec{F}$ and $\vec{P_s}$ = 120° $P_{xy} = 0.7 P_z$

$\tau_s = 700$ MPa
$t = 4$ mm, $V_c = 2$ m/s
$s = 0.25$ mm/rev
$a_1 = 0.25$ mm

determine U_c.

$\vec{P_{xy}} \parallel \vec{F} \Rightarrow \gamma_0 = 0°$

$90 + \beta = 120 \Rightarrow \beta = 30°$

$P_{xy} = 0.7 P_z$ or $F = 0.7 N \Rightarrow \eta = \tan^{-1} 0.7$
$= 35°$

$$P_s = \frac{\tau_s t s}{\sin\beta} = 1400 \text{ N}.$$

$$P_z = \frac{P_s \times \cos(\eta - \gamma_0)}{\cos(\eta + \beta - \gamma_0)} = 2713.59041$$

$$U_c = \frac{P_z}{s t} = 2.713 \text{ GJ/m}^3.$$

Qu) Angle b/w P_z & $F = 100°$
$a_1 = 0.25$ $s = 0.25$ mm/rev
$t = 4$ mm $V_c = 2$ m/s
$a_2 = 0.5$

assume high speed m/c ing

$\frac{F}{N} = 0.7$ | calculate U_c and τ_s, $P_z = 3000$N and σ_N = normal contact stress at the chip tool interface.

$90 - \gamma_0 = 100 \Rightarrow \gamma_0 = -10°$ $\eta = 35°$

$\xi = \frac{a_2}{a_1} = 2$ $\tan\beta = \frac{\cos\gamma_0}{\xi - \sin\gamma_0}$

$\beta = 24.4°$

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$$U_c = \frac{P_z}{st} = \frac{3000}{0.25 \times 4 \times 10^{-6}} = 3\, GJ/m^3$$

$$P_s = \frac{P_z \cdot \cos(\eta + \beta - \gamma_0)}{\cos(\eta - \gamma_0)} = 1500\, N$$

$$1500 = \frac{\tau_s t s}{\sin\beta} = \frac{\tau_s \times 4 \times 0.25 \times 10^{-6}}{\sin 24.4}$$

$$\tau_s = 618\, MPa$$

$$N = R \cdot \cos\eta = \frac{P_z \cos\eta}{\cos(\eta - \gamma_0)}$$

$$\Rightarrow N = 3475\, N$$

$$\sigma_N = \frac{N}{b_1 a_2 (1 + \tan(\beta - \gamma_0))}$$ ← From Abuladze's exp.

$$\sigma_N = 1030\, MPa$$

→ At such high Pressure, chip gets welded due to friction welding which is due to chemical reacn b/w chip & the the tool → micro-welding.

Qu.)

$\gamma_0 = -10°$

$\beta = 20°$

$\eta = 30°$

$t = 5mm$

$s = 0.25\, mm/rev$

$U_c = 2J/mm^3$

$V_c = 90\, m/min$

$\phi = 75°$

calculate :-

(i) ξ

(ii) cutting power

(iii) P_z

(iv) resultant thrust force

(v) P_s

(vi) P_n (vii) N

(viii) determine C_T without using Abuladze's Model

(i) $$\tan\beta = \frac{\cos\gamma_0}{\xi - \sin\gamma_0}$$

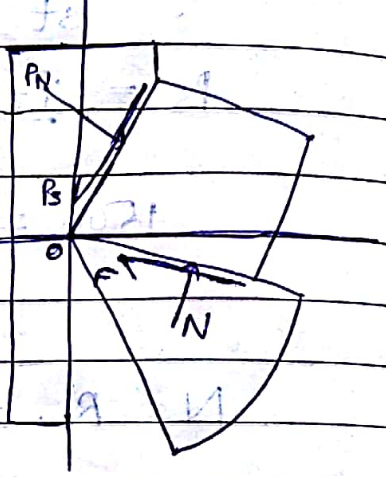
$$\tan 20 = \frac{\cos(-10)}{\xi + \sin(+10)}$$ $\xi = 2.53$

(viii) taking moment about 'O'

$$P_n \times \frac{a_i}{2\sin\beta} = N \times \frac{C_T}{2}$$

length of shear plane

contact length (total)



Assumption: P_n and N act at the mid-section of shear plane and total contact length → means they are uniformly distributed along the respective distances

ii) $U_c = \frac{P_z V_c}{V_c \cdot s \cdot t}$ ($P_z V_c$ = cutting power)

$\hat{E}_c = P_z V_c = U_c (V_c \cdot s \cdot t)$

$= 2 \times (1500 \times 0.25 \times 5 \ mm^3/s) = 3750 W$

iii) $P_z = \frac{3750}{1.5} = 2500 N$

iv) Resultant cutting force = R

Main cutting force = P_z

Resultant thrust force = P_{xy}

$P_{xy} = P_z \tan(\eta - \gamma_0)$

$= 2097 N$

v) $\frac{P_s}{\cos(\eta + \beta - \gamma_0)} = \frac{P_z}{\cos(\eta - \gamma_0)} \Rightarrow P_s = 1632 N$

vi) $\frac{P_n}{P_s} = \tan(\eta + \beta - \gamma_0)$

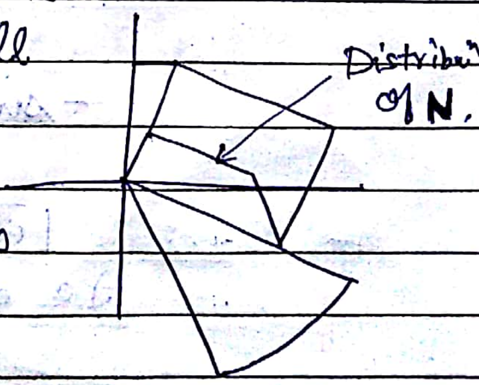
$\underline{P_n = 2826.7 N}$

vii) $N = R\cos\eta = \frac{P_Z}{\cos(\eta - \gamma_0)} \cos\eta$

$= 2826 N$

→ Along the principal shear plane, P_s is distributed uniformly.
→ Along Aux. cutting plane, F is uniform.

(viii) $C_T = 0.7$ mm ← This approach is better but it requires info. of Forces which may always not be available. However in Abuladze's Model, from tool geometry we can find C_T (no need of forces)
But in actual case distribution of N is not uniform, still first approach gives a better value of C_T compared to Abuladze's model.



→ $\underline{\eta, \beta, \gamma_0}$
↳ They were not related to one another.

Angle Relationships :

3 :-

→ brittle materials (cast Iron)
→ ductile materials (low carbon steel)
→ semi-ductile / Engg. Alloys.

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① For brittle materials

→ Ernst and Merchants' 1st Solution:

$\dot{E}_c$ = cutting power = $P_z V_c$

$$\dot{E}_c = \frac{\tau_s\, t s}{\sin\beta} \times \frac{\cos(\eta - \gamma_0)}{\cos(\eta + \beta - \gamma_0)} \times V_c$$

$\frac{d\dot{E}_c}{d\beta} = 0$ (There would be a β for which power will be min & nature will deform the material along that plane)

Assump^n → τ_s, $\cos(\eta - \gamma_0)$, V_c is constant

$$\frac{d\dot{E}_c}{d\beta} = -\frac{\tau_s t s \cos(\eta - \gamma_0) \cdot V_c}{\underbrace{D^2}_{\text{Denominator}}} \cdot \left(\frac{dD}{d\beta}\right)$$

$$-\sin\beta \cdot \sin(\eta + \beta - \gamma_0) + \cos(\eta + \beta - \gamma_0) \cdot \cos\beta = 0$$

$\boxed{\cos A \cos B - \sin A \sin B = 0}$ → $\boxed{\cos(A+B) = 0}$

$\Rightarrow$ ~~$\cot\beta = \tan(\eta + \beta - \gamma_0)$~~

$$\Rightarrow \cos(2\beta + \eta - \gamma_0) = 0$$

$$\Rightarrow \boxed{2\beta + \eta - \gamma_0 = \frac{\pi}{2}}$$

This eq^n is good only for brittle material

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$$P_z = \frac{\tau_s t_s}{\sin\beta} \times \frac{\cos(\eta - \gamma_0)}{\cos(\eta + \beta - \gamma_0)}$$

$$\frac{\tau_s t_s}{\sin\beta} \times \frac{\sin 2\beta}{\sin\beta}$$

$$= \frac{2\tau_s t_s}{\tan\beta} = 2\tau_s t_s \cot\beta$$

$$\boxed{P_z = 2\tau_s t_s \cot\beta}$$ ← check from book

$$P_{xy} = P_z \tan(\eta - \gamma_0) = P_z \cot 2\beta$$

$$P_{xy} = 2\tau_s t_s \cdot \frac{\cos\beta}{\sin\beta} \times \frac{\cos 2\beta}{\sin 2\beta}.$$

$$\boxed{P_{xy} = 2\tau_s t.s. \cot\beta . \cot 2\beta}$$

Primarily applicable for cast Iron & hard plastics

② Lee & Shaffer's angle relationship (for ductile W/m)

→ Slipline theory →

$$\boxed{\beta + \eta - \gamma_0 = \frac{\pi}{4}}$$ ← ductile W/m.

$$P_z = \frac{\tau_s . t . s}{\sin\beta} \times \frac{\cos(\eta - \gamma_0)}{\cos(\eta + \beta - \gamma_0)}$$

$$= \frac{\tau_s t.s}{\sin\beta} . \frac{\cos(\pi/4 - \beta)}{\cos(\pi/4)}$$

$$P_z = \tau_s t.s. (1 + \cot\beta)$$

β is not readily available.

$$\tan\beta = \left(\frac{\cos\gamma_0}{\xi - \sin\gamma_0}\right)$$

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$$\boxed{P_z = \tau_s\, t s \left\{ \frac{\zeta}{\cos\gamma_o} - \tan\gamma_o + 1 \right\}}$$ ← for ductile W/M Lee/Shaffer's

$$P_{xy} = P_z \tan(\eta - \gamma_o)$$

$$= \tau_s\, ts\,(1 + \cot\beta) \cdot \tan\left(\frac{\pi}{4} - \beta\right)$$

$$= \tau_s\, t.s.\,(1 + \cot\beta) \cdot \frac{1 - \tan\beta}{1 + \tan\beta}$$

$$= \tau_s\, t.s\left(\frac{\cancel{1+\tan\beta}}{\tan\beta}\right)\left(\frac{1-\tan\beta}{\cancel{1+\tan\beta}}\right)$$

$$= \tau_s\, t.s.\,(\cot\beta - 1)$$

$$\boxed{P_{xy} = \tau_s . t . s . \left(\frac{\zeta}{\cos\gamma_o} - \tan\gamma_o - 1 \right)}$$

→ LCS is most common material.
P_{xy} is mostly less than P_z.

for $\gamma_o = 0°$ $\quad \frac{P_z}{P_{xy}} = \left(\frac{\zeta + 1}{\zeta - 1}\right)$

Effect of cutting speed, feed and depth on cutting forces (P_z and P_{xy}):

for ductile w/m:

$$P_z = \tau_s t s \left\{ \frac{\zeta}{\cos\gamma_o} - \tan\gamma_o + 1 \right\}$$

→ form factor

as $t\uparrow$ P_z and P_{xy} $\uparrow\uparrow$ proportionately.

$$P_{xy} = \tau_s t s \left\{ \frac{\zeta}{\cos\gamma_o} - \tan\gamma_o - 1 \right\}$$

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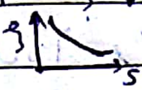
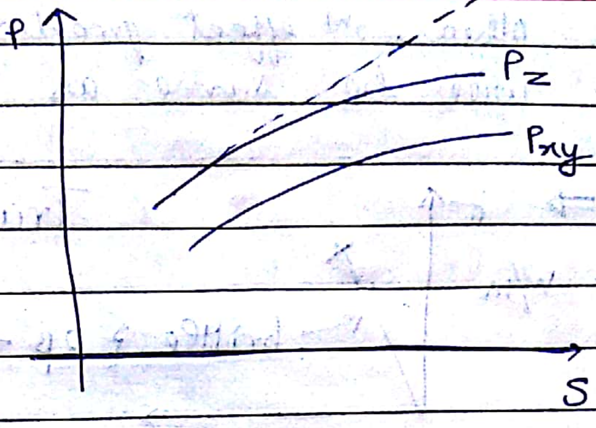
Effect of feed (S)

as $S\uparrow$ $\theta\uparrow$ (Temperature) $\tau_s\downarrow$ → W/m will easily deform

$S\uparrow$ $\xi\downarrow$ (due to effect of edge radius)



Thus P_z and P_{xy} do not increase linearly with S.



Effect of V_c

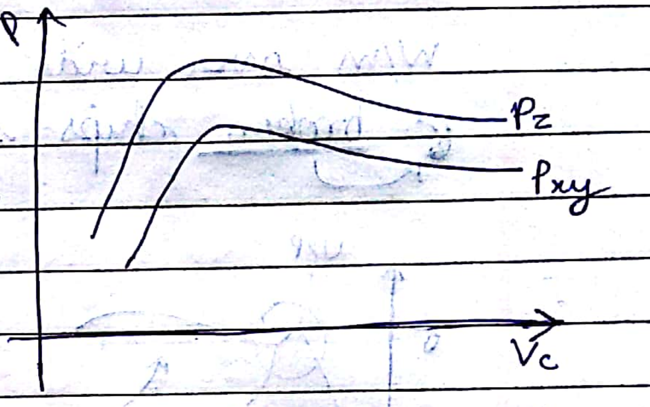
as $V_c\uparrow$ $\theta\uparrow$ $\tau_s\downarrow$

$V_c\uparrow$ $\xi\downarrow$

$V_c\uparrow$ $\dot{\epsilon}_c\uparrow$ $\tau_s\uparrow$

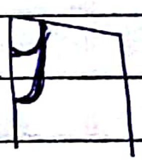
Material will harden

$\Rightarrow$



→ So, the nature of graph is dependent on nature of W/m. If it is temp. resistant, then last effect will be predominant.

→ Energy density (Analogy with magnifying glass)
As $t\uparrow\uparrow$, contact area $\uparrow\uparrow$; but Power also $\uparrow\uparrow$; Energy density is almost same.



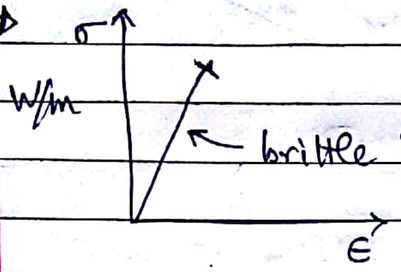
$$\text{Energy density} = \frac{F \cdot V_c}{b \cdot \xi \, a_1 (1+\tan(\beta-\gamma_0))}$$

(Here F and b $\uparrow\uparrow$ by same amount, So E.D. same ∴ Temp. same)

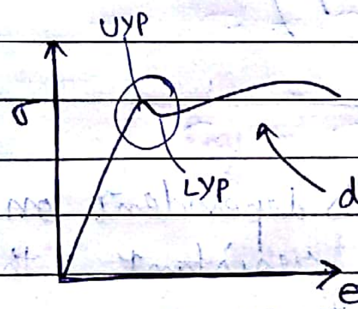
↳ When $S\uparrow$, but $[b \xi a_1 (1+\tan(\beta-\gamma_0))]$ (b inc., ξ decreases) doesn't increase linearly. ∴ Area of contact doesn't inc. linearly, ∴ Non-linear variation
In this case if S (2x) → F (2x) but say A (1.5x)
∴ Energy Density $\uparrow$ → Temp. $\uparrow$ → $\tau_s\downarrow\downarrow$

→ In P v/s V_c 1st & 3rd effect approx. cancels each other, 2nd effect predominant, ∴ variation of P will be same as ξ.

⇒ True Stress v/s True Strain



W/m over wide domain of s and V_c
broken chips are obtained (= brittle W/m)



(Differentiation with brittle material ~ % elongation at failure) — 10%
continous chips

Qu) Ductile W/m
$\gamma_0 = 0°$
find out the ratio b/w U_c and τ_s

$$U_c = \frac{P_z}{st} \qquad P_z = \frac{\tau_s t s}{\sin\beta} \frac{\cos(\eta - \gamma_0)}{\cos(\eta + \beta - \gamma_0)}$$

$$U_c \cdot st = \frac{\tau_s t s \cos\eta \times \sqrt{2}}{\sin\beta}$$

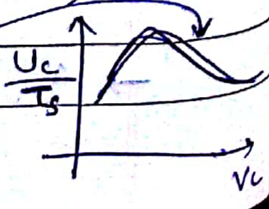
$$\frac{U_c}{\tau_s} = \frac{\sqrt{2}\cos\eta}{\sin\beta} = \frac{\sqrt{2}\cos(\pi/4 - \beta)}{\sin\beta}$$

$$= \frac{\cos\beta + \sin\beta}{\sin\beta}$$

$$\frac{U_c}{\tau_s} = \cot\beta + 1 = (\xi + 1)$$

$$\left[\tan\beta = \frac{\cos\gamma_0}{\xi - \sin\gamma_0}\right]$$

(same nature as ξ)



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Qu.) $v_c = 120$ m/min
$t = 4$ mm
$S = 0.25$ mm/rev
$a_1 = 0.25$ mm
$a_2 = 0.8$ mm
$U_c = 3$ J/mm^3

$\vec{P_s}$ and $\vec{F}$ are perpendicular to one another. Find $\beta, \gamma_0, \eta, \tau_s$. The W/m fails at an elongation of 40% during tensile testing.

$\Rightarrow$ Ductile $\rightarrow \eta + \beta - \gamma_0 = \frac{\pi}{4}$

$$\zeta = \frac{a_2}{a_1} = \frac{0.8}{0.25} = 3.2$$

$$\cancel{\eta} + \beta - \gamma_0 + \frac{\pi}{2} - \cancel{\eta} = \frac{\pi}{2}$$

$$\beta = \gamma_0 \qquad \eta = \pi/4$$

$$\tan\beta = \frac{\cos\beta}{3.2 - \sin\beta} \qquad \frac{\sin\beta}{\cos\beta} = \frac{\cos\beta}{3.2 - \sin\beta}$$

$$\Rightarrow 3.2 \sin\beta = 1 \qquad \underline{\beta = \gamma_0 = 18.2^\circ}$$

$$U_c = \frac{P_z}{s \cdot t} \Rightarrow P_z = 3 \times 0.25 \times 4 \times 10^3$$
$$P_z = 3000 \text{ N}$$

$$P_z = \tau_s t s \left(\frac{\zeta}{\cos\gamma_0} - \tan\gamma_0 + 1\right)$$

$$\Rightarrow \tau_s = \frac{3000}{4 \times 0.25 \times \left(\frac{3.2}{\cos 18.2} - \tan 18.2 + 1\right)} = 742.63 \text{ MPa}$$

#. • Ductile W/M : L & S : $\left\{\beta + \eta - \gamma_0 = \frac{\pi}{4}\right\}$

• Brittle W/m : E & M : $\left\{2\beta + \eta - \gamma_0 = \frac{\pi}{2}\right\}$

③ $\rightarrow$ Semiductile W/m $\rightarrow$ can be applied to all types of W/m.

P.T.O.

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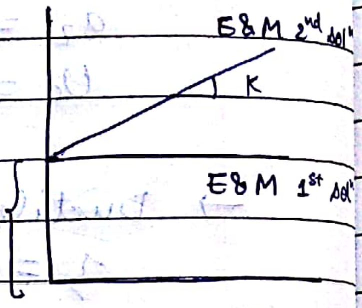
→ Ernst & Merchants 2nd Solⁿ

→ E & M 1st solⁿ assumes $\tau_s \neq f(\sigma_n)$
In 2nd solⁿ, it is assumed that :

$$\boxed{\tau_s = \tau_0 + K\sigma_n}$$



From Merchants circle diagram,

$$\tan(\beta+\eta-\gamma_0) = \frac{P_n}{P_s} = \frac{\sigma_n.A_s}{\tau_s.A_s} = \frac{\sigma_n}{\tau_s}$$

$$\Rightarrow \sigma_n = \tau_s \tan(\beta+\eta-\gamma_0)$$

$$\Rightarrow \tau_s = \tau_0 + K\sigma_n$$
$$= \tau_0 + K(\tau_s \tan(\beta+\eta-\gamma_0))$$

$$\Rightarrow \tau_s\{1 - K\tan(\beta+\eta-\gamma_0)\} = \tau_0$$

$$\Rightarrow \boxed{\tau_s = \frac{\tau_0}{\{1 - K\tan(\beta+\eta-\gamma_0)\}}}$$

Now, $P_z = \frac{\tau_s t s}{\sin\beta} \frac{\cos(\eta-\gamma_0)}{\cos(\eta+\beta-\gamma_0)}$

From MCD :
$$P_z = \frac{\tau_0 t s \cdot \cos(\eta-\gamma_0)}{\sin\beta \cos(\eta+\beta-\gamma_0)\{1-K\tan(\beta+\eta-\gamma_0)\}}$$

$$\frac{\partial(P_z V_c)}{\partial\beta} = 0 \Rightarrow \boxed{2\beta+\eta-\gamma_0 = C = \cot^{-1}K}$$

m/cing constant.

As normal ~~solⁿ~~ stress acting on the shear plane varies, τ_s varies (In 2nd solⁿ)

P.T.O.

$\rightarrow \cot^{-1}(0) = \frac{\pi}{2} = k \longrightarrow 1^{st}$ Sol^n.

Qu.3 (MTM qu. bank)

$F = 500\,N$ Semi ductile w/m ; $C = 78^\circ$

$\nu_o = 0^\circ$ $S = 0.25$ mm/rev.

$t = 4$ mm $a_1 = S \sin\phi = S = 0.25$

$V_c = 2$ m/sec $a_1 = 0.25$ mm

$\phi = 90^\circ$ $\xi = \frac{a_2}{a_1} = \frac{0.5}{0.25} = 2$

$a_2 = 0.5$ mm

$\gamma_o = 0^\circ$ $MRR = V_c \times t \times S$

$= 2 \times 4 \times 10^{-3} \times 0.25 \times 10^{-3}$

$MRR = 2 \times 10^{-6}\ m^3/sec.$

$\xi = \frac{\cos(\beta - \gamma_o)}{\sin\beta} = \frac{1}{\tan\beta}$

$\tan\beta = \frac{1}{\xi} \Rightarrow \boxed{\beta = 26.565^\circ}$

$\Rightarrow 2\beta + \eta - \gamma_o = C = 78^\circ$

$\Rightarrow \eta = 78 - 2 \times 26.565$

$\Rightarrow \boxed{\eta = 24.8^\circ}$

$F = \mu N$ $N = \frac{F}{\mu} = \frac{F}{\tan\eta} = \underline{1082.099\,N}$

$P_z = R\cos(\eta - \gamma_o)$

$F = R\sin\eta$

$\frac{P_z}{F} = \frac{\cos(\eta - \gamma_o)}{\sin\eta} \Rightarrow P_z = \frac{F}{\tan\eta}$

$\Rightarrow \underline{P_z = 1082.099\,N}$

$P_{xy} = P_z \tan(\eta - \gamma_o)$

$\underline{P_{xy} = 500\,N}$

$P_s = \frac{P_z \cos(\eta + \beta - \gamma_o)}{\cos(\eta - \gamma_o)}$ $\Rightarrow \underline{P_s = 743.68\,N}$

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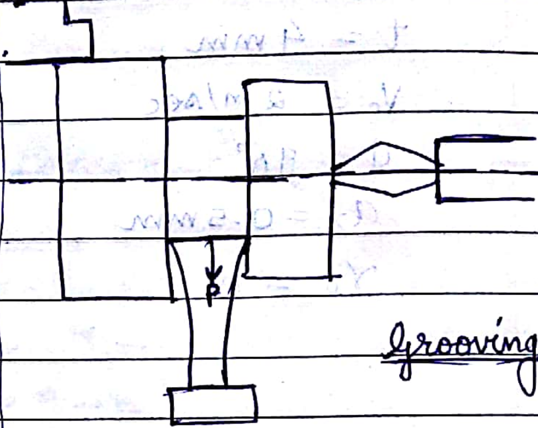
$\therefore P_s = \dfrac{\tau_s t s}{\sin\beta} \Rightarrow \tau_s = \dfrac{P_s \sin\beta}{t.s}$

$\Rightarrow \underline{\tau_s = 333\ MPa}$

Qu.) Here P is P_y as it is along the Y direction.



Grooving

$V_c = 90\ m/min.$
$S = 0.05\ mm$
$t = 5mm$
$\gamma_o = +20^\circ$
$\beta = 30^\circ$
W/m : Low C steel.
how much is P ?
$\tau_s = 700\ MPa$.

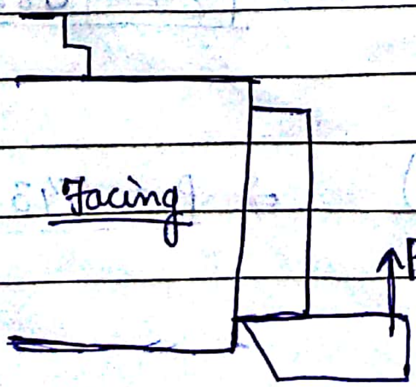
→ Ductile : $\beta + \eta - \gamma_o = 45^\circ$
$\Rightarrow \underline{\eta = 35^\circ}$

$P_{xy} = \tau_s . t . s . \{\cot\beta - 1\} = \underline{128.1\ N}$

$P_x = P_{xy} \sin\phi =$
$P_y = P_{xy} \cos\phi = 0^\circ$ which is not possible,

so as $\phi = 90^\circ$, the P force is P_{xy} force as the orthogonal plane is along the crossfeed direction. Thus P is called the feed force,
thus $P_{xy} =$.

Qu.)



Facing

$\tau_s = 600\ MPa$ $\beta = 25^\circ$
$V_c = 90\ m/min$ $C = 1.2$ radian
$S = 0.05\ mm$
$t = 5\ mm$ ~~Hoss~~
$\gamma_o = +5^\circ$

How much is P ?

↳ Here, P is the feed force, so $\boxed{P = P_{xy}\sin\phi}$ and if P is the thrust force, it would be $P_{xy}\cos\phi$.

Imp. ↳ From m/c tool point of view, P is the thrust force; but from machining point of view, it will be feed force.

$C = 1.2$ radian $= 68.75°$

$2\beta + \eta - \gamma_0 = 68.75°$

$\Rightarrow \eta = 23.8°$

$P_{xy} = \tau_s . t . s . (\cot\beta - 1) \rightarrow$ Why this formula can't be used?

$$P_s = \frac{\tau_s t s}{\sin\beta} = \frac{600\times10^6 \times 5\times0.05\times10^{-6}}{\sin 25}$$

$P_s = 355$ MPa

$$P_{xy} = \frac{P_s \sin(\eta - \gamma_0)}{\cos(\beta + \eta - \gamma_0)} = 158 \text{ N} \rightarrow \text{feed force}$$

$= P$

$= P_{xy}\sin\phi$

Qu.1) $F = 500$ N

$\gamma_0 = 0°$

$R = 1300$ N

$S = 0.25$ mm/rev

$t = 4$ mm

$V_c = 2$ m/s

$\phi = 90°$

$a_2 = 0.5$ mm

MRR ? β ? P_z ? P_N ?

P_{xy} ? P_s ? τ_s ?

E_c ? U_c ?

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Module-6

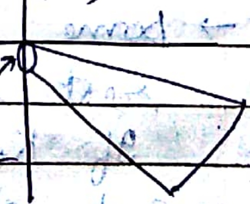
Temperature in Machining

Temperature in machining

→ W/p temp. after machining : $\theta_{w/p}$
→ chip temperature : θ_{chip}
→ shear plane temperature : θ_s
→ chip tool interface temp. : θ_c

This is characterized / noted as the cutting or machining temp.

⊛ If they are high
1) $\theta_{w/p}$ – M/cing thin w/p ($\theta_{w/p} \approx 200°C$ ← high)
II) θ_{chip}
III) θ_s → If high → $\tau_s \downarrow$
IV) θ_c → affect chip-tool interac^n (P.T.O.)



$P_s V_s + F V_f \approx P_z V_c$
→ can be proved using Merchant's circle diag.

1) → Basic purpose of m/cing – size & shape tolerances : (If you have high temp, tolerance would be poor due to thermal expans^n)

II) Chip is a waste material, if θ_{chip} is higher all other temp. would also be higher; it is not independent. It does not have any effect in general.

III) $\theta_c \uparrow \rightarrow \tau_s \downarrow$ forces $\downarrow$

Mat. which find applications in high strength intentionally θ_c is increased (process called hot machining).

- → gas torches
- → plasma torches
- → laser aided machine

} Temp ↑↑, material would deform easily.

We cannot comment on the energy efficiency of this process.

IV) If $\theta_c \uparrow$ → affects chip-tool interaction → thermally driven tool wear process. Tool wear would go up & Tool life ↓↓.
Chemical phenomenon – source is friction chances of attachement & detachment.

→ Some amount of heat enters the W/p & rest is given to chip. Chip is moving.
If K_t of tool is high, more heat would enter the tool (More in HSS less in Alumina)
① Whatever heat is absorbed is conducted throughout (K_t should be high)
② Very poor conductor which doesn't allow the heat to enter at all. (K_t very low)

→ Same way, energy entering the chip depends on its K_t. (Eg. Titanium $K_t = 7$ (very less))

→ If K_t of both ~~in~~ mat. is low, heat would be concenterated in the shear zone which will increase the interface temp.

$$\left[\underbrace{\underbrace{\underbrace{\rho\, V_c\, t\, s}_{MRR}}_{MRR(kg/s)}\, C_p}_{\text{Energy Rate}}\, \theta_s\right] = \eta\, P_s V_s\, (1-\Gamma)$$

θ_s → rise in shear plane temp.

↓

how much energy you require to raise temp. θ_s in the shear plane as particle passes by this heat is proportioned into tool & chip.

Γ → % or fraction of heat entering the w/p. from the primary shear zone

η → fraction of mechanical work '$P_s V_s$' converted to thermal energy ≈ 95%.

~~So~~ When mat. is deformed, plastic deformaⁿ are Introduced which brings up strain energy which remains locked inside the material ∴ whole energy not converted (Strain energy not released as thermal energy)

→ If β ↑↑ higher heat given to chip.

Qu) $c = 85°$ $\quad S = 0.25$ mm/rev $\quad C_p = 500$ J/kg-K

$a_1 = 0.25$ mm $\quad \rho = 7800$ kg/m³

$t = 4$ mm $\quad K = 50$ W-m/K

$U_c = 1.5$ GJ/m³

$a_2 = 0.7$ mm $\quad$ Estimate θ_s

$\gamma_0 = -10°$ $\quad \eta = 95\%$

10% of the heat enters the W/p from primary shear zone.

Sol^n $\zeta = \frac{a_2}{a_1} = \frac{0.7}{0.25} = 2.8$

$V_C = \frac{P_z}{s.t} \Rightarrow P_z = 1500\ N$

Ra $\tan\beta = \frac{\cos\gamma_0}{\zeta - \sin\gamma_0} \Rightarrow \underline{\beta = 18.32^\circ}$

$2\beta + \eta - \gamma_0 = C = 85^\circ$ ← semi ductile w/m.

$\Rightarrow \underline{\eta = 38.36^\circ}$

$P_s = P_z \cdot \frac{\cos(\eta + \beta - \gamma_0)}{\cos(\eta - \gamma_0)}$

$\Rightarrow \underline{P_s = 393.67\ N}$

$\theta_s = \frac{\eta P_s V_s (1 - \Gamma)}{\rho V_c t s\ C_p}$

$\frac{V_s}{V_c} = \frac{\cos\gamma_0}{\cos(\beta - \gamma_0)} = \underline{1.118}$

$\theta_s = \frac{0.95 \times 393.67 \times 1.118\ (1 - 0.1)}{7200 \times 4 \times 10^{-3} \times 0.25 \times 10^{-3} \times 500}$

$\underline{\theta_s = 219\ K}$

↳ rise in temp.

Absolute shear plane temp. = 219°C + 30 = 249°C
↓ inlet temp

→ When you m/c Ti, frac^n of energy entering w/p would be very less, ∴ θ_s would be quite high (∵ $\Gamma \approx 2$ to 3%)

Effect of process parameters on θ_s

$\theta_s \sim \frac{P_s V_s}{st} \longrightarrow$ heat flux

$\theta_c \sim$ heat flux $\sim \dfrac{F V_f}{b C_T} \sim \left(\dfrac{P_z V_c}{b C_T}\right)$

→ If $\left(\dfrac{P_z V_c}{b C_T}\right) \uparrow \Rightarrow \theta_c \uparrow$ (Temp. $\propto$ Heat flux)

→ If $t \uparrow\uparrow \Rightarrow P_z \uparrow\uparrow$, $b \uparrow\uparrow$

$\therefore \left(\dfrac{P_z V_c}{b C_T}\right)$ remains invaried

Thus $\theta_c \neq f(t)$ ← Cutting temp. is a weak funcn of 't'.

→ If $S \uparrow\uparrow \rightarrow P_z \uparrow$ (inc. but not proportionately)

$\rightarrow C_T = \{ a_1 (1 + \tan(\beta - \gamma_0)) \}$ inc. by small amount

Here length of arrow is also imp.

$\therefore \left(\dfrac{\uparrow P_z V_c}{b C_T \uparrow}\right) \uparrow \Rightarrow \theta_c \uparrow$ by small amount.

→ $V_c \uparrow \rightarrow P_z \downarrow$ (dec. by small amount)

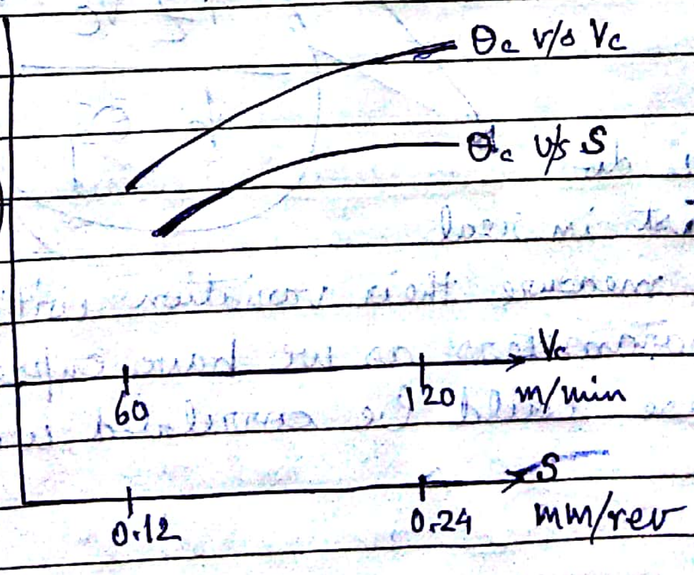
$C_T \downarrow$ (" " " ")

$\therefore \left(\dfrac{P_z^{\downarrow} V_c \uparrow}{b C_T^{\downarrow}}\right) \uparrow$ by large amount

$\Rightarrow V_c \uparrow\uparrow \Rightarrow \theta_c \uparrow\uparrow$ by large amount.

→

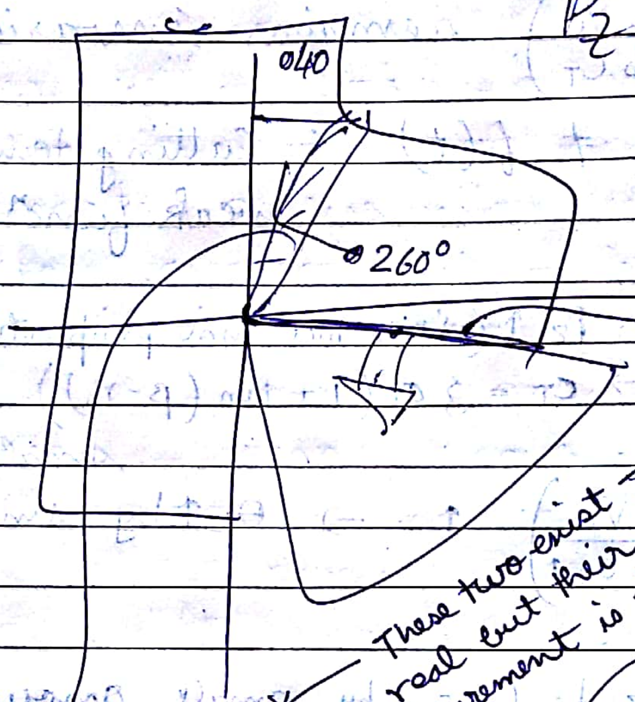
$\left(\dfrac{d\theta_c}{dV_c} > \dfrac{d\theta_c}{dS}\right)$



$\left(\frac{P_Z V_C}{b C_T}\right)$ is artificial heat flux (does not exist in machining.

$\left(\frac{F \times V_f}{b C_T}\right) \to$ Actual heat flux.

$$P_Z V_C = P_S V_S + F V_f$$



$$\frac{F V_f}{b C_T}$$

These two exist in real but their measurement is tough as we don't have any expressn relating them to the process parameters

$$\frac{P_S V_S}{b\, a_1/\sin\beta}$$

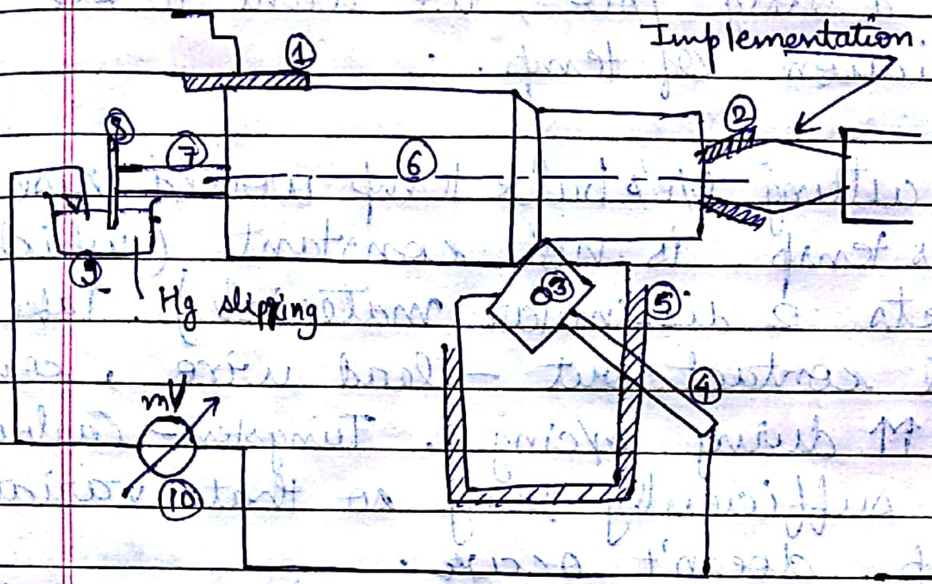
$$\frac{P_Z V_C}{t s} \to X$$

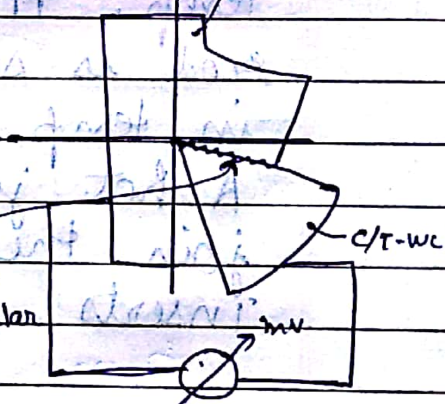
$$\frac{P_Z V_C}{b C_T}$$

These 2 do not exist in real but we measure their variation with change in process parameters as we have expressions for same. Also these could be correlated with real forces.

Temperature Measurement

a) Tool-work thermo-couple technique





- 1,2,5 → electrical insula^n
- 3 → WC cutting tool insert
- 4 – WC rod
- 6 – W/P
- 7 – extension of W/P
- 8 – slipring / disc
- 9 – Hg bath
- 10 – mV meter → milli-volt

WC → Tungsten Carbide

→ 2 dissimilar material – hot junc^n → Thermo EMF is generated : ① Both are electrically conductive. ② Thermo-EMF is generated (if lesser – difficult to separate). If you m/c LCS, HSS, chance of genera^n of Thermo-EMF would be less. Therotically, it should work for others. For LCS with C/T – alumina – won't work (∵ Alumina is not electrically conductive).

→ **Advantages** :-

① We can safely monitor ~~ing~~ the temperature. M/cing a long job, we want to see the variation of temp.

During cutting C/T's bulk temp would rise & junction's temp is not constant (which connects 2 dissimilar material). Take electrical contact out — lead wire, connecⁿ temp. ↑↑ during m/cing. Tungsten – Carbide rod is sufficiently long so that variation in temp. doesn't occur.

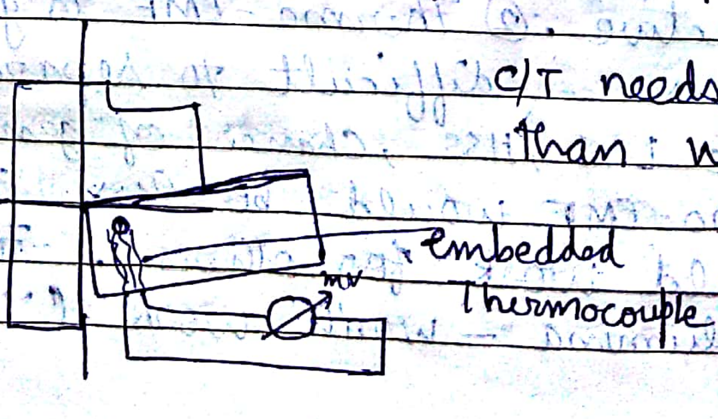
A hot junction where T is not constant join the dissimilar mat → Tungsten Carbide inserts. Fasten another rod of same mat.

→ Slip ring in Hg bath → from there take the 2nd lead out → measure the thermo emf → we need to know the calibraⁿ factor. When insert is engaged, T is ambient while juncⁿ T will change.

mV to Temp

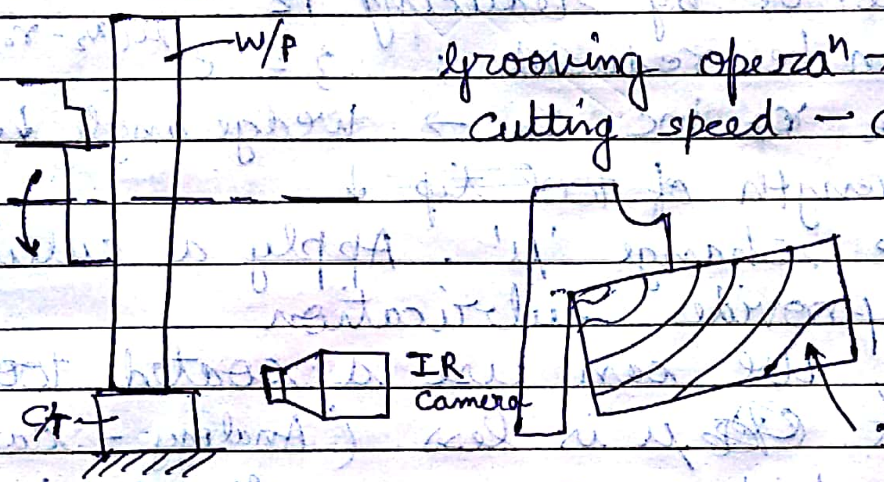
b) Embedded thermocouple technique

C/T needs to be harder than W/P.



No need to know calibration constant. You will not measure chip-tool interface temp. At particular location inside the tool, temp. would be measured. For comparitive method, this is good. No need for W/P and C/T to be dissimilar & electrically conductive. (Not for Alumina - hole not drilled).

use coolant

c) 

grooving operaⁿ → dia ↓↓
cutting speed - constant (CNC)

→ coolant can obstruct the view of IR camera
∴ Only possible for dry machining
→ Academic insti only.
→ Emissivity of W/P & C/T needs to be kept in mind; calibration adjusted for diff. material.
→ If the standoff distance change, intensity of radiaⁿ received by IR sensor changes.

Control of Temperature (Cutting Temp.)

↳ If W/P has less thermal inertia, so tolerances would change.

→ Tool wear (Wear ↑↑ as θ_c↑↑, life ↓↓)

→

$\theta_s \sim \frac{P_s V_c}{ts}$ $\theta_c \sim \frac{P_z V_c}{bC_T}$

$$\boxed{\theta_s = \frac{\eta(1-\Gamma) P_s V_s}{C_p V_c t s \rho}}$$

→ If $V_c \downarrow\downarrow$ MRR $\downarrow\downarrow$ → fire

→ V_c, t, s ~~or~~ cannot be changed (∵ they would change the MRR)

$$P_z = \tau_s t s \left\{ \frac{\xi}{\cos\gamma_0} - \tan\gamma_0 + 1 \right\}$$

→ Reduce θ_s by reducing P_z

ξ can be reduced ; $\xi = e^{\mu(\pi/2 - \gamma_0)}$

→ If we ~~γ₀~~ inc. γ_0 → wedge angle $\downarrow\downarrow$ → strength of tool tip $\downarrow$

→ ∴ We change 'μ'. Apply a cutting fluid that provides lubrication

→ Also, we can use a coated tool such that ~~Coe~~ μ is less. (Analogy - carrom)
(Boric Acid in carrom - solid lubricant)

↳ TiN (it brings down μ)

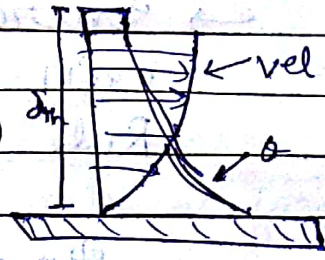
↳ MoS_2, WS_2 (same structure as graphite)
(graphite - slipping b/w 2 hexagonal planes)

Adhesion of the rear side of chip with the tool, μ is bought down → less energy to break → $\mu \downarrow\downarrow$

→ Cutting fluid also acts as a coolant remove heat from cutting zone.

→ Coolant having higher C_p value so its own Temp. inc. by lesser amount. That coolant would be favourable for cooling than the one having less C_p value.

cooling action
↳ forced convection
  ↳ K_{fe}
  ↳ C_p ↑



At base → conduction (∵ no slip)
↳ conductivity of fluid is imp.

$h_f \propto \frac{K_{fl} \leftarrow \text{more}}{\delta_{th} \leftarrow \text{less}}$

δ_{th} depends upon δ_v which depends upon velocity of coolant, ∴ use nozzle TO direct the coolant on W/P → vel ↑↑ at High pressure technology
→ Water is not used as coolant (will cause corrosion) → ∴ oil is used. (straight oil)
(20% water) → soluble oil → lubrication + cooling
(100% oil) → Straight oil → lubricates better

→ If thermal load is less, means $P_z V_c$ is less, then it requires lubrication, no such heat is developed, only straight oil can be used.

→ If tool wear is high, low tool life → do not spend so much of energy → reduce P_z by lubrication → cutting fluid that lubricates
Use coolant that reduce chip tool interface Temp. → ↑↑ velocity of the coolant by high pressure technology.

Can I change the K of fluid?
→ nano fluids (mix and 0.6% nano particles) → K goes up, particulate

matter is there.

→ Rule of mixture is satisfied.

→ Increase in thermal conductivity is more than normal mixture.

Rule of mixture →

$$\phi = x_1 \phi_1 + (1 - x_1)\phi_2$$

$\phi_1 \rightarrow$ material property

$x_1 \rightarrow$ mole fraction.

~~Tool wear & Tool life~~

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Tool Wear & Tool Life

→ The cutting tool gradually wears out (becomes blunt) due to
(i) sliding of the chip over the rake surface
(ii) Rubbing of finished w/p against the principal and aux. flank surface.

→ Further catastrophic failure of the C/T may as well take place particularly in interrupted m/cing operations like force milling. (Inherent defects in C/T)

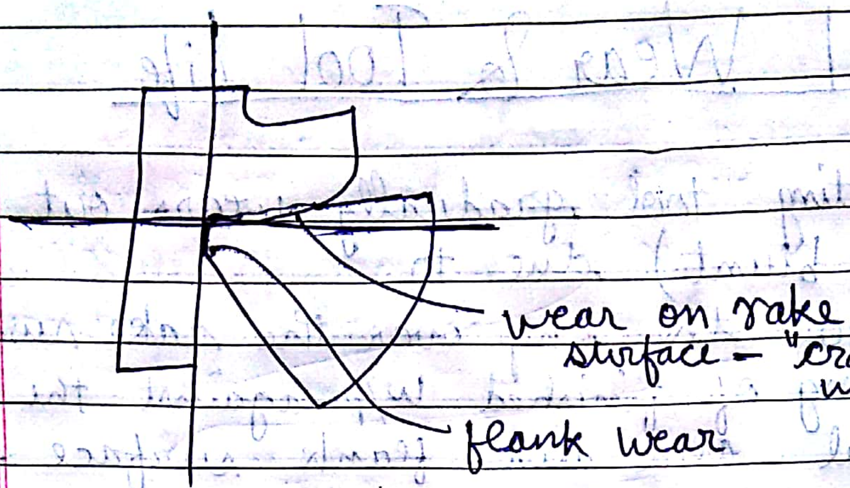
→ A blunt C/T is either to be replaced with a new C/T or resharpened.
$\boxed{\text{Time duration}}$* b/w resharpening or $\boxed{\text{time duration}}$ for which a C/T can be used without resharpening is Tool Life (TL or T)

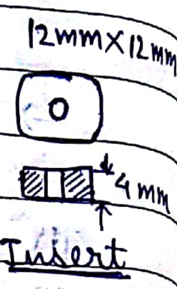
(*) → actual time duration

→ If you have spline shaft — interrupted cutting — catas. failure occur.
→ blunt C/T → Force & Temp. requirement is high ⇒ wear out quickly (cycle)

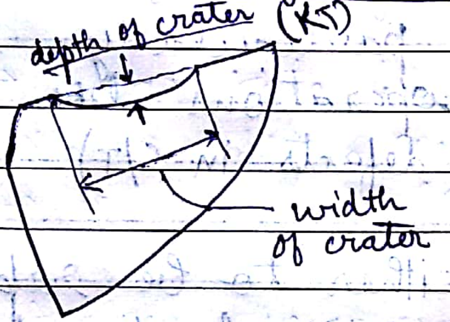
→ If you use C/T for 30 min ← TL

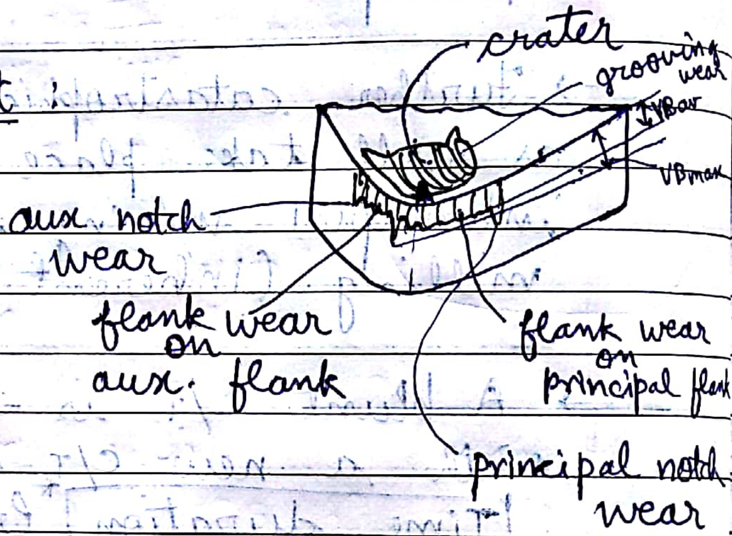
→ Product quality depends upon surface finish & tolerance achieved.





→ 3D view of insert :





crater wear does not start at cutting edge (some what away) ; depth is not constant and is max. at centre of crater. θ_c → avg. chip-tool interface Temp. – we measure avg. temp. ↘ it varies — max somewhat away from tip → ∴ Temp. driven wear will also be max. at that location.

Most tools fail as they reach limiting point of tool wear.

→ Tool-life

$VB_{av} \geq 0.3$ mm
or
$VB_{max} \geq 0.6$ mm
or
$KT \geq 0.14$ mm

} Tool has become blunt, not usable not giving expected surface finish or tolerance.

Mechanism of tool wear

flank wear – abrasive

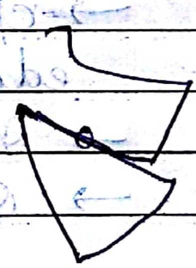
crater wear – adheso – diffusive + abrasive

groove + notch – chemical

steel – uncoated carbides – C/T bulk hardness >> W/P.

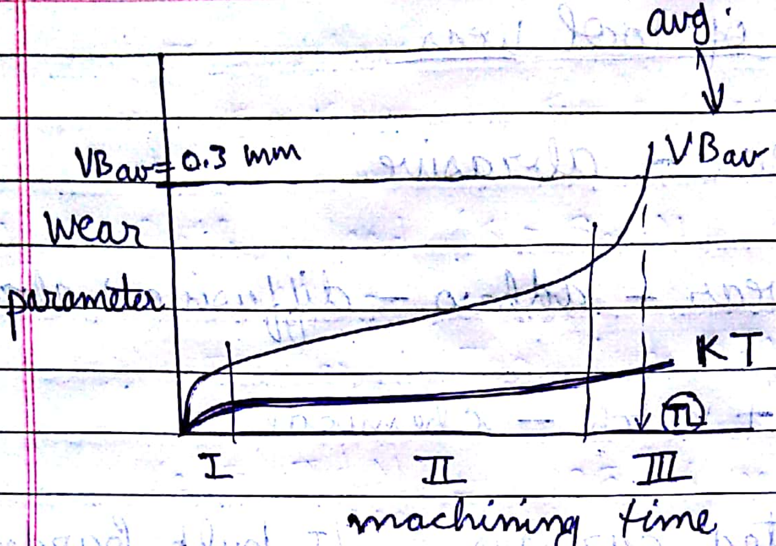
→ On microscopic scale – some carbide's hardness is same as that of W/P → abrasive. Abrasion due to W/P.

→ Adheso-diffusive → chip moving on rake surface – some microwelding will be there which breaks → some chip on crater surface, left on C/T redeposition. Adhesion followed by rupture. Temp. driven diffusion from C/T to W/P.

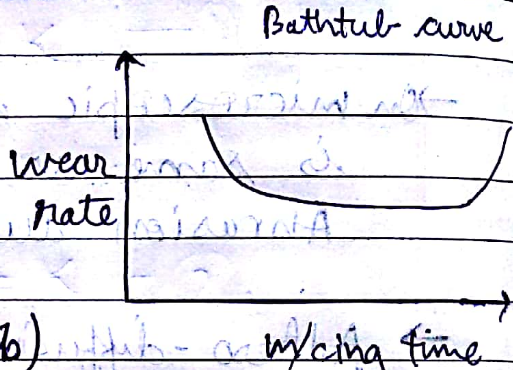


→ Chemical wears are enhanced → steep stress / Temp. gradient. At junc^n, Temp. gradient is present → chemical wear
Diamond being hardest cannot be used for m/c steel (not saturated with C)
↳ Steel will try to get C from Diamond
↳ crater well mechanism.

→ These valid for m/cing steel with uncoated carbide.



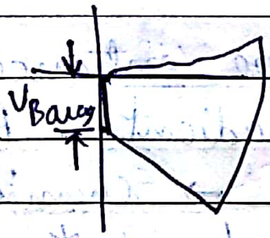
Because of improvement in crater properties



I → Wear rate ↓↓
II → " " const.
III → " " ↑↑

(Analogy – incandescent bulb)

→ C/T is sharp, some r_e is there, as edge becomes sharper, stress ↑↑ → blunt.
→ Once 150 μ → flank wear const.
→ Once 300 μ → you require higher temp. & higher forces to wear.



→ ~~TL get~~

As temp. ↑↑, all thermally driven wear ↑↑.

→ TL gets affected by θ_c
→ TL (time) gets affected by V_c, s and t.

can only be applied when t, s const.

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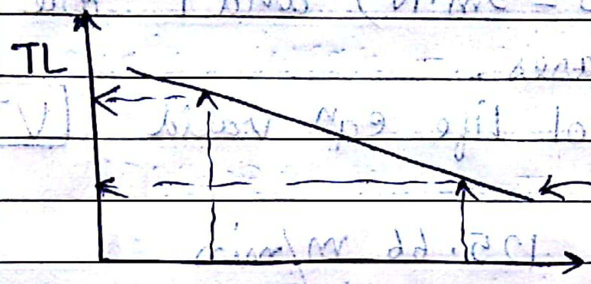
* → Taylor's Tool life eq^n → $\boxed{VT^n = K}$

V → V_c in m/min

T → TL in min



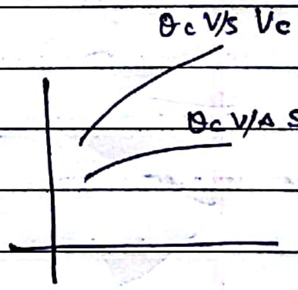
At low V_c high TL and vice-versa

Experimental curve-fitting

↳ $V_c \uparrow\uparrow \rightarrow \theta_c \uparrow\uparrow$ → Thermally driven wear mechanism ↑↑ → TL ↓↓

* → Modified Taylor's Tool life eq^n

$$\boxed{T = \frac{K_1}{V_c^{x} S^{y} t^{z}}}$$



T is strong $func^n$ of V_c
T is mild $func^n$ of S
T is weak $func^n$ of t
(based on heat flux)

→ TL of 15 min over 2 hr is profitable (life of 1 edge).

Q.) Straight turning:

i) d = 200 mm
N_1 = 200 rpm
S_m = 20 mm/min
t = 2 mm
T = 12 min

ii) n_2 = 100 rpm
S_m = 10 mm/min
t = 2 mm
T = 48 min

P.T.O.

find TL when

$\eta = 150$ rpm, $S_m = 15$ mm/min, $t = 2$mm.

Sol^n → Both feed ($s = S_m \cdot N$) and t are same in all 3 cases.

∴ Taylor's Tool life eq^n valid $\boxed{VT^n = K}$

I) $V_c = \frac{\pi d N}{1000} = 125.66$ m/min

$125.663 \times 12^n = K$ —— ①

II) $V_c = \frac{\pi d N}{1000} = 62.831$ m/min

$62.831 \times 48^n = K$ —— ②

① = ② ⇒ $\frac{125.663}{62.831} = 4^n$

⇒ $n = \frac{\log 2}{\log 4} = 0.5$ $\boxed{n = 0.5}$

⇒ $\boxed{K = 435.309}$

III) ∴ for $V_c = \frac{\pi d N}{1000} = 94.2477$

∴ $T = \left(\frac{435.309}{94.2477}\right)^2 = 21.33$ min.

Qu) $VT^{0.5} = 435$ —— Ⓐ

$VT^{1.5} = 4350$ —— Ⓑ

When should I use which one?

Ⓑ ÷ Ⓐ ⇒

$B \div A \Rightarrow T = 10$

$V\,10^{1.5} = 4350$ $\quad V = 137.5$

$V\,10^{0.5} = 435$ $\quad V = 137.5$

velocity at which transition taking place

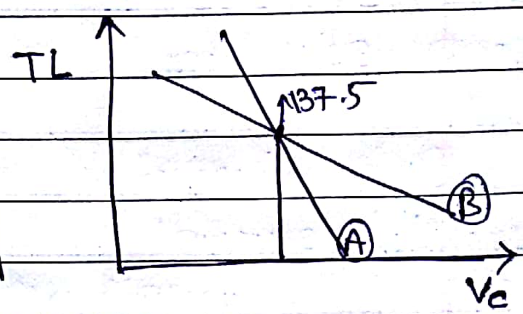
$VT^n = K$

$\cancel{dV}\; dV \cdot T^n + nT^{n-1}\,dT \cdot V = 0$

$\frac{dT}{dV} = \frac{T}{nV}$

$\frac{1}{0.5}$ $\quad \frac{1}{1.5}$

(A) 2 $\quad$ (B) 0.67



Eg. TL_A, $V_c = 150$ = 8.41

TL_B, $V_c = 150$ = 9.54 ✓

$V < (V_c)_{trans}$ → A would be used (TL more)

$V > (V_c)_{trans}$ → B would be used (TL more)