(c) Check for Keyways:

- critical location: gear (both bending & torsion are present)

$M = 240$ Nm $\quad T = 540$ Nm

$D = 60$ mm

$K_f = 5, \quad K_{fs} = 3$

factors: $k_a = 0.883$

$k_b = 1.51 d^{-0.157} = 0.794$

$S_e = 164.1$ MPa

stresses:

	alternating	midrange
Bending:	$\frac{32M}{\pi D^3} = 11.32$ MPa	0
Axial:	0	0
Torsion:	0	$\frac{16T}{\pi d^3} = 12.73$ MPa

$\sigma_a' = K_f \sigma_{bending} = 56.6$ MPa

$\sigma_m' = \sqrt{3} K_{fs} \times 12.73 = 66.15$ MPa

~~Check~~ $\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{56.6}{163.1} + \frac{66.15}{470}$

$\Rightarrow \boxed{n_f = 2.06}$

$n_y = \frac{S_y}{\sigma_a' + \sigma_m'} = \frac{390}{56.6 + 66.15} = \underline{3.18}$

Recheck: Factors: $K_a = 0.883$ (unchanged)

$K_b = 1.24 d^{-0.107}$

$= 0.816$

$K_c = K_d = K_e = K_f = 1.$

$S_e = \frac{0.816}{0.9} \times 186 = 168.64$ MPa.

Notch sensitivity $q = 0.75$ ($r = 2.5$ mm)

$q_{shear} = 0.8$

$K_f = 1.75$ (axial)

$K_f = 1.71$ (bending)

$K_{fs} = 1.48$ (shear)

Stresses:

	alternating	midrange
Bending:	$\frac{32M}{\pi d^3} = 23.22$ MPa	0
Axial:	0	$\frac{4F}{\pi d^2} = 11.4$ MPa
Torsion:	0	22 MPa

$\sigma_a' = K_f \sigma_{ben} = 39.7$ MPa

$\sigma_m' = \left((11.4 \times 1.75)^2 + 3 \times (1.48 \times 22)^2 \right)^{1/2} = 59.8$ MPa

Factor of safety: $\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$

$n_f = \left(\frac{39.7}{\cancel{390}\ 168.64} + \frac{59.8}{470} \right)^{-1}$

$\Rightarrow n_f = 2.75$

Yielding: $n_y = \frac{S_y}{(\sigma_a' + \sigma_m')} = 3.92$ $\Big\}$ > 2.5 (safe) stresses:

	alternating	Mid Range
Bending:	$\frac{32M}{\pi d^3} = \frac{2903}{d^3}$	0
Axial:	0	$\frac{4F}{\pi d^2} = \frac{28520.5}{d^2}$
Torsion:	0	$\frac{16T}{\pi d^3} = \frac{2750}{d^3}$

Von mises stress:

$$\sigma_a' = \left\{ \left(K_f \sigma_{bending} + \frac{K_f \cdot \sigma_{axial}}{0.85} \right)^2 + 3(K_{fs} \tau_a)^2 \right\}^{1/2}$$

$$= K_f \cdot \sigma_{bending} = \frac{5109.3}{d^3}$$

Ignoring this

$$\sigma_m' = \left\{ \left(K_f \sigma_{bending} + \cancel{K_f \cdot \sigma_{axial}} \right)^2 + 3(K_{fs} \tau_m)^2 \right\}^{1/2}$$

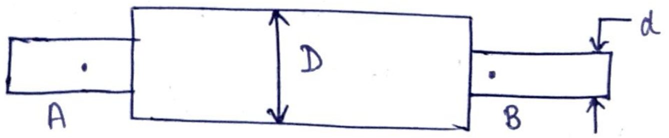
$$= \sqrt{3}\, K_{fs} \tau_m = \frac{4763.1}{d^3}$$

Modified Goodman:

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

$$\Rightarrow \frac{1}{2.5} = \frac{5109.3}{d^3 \times 186 \times 10^6} + \frac{4763.1}{d^3 \times 470 \times 10^6}$$

$$\Rightarrow d = 45.4\ mm$$



$D = 1.2d = 54.48\ mm$

choose

$$\boxed{\begin{array}{l} D = 60\ mm \ \ (\text{standard size}) \\ d = 50\ mm \\ r = 0.05d = 2.5\ mm \end{array}}$$

Part (b): <u>Critical location:</u>

<u>Right of B:</u> $M = 285\ Nm$

$T = 540\ N\text{-}m$

$F_{axial} = 22.4\ KN$

Factor of safety = 2.5

<u>Static SCF:</u> $\frac{D}{d} = 1.2, \quad \frac{r}{d} = 0.05$

<u>Shaft material</u>

1020 CD $\Rightarrow$ $\boxed{S_{ut} = 470\ MPa \quad S_y = 390\ MPa}$

$K_t = 2.0$ (fig A-15-7)

(~~Bending~~ (Axial)

$K_{ts} = 1.6$ (torsion) (fig A-15-8)

$K_t = 1.95$ (Bending) (fig A-15-9)

<u>Notch sensitivity factor:</u>

Fillet radius is unknown.

q, q_{shear} can not yet be determined

choose max. values

$q = 0.8, \quad q_{shear} = 0.85$

$K_f = 1 + q(K_t - 1) = 1.8$ (axial)

$K_{fs} = 1 + q_{shear}(K_{ts} - 1) = 1.51$ (torsion)

$K_f = 1.76$ (bending)

<u>Factors:</u>

$\boxed{S_e = K_a K_b K_c K_d K_e K_f S_e'}$

$S_e' = 0.5\,S_{ut}$

$K_a = a\,S_{ut}^{\,b} = 4.51\,S_{ut}^{-0.265} = 0.883$

$K_b = 0.9$ d is unknown (choose a starting value)

$K_c = K_d = K_e = K_f = 1.$

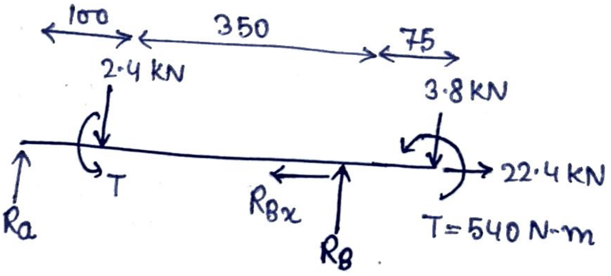
$\Rightarrow S_e = (0.883)(0.9)(0.5 \times 470) = 186\ MPa.$

$$P.E. = \sum W_i y_i$$

$$K.E. = \frac{1}{2} \sum \frac{W_i}{g} \omega^2 y_i^2$$

Problem → (from tutorial)

Part (a)



T is torsion force.

$T = 540$ N-m

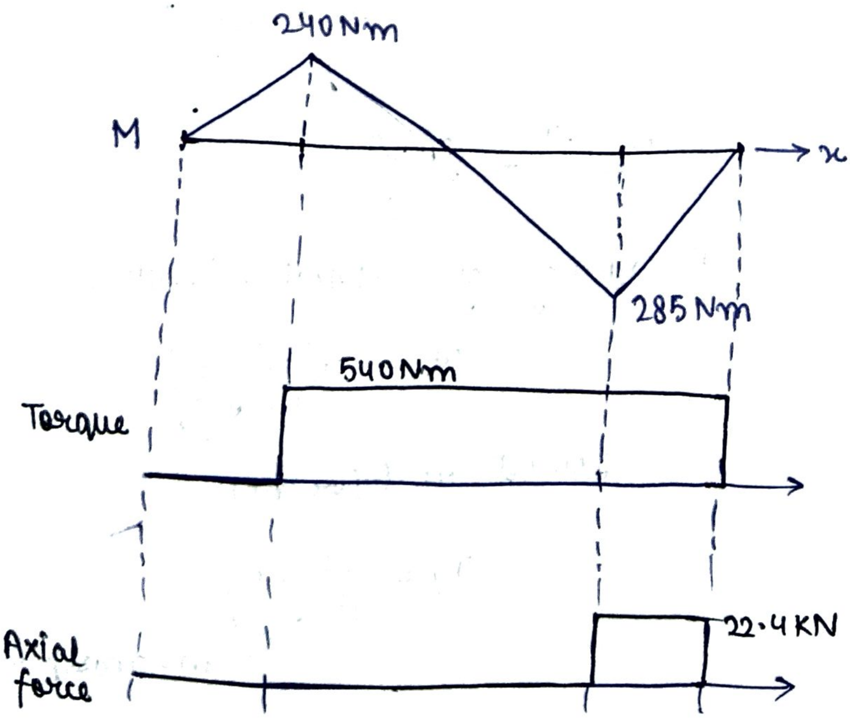
$R_{B_x} = 22.4$ KN

$R_A + R_B = 6.2$ KN

$\sum M_A = 0$: $R_B \times 0.45 - 2.4 \times 0.1 - 3.8 \times 0.525 = 0$

$\Rightarrow R_B = 4.967$ KN, $R_A = 1.233$ KN

$$\Rightarrow \boxed{\begin{array}{l} R_A = 1.233 \text{ KN} \\ R_B = 4.967 \text{ KN} \\ R_{Bx} = 22.4 \text{ KN} \\ T = 540 \text{ Nm} \end{array}}$$



$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^3 w}{\partial x^3} = -p$$

For free vibration, $p = 0$

$$\Rightarrow \rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^3 w}{\partial x^3} = 0.$$

$$w(0) = \frac{\partial^2 w}{\partial x^2}(0) = 0$$

$$w(l) = \frac{\partial^2 w}{\partial x^2}(l) = 0$$

(Simply supported)

Let $W(x,t) = e^{i\omega t} X(x)$

$$\Rightarrow -\rho A \omega^2 X + EI X''' = 0$$

$$\Rightarrow X''' - \left(\frac{\rho A \omega^2}{EI}\right) X = 0$$

Let $X = A \sin(\beta x)$ satisfies BC

$$\Rightarrow \beta^4 - \left(\frac{\rho A \omega^2}{EI}\right) = 0.$$

$$\Rightarrow \boxed{\beta^4 = \frac{\rho A \omega^2}{EI}}$$

apply BC @ l

$$w''(l) = w(l) = 0 \Rightarrow \sin(\beta l) = 0$$

$$\Rightarrow \boxed{\beta = \frac{n\pi}{l}}$$

$$\left(\frac{n\pi}{l}\right)^4 = \frac{\rho A \omega^2}{EI}$$

Frequency of natural vibration

$$\omega_n = \frac{n^2 \pi^2}{l^2} \sqrt{\frac{EI}{\rho A}}$$

Lowest ω (for $n=1$)

$$\omega_1 = \frac{\pi^2}{l^2} \sqrt{\frac{EI}{\rho A}}$$

m = mass per unit length

$$M = \frac{EI}{\rho}$$

Force Balance

$$(\rho A\, dx)\frac{\partial^2 w}{\partial t^2} = (V + dV) - V - p(x,t)\,dx$$

$$= dV - p(x,t)\,dx$$

$$\Rightarrow \boxed{\rho A \frac{\partial^2 w}{\partial t^2} = \frac{dV}{dx} - p} \rightarrow (1)$$

Moment Balance

$$(\rho I\, dx)\frac{\partial^2 \psi}{\partial t^2} = (M + dM) - M + (V + dV)\,dx - p\,dx \cdot \frac{dx}{2}$$

$$= dM + V\,dx \quad \text{(ignoring lower order terms)}$$

$$\Rightarrow \boxed{\rho I \frac{\partial^2 \psi}{\partial t^2} = \frac{dM}{dx} + V}$$

Rotary Inertia

Ignore rotary inertia, $V + \frac{dM}{dx} = 0$

$$\Rightarrow \boxed{V = -\frac{dM}{dx}} \rightarrow (2)$$

Combine (1) and (2)

$$\rho A \frac{\partial^2 w}{\partial t^2} = -\frac{dM}{dx} - p$$

or $$\boxed{\rho A \frac{\partial^2 w}{\partial t^2} + \frac{dM}{dx} = -p}$$

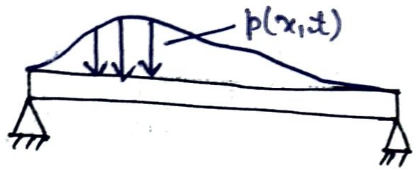
$$M = \frac{EI}{\rho} = EI \frac{\frac{\partial^2 w}{\partial x^2}}{\left(1 + \left(\frac{\partial w}{\partial x}\right)^2\right)^{3/2}} \approx EI \frac{\partial^2 w}{\partial x^2}$$

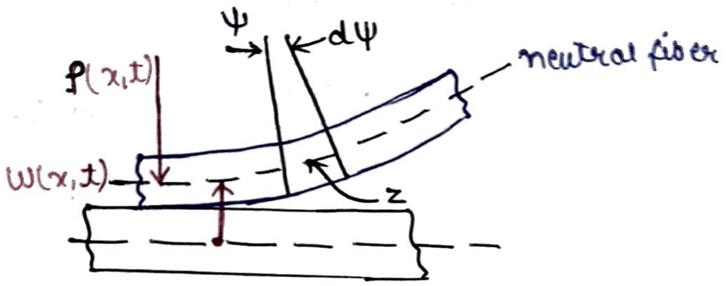
assume, $\frac{\partial w}{\partial x} \ll 1.$

Shafts

→ Introduction from the slides.

→ Design to be done in a similar manner as done in example (before mid-sem).





$\rho(x,t)$ = radius of curvature of deformed neutral axis

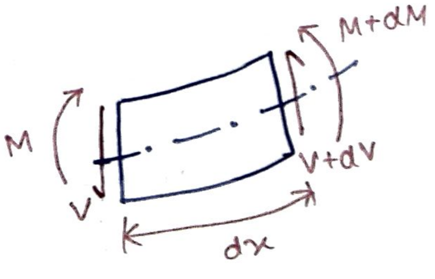
$w(x,t)$ = deflection along neutral fiber.

Strain: $\varepsilon_{xx}(x,z,t) = \dfrac{(\rho(x,t)-z)\,d\psi - \rho(x,t)\,d\psi}{\rho(x,t)\,d\psi}$

$$= -\frac{z}{\rho}$$

$\Rightarrow$ stress $= \sigma_{x} = E\epsilon_{xx} = -\dfrac{Ez}{\rho}$

Bending Moment



$$M = -\int_A \sigma\, z\, dA$$

$$= \int E\frac{z}{\rho}\, z\, dA$$

$$= \frac{E}{\rho}\int_A z^2\, dA = \frac{EI}{\rho}$$

$$F_e = VF_r\left(0.56 + 1.93 \times \frac{F_a}{VF_r}\right)$$

$$= 5.3\ kN = F_D.$$

$$\Rightarrow C_{10} = 110.2\ kN > 106\ kN$$

<u>Not satisfied</u>

Now, select bearing with bore dia 95 mm

$$\frac{F_a}{C_o} = \frac{2.47}{85} = 0.029$$

$$\Rightarrow e = 0.221,\ X_2 = 0.56$$

$$Y_2 = 1.98$$

$$\Rightarrow F_e = 5.425\ kN$$

$$\Rightarrow C_{10} = 112.8\ kN < 121\ kN$$

<u>Satisfied</u>

<u>Bearing at A:</u> 02-series, 95 mm bore dia Angular contact

<u>Bearing at B:</u> $R_D = 1$, straight roller

$$F_r = F_D = \sqrt{(0.16^2 + 0.3^2)} = 0.34\ kN$$

$$C_{10} = a_f F_D \left(\frac{x_D}{0.02}\right)^{1/a}$$ ($a = 10/3$ for roller bearing)

$C_{10} = 11\ kN$

See bearing from table 11-3.

Choose bore dia = 25 mm (02-series)

$$\Rightarrow \boxed{C_{10} = 16.8\ kN,\ C_o = 8.8\ kN}$$

or choose same bore dia as bearing at A

ie 95 mm $\Rightarrow C_{10} = 165\ kN,\ C_o = 112\ kN.$

$$\frac{F_a}{VF_r} = \frac{2.47}{0.954} = 2.59$$

Need to know: e from table 11-1.

→ Choose a starting value of $\frac{F_a}{C_0}$ at somewhere in the middle of table 11-1.

Let $\frac{F_a}{C_0} = 0.07 \Rightarrow e = 0.27 < \frac{F_a}{VF_r}$

$X = X_2 = 0.56, \quad Y = Y_2 = 1.63$

$$\Rightarrow F_e = VF_r\left(X + Y.\frac{F_a}{V.F_r}\right) = 4.57 \text{ KN}$$

$$C_0 = \frac{F_a}{0.07} = 35.3 \text{ KN}$$

$$C_{10} = a_f . F_D . \left\{\frac{x_D}{x_0 + (\theta - x_0)\{\ln(\frac{1}{R_D})\}^{1/b}}\right\}^{1/a}$$

$a = 3$ for ball bearing

$C_{10} = 96.93 \text{ KN}$

$L_D = 25\times10^3 \times 600 \times 60 = 900\times10^6$ cycles

$$x_D = \frac{L_D}{L_{10}} = \frac{900\times10^6}{10^6} = 900.$$

Now select a bearing from table 11-2.

Select: 02-series bearing with bore dia 90-mm

$\Rightarrow C_{10} = 106 \text{ KN} > 96.93 \text{ KN}$

$C_0 = 73.5 \text{ KN} > 35.3 \text{ KN}$

Check: $\frac{F_a}{C_0} = \frac{2.47}{73.5} = 0.034$

$\Rightarrow e = 0.23,$ (table 11-1) & linear interpolation)

$X_2 = 0.56, \quad Y_2 = 1.93$

$$F_{eq}^a = \frac{\sum F_{e_i}^a l_i}{\sum l_i} = \sum F_{e_i}^a \left(\frac{l_i}{\sum l_i} \right)$$

$$= \sum F_{e_i}^a f_i =$$

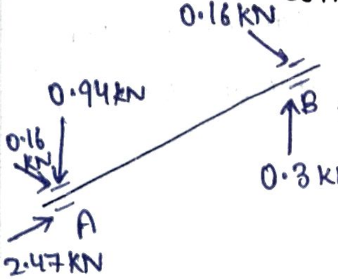
$f_i = \dfrac{l_i}{\sum l_i}$ = fraction of revolution run up load

$$\Rightarrow \boxed{F_{eq} = \left(\sum F_{e_i}^a f_i \right)^{1/a}}$$

Q→ from Tutorial →

Given: At A: Angular contact bearing, Takes all the axial thrust } 02-Series

At B: straight roller bearing



Combined reliability. 0.99,

Application factor: $a_f = 1.3$

Weibull parameters: $x_0 = 0.02$, $\theta - x_0 = 4.139$

$b = 1.483$

Reliability: $\underset{\text{combined}}{R} = R_A R_B \geq 0.99$

Either $R_A = R_B = \sqrt{0.99}$

In fig. loads are in lbf convert to KN.

or choose: $R_A = 0.99$

$R_B = 1$

Selection of Bearing at A:

$$F_r = \sqrt{(0.16^2 + 0.94^2)} = 0.954 \text{ KN}$$

$$F_a = 2.47 \text{ KN}$$

Find equivalent radial load: F_e

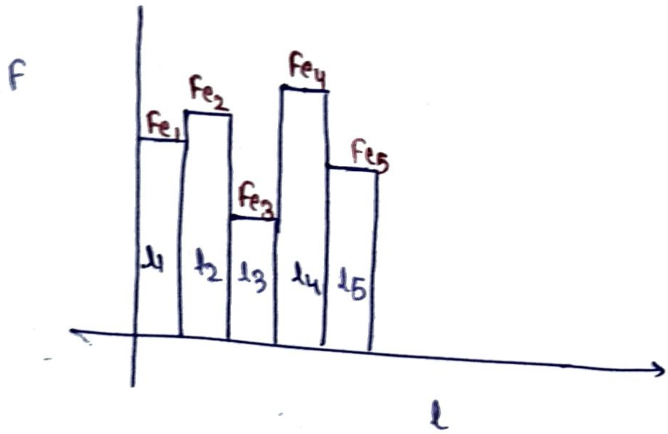
Inner race loading: $V = 1$

Variable Loading

$Fe_1 \leftarrow (F_{a_1}, F_{r_1})$ load for $- l_1$ cycles

$Fe_2 \leftarrow (F_{a_2}, F_{r_2})$ load for $- l_2$ cycles

$\vdots \qquad \vdots$



$$F L^{1/a} = \text{constant}$$

$L \rightarrow$ life to failure

$$\Rightarrow \quad Fe_1 L_1^{1/a} = Fe_2 L_2^{1/a} = \ldots \quad (\text{since bearing is same})$$

L_i is life at load Fe_i

$$l_1 < L_1, \ldots, l_i < L_i$$

$$\Rightarrow \quad Fe_i^a L_i = \text{constant}$$

Linear damage hypothesis:

$\Rightarrow$ Damage : $D_1 = Fe_1^a l_1$

$D_2 = Fe_2^a l_2, \ldots$

Total Damage

$$D = D_1 + D_2 + \ldots = \sum_{i=1}^{N} Fe_i^a l_i$$

Let bearing run for $(l_1 + l_2 + \ldots + l_N)$ cycles. at load Fe_q and cause same damage

$\Rightarrow L_{eq} = (l_1 + l_2 + \ldots + l_N)$

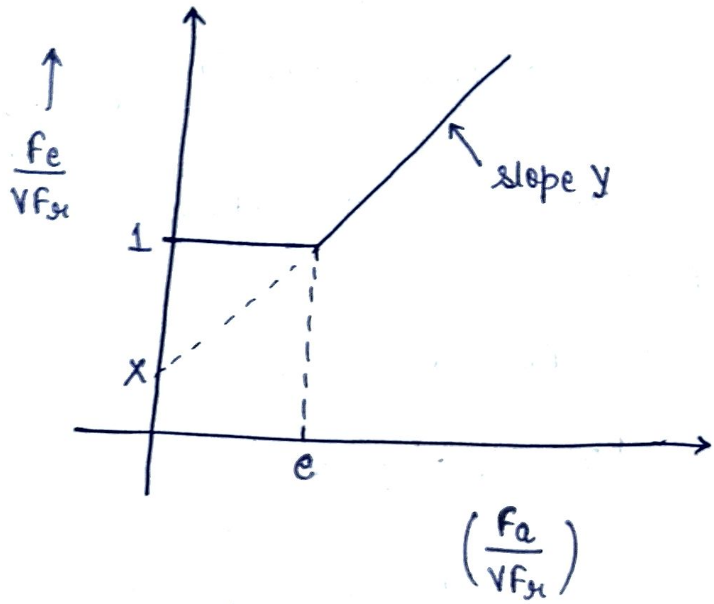
$$\Rightarrow \quad Fe_q^a L_{eq} = \sum Fe_q^a l_i = D.$$

Combined Radial + Thrust Load

F_a = Thrust load (along shaft axis)

F_r = Radial load

Find equivalent radial load → F_e



$$\frac{F_e}{VF_r} = 1 \quad \text{for} \quad \frac{F_a}{VF_r} \leq e$$

$$= X + Y\left(\frac{F_a}{VF_r}\right) \quad \text{for} \quad \frac{F_a}{VF_r} \geq e$$

$$V = \begin{cases} 1 & \text{if inner race rotates} \\ 1.2 & \text{if outer race rotates} \end{cases}$$

"e" depends on static load rating C_o

C_o:= basic static load rating

- A load that produces a total permanent deformation of 10^{-4} times that of rolling element size.

→ For e, X, Y see Table 11-1.

→ For C_{10}, C_o see table 11-2, 11-3.

→ load application factor a_f (table 11-5)

Known: R_D, f_D, x_D (or L_D)

Need to find: C_{10} (on $R = 0.9$ line)

To go from point D to B: (on $R = R_D$)

$$f_D \cdot L_D^{1/a} = f_B L_B^{1/a}$$

$$\Rightarrow f_B = f_D \left(\frac{L_D}{L_B} \right)^{1/a} = f_D \left(\frac{x_D}{x_B} \right)^{1/a}$$

x_B is still an unknown

Betn A & B

(or betn R_D & $R=0.9$ lines)

$$R_D = \exp\left[-\left(\frac{x_B - x_0}{\theta - x_0} \right)^b \right]$$

$$\Rightarrow x_B = x_0 + (\theta - x_0)\left[\ln\left(\frac{1}{R_D} \right) \right]^{1/b}$$

Now x_B is known.

Substitute in expn for f_B

$$\Rightarrow f_B = f_D \cdot \left[\frac{x_D}{\left[x_0 + (\theta - x_0)\left\{ \ln\left(\frac{1}{R_D} \right) \right\}^{1/b} \right]} \right]^{1/a}$$

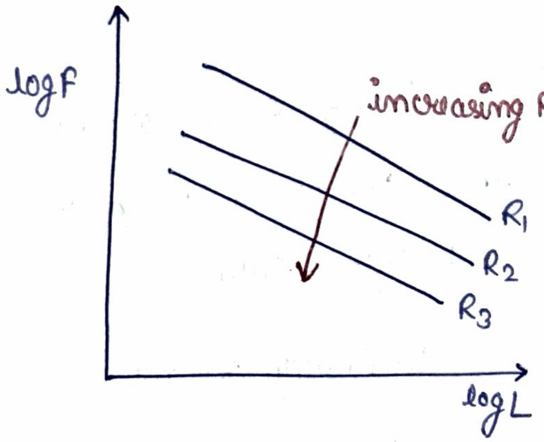
Choose: $f_B = f_A = C_{10}$ (use an application factor

$$\Rightarrow \boxed{C_{10} = a_f f_D \cdot \left(\frac{x_D}{x_B} \right)^{1/a} = a_f f_D \cdot \left[\frac{x_D}{x_0 + (\theta - x_0)\left\{ \ln\left(\frac{1}{R_D} \right) \right\}^{1/b}} \right]^{1/a}}$$

$$\boxed{\simeq a_f f_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}}$$

application factor
(kind of safety factor for bearing)

Desired reliability may differ from R = 0.9



Bearing survival:

At constant load reliability follows Weibull distribution

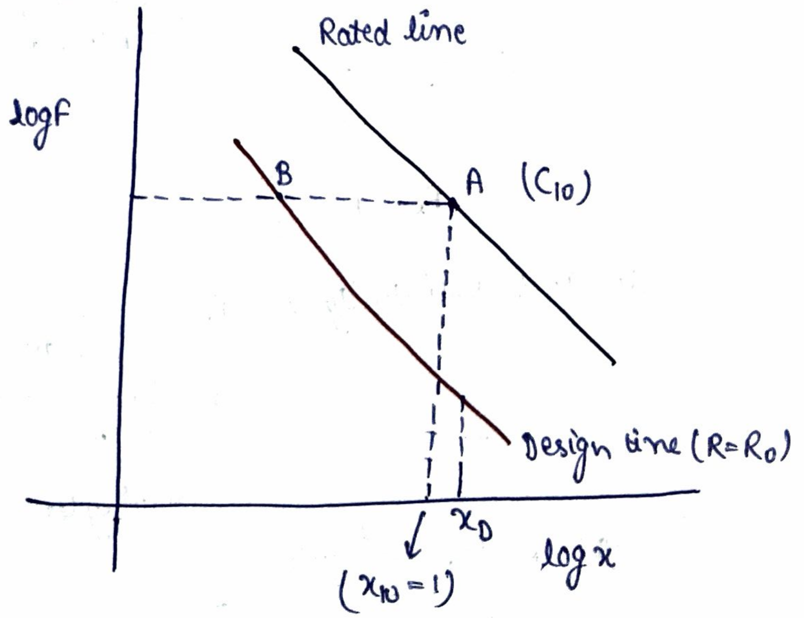
$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right]$$

$x = \frac{L}{L_{10}}$ dimensionless life

x_0 = guaranteed value of x

θ, b are parameters.

Rated load when reliability is different



Manufacturers Catalog:

specifies a rated life and a load rating.

Rated Life → No. of revs (or hours at constant speed) that 90% of a group of bearings attain or exceed before failure.

Reliability $R = 0.9$

$\Rightarrow$ Rated life $\sim 10^6$ cycles.

How to select a bearing?

- Catalog specifies rated load and life
- Compare with desired load and life

At constant reliability,

$$F_D L_D^{1/a} = F_R L_R^{1/a}$$

F_D, L_D ↑ design/desired; F_R, L_R ↑ Rated

$$F_R = F_D \left(\frac{L_D}{L_R}\right)^{1/a} = F_D \left\{\frac{\alpha_D n_d}{\alpha_R n_r}\right\}^{1/a}$$

Let $x_D = \dfrac{L_D}{L_R}$

$$\Rightarrow F_R = F_D \, x_D^{1/a}$$

At 90% Reliability: $\left.\begin{aligned} F_R &= C_{10} \\ L_R &= L_{10} \end{aligned}\right\}$ given in catalog

$L_{10} \rightarrow \sim 10^6$ cycles

If $L_D = L_R = L_{10} = 10^6$ cycles

$$\Rightarrow \boxed{x_D = x_{10} = 1}$$

Bearings

→ Introduction from slides.

Bearing Life

Failure criteria / scenario: Fatigue.

Measures of life: ① No of revs. of inner ring with outer ring fixed before failure.

② Hours of operation at constant angular speed.

③ Pitting or spalling of an area 0.01 in^2 (6.5 mm^2)

Load-Life relation (only radial load)

$$\boxed{F L^{1/a} = \text{constant}}$$

F in kN
L in Revs.

$a = \begin{cases} 3 & \text{ball bearings} \\ \frac{10}{3} & \text{roller bearings} \end{cases}$

at constant reliability

→ If life is measured in hours:

$\mathcal{L}$ = life in hrs.

n = constant rpm

$$\Rightarrow \boxed{\begin{array}{l} L = 60 \cdot \mathcal{L} \cdot n \\ \Rightarrow F\{(60 \cdot \mathcal{L} \cdot n)\}^{1/a} = \text{constant} \end{array}}$$