$\boxed{VA}$ TU+HW (I) (There are 3 pages)

Imp. → Obtain DEOM for each problem

(1) Determine the undamped natural frequency of small oscillations of the system shown in Figure 1. The uniform disc of mass m has two springs attached as indicated. Assume that the disc rolls without slipping. $\left[\omega_n = \sqrt{\frac{6k}{m}}\right]$

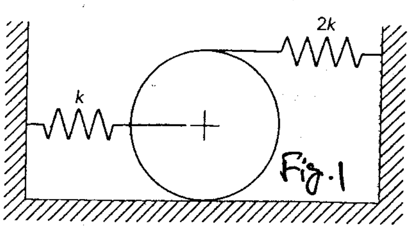


Fig. 1

(2) One end of a solid steel shaft is fixed and a uniform steel disc is attached at its centre to the other end of the shaft. Dimensions of the shaft and disc are shown in Figure 2. The modulus of rigidity G for steel is 8×10^{10} Pa and the density of steel is 7.8×10^3 kg/m^3.

Determine the natural frequency of torsional oscillations of the system. $[\omega_n = 450.13 \text{ rad/s}]$

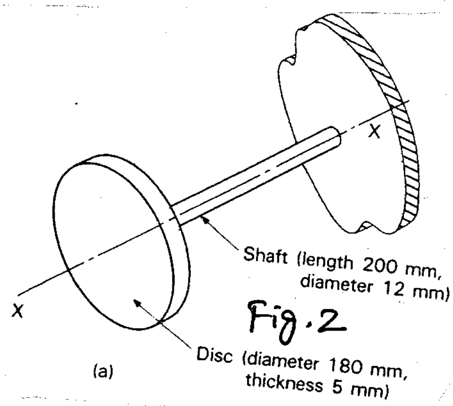


Fig. 2

(3) In Figure 3 the pulley has a mass of 20 kg and a radius of gyration about its centre k_G of 300 mm. The spring has a stiffness of 500 N/m. A load of 25 kg hangs from the centre of the pulley.

Determine the natural frequency of vertical vibrations when the load is released from a displaced position. $[f_n = 1.01 \text{ Hz}]$

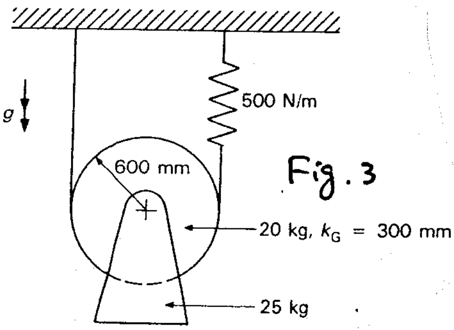


Fig. 3

(4) Figure 4 shows a uniform, rigid beam of mass m supported horizontally at its ends by two light, parallel links of equal lengths l. If the beam is given a small displacement and released, obtain an expression for the natural frequency of the resulting oscillations.

If the links oscillate with an amplitude of θ_0 radians, obtain approximate expressions for the tensions in the links when they are (a) in a vertical position and (b) inclined at an angle θ_0 to the vertical (Take into account second-order terms.)

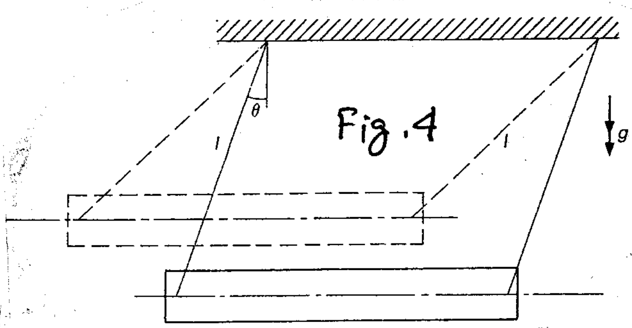


Fig. 4

(5) In the system shown in Figure 5 a belt A is attached to a fixed frame B, wrapped round the drums C and D and supports a load E of 20 kg. The drum C has a mass of 40 kg, radius 150 mm, and radius of gyration about its centre G of 70 mm. Drum D, which is of negligible mass, is attached at its centre to a spring F of stiffness 10 kN/m.

The load E is given a small downwards displacement and released from rest. Determine the natural frequency of the resulting oscillations. [Ans: 2.89 Hz

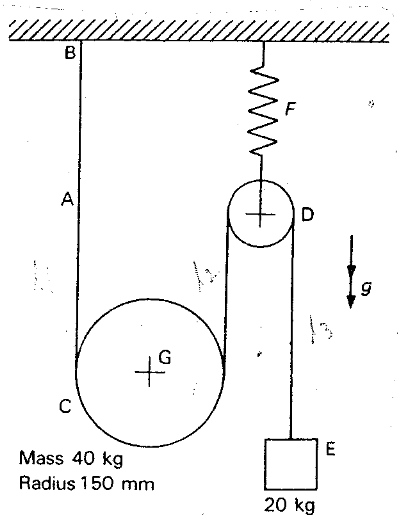


Fig. 5

(6) In the inverted pendulum (see Figure 6) a light, rigid rod OA is fixed at O and has a small body of mass M attached at the top. A spring of stiffness k is attached at B so that when the rod AO is in equilibrium in the vertical position the spring is unextended.

If body A is given a small, horizontal displacement, determine the circular frequency of the resulting oscillations.

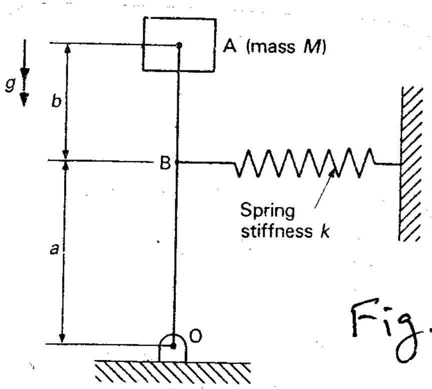


Fig. 6

1.1 Determine the natural frequency of small oscillations of the bell crank lever ABC shown in Figure 1.25. The lever is light but has a mass M fixed at C. BC is horizontal when the system is in the equilibrium position.

Answer $f_n = \{1/(2\pi)\}\{(c/d)(k/M)^{0.5}\}$.

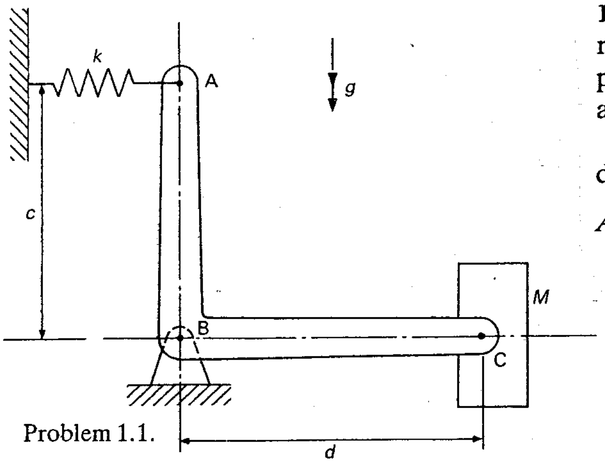


Figure 1.25 Problem 1.1.

1.2 A thin ring of 120 mm radius is placed on a frictionless pivot at O and given a small displacement. Determine the natural frequency of the oscillations. See Figure 1.26.

Answer $\omega_n = 6.39$ rad/s, $f_n = 1.018$ Hz.

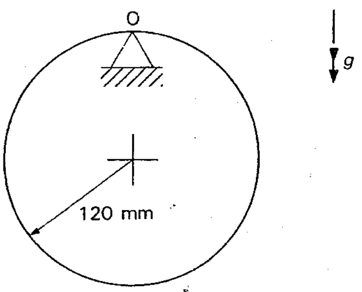


Figure 1.26 Problem 1.2.

1.3 A rotational system is formed by a solid steel shaft fixed at one end and a solid steel disc, as shown in Figure 1.27. In addition a

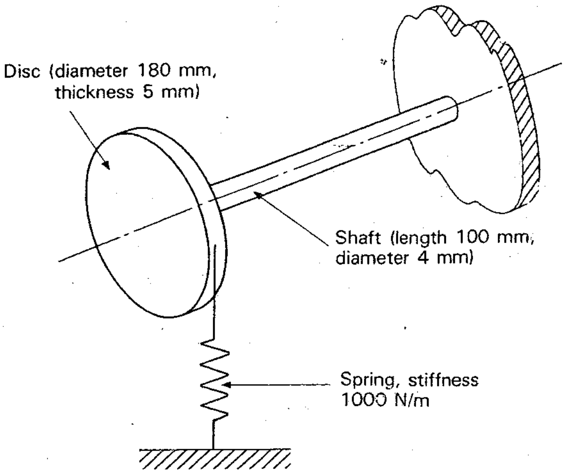


Figure 1.27 Problem 1.3.

inear spring is attached to the periphery of the disc so that its line of ıction is tangential to the disc. (G for steel is 8×10^{10} Pa, density of teel is 7.8×10^3 kg/m^3.)

Determine the natural frequency of small oscillations.

1nswer $\omega_n = 83.77$ rad/s ($f_n = 13.33$ Hz) – mass of shaft neglected.

1.4 A drum which is a solid cylinder of mass m and radius r can rotate in frictionless bearings at O as shown in Figure 1.28. A rope passes over the drum and carries a load of mass M. The rope is attached to a fixed support via a spring of stiffness k.

Given that the load is given a small displacement downwards determine the frequency of the resulting vibrations.

Answer $f_n = \{1/(2\pi)\}\{k/(M + m/2)\}^{0.5}$.

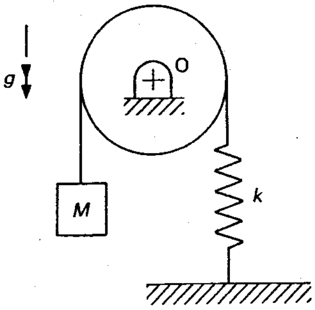


Figure 1.28 Problem 1.4.

1.5 In Figure 1.29 a belt is wrapped round a pulley A (mass 8 kg radius 120 mm and moment of inertia about an axis through th centre of 0.4 kg m^2) and a pulley B (of negligible mass) and i

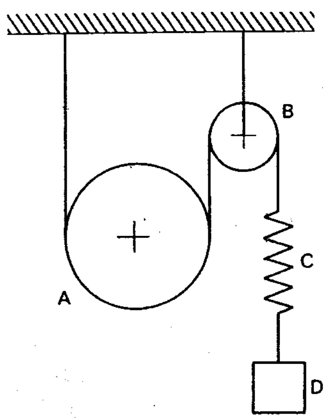


Figure 1.29 Problem 1.5.

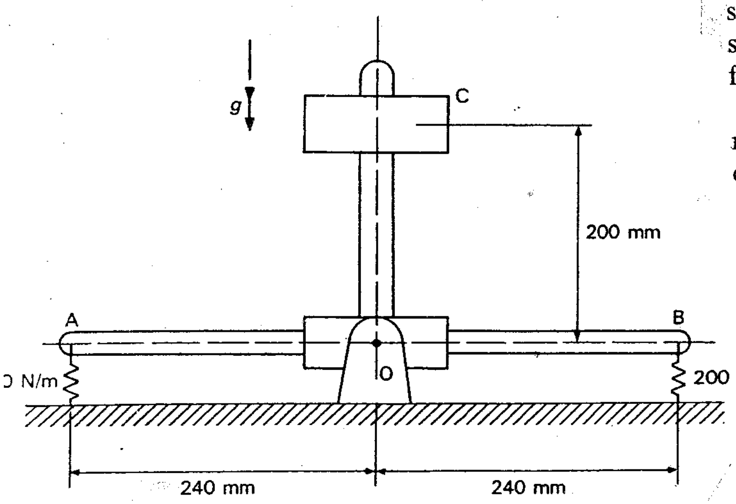
1.6 Figure 1.30 shows an inverted pendulum which is supported vertically in the equilibrium position by two springs, each of stiffness 200 N/m. The distance between the attachment point of each spring and the pivot point O is 240 mm. AB and OC form a rigid inverted T piece.

Determine the natural frequency of small oscillations when

(a) the T piece is of negligible mass and a concentrated mass of 3 kg is placed on OC, 200 mm from O, and

(b) the T piece, with OC equal to 240 mm, is made up of slender uniform rods of mass 1.2 kg/m and the concentrated mass of 3 kg remains at the same position on OC.

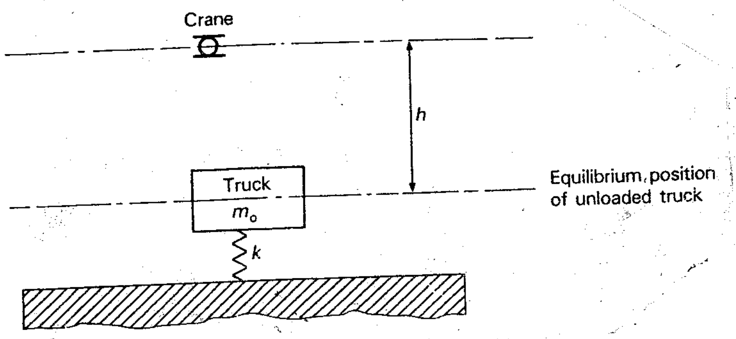
Answer (a) $f_n = 1.903$ Hz (b) $f_n = 1.766$ Hz.



1.8 Consider the system described in Problem 1.7. Assume that the stiffness of the truck suspension is 395 kN/m and that the truck mass is 9 tonnes. A load of 1000 kg impacts the truck and 5 s later a further load of 500 kg, released by the crane from the same height h of 0.5 m, impacts the truck. See Figure 1.31.

Determine the amplitude of the displacement of the final vibration.

Answer 81.9 mm.



1.12 The system shown in Figure 1.35 consists of a thin uniform rod OA, of mass m and length l, which is pivoted at the point O and has a small body of mass m_0 fixed to the top of the rod. A spring of stiffness k_1 is attached to the rod at a distance a from the pivot and a second spring of stiffness k_2 is attached to the rod at a distance b from O. The system is initially in equilibrium in the vertical position.

(a) Determine the natural frequency of the oscillations which result when the system is given a small displacement from its equilibrium position.

(b) Establish the condition for instability of the system.

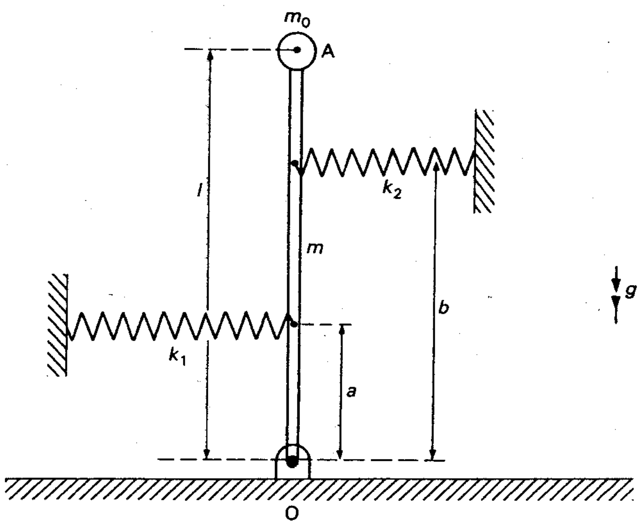


Figure 1.35 Problem 1.12.

Answer

(a) $\{1/(2\pi)\}[(k_1a^2 + k_2b^2 - m_0gl - 0.5mgl)/\{l^2(m_0 + m/3)\}]^{0.}$

(b) $k_1a^2 + k_2b^2 < m_0gl + 0.5mgl$.

TU/HW Sheet No. 2 (2 pages)

Page 1

(1) A body of mass 5.5 kg is hung on a spring of stiffness 1000 N/m. It is pulled down 50 mm below the position of static equilibrium and released so that it executes vertical vibrations. There is a viscous damping force acting on the body of 40 N when the velocity is 1 m/s.

(a) Determine the differential equation of the motion and obtain the expression for the displacement of the body as a function of time. [$y = \exp(-3.637t)(50\cos 12.984t + 14\sin 12.984t)$ mm]

(b) Calculate the distance the body moves from the instant of release until it is momentarily at rest at the highest point of its travel, and the time that has elapsed when it reaches that position. [70.7 mm, 0.242 s]

(c) Calculate the time that elapses for the body to pass through the equilibrium position for the first time after release. [0.142 s]

(2) The disc of a torsional pendulum has a moment of inertia of 0.6 kg m^2 and is immersed in a viscous fluid. The disc is suspended on a brass shaft which has a diameter of 9.0 mm and length of 400 mm. When the pendulum is oscillating the amplitudes that are observed on the same side of the mid-position for three successive cycles are 14.4°, 1.2° and 0.1°.

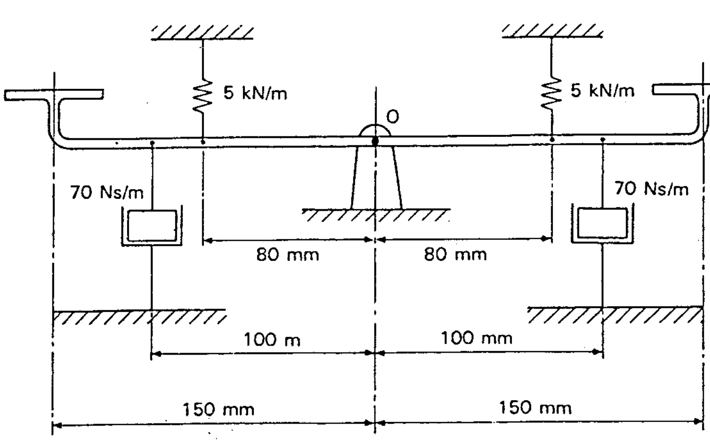
Take modulus of rigidity G to be 34.5 GN/m^2 for brass.

Determine:

(a) the logarithmic decrement, [2.49]
(b) the damping ratio, [0.37]
(c) the undamped circular natural frequency, [9.6 rad/s]
(d) the damped circular natural frequency, [8.9 rad/s]
(e) the damping torque at unit angular velocity. (4.2 Nms/rad)

(3) A schematic diagram of a balance for weighing purposes is shown in Figure 2.11. The beam and scale pans form a rigid body which has a moment of inertia of 0.01 kg m^2 about the rotation axis through the pivot point O. Springs and viscous dampers are shown. Determine the undamped natural frequency and the damping ratio of the system when it undergoes oscillations of small amplitude. [80 rad/s, 0.875]

If a mass of 5 kg is gently placed in contact with, for example, the right-hand scale pan and released, find the maximum deflection of the beam. [9.5°]



(4) The rotor of an indicating instrument is controlled by a torsional spring and viscous damper. The rotor has a mass of 50 g and its radius of gyration about the axis of rotation is 12 mm. When set in vibration the rotor makes one complete oscillation in 4.2 s and the amplitudes of consecutive swings, in opposite directions, are 90 : 12.

(a) Determine:

(1) the damping ratio, (0.54)
(2) the undamped natural frequency, (0.283 Hz)
(3) the stiffness of the torsional spring, (22.7×10^{-6} Nm/rad)
(4) the damping constant of the viscous damper. (13.8×10^{-6} N-m-s/rad)

(b) If the rotor, initially at rest, receives a sudden impulse, which causes it to swing through an angle of 25°, determine:

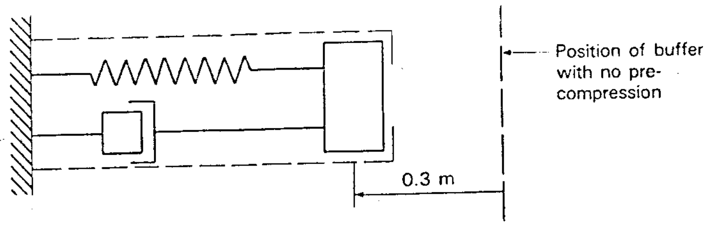
(1) the time taken to swing through 25°, (0.668 s)
(2) the initial angular velocity caused by the impulse, (1.47 rad/s)
(3) the energy supplied by the impulse. (7.8 μJ)

(5) A heavy-duty weighing machine consists of a platform of mass 800 kg which is supported by springs with an equivalent stiffness of 100 kN/m and a damper which provides viscous damping of damping constant 500 N s/m.

A sand bag of mass 70 kg is dropped from a height of 400 mm and strikes the platform without splitting. Determine the time required for the platform and bag to reach their lowest position during the subsequent vibration. Sketch the displacement of the platform against time. [0.174 s]

(PTO)

(6) A hydraulic buffer consists of two units each having an internal spring of stiffness 30 000 N/m in parallel with a viscous damper offering a resistance to motion of 120 000 N at 1 m/s velocity. The springs are initially pre-compressed an amount 0.3 m. If a train of mass 100 000 kg, moving at a velocity of 2 m/s, strikes the buffer without rebound, determine the distance the train moves before coming momentarily to rest. [0.646 m]



(7) An instrument consists essentially of a mass of 80 g whose movement is controlled by a spring and viscous damper. A free, damped vibration of periodic time 0.5 s gives the following readings for successive displacements on either side of the equilibrium position at which the reading is 60:

70 55 62.5 58.75 60.62.

Determine:
(a) the damping ratio,
(b) the stiffness of the spring, and
(c) the force exerted by the damper at a speed of 1 m/s.

Answers (a) 0.215 (b) 13.2 N/m (c) 0.44 N.

(8) A machine of mass 2 tonnes is supported on springs which have an equivalent stiffness of 25 kN/m. A dashpot with a damping constant of 1 kN s/m is attached between the machine and the ground in parallel with the springs. The machine is displaced downwards by 15 mm and then released.

Determine the maximum upward displacement from the equilibrium position, and the time taken to reach this position.

Answer 12.0 mm, 0.891 s.

(9) An instrument of mass 4 kg is mounted on four springs and a dashpot onto a base plate. Each spring has a stiffness of 350 N/m and the dashpot has a damping constant of 15.0 N s/m.

If the base plate is displaced downwards by 20 mm and then kept in this position, determine the lowest position of the instrument relative to its original equilibrium position.

Answer 34.6 mm below the equilibrium position.

(10) A steel ingot of mass 145 kg is ejected horizontally from a furnace at a speed of 0.75 m/s and is brought to rest by a hydraulic buffer incorporating a return spring. The spring exerts a force of 100 N for each metre of compression. The hydraulic piston exerts a force, proportional to the velocity, which has a value of 60 N at 1 m/s.

Determine the time taken and the distance travelled by the ingot from the instant of contact with the buffer until it first comes to rest.

Frictional resistances are to be neglected.

Answer 1.64 s, 643 mm.

(11) A gun of mass 1300 kg has recoil springs of total stiffness 500 kN/m. After firing, the gun recoils 450 mm, and for the return movement a viscous damper acts on it giving critical damping.

Show that the time for the return movement up to 20 mm from the position of static equilibrium is approximately 0.30 s.

2.9 The return mechanism of a swing door provides a torque proportional to an angle θ. The torque varies from 5 N m with the door closed ($\theta = 0°$) to 30 N m when the door is fully open ($\theta = 90°$).

When the door is closing, a viscous damping torque comes into effect at a rate of 40 N m at an angular velocity of 1 rad/s. The moment of inertia of the door about the axis of the hinge is 12.5 kg m^2.

Determine the angular position of the door after an elapsed time of 4 s.

Answer 2.21°.

(12) The rotor of an ammeter is controlled by a torsional spring and a damper which provides a linear damping torque. The rotor has a mass of 50 g and its radius of gyration about the axis of rotation is 13.5 mm. When the rotor is displaced by 20° and released the first complete oscillation is made in 6 s, after which time the displacement has reached a maximum at 1.6°. Determine

(a) the torsional stiffness of the spring and the damping constant at the damper.

If the rotor is initially at rest and receives a sudden impulse that causes it to swing through 10°, determine:
(b) the time taken to swing through 10°, and
(c) the energy supplied by the impulse.

Answers (a) 11.61×10^{-6} N m/rad, 7.67×10^{-6} N m s/rad
(b) 1.14 s
(c) 0.460 μJ.

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(1) A body of mass 100 kg is suspended by a spring of stiffness 30 kN/m and a dashpot of damping constant 1000 N s/m. Vibration is excited by a harmonic force of amplitude 80 N and frequency 3 Hz.

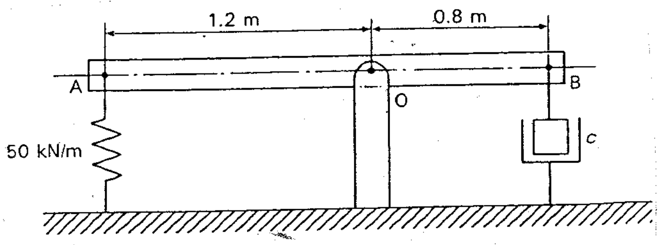
Calculate the amplitude of the displacement for the vibration and the phase angle between the displacement and the excitation force.

(2) In Figure 3.8 a slender rigid beam AB of mass 80 kg and length 2 m is shown supported in the horizontal position by a spring of stiffness 50 kN/m at A and by a frictionless pivot at O, 1.2 m away from the spring. A viscous damper with a damping constant c is positioned, as shown, at the other end B.

(a) Establish the differential equation which applies when the beam executes free damped oscillations of small amplitude.

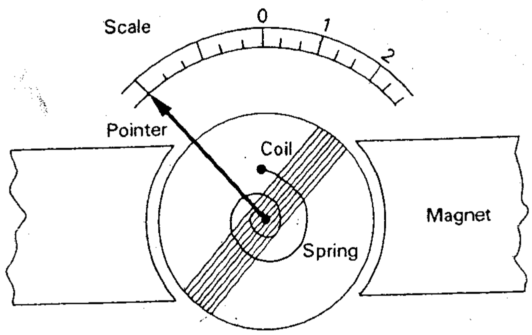
(b) Determine the value of the damping constant c in order to achieve a damping ratio ξ of 0.5.

(c) A sinusoidal force of amplitude 200 N and frequency 13 Hz is applied vertically at A. With the value of c obtained above determine the amplitude of the small steady-state oscillations of the beam.



(3) A moving coil d.c. ammeter has a rotor controlled by a torsion spring and a viscous damper. The angular displacement of the rotor is directly proportional to the current. The undamped natural frequency of the system is 4 Hz, and the damping is critical.

A current of constant amplitude is passed through the meter. When the current has zero frequency, the meter indication is 1 A. Determine the meter indication when the frequency of the current is 6 Hz.



(4) A mechanical shaker, illustrated in Figure 3.20, contains two shafts which rotate at the same speed in opposite directions. The out-of-balance of each shaft is equivalent to a mass m of 1.15 kg at a radius r of 50 mm. The shaker is attached to a rigid plate that is supported upon two vibration isolators, each of which has the combined characteristics of a linear spring and a dashpot for which the damping constant is c. The system can be assumed to have one degree of freedom in the vertical direction.

With the shaker running at a constant speed of 2000 rev/min, it is observed that the amplitude of steady-state forced vibration of the plate is 6.0 mm and that the phase angle between the vertical shaking force and the plate displacement is 30°.

Determine:

(a) the value of the constant c for each isolator, and

(5) A machine is of total mass 2 tonnes and operates at 240 strokes per minute. The excitation force is equivalent to a force induced by an out-of-balance mass of 12 kg rotating at a radius of 100 mm.

The machine is to be mounted on four vibration isolators with negligible damping. For (1) $\omega/\omega_n = 2$ and (2) $\omega/\omega_n = 4$ determine:

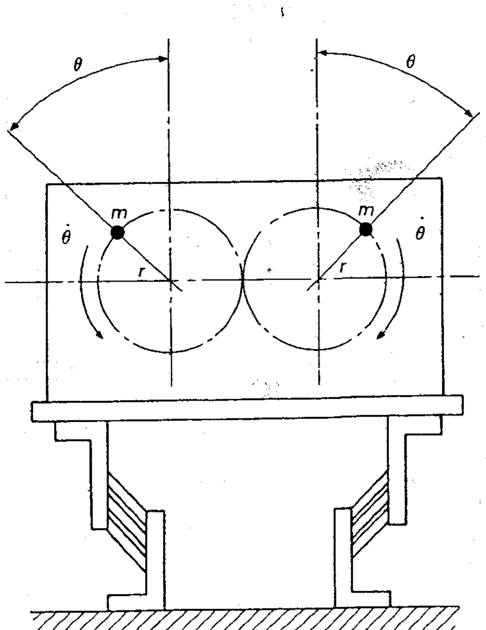
(a) the static deflection of the machine,

(b) the transmission ratio,

(c) the stiffness of each isolator, and

(d) the amplitude of the displacement of the machine.

Given that the machine is directly attached to a concrete base of mass 6 tonnes, determine for this new case the four quantities listed above when $\omega/\omega_n = 2$.



6) In Figure 3.29 is shown a rigid link AGB which is pivoted to the foundation by a pin at G. The centre of mass of the link is at G and the moment of inertia about an axis through G, perpendicular to the plane of motion, is I_G. A damper of rate C and a spring of stiffness k are pinned to the link at A and B as shown.

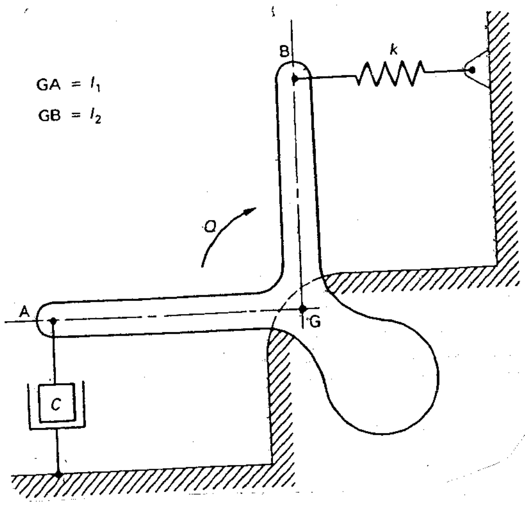
A couple has a moment Q which varies sinusoidally with time. The moment has an amplitude Q_0 and circular frequency ω. The couple is applied to the link and gives rise to a steady-state angular oscillation of the link.

Assuming that the amplitude of this oscillation is small, and neglecting the effects of gravity and the masses of the spring and dashpot, show that:

(a) there is no net force transmitted to the foundations,

(b) the net couple transmitted to the foundations has a moment with amplitude of

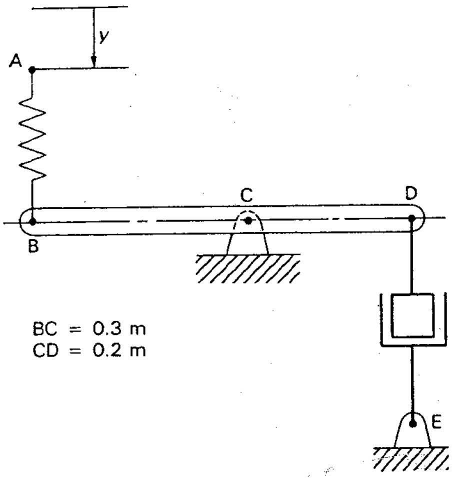
$$Q_0[\{k^2l_2^4 + C^2l_1^4\omega^2\}/\{(kl_2^2 - I_G\omega^2)^2 + C^2l_1^4\omega^2\}]^{0.5}.$$



7) Figure 3.36 shows a rigid link BCD which can pivot about the axis through C and has a moment of inertia about this axis of 0.25 kg m^2. The spring AB has a stiffness of 700 N/m and the damper DE has a rate of 60 N per m/s; both spring and damper may be assumed to be massless.

The forced displacement y of end A of the spring is sinusoidal with a circular frequency of 10 rad/s and an amplitude of 1 cm.

Find the amplitude of angular displacement, assuming this to be small, of the steady-state forced oscillation of link BCD.



8) A machine of mass 350 kg rests on supports with stiffness 160 kN/m and viscous damping 0.2 of the critical value. Part of the machine has a mass of 1 kg and moves vertically with simple harmonic motion of frequency 10 Hz and stroke 60 mm.

Calculate:

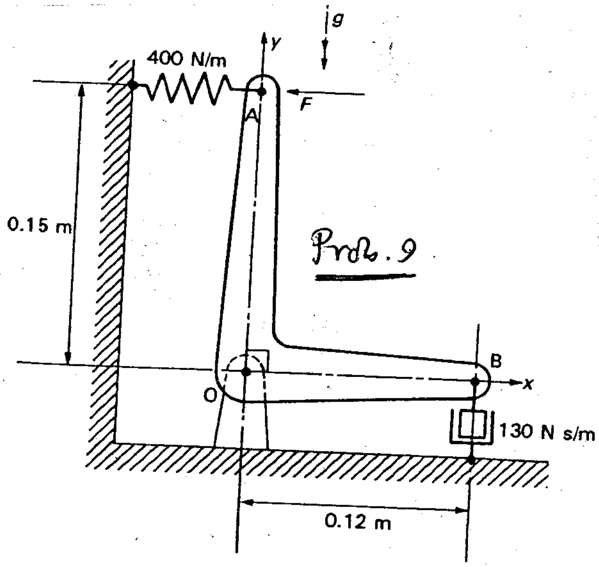
(a) the amplitude of forced vertical vibrations of the machine and the phase angle by which the displacement lags the excitation force,

(b) the magnitude of the vibratory force transmitted to the foundations and its phase angle relative to the applied force.

Answers (a) 0.096 mm, 171.3° (b) 23.8 N, 121.7°.

9) A bell crank lever is pivoted at O, as shown in Figure 3.45. The spring at A has a stiffness of 400 N/m and the viscous damper attached at B has a damping constant of 130 N s/m. The moment of inertia of the bell crank lever about an axis through O perpendicular to the xy plane is 0.1 kg m^2. In the equilibrium position OA is vertical. A sinusoidal force F of magnitude 50 N and frequency 3 Hz is applied horizontally at A. Calculate the amplitude of the resulting steady-state angular oscillations of the bell crank lever.

Answer 9.7°.



10) A machine of mass 23 kg is mounted on four springs each of which deflects 0.05 mm when a force of 1 N is applied to it. A damping device resists the motion of the machine with a force proportional to velocity such that the force is equal to 150 N at 1 m/s. A vertical force, of constant amplitude 55 N and variable frequency, is applied to the machine.

Determine the maximum amplitude of steady-state forced vibrations and the frequency at which it occurs.

Answers 6.2 mm, 9.36 Hz.

(11) [redacted] A punching machine of mass 510 kg is supported by a mounting which has a stiffness of 240 kN/m and which exerts a viscous damping force of 9600 N at 1 m/s. Part of the machine of mass 8 kg executes simple harmonic motion and moves vertically through a stroke of 40 mm.

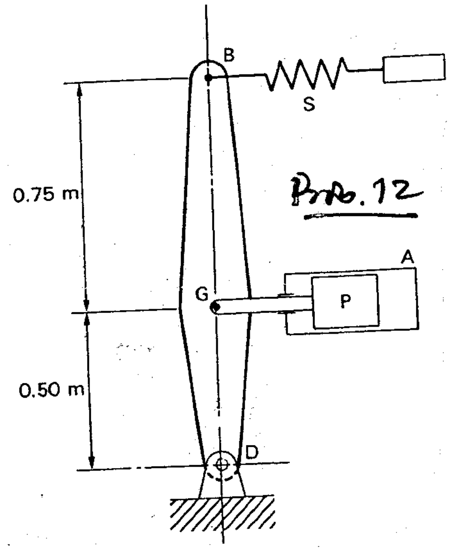
Calculate the amplitude of steady-state forced vibrations when the machine is running at 380 strokes/min.

Answer 0.37 mm.

(12) [redacted] A turbine control linkage is shown in Figure 3.48. A is the control valve cylinder and BGD is a lever of mass 18 kg and radius of gyration about G of 200 mm. D is a fixed pivot point. A light spring S of stiffness 2500 N/m is attached at B and the other end of the spring is given a continuous simple harmonic motion of amplitude 5 mm and frequency 4 Hz. The mass of the control valve and piston is 12 kg. The force resisting the motion of the piston is proportional to velocity and is equal to 175 N at 1 m/s.

Determine the amplitude of motion of the piston in the cylinder and the maximum force acting on the pin at G.

Answers 4.62 mm, 40.4 N.



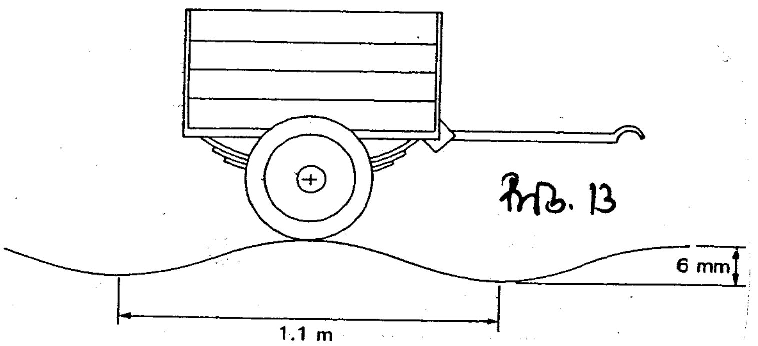
Prob. 12

(13) [redacted] A trailer of mass 500 kg forms a spring–mass–damper system, with a single degree of freedom, as shown in Figure 3.49. The equivalent stiffness of the trailer's springs and tyres is 5.25 MN/m and the damping constant is 20 kN/(m/s).

The trailer is pulled over a road surface that can be regarded as a sine wave with an amplitude of 6 mm and a wavelength of 1.1 m. The wheels always remain in contact with the road.

If the speed of the trailer is 17 m/s, what will be the amplitude of the vertical displacement of the trailer?

Answer 16.7 mm.

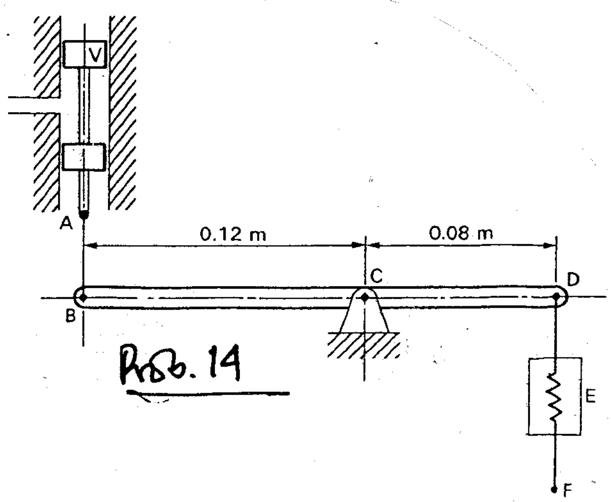


Prob. 13

(14) [redacted] Figure 3.50 shows a mechanism, lying in the horizontal plane, used to operate a hydraulic spool valve V. The link BCD is connected at B to the spool valve by means of the link AB, and at D to the input signal point F by means of the spring box E. The spool valve has a mass of 4 kg and viscous forces at the valve are equivalent to a damping coefficient of 10 N s/m. The spring at E has a stiffness of 1.6 kN/m and the moment of inertia of BCD about the fixed axis at C is 0.072 kg m^2.

The point F is subjected to a harmonic displacement of amplitude 2 mm at a frequency of 1 Hz. For the subsequent steady-state forced vibration of the system where the angular displacement of the lever BCD can be assumed to be small, determine:

(a) the amplitude of the motion of the spool valve, and
(b) the amplitude of the force in the link AB.



Prob. 14

Answers (a) 5.9 mm (b) 1.0 N