Assignment 2

1. Explain briefly the role of different operators, namely reproduction, crossover and mutation used in a GA.

2. The performance of a GA depends on the balance between selection pressure and population diversity – justify the statement.

3. Is GA a random search technique? Explain it.

4. Are uniform crossover with probability of 0.5 and bit-wise mutation with probability of 0.5 the same ? Explain it.

5. Explain elitist strategy used in GA.

6. Why do we prefer ranking selection to a Roulette-Wheel selection in GA ?

7. Why do we use a high value of crossover probability and a low value of mutation probability in GA-applications ?

8. Do you prefer a Gray-coded GA to a Binary-coded GA ? Explain it.

9. Explain the Building-Block Hypothesis of a binary-coded GA.

10. Can you declare GA a global optimizer ? Explain it.

11. State the advantages and disadvantages of a GA.

12. A binary-coded GA is to be used to solve an optimization problem involving one real and another integer variables. The real and integer variables are allowed to vary in the ranges of $(0.2, 10.44)$ and $(0, 63)$, respectively. Design a suitable GA-string to ensure a precision level of 0.01 for the real variable.

13. Use a binary-coded GA to minimize the function $f(x_1, x_2) = x_1 + x_2 - 2x_1^2 - x_2^2 + x_1x_2$, in the range of $0.0 \leq x_1, x_2 \leq 5.0$. Use a random population of size $N = 6$, tournament selection, a single-point crossover with probability $p_c = 1.0$ and neglect mutation. Assume 3 bits for each variable and thus, the GA-string will be 6-bits long. Show only one iteration by hand calculations.

14. An initial population of size $N = 10$ of a binary-coded GA is created at random as shown below, while maximizing a function.

$$\begin{matrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{matrix}$$

The fitness of a GA-string is assumed to be equal to its decoded value. Calculate the expected number of strings to be represented by the schema $H : \star\ 1\ \star\ \star\ 1\ \star$, at the end of first generation, considering a single-point crossover of probability $p_c = 0.9$ and a bit-wise mutation of probability $p_m = 0.01$. Make comments on the result.

15. A closed-coil helical spring (refer to Fig. 3.7) made up of a wire of circular cross-section is to be designed, so that it weighs as minimum as possible. Although the load is applied along the axis or parallel to the axis of the spring, shear stress is produced in it due to twisting and its value has to be less than the specified value S. The volume of the spring can be determined using the following expression:

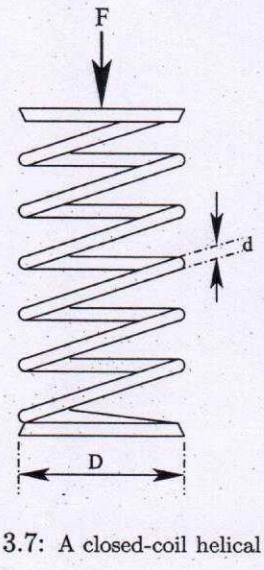

Figure 3.7: A closed-coil helical spring.

$$V = \frac{\Pi^2}{4}(N_c + 2)Dd^2,$$

where d indicates the diameter of the wire, D represents the coil diameter, N_c denotes the number of active turns. The developed shear stress in the spring can be calculated using the expression given below.

$$\tau_{developed} = \frac{8C_f FD}{\Pi d^3},$$

where $C_f = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ and $C = \frac{D}{d}$; F represents the maximum working load. The ranges of different variables are kept as follows:

$$0.5 \leq d \leq 0.8,$$
$$1.5 \leq D \leq 8.0,$$
$$16 \leq N_c \leq 31.$$

Assume that the density of spring material is represented by ρ.

- Formulate it as a constrained optimization problem.
- Use a binary-coded GA to solve the constrained (after assuming a suitable value of S) and corresponding un-constrained optimization problems. The real variables are assumed to have the accuracy level of 0.001 (approx).
  Develop a computer program of the binary-coded GA and solve the above problem using different forms of reproduction scheme (such as proportionate selection, ranking selection, tournament selection), crossover operator (namely single-point crossover, two-point crossover, uniform crossover) and a bit-wise mutation.
  (**Hints:** To determine the length of GA-substring (l) required for representing a variable x after ensuring a precision level of ϵ, follow the expression given below.

  $$\frac{x^{max} - x^{min}}{\epsilon} = 2^l.)$$

16. Let us consider a constrained optimization problem of two variables: x_1 and x_2 as given below.

$$\text{Minimize } f(x_1, x_2) = x_1 + x_2 - 2x_1^2 - x_2^2 + x_1 x_2$$

subject to

$$x_1 + x_2 < 9.5,$$
$$x_1^2 + x_2^2 - x_1 x_2 > 15.0$$

and

$$0.0 \leq x_1, x_2 \leq 5.0.$$

The above constrained optimization problem is to be solved by a GA using the concepts of static, dynamic and adaptive penalties. Assume suitable values for the constant terms. Show one set of calculations for the values of $x_1 = 2.0$ and $x_2 = 3.5$. (**Hints:** To implement static and dynamic penalties, put the penalty value equal to zero, if a functional constraint is satisfied).

17. A shaft of circular cross-section of diameter d m and length L' m supported at two ends, is subjected to both a concentrated load of P N acting at its mid-span and torque of T Nm, as shown in Fig. 3.8. The allowable shear stress of shaft material is denoted by $\tau_{allowable}$. The shaft should be as light as possible. However, it should be able to withstand the combined loading. Assume density of shaft material $\rho = 7.8 \times 10^3$ kg/m^3; $P = 200N$; $T = 400$ Nm; $\tau_{allowable} = 11MPa$. Formulate it as a constrained optimization problem, which is to be solved using a penalty function approach and a binary-coded GA. Show a suitable representation scheme of the variables. How do you calculate the fitness of a GA-string? Assume suitable values for the constant terms of penalty function approach. Take the ranges of the variables as given below.

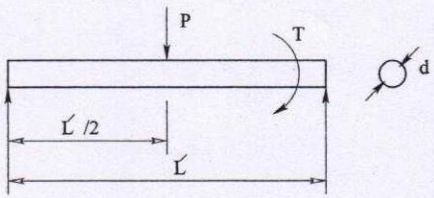

Figure 3.8: A schematic view showing combined loading of a shaft supported at its two ends.

$$0.05 \leq d \leq 0.15$$
$$0.5 \leq L' \leq 2.5.$$

(**Hints:** For this combined loading, equivalent torque $T_e = \sqrt{M^2 + T^2}$, where the maximum bending moment $M = \frac{PL'}{4}$ and shear stress $\tau = \frac{16T_e}{\pi d^3}$.)

18. In straight turning carried out on a Lathe by using a single-point tool, it is subjected to a cutting force of maginitude P. Fig. 3.9 shows the schematic view of a single-point cutting tool. Let us assume an orthogonal turning, in which the main component of the cutting force is denoted by P_z (neglect the other two components of the cutting force: P_x and P_y). Design an optimal cross-section of the cutting tool, so that it becomes as light in weight as possible after ensuring the condition of no mechanical breakage. Assume allowable stress of the tool material $\sigma_{allowable} = 170MPa$, density of the tool material $\rho = 7860kg/m^3$. Carry out optimization for a known value of $P_z = 250N$. Take a fixed length of the cutting tool $L' = 0.20m$. The design variables are allowed to vary in the ranges given below.

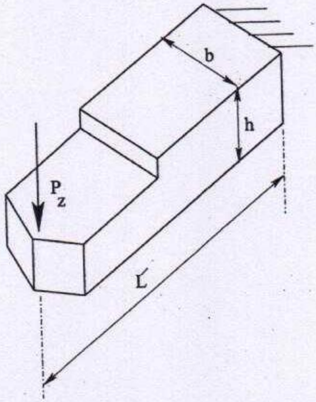

Figure 3.9: A single-point cutting tool.

$$0.015 \leq b \leq 0.07\ m,$$
$$0.01 \leq h \leq 0.06\ m$$

- Formulate it as a constrained optimization problem.
- Solve the constrained optimization problem using a binary-coded GA. Use the concept of static penalty. Show hand calculations for one generation of the GA run after assuming population size $N = 8$, $p_c = 1.0$, $p_m = 0.01$. Use 10 bits to represent each variable.

(**Hints:** Maximum bending moment $M = P_z L'$, as it can be assumed a cantilever beam. The developed stress due to this maximum bending moment is give by the expression: $\sigma = \frac{M}{I}y$, where I is the moment of inertia, and for a rectangular cross-section (of width b and hieght h), it is calculated as $I = \frac{1}{12}bh^3$; $y = \frac{h}{2}$.)

19. A beam of rectangular cross-section of height h m and width b m, and length $L' = 1.5$ m supported at two ends, is subjected to two equal concentrated loads of magnitude $P = 500\ N$ each, as shown in Fig. 3.10. The beam should be as light as possible.

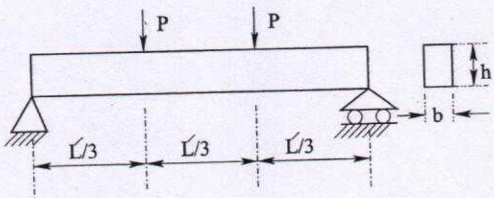

Figure 3.10: A rectangular crosssection beam supported at two ends.

However, it should be able to withstand the above loading. Assume, the density of beam material $\rho = 7.6 \times 10^3 kg/m^3$ and allowable bending stress $\sigma = 32000 N/m^2$. Formulate it as a constrained optimization problem, which is to be solved using the concept of dynamic penalty (for which assume $C = 10$, $\alpha = 1.5$, $\beta = 2$, the symbols carry usual meaning of dynamic penalty approach) in a binary-coded GA. The variables are allowed to vary in the ranges given below.

$$0.2 \leq b \leq 0.5\ m$$
$$0.4 \leq h \leq 0.7\ m$$

Show suitable representation of the variables to ensure the precision level of 0.001 m (approximately) each. Use a random population of the size $N = 4$, tournament selection, a single-point crossover with probability $p_c = 1.0$ and neglect mutation. Show only one iteration of the GA through hand calculations.
(**Hints:** Maximum bending moment $M = PL'/3$).