Indian Institute of Technology, Kharagpur

Mid-Autumn Semester Examination, 2016

Mechanical Engineering Department

Subject: Applied Elasticity (ME 60401 / ME40605)

Full Marks : 30 Time : 2 Hrs

Answer all questions. Symbols are self explanatory. Adopt reasonable assumption if it is required.

1. (a) A small straight line **OA** in an undeformed solid (Ω) has been mapped as **O′A′** in the same solid when it is deformed (Ω'). If the rectangular components of the vector represented by **OA** be dx_1, dx_2 and dx_3 while that of the vector represented by **O′A′** are dr_1, dr_2 and dr_3, prove that

$$dr_i = \left(\delta_{ik} + \frac{\partial u_i}{\partial x_k}\right) dx_k \; ; \; i, k = 1, 2, 3$$

in which u_i is the displacement field and x_k represents the material coordinate. (3)

(b) Prove that the principal shear strain axes are orthogonal to each other. (3)

2. (a) Show that the relation between the second Piola Kirchoff stress tensor ($[\Sigma]$) and the Cauchy stress tensor ($[\sigma]$) at a point in a solid is given by

$$[\Sigma] = J[F]^{-1}[\sigma][F]^{-T}$$

where $[F]$ is the deformation gradient matrix and $J = |[F]|$. (3)

(b) When the principal stresses are real? (1)

3. Show that the constitutive relation of a Green elastic solid is given by $\sigma_{ij} = \frac{\partial U}{\partial \epsilon_{ij}}$ where U is the strain energy density function. What is ϵ_{ij} called? (5+1)

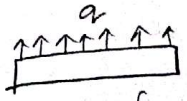
4. The elastic constant matrix of a transversely isotropic solid is given by

$$[C] = \begin{bmatrix} C_{1111} & C_{1122} & C_{1122} & 0 & 0 & 0 \\ C_{1122} & C_{2222} & C_{2233} & 0 & 0 & 0 \\ C_{1122} & C_{2233} & C_{2222} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{2323} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{1212} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{1212} \end{bmatrix}$$

Which is the axis of symmetry for this solid? Show that for isotropic solid $C_{1111} = C_{2222}$, $C_{1212} = C_{2323}$ and $C_{1122} = C_{2233}$. (5)

5. (a) What are the 15 equations of linear theories of elasticity? (3)

(b) A prismatic straight beam of rectangular transverse cross section is transversely loaded with a distributed force on its top surface. Using virtual work principle, derive the governing equations and the associated boundary conditions of the beam for obtaining the exact solutions for static response of the beam. (6)



$$\int \sigma_{xx} \delta \epsilon_{xx} \, dx + dV$$