Assignment 1

Transform Calculus (MA20101)

Section 4

To be submitted on or before 21st August (Monday) 2017

Q1) (Change of scale property)

If $\mathcal{L}\{f(t)\} = F(s)$, then show that

$$\mathcal{L}\{f(at)\} = \frac{1}{a} F(s/a).$$

Hence, evaluate the Laplace transform of the function $t\cos(6t)$.

Q2) Find $\mathcal{L}\{\sin\sqrt{t}\}$.

Q3)** Prove that if $F(s)$ indicates the Laplace transform of a piece-wise continuous function $f(t)$, then $\lim_{s\to\infty} F(s) = 0$.

Q4) Find the Laplace transform of the function $f(t)$, where $f(t)$ is given by

$$f(t) = \begin{cases} t, & 0 \le t \le 1, \\ 2-t, & 1 \le t \le 2, \\ 0, & \text{otherwise}. \end{cases}$$

Q5) Solve the initial value problem:

$$y'' - 5y' + 4y = e^{2t}, \quad y(0) = 19/12, \quad y'(0) = 8/3.$$

Q6)** Prove that if $\mathcal{L}\{f(t)\} = F(s)$ & $f(t)$ fails to be continuous at $t=a$, then

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0) - e^{-as}[f(a+) - f(a-)]$$

where $f(a+) - f(a-)$ is sometimes called the jump at the discontinuity $t = a$.

Q7) (a) Prove that $\int_0^\infty \frac{f(t)}{t}\,dt = \int_0^\infty F(u)\,du$, provided that the integrals converge.

(b) Show that $\int_0^\infty \frac{\sin t}{t}\,dt = \pi/2$.

Q8) Determine the inverse Laplace transform of the following:

(a) $\dfrac{2(2s+7)}{(s+4)(s+2)}, s > -2$,

(b) $\dfrac{s+9}{(s^2-9)}$.

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