Assignment - 1
Partial Differential Eqns.

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Q1. Find the power series solution of

$$y'' + xy' + x^2 y = 0 \quad \text{about } x = 0$$

[Ans: $y = c_0\left(1 - \frac{1}{12}x^4 + \frac{1}{90}x^6 - \cdots\right) + c_1\left(x - \frac{1}{6}x^3 - \frac{1}{40}x^5 - \cdots\right)$]

Q2. Find the power series solution of

$$y'' + (x-3)y' + y = 0 \quad \text{in powers of } x - 2.$$

[Ans; $y = c_0\left\{1 - \frac{1}{2}x(x-2)^2 - \frac{1}{6}(x-2)^3 - \frac{1}{12}(x-2)^4 - \cdots\right\}$
$+ c_1\left\{(x-2) + \frac{1}{2}(x-2)^2 - \frac{1}{6}(x-2)^3 - \frac{1}{6}(x-2)^4 - \cdots\right\}$]

Q3. Solve in series $x(1-x)y'' - 3xy' - y = 0$ near $x = 0$.

[Hints: Indicial eqn. $k(k-1) = 0$

Recurrence relation $c_m = \frac{k+m}{k+m-1} c_{m-1}$

$$y = c_0 x^k \left[1 + \frac{k+1}{k}x + \frac{k+2}{k}x^2 + \frac{k+3}{k}x^3 + \cdots\right] \quad \text{— (A)}$$

If $k = 0$, coeff. becomes infinite. So let $c_0 = k d_0$

$$\therefore y = d_0 x^k \left[k + (k+1)x + (k+2)x^2 + \cdots\right]$$

One solⁿ. $y = a(x + 2x^2 + 3x^3 + \cdots) = au$

2nd solⁿ by putting $k = 1$ in (A), $y = c_0(x + 2x + 3x^2 + \cdots)$
which is not L.I. to the previous one.

So $\left(\frac{\partial y}{\partial k}\right)_{k=0}$ is the other solⁿ.

$$\left(\frac{\partial y}{\partial k}\right)_{k=0} = b\left[u \ln x + (1 + x + x^2 + \cdots)\right] = bv$$]

*** The End ***