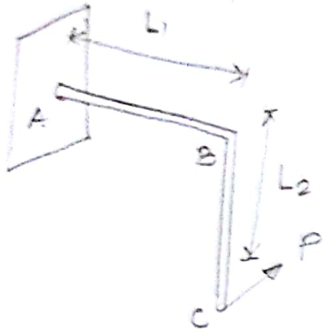
B/S-5.35

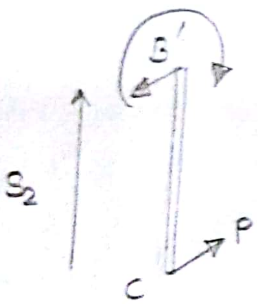
Que.



→ We consider 2 body fitted co-ordinate variables to track the lenght of the structure.

For CB : s_2 and for BA : s_1

For CB take a section at a distance s_2 from C.

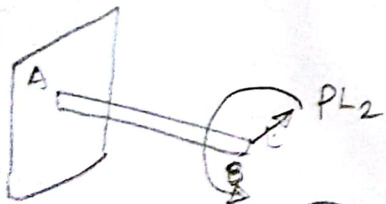


$M - Ps_2 = 0$

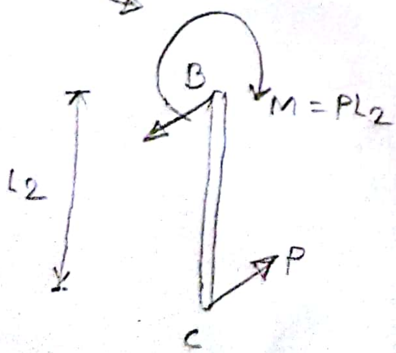
No axial force, no torsion.

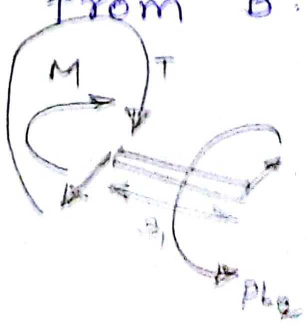
For BA :

- Step 1 : Make a cut @ B & consider BA only



- Step 2 : Take a section at a distance s_1 from B.





M = Bending moment

For deflection at C :

$$U = U_M + U_T$$

or

$$U = U_{CB} + U_{BA}$$

$$= \underbrace{\int_0^{L_2} \frac{M^2}{2EI}\,dx}_{CB} + \underbrace{\int_0^{L_1} \frac{M^2}{2EI}\,dx + \int_0^{L_2} \frac{T^2}{2GJ}\,dx}_{BA}$$

$$\therefore\ d = \frac{\partial U}{\partial P} = \int_0^{L_2} \frac{M}{EI}\frac{\partial M}{\partial P}\,ds_2 + \int_0^{L_1} \frac{M}{EI}\frac{\partial M}{\partial P}\,ds_1 + \int_0^{L_1} \frac{T}{GJ}\frac{\partial T}{\partial P}\,ds_1$$

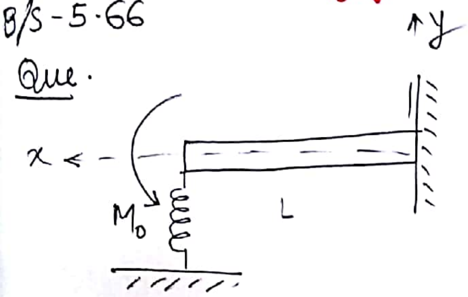
$$\therefore\ d = \int_0^{L_2} \frac{P s_2^2}{EI}\,ds_2 + \int_0^{L_1} \frac{P s_1^2}{EI}\,ds_1 + \int_0^{L_1} \frac{P b_2^2}{GJ}\,ds_1$$

$$\boxed{d = \frac{P L_2^3}{3EI} + \frac{P L_1^3}{3EI} + \frac{P L_2^2 L_1}{GJ}}$$

* Strain energy due to spring :—

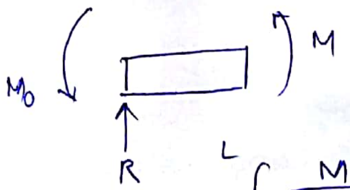
B/S-5.66

Que.



Find the force on the beam by the spring.

→ Take a cut section at a distanse 's' from the left end.



$$M + M_0 - Rs = 0$$

$$U = \int_0^L \frac{M^2}{2EI}\,dx + \underbrace{\frac{1}{2}k\,\nabla^2}_{\text{WD by force 'R'}}$$

$$d = \frac{\partial U}{\partial R} = \frac{\partial}{\partial R}\left[\int_0^L \frac{M^2}{2EI}\,dx + \frac{1}{2}k\frac{R^2}{k^2}\right]$$

$$= \frac{\partial}{\partial R}\left[\int_0^L \frac{(-M_0 + Rs)^2}{2EI}\,ds + \frac{R^2}{2k}\right]$$

$$0 = \frac{1}{EI}\int_0^L (-M_0 + Rs)\, s \cdot ds + \frac{R}{k}$$

$$0 = \frac{1}{EI}\left[-\frac{M_0 L^2}{2} + \frac{RL^3}{3} \right] + \frac{R}{k}$$

$$\therefore \quad R\left[\frac{L^3}{3EI} + \frac{1}{k} \right] = M_0 \frac{L^2}{2EI}$$

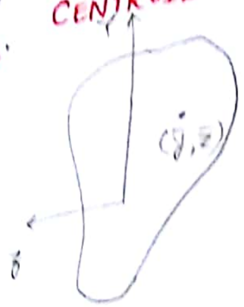
$$\therefore \quad \boxed{R = \frac{M_0 L^2 / 2EI}{L^3/3EI + 1/k}}$$

* Solve the example problems and then exercise from L.S. Srinath.
* Ex. problems from Boresi and Schmidt.

ASYMMETRICAL BENDING OF BEAMS

PRELIMINARY CONCEPTS –

1. CENTROID :



$A\bar{y} = \int_A y \cdot dA$

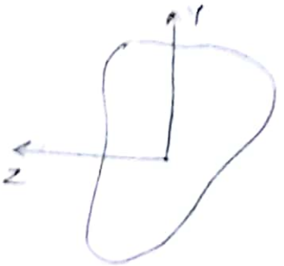
$A\bar{z} = \int_A z \cdot dA$

If we shift origin to the centroid.

$0 = \int_A y \cdot dA \quad ; \quad 0 = \int_A z \cdot dA$

* Centroidal axes are the ones which pass through centroid or are those about which the first moments of area are zero

• 2^{nd} Moment of Area:



$I_z = \int_A y^2 \cdot dA$

$I_y = \int_A z^2 \cdot dA$

$I_{yz} = \int_A yz \cdot dA$

⇒ If the axes are so chosen that $I_{yz} = 0$, then they are referred to as principal axes.

* Neutral Axis :

• The axis in a c/s along which the bending stress is zero.

• For pure bending, the neutral axis passes through the centroid.

For pure bending : $\boxed{\int_A \sigma_{xx} \cdot dA = 0}$

{ i.e the axial force over a c/s is zero }

• If we use $\sigma_{xx} = \frac{My}{I}$, $\int \frac{My}{I} dA = 0$

$\Rightarrow \int y \cdot dA = 0$ ✓

{ centroidal axis } ✓

{ works only if refred to neutral axis }

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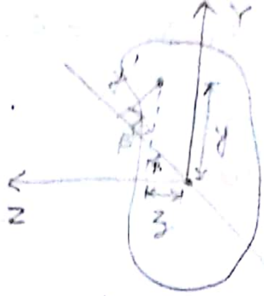
$\sigma_{xx} = ky'$

Represent y' in terms of y & z.

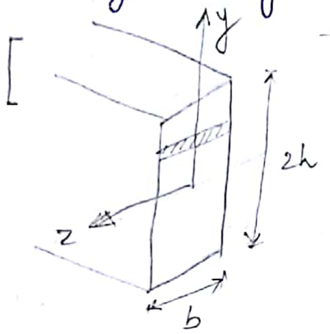


$$y = \frac{y'}{\sin\beta} + \frac{z}{\tan\beta}$$

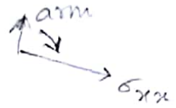
[origin is at centroid]

$$\Rightarrow \quad y' = y\sin\beta - z\cos\beta$$

[



$$M_z = -\int_{-h}^{h} y \cdot \sigma_{xx} b \cdot dy$$]



(Along -z)

$$\therefore \quad M_z = -\int \sigma_{xx} \cdot y^* \, dA$$

$$M_y = \int \sigma_{xx} \cdot z \cdot dA$$

$$\therefore \quad -M_z = \int_A k(y\sin\beta - z\cos\beta)\, y \, dA$$

$$= k\left[\int_A y^2 \sin\beta \, dA - \int_A yz\cos\beta \, dA\right]$$

$$= k\left[I_z \sin\beta - I_{yz}\cos\beta\right] \quad \text{——— (1)}$$

$$\therefore \quad M_y = \int_A k(y\sin\beta - z\cos\beta)\, z \, dA$$

$$= k\left[I_{yz}\sin\beta - I_y \cos\beta\right] \quad \text{——— (2)}$$

[M_y, M_z are given to us : They are the applied bending moment components]

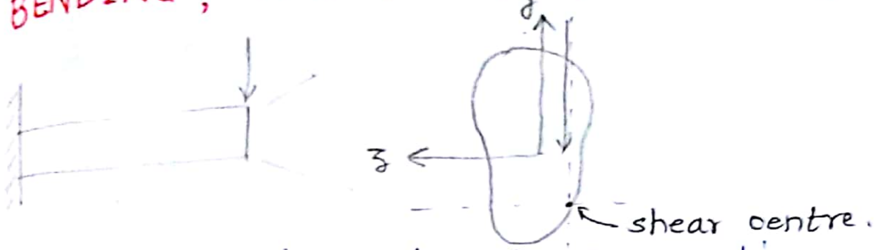
$$-\frac{M_y}{M_z} = \frac{I_{yz}\sin\beta - I_y\cos\beta}{I_z\sin\beta - I_{yz}\cos\beta}$$

$$= \frac{I_{yz}\tan\beta - I_y}{I_z\tan\beta - I_{yz}} \longrightarrow \tan\beta = ✓$$

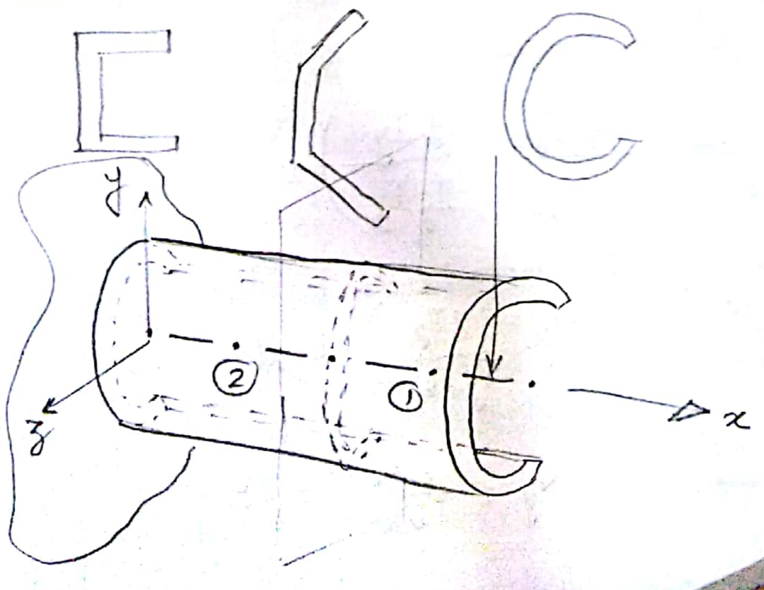
After Finding β, find k from M_y/M_z.
Then use $\sigma_{xx} = ky'$ to obtain general flexure formula:

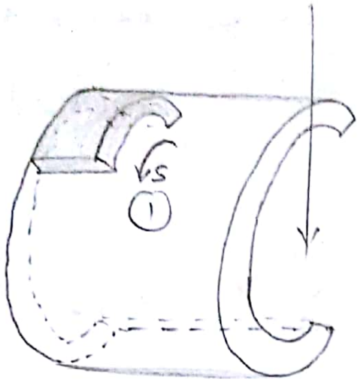
$$\sigma_{xx} = \frac{M_y(y I_{yz} - z I_z) - M_z(z I_{yz} - y I_y)}{I_{yz}^2 - I_y I_z}$$

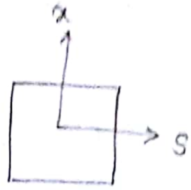
BENDING, TWISTING AND SHEAR CENTRE :—



- It is possible to find a line of application for each of the horizontal and vertical components of a general oblique load such that the twisting induced is zero. The point of intersection of there 2 lines of application is called the shear centre.
- For irregular c/s it is very difficult to find stress distribution and the shear centre.
- It is relatively easier to find shear centre for thin walled, open sections.

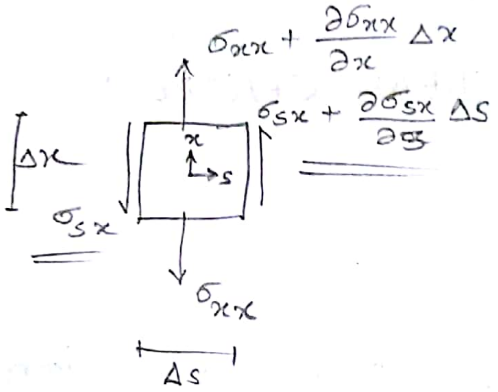






Top view

t is an avg.
t_s is thickness along the c/s @ particular.



$$\underline{x\text{-dir}}: \ (-\sigma_{xx} t \Delta s) + \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\Delta x\right) t \Delta s$$

$$- \sigma_{sx} t_s \Delta x$$

~~s-dir~~:

$$\cancel{-\sigma_{sx} t_s \Delta x} \quad + \left(\sigma_{sx} t_s + \frac{\partial (\sigma_{sx} t_s)}{\partial s}\Delta s\right)\Delta x \quad = 0$$

$$\Rightarrow \frac{\partial \sigma_{xx}}{\partial x}\Delta x \cdot t \cdot \Delta s + \frac{\partial(\sigma_{sx} t_s)}{\partial s}\Delta s \, \Delta x = 0$$

$$\Rightarrow \frac{\partial(\sigma_{sx} t_s)}{\partial s} = -\frac{\partial \sigma_{xx}}{\partial x} t \qquad \text{—— (1)}$$

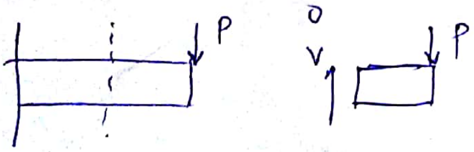
- From the flexure formula:

$$\sigma_{xx} = -\frac{M_z (z I_{yz} - y I_y)}{I_{yz}^2 - I_y I_z} \qquad \text{—— (2)}$$

$$\therefore \frac{\partial(\sigma_{sx} t_s)}{\partial s} = \left(\frac{\partial M_z}{\partial x}\right)\left(\frac{z I_{yz} - y I_y}{I_{yz}^2 - I_y I_z}\right) t$$

Integrate from 0 to s w.r.t s:

$$\therefore \sigma_{sx} t_s = \int_0^s \left(\frac{\partial M_z}{\partial x}\right)\left(\frac{z I_{yz} - y I_y}{I_{yz}^2 - I_y I_z}\right) \cdot t \cdot ds$$



$$\boxed{V_y = \frac{\partial M}{\partial x} = P}$$

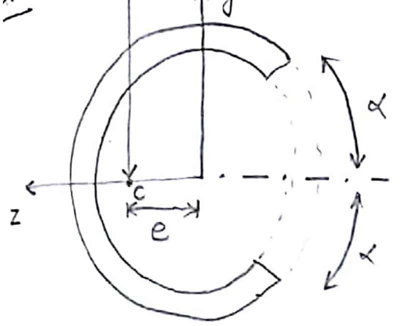
$$\sigma_{sx} t_s = \int_0^s V_y \left(\quad \right) \cdot t \cdot ds$$

$$= \frac{P}{I_{yz}^2 - I_y I_z} \int_0^s \left(z I_{yz} t - y I_y t \right) \cdot ds$$

$$= \frac{P}{I_{yz}^2 - I_y I_z} \left[I_{yz} Q_y - I_y Q_z \right]$$

where, $Q_y = \int_0^s z t \cdot ds$; $Q_z = \int_0^s y t \cdot ds$

Que. Find the shear centre for the c/s

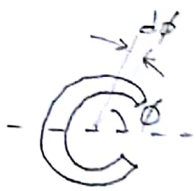


- For symmetric c/s, one line of action coincides with the axis of symmetry. (here, z axis)

- Then, we assume a loading and consider pt. C as the shear centre.
- Choice of co-ordinate axis - ensures $I_{yz} = 0$

$$\therefore \sigma_{sx} = \frac{P}{-I_y I_z t_s} \left(- I_y Q_z \right)$$

$$= \frac{P Q_z}{I_z t_s}$$



$$Q_z = \int_0^s y t \cdot ds$$

$$= \int_\alpha^\phi (t \cdot \underbrace{R d\phi}_{ds}) \cdot R \sin\phi$$

$$I_z = \int_\alpha^{2\pi - \alpha} (R \sin\phi)^2 \cdot \underbrace{(t R d\phi)}_{dA}$$

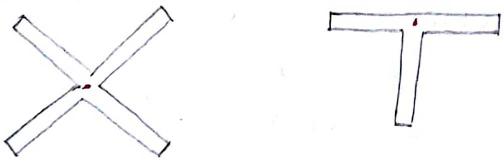
$I_z = \int y^2 dA$

To find the location of shear centre, balance the twisting moment due to P by the twisting moment due to the distribution of σ_{sx}.

$$\Rightarrow \quad P\cdot e = \int_{\alpha}^{2\pi-\alpha} \underbrace{\sigma_{sx}\,(\underbrace{t\cdot R\,d\phi}_{dA})}_{dF} \cdot R$$

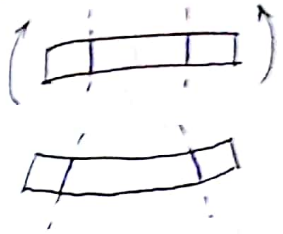
[Go through example problem on channel section]

For such c/s, intersection of rectangles is the location of the shear centre.

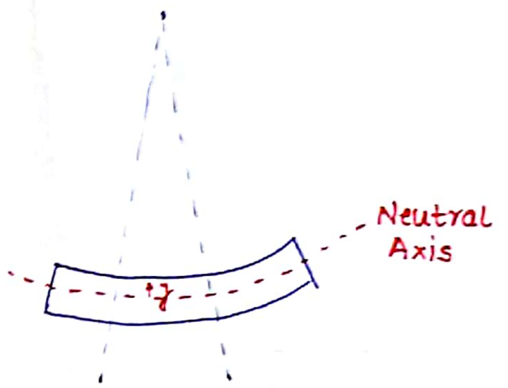


Friday
26/10/2018

Euler, Bernoulli Hypothesis :



- Planes remain planes
- Lenght of vertical lines remain same.



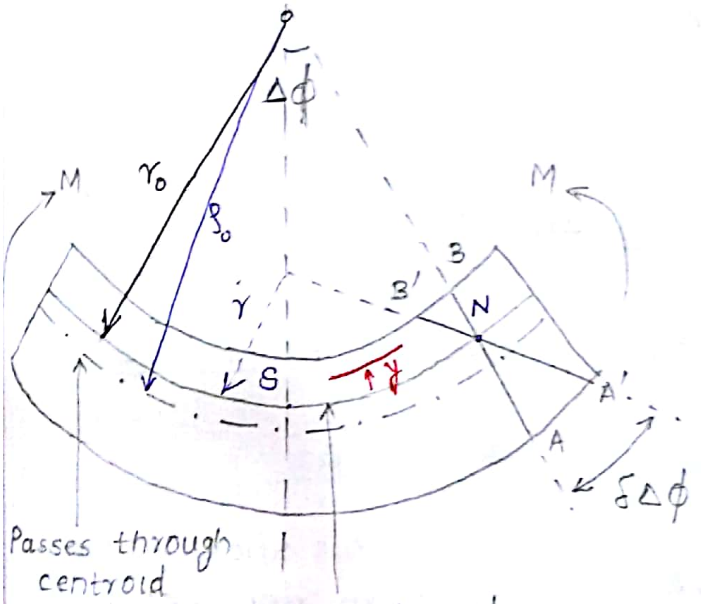
$$l' = (R - y)\theta$$

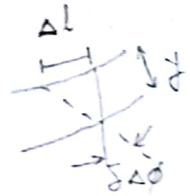
$$l = R\theta$$

$$\varepsilon = \frac{l' - l}{l}$$

$$= \frac{-y}{R}$$

y : Distance from neutral axis ...





$$\Delta l = y\, \delta\Delta\phi$$

$$\therefore \quad \varepsilon = -\frac{y(\delta\Delta\phi)}{(r_0 - y)(\Delta\phi)}$$

Now, we want to $\frac{\delta\Delta\phi}{\Delta\phi}$.

Consider SN in 2 different ways :

$$SN = r_0 \Delta\phi \quad \text{—— ①}$$

$$SN = r(\Delta\phi + \delta\Delta\phi) \quad \text{—— ②}$$

$$\therefore \quad r_0 \Delta\phi = r(\Delta\phi + \delta\Delta\phi)$$

$$(r_0 - r)\Delta\phi = r\,\delta\Delta\phi$$

$$\Rightarrow \quad \frac{\delta\Delta\phi}{\Delta\phi} = \frac{r_0 - r}{r} = r_0\left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\therefore \quad \varepsilon = -\left(\frac{y}{r_0 - y}\right) r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) \quad \text{—— (1)}$$

$$\Rightarrow \quad \sigma_{xx} = E\,\varepsilon_{xx}$$

$$= -\frac{Ey}{r_0 - y}\, r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right)$$

$$\Rightarrow \quad \lim_{r_0 \to \infty} \sigma_{xx} = \frac{-Ey}{\left(1 - \frac{y}{r_0}\right)}\left(\frac{1}{r} - \frac{1}{r_0}\right) = -\frac{Ey}{r}$$

[Remember : $EI = MR$

$\sigma_{xx} = -\frac{My}{I}$]

$$\rightarrow \quad \int \sigma_{xx} \cdot y \cdot dA = -M$$

* For equilibrium : $\int_A \sigma_{xx} \cdot dA = 0$ —— (1)

$$\int_A \sigma_{xx} \cdot y \cdot dA = 0 \quad \text{—— (2)}$$

From (1) : $\int_A \left(\frac{-Ey}{r_0 - y}\right) r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) dA = 0$

$\therefore \int_A \left(\frac{y}{r_0 - y}\right) dA = 0$ } This shows that NA does not pass through the centroid ...

From (2) : $\int_A \left(\frac{-Ey}{r_0 - y}\right) r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) y \cdot dA = \cancel{-}M$

$$\therefore \quad \cancel{-}E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) \int_A \left(\frac{y^2}{r_0 - y}\right) dA = \cancel{-}M$$

$$\therefore \quad E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) \int_A \left(\frac{y^2 - r_0 y + r_0 y}{r_0 - y}\right) dA = M$$

$$\therefore \quad E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) \left[\int_A (-y)\, dA + r_0 \underbrace{\int \frac{y}{r_0 - y}\, dA}_{0}\right] = M$$

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$$E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) \int_A (-y)\, dA = M$$

$$E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) A(-e_n) = M$$

$y' = y + e_n$
$y = y' - e_n$
$\int (y' - e_n)\, dA$
$= \int y'\, dA - \int e_n\, dA$
About centroidal axis

$$M = E r_0 \left(\frac{1}{r} - \frac{1}{r_0}\right) A e_n$$

$$\therefore \boxed{\sigma_{xx} = \frac{-y}{r_0 - y} \frac{M}{A e_n}} \longrightarrow \text{Winkler Bach Formula ...}$$

- This formula is derieved by taking the origin on the neutral axis.
- If we shift origin to the centroid :

$$\rho_0 = r_0 + e_n$$

$$y' = y + e_n$$

$$\therefore \sigma_{xx} = -\frac{y' - e_n}{\rho - y'} \frac{M}{A e_n}$$

Find r_0

- We have been given with ρ_0.

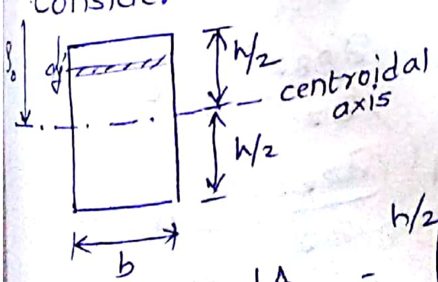
Let $u = r_0 - y$

$$\int_A \frac{y}{r_0 - y}\, dA = 0$$

$$\therefore \int \frac{r_0 - u}{u}\, dA = 0$$

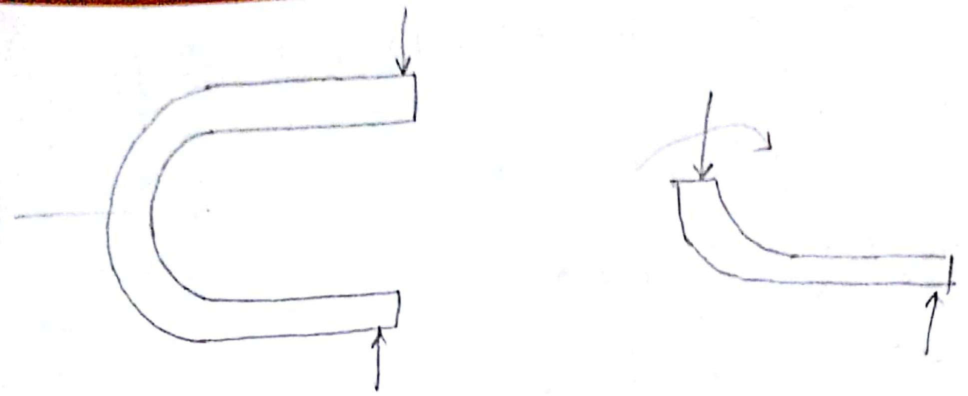
$$\Rightarrow r_0 \int \frac{dA}{u} = A \implies r_0 = \frac{A}{\int \frac{dA}{u}}$$

Consider a rectangular c/s



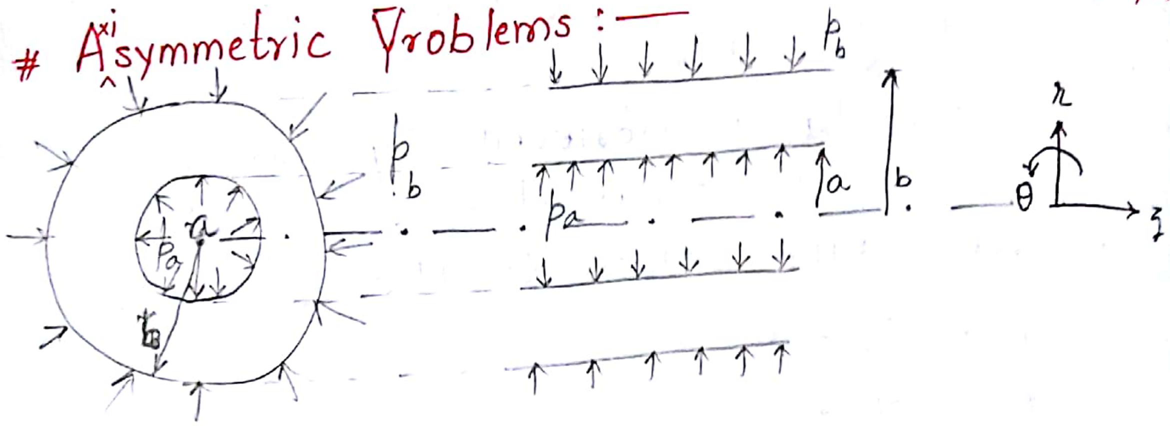
$u = r_0 - y$
$= \rho_0 - y'$ $=$
$dA = b \cdot dy'$
$dy' = -du$

Now $$\int \frac{dA}{u} = \int_{-h/2}^{h/2} \frac{b \cdot dy'}{u} = \int_{\rho_0 + h/2}^{\rho_0 - h/2} \frac{-b \cdot du}{u} = \int_{\rho_0 - \frac{h}{2}}^{\rho_0 + \frac{h}{2}} \frac{b \cdot du}{u}$$



Monday
29/10/2018

Axisymmetric Problems :—



End-on view

- open end pressure vessel — closed end pressure vessel

- axisymmetric — $\frac{\partial (\,)}{\partial \theta} = 0$
- no θ-comp. of displacement : $u_\theta = 0$
- $\nabla \cdot \underset{\approx}{\sigma} = 0$

$$(a)\quad \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\left[\cancel{\frac{\partial \sigma_{rr}}{\partial \theta}} + \sigma_{rr} - \sigma_{\theta\theta}\right] + \cancel{\frac{\partial \sigma_{rr}}{\partial z}} = 0 \quad —①$$

$$(b)\quad \cancel{\frac{\partial \sigma_{\theta r}}{\partial r}} + \frac{1}{r}\left[\cancel{\frac{\partial \sigma_{\theta\theta}}{\partial \theta}} + 2\cancel{\sigma_{r\theta}}\right] + \cancel{\frac{\partial \sigma_{\theta z}}{\partial z}} = 0 \quad —②$$

$$(c)\quad \cancel{\frac{\partial \sigma_{zr}}{\partial r}} + \frac{1}{r}\left[\cancel{\frac{\partial \sigma_{zz}}{\partial \theta}} + \sigma_{zr}\right] + \frac{\partial \sigma_{zz}}{\partial z} = 0 \quad —③$$

- $\sigma_{r\theta} = 2G\epsilon_{r\theta}$

$$= 2G\left[\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right)\right] = 0$$

- $\sigma_{z\theta} = 2G\epsilon_{\theta z}$

$$= 2G\,\frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) = 0$$