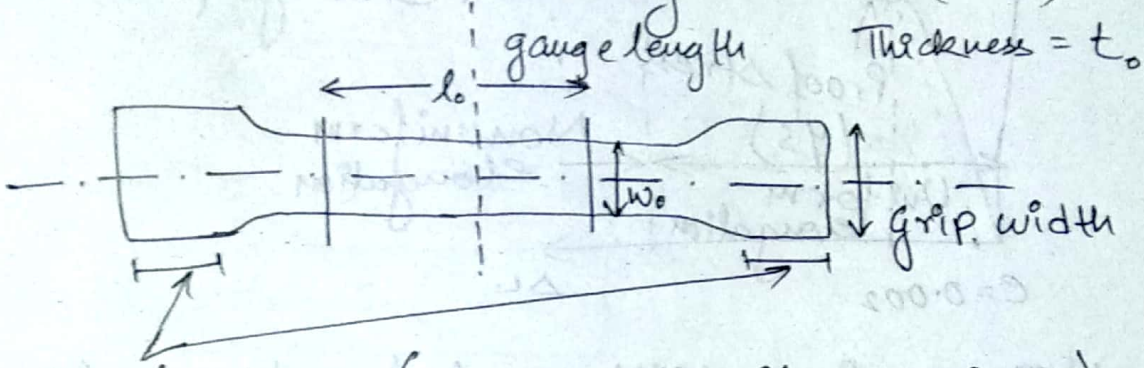
Prof. Sushanta Kumar Panda (SKP)

1) Mechanical Properties — 23rd July

2) Casting 3) Sheet metal forming

* Stress - strain

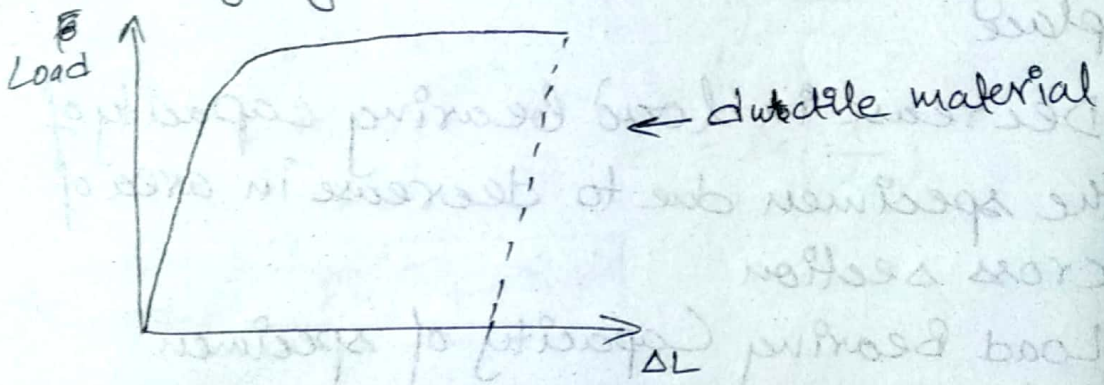
Universal Tensile testing machine (UTM)



(ASTM, tensile specimen)

Load Cell ⇒ measures the load applied

Strain Gauge ⇒ measures the strain.



initial Area = $A_0 = w_0 \times t_0$

Engineering Stress (S) = $\frac{F}{A_0}$

Engineering Strain (e) = $\frac{\Delta l}{l_0}$

Even for S & e the curve holds

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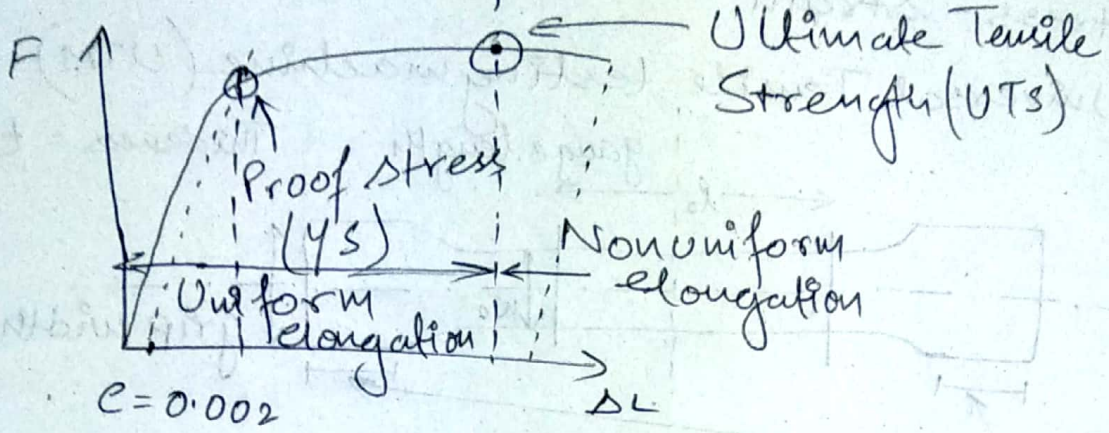
At 0.2% strain i.e. strain = 0.002
Draw a line ∥ to linear regime line and the point where it intersects the curve is known as Proof Stress



UTS is instability point where necking takes place.

*) During Tensile test, two phenomenon take place

1) Decrease in load bearing capacity of the specimen due to decrease in area of cross section
2) Load Bearing Capacity of specimen increases due to strain hardening

From YS to UTS ⇒ strain hardening dominates

after UTS ⇒ Reducing Area of cross-section dominates.

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* True Stress

$$\sigma = \frac{F}{A}$$ A = instantaneous area of cross-section.

$$\Rightarrow \sigma = \frac{F}{A_0} \times \frac{A_0}{A}$$

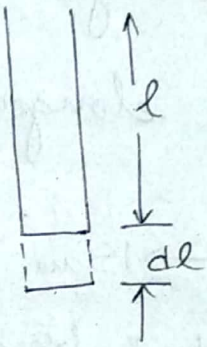
$$A_0 l_0 = A l$$

$$\Rightarrow \frac{A_0}{A} = \frac{l}{l_0}$$

$$\Rightarrow \sigma = \frac{F}{A_0} \times \frac{l}{l_0} = S \times \left(\frac{l_0 + \Delta l}{l_0}\right)$$

$$\Rightarrow \boxed{\sigma = S(1+e)}$$

* True Strain



$$d\epsilon = \frac{dl}{l}$$

$$\therefore \epsilon = \int_{l_0}^{l} \frac{dl}{l}$$

$$\Rightarrow \epsilon = \ln\left(\frac{l}{l_0}\right)$$

$$\Rightarrow \boxed{\epsilon = \ln(1+e)}$$

Q1) A bar of length 'l_0' is uniformly extended until it's length $l = 2l_0$

a) Compute value of engineering strain & true strain

b) To what final length must a bar of length 'l_0' be compressed if strains are to be same except the sign in a)

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a) $e = \frac{l_0}{l_0} = 1$

$\epsilon = \ln 2 = 0.693$

b) Now, $\epsilon = -\ln 2$

$\Rightarrow \ln(1+e) = \ln(1/2)$

$e = -1/2$

True strain for equivalent amount of tension & compression are same except the sign.

Q2) A 10cm long bar is elongated to 20 cm in 3 steps.

1) $10 \rightarrow 12$cm 2) 12cm $\rightarrow 15$cm 3) $15 \rightarrow 20$cm

a) Calculate engineering strain for each step & compare it with overall engineering strain.

b) Repeat for true strain

a) $e_{overall} = 1$

$e_1 = \frac{2}{10}$, $e_2 = \frac{3}{12}$, $e_3 = \frac{5}{15}$

$e_1 + e_2 + e_3 = 0.2 + 0.25 + 0.33$

$= 0.78$

Sum is less than overall

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b) $\epsilon_{overall} = \ln(1+1) = \ln 2 = 0.693$

$\epsilon_1 = \ln(1+0.2)$

$\epsilon_2 = \ln(1+0.25)$

$\epsilon_3 = \ln(1+0.33)$

$\epsilon_1 + \epsilon_2 + \epsilon_3 = \ln\left(\frac{12}{10} \times \frac{15}{12} \times \frac{20}{15}\right)$

$= \ln(2) = \epsilon_{overall}$

$\therefore$ $\boxed{\text{True strain is additive in nature}}$

Let a cuboid $\rightarrow$ $l_0 \times w_0 \times t_0$

$\Downarrow$ After deformation

$l \times w \times t_0$

Volume remains same

$\Rightarrow l_0 \times w_0 \times t_0 = l \times w \times t$

$\Rightarrow \frac{l}{l_0} \times \frac{w}{w_0} \times \frac{t}{t_0} = 1$

$\Rightarrow \ln\left(\frac{l}{l_0}\right) + \ln\left(\frac{w}{w_0}\right) + \ln\left(\frac{t}{t_0}\right) = 0$

$\Rightarrow \boxed{\epsilon_l + \epsilon_w + \epsilon_t = 0}$

5) Express Volume constancy condition in engineering strain.

$(1+e_l) \times (1+e_w) \times (1+e_t) = 1$

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* Hollomon Power hardening law,

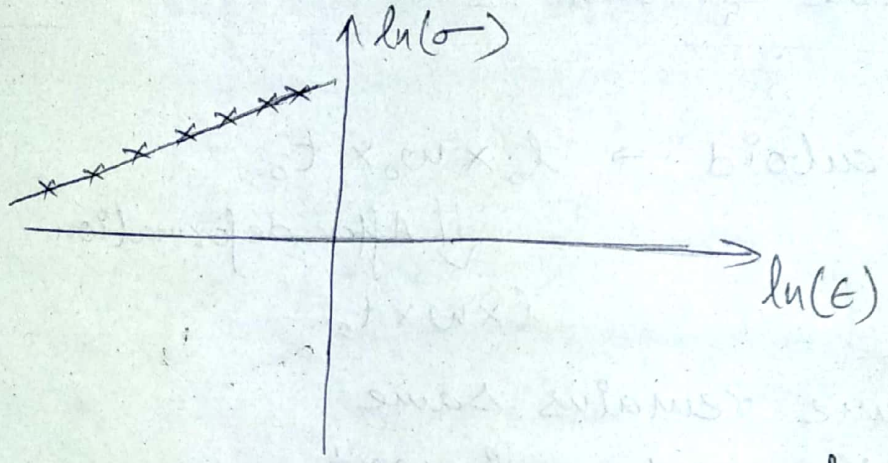
From YS to UTS

$$\sigma = K\epsilon^n$$

K = strength coefficient

n = strain hardening exponent.

$$\ln(\sigma) = n\ln(\epsilon) + \ln(K)$$



Strain hardening = strengthening of ~~material~~ metal due to plastic deformation.

Strain hardening :-

It occurs due to dislocation movements & dislocation ~~movements~~ generation within the crystal structure of material.

Sometimes it may be undesirable as in machining, the early passes of the cutter harden the workpiece causing damage to cutter in later stages.

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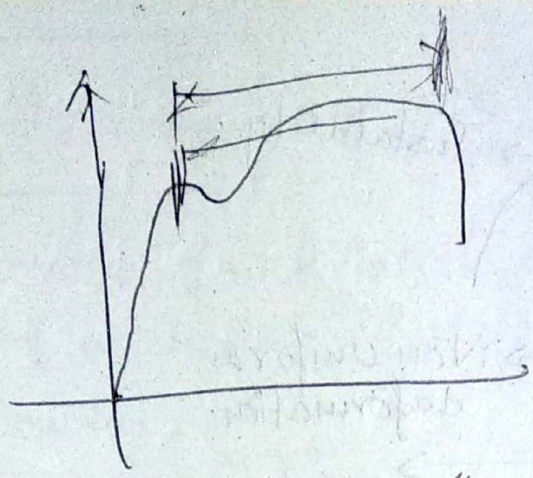

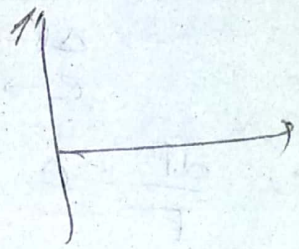

n- value

30th July 18

YS to UTS

$$\sigma = K\epsilon^n$$

Physical Significance of n:-

$$F = \sigma A$$

$$\Rightarrow dF = \sigma dA + A d\sigma$$

$\Rightarrow$ Dividing by F bothsides

$$\frac{dF}{F} = \frac{\sigma dA + A d\sigma}{F} = \frac{\sigma dA + A d\sigma}{\sigma A}$$

$$\Rightarrow \boxed{\frac{dF}{F} = \frac{dA}{A} + \frac{d\sigma}{\sigma}}$$

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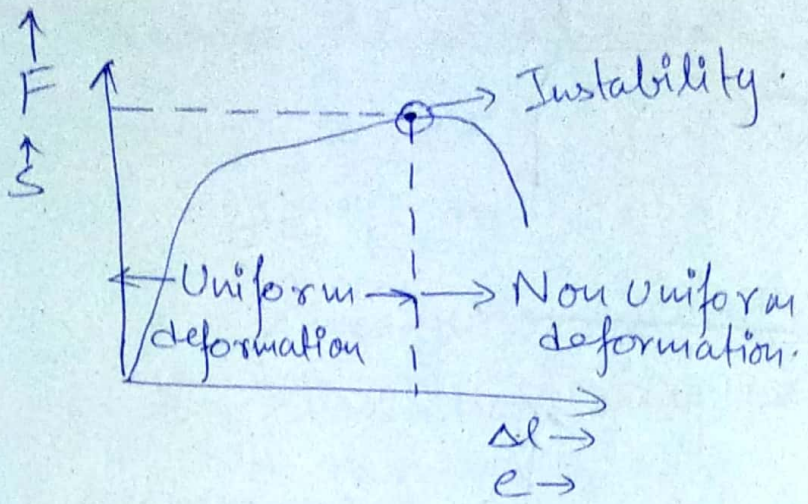

at UTS, $\frac{dF}{F} = 0$ [$\because F = \text{max}^{\underline{m}}$]

$$\Rightarrow \boxed{\frac{dA}{A} + \frac{d\sigma}{\sigma} = 0}$$

Also $V = Al$

$$\Rightarrow \underbrace{\cancel{\frac{dV}{V}}}_{0} = \frac{dA}{A} + \frac{dl}{l}$$

[$\because$ volume is constant]

$$\Rightarrow \frac{d\sigma}{\sigma} + \left(-\frac{dl}{l}\right) = 0$$

$$\Rightarrow \frac{d\sigma}{\sigma} = \frac{dl}{l} = d\epsilon$$

$$\Rightarrow \boxed{\frac{d\sigma}{d\epsilon} = \sigma} \text{ at UTS}$$

$\sigma = K\epsilon^n$

$\Rightarrow K n \cdot \epsilon^{n-1} = K\epsilon^n$

Let $\epsilon = \epsilon_u$ at UTS

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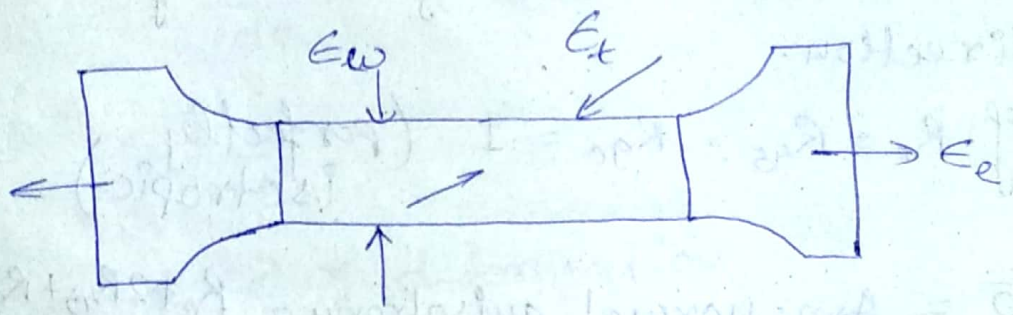
$\Rightarrow \boxed{n = \epsilon_u}$ at UTS

The n-value of materials is equal to uniform strain of material (if Holoman hardening law is followed)

if material has higher n-value, its uniform elongation (ductility) will be more.

Anisotropy



$\epsilon_l + \epsilon_w + \epsilon_t = 0$

There is no loading in width or thickness

Hence the state of strain is same

So $\epsilon_w = \epsilon_t$ [for isotropic material]

$\epsilon_l + 2\epsilon_w = 0$

$\boxed{\epsilon_w = -\frac{\epsilon_l}{2}}$ for isotropic materials only.

Lankford anisotropy parameter $= R = \frac{\epsilon_w}{\epsilon_t}$

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or $R = \dfrac{\epsilon_w}{-(\epsilon_w + \epsilon_l)}$

Rolled sheets have high anisotropy.

$R_0 \rightarrow$ Rolling direction

$R_{45°} \rightarrow$ Diagonal direction

$R_{90} \rightarrow$ Transverse direction

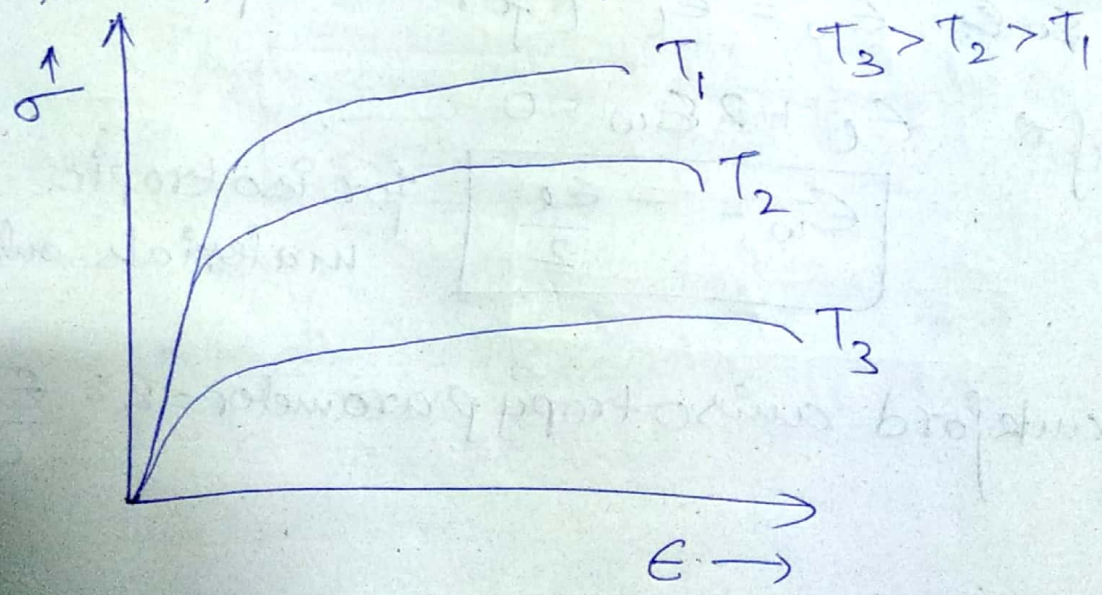
The Lankford anisotropy parameter, R are determined along Rolling direction, Transverse direction & 45° Diagonal direction.

if $R_0 = R_{45} = R_{90} = 1$ (perfectly isotropic)

$\bar{R}$ = Avg. normal anisotropy $= \dfrac{R_0 + 2R_{45} + R_{90}}{4}$

ΔR = Planar anisotropy $= \dfrac{R_0 - 2R_{45} + R_{90}}{2}$

*Effect of temperature & strainrate



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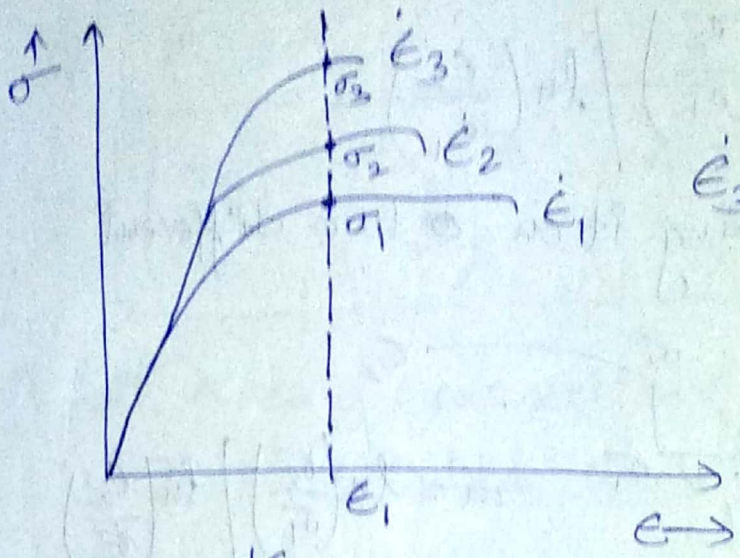

$\dot{\epsilon}_3 > \dot{\epsilon}_2 > \dot{\epsilon}_1$

$$\dot{\epsilon} = \frac{d\epsilon}{dt}$$

$$= \frac{\frac{dl}{l}}{dt} = \frac{1}{l}\frac{dl}{dt} = \frac{u}{l}$$

$$\Rightarrow \dot{\epsilon} = \frac{u}{l}$$

u = speed of deformation,

So we have two types of UTM,

1) speed constant
2) $\dot{\epsilon}$ constant

if material strain changes due to strain rate, then it is called strain rate sensitive material.

So $\boxed{\sigma = C\,\dot{\epsilon}^m}$ at a constant strain

where C = strain coefficient

m = strain rate sensitivity index.

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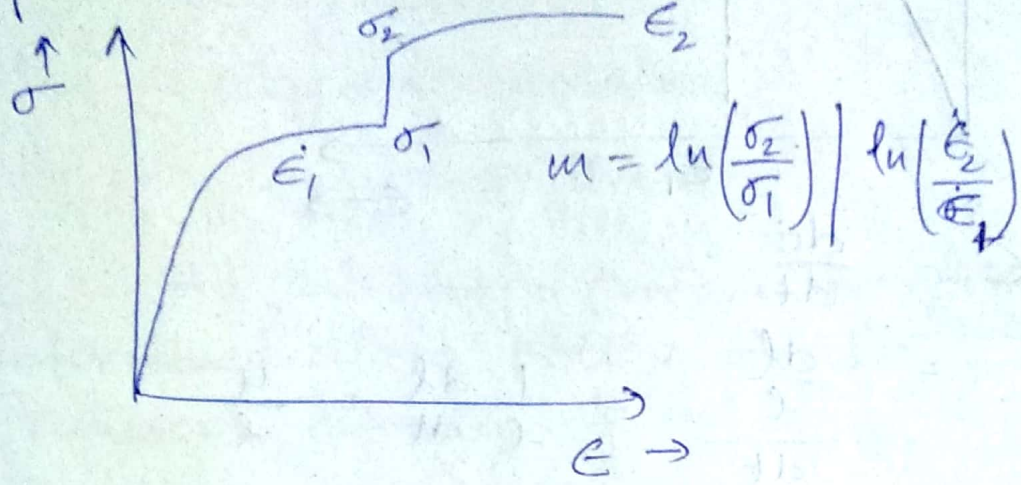
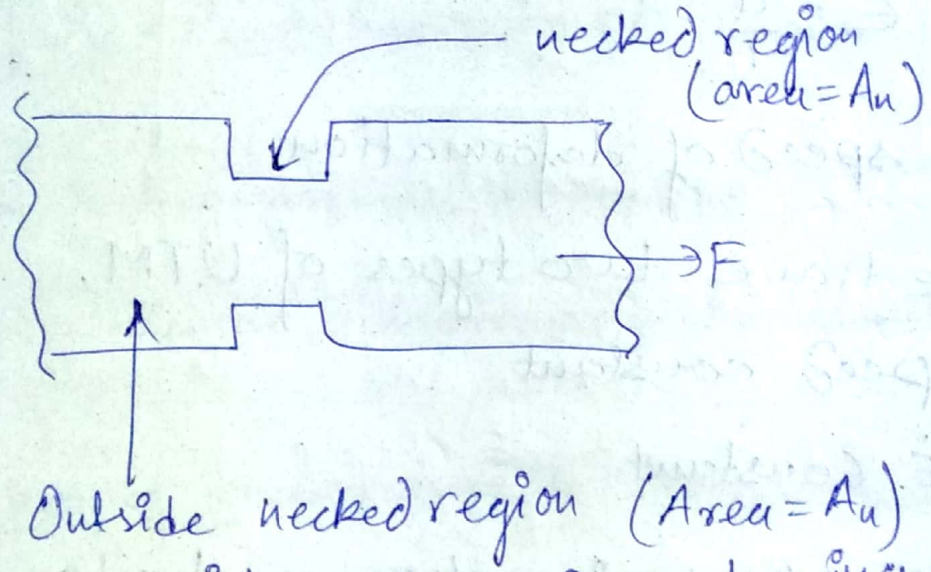
$$m = \ln\left(\frac{\sigma_2}{\sigma_1}\right) \Big/ \ln\left(\frac{\dot{\epsilon}_2}{\dot{\epsilon}_1}\right)$$

So rather doing it on two different specimen





Let us consider a specimen having necked region (A_n). Outside Area of cross-section is A_u.

$$F = \sigma_u A_u = \sigma_n A_n$$

$$\boxed{\frac{\sigma_u}{\sigma_n} = \frac{A_n}{A_u}}$$

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& $\sigma = C\dot{\epsilon}^m$

$$\Rightarrow \boxed{\left(\frac{\dot{\epsilon}_u}{\dot{\epsilon}_n}\right) = \left(\frac{\sigma_u}{\sigma_n}\right)^{1/m} = \left(\frac{A_n}{A_u}\right)^{1/m}}$$

Q) Let Area of Cross section of necked region is 90% of Outside neck.

<u>Case I</u> Let $m = 0.02$

$$\frac{\dot{\epsilon}_u}{\dot{\epsilon}_n} = (0.9)^{50}$$

<u>Case II</u> Let $m = 0.5$

$$\frac{\dot{\epsilon}_u}{\dot{\epsilon}_n} = (0.9)^{\frac{1}{0.5}} = 0.81$$

if m value is higher, then post uniform elongation will be higher.

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6th Aug 2018

Q1) The plastic behaviour of a metal can be expressed as

$$\sigma = 500\,\epsilon^{0.5} \quad MPa$$

estimate the strength of a bar of this material, if it is uniformly cold washed to a reduction of $r = 0.3$ (Reduction in area of crossection)

$$\frac{A_0 - A}{A_0} = r = 0.3$$

$$1 - \left(\frac{R}{R_0}\right)^2 = 0.3$$

$$\Rightarrow \quad \frac{R}{R_0} = \sqrt{0.7}$$

$$\pi R^2 l = \pi R_0^2 l_0$$

$$\frac{l}{l_0} = \left(\frac{R_0}{R}\right)^2 = \frac{1}{0.7} \Rightarrow \frac{l}{l_0} = \frac{10}{7}$$

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$$\epsilon = \ln\left(\frac{l}{l_0}\right) = \ln\left(\frac{10}{7}\right)$$

$$\sigma = 500\left(\ln\left(\frac{10}{7}\right)\right)^{0.5}$$

$$\boxed{\sigma = 298.6 \text{ MPa}}$$

Q2) (a) Strain hardening behaviour of a low carbon steel is given by ~~σ=~~

$$\sigma = 700\,\epsilon^{0.2} \text{ MPa}$$

estimate strength of bar if it is cold worked to 50% reduction.

(b) Suppose another bar of same steel was cold worked an unknown amount & then further cold worked 15% more; & found to have yeild strength of 525 MPa. What was the unknown amount of cold worked

a) $\epsilon = \ln\left(\frac{1}{1-r}\right) = \ln(2)$

$$\sigma = 700(\ln 2)^{0.2} = \cancel{464.66 \text{ MPa}}$$

650.52 MPa

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(b) $\sigma = 700\,\epsilon^{0.2}$

$$\frac{525}{700} = \epsilon^{0.2}$$

$$\epsilon = (0.75)^5$$

$$\epsilon = 0.237$$

$$\ln\left(\frac{1}{1-r}\right) = 0.237$$

$$1 - r = \frac{1}{e^{0.237}} = 0.789$$

$$r = 0.211$$

$$\frac{A_0 - A_1}{A_0} = x$$

$$\phi \quad \frac{A_1 - A_2}{A_1} = 0.15$$

$$\frac{A_0 - A_2}{A_0} = 0.211$$

$$\frac{A_2}{A_0} = 1 - 0.211 = 0.789$$

$$\frac{A_2}{A_1} = 0.85$$

$$\frac{A_1}{A_0} = 1 - x$$

$$0.789 = (0.85)(1-x)$$

$$x = 1 - 0.928 = 0.072$$

$$\boxed{x = 7.2\%}$$

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Q3) A cylindrical test specimen of diameter $d_0 = 10$ mm, and gauge length = 50 mm is extended to 65 mm. a) Determine true strain

b) if UTS occurs at a force of 25 KN & extension of 70 mm, Determine strain hardening exponent n, & UTS of material

a) $\epsilon = \ln\left(\frac{65}{50}\right) = \ln(1.3)$

b) $\frac{l}{l_0} = \left(\frac{R_0}{R}\right)^2$

$R^2 = R_0^2 \times \left(\frac{l_0}{l}\right) = (5)^2 \times \frac{50}{70}$

$R = 5 \times \sqrt{\frac{5}{7}}$

$UTS = \frac{F}{A} = \frac{25\ KN}{\pi\left(5^2 \times \frac{50}{70}\right) mm^2}$

$\Rightarrow UTS = 0.445$ GPa

$UTS = 445$ MPa

$\boxed{\epsilon_u = \ln\left(\frac{7}{5}\right) = n}$ $\boxed{n = 0.336}$

$UTS_E = \frac{25\ KN}{\pi\ 5^2\ mm^2}$

$UTS_I = 318$ MPa

Q4) Aluminium test specimen 100mm long, 200 mm wide & 2mm thick is elongated to 130 mm. if anisotropy ratio R value R = 2, determine true strain in length, width, thickness.

$$\epsilon_l = \ln(1.3) = 0.262$$

$$R = \frac{\epsilon_w}{-(\epsilon_w + \epsilon_l)}$$

$$\Rightarrow 2 = \frac{\epsilon_w}{-(\epsilon_w + \ln(1.3))}$$

$$2\epsilon_w + 2\ln(1.3) = -\epsilon_w$$

$$\Rightarrow \epsilon_w = -\frac{2}{3}\ln(1.3) = -0.175$$

& $\epsilon_l + \epsilon_w + \epsilon_t = 0$.

$$\Rightarrow \epsilon_t = -\epsilon_w - \epsilon_l$$

$$= \frac{2}{3}\ln(1.3) - \ln(1.3)$$

$$\epsilon_t = -\frac{1}{3}\ln(1.3) = -0.087$$

if R is more, material ~~is~~ will squeeze more hence resistence to ~~thickness~~ thinning is more.

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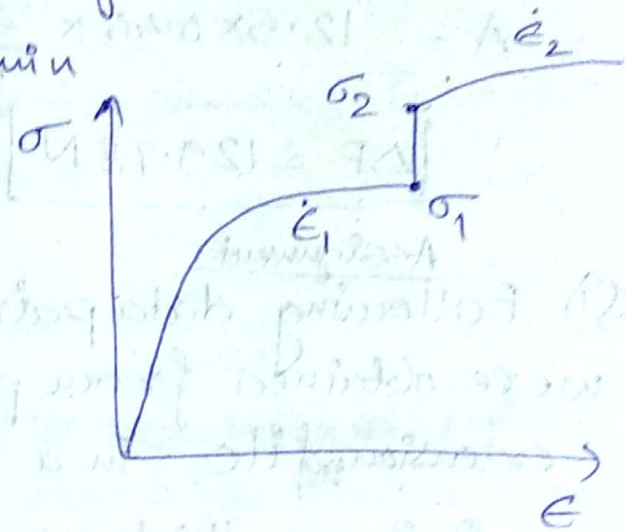
Q5) The K, n, m values of stainless steel sheet material are 1140, 0.35 & 0.01 resp. a test piece has initial width, thickness & gauge length of 12.5, 0.45, & 50 mm resp. Determine the increase in load when extension is 10% & extension rate of gauge length is increased to 0.5 to 50 mm/min



$$\sigma = K\epsilon^n$$

$$\& \ \sigma = C\dot{\epsilon}^m$$

$$\sigma = K\epsilon^n \dot{\epsilon}^m$$

$$\sigma_1 = 1140\,(\ln(1.1))^{0.35} \cancel{\times (0.5} \ \dot{\epsilon}_1^{0.01}$$

$$\dot{\epsilon}_1 = \frac{0.5\ \text{mm/min}}{55\ \text{mm}} = \frac{1}{110}$$

$$\sigma_1 = 1140\,(\ln(1.1))^{0.35}\left(\frac{1}{110 \times 60}\right)^{0.01}$$

$$\boxed{\sigma_1 = \cancel{477\ \text{MPa}}\ 458\ \text{MPa}}$$

$$\dot{\epsilon}_2 = \frac{50\ \text{mm/min}}{55} = \frac{10}{11}$$

$$\sigma_2 = 1140\,(\ln(1.1))^{0.35}\left(\frac{10}{11 \times 60}\right)^{0.01}$$

$$\boxed{\sigma_2 = 480.18\ \text{MPa}}$$

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$\therefore \Delta\sigma = \sigma_2 - \sigma_1$

Change in load = $(\Delta\sigma)$ (instant area)

$\Delta\sigma = 21 MPa$

Instant Area = $A = \frac{A_0}{l_0} \times l$

$A = 12.5 \times 0.45 \times \frac{55}{50} = 6.18 \ mm^2$

$\therefore \boxed{\Delta F = 129.78 N}$

Assignment

Q1) Following data pairs (L (in KN), δ (in mm)) were obtained from plastic part of load extension profile in a tensile test

0.8 mm thickness
12.5mm width
50 mm gauge length
} initial dimensions.

Load (KN)	Extension (mm)
1.57	0.080
1.90	0.760
2.24	1.85
2.57	3.66
2.78	5.84
2.90	8.92
2.93	11.06
2.94	13.49

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Manufacturing Science by A Ghosh & AK Malik.

2.92	16.59
2.86	19.48
2.61	21.82
2.18	22.69

1) Plot E.Stress vs E. Strain, T. Stress vs ~~2) Plo~~ T. Strain, $\ln\sigma$ vs $\ln\epsilon$

2) Determine UTS & true strain at max^m load

3) Determine the K & n value of material, if Holoman Power hardening law is followed.

Q2) Determine work done if a bar of 10 mm dia & 20 mm length is extended to 22 mm. The true stress strain respons of material is given by

$$\sigma = 250\,\epsilon^{0.3} \quad N/mm^2$$

$$\cancel{\epsilon = \ln\frac{l}{l_0}}$$

$$dW = \sigma A\, dl$$

$$= \sigma A\, l\, d\epsilon$$

$$W = \int_0^{\epsilon} (Al)\sigma\, d\epsilon$$

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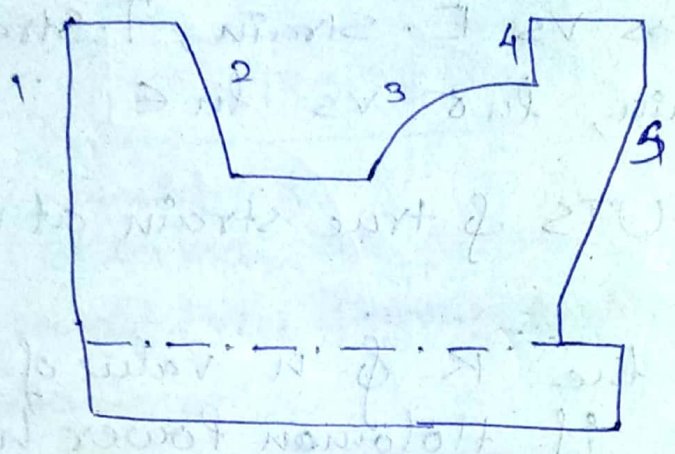
Expendable Mould Casting } types of
Permanent mould Casting } Casting

* Patterns & Designs

Undercuts are those features that are difficult to take out.



2 & 3 are natural drafts
1 & 4 need artificial draft
5 is an undercut.

Q) For bracket casting shown, compare directions X, Y, Z (parting line at X = 80mm, Y = 40mm, Z = 20mm) in terms of draw distance, undercut volume (with number of core) Show the core & pattern with parting line.

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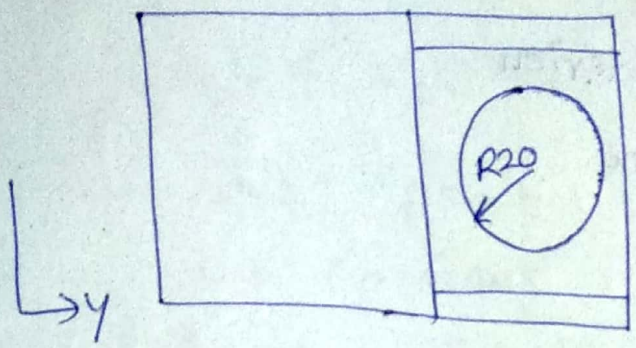
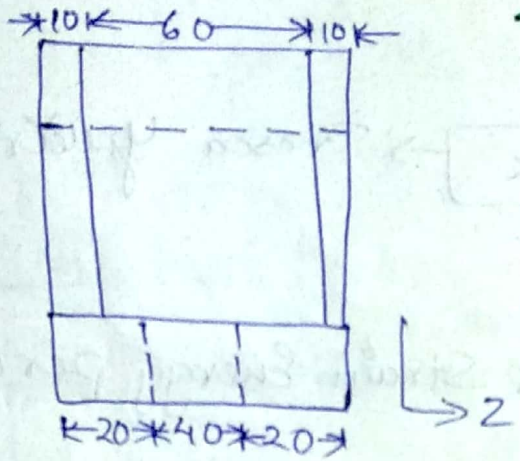
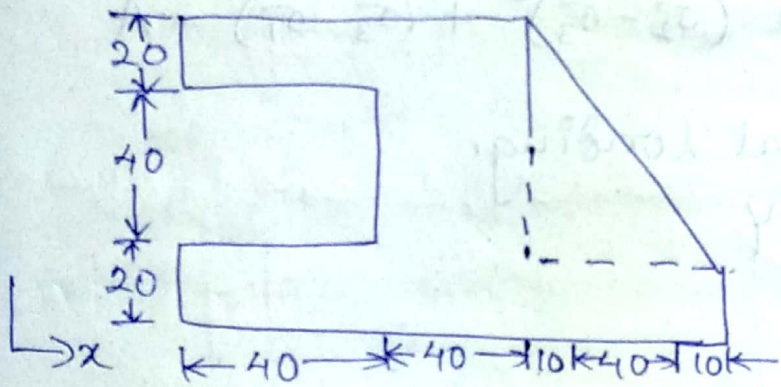


This is important





Parting Position	No. of Cores	Core Volume	Draw distance
X = 80mm	1	$\pi(20)^2 20$	80 mm
y = 40 mm	1		40mm
z = 20mm	2		60 mm

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Casting

20/8/18

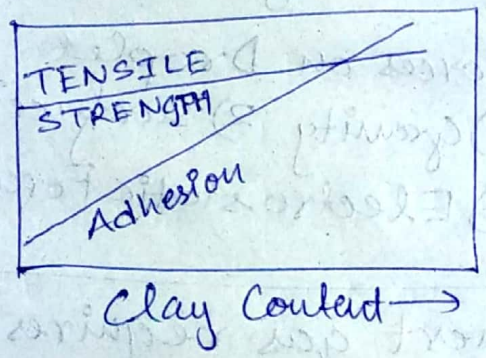
AFS grain fineness no:-

with increase in AFS GFN, grain size will decrease.

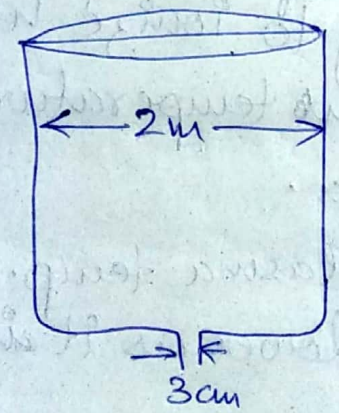
AFS#GFN-58 is a 3 Screen sand.

3 Screen Sand ⇒ for 3 consecutive sieve % weight retained > 10%.

↑ Clay Content ⇒ ↑ Adhesiveness
⇒ ↑ Strength of Sand



Q)



70 K kg

$\rho_{steel} = 7.8 \times 10^3 \, kg/m^3$

$$v_2 = \sqrt{2gh}$$

$$h_o = \frac{m}{\pi r^2} = \frac{70/7.8}{\pi (1)^2}$$

$$h_o = 2.85 m$$

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$$-\frac{dh}{dt} = \sqrt{2gh}\,\frac{A_2}{A_1}$$

$$V_1 = \frac{A_2}{A_1}$$

$$-\int_{h_0}^{0}\frac{dh}{\sqrt{h}} = \int_0^t \sqrt{2g}\,dt\left(\frac{D_2}{D_1}\right)^2$$

$$-\left[2\sqrt{h}\right]_{h_0}^{0} = \sqrt{2g}\,t$$

$$\Rightarrow \frac{2\sqrt{2.85}}{\sqrt{2\times 9.81}} = t\times\left(\frac{3\times10^{-2}}{2}\right)^2$$

$$t = 0.76\ \text{sec}\times\frac{4\times10^4}{9}$$

$$t = 3387.8\ \text{sec}$$

Let us assume few atoms arrange themselves to a small nucleus or embryo of a spherical shape of solid phase.

Let ΔG_v is the free energy per unit volume & γ is interfacial surface energy.

Change in volume free energy $= \frac{4}{3}\pi r^3 \Delta G_v$

Energy required to create new interface $= 4\pi r^2 \gamma$

$\therefore$ Total Gibbs energy

$$= \frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma$$

1st term is -ve since it is reduction in volume but 2nd term is +ve since new surface is being created.

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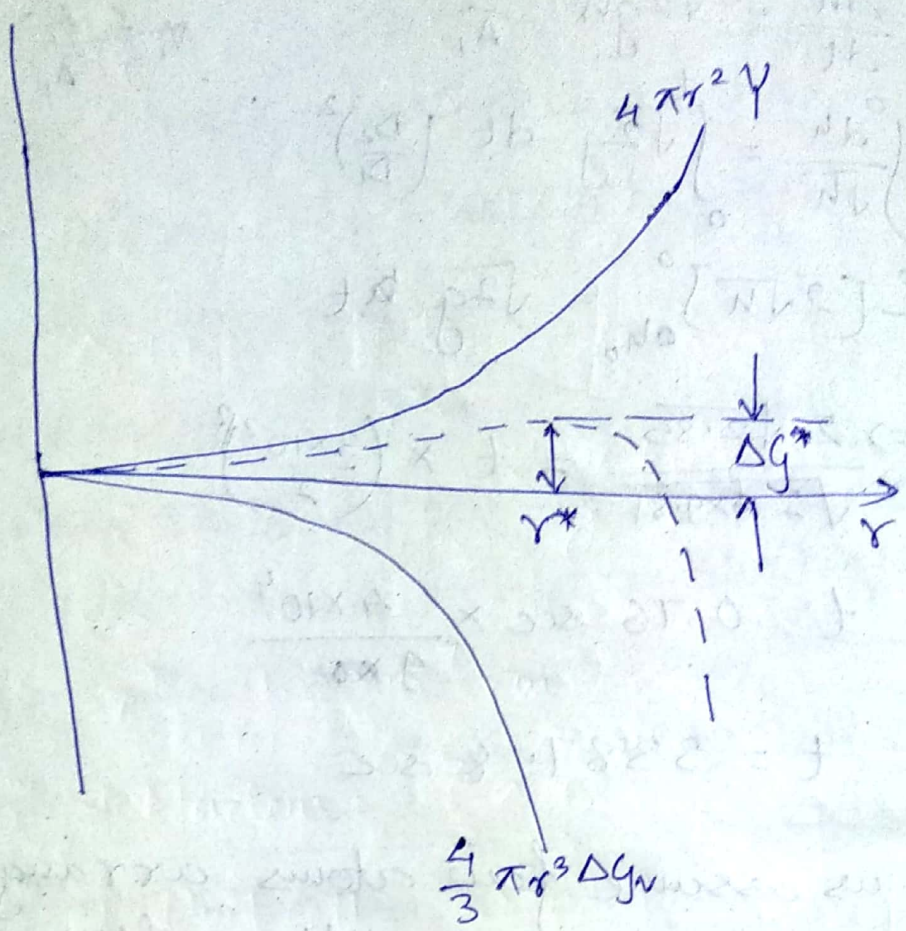

There exist a critical radius where barrier of solidification is max^m & it is given as r^*

$$\Delta G^T = \frac{4}{3}\pi r^3 \Delta G_v + 4\pi r^2 \gamma$$

Now, $4\pi r^{*2} \Delta G_v + 8\pi r^* \gamma = 0$

$$\Rightarrow \boxed{r^* = \frac{-2\gamma}{\Delta G_v}}$$

$$\Delta G_T = \Delta G^* = \frac{16\pi \gamma^3}{3\Delta G_v^2}$$

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$$\Delta G_v = \frac{-2\gamma}{r^*} = -L\frac{\Delta T}{T_E}$$

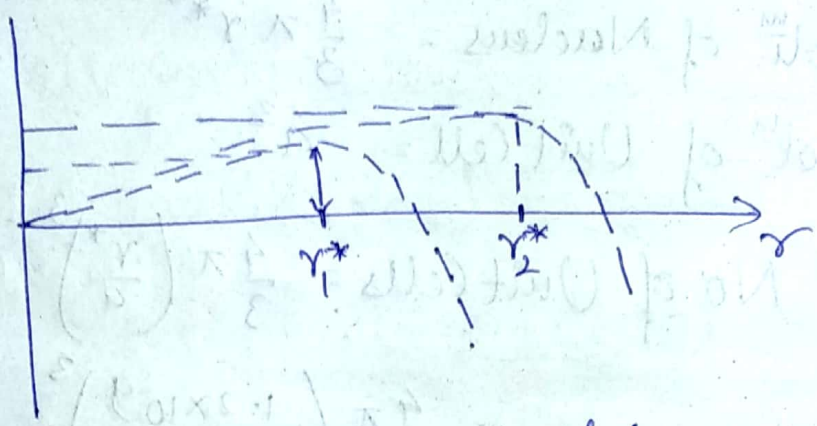
$$\Delta T = T_E - T = \frac{2\gamma T_E}{L r^*}$$

$$\boxed{T = T_E\left(1 - \frac{2\gamma}{L r^*}\right)} \rightarrow \text{Gibbs-Thompson Eqn.}$$

$$T_1 < T_2 < T_E$$

$$\frac{2\gamma_1}{L r_1^*} > \frac{2\gamma_2}{L_2 r_2^*}$$

$$r_1^* < r_2^*$$



Q) Homogeneous nucleation of Cu

a) Cal. r^*

b) No. of atoms in nucleus

$a = 0.3615\,nm$ (FCC)

$T_{freezing} = 1085°C$

L of solidification $= 1628\ J/cm^3$

~~Sur~~ $\gamma = 177 \times 10^{-7}\ J/cm^2$

ΔT = Undercooling $= 236°C$

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a) $r^* = \frac{-2\gamma}{\Delta G_v}$

$$r^* = \frac{-2 \times 177 \times 10^{-7}}{\Delta T}$$

$$\Delta T = \frac{2 \gamma T_E}{L r^*}$$

$$\Rightarrow 236 = \frac{2 \times 177 \times 10^{-7}}{1628\, r^*} (1085+273)$$

$$r^* = 1.2 \times 10^{-7} \text{ cm}$$

Volⁿ of Nucleus $= \frac{4}{3} \pi r^{*3}$

Volⁿ of Unit Cell $= a^3$

$\therefore$ No of Unit Cells $= \frac{4}{3} \pi \left(\frac{r^*}{a}\right)^3$

$$= \frac{4}{3} \pi \left(\frac{1.2 \times 10^{-9}}{0.3615 \times 10^{-9}}\right)^3$$

No of Unit Cells = ~~153×10^8~~ 153

$\therefore$ No of atoms = ~~$4 \times 153 \times 10^8$~~ 4×153

~~$= 6.12 \times 10^8$ atoms~~

612 atoms

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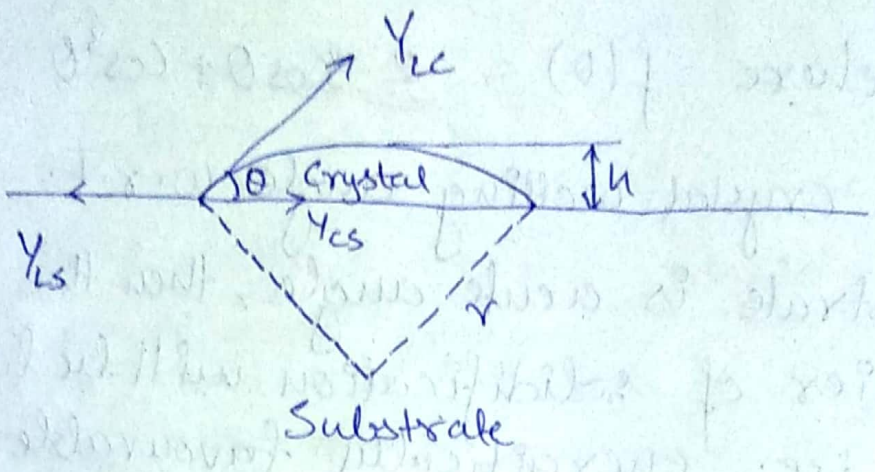
* Nucleation due to Nucleating agents.



$$\cos\theta = \frac{\gamma_{LS} - \gamma_{CS}}{\gamma_{LS}}$$

$$h = r(1-\cos\theta)$$

$$V = \frac{1}{3}\pi h^2(3r-h)$$

$$= \frac{\pi}{3} r^3 (2 - 3\cos\theta + \cos^3\theta)$$

$$A = 2\pi r h = 2\pi r^2 (1-\cos\theta)$$

$$\Delta G_T = \gamma_{LC} \cdot 2\pi r^2 (1-\cos\theta) + \frac{\pi}{3} r^3 (2 - 3\cos\theta + \cos^3\theta)\Delta G_V + (\gamma_{CS} - \gamma_{SL})\pi r^2 (1-\cos^2\theta)$$

$$r^* = \frac{2\gamma_{LC}}{\Delta G_V}$$

$$\Delta G_T^* = \frac{4\pi \gamma_{LC}^3 (2 - 3\cos\theta + \cos^3\theta)}{3\Delta G_V^2}$$

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$$\left(\Delta G_T^*\right)_{hetro} = \frac{1}{4}\left(\Delta G_T^*\right)_{homo} \cdot f(\theta)$$

where $f(\theta) = 2 - 3\cos\theta + \cos^3\theta$

if the crystal wetting angle w.r.t. substrate is acute angle, then the barrier of solidification will be lower ~~that~~ i.e. energetically favourable.

Lattice mismatch parameter $= \delta = \frac{\delta a}{a}$

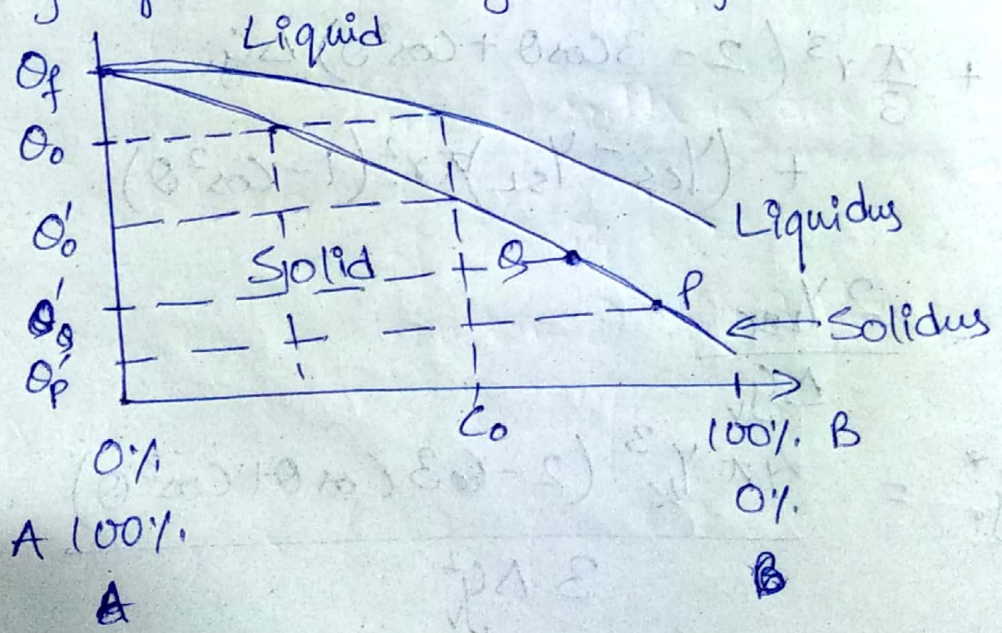
where a = lattice parameter of crystal

δa = difference of lattice parameter ~~bet~~ of crystal & substrate.

Eg:

Crystal	Substrate	δ
Al	VC	0.014

Segregation during Solidification



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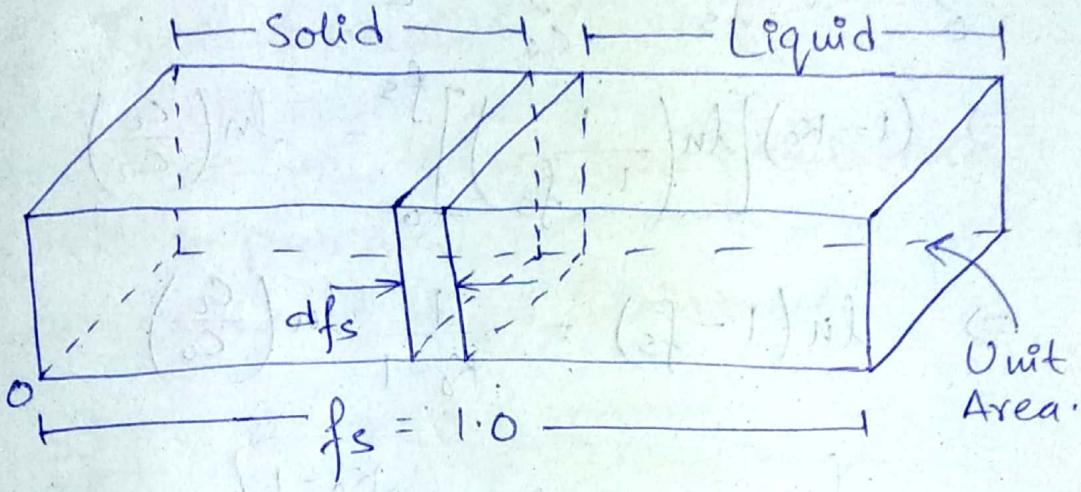
K_0 = Equilibrium Partition Coefficient

$$\frac{C_s}{C_L} = K_0$$



Let liquid metal of initial conc. C_0 is solidifying within the rectangular element. And Solidification front is propagating in one direction from left to right. let the fraction solidified be f_s. Now Composition of liquid is C_m.

At any instant of time let solidification front of df_s has solidified.

Let Concentration of this small elemental front is C_s^* and Concentration of Liquid Changed by dC_L

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$$(C_L - C_s^*)\, df_s = (1-f_s)\, dC_L$$

$$(C_L - K_0 C_L)\, df_s = (1-f_s)\, dC_L$$

$$\Rightarrow \int_0^{f_s} \frac{(1-K_0)\, df_s}{1-f_s} = \int_{C_0}^{C_L} \frac{dC_L}{C_L}$$

$$\Rightarrow (1-K_0)\left[\ln\left(\frac{1}{1-f_s}\right)\right]_0^{f_s} = \ln\left(\frac{C_L}{C_0}\right)$$

$$\Rightarrow \ln(1-f_s) = \frac{1}{K_0-1}\ln\left(\frac{C_L}{C_0}\right)$$

$$\Rightarrow \boxed{C_L = C_0(1-f_s)^{K_0-1}}$$

$$\boxed{C_s^* = K_0 C_L = K_0 C_0 (1-f_s)^{K_0-1}}$$

Scheil Eqn

27th Aug

Nucleating agents can help in early nucleation

Dendritic structure results due to undercooled surface.

Disadvantages of Dendrites:-

1) Results in hindered fluid flow.

2) Start & end of solidification is same.

Chill Mold ⇒ Made of metal

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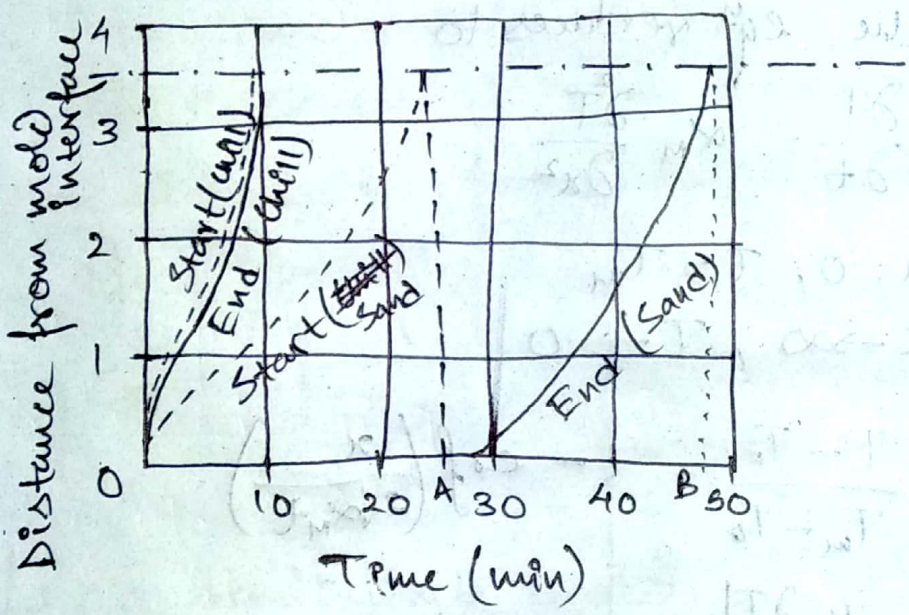
in Chill Mold, two fronts → start of solidification & end of solidification front, propagate & between them, metal is both solid & liquid

Dendritic structures depend upon
1) % Composition of metal
2) Type of mold.

For dendrites, these two fronts are more separated.

Centreline Fatigue Resistance



AB = time between start & end of freezing at centre line of mold

OB = total freezing time

$$CFR = \frac{AB}{OB}$$

if CFR ≥ 70% ⇒ ~~flow~~ feeding is difficult.

Tangling of Dendrites will lead to ↑ in tangling surface area which may ↑ strength.

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Assumption:-

1) Mold is semi-infinite, one dimensional
2) The temp. difference across the solidified metal is negligible.
3) There is no air gap between solidified metal & mold wall.
4) Due to 2) & 3) The temp. maintained at mold-metal interface is equal to melting point of metal T_M.

Now the eqn reduces to

$$\frac{\partial T}{\partial t} = \alpha_M \frac{\partial^2 T}{\partial x^2}$$

at $x = 0$, $T = T_M$

as $x \to \infty$, $T \to 0$

$$\therefore \frac{T - T_0}{T_M - T_0} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_M t}}\right)$$

$$q = -K \left.\frac{\partial T}{\partial x}\right|_{x=0}$$

$$\frac{\partial T}{\partial x} = (T_M - T_0)\left(-\frac{2}{\sqrt{\pi}}\left(e^{-\frac{x^2}{4\alpha_M t}} \times \frac{1}{2\sqrt{\alpha_M t}}\right)\right)$$

$$q = \frac{K(T_M - T_0)}{\sqrt{\pi \alpha_M t}}$$

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$$Q = \int_0^{t_s} qA\,dt = \frac{2KA(T_M - T_0)}{\sqrt{\pi \alpha_M}} \sqrt{t_s}$$

Heat balance

$$\frac{2KA(T_M - T_0)}{\sqrt{\pi \alpha_M}} \sqrt{t_s} = \rho_M V \left[\underbrace{L}_{\text{Latent Heat}} + \overbrace{C_M (T_p - T_M)}^{\text{Super Heat}} \right]$$

$$t_s \propto \left(\frac{V}{A}\right)^2 \longleftrightarrow \text{Chvorinov's Rule.}$$

Q) metal with 700°C melting temp. is poured into sand mould at 30°C
find thickness of mold to be considered semi infinite for 1 min 15 sec time after pouring the metal.

$$\left[\begin{array}{l} C_p = 1.16\ J/g\cdot°C,\ K = 0.6\ W/m\cdot°C \\ \rho = 1.5\ g/cm^3 \quad \text{for sand mold} \end{array} \right]$$

for a semiinfinite mold, $T(x,t) = T_0$

$$\Rightarrow 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_M t}}\right) = 0$$

$$\Rightarrow \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_M t}}\right) = 1$$

$$\frac{x}{2\sqrt{\alpha_M t_s}} = 2$$

$$\Rightarrow x = 4\sqrt{\alpha_M t_s} = 4\sqrt{\frac{0.6 \times 75}{1.5\times10^6 \times 1.16}}$$

$$x = 2\ cm$$

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Casting 28th Aug

7 principles of feeder design.

1) Don't provide feeder until necessary. Since casting yield will ↓.

$$\text{Casting yield} = \frac{\text{Wt. of Casting Shipped}}{\text{Wt. of liquid metal poured}} \times 100$$

2) feeder must solidify after casting. if the experiment is done in vaccum, air porosity would be there hence the min^m will come to 0.

$M_r = (1.1 - 1.2) M_c$ ensures that shrinkage is totally avoided.

3) Using insulating sleeves, we can increase time of freezing by minimizing the volume.

Caine's Method

Volume ratio $\left(\frac{V_r}{V_c}\right)$ vs Freezing ratio $\left(\frac{A_c/V_c}{A_r/V_r}\right)$

3) $e \times V_r \geq \alpha (V_r + V_c)$

e = feeder efficiency = 14 %

α = Contraction coefficient

α = 7% for Al, 4% for steel.

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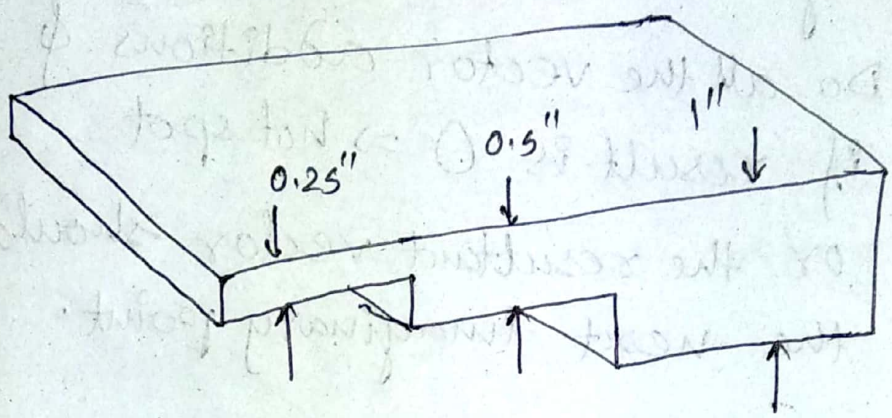
4) Riser must be at thickest part.

Eg:



5) Junction ~~region~~ requirement

6) Presence requirement.

7) Temperature requirement.

if temp. gradient falls below 2-3°C/cm there will be lot of porosity.

$$\text{No. of riser} = \frac{\text{length} - 2EE}{2RE + d}$$

EE ← End effect

d ← diameter of riser.

RE ← riser effect

Algorithm:

1) Give coordinates & draw a 2D section
2) Imagine a point inside section.
3) Join corners of the section to the point i.e. discretize the section.
4) Evaluate Volume (Area of section) & surface area (visible edge length)

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of each element

5) Draw a vector of magnitude $\frac{V}{A}$ & direction is bisecting the internal angle made at the point.

6) Do all the vector additions & if result is 0 => hot spot or the resultant vector should be the next imaginary point.

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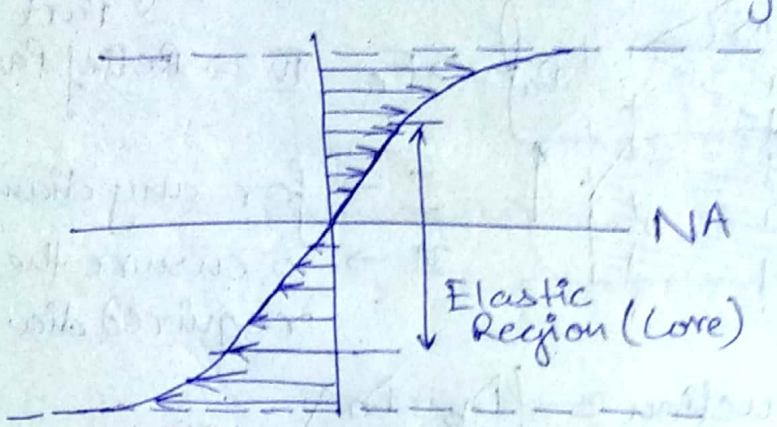
Casting 4th Sept

Formability → ease with which sheet can be deformed, without fracture/ excessive thinning.

Stress distribution during bending process



if Elastic Core is more, springback will be higher.

Ironing is done ~~to~~ for uniform diameter.

the above curve depends on plastic flow curve.

Ironing is different from Deep drawing since workpiece isn't flat & nothing is holding it.

In plane stretching ⇒ Biaxial stresses are in a plane.

Out of plane stretching ⇒ Biaxial stresses are out of plane.

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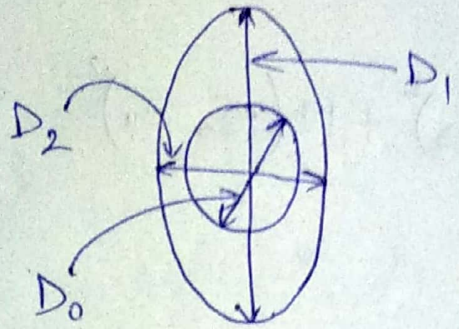
for 100 x 100 case



$$\epsilon_1 = \ln\left(\frac{D_1}{D_0}\right)$$

$$\epsilon_2 = \ln\frac{D_2}{D_0}$$

for 100 x 20 case

$\epsilon_1 > 0$ but $\epsilon_2 < 0$

Forming limit diagram.

*) Deep Drawing

if there is no difference in die corner radius & Punch radius, drawing of metal can't be facilitated.

Flange experiences circumferential compressive stress which leads to thickenin

$$(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2) = 2Y^2$$

in terms of Principal stresses

$$(\sigma_{1} - \sigma_{2})^2 + (\sigma_{2} - \sigma_{3})^2 + (\sigma_{3} - \sigma_{1})^2 = 2Y^2$$

Von-Mises criterion.

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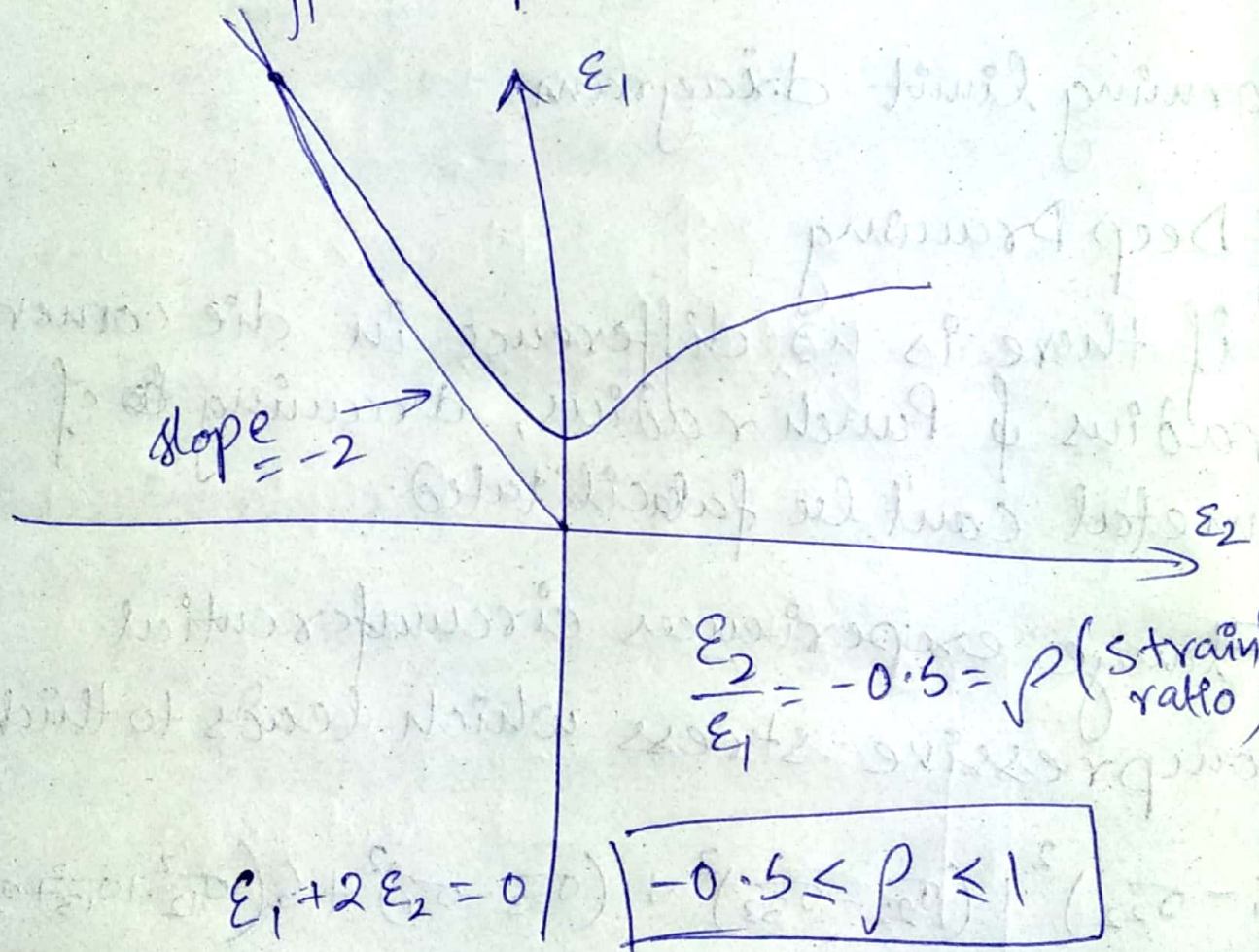
Casting

10th Sept

Forming limit diagram represents limiting strain (major & minor surface strain) which a sheet metal can be reformed under different plane ~~sp~~ stress condition.



$$\frac{\varepsilon_2}{\varepsilon_1} = -0.5 = \rho \text{ (strain ratio)}$$

$$\varepsilon_1 + 2\varepsilon_2 = 0$$

$$\boxed{-0.5 \leq \rho \leq 1}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = -2$$

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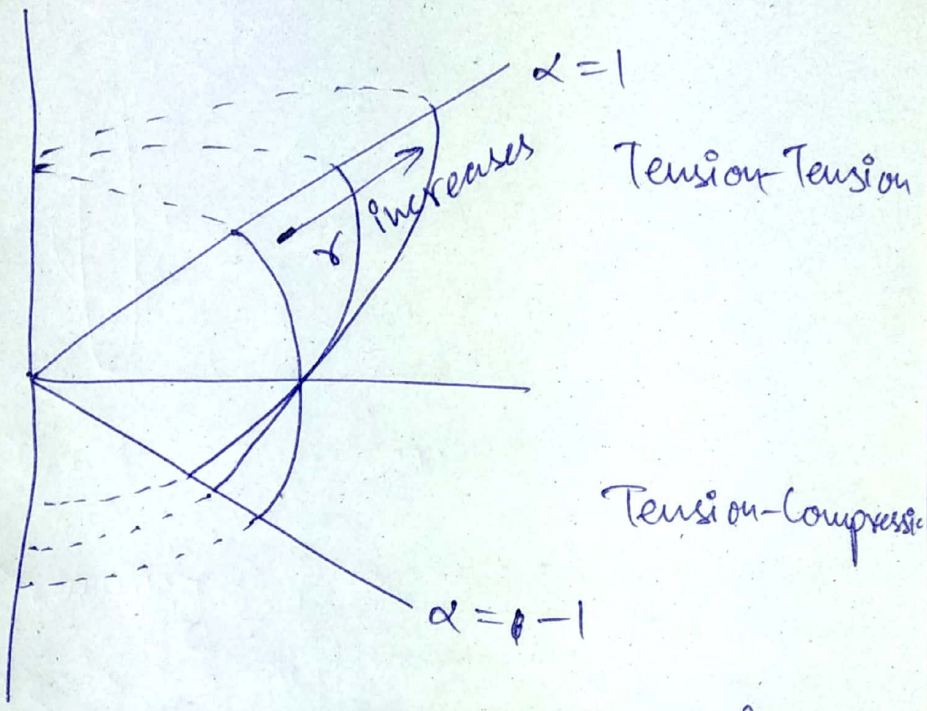
Under plane stress condition $\sigma_3 = 0$

$$\bar{\sigma}^2 = \frac{1}{1+r}\left((1+r)(\sigma_1^2+\sigma_2^2) - 2r\,\sigma_1\sigma_2\right)$$

$$= \sigma_1^2 + \sigma_2^2 - \frac{2r}{1+r}\sigma_1\sigma_2$$



↑ r value ⇒ deeper cups could be drawn.

For Exam

→ Material Properties
→ Pattern & Core Design
→ Solidification
→ Riser Design

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