

Assignment - 6
PDE (MA 20103)

Course Teacher
Koeli Ghoshal

Q1. Classify the PDE

$$\frac{\partial^2 z}{\partial x^2} - y^4 \frac{\partial^2 z}{\partial y^2} = 2y^3 \frac{\partial z}{\partial y}$$

and reduce it to its canonical form.

[Ans: $\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$]

Q2. Classify and reduce to canonical form

$$4 \frac{\partial^2 z}{\partial x^2} - y^6 \frac{\partial^2 z}{\partial y^2} = 3y^5 \frac{\partial z}{\partial y}$$

[Ans: $\frac{\partial^2 u}{\partial \xi \partial \eta} = 0$]

Q3. Classify: $x^2(y-1)z_{xx} - x(y^2-1)z_{xy} + y(y-1)z_{yy}$
 $+ xy z_x - z_y = 0$

[Everywhere hyperbolic except for $x=0, y=1$ where it is parabolic]

Q4. Classify & reduce to canonical form

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$

[Ans: $\frac{\partial^2 u}{\partial \eta^2} = 0$ Hint: Here $\xi = x^2 + y^2$; you can take $\eta = x^2 - y^2$]

* * * * The End * * * *