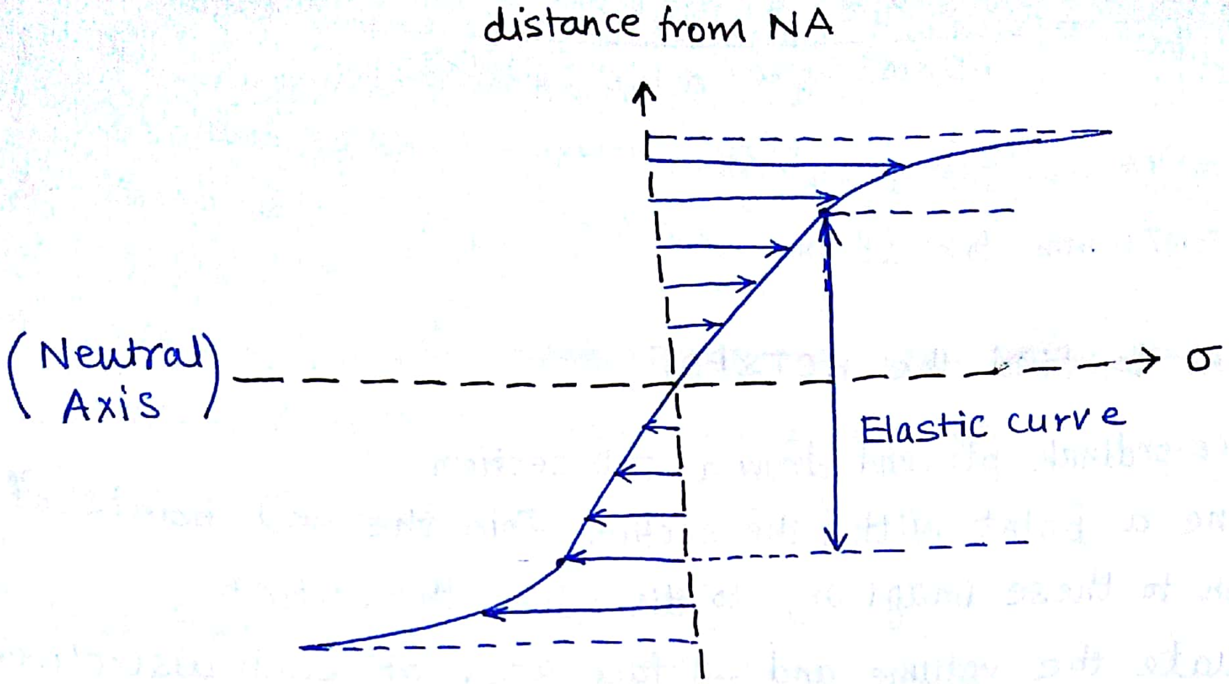
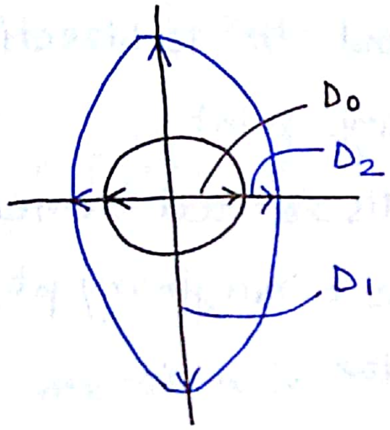
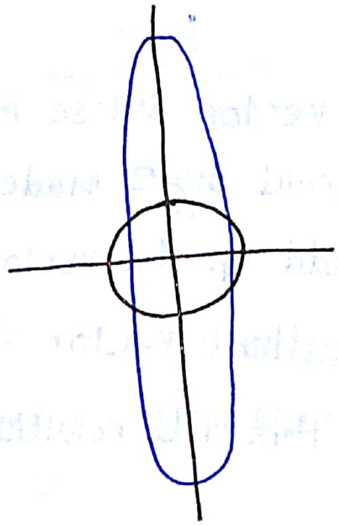






$$\epsilon_1 = \ln \frac{D_1}{D_o} = +ve$$

$$\epsilon_2 = \ln \frac{D_2}{D_o} = +ve$$

$$\epsilon_1 = \ln \frac{D_1}{D_o} = +ve$$

$$\epsilon_2 = \ln \frac{D_2}{D_o} = -ve$$

Von-Moses stress:

$$(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6\sigma_{12}^2 + 6\sigma_{23}^2 + 6\sigma_{31}^2 = 2Y^2$$

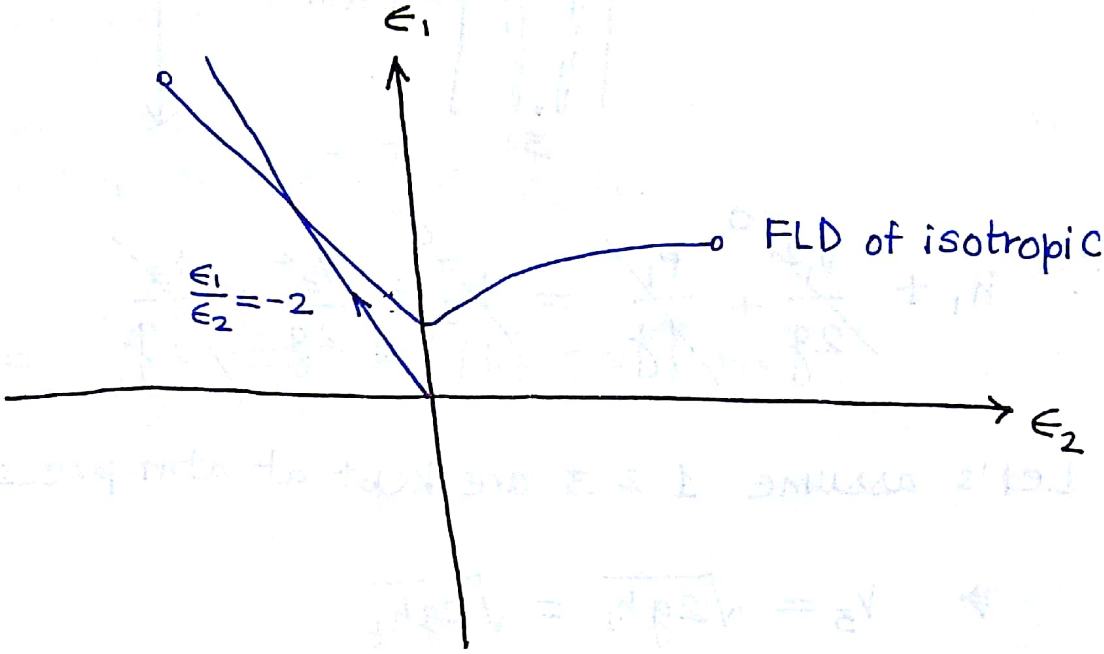
For principal planes:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$$

Hill

$$F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = 1$$

Forming limit diagram represents the limiting strength (in the form of major & minor ~~str~~ surface strength) which a sheet metal can be deformed under diff. plane stress conditions.



$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$ $\quad$ $\epsilon_2 + \epsilon_3 = -\epsilon_1 \Rightarrow 2\epsilon_2 = -\epsilon_1$

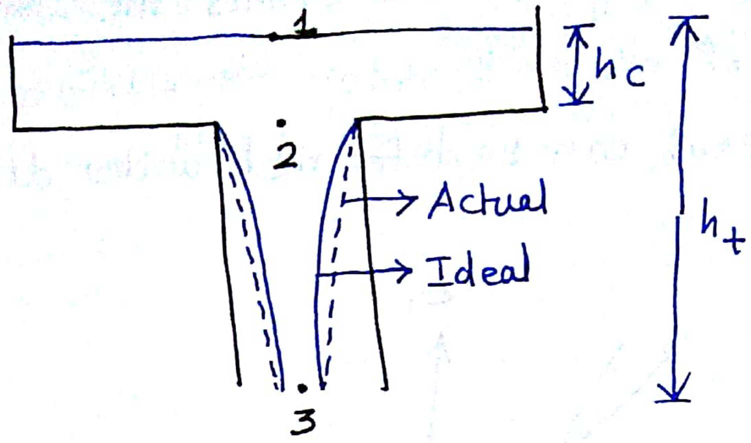
$\Rightarrow \epsilon_1 + 2\epsilon_2 = 0$

Mid-sem: Material Properties, Pattern & Core Design, Solidification, Riser design

* Strainer gate must be used only for small castings and never for larger castings because impurities would start obstructing the flow of liquid.

* If sprue is not designed very well, then air gets sucked into the mould cavity which is called Aspirition Effect.

If sprue is of vertical cylindral design



$$\because \quad h_1 + \cancel{\frac{V_1^2}{2g}}^{\,0} + \cancel{\frac{P_1}{\rho g}} = \cancel{h_3}^{\,0} + \frac{V_3^2}{2g} + \cancel{\frac{P_3}{\rho g}}$$

Let's assume 1 & 3 are kept at atm. pressure.

$$\Rightarrow \quad V_3 = \sqrt{2gh_1} = \sqrt{2gh_t}$$

[$A_2 = A_3$ for vertical sprue

$\Rightarrow V_2 = V_3$]

$$\because \quad h_2 + \cancel{\frac{V_2^2}{2g}} + \frac{P_2}{\rho g} = \cancel{h_3}^{\,0} + \cancel{\frac{V_3^2}{2g}} + \frac{P_3}{\rho g}$$

$$\Rightarrow \quad P_3 - P_2 = \rho g h_2 = \rho g (h_t - h_c)$$

To keep 2 & 3 at atm. conditions, $V_2 = \sqrt{2gh_c}$

$V_3 = \sqrt{2gh_t}$

$$R = \frac{A_3}{A_2} = \sqrt{\frac{h_c}{h_t}}$$: equation of parabola

↓

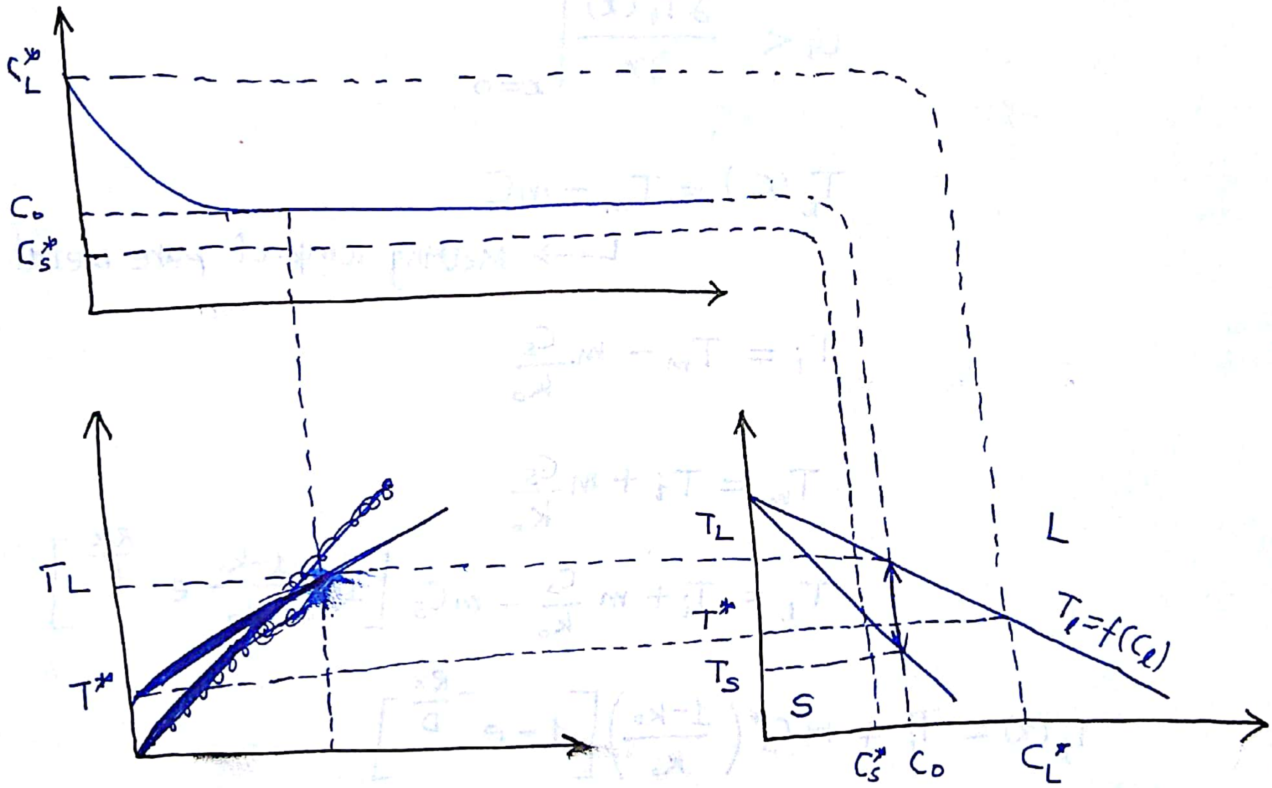
difficult to produce (hence, the actual & ideal shown in fig.)

$$\frac{1}{\sqrt{2g}} \int_0^{h_m} \frac{dh}{\sqrt{h_t - h}} = \frac{A_g}{A_m} \int_0^{t_f} dt$$

$$\Rightarrow \frac{1}{\sqrt{2g}} \left[-2\sqrt{h_t - h} \right]_0^{h_m} = \frac{A_g}{A_m} \left[t \right]_0^{t_f}$$

$$\Rightarrow \frac{-2}{\sqrt{2g}} \left[\sqrt{h_t - h_m} - \sqrt{h_t} \right] = \frac{A_g}{A_m} t_f$$

$$\Rightarrow t_f = \frac{A_m}{A_g} \cdot \frac{1}{\sqrt{2g}} \cdot 2\left(\sqrt{h_t} - \sqrt{h_t - h_m}\right)$$



A composition profile is developed ahead of solid-liquid interface

$$C_l(x) = C_s^* \left[1 + \frac{1-k_o}{k_o} e^{-\frac{Rx}{D}} \right]$$

Due to these composition change depending upon the phase diagram fig.(b), the freezing temp. profile is shown in fig.(c)

Reasons for formation of Dendrites :

1. Constitution under cooling. ahead of solid-liquid interface which will create instability of the solid-liquid interface and any pertubaration can be stable.

2. There is a preferential direction of growth (it will be in the dir^n of higher temp. gradient). There exists also a preferred dir^n of growth in crystal structure as well. [100] for BCC.

3. Also the latent heat of solidification is liberated at solid-liquid interface. Hence, the growth is not planar and hence, a tree-like structure or arms stretch.

$$G < \left.\frac{\partial T_l(x)}{\partial x}\right|_{x=0}$$

$$T_l(C) = T_m - mC$$

$\longrightarrow$ Melting temp. of pure metal

$$T_i = T_m - m.\frac{C_s}{K_o}$$

$$T_m = T_i + m\frac{C_s}{K_o}$$

$$T_l = T_i + m\frac{C_s}{K_o} - mC_s\left[1 + \frac{1-K_o}{K_o}e^{-\frac{Rx}{D}}\right]$$

$$T_l(x) = T_i + mC_s^*\left(\frac{1-K_o}{K_o}\right)\left[1 - e^{-\frac{Rx}{D}}\right]$$

$$G < \left.\frac{\partial T_l(x)}{\partial x}\right|_{x=0} = \left.\frac{RmC_s^*}{D}\left(\frac{1-K_o}{K_o}\right)e^{-\frac{Rx}{D}}\right|_{x=0} = \frac{mC_s^*R}{D}\left(\frac{1-K_o}{K_o}\right)$$

$$\therefore \boxed{\frac{G}{R} < \frac{mC_s^*}{D}\left(\frac{1-K_o}{K_o}\right)}$$

Condition for Constitutional under cooling

To avoid constitutional undercooling

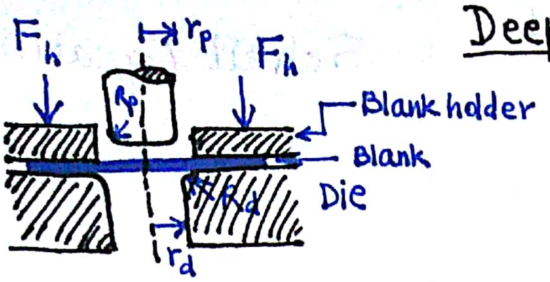
$$\frac{G}{R} < \frac{m C_s^*}{D} \left(\frac{1-k_o}{k_o} \right)$$

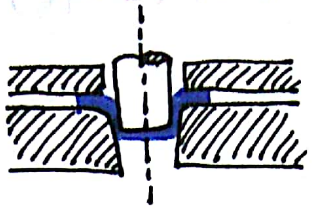
- Higher Temp. Gradient (G)
- Low Growth Rate
- Lower slope of the liquidus line
- Low alloying content
- Higher Diffusability
- Higher partioning coeff. (k_o) i.e. close to 1 or
- Lower freezing range

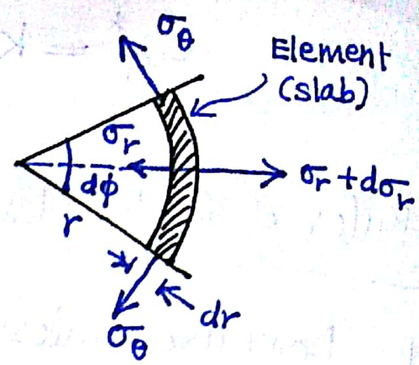
Reasons for Hot Tears:

1. Development of stresses due to constraints encountered in cooling.
2. Higher freezing range

Deep Drawing Analysis







- To evaluate the force required for deep drawing
- Assumptions:
  - increase in flow strength (Yield strength at each pt.) during deformation is negligible. (rigid + perfectly plastic)

  - material is homogeneous and isotropic.
  - change in thickness is neglected (plane strain only)
  - von-Mises criteria: $(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2 = 2Y^2$ — (1)

    Leny-Mises criteria: $\dfrac{d\epsilon_{ij}^p}{\sigma_{ij}'} = d\lambda \qquad \dfrac{d\epsilon_1}{\sigma_1'} = \dfrac{d\epsilon_2}{\sigma_2'} = \dfrac{d\epsilon_3}{\sigma_3'} = d\lambda \Rightarrow d\lambda \cdot \sigma_3' = 0$ [for plane strain]

    $$d\lambda\left(\sigma_3 - \frac{\sigma_1+\sigma_2+\sigma_3}{3}\right) = 0$$

    $$\Rightarrow \frac{2}{3} d\lambda\left(\sigma_3 - \frac{\sigma_1+\sigma_2}{2}\right) = 0 \Rightarrow \sigma_3 = \frac{\sigma_1+\sigma_2}{2} \quad \text{— (2)}$$

    Put (2) in (1), $\frac{3}{2}(\sigma_1-\sigma_2)^2 = 2Y^2 \Rightarrow \boxed{\sigma_1 - \sigma_2 = \frac{2}{\sqrt{3}} Y}$
  - Blank holder force is only acting at the rim.
  - Friction b/w the die wall and cup is not considered.

Slab Analysis

$$(\sigma_r + d\sigma_r)(r+dr)\cancel{d\phi}\, t - \sigma_r \cdot r\cancel{d\phi}\, t - \cancel{2}\sigma_\theta \cdot dr \cdot t \cdot \sin\frac{d\phi}{2} = 0 \qquad \left(\sin\frac{d\phi}{2} \to \frac{d\phi}{2}\right)$$

$$\Rightarrow (\sigma_r + d\sigma_r)(r+dr) - \sigma_r \cdot r - \sigma_\theta\, dr = 0 \Leftrightarrow \cancel{\sigma_r \cdot r} + \sigma_r\, dr + d\sigma_r \cdot r + \cancel{d\sigma_r \cdot dr}^{\,0} - \cancel{\sigma_r \cdot r} - \sigma_\theta \cdot dr = 0$$

$$\Rightarrow d\sigma_r \cdot r + (\sigma_r - \sigma_\theta)\, dr = 0 \quad (\text{Von-Mises} \Rightarrow \sigma_r - \sigma_\theta = \sigma_o')$$

$$\Rightarrow d\sigma_r \cdot r + \sigma_o'\, dr = 0$$

$$\Rightarrow \frac{d\sigma_r}{\sigma_o'} = -\frac{dr}{r} \Rightarrow \boxed{\frac{\sigma_r}{\sigma_o'} = -\ln r + C}$$

at $r = r_o$,

$$\sigma_r = \frac{\cancel{2}\mu F_h}{\cancel{2}\pi r_o t} = \frac{\mu F_h}{\pi r_o t}$$

$$\frac{\mu F_h}{\sigma_o' \times \pi r_o t} = -\ln r_o + C \Rightarrow C = \frac{1}{\sigma_o'} \cdot \frac{\mu F_h}{\pi r_o t} + \ln r_o$$

$$\boxed{\sigma_r = \sigma_o' \ln\frac{r_o}{r} + \frac{\mu F_h}{\pi r_o t}}$$

- The stress required to draw the material at die wall $\boxed{\sigma_W = \sigma_r e^{\mu\pi/2}}$

  The force required to draw the material: $\boxed{F = \sigma_W \times 2\pi r_p t}$