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MECHANICS & ROBOT KINEMATICS TEST

Ans 1.(i) $R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix}$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_c(\phi, \theta) = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_c = \begin{bmatrix} c\phi & s\theta s\phi & s\phi c\theta \\ 0 & c\theta & -s\theta \\ -s\phi & s\theta c\phi & c\theta c\phi \end{bmatrix} \text{ — } \underline{\text{Ans (i)}}$$

For inverse kinematics step, we will use this simultaneous equation for angle calcutn

$\boxed{r_{22} = \cos\theta, \; r_{11} = \cos\phi} \Rightarrow \phi = \cos^{-1}(r_{11}) \quad \theta = \cos^{-1}(r_{22})$

An 4(ii) $K = [a_x \ a_y \ a_z]^T$

$\vec{K} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$ is a vector in 3D co-ordinate system.

$\hat{K} = \frac{\vec{K}}{|\vec{K}|}$ is the unit vecter corresponding to $\vec{K}$

Two properties of $\hat{k}$ are

(A) Unit magnitude : Magnitude of $|\hat{k}| = 1$

$$\hat{k} = \left[\frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}} \quad \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}} \quad \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}} \right]^T$$

(B) Direction of $\hat{K}$ is same as that of $\vec{k}$

$$\vec{K} \times \hat{K} = 0$$

An 3 Unit quaternions provide a convenient mathematical notation for representing spatial orientations and rotations of elements in three dimensional space. They encode information about axis - angle rotation about an arbitary axis

Compared to rotation matrices, quaternions are more compact, efficient and numerically stable. Compared to Euler angles they are simpler to compose and avoid the problem of gimbal lock.

$R_y(\phi) \Rightarrow \quad q_y = e^{\frac{\phi}{2}(j)} = \cos\frac{\phi}{2} + j\sin\frac{\phi}{2}$

$R_x(\theta) \Rightarrow \quad q_x = e^{\theta/2(i)} = \cos\frac{\theta}{2} + i\sin\frac{\theta}{2}$

$$R_{yx}(\phi,\theta) = q_y \cdot q_x = \cos\frac{\theta}{2}\cos\frac{\phi}{2} - \sin\frac{\theta}{2}\sin\frac{\phi}{2}$$
$$i\sin\frac{\theta}{2}\cos\frac{\phi}{2} + j\sin\frac{\phi}{2}\cos\frac{\theta}{2}$$

In terms of equivalent axis $\hat{k} = [k_x \; k_y \; k_z]^T$, the equivalent axis is given by

$\varepsilon_1 = k_x \sin\theta/2$

$\varepsilon_2 = k_y \sin\theta/2 \qquad \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1$

$\varepsilon_3 = k_z \sin\theta/2$

$\varepsilon_4 = \cos\theta/2 \qquad \varepsilon_4 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$

$$R_\varepsilon = \begin{bmatrix} 1-2\varepsilon_2^2-2\varepsilon_3^2 & 2(\varepsilon_1\varepsilon_2-\varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3+\varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2+\varepsilon_3\varepsilon_4) & 1-2\varepsilon_1^2-2\varepsilon_3^2 & 2(\varepsilon_2\varepsilon_3-\varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3-\varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3+\varepsilon_1\varepsilon_4) & 1-2\varepsilon_1^2-2\varepsilon_2^2 \end{bmatrix}$$

$$\varepsilon_1 = \frac{r_{32}-r_{23}}{4\varepsilon_4} \qquad \varepsilon_2 = \frac{r_{13}-r_{31}}{4\varepsilon_4} \qquad \varepsilon_3 = \frac{r_{21}-r_{12}}{4\varepsilon_4}$$

Ans 1(ii) Fixed angle & Euler angles.

X-Y-Z fixed angles

$$R_{XYZ}(\gamma, \beta, \alpha) = R_Z(\alpha) \cdot R_Y(\beta) \cdot R_Z(\gamma)$$

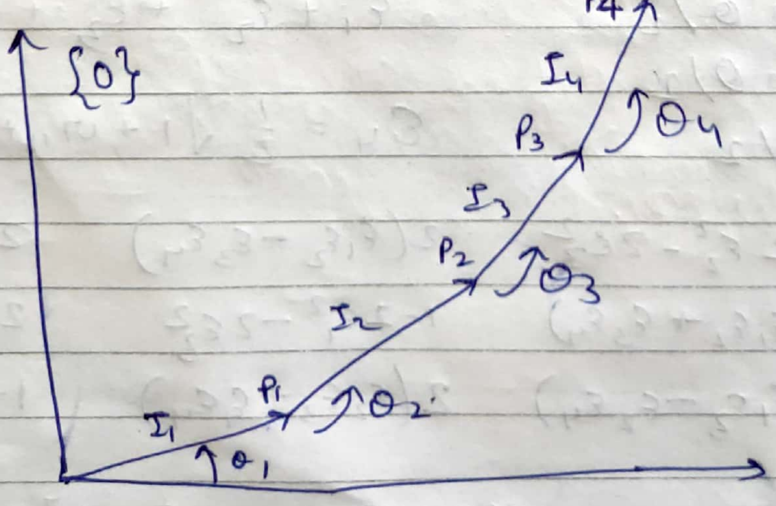
Z-Y-X Euler angles

$$R_{Z'Y'X'}(\alpha, \beta, \gamma) = R_Z(\alpha)\, R_Y(\beta)\, R_X(\gamma)$$

$$R_{Z'Y'X'}(\alpha, \beta, \gamma) = R_{XYZ}(\gamma, \beta, \alpha)$$

There are 24 conventions, of these, 12 conventions are for fixed angle sets and 12 are for Euler angle set.

Ans 4



$$v_{r_1} = 0$$

$$v_{p_2} = v_{p_1} + \omega_1 \times p_2$$ OR

$$v_{p_3} = v_{p_2} + \omega_2 \times r_3$$

$$^0 v_{p_2} = 0 + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

and

$$^0 v_{p_3} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2)\, {}^0 P_3$$

$$^0 v_{p_3} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} -l_2 s_{12} \\ l_2 c_{12} \\ 0 \end{bmatrix} (\dot{\theta}_1 + \dot{\theta}_2)$$

The angular velocities are simple since the are all rotations about the z-axis perpendicular to plane of the paper

$$^0 \omega_3 = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\, {}^0 z_0.$$

In matrix form,

$$ {}^{0}v_{P_3} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

and

$$ {}^{0}\omega_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

and from which, we obtain ~~Jac~~ Jacobi

$$ \begin{pmatrix} v \\ \omega \end{pmatrix} = J \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{pmatrix} $$

Most manipulators have values of θ where the Jacobian becomes singular. Such locations are called singularities for short. All manipulators have singular at boundary of workspace and most have loci of singularities insid their workspace.

Ans 2. Coincident with {A}
About Y_B by 30°
Then Z_B by 45°.

${}^{A}P$ to ${}^{B}P$

$${}^{A}_{B}P = ROT(\hat{z}, 45°)\ ROT(\hat{y}, 30°)$$

$$ROT(\hat{z}, 45°) = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$ROT(\hat{y}, 30°) = \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix}$$

$${}^{A}_{B}P = \begin{bmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.866 & 0 & 0.5 \\ 0 & 1 & 0 \\ -0.5 & 0 & 0.866 \end{bmatrix}$$

$${}^{A}_{B}P = \begin{bmatrix} 0.6122 & -0.707 & 0.3535 \\ 0.6122 & 0.707 & 0.3535 \\ -0.5 & 0 & 0.8666 \end{bmatrix}$$