Application of Newton's Equations of Motion: Inverse Dynamics and Postural Stability

Outline

- effect of inertial forces on joint moments during movement
- use of inverse dynamics to estimate joint moments and forces during movement
- introduction to postural stability

Force-acceleration relationships are described by differential equations.

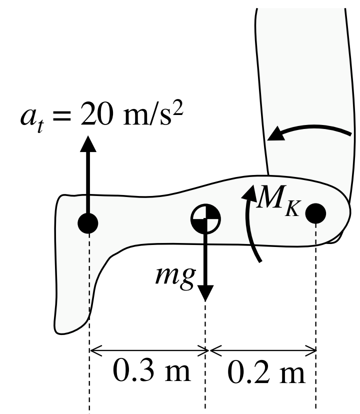
Some problems require us to determine the relations between the forces applied to body segments, and the corresponding accelerations, velocities, and positions of those segments. These relations cannot be described by conservation of energy, or the impulse-momentum (integral) form of Newton's second law. We must instead use the derivative form of Newton's second law:

$$\sum F_x = m(a_{CG})_x$$
$$\sum F_y = m(a_{CG})_y$$
$$\sum M_{CG} = I_{CG}\alpha$$

These differential equations are sometimes referred to as the system's "equations of motion."

Example: effect of inertial forces on joint moments during movement

Find the magnitude and direction of the moment M_K that must be generated by the knee muscles to produce a tangential acceleration of 20 m/s2 at the ankle in the figure at right. The mass m of the lower leg is 5 kg and its moment of inertia I_{CG} about the centre of gravity is 0.08 kg·m^2.



Example: effect of inertial forces (cont)

Calculate shin's moment of inertia about the knee

$$I_0 = I_{CG} + md^2 = 0.08 + 5 \cdot (0.2)^2 = 0.28 \text{ kg} \cdot \text{m}^2$$

Calculate the shin's angular acceleration

$$\alpha = \frac{a_t}{r} = \left(\frac{20}{0.5}\right) = 40 \text{ rad/s}^2 \text{ (clockwise)}$$

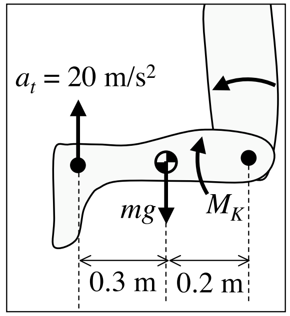
Write the equation of motion for rotation

$$\sum M_0 = I_0 \alpha$$

$$5 \cdot 9.81 \cdot (0.2) - M_K = (0.28) \cdot (\text{-}40) = \text{-}11.2$$

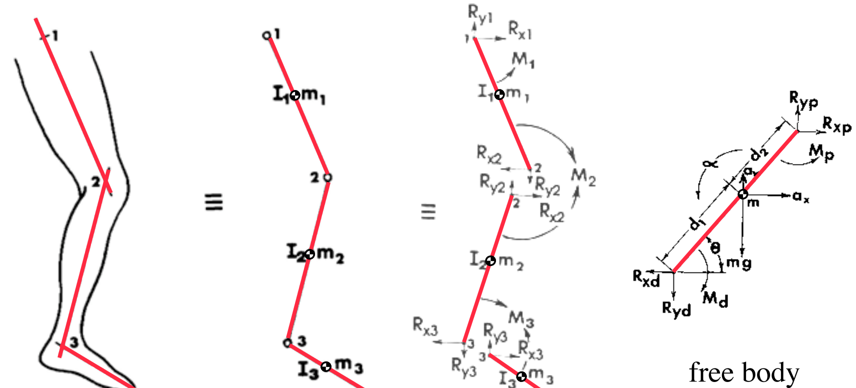
$$M_K = 9.81 + 11.2 = 21.01 \text{ N} \cdot \text{m}$$

"gravitational" moment "inertial" moment



In this example, more than twice the knee flexor moment (and hamstring force) is required to accelerate the limb than to hold it stationary.

Link segment ("inverse dynamics") analysis of net joint forces and muscle moments during movement



anatomical model ≡ link segment model ≡ free body diagram; free body diagram (with accelerations)

Clinical gait analysis

Clinical gait analysis usually involves examining both joint rotations (kinematics) and the muscle moments and net forces at joints (kinetics).

Joint rotations are calculated from the 3D positions of skin surface markers, measured by a multi-camera system.

A force platform is used to acquire simultaneous measures of the magnitude and point of application of foot reaction forces $(F_{APP})_X$ and $(F_{APP})_Y$.

These are input with body segment positions, velocities, and accelerations into an inverse dynamics routine to determine joint muscle moments and net joint forces.

Example: Ankle moments during gait.

Consider the free body diagram at right, which shows forces and accelerations applied to the foot during the terminal stance phase of gait.

(a) Fill in the blanks:

these parameters are directly measured:

____

these parameters are estimated from anthropometry:

____

these parameters are calculated from numerical differentiation:

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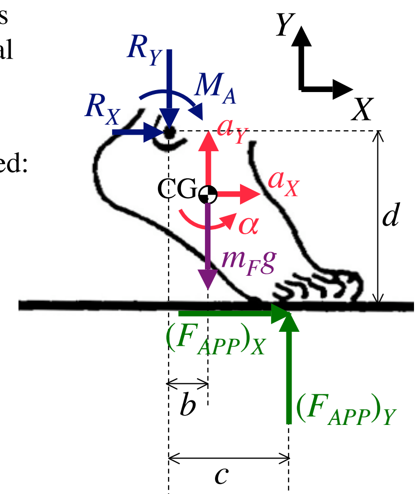
these parameters are calculated from inverse dynamics:

____



Ankle moments during gait (cont)

(b) Determine the equations for calculating R_X, R_Y, and M_A.

$$\sum F_X = m_F a_X$$

$$R_X + (F_{APP})_X = m_F a_X$$

$$R_X = m_F a_X - (F_{APP})_X$$

$$\sum F_Y = m_F a_Y$$

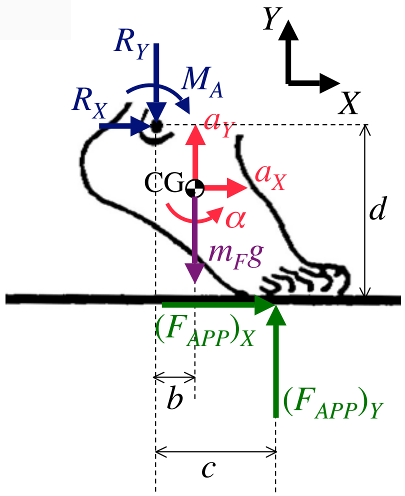
$$-R_Y - m_F g + (F_{APP})_Y = m_F a_Y$$

$$R_Y = (F_{APP})_Y - m_F g - m_F a_Y$$

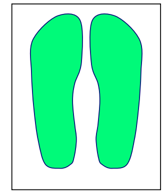
$$\sum M_0 = I_0 \alpha$$

$$-M_A - m_F g \cdot b + (F_{APP})_Y \cdot c + (F_{APP})_X \cdot d = I_0 \alpha$$

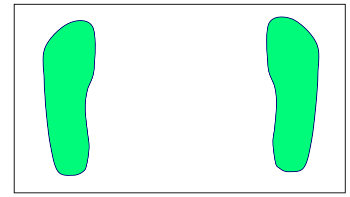
$$M_A = (F_{APP})_Y \cdot c + (F_{APP})_X \cdot d - m_F g \cdot b - I_0 \alpha$$



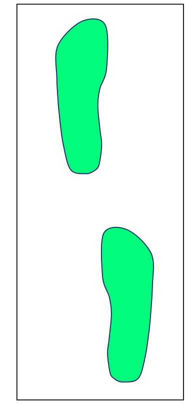
Human Balance



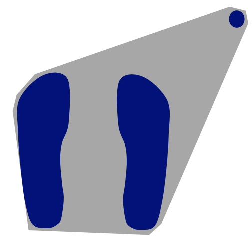
poor stability



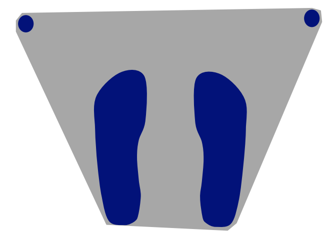
good ML stability



good AP stability



cane

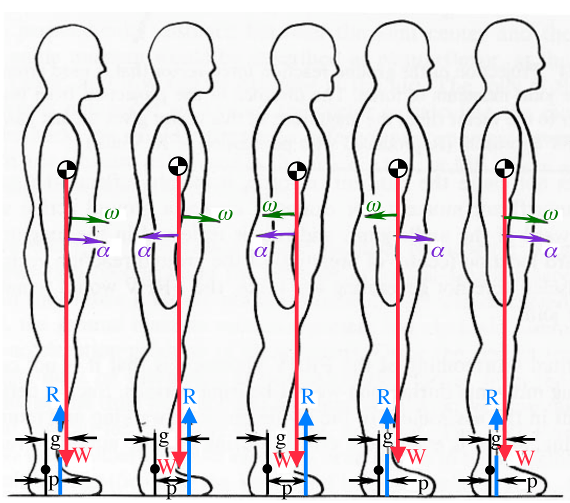


crutch

Sway during quiet stance: cat and mouse between COP and COG

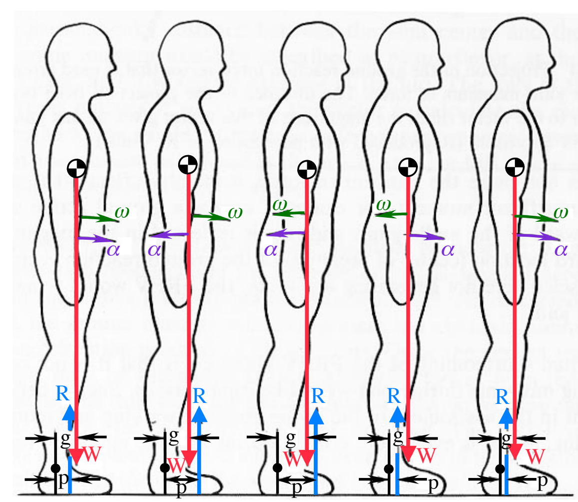
Even during quiet stance, the body's COG is never stationary.

Postural stability requires that we constantly move the foot COP (distance "p" in the figure at right) to catch the COG (distance "g" in the figure at right)



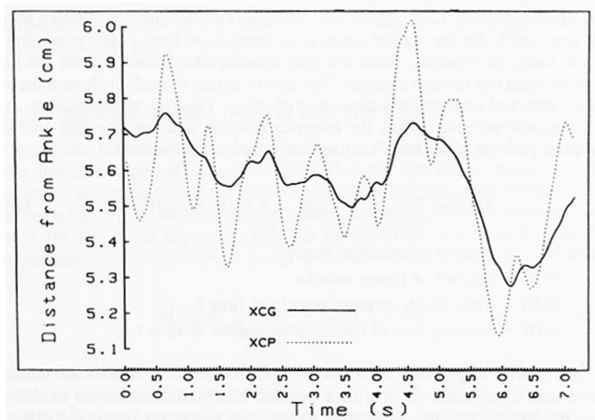
Sway during quiet stance (cont)

- if p < g, then α will be cw
- if p > g, then α will be ccw
- if α is cw and ω is cw, then p must be moved to be > g
- if α is ccw and ω is ccw, then p must be moved to be < g



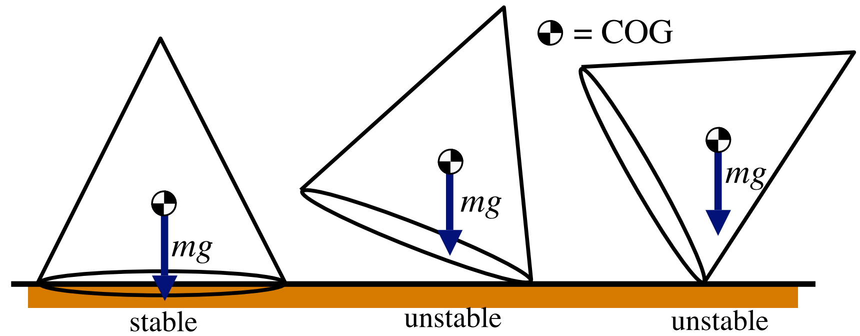
Sway during quiet stance (cont)

- the COP continuously moves anterior and posterior to the COG to maintain balance
- therefore, COP excursions are of higher frequency and greater amplitude, when compared to COG excursions



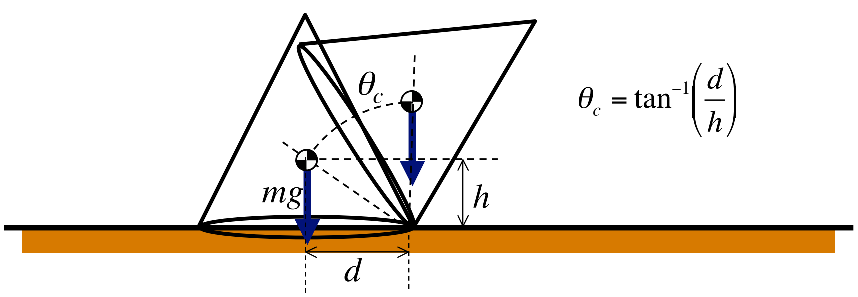
Postural stability requires that the COG is within the base of support

A *stable posture* exists if the vertical line of action of the force of gravity passing through the whole-body COG is within the base of support provided by the feet



Stability is decreased if the size of the base of support is reduced

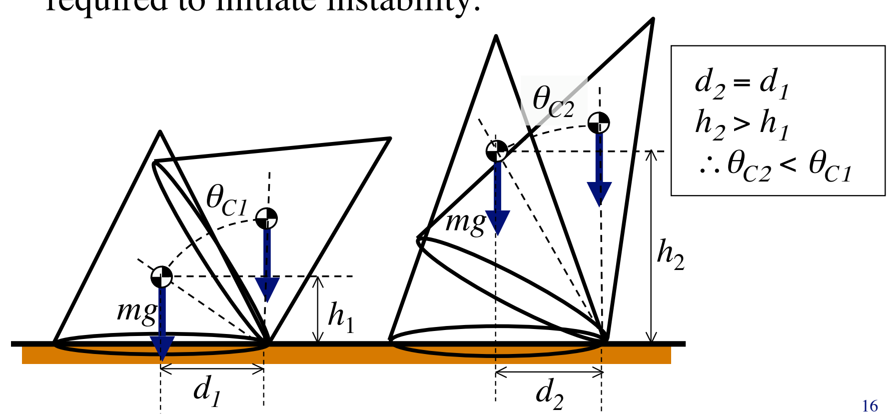
If the size of the base of support (d) is reduced, a smaller critical angle (θ_c) is required to initiate instability. (Does strength influence the size of the base of support?)



$$\theta_c = \tan^{-1}\left(\frac{d}{h}\right)$$

Stability is decreased if the height of the COG is increased

For a given base of support (d), an increase in the height (h) of the COG above the ground reduces the angle θ required to initiate instability.

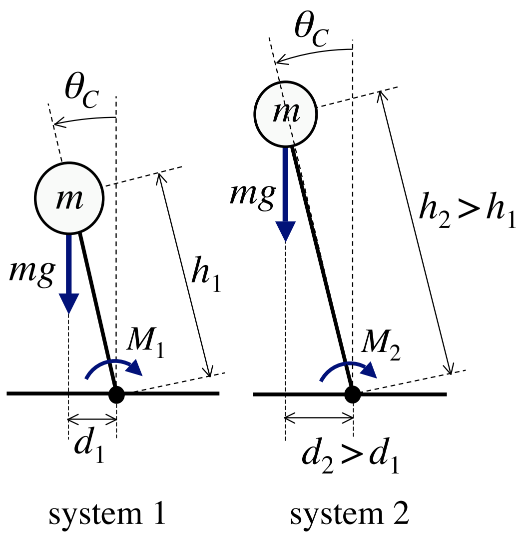


COG height and body mass affect stability

The moment required to recover stability (from a given lean angle θ) increases with the height of the COG.

Both systems at right have the same θ_C. However, the moment required to stabilize the system M_2 is greater than M_1.

Furthermore, if the mass (m) increases, and h and d remain constant, a larger force is required to displace the COG by an angle θ_C.



Balance Challenge #1

- Can you stand with your heels against the wall, then bend over (at hips, not knees) to pick up a dollar bill at your feet?
- Why not?

Balance Challenge #2

- Can you stand in a doorway with your nose and abdomen touching the door jamb, then rise up on your toes?
- Why not?

Review Questions

- what equations are used to determine the relations between segment accelerations and joint forces and moments?
- in gait analysis, what data is acquired from a force plate? from a motion measurement system?
- what is "inverse dynamics"? how is this procedure used in gait analysis to estimate joint forces and moments?
- what is meant by "sway during quiet stance"?
- how does the position of the COP with respect to the COG affect whole-body angular acceleration? how does it affect postural stability?
- how is postural stability affected by the size of the base of support, the height of the COG, and the magnitude of body mass?
- how does strength affect balance?