Assignment - 4

1. Using Fourier transform (FT) express the solution of the following initial value problem in terms of difference of two error functions:

$$u_t = c^2 u_{xx}; \quad -\infty < x < \infty; \quad t > 0.$$

$$u(x,0) = \begin{cases} 0, & x < a \\ L, & a \le x \le b \\ 0, & x > b. \end{cases}$$

2. Solve by applying FT,

$$u_t = t^2 u_{xx}; \quad -\infty < x < \infty, \ t > 0; \quad u(x,0) = f(x); \ -\infty < x < \infty.$$

3. Use appropriate FT to solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $0 < x < \infty$, $t > 0$, where $u(x,t)$ satisfies the conditions.

1) $u_x(0,t) = 0, \ t > 0$ 2) $u(x,0) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$

3) $u(x,t)$ is bounded.

4. Using Laplace transform (LT) solve

$$u_{tt} = u_{xx}; \quad 0 < x < 1, \ t > 0; \quad u(x,0) = \sin \pi x;$$

$$u_t(x,0) = -\sin \pi x; \quad 0 < x < 1.$$

5. Using LT solve $u_{xx} = \frac{1}{\alpha^2} u_{tt} - \cos \omega t$; $0 \le x < \infty$, $0 \le t < \infty$; $u(0,t) = 0$, $u(x,0) = 0$, $u_t(x,0) = 0$.

6. Solve using LT, $u_{tt} = a^2 u_{xx}$; $x > 0, \ t > 0$; $u(x,0) = 0, \ x > 0$; $u_t(x,0) = 0$; $x > 0$; $u(0,t) = \sin \omega t$, $\lim_{x \to \infty} (u(x,t)) = 0$.

P. T. O.

7. Solve by applying FT,

$$u_{xx} + u_{yy} = 0; \quad -\infty < x < \infty, \; y > 0.$$

$$u(x,0) = \begin{cases} 1, & a < x < b \\ 0, & \text{otherwise}. \end{cases}$$

8. Solve, employing appropriate transform technique w.r. to y, the following BVP:

$$u_{xx} + u_{yy} = 0; \quad 0 < x < l, \; y > 0.$$

$$u(0,y) = e^{-2y}, \quad u(l,y) = 0; \; y > 0; \quad u_y(x,0) = 0, \; 0 < x < l.$$

9. Solve the simultaneous PDE's

$$\frac{\partial u}{\partial x} = -2v, \quad \frac{\partial v}{\partial x} = \frac{1}{2}\frac{\partial u}{\partial t}.$$

Here $u = u(x,t)$, $v = v(x,t)$: given conditions are, $u(x,0) = 0$, $v(x,0) = 0$, $u(0,t) = u_0$.

10. Solve $x y_x + y_t - y = x^2$; $x > 0, t > 0$; $y \equiv y(x,t)$, subject to the boundary conditions $y(0,t) = 0$, $y(x,0) = 0$.