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INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: 21-09-2017 AN, **Time: 2 Hrs,** **Full Marks: 100** **Dept. : ME**
No. of Students: 182 **Mid Autumn Semester Examination**
Sub. No.: ME31013 **Sub. Name: Mechanics of Solids**
3rd Year B.Tech.(H)/M.Tech.(Dual Degree)

Instructions: Attempt all questions. Symbols have their usual meanings. Please explain your work carefully. Make suitable assumptions wherever necessary. Please state your assumptions clearly.

Q1. Consider a state of stress at a point, referred to a xyz - coordinate system, and represented in matrix form as

$$[\sigma]=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

Find the principal stresses and the corresponding principal directions for this state of stress. **[20 marks]**

Q2. Consider a state of stress at a point, referred to a xyz - coordinate system, and represented in matrix form as

$$[\sigma]=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(a) Find the unit normal to a plane that is parallel to the z - axis and on which the resultant stress (or, traction) vector is tangential to the plane itself.
(b) Determine the normal stress on a plane that passes through the same point and is parallel to the plane given by the equation $x+2y-2z=-4$.

[20 marks]

Q3. A body has the following displacement field

$$u(x,y,z)=30x^2z-10x^3y+20y^3$$
$$v(x,y,z)=10x^3+20xy^3+5y^2z$$
$$w=0$$

(a) Given that the material constituting the body is linear, elastic, and isotropic with Young's modulus $E=300\times10^9$ Pa and Poisson's ratio $\nu=0.3$, determine stresses at a point P $(0.05,0.02,0.01)$ on the body.
(b) Determine the octahedral shearing stress at the same point P.

[20 marks]

Q4. A state of strain is given by

$$[\varepsilon]=\begin{bmatrix} Az & 0 & 0 \\ 0 & Az & 0 \\ 0 & 0 & Bz \end{bmatrix}.$$

From the strain-displacement relations for normal strains, show that the three components (u, v, w) of the displacement vector are given by:

$$u = Axz + f(y, z),$$
$$v = Ayz + g(x, z),$$
$$w = \frac{1}{2}Bz^2 + h(x, y),$$

where $f(y,z)$, $g(x,z)$, and $h(x,y)$ are arbitrary functions. Now, use the strain-displacement relations for the shear strains to obtain $f(y,z)$ and $g(x,z)$ in terms of some arbitrary constants if it is given that

$$h(x,y) = -\frac{1}{2}Ax^2 - \frac{1}{2}Ay^2 - C_1x + C_2 + C_3y,$$

where C_1, C_2, and C_3 are also arbitrary constants. Thus obtain expressions for u_x, u_y, and u_z (involving additional arbitrary constants). **[20 marks]**

Q5. Consider the bending of a narrow beam of length ($2l$), height ($2h$), and unit width (into the plane of the paper) due to the uniform loading (q per unit area) as shown in Fig. 1 (disregard body forces)

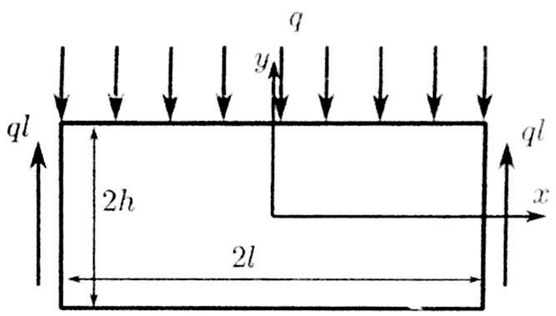

Fig. 1 for Q5.

(a) Mention the boundary conditions on each of the vertical and horizontal surfaces, i.e. at $x = \pm l$ and $y = \pm h$ in terms of the appropriate stress components.

(b) To solve the problem of finding the stress field in the beam using the Airy stress function approach, it is suggested that we start with the following polynomial form:

$$\varphi = Axy + Bx^2 + Cx^2y + Dy^3 + Exy^3 + Fx^2y^3 + Gy^5,$$

which, in general, will not satisfy the biharmonic equation $\nabla^4\varphi = 0$, i.e.

$$\frac{\partial^4\varphi}{\partial x^4} + 2\frac{\partial^4\varphi}{\partial x^2\partial y^2} + \frac{\partial^4\varphi}{\partial y^4} = 0.$$

Find the conditions involving the coefficients that will ensure the satisfaction of the biharmonic equations. Then find the conditions that will satisfy the boundary conditions. Finally, find expressions for the stress components.

[20 marks]