

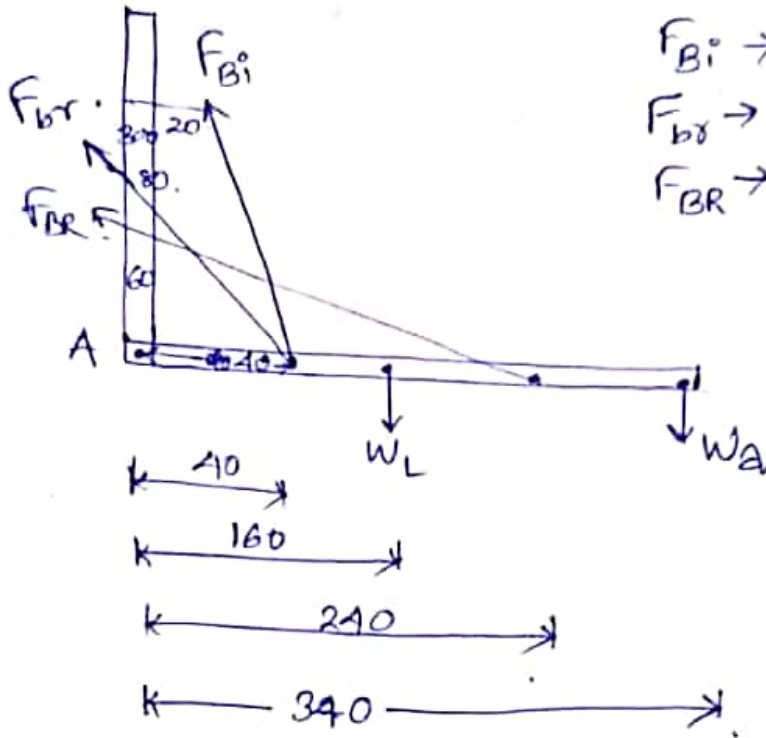
2016

$W_a \rightarrow$ Applied load.
 $W_L \rightarrow$ Weight of limb.

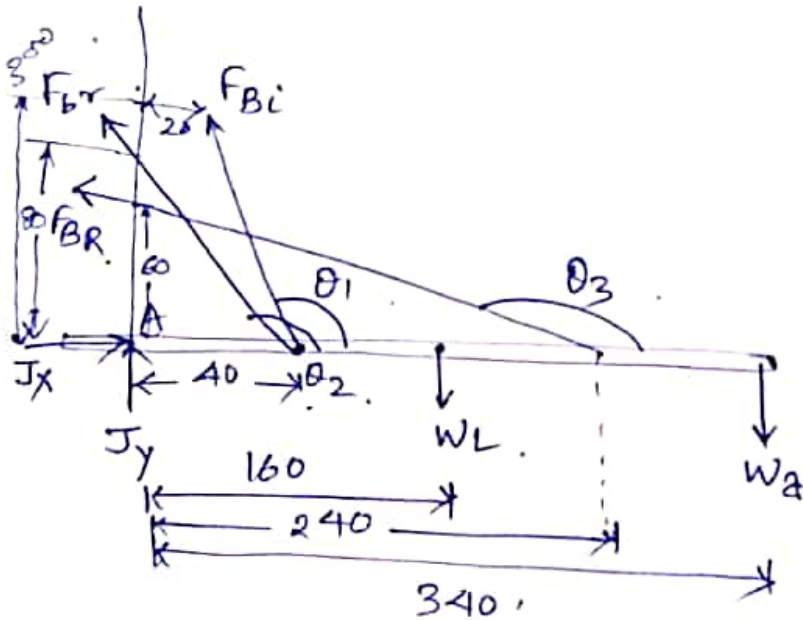
$F_{Bi} \rightarrow$ Force on Bicep

$F_{bx} \rightarrow$ Force on Brachialis

F_{BR} → " " Brachio-radialis



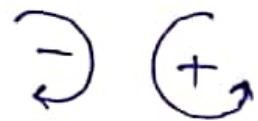
Free Body diagram

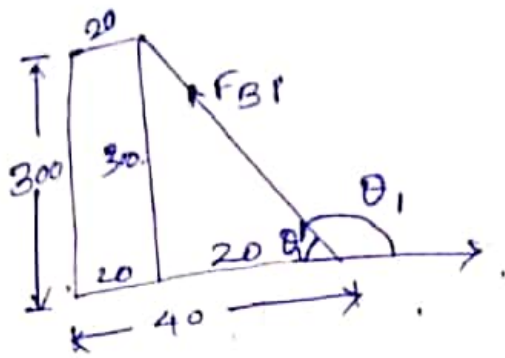


$$\sum F_x = 0$$

$$\Sigma F_y = 0$$

$$\sum M_A = 0$$





$$\tan \theta_1' = \frac{300}{20}$$

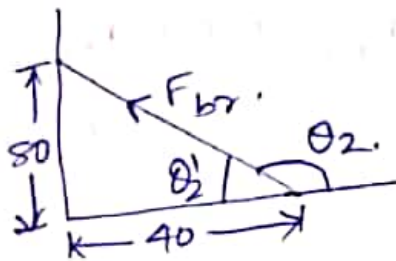
$$\Rightarrow \theta_1' = 86.1859^\circ$$

$$\theta_1 = 180^\circ - 86.1859^\circ$$

$$\Rightarrow \boxed{\theta_1 = 93.8141^\circ}$$

$$\sin \theta_1 = 0.9978$$

$$\cos \theta_1 = -0.0665$$



$$\tan \theta_2' = \frac{80}{40}$$

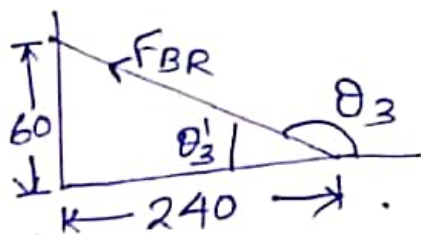
$$\Rightarrow \theta_2' = 63.4349^\circ$$

$$\theta_2 = 180^\circ - 63.4349^\circ$$

$$\Rightarrow \boxed{\theta_2 = 116.5651^\circ}$$

$$\sin \theta_2 = 0.8944$$

$$\cos \theta_2 = -0.4472$$



$$\tan \theta_3' = \frac{60}{240}$$

$$\Rightarrow \theta_3' = 14.0362^\circ$$

$$\theta_3 = 180^\circ - 14.0362^\circ$$

$$\Rightarrow \boxed{\theta_3 = 165.9638^\circ}$$

$$\sin \theta_3 = 0.2425$$

$$\cos \theta_3 = -0.9701$$

Now

$$\sum F_x = 0$$

$$\Rightarrow J_x + F_{Bi} \cos \theta_1 + F_{Br} \cos \theta_2 + F_{BR} \cos \theta_3 = 0$$

$$\Rightarrow J_x + F_{Bi} (-0.0665) + F_{br} (-0.4472) + F_{BR} (-0.9701) = 0$$

$$\Rightarrow J_x =$$

$$\Rightarrow J_x - 0.0665 F_{Bi} - 0.4472 F_{br} - 0.9701 F_{BR} = 0$$

————— (1)

$$\sum F_y = 0$$

$$\Rightarrow J_y + F_{Bi} \sin \theta_1 + F_{br} \sin \theta_2 + F_{BR} \sin \theta_3 - W_L - W_a = 0$$

$$\Rightarrow J_y + 0.9978 F_{Bi} + 0.8944 F_{Br} + 0.2425 F_{BR} - W_L - W_a = 0$$

————— (2)

$$\sum M_A = 0$$

$$\Rightarrow F_{Bi} \sin \theta_1 \times 40 + F_{Br} \sin \theta_2 \times 40 + F_{BR} \sin \theta_3 \times 240 - W_L \times 160 - W_a \times 340 = 0$$

$$\Rightarrow 0.9978 \times 40 \times F_{Bi} + 0.8944 \times 40 \times F_{Br} + 0.2425 \times 240 \times F_{BR} - 160 W_L - 340 W_a = 0$$

$$\Rightarrow 39.912 F_{Bi} + 35.776 F_{Br} + 58.2 F_{BR} - 160 W_L - 340 W_a = 0$$

————— (3)

We have 3 equations and no. of unknowns are 5 $\rightarrow J_x, J_y, F_{Bi}, F_{BR}, F_{br}$ (Indeterminate situation)

Assume that all 3 muscles are stressed to the same intensity. Then the force produced by each muscle will be proportional to its cross-sectional area.

$$\begin{aligned}\text{Given that } A_{Bi} &= 500 \text{ mm}^2 \\ A_{br} &= 480 \text{ mm}^2 \\ A_{BR} &= 100 \text{ mm}^2\end{aligned}$$

So according to our assumption,

$$\frac{F_{Bi}}{A_{Bi}} = \frac{F_{br}}{A_{br}} = \frac{F_{BR}}{A_{BR}}$$

$$\Rightarrow \frac{F_{Bi}}{500} = \frac{F_{br}}{480}$$

$$\Rightarrow F_{br} = 0.96 F_{Bi} \quad \text{--- (4)}$$

$$\frac{F_{Bi}}{500} = \frac{F_{BR}}{100}$$

$$\Rightarrow F_{BR} = 0.2 F_{Bi} \quad \text{--- (5)}$$

Now no. of eqⁿ = no. of unknowns = 5.
(determinate situation).

Rewriting the eqⁿ ①, ②, ③ in terms of F_{Bi}

Again we can find that

$$J_x - 0.0665 F_{Bi} - 0.4472 \times 0.96 F_{Bi} - 0.9701 \times 0.2 F_{Bi} = 0$$

$$\Rightarrow J_x - 0.6898 F_{Bi} = 0 \text{ ————— (6)}$$

$$J_y + 0.9978 F_{Bi} + 0.8949 \times 0.96 F_{Bi} + 0.2425 \times 0.2 F_{Bi} - 25 - 120 = 0$$

$$(\because W_L = 25 \text{ N} \text{ Given})$$

$$W_2 = 120 \text{ N}$$

$$\Rightarrow J_y + 1.9049 F_{Bi} - 145 = 0 \text{ ————— (7)}$$

$$39.912 F_{Bi} + 35.776 \times 0.96 F_{Bi} + 58.2 \times 0.2 F_{Bi} - 160 \times 25 - 340 \times 120 = 0$$

$$\Rightarrow 85.8969 F_{Bi} = 44800$$

$$\Rightarrow \boxed{F_{Bi} = 521.5554 \text{ N}}$$

$$F_{br} = 0.96 F_{Bi}$$

$$\Rightarrow F_{br} = 0.96 \times 521.5554$$

$$\Rightarrow \boxed{F_{br} = 500.6932 \text{ N}}$$

$$F_{BR} = 0.2 F_{Bi}$$

$$\Rightarrow F_{BR} = 0.2 \times 521.5554 \Rightarrow \boxed{F_{BR} = 104.3111 \text{ N}}$$

② Force produced by each muscle

$$F_{Bi} = 521.5554 \text{ N}$$

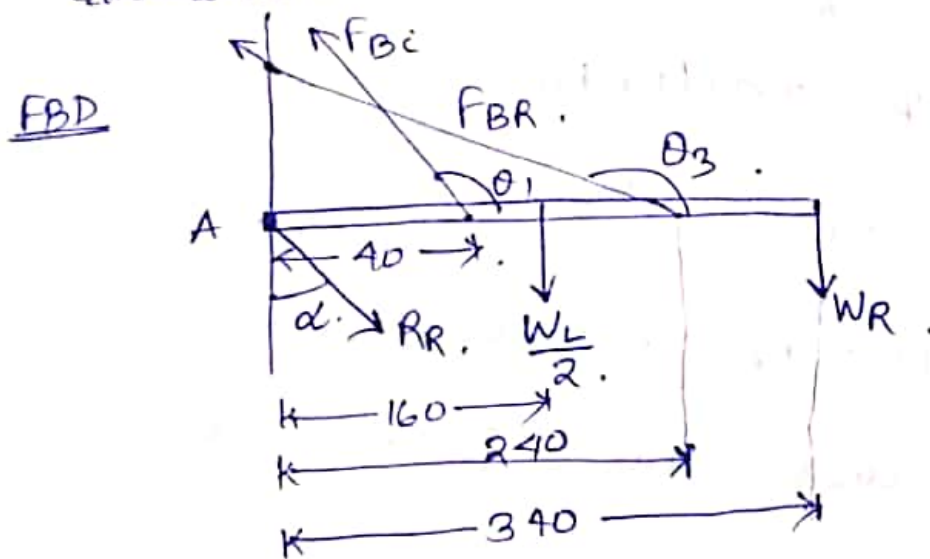
$$F_{Br} = 500.6932 \text{ N}$$

$$F_{BR} = 104.3111 \text{ N}$$

Humero-radial Joint force R_R \rightarrow Forces acting on the radius are

a portion of the applied load, say W_R
half of the weight of forearm $\frac{W_L}{2}$
 F_{Bi} , F_{Br} , R_R

Ligament forces (F_L) \rightarrow which will be ignored for the first stage stage of calculation.



$$\sum F_x = 0$$

$$\Rightarrow R_R \sin \alpha + F_{Bi} \cos \theta_1 + F_{BR} \cos \theta_3 = 0$$

$$\Rightarrow R_R \sin \alpha + 521.5554 \times (0.0665) + (-0.9701) 104.3111 = 0$$

$$\Rightarrow R_R \sin \alpha = 135.8756 \quad \text{--- (8)}$$

$$\sum F_y = 0$$

$$\Rightarrow W_R + \frac{W_L}{2} + R_R \cos \alpha = F_{Bi} \sin \theta_1 + F_{BR} \sin \theta_3$$

$$\Rightarrow W_R + \frac{25}{2} + R_R \cos \alpha = 521.5554 \times 0.497 + 104.3111 \times 0.292$$

$$\Rightarrow W_R + R_R \cos \alpha = 533.2034 \quad \text{--- (9)}$$

$$\Sigma M_A = 0$$

$$\Rightarrow F_{Bi} \sin \theta_1 \times 40 + F_{BR} \sin \theta_3 \times 240 - \frac{W_L}{2} \times 160 - W_R \times 340 = 0$$

$$\Rightarrow -2000 - 340 W_R = 0$$

$$\Rightarrow 340 W_R = 24887.2251$$

$$\Rightarrow \boxed{W_R = 73.1977 \text{ N}}$$

Putting W_R in eqⁿ (9)

$$73.1977 + R_R \cos \alpha = 533.2034$$

$$\Rightarrow R_R \cos \alpha = 460.0057 \quad \text{--- (10)}$$

From eqⁿ (8) & (10)

$$\tan \alpha = \frac{135.8756}{460.0057}$$

$$\Rightarrow \boxed{\alpha = 16.4559^\circ}$$

Normal force $R_N =$

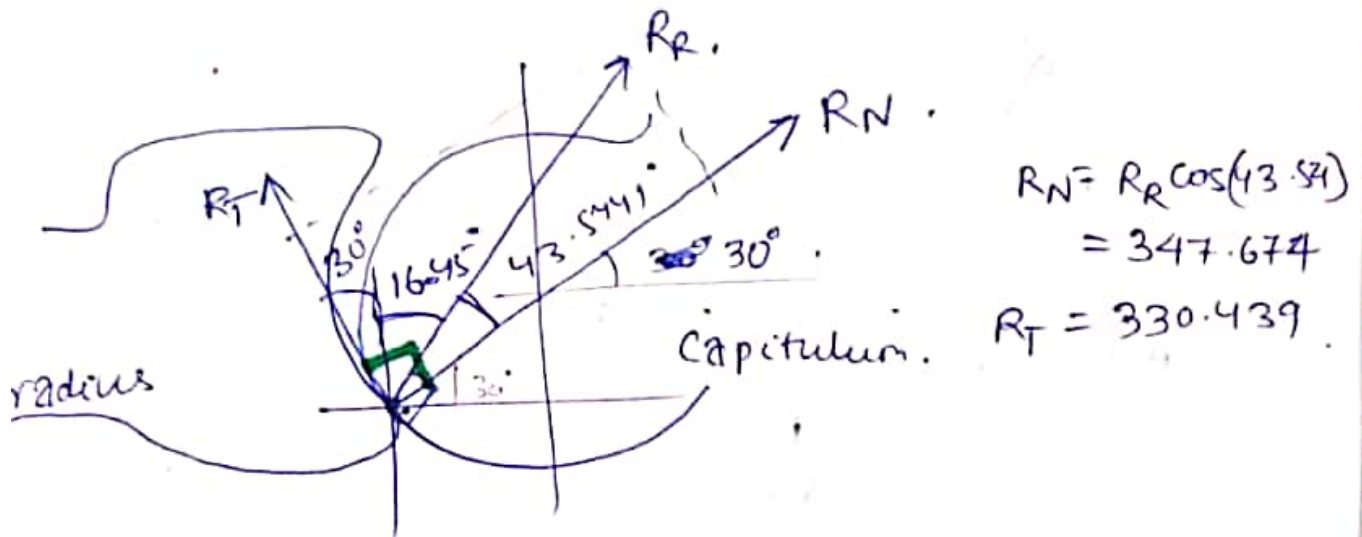
$$R_R \sin \alpha$$

$$R_R \cos \alpha = 460.0057$$

$$\Rightarrow \boxed{R_R = 479.653 \text{ N}}$$

Normal force $R_N = R_R \cos \alpha = 460.0057 \text{ N}$

Transverse force $= R_T = R_R \sin \alpha = 135.87$



Assume the interface friction coeff. = 0.02 (μ).

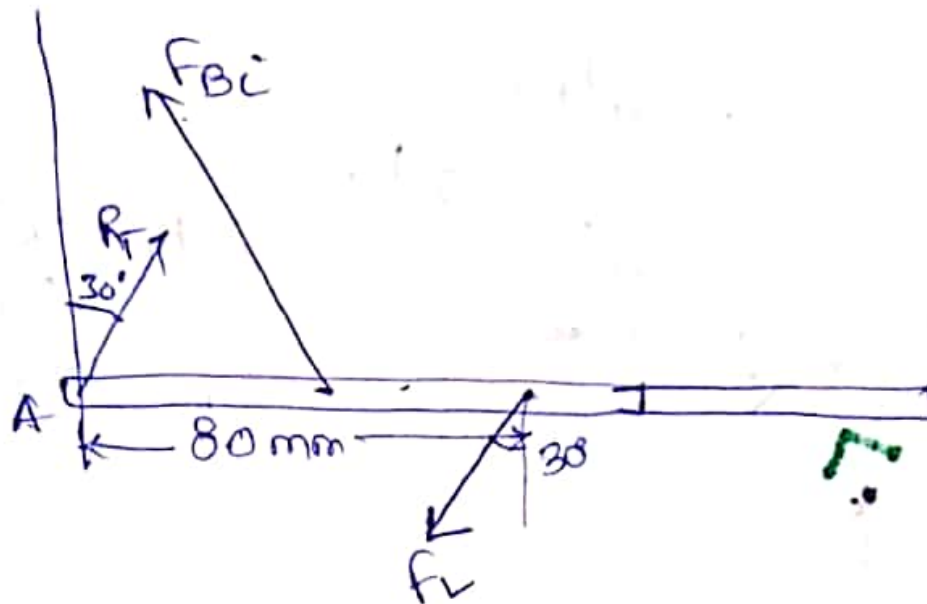
$$\text{Frictional force} = \mu R_N = 0.02 \times 347.674 = 6.9535 \text{ N}$$

So subtracting frictional force from the transverse reaction force remainder will be 323.4855 N which will cause sliding and the radial head may be pulled away across the capitulum. But to stop this this force is absorbed by the ligaments.

The F_L & R_T will make a couple.

F_L acts 80 mm from 'A'.

$$F_L = 323.4855 \text{ N}$$



$$\begin{aligned}
 \text{Moment of } F_L \text{ at 'A'} &= F_L \cos 30^\circ \times 80. \\
 &= 323.4855 \cos 30^\circ \times 80 \\
 &= 22411.733 \text{ Nmm}
 \end{aligned}$$

This moment opposes the flexion muscular and so decreases the load which can be carried.

$$\begin{aligned}
 \text{The moment created by applied load} \\
 &= 120 \times 340 = 40800 \text{ Nmm.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Subtracting the Moment by } F_L \text{ from} \\
 \text{the above} &= 40800 - 22411.733 \\
 &= 18388.267 \text{ Nmm.}
 \end{aligned}$$

$$\begin{aligned}
 \text{So the load to be carried} &= \frac{18388.267}{340} \\
 &= 54.083 \text{ N}
 \end{aligned}$$

Since we are considering ligaments now so the R_R need to be recalculated.

$R_N = 347.674 \text{ N}$ at 60° to Humeral axis.

$R_T = \text{sliding force} = \text{frictional force} + \text{force carried by ligaments.}$

So ~~R_T~~ only frictional force $= 6.9535 \text{ N}$ will be there.

$$\text{So } R_R = \sqrt{6.9535^2 + 347.674^2}$$

$$\Rightarrow R_R = 347.744 \text{ N}$$

$$\tan \alpha = \frac{R_T}{R_N}$$

$$\tan \alpha = \frac{R_T}{R_N}$$

$$\Rightarrow \text{or } \tan \alpha = \frac{6.9535}{347.674}$$

$$\Rightarrow \alpha = 1.145^\circ$$

