Indian Institute of Technology, Kharagpur

Mid-Autumn Semester Examination, 2016

Mechanical Engineering Department

Subject: Mechanics of Solids Subject No. : ME 31013

Full Marks : 30 Time : 2 Hrs

Adopt appropriate assumption if it is necessary and the stresses mentioned here refer to the Cauchy stresses. Unless otherwise mentioned, all dimensions in Figs. 2 and 3 are in mm.

1. (a) When and why principal stresses are real? Prove that the principal planes for the distinct principal stresses are orthogonal to each other. (1+2)

   (b) Figure 1 shows the normal and shear stresses at a point (**O**) of a deformed solid body based on the two planes **AB** and **CD**. The solid does not have body couples and is in equilibrium. Assuming the state of stress at the point as the state of plane stress problem, compute the shear stress τ_{xy} at the point as shown in the figure. (3)

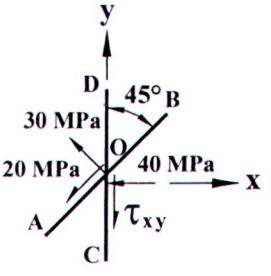


Fig. 1

2. The displacement field in a deformed solid is given by

$$\mathbf{u} = (3x^2y + 6)\times 10^{-2},\ \mathbf{v} = (6xz + y^2)\times 10^{-2} \text{ and } \mathbf{w} = (6z^2 + 2yz + 10)\times 10^{-2}$$

   (a) Calculate the state of strain at a point (1, 0, 2) and the longitudinal strain at the same point along the line $2x + \sqrt{3}y - 3z + 4 = 0$. (3+2)

   (b) If the solid is homogeneous and isotropic and its Young's modulus and Poisson's ratio are 200**GPa** and 0.3, respectively, determine the octahedral stresses at the same point. (3)

3. A straight prismatic beam made of homogeneous isotropic solid is loaded as shown in Fig. 2. The line of action of the load passes through the centroid of the cross-section at the mid-span of the beam and is tangential to this cross-section. Find the neutral axis in the cross-section and compute the normal stress at point **A**. Assume that the beam is simply supported. (6)

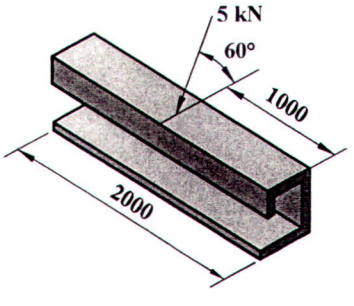
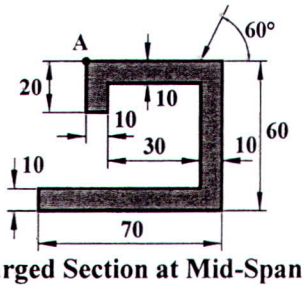


Fig. 2



Enlarged Section at Mid-Span

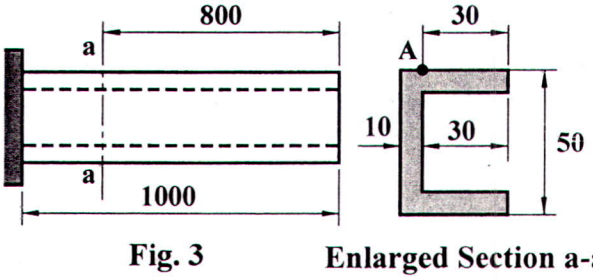


Fig. 3

Enlarged Section a-a

4. (a) Why the formula $\tau = VQ/(I_y t)$ is used for computing the transverse shear stresses in a beam instead of using the stress-strain relation? (2)

   (b) A cantilevered homogenous isotropic prismatic beam is shown in Fig. 3. The beam is subjected to an upward transverse load of 10 **kN** at the free end of the beam. Find the line of action of the load such that the bending-twisting coupling does not occur in the beam. Also, determine the principal stresses at a point **A**. The point **A** is located at section **a-a** as shown in Fig. 3. Wall thickness of the beam is uniform. (5+3)