The Compressive Behavior of Bone as a Two-Phase Porous Structure*†

BY DENNIS R. CARTER, PH.D.‡, SEATTLE, WASHINGTON AND WILSON C. HAYES, PH.D. §, PHILADELPHIA, PENNSYLVANIA

From the Department of Orthopaedics, University of Washington, Seattle, and the Department of Orthopaedic Surgery, University of Pennsylvania, Philadelphia

ABSTRACT: Compression tests of human and bovine trabecular bone specimens with and without marrow *in situ* were conducted at strain rates of from 0.001 to 10.0 per second. A porous platen above the specimens allowed the escape of marrow during testing. The presence of marrow increased the strength, modulus, and energy absorption of specimens only at the highest strain rate of 10.0 per second. This enhancement of material properties at the highest strain rate was due primarily to the restricted viscous flow of marrow through the platen rather than the flow through the pores of the trabecular bone. In specimens without marrow, the strength was proportional to the square of the apparent density and the modulus was proportional to the cube of the apparent density.

Both strength and modulus were approximately proportional to the strain rate raised to the 0.06 power. These power relationships, which were shown to hold for all bone in the skeleton, allow meaningful predictions of bone tissue strength and stiffness based on *in vivo* density measurements.

Whole bones of the skeleton are composed of bone tissue exhibiting two forms of structural organization. Compact bone tissue forms the structural shell of most bones and nearly all of the diaphyses of long bones. Trabecular bone is continuous with the inner surface of the cortical shell and exists as a three-dimensional lattice composed of plates and columns of bone. The trabeculae divide the interior volume of bone into intercommunicating pores of different dimensions, producing a structure of variable porosity and apparent density [10,20]. The classification of bone tissue as compact or trabecular is based on bone porosity, which is the proportion of the volume occupied by non-mineralized tissue. Compact bone has a porosity of approximately 5 to 30 per cent; trabecular-bone porosity may range from approximately 30 to more than 90 per cent. However, the distinction between very porous compact bone and very dense trabecular bone is somewhat arbitrary [21]. The porosity of normal bone tissue shows an approximately linear relationship to the apparent density (mineralized tissue mass per total tissue volume) and ash density (ash weight per total tissue volume) [20,23]. Therefore porosity, ash density, and apparent density are all reasonable measures of the amount of mineralized tissue.

Weaver and Chalmers [34] investigated the influence of ash density and age on the ultimate compressive strength of trabecular bone specimens from vertebral bodies. They found that compressive strengths ranged from 3.81 meganewtons per square meter in an eighty-one-year-old man to 8.25 meganewtons per square meter in a thirty-year-old woman, and derived positive correlation coefficients relating strength to ash density. Similar results were obtained by Bartley and associates [1]. Bell and co-workers [2] tested intact human vertebrae in compression and noted ultimate compressive strengths of from 1.37 to 13.7 meganewtons per square meter. Vertebral ash density was shown to decline with age and appeared to be non-linearly related to the vertebral compressive strength.

Galante and co-workers [10] examined the influence of apparent density and trabecular orientation on the compressive strength of vertebral bone specimens. They and other workers [16,19] found a positive correlation between apparent density and compressive strength. Specimens loaded in the superior-inferior direction were more than twice as strong as those loaded in the lateral-medial direction. In addition, specimens loaded at a rate of 0.1 centimeter per minute were approximately 30 per cent stronger than those tested at 0.01 centimeter per minute.

McElhaney and associates [19] observed positive correlations between the apparent density of vertebral bone specimens and compressive strength and modulus of elasticity. Using a porous block model for bone, they hypothesized that the compressive modulus was proportional to the cube of the apparent density and that the compressive strength was proportional to the fourth power of the apparent density [19,20].

Bone as a Two-Phase Porous Material

The compositions and true tissue densities (mass of mineralized tissue per volume of mineralized tissue) of normal trabecular and compact bone tissue [10,12] are similar. In addition, recent work has suggested that the microscopic material properties of trabecular bone are similar to those of compact bone [20,21,27,28,32]. These observations suggest that whole bones can be modeled as composite structures consisting of both a solid and a fluid phase. The solid phase is the mineralized bone tissue, and the fluid phase is composed

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‡ School of Medicine, University of Washington, Seattle, Washington 98195.

§ School of Medicine, University of Pennsylvania, Philadelphia, Pennsylvania 19174.

of blood vessels, blood, red and yellow marrow, nerve tissue, miscellaneous cells, and interstitial fluid [19,20]. The distribution of pores throughout the whole bone contributes to the apparent density distributions which are observed roentgenographically. The pore geometry and orientation contribute to the structural anisotropy, which is evident in both trabecular and compact bone.

The structure of trabecular bone is similar to that of open-celled rigid plastic foams and aerated (porous) concrete. In such materials the apparent density is generally the most important factor affecting the material properties [25]. Empirically derived expressions for relating the elastic constants or strength of these porous materials to their apparent densities are power relations of the form [20,25]

$$\gamma = A\rho^{B} \tag{1}$$

where γ = material property (for example, strength, modulus), ρ = apparent density (mass/bulk volume), and A and B = experimentally derived constants. This equation could also be written as

$$\log \gamma = \log A + B \log \rho$$

The relationship between γ and ρ is thus linear on log-log plots. The regression slope of data presented in this manner is the value of B and the intercept is log A.

Additional factors affecting the material properties include structural anisotropy (direction of so-called foam rise or trabecular orientation in the case of bone), the material properties of the solid phase, and the size of the test specimens.

The objectives of this investigation were to explore further the model of bone as a two-phase composite material. Compression tests of human and bovine trabecular bone which spanned a wide range of apparent densities were conducted over a wide range of strain rates. By analyzing these data with reference to the values for the mechanical behavior of compact bone reported in the literature, we hoped to obtain empirical relationships between the apparent density and the compressive strength, modulus, and strain rate of bone.

Materials and Methods

One hundred cylindrical specimens of human trabecular bone and twenty-four specimens of bovine trabecular bone were machined under continuous irrigation. The specimens, which were approximately five millimeters thick with a radius of 10.3 millimeters, were removed from human tibial plateaus and bovine femoral condyles and were oriented with their axes parallel to the long axis of the bone. After the machining, the specimens were stored at −20 degrees centigrade until testing [30]. Prior to testing, they were thawed and kept fully moist.

Several additional specimens prepared in an identical manner were sectioned, embedded in acrylic resin, and examined in a reflected light microscope for evidence of damage during preparation. Fracture and tearing of trabeculae were occasionally observed on the exposed surfaces. However, the specimens appeared to be undamaged throughout most of their volume.

TABLE I

EXPERIMENTAL DESIGN

Specimen Type	Strain Rate per Second* 0.001	0.01	0.1	1.0	10.0
Human with marrow *in situ*	10	10	10	10	10
Human without marrow	10	10	10	10	10
Bovine without marrow	..	12	12	..	..

* Number of specimens at each strain rate.

A random allocation scheme was used to divide the specimens into twelve test groups (Table I). Groups of twenty human bone specimens were tested at constant strain rates of 0.001, 0.01, 0.1, 1.0, and 10.0 per second. Half of the human specimens were tested with marrow *in situ*, and half were tested after the marrow had been removed with an air jet and running water. The bovine specimens were tested primarily to study bone with greater apparent densities than those normally found in human tibial metaphyseal bone. All of the bovine specimens were tested without marrow *in situ*. Twelve bovine specimens were compressed at a strain rate of 0.01 per second, and twelve were compressed at a rate of 0.1 per second.

Testing was done in uniaxial strain by confining the specimens in a rigid stainless-steel annulus and compressing them by more than 50 per cent of the original specimen thickness in an electrohydraulic materials testing machine (MTS Systems). A porous compression platen above the specimen allowed the escape of marrow from the specimens during testing. A load-deformation curve for each specimen was recorded on a storage oscilloscope and photographed for later data analysis. Load was measured by the calibrated load cell of the test system, and deformation was measured by the built-in linear variable differential transformer of the loading piston. The stiffness of the test system was also recorded during the application of direct platen-to-platen loading. This machine stiffness was later used to adjust the load-deformation data for each specimen based on a correction for the loading system compliance [26].

After testing, the marrow was removed from those specimens tested with the marrow *in situ* by an air jet and running water. All specimens were degreased with ethanol. After immersion in distilled water and vacuum degassing, the specimens were weighed suspended from an analytical balance to determine *submerged weight*. They were then centrifuged at 8,000g on blotting paper for fifteen minutes to remove residual water from the pores and weighed in air to obtain *hydrated tissue weight*. The volume of bone tissue in cubic centimeters (excluding pores) was calculated as the difference between the hydrated tissue weight and submerged weight expressed in grams [10]. *Tissue density* of the specimen was found by dividing hydrated tissue weight by bone tissue volume. *Apparent density* of the specimen was calculated by dividing hydrated tissue weight by bulk volume of the specimen as determined by micrometer measurements prior to testing [10,23].

Results and Analysis

Stress-strain curves for specimens tested without marrow *in situ* showed that yield occurred when the strain was between 3 and 20 per cent (Fig. 1). These curves also showed that yield was generally followed by a slight softening and then a large region of nearly constant stress. This post-yield behavior indicates progressive trabecular failure and pore collapse until the specimens were compressed by more than half their original thickness. Then as the pores became closed, the stiffness of the specimens increased markedly. Specimens with low apparent densities tolerated higher strains before the onset of pore closure [13].

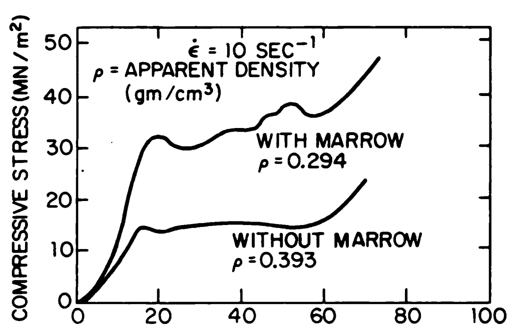

FIG. 1

Representative stress-strain curves for one specimen of trabecular bone with marrow and one without marrow tested at a strain rate of 10.0 per second.

The stress-strain curves of the human bone specimens tested with marrow *in situ* at strain rates of 0.001, 0.01, 0.1, and 1.0 per second could not be distinguished from the curves of the specimens tested without marrow (Table II). The viscous flow of marrow out of the pores in these specimens did not cause a significant increase in bone stiffness, strength, or energy absorption at the four lower strain rates. The ten human specimens with marrow which were tested at a strain rate of 10.0 per second, on the other hand, showed highly significant increases in stiffness, strength, and energy absorption when compared with the specimens tested without marrow at the same rate ($p < 0.01$ in all cases) (Fig. 1, Table III).

Compressive Strength

The compressive strength was taken to be the maximum stress achieved prior to pore collapse. The influence of specimen density on the compressive strength of specimens tested at the various strain rates is shown in Figures 2, *A* through 3, *B*. Compressive strength was plotted against apparent density of the specimen on log-log scales, since it was anticipated that the compressive strength of bone could be expressed approximately as a power function of the specimen density. The figures illustrate a strong relationship between bone density and compressive strength for all specimens except those tested with marrow *in situ* at a strain rate of 10.0 per second. For those ten specimens (Fig. 2, *E*), the compressive strength was independent of the bone density, thus suggesting that the viscous flow of marrow during compression was the dominant factor controlling the mechanical behavior of these specimens. Eliminating these specimens, the data show that density of the specimen has a much greater influence than strain rate on bone-tissue compressive strength. Adjusting for the influence of the apparent density of the specimens, we established the relationship between strength and strain rate (disregarding the effect of marrow flow) by using a power curve fit (linear regression on log-log plots) of strength versus apparent density for all the human specimens at each strain rate (Figs. 2, *A* through 3, *E*). The curve fit for specimens tested at a strain rate of 10.0 per second, however, was done only with the specimens tested without their marrow. These curve fits were then used to establish an adjusted mean compressive strength (adjusted to a mean density of 0.31 gram per cubic centimeter) for specimens at each of the five strain rates (Table II).

The adjusted mean compressive strengths for the human bone specimens (disregarding the specimens tested with marrow *in situ* at a strain rate of 10.0 per second) are plotted against strain rate on log-log scales in Figure 4-A [5]. Data from previous studies on the effect of strain rate on compact bone strength are also shown [6,18,35]. The regression slopes for the human trabecular bone specimens of this investigation and for the compact bone from other studies [6,7,18,35] are both approximately 0.06. This finding suggests that the strength of all bone tissue is approximately proportional to the strain rate raised to the 0.06 power.

The effect of specimen density on compressive strength for human and bovine bone at a strain rate of 0.01 per second is shown in Figure 4-B [5] in which the results for specimens in this investigation and for specimens of trabecular [10] and compact bone [18] from other studies are shown. The data are well described by a straight line with

TABLE II

SUMMARY OF HUMAN TEST DATA

Strain Rate (*per second*)	Effect of Marrow*	Adjusted Apparent Density (g/cm^3)	Adjusted Strength (S.E.) (MN/m^2)	Adjusted Modulus (S.E.) (MN/m^2)
0.001	N.S.	0.31	4.2 (0.6)	56.6 (9.7)
0.01	N.S.	0.31	4.1 (0.7)	75.5 (11.8)
0.1	N.S.	0.31	5.8 (0.7)	81.5 (8.0)
1.0	N.S.	0.31	6.7 (0.8)	81.2 (17.1)
10.0	S.	0.31	9.13† (1.32)	83.7† (13.8)

* N.S. = not significant; S. = significant.

† Data based on specimens *without* marrow *in situ* only.

TABLE III

HUMAN BONE AT A STRAIN RATE OF 10.0 PER SECOND

	Apparent Density (S.D.) (g/cm^3)	Compressive Strength (S.D.) (MN/m^2)	Compressive Modulus (S.D.) (MN/m^2)	Energy Absorbed at 50 Per Cent Strain ($MN\text{-}m/m^3$)
Marrow *in situ*	0.30 (0.09)	27.0 (9.8)	211 (78)	11.0 (3.3)
Without marrow	0.24 (0.09)	5.9 (4.2)	54 (37)	2.3 (1.6)

a slope of 2.0. This finding suggests that the compressive strength of bone over a very wide range of apparent densities is approximately proportional to the square of the apparent density of the bone. When the data from this investigation for specimens tested at a strain rate of 0.1 per second were plotted, the curve was similar.

The effects of strain rate and density on the mean compressive strength of the test specimens (Figs. 4-A and 4-B) suggest that the strength can be approximated by the relation

$$S = S_c \dot{\epsilon}^{0.06} \left(\frac{\rho}{\rho_c}\right)^2 \qquad (2)$$

where S = compressive strength (meganewtons per square meter) of a bone specimen of apparent density ρ (grams per cubic centimeter) tested at a strain rate of $\dot{\epsilon}$ (per second), and S_c = compressive strength of compact bone with an apparent density of ρ_c (grams per cubic centimeter) tested at a strain rate of 1.0 per second. Human compact bone tested at a strain rate of 1.0 per second has a compressive strength of approximately 221 meganewtons per square meter [18] and an apparent density of approximately 1.8 grams per cubic centimeter [17]. Using these values, Equation 2 can be simplified to

$$S = 68\ \dot{\epsilon}^{0.06} \rho^2 \qquad (3)$$

Compressive Modulus

The modulus of each specimen was taken to be the slope of the stress-strain curve in the most linear portion of the pre-yield loading region [8]. With the modulus defined in this manner, the small, initially non-linear behavior due to surface irregularities of the specimen and settling of the platen is disregarded [33]. The effect of the apparent density of the specimens on compressive modulus is summarized in Figures 5, *A* through 6, *B*. Again, since a power relationship of the modulus with density was anticipated, the data were plotted on log-log scales. As seen in Figures 5, *A* through 6, *B*, there was a strong relationship between bone density and compressive modulus for all specimens except those tested with marrow *in situ* at a strain rate of 10.0 per second. For these ten specimens (Fig. 5, *E*), the modulus was independent of the apparent density of the bone, a finding that again suggests that in these specimens the viscous flow of marrow was the dominant factor controlling the mechanical behavior. After eliminating these ten specimens from consideration, we examined the influence of strain rate on the modulus of human bone (disregarding the marrow effects). To determine an adjusted mean modulus at each strain rate, we made power curve fits for all the human specimens shown in Figures 5, *A* through 5, *E*. The curve fit for the specimens tested at 10.0 per second was done using only those specimens tested without marrow *in situ*. Adjusted mean compressive

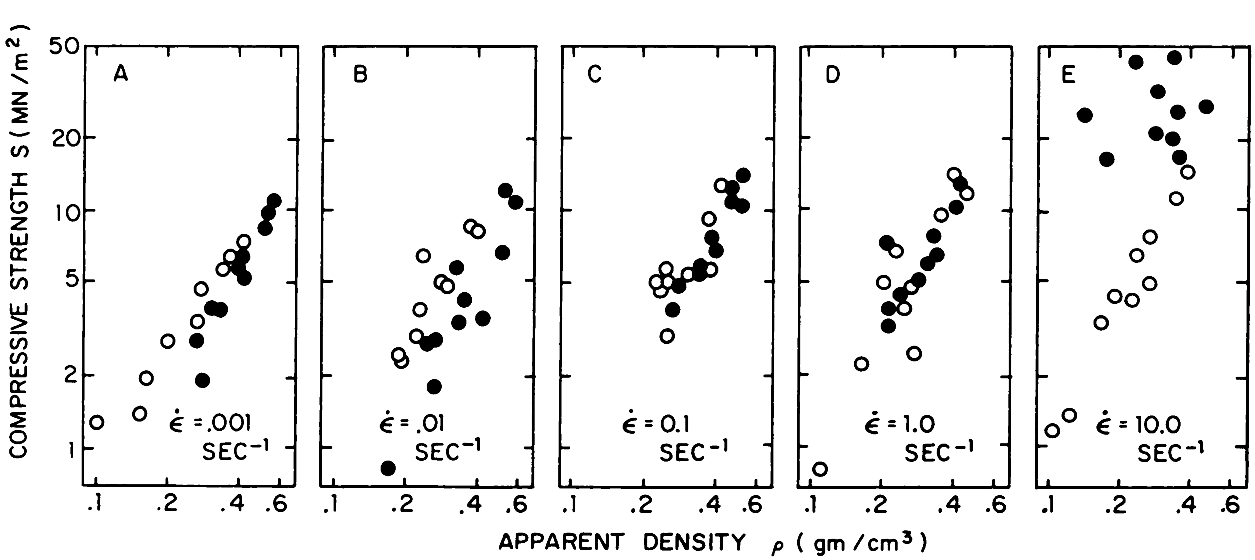

FIG. 2

Compressive strength versus apparent density of human trabecular bone tested at each of five strain rates. Open circles represent specimens without marrow; filled circles represent specimens with marrow.

moduli corresponding to a specimen density of 0.31 gram per cubic centimeter were then computed for specimens tested at each of the five strain rates (Table II).

In Figure 7-A the adjusted mean compressive moduli for the human trabecular bone specimens (disregarding the specimens tested with marrow *in situ* at a strain rate of 10.0 per second) are plotted against strain rates on log-log scales. Data from previous studies on the effect of strain rate on the modulus of compact bone are also shown [18,35]. In both types of bone a progressive increase in bone stiffness accompanies the increase in strain rate. The regression slopes for the human trabecular bone specimens of this investigation and for the compact bone of other studies are both approximately 0.06. This finding suggests that the modulus (as well as the strength) is approximately proportional to the strain rate raised to the 0.06 power.

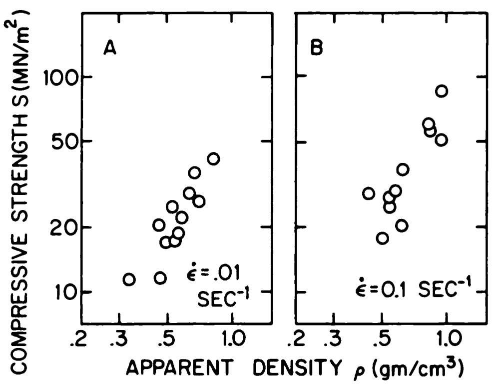

FIG. 3

Compressive strength versus apparent density of bovine trabecular bone tested at strain rates of 0.01 and 0.1 per second.

The influence of specimen density on compressive modulus for human and bovine bone at a strain rate of 0.01 per second is shown in Figure 7-B. The data plotted on this graph are well described by a regression line with a slope of about 3.0. This finding suggests that the compressive modulus of bone is approximately proportional to the cube of the apparent density. A similar curve was produced when the data for specimens tested at a strain rate of 0.1 per second were plotted. Figures 7-A and 7-B suggest that the compressive modulus of trabecular bone at a particular strain rate can be approximated by the relation

$$E = E_c \dot{\epsilon}^{0.06} \left(\frac{\rho}{\rho_c} \right)^3 \tag{4}$$

where E = compressive modulus (meganewtons per square meter) of a bone specimen of apparent density ρ (grams per cubic centimeter) tested at a strain rate of $\dot{\epsilon}$ (per second), and E_c = compressive modulus (meganewtons per square meter) of compact bone with an apparent density of ρ_c (grams per cubic centimeter) tested at a strain rate of 1.0 per second. Human compact bone tested at a strain rate of 1.0 per second has a compressive modulus of approximately 2.21×10^4 meganewtons per square meter [18]. Again assuming an apparent density for compact bone of 1.8 grams per cubic centimeter, Equation 4 can be simplified to

$$E = 3790\ \dot{\epsilon}^{0.06}\ \rho^3 . \tag{5}$$

Discussion

The results of this study indicate that the mechanical behavior of bone is similar to that of fluid-filled porous engineering materials. The compressive strength of bone was approximately proportional to the square of the apparent density, and the compressive modulus was approximately proportional to the cube of the apparent density. The strength and modulus of bone were shown to be approximately proportional to the strain rate raised to the 0.06 power. The viscous flow of marrow influenced the mechanical properties of trabecular bone specimens only at a high strain rate (10.0 per second) with very confined boundary conditions.

It should be recognized that Equations 3 and 5 are empirically derived expressions which may be used to estimate bone tissue compressive strength and modulus when considerable variations in the apparent density of bone exist. These relations have been derived to describe the behavior of *bone tissue* when the influence of viscous marrow flow is neglected. A more rigorous analysis accounting for factors such as bone ultrastructure, mineralization, trabecular orientation (anisotropy), and disease state is needed to describe more exactly the compressive strength and modulus of bone tissue over a narrow range of densities. It should be noted also that the influence of strain rate is relatively small compared with the influence of apparent density.

The wide range of strain rates used in this study encompasses those strain rates encountered in daily activities as well as those associated with severe impact fractures. Estimates of *in vivo* bone strain rates made by other researchers were based on diaphyseal compact bone rather than on trabecular bone. Due to the more compliant behavior of trabecular bone, the strain rates encountered in this bone type may be slightly higher than those in compact bone. However, the orders of magnitude of the strain rates produced in trabecular bone and in compact bone are probably comparable.

Using strain gauges bonded to a human tibial diaphysis, Lanyon and associates [15] recorded maximum strain rates in the range of 0.002 per second during walking (1.0 meter per second) and 0.01 per second during running (2.2 meters per second). Burstein and Frankel [3] estimated that most torsional impact fractures of the tibia occur during a total loading period of about 0.1 to 0.01 second. If one assumes that the ultimate strain of the diaphyseal bone at failure is approximately 0.01, these impact fractures correspond to strain rates in the range of 0.1 to 1.0 per second. In high-speed automobile accidents, higher strain rates may be encountered.

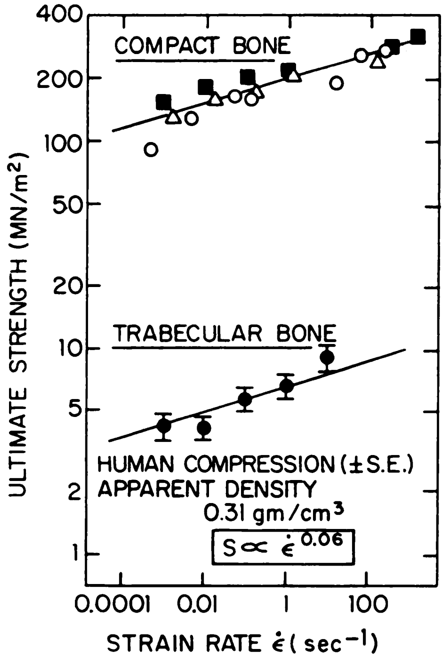

FIG. 4-A

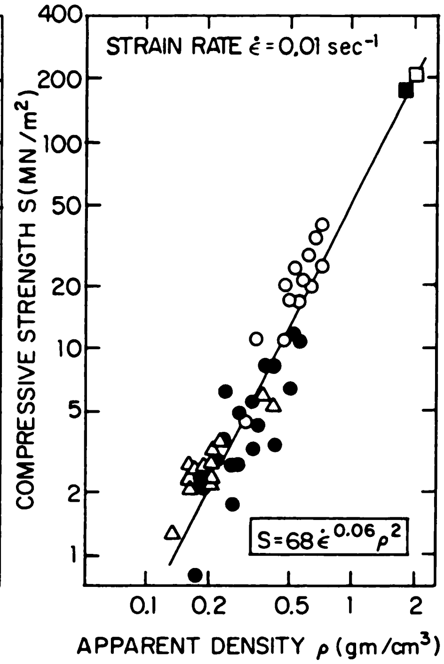

FIG. 4-B

Fig. 4-A: Ultimate strength versus strain rate of the human trabecular bone tested in this investigation and of compact bone tested by previous investigators [6,18,35]. The squares represent embalmed human specimens tested in compression [18]; the triangles, fresh bovine specimens tested in tension [6]; and the open circles, fresh bovine specimens tested in tension [35].

Fig. 4-B: Compressive strength versus apparent density of compact and trabecular bone. The filled circles represent the human and the open circles represent the bovine trabecular bone tested in this investigation. The triangles represent fresh human compact bone [10]; the filled and open squares represent embalmed human and fresh bovine compact bone, respectively [18].

The Influence of Strain Rate on Specimens without Marrow

The influence of strain rate on stiffness and strength was similar in the trabecular bone tested in this study and in the compact bone tested by previous investigators. The compressive strength and modulus of both compact and trabecular bone tissue are approximately proportional to the strain rate raised to the 0.06 power (Figs. 4-A and 7-A). The strength and stiffness for specimens without marrow *in situ* approximately doubled when the strain rate was increased from 0.001 to 10.0 per second. The effects of strain rate on the mechanical behavior of trabecular bone tissue were evident when adjustments were made for specimen densities. Considering the amount of unexplained scatter of the data, however, it appears that the influence of strain rate on bone tissue was of the same order of importance as the composition of the bone tissue and the orientation of the trabeculae. Apparent density was

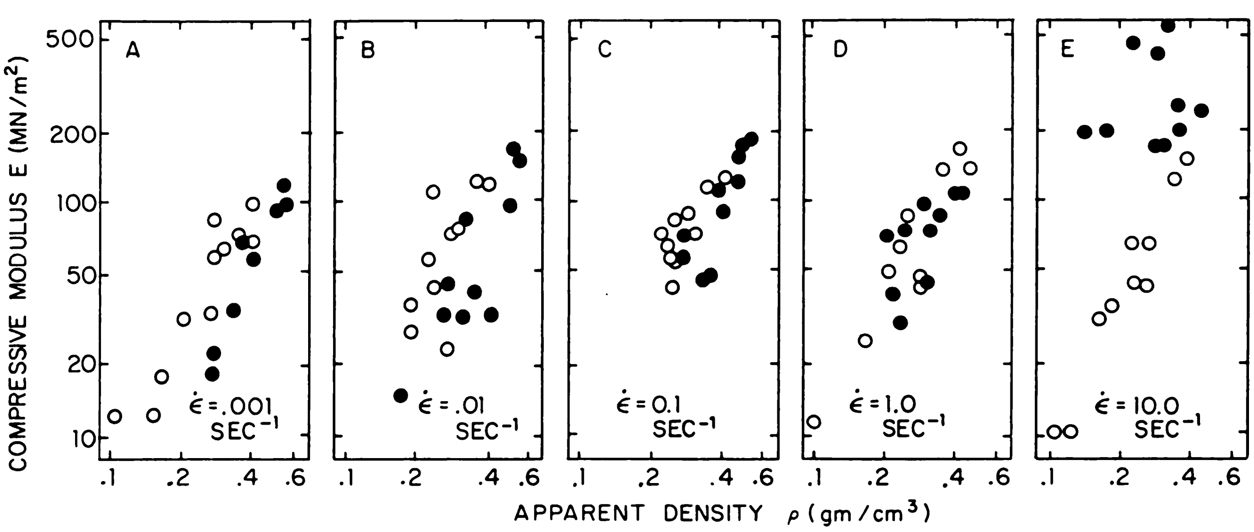

FIG. 5

Compressive modulus versus apparent density of human trabecular bone tested at each of five strain rates. The open circles represent specimens without marrow and the filled circles, specimens with marrow.

a much more important factor than strain rate.

The Influence of Strain Rate on Specimens with Marrow in Situ

The loading configuration in this study imposed uniaxial compressive strains on the test specimens, allowing restricted flow of marrow out through the porous platen. Since the characteristic dimensions of the platen pores were much smaller than those of trabecular bone, the influence of marrow on the recorded mechanical properties is due to marrow flow out through the platen rather than through the pores of the trabecular bone specimens. In spite of these restrictive boundary conditions during testing, the specimens tested at strain rates between 0.001 and 1.0 per second showed no enhancement of strength, stiffness, or energy absorption. Specimens tested at a strain rate of 10.0 per second, however, demonstrated a marked increase in strength, stiffness, and energy absorption.

These results are consistent with the finding of Swanson and Freeman[31] that trabecular bone is not hydraulically strengthened by the presence of marrow under moderate, physiological loading conditions such as normal walking. The findings in this study are also consistent with those of Pugh and co-workers[27], who found that the presence of marrow in *unconfined* specimens did not influence the viscoelastic behavior of trabecular bone. The results from specimens tested at a strain rate of 10.0 per second, however, suggest that the presence of marrow during severe, traumatic, compressive loading *in vivo* may serve to absorb considerable energy.

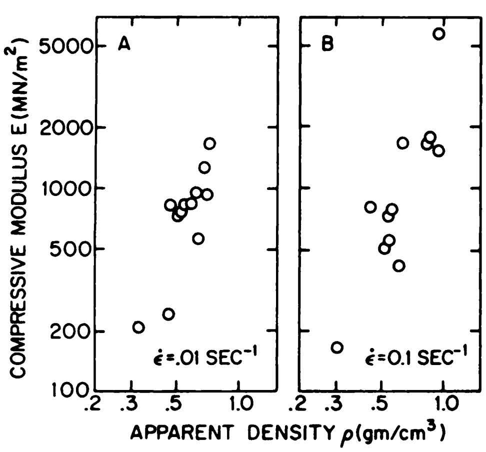

FIG. 6

Compressive modulus versus apparent density of bovine trabecular bone tested at strain rates of 0.01 and 0.1 per second.

It is important to recognize that the observed enhancement of the properties of trabecular bone by the presence of marrow is dependent primarily on the flow of marrow through restrictive boundaries rather than through the trabecular pores themselves. Trabecular bone in the skeleton (for example, in the vertebral bodies or in the ends of long bones) is normally surrounded by a shell of compact bone. When traumatic compression fractures of trabecular bone occur, the surrounding compact bone itself presents very restrictive boundaries limiting the outflow of marrow. The boundary conditions used in the tests in this study imposed similar restrictions on the outflow of marrow. A more rigorous analytical or experimental study which attempts to simulate *in vivo* whole-bone geometry is needed to quantify the practical significance of marrow flow in high-strain-rate compression fractures.

The Influence of Apparent Density on Strength and Modulus

Bell and associates[2] found that the total ash density (and therefore the apparent bone density) of human vertebrae decreased with age. Compression tests of whole vertebrae resulted in a positive, non-linear correlation between total ash density and compressive strength, which ranged from 1.37 to 13.7 meganewtons per square meter. A simplified mathematical analysis based on Euler buckling of trabeculae suggested that the compressive strength of the vertebrae was approximately proportional to the square of the ash content. The semi-empirical model of bone as a porous material developed by McElhaney and associates[20] predicted that the compressive strength of bone tissue was proportional to the fourth power of the apparent density.

In the present investigation the compressive strength of bone was found to be approximately proportional to the apparent density squared (Fig. 4-B). The most striking feature of this finding is that the relationship is valid for the entire range of bone density in the skeleton, from compact bone to the most porous trabecular bone. The squared relationship between compressive strength and apparent density is consistent with theoretical and experimental work on rigid cellular plastics[25]. The basic assumption of the theoretical analysis is that the cellular struts and walls (trabeculae in bone) fail by buckling and bending. Previous studies of the compressive behavior of trabecular bone indicate that buckling and bending of trabeculae are indeed major failure modes[13,28,32].

The compressive modulus of bone tissue was approximately proportional to the cube of the bone density (Fig. 7-B). Significant scatter of the data was present, however, suggesting that other factors may play an important role in establishing bone stiffness. Such factors may include the ultrastructure of bone tissue and microscopic properties (such as true tissue density) as well as trabecular orientation[28] or anisotropy[8,10,33]. The relationship of the compressive modulus to the cube of the bone density found in this study is consistent with that predicted by the porous model of cranial bone derived by McElhaney and associates[20]. This finding suggests that analytical or finite-element models based on the porous-structure concept may be extremely useful in describing whole-bone mechanical behavior. In such models it would be appropriate to approximate the compressive modulus of bone tissue as a cubic function (Equation 5) and the compressive strength as a squared function of the apparent density of the local

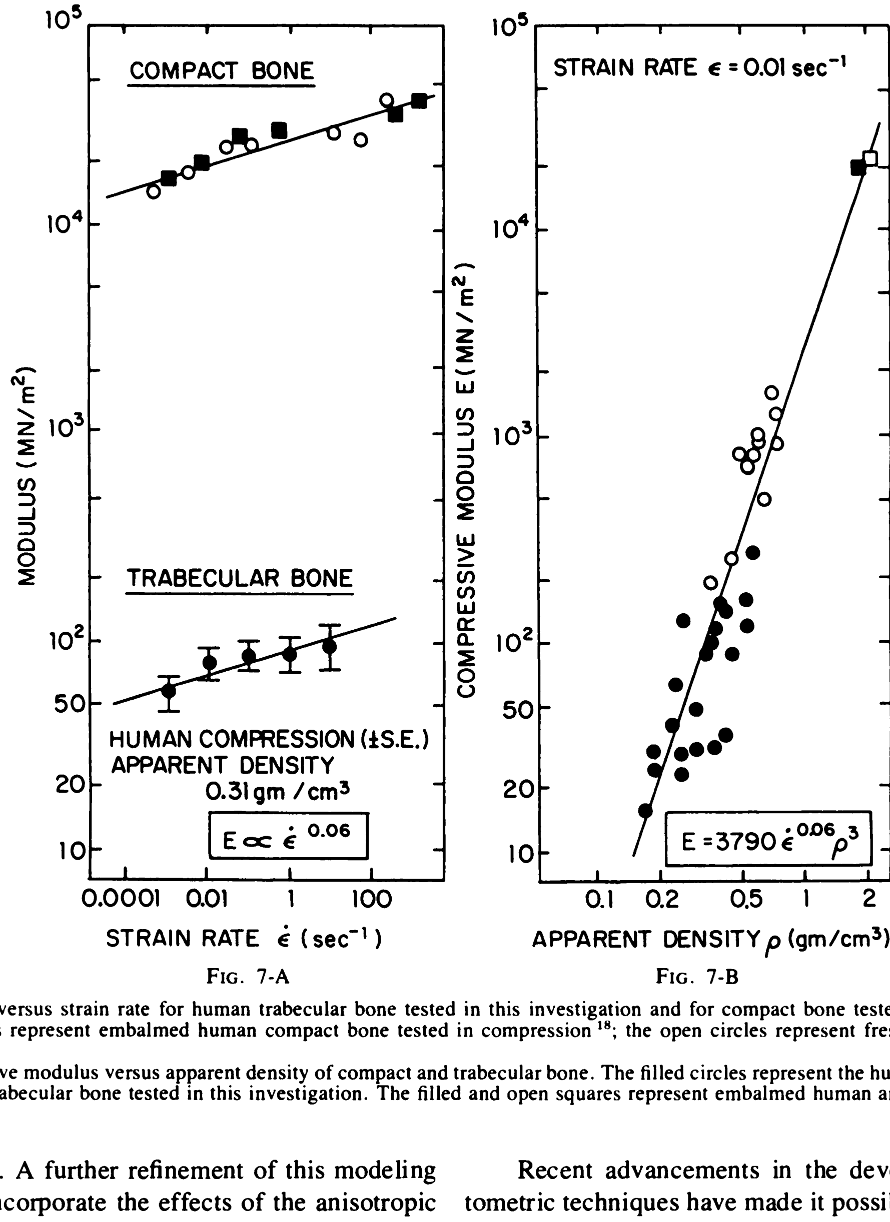

FIG. 7-A FIG. 7-B

Fig. 7-A: Modulus versus strain rate for human trabecular bone tested in this investigation and for compact bone tested in previous investigations [18,35]. The squares represent embalmed human compact bone tested in compression [18]; the open circles represent fresh bovine compact bone tested in tension [35].

Fig. 7-B: Compressive modulus versus apparent density of compact and trabecular bone. The filled circles represent the human and the open circles represent the bovine trabecular bone tested in this investigation. The filled and open squares represent embalmed human and fresh bovine compact bone, respectively [18].

bone (Equation 3). A further refinement of this modeling approach would incorporate the effects of the anisotropic behavior of bone and the strain rate.

The results of this investigation provide a basis for predicting bone strength in patients with decreased skeletal mass. Rarefaction of bone (osteoporosis) is present in many pathological conditions and is commonly found in bedridden or elderly patients. In all types of osteoporosis, the earliest and most striking change occurs in trabecular bone, where the trabeculae become thin and sparse [29]. Later clinical features may include compressed vertebral bodies, chronic back pain, dorsal kyphosis, and general skeletal pain. Gross pathological fractures, including fractures of the femoral neck, are subsequent complications [9,29].

Recent advancements in the development of densitometric techniques have made it possible to determine the *in vivo* distribution of bone density and the loss of skeletal mass [4,11,14,22]. For example, photon absorption densitometry studies have shown that the bone density of the ulna in osteoporotic patients may be reduced to as little as one-third of normal [24]. The results of our investigation suggest that in these patients the bone compressive strength is reduced to one-ninth of normal. The increase in the incidence of ulnar crush fractures [24] and Colles' fractures of the distal end of the radius in osteoporotic patients is consistent with this predicted reduction in strength.

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