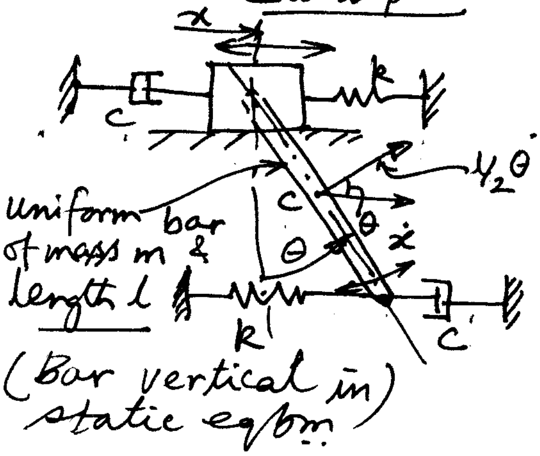
Lagrange's Equations (Continued)

Example:- Obtain the DEOM of the system in the figure using Lagrange's equations.



The system has 2 DOF. Let $x(t)$ & $\theta(t)$ be the generalized coordinates.

The Lagrange Equations are:-

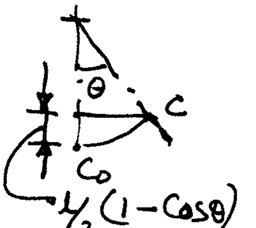
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial V}{\partial x} + \frac{\partial D}{\partial \dot{x}} = 0 \quad \text{--- (1)}$$

$$\& \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} + \frac{\partial D}{\partial \dot{\theta}} = 0 \quad \text{--- (2)}$$

$$T = T_{block} + T_{bar} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}mv_c^2 + \frac{1}{2}I_c\dot{\theta}^2$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\left[\dot{x}^2 + \frac{l^2}{4}\dot{\theta}^2 + 2\dot{x}\frac{l}{2}\dot{\theta}\cos\theta\right]$$

$$+ \frac{1}{2} \times \frac{1}{12}ml^2\dot{\theta}^2$$

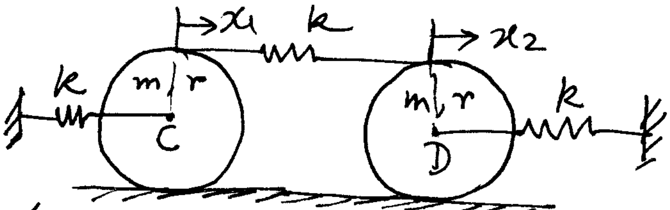


$$V = \frac{1}{2}kx^2 + \frac{1}{2}k'(x + l\theta)^2 + mg\frac{l}{2}(1-\cos\theta)$$

$$D = \frac{1}{2}c\dot{x}^2 + \frac{1}{2}c'(\dot{x} + l\dot{\theta}\cos\theta)^2$$

→ Compute the derivatives & complete the problem.

Example:-



If the cylinders roll w/o slipping, obtain the DEOM using x_1 & x_2 as the gen. coords. where these represent displacements of the centres of cylinders.

The angular velocities are $\frac{\dot{x}_1}{r}$ & $\frac{\dot{x}_2}{r}$

$$T = \frac{1}{2} I_C \left(\frac{\dot{x}_1}{r}\right)^2 + \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} I_D \left(\frac{\dot{x}_2}{r}\right)^2 + \frac{1}{2} m \dot{x}_2^2$$

where $I_C = \frac{1}{2} m r^2 = I_D$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} k [2x_2 - 2x_1]^2 + \frac{1}{2} k x_2^2$$

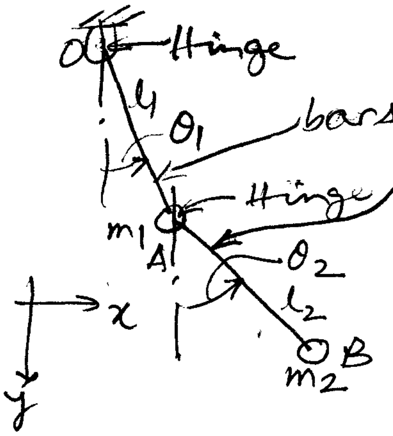
Lagrange's Equations are:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) - \frac{\partial T}{\partial x_1} + \frac{\partial V}{\partial x_1} = 0$$

$$\& \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) - \frac{\partial T}{\partial x_2} + \frac{\partial V}{\partial x_2} = 0.$$

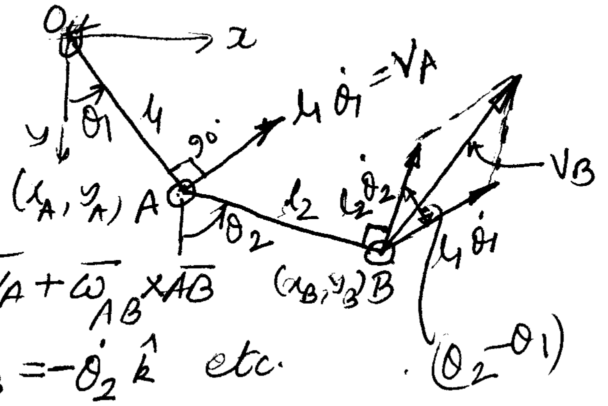
Complete the problem.

Example:- The Double Pendulum Problem



Obtain the nonlinear DEOM of the double pendulum using Lagrange's equations. Linearize the DEOM.

$\theta_1, \theta_2 \to$ gen. coords.



$$V_B^2 = l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1)$$

OR

$$\boxed{\bar{V}_B = \bar{V}_A + \bar{\omega}_{AB} \times \bar{AB} \qquad \bar{\omega}_{AB} = -\dot{\theta}_2 \hat{k} \text{ etc.}}$$

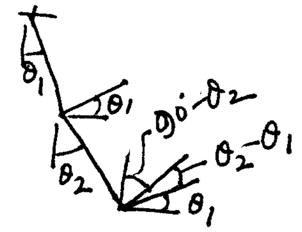
$$\therefore T = \frac{1}{2} m_1 (l_1 \dot{\theta}_1)^2 + \frac{1}{2} m_2 \left[l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1) \right]$$

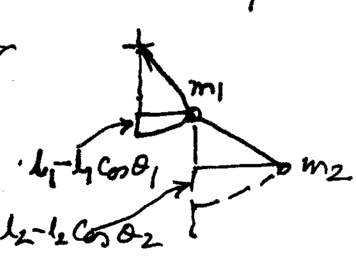
OR

$$\left.\begin{aligned} x_A &= l_1 \sin\theta_1 \\ y_A &= l_1 \cos\theta_1 \\ x_B &= l_1 \sin\theta_1 + l_2 \sin\theta_2 \\ y_B &= l_1 \cos\theta_1 + l_2 \cos\theta_2 \end{aligned}\right\}$$

$$\therefore \bar{V}_B = \dot{x}_B \hat{i} + \dot{y}_B \hat{j}$$

$$V_B^2 = |\bar{V}_B|^2 = \dot{x}_B^2 + \dot{y}_B^2 \text{ etc.}$$

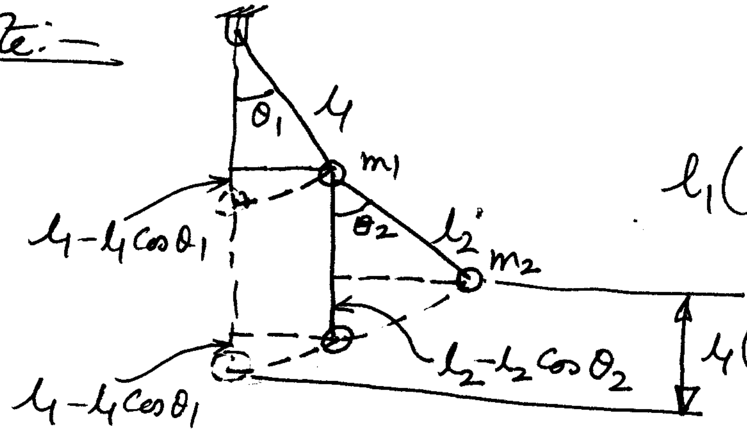




$$V = m_1 g l_1 (1 - \cos\theta_1) + m_2 g l_1 (1 - \cos\theta_1) + m_2 g l_2 (1 - \cos\theta_2)$$

(check)

Note:—



$l_1(1-\cos\theta_1)$ = height through which m_1 is raised.

$l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)$ = ~~Amount~~ Height through which m_2 is raised.

Now, the Lagrange equns are:—

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_1}\right) - \frac{\partial T}{\partial \theta_1} + \frac{\partial V}{\partial \theta_1} = 0 \quad \text{—(1)}$$

$$\& \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_2}\right) - \frac{\partial T}{\partial \theta_2} + \frac{\partial V}{\partial \theta_2} = 0 \quad \text{—(2)}$$

$$\therefore \quad \frac{\partial T}{\partial \dot\theta_1} = m_1 l_1^2 \dot\theta_1 + m_2 l_1^2 \dot\theta_1 + m_2 l_1 l_2 \dot\theta_2 \cos(\theta_2 - \theta_1)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_1}\right) = (m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1) + m_2 l_1 l_2 \dot\theta_2 [-\sin(\theta_2-\theta_1)](\dot\theta_2 - \dot\theta_1)]$$

$$\frac{\partial T}{\partial \theta_1} = m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \left[\frac{\partial}{\partial\theta_1}\cos(\theta_2-\theta_1)\right]$$

$$= + m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2 - \theta_1)$$

$$\frac{\partial V}{\partial \theta_1} = (m_1 g l_1 + m_2 g l_1)\sin\theta_1$$

Hence, from (1), we get

$$(m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1) - m_2 l_1 l_2 \dot\theta_2^2 \sin(\theta_2-\theta_1) + \cancel{m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)} - \cancel{m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \sin(\theta_2-\theta_1)} + (m_1+m_2) g l_1 \sin\theta_1 = 0$$

or $$(m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \cos(\theta_2-\theta_1) - m_2 l_1 l_2 \dot\theta_2^2 \sin(\theta_2-\theta_1) + (m_1+m_2) g l_1 \sin\theta_1 = 0 \quad \text{—(3)}$$

This is the first DEOM.

Again, $\dfrac{\partial T}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_2 - \theta_1)$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1 \left[\frac{d}{dt}\cos(\theta_2 - \theta_1)\right]$$

$$= m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) - m_2 l_1 l_2 \dot{\theta}_1 (\dot{\theta}_2 - \dot{\theta}_1) \sin(\theta_2 - \theta_1)$$

$$= m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) - m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1)$$

$$\frac{\partial T}{\partial \theta_2} = -m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2 - \theta_1)$$

$$\frac{\partial V}{\partial \theta_2} = m_2 g l_2 \sin\theta_2$$

From (2), we get

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + m_2 l_1 l_2 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) + m_2 g l_2 \sin\theta_2 = 0 \quad \text{--- (4)}$$

(4) is the other DEOM required.

→ For linearization, we assume both θ_1 & θ_2 to be small. Then, $\cos(\theta_2 - \theta_1) \approx 1$, $\sin(\theta_2 - \theta_1) \approx (\theta_2 - \theta_1)$, $\sin\theta_1 \approx \theta_1$, $\sin\theta_2 \approx \theta_2$. But these are not sufficient. We also have to neglect $\dot{\theta}_1^2(\theta_2 - \theta_1)$ & $\dot{\theta}_2^2(\theta_2 - \theta_1)$. (How do we justify these?)

Then the reqd linear DEOM are:

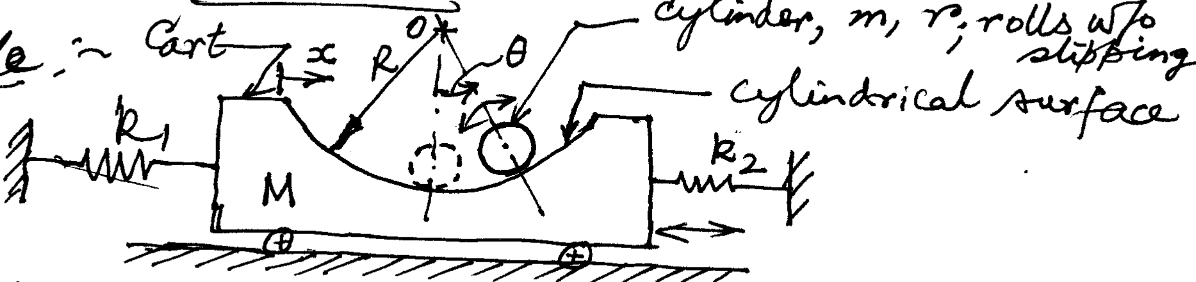
$$(m_1 + m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 + (m_1 + m_2) g l_1 \theta_1 = 0$$

& $$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_1 l_2 \ddot{\theta}_1 + m_2 g l_2 \theta_2 = 0$$

OR:- $\begin{bmatrix}(m_1+m_2)l_1^2 & m_2 l_1 l_2 \\ m_2 l_1 l_2 & m_2 l_2^2\end{bmatrix}\begin{Bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{Bmatrix}+\begin{bmatrix}(m_1+m_2)g l_1 & 0 \\ 0 & m_2 g l_2\end{bmatrix}\begin{Bmatrix}\theta_1\\ \theta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix}$

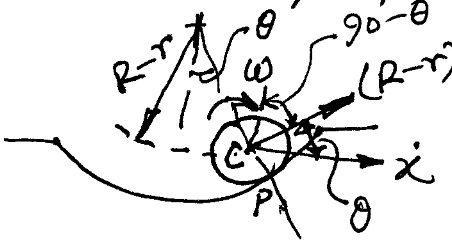
(Check)

Example:-



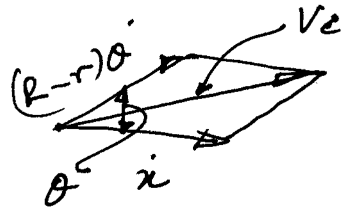
The cart oscillates, so does the cylinder. Obtain the (Linearize) DEOM using Lagrange's equns.

→ Let x & θ be the generalized coordinates chosen for this 2-DOF system. ($x=0, \theta=0$ at static equilibrium)



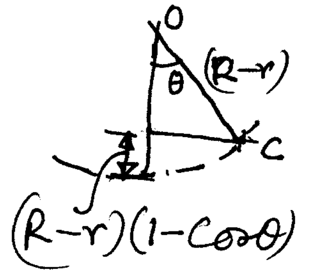
$r\omega=(R-r)\dot{\theta} \Rightarrow \boxed{\omega=\left(\frac{R-r}{r}\right)\dot{\theta}}$

$T = T_{cart}+T_{cylinder}$



$T_{cart}=\frac{1}{2}M\dot{x}^2+\frac{1}{2}I_c\omega^2+\frac{1}{2}mV_c^2$

where $V_c^2=\dot{x}^2+(R-r)^2\dot{\theta}^2$
$+2\dot{x}(R-r)\dot{\theta}\cos\theta$



So, $T=\frac{1}{2}M\dot{x}^2+\frac{1}{2}\times\frac{1}{2}m\cancel{r^2}\frac{(R-r)^2}{\cancel{r^2}}\dot{\theta}^2$
$+\frac{1}{2}m\left[\dot{x}^2+(R-r)^2\dot{\theta}^2+2\dot{x}(R-r)\dot{\theta}\cos\theta\right]$

$V=\frac{1}{2}k_1x^2+\frac{1}{2}k_2x^2+mg(R-r)(1-\cos\theta)$

The Lagrange equations are:-

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right)-\frac{\partial T}{\partial x}+\frac{\partial V}{\partial x}=0$ --- ①

& $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial V}{\partial \theta}=0$ --- ②

Now, $\frac{\partial T}{\partial \dot{x}}=M\dot{x}+m\dot{x}+m(R-r)\dot{\theta}\cos\theta$ →

So, $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = (M+m)\ddot{x} + m(R-r)\ddot{\theta} - m(R-r)\dot{\theta}^2 \sin\theta$

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial V}{\partial x} = (k_1+k_2)x$$

So, (1) becomes:

$$(M+m)\ddot{x} + m(R-r)\ddot{\theta} - m(R-r)\dot{\theta}^2\sin\theta + (k_1+k_2)x = 0 \quad \text{--- (3)}$$

which is a DEOM of the system

Again, $\frac{\partial T}{\partial \dot{\theta}} = \frac{1}{2}m(R-r)^2\dot{\theta} + m(R-r)^2\dot{\theta} + m\dot{x}(R-r)\cos\theta$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{3}{2}m(R-r)^2\ddot{\theta} + m(R-r)\ddot{x}\cos\theta - m(R-r)\dot{x}\dot{\theta}\sin\theta$$

$$\frac{\partial T}{\partial \theta} = -m\dot{x}(R-r)\dot{\theta}\sin\theta$$

$$\frac{\partial V}{\partial \theta} = mg(R-r)\sin\theta.$$

Thus, using (2), we get the second DEOM as:

$$\frac{3}{2}m(R-r)^2\ddot{\theta} + m(R-r)\ddot{x}\cos\theta - \cancel{m(R-r)\dot{x}\dot{\theta}\sin\theta} + \cancel{m(R-r)\dot{x}\dot{\theta}\sin\theta} + mg(R-r)\sin\theta = 0$$

Or, $\frac{3}{2}m(R-r)^2\ddot{\theta} + m(R-r)\ddot{x}\cos\theta + mg(R-r)\sin\theta = 0$ --- (4)

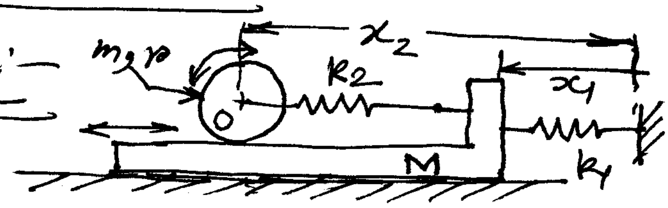
The Linearized DEOM are:

$$(M+m)\ddot{x} + m(R-r)\ddot{\theta} + (k_1+k_2)x = 0 \quad \text{--- (5)}$$

& $m(R-r)\ddot{x} + \frac{3}{2}m(R-r)^2\ddot{\theta} + mg(R-r)\theta = 0$ --- (6)

OR, $\begin{bmatrix} (M+m) & m(R-r) \\ m(R-r) & \frac{3}{2}m(R-r)^2 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & 0 \\ 0 & mg(R-r) \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

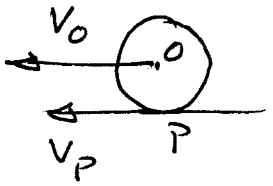
Example:—



The cylinder rolls w/o slipping on the cart.

Using x_1 & x_2 as generalized coordinates, obtain the DEOM using Lagrange's equns.

Solution:- clearly, v_0 = velocity of the centre O of cylinder $= \dot{x}_2$. Hence ω = angular velocity of cylinder $= \frac{v_0 - v_P}{r} = \frac{(\dot{x}_2 - \dot{x}_1)}{r}$



$$\therefore T = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{2}\times\frac{1}{2}m\cancel{r^2}\frac{(\dot{x}_2-\dot{x}_1)^2}{\cancel{r^2}}$$

$$= \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{1}{4}m(\dot{x}_2-\dot{x}_1)^2$$

$$V = \frac{1}{2}k_1(x_1-l_1)^2 + \frac{1}{2}k_2\left[(x_2-x_1)-l_2\right]^2$$

where l_1 = Free-length of spring k_1

l_2 = " " " " k_2

The Lagrange equns are:-

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) - \frac{\partial T}{\partial x_1} + \frac{\partial V}{\partial x_1} = 0 \quad \text{— (1)}$$

& $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) - \frac{\partial T}{\partial x_2} + \frac{\partial V}{\partial x_2} = 0 \quad \text{— (2)}$$

$$\frac{\partial T}{\partial \dot{x}_1} = M\dot{x}_1 - \frac{1}{2}m(\dot{x}_2-\dot{x}_1) = \left(M+\frac{m}{2}\right)\dot{x}_1 - \frac{m}{2}\dot{x}_2$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = \left(M+\frac{m}{2}\right)\ddot{x}_1 - \frac{m}{2}\ddot{x}_2, \quad \frac{\partial T}{\partial x_1} = 0$$

$$\frac{\partial V}{\partial x_1} = k_1(x_1-l_1) - k_2\left[(x_2-x_1)-l_2\right]$$

$\therefore$ from (1), the 1st DEOM is:

Check

$$\left(M+\frac{m}{2}\right)\ddot{x}_1 - \frac{m}{2}\ddot{x}_2 + (k_1+k_2)x_1 - k_2x_2 + k_2l_2 - k_1l_1 = 0 \quad \text{--(3)}$$

Also, $$\frac{\partial T}{\partial \dot{x}_2} = m\dot{x}_2 + \frac{1}{2}m(\dot{x}_2-\dot{x}_1)$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m\ddot{x}_2 + \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) \longrightarrow$$

$$\frac{\partial T}{\partial x_2} = 0 \; ; \quad \frac{\partial V}{\partial x_2} = k_2\left[(x_2 - x_1) - l_2\right]$$

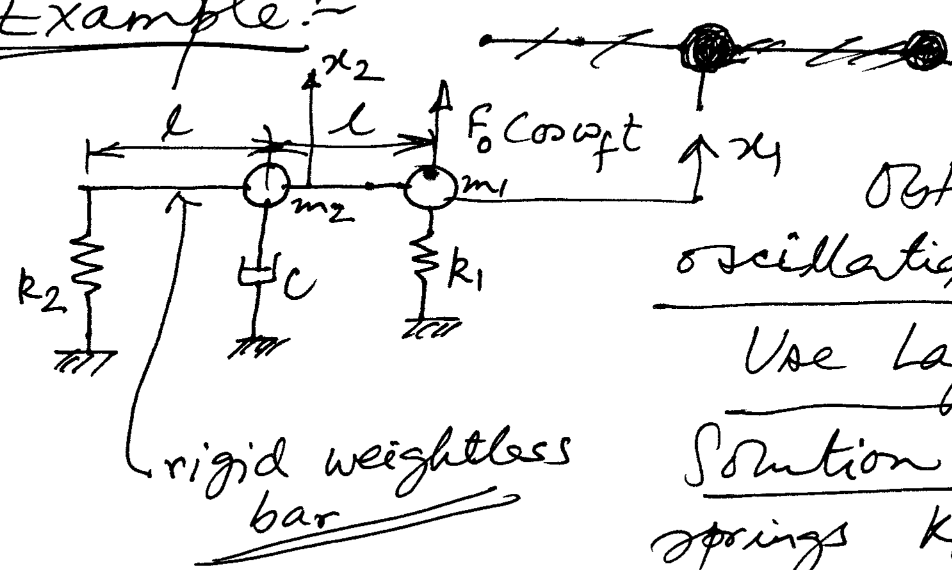
So, from (1), the 2nd DEOM are:–

$$-\frac{1}{2} m \ddot{x}_1 + \frac{3}{2} m \ddot{x}_2 - k_2 x_1 + k_2 x_2 - k_2 l_2 = 0 \quad \text{---} (4)$$

In matrix form, the DEOM are:

$$\begin{bmatrix} \left(M + \frac{m}{2}\right) & -\frac{m}{2} \\ -\frac{m}{2} & \frac{3}{2} m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} (k_1 l_1 - k_2 l_2) \\ k_2 l_2 \end{Bmatrix}$$

Example:–



Obtain, for small oscillations, the DEOM.

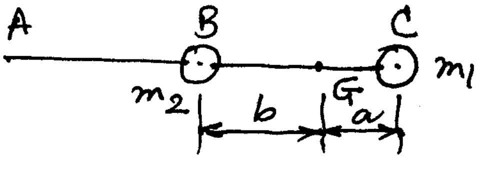
Use Lagrange's Equns.

Solution :– Let the springs k_1 & k_2 are so adjusted that the bar was horizontal in static equilibrium. The system has 2–DOF (Why?)

→ One way to choose the 2 generalized coordinates is to consider vertical (approx.) displacements x_1 & x_2 of m_1 & m_2 respectively.

→ We could also take vertical displacement x of the CG as well as angle of rotation of the bar θ as the gen. coords.

→ We shall use x & θ. (You try out both sets.)

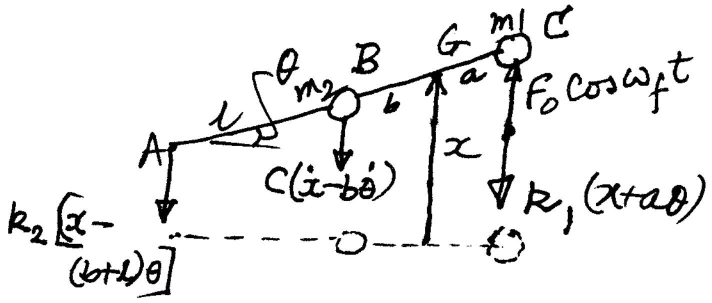


Let G be the CG of the bar plus the particles. →

$\therefore$ $a+b=l$ —(1) & $am_1 = bm_2$ —(2)

So, $b = \dfrac{am_1}{m_2}$ & $a + \dfrac{am_1}{m_2} = l$ or, $\dfrac{a(m_1+m_2)}{m_2} = l$

$$\Rightarrow a = \frac{m_2 l}{(m_1+m_2)} \Rightarrow b = l - \frac{m_2 l}{(m_1+m_2)} = \frac{m_1 l}{(m_1+m_2)}$$



Vertical (approx) Deflection of left spring (above eqlbm value) $= x - (b+l)\theta$.

That of right spring $= x + a\theta$.

Deflection at B $= x - b\theta$

velocity at B $= \dot{x} - b\dot{\theta}$

The relevant FBD is shown above.

→ Applying Newton's 2nd law to the CG in vertical direction, we get:

$$(m_1+m_2)\ddot{x} = F_0 \cos\omega_f t - k_1(x+a\theta) - c(\dot{x} - b\dot{\theta}) \cancel{- c(\dot{x}-b\dot{\theta})} - k_2[x-(b+l)\theta]$$

$$\Rightarrow (m_1+m_2)\ddot{x} + c\dot{x} - bc\dot{\theta} + (k_1+k_2)x + [k_1 a - k_2(b+l)]\theta \cancel{+ k_1 x + k_1 a\theta} = F_0 \cos\omega_f t \quad \text{—(1)}$$

(check) which is the first DEOM reqd.

→ Applying MOM theorem (moment balance ~~method~~ method), we get, taking moments about G,

$$I_G \ddot{\theta} = k_2[x-(b+l)\theta](b+l) + c(\dot{x}-b\dot{\theta})b - k_1(x+a\theta)a + (F_0 \cos\omega_f t)a$$

$\boxed{I_G = m_1 a^2 + m_2 b^2}$

$$= k_2(b+l)x - k_2(b+l)^2\theta + bc\dot{x} - b^2 c\dot{\theta} - k_1 a x - k_1 a^2\theta + F_0 a \cos\omega_f t$$

or,

$$I_G\ddot{\theta} + b^2 c\dot{\theta} - bc\dot{x} + [k_1 a - k_2(b+l)]x + [k_1 a^2 + k_2(b+l)^2]\theta = F_0 a \cos\omega_f t \quad \text{—(2)}$$

(Check)

(2) is the reqd. 2nd DEOM. →

In matrix form :

$$\begin{bmatrix} (m_1+m_2) & 0 \\ 0 & I_G \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & -bc \\ -bc & b^2c \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix}$$

$$+ \begin{bmatrix} (k_1+k_2) & [k_1 a - k_2(b+l)] \\ [k_1 a - k_2(b+l)] & (k_1 a^2 + k_2(b+l)^2) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F_0 \cos\omega_f t \\ F_0 a \cos\omega_f t \end{Bmatrix}$$

check

Tu/HW (Lagrange's Equns)

Tu-5

19. Use Lagrange's equation to derive the equations of motion for the coupled pendulum as shown in Fig. 2-23.

21. A double pendulum of lengths L_1 and L_2, masses m_1 and m_2 is shown in Fig. 2-25. Use Lagrange's equation to derive the equations of motion.

26. A circular cylinder of radius r and mass m rolls without slipping inside a semi-circular groove of radius R. Block M is supported by a spring of constant k and constrained to move without friction in the vertical guide as shown in Fig. 2-30. By the use of Lagrange's equation, find the equations of motion of the system.

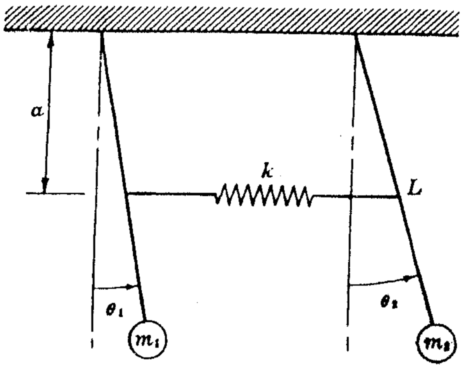


Fig. 2-23

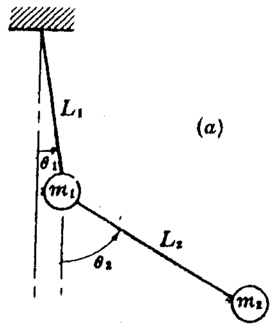


Fig.2.25

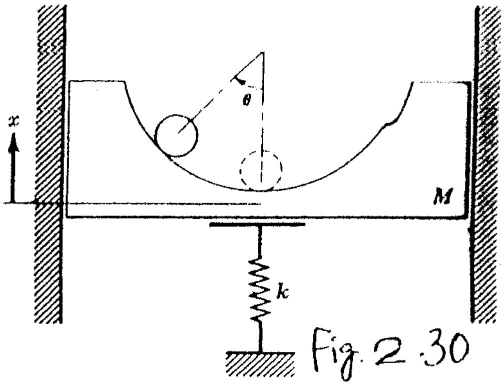
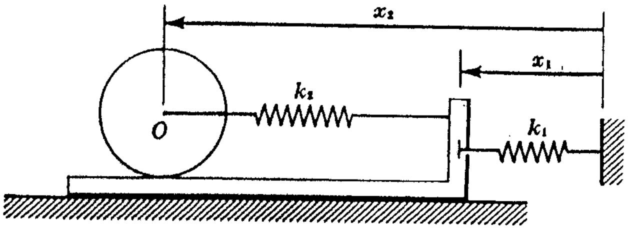


Fig 2.30

25. A solid homogeneous cylinder of mass M and radius r rolls without slipping on a cart of mass m as shown in Fig. 2-29. The cart, connected by springs of constants k_1 and k_2, is free to slide on a horizontal surface. By the use of Lagrange's equation, find the equations of motion of the system.



27. Fig. 2-32 shows a two-degree-of-freedom spring-mass system with damping. Determine the equations of motion of the system by the use of Lagrange's equation.

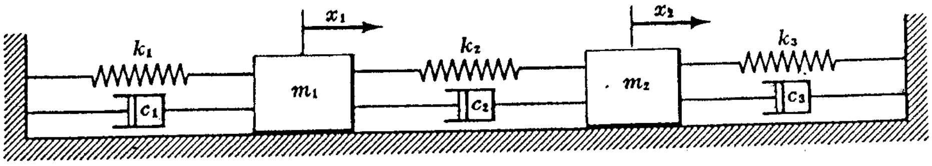
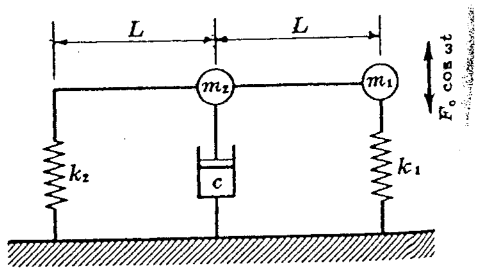


Fig. 2-32

29. Two masses m_1 and m_2 are attached to a rigid weightless bar which is supported by springs k_1 and k_2 and dashpot c as shown in Fig. 2-34. If the motion of the bar is restricted to the plane of the paper, determine the equations of motion of the system by the use of Lagrange's equation.



(PTO)

Contd. Tu/HW (Lagrange's Eqns)

71. A double pendulum is connected by four springs of equal stiffness as shown in Fig. 2-73 below. For small angles of oscillation, find its frequencies by the use of Lagrange's equation.
Ans. $\omega_2 = \sqrt{2k/m + 3.12g/L}$, $\omega_1 = \sqrt{2k/m + 0.58g/L}$ rad/sec

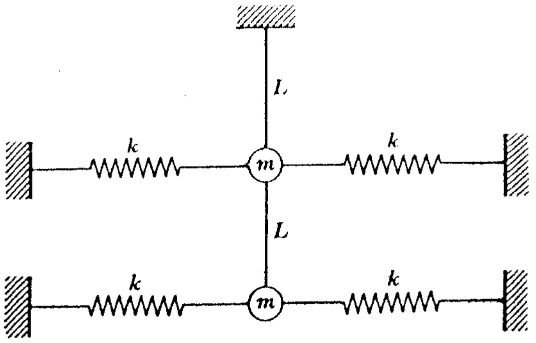


Fig. 2-73

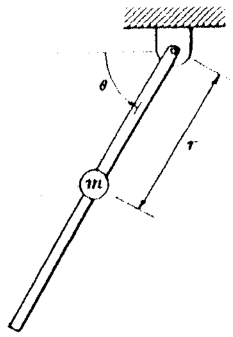


Fig. 2-74

72. A small mass m is free to slide on a homogeneous uniform rod of mass M and length L which is pivoted at one end as shown in Fig. 2-74 above. Derive the equations of motion by the use of Lagrange's equation.
Ans. $(ML^2 + mr^2)\ddot{\theta} + 2mr\dot{r}\dot{\theta} - (mr + ML)g \cos\theta = 0$
$m\ddot{r} - m\dot{\theta}^2 r + mg(1 - \sin\theta) = 0$

73. A circular homogeneous cylinder of mass M and radius R rolls without slipping inside a circular surface of radius $3R$. A small mass m, connected by two equal springs of modulus k, is initially at the center of the cylinder at the equilibrium position as shown in Fig. 2-75 below. Derive expressions for the equations of motion of the system by the use of Lagrange's equation.
Ans. $4(MR^2 + J_0 + mR^2)\ddot{\theta} + 2(M + m)gR\theta + 2mR\ddot{r} - 2mgr = 0$
$m\ddot{r} + 2kr + 2mR\ddot{\theta} - 2mg\theta = 0$
where J_0 is the moment of inertia of the cylinder.

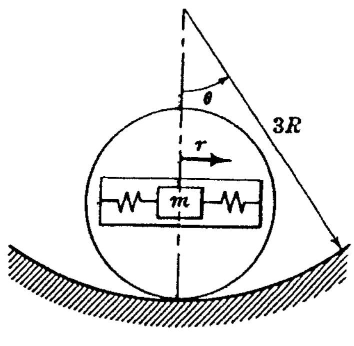


Fig. 2-75

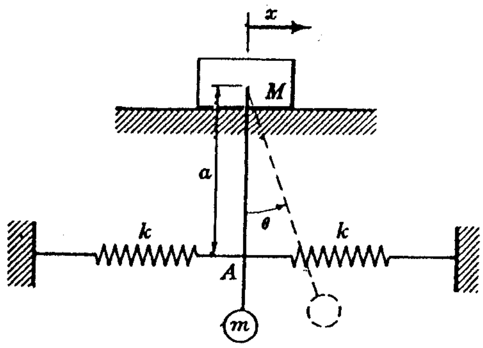


Fig. 2-76

74. A particle of mass m is moving on a horizontal plane under the action of an attractive force which is a function of the displacement, i.e. $F(t) = f(1/r^2)$. Determine the equations of motion of the particle by the use of Lagrange's equation.
Ans. $r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$ H.W
$m\ddot{r} + k/r^2 - mr\dot{\theta}^2 = 0$

75. The block of mass M moves along a smooth horizontal plane, and carries a simple pendulum of length L and mass m as shown in Fig. 2-76 above. Two equal springs of modulus k are connected to the pendulum at point A. Determine the equations of motion describing small oscillation of the system about the equilibrium position by the use of the Lagrange's equation.
Ans. $(M + m)\ddot{x} + 2kx + mL\ddot{\theta} + 2ak\theta = 0$
$mL^2\ddot{\theta} + (mgL + 2ka^2)\theta + mL\ddot{x} + 2akx = 0$

(PTO)