Applications of Fuzzy Systems

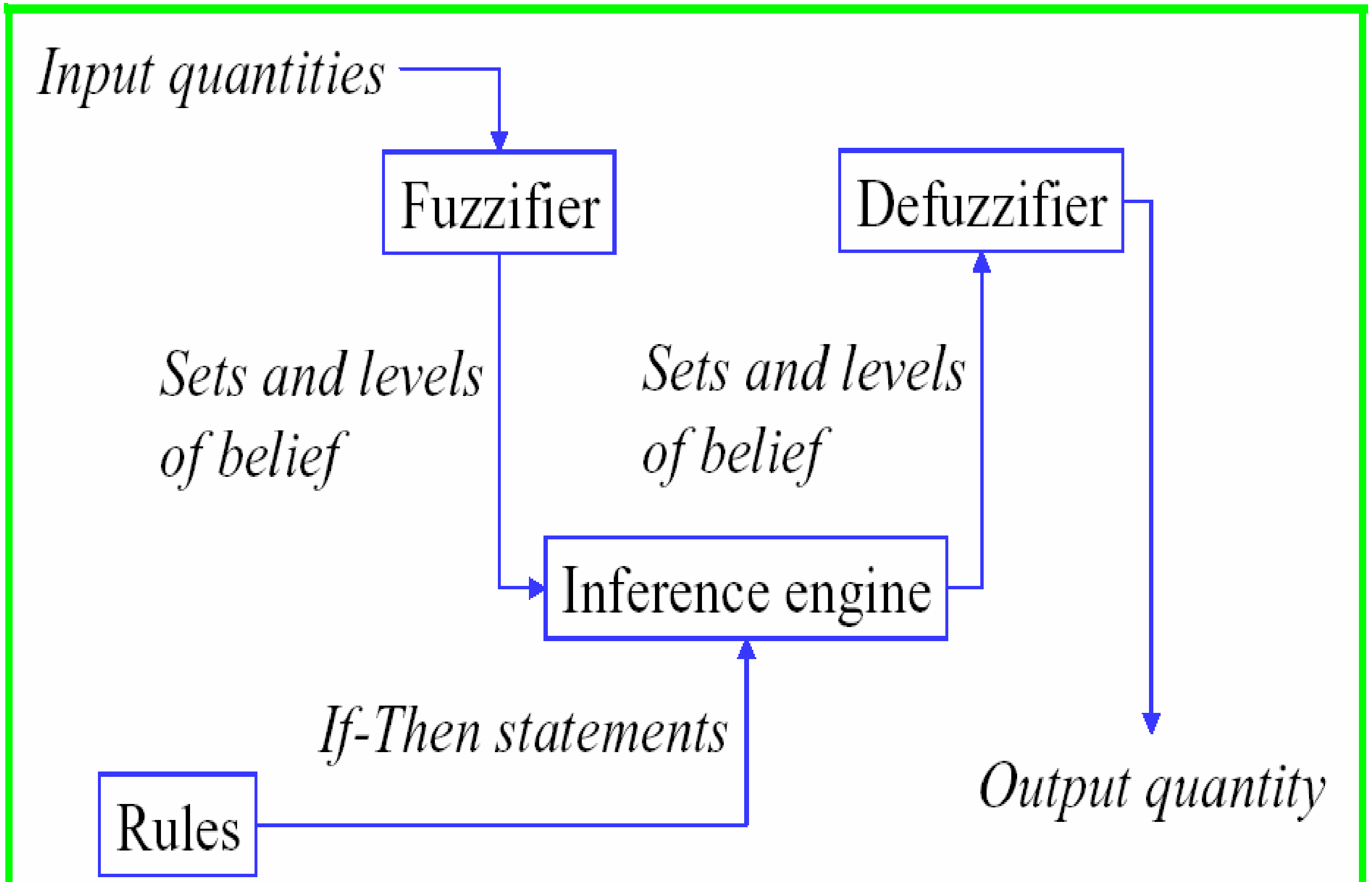
Examples and Procedures

IMS 2014

Steps by Step Approach

- The above steps are summarized into three main stages
  - Fuzzification
    - Membership functions used to graphically describe a situation
  - Evaluation of Rules
    - Application of the fuzzy logic rules
  - Diffuzification
    - Obtaining the crisp results

Steps by Step Approach



Other Applications

- Coal Power Plant
- Refuse Incineration Plant
- Water Treatment Systems
- AC Induction Motor
- Fraud Detection
- Customer Targeting
- Quality Control
- Speech Recognition
- Nuclear Fusion
- Truck Speed Limiter
- Sonar Systems
- Toasters
- Photocopiers
- Creditworthiness Assessment
- Stock Prognosis
- Mortgage Application
- Hi-Fi Systems
- Humidifiers
- Domestic Goods - Washing Machines/Dryers
- Microwave Ovens
- Consumer Electronics – Television
- Still and Video Cameras - Auto focus, Exposure and Anti-Shake
- Vacuum Cleaners

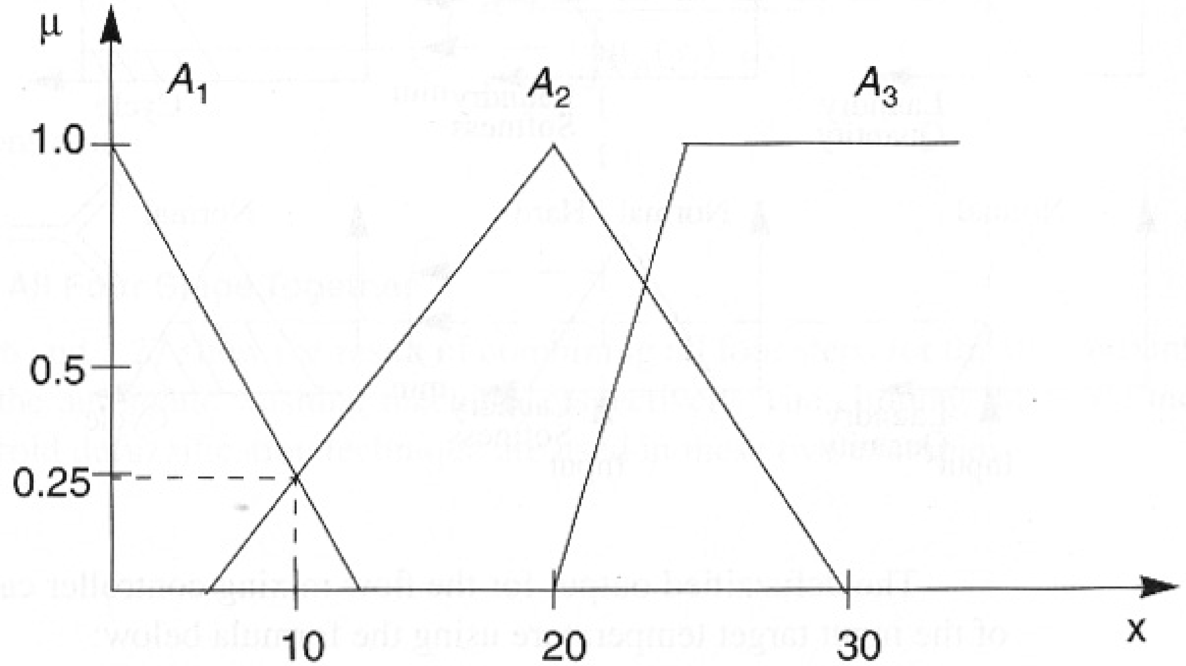
Designing Antecedent Membership Functions

- Recommend designer to adopt the following design principles:
  - Each Membership function overlaps only with the closest neighboring membership functions;
  - For any possible input data, its membership values in all relevant fuzzy sets should sum to 1 (or nearly)

* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Designing Antecedent Membership Functions

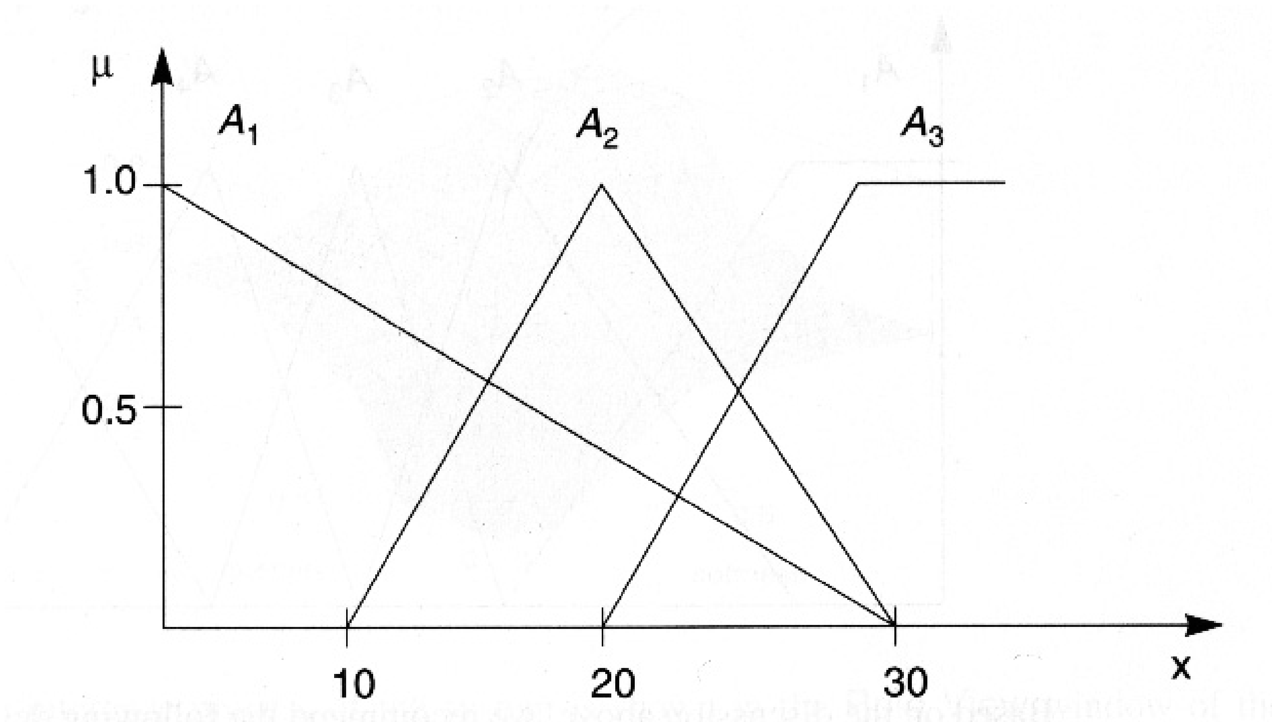
A Membership Function Design that violates the second principle



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Designing Antecedent Membership Functions

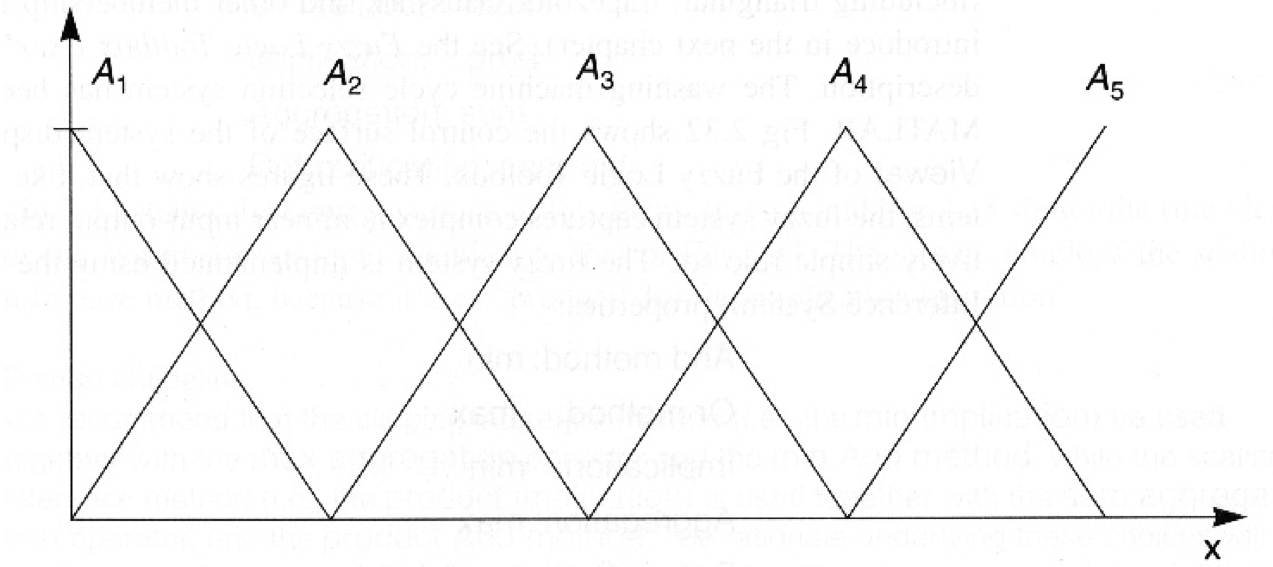
A Membership Function Design that violates both principle



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Designing Antecedent Membership Functions

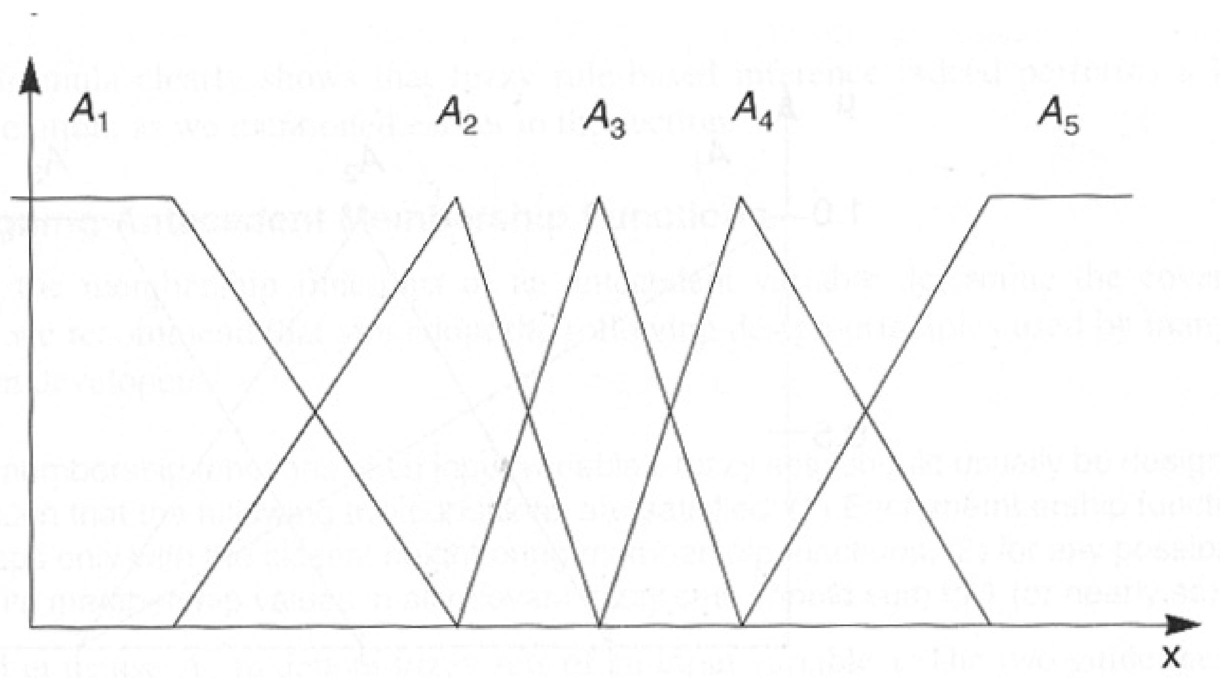
A symmetric Function Design Following the guidelines



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Designing Antecedent Membership Functions

An asymmetric Function Design Following the guidelines



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Furnace Temperature Control

- Inputs
  - Temperature reading from sensor
  - Furnace Setting
- Output
  - Power control to motor

* Fuzzy Systems Toolbox, M. Beale and H Demuth

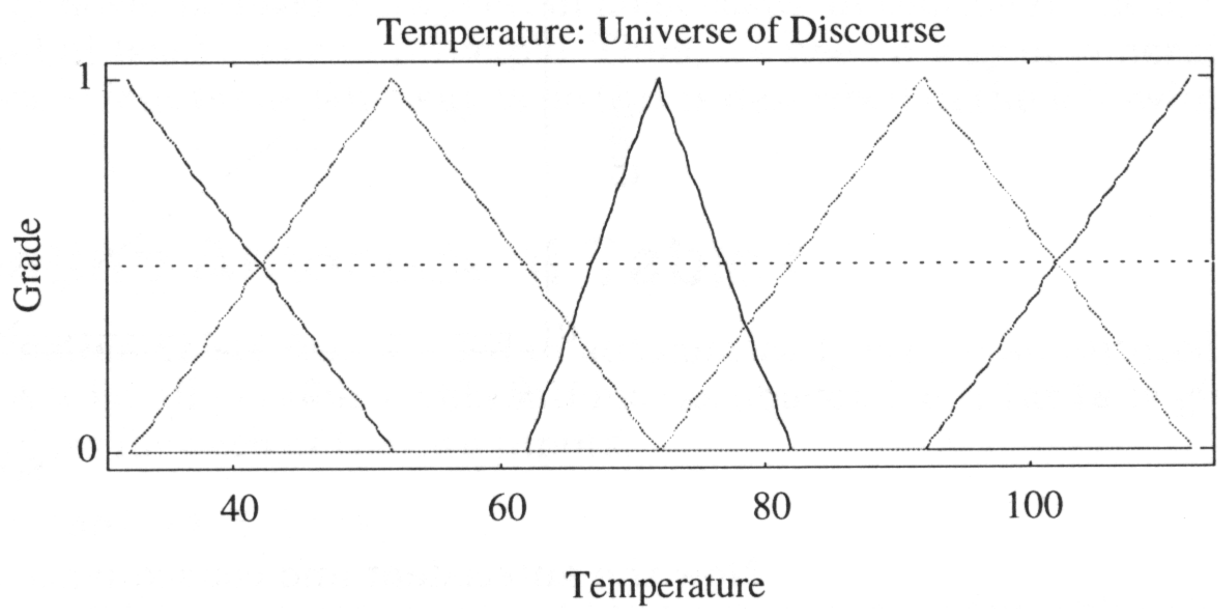
MATLAB: Create membership functions - Temp

```
tempS = 32:0.5:112;

tempG = triangle(tempS,[32 32 52; 32 52 72; 62 72 82;...

        72 92 112; 92 112 112]);
```

Here is a plot of the universe of discourse for temperature:



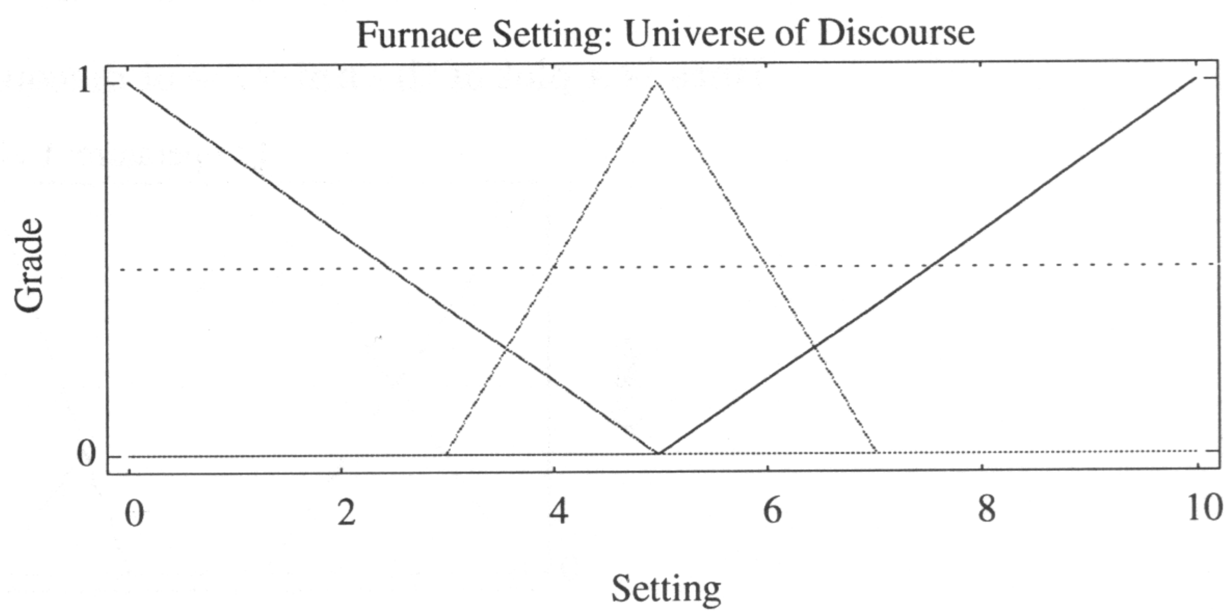
* Fuzzy Systems Toolbox, M. Beale and H Demuth

MATLAB: Create membership functions - Setting

```
settingS = 1:10;

settingG = triangle(settingS,[0 0 5; 3 5 7; 5 10 10]);
```

These fuzzy sets can also be thought of as "low," "medium," and "high":



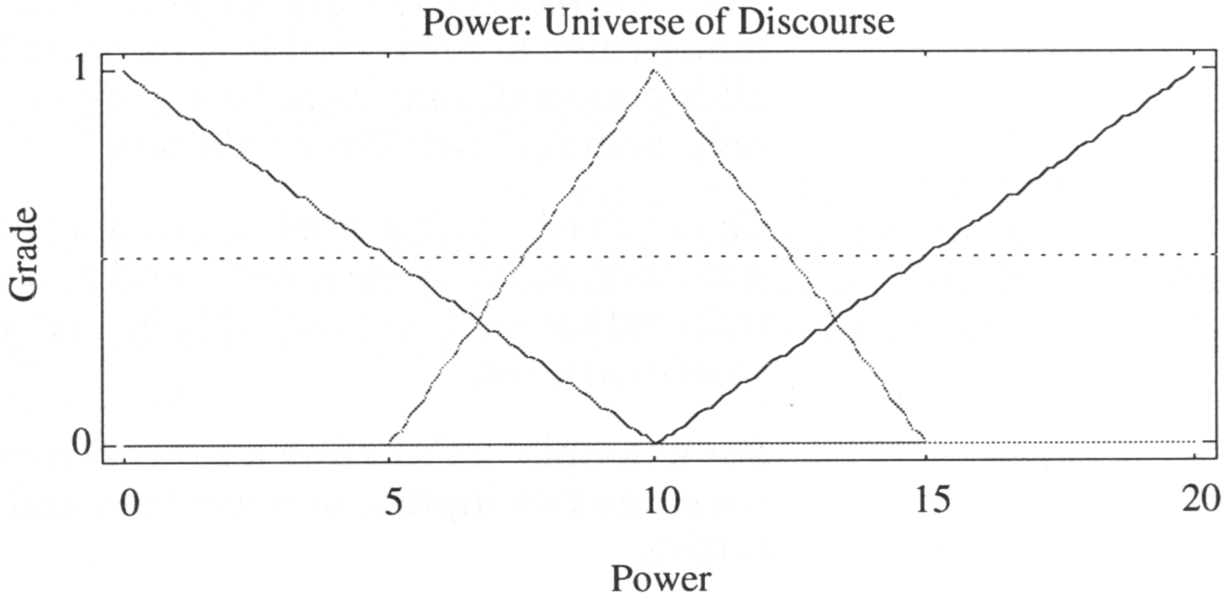
* Fuzzy Systems Toolbox, M. Beale and H Demuth

MATLAB: Create membership functions - Power

```
powerS = 0:0.1:20;

powerG = triangle(powerS,[0 0 10; 5 10 15; 10 20 20]);
```

These fuzzy sets can be thought of as “low,” “medium,” and “high”:



* Fuzzy Systems Toolbox, M. Beale and H Demuth

If - then - Rules

Fuzzy Rules for Furnace control

Temp \ Setting	Low	Medium	High
Cold	Low	Medium	High
Cool	Low	Medium	High
Moderate	Low	Low	Low
Warm	Low	Low	Low
Hot	low	Low	Low

* Fuzzy Systems Toolbox, M. Beale and H Demuth

Antecedent Table

The antecedent table has 15 rows to represent each unique pair of input conditions. Each row contains a different combination of indices for the two input variables:

```
A = [1    1;
     2    1;
     3    1;
     4    1;
     5    1;
     1    2;
   ...etc...
     3    3;
     4    3;
     5    3];
```

For example, row 3 ([3 1]) represents the case where the first variable, temperature, occurs in its third fuzzy set, "moderate," and the second variable, power, is in its first fuzzy set "low." Thus, this row represents the antecedent part of the third rule:

"If temperature is moderate and power is low, then..."

* Fuzzy Systems Toolbox, M. Beale and H Demuth

Antecedent Table

- MATLAB
  - A = table(1:5,1:3);
    - Table generates matrix represents a table of all possible combinations

* Fuzzy Systems Toolbox, M. Beale and H Demuth

Consequence Matrix

```
C = [1 1 1 1 1 2 2 1 1 1 3 3 1 1 1]';
```

Each element in the column indicates which fuzzy set the output, the fan setting, should fall into, given that its associated input conditions are true. Thus, the consequence for the third rule can be found by looking at the third element. The third element is 1, so the consequence for the third rule

"If temperature is moderate and power is low, then..."

is:

"...set the fan low."

Combining row 3 of the antecedent and consequence tables results in the rule:

"If temperature is moderate and power is low, then set the fan low."

---

* Fuzzy Systems Toolbox, M. Beale and H Demuth

Evaluating Rules with Function FRULE

The function **frule** can be used to evaluate a fuzzy system's output for any input. In the following paragraphs, we will apply **frule** to the temperature/power/settings system discussed above.

First, we will use the function **group** to arrange the supports and grades for each input (temperature, power) and output (fan setting) in pairs.

```
Z = group(tempS,tempG,powerS,powerG,settingS,settingG);
```

Then we will define the current input vector. For example, if temperature is 70 (moderate) and the power is 2 (low) the input vector **x** is:

```
x = [70; 2];
```

We can now apply **frule** to this input. (This action tests the third rule of the antecedent consequence table.)

```
y = frule(A,C,Z,x)

y =

        2.333
```

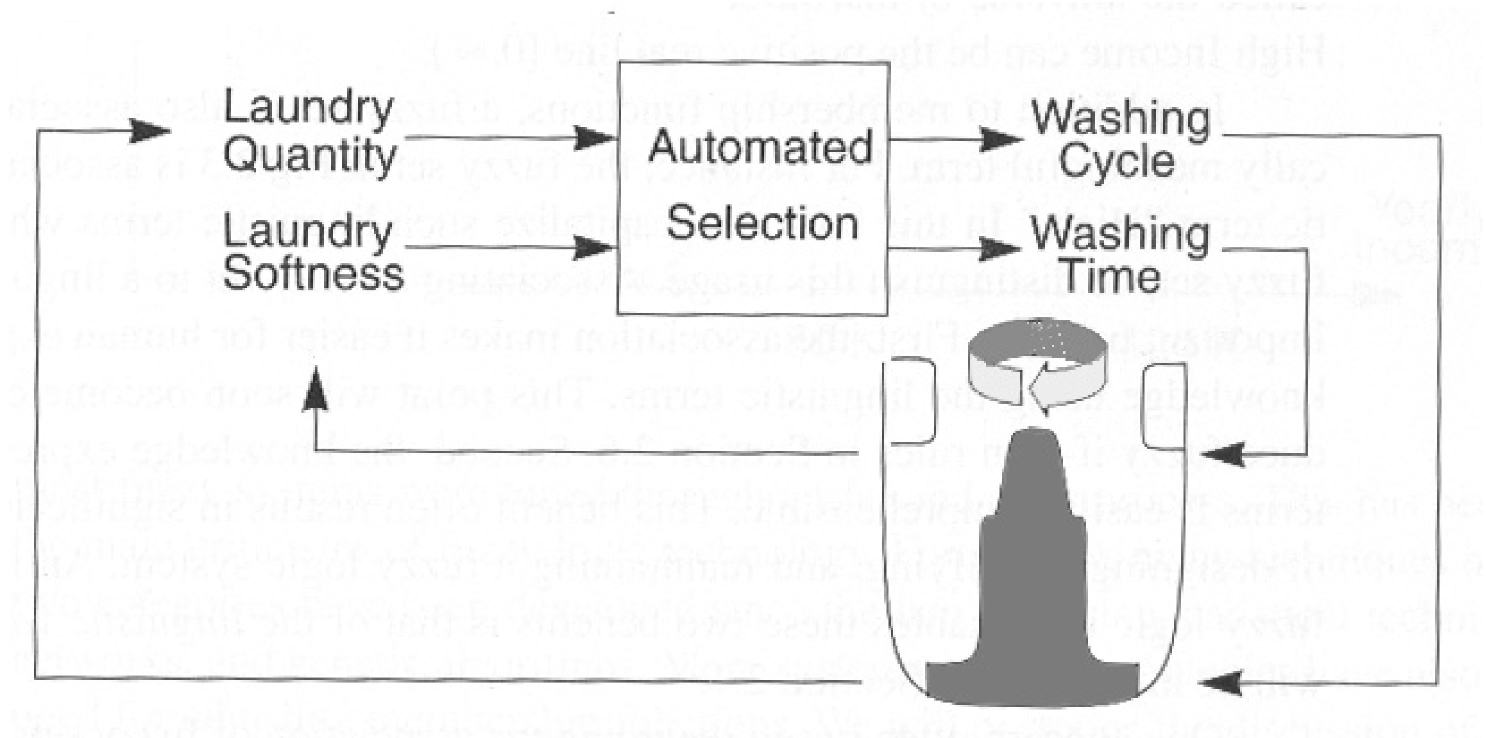
* Fuzzy Systems Toolbox, M. Beale and H Demuth

Design Guideline (Inference)

- Recommend
  - Max-Min (Clipping) Inference method be used together with the MAX aggregation operator and the MIN AND method
  - Max-Product (Scaling) Inference method be used together with the SUM aggregation operator and the PRODUCT AND method

* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Fully Automatic Washing Machine



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

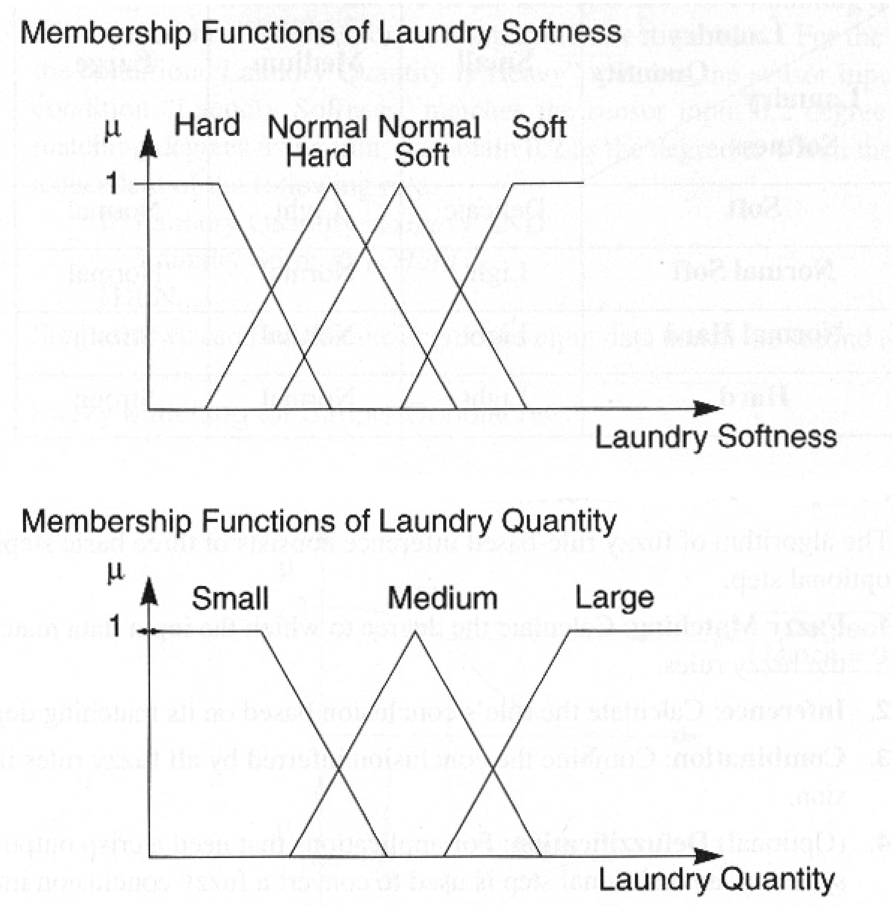
Example: Fully Automatic Washing Machine

- Inputs
  - —Laundry Softness
  - —Laundry Quantity
- Outputs
  - —Washing Cycle
  - —Washing Time

---

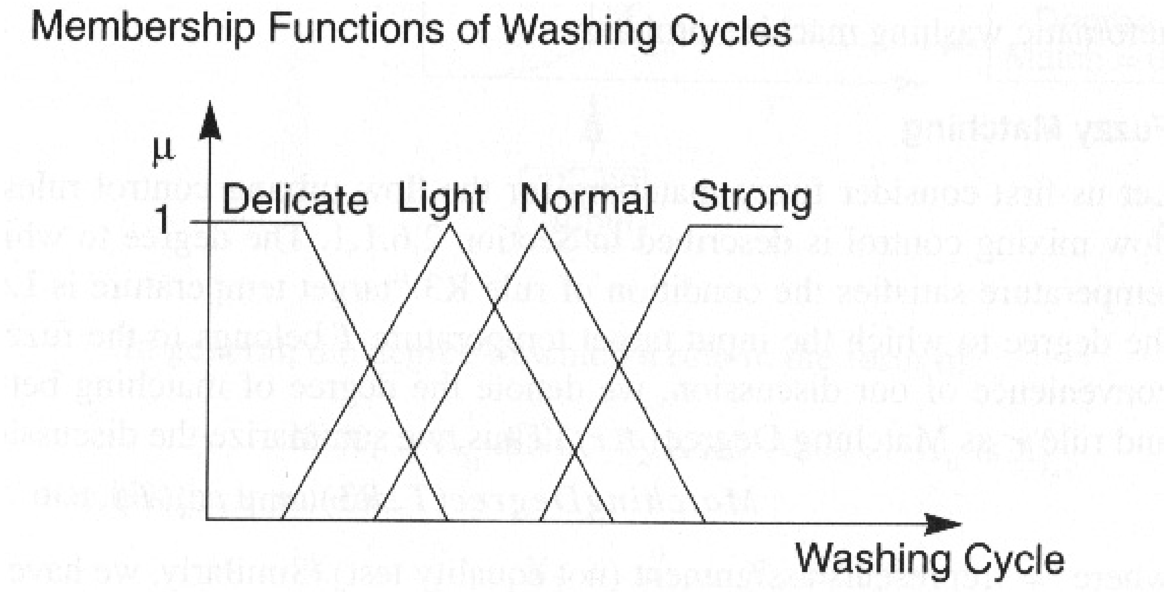
* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Input Membership functions



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Output Membership functions



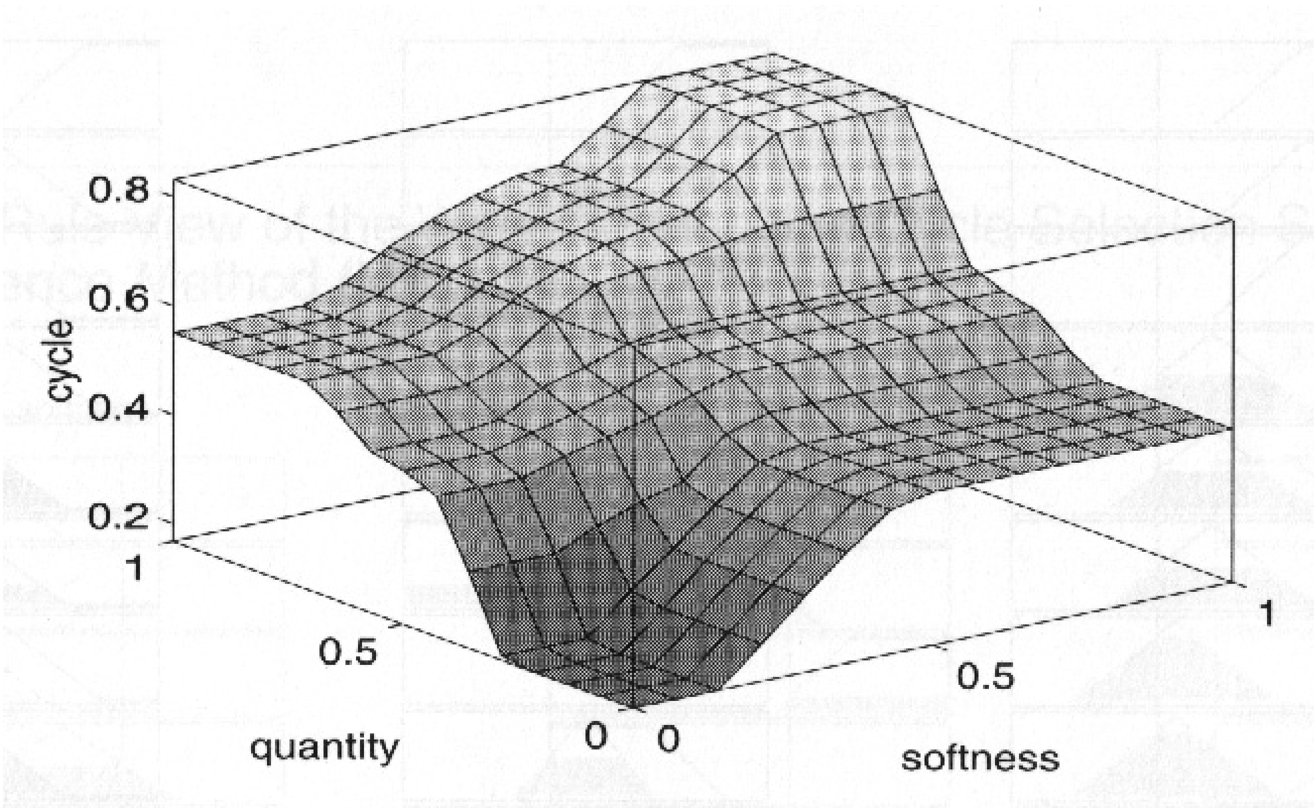
* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Fuzzy Rules for Washing Cycle

Softness \ Quantity	Small	Medium	Large
Soft	**Delicate**	**Light**	**Normal**
Normal Soft	**Light**	**Normal**	**Normal**
Normal Hard	**Light**	**Normal**	**Strong**
Hard	**Light**	**Normal**	**Strong**

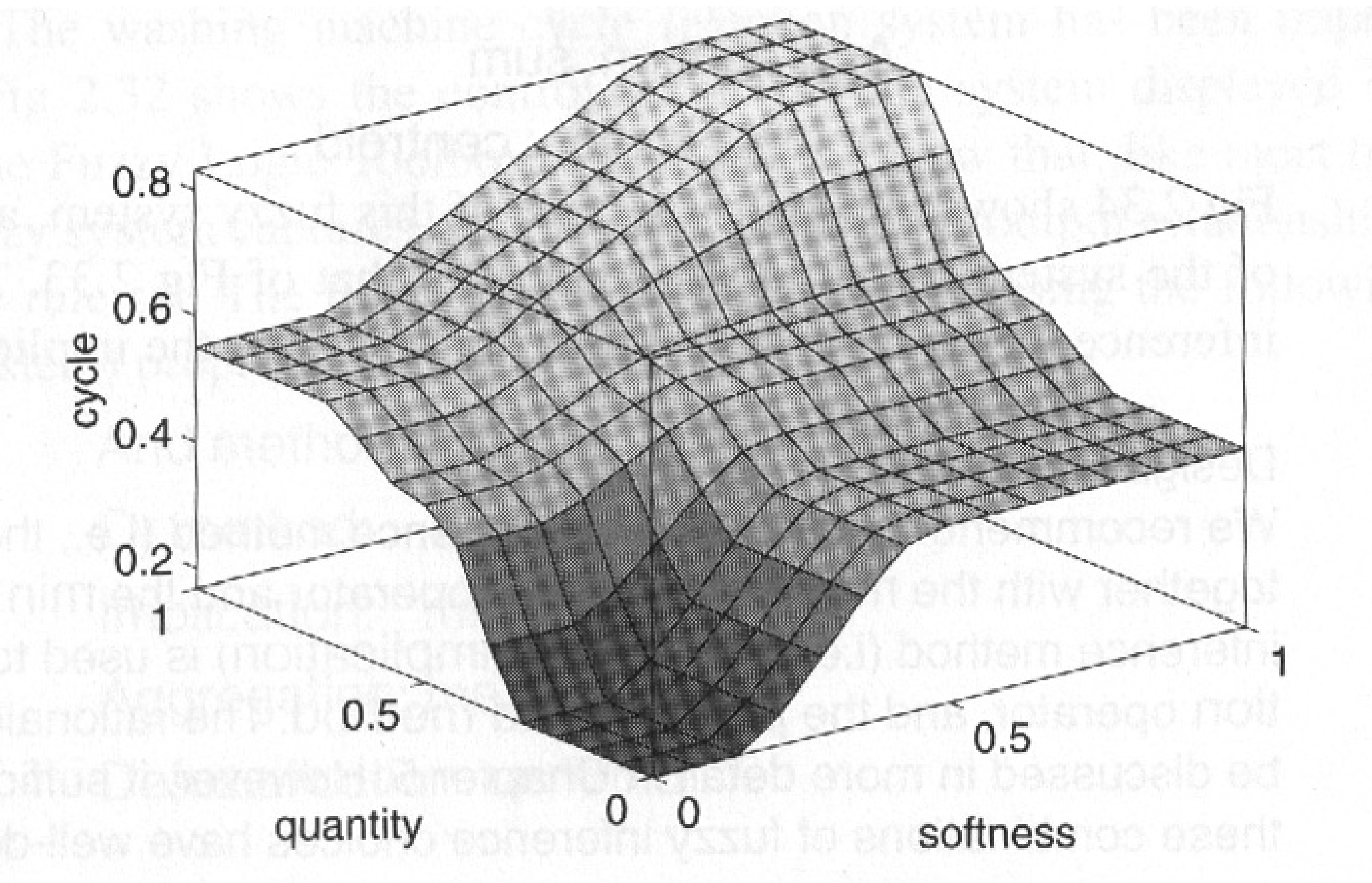
* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Control Surface View (Clipping)



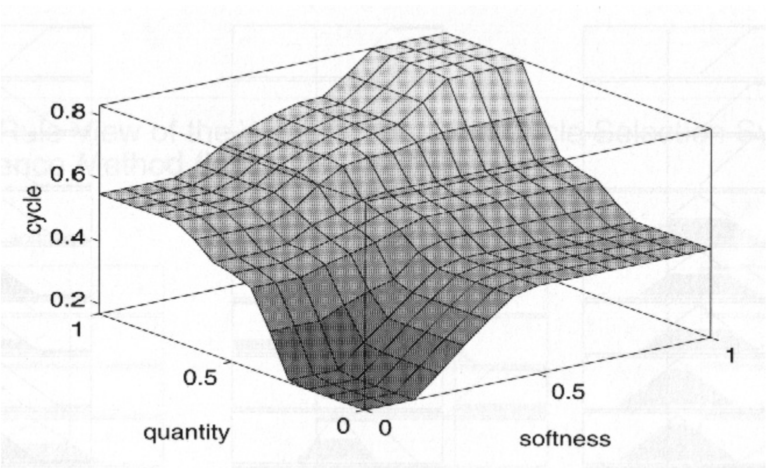
* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Control Surface View (Scaling)

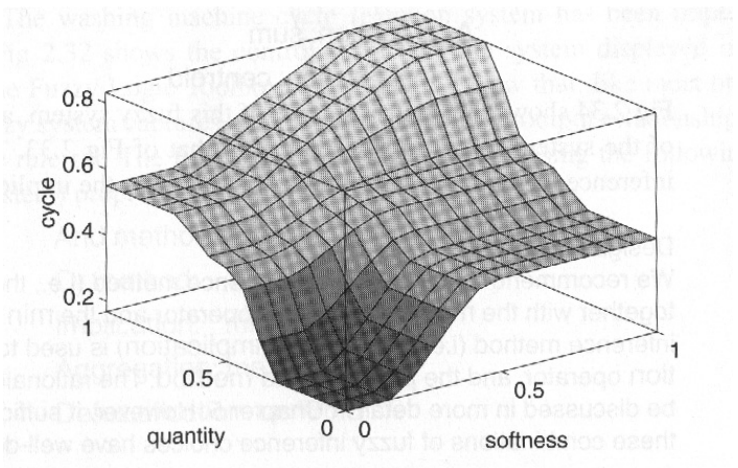


* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Control Surface View



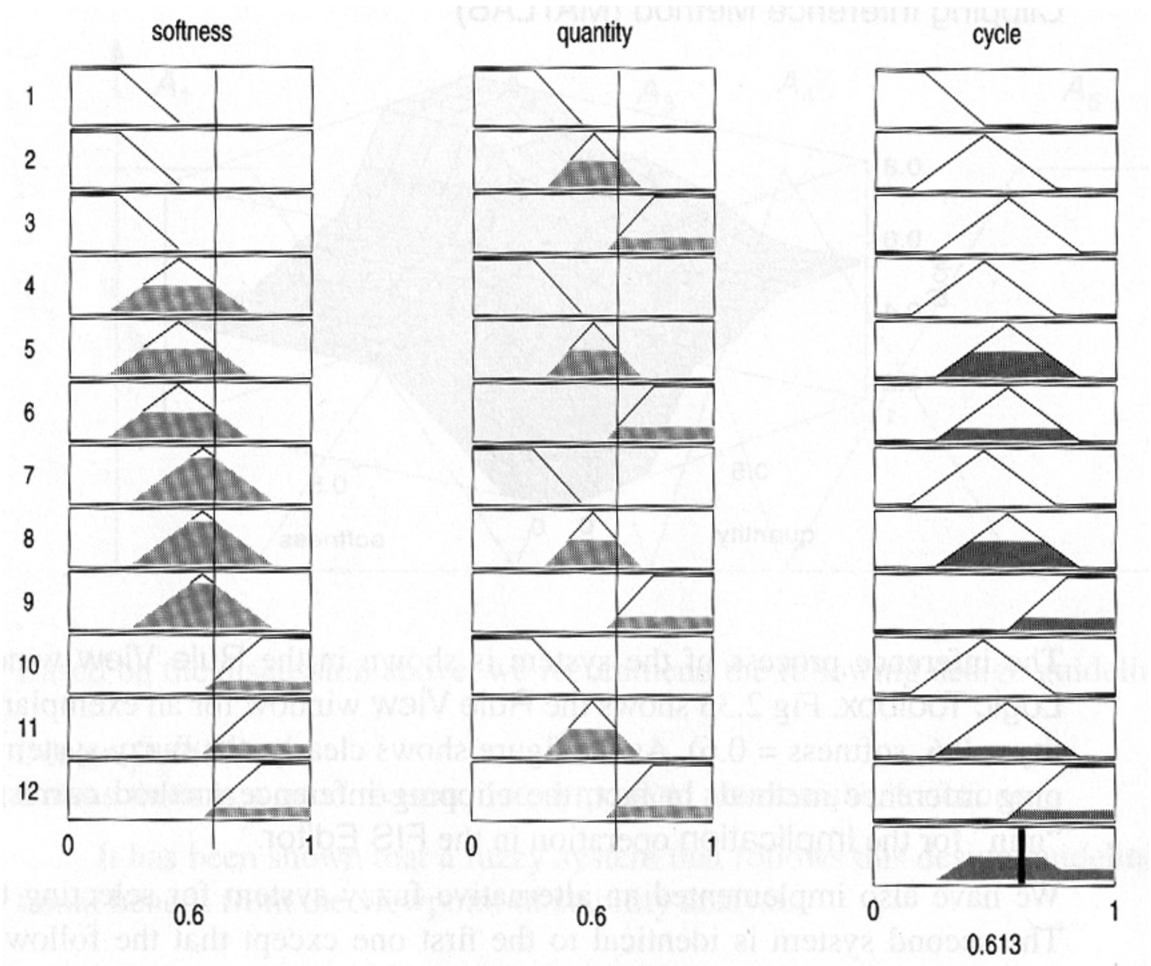
Clipping



Scaling

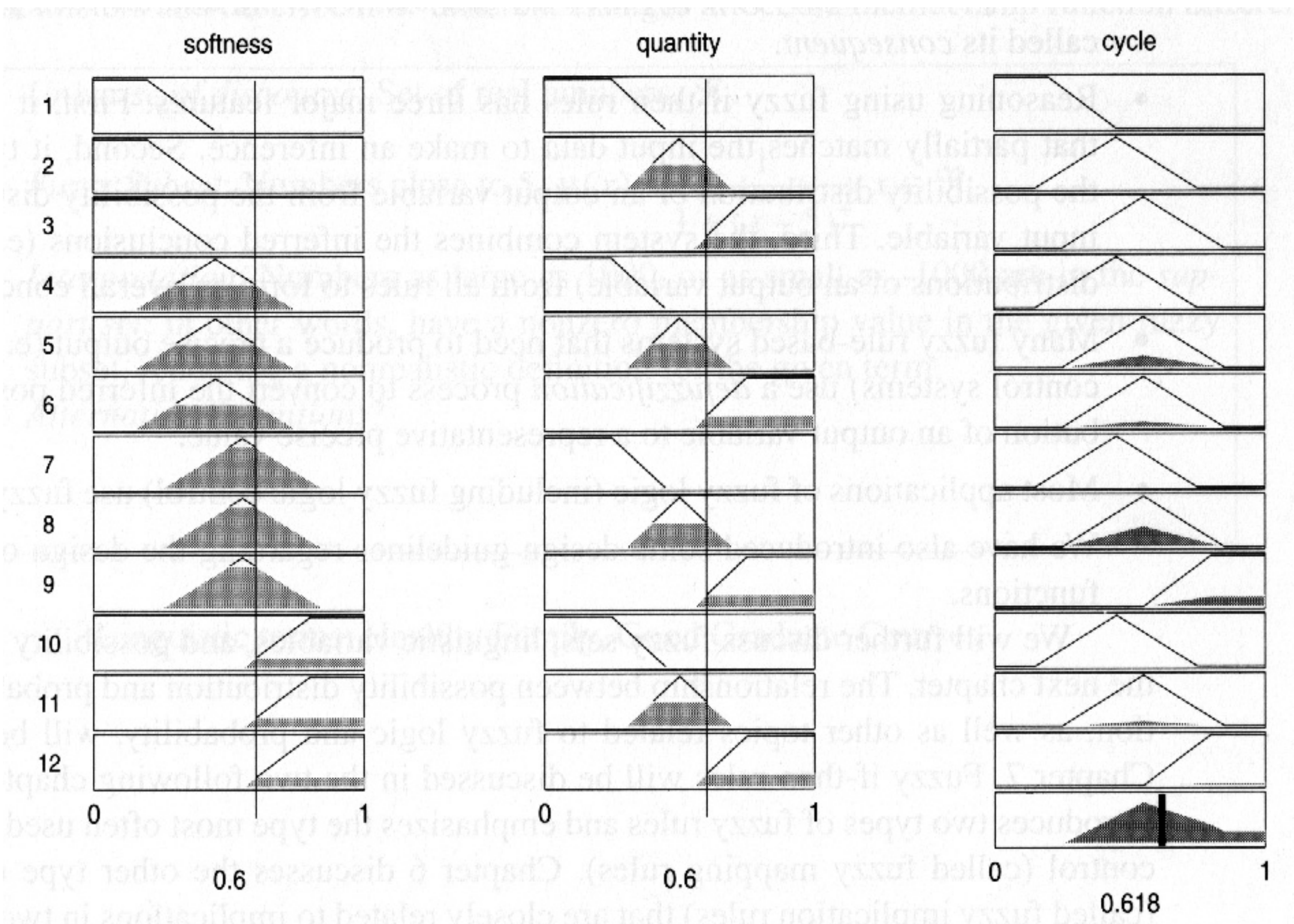
* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Rule View (Clipping)



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Example: Rule View (Scaling)



* Fuzzy Logic: Intelligence, control, and Information, J. Yen and R. Langari, Prentice Hall

Pattern Recognition

- Task is to classify a pattern based on certain measurements of that pattern
- Inexact
- Ambiguous
- Corrupted data
- High variability

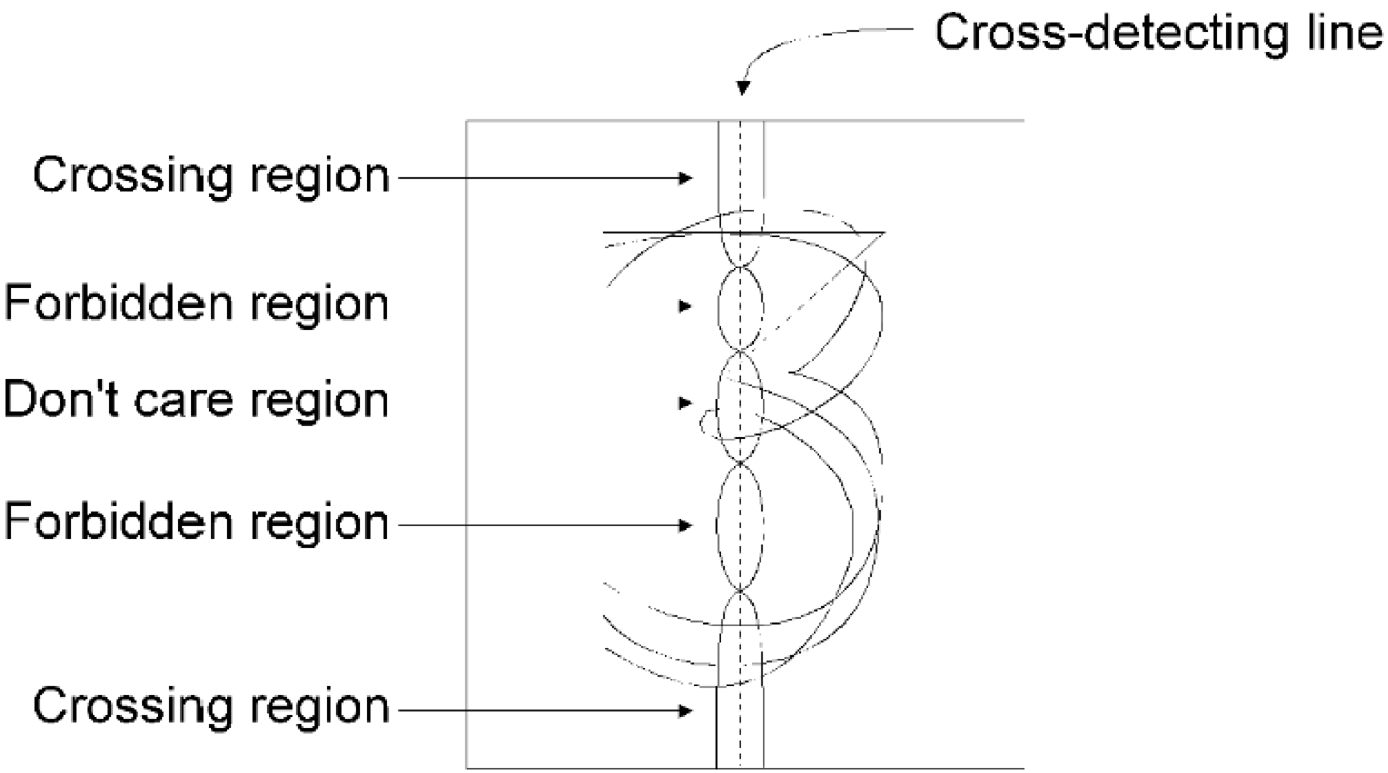
Pattern Recognition

- Examples
  - handwriting recognition
  - iris classification

Handwriting Recognition

- Identifying handwritten characters
- Much variation between writers
- Much variation from the same writer
- Track where character “crosses” specific zones

Handwriting Recognition



Iris Classification

- Classic classification problem
- Problem is to identify the species of an iris flower
- Input variables are four measurements of the flower
- Output variable is the species of the iris

Iris Classification

- Why is this a difficult problem?
- Uncertainty
  - wide variety in the species
  - living things have a wide variation
  - uncertainty about species

Iris Classification

- Accurate classification can be performed with the appropriately defined MF and rules
  - about 12 rules will do
  - optimal MF are harder to define
  - Lab 6

Control Systems

- Use mathematical systems to transform the current state of a system and the desired state of a system into a future state of a system
- Current State + Desired State = Influences
- Behaviour of the system must be modelable by a mathematical function

Control Systems

- Don’t scale well
- Don’t handle non-monotonic functions well

Fuzzy Control

- Replaces mathematical approximations with a fuzzy system
- Rules define actions for specific conditions
- Biggest application area of fuzzy logic
- Examples
  - Inverted pendulum
  - Sendai subway system
  - Washing machines

Sendai Subway

- Underground train in Sendai, Japan
- Fuzzy system controls
  - train accelerator
  - brakes
- Fuzzy controller must
  - accelerate to target speed
  - maintain speed
  - stop accurately

Sendai Subway

- Rules obtained from experienced train drivers
- Very efficient system
- Not portable

Washing Machines

“Bosch - Washing Machine - WOL2200 - Free Standing

Premium Fuzzy Logic, Top loader washing machine with 1100 rpm spin and 15 Clothes Care wash programmes. “

- http://www.itlocal.co.uk/rdo/acatalog/Online_Catalogue_FREESTANDING_273.html

Washing Machines

- Inputs
  - amount of dirt
  - type of dirt
- Outputs
  - wash time
  - can also include amount of detergent

Washing Machines

- Commercial success
- Possibly the most widely known application of fuzzy logic / fuzzy systems
- How much is due to marketing techno babble?

Decision Making

- Given measurements of a specific situation, decide what to do
- Many such measurements are not clear-cut
  - boundaries
- Examples
  - loan approval
  - insurance evaluation

Summary

- Advantages of fuzzy systems make them applicable to many different applications
- Biggest application area is fuzzy control
- Fuzzy control applies fuzzy systems to control systems
- Largest commercial use of fuzzy logic

Summary

- Other applications include
  - pattern recognition
  - decision making
  - fuzzy databases / information retrieval
- Specialised applications

Fuzzy Logic Example

Automotive Speed Controller

3 inputs:

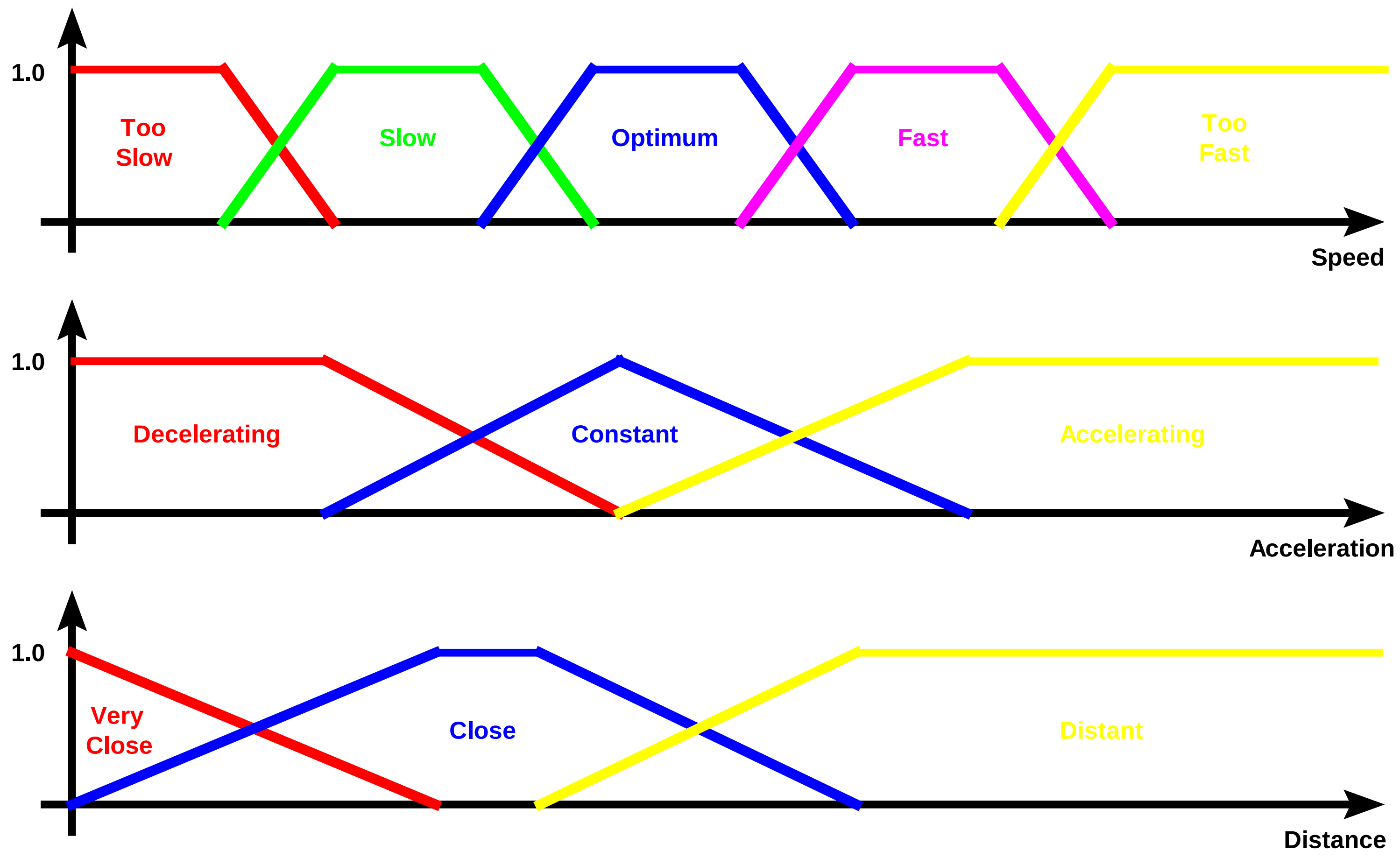
- speed (5 levels)
- acceleration (3 levels)
- distance to destination (3 levels)

1 output:

- power (fuel flow to engine)

Set of rules to determine output based on input values

Fuzzy Logic Example



Fuzzy Logic Example

Example Rules

IF speed is TOO SLOW and acceleration is DECELERATING,
THEN INCREASE POWER GREATLY

IF speed is SLOW and acceleration is DECREASING,
THEN INCREASE POWER SLIGHTLY

IF distance is CLOSE,
THEN DECREASE POWER SLIGHTLY

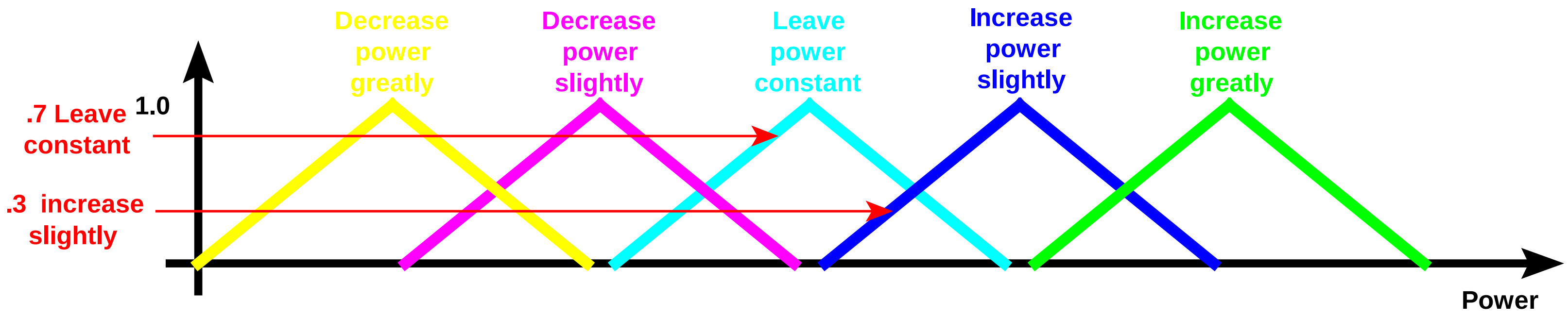
. . .

Fuzzy Logic Example

Output Determination

Degree of membership in an output fuzzy set now represents each fuzzy action.

Fuzzy actions are combined to form a system output.



Fuzzy Logic Example

Steps

Fuzzification: determines an input's % membership in overlapping sets.

Rules: determine outputs based on inputs and rules.

Combination/Defuzzification: combine all fuzzy actions into a single fuzzy action and transform the single fuzzy action into a crisp, executable system output. May use centroid of weighted sets.

Fuzzy Logic Example

Note there would be a total of 95 different rules for all combinations of inputs of 1, 2, or 3 at a time.

(5x3x3 + 5x3 + 5x3 + 3x3 + 5 + 3 + 3)

In practice, a system won't require all the rules.

System tweaked by adding or changing rules and by adjusting set boundaries.

System performance can be very good but not usually optimized by traditional metrics (minimize RMS error).

Fuzzy Logic Summary

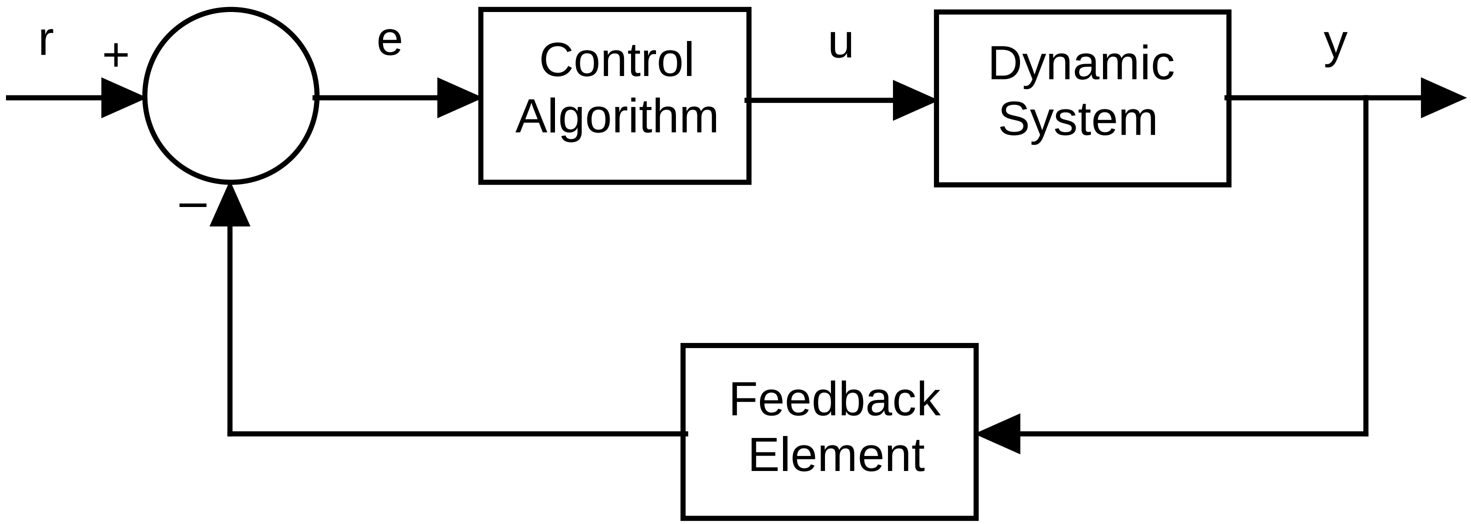
Doesn't require an understanding of process but any knowledge will help formulate rules.

Complicated systems may require several iterations to find a set of rules resulting in a stable system.

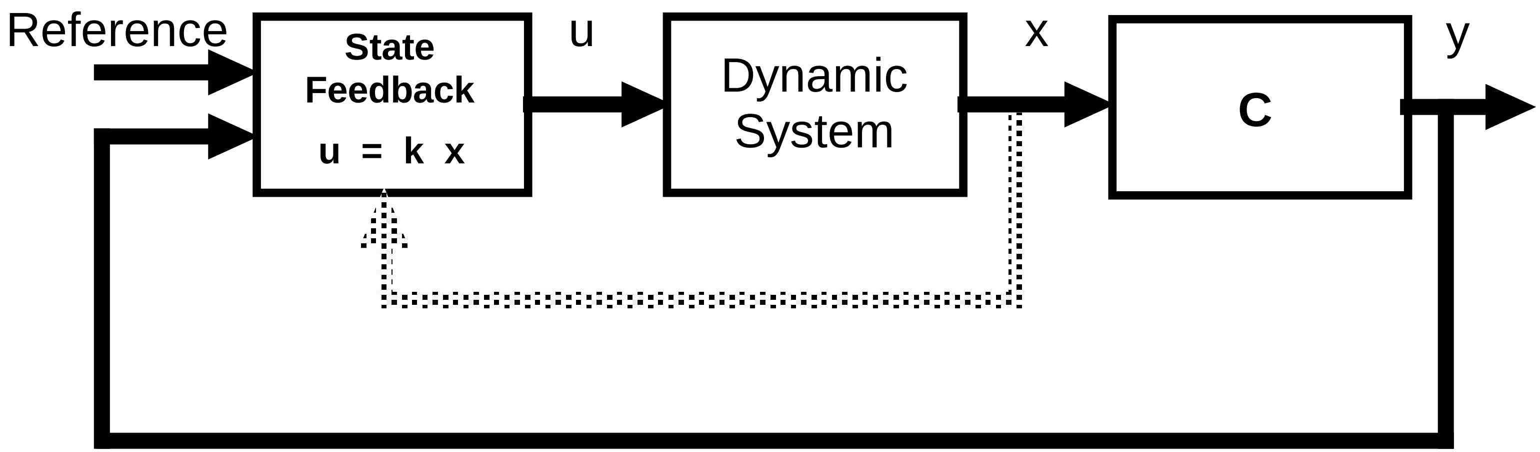
Combining Neural Networks with fuzzy logic reduces time to establish rules by analyzing clusters of data.

Possible applications: Master Production Schedule, Material Requirements Planning, Inventory Capacity Planning

Classical Feedback Control



Modern Control

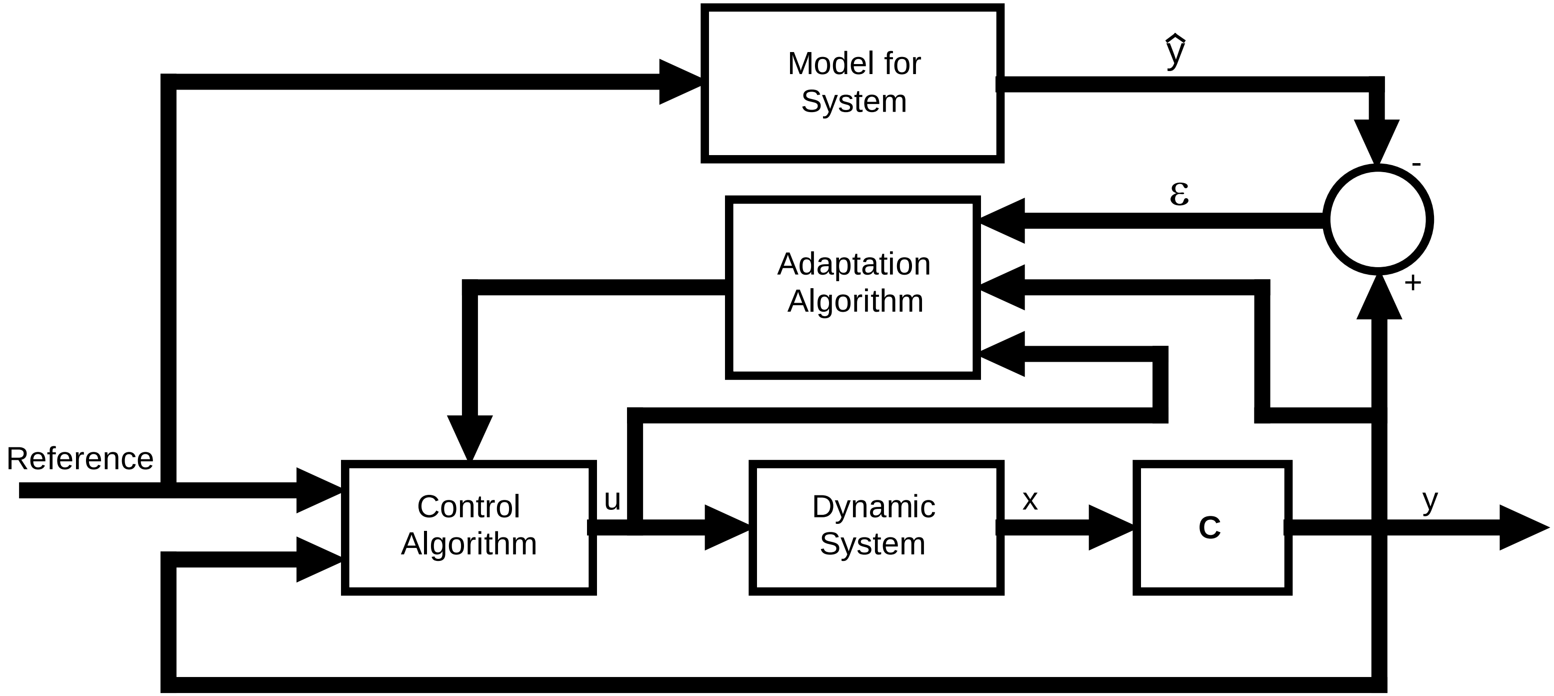


State: $\frac{d\mathbf{x}(t)}{dt} = \mathbf{Ax}(t) + \mathbf{Bu}(t)$, Output: $\mathbf{y}(t) = \mathbf{Cx}(t)$

Performance Index: $J = \int_0^t \mathbf{x}'(\tau)\mathbf{Rx}(\tau) + \mathbf{u}'(\tau)\mathbf{Qu}(\tau)\, d\tau$

Control: **u = k x ; k = f(A,B,C,J,Q,R)**

Model Reference Adaptive Control



State: $\dfrac{d\mathbf{x}(t)}{dt} = \mathbf{Ax}(t) + \mathbf{Bu}(t)$, Output: $\mathbf{y}(t) = \mathbf{Cx}(t)$

Performance Index: $J = \int_0^t \mathbf{x}'(\tau)\mathbf{Rx}(\tau) + \mathbf{u}'(\tau)\mathbf{Qu}(\tau)\, d\tau$

Control: $\mathbf{u} = \mathbf{k}(\boldsymbol{\theta}, \mathbf{R}, \mathbf{Q})\, \mathbf{x}$

Prediction: $\hat{y} = \boldsymbol{\theta}^T\boldsymbol{\phi}$, $\boldsymbol{\phi}^T = [y(k), y(k-1), .. u(k), u(k-1) ...]$

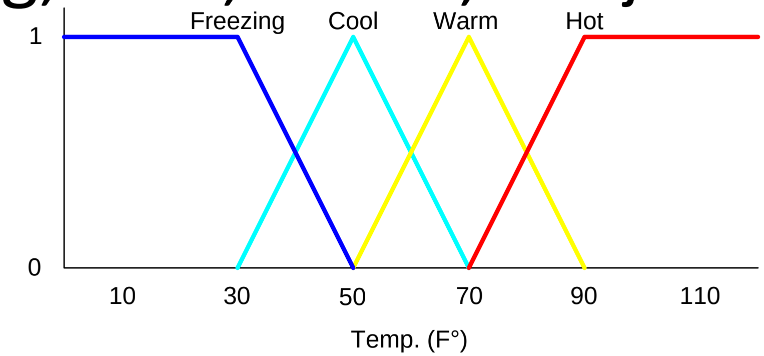
Adaptation: $\boldsymbol{\theta}^i = \boldsymbol{\theta}^{i-1} + \mathbf{P}[y - \hat{y}]$, $\boldsymbol{\theta}^T = [a_1, a_2, ... b_1, b_2, ...]$

Fuzzy Control

- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.
- Disjunction of Conjunctions

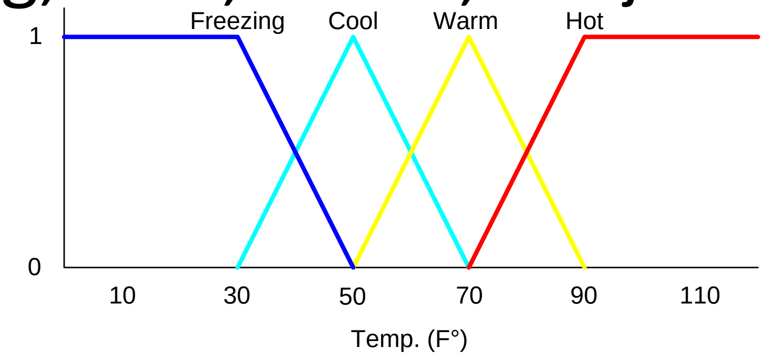
Inputs: Temperature

- Temp: {Freezing, Cool, Warm, Hot}

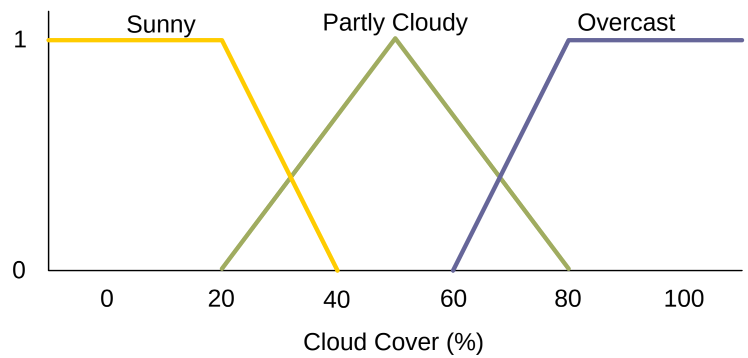


Inputs: Temperature, Cloud Cover

- Temp: {Freezing, Cool, Warm, Hot}

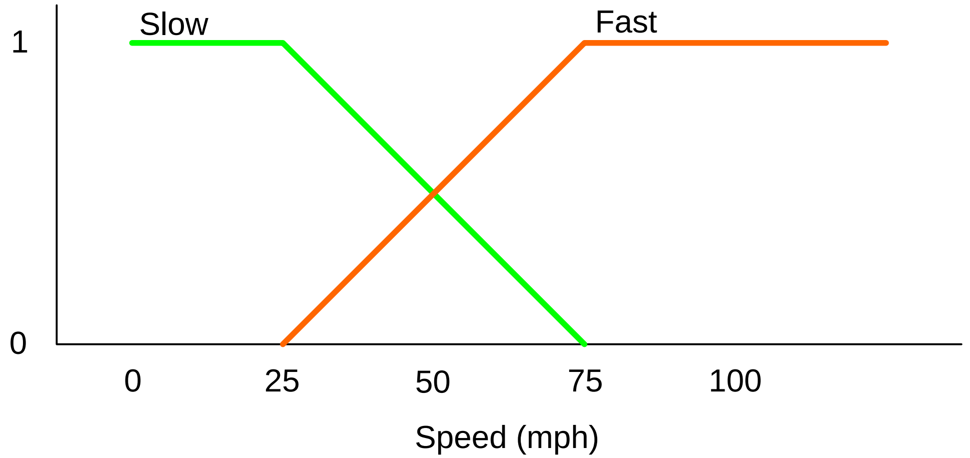


- Cover: {Sunny, Partly, Overcast}



Output: Speed

- Speed: {Slow, Fast}



Rules

- If it's Sunny and Warm, drive Fast

  $\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$

- If it's Cloudy and Cool, drive Slow

  $\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$

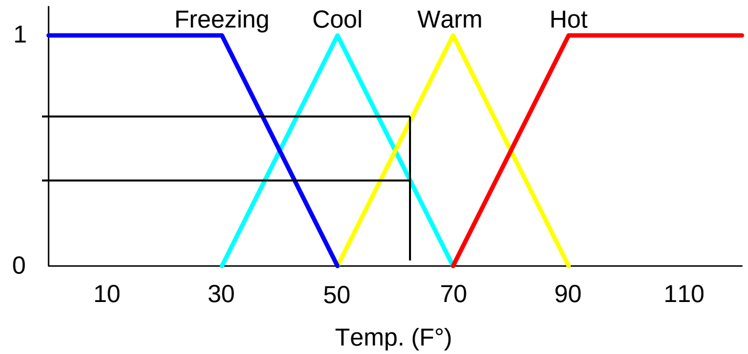
- Driving Speed is the combination of output of these rules...

Example Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

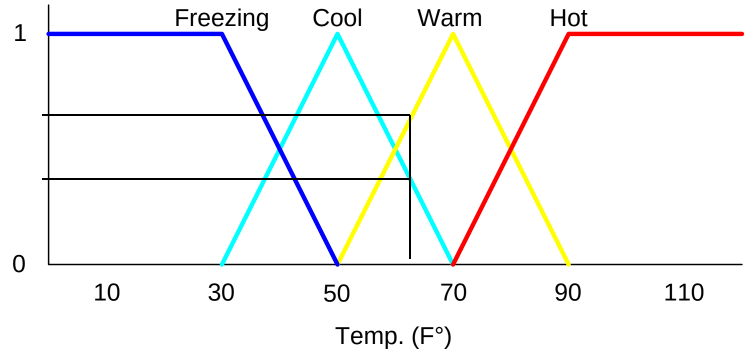
Fuzzification:
Calculate Input Membership Levels

- 65 F° $\Rightarrow$ Cool = 0.4, Warm= 0.7

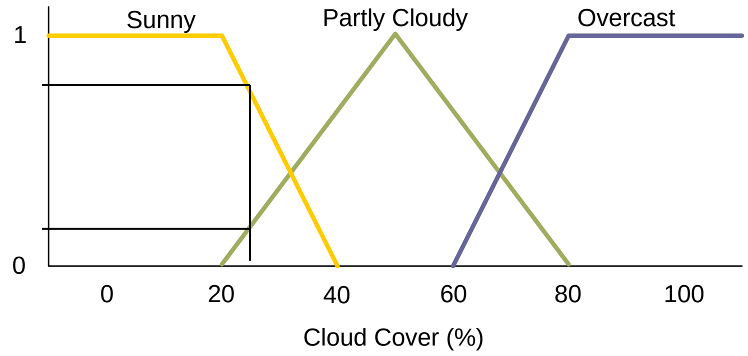


Fuzzification:
Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm= 0.7



- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$\Rightarrow$ **Fast = 0.7**

- If it's Cloudy and Cool, drive Slow

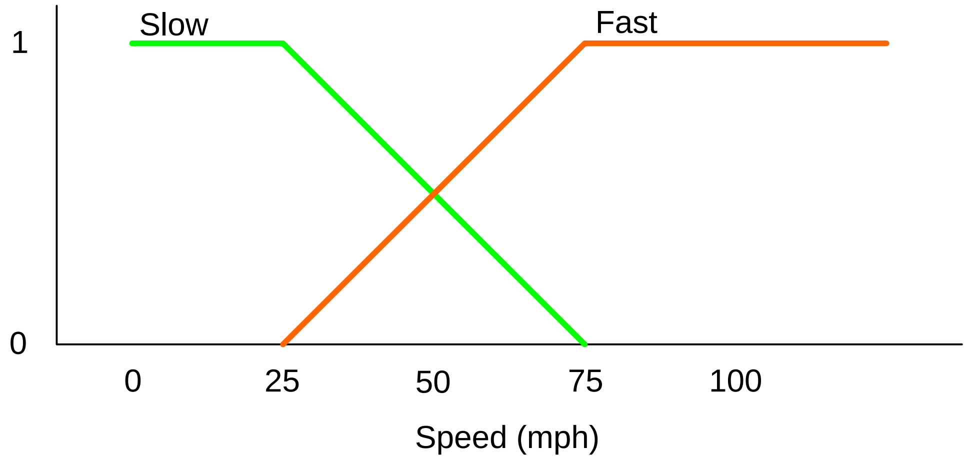
$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

$$0.2 \wedge 0.4 = 0.2$$

$\Rightarrow$ **Slow = 0.2**

Defuzzification: Constructing the Output

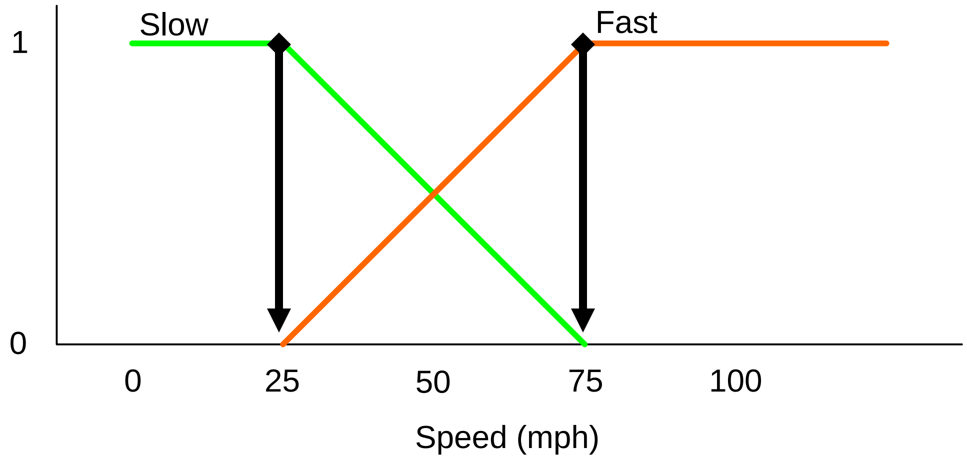
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

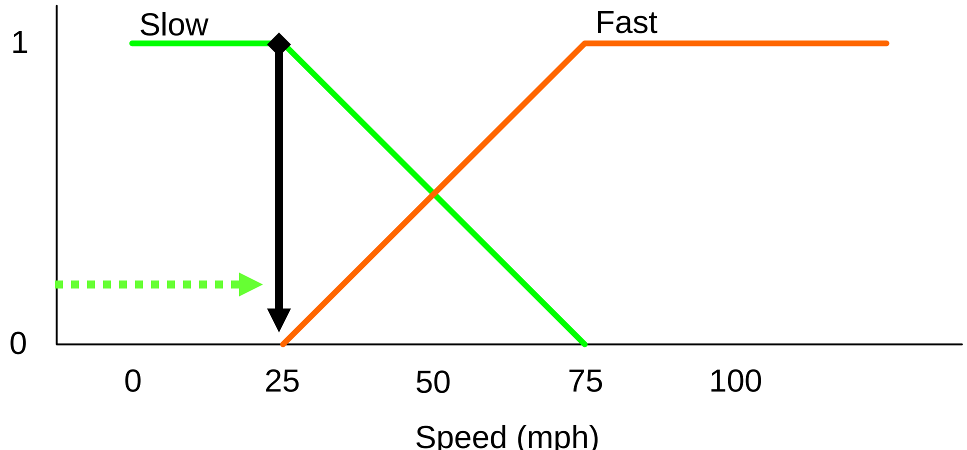
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

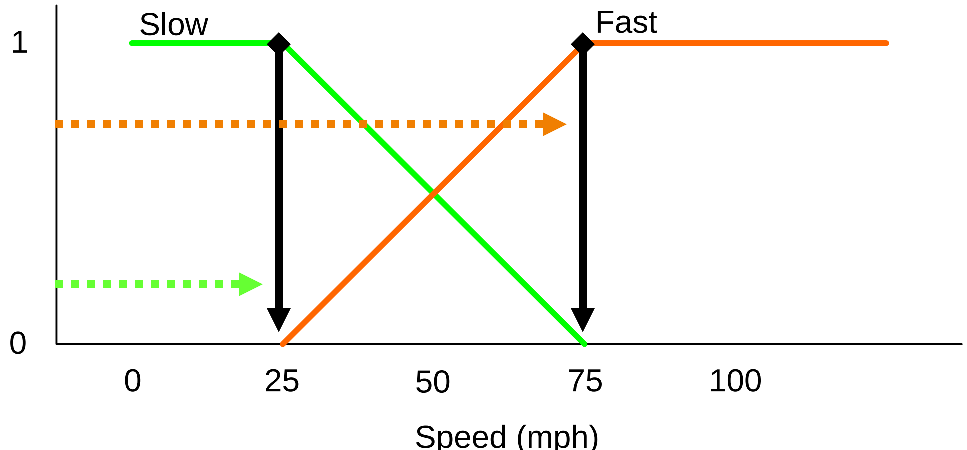
- Speed is 20% Slow and 70% Fast



- Speed = weighted mean
  = (2*25+...

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean
  = (2*25+7*75)/(9)
  = 63.8 mph

Notes: Follow-up Points

- Fuzzy Logic Control allows for the smooth interpolation between variable centroids with relatively few rules
- This does not work with crisp (traditional Boolean) logic
- Provides a natural way to model some types of human expertise in a computer program

Notes: Drawbacks to Fuzzy logic

- Requires tuning of membership functions
- Fuzzy Logic control may not scale well to large or complex problems
- Deals with imprecision, and vagueness, but not uncertainty

Summary

- Fuzzy Logic provides way to calculate with imprecision and vagueness
- Fuzzy Logic can be used to represent some kinds of human expertise
- Fuzzy Membership Sets
- Fuzzy Linguistic Variables
- Fuzzy AND and OR
- Fuzzy Control