Conduction Heat Transfer

Practice problems

1. A steel pipe of conductivity $50\ \text{W/m-K}$ has inside and outside surface temperature of $200\ ^{o}C$ and $160\ ^{o}C$ respectively. Find the heat flow rate per unit pipe length and flux per unit inside and per unit outside area. The ID and OD of the pipe are $40\ \text{mm}$ and $25\ \text{mm}$ respectively.

[Ans: 26.73 kW/m]

2. Consider a cylindrical element of inside and outside diameter $'a'$ and $'b'$ with the inside and outside temperature levels T_1 and T_2. For a linear variation of $'k'$ with T

$$k = k_0\left(1+\beta T\right),$$

find the average value k_m which satisfies

$$q' = \frac{2\pi k_m\left(T_1 - T_2\right)}{\ln\left(b/a\right)}.$$

[Ans: $\mathrm{k_m = k_o\left(1+\frac{\beta}{2}(T_1+T_2)\right)}$]

3. A plane wall of a composite of two material A and B. The wall of material A has $q''' = 1.5\times10^{6}\ \text{W/m}^3$, $k_A = 75\ \text{W/m-K}$ and thickness $L_A = 50\ \text{mm}$. The material B has no energy generation with $k_B = 150\ \text{W/m-K}$ and thickness $L_B = 20\ \text{mm}$. The inner surface of A is well insulated, while the outer surface of B is cooled by a water stream $T_\infty = 30\ ^{o}C$ and $h = 1000\ \text{W/m}^2\text{-K}$.

a) Sketch the temperature distribution of the composite under steady state condition,

b) Determine the temperature T_o of the insulated surface and temperature T_2 of the cooled surface.

[Ans: To=140°C, T2=105°C]

4. A silicon chip is encapsulated such that, under steady state conditions, all the power it dissipates is transferred by convection to a fluid stream for which $h = 1000\ \text{W/m}^2\text{-K}$ and $T_\infty = 25\ ^{o}C$. The chip is separated from the fluid by a $2\ \text{mm}$ thick aluminum cover plate and the contact resistance of the clip/aluminum surface is $0.5\times10^{-4}\ \text{m}^2\text{-K/W}$. If the chip area is $100\ \text{mm}^2$ and its maximum allowable temperature is $85\ ^{o}C$, what is the maximum allowable power dissipation in the chip ?

[Ans: 5.67W]

5. Find the temperature distribution in a 2D rectangular plate (at steady state) of sides $[a,b]$ subject to $T(0,y)=0$, $T(x,0)=0$, $T(a,y)=0$ and $T(x,b)=Ax$.

$$[\text{Ans: } T(x) = -\frac{2Aa}{\pi}\sum_{n=1}^{\infty}\frac{n}{\sinh\left(\frac{n\pi b}{a}\right)}\sin\left(\frac{n\pi x}{a}\right)\sinh\left(\frac{n\pi y}{a}\right)]$$

6. An exterior wall of a house may be approximated by a $4\,\text{in}$ layer of common brick $[k=0.7\ \text{W/m-K}]$ followed by a $1.5\,\text{in}$ layer of gypsum plaster $[k=0.48\ \text{W/m-K}]$. What thickness of rock-wood insulation having $k=0.065\ \text{W/m-K}$ should be added to reduce the heat loss (or gain) from the wall by $80\,\%$?

[Ans: 58.34mm]

7. An aluminum $(k=200\ \text{W/m-K})$ protrudes from a wall. It is $3\,\text{mm}$ thick and $75\,\text{cm}$ long. The back is maintained at $300\ ^oC$ and the ambient temperature is $50\ ^oC$ with $h=10\ \text{W/m}^2\text{-K}$. Calculate the heat loss per unit depth of the fin.

[Ans: 866W/m]

8. A certain material has a thickness of $30\,\text{cm}$ and a thermal conductivity of $0.04\ \text{W/m-K}$. At a particular instant in time, the temperature distribution with x, the distance from the left face is $T=150x^2-30x$, where x is in meters. Calculate the heat flow rates at $x=0$ and $x=30\,\text{cm}$. Is the solid heating up or cooling down ?

[Ans: 1.2W/m^2, 14.35W/m^2]

9. Derive a relation for the critical radius of insulation for a sphere.

[Ans: 2k/h]

10. A plane wall $6\,\text{cm}$ thick generates heat internally at the rate of $0.3\ \text{MW/m}^3$. One side of the wall is insulated and the other side is exposed to an environment at $93\ ^oC$. The $'h'$ between the wall and the environment is $570\ \text{W/m}^2\text{-K}$. The thermal conductivity of the wall is $21\ \text{W/m-K}$. Calculate the maximum temperature in the wall.

[Ans: 150.28^{o}C]

11. Derive an expression for the temperature distribution in a hollow cylinder with heat source which vary according to the linear relation

$$q''' = a + br$$

with q''' is the heat generated per unit volume at $r = r_i$. The inside and outside temperature are $T = T_i$ at $r = r_i$ and $T = T_o$ at $r = r_o$.

[Ans:

$$T(r) = -\left(\frac{b}{9k}\right)r^3 - \left(\frac{a}{4k}\right)r^2 + C_1 \ln(r) + C_2, \text{ where}$$

$$C_1 = \frac{36k(T_i - T_o) + 9a(r_i^2 - r_o^2) + 4b(r_i^3 - r_o^3)}{36k \log\left(\dfrac{r_i}{r_o}\right)},$$

$$C_2 = \frac{36\ln(r_i)T_o k + 9\ln(r_i)ar_o^2 + 4\ln(r_i)br_o^3 - 36T_i k\ln(r_o) - 9ar_i^2\ln(r_o) - 4br_i^3\ln(r_o)}{36k \log\left(\dfrac{r_i}{r_o}\right)}.$$

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12. A very long thin copper rod 6.4 mm in diameter is exposed to an environment at 20 ^oC. The base temperature is 150 ^oC and 'h' between the rod and surrounding is 24 W/m^2-K. Calculate heat loss from the rod.

[Ans: 10.1 W assuming k = 390W/m-K]

13. For a strip of infinite y-dimension and length $2L$ in x-dimension, prove that the slope of the isothermals is zero at $x = L$ for all θ/θ_1; and zero and infinite at $x = 0$ and $2L$, for $\theta/\theta_1 = 0$ and $\theta/\theta_1 = 1$ respectively. Here $\theta = T - T_0$ and the boundary conditions are $T(x,0) = T_1$; $T(0,y) = T_0$ and $T(2L,y) = T_0$.

14. Consider a long metal rod of square cross-section $(L \times L)$. The upper and lower side faces are at T_0. The left face is induction heated; assume that the effect is a uniform flux input at the surface. The right face is at T_1.

a. Find temperature distribution in the rod-material.

b. Obtain a solution if the left face is adiabatic.

15. A steel ball $\left(C_p = 0.46\,\text{kJ/kg-K},\ k = 35\,\text{W/m-K}\right)$, $5\,\text{cm}$ in diameter and initially at a uniform temperature of $450\,^{\circ}\text{C}$ is suddenly placed in a controlled environment where the temperature is maintained at $100\,^{\circ}\text{C}$. The convective heat transfer co-efficient is $10\,\text{W/m}^2\text{-K}$. Calculate the time required for the ball to attain $150\,^{\circ}\text{C}$. Assume specific gravity of steel to be 7.85.

[Ans: 1.6 hrs]

16. A large block of steel $\left(k = 45\ \text{W/m-K},\ \alpha = 1.4\times10^{-5}\ \text{m}^2/\text{s}\right)$, is initially at a uniform temperature of $35\,^{\circ}\text{C}$. The surface is exposed to a heat flux

a. by suddenly raising the surface temperature to $250\,^{\circ}\text{C}$.

b. through a constant surface flux of $3.2\times10^5\ \text{W/m}^2$.

Calculate the temperature at a depth of $2.5\,\text{cm}$ after a time of $0.5\,\text{min}$ in both cases.

[Ans: a) 118.506°C b) 79.34°C]

17. A large plate of aluminium $\left(k = 215\,\text{W/m-K}; \alpha = 8.4\times10^{-5}\ \text{m}^2/\text{s}\right)$ $5.0\,\text{cm}$ thick and initially at $200\,^{\circ}\text{C}$ is suddenly exposed to a convection surface environment of $70\,^{\circ}\text{C}$ with $h = 525\,\text{W/m}^2\text{-K}$. Calculate the temperature at a depth of $1.25\,\text{cm}$ from one of the faces one minute after the plate has been exposed to the environment. How much energy per unit area has been removed per unit area of the plate during this time.

[Ans: 203.57 °C]

18. A plate of thickness δ initially has a sinusoidal temperature distribution varying from T_o at $x = 0$ to T_m at $x = \delta/2$ to T_0 at $x = \delta$. If the surfaces of the plate are held at T_0 for subsequent times, find the temperature distribution as a function of time.

Ans: $\theta(x,t) = \sin(\pi x)\, e^{-\pi^2 t}$

19. Consider two semi-infinite solids, one initially at $0\,^{\circ}\text{C}$ and the other at θ_o, which are suddenly at $(\tau = 0)$ placed in intimate contact at their surfaces. Assuming no contact resistance, calculate the subsequent contact surface temperature. Qualitatively show the temperature plots in the two solids at different time instants.

[Ans: $\theta_s = \frac{\sqrt{k_A \rho_A C_A}\,\theta_o}{\sqrt{k_A \rho_A C_A} + \sqrt{k_B \rho_B C_B}}$]

20. A large slab of copper having a thickness of $3.0\,\text{cm}$ is initially at $300\,^{\circ}\text{C}$. It is suddenly exposed to a convection environment on the top surface at $80\,^{\circ}\text{C}$ while the bottom surface is insulated. In

6 minutes, the surface temperature drops to 140 °C. Calculate the value of 'h'. Assume for copper, $k = 400$ W/m-K; $\alpha = 1.12\times10^{-4}$ m^2/s.

[Ans: 308.3 W/m^2-K]

21. A steel sphere 10 cm in diameter is suddenly immersed in a tank of oil at 10 °C. The initial temperature of the sphere is 220 °C, $h = 5000$ W/m^2-K. How long will it take the centre of the sphere to cool down to 120 °C? For steel, $k = 45$ W/m-K; $\alpha = 1.4\times10^{-5}$ m^2/s.

[Ans: 26.8 s]

22. A very thick low carbon steel plate ($k = 34.96$ BTu/hr-ft °F) initially at 60 °F, is placed in a heat treatment bath at 860 °F. The surface process is characterized by a convection coefficient of 150Btu/ft^2-hr-°F. Find the surface temperature variation with time for short times. Assume $\alpha = 0.688$ ft^2/hr.

[Ans: $T(x,t) = 860 - 800\left(exp(12.66t)erfc(3.56\sqrt{t})\right)$ where t is in hrs]

23. Steel plates being thinned by rolling must be periodically reheated. How long must plates 3in thick, which are at 1000 °F, be kept in a furnace surrounding at 1600 °F, $h = 30$ BTu/hr-ft^2 °F, in order to reach a minimum metal temperature of 1300 °F ? Assume $k = 20$ BTu/hr-ft °F; α=0.37 ft^2/hr.

[Ans: 152 s]

24. A semi-infinite solid initially at T_o throughout is in contact at its face with an environment at T_e. A surface coefficient 'h' may be used at the exposed surface. If T_e is a function of time, determine if there is any such function or functions, which will permit a 'similarity' type of solution, that is in terms of $\frac{x}{\sqrt{\alpha\tau}}$.

[Ans: *deliberately withheld*]