(5) The Matrix Iteration Method :-

The DEOM for free vibration of an undamped 2-DOF system in the matrix form is:

$$[m]\{\ddot{x}\}+[K]\{x\}=\{0\}, \text{ say. } \quad \text{---(1)}$$

For solving (1), We take $\{x\}=\{X\}\sin(\omega t+\phi)$ --- (2)

Then $\{\ddot{x}\}=-\omega^2\{X\}\sin(\omega t+\phi)$ --- (3)

where $\{X\}=\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$ is the amplitude vector.

Substitution of (2) & (3) in (1) leads to:

$-\omega^2[m]\{X\}+[K]\{X\}=\{0\}$ --- (4) which is ~~nothing~~ but the amplitude equations in matrix form. Premultiplying both sides of (4) by $[K]^{-1}$, we get,

$$-\omega^2[K]^{-1}[m]\{X\}+\{X\}=\{0\}$$

or, $-\omega^2[D]\{X\}=-\{X\}$

or, $[D]\{X\}=\frac{1}{\omega^2}\{X\}$ --- (5)

(Since $[K]^{-1}[K]=[I]$ & $[I]\{X\}=\{X\}$. Also, $[K]^{-1}\{0\}=\{0\}$, $[I]=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ is the unit or identity matrix)

where $[D]=[K]^{-1}[m]$ is a dynamic matrix (or, dynamical matrix) for our system, since ~~it controls the~~ it determines the dynamic characteristics of the system in free vibration.

Now, relation (5) basically means that

(6) $\cdots [D]\{X\}_1=\frac{1}{\omega_1^2}\{X\}_1$ & $[D]\{X\}_2=\frac{1}{\omega_2^2}\{X\}_2$ --- (7)

Actually, (5) represents an eigenvalue problem like $[A]\{x\}=\lambda\{x\}$, which you have studied in an engineering mathematics course. In (5), $\frac{1}{\omega^2}$ is like λ, the eigenvalue sought.

From 6 & 7, it is clear that if an eigenvector $\{X\}_1$ or $\{X\}_2$ is premultiplied by $[D]$, we get back the same eigenvector multiplied by a constant which is $\frac{1}{\omega_1^2}$ in case of $\{X\}_1$ & $\frac{1}{\omega_2^2}$ in case of $\{X\}_2$.

→ Relation 5 is the basis for the Matrix Iteration (MI) method.

Suppose we start with an arbitrary vector $\{u\} = \left\{ \begin{matrix} u_1 \\ u_2 \end{matrix} \right\}$ & premultiply it by $[D]$. Unless, by luck (& this is very very unlikely to happen), $\{u\}$ happens to be a modal vector, its premultiplication by $[D]$ won't give a vector proportional to $\{u\}$. But we can show that $[D]\{u\}$ will be a better approximation to $\{x\}_1$ than $\{u\}$ is. If we keep on premultiplying the resulting vectors by $[D]$, we can get a vector very, very close to $\{x\}_1$, the first modal vector.

We can prove this convergence as follows:- By the expansion theorem, we can write the arbitrary trial vector $\{u\}$ as: $\{u\} = c_1\{x\}_1 + c_2\{x\}_2$ (c_1, c_2 constants)

Premultiplying both sides by $[D]$, we get

$\{u\}_1 = [D]\{u\} = c_1[D]\{x\}_1 + c_2[D]\{x\}_2$ ⟶

or, $\{u\}_1 = \frac{c_1}{\omega_1^2}\{X\}_1 + \frac{c_2}{\omega_2^2}\{X\}_2$ $\left[\text{Since } [D]\{X\}_1 = \frac{1}{\omega_1^2}\{X\}_1 \text{ \& } [D]\{X\}_2 = \frac{1}{\omega_2^2}\{X\}_2\right.$

Premultiplying again, we get

$$\{u\}_2 = [D]\{u\}_1 = \frac{c_1}{\omega_1^2}[D]\{X\}_1 + \frac{c_2}{\omega_2^2}[D]\{X\}_2$$

$$= \frac{c_1}{(\omega_1^2)^2}\{X\}_1 + \frac{c_2}{(\omega_2^2)^2}\{X\}_2$$

→ Continuing this way, after p premultiplications, we get

$$\{u\}_p = \frac{c_1}{(\omega_1^2)^p}\{X\}_1 + \frac{c_2}{(\omega_2^2)^p}\{X\}_2 \quad \text{— (a)}$$

& after one more step,

$$\{u\}_{p+1} = \frac{c_1}{(\omega_1^2)^{p+1}}\{X\}_1 + \frac{c_2}{(\omega_2^2)^{p+1}}\{X\}_2 \quad \text{— (b)}$$

Since $\underline{\omega_1 < \omega_2}$, $\frac{1}{\omega_1^2} > \frac{1}{\omega_2^2}$,

much greater

$\frac{1}{(\omega_1^2)^2} >> \frac{1}{(\omega_2^2)^2}$ & if p is large enough,

very very much greater

$\frac{1}{(\omega_1^2)^p} >>--> \frac{1}{(\omega_2^2)^p}$; $\frac{1}{(\omega_1^2)^{p+1}} >>--> \frac{1}{(\omega_2^2)^{p+1}}$

& in (a) & (b), we may neglect the 2nd terms on RHS compared with the first terms. So, $\{u\}_p \simeq \frac{c_1}{(\omega_1^2)^p}\{X\}_1$;

$\{u\}_{p+1} \simeq \frac{c_1}{(\omega_1^2)^{p+1}}\{X\}_1$. So, $\{u\}_{p+1}$ represents $\{X\}_1$ very ~~approx~~ accurately and

the ratio $\left[\frac{c_1}{(\omega_1^2)^p}\right] \Big/ \left[\frac{c_1}{(\omega_1^2)^{p+1}}\right]$ gives ω_1^2

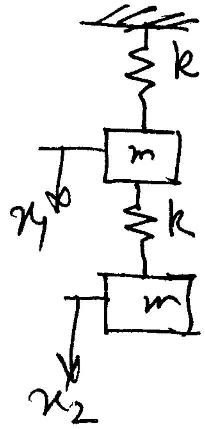
& hence ω_1 very accurately if p is large enough. →

This is matrix iteration method for ω_1 & $\{X\}_1$.

We now take up an example.

MI method is also called Power method

Ex.



Obtain ω_1 & $\{X\}_1$ approximately by the MI method. Convergence upto 3 places after decimal for $\{X\}_1$ would do.

(Note:– We have to find $[D] = [k]^{-1}[m]$. If you make a mistake here, everything done subsequently goes wrong. So, be very careful to get the correct dynamic matrix. Also, in the following, we shall implement the method discussed above in a slightly different way. At each step, we shall take a normalized trial vector discarding factors. This will result in neat expressions easier to handle)

→ We obtain the DEOM first to get $[m]$ & $[k]$.
(There are alternative ways to get $[m]$ & $[k]$ without deriving the DEOM, note)

→ We already obtained these & we know that $[m] = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$ & $[k] = \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}$.

$\det[k] = k^2$, $\operatorname{adj}[k] = \begin{bmatrix} k & k \\ k & 2k \end{bmatrix}$. So, $[k]^{-1} = \frac{1}{k^2}\begin{bmatrix} k & k \\ k & 2k \end{bmatrix}$

Check

$\therefore [D] = [k]^{-1}[m] = \frac{1}{k^2}\begin{bmatrix} k & k \\ k & 2k \end{bmatrix}\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} = \frac{1}{k^2}\begin{bmatrix} km & km \\ km & 2km \end{bmatrix} = \frac{m}{k}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

→ We start iteration by taking an arbitrary trial vector $\{u\} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$, say. (Any non-null trial vector would do like $\begin{Bmatrix} -10^4 \\ +10^5 \end{Bmatrix}$ etc. but

$\begin{Bmatrix}1\\1\end{Bmatrix}$ works well, usually. [After all, with our experience with example problems, we know that the elements of a normalized modal vector are not very far apart, like $\{X\}_1 = \begin{Bmatrix}1\\1.618\end{Bmatrix}$ in this problem. We are trying to get this $\{X\}_1$ as well as ω_1 (which is $0.618\sqrt{\frac{k}{m}}$) by the MI method.]

1st iteration: $[D]\{u\} = \frac{m}{k}\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}1\\1\end{Bmatrix} = \frac{m}{k}\begin{Bmatrix}2\\3\end{Bmatrix} = \frac{2m}{k}\begin{Bmatrix}1\\1.5\end{Bmatrix}$

We take $\begin{Bmatrix}1\\1.5\end{Bmatrix}$ as the next trial vector $\{u\}_1$. [Note that after only one iteration we got $\begin{Bmatrix}1\\1.5\end{Bmatrix}$ which is not far from $\begin{Bmatrix}1\\1.618\end{Bmatrix}$]

2nd iteration: $[D]\{u\}_1 = \frac{m}{k}\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}1\\1.5\end{Bmatrix} = \frac{m}{k}\begin{Bmatrix}2.5\\4\end{Bmatrix}$

$= \frac{2.5m}{k}\underbrace{\begin{Bmatrix}1\\1.6000\end{Bmatrix}}_{\{u\}_2}$

3rd iteration: $[D]\{u\}_2 = \frac{m}{k}\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}1\\1.6\end{Bmatrix}$

Check

$= \frac{m}{k}\begin{Bmatrix}2.6\\4.2\end{Bmatrix} = \frac{2.6m}{k}\underbrace{\begin{Bmatrix}1\\1.6154\end{Bmatrix}}_{\{u\}_3}$

(For convergence upto 3 places after decimal, keep at least 4 places after decimal in the trial vectors)

4th iteration: $[D]\{u\}_3 = \frac{m}{k}\begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}1\\1.6154\end{Bmatrix} = \frac{m}{k}\begin{Bmatrix}2.6154\\4.2308\end{Bmatrix}$

$= \frac{2.6154m}{k}\underbrace{\begin{Bmatrix}1\\1.6176\end{Bmatrix}}_{\{u\}_4}$

5th iteration: $[D]\{u\}_4 = \cancel{\frac{2.6180m}{k}\{\ \}}$

$= \frac{2.6176m}{k}\underbrace{\begin{Bmatrix}1\\1.6180\end{Bmatrix}}_{\{u\}_5}$ (Check)

6th iteration:

$[D]\{u\}_5 = \frac{2.6180m}{k}\underbrace{\begin{Bmatrix}1\\1.6180\end{Bmatrix}}_{\{u\}_6}$. Comparing $\{u\}_5$ & $\{u\}_6$, We see that convergence upto 3 places (actually upto 4 places!) has been achieved.

So, we take $\{u\}_5 \cong \{X\}_1$, ~~that is~~, $\{X\}_1$ req^d.

$= \begin{Bmatrix} 1 \\ 1.6180 \end{Bmatrix}$. Then, $\frac{2.6180m}{K} = \frac{1}{\omega_1^2}$ (Why?)

$$\Rightarrow \omega_1 = \sqrt{\frac{K}{2.6180m}} = 0.6180\sqrt{\frac{K}{m}}$$

We know that $(\omega_1)_{exact} = \cancel{0.618}\ \sqrt{\left(\frac{3-\sqrt{5}}{2}\right)\frac{K}{m}} = 0.6180\sqrt{\frac{K}{m}}$

(done a long ago) (upto 4 places)

See VA-4-Part 1

So, % error $= \frac{[\omega_1 - (\omega_1)exact]}{(\omega_1)exact} \times 100 = 0.0$!

$[(\omega_1)_{exact} = (\omega_1)_{MI}]$. (Check)

Also, $\{X\}_{1\,exact} = \begin{Bmatrix} 1 \\ 1.6181 \end{Bmatrix}$ is very close to $\{X\}_1$ obtained above. Thus, the MI method works wonderfully well, it seems. [This method fails if $\omega_1 = \omega_2$, note]

→ Once ω_1 & $\{X\}_1$ have been obtained, our next task is to get ω_2 & $\{X\}_2$.

→ Now note the following carefully. If we start with any new trial vector $\{v\} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$, the convergence will be to ω_1 & $\{X\}_1$ only, as the proof on page 3 using the expansion theorem shows. So, to achieve convergence to ω_2 & $\{X\}_2$, we must make $c_1 = 0$ in the expansion theorem

$$\{v\} = c_1\{X\}_1 + c_2\{X\}_2 \quad \text{—— (2)}$$

Now, $C_1 = \dfrac{\{X\}_1^T [m] \{v\}}{\{X\}_1^T [m] \{X\}_1}$ as we had obtained earlier. So, if the trial vector $\{v\}$ is such that $\{X\}_1^T [m] \{v\} = 0$, i.e., $\{v\}$ is orthogonal to $\{X\}_1$ w.r.t. $[m]$, then $C_1 = 0$ & the convergence of the MI method would be to ω_2 & $\{X\}_2$.

→ Thus, for an arbitrary ~~start~~ trial vector $\{v\} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$, the condition

$$\{1 \;\; 1.618\} \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = 0$$ must be satisfied. This condition can be explicitly written as: $m v_1 + 1.618 m v_2 = 0$ or,

$$1.618 v_2 = -v_1 \quad \text{or} \quad v_2 = -\frac{v_1}{1.618} \quad \text{---} (\beta)$$

Hence, after taking an arbitrary trial vector, say, $\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$, (with one sign change, from + to minus) we actually have to change the element -1 to $-\dfrac{1}{1.618}$, see the relation (β) above.

$\boxed{-\dfrac{1}{1.618} = -0.618!}$

Instead of doing this for every iteration, an algebraic trick can be applied. This is done as follows:– (This is especially useful for higher DOF systems)

$$1.618 v_2 = -v_1 \quad \text{or,} \quad -1.618 v_2 = v_1$$

$$\left.\begin{aligned} \text{or,} \quad & 0.v_1 - 1.618 v_2 = v_1 \\ \text{Also,} \quad & 0.v_1 + v_2 = v_2 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

Hence, every trial vector $\begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$, if pre-multiplied by the matrix $\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix}$, would result in convergence to ω_2 & $\{X\}_2$.

→ Instead of pre-multiplying the trial vector by $\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix}$, we can obtain a new dynamic matrix $[D]\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix} = [D_2]$ & use $[D_2]$ for iteration instead of $[D]$.

This way, every trial vector would be ~~automatically~~ automatically premultiplied by $\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix}$.

This matrix $\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix}$ is called a <u>sweeping matrix</u>, denoted by $[S_1]$.

So, $[S_1] = \begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix}$ & its postmultiplication to $[D]$ <u>sweeps away</u> (removes) the first mode.

→ Now, $[D_2] = \underbrace{\frac{m}{k}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}}_{[D]}\begin{bmatrix} 0 & -1.618 \\ 0 & 1 \end{bmatrix} = \frac{m}{k}\begin{bmatrix} 0 & -0.618 \\ 0 & 0.382 \end{bmatrix}$

→ <u>Start of iteration</u> for ω_2 & $\{X\}_2$:-

Let $\{v\} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$ be the starting trial vector. (We have incorporated <u>one</u> sign change, note)

<u>1st iteration</u> :-

$$[D_2]\{v\} = \frac{m}{k}\begin{bmatrix} 0 & -0.618 \\ 0 & 0.382 \end{bmatrix}\begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = \frac{m}{k}\begin{Bmatrix} 0.618 \\ -0.382 \end{Bmatrix} = \frac{0.618m}{k}\underbrace{\begin{Bmatrix} 1 \\ -0.6181 \end{Bmatrix}}_{\{v\}_1}$$

<u>2nd iteration</u> :-

$$[D_2]\{v\}_1 = \frac{m}{k}\begin{bmatrix} 0 & -0.618 \\ 0 & 0.382 \end{bmatrix}\begin{Bmatrix} 1 \\ -0.6181 \end{Bmatrix}$$

$$= \frac{0.3820m}{k}\underbrace{\begin{Bmatrix} 1 \\ -0.6181 \end{Bmatrix}}_{\{v\}_2}$$

Comparing, $\{v\}_2$ with $\{v\}_1$, we see that convergence is achieved!

So, $\frac{1}{\omega_2^2} = \frac{0.382m}{K} \Rightarrow \omega_2 = \sqrt{\frac{K}{0.382m}} = 1.618\sqrt{\frac{k}{m}}$.

check & we have achieved exact values!

→ Actually, one need not iterate for the highest natural frequency and corresponding modal vectors. Using the already obtained lower modes & orthogonality principle, one can obtain the highest natural frequency & corresponding modal vector. We now show this for our example problem.

→ Suppose we have ~~at~~ already obtained ω_1 & $\{X\}_1$ by the MI method (we actually did.)

So, $\{X\}_1 = \begin{Bmatrix} 1 \\ 1.6180 \end{Bmatrix}$. Let $\{X\}_2 = \begin{Bmatrix} 1 \\ \mu_2 \end{Bmatrix}$.

Then, $\{X\}_1^T [m] \{X\}_2 = 0$ (by mass orthogonality)

or, $\{1 \quad 1.618\} \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ \mu_2 \end{Bmatrix} = 0$

or, $\{m \quad 1.618m\} \begin{Bmatrix} 1 \\ \mu_2 \end{Bmatrix} = 0$

$\cancel{\Rightarrow m(1+1.618\mu_2) = 0 \Rightarrow \mu_2 = \frac{-1}{1.618}}$

$\Rightarrow m(1+1.618\mu_2) = 0 \Rightarrow \mu_2 = \frac{-1}{1.618} = -0.6180$

So, $\{X\}_2 = \begin{Bmatrix} 1 \\ -0.6180 \end{Bmatrix}$.

→ To get ω_2, note that $\omega_2^2 = \frac{\{X\}_2^T [K] \{X\}_2}{\{X\}_2^T [m] \{X\}_2}$

or, $\omega_2^2 = \frac{\{1 \; -0.618\} \begin{bmatrix} 2K & -K \\ -K & K \end{bmatrix} \begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix}}{\{1 \; -0.618\} \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} 1 \\ -0.618 \end{Bmatrix}} \Rightarrow \omega_2 =$ etc. (Complete it).

Hence, ω_2 is also obtained. →

→ We now discuss how we get the formula

$$\omega_2^2 = \frac{\{X\}_2^T [k] \{X\}_2}{\{X\}_2^T [m] \{X\}_2}.$$

We start, once again, with $[m]\{\ddot{x}\}+[k]\{x\}=\{0\}$ — ①

Assume $\{x\} = \{X\} \sin(\omega t+\phi)$ — ②

$\Rightarrow \{\ddot{x}\} = -\omega^2 \{X\} \sin(\omega t+\phi)$ — ③

Substitution of ② & ③ in ① results in:

$$-\omega^2 [m]\{X\} + [k]\{X\} = \{0\}$$

or, $-\omega^2 \{X\}^T [m]\{X\} + \{X\}^T [k]\{X\} = \{X\}^T\{0\} = \{0\}$

or, $\omega^2 \{X\}^T [m] \{X\} = \{X\}^T [k]\{X\}$

or, $$\omega^2 = \frac{\{X\}^T [k] \{X\}}{\{X\}^T [m] \{X\}}.$$

So, when $\omega = \omega_1$, $\{X\} = \{X\}_1$, we get

$$\omega_1^2 = \frac{\{X\}_1^T [k] \{X\}_1}{\{X\}_1^T [m] \{X\}_1}$$ & when $\omega=\omega_2$, $\{X\}=\{X\}_2$,

we have $$\omega_2^2 = \frac{\{X\}_2^T [k] \{X\}_2}{\{X\}_2^T [m] \{X\}_2}.$$

So, remember that $$\boxed{\omega_r^2 = \frac{\{X\}_r^T [k] \{X\}_r}{\{X\}_r^T [m] \{X\}_r}}$$

|| $r = 1, 2$ for a 2-DOF system.
|| $r = 1, 2, 3, \ldots, n$ for an n-DOF system.

(Be careful a little. From $-\omega^2[m]\{X\}+[k]\{X\}=\{0\}$
we could be tempted to write $\omega^2 [m]\{X\} = [k]\{X\}$

or, $$\omega^2 = \frac{[k]\{X\}}{[m]\{X\}}.$$ ← This is absurd,

$\underset{n\times n}{[k]}\underset{n\times 1}{\{X\}}$ since $[k]\{X\}$ is an $(n\times 1)$ vector [(2×1) vector for 2-DOF system)

& $[m]\{x\}$ is also an $(n\times1)$ vector & division of such vectors is <u>undefined</u>. So, to make a scalar, we do operations as mentioned above. $\overset{1\times n}{\{x\}^T}\overset{n\times n}{[K]}\overset{n\times1}{\{x\}}$ is 1×1 (scalar) etc.; note.

→ Now note an interesting feature of the iteration process. If you commit a calculation mistake in getting a trial vector at a certain stage of iteration process, the wrong vector will once again lead to correct convergence but naturally, number of iterations will increase usually causing a waste of time. Of course, repeated mistakes will get you nowhere!

→ We had written our eigenvalue problem as $[D]\{X\}=\frac{1}{\omega^2}\{X\}$. [See (5), page 1]
This can also be written as

$$\omega^2\{X\} = [D]^{-1}\{X\} \text{ or } \boxed{[D']\{X\}=\omega^2\{X\}}$$
$$\left([D']=([K]^{-1}[m])^{-1}=[m]^{-1}[K]\right)$$

This form of the eigenvalue problem (with eigenvalue $\lambda=\omega^2$) leads to convergence to the <u>highest</u> natural frequency (ω_2 for our example) & corresponding modal vector [$\{X\}_2$ here] rather than ω_1 & $\{X\}_1$ first. This happens

because $\omega_2^2 > \omega_1^2$, $(\omega_2^2)^2 >> (\omega_1^2)^2$ & so on.
(See proof on page 3 & proceed similarly to prove this)

[Actually, now, $\{u\} = c_1 \{X\}_1 + c_2 \{X\}_2$.

$$\text{So, } \{u\}_1 = [D'] \{u\} = c_1 [D'] \{X\}_1 + c_2 [D'] \{X\}_2$$

$$\text{or, } \{u\}_1 = c_1 \omega_1^2 \{X\}_1 + c_2 \omega_2^2 \{X\}_2$$

$$\{u\}_2 = [D'] \{u\}_1 = c_1 (\omega_1^2)^2 \{X\}_1 + c_2 (\omega_2^2)^2 \{X\}_2$$

$$- - - - - - - - -$$

$$\{u\}_p = [D'] \{u\}_{p-1} = \cancel{c_1 (\omega_1^2)^p + c_2 (\omega_2^2)}$$

$$= c_1 (\omega_1^2)^p \{X\}_1 + c_2 (\omega_2^2)^p \{X\}_2$$

Since $(\omega_2^2)^p >>> \cdots > (\omega_1^2)^p$,

$$\{u\}_p \simeq c_2 (\omega_2^2)^p \{X\}_2$$

$$\{u\}_{p+1} \cong c_2 (\omega_2^2)^{p+1} \{X\}_2 \quad \text{etc. \&}$$

convergence will be to $\{X\}_2$ & ω_2.

After this, using sweeping matrix, $\{X\}_1$ & ω_1 can be obtained (or, you may invoke the orthogonality).
(for a 2-DOF system)

So, remember the following:

→ If you are asked to obtain the lowest natural frequency & associated modal vector by MI, start with $[D] = [k]^{-1}[m]$.

→ If you are asked to get the highest natural frequency & associated modal vector by MI, start with $[D'] = [m]^{-1}[k]$.

END OF VA-4, Part 7