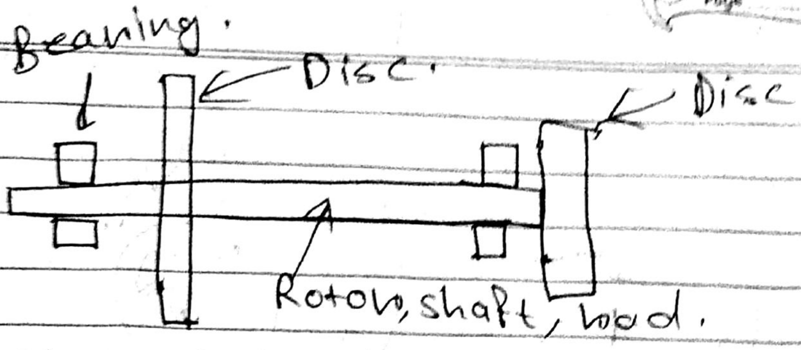


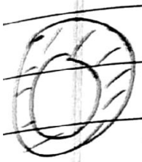
impeller vs propeller

Rolling Element Bearing. (REB)

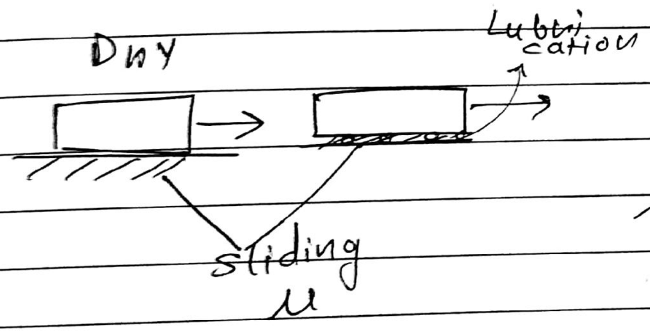
→ Non conformal (only line contact)

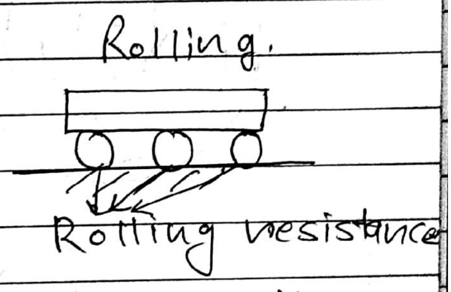
Journal Bearing

→ conformal (arc contact)
much less pressure that REB as surface area taking the load is large.



→ Load = P = $\frac{W}{\text{Projected area}}$ ↪ (DxL)

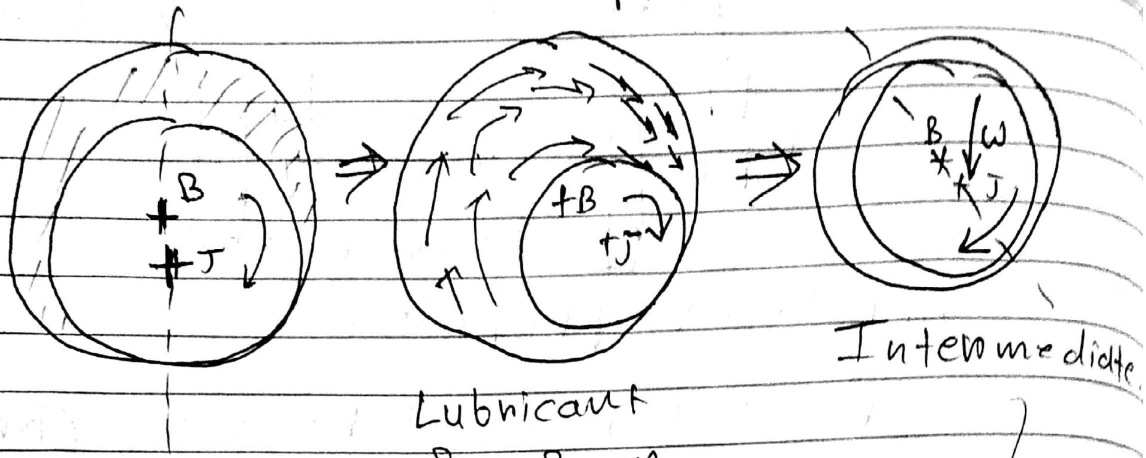




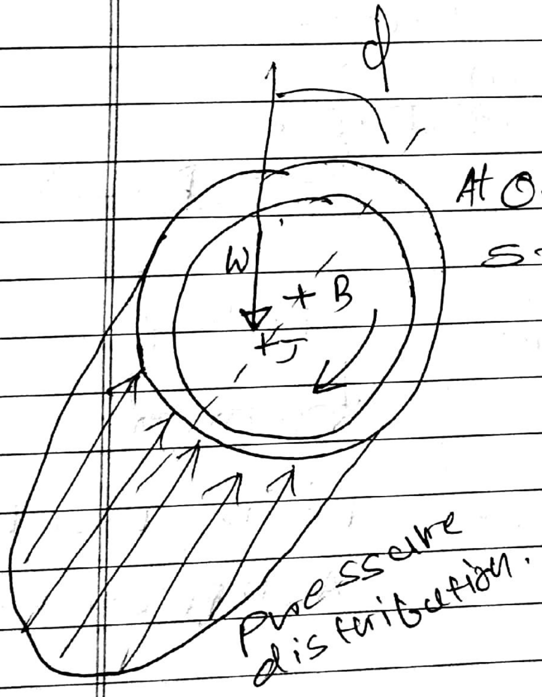
Journal Bearing → starting friction is ~~low~~ high but afterwards it becomes very less as journal loses contact with bearing.

Hydrodynamic → pressure is developed. automatically by rotating motion

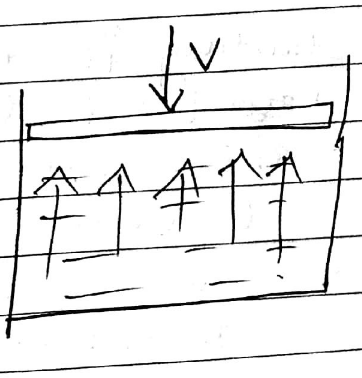
Hydrostatic → pressure is artificially provided.



Lubricant flow from diverging to converging zone and high pressure will be developed in converging zone



At Operating speed stable configuration



Squeeze-film

Hybrid bearing (Based on loading on type of lubrication)

1. Hydrostatic + Hydrodynamic
2. Axial + Thrust

Tilting pad makes more stable bearing.

Herringbone grooved journal bearing.

Textured thrust pad.

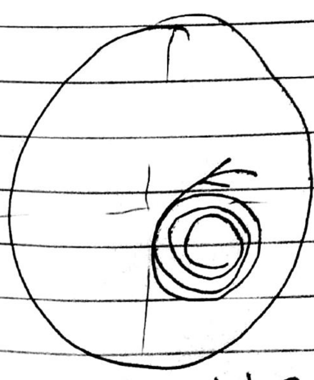
For gas/air lubrication:
requires very high RPM to ~~we~~ developed a pressure.
Surface finish needs to be very high ~~a~~ in nanometers (nm)
Surface finish on lathe is ~~[illegible]~~ 5 micron.

spinning

whirling.

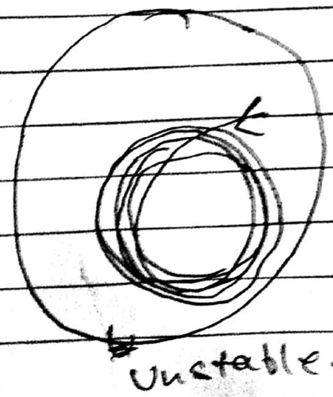
whole shaft rotates about a line other than its centerline.

Journal will spin about it's centre. and whirl about bearing centre.



Stable

marginally stable



Unstable.

Tounnal bearing - Megapascal pressure.
Rolling → Giga pascal pressure

Deep groove Ball	Angular contact ball
① Contact angle upto 5°	① contact angle upto 30°
② No axial load	② can bear axial load.

Salome-Meca

Assignment

What is natural frequency
resonating frequency
resonance
critical speed
modal frequency.

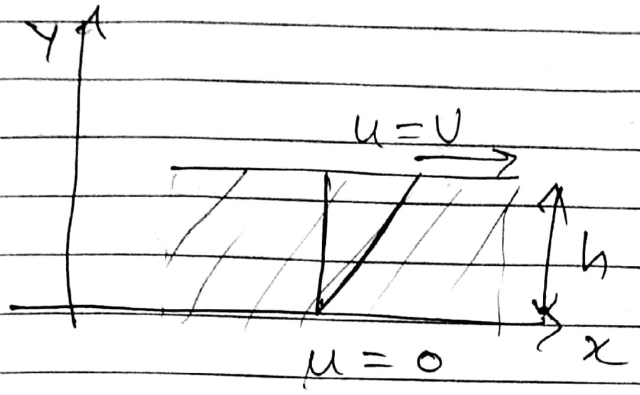
Properties of lubricant

- Viscosity
- Density

- Specific heat
- Thermal ~~Cod~~ Coductivity
- Acidity and Alkalinity
- Oxidation stability
- Flash Point → temp at which catches fire

- Foaming
- Pour point
- Demulsibility

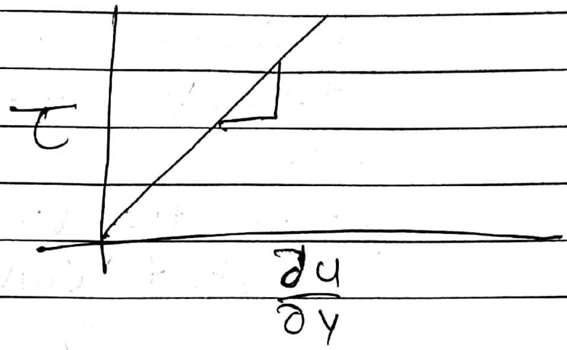
Viscocity →



For a newtonian fluid

$$\tau \propto \frac{\partial u}{\partial y}$$

shear stress — Shear Strain rate



$$\tau = \eta \frac{\partial u}{\partial y}$$

↓ Coefficient of viscocity.
absolute (dynamic)

$$\tau = \eta \frac{\partial u}{\partial y} = \eta \frac{U}{h}$$

S.I. — C.G.S

$$\eta = \frac{\tau h}{U} = \left[\frac{Pa \cdot m}{m/s}\right] = \boxed{Pa \cdot s} \qquad \frac{dyne \cdot s}{cm^2}$$

↓

CP (centi Poise) ⟵ Poise.

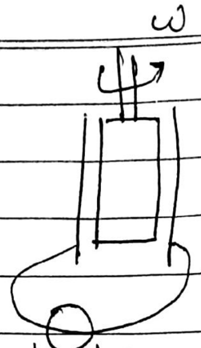
Lubricant

Kinematic Viscosity

$$\nu = \frac{\eta}{\rho} = \frac{m^2}{s} \Rightarrow \frac{cm^2}{s} = \boxed{stoke}$$

⇓

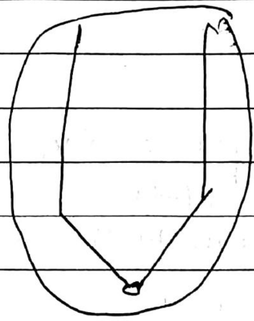
(centi Stoke) cS



Inner rotates outer which restricted by a Loadcell and measures Torque to overcome shear stress.

Kinematic Viscosity measurement

Saybolt Universal Seconds



$$\nu = \left(0.22\,t - \frac{180}{t}\right) cS$$

time required to fluid come out t

Density measurement

Mesure weight, Mesure gravity (using pendulum)

Then volume.

Effect of temp on Viscosity

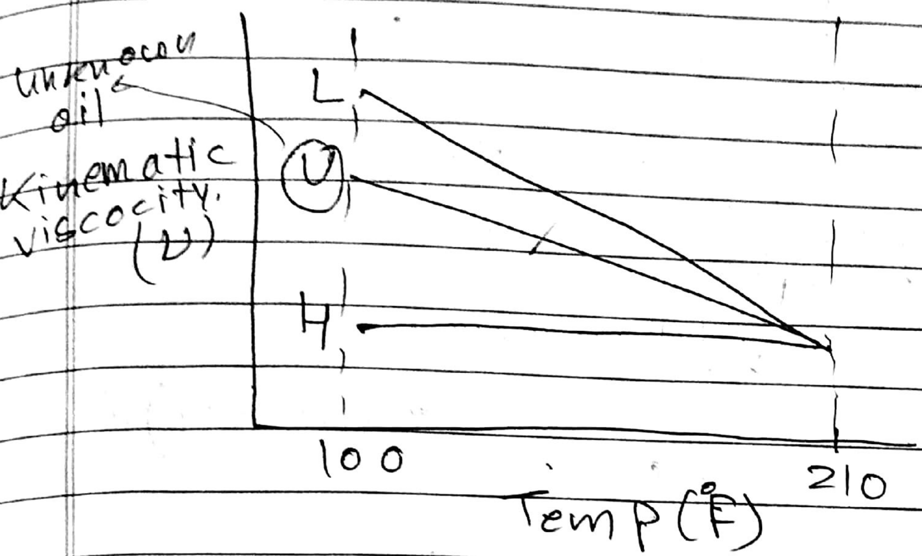
Temp ↑ Oil ↓ Gas ↑

Oil: Viscosity is because of inter molecular interaction.

Gas: Viscosity because of random movement of molecules.

ASTM

$$\log_{10}\log_{10}(\nu+0.8) = n\log_{10}T + c$$



L → Gulf Coast oil

H → Pennsylvania oil

$$\boxed{V.I|_U = \frac{U-L}{H-L} \times 100}$$

Viscosity Index.

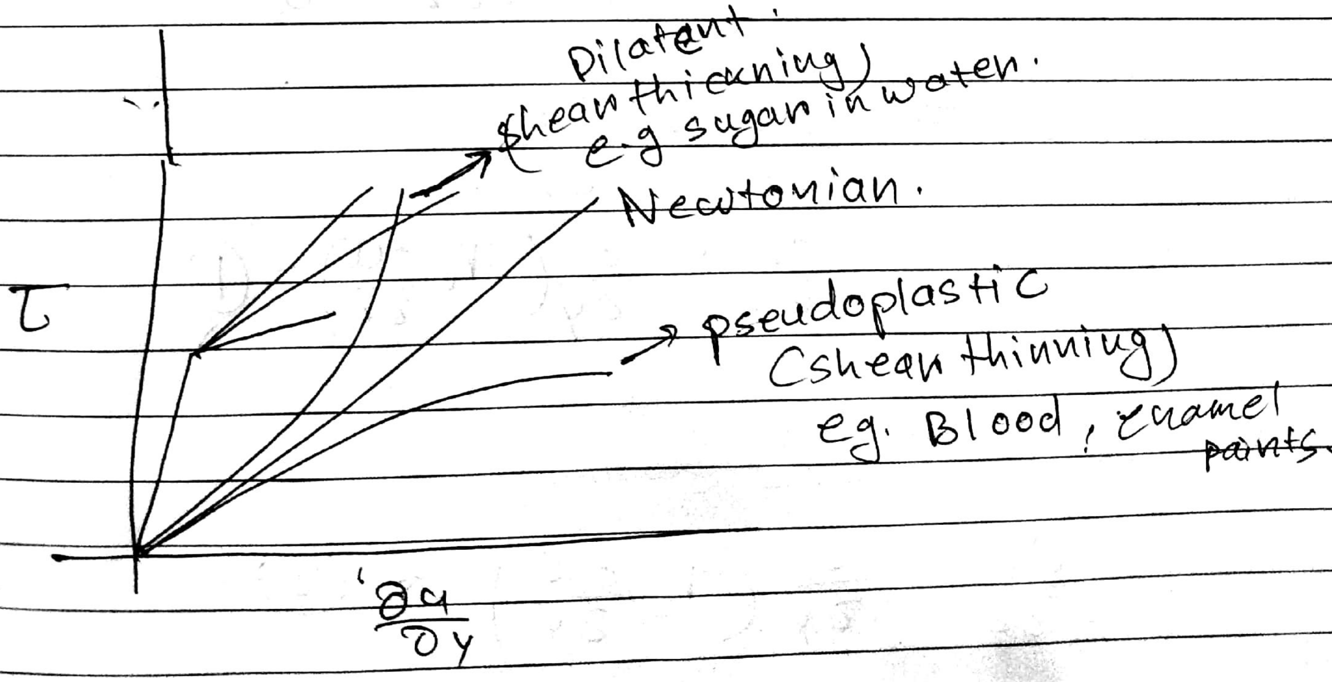
Effect of pressure on Viscosity

Pressure ↑ oil ↑ gas ↑

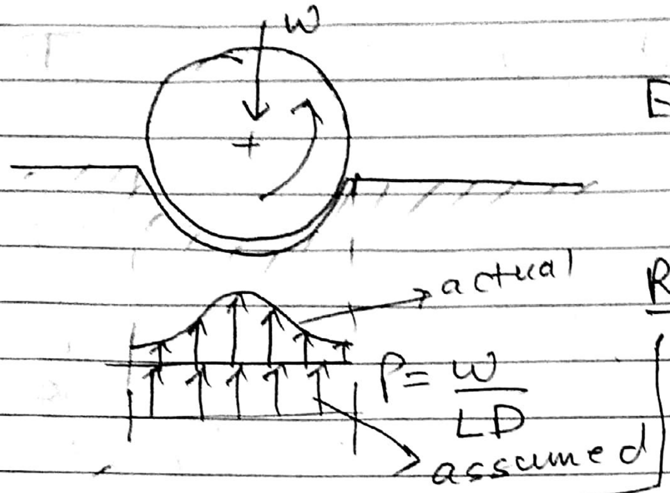
Barus Relationship $\eta = \eta_0 e^{\alpha P}$

η_0 → Viscocity at ambient temp

α → pressure viscocity coefficient

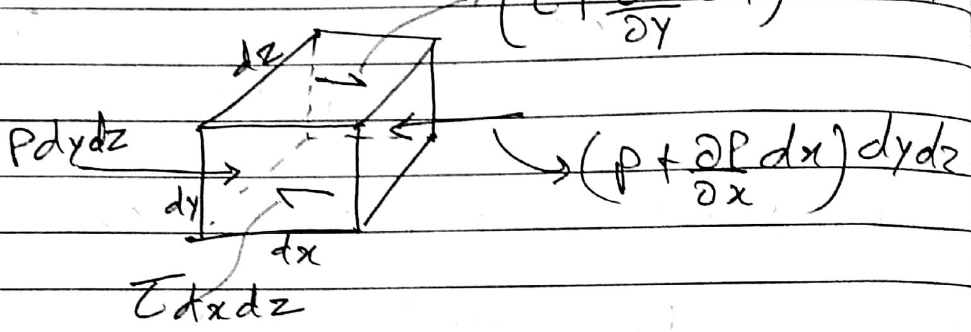


Lubrication Technology



Experiment of Tower (1883)

Reynolds equn (1886)



x-direction

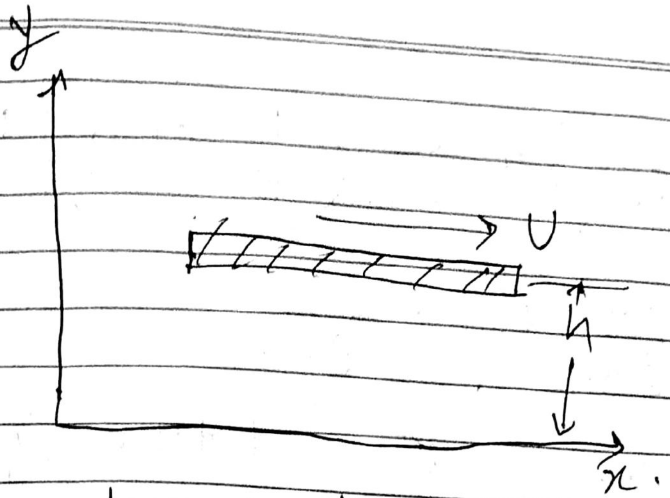
$$P\,dy\,dz + \left(\tau \frac{\partial \tau}{\partial y} dy\right) dx\,dz - \left(p + \frac{\partial p}{\partial x} dx\right) dy\,dz - \tau\, dx\,dz = 0$$

$$\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}.$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\eta \frac{\partial u}{\partial y}\right) \quad - (1)$$

z-direction

$$\frac{\partial p}{\partial z} = \frac{\partial}{\partial y}\left(\eta \frac{\partial w}{\partial y}\right) \quad - (2)$$



Boundary conditions

at $y=0$, $u=0$, $w=0$

at $y=h$, $u=U$, $w=0$

Integrating eqn (1) and (2)

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y(y-h) + \frac{U}{h} y$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y(y-h)$$

Flow in x and z directions per unit width.

$$q_x = \int_0^h u\,dy, \quad q_z = \int_0^h w\,dy$$

$$q_x = -\frac{h^3}{12\eta}\frac{\partial p}{\partial x} + \frac{Uh}{2}$$

$$q_z = -\frac{h^3}{12\eta}\frac{\partial p}{\partial z}$$

For steady flow

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial z}\right)$$ } Pressure induced flow Poiseuille term.

$$= \underbrace{6U\frac{\partial h}{\partial x}}_{\text{Velocity induced flow (couette term)}} + \underbrace{6h\frac{\partial U}{\partial x}}_{\text{Stretch term}}$$

Stretch term. (most of the time is zero as U is cons term)

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial z}\right)=6U\frac{\partial h}{\partial x}}$$

→ steady state Reynolds eq^n.

Navien-stokes equation,

① $$\rho\frac{Du}{dt} = \rho X - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\eta\left[2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)\right]\right)$$

$$+\frac{\partial}{\partial y}\left[\eta\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right]$$

② $\rho\frac{Dv}{dt} = - - - - -$ ③ $\rho\frac{Dw}{dt} = - -$

$$\frac{Du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial u}{\partial t}$$

④ $$\frac{\partial \rho}{dt} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

continuity equ^n

Navien stokes → Reynolds Assumptions

Assumptions of thin film lubrication.

① Inertia and body forces ~~neg~~ are negligible as compared to pressure and viscous terms

② The film is thin, so there is no variation of pressure across the fluid film.

③ There is no slip across the fluid boundaries.

④ No external forces act on the film

⑤ The flow is viscous and laminar.

⑥ The derivatives of u and w w.r.t y are much larger that other derivatives of velocity component.

⑦ height of the fluid film should be very small compared to the length of contact $(h/l) \leq 10^{-3}$

$+\frac{\partial w}{\partial z}$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\eta \frac{\partial u}{\partial y}\right)$$

$$0 = -\frac{\partial p}{\partial y}$$

Momentum

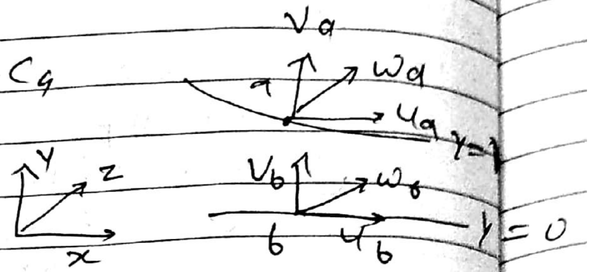
$$0 = -\frac{\partial p}{\partial z} + \frac{\partial}{\partial y}\left(\eta \frac{\partial w}{\partial y}\right)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Continuity.

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y^2 + c_1 y + c_3$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y^2 + c_2 y + c_4$$



at $y = 0$

$u = u_b,\ w = w_b$

at $y = h$

$u = u_a,\ w = w_a$

$$u = \frac{1}{2\eta}\frac{\partial p}{\partial x} y(y-h) + \left(\frac{h-y}{h}\right)u_b + \frac{y}{h} u_a$$

$$w = \frac{1}{2\eta}\frac{\partial p}{\partial z} y(y-h) + \left(\frac{h-y}{h}\right)w_b + \frac{y}{h} w_a$$

$$\frac{\partial}{\partial y}(\rho v) = -\frac{1}{2}\left\{\frac{\partial}{\partial x}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial x} y(y-h)\right] + \frac{\partial}{\partial z}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial z} y(y-h)\right]\right\} - \frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-y}{h}\right)u_b + \frac{y}{h}u_a\right\}\right] - \frac{\partial}{\partial z}\left[\rho\left\{\left(\frac{h-y}{h}\right)w_b + \frac{y}{h}w_a\right\}\right] - \frac{\partial \rho}{\partial t}$$

$$\rho(v_a - v_b) = -\frac{1}{2}\left\{\int_0^h \frac{\partial}{\partial x}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial x} y(y-h)\right] dy + \int_0^h \frac{\partial}{\partial z}\left[\frac{\rho}{\eta}\frac{\partial p}{\partial z} y(y-h)\right] dy\right\} - \int_0^h \frac{\partial}{\partial x}\left[\rho\left\{\left(\frac{h-y}{h}\right)u_b + \frac{y}{h}u_a\right\}\right] dy - \int_0^h \frac{\partial}{\partial z}\left(\rho\left\{\left(\frac{h-y}{h}\right)w_b + \frac{y}{h}w_a\right\}\right) dy$$

$$-h\frac{\partial \rho}{\partial t}.$$

$=0$

Leibnitz rule.

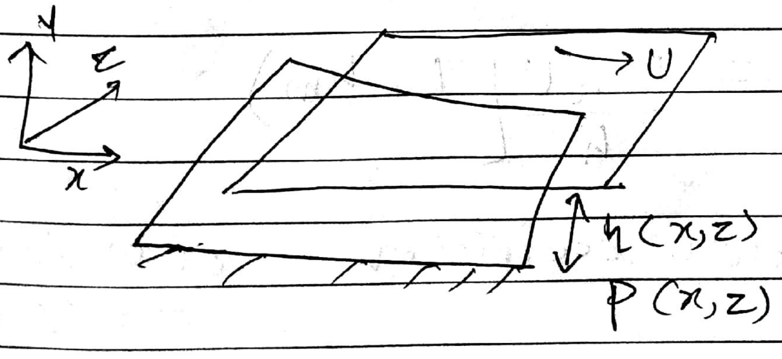
$$\int_0^h \frac{\partial}{\partial x} f(x,y,z)\,dz = \frac{\partial}{\partial x}\int_0^h f(x,y,z)\,dz - f(x,y,h)\frac{\partial h}{\partial x}.$$

Generalized Reynolds Equation

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\eta}\frac{\partial P}{\partial z}\right)}_{\text{Poiseuille Term (Pressure induced)}} = \underbrace{\left(\frac{u_a+u_b}{2}\right)\frac{\partial}{\partial x}(\rho h) + \left(\frac{w_a+w_b}{2}\right)\frac{\partial}{\partial z}(\rho h)}_{\text{Couette term (velocity induced)}} + \underbrace{\frac{\partial}{\partial t}(\rho h)}_{\text{Squeeze term}}$$

If density variation can be neglected ~~in a~~ during bearing operation and assuming boundary velocities in only one direction the above equation reduces to:
(and steady state is assumed)

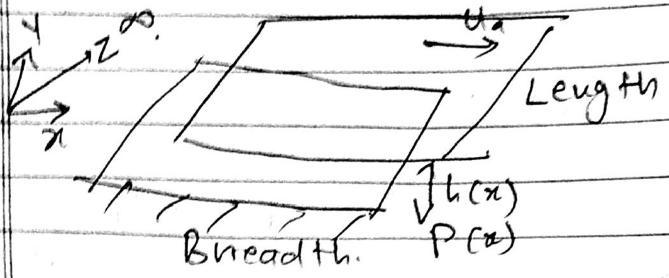
$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial P}{\partial z}\right) = \underbrace{U}_{\left(\frac{u_a+u_b}{2}\right)}\frac{\partial h}{\partial x}}$$



classmate

Date ______
Page ______

~~the~~ Assuming ~~zo~~ no-variation is z-direction i.e. in z direction ~~is~~ is assumed to be infinite. Length



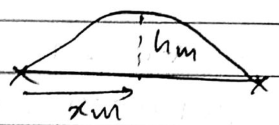
$L > B$

$\frac{L}{B} > 1.5$ } for one-D analysis.

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\eta}\frac{\partial P}{\partial x}\right) = 12U\frac{\partial h}{\partial x}.$$

$$\frac{\partial P}{\partial x} = \frac{12\eta U}{h^2} + \frac{\eta}{h^3}C_1$$

At both open ends, pressure will be atmospheric thus gauge pressure at both the ends will be zero.



Thus pressure will reach a maximum value in between the two ends.

$$\therefore \frac{\partial P}{\partial x} = 0, \quad x = x_m, \; h = h_m$$

$$0 = \frac{12\eta U}{h_m^2} + \frac{\eta}{h_m^3}C_1$$

$$C_1 = -12Uh_m$$

$$\frac{\partial P}{\partial x} = \frac{12\eta U}{h^3}\left[h - h_m\right)$$

Friction coefficient is of order 10^{-2}
$\rightarrow (0.02)$

Force (load)
friction
flow

Flow of oil between plates along their length will carry heat generated because of friction.
Depending on flow output flow input needs to be controlled.

$$u = \frac{1}{2\eta}\frac{\partial P}{\partial x} y(y-h) + \left(\frac{h-y}{h}\right) u_b + \frac{y}{h} u_a$$

$$w = \cancel{\frac{1}{2\eta}\frac{\partial P}{\partial x} y(y-h)} + \frac{1}{2\eta}\frac{\partial P}{\partial z} y(y-h)$$

$(w_a = w_b = 0)$

$$q_x = \int_0^h u\,dy, \quad q_z = \int_0^h w\,dy.$$

$$q_x = -\frac{h^3}{12\eta}\frac{\partial P}{\partial x} + \frac{h}{2}(u_a + u_b)$$

$$q_z = -\frac{h^3}{12\eta}\frac{\partial P}{\partial z}$$

$$Q_z = \int_0^B q_z\,dx.$$

Shear stress

$$\tau_x = \eta\frac{\partial u}{\partial y}, \quad \tau_z = \eta\frac{\partial w}{\partial y}$$

✓ $$\tau_x = \frac{1}{2}\frac{dP}{dx}(2y-h) + \frac{\eta}{h}(u_a - u_b)$$

× $$\tau_z = \frac{1}{2}\frac{\partial P}{\partial z}(2y-h)$$

z-direction friction force ~~is not~~ does not come into picture as the upper surface is moving only in x-direction.

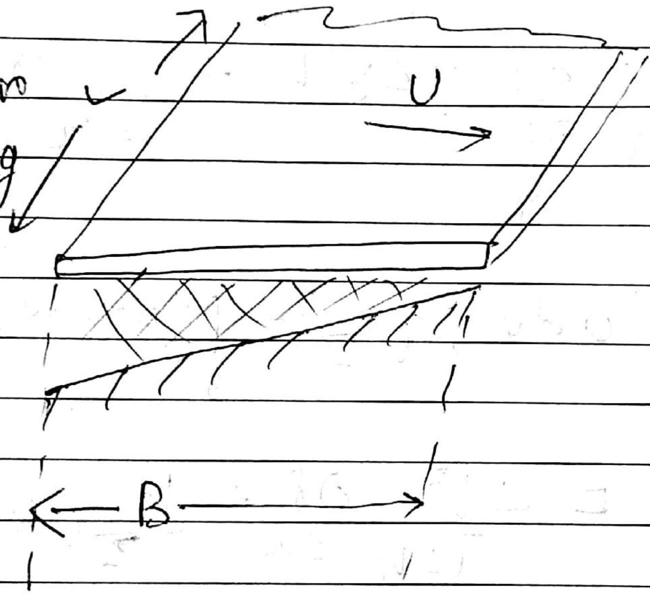
Friction Force

$$F = \iint \tau_x \, dA \, (dx\,dz)$$

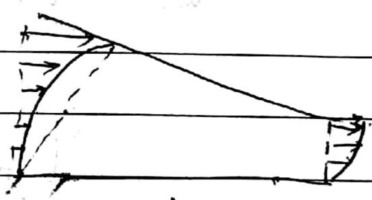
$\tau_x|_{y=0}$ and $\tau_x|_{y=h}$

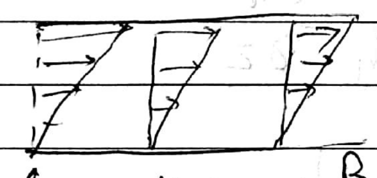
$$F\Big|_{y=0,h} = \int_0^z \int_0^x \left[\mp \frac{h}{2} \frac{\partial P}{\partial x} + \frac{\mu}{h}(u_a - u_b) \right] dx\,dz$$

Slider Bearing



converging, thus pressure is developed.

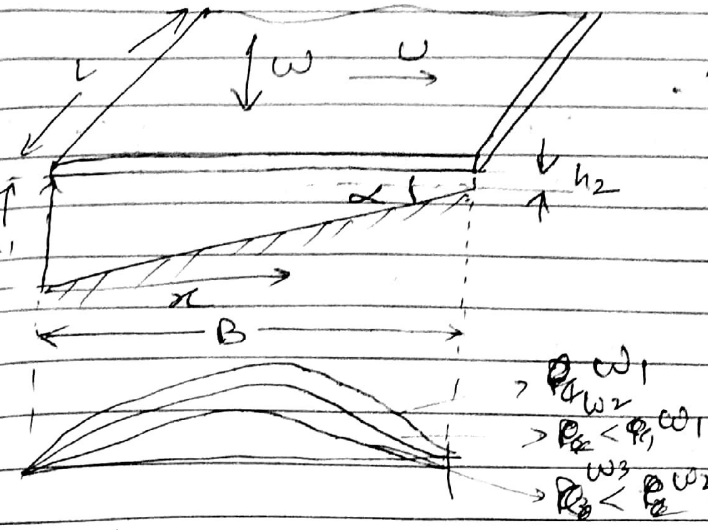




Parallel

Profile ~~Pressure~~ will be linear in-between two ends where $\frac{\partial P}{\partial x} = 0$

Slider Bearing



$L \to \infty$

∴ One-dimentional Analysis.

Applied Load controls the pressure distribution and gap between the plates.

Here we are doing reverse analysis where we assume ~~sem~~ certain geometry (gap) and find out the load carrying capacity.

$$\frac{\partial p}{\partial x} = \frac{6\eta U}{h^3}(h - h_m)$$

$$\tan\alpha = \frac{h_1 - h_2}{B} \qquad \alpha = \frac{h_1 - h_2}{B} \ (\alpha \text{ is small})$$

$$h = h_1 - \alpha x$$

$$\int dp = \int_0^B \frac{6\eta U}{(h_1 - \alpha x)^3}(h_1 - \alpha x - h_m)\,dx$$

$$p = 6\eta U\left[\frac{1}{\alpha(h_1 - \alpha x)} - \frac{h_m}{2\alpha(h_1 - \alpha x)^2}\right] + C_1$$

$$p = 0 \text{ at } x = 0 \text{ and } B$$

$$0 = 6\eta U\left[\frac{1}{\alpha h_1} - \frac{h_m}{2\alpha h_1}\right] + C_1$$

$$0 = 6\eta U\left[\frac{1}{\alpha(h_1 - \alpha B)} - \frac{h_m}{2\alpha(h_1 - \alpha B)}\right] + C_1$$

$$\Rightarrow h_m = \frac{2n h_2}{n+1} \qquad C_1 = -\frac{6\eta U}{\alpha h_2}\left(\frac{1}{n+1}\right)$$

$$n = h_1/h_2 \ (\text{attitude})$$

∴ Pressure distribution is.

$$P = \frac{6\eta U B}{h_2^2}\left(\frac{1}{n^2-1}\right)\left[\left(\frac{h_1}{h_1-\alpha x}-1\right)\left(1-\frac{h_2}{h_1-\alpha x}\right)\right]$$

Load carrying capacity.

$$W = L\int_0^B p\,dx$$

$$= \frac{6\eta U B^2 L}{h_2^2}\frac{1}{(n-1)^2}\left(\ln(n) - \frac{2(n-1)}{n+1}\right)$$

Shear stress

$$\tau = \frac{h}{2}\frac{\partial P}{\partial x} + \frac{\eta U}{h}.$$

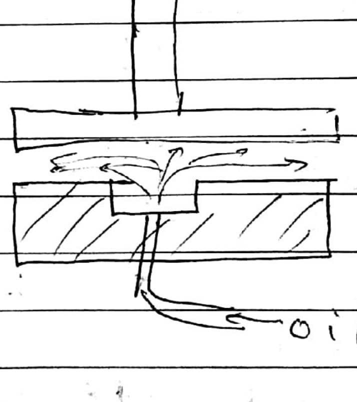
Friction Force

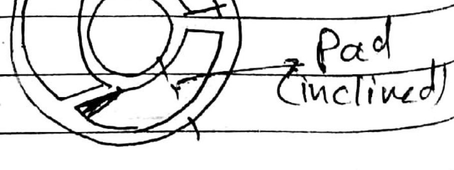
$$F = L\int_0^B \tau\,dx$$

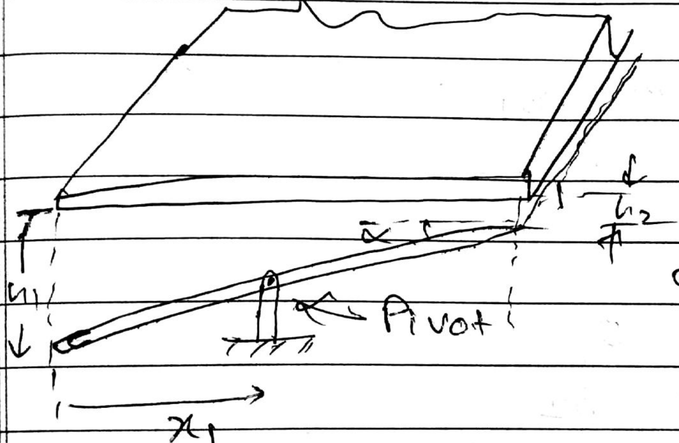
$$F = \frac{\eta U B L}{h_2}\left(\frac{1}{n-1}\right)\left[4\ln(n) - \frac{6(n-1)}{n+1}\right]$$

Coefficient of friction

$$\mu = \frac{F}{W} = \frac{h_2}{B}\left[\frac{2(n^2-1)\ln(n) - 3(n-1)^2}{3(n+1)\ln(n) - 6(n-1)}\right]$$







α can change if the bottom plate is pivoted.

~~so~~. Centre of pressure falls on pivot so as to balance the bottom plate about it.

Position of pivot (x_1)

$$x_1 = \frac{L}{W}\int_0^B p\,dx$$

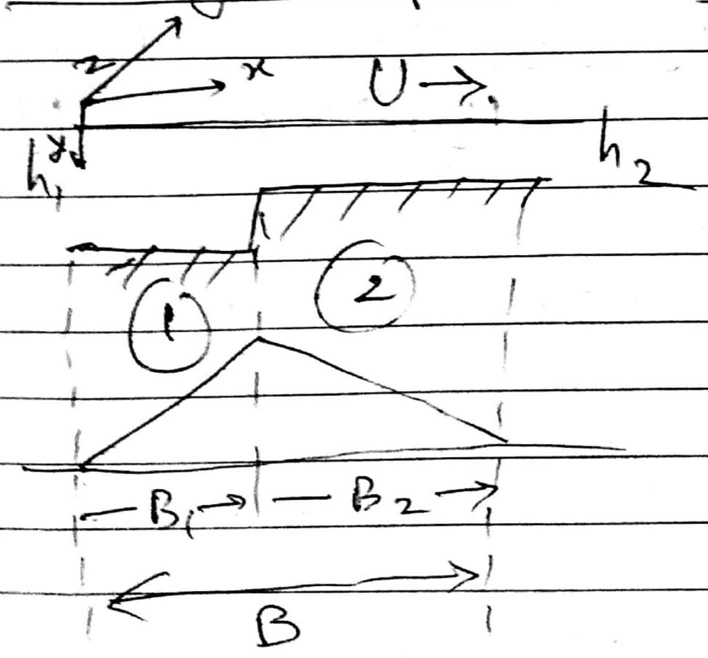
$$\frac{x_1}{B} = \frac{n(2+n)\ln(n) - (n-1)[2.5(n-1)+3]}{(n^2-1)(\ln(n)) - 2(n-1)^2}$$

Max Load

$$\frac{dW}{dn} = 0 \qquad n = 2.18$$

$$\boxed{\frac{x_1}{B} = \frac{5}{9}} \leftarrow \text{For optimum Load.}$$

Rayleigh Step Bearing



$$\frac{\partial P}{\partial x} = 6\eta U\left[\frac{h-h_m}{h^3}\right]$$

$$P = 6\eta U\left(\frac{h-h_m}{h^3}\right)x + C_1$$

B.C.S w.r.t. 1

$P = 0$ at $x = 0$

$P = P_c$ at $x = B_1$

P

$$C_1 = 0, \; P_c = 6\eta U \left[\frac{h_1 - h_m}{h_1^3}\right] B_1 \quad \text{--- (1)}$$

B.C.S for (2)

$P = 0$ at $x = 0$

$P = P_c$ at $x = B_2$

$$C_1 = 0 \quad P_c = 6\eta U \left(\frac{h_m - h_2}{h_2^3}\right) B_2 \quad \text{--- (2)}$$

equating (1) and (2)

$$\boxed{h_m = \frac{h_1 h_2 (B_1 h_2^2 + B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3}}$$

$$\frac{2(4+1)}{8+\cancel{8}}$$

$$\frac{10}{\cancel{16}}$$

$$P\big|_{(1)} = \frac{6\eta U}{h_1^2}\left[1 - \frac{h_2 (B_1 h_2^2 + B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3}\right] x$$

$x \rightarrow$ from Left

$$P\big|_{(2)} = \frac{6\eta U}{h_2^2}\left[\frac{h_1 (B_1 h_2^2 + B_2 h_1^2)}{B_1 h_2^3 + B_2 h_1^3} - 1\right] x$$

$x \downarrow$ from Right

Load carrying capacity

$$W = L\int_0^{B} P_1\, dx + L\int_0^{B_2} P_2\, dx.$$

$$W = \frac{3\eta U L B B_2 (B - B_2)(n-1)}{(B_2 n^3 + B - B_2)\, h_2^2}$$

$$n = \frac{h_1}{h_2}$$

Optimum Load carrying capacity

Variables: B_2, n (should be adjusted)

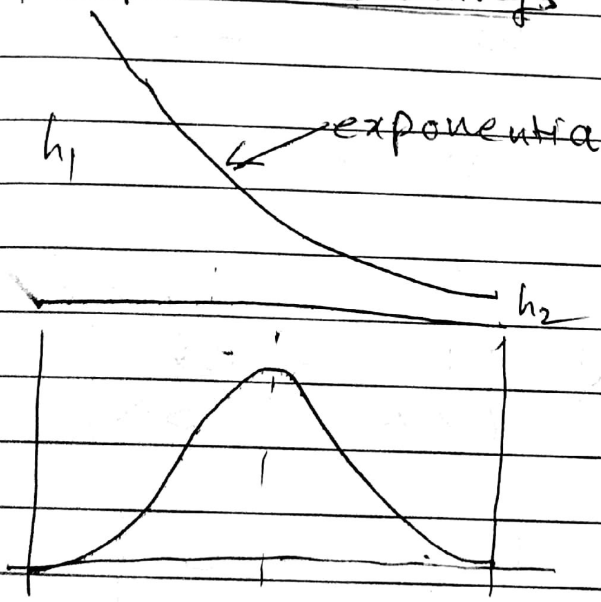
$$\frac{\partial W}{\partial B_2} = 0 \text{ and } \frac{\partial W}{\partial n} = 0.$$

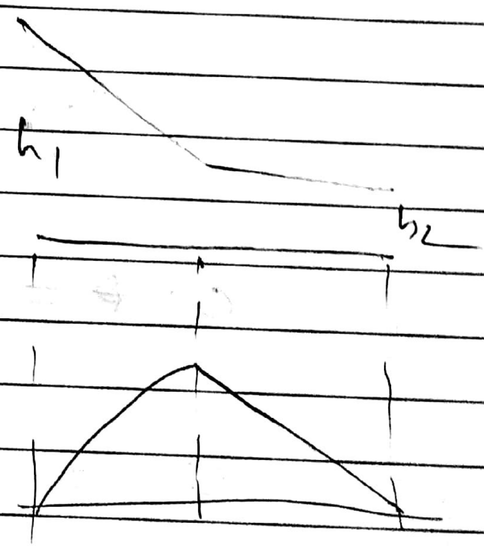
$$B_2 = \frac{B}{2n^3 - 3n^2 + 1} \qquad \boxed{n = 1.866}$$

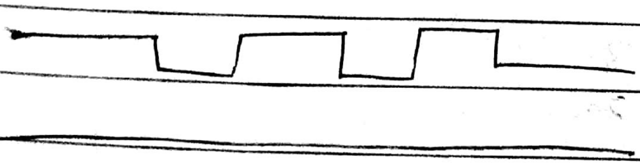
$$\boxed{\frac{B_1}{B_2} = 2.549}$$

HW Find friction and coefficient of friction. Centre of pressure etc.

Composite Bearings

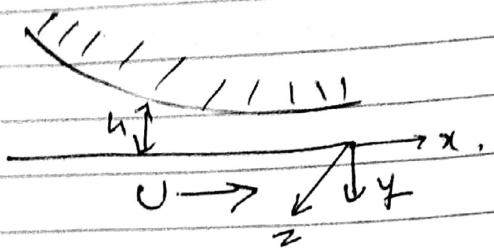






h_1, h_2 are in order of microns (30, 40, 50 etc)

Exponential film



$$\frac{\partial P}{\partial x} = 6\eta U\left[\frac{h-h_m}{h^3}\right]$$

$$h = h_o e^{-\alpha x}$$

$$\frac{\partial P}{\partial x} = 6\eta U\left(\frac{h_o e^{-\alpha x} - h_m}{h_o^3 e^{-3\alpha x}}\right]$$

$$P = 6\eta U\left[\frac{h_o e^{2\alpha x}}{2\alpha h_o^3} - \frac{h_m e^{3\alpha x}}{h_o^3 3\alpha}\right] + C_1$$

$$P = 6\eta U\left[\frac{\cancel{h_o} e^{2\alpha x}}{2\alpha h_o^2} - \frac{h_o e^{-\alpha x_m} e^{3\alpha x}}{h_o^3 3\alpha}\right] + C_1$$

$$P = \frac{6\eta U}{h_o^2}\left[\frac{e^{2\alpha x}}{2\alpha} - \frac{e^{-\alpha x_m} e^{3\alpha x}}{3\alpha}\right] + C_1$$

at $x = 0$ $P = 0$

at $x = -\infty$ $P = 0$

$$0 = \frac{6\eta U}{h_o^2}\left[\frac{1}{2\alpha} - \frac{e^{-\alpha x_m}}{3\alpha}\right] + C_1$$

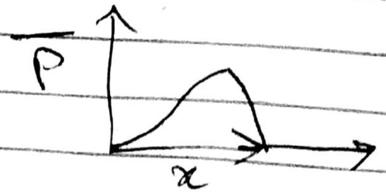
$$0 = \frac{6\eta U}{h_o^2}\left[0 - 0\right] + C_1$$

$$C_1 = 0$$

$$e^{-\alpha x_m} = \frac{3}{2}$$

$$\boxed{P = \frac{3\eta U}{\alpha h_o^2}\left(e^{2\alpha x} - e^{3\alpha x}\right)}$$

$$\bar{P} = \frac{P \alpha h_0^2}{3 \eta U} = e^{2\alpha x} - e^{3\alpha x}$$ ← Non dimentional pressure.



Nondimensional solution independent of viscocity, plate velocity, initial film thickness(h_0)

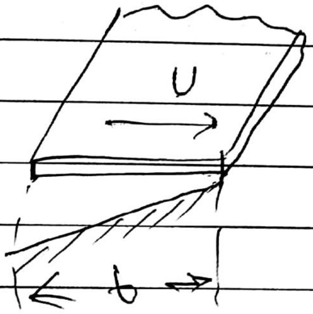
Load carrying capacity of exponential film:

$$W = L \int_{-\infty}^{0} p\, dx.$$

$$W = \frac{\eta U L}{2 \alpha h_0^2}$$

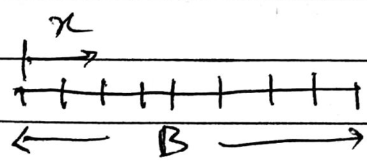
$$\boxed{\bar{W} = \frac{W h_0^2 \alpha}{\eta U L} = \frac{1}{2}}$$ Non-dimensional weight carrying capacity.

Numerical solution of slider bearings. (FDM)



$$\frac{\partial}{\partial x}\left[h^3 \frac{\partial P}{\partial x} \right) = 6 \eta U \frac{\partial h}{\partial x}.$$

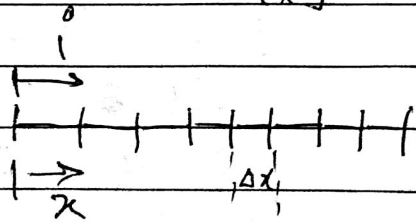
$$\frac{\partial^2 P}{\partial x^2} = \left(\frac{6 \eta u}{h^3} - \frac{3}{h} \frac{\partial P}{\partial x} \right) \frac{\partial h}{\partial x}.$$



No. of nodes $N_x = 9$

$x = [\qquad\qquad]_{1 \times 9}$ $\quad h = h_1 - \alpha x.$

$h = [\qquad\qquad]_{1 \times 9}$ $\quad$ (CDM)



$$\left.\frac{\partial P}{\partial x}\right|_i = \frac{P_{i+1} - P_{i-1}}{2 \Delta x}.$$

$$\left.\frac{\partial^2 P}{\partial x^2}\right|_i = \frac{\left.\frac{\partial P}{\partial x}\right|_{i+1/2} - \left.\frac{\partial P}{\partial x}\right|_{i-1/2}}{\Delta x}$$

$$= \frac{\frac{P_{i+1} - P_i}{\Delta x} - \frac{P_i - P_{i-1}}{\Delta x}}{\Delta x}$$

$$= \frac{P_{i+1} + P_{i-1} - 2P_i}{(\Delta x)^2}$$

$$\frac{P_{i+1} - 2P_i + P_{i-1}}{\Delta x^2} = \left(\frac{6\eta U}{h_i^3} - \frac{3}{h_i}\left(\frac{P_{i+1} - P_{i-1}}{2\Delta x}\right)\right)\left(\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right)$$

$$\frac{P_{i+1}}{\Delta x^2} - \frac{2P_i}{\Delta x^2} + \frac{P_{i-1}}{\Delta x^2} = \frac{6\eta U}{h_i^3}\left(\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right) - \frac{3}{h_i}\left[\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right](P_{i+1} - P_{i-1})$$

$$\frac{2P_i}{\Delta x^2} = P_{i+1}\left[\frac{1}{\Delta x^2} + \frac{3}{h_i}\left[\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right]\right] + P_{i-1}\left[\frac{1}{\Delta x^2} - \frac{3}{h_i}\left[\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right]\right] - \frac{6\eta U}{h_i^3}\left[\frac{h_{i+1} - h_{i-1}}{2\Delta x}\right]$$

while conv $> 10^{-5}$

for $i = 2, nx-1$

$E = RHS - P(i) \rightarrow P(i) = P(i) + 1.25\,E$

end

$conv = \frac{\Sigma P - \Sigma P_{old}}{\Sigma P_{old}}$

$P_{old} = P$

end

0.8 − 1.3

< 1 underrelaxation

> 1 over

Homework.

Take, (B, L, U, η, α, h_1, h_2) values

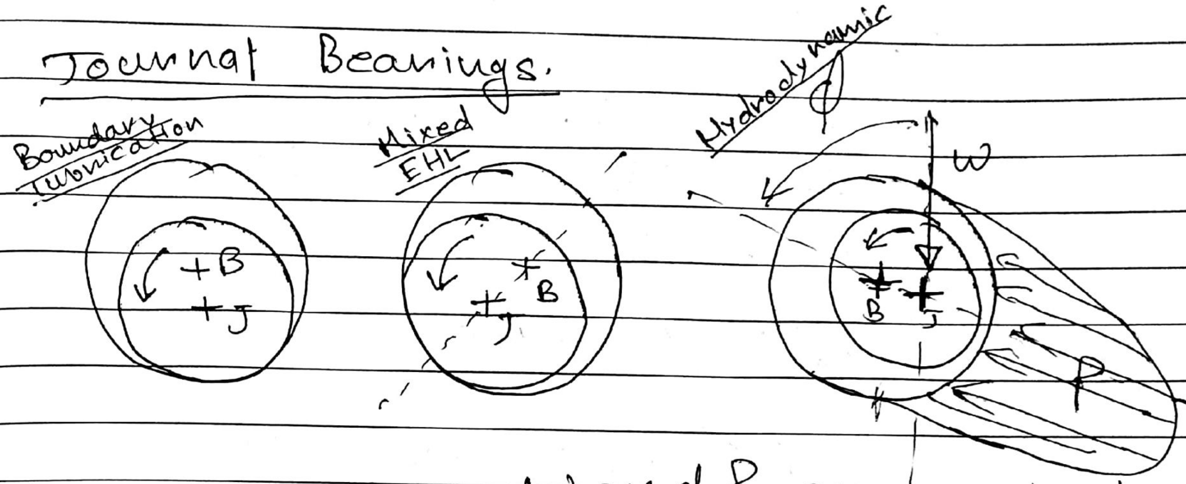
Solve using MATLAB

Plot Pressure vs B,

Calculate $W = L\int_0^B p\,dx$.

plot W vs n (h_1/h_2)
(1 to 2)

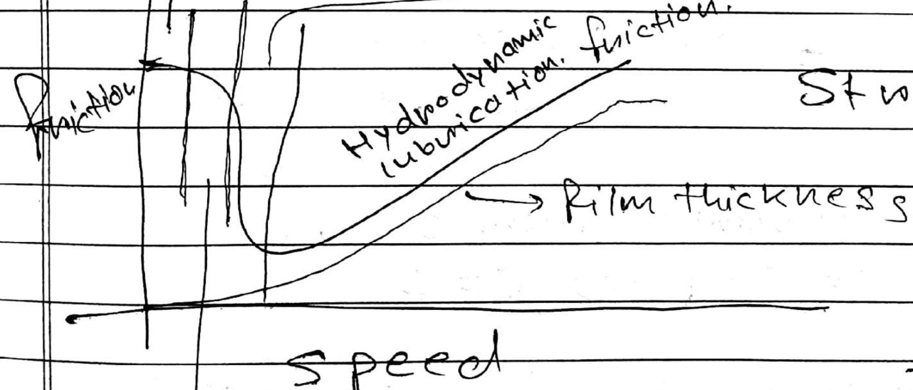
Journal Bearings.



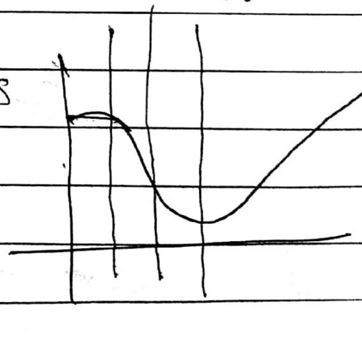
W and P are not along same line still equilibrium is achieved because of friction acting on the surface of journal.

Boundary lubrication (gap is less than a nanometer as the journal is in contact with bearing)

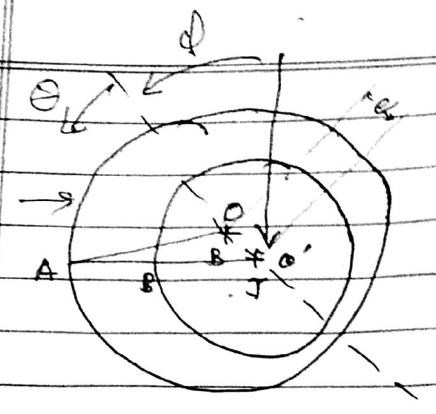
→ Elasto Hydrodynamic lubrication.



Stribeck Curve.



Mixed lubrication
(contact will be there as well as ~~of~~ fluid film in microns)



Indefinitely long

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial P}{\partial x}\right) = 6\eta U\frac{\partial h}{\partial x}.$$

h =

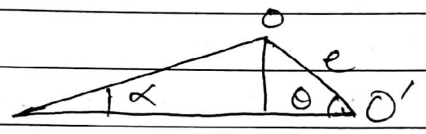
$x = R\theta \quad dx = R\,d\theta.$

$$\frac{\partial}{\partial \theta}\left(h^3\frac{\partial P}{\partial \theta}\right) = 6\eta U R\frac{\partial h}{\partial \theta}.$$

$\because U = \omega R.$

$h(\theta) = AB = AO' - R$

$$AO' = AO\cos\alpha + e\cos\theta$$
$$= R + \underbrace{c}_{\text{bearing clearance.}} + e\cos\theta.$$



$h = c + e\cos\theta.$

$= c(1 + \underbrace{\epsilon}_{\text{eccentricity ratio.}}\cos\theta)$

$\epsilon \rightarrow$ varies from 0 to 1

when $\epsilon = 1 \Rightarrow c = e$

$c \rightarrow$ eccentricity when ~~bearing~~ journal is in contact with bearing.

at h_m $\partial P/\partial\theta = 0$

$$\frac{\partial P}{\partial \theta} = 6\eta U R\,\frac{h - h_m}{h^3}$$

$$P = \frac{6\eta U R}{C^2}\left[\underbrace{\int \frac{d\theta}{(1+\epsilon\cos\theta)^2}}_{I_2} - \frac{h_m}{C}\underbrace{\int \frac{d\theta}{(1+\epsilon\cos\theta)^3}}_{I_3}\right] + C_1$$

$1 + \epsilon^2\cos^2\theta + 2\epsilon\cos\theta$.

Sommerfield substitution for I_2, I_3

$$\cos\gamma = \frac{\epsilon + \cos\theta}{1 + \epsilon\cos\theta}.$$

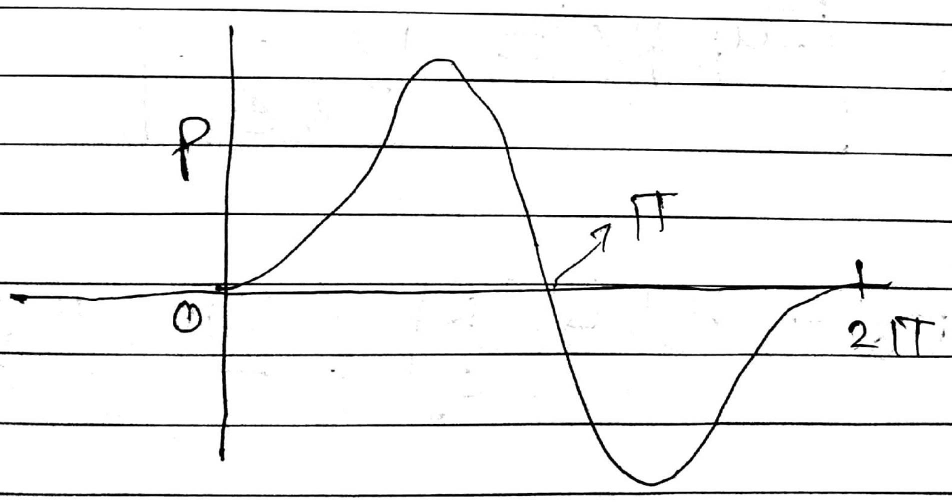
$$\sin\theta = \frac{(1-\epsilon^2)^{1/2}\sin\gamma}{1+\epsilon\cos\gamma}.$$

$$\cos\theta = \frac{\cos\gamma - \epsilon}{1 - \epsilon\cos\gamma} \qquad d\theta = \frac{(1-\epsilon^2)^{1/2}\,d\gamma}{1-\epsilon\cos\gamma}$$

$P = 0$ at $\theta = 0$ | $C_1 = 0$

$P = 0$ at $\theta = 2\pi$ | $h_m = \dfrac{2C(1-\epsilon^2)}{2+\epsilon^2}$

$$P = \frac{6\eta U R\epsilon}{C^2}\,\frac{(2+\epsilon\cos\theta)\sin\theta}{(2+\epsilon^2)(1+\epsilon\cos\theta)^2}$$



$\phi \rightarrow$ Attitude Angle.

$$\omega_r = \omega\cos\phi = -L\int_0^{2\pi} p\cos\theta \, R\, d\theta.$$

$$\omega_\phi = \omega\sin\phi = L\int_0^{2\pi} p\sin\theta \, R\, d\theta.$$

$$\omega_r = \omega\cos\phi = 0 \Rightarrow \phi = \pi/2.$$

$$\omega_\phi = \frac{12\eta\pi U L \epsilon (R/C)^2}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}} = \omega$$

shear stress

$$\tau_j = \frac{\eta U}{h} + \frac{h}{2R}\frac{dp}{d\theta}.$$

$$F_j = \int_0^{2\pi} \tau_j \, L R \, d\theta.$$

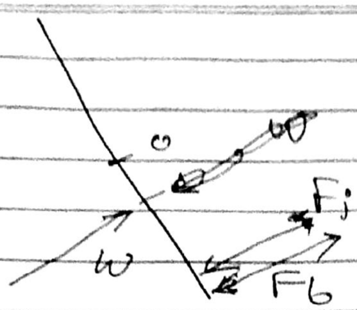
$$F_j = \eta U L\left(\frac{R}{C}\right)\frac{4\pi(1+2\epsilon^2)}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$

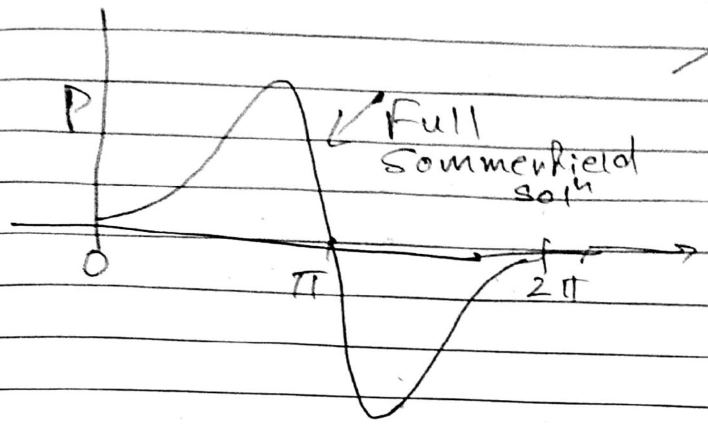
$$\mu_j = \frac{F_j}{\omega} = \frac{C}{R}\left(\frac{1+2\epsilon^2}{3\epsilon}\right)$$

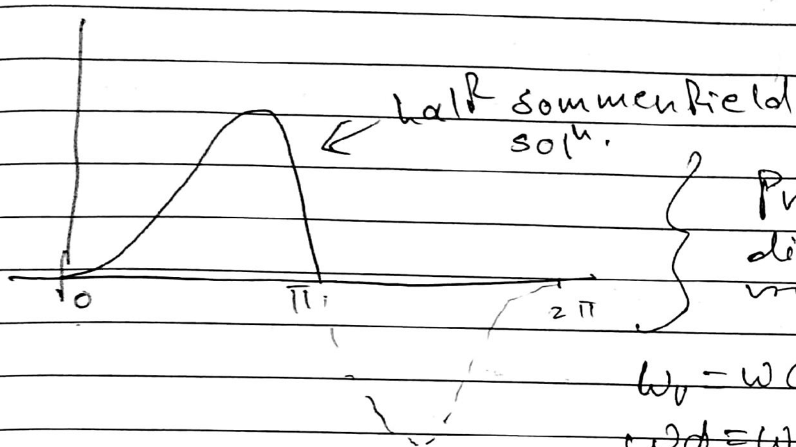
$$\boxed{\mu_j\left(\frac{R}{C}\right) = \frac{1+2\epsilon^2}{3\epsilon}}$$

$$F_b = \eta U L\left(\frac{R}{C}\right)\left(\frac{4\pi(1-\epsilon^2)}{(2+\epsilon^2)}\right)$$

$$RF_j = RF_b + We$$







Pressure distribution remains same.

$$W_r = W\cos\phi = L\int_0^{\pi} p\cos\alpha R\,d\alpha$$

$$W_\phi = W\sin\alpha = L\int_0^{\pi} p\sin\alpha R\,d\alpha$$

$$W_r = 12\eta UL\left(\frac{R}{C}\right)^2 \frac{\epsilon^2}{(2+\epsilon^2)(1-\epsilon^2)}$$

$$W_\phi = 6\eta UL\left(\frac{R}{C}\right) \frac{\pi\epsilon}{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}$$

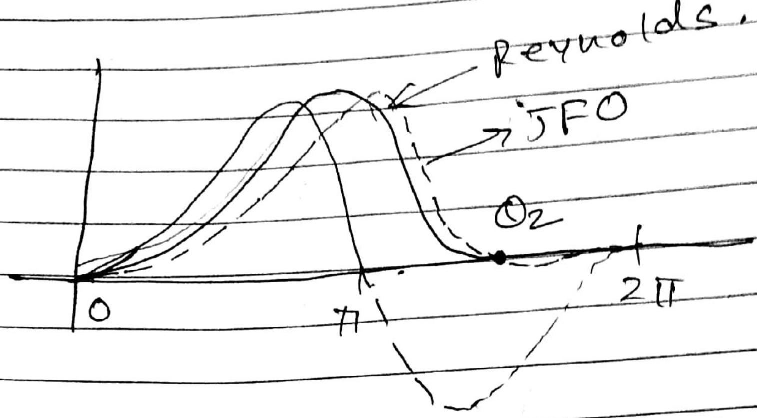
$$W = \sqrt{W_r^2 + W_\phi^2}$$

$$= 6\eta UL\left(\frac{R}{C}\right)^2 \frac{\epsilon\left(\pi^2 - \epsilon^2(\pi^2-4)\right)^{1/2}}{(2+\epsilon^2)(1-\epsilon^2)}$$

$$\phi = \tan^{-1}\left(\frac{W_\phi}{W_r}\right) = \tan^{-1}\left(\frac{\pi}{2\epsilon}\sqrt{1-\epsilon^2}\right)$$

Sommerfield No. $S = \frac{\eta N}{P}\left(\frac{R}{C}\right)^2 \quad \Big| \quad N = \text{rps}, \; P = \frac{W}{2LR}$

$$S = \frac{(2+\epsilon^2)(1-\epsilon^2)^{1/2}}{12\pi^2\epsilon}$$



$C_1, h_m, \theta_2 \leftarrow$ constants to find.

$p = 0$ at $\theta = 0$

$p = 0$ at $\theta = \theta_2$ | Reynolds

$\frac{\partial p}{\partial \theta} = 0$ at $\theta = \theta_2$ | Cavitation,

Petroff Solution (Lightly Load)

$\epsilon <<$ $\qquad h = c$

$$\tau = \frac{\eta U}{h} \qquad F = \tau A.$$

$U = 2\pi RN$, $A = 2\pi RL$, N in rps

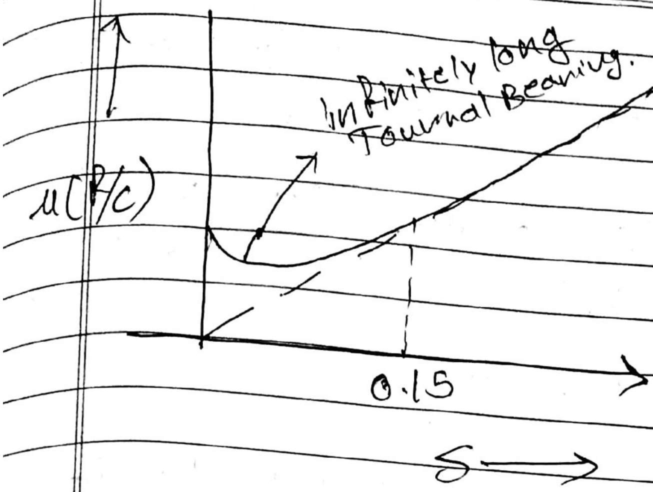
$$F = \frac{4\pi^2 \eta R^2 N L}{c}$$

$$\mu = \frac{F}{W} = 2\pi^2 \frac{\eta N}{P}\left(\frac{R}{c}\right) \qquad P = \frac{W}{2LR}.$$

$$\mu\left(\frac{R}{c}\right) = 2\pi^2 S$$

frugality

miserliness.

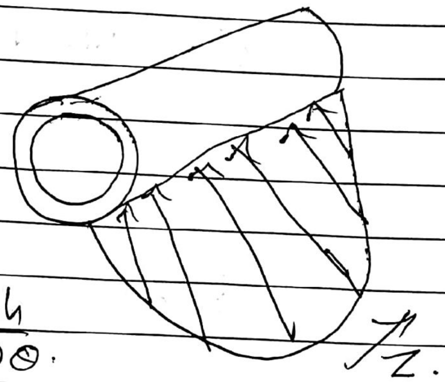


Infinitely short Journal Bearings
(Narrow Bearings)

$$\frac{\partial p}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = 6\eta U\frac{\partial h}{\partial x}.$$

~~Put~~

$x = R\theta \quad dx = R\,d\theta.$



$$\cancel{h^3\partial p}\quad \frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right) = \frac{6\eta U}{R}\frac{\partial h}{\partial\theta}.$$

$$p = \frac{6\eta U}{Rh^3}\frac{\partial h}{\partial\theta}\frac{z^2}{2} + C_1 z + C_2$$

B.C.S $\quad p = 0$ at $z = \pm L/2$

$$p = \frac{3\eta U}{RC^2}\left[\frac{L^2}{4} - z^2\right)\frac{\epsilon\sin\theta}{(1+\epsilon\cos\theta)^3}$$

$$W_r = -2\int_0^{2\pi}\int_0^{L/2} p\cos\theta\,R\,d\theta\,dz$$

$$W_\phi = 2\int_0^{2\pi}\int_0^{L/2} p\sin\theta\,R\,d\theta\,dz.$$

$$W_r = \frac{\eta U L^3}{C^2}\frac{\epsilon^2}{(1-\epsilon^2)^2} \qquad W_\phi = \frac{\eta U L^3}{4C^2}\frac{\pi\epsilon}{(1-\epsilon^2)^{3/2}}$$

$$w = \sqrt{w_n^2 + w_\phi^2} = \frac{\eta U L^3 \epsilon}{4c^2(1-\epsilon^2)^2}\left[\pi^2(1-\epsilon^2)+16\epsilon^2\right]^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{w_\phi}{w_n}\right) = \tan^{-1}\left(\frac{\pi}{4}\frac{(1-\epsilon^2)^{1/2}}{\epsilon}\right)$$

Shear stress $\tau = \frac{\eta U}{h}$

Friction force. $F = \int_0^{2\pi} \eta \frac{U}{h} LR\, d\theta$

$$= \frac{\eta U L R}{c}\left(\frac{2\pi}{(1-\epsilon^2)^{1/2}}\right)$$

Coefficient of friction $\mu = \frac{F}{w}$

$$\mu(R/c) = \frac{2\pi^2 S}{(1-\epsilon^2)^{1/2}}$$

Flow $dq_z = \frac{h^3}{12\eta}\frac{\partial P}{\partial z}$

$$Q_z = -2\int_0^{2\pi} \frac{Rh^3}{12\eta}\left.\frac{\partial P}{\partial z}\right|_{z=L/2} d\theta.$$

$$Q_z = \epsilon U L C$$

$L/D > 1 \rightarrow$ infinitely long bearing.

$L/D < 1 \rightarrow$ Narrow bearing

Finite Bearings (Numerical Solⁿ, FDM)

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial P}{\partial z}\right)=\frac{U_a+U_b}{2}\frac{\partial h}{\partial x}$$

$$\theta=\frac{x}{R},\quad \bar{z}=\frac{2z}{L},\quad \bar{h}=\frac{h}{c},\quad U=\frac{U_a+U_b}{\cancel{2}}$$

$$\boxed{\frac{\partial}{\partial \theta}\left(\bar{h}^3\frac{\partial \bar{P}}{\partial \theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial \bar{z}}\left(\bar{h}^3\frac{\partial \bar{P}}{\partial \bar{z}}\right)=\frac{\partial \bar{h}}{\partial \theta}}$$

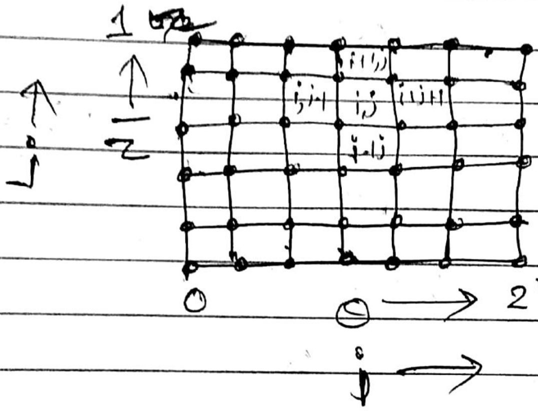
$D=2R$

$$\boxed{\bar{P}=\frac{Pc^2}{6\eta U R}}$$

$\frac{L}{D}=?$

$\epsilon=?$

$$\frac{\partial^2 \bar{P}}{\partial \theta^2}+\left(\frac{D}{L}\right)^2\frac{\partial^2 \bar{P}}{\partial \bar{z}^2}+\frac{3}{\bar{h}}\frac{\partial \bar{P}}{\partial \theta}\frac{\partial \bar{h}}{\partial \theta}=\frac{1}{\bar{h}^3}\frac{\partial \bar{h}}{\partial \theta}$$



$$\left.\frac{\partial P}{\partial \theta}\right|_{i,j}=\frac{P_{i+1,j}-P_{i-1,j}}{2\Delta\theta}$$

$$\left.\frac{\partial^2 P}{\partial \theta^2}\right|_{i,j}=\frac{\left.\partial P/\partial\theta\right|_{i+\frac{1}{2},j}-\left.\frac{\partial P}{\partial \theta}\right|_{i-\frac{1}{2},j}}{\Delta\theta}$$

$$=\frac{\frac{P_{i+1,j}-P_{i,j}}{\Delta\theta}-\frac{P_{i,j}-P_{i-1,j}}{\Delta\theta}}{\Delta\theta}$$

$$=\frac{P_{i+1,j}+P_{i-1,j}-2P_{i,j}}{\Delta\theta^2}$$

$$\bar{h} = \epsilon\,[1 + \cos \bar{x})$$

$$\left.\frac{\partial P}{\partial z}\right|_{(i,j)} = \frac{P_{i,j+1} - P_{i,j-1}}{2\,\Delta z}$$

$$\left.\frac{\partial^2 P}{\partial z^2}\right|_{i,j} = \frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta z^2}$$

$\epsilon = ?$, $L/D = ?$

$nx = 10$ $nz = 12$ $P = C$ $(nz \times nx)$

$P_{old} = P$

x

$$\frac{P(i+1, j) - 2P(i,j) + P(i-1, j)}{\Delta \theta^2}$$

$$+\left(\frac{D}{L}\right)^2 \frac{P(i, j+1) + P(i, j-1) - 2P(i,j)}{\Delta z^2}$$

$$+\frac{3}{h(i)} \frac{P(i+1,j) - P(i-1,j)}{\Delta \theta} \left(\frac{h_{i+1} - h_{i-1}}{\Delta \theta}\right)$$

$$= \frac{1}{[h(i)]^3} \left[\frac{h_{i+1} - h_{i-1}}{\Delta \theta}\right]$$

Load carrying capacity

$$W = \int_0^L \int_0^{2\pi R} p\, dx\, dz$$

$$= 2\int_0^{1} \int_0^{2\pi} \bar{p}\,\frac{6\eta U R}{C^2} R\, d\theta\, d\bar{z}\,\frac{L}{2}$$

$$= \frac{6\eta U R^2 L}{C^2} \int_0^1 \int_0^{2\pi} \bar{p}\, d\theta\, d\bar{z}$$

$$\bar{W} = \frac{WC^2}{6\eta U R^2 L} = \int_0^1 \int_0^{2\pi} \bar{p}\, d\theta\, d\bar{z}$$

Friction

$$F = \int_0^L \int_0^{2\pi R} \tau_x \, dx\, dz$$

$$= \int_0^L \int_0^{2\pi R} \left(\frac{h}{2} \frac{\partial P}{\partial x} + \frac{\eta U}{h} \right) dx\, dz$$

$$= \frac{RL\eta U}{C} \int_0^1 \int_0^{2\pi} \left[3\bar{h} \frac{\partial \bar{P}}{\partial \theta} + \frac{1}{\bar{h}} \right] d\theta\, d\bar{z}$$

$$\bar{F} = \frac{FC}{\eta URL} = \int_0^1 \int_0^{2\pi} \left[3\bar{h} \frac{\partial \bar{P}}{\partial \theta} + \frac{1}{\bar{h}} \right] d\theta\, d\bar{z}$$

$$\mu = \frac{F}{W} = \frac{\bar{F} \eta U R L C^2}{C \bar{W} 6 \eta U R^2 L}$$

$$\mu \left(R/C \right) = \frac{\bar{F}}{6\bar{W}}$$

End Flow

$$dQ_z = -\frac{h^3}{12\eta} \frac{\partial P}{\partial z} \Big|_{z = L/2}$$

$$Q_z = \int_0^{2\pi R} -\frac{h^3}{12\eta} \frac{\partial P}{\partial z} \Big|_{z = L/2} dx.$$

$$\bar{Q}_z = \frac{Q_z L}{U C^3 R^2} = \int_0^{2\pi} -\bar{h}^3 \frac{\partial \bar{P}}{\partial \bar{z}} \Big|_{\bar{z}=1} d\theta.$$

HW

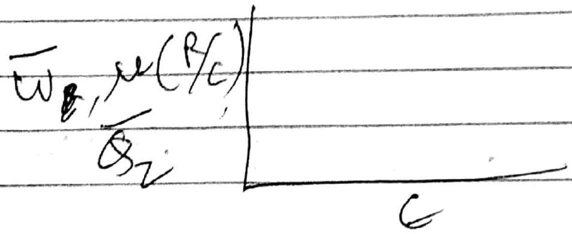
~~ϵZ~~

$L/D = 1 \quad \epsilon = 0.4$

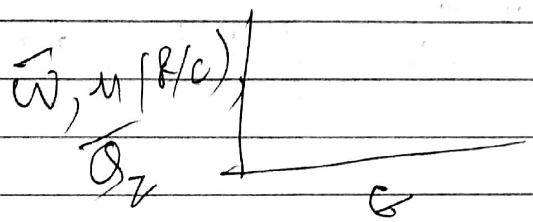
~~Case I~~: Solve & plot P

~~Case~~

$\epsilon = 0.2, 0.4, 0.6, 0.8$



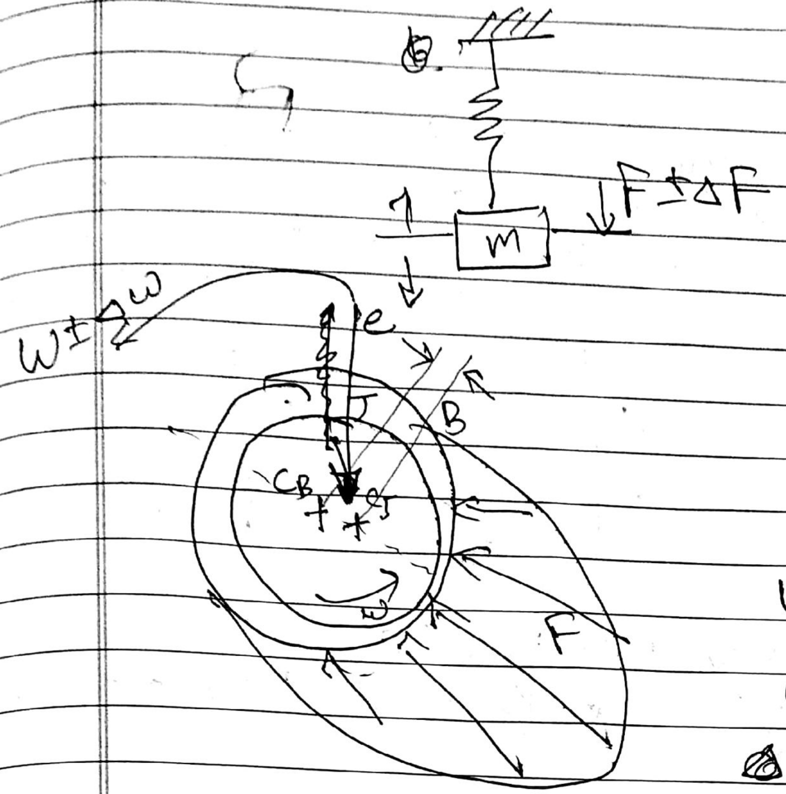
Case II: $\epsilon = 0.4, \quad L/D = 0.5, 1, 1.5, 2$



$$P(i,j)\left[\frac{2}{\Delta\theta^2} + \frac{2}{\Delta Z^2}\left(\frac{D}{L}\right)^2\right]$$

$$= \left(\frac{P(i+1,j) + P(i-1,j)}{\Delta\theta^2}\right) + \left(\frac{D}{L}\right)^2\left(\frac{P(i,j+1) + P(i,j-1)}{\Delta Z^2}\right) + \frac{3}{h(i)}\left(\frac{P(i+1,j) - P(i-1,j)}{\Delta\theta^2}\right)\left(h(i+1) - h(i-1)\right) - \frac{1}{[h(i)]^3}\left(\frac{h(i+1) - h(i-1)}{\Delta\theta}\right)$$

Hydrodynamic Instability



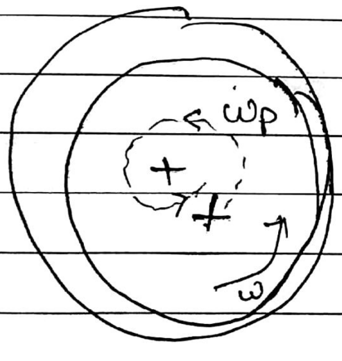
$(m\omega^2 r)$
↓
because of ecentric load in running conditn the unbalace chages with ω

$W = F$

$W \pm \Delta W$

ω chages (reduces) suddenly, so it will take time for hydrodynamic force (F) to change and balance modified weight.

In mean time centre of journal revolves around bearing centre. because of this unbalance.

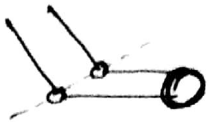


$\omega_p \rightarrow$ whirling frequency
$\omega \rightarrow$ spining frequency

$-h(1-)$

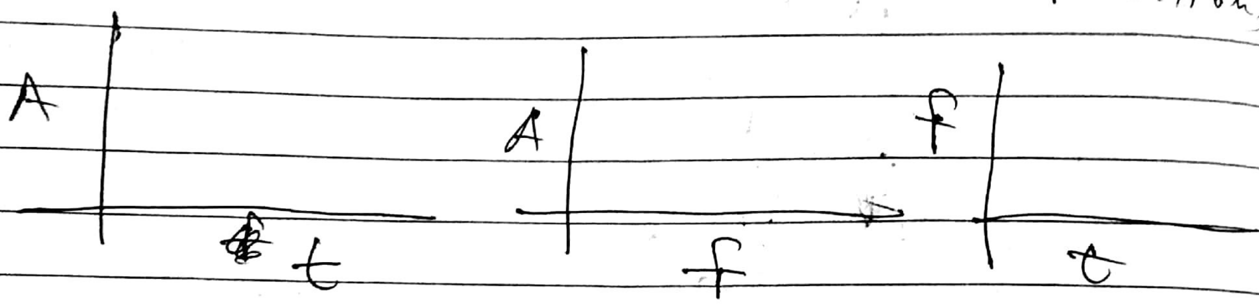
ω_p depends of ω. and thus it is inheniantly present in rotating system.

$$\frac{\omega_p}{\omega} \simeq \frac{1}{2}.$$



$x(t)$
$\dot{x}(t)$
$\ddot{x}(t)$

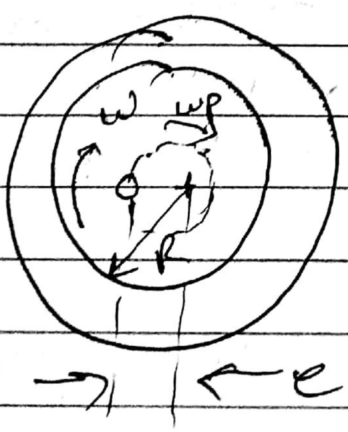
Measuring $\ddot{x}(t)$ involves less noise as to obtain other two ($\dot{x}(t)$, $x(t)$) we need to integrate (less noise amplication)



Frequency varies with time during run up and run down.

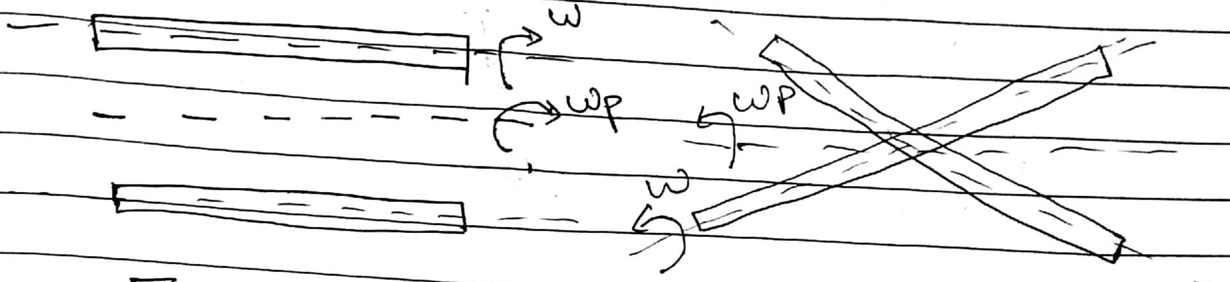
Cause of Instability

Theoretical prediction



$m\omega^2 e$.

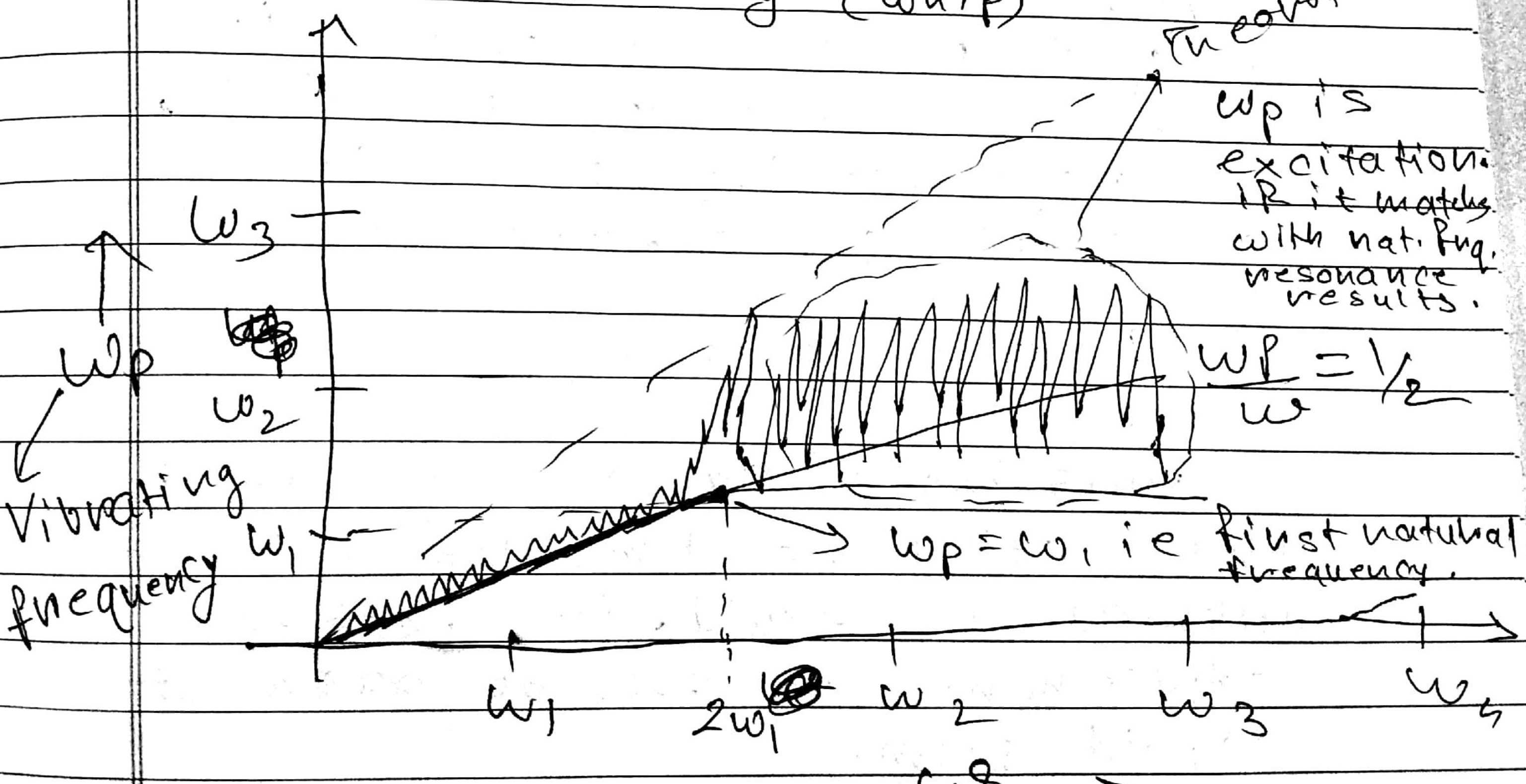
Tuype of Instability



Translatory whirl — L/D is small

Conical whirl — L/D is large.

1. Subsynchronous (whirl)
2. Resonating (whip)

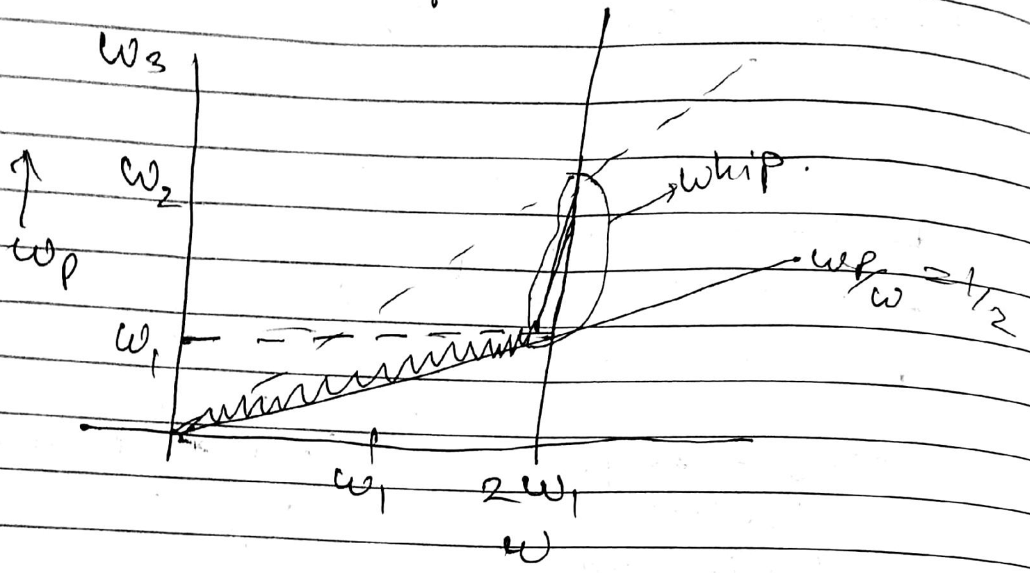


$\omega_1, \omega_2, \omega_2 \ldots$ are natural frequency of the system.

Practically, after ω_p reaches $2\omega_1$, no further increase in ω is possible.

$\therefore$ journal bearing can reach maximum up to $2\omega_1$ speed.

$2\omega_1$ is called as stability threshold speed.



→ $0.85 \times (2\omega_1)$ design

To increase threshold (stability) speed for given system i.e ω_1, ω_2 ... nat freq. are fixed, slope of ω_p vs ω line should be decreased.

→ How, to do this?

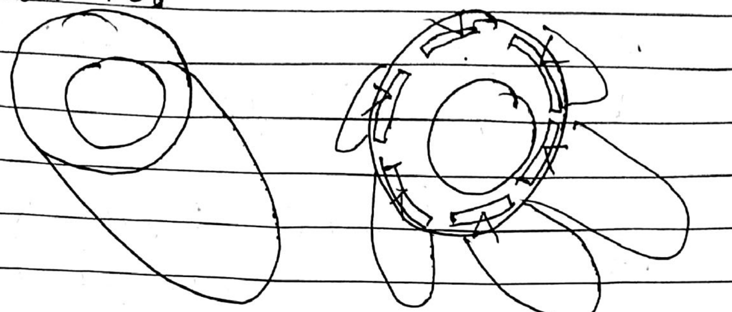
This instability occurs due to the settling time because of lubricating fluid. i.e pressure adjustment is slow because of liquid film.

$x = X e^{i\omega t}$ $\qquad \frac{X}{F} = \frac{1/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$

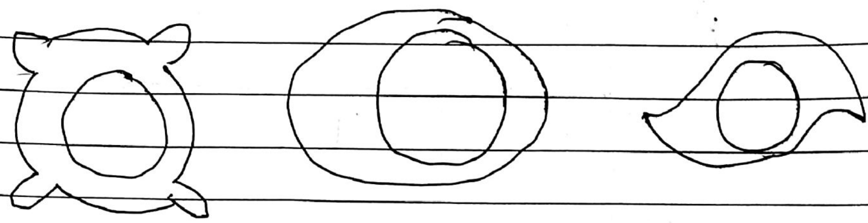
$F = F e^{i\omega t}$

To tackle this, pivoted pads are introduced so that pressure gets distributed over number of small sections and thus pressure adjustment is quid in such system.

~~Thus For~~

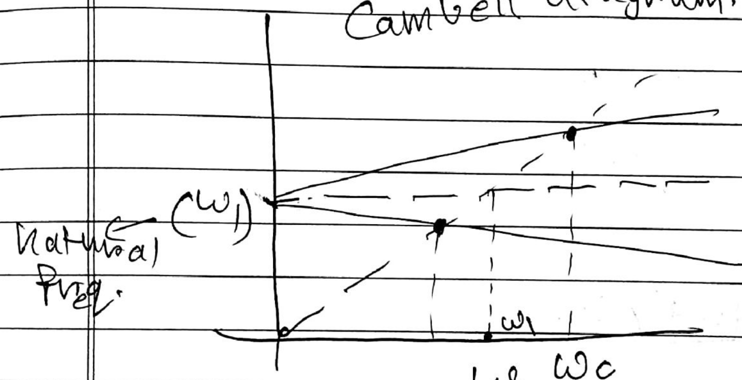


Other solutions



Cambell diagram.

$\zeta = \frac{c}{2m\omega_n}$



$m\ddot{x} + c\dot{x} + kx = 0$

natural freq. (ω_1)

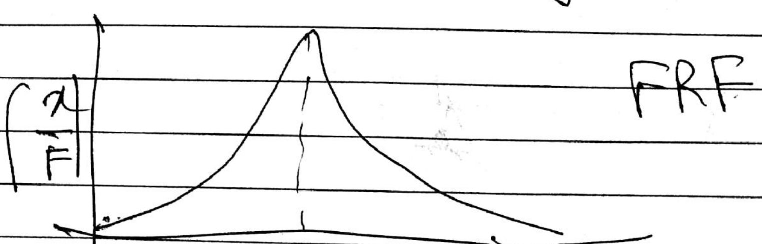
natural freq ω_1

ω_{d_1} damped natural frequency.

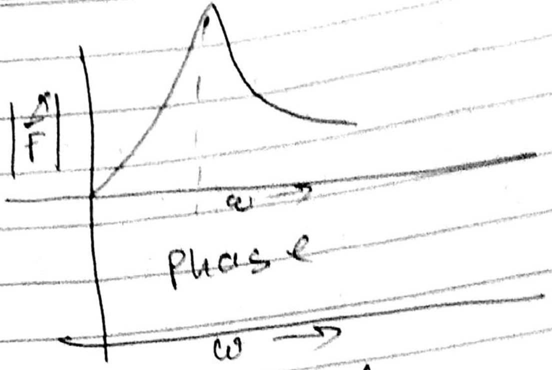
ω ω_c

running freq^n

at zero running speed $\omega_{d_1} = \omega_1$



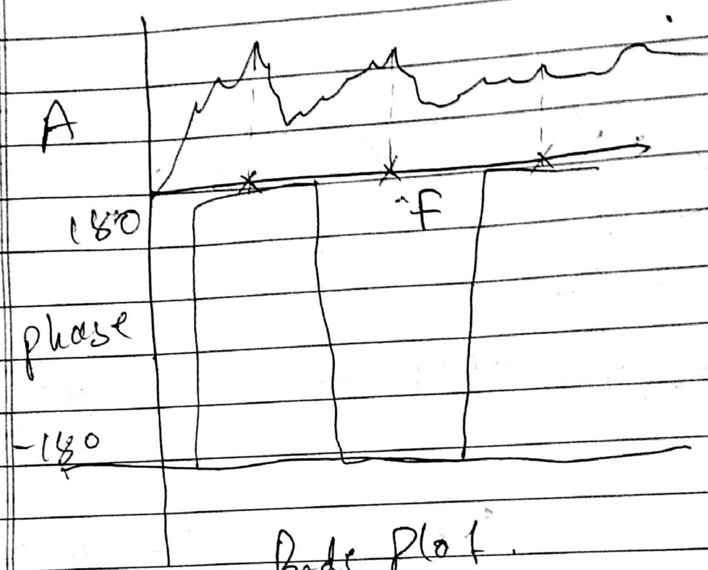
FRF



$$\cos(\omega t + \beta + \phi)$$

$$\frac{x}{F} = \frac{1/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

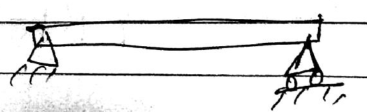
$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$



Bode Plot.

Modal Analysis.

Theoretical ↓ ← → Experimental



$M\ddot{x} + c\dot{x} + kx = 0.$

$M\ddot{x} + kx = 0$

$M\omega^2 + K = 0$

$M\omega^2 = -K.$

$[M^{-1}K]$

$\downarrow$

eigenvalues eigenvectors.

$[R]_{10 \times 10}$ $[V]_{10 \times 10}.$

Operating deflection shape (shape for frequencies in between natural frequencies)

operating deflection shapes are functions of all mode shapes.

$V = f(v_1, v_2, v_3, v_4 \ldots)$

modeshape corresponding to ω_1 and so on.

$V = \underbrace{[V]}_{\text{mode shapes}} \times \underbrace{\{\omega_x\}}_{\text{weightage.}}$