International Journal of Control, Automation and Systems 19(9) (2021) 2940-2949
http://dx.doi.org/10.1007/s12555-020-0343-7
ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

Event-trigger Optimal Consensus for Multi-agent System Subject to Differential Privacy

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Abstract: Optimal consensus algorithm is a very useful consensus algorithm for distributed cooperative control, which makes all the agents not only achieve consensus but also minimize the cost function. However, to achieve consensus, agents need to exchange their state with each other on public channel. If attackers want to obtain the privacy information of agents, they only need to monitor the public channel. To solve this problem, a novel event-triggered differentially privacy optimal consensus algorithm is proposed to preserve the privacy of the cost function of each agent in the whole process of consensus computation. Based on event-trigger condition, we analyze the consensus of our algorithm in detail, including the accuracy and consensus conditions. In addition, the privacy-preserving analysis are also given, which exhibits that privacy of the states of all agents can be preserved. The privacy level and the sensitivity of the differential privacy are also obtained. Finally, a numerical simulation is given to illustrate the effectiveness of the theoretical results.

Keywords: Consensus, differential privacy, event-trigger, zero-gradient-sum.

1. INTRODUCTION

In the past few years, the optimal consensus problems for multi-agent systems (MASs) have received attractive attentions for their extensive applications such as formation control [1], robots network [2], flocking [3], mobile sensor area coverage [4–6] and so on. The goal of optimal consensus is to make agents not only achieve consensus but also minimize the cost such as energy consumption, bandwidth requirement and so on. Up to date, many gradient-based optimal consensus algorithms have been proposed in multi-agent network [7–16].

As known to all, to achieve consensus, agents need to exchange their state with each other on public channel. If attackers want to obtain the privacy information of agents, they only need to monitor the public channel. To solve these problems, many privacy preserving algorithms are proposed. Among these algorithms, differential privacy is widely concerned because it is easy to implement [17,18] by adding Laplace noise conformed to the proper distribution. To date, several differential privacy consensus algorithms have been proposed [19–28,31–36]. However, it can be seen few works investigate the optimal consensus problem.

Motivated by the discussion, in this paper, based on differential privacy scheme and event-trigger scheme, a novel event-triggered differential privacy optimal consensus algorithm is proposed to preserve the privacy of the state of each agent in the whole process of consensus computation. The major contributions of this paper are summarized as follows:

1) The proposed algorithm not only ensures that the agent can reach the optimal consensus point, but also preserves the privacy of cost function in the whole process of achieving consensus.

2) The event-triggered control strategy is used in the controller, which can reduce the communication load and save resource.

3) The convergence rate, privacy level and the sensitivity of the differential privacy are also obtained.

The rest of this paper is organized as follows: in Section 2, some basic mathematical concepts and relevant definitions are given. Problem formulating, differential privacy and algorithm description are given in Section 3. In Sec-

Manuscript received June 17, 2020; revised November 25, 2020; accepted December 18, 2020. Recommended by Associate Editor Niket Kaisare under the direction of Editor Jessi (Ju H.) Park. This work was supported in part by the National Key Research and Development Project of China under Grant 2018AAA0100101, in part by Fundamental Research Funds for the Central Universities under Grant XDJK2020B009, in part by Chongqing Basic and Frontier Research Project under Grant cstc2019jcyj-msxm2105 and cstc2020jcyj-msxmX0470, in part by the Chongqing Technological Innovation and Application Project under Grant cstc2018jszx-cyzdX0171, in part by the Science and Technology Research Program of Chongqing Municipal Education Commission under Grant KJQN201900816, in part by Chongqing Social Science Planning Project under Grant 2019BS053.

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tion 4 the main results are given. Section 5 presents a numerical simulation to illustrate the consensus and privacy preserving performance.

Notations: The main notations used in this paper are introduced as followed. Denote the $\|x\|_p$ as p-norm of vector $x=[x_1,x_2,\ldots,x_n]^T \in R^n$, $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}}$. $f(x)$ is the distribution function of variable x and $E(f(x))$ is the expectation of $f(x)$. Define ∇f is the gradient of function f.

2. PRELIMINARIES

2.1. Graph description and Laplace distribution

Let $\mathcal{G}(\mathcal{V},\mathcal{E})$ be a weighted digraph with the set of vertices $\mathcal{V}=\{1,2,...,N\}$ and the set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. $(i,j)\in\mathcal{E}$ is the edge , when agent i and j can exchange information with each other. We define $\mathrm{N}(i)\triangleq\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$ as the neighborhood of the agent i . In adjacency matrix $\mathcal{A}$, $a_{ij}>0$ if and only if $(i,j)\in\mathcal{E}$, else $a_{ij}=0$.

Let x be subject to Laplace distribution. $Laplace(\mu,b)=\frac{1}{2b}e^{-\frac{|x-\mu|}{b}}$ is the probability density function of x, whose variance is $2b^2$ and mathematical expectation is μ. $\mathrm{P}(X\leq x)=\int_{-\infty}^{x} Laplace(\mu,b)\,dx=\int_{-\infty}^{x}\frac{1}{2b}e^{-\frac{|x-\mu|}{b}}dx$ is probability distributed function.

2.2. Convex function

A twice continuously differential function h is convex if there exists a constant $\psi>0$, $\Psi>0$ such that the following conditions hold: $h(b)-h(a)-\nabla h(a)^T(b-a)\geq\frac{\psi}{2}\|b-a\|^2$, $\nabla^2 h(x)\geq\psi I_n$, $h(b)-h(a)-\nabla h(a)^T(b-a)\leq\frac{\Psi}{2}\|b-a\|^2$, $(\nabla h(b)-\nabla h(a))^T(b-a)\leq\Psi\|b-a\|^2$, $\nabla^2 h(x)\leq\Psi I_n$.

2.3. Differential privacy

In this section, we give some concepts of differential privacy based on [19,20].

Definition 1 (Adjacent databases)**:** Two databases $D=\{x_i\}_{i=1}^N$ and $D'=\{x_i{}'\}_{i=1}^N$ are said to be adjacent, if only one data of two databases is different, which can be written as $x_i\neq x_i{}'$ for all $i\neq j$, $x_j=x_j{}'$.

Definition 2 (ε-differential privacy)**:** For any two adjacent databases $D=\{x_i\}_{i=1}^N$ and $D'=\{x_i{}'\}_{i=1}^N$ and output $\forall M\subseteq Range(F)$, algorithm F is $\varepsilon-$differential privacy if there is a parameter $\varepsilon>0$ such that F satisfies

$$\Pr\left[F(D)\in M\right]\leq e^{\varepsilon}\Pr\left[F(D')\in M\right],\ \forall M\subseteq range(F),$$

where $range(F)$ represents all possible outputs of algorithm F, ε is the level of privacy preserving.

Definition 3 (Sensitivity)**:** For Laplace mechanism, the sensitivity of the query function f with two adjacent databases D and D' is defined as follows:

$$\Delta=\max_{D\simeq D'}\|f(D)-f(D')\|_1.$$

2.4. Mean square consensus

Definition 4 (Mean square consensus)**:** The multi-agent system is said to achieve mean square consensus if there exists a random value x^* satisfying the following conditions:

1) $\lim_{k\to\infty} E|x_i(k)|$ is bounded.
2) $\lim_{k\to\infty} E|x_i(k)-x^*|^2=0$.

3. PROBLEM FORMULATION

3.1. Distributed optimal consensus algorithm

Consider a multi-agent network with N agents where each agent i has its own objective function h_i, the distributed optimal consensus problem of this group can be described as follows:

$$\begin{aligned}&\min_{x\in R^+} h(x)=\sum_{i=1}^{N}h_i(x_i)\\ &\text{s.t. } x_1=x_2=...=x_n,\end{aligned}\tag{1}$$

where $x=\{x_1,x_2,...,x_n\}$ is a decision vector; h_i is continuously differentiable local cost function of agent i, which is convex; $h(x)$is the global cost function defined as the sum of local functions. To solve this optimal problem, based on the ZGS [23], a distributed consensus algorithm is proposed as follows:

$$\begin{cases}y_i(k)=\nabla h_i(x_i(k)),\\ u_i(k)=\gamma\sum_{j\in \mathrm{N}_i}a_{ij}(\hat{x}_j(k)-\hat{x}_i(k)),\\ \nabla h_i(x_i(k+1))=y_i(k)+u_i(k),\\ x_i(0)=x_i^*,\end{cases}\tag{2}$$

where $\hat{x}_i(k)=x_i\left(t_{m_i}^i\right)$ for $t_{m_i}^i<k<t_{m_i+1}^i$, $t_{m_i}^i$ is the trigger time, $\nabla h_i(x_i(k))$ is the derivatives of h, x_i^* is the optimal solution of locally cost function.

3.2. Privacy in optimal consensus algorithm

The initial state of agent $x_i(0)\in D$, $i=1,\cdots,N$, is the optimal solution of local cost function, which cannot be disclosed to the public. In the study of privacy protection, it is generally assumed that the adversary has ability to collect agent states on the public channel and infer the agent's privacy information by using the collected information. Under this condition, the algorithm (2) may lead to $x_i(0)$ disclosure for reasons described blow. From (2), it can be seen that $y_i(k)$ is determined by $x_i(0)$ and is published to the public, as a result, the adversary can collect $y_i(k)$ easily. With enough information, the adversary can infer the information of $x_i(0)$.

3.3. Problem formulation

Our goal is to protect the agent's initial state $x_i(0)$, even if the attacker can collect all signal $\{y_i(k)\}$. Based on the definition of deferential privacy, we define database $D = \{x_i(0)\}_{i=1}^{N}$, and query $y = (y_1(1), y(2), \dots, y(k))$.

Definition 5 (Adjacent relationship for optimal consensus)**:** For two databases $D = \{x_i(0)\}_{i=1}^{N}$ and $D' = \{x_i{}'(0)\}_{i=1}^{N}$, they are said to be adjacent if and only if there is $i \in [N]$, such that $|x_i(0) - x_i{}'(0)| \le a$, $x_j(0) = x_j{}'(0)$ for all $i \ne j$, where a depends on the privacy requirement.

According to Definition 5, we give the problem of differential privacy optimal consensus algorithm.

Problem 1: (Differential privacy optimal consensus). The goal of differential privacy optimal consensus algorithm is to design a distributed optimal consensus algorithm F that not only solves the optimal problem (1), but also approximates $y = (y_1(1), y(2), \dots, y(k))$ and realize ε-differential privacy based on $D = \{x_i(0)\}_{i=1}^{N}$ and $D' = \{x_i{}'(0)\}_{i=1}^{N}$, which means that F needs to satisfy

$$\Pr[F(D) \in M] \le e^{\varepsilon} \Pr[F(D') \in M],\ \forall M \subseteq range(F).$$

3.4. Differential privacy optimal consensus algorithm

To solve problem 1, we propose an event-triggered differential privacy optimal consensus algorithm:

$$\begin{cases} y_i(k) = \nabla h_i(x_i(k)) + \omega_i(k), \\ u_i(k) = \gamma \sum\limits_{j \in \mathrm{N}_i} a_{ij}(\hat{x}_j(k) - \hat{x}_i(k)), \\ \nabla h_i(x_i(k+1)) = y_i(k) + u_i(k), \\ x_i(0) = x_i^*, \end{cases} \tag{3}$$

where $\hat{x}_i(k) = x_i(t_{m_i}^i)$ for $t_{m_i}^i < k < t_{m_i+1}^i$, $t_{m_i}^i$ is the trigger time, $\nabla h_i(x_i(k))$ is the derivatives of h, $\omega_i(k) \sim Lap\left(0, \frac{1}{k^q}\right)$, $0.5 < q < 1$, $\omega_i(0) \sim Lap(0,1)$ is Laplace noise with mathematical expectation 0.

Define the error measurement as

$$r_i(k) = \hat{x}_i(k) - x_i(k).$$

The trigger function is

$$\begin{aligned} H_i(k, x_i(k)) = &\|r_i(k)\| - \beta \sqrt{\sum_{j=1}^{N} a_{ij}(\hat{x}_j(k) - \hat{x}_i(k))} \\ &- \alpha\sqrt{\exp(-k)}, \end{aligned} \tag{4}$$

where $\alpha > 0, 0 < \beta < 1$. Once $H_i(k, x_i(k)) > 0$, agent i updates the sampled value $\hat{x}_i(k)$ using its own current agent state $x_i(k)$. By (3), one can obtain

$$\begin{aligned} &\nabla h_i(x_i(k+1)) \\ &= \gamma \sum_{j \in \mathrm{N}_i} a_{ij}(\hat{x}_j(k) - \hat{x}_i(k)) + \nabla h_i(x_i(k)) + \omega_i(k) \\ &= \gamma \sum_{j \in \mathrm{N}_i} a_{ij}(x_j(k) + r_j(k) - r_i(k) - x_i(k)) \\ &\quad + \nabla h_i(x_i(k)) + \omega_i(k) \\ &= \gamma \sum_{j \in \mathrm{N}_i} a_{ij}(x_j(k) - x_i(k) + r_j(k) - r_i(k)) \\ &\quad + \nabla h_i(x_i(k)) + \omega_i(k). \end{aligned} \tag{5}$$

Writing (5) in impact form, one has

$$\nabla h(x(k+1)) = \nabla h(x(k)) - \gamma Lx(k) - \gamma Lr(k) + \omega(k). \tag{6}$$

Then, we have

$$x(k+1) = x(k) - \gamma P(k)L(x(k) + r(k)) + P(k)\omega(k), \tag{7}$$

where $P(k) = diag\{p_i(k)\}$, $p_i(k) = \frac{x_i(k+1) - x_i(k)}{\nabla h_i(x(k+1)) - \nabla h_i(x(k))}$, $P(k) \le \frac{1}{\underline{\psi}} I_N$, $\underline{\psi} = \min\{\psi_i\}_{i=1}^{N}$, L is Laplace matrix, $\mathcal{A} = [a_{ij}]_{N \times N}$ is adjacent matrix, $D_{ii} = \sum\limits_{j \in \mathrm{N}_i} a_{ij}$.

4. MAIN RESULT

4.1. Convergence analysis

Before analyzing the convergence of algorithm, we give the following lemma:

Lemma 1: Consider a system $\chi(n+1) = h(\chi(n), n, \omega(n))$, where $\chi(n) \in R^N$ is the system state and $h(\chi(n), n, \omega(n))$ is bounded on $n \in [0, \infty)$. If there exists a nonnegative function $\Upsilon(\chi(n))$ satisfying the following conditions:

1) $\Upsilon(n)$ boundedness leads to the boundedness of $\chi(n)$.

2) $\Upsilon(\chi(n+1)) - \Upsilon(n) \le -W(n) + \delta(n) + \omega(n)$ then we can obtain $E(\Upsilon(n)) \le E(\Upsilon(\chi(0)) + \bar{\delta})$ and $\lim\limits_{k \to \infty} E(W(\chi(k))) = 0$, where $\omega(n) \sim Lap\left(0, \frac{1}{n^q}\right)$, $0.5 < q < 1$, $\omega(0) \sim Lap(0,1)$, and $\bar{\delta} = \sum\limits_{m=0}^{n-1} \delta(m)$ are bounded, $W(x)$ is a nonnegative function.

Proof: As $W(x)$ is nonnegative, we have

$$\begin{aligned} &\Upsilon(n) - \Upsilon(\chi(n-1)) \\ &\le -W(\chi(n-1)) + \delta(n-1) + \omega(n-1) \\ &\le \delta(n-1) + \omega(n-1). \end{aligned} \tag{8}$$

Let $\bar{\delta} = \sum\limits_{m=0}^{n-1} \delta(m)$, take the summation of (8) from 1 to n, we have

$$\begin{aligned} \Upsilon(n) - \Upsilon(\chi(0)) &\le \sum_{m=0}^{n-1} \delta(m) + \sum_{n=0}^{\infty} \omega(n) \\ &\le \bar{\delta} + \sum_{n=0}^{\infty} \omega(n). \end{aligned} \tag{9}$$

As $\omega(n) \in Lap\left(0, \frac{1}{n^q}\right)$, one can obtain $E(\omega(n)) = 0$. Taking the expectation of (9), one can obtain

$$E(\Upsilon(n)) \le E(\Upsilon(\chi(0))) + \bar{\delta}. \tag{10}$$

Next, we prove $\lim_{k\to\infty} E(W(n)) = 0$. Based on (8) and (9), one can obtain

$$\sum_{n=0}^{\infty} W(n) \le \Upsilon(\chi(0)) - \Upsilon(n) + \bar{\delta} + \sum_{n=0}^{\infty}\omega(n) \le \Upsilon(\chi(0)) + \bar{\delta} + \sum_{n=0}^{\infty}\omega(n). \tag{11}$$

As $E(\omega(n))$ is 0, taking the expectation of (11), we have

$$E\left(\sum_{n=0}^{\infty} W(n)\right) \le E\left(\Upsilon(\chi(0)) + \bar{\delta}\right). \tag{12}$$

By (12), we can obtain that $\sum_{n=0}^{\infty} E(W(n))$ is bounded and $\lim_{n\to\infty} E(W(n)) = 0$, which completes the proof.

Lemma 2: For $\forall k$, the algorithm (3) satisfies

$$E\left(\sum_{i=1}^{N} \nabla h_i(x_i(k))\right) = 0.$$

Proof: From (3), one can obtain

$$\sum_{i=1}^{N} [\nabla h_i(x_i(k+1)) - \nabla h_i(x_i(k))] = \gamma \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\hat{x}_j(k) - \hat{x}_i(k)) + \sum_{i=1}^{N}\omega_i(k). \tag{13}$$

As communication graph is undirected and connected, we have $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$ and $\gamma\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{x}_j(k) - \gamma\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\hat{x}_i(k) = 0$, by (13), one can obtain

$$\sum_{i=1}^{N} \nabla h_i(x_i(k+1)) = \sum_{i=1}^{N}\omega_i(k) + \sum_{i=1}^{N}\nabla h_i(x_i(k)). \tag{14}$$

Taking the expectation of (14), we have

$$E\left(\sum_{i=1}^{N} \nabla h_i(x_i(k+1))\right) = E\left(\sum_{i=1}^{N} \nabla h_i(x_i(k))\right). \tag{15}$$

According to (15), it draws $E\left(\sum_{i=1}^{N} \nabla h_i(k+1)\right) = E\left(\sum_{i=1}^{N} \nabla h_i(k)\right) = E\left(\sum_{i=1}^{N} \nabla h_i(x_i(0))\right) = 0$. As $x_i(0) = x_i^*$, we can deduce $E\left(\sum_{i=1}^{N} \nabla h_i(x_i(0))\right) = E\left(\sum_{i=1}^{N} \nabla h_i(x_i^*)\right) = 0$, which completes the proof. □

Theorem 1: The mean square consensus based on algorithm (3) is achieved and the convergence rate of (3) is $rate = \left(1 - \rho\left(\frac{\gamma}{4} - \frac{2\gamma^2}{\underline{\psi}\mu}\right)\right)$, where $0 < \gamma < \frac{\underline{\psi}\mu}{8}$, $\mu = \sup\left\{\varepsilon : \varepsilon LL^T \le \left(L + L^T\right)\right\}$, $\underline{\psi}$ is defined in (6).

Proof: Define the following Lyapunov function:

$$V(k) = \sum_{i=1}^{N} \begin{pmatrix} h_i(x^*) - h_i(x_i(k)) \\ -\nabla h_i(x_i(k))^T(x^* - x_i(k)) \end{pmatrix}. \tag{16}$$

Using the convex function properties $h_i(b) - h_i(a) - \nabla h_i(a)(b-a) \ge \frac{\psi_i}{2}|b-a|^2$, one can obtain $V(k) \ge \sum_{i=1}^{N}\frac{\psi_i}{2}|x - x^*|^2 > 0$. Taking the expectation of $\Delta V(k+1)$, then we can obtain

$$\begin{aligned} &E(\Delta V(k+1)) \\ &= E(V(k+1) - V(k)) \\ &= E\begin{pmatrix} -\sum_{i=1}^{N}(h_i(x_i(k+1)) - h_i(x_i(k))) \\ +\sum_{i=1}^{N}\left(-\nabla h_i(x_i(k+1))^T + \nabla h_i(x_i(k))^T\right)x^* \end{pmatrix} \\ &\quad + E\left(\sum_{i=1}^{N}\begin{pmatrix} \nabla h_i(x_i(k+1))^T x_i(k+1) \\ -\nabla h_i(x_i(k))^T x_i(k) \end{pmatrix}\right). \end{aligned} \tag{17}$$

As $E\left(\sum_{i=1}^{N} \nabla h_i(x_i(k+1))\right) = 0$, one has

$$E\left(\sum_{i=1}^{N}\left(-\nabla h_i(x_i(k+1))^T + \nabla h_i(x_i(k))^T\right)x^*\right) = 0. \tag{18}$$

Substituting (18) into (17), we can obtain

$$\begin{aligned} &E(\Delta V(k+1)) \\ &= E\left(-\sum_{i=1}^{N}\begin{pmatrix} h_i(x_i(k+1)) - h_i(x_i(k)) \\ -\nabla h_i(x_i(k))^T(x_i(k+1) - x_i(k)) \end{pmatrix}\right) \\ &\quad + E\Big(\sum_{i=1}^{N}\left(\nabla h_i(x_i(k+1))^T - \nabla h_i(x_i(k))^T\right) \\ &\quad \times x_i(k+1)\Big). \end{aligned} \tag{19}$$

Based on the definition of convex function in Section 2, we have

$$\begin{aligned} &E(\Delta V(k+1)) \\ &\le -\frac{\psi_i}{2}|x_i(k+1) - x_i(k)|^2 \\ &\quad + E\Big(\sum_{i=1}^{N}\left(\nabla h_i(x_i(k+1))^T - \nabla h_i(x_i(k))^T\right) \\ &\quad \times x_i(k+1)\Big) \\ &\le E\Big(\sum_{i=1}^{N}\left(\nabla h_i(x_i(k+1))^T - \nabla h_i(x_i(k))^T\right) \\ &\quad \times x_i(k+1)\Big). \end{aligned} \tag{20}$$

Combining (18) and (20), we can infer that

$$E(\Delta V(k+1))$$

$$\begin{aligned}
&\leq E\Big(\sum_{i=1}^{N}\Big(\nabla h_i(x_i(k+1))^T - \nabla h_i(x_i(k))^T\Big)\\
&\quad \times x_i(k+1)\Big)\\
&= E\left(\left(\nabla h\left(x(k+1)\right) - \nabla h(x(k))\right)^T x(k+1)\right)\\
&= E\left(-\gamma(x(k)+r(k))^T L^T x(k+1) + \omega(k)^T x(k+1)\right).
\end{aligned} \tag{21}$$

Substituting (7) into (21), one has

$$\begin{aligned}
&E\left(\Delta V\left(k+1\right)\right.\\
&\leq E\left(-\gamma(x(k)+r(k))^T L^T x(k+1)+\omega(k)^T x(k+1)\right)\\
&= \underbrace{E\Big(\gamma(x(k)+r(k))^T L^T (x(k)-\gamma P(k)L(x(k)+r(k)))\Big)}_{V_1(k+1)}\\
&+ \underbrace{E\begin{pmatrix}-\gamma(x(k)+r(k))^T L^T P(k)\omega(k)+\omega(k)^T\\ \times\left(x(k)-\gamma P(k)L(x(k)+r(k))+P(k)\omega(k)\right)\end{pmatrix}}_{V_2(k+1)}.
\end{aligned} \tag{22}$$

Firstly, we compute $V_1(k+1)$. As $\gamma x(k)^T L^T x(k) > 0$, one can obtain

$$\begin{aligned}
&V_1(k+1)\\
&= E\Big(-\gamma(x(k)+r(k))^T L^T \left(x(k)-\gamma P(k)\right.\\
&\quad \times L(x(k)+r(k)))\Big)\\
&= E\left(-\gamma x(k)^T L^T x(k) - \gamma r(k)^T L^T x(k)\right)\\
&+ E\left(\gamma^2 (x(k)+r(k))^T L^T P(k) L(x(k)+r(k))\right)\\
&\leq E\Big(-\gamma r(k)^T L^T x(k) + \gamma^2 (x(k)+r(k))^T\\
&\quad \times L^T P(k) L(x(k)+r(k))\Big).
\end{aligned} \tag{23}$$

Let $\mu = \sup\left\{\varepsilon : \varepsilon LL^T \leq \left(L+L^T\right)\right\}$, using Young's inequality, one has

$$\begin{aligned}
\gamma r(k)^T L^T x(k) &\leq \gamma\frac{1}{\mu} r(k)^T r(k) + \gamma\frac{\mu}{4} x(k)^T LL^T x(k)\\
&\leq \gamma\frac{1}{\mu} r(k)^T r(k) + \gamma\frac{1}{4} x(k)^T (L+L^T) x(k).
\end{aligned} \tag{24}$$

Using the inequality of arithmetic and geometric means [30,36] and $P(k) \leq \frac{1}{\underline{\psi}} I_N$, we can obtain

$$\begin{aligned}
&\gamma^2 (x(k)+r(k))^T L^T P(k) L(x(k)+r(k))\\
&\leq 2\gamma^2 x(k) L^T P(k) L x(k) + 2\gamma^2 r(k) L^T P(k) L r(k)\\
&\leq 2\gamma^2 \frac{1}{\underline{\psi}} x(k) L^T L x(k) + 2\gamma^2 \frac{1}{\underline{\psi}} r(k) L^T L r(k)\\
&\leq \frac{2\gamma^2}{\underline{\psi}\mu} x(k)\left(L^T+L\right)x(k) + \frac{2\gamma^2 \|L\|^2}{\underline{\psi}} r(k)^T r(k).
\end{aligned} \tag{25}$$

Substituting (24) and (25) into (23), we can obtain

$$\begin{aligned}
&V_1(k+1)\\
&\leq E\left(\frac{2\gamma^2}{\underline{\psi}\mu} x(k)\left(L^T+L\right)x(k) + \frac{2\gamma^2\|L\|^2}{\underline{\psi}} r(k)^T r(k)\right)\\
&+ E\left(\gamma\frac{1}{\mu} r(k)^T r(k) - \gamma\frac{1}{4} x(k)^T \left(L+L^T\right) x(k)\right)\\
&\leq E\begin{pmatrix} -\left(\frac{\gamma}{4} - \frac{2\gamma^2}{\underline{\psi}\mu}\right) x(k)\left(L^T+L\right)x(k)\\ +\left(\frac{2\gamma^2\|L\|^2}{\underline{\psi}} + \frac{\gamma}{\mu}\right) r(k)^T r(k)\end{pmatrix}.
\end{aligned} \tag{26}$$

In the following, we compute $V_2(k+1)$. As $\omega_i(k) \sim Lap(0,\, b_i(k))$, $b_i(k) = \frac{1}{k^q}$, $0.5 < q < 1$, $w_i(0) \sim Lap(0,\, 1)$ and $E\left(\omega(k)\right) = 0$, $V_2(k+1)$ can be rewritten as

$$\begin{aligned}
V_2(k+1) &= E\left(\omega(k)^T P(k) \omega(k)\right)\\
&\leq \frac{1}{\underline{\psi}} NE\left(\omega_i(k)^T \omega_i(k)\right)\\
&= \frac{1}{\underline{\psi}} N\mathrm{var}\left(\omega_i(k)\right) = \frac{2N}{\underline{\psi}} {b_j}^2 = \frac{2N}{\underline{\psi} k^{2q}}.
\end{aligned} \tag{27}$$

Based on trigger condition (4), we have $\|r_i(k)\| \leq \beta\sqrt{\sum_{j=1}^{N} a_{ij}\left(\hat{x}_j(k)-\hat{x}_i(k)\right)} + \alpha\sqrt{\exp\left(-k\right)}$. As the graph is undirected, $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\hat{x}_j(k)-\hat{x}_i(k)\right) = 0$, we have

$$\sum_{i=1}^{N}\|r_i(k)\|^2 \leq 2\sum_{i=1}^{N}\alpha^2\exp(-k) = 2N\alpha^2\exp(-k). \tag{28}$$

Combining (26), (27) and (28), we have

$$\begin{aligned}
&E\left(\Delta V\left(k+1\right)\right)\\
&\leq E\left(-\left(\frac{\gamma}{4} - \frac{2\gamma^2}{\underline{\psi}\mu}\right) x(k)\left(L^T+L\right)x(k)\right)\\
&\quad + E\left(\bar{e} + \frac{2N}{\underline{\psi}}{b_j}^2\right),
\end{aligned} \tag{29}$$

where $\bar{e} = \sum_{j=0}^{k}\left(\frac{2\gamma^2\|L\|^2}{\underline{\psi}} + \frac{\gamma}{\mu}\right) 2N\alpha^2\exp\left(-k\right)$.

As the boundedness of $V(k+1)$ ensures the boundedness of $x(k+1)$ and from (29), $E\left(\Delta V(k+1)\right)$ can be rewritten as the form of $E\left(V(k+1)-V(k)\right) \leq E(-W(x(k))+\delta(n)+\omega(n))$, where $W(x(k)) = \left(\frac{\gamma}{4} - \frac{2\gamma^2}{\underline{\psi}\mu}\right)$

$x(k)\left(L^T+L\right)x(k)$, $0<\gamma<\frac{\underline{\psi}\mu}{8}$ is a nonnegative function, based on Lemma 1, we have

$$\begin{aligned}E(V(k+1)) &\le E(V(0)+\bar{e})+1+\sum_{j=0}^{k}\frac{2N}{\underline{\psi}}b_j{}^2\\ &\le E(V(0)+\bar{e})+\eta+1\\ &\le E(V(0)+\bar{\delta}).\end{aligned}\tag{30}$$

For $2q>1$, we can obtain that $\sum_{j=1}^{k}\frac{1}{j^{2q}}\le\eta$ is bounded and $\sum_{j=0}^{k}b_j{}^2=1+\sum_{j=1}^{k}\frac{1}{j^{2q}}\le\eta+1$ is bounded, so $\bar{\delta}=\bar{e}+\eta+1$ is bounded. It is easy to get $\lim_{k\to\infty}W(x(k))=\lim_{k\to\infty}-\left(\frac{\gamma}{4}-\frac{2\gamma^2}{\underline{\psi}\mu}\right)x(k)\left(L^T+L\right)x(k)=0$, namely, $\lim_{k\to\infty}E(Lx(k))=0$, $\lim_{k\to\infty}E(x_i(k))=E(x_\infty)=x_\infty=x^*$. Next, we compute the convergence rate. Based on (29) and [23] Proposition 2, there exists a positive constant ρ such that

$$\begin{aligned}&E(\Delta V(k+1))\\ &=E(V(k+1)-V(k))\\ &\le E\left(-\rho\left(\frac{\gamma}{4}-\frac{2\gamma^2}{\underline{\psi}\mu}\right)V(k)\right)\\ &\quad+E\left(\left(\frac{2\gamma^2\|L\|^2}{\underline{\psi}}+\frac{\gamma}{\mu}\right)2N\alpha^2\exp(-k)\right)\\ &\quad+\frac{1}{\underline{\psi}}2N\sum_{j=0}^{k}b^2,\end{aligned}\tag{31}$$

$$\begin{aligned}&E(V(k+1))\\ &\le E\left(\left(1-\rho\left(\frac{\gamma}{4}-\frac{2\gamma^2}{\underline{\psi}\mu}\right)\right)V(k)\right)\\ &\quad+E\left(\left(\frac{2\gamma^2\|L\|^2}{\underline{\psi}}-\frac{\gamma}{\mu}\right)2N\alpha^2\exp(-k)\right)\\ &\quad+\frac{1}{\underline{\psi}}2N\sum_{k=1}^{k}\frac{1}{k^{2q}}.\end{aligned}\tag{32}$$

Let $\varphi=\left(1-\rho\left(\frac{\gamma}{4}-\frac{2\gamma^2}{\underline{\psi}\mu}\right)\right)<1$, $\chi=\left(\frac{2\gamma^2\|L\|^2}{\underline{\psi}}-\frac{\gamma}{\mu}\right)2N\alpha^2$, $\eta(k)=\frac{1}{\underline{\psi}}2N\sum_{j=0}^{k}\frac{1}{k^{2q}}=1+\sum_{j=1}^{k}\frac{1}{j^{2q}}$, one has

$$\begin{aligned}&E(V(k+1))\\ &\le E(\varphi V(k)+\chi\exp(-k)+\eta(k))\\ &\le E\left(\varphi^{k+1}V(x(0))+\sum_{j=0}^{k}\varphi^{k-j}\eta(j)+\chi\sum_{j=0}^{k}\frac{\varphi^{k-j}}{e^j}\right).\end{aligned}\tag{33}$$

Combining the $\sum_{i\in\mathcal{V}}\frac{\psi_i}{2}\|x_i-x^*\|^2\le V(k)$ [23, Proposition 2] and (33), we have

$$\begin{aligned}&E\left(\sum_{i\in\mathcal{V}}\|x_i(k+1)-x^*\|^2\right)\\ &\le E\left(\varphi^{k+1}V(x(0))+\sum_{j=0}^{k}\varphi^j\eta(k-j)\right.\\ &\quad\left.+\chi\sum_{j=0}^{k}\frac{\varphi^j}{e^{k-j}}\right).\end{aligned}\tag{34}$$

The convergence rate is

$$\begin{aligned}rate&=\frac{\sum_{i\in V}E\|x_i(k+1)-x^*\|^2}{\sum_{i\in V}E\|x_i(k)-x^*\|^2}\\ &=E\left(\frac{\varphi^{k+1}V(x(0))+\sum_{j=0}^{k}\varphi^{k-j}\eta(j)+\chi\sum_{j=0}^{k}\frac{\varphi^{k-j}}{e^j}}{\varphi^kV(x(0))+\sum_{j=0}^{k-1}\varphi^{k-j}\eta(j)+\chi\sum_{j=0}^{k-1}\frac{\varphi^{k-j}}{e^j}}\right)\\ &=\left(1-\rho\left(\frac{\gamma}{4}-\frac{2\gamma^2}{\underline{\psi}\mu}\right)\right).\end{aligned}\tag{35}$$

The proof is completed. □

4.2. Privacy analysis

In this subsection, the differential private analysis is presented.

Lemma 3: If $0<q<1$ and $0<a<1$ are satisfied, one can obtain $\lim_{K\to\infty}\sum_{k=1}^{K}k^qa^K\le\frac{a}{(1-a)^2}$.

Proof: Expanding $\sum_{k=1}^{K}k^qa^k$, one has

$$\begin{aligned}\sum_{k=1}^{K}k^qa^k=&1^qa^1+2^qa^2+\ldots+(K-1)^qa^{K-1}\\ &+K^qa^K.\end{aligned}\tag{36}$$

By (36), one has

$$\begin{aligned}a\sum_{k=1}^{K}k^qa^k=&1^qa^2+2^qa^3+\ldots+(K-1)^qa^K\\ &+K^qa^{K+1}.\end{aligned}\tag{37}$$

Let $m(x)=(x+1)^q-x^q$, where $0<q<1$. It is easy to see $m(0)=1$. Taking the derivative of $m(x)$, one has

$$m'(x)=q(x+1)^{q-1}-qx^{q-1}.\tag{38}$$

As $0<q<1$ and $\frac{x}{x+1}<1$, we have $\frac{1}{(x+1)^{1-q}}-\frac{1}{x^{1-q}}<0$ and $m'(x)<0$, which indicates the function $m(x)$ is a decreasing function. It is not hard to obtain that the maximum value of $m(x)$ in $x \geq 0$ is $m(0) = 1$. For $0 < q < 1$, based on the property of $m(x)$, we can obtain $0 < (k+1)^q - k^q < 1$, then we have

$$\begin{aligned}
&\sum_{k=1}^{K} k^q a^k - a \sum_{k=1}^{K} k^q a^k \\
&= 1^q a^1 + \left(2^q a^2 - 1^q a^2\right) + \ldots \\
&\quad + \left((k-1)^q a^{K-1} - (k-2)^q a^{K-1}\right) \\
&\quad + \left(k^q a^K - (k-1)^q a^K\right) - k^q a^{K+1} \\
&\leq a^1 + a^2 + \ldots + a^K = \frac{a\left(1-a^K\right)}{1-a}.
\end{aligned} \tag{39}$$

For $0 < a < 1$, one can obtain

$$\lim_{K\to\infty} \frac{a\left(1-a^K\right)}{1-a} = \frac{a}{(1-a)}. \tag{40}$$

Then, we have

$$\lim_{K\to\infty} \sum_{k=1}^{K} k^q a^K = \frac{a}{(1-a)^2}. \tag{41}$$

which completes the proof.

Theorem 3: If $\omega_i(k) \sim Lap(0, b_i(k))$, where $b_i(k) = \frac{1}{k^q}, b_i(0) = 1$, $0.5 < q < 1$, the algorithm (2) satisfies ε-differential privacy. Moreover, the sensitivity is $\Delta(k) = a\Psi_i\left(1-\gamma\frac{1}{\underline{\psi}}l_{ii}\right)^k$ and the level of privacy preserving is $\varepsilon = \frac{1}{1-\eta}a(\Psi_i - \gamma l_{ii})$, where $\eta = \left(1-\gamma\frac{1}{\underline{\psi}}l_{ii}\right)$, $0 < \gamma < \frac{\underline{\psi}}{l_{ii}}$, $a = |x_i(0) - x_i{}'(0)|$, $l_{ii} = L_{ii}$.

Proof: The sensitivity of (3) as follows:

$$\Delta = \max_{D \simeq D'} \|f(D) - f(D')\|_1.$$

As $f(D)$ is $\nabla h_i(x_i)$ and $\nabla h(y)$ is convex function, based on the property of convex function, one can obtain

$$\begin{aligned}
\Delta(k+1) &= \|\nabla h_i(x_i(k+1)) - \nabla h_i(x_i{}'(k+1))\|_1 \\
&\leq \Psi_i \|x_i{}'(k+1) - x_i(k+1)\|_1 \\
&= \Psi_i \left\| x_i{}'(k) - x_i(k) - \gamma\frac{1}{\underline{\psi}}l_{ii}(x_i{}'(k) - x_i(k)) \right\|_1 \\
&= a\Psi_i \left(1 - \gamma\frac{1}{\underline{\psi}}l_{ii}\right)^{k+1},
\end{aligned} \tag{42}$$

where $a = |x_i{}'(0) - x_i(0)|$.

As $x_i(k)$ and $x_j(k)$ are independent of each other, according to the definition of differential privacy, one can obtain

$$e^{\varepsilon_k} = \frac{P[y(k) \in S]}{P[y'(k) \in S]} = \frac{P[y_i(k) \in S]}{P[y'_i(k) \in S]}. \tag{43}$$

As

$$\begin{aligned}
&P[y_i(k) \in S] \\
&= P[S - \nabla h_i(x_i(k)) + \gamma l_{ii}(x_i(k) + r_i(k)) \in \omega(k)] \\
&= \lim_{k\to\infty} \int (S - \nabla h_i(x_i(k)) + \gamma l_{ii}(x_i(k) + r_i(k)))\, d\omega(k),
\end{aligned} \tag{44}$$

and

$$\begin{aligned}
&P[y_i{}'(k) \in S] \\
&= P[S - \nabla h(x_i(k)) + \gamma l_{ii}(x_i{}'(k) + r_i(k)) \in \omega(k)] \\
&= \lim_{k\to\infty} \int (S - \nabla h(x_i{}'(k)) + \gamma l_{ii}(x_i{}'(k) + r_i(k)))\, d\omega(k).
\end{aligned} \tag{45}$$

Combining (43), (44) and (45), the following equality can be obtained

$$\begin{aligned}
&\prod_{k=0}^{\infty} \frac{P[y_i(k) \in S]}{P[y'_i(k) \in S]} \\
&= \prod_{k=0}^{\infty} \frac{\int (S - \nabla h_i(x_i(k)) + \gamma l_{ii}(x_i(k) + r_i(k)))\, d\omega(k)}{\int (S - \nabla h_i(x_i{}'(k)) + \gamma l_{ii}(x_i{}'(k) + r_i(k)))\, d\omega(k)} \\
&= e^{\sum_{k=0}^{\infty}\left(\frac{\left|-\nabla h_i\left(x_i(k)\right) + \gamma l_{ii} x_i(k) - \left(-\nabla h_i\left(x_i{}'(k)\right) + \gamma l_{ii} x_i{}'(k)\right)\right|}{b_k}\right)} \\
&\leq e^{\sum_{k=0}^{\infty} \varepsilon_k} = e^{\varepsilon},
\end{aligned} \tag{46}$$

where $L = D - W$, $l_{ii} = L_{ii}$, $0 < l_{ii} < 1$. From [20] (adaptive sequential composition), we can obtain $\varepsilon = \sum_k \varepsilon_k$, then, we have

$$\begin{aligned}
\varepsilon &= \sum_{k=0}^{\infty} \varepsilon_k \\
&= \sum_{k=0}^{\infty} k^q |\nabla h(x_i{}'(k)) - \nabla h(x_i(k)) - \gamma l_{ii}(x_i{}'(k) - x_i(k))| \\
&\leq \sum_{k=0}^{\infty} k^q \left| \left(1 - \gamma\frac{1}{\underline{\psi}}l_{ii}\right)^k (\Psi_i - \gamma l_{ii})\left((x_i{}'(0) - x_i(0))\right) \right|.
\end{aligned} \tag{47}$$

Let $\eta = \left(1 - \gamma\frac{1}{\underline{\psi}}l_{ii}\right)$, where $0 < \gamma < \frac{\underline{\psi}}{l_{ii}}$. From Lemma 3, we can obtain $\lim_{K\to\infty} \sum_{k=1}^{K} k^q \eta^K = \frac{\eta}{(1-\eta)^2}$, then, we have

$$\varepsilon = \lim_{k\to\infty} \sum_{0}^{k} \varepsilon_k = a(\Psi_i - \gamma l_{ii}) \frac{\eta}{(1-\eta)^2}. \tag{48}$$

The proof is completed. □

5. SIMULATION

In this section, a example is given to verify the obtained result. Consider a mutli-agent network with seven nodes. The topology is shown in Fig. 1. The adjacent matrix is

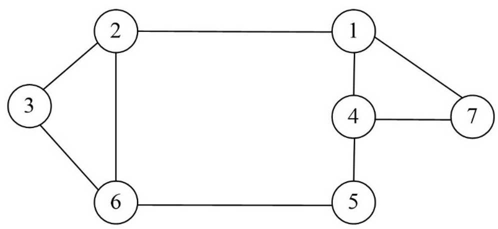

Fig. 1. The communication topology of multi-agent network.

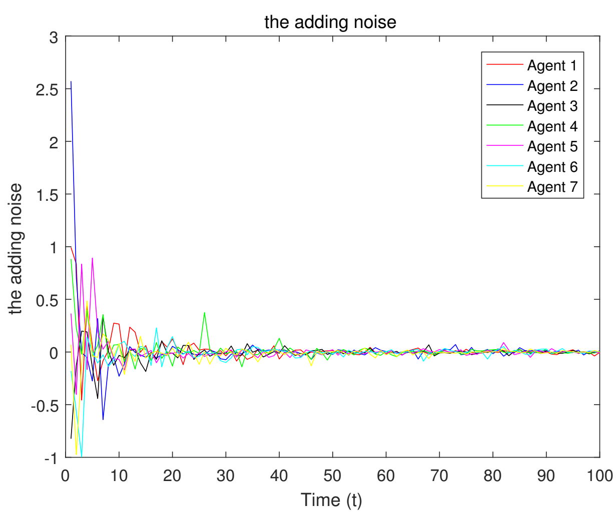

Fig. 2. The Laplace noise.

$$\mathcal{A}=[a_{ij}]=\frac{1}{7}\begin{bmatrix}0&1&0&1&0&0&1\\1&0&1&0&0&1&0\\0&1&0&0&0&1&0\\1&0&0&0&1&0&1\\0&0&0&1&0&1&0\\0&1&1&0&1&0&0\\1&0&0&1&0&0&0\end{bmatrix}.$$

Let $i=1$, $l_{ii}=0.43$ parameters $\psi=1$, $\Psi_i=1.1$, $q=0.9$, $\gamma=1$, $a=0.25$, $\bar{a}=0.25$, $yinital=[5.43, 3.98, 2.45, 3.68, 6.20, 1.79, 2.1]$, $h_i(x)=\frac{1}{2}(x-y_i)^2$. By calculation, one can obtain that the optimal point $x^*=3.66$. The noise distribution is shown in Fig. 2. The trajectory of agents is presented in Fig. 3, which illustrates that agents converge to the optimal value x^*. Fig. 4 indicates the trajectory of $\varepsilon=\sum_k \varepsilon_k$. Fig. 5 is the trigger instant. Fig. 6 is privacy level. It is found that when $k\to\infty$, the value of ε tends to stable and $\lim_{k\to\infty}\varepsilon_k=0$, which means the differential privacy preserving is achieved.

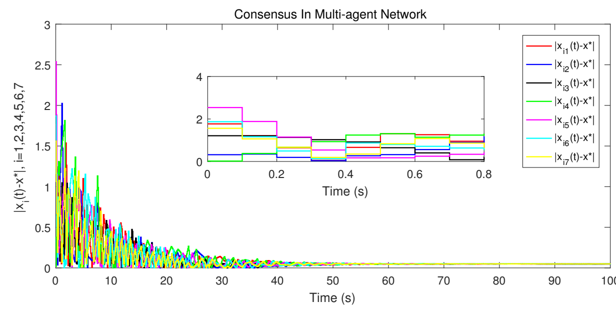

Fig. 3. Consensus in multi-agent network.

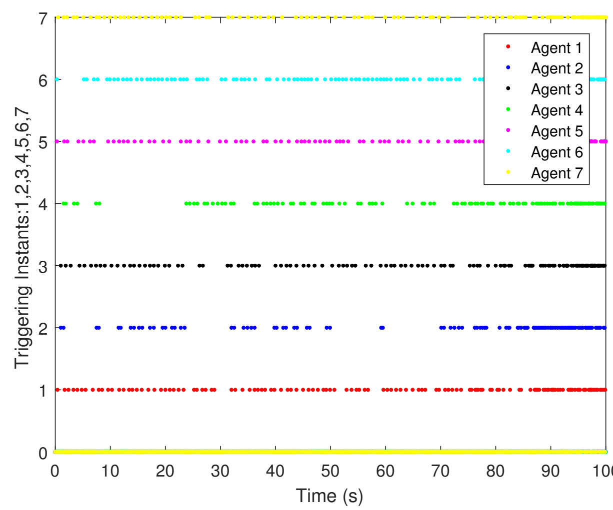

Fig. 4. Trigger instant.

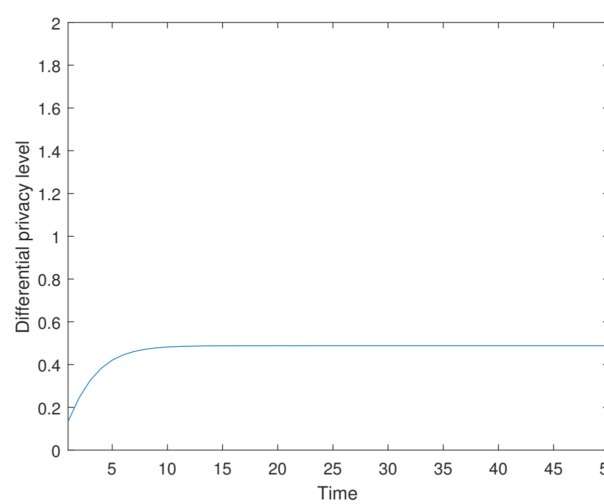

Fig. 5. Privacy preserving level.

6. CONCLUSION

The security problem of cooperative control is that the adversary can obtain agents privacy information easily because agents in the group exchange their information on the public channel, which may lead to sensitive information disclosure. To handle this problem, in this paper, based on differential privacy scheme, a novel event-triggered differentially private optimal consensus algorithm is proposed to preserve the privacy of the cost function of each agent in the whole process of consensus computation. Based on the event-trigger condition, we analyze the consensus of our algorithm in detail, including the accuracy and consensus conditions. In addition, the privacy-preserving analysis is also given, which exhibits that privacy of the states of all agents is guaranteed to preserve. The privacy level and the sensitivity of the differential privacy are also obtained. Finally, simulations are presented to illustrate the results.

Our future work is to study on choosing the privacy budget parameter in differential privacy consensus algorithm.

CONFLICTS OF INTEREST

No conflict of interest exits in the submission of this manuscript, and manuscript is approved by all authors for publication.

DATA AVAILABILITY STATEMENT

All data, models, and code generated or used during the study appear in the submitted article.

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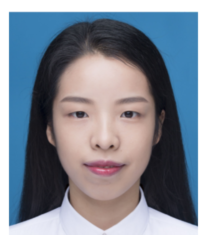

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