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Periodic Event-Trigger Control for Continuous-Time LPV Systems*

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Abstract— **This paper proposes a periodic event-triggered control (PETC) strategy for continuous-time linear parameter-varying (LPV) systems. Considering a sampled-data LPV control law and a parameter-dependent looped-functional to deal with the sampling effects, LMI conditions to design the parameters of the triggering criterion leading to the stability of the closed-loop system under the PETC strategy are proposed. These conditions are then incorporated into a convex optimization problem in order to design the triggering rule parameters aiming at reducing the frequency of control signal updates. A numerical example illustrates the proposed method.**

Index Terms— **LPV systems, periodic event-triggered control, looped-functional, sampled-data control.**

I. Introduction

The study of linear parameter-varying (LPV) systems has got great attention from the control community as can be seem for instance in [1], [2] and the references therein. LPV systems consist of a linear system with some parameters that change over time and this setup can naturally model many physical systems. Besides that, an LPV system can be used to represent some uncertain systems and as approximations to some classes of non-linear systems via the so-called quasi-LPV form.

On the other hand, the new paradigm of Networked Control Systems (NCS), i.e. the implementation of the control loop over communication networks, leads to reductions in cost and increases flexibility ([3], [4]), but it brings issues regarding bandwidth consumption ([5], [6]). One approach to deal with these issues is the event-triggered control (ETC), which consists of sending information over the network and updating the control signal only when a criterion based on the system states or outputs is verified ([3], [7], [8]). This criterion is called the triggering criterion and it can be monitored continuously as in [8] or periodically as in [9]–[14]. This second approach, called as periodic event-triggered control (PETC), is more suitable to be implemented in digital platforms.

PETC, as well as ETC, effectively reduces the number of control updates and in consequence, the frequency and amount of messages transmitted over the network, but it introduces challenges regarding the stability of the closed-loop system. Thus, one central question about designing event-triggered controllers is to establish conditions that ensure the stability of the closed-loop system under the aperiodic updates imposed by the triggering criterion.

The literature on PETC seems to start with [9]. More recently, [10] deals with output-based PETC for perturbed nonlinear systems in an emulation design context and employs a hybrid system formalism to obtain stability conditions in linear matrix inequality (LMI) form. Also considering emulation design, [11] presents an output-based PETC solution with performance guarantees expressed as an average quadratic cost. In [15], a dynamic triggering criterion is proposed as means to reduce the number of events and non-monotonic Lyapunov functions are employed to certify the closed-loop stability, but requiring an upper-bound for the inter-event times. Again, only emulation design is covered. Experimental results for a robotic manipulator are shown in [16]. This work takes into account the nonlinear dynamics of the system and achieves practical stabilization of the trajectories, i.e., the trajectory errors are guaranteed to converge to a ball around the origin. Wang et al. [17] investigate the problem of PETC when there are multiple asynchronous networks involved, e.g. separate networks for sensors-controller and controller-plant paths. A hybrid system formalism and a novel hybrid Lyapunov function are employed to obtain stability conditions in LMI form. On the other hand, [12] and [13] address PETC in the context of systems subject to input saturation and with limited information, respectively. These two works propose LMI-based conditions to certify asymptotic stability and convex optimization problems as means of tuning the controller and the trigger criterion.

In the present paper, we address periodic event-triggered controllers for LPV systems. A key issue in this case regards the fact that the parameter in the plant evolves continuously, while the controller parameter is kept constant at the value measured at the last event instant. The literature on this subject is scarce and only a few results are available, as can be seen in [14], [18]–[23] and the references therein. Works [14] and [18] address the co-design of state-feedback controllers and also provide insights on the tracking of constant references with the addition of integral action to the controller. On the other hand, [19] studies the use of PETC in the scenario of fault detection and isolation for LPV systems. The emulation and co-design problems are addressed in [20], considering a class of switched LPV systems suitable for aircraft engines. The use of PETC and linear parameter-varying model predictive control to solve the consensus problem

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in a multi-agent mobile robot system is explored in [21]. The co-design problem is addressed in [23], which employs parameter-dependent Lyapunov-Krasovskii functionals to obtain stability conditions and proposes convex optimization problems as means to tune the trigger rule. One important limitation present in all these references is that, differently from the present work, they consider only discrete-time LPV systems. This is not realistic in many practical cases, where the system dynamics evolves in continuous-time. In this case, the mismatch between the value of the parameter varying continuously in the plant and the one used in the controller, which is kept constant between two consecutive sampling instants, should be explicitly considered.

In this work, we are particularly interested in the emulation design of periodic event-triggered controllers under LPV state feedback control laws, considering that the plant state evolves continuously in time. We assume that the time-varying parameters are measurable and bounded in amplitude and rate. The approach is based on a parameter dependent looped-functional to deal with the periodic sampling issue and a relative error event-triggering mechanism. From these elements, we derive LMI conditions to ensure the asymptotic stability of the closed-loop system under the ETC strategy. These conditions are then incorporated into a convex optimization problem proposed as means to compute the triggering criterion parameters aiming at reducing the number of control updates with respect to a periodic (time-triggered) implementation of the controller. The potentialities of the proposed method are illustrated by numerical examples.

Notation. The sets $\mathbb{N}$, $\mathbb{R}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{S}^n$ denote respectively the set of non-negative integer numbers, n-dimensional vectors, $m \times n$ matrices with real entries and symmetric matrices of $\mathbb{R}^{n\times n}$. For a given positive scalar, T, $\mathcal{F}^n_{[0,T]}$ is the set of continuous functions from an interval $[0,\ T]$ to $\mathbb{R}^n$. $P > 0$ for $P \in \mathbb{S}^n$ means that P is positive definite and He$\{A\}$ refers to $A + A'$. $\otimes$ denotes Kronecker product and the shortcut of $\phi\otimes I$ is represented by $\Lambda(\phi)$. I and 0 represent the identity and the zero matrices of appropriate dimension. $Co\{\cdot\}$ and Ver$(\mathcal{B})$ denote a convex hull and the set of the vertices of set $\mathcal{B}$, respectively. $A_{(i)}$ and $x_{(i)}$ represent the i-th line of the matrix A and the i-th element of the vector x.

II. Problem Statement

The following continuous-time LPV system is considered:

$$\dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the vectors of state and control input, respectively. The vector of N time-varying parameters is represented by $\sigma(t) = [\sigma_1(t)\ \sigma_2(t)\ \ldots\ \sigma_N(t)]' \in \mathbb{R}^N$, where each parameter is bounded in magnitude and in time-derivative, such that:

$$\begin{aligned}\sigma(t) &\in \mathcal{B}_\sigma = \{\sigma \in \mathbb{R}^N; \underline{\sigma}_{(j)} \le \sigma_{(j)} \le \overline{\sigma}_{(j)}, j = 1,\ldots,N\},\\ \dot{\sigma}(t) &\in \mathcal{B}_{\dot{\sigma}} = \{\dot{\sigma} \in \mathbb{R}^N; \underline{\xi}_{(j)} \le \dot{\sigma}_{(j)} \le \overline{\xi}_{(j)}, j = 1,\ldots,N\}.\end{aligned} \tag{2}$$

It is assumed that u is given by an LPV state-feedback control law. Moreover, we suppose that states x and parameters σ are available for measurement only at periodic sampling instants t_k, where $t_k = kT$, with $k = 0, 1, 2, \ldots, \infty$ and $T > 0$ being the sampling period.

Since we consider a PETC strategy to implement the control loop, at each sampling instant $t = t_k$, the occurrence of an event is determined by an event-trigger generator. The triggering times are defined as $\tilde{t}_e$, with $e \in \mathbb{N}$ and $\tilde{t}_0 = 0$. If an event occurs at t_k, we define $\tilde{t}_e = t_k$ and the control signal applied to the plant is updated and kept constant until the next event. Note that $\tilde{t}_{e+1} = \tilde{t}_e + cT$, with some $c \in \mathbb{N}$, where $c \ge 1$.

Then, the control law is assumed to be given by

$$u(t) = u(\tilde{t}_e) = K(\sigma(\tilde{t}_e))x(\tilde{t}_e), \forall t \in [\tilde{t}_e, \tilde{t}_{e+1}), \forall e \in \mathbb{N}, \tag{3}$$

where $K(\sigma(\tilde{t}_e)) : \mathbb{R}^N \to \mathbb{R}^{m\times n}$ is a parameter-dependent gain matrix.

In this paper, we consider the following assumption.

Assumption 1: Matrices $A(\sigma(t))$ and $B(\sigma(t))$ depend affinely on $\sigma(t)$, while $K(\sigma(\tilde{t}_e))$ depends affinely on the sampled parameter.

From (1) and (3), the closed-loop system dynamics is given by

$$\dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))K(\sigma(\tilde{t}_e))x(\tilde{t}_e), \forall t \in [\tilde{t}_e, \tilde{t}_{e+1}). \tag{4}$$

From the above setup, the following problem is addressed in this paper.

Problem 1: Considering the LPV system (1) and the LPV control law (3), devise a periodic event-trigger control strategy such that closed-loop system (4) is asymptotically stable, while the control updates are reduced in comparison to a periodic time-triggered implementation.

III. Event-trigger Strategy

In order to address Problem 1, recalling that $t_k = kT$, we define the triggering function $l : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ as

$$l(x(t_k), \delta(t_k)) = \delta'(t_k)Q_\delta\delta(t_k) - x'(t_k)Q_x x(t_k), \tag{5}$$

where Q_δ and $Q_x \in \mathbb{S}^n$ are positive definite matrices and $\delta(t_k)$ is the difference between the state value at the last event $\tilde{t}_e$ and the current sampled value at t_k, i.e.

$$\delta(t_k) = x(\tilde{t}_e) - x(t_k), \text{ for } t_k \in [\tilde{t}_e, \tilde{t}_{e+1}). \tag{6}$$

Then the triggering times are given by the following rule:

$$\tilde{t}_{e+1} = min\{t_k > \tilde{t}_e;\ l(\delta(t_k), x(t_k)) > 0\}. \tag{7}$$

Considering $\delta(t_k)$ defined in (6), it follows that the closed-loop system (4) can be rewritten as:

$$\begin{aligned}\dot{x}(t) = &\ A(\sigma(t))x(t) + B(\sigma(t))K(\sigma(\tilde{t}_e))x(t_k)\\ &+ B(\sigma(t))K(\sigma(\tilde{t}_e))\delta(t_k),\ \forall t \in [t_k, t_{k+1}).\end{aligned} \tag{8}$$

In the following section, considering a looped-functional approach [24], conditions are proposed to ensure the asymptotic stability of the closed-loop system (8) under the PETC strategy (7). In this formulation, the continuous-time evolution of states and parameters in the plant are taken into account, while their sampled values are considered for control purposes. Differently from the approaches in the literature [14], [18], [25], note that we do not consider a discrete-time model of the LPV system (1).

A. Looped-functional Approach

As in [24] and [26], define $x_k(\tau) = x(t_k+\tau)$ and $\sigma_k(\tau) = \sigma(t_k+\tau)$ with $\tau \in [0,T]$. Moreover, denote $\delta_k(0) = \delta(t_k)$ and the latest sampled parameter used to update the control signal as $\sigma_e(0) = \sigma(\tilde{t}_e)$, for $t \in [\tilde{t}_e, \tilde{t}_{e+1})$. Hence, for $t \in [t_k, t_{k+1})$ the closed-loop dynamics is given by:

$$\begin{aligned}\dot{x}_k(\tau) = A(\sigma_k(\tau))x_k(\tau) + B(\sigma_k(\tau))K(\sigma_e(0))x_k(0) \\ +B(\sigma_k(\tau))K(\sigma_e(0))\delta_k(0),\ \tau \in [0,T).\end{aligned} \tag{9}$$

Based on a looped-functional approach (see [24] and [27] for more details), the following theorem is proposed to assess the asymptotic stability of closed loop system (8) considering the proposed event-triggering strategy with the triggering rule (7).

Theorem 1: Consider a parameter dependent function (PDF) $V : \mathbb{R}^n \times \mathbb{R}^N \to \mathbb{R}^+$ and a parameter dependent looped-functional (PDLF) $\mathcal{V}_0 : [0,T] \times \mathcal{F}^n_{[0,T]} \times \mathcal{F}^N_{[0,T]}$, that satisfy

$$\mu_1||x||^2 \leq V(x,\sigma) \leq \mu_2||x||^2, \tag{10}$$

$$\mathcal{V}_0(0, x_k, \sigma_k) = \mathcal{V}_0(T, x_k, \sigma_k), \tag{11}$$

for all σ satisfying (2), with μ_1 and $\mu_2 > 0$.

Define the functional

$$\mathcal{W}(\tau, x_k, \sigma_k) = V(x_k(\tau), \sigma_k(\tau)) + \mathcal{V}_0(\tau, x_k, \sigma_k),$$

and let $\dot{\mathcal{W}}$ be the derivative of $\mathcal{W}$ with respect to τ. If the following inequality

$$\dot{\mathcal{W}}(\tau, x_k, \sigma_k) - l(x_k(0), \delta_k(0)) \leq -\beta||x_k(0)||^2 \tag{12}$$

is satisfied along the trajectories of (9) for $\tau \in [0,T]$, $\forall k \in \mathbb{N}$, for some scalar $\beta \geq 0$, then

(i) $\Delta V_k = V(x(t_{k+1}), \sigma(t_{k+1})) - V(x(t_k), \sigma(t_k)) \leq -\beta T||x(t_k)||^2$, $\forall k \in \mathbb{N}$;

(ii) the closed-loop system (8) is asymptotically stable under event-triggering strategy given by (7).

Proof: Recall that if at instant t_k an event is not generated, it means that $l(x(t_k), \delta(t_k)) = l(x_k(0), \delta_k(0)) \leq 0$ and, from (12), we have that $\dot{\mathcal{W}}(\tau, x_k, \sigma_k) \leq -\beta||x_k(0)||^2$. Then integrating over any interval $[t_k, t_{k+1}]$, it follows that $\Delta V_k = V(x(t_{k+1}), \sigma(t_{k+1})) - V(x(t_k), \sigma(t_k)) \leq -\beta T||x(t_k)||^2$, since $\mathcal{V}_0$ satisfies (11). Consider now that at instant t_k an event is generated. In this case $\delta_k(0)$ is set to zero and (12) reads $\dot{\mathcal{W}}(\tau, x_k, \sigma_k) \leq -x_k(0)Q_x x_k(0) - \beta||x_k(0)||^2 \leq -\beta||x_k(0)||^2$. Then, applying the same reasoning above, i.e. integrating over any interval $[t_k, t_{k+1}]$ and considering (11), it follows that item (i) is verified $\forall k \in \mathbb{N}$. Thus $\lim_{k\to\infty} x_k(0) \to 0$. Since system (1) is a linear time varying one, we can use a development similar to the one in [26] to conclude that $\lim_{t\to\infty} x(t) \to 0$, which implies item (ii). ∎

IV. Main Results

Considering the sets defined in (2), note that $\mathcal{B}_\sigma$ and $\mathcal{B}_{\dot{\sigma}}$ are convex polytopes in $\mathbb{R}^N$ with 2^N vertices, i.e.

$$\mathcal{B}_\sigma = Co\{s_1, s_2 \ldots, s_{2^N}\}, \mathcal{B}_{\dot{\sigma}} = Co\{d_1, d_2 \ldots, d_{2^N}\}. \tag{13}$$

Then $\sigma(t)$, $\sigma(\tilde{t}_e)$ and $\dot{\sigma}(t)$ can be represented by a convex combination of the vertices of polytopes defined in (13), that means:

$$\begin{gathered}\sigma(t) = \sum_{f=1}^{2^N} \lambda_f(t)s_f, \quad \sigma(\tilde{t}_e) = \sum_{g=1}^{2^N} \eta_g(\tilde{t}_e)s_g, \\ \dot{\sigma}(t) = \sum_{h=1}^{2^N} \mu_h(t)d_h,\end{gathered} \tag{14}$$

with $\lambda_f(t)$, $\eta_g(\tilde{t}_e)$, $\mu_h(t) \geq 0$ and $\sum_{f=1}^{2^N} \lambda_f(t) = \sum_{g=1}^{2^N} \eta_g(\tilde{t}_e) = \sum_{h=1}^{2^N} \mu_h(t) = 1$.

Therefore, from Assumption 1, it is possible to re-write matrices $A(\sigma(t))$, $B(\sigma(t))$ and $K(\sigma(\tilde{t}_e))$ as follows:

$$A(\sigma(t)) = A_0 + \mathbf{A}\Lambda(\sigma_k(\tau)) = A_0 + \mathbf{A}\sum_{f=1}^{2^N} \lambda_f \Lambda(s_f),$$

$$B(\sigma(t)) = B_0 + \mathbf{B}\Lambda(\sigma_k(\tau)) = B_0 + \mathbf{B}\sum_{f=1}^{2^N} \lambda_f \Lambda(s_f),$$

$$K(\sigma(\tilde{t}_e)) = K_0 + \mathbf{K}\Lambda(\sigma_e(0)) = K_0 + \mathbf{K}\sum_{g=1}^{2^N} \eta_g \Lambda(s_g),$$

where $\mathbf{A} = [A_1 \ldots A_N]$, $\mathbf{B} = [B_1 \ldots B_N]$, $\mathbf{K} = [K_1 \ldots K_N]$, with $A_j \in \mathbb{R}^{n\times n}$, $B_j \in \mathbb{R}^{n\times m}$ and $K_j \in \mathbb{R}^{m\times n}$ for $j = 0, \ldots, N$ and $\Lambda(s)$ is a shortcut to $s \otimes I$.

A. Stability Assessment Conditions

Based on the result of Theorem 1, we now propose LMI conditions to compute the triggering criterion parameters, i.e. the matrices Q_δ and Q_x, such that the closed-loop system (8) considering the event-triggering strategy proposed in (7) is asymptotically stable. With this aim, we consider a quadratic PDF defined as

$$V(x_k(\tau), \sigma_k(\tau)) = x_k'(\tau)P(\sigma_k(\tau))x_k(\tau), \tag{15}$$

with $P(\sigma_k(\tau)) = P_0 + \mathbf{P}\Lambda(\sigma_k(\tau))$, where $\mathbf{P} = [P_1 \ \ P_2 \ \ldots P_N]$ and $P(\sigma_k(\tau)) = P'(\sigma_k(\tau)) > 0$ $\forall \sigma_k(\tau) \in \mathcal{B}_\sigma$, and the following PDLF

$$\begin{aligned}\mathcal{V}_0(\tau, x_k, \sigma_k) = (T-\tau)\Big\{\tau x_k(0)'Xx_k(0) + (x_k(\tau) \\ - x_k(0))'[2G(\sigma_k(\tau))x_k(0) + F(\sigma_k(\tau))(x_k(\tau) - x_k(0))] \\ + \int_0^\tau \begin{bmatrix}\dot{x}_k(\theta) \\ x_k(0)\end{bmatrix}' R \begin{bmatrix}\dot{x}_k(\theta) \\ x_k(0)\end{bmatrix} d\theta\Big\},\end{aligned} \tag{16}$$

with $R \in \mathbb{S}^{2n}$, $R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}' & R_{22}\end{bmatrix} > 0$ and parameter dependent matrices $F(\sigma_k(\tau)) = F_0 + \mathbf{F}\Lambda(\sigma_k(\tau))$, $G(\sigma_k(\tau)) = G_0 + \mathbf{G}\Lambda(\sigma_k(\tau))$, where $\mathbf{F} = [F_1 \ \ldots \ F_N]$,

$\mathbf{G} = [G_1 \ \dots \ G_N]$, F_j and $X \in \mathbb{S}^n$, $G_j \in \mathbb{R}^{n\times n}$, for $j = 0, \dots, N$. Note that (15) and (16) satisfy (10) and (11), respectively.

Theorem 2: If there exist a symmetric positive definite matrix $R \in \mathbb{S}^{2n}$, symmetric matrices P_j, F_j, X, Q_δ and $\bar{Q}_x \in \mathbb{S}^n$, matrices $G_j \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{n\times 3n}$, $J_j \in \mathbb{R}^{3n\times n}$, $j = 0, 1, \dots, N$ satisfying

$$\begin{bmatrix} \Psi_1(s_f, s_g, d_h) & M_2' & \Psi_3(s_f, s_g) \\ * & -\bar{Q}_x & 0 \\ * & * & -Q_\delta \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} \Psi_2(s_f, s_g, d_h) & T(J_0 + \mathbf{J}\Lambda(s_f)) & M_2' & \Psi_3(s_f, s_g) \\ * & -TR_{11} & 0 & 0 \\ * & * & -\bar{Q}_x & 0 \\ * & * & * & -Q_\delta \end{bmatrix} < 0, \quad (18)$$

$$P_0 + \mathbf{P}\Lambda(s_f) > 0, \quad (19)$$

$\forall (s_f, s_g, d_h) \in \text{Ver}(\mathcal{B}_\sigma) \times \text{Ver}(\mathcal{B}_\sigma) \times \text{Ver}(\mathcal{B}_{\dot\sigma})$, with

$$\begin{aligned} \Psi_1(s_f, s_g, d_h) &= \Pi_1(s_f, d_h) + T\Pi_2(s_f, d_h) + TM_2'XM_2 \\ &\quad + \Pi_3(s_f, s_g) - He\{(J_0 + \mathbf{J}\Lambda(s_f))M_{12}\}, \\ \Psi_2(s_f, s_g, d_h) &= \Pi_1(s_f, d_h) - TM_2'XM_2 - TM_2'R_{22}M_2 \\ &\quad + \Pi_3(s_f, s_g) - He\{(J_0 + \mathbf{J}\Lambda(s_f))M_{12}\}, \\ \Psi_3(s_f, s_g) &= Y'(B_0 + \mathbf{B}\Lambda(s_f))(K_0 + \mathbf{K}\Lambda(s_g)), \end{aligned} \quad (20)$$

where

$$\begin{aligned} \Pi_1(s_f, d_h) &= He\{M_3'(P_0 + \mathbf{P}\Lambda(s_f))M_1 - M_{12}'R_{12}M_2 \\ &\quad - M_{12}'(G_0 + \mathbf{G}\Lambda(s_f))M_2\} - M_{12}'(F_0 + \mathbf{F}\Lambda(s_f))M_{12} \\ &\quad + M_1'\mathbf{P}\Lambda(d_h)M_1, \\ \Pi_2(s_f, d_h) &= M_{32}'RM_{32} + He\{M_3'(F_0 + \mathbf{F}\Lambda(s_f))M_{12} \\ &\quad + M_3'(G_0 + \mathbf{G}\Lambda(s_f))M_2 + M_{12}'\mathbf{G}\Lambda(d_h)M_2\} \\ &\quad + M_{12}'\mathbf{F}\Lambda(d_h)M_{12}, \\ \Pi_3(s_f, s_g) &= He\{Y'((A_0 + \mathbf{A}\Lambda(s_f))M_1 \\ &\quad + (B_0 + \mathbf{B}\Lambda(s_f))(K_0 + \mathbf{K}\Lambda(s_g))M_2 - M_3)\}, \end{aligned} \quad (21)$$

with the auxiliary matrices

$$\begin{aligned} M_1 = [I\ 0\ 0], \quad M_2 = [0\ I\ 0], \quad M_3 = [0\ 0\ I], \\ M_{12} = M_1 - M_2, \quad M_{32} = [M_3'\ M_2']', \end{aligned} \quad (22)$$

then, considering (5) with Q_δ and $Q_x = \bar{Q}_x^{-1}$, the closed-loop system (8) with periodic event-triggering strategy given by (7) is asymptotically stable.

Proof: Consider the PDF V and the PDLF $\mathcal{V}_0$ defined in (15) and (16), respectively. From convexity arguments, condition (19) ensures that $P(\sigma(t)) > 0$ for all $\sigma \in \mathcal{B}_\sigma$. Defining $\mathcal{X}(\tau) = [x_k'(\tau) \ \ x_k'(0) \ \ \dot{x}_k'(\tau)]'$ and differentiating $\mathcal{W}(\tau, x_k, \sigma_k) = V(x_k(\tau), \sigma_k(\tau)) + \mathcal{V}_0(\tau, x_k, \sigma_k)$ with respect to τ, we obtain[1]

$$\begin{aligned} \dot{\mathcal{W}} &= \mathcal{X}'(\tau)\Big[\Pi_1(\sigma_k(\tau), \dot\sigma_k(\tau)) + (T-\tau)\Pi_2(\sigma_k(\tau), \dot\sigma_k(\tau)) \\ &\quad + (T - 2\tau)M_2'XM_2\Big]\mathcal{X}(\tau) - \tau x_k'(0)R_{22}x_k(0) \\ &\quad - \int_0^\tau \dot{x}_k'(\theta)R_{11}\dot{x}_k(\theta)d\theta, \end{aligned} \quad (23)$$

with matrices $\Pi_1(\sigma_k(\tau), \dot\sigma_k(\tau))$ and $\Pi_2(\sigma_k(\tau), \dot\sigma_k(\tau))$ given in (21), and auxiliary matrices M_1, M_2, M_3, M_{12} and M_{32} defined in (22). Note that $\dot{P}(\sigma_k(\tau)) = \mathbf{P}\Lambda(\dot\sigma_k(\tau))$, $\dot{F}(\sigma_k(\tau)) = \mathbf{F}\Lambda(\dot\sigma_k(\tau))$ and $\dot{G}(\sigma_k(\tau)) = \mathbf{G}\Lambda(\dot\sigma_k(\tau))$.

Consider now a matrix $J(\sigma_k(\tau)) = J_0 + \mathbf{J}\Lambda(\sigma_k(\tau))$, where $\mathbf{J} = [J_1 \ \dots \ J_N]$, with $J_j \in \mathbb{R}^{3n\times n}$ for $j = 0, 1, \dots, N$. Since $R_{11} > 0$, we have that [27]

$$\begin{aligned} \int_0^\tau \dot{x}_k'(\theta)R_{11}\dot{x}_k(\theta)d\theta &\geq 2\mathcal{X}'(\tau)J(\sigma_k(\tau))(x_k(\tau) - x_k(0)) \\ &\quad - \tau\mathcal{X}'(\tau)J(\sigma_k(\tau))R_{11}^{-1}J'(\sigma_k(\tau))\mathcal{X}(\tau). \end{aligned} \quad (24)$$

We aim at providing LMI conditions to ensure (12). Then, considering the triggering function (5), the derivative of $\mathcal{W}$ in (23) and inequality in (24), it follows that

$$\begin{aligned} \dot{\mathcal{W}} - l(x_k(0), \delta_k(0)) &\leq \mathcal{X}'(\tau)\Big[\Pi_1(\sigma_k(\tau), \dot\sigma_k(\tau)) \\ &\quad + (T-\tau)\Pi_2(\sigma_k(\tau), \dot\sigma_k(\tau)) + (T - 2\tau)M_2'XM_2 \\ &\quad + \tau\big(J(\sigma_k(\tau))R_{11}^{-1}J'(\sigma_k(\tau)) - M_2'R_{22}M_2\big) \\ &\quad - He\{J(\sigma_k(\tau))M_{12}\} + M_2'Q_xM_2\Big]\mathcal{X}(\tau) - \delta_k'(0)Q_\delta\delta_k(0). \end{aligned} \quad (25)$$

From (9), for any matrix $Y \in \mathbb{R}^{n\times 3n}$ we have that

$$\begin{aligned} \mathcal{X}'(\tau)Y'(A(\sigma_k(\tau))x_k(\tau) + B(\sigma_k(\tau)K(\sigma_e(0))x_k(0) \\ + B(\sigma_k(\tau))K(\sigma_e(0))\delta_k(0) - \dot{x}_k(\tau)) = 0. \end{aligned} \quad (26)$$

Thus considering (25) and (26), it follows that

$$\dot{\mathcal{W}} - l(x_k(0), \delta_k(0)) \leq \begin{bmatrix} \mathcal{X}(\tau) \\ \delta_k(0) \end{bmatrix}' \Gamma(\tau, \sigma_k, \sigma_e, \dot\sigma_k) \begin{bmatrix} \mathcal{X}(\tau) \\ \delta_k(0) \end{bmatrix},$$

with

$$\Gamma(\tau, \sigma_k, \sigma_e, \dot\sigma_k) = \begin{bmatrix} \hat\Psi(\tau, \sigma_k, \sigma_e, \dot\sigma_k) + \Phi(\tau, \sigma_k) & \Psi_3(\sigma_k(\tau), \sigma_e(0)) \\ * & -Q_\delta \end{bmatrix},$$

where

$$\begin{aligned} \hat\Psi(\tau, \sigma_k, \sigma_e, \dot\sigma_k) &= \Pi_1(\sigma_k(\tau), \dot\sigma_k(\tau)) + \Pi_3(\sigma_k(\tau), \sigma_e(0)) \\ &\quad + (T-\tau)\Pi_2(\sigma_k(\tau), \dot\sigma_k(\tau)) - \tau M_2'R_{22}M_2 \\ &\quad + (T - 2\tau)M_2'XM_2 - He\{J(\sigma_k(\tau))M_{12}\}, \\ \Phi(\tau, \sigma_k) &= \tau J(\sigma_k(\tau))R_{11}^{-1}J'(\sigma_k(\tau)) + M_2'Q_xM_2, \end{aligned}$$

[1]For simplicity, the argument of $\mathcal{W}(\tau, x_k, \sigma_k)$ will be omitted.

with $\Pi_3(\sigma_k(\tau), \sigma_e(0))$ and $\Psi_3(\sigma_k(\tau), \sigma_e(0))$ defined in (21) and (20), respectively. Hence, by applying Schur's complement and defining $\bar{Q}_x = Q_x^{-1}$, if the matrix inequality

$$\begin{bmatrix} \hat{\Psi}(\tau, \sigma_k, \sigma_e, \dot{\sigma}_k) & \tau J(\sigma_k) & M_2' & \Psi_3(\sigma_k, \sigma_e) \\ * & -\tau R_{11} & 0 & 0 \\ * & * & -\bar{Q}_x & 0 \\ * & * & * & -Q_\delta \end{bmatrix} < 0, \tag{27}$$

is verified for all $\tau \in [0, T]$, $\sigma_k(\tau) \in \mathcal{B}_\sigma$, $\sigma_e(0) \in \mathcal{B}_\sigma$ and $\dot{\sigma}_k(\tau) \in \mathcal{B}_{\dot{\sigma}}$, it follows that $\Gamma(\tau, \sigma_k, \sigma_e, \dot{\sigma}_k) < 0$ and, consequently, (12) is ensured. Since $\sigma_k(\tau) \in \mathcal{B}_\sigma$, $\sigma_e(0) \in \mathcal{B}_\sigma$ and $\dot{\sigma}_k(\tau) \in \mathcal{B}_{\dot{\sigma}}$, a necessary and sufficient condition to ensure (27) consists in satisfying it for all combinations of the vertices of $\mathcal{B}_\sigma$, $\mathcal{B}_\sigma$ and $\mathcal{B}_{\dot{\sigma}}$, that is

$$\begin{bmatrix} \hat{\Psi}_1(\tau, s_f, s_g, d_h) & \tau J(s_f) & M_2' & \Psi_3(s_f, s_g) \\ * & -\tau R_{11} & 0 & 0 \\ * & * & -\bar{Q}_x & 0 \\ * & * & * & -Q_\delta \end{bmatrix} < 0, \tag{28}$$

$\forall (s_f, s_g, d_h) \in \text{Ver}(\mathcal{B}_\sigma) \times \text{Ver}(\mathcal{B}_\sigma) \times \text{Ver}(\mathcal{B}_{\dot{\sigma}})$ and $\forall \tau \in [0, T]$.

Now, noting that (28) is affine on $\tau \in [0, T]$, from convexity arguments, it suffices to ensure (28) for $\tau = 0$ and $\tau = T$, which can be accomplished by verifying (17) and (18). Consequently, (17) and (18) imply that (12) holds, thus all conditions in Theorem 1 are fulfilled and we conclude that $x(t) \to 0$ as $t \to \infty$.

■

B. Optimization problem

Theorem 2 provides LMI stability conditions for the closed-loop system (8) with the event-triggering strategy described in (7). As described in Problem 1, another goal is to reduce the occurrence of events. For this, the following optimization problem, solved off-line, can be considered to select $\bar{Q}_x$ and Q_δ

$$\begin{aligned} \min : & \; tr(\bar{Q}_x) + tr(Q_\delta) \\ \text{subject to} : & \; (17) \text{ to } (19). \end{aligned} \tag{29}$$

Note that no event occurs while $l(x(t_k), \delta(t_k)) \leq 0$ and, from the definition of $l(x(t_k), \delta(t_k))$ in (5), we have that

$$\frac{\delta'(t_k) Q_\delta \delta(t_k)}{x'(t_k) Q_x x(t_k)} \leq \frac{\lambda_{max}(Q_\delta) ||\delta(t_k)||^2}{\lambda_{min}(Q_x) ||x(t_k)||^2} \leq 1, \tag{30}$$

where λ_{max} and λ_{min} denote, respectively, the maximum and minimum eigenvalues of the matrix. Then, in order to maximize the time between events, we aim at enlarging the time while (30) holds, which can be accomplished by reducing $\frac{\lambda_{max}(Q_\delta)}{\lambda_{min}(Q_x)}$. This can be indirectly obtained from the minimization of $tr(\bar{Q}_x) + tr(Q_\delta)$.

V. Numerical Example

The following LPV system is considered:

$$\dot{x}(t) = \begin{bmatrix} -4.1 - 3\sigma(t) & 1 \\ 0 - 2\sigma(t) & 2 - 3.2\sigma(t) \end{bmatrix} x(t) + \begin{bmatrix} 3 \\ 2 \end{bmatrix} u(t), \tag{31}$$

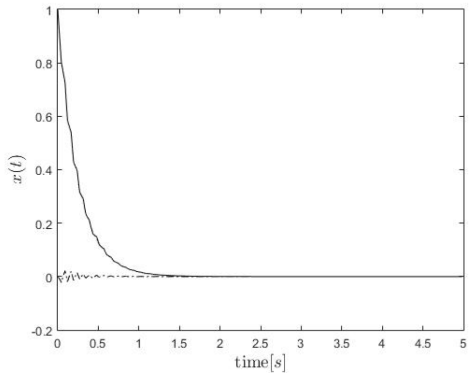

Fig. 1. Evolution of plant states $x_1(t)$ and $x_2(t)$ in solid and dashed lines, respectively.

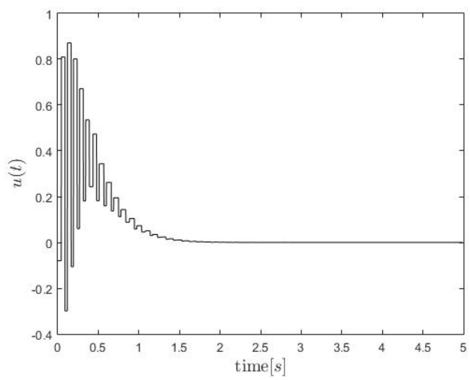

Fig. 2. Control signal $u(t)$.

with $|\sigma(t)| \leq 4$, $|\dot{\sigma}(t)| \leq 5$. We assume that the control law (3) is given with gain matrices $K_0 = [-0.0800 \;\; -28.7051]$, $K_1 = [0.9907 \;\; 1.5011]$ computed from conditions presented in [26] assuming a periodic sampling period $T = 15$ ms.

The goal is to determine the triggering function parameters Q_x and Q_δ ensuring the asymptotic stability of the closed-loop system while reducing the number of events, i.e. reducing the number of updates of the control signal. With this aim, optimization problem (29) is solved leading to:

$$Q_x = \begin{bmatrix} 0.0918 & -0.0400 \\ -0.0400 & 0.1987 \end{bmatrix}, \; Q_\delta = \begin{bmatrix} 1.8970 & -1.7025 \\ -1.7025 & 15.5516 \end{bmatrix}.$$

For the initial condition $x(0) = [1 \;\; 0]'$ and $\sigma(t) = 4\sin(1.25t)$ for $0 \leq t < 2.2$ s and $\sigma(t) = 1.527$ for $t \geq 2.2$ s, simulation results are illustrated in Figures 1-3, where the time evolution of the states x, the control signal u and the time between events $\tilde{t}_e - \tilde{t}_{e-1}$ are depicted. With the proposed periodic event-triggering strategy, there is a reduction of 64.07% in the control updates compared to the classic time-triggered control. The reduction of events with respect to the time-triggered implementation can be seen in Figure 3, where the intervals between two successive events are larger than the period $T = 15$ ms.

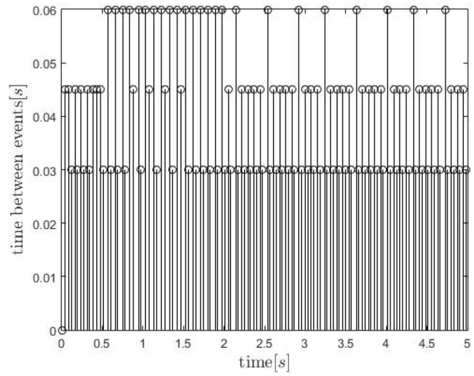

Fig. 3. Event triggering times $\tilde{t}_e$ and respective inter-event times $\tilde{t}_e - \tilde{t}_{e-1}$, with $T = 0.015$ s.

VI. Conclusion

In this paper, the problem of periodic event trigger control for continuous-time LPV systems has been addressed. Considering a sampled-data framework and a parameter-dependent looped-functional approach, LMI conditions to compute the triggering function parameters that ensure the closed-loop asymptotic stability under the PETC strategy are proposed. Differently from previous works dealing with event-trigger control for LPV systems, the possible continuous variation of the plant parameter and the fact that the value of the parameter in the control law is kept constant between two consecutive events are explicitly taken into account. Based on the derived LMI conditions, an optimization problem has been proposed to compute the triggering function parameters aiming at reducing the frequency of control signal updates.

Moreover, many practical systems require also robustness in the presence of time-invariant parametric uncertainties. These uncertainties can be modeled directly in the considered LPV model, and the same conditions can be applied. Another interesting issue regards to exogenous disturbances attenuation which needs more attention and it is the subject of future works.

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