CS 540 - Homework 5
Solution (written part)

Spring 2016

1 - Neural Nets (10 points)

Given `ex = [0.1, 0.9]` and `label(ex) = 1`.

(4 points) First, we propagate inputs forward to compute the outputs:

$$\begin{aligned} output(h_1) &= Sigmoid(0.05 \cdot 0.1 + 0.13 \cdot 0.9 + 0.3 \cdot 1) = Sigmoid(0.422) = 0.60396 \\ output(h_2) &= Sigmoid(-0.43 \cdot 0.1 + 0.22 \cdot 0.9 + 0.2 \cdot 1) = Sigmoid(0.355) = 0.58783 \\ output(out) &= Sigmoid(-0.27 \cdot 0.60396 + 0.11 \cdot 0.58783 - 0.11 \cdot 1) = Sigmoid(-0.20840) = 0.44809 \end{aligned}$$

(4 points) Next, we propagate deltas backward from output layer to input layer:

$$\begin{aligned} \Delta(out) &= (output(out) - \text{label}(\texttt{ex})) \cdot output(out) \cdot (1 - output(out)) \\ &= (0.44809 - 1) \cdot 0.44809 \cdot (1 - 0.44809) = -0.13649 \end{aligned}$$

$$\begin{aligned} \Delta(h_1) &= (w_5 \cdot \Delta(out)) \cdot output(h_1) \cdot (1 - output(h_1)) \\ &= (-0.27 \cdot (-0.13649)) \cdot 0.60396 \cdot (1 - 0.60396) = 0.00881 \end{aligned}$$

(2 points) Finally, we update weights w_1 and w_6:

$$\begin{aligned} w_1 &= w_1 - \alpha \cdot 0.1 \cdot \Delta(h_1) = 0.05 - 0.1 \cdot 0.1 \cdot 0.00881 = 0.04991 \\ w_6 &= w_6 - \alpha \cdot output(h_2) \cdot \Delta(out) = 0.11 - 0.1 \cdot 0.58783 \cdot (-0.13649) = 0.11802 \end{aligned}$$

2 - Uncertainty (10 points)

a) (2 points) $\frac{1}{2}$

b) (2 points) $\frac{1}{2}$

c) (2 points) $\frac{1}{3}$

d) (2 points) $\frac{4}{7}$

e) (2 points) $P(U_1|R) = \frac{P(R|U_1) \cdot P(U_1)}{P(R|U_1) \cdot P(U_1) + P(R|U_2) \cdot P(U_2)} = \frac{7}{19} = 0.368$

3 - Bayesian Networks (20 points)

a) (7 points) Using Addition/Conditioning rule, we get:

$$P(R) = P(R|M) \cdot P(M) + P(R|\neg M) \cdot P(\neg M) = 0.42$$

b) (7 points) Given the network, we know that the probability P(S, M, T, L, R) for any values of S, M, T, L and R can be calculated as

$$P(S, M, T, L, R) = P(S) \cdot P(M) \cdot P(T|L) \cdot P(L|S, M) \cdot P(R|M),$$

where numerical values for each term can be found directly in the Bayes Net diagram. Then:

$$P(L, R) = \sum_{s,m,t} P(L, R, s, m, t) = 0.0561$$

c) (6 points) Using the definition of conditional probability:

$$P(L|R) = \frac{P(L, R)}{P(R)} = \frac{0.0561}{0.42} = 0.1336$$