

# Control Systems

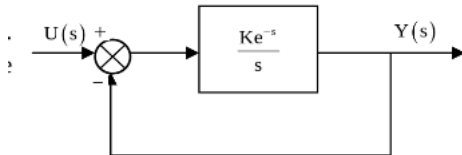
## Gate Question Presentation

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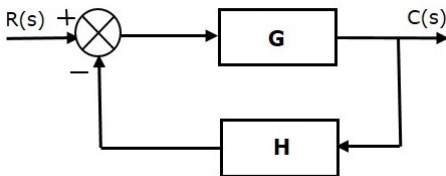
## Question



Q. Consider the unity feedback control system shown. The value of  $K$  the results in the phase margin of system to be  $30^\circ$  is

Ans: 1.047

## Definition



Phase margin is the difference between phase of  $G(s)H(s)$  and  $-180^\circ$  evaluated at gain crossover frequency ( $\omega_{gc}$ ).

Where  $\omega_{gc}$  is defined as the frequency at which magnitude of  $G(s)H(s)$  is unity.

$$PM = \phi - (-180^\circ) = \phi + 180^\circ$$

In our case  $H(s) = 1$ ,  $G(s) = Ke^{-s}/s$ .

Put  $s = j\omega$  for frequency domain analysis, and equate  $|G(j\omega_{gc})|$  to 1.  
We have,

$$\left| \frac{Ke^{-j\omega_{gc}}}{j\omega_{gc}} \right| = 1 \implies \omega_{gc} = K \text{ (assuming positive } K)$$

$$\text{then, } \angle G(j\omega_{gc}) H(j\omega_{gc}) = \angle \frac{Ke^{-j\omega_{gc}}}{j\omega_{gc}} = \angle \frac{Ke^{-jK}}{jK}$$

$$\implies -90^\circ - K * 180/\pi \quad [= \phi]$$

$$PM = 30^\circ = \phi + 180^\circ$$

On solving, we get  $K = \frac{\pi}{3} = 1.047$

## Verification

