

Control Systems

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Abstract—The objective of this manual is to introduce control system design at an elementary level.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/ketan/codes>

1 NODE PLOT

1.1 Gain and Phase Margin

- 1.1.1. For a unity feedback system shown in Fig. 1.1.1, having transfer function given below in eq 1.1.1.1. Design the value of gain K for (i) a gain margin of 38 dB. (ii) Phase margin of 40° . (iii) to yield maximum peak overshoot of 20 percent for a step input.

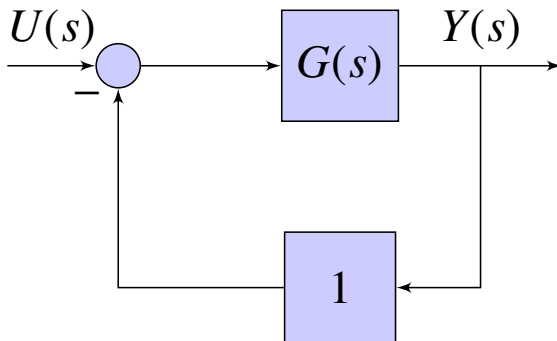


Fig. 1.1.1

$$G(s) = \frac{K(s+2)}{s(s+3)(s+4)(s+5)} \quad (1.1.1.1)$$

Solution:

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$$G(s)H(s) = \frac{K(s+2)}{s(s+3)(s+4)(s+5)} \quad (1.1.1.2)$$

Name-

$$T(s) = \frac{(s+2)}{s(s+3)(s+4)(s+5)} \quad (1.1.1.3)$$

Assuming positive value of K. Gain -

$$= 20 \log(|G(s)H(s)|) \quad (1.1.1.4)$$

$$\Rightarrow 20 \log(K) + 20 \log|T(s)| \quad (1.1.1.5)$$

Phase-

$$= \angle G(s)H(s) \quad (1.1.1.6)$$

$$\Rightarrow \angle T(s) \quad (1.1.1.7)$$

Thus value of K has - a) no effect on phase. b) linear effect on gain.

(i) Given gain = 38dB

Solution: The following code generates Bode plot of $T(s)$ as shown in Fig 1.1.2

codes/ee18btech11038_a.py

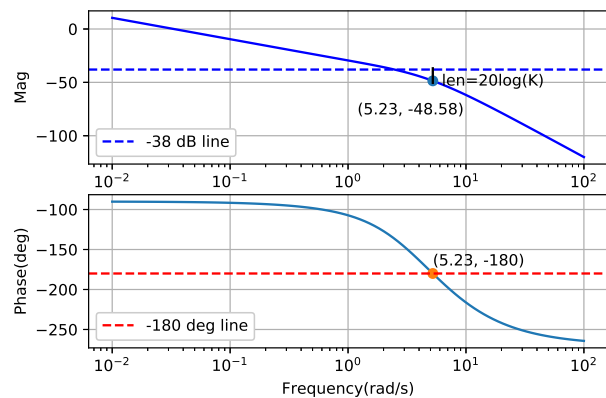


Fig. 1.1.2: Bode Plot of $T(s)$

Fig 1.1.2 shows how much the gain graph be slided to get -38 dB gain at ω_{pc} . From the graph $K = 3.38$

1.1.3. Verify by substituting value of K obtained 1.1.5. Verify by substituting value of K obtained above.

Solution: The following code generates Fig 1.1.3.

```
codes/ee18btech11038_vera.py
```

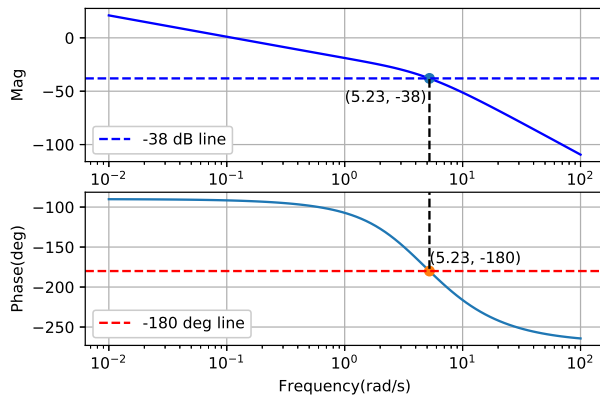


Fig. 1.1.3: Bode Plot of $G(s)$ with $K = 3.38$

1.1.4. (i) Given $PM = 40^\circ$

Solution:

$$\text{phase at } \omega_{gc} = -180^\circ + PM \quad (1.1.4.1)$$

$$\Rightarrow -140^\circ \quad (1.1.4.2)$$

The following code generates Bode plot of $T(s)$ to obtain ω_{gc} as shown in Fig 1.1.4

```
codes/ee18btech11038_b.py
```

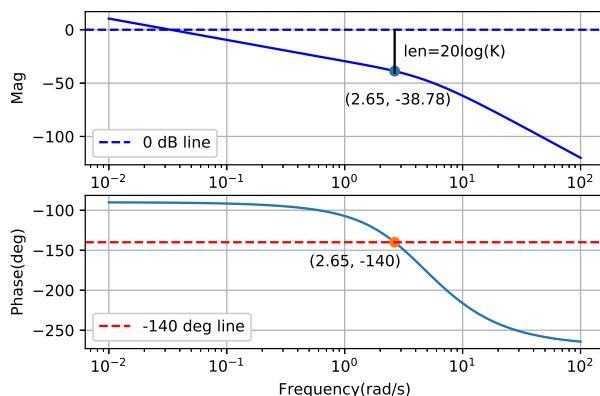


Fig. 1.1.4: Bode Plot of $T(s)$

Fig 1.1.4 shows how much the gain graph be slided to get 0 dB gain at ω_{gc} . From the graph $K = 86.87$

Solution: The following code generates Fig 1.1.5.

```
codes/ee18btech11038_verb.py
```

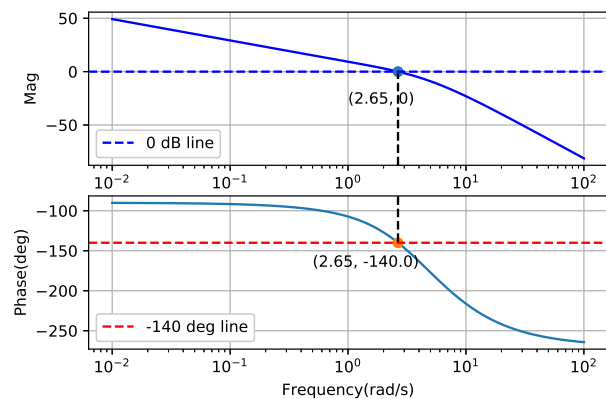


Fig. 1.1.5: Bode Plot of $G(s)$ with $K = 86.87$

1.1.6. (iii) 20 percent peak overshoot in step response.

Solution:

$$\frac{G(s)}{1 + G(s)} \quad (1.1.6.1)$$

$$\Rightarrow \frac{K(s+2)}{s^4 + 12s^3 + 47s^2 + (60+K)s + 2K} = C(s) \quad (1.1.6.2)$$

Step response-

$$\frac{K(s+2)}{(s)[s^4 + 12s^3 + 47s^2 + (60+K)s + 2K]} \quad (1.1.6.3)$$

By final value theorem, steady state value-

$$\lim_{s \rightarrow 0} sC(s) = \lim_{t \rightarrow \infty} c(t) = 1 \quad (1.1.6.4)$$

So the value at peak should be 1.2. Now it is extremely difficult to find K from the given data. Routh Hurwitz criteria only reveals that positive $K < 156$. Since it is a fourth order system, there exist no explicit formula for peak time. Thus, trying a random value of K under the bound, then taking inverse Laplace and differentiating to get peak time and thus overshoot, is the only method that remains.

Using trial and error $K = 69.2$ and $T_{peak} = 1.19s$

1.1.7. Verify by substituting value of K .

Solution: The following code generates Fig 1.1.7.

```
codes/ee18btech11038_os.py
```

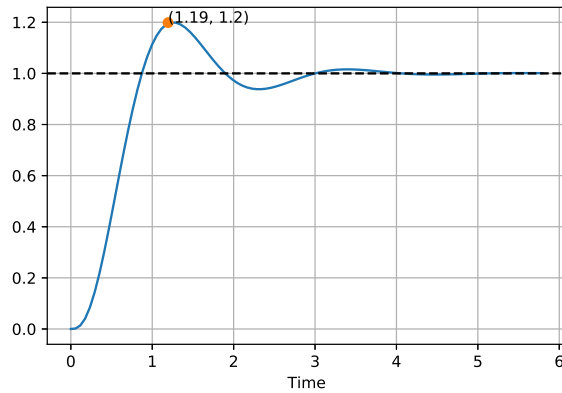


Fig. 1.1.7: Step Response of the system for $K = 69.2$

1.1.8. Results 1.1.8

Specification	Proposed	Actual
Gain Margin	38	38.003
Phase Margin	40°	40.001°
OS%	20%	19.86%

TABLE 1.1.8: Comparing the Proposed and Actual results