Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 Bode Plot

1.1 Gain and Phase Margin

1.1.1. For a unity feedback system shown in Fig. 1.1.1, having transfer function given below in eq 1.1.1.1. Design the value of gain K for (*i*) a gain margin of 38 dB. (*ii*) Phase margin of 40°. (*iii*) to yield maximum peak overshoot of 20 percent for a step input.

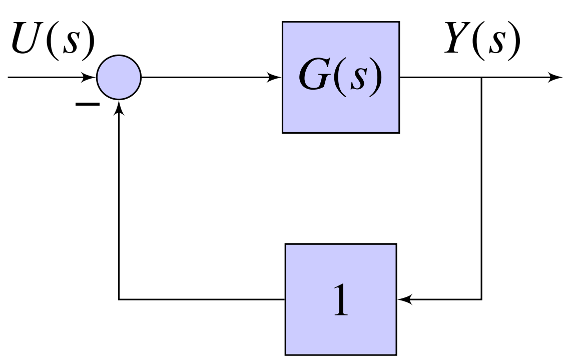

Fig. 1.1.1

$$G(s) = \frac{K(s+2)}{s(s+3)(s+4)(s+5)} \tag{1.1.1.1}$$

Solution:

$$G(s)H(s) = \frac{K(s+2)}{s(s+3)(s+4)(s+5)} \tag{1.1.1.2}$$

Name-

$$T(s) = \frac{(s+2)}{s(s+3)(s+4)(s+5)} \tag{1.1.1.3}$$

Assuming positive value of K. Gain -

$$= 20log(|G(s)H(s)|) \tag{1.1.1.4}$$

$$\implies 20log(K) + 20log|T(s)| \tag{1.1.1.5}$$

Phase-

$$= \angle G(s)H(s) \tag{1.1.1.6}$$

$$\implies \angle T(s) \tag{1.1.1.7}$$

Thus value of K has - a) no effect on phase. b) linear effect on gain.

1.1.2. (*i*) Given gain = 38dB

Solution: The following code generates Bode plot of $T(s)$ as shown in Fig 1.1.2

```
codes/ee18btech11038_a.py
```

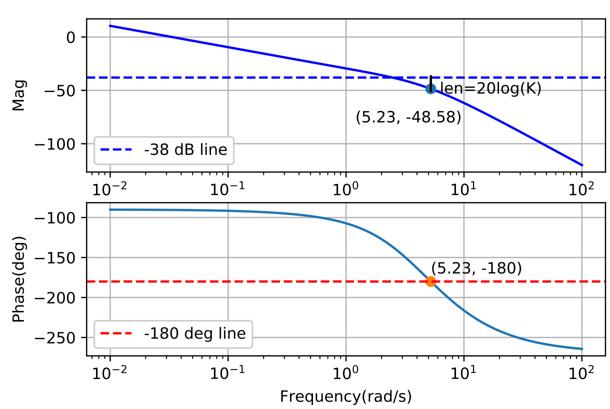

Fig. 1.1.2: Bode Plot of $T(s)$

Fig 1.1.2 shows how much the gain graph be slided to get -38 dB gain at ω_{pc}. From the graph $K = 3.38$

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1.1.3. Verify by substituting value of K obtained above.
Solution: The following code generates Fig 1.1.3.

```
codes/ee18btech11038_vera.py
```

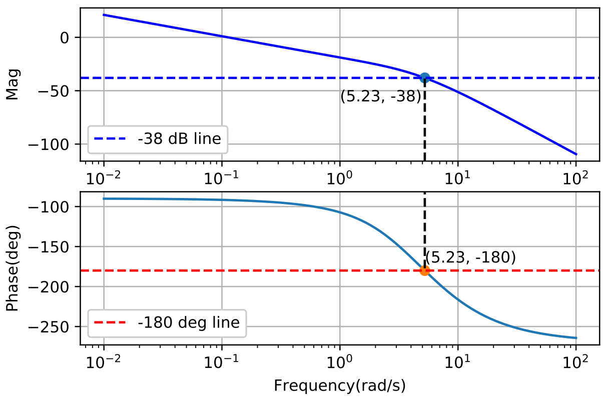

Fig. 1.1.3: Bode Plot of $G(s)$ with $K = 3.38$

1.1.4. (i) Given PM = 40°
Solution:

$$phase\ at\ \omega_{gc} = -180^\circ + PM \tag{1.1.4.1}$$
$$\implies -140^\circ \tag{1.1.4.2}$$

The following code generates Bode plot of $T(s)$ to obtain ω_{gc} as shown in Fig 1.1.4

```
codes/ee18btech11038_b.py
```

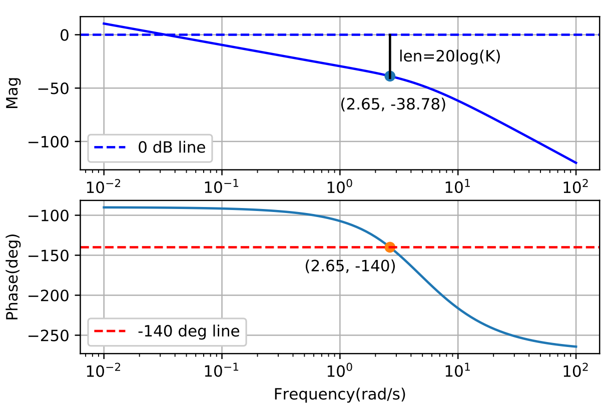

Fig. 1.1.4: Bode Plot of $T(s)$

Fig 1.1.4 shows how much the gain graph be slided to get 0 dB gain at ω_{gc}. From the graph $K = 86.87$

1.1.5. Verify by substituting value of K obtained above.
Solution: The following code generates Fig 1.1.5.

```
codes/ee18btech11038_verb.py
```

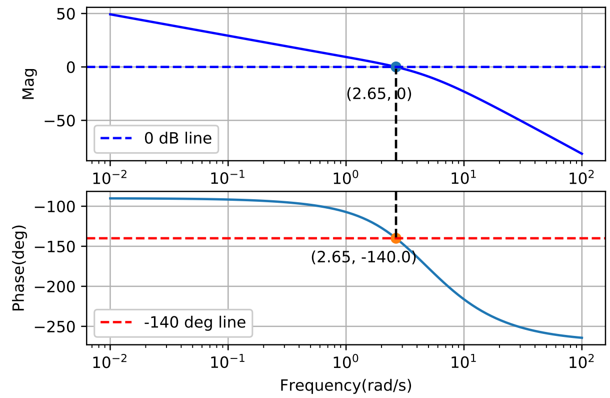

Fig. 1.1.5: Bode Plot of $G(s)$ with $K = 86.87$

1.1.6. (iii) 20 percent peak overshoot in step response.
Solution:

$$\frac{G(s)}{1 + G(s)} \tag{1.1.6.1}$$

$$\implies \frac{K(s+2)}{s^4 + 12s^3 + 47s^2 + (60+K)s + 2K} = C(s) \tag{1.1.6.2}$$

Step response-

$$\frac{K(s+2)}{(s)\left[s^4 + 12s^3 + 47s^2 + (60+K)s + 2K\right]} \tag{1.1.6.3}$$

By final value theorem, steady state value-

$$\lim_{s\to 0} sC(s) = \lim_{t\to\infty} c(t) = 1 \tag{1.1.6.4}$$

So the value at peak should be 1.2. Now it is extremely difficult to find K from the given data. Routh Hurwitz criteria only reveals that $K < 268.5$. Since it a fourth order system, there exist no explicit formula for peak time. Thus, trying a random value of K under the bound, then taking inverse Laplace and differentiating to get peak time and thus overshoot, is the only method that remains.