AI and ML project

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Problem -

Q) Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, b cos t) and (1, 0), where t is a parameter, is

(A) (3x-1)$^2 + (3y)^2 = a^2b^2$
$(B)(3x-1)^2 + (3y)^2 = a^2 + b^2$
$(C)(3x+1) + (3y) = a + b$
$(D)(3x+1)^2 + (3y)^2 = a^2b^2$

Q) $A = \begin{bmatrix} acost \\ asint \end{bmatrix}$ $B = \begin{bmatrix} asint \\ -acost \end{bmatrix}$ $C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

where $t = pi* \begin{bmatrix} 0 & \frac{2}{100} & \frac{4}{100} & . & . & . & . & . & . & 2 \end{bmatrix}$

Find matrix G that is the centroid?

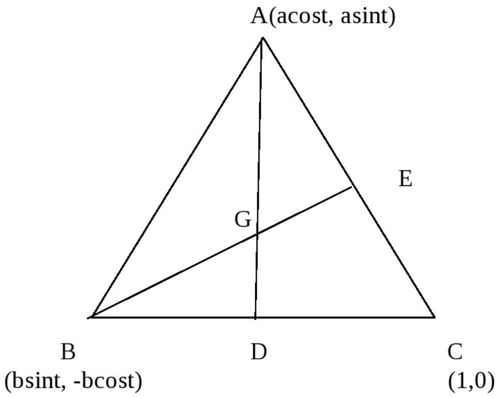

Figure: Figure

Solution through Matrix

$$A = \begin{bmatrix} acost \\ asint \end{bmatrix} \qquad B = \begin{bmatrix} asint \\ -acost \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$D = \text{mid point of BC} = \begin{bmatrix} (asint + 1)/2 \\ -acost/2 \end{bmatrix}$$

$$E = \text{mid point of AC} = \begin{bmatrix} (acost + 1)/2 \\ asin/2 \end{bmatrix}$$

$$AD = \begin{bmatrix} (2acost - bsint - 1)/2 \\ (2asint + bcost)/2 \end{bmatrix}$$

$$BE = \begin{bmatrix} (2bsint - acost - 1)/2 \\ (-2bcost - asint)/2 \end{bmatrix}$$

N = (n1,n2) $\begin{bmatrix} -(2asint + bcost)/2 & (2bcost + asint)/2 \\ (2acost - bsint - 1)/2 & (2bsint - acost - 1)/2 \end{bmatrix}$

$N^T = \begin{bmatrix} -(2asint + bcost)/2 & (2acost - bsint - 1)/2 \\ (2bcost + asint)/2 & (2bsint - acost - 1)/2 \end{bmatrix}$

P = [P1,P2] P1 = n$^T_{AD}(A) P2 = n^T_{BE}(B)$

On calculating P1, P2 and P

P1 = -1/2 (ab+asint) P2 = 1/2(ab+bcost)

P = $\begin{bmatrix} -a/2(b + sint) \\ b/2(a + cost) \end{bmatrix}$

Final Answer

$$Centroid = G = (N^T)^{-1}(P) = \begin{bmatrix} (acost + bsint + 1)/3 \\ (asint - bcost)/3 \end{bmatrix}$$

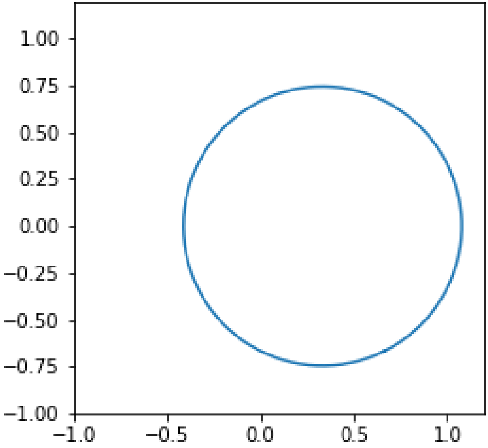

Figure: Locus Plot $(3x-1)^2+(3y)^2=5$