

# Control Systems

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**Abstract**—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/control/codes
```

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## 1 MASON'S GAIN FORMULA

### 2 BODE PLOT

#### 2.1 Introduction

#### 2.2 Example

## 3 SECOND ORDER SYSTEM

#### 3.1 Damping

#### 3.2 Example

## 4 ROUTH HURWITZ CRITERION

#### 4.1 Routh Array

#### 4.2 Marginal Stability

#### 4.3 Stability

## 5 STATE-SPACE MODEL

#### 5.1 Controllability and Observability

#### 5.2 Second Order System

## 6 NYQUIST PLOT

## 7 PHASE MARGIN

7.0.1. Consider a unity feedback control system as shown in Fig. 7.0.1. Find the value of  $K$  that results in a phase margin of the system to be  $30^\circ$ .

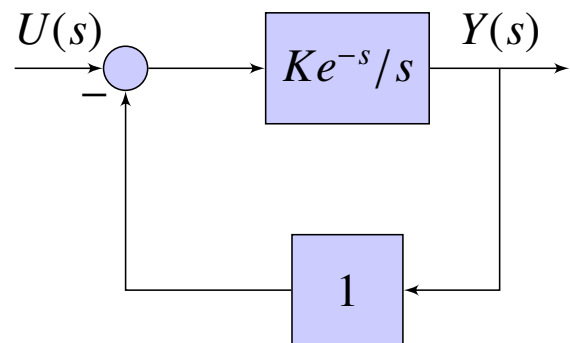


Fig. 7.0.1

**Solution:** Given  $H(s) = 1$  and  $G(s) = Ke^{-s}/s$   
Phase Margin( $PM$ ) is defined as-

$$PM = \phi - (-180^\circ) \quad (7.0.1.1)$$

$$\Rightarrow \phi + 180^\circ \quad (7.0.1.2)$$

where,

$$\phi = \angle G(j\omega_{gc}) H(j\omega_{gc}) \quad (7.0.1.3)$$

$\omega_{gc}$  is the gain-cross over frequency.

7.0.2. Find  $\omega_{gc}$ .

**Solution:**

$$\left| G(j\omega_{gc}) H(j\omega_{gc}) \right| = 1 \quad (7.0.2.1)$$

$$\Rightarrow \left| \frac{K e^{-j\omega_{gc}}}{j\omega_{gc}} \right| = 1 \quad (7.0.2.2)$$

$$\Rightarrow \omega_{gc} = K \quad \text{Assuming } K > 0 \quad (7.0.2.3)$$

7.0.3. Find  $\phi$ .

**Solution:**

$$\phi = \angle G(j\omega_{gc}) H(j\omega_{gc}) \quad (7.0.3.1)$$

$$\Rightarrow \angle \frac{K e^{-jK}}{jK} \quad (7.0.3.2)$$

$$\Rightarrow -90^\circ - K(180/\pi) \quad (7.0.3.3)$$

7.0.4. By (7.0.1.1)

$$PM = 30^\circ \quad (7.0.4.1)$$

$$\text{by(7.0.3.1)} \quad K = \pi/3 \quad (7.0.4.2)$$

7.0.5. Verify result by plotting the gain and phase plots of  $G(j\omega)$

**Solution:** The following code plots Fig. 7.0.5

codes/ee18btech11038\_plot.py

The Phase plot is as shown-

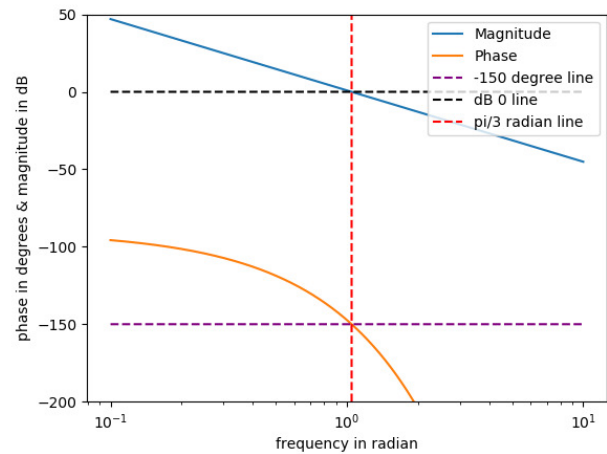


Fig. 7.0.5

8 GAIN MARGIN

9 COMPENSATORS

9.1 Phase Lead

10 OSCILLATOR