

# Op-Amp Stability

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An op amp having a low-frequency gain of  $10^3$  and a single-pole rolloff at  $10^4$  rad/s is connected in a negative feedback loop via a feedback network having a transmission  $k$  and a two-pole rolloff at  $10^4$  rad/s. Find the value of  $k$  above which the closed-loop amplifier becomes unstable.

### 1. Find Open Loop Gain, $G(s)$

**Solution:** Gain for an amplifier whose transfer function is characterised with a single pole is-

$$G(s) = \frac{G_o}{1 + \frac{s}{w_p}} \quad (1.1)$$

Here,  $G_o$  is low frequency gain and  $w_p$  is pole frequency. Thus for our problem,

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \quad (1.2)$$

### 2. Find Feedback factor, $H(s)$

**Solution:** Given transmission(=low freq. gain) and two pole roll-off at  $10^4$  rad/s.

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \quad (2.1)$$

### 3. Loop Gain, $G(s)H(s)$

**Solution:**

$$G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \quad (3.1)$$

### 4. Stability of the Closed Loop Amplifier

**Solution:** Closed loop systems stability can be tested by examining the Gain Margin, or Phase Margin alternatively. First, let's find  $\omega_{180}$ , the

phase cross-over frequency

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} \quad (4.1)$$

$$\Rightarrow -3\tan^{-1}\left(\frac{\omega}{10^4}\right) \quad (4.2)$$

At  $\omega = \omega_{180}$ ,

$$-180^\circ = -3\tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \quad (4.3)$$

$$\Rightarrow \omega_{180} = \sqrt{3} \times 10^4 \quad (4.4)$$

For a stable system,  $GM_{dB} < 0$ ,

$$\Rightarrow |G(j\omega_{180})H(j\omega_{180})| < 1 \quad (4.5)$$

$$\Rightarrow \left| \frac{10^3 k}{\left(1 - \sqrt{3}j\right)^3} \right| < 1 \quad (4.6)$$

$$\Rightarrow \frac{10^3 |k|}{8} < 1 \quad (4.7)$$

$$\Rightarrow |k| < 0.008 \quad (4.8)$$

### 5. Analyze stability using phase margin.

**Solution:** For a stable closed loop amplifier, phase(absolute value) at gain cross-over frequency( $\omega_1$ ) must be less than  $180^\circ$  i.e.  $PM > 0$ . Let's first find  $\omega_1$ .

$$|G(j\omega_1)H(j\omega_1)| = 1 \quad (5.1)$$

$$\Rightarrow \left| \frac{10^3 k}{\sqrt{1 + \frac{\omega_1^2}{10^8}}^3} \right| = 1 \quad (5.2)$$

$$\Rightarrow \omega_1 = \sqrt{10^8 \left(10^2 k^{\frac{2}{3}} - 1\right)} \quad (5.3)$$

For stability,

$$180^\circ > 3\tan^{-1}\left(\frac{\sqrt{10^8 \left(10^2 k^{\frac{2}{3}} - 1\right)}}{10^4}\right) \quad (5.4)$$

$$(5.5)$$

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$$\Rightarrow \sqrt{10^8 (10^2 k^{\frac{2}{3}} - 1)} < \sqrt{3} \times 10^4 \quad (5.6)$$

$$\Rightarrow 10^2 k^{\frac{2}{3}} < 4 \quad (5.7)$$

$$\Rightarrow |k| < 0.008 \quad (5.8)$$

Thus the closed loop amplifier is unstable for  $k > 0.008$ .

#### 6. Design the Feedback Circuit

**Solution:**

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \quad (6.1)$$

$$\Rightarrow k \left( \frac{1}{1 + \frac{2s}{10^4} + \frac{s^2}{10^8}} \right) \quad (6.2)$$

To realise the transfer function above, consider the following circuit-

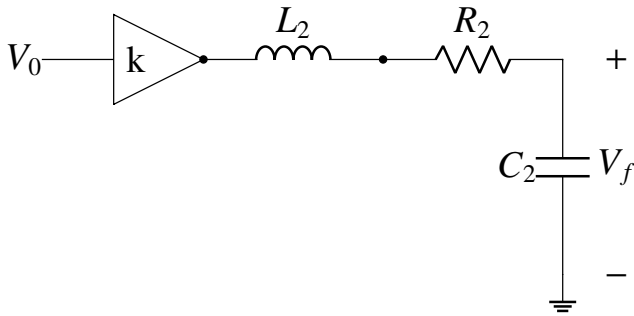


Fig. 6.1: Feedback Circuit

Transfer function for the circuit in Fig 6.1 is,

$$k \left( \frac{\frac{1}{sC_2}}{R_2 + sL_2 + \frac{1}{sC_2}} \right) \quad (6.3)$$

$$\Rightarrow k \left( \frac{1}{1 + sR_2C_2 + s^2L_2C_2} \right) \quad (6.4)$$

Comparing it with 6.2, one possible set of values for  $R_2$ ,  $L_2$  and  $C_2$  is-

$$R_2 = 2k\Omega \quad (6.5)$$

$$L_2 = 100mH \quad (6.6)$$

$$C_2 = 100nF \quad (6.7)$$

#### 7. Design the closed loop circuit.

**Solution:** The closed loop circuit looks as shown in the Fig 7.2.

Here, the op-amp has a gain given by

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \quad (7.1)$$

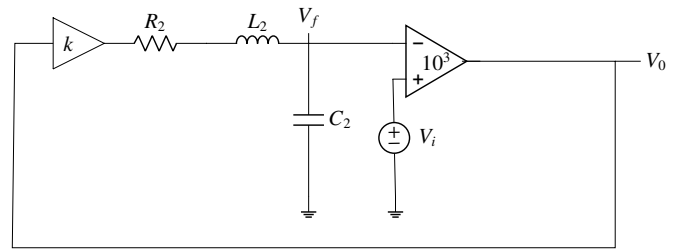


Fig. 7.2: Closed Loop Circuit

#### 8. Find the closed loop transfer function, $T(s)$ .

**Solution:**

$$T(s) = \frac{\frac{10^3}{1 + \frac{s}{10^4}}}{1 + \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3}} \quad (8.1)$$

$$= \frac{10^7 s^2 + 2 \times 10^{11} s + 10^{15}}{s^3 + 3 \times 10^4 s^2 + 3 \times 10^8 s + 10^{12} + 10^{15} \times k} \quad (8.2)$$

#### 9. Sketch the bode plot of $T(s)$ .

**Solution:** Assuming  $k = 0.001$  for numerical simplicity. The bode plot looks as shown in Fig 9.3 You can find the python script used to generate the bode plot here:

codes/ee18btech11038\_bode.py

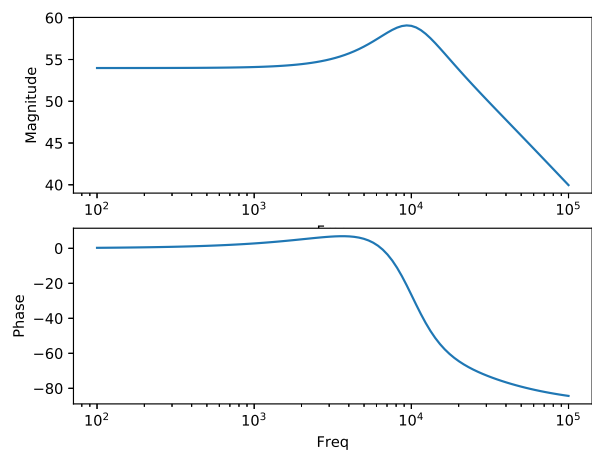


Fig. 9.3: Bode Plot for  $T(s)$

#### 10. Simulate the circuit using spice.

**Solution:** For simulation purpose, we will realise the closed loop amplifier as shown in Fig 10.4 in spice.

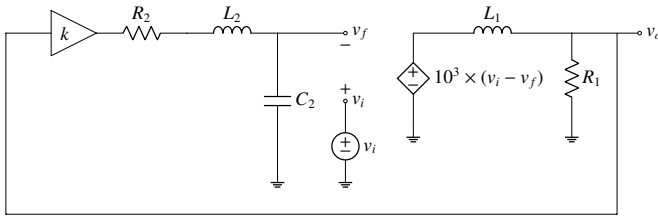


Fig. 10.4: Circuit used for Simulation

$$R_1 = 1k\Omega \quad (10.1)$$

$$L_1 = 100mH \quad (10.2)$$

11. Verify by results obtained using simulation

**Solution:** Fig 11.5 shows the frequency response plot for the circuit in Fig 11.5 obtained using an A.C. sweep in spice. You can find the netlist for the simulated circuit here:

spice/ee18btech11038\_opamp.cir

You can find the python script used to generate the output here:

spice/ee18btech11038\_opamp.py

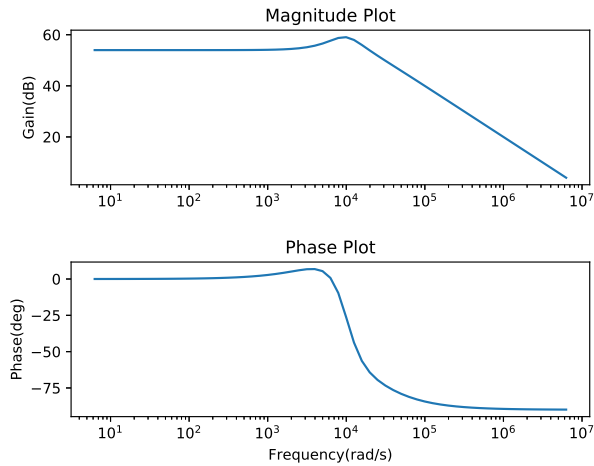


Fig. 11.5: Frequency Response from Simulation

Clearly, the bode plot in Fig 9.3 and plots in Fig 11.5 match. Hence verified.