Op-Amp Stability

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Contents

An op amp having a low-frequency gain of 10^3 and a single-pole rolloff at 10^4 rad/s is connected in a negative feedback loop via a feedback network having a transmission k and a two-pole rolloff at 10^4 rad/s. Find the value of k above which the closed-loop amplifier becomes unstable.

1. Find Open Loop Gain, $G(s)$
Solution: Gain for an amplifier whose transfer function is characterised with a single pole is-

$$G(s) = \frac{G_o}{1 + \frac{s}{w_p}} \tag{1.1}$$

Here, G_o is low frequency gain and w_p is pole frequency. Thus for our problem,

$$G(s) = \frac{10^3}{1 + \frac{s}{10^4}} \tag{1.2}$$

2. Find Feedback factor, $H(s)$
Solution: Given transmission(=low freq. gain) and two pole roll-off at 10^4rad/s.

$$H(s) = \frac{k}{\left(1 + \frac{s}{10^4}\right)^2} \tag{2.1}$$

3. Loop Gain, G(s)H(s)
Solution:

$$G(s)H(s) = \frac{10^3 k}{\left(1 + \frac{s}{10^4}\right)^3} \tag{3.1}$$

4. Stability of the Closed Loop Amplifier
Solution: Closed loop systems stability can be tested by examining the Gain Margin, or Phase Margin alternatively. First, let's find ω_{180}, the phase cross-over frequency

$$\angle G(j\omega)H(j\omega) = \angle \frac{10^3 k}{\left(1 + \frac{j\omega}{10^4}\right)^3} \tag{4.1}$$

$$\implies -3tan^{-1}\left(\frac{\omega}{10^4}\right) \tag{4.2}$$

At $\omega = \omega_{180}$,

$$-180^\circ = -3tan^{-1}\left(\frac{\omega_{180}}{10^4}\right) \tag{4.3}$$

$$\implies \omega_{180} = \sqrt{3} \times 10^4 \tag{4.4}$$

For a stable system, $GM_{dB} < 0$,

$$\implies |G(j\omega_{180})H(j\omega_{180})| < 1 \tag{4.5}$$

$$\implies \left|\frac{10^3 k}{\left(1 - \sqrt{3}j\right)^3}\right| < 1 \tag{4.6}$$

$$\implies \frac{10^3 |k|}{8} < 1 \tag{4.7}$$

$$\implies |k| < 0.008 \tag{4.8}$$

5. Analyze stability using phase margin.
Solution: For a stable closed loop amplifier, phase(absolute value) at gain cross-over frequency(ω_1) must be less than 180° i.e. $PM > 0$. Let's first find ω_1.

$$|G(j\omega_1)H(j\omega_1)| = 1 \tag{5.1}$$

$$\implies \left|\frac{10^3 k}{\sqrt{1 + \frac{\omega_1^2}{10^8}}^3}\right| = 1 \tag{5.2}$$

$$\implies \omega_1 = \sqrt{10^8 \left(10^2 k^{\frac{2}{3}} - 1\right)} \tag{5.3}$$

For stability,

$$180^\circ > 3tan^{-1}\left(\frac{\sqrt{10^8 \left(10^2 k^{\frac{2}{3}} - 1\right)}}{10^4}\right) \tag{5.4}$$

$$\tag{5.5}$$

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$$\implies \sqrt{10^8\left(10^2k^{\frac{2}{3}}-1\right)} < \sqrt{3}\times 10^4 \tag{5.6}$$

$$\implies 10^2k^{\frac{2}{3}} < 4 \tag{5.7}$$

$$\implies |k| < 0.008 \tag{5.8}$$

Thus the closed loop amplifier is unstable for $k > 0.008$.

6. Design the Feedback Circuit
Solution:

$$H(s) = \frac{k}{\left(1+\frac{s}{10^4}\right)^2} \tag{6.1}$$

$$\implies k\left(\frac{1}{1+\frac{2s}{10^4}+\frac{s^2}{10^8}}\right) \tag{6.2}$$

To realise the transfer function above, consider the following circuit-

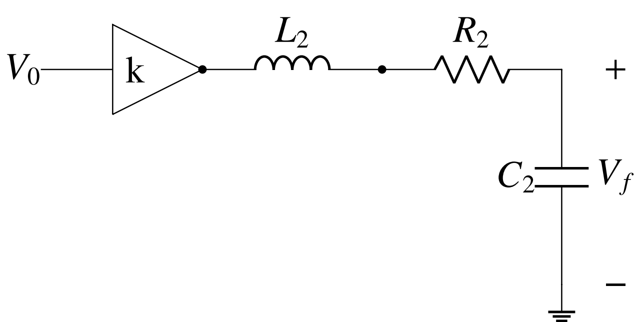

Fig. 6.1: Feedback Circuit

Transfer function for the circuit in Fig 6.1 is,

$$k\left(\frac{\frac{1}{sC_2}}{R_2+sL_2+\frac{1}{sC_2}}\right) \tag{6.3}$$

$$\implies k\left(\frac{1}{1+sR_2C_2+s^2L_2C_2}\right) \tag{6.4}$$

Comparing it with 6.2, one possible set of values for R_2, L_2 and C_2 is-

$$R_2 = 2k\Omega \tag{6.5}$$

$$L_2 = 100mH \tag{6.6}$$

$$C_2 = 100nF \tag{6.7}$$

7. Design the closed loop circuit.
Solution: The closed loop circuit looks as shown in the Fig 7.2.
Here, the op-amp has a gain given by

$$G(s) = \frac{10^3}{1+\frac{s}{10^4}} \tag{7.1}$$

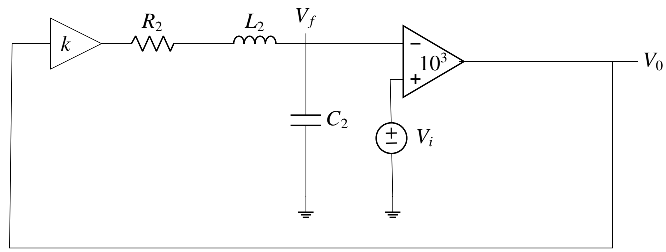

Fig. 7.2: Closed Loop Circuit

8. Find the closed loop transfer function, $T(s)$.
Solution:

$$T(s) = \frac{\frac{10^3}{1+\frac{s}{10^4}}}{1+\frac{10^3k}{\left(1+\frac{s}{10^4}\right)^3}} \tag{8.1}$$

$$= \frac{10^7s^2+2\times 10^{11}s+10^{15}}{s^3+3\times 10^4s^2+3\times 10^8s+10^{12}+10^{15}\times k} \tag{8.2}$$

9. Sketch the bode plot of T(s).
Solution: Assuming $k = 0.001$ for numerical simplicity. The bode plot looks as shown in Fig 9.3 You can find the python script used to generate the bode plot here:

```
spice/ee18btech11038_bode.py
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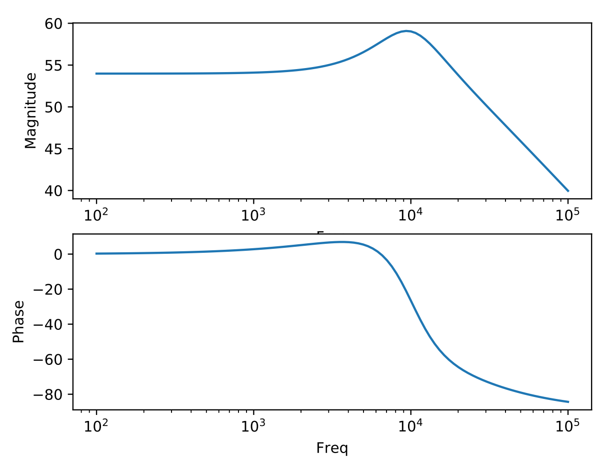

Fig. 9.3: Bode Plot for $T(s)$

10. Simulate the circuit using spice.
Solution: For simulation purpose, we will realise the closed loop amplifier as shown in Fig 10.4 in spice.

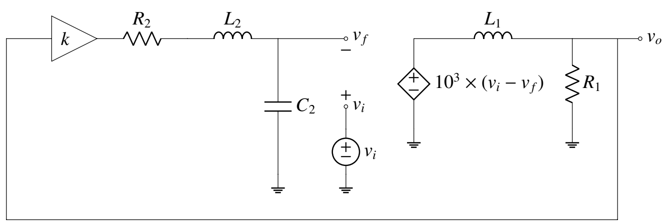

Fig. 10.4: Circuit used for Simulation

$$R_1 = 1k\Omega \tag{10.1}$$
$$L_1 = 100mH \tag{10.2}$$

11. Verify by results obtained using simulation
Solution: Fig 11.5 shows the frequency response plot for the circuit in Fig 11.5 obtained using an A.C. sweep in spice. You can find the netlist for the simulated circuit here:

spice/ee18btech11038_opamp.cir

You can find the python script used to generate the output here:

spice/ee18btech11038_opamp.py

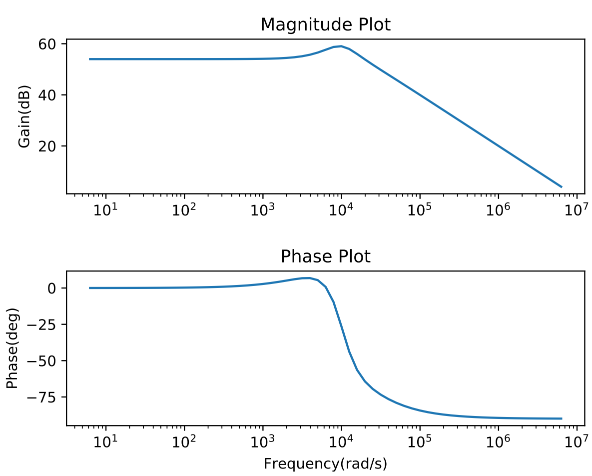

Fig. 11.5: Frequency Response from Simulation

Clearly, the bode plot in Fig 9.3 and plots in Fig 11.5 match. Hence verified.