Date: / /
Page No.

Assignment – 6

Ans 1. Given :

X	-10	-5	0	5	10
Y	5	9	7	11	13

X	$X-\mu_x$	Y	$Y-\mu_y$	u^2	v^2	UV
-10	-10	5	-4	100	16	40
-5	-5	9	0	25	0	0
0	0	7	-2	0	4	0
5	5	11	2	25	4	10
10	10	13	4	100	16	40
Total :	0		0	250	40	90

$\overline{U} = 0$

$\overline{V} = 0$

$\text{Cov}(u,v) = \frac{1}{n}\Sigma UV - \overline{U}\overline{V}$

$= \frac{1}{5} \times 90 - 0$

$= 18$

$\sigma_u^2 = \frac{1}{5} \times 250 - 0$

$= 50$

$\sigma_v^2 = \frac{1}{5} \times 40 - 0$

$= 8$

Date: / /
Page No.

$r(u,v) = \frac{18}{\sqrt{50}\sqrt{8}}$

$= \frac{18}{5 \times 2 \times 2}$

$= 18/20$

$= 0.9$

Q2.

Assignment - 6

Ans1.

X	-10	-5	0	5	10	$\overline{X} =$	0
Y	5	9	7	11	13	$\overline{Y} =$	$\frac{45}{5} = 9$

X	Y	U	V	UV	U^2	V^2
-10	5	-10	-4	40	100	16
-5	9	-5	0	0	25	0
0	7	0	-2	0	0	4
5	11	5	2	10	25	4
10	13	10	4	40	100	16
0	45	0	0	90	250	40

$\overline{U} = 0 \qquad U_i = X_i - \overline{X}$

$\overline{V} = 0 \qquad V_i = Y_i - \overline{Y}$

$$Cov(U,V) = \frac{1}{n}\Sigma UV - \overline{U}\overline{V}$$

$$= \frac{1}{5} \times 90 - 0$$

$$= 18$$

$$\sigma_u^2 = \frac{1}{n}\Sigma u^2 - (\overline{U})^2$$

$$= \frac{250}{5} - 0$$

$$= 50$$

$$\sigma_v^2 = \frac{1}{n}\Sigma v^2 - (\overline{V})^2 .$$

$$= \frac{1}{5} \times 40 - 0$$

$$= 8$$

$$r(u,v) = \frac{cov(u,v)}{\sigma_u \cdot \sigma_v}$$

$$= \frac{18}{\sqrt{8} \times \sqrt{50}}$$

$$= \frac{18}{20}$$

$$= \underline{\underline{0.9}}$$

$$r(u,v) = r(X,Y)$$

$$= \underline{\underline{0.9}}$$

Ans2.

X\Y	10	20	F(X).
5	30	20	50/100
10	20	30	50/100
F(Y)	50/100	50/100	100/100

$$E(X) = \sum_{\forall x} x.f(x) = 5 \times 50 + 10 \times 50$$

$$= \frac{750}{100} = \underline{\underline{7.5}}$$

Date: / /
Page No.

$$E(X^2) = \sum_{\forall x} x^2 f(x)$$
$$= \left(25 \times \frac{1}{2}\right) + 100 \times \frac{1}{2}$$
$$= \frac{125}{2}$$
$$= \underline{\underline{62.5}}$$

$$\sigma_x^2 = 62.5 - (7.5)^2$$
$$= 62.5 - 56.25$$
$$= \underline{\underline{6.25}}$$

$$\sigma_x = \underline{\underline{2.5}}$$

$$E(Y) = \sum_{\forall y} y f(y)$$
$$= 10 \times \frac{1}{2} + 20 \times \frac{1}{2}$$
$$= \underline{\underline{15}}$$

$$E(Y^2) = \sum_{\forall y} y^2 f(y)$$
$$= \frac{100}{2} + \frac{400}{2}$$
$$= \underline{\underline{250}}$$

$$\sigma_y^2 = 250 - 225$$
$$= 25$$

Date: / /
Page No.

$$\sigma_y = \underline{\underline{5}}$$

$$Cov(X,Y) = E(XY) - E(X).E(Y).$$

$$E(XY) = \sum_{\forall x}\sum_{\forall y} xy\, f_{XY}(x,y).$$
$$= \frac{\cancel{50} \times \overset{15}{\cancel{30}}}{\cancel{100}} + \frac{\cancel{100} \times 20}{\cancel{100}} + \frac{\cancel{100} \times 20}{\cancel{100}} + 2\,\cancel{200} \times \frac{30}{\cancel{100}}$$
$$= 15 + 20 + 20 + 60$$
$$= 115$$

$$Cov(X,Y) = 115 - (7.5 \times 15)$$
$$= 115 - 112.5$$
$$= 2.5$$

$$r(X,Y) = \frac{cov(X,Y)}{\sigma_x . \sigma_y}$$
$$= \frac{2.5}{2.5 \times 5}$$
$$= \underline{\underline{0.2}}$$

Ans 3.

$n = 25$

$\Sigma X = 125$ $\quad \Sigma X^2 = 650$

$\Sigma Y = 100$ $\quad \Sigma Y^2 = 460$

$\Sigma XY = 508$

Date: / /
Page No.

Incorrect :

X	6	9
Y	14	6

Correct :

X	8	~~12~~ 6
Y	~~6~~ 12	8

$\Sigma X = 125$

Actual $\Sigma X = 125 - 15 + 20$

$= 125 + 5$

$= \cancel{130}\ 124$

$\Sigma Y = 100$

Actual $\Sigma Y = 100 \cancel{+} - 20 + 14\ \cancel{20}$

$= \underline{\underline{94}}\ \cancel{100}$

$\Sigma X^2 = 650$

$\Sigma X^2 = 650 - 81 - 36 + 64 + 36$

$= 650 - 17$

$= 633$

$\Sigma Y^2 = 460$

$\Sigma Y^2 = 460 - 196 - 36 + 144 + 64$

$= 436$

$\Sigma XY = 508$

$\Sigma XY = 508 - 84 - 36 - 126 - 54 +$
$96 + 64 + 72 + 48$

$= 488$

Date: / /
Page No.

$$Cov(X,Y) = \frac{1}{n}\Sigma XY - \bar{X}\bar{Y}$$

$$= \frac{488}{25} - \frac{124 \times 100}{25 \times 25}$$

$$= -0.32$$

$$\sigma_X^2 = \frac{1}{n}\Sigma X^2 - (\bar{X})^2$$

$$= \frac{633}{25} - \left(\frac{124}{25}\right)^2$$

$$= 0.7184$$

$$\sigma_Y^2 = \frac{436}{25} - (4)^2$$

$$= 1.44$$

~~$r(X,Y) =$~~

ΣXY given $= 508$

For actual,

$\Sigma XY = 508 - 6 \times 100 - 9 \times 100 +$
$8 \times 100 + 6 \times 100 -$
$14 \times 125 - 6 \times 125 +$

Date: / /
Page No.

X	6	9		X	8	6
Y	14	6		Y	12	8

Incorrect — Correct

1. $\Sigma X = 125$

$(\Sigma X)_{New} = 125-6-9+8+6$

$= 124$

2. $\Sigma Y = 100$

$(\Sigma Y)_{new} = 100-6-14+12+8$

$= 100$

3. $\Sigma X^2 = 650$

$(\Sigma X^2)_{new} = 650-81-36+64+36$

$= 633$

4. $\Sigma Y^2 = 460$

$(\Sigma Y^2)_{new} = 460-196-36+144+64$

$= 436$.

5. $\Sigma XY = 508$

$(\Sigma XY)_{new} = 508 - 6(\Sigma Y) - 9(\Sigma Y) - (\Sigma X-6-9)(14) - (\Sigma X-6-9)(6) + 8(\Sigma Y)_{new} + 6(\Sigma Y)_{new} + (\Sigma X_{new}-8-6)(12) + (\Sigma X_{new}-8-6)8$

$= 508 - 600 - 900 - 1540 - 660 + 800 + 600 + 1320 + 880$

Date: / /
Page No.

5. $\Sigma XY = 508$

$(\Sigma XY)_{new} = 508 - 84 - 54 + 96 + 48$

$= 514$

$cov(X,Y) = \frac{1}{n}\Sigma XY - \bar{X}\bar{Y}$

$= \frac{1}{25}(514) - \frac{124 \times \cancel{100}\,4}{25 \times \cancel{25}}$

$= 20.56 - 19.84$

$= 0.72$

$\sigma_X = \sqrt{\frac{1}{n}\Sigma X^2 - (\bar{X})^2}$

$= \sqrt{\frac{633}{25} - \left(\frac{124}{25}\right)^2}$

$= \sqrt{\frac{449}{25}}$

$= 0.8475$

$\sigma_Y = \sqrt{\frac{1}{25} \times 436 - \left(\frac{100}{25}\right)^2}$

$= 1.2$

$r(X,Y) = \frac{0.72}{1.2 \times 0.8475}$

$= \underline{\underline{0.70}}$

Date: / / Page No.

4

Ans 4.

$$U = X\cos\alpha + Y\sin\alpha$$
$$V = -X\sin\alpha + Y\cos\alpha$$
$$\tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}.$$

$$\overline{U} = \overline{x}\cos\alpha + \overline{y}\sin\alpha$$
$$\overline{V} = \overline{Y}\cos\alpha - \overline{x}\sin\alpha$$

U and V are uncorrelated

$\Rightarrow Cov(U,V) = 0$

$\Rightarrow E[(U-\overline{U})(V-\overline{V})] = 0$

$\Rightarrow E\{[(x-\overline{x})\cos\alpha + (y-\overline{y})\sin\alpha)]$
$[(Y-\overline{Y})\cos\alpha - (X-\overline{X})\sin\alpha]\} = 0$

$\Rightarrow E\{(x-\overline{x})(y-\overline{y})(\cos^2\alpha - \sin^2\alpha) + [(y-y)^2 - (x-\overline{x})^2]\sin\alpha\cos\alpha\} = 0$

$\Rightarrow E\{[(Y-\overline{Y})^2 - (X-\overline{X})^2]\sin\alpha\cos\alpha + \cos 2\alpha\,[(X-\overline{X})(Y-\overline{Y})]\} = 0$

$\Rightarrow E\{(y-\overline{y}+x-\overline{x})(y-\overline{y}-x+\overline{x})(\sin\alpha\cos\alpha) + \cos 2\alpha\,[(X-\overline{X})(Y-\overline{Y})]\} = 0$

$\Rightarrow E\{[y^2 - x^2 - 2y\overline{y} - 2x\overline{x} + (\overline{y})^2 - (\overline{x})^2]\sin\alpha\cos\alpha + \cos 2\alpha\,(xy - x\overline{y} - \overline{x}y + \overline{x}\overline{y})\} = 0$

Date: / / Page No.

$\Rightarrow \{\frac{1}{2}\,E(y^2) - E(x^2) - 2(\overline{y})E(y) - 2\overline{x}E(x) + E[(\overline{y})^2] - E[(\overline{x})^2]\}\sin 2\alpha +$
$\cos 2\alpha\,[E(xy) - E(x)E(y) - E(x)E(y) + E(\overline{x}\overline{y})]$

Since they are normally distributed,
$\mu = \overline{X} = \overline{Y} = 0.$

$\therefore \frac{1}{2}[E(y^2) - E(x^2)]\sin 2\alpha + \cos 2\alpha\,E(xy) = 0$

Now, $E[(X-\overline{X})(Y-\overline{Y})]$
$= E(XY)$ [since $\overline{X} = \overline{Y} = 0$]

$\therefore E[(X-\overline{X})(Y-\overline{Y})] = Cov(X,Y)$
$= E(XY)$
$= r\sigma_X\sigma_Y$

$E(Y^2) - E(X^2)$
$= E[(Y-\overline{Y})^2] - E[(X-\overline{X})^2]$
$= |\sigma_Y^2 - \sigma_X^2|$

$\therefore \frac{1}{2}|\sigma_Y^2 - \sigma_X^2|\sin 2\alpha + \cos 2\alpha\,E(xy) = 0$

$\Rightarrow -\frac{1}{2}(\sigma_X^2 - \sigma_Y^2)\sin 2\alpha + r\sigma_X\sigma_Y\cos 2\alpha = 0$

$\Rightarrow \tan 2\alpha = \frac{2r\sigma_X\sigma_Y}{\sigma_X^2 - \sigma_Y^2}.$

Ans5

Ans5. $f(x) = \begin{cases} 4ax & 0 \le x \le r \\ 0 & \text{otherwise} \end{cases}$

$g(y) = \begin{cases} 4by & 0 \le y \le s \\ 0 & \text{otherwise} \end{cases}$

$r(U,V) = \dfrac{b-a}{b+a}$

$U = X+Y$

$V = X-Y.$

Ans) $X = \dfrac{U+V}{2}$ $\quad Y = \dfrac{U-V}{2}.$

$$\frac{\partial(X,Y)}{\partial(U,V)} = J = \begin{vmatrix} 1/2 & +1/2 \\ 1/2 & -1/2 \end{vmatrix}$$

$$= \left| -\frac{1}{4} - \frac{1}{4} \right|$$

$$= \underline{\underline{1/2}}.$$

Since x, y are independent,

$f_{XY}(x,y) = 4ax \cdot 4by$

$= 16abxy.$

$g_{UV}(u,v) = f_{XY}(x,y)\,(J)$

$= 16ab(xy)(1/2)$

$= 8ab\left(\dfrac{U+V}{2}\right)\left(\dfrac{U-V}{2}\right)$

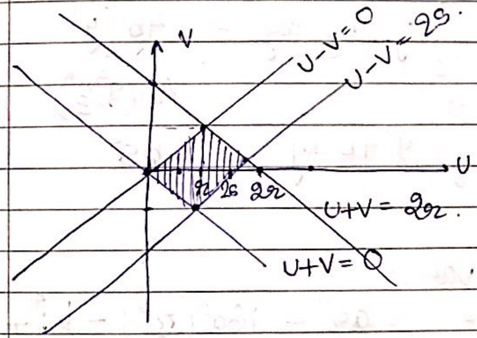
$= 2ab\,(u^2 - v^2).$

$r(u,v) = \dfrac{\text{cov}(u,v)}{\sigma_u \sigma_v}$

$g_{UV}(u,v) = 2ab\,(u^2 - v^2) \quad 0 \le U+V \le 2r$

$\quad 0 \le U-V \le 2s$

$= 0 \quad \text{otherwise}$



$E(UV) = E[(X+Y)(X-Y)]$

$= E(X^2 - Y^2)$

$= E(X^2) - E(Y^2) \quad \boxed{\text{Independent}}$

$E(U).E(V) = E(X+Y).E(X-Y)$

$= [E(X) + E(Y)][E(X) - E(Y)]$

$= [E(X)]^2 - E(X)E(Y) + E(X)(E(Y))$

$- + [E(Y)]^2$

$= [E(X)]^2 + [E(Y)]^2$

Date: / /
Page No.

$$Cov(U,V) = E(UV) - E(U).E(V)$$
$$= E(X^2) - [E(X)]^2 - E(Y^2) + [E(Y)]^2$$
$$= Var(X) - Var(Y).$$

$$E(X) = \int_0^r x.4ax = (4a)(r^3/3)$$

$$E(X^2) = \int_0^r x^2 4ax = ar^4$$

$$E(y) = \int_0^s y.4by.dy = 4b\int_0^s y^2$$
$$= 4b(s^3/3)$$

$$E(y^2) = \int_0^s y^3 4b\,dy = bs^4$$

$$Var(X) - Var(Y)$$
$$= ar^4 - 16a^2\left(\frac{r^6}{9}\right) - bs^4 + \frac{16b^2s^6}{9}$$
$$= ar^4\left(1 - \frac{16ar^2}{9}\right) - bs^4\left(1 - \frac{16bs^2}{9}\right)$$

Date: / /
Page No.

Ans 6

$$f(X) = 1 \qquad -\tfrac{1}{2} < X < 1/2$$
$$f(Y/X) = 1 \qquad X < Y < X+1$$
$$-1/2 < X < 0$$
$$= 1 \qquad -X < Y < 1-X$$
$$0 < X < 1/2$$
$$= 0 \qquad \text{otherwise}$$

$$E(X) = \int_{-1/2}^{1/2} x\,dx$$
$$= \left[\frac{x^2}{2}\right]_{-1/2}^{1/2} = \underline{\underline{0}}$$

$$E(XY) = \int_{-1/2}^{0} x\left[\int_x^{x+1} y\,dy\right]dx +$$
$$\int_0^{1/2} x\left[\int_{-x}^{1-x} y\,dy\right]dx$$

$$= \int_{-1/2}^{0} x\left(\frac{(x+1)^2}{2} - \frac{x^2}{2}\right)dx +$$
$$\int_0^{1/2} x\left(\frac{(1-x)^2}{2} - \frac{(-x)^2}{2}\right)dx$$

$$= \int_{-1/2}^{0} \frac{\cancel{2}x^2}{\cancel{2}} + \frac{1.x}{2}\,dx +$$
$$\int_0^{1/2} -\frac{\cancel{2}x^2}{\cancel{2}} + \frac{1}{2}x\,dx$$

$$= \left.\frac{x^3}{3} + \frac{x^2}{4}\right|_{-1/2}^{0} + \left.-\frac{x^3}{3} + \frac{x^2}{4}\right|_0^{1/2}$$

Date: / /
Page No.

$$= 0+0-\left(\frac{-1}{24}+\frac{1}{16}\right) + \left(\frac{-1}{24}\right)+\frac{1}{16}$$

$$= 0+0+\frac{1}{24}-\frac{1}{16}-\frac{1}{24}+\frac{1}{16}$$

$$= \underline{\underline{0}}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$
$$= 0 - 0$$
$$= 0$$

$\therefore$ $r(X,Y) = 0$

$\Rightarrow$ X and Y are uncorrelated.

Ans5
Ans 5. $E(W) = E[(X+Y)(X-Y)]$

$$= E(X^2 - Y^2)$$
$$= E(X^2) - E(Y^2)$$

$$E(U).E(V) = [E(X)+E(Y)][E(X)-E(Y)]$$
$$= [E(X)]^2 - [E(Y)]^2$$

$$E(UV) - E(U).E(V)$$
$$= \sigma_X^2 - \sigma_Y^2$$

$$E(U^2) = E[(X+Y)^2]$$
$$= E(X^2) + 2E(X).E(Y) + E(Y^2)$$
$$E(V^2) = E(X^2) - 2E(X)\cdot E(Y) + E(Y^2)$$

Date: / /
Page No.

$$E(U) = E(X)+E(Y)$$

$$E(U^2) - [E(U)]^2$$
$$= E(X^2) + 2E(X)E(Y) + E(Y^2) - [E(X)]^2 - [E(Y)]^2 - 2E(X)E(Y)$$
$$= Var(X) + Var(Y).$$

$$E(V) = E(X) - E(Y)$$

$$E(V^2) - [E(V)]^2 = E(X^2) - 2E(X)E(Y) + E(Y^2) - [E(X)]^2 - [E(Y)]^2 + 2E(X)E(Y)$$
$$= Var(X) + Var(Y).$$

$$Var(U) = \sigma_u^2$$
$$\sigma_u = \sqrt{\sigma_X^2 + \sigma_Y^2}$$
$$\sigma_V = \sqrt{\sigma_X^2 + \sigma_Y^2}$$

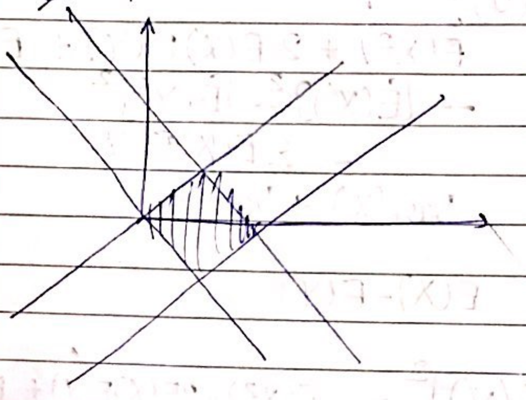
$$r(U,V) = \frac{\sigma_X^2 - \sigma_Y^2}{\sigma_X^2 + \sigma_Y^2}$$

$$= \frac{\frac{ar^4 - 16a^2r^6}{9} - bs^4 + \frac{16b^2s^6}{9}}{\frac{ar^4 - 16a^2r^6}{9} + bs^4 - \frac{16b^2s^6}{9}}$$

$$= \frac{1 - \frac{\sigma_Y^2}{\sigma_X^2}}{1 + \frac{\sigma_Y^2}{\sigma_X^2}}$$

$$= \frac{9ar^4 - 16a^2r^6 + 16b^2s^6 - 9bs^4}{9ar^4 - 16a^2r^6 + 9bs^4 - 16b^2s^6}$$

7.



$$\int \frac{f(x)}{dx} = \int 4ax\,dx \qquad x \in (0, r)$$

$$\Rightarrow \int_0^r 4ax\,dx = 1$$

$$\Rightarrow \frac{4ax^2}{2} = 1$$

$$\Rightarrow \Big|_0^r x^2 = \frac{2}{4a}$$

$$\Rightarrow r = \sqrt{\frac{1}{2a}}$$

$$\int g(y)\,dy = \int_0^s 4by\,dy = 1$$

$$\Rightarrow \frac{2\cancel{4}by^2}{\cancel{2}}\Big|_0^s = 1$$

$$\Rightarrow s = \sqrt{\frac{1}{2b}}$$

Assignment 7

$$\therefore r(u,v) = \frac{9 \times a \times \frac{1}{4a^2} - 16a^2 \times \frac{1}{8a^3} + 16b^2\left(\frac{1}{8b^3}\right) - 9b \times \frac{1}{4b^2}}{\frac{9a}{4a^2} - \frac{16a^2}{8a^3} + \frac{9b}{4b^2} - \frac{16b^2}{8b^3}}$$

$$\Rightarrow \frac{\frac{9}{4a} - \frac{2}{a} + \frac{2}{b} - \frac{9}{4b}}{\frac{9}{4a} - \frac{\cancel{2}}{\cancel{a}} + \frac{9}{4b} - \frac{\cancel{2}}{\cancel{b}}} = r(u,v)$$

$$\Rightarrow r(u,v) = \frac{\frac{1}{4a} + \frac{(-b)}{4b^2}}{\frac{1}{4a} + \frac{1}{4b}} = \frac{\frac{4b-4a}{\cancel{16ab}}}{\frac{4a+4b}{\cancel{16ab}}}$$

$$= \frac{4b - 4a}{4b + 4a}$$

$$= \frac{4}{4}\frac{(b-a)}{(b+a)}$$

$$\therefore \boxed{r(u,v) = \frac{b-a}{b+a}}$$

Date: / /
Page No.

Assignment - 7

Ans 1.

X	Y	U	V	UV	U^2	V^2
65	67	-3	-2	6	9	4
66	68	-2	-1	2	4	1
67	65	-1	-4	4	1	16
67	68	-1	-1	1	1	1
68	72	0	3	0	0	9
69	72	1	3	3	1	9
70	69	2	0	0	4	0
72	71	4	2	8	16	4
544	552	0	0	24	36	44

$\overline{X} = 68$ $\quad \overline{U} = 0$

$\overline{Y} = 69$ $\quad \overline{V} = 0$

$$Cov(U,V) = \frac{1}{n}\Sigma UV - \overline{U}\overline{V}$$

$$= \frac{1}{8} \times 24 - 0$$

$$= 3$$

$$\sigma_u^2 = \frac{1}{n}\Sigma u^2 - (\overline{U})^2$$

$$= \frac{1}{8} \times 36 - 0$$

$$= 4.5$$

Date: / /
Page No.

$$\sigma_v^2 = \frac{1}{8} \times 44 - 0^2$$

$$= \underline{\underline{5.5}}$$

$$r(U,V) = \frac{Cov(U,V)}{\sigma_u \sigma_v}$$

$$= \frac{3 \times 10}{\sqrt{45}\sqrt{55}}$$

$$= 0.6030.$$

$$r(U,V) = r(X,Y).$$

$$\sigma_u^2 = \sigma_x^2$$

$$\sigma_v^2 = \sigma_y^2$$

$$b_1 = \frac{r.\sigma_y}{\sigma_x}$$

$$= 0.60 \times \frac{\sqrt{5.5}}{\sqrt{4.5}}$$

$$= 0.6667$$

$$b_2 = \frac{r\sigma_x}{\sigma_y}$$

$$= 0.603 \times \frac{\sqrt{4.5}}{\sqrt{5.5}}$$

$$= 0.5454$$

Lines of regression.

Date: / /
Page No.

$(y-\bar{y}) = b_1(x-\bar{x})$

$(x-\bar{x}) = b_2(y-\bar{y})$.

$y - 69 = 0.6667(x-68)$

$\Rightarrow y - 69 = 0.6667x - 45.333$

$\Rightarrow 0.6667x - y = -69 + 45.333$

$\Rightarrow y = 0.6667x + 23.6667$.

$(x-68) = 0.5454(y-69)$.

$x - 68 = 0.5454y - 37.6326$

$x = 0.5454y + 30.3674$.

For $y = 70$,

$x = 0.5454 \times 70 + 30.3674$

$= \underline{68.5454}$

Ans 2.

Overtime (X)	1	1	2	2	3	3	4	5	6	7
Additional (Y) Units	2	7	7	10	8	12	10	14	11	14

$\bar{X} = 34/10 = 3.4$

$\bar{Y} = 95/10 = 9.5$.

Date: / /
Page No.

$Y = a + b_1X + b_2X^2$.

$\sum_{i=01}^{n} y_i = a_1 n + b_1 \sum_{i=1}^{n} x_i + b_2 \sum_{i=1}^{n} x_i^2$

$\sum_{i=1}^{n} x_i y_i = a_1 \Sigma x_i + b_1 \Sigma x_i^2 + b_2 \Sigma x_i^3 \ldots$

$\Sigma x_i^2 y_i = a \Sigma x_i^2 + b_1 \Sigma x_i^3 + b_2 \Sigma x_i^4 \ldots$

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i y_i^2$
1	2	1	1	1	2	4
1	7	1	1	1	7	49
2	7	4	8	16	14	98
2	10	4	8	16	20	200
3	8	9	27	81	24	192
3	12	9	27	81	36	432
4	10	16	64	256	40	400
5	14	25	125	625	70	980
6	11	36	216	1296	66	726
7	14	49	343	2401	98	1372
34	95	154	820	4774	377	4453

$95 = 10a + 34b_1 + 154b_2$

$377 = 34a + 154b_1 + 820b_2$

$4453 = 154a + 820b_1 + 4774b_2$

$a = 248.28$

$b_1 = -171.96$

$b_2 = 22.460$

WRONG

Date: / /
Page No.

$\Sigma x_i y_i = 377$
$\Sigma x_i^2 y_i = 1849$
$\Sigma x_i^3 y_i = 10253$

$95 = 10a + 34b_1 + 154b_2$
$377 = 34a + 154b_1 + 820b_2$
$1849 = 154a + 820b_1 + 4774b_2$

$a = 1.8$
$b_1 = 3.48$
$b_2 = -0.268$.

$\therefore \quad Y = 1.8 + 3.48X + (-0.27)X^2$.

Ans 3) $Y = ab^X$

X	Y	$\log Y = U$	UX	X^2	U^2
1	1.0	0	0	1	0
2	1.2	0.079	0.158	4	6.27×10^{-3}
3	1.8	0.255	0.765	9	0.065
4	2.5	0.398	1.59	16	0.158
5	3.6	0.556	2.781	25	0.309
6	4.7	0.67	4.082	36	0.451
7	6.6	0.82	5.79	49	0.67
8	9.1	0.96	7.67	64	0.919
36	30.5	3.7393	22.74	204	2.582

Date: / /
Page No.

$\log Y = \log a + X \log b$.
$U = \log a + X \log b$.

$Cov(U, X) = \frac{1}{n} \Sigma UX - \bar{U}\bar{X}$

$= \frac{1}{8} \times 22.74 - \frac{3.739 \times 36}{8 \times 8}$

$= 0.739$.

$\sigma_U^2 = \frac{1}{n} \Sigma U^2 - (\bar{U})^2$

$= \frac{1}{8} \times 2.582 - \left(\frac{3.7393}{8}\right)^2$

$= 0.104$

$\sigma_X^2 = \frac{1}{8} \times 204 - \left(\frac{36}{8}\right)^2$

$= 5.25$

$r(U, X) = \frac{0.739}{\sqrt{5.25}\sqrt{0.104}}$

$= \underline{\underline{1}}$.

$b_1 = r \cdot \frac{\sigma_U}{\sigma_X}$

$= \frac{\sqrt{0.104}}{\sqrt{5.25}} = 0.1407$

Date: / /
Page No.

$b_2 = \dfrac{r\sigma_x}{\sigma_u}$

$= r\sqrt{\dfrac{5.25}{0.104}}$

$= 7.1049$.

$(U-\bar{U}) = 0.1407(x-\bar{x})$

$(x-\bar{x}) = 7.1049(U-\bar{U})$.

$\bar{U} = 0.467$

$\bar{X} = 36/8 = 4.5$

$U - 0.467 = 0.1407(X-4.5)$

$U = 0.1407X - 0.16615$

$X - 4.5 = 7.1049U - 3.3179$

$7.1049U = X - 1.182$

$U = 0.1407X - 0.16615$.

$\therefore \log b = 0.1407$

$b = 1.3826$

$\log a = -0.1$

$a = 0.682$

$\therefore Y = (0.682)(1.3826)^X$.

Date: / /
Page No.

Ans 1) $n = 10$

X	Y	X'Y'	$(X')^2$	$(Y')^2$	X'	Y'
59	75	25.48	7.84	8281	2.8	9.1
65	70	36.08	77.44	1681	8.8	4.1
45	55	1.22.08	125.44	118.81	-11.2	-10.9
52	65	3.78	17.64	0.81	-4.2	-0.9
60	60	22.42	14.44	34.81	3.8	-5.9
62	69	17.98	33.64	9.61	5.8	3.1
70	80	194.58	190.44	198.81	13.8	14.1
55	65	1.08	1.44	0.81	-1.2	-0.9
45	59	77.28	125.44	47.61	-11.2	-6.9
49	61	35.28	51.84	24.01	-7.2	-4.9
562	659	536.04	645.6	534.9	0	0

$\bar{X} = 56.2$

$\bar{Y} = 65.9$.

$Cov(X', Y') = \dfrac{1}{10}\Sigma X'Y' - (\bar{X'})(\bar{Y'})$

$= \dfrac{1}{10} \times 536.04$

$= 53.604$

$\sigma_x^2 = \left(\dfrac{1}{10}\right)(645.6) - 0$

$= 64.56$

Date: / /
Page No.

$\sigma_Y^2 = \frac{1}{10}(534.9) - 0^2$

$= 53.49$

$r(x', y') = r(x, y)$

$= \frac{53.604}{\sqrt{64.56} \times \sqrt{53.49}}$

$= 0.9025$

$b_1 = 0.9025 \times \sqrt{\frac{53.49}{64.56}}$

$= 0.821$

$b_2 = 0.9025 \times \sqrt{\frac{64.56}{53.49}}$

$= 0.991$

$(Y - \bar{Y}) = 0.821(X - \bar{x})$

$\Rightarrow Y - 65.9 = 0.821 (X - 56.2)$

$\Rightarrow Y = 0.821X + 19.759$

$(X - \bar{x}) = 0.991 (Y - \bar{Y})$

$X - \bar{X} = 0.991 (Y - 65.9)$

$\Rightarrow X = 0.991Y - 9.1069$

$Y = 61, X = ?$

Date: / /
Page No.

$X = 0.991 \times 1 - 9.1069$

$= \underline{\underline{51.3441}}$

Assignment - 8

Ans1) $P = 1/2$ $= \mu$. $\sigma = \sqrt{\frac{PQ}{n}}$

$Q = 1/2$

$Z = \frac{X - \mu}{\sigma}$ $X : 40\%$

$= \frac{0.4 - 0.5}{\sqrt{1/4}} \times \sqrt{120}$

$Z_c = -2.190$.

probability $= 2 \times 0.4857$

$= \underline{\underline{0.9714}}$

$Z = \frac{0.6 - 0.5}{\sqrt{\frac{1/2 \times 1/2}{120}}} = \underline{\underline{2.190}}$.

Ans2) $S_1 : \{3, 7, 8\}$

$S_2 : \{2, 4\}$.

$\mu_{S_1} = \frac{18}{3}$ $\mu_{S_2} = \frac{6}{2}$

$= 6$ $= 3$

Date: / /
Page No.

$\sigma_{s_1} = \sqrt{\Sigma(x_i - \mu)^2 / n}$

$= \sqrt{(3-6)^2 + (7-6)^2 + (8-6)^2 / n}$

$= \sqrt{9+1+4/3}$

$= \sqrt{\frac{14}{3}} = 2.1602$

$\sigma_{s_2} = \sqrt{\frac{\Sigma(x_i-\mu)^2}{n}}$

$= \sqrt{\frac{(2-3)^2 + (4-3)^2}{2}}$

$= \sqrt{2/2} = \underline{\underline{1}}$

$\mu_{s_1-s_2}, \sigma_{s_1-s_2}$

$s_1 - s_2 : \{1, -1, 5, 3, 6, 4\}$.

$\mu_{s_1-s_2} = 18/6 = 3.$

$\sigma_{s_1-s_2} = \sqrt{\frac{4+16+4+0+9+1}{6}}$

$= \sqrt{34/6}$

$= \underline{\underline{2.380}}$

Date: / /
Page No.

Ans 3) $\mu_A = 1400$ $\mu_B = 1200$

$\sigma_A = 200$ $\sigma_B = 100$

$n = 125$.

Mean $= \mu_A \pm Z_c \frac{\sigma}{\sqrt{n}}.$

~~$160 = 1400 \pm Z_c \times \frac{200}{\sqrt{125}}$~~

~~$\Rightarrow \pm Z_c = \frac{(1400-160)\sqrt{125}}{200}$~~

~~$=$~~

$\bar{X}_1 - \bar{X}_2 \pm Z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$

(i) $160 = 200 \pm Z_c \sqrt{\frac{(200)^2}{125} + \frac{(100)^2}{125}}$

$\Rightarrow -40 = \pm Z_c(20)$

$\Rightarrow Z_c = \pm 2$.

Ans) 2×0.4772

$= \underline{\underline{0.9544}}$

(ii) $250 = 200 \pm \sqrt{400} \times Z_c$.

$\Rightarrow +50 = \pm Z_c(20)$

$\Rightarrow Z_c = \pm 2.5$

Probability $= 2 \times 0.4938$

$= \underline{\underline{0.9876}}$

Date: / /
Page No.

Ans4) $n = 50$

A tosses 5 or more heads than B

$P = 1/2$

$Q = 1/2$

$\sigma = \sqrt{\frac{1/2 \times 1/2}{50}}$

$\mu = 1/2$

$\frac{5}{50} = P_1 - P_2 \pm Z_c \sqrt{\frac{1/2 \times 1/2}{50} + \frac{1/2 \times 1/2}{50}}$

$\Rightarrow \frac{5}{50} = 1/2 - 1/2 \pm Z_c (0.1)$

$\Rightarrow 5 = Z_c \times 0.1 \times 50$

$\Rightarrow Z_c = \frac{5}{0.1 \times 50}$

$= \pm 1.$

$5 \times \left(\frac{1}{2}\right)^5$

$\therefore$ Probability

$= 2 \times 0.3413$

$= \underline{\underline{0.6826}}.$

Date: / /
Page No.

Ans5. $\sigma_A = 0.16 cm$ 27.3 cm

$\sigma_B = 0.08 cm$ 15.6 cm

$\sqrt{\sigma_A^2 + \sigma_B^2} = \sigma_{sum} = 0.1788$

Scanned by CamScanner

Date: / / Page No.

Assignment - 9

Ans1) $n = 50$, $\mu = 75$, $\sigma = 10$

95% confidence level for estimates of mean of 200 grades ?

(i) $75 \pm z_c \times \frac{\sigma}{\sqrt{n}}$

$\frac{0.95}{2} = 0.475$

$\therefore z_c = 1.96$.

$\therefore$ Mean $= 75 \pm \frac{1.96 \times 10}{\sqrt{50}}$

$= 75 \pm 2.7718$

$= (72.228, 77.7718)$

Probability that $\mu \in [72.228, 77.771]$ is 0.95.

(ii) $75 \pm 1 \Rightarrow z_c \times \frac{\sigma}{\sqrt{n}} = 1$

$\Rightarrow z_c \times \frac{10}{\sqrt{50}} = 1$

$\Rightarrow z_c = 0.707$.

Probability $= 2 \times 0.2580$

$= 0.516$

Date: / / Page No.

$\therefore$ 51.6 % confidence level

Ans2) $\mu_A = 1400$, $\sigma_A = 120$, $n_A = 150$

$\mu_B = 1200$, $\sigma_B = 80$, $n_B = 200$

(i) $95\% \Rightarrow z_c = 1.96$.

$X_1 - X_2 \pm z_c \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$= 1400 - 1200 \pm 1.96 \sqrt{\frac{(120)^2}{150} + \frac{(80)^2}{200}}$

$= 200 \pm 22.1748$

Confidence limits are $(177.825, 222.174)$

(ii) $99\% \Rightarrow z_c = 2.58$

$1400 - 1200 \pm 2.58 \times \sqrt{\frac{(120)^2}{150} + \frac{(80)^2}{200}}$

$= 200 \pm 29.189$

Limits are $(170.81, 229.189)$

Ans3) $\sigma = 100$, $n = 200$

$S \pm z_c \left(\frac{\sigma}{\sqrt{2n}}\right)$.

Date: / /
Page No.

(i) 95% $Z_c = 1.96$.

$= \cancel{100 \pm 1.96 \times \frac{100}{\sqrt{200}}}$

$= \cancel{100 \pm 13.86}$.

$\sigma = S \pm Z_c\left(\frac{\sigma}{\sqrt{2n}}\right)$.

$\sigma = 100 \pm \frac{1.96 \times \sigma}{\sqrt{2 \times 200}}$

$\sigma \pm \frac{1.96\sigma}{20} = 100$.

$\Rightarrow \sigma\left(1 \pm \frac{1.96}{20}\right) = 100$

$\Rightarrow \sigma = \frac{100}{1 \pm \frac{1.96}{20}}$

$= (91.07, 110.86)$

Limits are $(91.07, 110.86)$

(ii) 99% $Z_c = 2.58$

$\sigma = S \pm Z_c\left(\frac{\sigma}{\sqrt{2n}}\right)$

$\sigma = 100 \pm \frac{2.58 \times \sigma}{20}$

Date: / /
Page No.

$100 = \sigma\left(1 \pm \frac{2.58}{20}\right)$

$100 = \sigma\left(\frac{20 \pm 2.58}{20}\right)$

$\sigma = \frac{100 \times 20}{20 \pm 2.58}$

$= (88.574, 114.810)$.

Limits are $(88.574, 114.81)$

Ans 4) $\mu = 18.2V$ $n = 50$.
$\sigma = 0.5V$

(i) Probable error of mean

$= \frac{\sigma}{\sqrt{n}}$

$= \frac{0.5}{\sqrt{50}}$

$= 0.0707\ V$

(ii) $Z_c = 0.68$.

50% confidence.
$= 0.50/2$
$= 0.25$.

$\mu \pm Z_c \times \frac{\sigma}{\sqrt{n}}$.

$= 18.2 \pm 0.68 \times 0.0707$

$= 18.2 \pm 0.048$

$= (18.152, 18.248)$

Date: / /
Page No.

Ans 5. $(216.80 \pm z_c 0.272)$ gm

$z_c = 1.96$.

$\therefore$ Limits are
(216.80 ± 0.53312)

$= (216.266, 217.33)$.