EE698G Assignment 2

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1 Q1

Output Images in Figure 1

2 Q2

See Figure 2 for data

Principal Components obtained to corresponding randomly generated data were :$[0.1912\ 0.9816]^T$ and $[-0.9816\ 0.1912]^T$ which is very close to the eigen vectors of covariance matrix $[1\ 5]^T$, $[-5\ 1]^T$ as the eigen vectors and principal components are roughly in the same direction.

3 Q3

Answers to theory questions in handwritten form appended after Figure 5.

Output Image on Figure 3 (1.b.1) and Figure 4 (1.b.2 and 1.b.3)

4 Q4

Output Images on Figure 5

5 Q5

See attached handwritten document in the end.

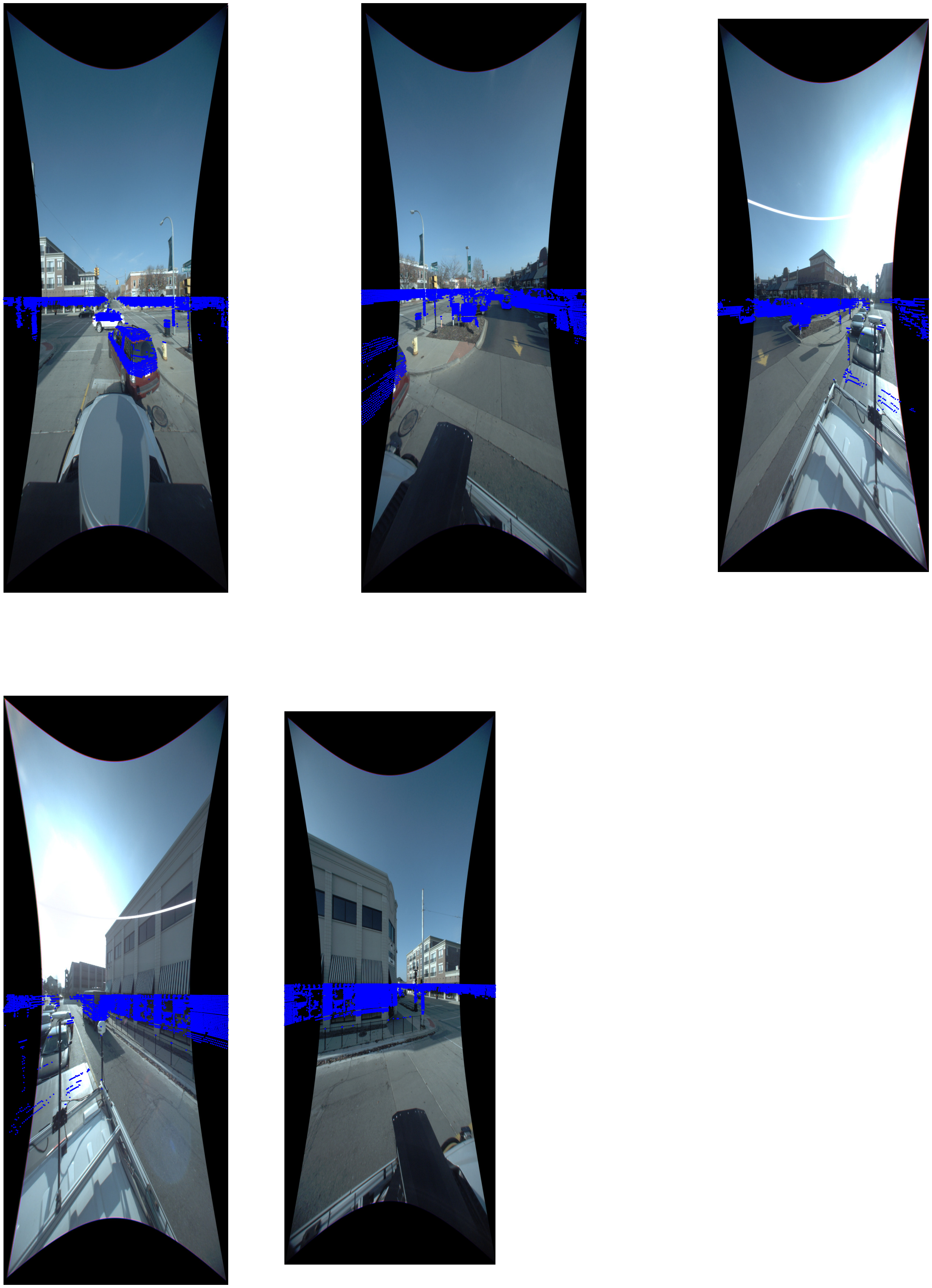


Figure 1: Top Row: 1-3 (Left to Right) Bottom Row 4-5 (Left to Right)

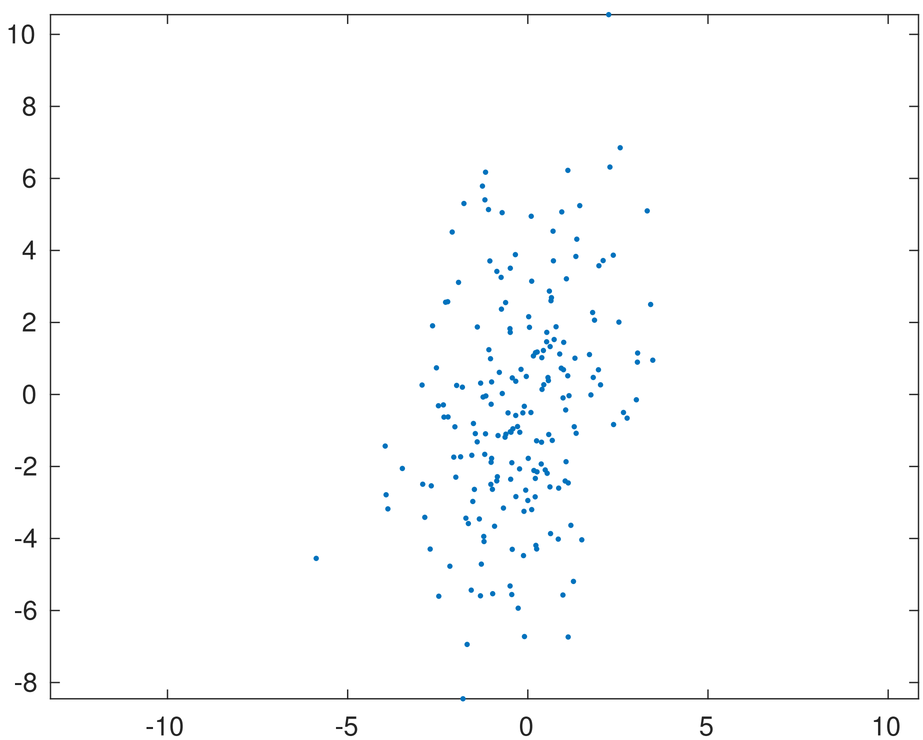


Figure 2: 200 randomly generated points

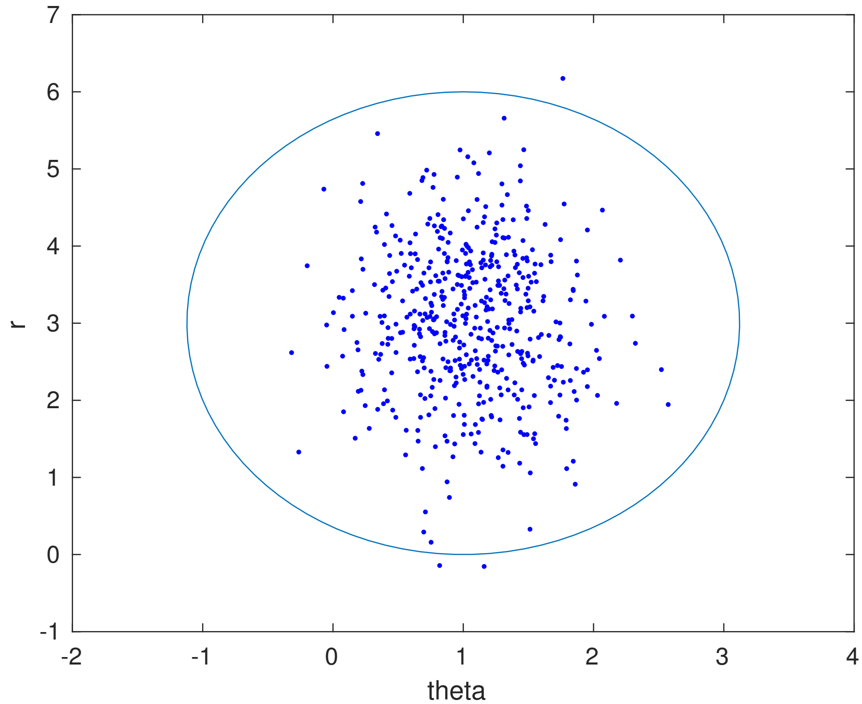


Figure 3: Error ellipse for points in θ-r axis

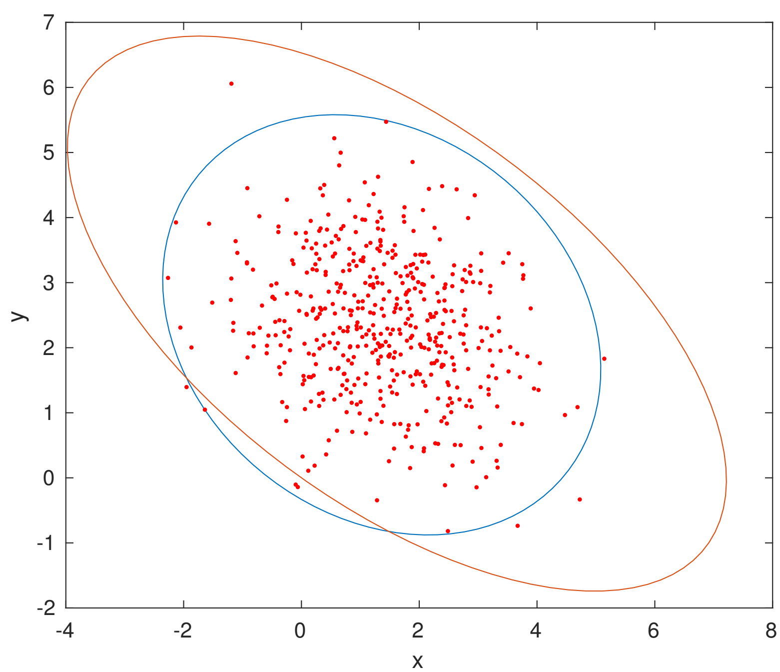


Figure 4: Blue-error ellipse with sample data calculated mean and variance. Red-error ellipse calculated with jacobian (linearisation).

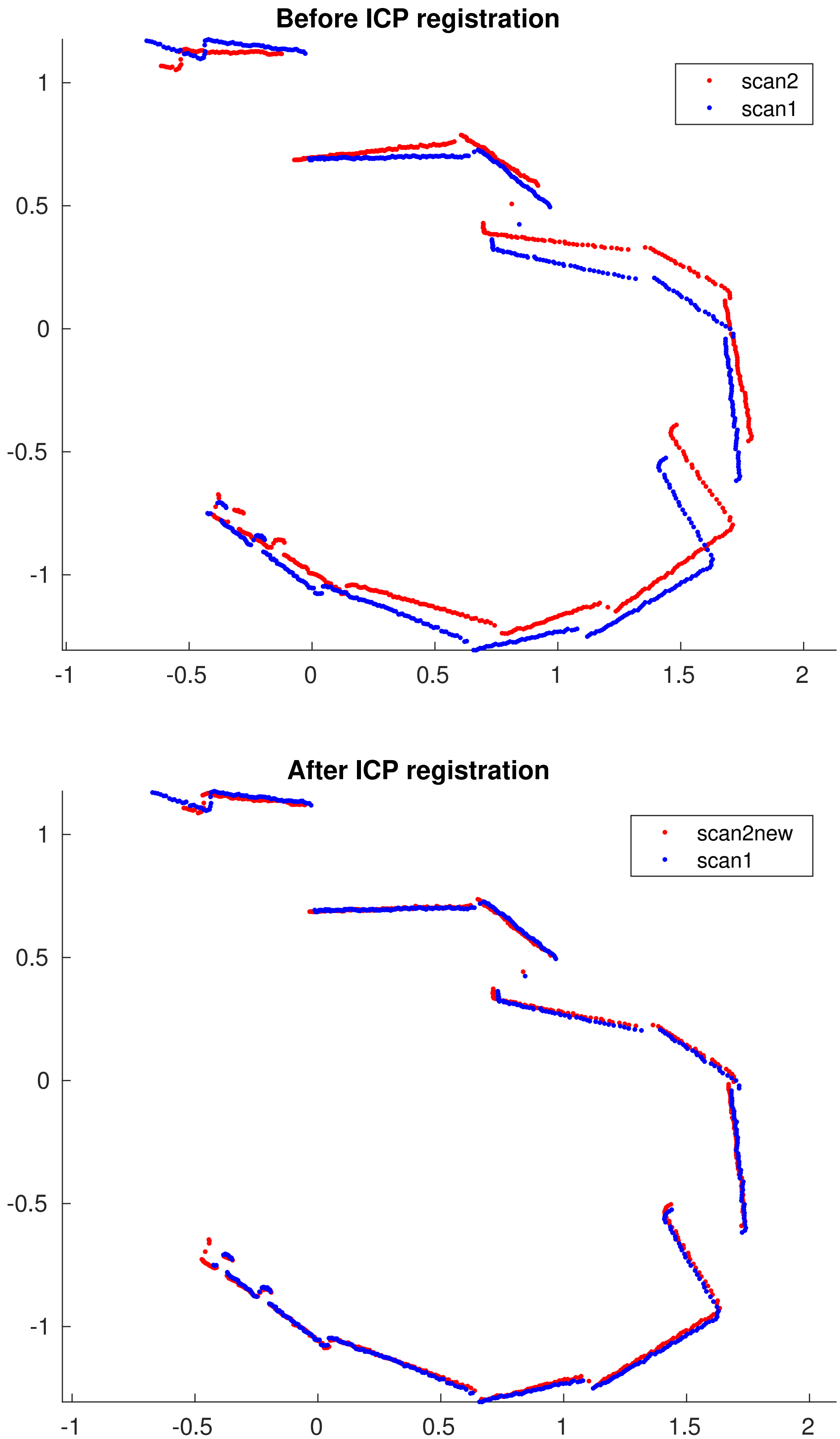


Figure 5: ICP output

Q3

1a.1 No, euclidean coordinates need not be normally distributed also, since they are not strictly linear functions of θ and r.

1a. 2

$$J = \begin{bmatrix} \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial r} \\ \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial r} \end{bmatrix} = \begin{bmatrix} -r\sin\theta & \cos\theta \\ r\cos\theta & \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = J \cdot \left(\begin{bmatrix} \theta \\ r \end{bmatrix} - \begin{bmatrix} \hat{\theta} \\ \hat{r} \end{bmatrix} \right) + \left(\begin{bmatrix} \hat{r}\cos\hat{\theta} \\ \hat{r}\sin\hat{\theta} \end{bmatrix} \right) \quad \left\{ \begin{matrix} \hat{r} = \mu_r \\ \hat{\theta} = \mu_\theta \end{matrix} \right\}$$

$\because (\theta, r) \rightarrow (x, y)$ is highly non-linear

we choose $\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix} = \begin{bmatrix} \hat{r}\cos\hat{\theta} \\ \hat{r}\sin\hat{\theta} \end{bmatrix} = \mu_e$

$$\Sigma_e = J \Sigma_p J^T$$

$$= \begin{bmatrix} -r\sin\theta & \cos\theta \\ r\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix} \begin{bmatrix} -r\sin\theta & r\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

Joint pdf is given by

$$f(x,y) = \frac{1}{\sqrt{(2\pi)\det(\Sigma_e)}} \exp\left(\frac{-1}{2} \left(\bar{x} - \mu_e \right)^T \Sigma_e^{-1} \left(\bar{x} - \mu_e \right) \right)$$

where $\bar{x} = [x \;\; y]^T$ & μ_e, Σ_e as defined above

1.a.3 ~~σ_e~~ Σ_e is defined by linearisation as shown in 1.a.2

$$\Sigma_c = J \Sigma J^T$$

$$= \begin{bmatrix} -r\sin\theta & \cos\theta \\ r\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix} \begin{bmatrix} -r\sin\theta & r\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

1.b.3 This is same as question 1.a.3

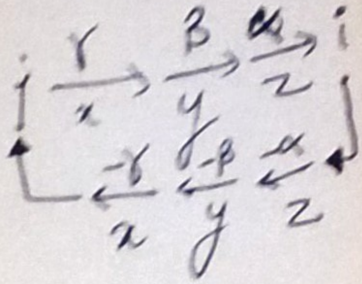
1.b.4 The error ellipse obtained after linearisation is significantly different due to a different covariance matrix. The covariance matrix obtained from linearisation is not accurate because mapping from polar to euclidean coordinates is a highly non-linear one.

If ~~plot~~ was made by choosing mean from linearisation the ellipse would have been offset more and more inaccurate

Q 5.

To prove : $H_{ij} = inv(H_{ji})$

Rotation sequences



$$H_{ij} = \begin{bmatrix} R_{ij}\big|_{3\times3} & T_{ij}\big|_{3\times1} \\ 0 & 1 \end{bmatrix}$$

$$H_{ji} = \begin{bmatrix} R_{ji}\big|_{3\times3} & T_{ji}\big|_{3\times1} \\ 0 & 1 \end{bmatrix}$$

$R_{ij} = R_{ji}^{-1} = R_{ji}^{T}$ [Proof shown below]

$R_{ij} = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) \Rightarrow R_{ji} = R_x(-\gamma)\, R_y(-\beta)\, R_z(-\alpha)$

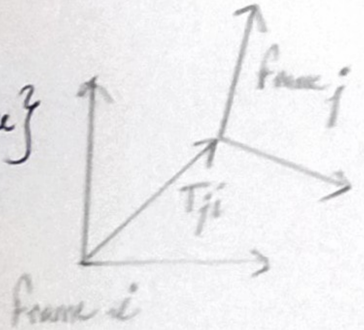
{Euler rotations in reverse}

$\Rightarrow R_{ji}^{T} = R_z(-\alpha)^T R_y(-\beta)^T R_x(-\gamma)^T$

$= \left[R_z(\alpha)^T\right]^T \left[R_y(\beta)^T\right]^T \left[R_x(\gamma)^T\right]^T = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma)$

$\Rightarrow R_{ji}^{T} = R_{ij}$

$T_{ji} = -R_{ij}^{T} T_{ij}$ {from figure}



$$\therefore H_{ji} = \begin{bmatrix} R_{ij}^{T} & -R_{ij}^{T} T_{ij} \\ 0 & 1 \end{bmatrix}$$

$$\therefore H_{ij} \cdot H_{ji} = \begin{bmatrix} R_{ij} \cdot R_{ij}^{T} & R_{ij} T_{ji} + T_{ij} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I_{3\times3} & -R_{ij} R_{ij}^T T_{ij} + T_{ij} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} I & -T_{ij} + T_{ij} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I_{3\times3} & 0_{3\times1} \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow H_{ij} H_{ji} = I$$

$$\Rightarrow H_{ji} = H_{ij}^{-1}$$