# MODULE - I

## Multiple Choice Type Questions

1. The CPU (or CCU) performs the tasks necessary to fulfill the PLC function such as [WBUT 2009, 2016]

a) scanning
b) I/O bus traffic control
c) device communication
d) all of these

**Answer:** (d)

2. A transfer function in z-domain is obtained from the s-transfer function $G(s)=\frac{1}{s+2}$ as $G(z)=\frac{\lambda(z+1)}{z-e^{-2T}}$ using pole-matching technique. The steady-state gain $\lambda$ will be [WBUT 2009]

a) $-\frac{-e^{-2T}}{2}$
b) $\frac{1-e^{-2T}}{4}$
c) $-\frac{-e^{-2T}}{3}$
d) $\frac{1-e^{-2T}}{6}$

**Answer:** (b)

3. The discrete-time system has the transfer function $M(z)=\frac{(z-0.5)}{\{z-(0.8+j0.8)\}\{z-(0.8+j0.8)\}}$. The system is [WBUT 2009]

a) stable
b) unstable
c) marginally stable
d) none of these

**Answer:** (a)

4. A transfer function $G(z)$ will have an ideal pole placement in $z$-plane when it is obtained from $G(S)$ using differential mapping method by setting [WBUT 2009]

a) $s=\frac{In\,z}{T}$
b) $s=\frac{z-1}{T}$
c) $s=\frac{z-1}{zT}$
d) $s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)$

**Answer:** (d)

5. Backward difference approximation method of discretization transforms the left-half of the $s$-plane into a circle in the z-plane of radius [WBUT 2009, 2016]

a) $\frac{1}{\sqrt{2}}$
b) $\frac{1}{4}$
c) $\frac{1}{2}$
d) 1

**Answer:** (d)

6. A real sampler when sampling an analog signal acts as an/a [WBUT 2010]

a) impulse modulator
b) pulse amplitude modulator
c) pulse width modulator
d) pulse code modulator

**Answer:** (b)

7. The output of a closed-loop sampled-data system is given by $y(z)=\dfrac{z(z+0.2)}{(z-0.2)(z-1)}$. The steady-state output is [WBUT 2010]

a) 0 b) 1.25 c) 1.5 d) infinite

**Answer:** (d)

8. The minimum sampling interval required to avoid aliasing for the analog signal $x(t)=5\sin(200\pi)t+3\cos(100\pi)t$ is [WBUT 2010]

a) 10 msec b) 5 msec c) 2.5 sec d) 20 sec

**Answer:** (b)

9. To start reconstructing the continuous-time counterpart from its sampled values, the number of past values required for an *n*-th order device is [WBUT 2010, 2012]

a) $n-2$ b) $n-1$ c) n d) $n+1$

**Answer:** (d)

10. The selection of sampling rates can be based on the bandwidth of the closed-loop [WBUT 2010]

a) 40 to 50 times bandwidth b) 10 to 30 times bandwidth
c) 50 to 60 times bandwidth d) 1 to 10 times of bandwidth

**Answer:** (b)

11. Ideal sampler output for a continuous signal $f(t)$, for a sampling period *T* can be represented as [WBUT 2010]

a) $f^{*}(t)=\sum_{k=0}^{\infty} f(KT)\partial_{T}(KT)$ b) $f^{*}(t)=\sum_{k=0}^{\infty} f(kt)$
c) $f^{*}(t)=\sum_{k=0}^{\infty} f(KT)\partial_{T}(t-KT)$ d) $f^{*}(t)=\sum_{k=0}^{\infty} \partial_{T}(t-KT)$.

**Answer:** (c)

12. For a sampled-data system to be stable, the z-domain poles must be [WBUT 2010, 2014]

a) within the unit circle b) outside the unit circle
c) exactly on the perimeter of the unit circle d) anywhere in the z-plane

**Answer:** (a)

13. The equivalent Z-transform expression for the discrete function $x(k-3)$ is [WBUT 2011]

a) $z\times(z)$ b) $z^{3}\times(z)$ c) $z^{-3}\times(z)$ d) $3\times(z)$

**Answer:** (b)

14. Z-transform of the sequence $\{3,2,1,0,1,3\}$, is [WBUT 2011]

a) $2z^{-2}+2z^{-1}+1+2z^{2}+3z^{3}$
b) $3z^{2}+2z+1+2z^{-2}+3z^{-3}$
c) $3+2z^{-1}+z^{-2}+2z^{-4}+3z^{-5}$
d) $3+2z^{1}+z^{2}+2z^{4}+3z^{5}$.

**Answer:** (b)

15. The steady-state value of a function in Z-domain, F(z) can be obtained by computing [WBUT 2011]

a) $\lim_{z\to a} F(z)$
b) $\lim_{z\to a} F(1-z^{-1})F(z)$
c) $\lim_{z\to 0} F(z-1)F(z)$
d) $\lim_{z\to 1} F(1-z^{-1})F(z)$.

**Answer:** (d)

16. An ideal signal is composed of frequencies $\leq 200Hz$. The minimum sampling frequency recommended is [WBUT 2011]

a) 200 Hz
b) 100 Hz
c) 400 Hz
d) 1 kHz

**Answer:** (c)

17. The adaptive control technique refers to [WBUT 2011, 2016]

a) autotuning
b) gain scheduling
c) both (a) and (b)
d) none of these

**Answer:** (c)

18. In a first-order hold device, for reconstruction of signal [WBUT 2011, 2012, 2014, 2018]

a) last sampled data is used
b) last two sampled data are used
c) last three sampled data are used
d) more than three sampled data are used

**Answer:** (b)

19. The z-transform of a unit step function is [WBUT 2012, 2014, 2018]

a) $\frac{1}{(1+z^{-1})}$
b) $\frac{1}{(1-z^{-1})}$
c) $\frac{1}{(1+z)}$
d) $\frac{1}{(1-z)}$

**Answer:** (b)

20. The z-transform can be used for [WBUT 2012, 2014]

a) any system
b) any continuous time system
c) any discrete data system
d) any linear time-invariant discrete data system

**Answer:** (d)

**21. A signal has frequency 20 Hz. The minimum sampling frequency is** **[WBUT 2012, 2014, 2016, 2018]**

**a) 10Hz** **b) 20Hz** **c) 40Hz** **d) none of these**

**Answer:** (a)

**22. The minimum sampling interval required to avoid aliasing for the analog signal $x(t)=5\sin(200\pi)t+3\cos(100\pi)t$ is** **[WBUT 2012]**

**a) 10 msec** **b) 5 msec** **c) 2.5 sec** **d) 20 sec**

**Answer:** (b)

**23. The w-transform is given by** **[WBUT 2014, 2018]**

**a)** $w=[\ln(z)]/T$ **b)** $w=T[\ln(z)]$ **c)** $z=[\ln(w)]/T$ **d)** $z=T[\ln(w)]$

**Answer:** (a)

**24. The antialiasing filter is used in a digital control loop is** **[WBUT 2014]**

**a) discretize the continuous time signal**
**b) reconstruction of the discrete signal**
**c) suppress the unwanted noise**
**d) reduce the quantization effect**

**Answer:** (c)

**25. For a sampled data system to be stable, the Z domain poles must be** **[WBUT 2015, 2018]**

**a) within the unit circle**
**b) outside the unit circle**
**c) anywhere in the Z plane**
**d) exactly on the perimeter of the unit circle**

**Answer:** (a)

**26. The CPU (or CCU) performs the tasks necessary to fulfill the PLC function such as** **[WBUT 2015]**

**a) scanning** **b) I/O bus traffic control**
**c) device communication** **d) all of these**

**Answer:** (d)

**27. The characteristic equation of a linear digital control system is given by $F(z)=z^4-8z^3+2z^2-z+1$. The system is** **[WBUT 2015, 2017]**

**a) stable** **b) unstable**
**c) marginally stable** **d) none of these**

**Answer:** (c)

**28. The abbreviations NO and NC represent the electrical state of the switch contact when** **[WBUT 2016]**

**a) power is applied** **b) power is not applied**
**c) the switch is activated** **d) the switch is not activated**

**Answer:** (d)

29. A transfer function $G(z)$ will have an ideal pole placement in z-plane when it is obtained from $G(s)$ using differential mapping method by setting [WBUT 2016]

a) $s = \ln z / T$
b) $s = z - 1/T$
c) $s = z - 1/zT$
d) $s = 2/T(z - 1/z + 1)$

Answer: (a)

30. If z-transform of a function is $\frac{z \sin \omega t}{z^2 - 2z\cos \omega t + 1}$, its corresponding Laplace transform will be [WBUT 2017]

a) $\frac{\omega}{s^2 + \omega^2}$
b) $\frac{s}{s^2 + \omega^2}$
c) $\sin \omega t$
d) $\frac{\omega}{s + \omega}$

Answer: (a)

31. Number of past samples required for a 2nd order hold for data reconstruction is [WBUT 2017]

a) 3
b) 1
c) 2
d) 4

Answer: (a)

32. An ideal sampler acts as [WBUT 2017]

a) pulse amplitude modulator
b) pulse width modulator
c) pulse code modulator
d) impulse modulator

Answer: (d)

33. The mapping of the s-plane to the z-plane using the impulse invariant transformation is [WBUT 2017]

a) many to 1
b) 1 to 1
c) 1 to many
d) none of these

Answer: (b)

34. For bilinear transformation, the relationship between the w-domain and z-domain is given by [where $T$ is the sampling period] [WBUT 2017]

a) $w = \frac{z-1}{z+1}$
b) $w = \frac{z+1}{z-1}$
c) $w = \frac{2}{T} \cdot \frac{z-1}{z+1}$
d) $w = \frac{2}{T} \cdot \frac{z+1}{z-1}$

Answer: (c)

35. An anti-aliasing filter is a [WBUT 2017]

a) high pass filter
b) band pass filter
c) low pass filter
d) band stop filter

Answer: (c)

36. The w-transform can be used for [WBUT 2018]

a) any system
b) any continuous time system
c) any discrete data system
d) any linear time-invariant discrete data system

Answer: (d)

**37. In r-transform, the perimeter of the unit circle in z-plane is mapped onto** **[WBUT 2018]**

a) the real axis in r-plane.
b) the imaginary axis in r-plane
c) another circle in r-plane
d) None of these

**Answer:** (b)

**38. Pulse transfer function in derived in** **[WBUT 2018]**

a) t-plane
b) s-plane
c) z-plane
d) w-plane

**Answer:** (c)

## Short Answer Type Questions

**1. Find the *z*-transformation of a unit step function and a unit ramp function. What are the shortcomings of *z*-transform?** **[WBUT 2009, 2015]**

**Answer:**

Z transform of unit step function:

$$X(z)=\sum_{n=-\alpha}^{\alpha} u(n)z^{-n}$$

$$=\sum_{n=0}^{n=\alpha} z^{-n}=\sum_{n=0}^{n=\alpha}(z^{-1})^{n} \quad [u(n) = 0, \text{ for } n<0, =1 \text{ for } n>= 0]$$

$$=\frac{1}{1-z^{-1}}=\frac{z}{z-1}$$

Z transform of unit ramp function:

$$x(n)=nu(n)$$

$$X(z)=\sum_{n=-\alpha}^{n=\alpha} nu(n)z^{-n}$$

$$=-z\frac{d}{dz}\sum_{n=-\alpha}^{n=\alpha} u(n)z^{-n}$$

$$=-z\frac{d}{dz}\sum_{n=0}^{n=\alpha} z^{-n}$$

$$=-z\frac{d}{dz}\left(\frac{z}{z-1}\right)$$

$$=-z\frac{(z-1)-z(1)}{(z-1)^{2}}$$

$$=\frac{z}{(z-1)^{2}}$$

The z Transform is a powerful mathametical tool for the analysis of linear-time invariant discrete time systems in frequency domain. In z domain the convolution of two time domain signal is equivalent to multiplivcation of their corresponding z transform.This property simplifies the analysis of response of an LTI system of various signals.

But it has some drawbacks like-

- It cannot aaply in continious signal.
- It can not analyse analog filter

**2. Check for stability of a sampled-data system that has characteristic equation given by $F(z)=z^4+1.2z^3-2z^2-3z+1=0$** **[WBUT 2009, 2016]**

**Answer:**

By Jury's stability we can check the stability of this system-

$$F(z)=z^4+1.2z^3-2z^2-3z+1$$

*now,*

$$\Delta(1)=1+1.2-2-3+1$$
$$=1+1.2-5+1$$
$$=2-3.8$$
$$=-1.8$$

*again,*

$$\Delta(-1)=1-1.2-2+3+1$$
$$=1-3.2+4$$
$$=1-0.8$$
$$=0.2>0$$

It doesn't satisfy the basic condition of Jury's stability, so the system is unstable.

**3. What is an ideal sampler? Derive the expression for the output of an ideal sampler when a continuous time signal $f(t)$ is sampled by it.**

**[WBUT 2009, 2014, 2016]**

**Answer:**

The sampler used for the purpose of instantaneous sampling or ideal sampling is called ideal sampler.

In this type of sampling, the sampling function is a train of impulses. Here the input signal x(t) is to be sampled, The figure below shows the circuit of the ideal sampler.This circuit is known as a switching sampler. This circuit simply consists of a switch. Now, if we consider that the closing time 't' of the switch approaches to zero, the output g(t) of this circuit will contain only instantaneous sampling gives a train of impulses of height equal to the instantaneous value of the input signal x(t) at the sampling instant.

We know that the train of impulses may be represented as-

$$\delta T_s(t) = \sum_{n=-a}^{a} \delta(t-nTs)$$

This is known as sampling function and its waveform is shown in the figure below.

The sampled signal g(t) can be expressed as the multiplication function of x(t) and $\delta T_s(t)$

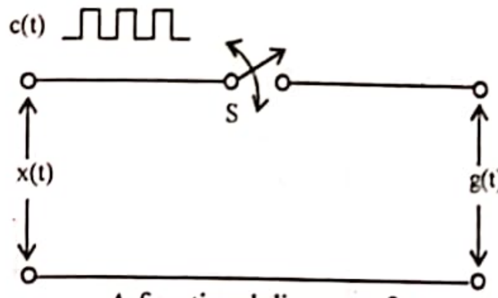
$$g(t) = x(t).\delta T_s(t)$$

$$= x(t).\sum_{n=-\alpha}^{\alpha} \delta(t - nT_s)$$

$$or, \quad g(t) = \sum_{n=-\alpha}^{\alpha} x(nT_s).\delta(t - nT_s)$$

The Fourier transform of the ideally sampled signal given by the above equation may be expressed as-

$$G(f) = f_s \sum_{n=-\alpha}^{\alpha} X(f - nf_s)$$




A functional diagram of a Natural Sampler

**4. What is a sampler with zero order hold (ZOH)? Derive the transfer function of ZOH in S-domain and Z-domain.** **[WBUT 2010, 2014]**

**OR,**

**Find the pulse transfer function of a zero Order Hold (ZOH) device. What does the value convey?** **[WBUT 2011]**

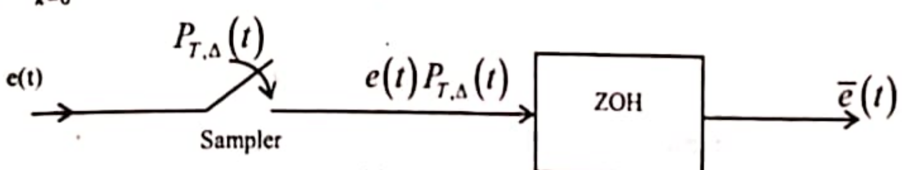
**Answer:**

**Sampler with zero-order hold**

***Sampler and zero-order hold:*** Figure below shows a sampler with a ZOH. The ZOH clamps the output signal to a value equal to that of the input signal at the sampling instant.

In the analysis of the operation of the sampler, if the sampling duration $\Delta$ is very much smaller than the sampling period $T$ and the smallest time constant of the input signal $e(t)$, then the output of the sampler can be approximated by a sequence of flat topped pulses. Figure below shows the sampler output (ZOH input) and ZOH output at $k$th sampling instant. Sampling operation is thus described by the equation

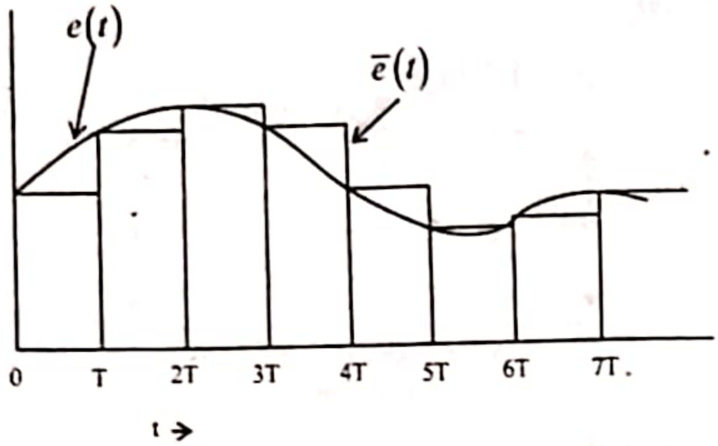
$$e(t)P_{T,\Delta}(t) = \sum_{k=0}^{\infty} e(kT)P_{T,\Delta}(t - kT)$$




(a)

(Sampler with zero-order hold system)


zoH i/P zoH o/P




Sampler and zero-order-hold

The ZOH output pulse appearing at $K$th sampling instant can be expressed as

$$e(kT)\left[u(t-kT)-u\left(t-\overline{k+1}T\right)\right]$$

Where $u(t)$ is a unit-step function

We can therefore write the ZOH output as

$$\overline{e}(t)=\sum_{k=0}^{\infty}e(kT)\left[u(t-kT)-u\left(t-\overline{k+1}T\right)\right]$$

ZOH holds the sampled value of the signal e(kT) for the period $kT \le t \le (k+1)T$ arrive. The impulse response of ZOH can be expressed as $g_{HO}$= u(t)-u(t-T), where u(t) is the unit-step function.

The transfer function $G_{HO}$ of ZOH can therefore be obtained as-

$$G_{HO}(s)=L[g_{HO}(t)]=\int_{0}^{\infty}[u(t)-u(t-T)e^{-st}dt$$

$$=\frac{(1-e^{-sT})}{s}$$

Suppose a the transfer function G(s) follows a ZOH.

Then,

$$X(s)=\frac{1-e^{-Ts}}{s}G(s)$$

Now,

$$X(z)=z\left|X(s)\right|=(1-z^{-1})z\left|\frac{G(s)}{s}\right|$$

**5. What is Nyquist frequency? Explain aliasing phenomenon indicating the difference between the aliasing frequency and the Nyquist frequency. [WBUT 2010]**

**Answer:**

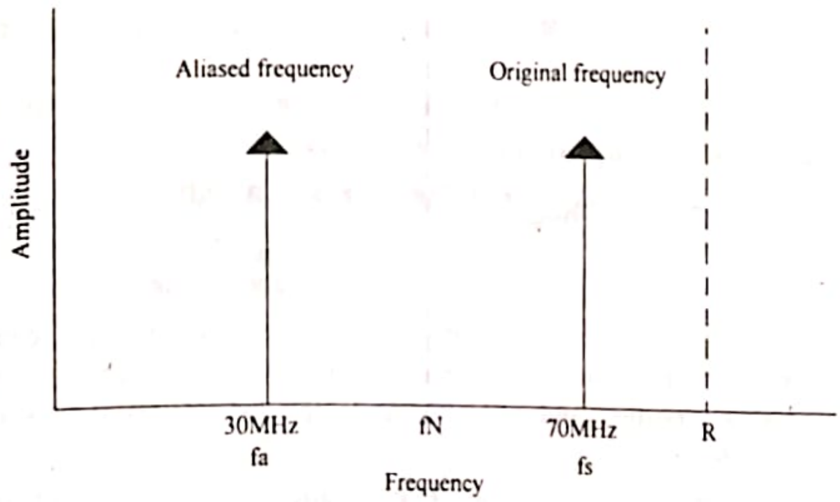
**1st Part:**

The unit of analog frequency is rad/s and the unit of digital frequency is rad/sampling interval. Thus, the highest digital frequency is $\pi$ rad/sampling interval & it is called the Nyquist frequency.In different units ,it is equal to

$$\text{Nyquist frequency} = \pi \text{ rad/sampling interval}$$
$$=\frac{\pi}{T_s}\text{rad/s} = \frac{1}{2T_s}\text{Hz} = \frac{1}{2}\text{revolution/s}$$

**2nd Part:**

One of the limitations of discrete-time sampling is an effect called aliasing. The Nyquist theorem states that one needs 2 samples per "cycle" of your input signal to define it. Thus, one can accurately measure the frequency of a signal with frequency $f$ as long as he/she is sampling it at greater than $2f$. If it is tried to measure the frequency of signals having a frequency above $f$ with a sampler operating at $2f$, then it will alias the signal, or create false images of this signal at frequencies below $f$. These false frequencies will appear as mirror images of the original frequency around the Nyquist frequency. This situation is called "aliasing back" or "folding back" and can be seen in Figure below.




**6. How does a PID type controller different from a on-off controller? What is integral saturation? [WBUT 2010, 2014]**

**Answer:**

**1st Part:**

**On-off controller** is a discontinuous type of controller.

If we consider the most common type two position controller then, it works on a very simple principle.It compares the value of an analog variable input with the set point and generates a digital (two position) output. Thus, whenever the signal to the controller is bellow the set point. the controller output will be 100%. As the measurement crosses the set point. the controller output goes to 0%.

On the other hand **PID controller** is a continuous type of controller. In this mode of control. the relationship between the controller input and the error signal is a continuous mathematical function.

The word PID controller stands for Proportional-integral-derivative controller. PID controller achieves its control goal by utilizing a feedback signal that reflects the actual operational state of the plant being controlled. When a PID controller receives a control command. the controller first compare it with the feedback signal to identify the difference between the desired set point and the actual state. it then correct the result. The ability to make control adjustment based on the actual plant state. a PID controller can also correct the undesirable behavior induced by external disturbances during plant operation.

Compared to a continuous controllers. discontinuous controllers operate on very simple. switching final control element. If the process time constants are large enough. good quality of control with small error can be attained even with discontinuous controllers and simple control elements.

**2nd Part:**

**Integral Saturation**

Integration is a continual summing. Integration of error means that we continually sum controller error up to the present time. The integral sum starts accumulating when the controller is first put in automatic and continues to change as long as controller error exists.

If an error is large enough and/or persists long enough, it is mathematically possible for the integral term to grow very large (either positive or negative):

$$CO = CO_{bias} + Kc \cdot e(t) + Ki \underbrace{\int e(t)dt}_{\text{can grow very large}}$$

This large integral, when combined with the other terms in the equation, can produce a CO value that causes the final control element (FCE) to saturate. That is, the CO drives the FCE (e.g. valve, pump and compressor) to its physical limit of fully open/on/maximum or fully closed/off/minimum. And if this extreme value is still not sufficient to eliminate the error, the simple mathematics of the controller algorithm, if not jacketed with protective logic, permits the integral term to continue growing.If the integral term grows unchecked, the equation above can command the valve, pump or compressor to move to 110%, then 120% and more. Clearly, however, when an FCE reaches its full 100% value, these last commands have no physical meaning and consequently, no impact on the process.

**7. Using bilinear transformation, the entire left half of the s-plane can be mapped inside a unit circle in the z-plane, Prove. [WBUT 2011]**

**Answer:**

**Bilinear transform**

The bilinear transformation defined by $z = \frac{w+1}{w-1}$

which, when solved for w, gives $w = \frac{z+1}{z-1}$

maps the inside of the unit circle in the Z plane into the left half of the w plane. This can be seen as follows. Let the real part of w be called s and the imaginary part $\omega$, so that

$$w = \sigma + j\omega$$

Since the inside of the unit circle in the Z plane is $|z| = \left|\frac{w+1}{w-1}\right| = \left|\frac{\sigma + j\omega + 1}{\sigma + j\omega - 1}\right| < 1$

$$\frac{(\sigma+1)^2 + \omega^2}{(\sigma-1)^2 + \omega^2} < 1$$

we get $(\sigma+1)^2 + \omega^2 < (\sigma-1)^2 + \omega^2$

which yields $\sigma < 0$

Thus , the inside of the unit circle in the Z plane ($|z| < 1$) corresponds to the left half of the w plane. The unit circle in the Z plane is mapped into the imaginary axis in the w plane, and the outside of the unit circle in the Z plane is mapped into the right half of the w plane. (It is pointed out that, although the w plane is similar to the s plane in that it maps the inside of the unit circle to the left half-plane, it is by no means quantitatively equivalent to the s plane. Therefore, estimating the relative stability of the system from the pole locations in the w plane is difficult.)

**8. The characteristics polynomial of a system is given by [WBUT 2011]**

$$F(z) = 2z^4 + 7z^3 + 10z^2 + 4z + 1 = 0$$

**Determine the stability of the system using Jury's stability test. Comment on the stability.**

**Answer:**

$2z^4 + 7z^3 + 10z^2 + 4z + 1 = 0$

$\Delta(1) = 2 + 7 + 10 + 4 + 1 = 24 > 0$ satisfied

$\Delta(-1) = 2 - 7 + 10 - 4 + 1 = 13 - 8 = 5 > 0$ satisfied.

| $z^0$ | $z^1$ | $z^2$ | $z^3$ | $z^4$ |
|---|---|---|---|---|
| 1 | 4 | 10 | 7 | 2 |
| 2 | 7 | 10 | 4 | 1 |
| 0 | –3 | –10 | –20 | –8 |
| –8 | –3 | –10 | –20 | 0 |
| 0 | 336 | 176 | –80 | |

$$b_0 = \begin{vmatrix} \alpha_n & \alpha_{n-1-k} \\ \alpha_0 & \alpha_{k+1} \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_{4-1-0} \\ \alpha_0 & \alpha_{4+1} \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_3 \\ \alpha_0 & \alpha_5 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix} = -8$$

$$b_1 = \begin{vmatrix} \alpha_4 & \alpha_{4-1-1} \\ \alpha_0 & \alpha_{1+1} \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_2 \\ \alpha_0 & \alpha_2 \end{vmatrix} = \begin{vmatrix} 1 & 10 \\ 2 & 10 \end{vmatrix} = -20$$

$$b_2 = \begin{vmatrix} \alpha_n & \alpha_{4-1-2} \\ \alpha_0 & \alpha_{2+1} \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_1 \\ \alpha_0 & \alpha_3 \end{vmatrix} = \begin{vmatrix} 1 & 7 \\ 2 & 4 \end{vmatrix} = -10$$

$$b_3 = \begin{vmatrix} \alpha_4 & \alpha_{4-1-3} \\ \alpha_0 & \alpha_{3+1} \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_0 \\ \alpha_0 & \alpha_4 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$$

$$b_4 = \begin{vmatrix} \alpha_4 & \alpha_{4-1-4} \\ \alpha_0 & \alpha_5 \end{vmatrix} = \begin{vmatrix} \alpha_4 & \alpha_{-1} \\ \alpha_0 & \alpha_5 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$$

$$c_k = \begin{vmatrix} b_{n-1} & b_{n-2-k} \\ b_0 & b_{k+1} \end{vmatrix}$$

$$c_0 = \begin{vmatrix} b_3 & b_{4-2-0} \\ b_0 & b_{4+1} \end{vmatrix} = \begin{vmatrix} b_3 & b_2 \\ b_0 & b_5 \end{vmatrix} = \begin{vmatrix} -20 & -10 \\ -8 & 0 \end{vmatrix} = -80$$

$$c_1 = \begin{vmatrix} b_3 & b_1 \\ b_0 & b_2 \end{vmatrix} = \begin{vmatrix} -20 & -3 \\ -8 & -10 \end{vmatrix} = 176$$

$$c_2 = \begin{vmatrix} b_3 & b_0 \\ b_0 & b_3 \end{vmatrix} = \begin{vmatrix} -20 & -8 \\ -8 & -20 \end{vmatrix} = 336$$

$$c_3 = \begin{vmatrix} b_3 & b_{-1} \\ b_0 & b_4 \end{vmatrix} = \begin{vmatrix} -20 & 0 \\ -8 & 0 \end{vmatrix} = 0$$

$$-8-(-20) = -8+20 = 12 > 0$$

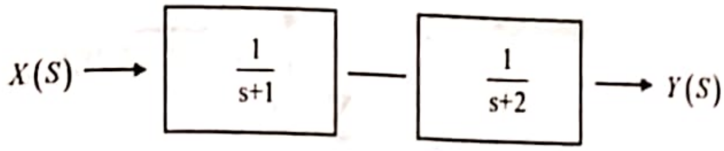
$$-20-(-8) = -20+8 = -12 < 0$$

$|b_{n-1}| > |b_0|$ (satisfied)

$|c_{n-2}| > |c_0|$ (satisfied)

$\therefore$ System is stable.

**9. a) What is pulse transfer function?** **[WBUT 2011, 2015, 2016]**
**b) Find the pulse T.F. of the system shown below:**




**Answer:**

**a)** If the test signal input is a step function applied by a digital computer via a DAC and the output is sampled by an ADC, long dividing the output sequence by the input sequence, results in the pulse response of the system. If this function is approximated by the difference equation method, the result is the pulse transfer function $G_hG(z)$.

**b)** $$L^{-1}\left\{\left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)\right\}$$

$$L^{-1}\left(\frac{1}{s+1}-\frac{1}{s+2}\right)=e^{-t}-e^{-2t}$$

Now discretising the function, we get,

$=\left(e^{-n}-e^{-2n}\right)u(n)$ Taking z transform of the above function, we get,

$$\frac{1}{1-e^{-1}z^{-1}}-\frac{1}{1-e^{-2}z^{-1}}=\frac{e^{-1}z^{-1}-e^{-2}z^{-1}}{\left(1-e^{-1}z^{-1}\right)\left(1-e^{-2}z^{-1}\right)}$$

$$\frac{z^{-1}\left(e^{-1}-e^{-2}\right)}{1-z^{-1}\left(e^{-2}+e^{-1}\right)+e^{-3}z^{-2}}$$

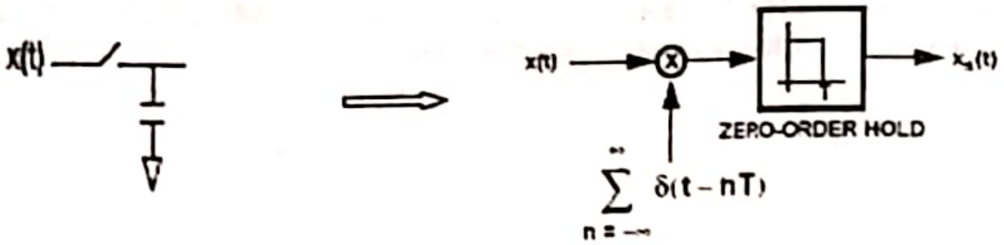
$$\therefore \quad \frac{Y(z)}{X(z)}=\frac{0.232z^{-1}}{1-0.232z^{-1}+0.049z^{-2}}$$

**10. Draw the circuit diagram of a practical sample and hold circuit. Also explain how its characteristics differs from ideal sample and hold circuit with reference to the following parameters: Acquisition time, Aperture time and Settling time.**
**[WBUT 2012]**

**Answer:**

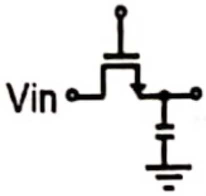
**1st Part:**

Generally, it is necessary to "hold" the sampled signal for some period of time. A simple SAMPLE &HOLD circuit is formed by sampling switch followed by a hold capacitor. Ideally, this is equivalent to impulse sampling followed by a zero-order hold as shown in fig. below.




The switch in a practical sample and hold circuit is MOS having some finite conducting resistance so providing a non-zero time constant. Ideal switch has zero resistance showing a zero time constant thus signal to be sampled is replicated instantly at the output.

**2nd Part:**

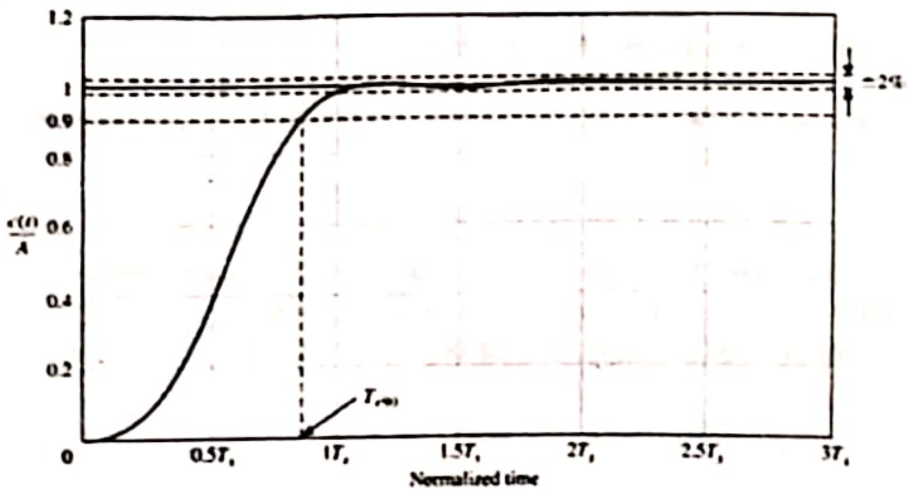



Acquisition time is the time for instant switch closes until Vi within defined % of input. Determined by input time constant $\tau$ = RinC $5\tau$ value = 99.3% of final value.
After the hold command, aperture time is the time after which changes of the input voltage no longer affect the output voltage.

**11. What is a deadbeat response?** **[WBUT 2012]**

**Answer:**

It is a response that proceeds rapidly to the desired level and holds at that level with minimal overshoot.2% of variation from the desired response is used as the acceptable range at the desired level as shown in the figure below.




The characteristics of a deadbeat response are:

1. For the specified input signal the steady state error is zero at the sampling instants.

2. Minimum settling time, i.e., minimum number of samples N should be required for the response to reach the steady state.
3. The ripples do not appear in between sampling instants at steady state.

**12. A unit step signal is sampled by an ideal sampler and then passes through a zero order hold (ZOH). Find the z-transform of the signal at the output of ZOH. What are the drawbacks of z-transform? [WBUT 2015]**

**Answer:**

**1st Part:** ***Refer to Question No. 7(a) of Long Answer Type Questions.***

**2nd Part:**

**Drawbacks of z-transform**

Z- transform is

a) not applicable to continuous system.
b) Z-transform is not unique.
c) accuracy is sampling frequency dependent
d) sampler should have ideal characteristics.

**13. What is signal discretization? Explain trapezoidal technique for discretization. [WBUT 2016]**

**Answer:**

***Refer to Question No. 1(a) of Long Answer Type Questions.***

**14. Explain Zero Hold (ZOH) & show the output of the transfer function of ZOH unit as input unit impulse. [WBUT 2016]**

**Answer:**

***Refer to Question No. 7(a) of Long Answer Type Questions.***

**15. Solve the difference equation: [WBUT 2017]**

$x(k+2)-3x(k+1)+2x(k)=4^k$ **, with the initial conditions** $x(0)=0$ **and** $x(1)=1$ **.**

**Answer:**

Taking z-transform at both sides of the equation

$$z^2X(z)-z^2x(0)-zx(1)-3zx(z)-3x(0)+2X(z)=\frac{1}{1-4z^{-1}}=\frac{z}{z-4}$$

Putting the values of $x(0)$ and $x(1)$

$$\Rightarrow \quad X(z)(z^2-3z+2)=\frac{z}{z-4}+z$$

$$\Rightarrow \quad X(z)(z-2)(z-1)=\frac{z(z-3)}{z-4}$$

$$\Rightarrow \quad X(z)=\frac{z(z-3)}{(z-1)(z-2)(z-4)}$$

$$\Rightarrow \quad \frac{X(z)}{z}=\frac{(z-3)}{(z-1)(z-2)(z-4)}=\frac{A}{(z-1)}+\frac{B}{(z-2)}+\frac{C}{(z-4)}$$

$$\Rightarrow \quad \frac{X(z)}{z}=\frac{-2}{3(z-1)}+\frac{1}{2(z-2)}+\frac{1}{6(z-4)}$$

$$\Rightarrow \quad X(z)=\frac{-2z}{3(z-1)}+\frac{z}{2(z-2)}+\frac{z}{6(z-4)}$$

Taking inverse Z-transform

$$x(n)=-\frac{2}{3}(n)+\frac{1}{2}(n)+\frac{1}{6}(n)$$

**16. What are the drawbacks of z-transform? State and prove the final value theorem of z-transform.** **[WBUT 2018]**

**Answer:**

**1st part:** ***Refer to Question no. 1(2nd part) of Short Answer Type Questions.***

**2nd part:** ***Refer to Question No. 2(c) of Long Answer Type Questions.***

**17. A unit step signal is sampled by an ideal sampler and then passes through a zero order hold (SOH). Find the z-transform of the signal at the output of ZOH. Compare zero order hold and first order hold.** **[WBUT 2018]**

**Answer:**

**1st part:** ***Refer to Question No. 7(a) of Long Answer Type Questions.***

**2nd part:** ***Refer to Question No. 7(b) of Long Answer Type Questions.***

**18. Map the region of stability in s-plane into z-plane.** **[WBUT 2018]**

**Answer:**

**Relationship between s plane and z-plane**

In the analysis and design of continuous time control systems, the pole-zero configuration of the transfer function in s-plane is often referred. We know that:

- Left half of s-plane $\Rightarrow$ Stable region.
- Right half of s-plane $\Rightarrow$ Unstable region.

For relative stability again the left half is divided into regions where the closed loop transfer function poles should preferably be located.

Similarly the poles and zeros of a transfer function in z-domain govern the performance characteristics of a digital system.

One of the properties of $F^*(s)$ is that it has an infinite number of poles, located periodically with intervals of $\pm m\omega_s$ with $m = 0, 1, 2, \cdots\cdots\cdots$, in the s-plane where $\omega_s$ is the sampling frequency in rad/sec.

If the primary strip is considered, the path, as shown in Fig. 1, will be mapped into a unit circle in the z-plane, centered at the origin.

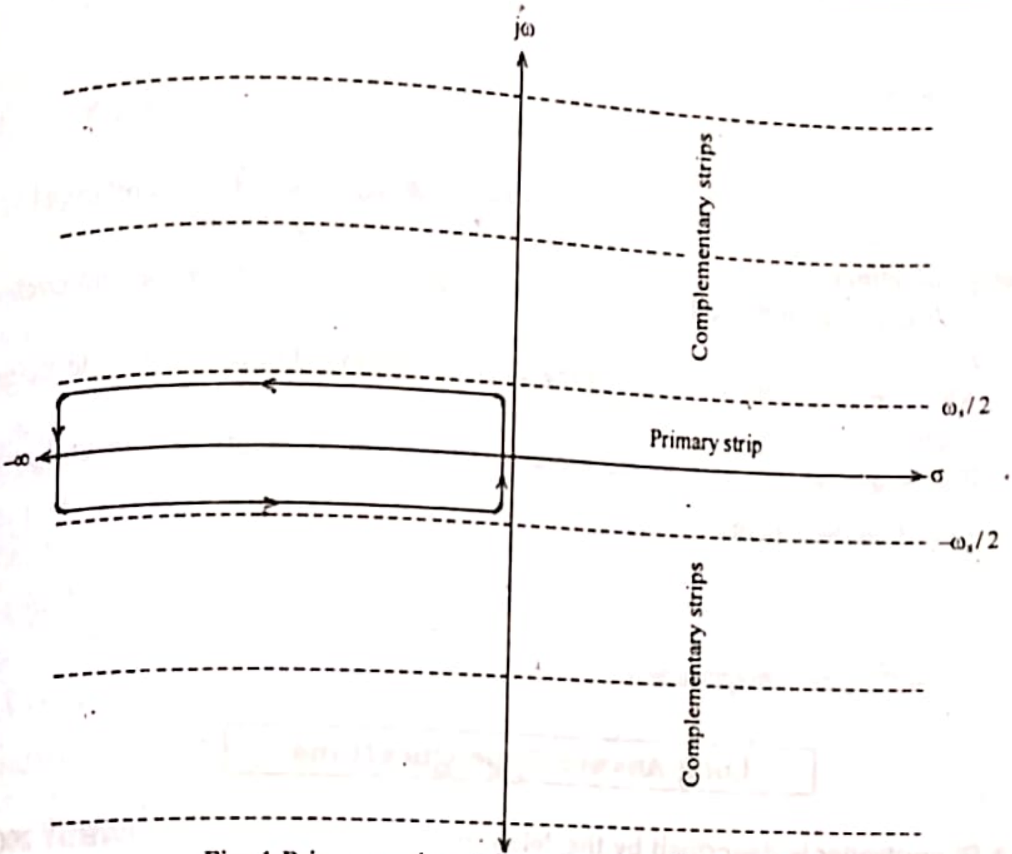



Fig: 1 Primary and complementary strips in s-plane

The mapping is shown in Fig. 2.

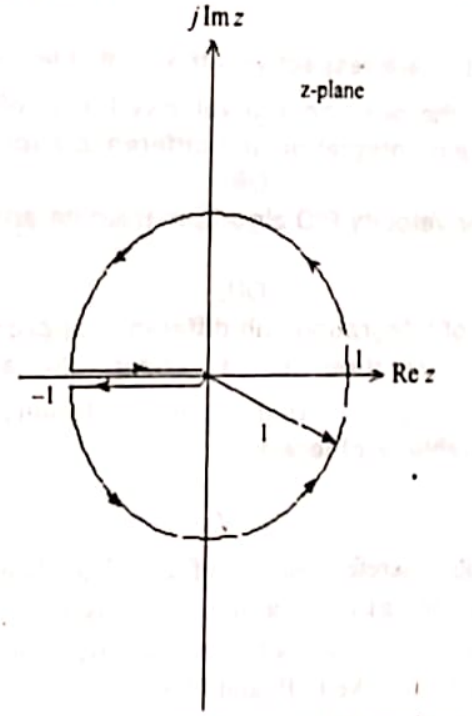



Fig: 2 Mapping of primary strip in z-plane

Since

$$e^{(s+jm\omega_s)T} = e^{T_s} e^{j2\pi m}$$
$$= e^{T_s}$$
$$= z$$

where $m$ is an integer, all the complementary strips will also map into the unit circle.

***Mapping guidelines***

1. All the points in the left half s-plane correspond to points inside the unit circle in z-plane.
2. All the points in the right half of the s-plane correspond to points outside the unit circle.
3. Points on the $j\omega$ axis in the s-plane correspond to points on the unit circle $|z|=1$ in the z-plane.

$s = j\omega$

$z = e^{T_s}$

$= e^{j\omega T} \Rightarrow$ magnitude $=1$

## Long Answer Type Questions

**1. a) A PI controller is described by the relation** **[WBUT 2009]**

$$p(t) = K_p\left(e(t) + \frac{1}{\tau_i}\int_0^t e(t)dt + \tau_d \frac{de(t)}{dt}\right)$$

**where $p(t)$, $e(t)$, $\tau_i$ and $\tau_d$ are respectively the controller output, error input, reset time and rate time. Find the position and velocity forms of the control algorithms using the trapezoidal rule of integration and difference approximation.**

**OR,**

**Derive the expression for velocity PID algorithm from its analog counterpart.** **[WBUT 2011, 2017]**

**OR,**

**What is trapezoidal rule of integration and difference approximation? [WBUT 2016]**

**b) The characteristic equation of a linear digital control system is $z^4 - 2z^3 + 1.75z^2 - 0.75z + 0.125 = 0$. Using Jury's stability criteria determination whether the system is stable or otherwise.** **[WBUT 2009, 2016]**

**Answer:**

**a) 1st Part:**

A digital controller accepts discrete sequence of error signals e(k) and computes discrete sequence of control action u(k), according to a discrete control law that may be represented in time domain or in Z domain. There are two commonly used algorithm for programming analog controllers like P, PI and PID.

Equation for a PID control action in continuous time domain is given by:

$$m = K_p e + \frac{K_p}{\tau_i}\int e dt + K_p \tau_d \frac{de}{dt} + m_i \qquad \text{(where } m_i \text{ is the initial valve position)}$$

In each sampling period (T), a sampled value of the processor output enters the computer. Let y(nT) be the sampled value at the nth sampling instant. y(nT) is compared with set point value at the same instant which yields the value of discrete-time error, e(nT) expressed as e(nT) = $y_{sp}(nT) - y(nT)$. In terms of error at sampling instants, e(nT) may be written as $e_n$

Then the discrete time control action produced by the proportional mode based on error present at e(nT) is $K_p e(nT)$ or $K_p e_n$

The control action produced bt the integral mode is based on the integration of errors over a time period. Science error values are available on a discrete-time basis, the integral $\int e(t)dt$ can be approximated by summation.

$$\int_0 e(t)dt = T\sum_{k=0}^{n} e(k)$$

Therefore, the integral mode control action is given by:

$$m_n = \frac{T}{T_i}\sum_{k=0}^{n} e(k) = m_{n-1} + \frac{T}{T_i} e_n$$

For the derivative mode action, we needed a numerical evaluation of the derivative term $\frac{de}{dt}$. Therefore,

$$T_d \frac{de}{dt} = \frac{T_d}{T}[e(nT) - e(n-1)T] = \frac{T_d}{T}[e_n - e_{n-1}]$$

$$m_n = K_p[e_n + \frac{T}{T_i}\sum_{k=0}^{n} e_k + \frac{T_d}{T}(e_n - e_{n-1})] + m_i \qquad (1)$$

This is called **position algorithm** because the value calculated is the actual manipulation variable, which in many case represents an actuator(valve, piston, etc.) position. The necessity to initialize the position algorithm (i.e., to somehow supply the initial actuator position, $m_i$, to the computer)can be a disadvantage for the controller. Knowledge of $m_i$ is necessary to ensure a smooth transition (bumpless transfer) from manual or no control to automatic control. Additionally, if the computer fails to respond, the the resulting zero output could imply zero position, to which the actuator would respond perhaps with undesirable consequences. With similar analogy, the control valve position at the (n-10)$^{th}$ instant can be written as below-

$$m_{n-1} = K_p[e_{n-1} + \frac{T}{T_i}\sum_{k=0}^{n-1} e_k + \frac{T_d}{T}(e_{n-1} - e_{n-2}) + m_i \qquad (2)$$

If equation (2) is subtracted from equation (1) the **velocity form** of the PID algorithm is obtained. The change in control valve position is better because there is no need to compute the initial position of valve.

$$m_n - m_{n-1} = K_p[(e_n - e_{n-1}) + \frac{T}{T_i}e_n + \frac{T_d}{T}(e_n - 2e_{n-1} + e_{n-2})]$$

$$= K_p[e_n - e_{n-1} + \frac{T}{T_i}e_n + \frac{T_d}{T}e_n - 2\frac{T_d}{T}e_{n-1} + \frac{T_d}{T}e_{n-2}]$$

$$= K_p[(1 + \frac{T}{T_i} + \frac{T_d}{T})e_n - (1 + 2\frac{T_d}{T})e_{n-1} + \frac{T_d}{T}e_{n-2}]$$

$$= (K_p + \frac{TK_p}{T_i} + \frac{K_pT_d}{T})e_n - (K_p + \frac{2T_dK_p}{T})e_{n-1} + \frac{K_pT_d}{T}e_{n-2}$$

$$= Ae_n + Be_{n-1} + Ce_{n-2}$$

$$where, A = (K_p + \frac{TK_p}{T_i} + \frac{K_pT_d}{T}), B = (K_p + \frac{2T_dK_p}{T}), C = \frac{K_pT_d}{T}$$

The term A,B,C depend on the control parameter and sampling period.

**2nd Part:**

There are two alternative forms of the digital PID control equation, the position form and the velocity form. A straightforward way of deriving a digital version of the parallel form of the PID controller is to replace the integral and derivative terms by finite difference approximations,

$$\int_0^t e(t^\theta)dt^\theta \approx \sum_{j=1}^{k} e_j\Delta t \qquad \text{... (1)}$$

$$\frac{de}{dt} \approx \frac{e_k - e_{k-1}}{\Delta t} \qquad \text{... (2)}$$

where

$\Delta t$ = the sampling period (the time between successive measurements of the controlled variable)

$e_k$ = error at the kth sampling instant for k = 1, 2, ...

Substituting Eqs. 1and 2 gives the position form.

$$p_k = \overline{p} + K_c\left[e_k + \frac{\Delta t}{\tau_I}\sum_{j=1}^{k} e_j + \frac{\tau_D}{\Delta t}(e_k - e_{k-1})\right] \qquad \text{... (3)}$$

where $p_k$ is the controller output at the $k$th sampling instant. The other symbols in Eq. 3 have the same meaning as previous. Equation 3 is referred to as the position form, because the actual value of the controller output is calculated.

In the velocity form, the change in controller output is calculated. The velocity form, the change in controller output is calculated. The velocity form can be derived by writing Eq. 3 for the $(k-1)$ sampling instant:

$$p_{k-1} = \bar{p} + K_c\left[e_{k-1} + \frac{\Delta t}{\tau_I}\sum_{j=1}^{k} e_j + \frac{\tau_D}{\Delta t}(e_{k-1} - e_{k-2})\right] \qquad \ldots (4)$$

Note that the summation still begins at $j=1$, because it is assumed that the process is at the desired steady state for $j \le 0$, and thus $e_j = 0$ for $j \le 0$. Subtracting Eq. 4 from (3) gives the velocity form of the digital PID algorithm:

$$\Delta p_k = p_k - p_{k-1} = K_c\left[(e_k - e_{k-1}) + \frac{\Delta t}{\tau_I}e_k + \frac{\tau_D}{\Delta t}(e_k - 2e_{k-1} + e_{k-2})\right] \quad \ldots (5)$$

The velocity form has three advantages over the position form:

1. It inherently contains antireset windup, because the summation of errors is not explicity calculated.
2. This output is expressed in a form, $\Delta p_k$, that can be utilized directly by some final control elements, such as a control valve driven by a pulsed stepping motor.
3. For the velocity algorithm, transferring the controller from manual to automatic model does not require any initialization of the output ( $\bar{p}$ in Eq. 3). However, the control valve (or other final control element) should be placed in the appropriate position prior to the transfer.

Certain types of advanced control strategies, such as cascade control and feed-forward control, require that the actual controller output $p_k$ be calculated explicitly.. However, $p_k$ can easily be calculated by rearranging Eq. 5:

$$p_k = p_{k-1} + K_c\left[(e_k - e_{k-1}) + \frac{\Delta t}{\tau_I}e_k + \frac{\tau_D}{\Delta t}(e_k - 2e_{k-1} + e_{k-2})\right] \qquad \ldots (6)$$

A minor disadvantage of the velocity form is that the integral mode must be included. When the set point is constant, it cancels out in both the proportional and derivative error terms. Consequently, if the integral mode were omitted, the process response to a disturbance would tend to drift away from the set point.

The position form of the PID algorithm (Eq. 3) requires a value of $\bar{p}$, while the velocity form in Eq. 5 does not. Initialization of either algorithm is straight-forward, because manual operation of the control system usually precedes the transfer to auto.natic control.

**b)** $z^4 - 2z^3 + 1.75z^2 - 0.75z + 0.125$

From the equation we get

$a_0 = 1$

$a_1 = -2$

$a_2 = 1.75$

$a_3 = -0.75$

$a_4 = 0.125$

We know that,

$$b_k = \begin{vmatrix} a_n & a_{n-1-k} \\ a_0 & a_{k+1} \end{vmatrix}$$

$$c_k = \begin{vmatrix} b_{n-1} & b_{n-2-k} \\ b_0 & b_{k+1} \end{vmatrix}$$

| $z_0$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|---|---|---|---|---|
| 0.125<br>0.9843<br>0.5<br>0.718 | −0.75<br>1.062<br>−1.531<br>1.81 | 1.75<br>−1.531<br>1.062<br>−2.037 | −2<br>0.5<br>0.9843 | 1 |

$$b_0 = \begin{vmatrix} a_4 & a_3 \\ a_0 & a_1 \end{vmatrix} = \begin{vmatrix} 0.125 & -0.15 \\ 1 & -2 \end{vmatrix} \quad = -0.25 - (-0.75) = -0.25 + 0.75 = 50$$

$$b_1 = \begin{vmatrix} a_4 & a_2 \\ a_0 & a_2 \end{vmatrix} = \begin{vmatrix} 0.125 & 1.75 \\ 1 & 1.75 \end{vmatrix} \quad = -1.531$$

$$b_2 = \begin{vmatrix} a_4 & a_1 \\ a_0 & a_3 \end{vmatrix} = \begin{vmatrix} 0.125 & -2 \\ 1 & -0.75 \end{vmatrix} \quad = -0.9375 - (-2) = -0.9375 + 2 = 1.0625$$

$$b_3 = \begin{vmatrix} a_4 & a_0 \\ a_0 & a_4 \end{vmatrix} = \begin{vmatrix} 0.125 & 1 \\ 1 & 0.125 \end{vmatrix} \quad = 0.9843$$

$$b_4 = \begin{vmatrix} a_4 & 0 \\ a_0 & 0 \end{vmatrix} \quad = 0$$

$\therefore$
$$b_0 = 0.5$$
$$b_1 = -1.531$$
$$b_2 = 1.062$$
$$b_3 = 0.9843$$

Now,

$$c_0 = \begin{vmatrix} b_3 & b_2 \\ b_0 & b_1 \end{vmatrix} = \begin{vmatrix} 0.984 & 1.062 \\ 0.5 & -1.531 \end{vmatrix} \quad = -1.506504 = 0.531 = -2.037$$

$$c_1 = \begin{vmatrix} b_3 & b_1 \\ b_0 & b_2 \end{vmatrix} = \begin{vmatrix} 0.984 & -1.531 \\ 0.5 & 1.062 \end{vmatrix} \quad = 1.045 + 0.765 = 1.810$$

$$c_2 = \begin{vmatrix} b_3 & b_0 \\ b_0 & b_3 \end{vmatrix} = \begin{vmatrix} 0.984 & 0.5 \\ 0.5 & 0.984 \end{vmatrix} \quad = 0.968256 - 0.25 = 0.718$$

Now, from the above, we can get,

$$\Delta(1) = 1 - 2 + 1.75 - 0.75 + 0.125 = 0.125 > 0$$
$$\Delta(-1) = 1 - 2 + 1.75 + 0.75 + 0.125 = 1 + 1.75 + 0.75 + 0.125 - 2 = 1.625 > 0$$

$|a_n| < |a_0|$

$= (0.125) < (1)$ (satisfied)

$|0.9843| > |0.5|$

$|0.718| > |-2.037|$ (satisfied)

$\therefore$ System is stable.

**2. a) What device is used for signal reconstruction? How can a signal be constructed from a sequence of data points? Compare zero-order hold and first-order hold devices.** **[WBUT 2010, 2014]**

**OR,**

**Explain the principle of reconstruction of a continuous time output using (i) Zero Order Hold (ZOH) and (ii) First Order Hold (FOH) devices. Sketch the nature of output response to a unit impulse input in both the cases.** **[WBUT 2011]**

**b) Why is z-transform required for analysis of discrete-data systems?**
**[WBUT 2010, 2012, 2014]**

**c) State and prove the initial value and final value theorems of z-transform.**
**[WBUT 2010, 2014]**

**d) Find inverse z-transform of the function** $F(z) = z/(z^2 - 4z + 2)$.
**[WBUT 2010, 2014]**

**Answer:**

**a)** In a mixed-signal system(analog and digital) a reconstruction filter (or anti-imaging filter) is used to construct a smooth analogue signal. A low pass reconstructions filter may also be used which can remove the harmonics above the nyquist unit and smoother the signal.

Let $x[n]$ a set of samples that came from some continuous-time signal $x(t)$. Let also assume that the sampling rate is T, so, $x(nT) = x[n]$. We have to recover $x(t)$ from the sampled signals.

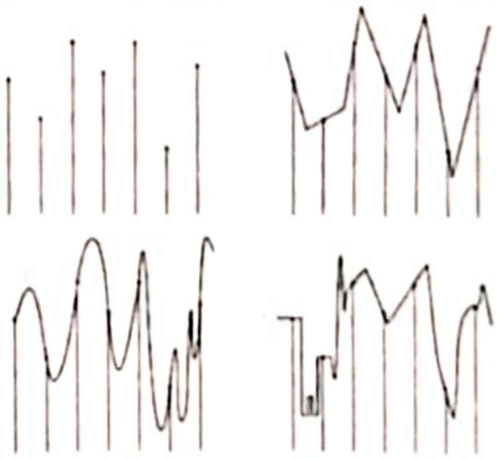


Fig: Possible continuous-time functions corresponding to samples



Scanned by CamScanner

Figure above shows some discrete-time samples and some possible continuous-time functions from which these samples could have been obtained. There are some simple estimates might made to approximately reconstruct $x(t)$.

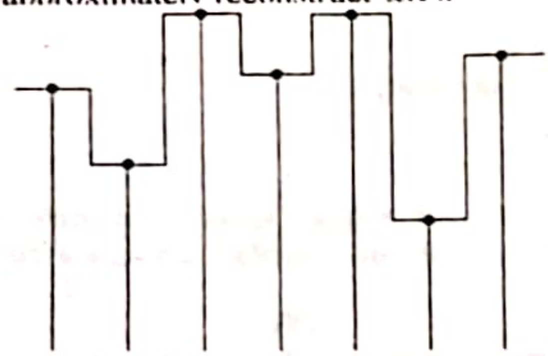


Fig: Nearest-neighbour reconstruction

The first estimate one might think of is to just assume that the value at time t is the same as the value of the sample at some time nT that is closest to t. This nearest-neighbor interpolation results in a piecewise-constant (staircase-like) reconstruction as shown in Figure above.

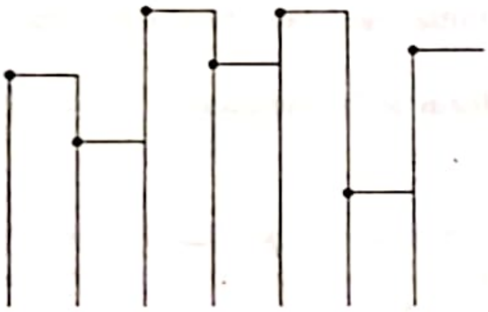


Fig: Zero-order hold reconstruction

Actually, instead of nearest-neighbor interpolation, most devices implement a similar type of interpolation known as **zero-order-hold interpolation** shown in Figure above. This is one of the most widely used methods and is easy to implement. As with nearest-neighbor interpolation, this results in a piecewise-constant reconstruction, but the discontinuities are at the sample points instead of between sample points.

Another conceptually simple method is linear interpolation, which is also called **first-order-hold** interpolation and is shown in Figure . With this method, the reconstruction is a continuous function that just connects the sample values with straight lines. Higher order interpolation schemes also exist that pass smoother functions through the samples. The higher the order, the more samples that need to be used to reconstruct a value at each time. One technique that actually uses all the samples will be mentioned in the next section.

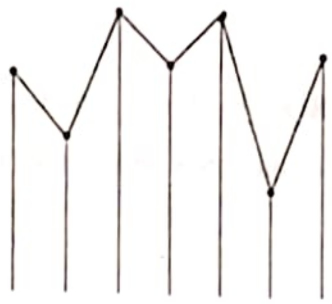


Fig: First-order hold reconstruction

Among zero order hold and higher order hold, zero order hold is the simplest and is used most frequently in practice. The term zero order refers to the capability of the device to pass without distortion a constant, i.e a zero-order polynomial. On the other hand, a first order hold device can pass without distortion a first-order polynomial signal. Basically, the first-order hold is a signal extrapolation using the first difference equation. Then the n-th order hold device then can pass without distorting nth order polynomials.

In general only zero-order hold devices implemented in hardware. Higher-order hold algorithms are implemented in computer software since it is most difficult to physically implement higher order hold devices.

**b)** The z Transform is a powerful mathematical tool for the analysis of linear-time invariant discrete time systems in frequency domain for example- for digital control or for filtering and may be compared to the Laplace transform, used for the analysis of continuous time signals and systems. The z transform is important as a means to characterize a linear time-invariant system in terms of its pole-zero locations, its transfer function, Bode diagram and its response to a large variety of signals. In addition it provides important relationship between temporal and spectral properties of signals. In z domain the convolution of two time domain signal is equivalent to multiplication of their corresponding z transform. This property simplifies the analysis of response of an LTI system of various signals. The z transform generally appears in the analysis of difference equations as used in many branches of engineering and applied mathematics.

**c) Initial value theorem**

If $X_+(z)=Z\{x(n)\}$, then

$$x(0)=\lim_{z\to\infty} X_+(z) \qquad \dots (1)$$

Where x(n) is a discrete time sequence

Proof:

$$X_+(z)=\sum_{n=0}^{\infty} x(n)z^{-n} = x(0)+x(1)z^{-1}+x(2)z^{-2}+\dots$$

As $z\to\infty$, all the terms vanish except $x(0)$. Therefore,

$$\lim_{z\to\infty} X_+(z)=\lim_{z\to\infty}\sum_{n=0}^{\infty} x(n)z^{-n}=x(0)$$

That is, $\qquad x(0)=\lim_{z\to\infty} X_+(z)$

**Final value theorem**

If $X_+(z)=Z\{x(n)\}$, where the ROC for $X_+(z)$ includes, but is not necessarily confined to $|z|>1$ and $(z-1)X_+(z)$ has no poles on or outside the unit circle, then

$$x(\infty)=\lim_{z\to1}(z-1)X_+(z) \qquad \dots (2)$$

Proof:

$$Z\{x(n+1)\}-Z\{x(n)\}=\lim_{k\to\infty}\sum_{n=0}^{k}[x(n+1)-x(n)]z^{-n}$$

$$zX_+(z)-zx(0)-X_+(z)=\lim_{k\to\infty}\sum_{n=0}^{k}[x(n+1)-x(n)]z^{-n}$$

$$(z-1)X_+(z)-zx(0)=\lim_{k\to\infty}\sum_{n=0}^{k}[x(n+1)-x(n)]z^{-n}$$

Now let $z\to1$

$$\lim_{z\to1}(z-1)X_+(z)-x(0)=\lim_{k\to\infty}\{[x(1)-x(0)]+[x(2)-x(1)]+[x(3)-x(2)] +\dots\dots+[x(k+1)-x(k)]\}$$

$$=x(\infty)-x(0)$$

Therefore

$$x(\infty)=\lim_{z\to1}(z-1)X_+(z)$$

or $$x(\infty)=\lim_{z\to1}(1-z^{-1})X(z)$$

d) $F(z)=\dfrac{z}{z^2-4z+2}$

$$\frac{F(z)}{z}=\frac{1}{(z^2-4z+2)}$$

$$\frac{A}{z-(2+\sqrt{2})}+\frac{B}{z-(2-\sqrt{2})}$$

$$A=\left[\left(z-(2+\sqrt{2})\frac{1}{(z^2-4z+2)}\right)\right]_{\text{at } z=2+\sqrt{2}}=\frac{1}{z-(2-\sqrt{2})}=\frac{1}{2+\sqrt{2}-2+\sqrt{2}}=\frac{1}{2\sqrt{2}}$$

$$B=\left[\left(z-(2-\sqrt{2})\frac{1}{(z^2-4z+2)}\right)\right]_{\text{at } z=2-\sqrt{2}}=\frac{1}{z-(2+\sqrt{2})}=\frac{1}{2-\sqrt{2}-2-\sqrt{2}}=-\frac{1}{2\sqrt{2}}$$

$$\therefore \frac{F(z)}{z}=\frac{1}{2\sqrt{2}}\left[\frac{z}{z-(2+\sqrt{2})}-\frac{z}{z-(2-\sqrt{2})}\right]$$

Taking inverse transform,

$$F(n)=\frac{1}{2\sqrt{2}}\left[\left(2+\sqrt{2}\right)^n u(n)-\left(2-\sqrt{2}\right)^n u(n)\right]$$

$$=0.3535\left[(3.414)^n u(n)-(0.5857)^n u(n)\right],\ |z|>3.414$$

$$=0.3535\left[-(3.414)^n u(-n-1)-(0.5857)^n u(-n-1)\right],\ |z|>0.5857$$

$$=0.3535\left[-(3.414)^n u(-n-1)-(0.5857)^n u(n)\right],\ 0.5857<|z|<3.414$$

**3. For the following discrete-time open loop transfer function for a unity feedback control system find the minimum value of K such that the steady-state error due to ramp input is** $K_v \geq 4/\text{sec}$.

$G_{zoh}G_p(z)=K(z+0.76)/\left[16(z-1)(z-0.45)\right]$. **[WBUT 2010, 2012]**

**Answer:**

From the above we can write that,

$$k_v=\frac{1}{T}\lim_{z\to 1}\left[(z-1)GH(z)\right]$$

$$G_{ZOH}G_P(z)=\frac{k(z+0.76)}{16(z-1)(z-0.45)}$$

$$\therefore \quad k_v=\frac{1}{T}\lim_{z\to 1}\left[\frac{(z-1)k(z+0.76)}{16(z-1)(z-0.45)}\right]=\frac{1}{T}\left[\frac{k(1.76)}{16(0.55)}\right]$$

$T$ is rate per unit time with which the ramp input increases.

Assuming $T$ to be 1, we get,

$$k_v=\frac{K(1.76)}{16(0.55)}$$

If $\quad k_v=4$,

$$k=\frac{4\times 0.55\times 16}{1.76}=20$$

So, the minimum value of $k$ for $k_v$ to be greater than $4=20$.

**4. What is the difference between a real and an ideal sampler? Under what conditions an ideal sampler will have identical sampled values from the two sinusoidal waves–**

$\sin(\omega_1 t)$ and $\sin(\omega_2 t)$? **[WBUT 2011]**

**Answer:**

Sampling is obtained by applying a continuous time signal to an ADC whose output is a series of digital values.

In this case let us consider, T is the time interval between two successive samples. Then the discrete-time signal can be represented by the relation-

$$x(nT) = x(n) \quad -\infty < n < \infty \qquad \ldots (1)$$

Then the time interval between successive samples is called the sampling period and its reciprocal $1/T_s = F_s$ is called sampling rate.

Let us consider a sinusoidal signal

$$x(t) = \sin \Omega t \qquad \ldots (2)$$

The sampled sinusoidal sequence can be obtained by substituting t=nT in eq.(2). That is,

$$x(nT) = \sin \Omega nT \qquad \ldots (3)$$

$$= \sin \omega n \text{ where, } \omega = \Omega T$$

The range of frequency variable $\Omega$ in a continuous-time signal is given by $-\infty < \Omega < \infty$, where as the range of frequency variable $\omega$ in a discrete-time signal is given by $-\pi < \omega < \pi$. That is, the periodic sampling of a continuous-time signal maps the infinite frequency variable $\Omega$ into a finite frequency variable $\omega$.

If we imagine that the sinusoid of eq. (2) is the input of an ADC, then the output of the ADC is given by eq.(3). where the sampling frequency of the ADC is given by $\Omega_s = 2\pi/T$. Let us study the output of ADC as a function of $\Omega$. The output of ADC at frequency that differs by multiple of $2\pi/T$ is given by-

$$x(n) = \sin[(\Omega + \frac{2\pi M}{T})nT] \qquad M=1,2\ldots\ldots$$

$$= \sin[(\Omega nT + 2\pi Mn)] = \sin\Omega nT \qquad \ldots (4)$$

Which is identical to (3), that is the signals $\sin\Omega t$ and $\sin(\Omega+\Omega_s)t$ produce the same output samples. It means that a sequence x(n) obtained by sampling the sinusoidal signal of $\Omega$ rad/sec (or, f Hz), also exactly represents sine waves at other frequencies namely $(\Omega + 2\pi k/T)$ or $(f + kf_s)$, that is when the signal is sampled at a rate of $f_s$ sample/sec, and if k is a positive or negative integer, we cannot distinguish between the sampled values of a sine wave of frequencies f Hz and $(f + kf_s)$Hz. Thus an infinite number of continuous-time sinusoids are represented by the same set of samples. As a result an ambiguity may exists.

**5. The characteristic polynomial of a system is given by $F(z) = z^4 - 2z^3 + 1.75z^2 - 0.75z + 0.125 = 0$. Determine the stability of the system using Jury's Stability test. [WBUT 2012, 2015, 2017]**

**Answer:**

$z^4 - 2z^3 + 1.75z^2 - 0.75z + 0.125$

From the equation we get

$$a_0 = 1$$
$$a_1 = -2$$
$$a_2 = 1.75$$
$$a_3 = -0.75$$
$$a_4 = 0.125$$

We know that,

$$b_k = \begin{vmatrix} a_n & a_{n-1-k} \\ a_0 & a_{k+1} \end{vmatrix}$$

$$c_k = \begin{vmatrix} b_{n-1} & b_{n-2-k} \\ b_0 & b_{k+1} \end{vmatrix}$$

| $z_0$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ |
|---|---|---|---|---|
| 0.125 | –0.75 | 1.75 | –2 | 1 |
| 0.9843 | 1.062 | –1.531 | 0.5 | |
| 0.5 | –1.531 | 1.062 | 0.9843 | |
| 0.718 | 1.81 | –2.037 | | |

$$b_0 = \begin{vmatrix} a_4 & a_3 \\ a_0 & a_1 \end{vmatrix} = \begin{vmatrix} 0.125 & -0.15 \\ 1 & -2 \end{vmatrix} \qquad = -0.25 - (-0.75) = -0.25 + 0.75 = 50$$

$$b_1 = \begin{vmatrix} a_4 & a_2 \\ a_0 & a_2 \end{vmatrix} = \begin{vmatrix} 0.125 & 1.75 \\ 1 & 1.75 \end{vmatrix} \qquad = -1.531$$

$$b_2 = \begin{vmatrix} a_4 & a_1 \\ a_0 & a_3 \end{vmatrix} = \begin{vmatrix} 0.125 & -2 \\ 1 & -0.75 \end{vmatrix} \qquad = -0.9375 - (-2) = -0.9375 + 2 = 1.0625$$

$$b_3 = \begin{vmatrix} a_4 & a_0 \\ a_0 & a_4 \end{vmatrix} = \begin{vmatrix} 0.125 & 1 \\ 1 & 0.125 \end{vmatrix} \qquad = 0.9843$$

$$b_4 = \begin{vmatrix} a_4 & 0 \\ a_0 & 0 \end{vmatrix} \qquad = 0$$

$\therefore$ $b_0 = 0.5$

$b_1 = -1.531$

$b_2 = 1.062$

$b_3 = 0.9843$

Now,

$$c_0 = \begin{vmatrix} b_3 & b_2 \\ b_0 & b_1 \end{vmatrix} = \begin{vmatrix} 0.984 & 1.062 \\ 0.5 & -1.531 \end{vmatrix} \qquad = -1.506504 = 0.531 = -2.037$$

$$c_1 = \begin{vmatrix} b_3 & b_1 \\ b_0 & b_2 \end{vmatrix} = \begin{vmatrix} 0.984 & -1.531 \\ 0.5 & 1.062 \end{vmatrix} \qquad = 1.045 + 0.765 = 1.810$$

$$c_2 = \begin{vmatrix} b_3 & b_0 \\ b_0 & b_3 \end{vmatrix} = \begin{vmatrix} 0.984 & 0.5 \\ 0.5 & 0.984 \end{vmatrix} \qquad = 0.968256 - 0.25 = 0.718$$

Now, from the above, we can get,

$$\Delta(1) = 1 - 2 + 1.75 - 0.75 + 0.125 = 0.125 > 0$$

$$\Delta(-1) = 1 - 2 + 1.75 + 0.75 + 0.125 = 1 + 1.75 + 0.75 + 0.125 - 2 = 1.625 > 0$$

$$|a_n| < |a_0|$$
$$= (0.125) < (1) \text{ (satisfied)}$$
$$|0.9843| > |0.5|$$
$$|0.718| > |-2.037| \text{ (satisfied)}$$

$\therefore$ System is stable.

**6. a) What is meant by HART protocol? Describe the typical HART architecture with principle technical data of HART.**
**b) Describe the Ethernet TCP/IP architecture of PROFIBUS DP as control network. Discuss the main features of the network. [WBUT 2014, 2016, 2018]**

**Answer:**

**a) 1st Part:** ***Refer to Question No 12(a) of Long Answer type Questions.***

**2nd Part:**

Figure (i) shows three typical ways in which HART devices are integrated into control system architectures. Where a controller uses the HART digital signal, the HART devices are connected either point-to-point or as HART multi-drop bus to a HART I/O controller card. In this configuration, the full parameter set of the device will normally be available to the operator via, for example, an OPC server and commissioning/diagnosis of the HART devices is possible from a central maintenance station.

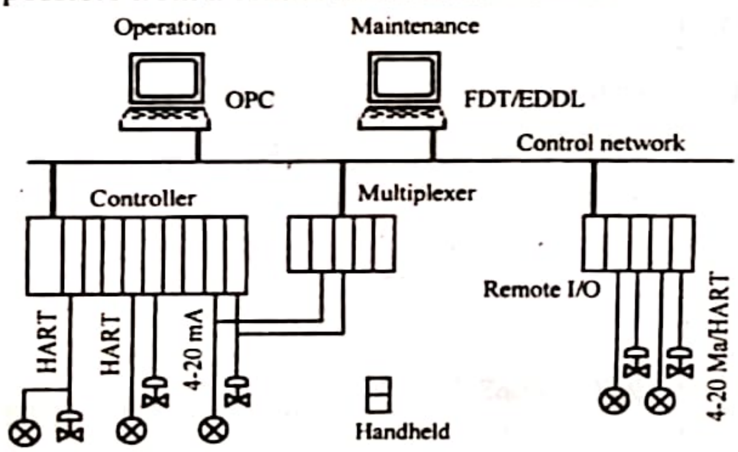



Fig: (i) Typical HART architecture

If the HART signal is used only for commissioning/diagnosis, it is more likely that the devices will be connected to a remote or intelligent I/O. Remote I/Os are wired point-to-point and often provide loop power for the 4-20 mA devices. Intelligent I/Os may also contain processing abilities, for example, the ability to link signals from several sensors to produce a secondary process variable, for example, density or mass or be able to perform simple control actions such as closing valve. Device information is collected by the controller or application software via the control network. Remote I/Os are often transparent, allowing direct configuration of the devices by an appropriate maintenance tool.

b) Ethernet has become the dominant technology at the operations and in some cases the control network level. A typical system architecture is shown in figure below. Unlike PROFIBUS and FOUNDATION Fieldbus, it is not a designer solution for Process Automation; rather, it has been found to have a lot of characteristics required for the H2 layer: openness, inexpensive components and support from major systems and software manufacturers.

Unfortunately, there is no "Industrial Ethernet" standard as such, although there is a supporting association. Rather there is a collection of protocols that use the Ethernet physical layer, IP and TCP. The most fully defined protocol is FOUNDATION Fieldbus HSE. MODBUS TCP is not a standard but is popular and in widespread use. Ethernet IP has some support and there are a number of proprietary systems with Ethernet TCP/IP backbones.

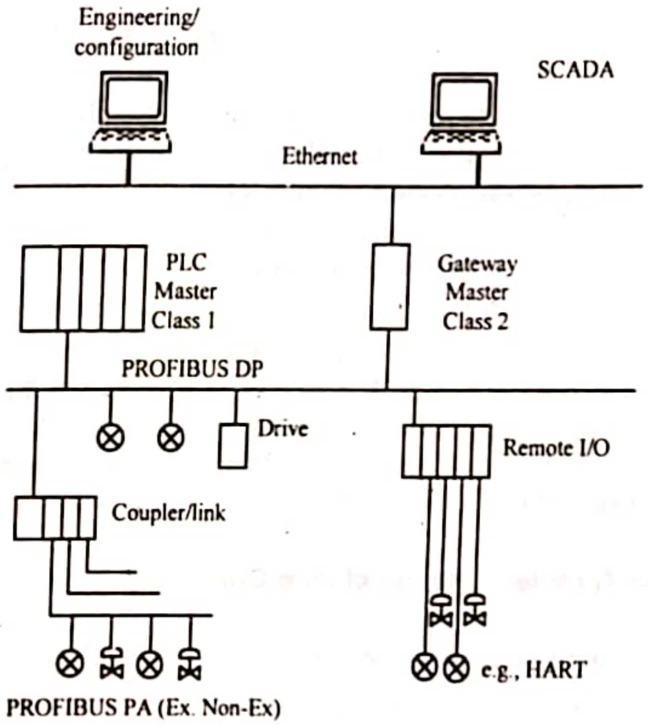



Fig: (i) Typical Ethernet TCP/IP architecture using the example of PROFIBUS DP as control network

The protocols function by either packing their information in the Ethernet data frame or mapping proprietary applications on the TCP and IP protocols. Devices using different protocols cannot understand each other, but in some cases, e.g., with HSE and MODBUS TCP/IP, it is possible to use the same network to transport data. The matter is further complicated by the existence of two different Ethernet frames: devices receiving the wrong kind simply assume that a collision has occurred and reject the data. On the other hand, it is a fairly simple matter to map compatible systems and for example, FF Function blocks for the interpretation of MODBUS data already exist, providing a excellent means of communication between FF and legacy systems.

Ethernet is accessed by means of the CSMA/CD method. This means that it is ideal for the transfer of non-time-critical information but requires as additional means of controlling medium access if it is to be used in deterministic control applications. Current equipment is usually capable of transmitting at 10 or 100 Mbit/s as required over 4-wire twisted pairs as well as fiber-optic cables. Commercial equipment is sufficiently rugged for a control room environment.

**Cost and availability of suitable chips:** This is probably a question of supply and demand; if there is a move toward Ethernet devices, the costs will decrease in inverse proportion to the numbers produced. In the long term, therefore, no hindrance will occur.

**Information content and response times:** The standard TCP/IP frame has a minimum length of 72 bytes. Although the transmission of information across the network is fast, the protocol overhead increases the turnaround time and hence the response time of the system. At present, Ethernet is no match to a binary protocol such as AS-i and in this case, would be definitely more expensive. The increase in transmission speed provides little advantage where process variables such as temperature and level are concerned, because the sensor response time places a lower limit on the polling interval.

**(Intrinsic) powering of the devices:** Power supply over Ethernet is not an issue, but intrinsically safe power is. This is the main reason why IEC 61158-2 was developed for process automation. Any substitute must solve this problem before it can penetrate at field level. Since intrinsic power is not of interest to the home and office applications, it is unlikely that there will be much pressure in this direction.

**Ruggedness:** Industrial connectors do exist, but other Ethernet equipment, such as cables and switches, is seldom designed for an industrial environment. An adaption to e.g., IP 67 or the equivalent NEMA rating would cut off the user from the benefits of a mass market and lead to a considerable increase in price.

**7. a) Derive the Transfer Function of Zero Order Hold.** **[WBUT 2015, 2016]**

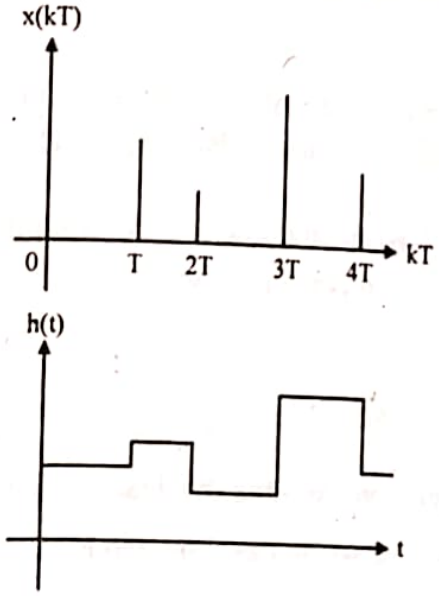
**Answer:**

If the data hold circuit is an $n$th-order polynomial extrapolator, it is called an $n$th-order hold. It uses the past $n+1$ discrete data $x\left(\left(k-n\right)T\right)$, $x\left(\left(k-n+1\right)T\right)$. ...., $x\left(kT\right)$ to generate $h\left(kT+\tau\right)$.

**Zero-order hold:**

If $n=0$ in the above equation, we have a zero order hold so that

$$h\left(kT+\tau\right)=x\left(kT\right) \qquad 0\le\tau<T, \qquad k=0,\ 1;\ 2,\ \ldots\ldots$$




**Transfer Function of ZOH:**

$$h(t)=x(0)\left[u(t)-u(t-T)\right]+x(T)\left[u(t-T)\right]-u(t-2T)$$
$$+x(2T)\left[u(t-2T)-u(t-3T)\right]+\ldots\ldots$$
$$=\sum_{k=0}^{\infty}x(kT)\left[u(t-kT)-u(t-(k+1)T)\right]$$

Now, $L\left[u(t-kT)\right]=\frac{e^{-kTs}}{s}$

Thus, $L\left[h(t)\right]=H(s)=\sum x(kT)\frac{e^{-kTs}-e^{-(k+1)Ts}}{s}=\underbrace{\frac{1-e^{-Ts}}{s}}_{G_{ho}(s)}\underbrace{\sum_{k=0}^{\infty}x(kT)e^{-kTs}}_{X^{*}(s)}$

Thus, transfer function of ZOH

$$=\frac{H(s)}{X^{*}(s)}=\frac{1-e^{-Ts}}{s}$$

**b) Differentiate between Zero Order Hold & First Order Hold? [WBUT 2015, 2016]**

**OR,**

**Compare zero order hold and first order hold devices. [WBUT 2018]**

**Answer:**

Whereas the **Zero-Order Hold** circuit generates a continuous input signal u(t) by holding each sample value u[k] constant over one sample period, a **First-Order Hold** circuit uses linear interpolation between samples.

First order gives better sampled data information than zero order hold. In zero order hold the information between two consecutive samples remain same.

**8. "A finite pulse width sampler can be approximated as an ideal sampler". Justify the statement mathematically.** **[WBUT 2017]**

**Answer:**

A sampler converts a continuous time signal into a pulse modulated or discrete signal. The most common type of modulation is the pulse amplitude modulation using the sample and hold operation.

The symbolic representation, block diagram and operation of a sampler are shown in Fig. 1. Where $p(t)$ is a unit pulse train with period $T$ and is expressed mathematically as shown in Eqn. (1).

$$p(t)=\sum_{k=-\infty}^{\infty}\left[u_s(t-kT)-u_s(t-kT-p)\right] \qquad \text{.... (1)}$$

where, $u_s(t)$ represents unit step function. Assume that leading edge of the pulse at $t=0$ coincides with $t=0$. Thus $f_p^*(t)$ can be written as shown in Eqn. (2)

$$f_p^*(t)=f(t)\sum_{k=-\infty}^{\infty}\left[u_s(t-kT)-u_s(t-kT-p)\right] \qquad \text{.... (2)}$$

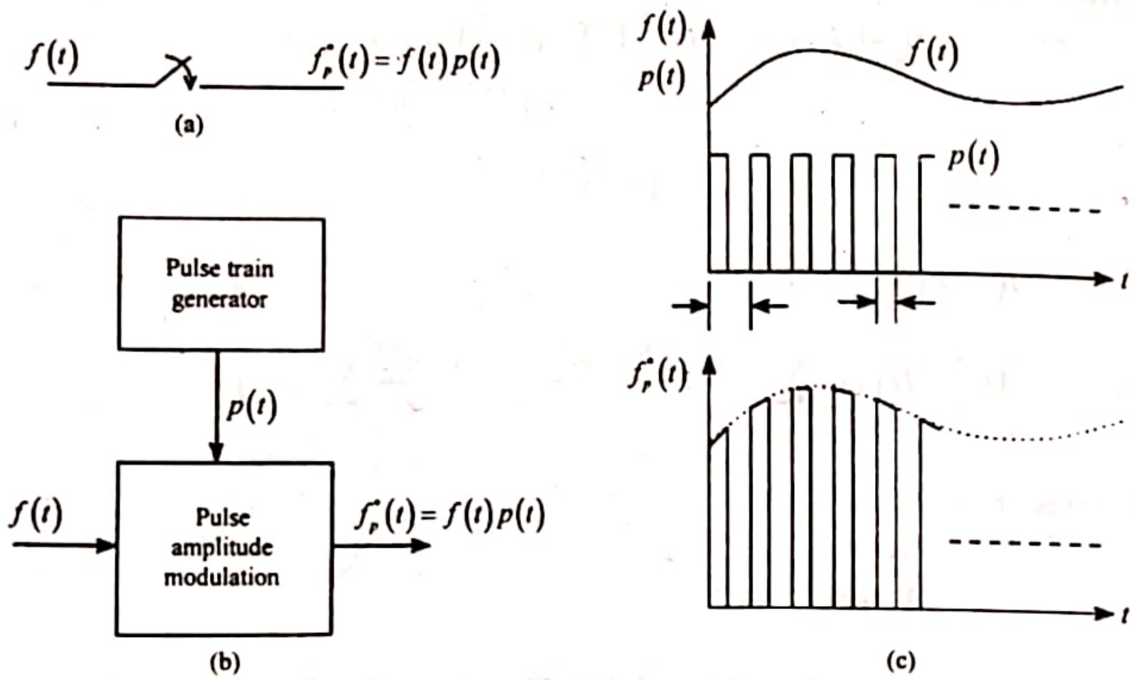



Fig: 1 Finite pulse width sampler: (a) Symbolic representation (b) Block diagram (c) Operation

The Laplace transform of $f_p^*(t)$ can be written as

$$F_p^*(s)=\sum_{n=-\infty}^{\infty}\frac{1-e^{-jn\omega_s p}}{jn\omega_s T}F(s+jn\omega_s)$$

If the sampling duration $p$ is much smaller than the sampling period $T$ and the smallest time constant of the signal $f(t)$, the sampler output can be approximated by a sequence of rectangular pulses since the variation of $f(t)$ in the sampling duration will be less significant. Thus for $k = 0, 1, 2, \ldots\ldots$, $f_p^*(t)$ can be expressed as an infinite series

$$f_p^*(t) = \sum_{k=0}^{\infty} f(kT)\left[u_s(t-kT) - u_s(t-kT-p)\right]$$

Taking Laplace transform, $F_p^*(s) = \sum_{k=0}^{\infty} f(kT)\left[\frac{1-e^{-ps}}{s}\right]e^{-kTs}$

Since $p$ is very small, $e^{-ps}$ can be approximated by taking only the first 2 terms, as

$$1-e^{-ps} = 1 - \left[1 - ps + \frac{(ps)^2}{2!}\ldots\ldots\right] \cong ps$$

Thus, $F_p^*(s) \cong p\sum_{k=0}^{\infty} f(kT)e^{-kTs}$

In time domain, $f_p^*(t) = p\sum_{k=0}^{\infty} f(kT)\delta(t-kT)$

where, $\delta(t)$ represents the unit impulse function. Thus the finite pulse width sampler can be viewed as an impulse modulator or an ideal sampler connected in series with an attenuator with attenuation $p$.

**The ideal sampler**

In case of an ideal sampler, the carrier signal is replaced by a train of unit impulse as shown in Fig. 2. The sampling duration $p$ approaches 0, i.e., its operation is instantaneous.

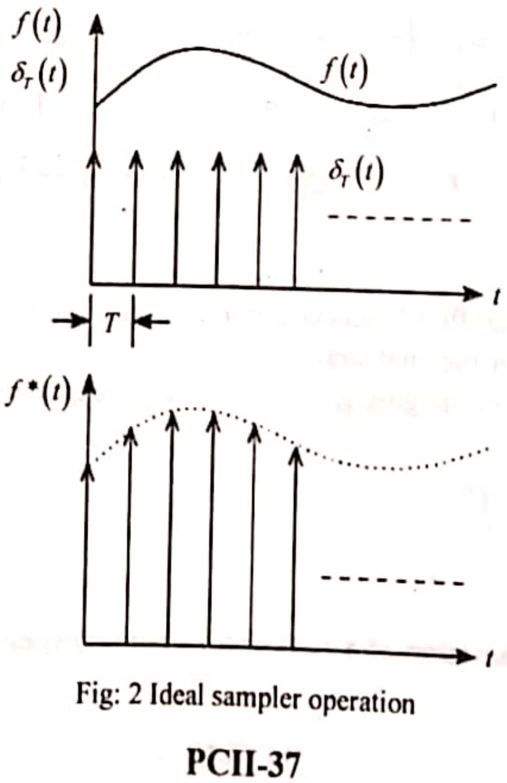



Fig: 2 Ideal sampler operation

The output of an ideal sampler can be expressed as

$$f^*(t)=\sum_{k=0}^{\infty} f(kT)\delta(t-kT)$$

$$\Rightarrow \qquad F^*(s)=\sum_{k=0}^{\infty} f(kT)e^{-kTs}$$

The output of a sampler is always followed by a hold device which is the reason behind the name sample and hold device. Now, the output of a hold device will be the same regardless the nature of the sampler and the attenuation factor $p$ can be dropped in that case. Thus the sampling process can be always approximated by an ideal sampler or impulse modulator.

**9. a) Compute the Z-transform of a parabolic function $P(k)$ where** **[WBUT 2017]**

$$P(k)=k^2\text{, for } k\geq 0$$
$$=0\text{, for } k<0$$

**Answer:**

The equation of a parabola and its samples are given below:

$$x(t)=t^2 \qquad x(k)=k^2T^2$$

Its Z-transform is formally given as:

$$Z\left[k^2T^2\right]=T^2\left[z^{-1}+4z^{-2}+9z^{-3}+\ldots\ldots\right] \qquad \ldots.(1)$$

Multiplying by $z^{-1}$ gives:

$$z^{-1}Z\left[k^2T^2\right]=T^2\left[z^{-2}+4z^{-3}+9z^{-4}+\ldots\ldots\right] \qquad \ldots.(2)$$

Subtracting Eqn. (2) from Eqn. (1) one may have,

$$\left(1-z^{-1}\right)Z\left[k^2T^2\right]=T^2\left[\left(z^{-1}+4z^{-2}+9z^{-3}+\ldots\ldots\right)-\left(z^{-2}+4z^{-3}+9z^{-4}\right)\right]$$

$$=T^2\left[\left(-z^{-1}+2z^{-1}+2.2z^{-2}+2.3z^{-3}+\ldots\ldots\right)-\left(z^{-2}+z^{-3}+z^{-4}+\ldots\ldots\right)\right]$$

$$\Rightarrow \qquad \left(1-z^{-1}\right)Z\left[k^2T^2\right]=T^2\left[-z^{-1}+2\left(z^{-1}+2z^{-2}+3z^{-3}+4z^{-4}+\ldots\ldots\right)\right.$$
$$\left.-\left(z^{-2}+z^{-3}+z^{-4}+\ldots\ldots\right)\right]$$

The first expression in the first brackets can be summed using the result from the ramp and second expression in the first brackets is a delayed step which can also be readily summed. Summing and rearranging gives the following expression for the Z transform of the parabola.

$$Z\left[k^2T^2\right]=\frac{T^2z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$$

**b) Derive the transfer function of a zero-order hold (ZOH) circuit.** **[WBUT 2017]**

**Answer:**

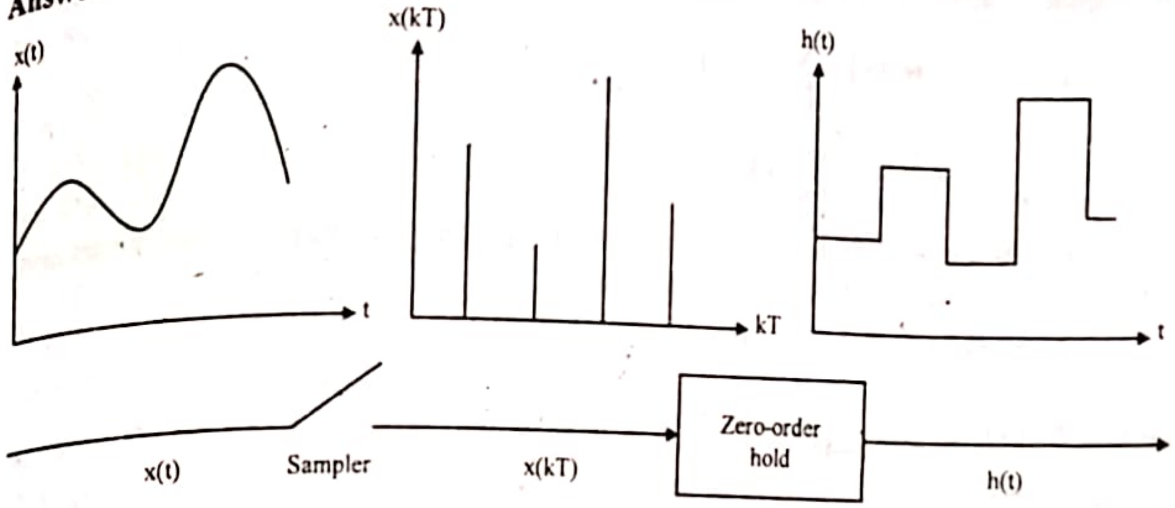



Fig: 1 Sampler and zero-order hold

Fig. 1 shows a sampler and a zero-order hold. The input signal $x(t)$ is sampled at discrete instants and the sampled signal is passed through the zero-order hold. The zero-order hold circuit smoothes the sampled signal to produce the signal $h(t)$, which is constant from the last sampled value until the next sample is available. That is,

$$h(kT+t)=x(kT),\ \text{for } 0\le t<T \qquad \text{....(1)}$$

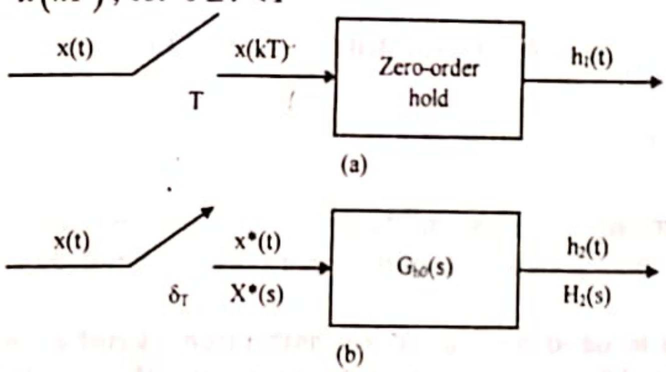



Fig: 2(a) A real sampler and zero-order hold,
(b) Mathematical model that consists of an
impulse sampler and transfer function $G_{h0}(s)$

Consider the sampler and zero-order hold shown in Fig. 2(a). Assume that the signal $x(t)$ is zero for $t<0$. Then the output $h_1(t)$ is related to $x(t)$ as follows:

$$h_1(t)=x(0)\left[1(t)-1(t-T)\right]+x(T)\left[1(t-T)-1(t-2T)\right]$$
$$+x(2T)\left[1(t-2T)-1(t-3T)\right]+\ldots\ldots$$

$$=\sum_{k=0}^{\infty}x(kT)\left[1(t-kT)-1(t-(k+1)T)\right] \qquad \text{....(2)}$$

Since $L\left[1(t-kT)\right]=\dfrac{e^{-kTs}}{s}$

The Laplace transform of Eqn. (2) becomes

$$L[h_1(t)] = H_1(s) = \sum_{k=0}^{\infty} x(kT)\frac{e^{-kT_s} - e^{-(k+1)T_s}}{s}$$

$$= \frac{1-e^{-T_s}}{s}\sum_{k=0}^{\infty} x(kT)e^{-kT_s} \quad \text{.... (3)}$$

Next, consider the mathematical model shown in Fig, 2(b). The output of this model must be the same as that of the real zero-order hold, or

$$L[h_2(t)] = H_2(s) = H_1(s)$$

Thus, $$H_2(s) = \frac{1-e^{-T_s}}{s}\sum_{k=0}^{\infty} x(kT)e^{-kT_s} \quad \text{.... (4)}$$

From Fig. 2(b), we have

$$H_2(s) = G_{h0}(s)X^*(s) \quad \text{.... (5)}$$

Since $$X^*(s) = \sum_{k=0}^{\infty} x(kT)e^{-kT_s}$$

Eqn. (4) may be written as

$$H_2(s) = \frac{1-e^{-T_s}}{s}X^*(s) \quad \text{.... (6)}$$

By comparing Eqns. (5) and (6), we see that the transfer function of the zero-order hold may be given by

$$G_{h0}(s) = \frac{1-e^{-T_s}}{s}$$

Note that, mathematically, the system shown in Fig. 2.(a) is equivalent to the system shown in Fig. 2(b) from the viewpoint of the input-output relationship.

**10. a) What device is used for signal reconstruction? Briefly describe how can a signal be constructed from a sequence of data points. What is the effect of adding a zero order hold to a discrete-time control system?**
**b) Why is z-transform required for analysis of discrete-data systems?**
**c) State and prove the real convolution theorem of z-transform.**
**d) Find inverse z-transform of the function** $\dfrac{F(z)=z}{(z^2-4z+2)}$ **[WBUT 2018]**

**Answer:**

**a) 1st & 2nd part:**

***Refer to Question No. 2(a) (1st & 2nd part) of Long Answer Type Questions.***

**3rd part:**

ZOH is an interface between continuous and discrete domains in hybrid system. This is the most convenient means for representing a hybrid system in transfer function form. However, there is no z transform for ZOH, since it is not a discrete-time system, it is

continuous time system meant to model reconstruction of discrete time signal into continuous time signal.
Most ADCs (Analog to Digital converters) need a constant input while converting, therefore some Sample-and-Hold is necessary. Because during the conversion from analog to digital the analog input's magnitude should remain constant. Hold circuit ensures that. Similarly, during DAC (digital to analog conversion), to have a speed match between the high speed computational device and slow speed device a hold circuit is needed.

b) ***Refer to Question No. 2(b) of Long Answer Type Questions.***

c) The convolution theorem for z-transforms states that for any (real or) complex causal signals $x$ and $y$, convolution in time domain is multiplication in the z-domain, i.e.,

$$x * y \leftrightarrow X \cdot Y$$

or, using operator notation,

$$Z_z(x * y) = X(z)Y(z)$$

where, $X(z) \triangleq Z_z(x)$, and $Y(z) \triangleq Z_z(y)$

**Proof:**

$$Z_z(x * y) \triangleq \sum_{n=0}^{\infty}(x * y)_n z^{-n}$$

$$\triangleq \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} x(m)y(n-m)z^{-n}$$

$$= \sum_{m=0}^{\infty} \dot{x}(m)\sum_{x=0}^{\infty} y(n-m)Z^{-n} \qquad \left[\text{Here, } \sum_{x=0}^{\infty} y(n-m)z^{-n} = z^{-m}Y(z)\right]$$

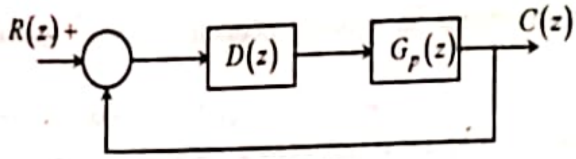
$$= \left(\sum_{m-0}^{\infty} x(m)z^{-m}\right)Y(z) \qquad \text{(by the Shift theorem)}$$

$$\triangleq X(z)Y(z)$$

The convolution theorem provides a major cornerstone of linear systems theory.

d) ***Refer to Question No. 2(d) of Long Answer Type Questions.***

**11. a) Find the closed-loop transfer function in z-domain for a sampled-data linear time-invariant system, where the plant is preceded by an ideal sampler and zero-order hold assembly.**
**b) What is a deadbeat response? Design a deadbeat controller for the all-digital system given below, for unit step, where $G_p(z) = (z+0.5)/(z^2 - z - 1)$. Verify the physical reliability of the controller.**




**c) Why is w-domain transfer function preferred to z-domain transfer function for plotting of Bode plots for discrete time control systems?** **[WBUT 2018]**

**Answer:**

**a)** ***Refer to Question No. 7(a) of Long Answer Type Questions.***

**b) 1st part:** ***Refer to Question No. 11 of Short Answer Type Questions.***

**2nd part:**

The transfer function of all-digital system is given by

$$M(z)=\frac{C(z)}{R(z)}=\frac{D(z)G_p(z)}{1+D(z)G_p(z)}$$

$$\frac{M(z)}{1-M(z)}=\frac{D(z)G_p(z)}{\left[1+D(z)G_p(z)\right]-D(z)G_p(z)} \text{ (Dividendo rule)}$$

$$\Rightarrow\frac{M(z)}{1-M(z)}=D(z)G_p(z)$$

$$\Rightarrow D(z)=\frac{1}{G_p(z)}\left[\frac{M(z)}{1-M(z)}\right] \qquad \text{.... (1)}$$

**As per the dead beat response,** $M(z)=1/z$

$$\therefore \quad \frac{M(z)}{1-M(z)}=\frac{\frac{1}{z}}{1-\frac{1}{z}}=\frac{1}{z-1} \qquad \text{.... (2)}$$

Given, $G_p(z)=(z+0.5)/(z^2-z-1)$ .... (3)

Combining equations (1), (2) and (3),

$$D(z)=\frac{1}{G_p(z)}\left[\frac{M(z)}{1-M(z)}\right]=\frac{(z^2-z-1)}{(z+0.5)}\times\frac{1}{(z-1)}$$

$$\Rightarrow D(z)=\frac{(z^2-z-1)}{(z+0.5)(z-1)}=\frac{(z-1.6)(z-0.1)}{(z+0.5)(z-1)}$$

**c) W Domain:**

It is known that the z transformation maps the primary and complementary strips of the left half of the s plane into the unit circle in the z plane. Thus conventional frequency response methods that deal with the entire left-half of the s-plane can not be applied to the z plane.

Therefore, when a the pulse transfer function of a system is $G(z)$ and the frequency response of the system is given by $G(z)\big|_{z=e^{j\omega T}} = G(e^{j\omega T})$, if one wants to have the frequency response in the $z$–plane, the simplicity of logarithmic plots will be lost. Such a problem is addressed by transforming the pulse transfer function in the $z$–plane into one in the w-plane.

The w-transformation is a bilinear transformation given by

$$z = \frac{1+\frac{T}{2}w}{1-\frac{T}{2}w}$$

where, $T$ is the sampling time.

The inverse transformation is given as

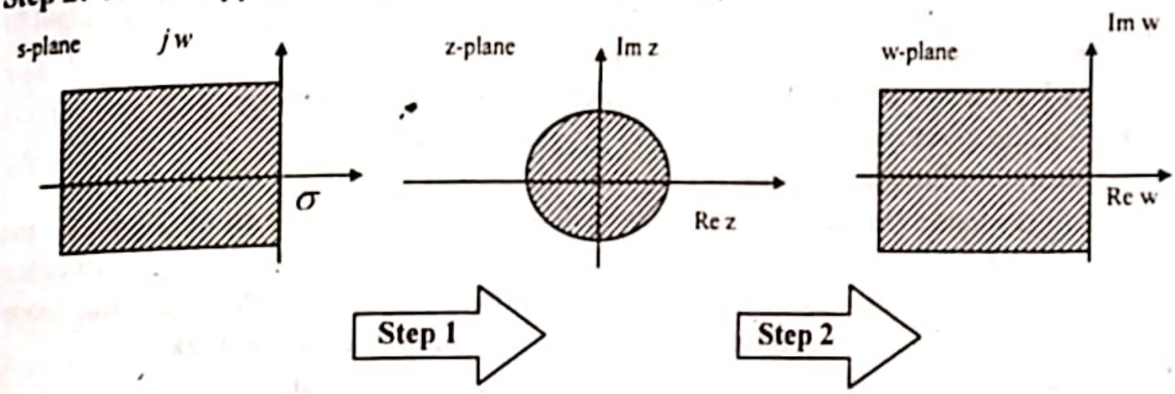
$$w = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

The origin in the z-plane maps into the point $w = -2/T$ in the w-plane.

The transformation steps are as follows

**Step 1:** Through the z transformation and the w transformation, the primary strip of the left half of the s plane is mapped into the inside of the unit circle in the z plane.

**Step 2:** Then mapped into the entire left half of the w plane.




**12. Write short notes of the followings:**

**a) HART protocol** **[WBUT 2009, 2011]**

**b) Nyquist frequency and aliasing** **[WBUT 2009, 2010, 2015, 2016]**

**OR,**

**Aliasing** **[WBUT 2011]**

**c) Trapezoidal technique for discretization** **[WBUT 2009]**

**d) Jury's stability test** **[WBUT 2010, 2014, 2015, 2018]**

**e) Bode plot analysis** **[WBUT 2010]**

**f) Quantization error** **[WBUT 2012]**

**g) Bilinear Transformation** **[WBUT 2012, 2014, 2016, 2017]**

**h) Components of digital control loop** **[WBUT 2014]**

**i) Pole zero matching for discretization** **[WBUT 2014, 2015, 2016]**

**j) Signal reconstruction using hold device** **[WBUT 2017]**

**Answer:**

**a) HART Protocol**

This protocol was originally developed by Rosemount and is regarded as an open standard, available to all manufacturers. Its main advantage is that it enables an instrumentation engineer to keep the existing 4-20 mA instrumentation cabling and to use, simultaneously, the same wires to carry digital information superimposed on the analog signal. This enables most companies to capitalize on their existing investment in 4-20 mA instrumentation cabling and associated systems and to add further capability of HART without incurring major costs.

HART is a hybrid analog and digital protocol, as apposed to most fieldbus systems, which are purely digital.

The Hart protocol uses the frequency shift keying (FSK) technique based on the Bell 202 communications standard. Two individual frequencies of 1200 and 2200 Hz, representing digits 1 and 0 respectively, are used. The average value of the sine wave (at the 1200 and 2200 Hz frequencies) which is superimposed on the 4-20 mA signal, is zero. Hence, the 4-20 mA analog information is not affected.

The HART protocol can be used in three ways:

- In conjunction with the 4-20 mA current signal in point-to-point mode.
- In conjunction with other field devices in multi drop mode.
- In point-to-point mode with only one field device broadcasting in burst mode.

Traditional point-to-point loops use zero for the smart device polling address to a number greater then zero creates a multidrop loop. The smart device then sets its analog output to a constant 4 mA and communicates only digitally.

The HART protocol has two formats for digital transmission of data:

- Poll/response mode
- Burst (or broadcast) mode

In the poll/response mode the master polls each of the smart devices on the highway and requests the relevant information. In burst mode the field device continuously transmits process data without thye need for the host to send request messages. Although this mode is fairly fast (up to 3.7 times/second), it cannot be used in multidrop networks.

The protocol is implemented with the OSI mode using layers 1, 2 and 7.

**b) Nyquist frequency and aliasing:**

When the sampling rate becomes exactly equal to $2f_m$ samples per second, then it is called Nyquist rate. Nyquist rate is also called the minimum sampling rate. It is given by-

$$f_s = 2f_m$$

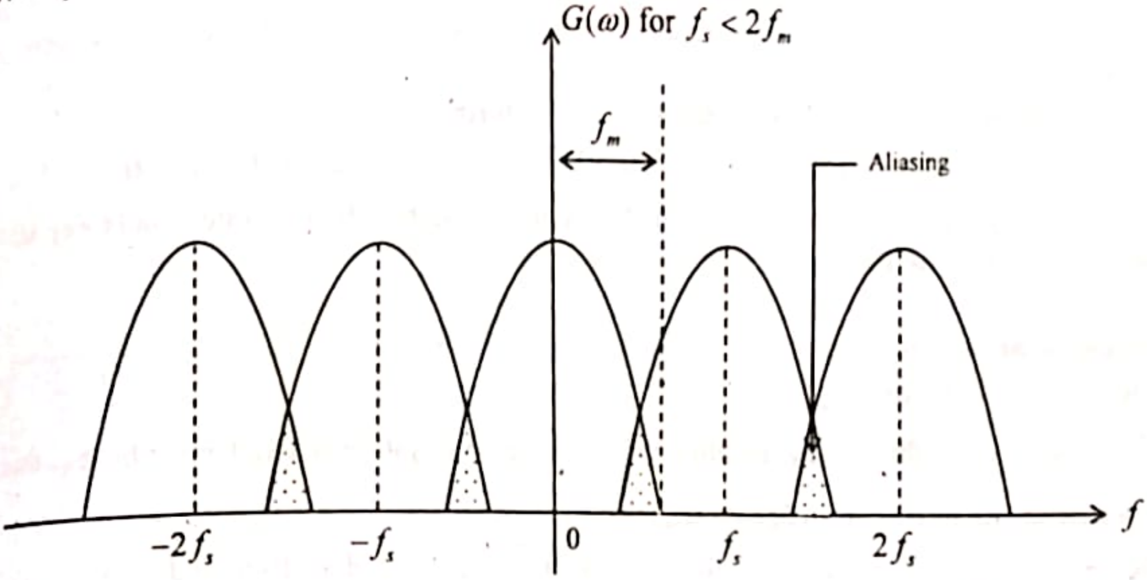
Similarly maximum sampling interval is called Nyquist interval. It is given by-

$$T_s = \frac{1}{2f_m}$$

When the continuous time band limited signal is sampled at Nyquist rate($f_s = 2f_m$), the sampled-spectrum $G(\omega)$ contains non overlapping $G(\omega)$ repeatibng periodically. But the successive cycles of $G(\omega)$ touch each other. Therefore the original spectrum $X(\omega)$

can be recovered from the sampled spectrum by using a low pass filter with a cut-off frequency $\omega$.

When a continuous time band limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the spectrum $G(\omega)$ of the sampled signal $g(t)$ overlap with each other as shown in the figure below.




Spectrum of the sampled signal for the case $f_s < 2f_m$

Hence, the signal is under sampled in this case ($f_s < 2f_m$) and some amount of aliasing is produced in this under-sampling process. In fact, aliasing is the phenomenon in which a high frequency component in the frequency-spectrum of the signal takes identity of a lower-frequency component in the spectrum of the sampled signal. From the above figure it is clear that because of the overlap due to aliasing phenomenon, it is not possible or recover original signal $x(t)$ from sampled signal $g(t)$ by low-pass filtering science the spectral components in the overlap region add and hence the signal is distorted. Since any information signal contains a number of frequencies, so, to decide a sampling frequency is always a problem. Therefore, a signal is first passed through a low-pass filter. This low-pass filter blocks all the frequencies which are above $f_m$Hz. This process is known as band limiting of the original signal $x(t)$. This low-pass filter is called pre alias filter because it is used to prevent aliasing effect. After band-limiting, it becomes easy to decide sampling frequency since the maximum frequency is fixed at $f_m$ Hz.

i.e. to avoid aliasing,

(i) Prealias filter must be used to limit band of frequencies of signal to $f_m$ Hz.

(ii) Sampling frequency '$f_s$' must be selected such that $f_s > 2f_m$

**c) Trapezoidal technique for discretization:**

***Refer to Question No 1(a) of Long Answer Type Questions.***

**d) Jury's stability test:**

**Jury's Stability Criterion**, an algebraic criterion similar to the Routh-Hurwitz criterion, is used to analysis the stability of an LTI digital system in the Z domain. Jury's test is a necessary and sufficient test for stability of digital systems. It focuses exclusively on D(z), the characteristic polynomial of the system for determining whether its roots lie within the unit circle or not. If the roots (poles) are outside the unit circle, the system is unstable.

Given a characteristic equation of degree N in the form,

$$D(z) = a_0 z^N + a_1 z^{N-1} + a_2 z^{N-2} + .... + a_{N-1} z + a_N = 0, \quad a_0 > 0$$

The following rules determine whether the system is stable. If the system fails any test, one need not try any further tests.

**For necessary conditions**

Rule: 1 $D(1) > 0 \;\; for \; z-1$

Rule: 2 $|a_N| < a_0$, that is, the absolute value of the constant term $(a_N)$ must be less than the value of the highest coefficient $(a_0)$.

If Rule 1 and Rule 2 are satisfied, the Jury Array is constructed as discussed below.

***The Jury Array:*** The Jury array is constructed by first writing a row of coefficients in ascending order, and then writing the second row with the same coefficients in reverse order. We add other rows of coefficient ***b*** for third row, and ***c*** for the fourth row, and so on. The coefficients of the lower rows are calculated from the coefficients of the upper rows. Each new row that we add will have one fewer coefficient than the row before it.

**e) Bode plot analysis:**

The Bode plot is the frequency response plot of the transfer function of a system.

A Bode diagram consists of two graphs: one is a plot of the magnitude of a ***sinusoidal transfer function in dB*** and the other one is the phase angle. Both are plotted against the frequency on a logarithmic scale.

In the logarithmic representation, the plots are drawn on the semilog paper, using the log scale for frequency and the linear scale for either magnitude (in dB) or phase angle (in degrees) as shown in Fig. 1.

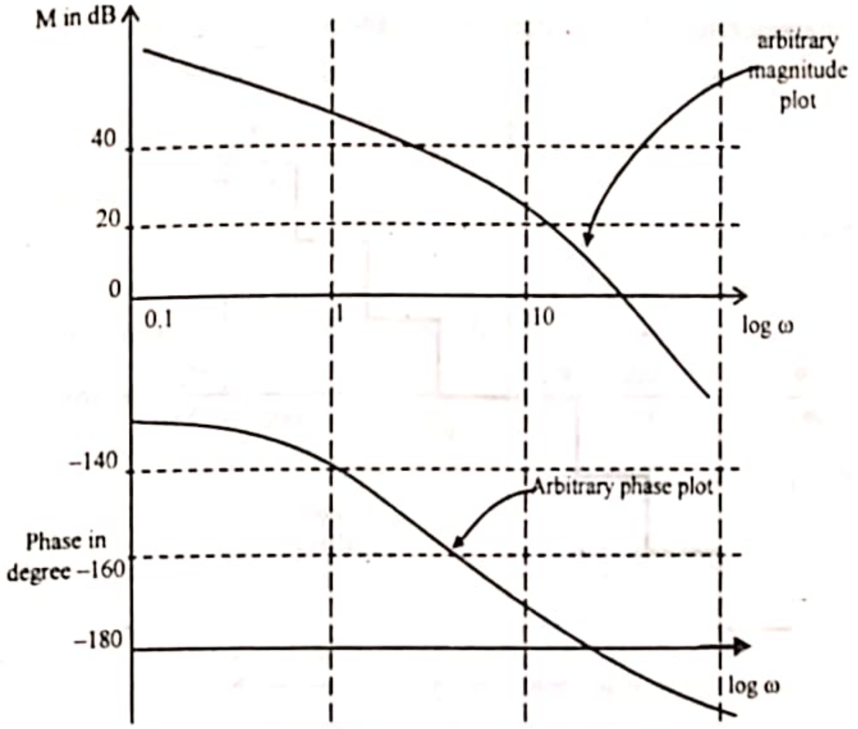



Fig: 1 An arbitrary magnitude and phase plots

The main advantages are:

i) Multiplication of magnitude can be in to addition.
ii) A simple method for sketching an approximate log curve is available.
iii) It is based on asymptotic approximation. Such approximation is sufficient if rough information on the frequency response characteristic is needed.

The phase angle curves can be easily drawn if a template for the phase angle curve of $1+j\omega$ is available

**f) Quantization error:**

There are basically two types of quantizer, namely, (i) uniform quantizer and (ii) non-uniform quantizer.

A uniform quantizer is that type of quantizer in which the step size is uniform throughout the input signal range. Quantization process involved in such quantizer is called uniform quantization. A uniform quantizer is also called linear quantizer.

The type of quantizer in which the step size varies according to the input signal value is called non-uniform quantizer and the process involved is called non-uniform quantization. A non-uniform quantizer is also called non-linear quantizer.

***Transfer Characteristics of Quantizers***

In quantizing process a straight line representing the relation between input and output of a linear analog system is replaced by a transfer characteristic that looks like a staircase. Two types of transfer characteristics are possible. One is for mid- tread type quantizer and the other is for mid-riser type quantizer.

The transfer characteristic of a mid-tread type quantizer is shown below.

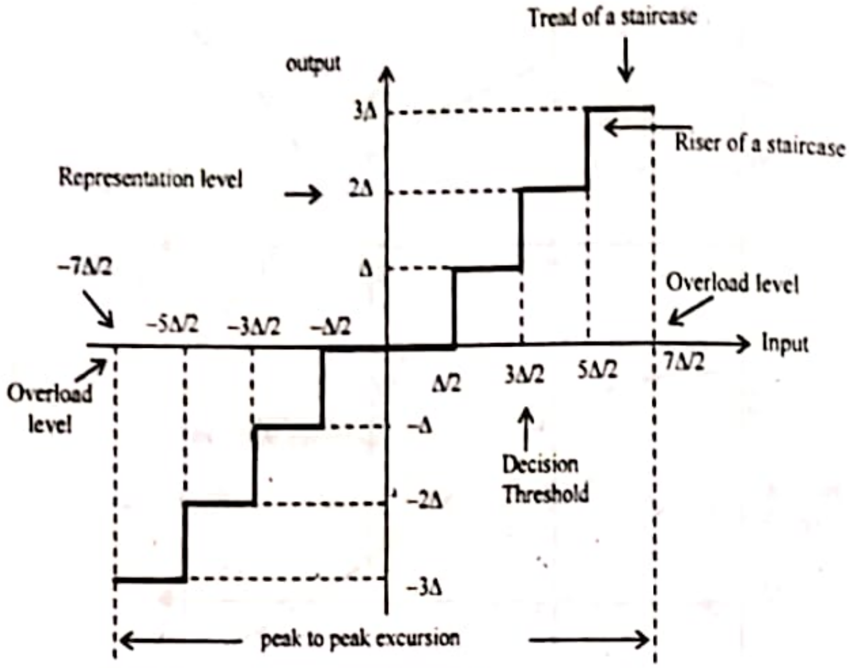



Fig: Transfer characteristic of mid-tread type quantizer

The decision thresholds of the mid-tread type quantizer are located at $\pm\frac{\Delta}{2}$, $+\frac{3\Delta}{2}, \pm 5\frac{\Delta}{2}$ and so on. The representation levels are located at 0, $\pm\Delta$, $\pm 2\Delta$ ....where $\Delta$ is the step size.

The diagram below shows the transfer characteristic of a mid- riser type quantizer.

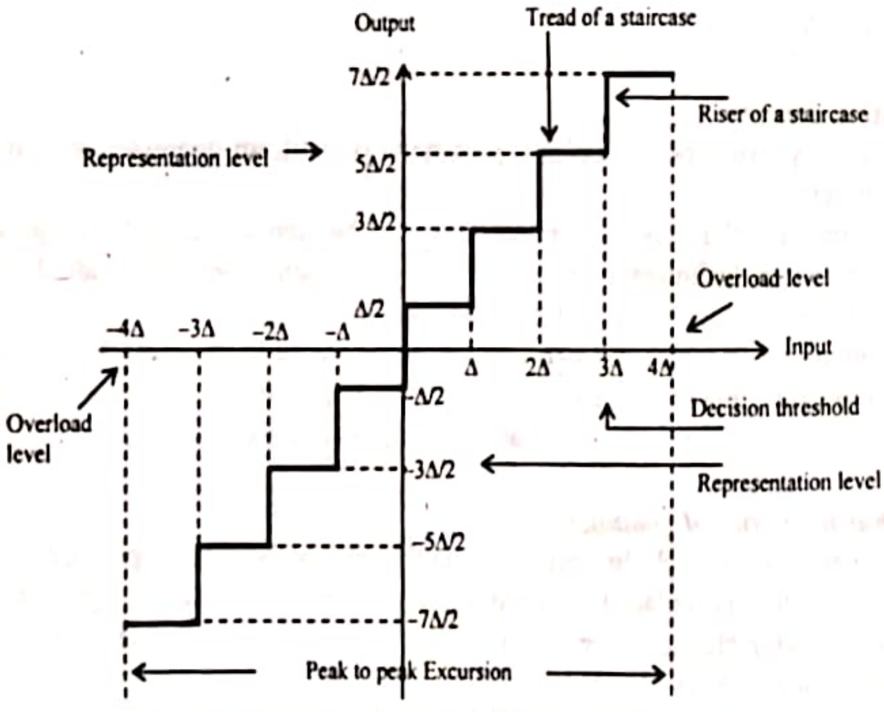



Fig: Transfer characteristic of a mid-riser type quantizer

In the mid-riser type quantizer the decision thresholds are located at 0, $\pm\Delta$, $\pm 2\Delta$....etc. and the representation levels are located at $\pm \Delta/2$, $\pm 3\Delta/2$, $\pm 5\Delta/2$...etc.

In the case of a mid-tread type of quantizer the origin lies in the middle of a tread of a staircase and for a mid-riser type of quantizer the origin lies in the middle of a riser of a staircase.

It may be noted that the absolute value of the overload level is one half of the peak to peak range of input sample values. The number of representation levels which is the number of intervals into which the peak-to-peak excursion is divided is equal to twice the absolute value of the overload level divided by the step size. Peak to peak excursion is also called dynamic range. The amplitude of the quantizer input must not exceed the overload level, otherwise, overload distortion results.

***Quantization Error or Quantization Noise***

The quantized signal is an approximation to the original signal. An error is introduced in the signal due to this approximation. The instantaneous error $e = V - V_q$ is randomly distributed within the range $\Delta/2$ and is called the quantization error or noise. The variations of quantization error with input signal for mid-tread and mid-riser types of quantizers are shown in the diagrams below.

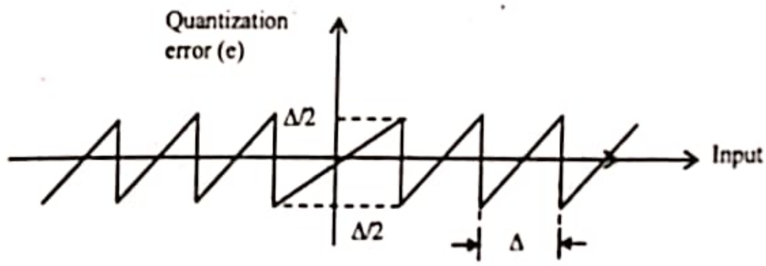



Fig: Variation of quantization error with input of a mid-tread quantizer

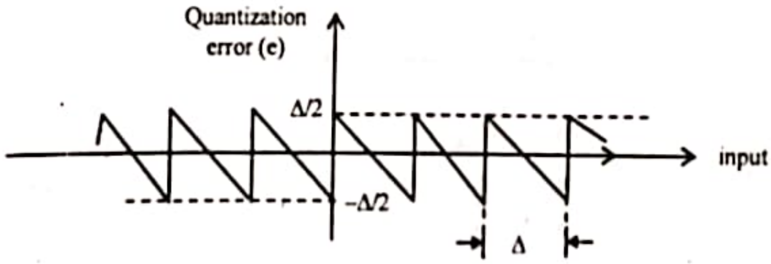



Fig: Variation of quantization error with input of a mid-riser quantizer

It is obvious that the values of the quantization error equal the differences between the output and input values of the quantizer. In both the cases above, the maximum instantaneous value of the error is half of the step size. i.e. $\pm\Delta/2$ and the total range of variation is from $-\Delta/2$ to $+\Delta/2$.

***Calculation of Quantization Error***

For linear quantization, the probability distribution of error can be assumed to be constant within the range $\pm \Delta/2$. The variation of probability p(e) with error is shown in the figure below.

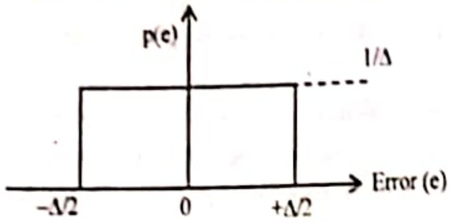



Fig: Probability distribution of quantization error due to linear quantization

The average quantization noise output power is given by the variance

$$\sigma^2 = \int_{-\infty}^{\infty} (e-\mu)^2 \; p(e)\, de$$

where $\mu$ is the mean value of the error, e.

In this case, the limits of integration are $+\Delta/2$ and $-\Delta/2$ and $\mu = 0$. Also $p(e) = \frac{1}{\Delta}$ and thus

$$\sigma^2 = \int_{-\Delta/2}^{+\Delta/2} e^2 p(e) de = \int_{-\Delta/2}^{+\Delta/2} e^2 \left(\frac{1}{\Delta}\right) de$$

$$= \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} e^2 de = \frac{1}{\Delta}\left[\frac{e^3}{3}\right]_{-\Delta/2}^{+\Delta/2} = \frac{\Delta^2}{12}$$

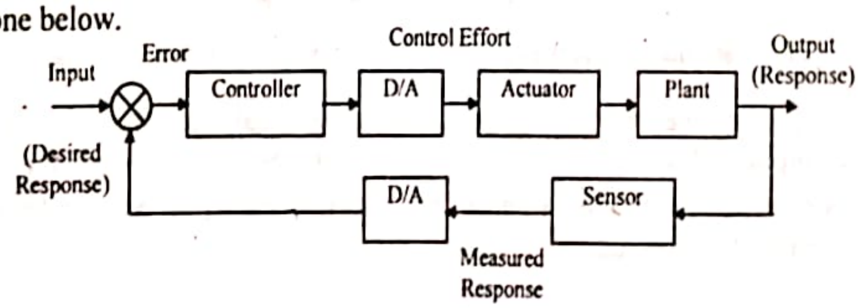
Thus the average power of the quantization noise is proportional to the square of the step size, $\Delta$. The signal distortion due to quantizing noise can be minimized by reducing the step size to a very small value.

**g) Bilinear Transformation:**

***Refer to Question No. 7 of Short Answer Type Questions.***

**h) Components of digital control loop:**

When you implement a control system using a digital computer as part of the control loop, there is a generic structure to the control loop that results. Most control loops look like the one below.




In this system, there is an input - the desired response - and an output - the actual response. Let's follow the loop around from the output.

- First, the output of the system is measured with a sensor, producing a measured response. In many sensors, the measured response is actually a voltage proportional to the actual response.
- The measured response is converted to digital from in an A/D. Once that conversion takes place, the signal is in the digital part of the loop - shown in blue.
- Inside the digital part of the loop, the measured response - now in digital form - is compared to the input - the desired response to form an error.
- The controller looks at the error and uses that information to generate a control signal - also called the control effort. That might be done using an ON-OFF controlmethod, proportional control or some other control algorithm.
- Once the control signal/control effort has been computed, the control effort is applied to the actuator by converting the digital signal to an analog voltage using a D/A.
- The actuator is a device that takes the control effort and applies it to the system being controlled - the plant. The actuator might be a power amplifier (if, for example, you are trying to control the speed of a large motor, like on a subway car) or a hydraulic system (if, for example, you are trying to control the position of an aileron on an airplane).
- The plant is the thing you are trying to control.

**i) Pole zero matching:**

***Pole-Zero Matching Control Technique***

The pole-zero matching design technique is just an application of the continuous-time design methods in $s$-domain to the discrete-time systems in the z-domain. This technique allows the controller to be designed in the $s$-plane and then transferred in the $z$-plane. In some application, this technique is known as the emulation technique. Basically, when the controller is designed in the continuous-time domain, it is then digitized by using relationship

$$z = e^{sT}$$

The technique requires the continuous-time poles and zeros of the controller to be preserved by the discrete-time controller and hence the name "POLE-ZERO MATCHING". The technique assumes that for every continuous-time controller there is an equal number of poles as the zeros. Therefore, if the degree of the numerator is less than that of the denominator, then some of the zeros are infinity. Thus, for the continuous-time controller

$$D_c(s) = K_c \frac{\prod_{i=1}^{n}(s - z_i)}{\prod_{j=1}^{m}(s - p_j)} \qquad n < m \qquad \ldots (1)$$

where $\{z_i\}_{i=1}^{n}$ and $\{p_j\}_{j=1}^{m}$ are the zeros and poles respectively, there are $(m - n)$ zeros at infinity. Now, the discrete-time controller which emulates this continuous-time controller must map all poles according to

$$P_j = e^{p_j T}$$

and zeros according to

$$Z_i = e^{z_i T},$$

whereas those zeros and poles (if any) that are at infinity are mapped to digital zeros at $z = -1$ and the zeros at $s = 0$ are mapped into corresponding digital zeros at $z = 1$. Finally, the DC gains for both the continuous-time controller and the discrete-time controller must be matched according to

$$D_c(s)\big|_{s=0} = D_D(z)\big|z = 1$$

Thus, the digitized version of the continuous controller in eqn. (1) is given by

$$D_D(z) = K_D(z+1)^{n-m}\left[\frac{\prod_{i=1}^{n}(z - e^{z_i T})}{\prod_{j=1}^{m}(z - e^{p_j T})}\right]$$

While this method is the fastest and simplest of all because of the fact that most systems found in real life are continuous-time in nature, so that analysis and design in the continuous-time domain seen more appealing. The presence of the ZOH element in the actual digital system, which is not taken into account when designing the continuous-time controller and hence the resulting digitized controller, causes functional problems with controllers designed this way. For most applications, this approach is only suitable if the sampling rate is very high to the level that the whole system can be considered as a continuous-time system.

**j) Signal reconstruction using hold device:**

The process of reconstructing a continuous time signal $x(t)$ from its samples is known as interpolation. In the sampling theorem a signal $x(t)$ band limited to $X$ Hz can be reconstructed from its samples. This reconstruction is accomplished by passing the sampled signal through an ideal low pass filter of bandwidth $X$ Hz.

There are many techniques that can be used to reconstruct a signal and the selection of a technique depends on

i) what accuracy of reconstruction is required
ii) how oversampled the signal is

In its simplest form, the analog signal (continuous-time signal) is multiplied with a periodic impulse train, referred to as Sampling Function to obtain a sampled signal as shown in Fig. (a).

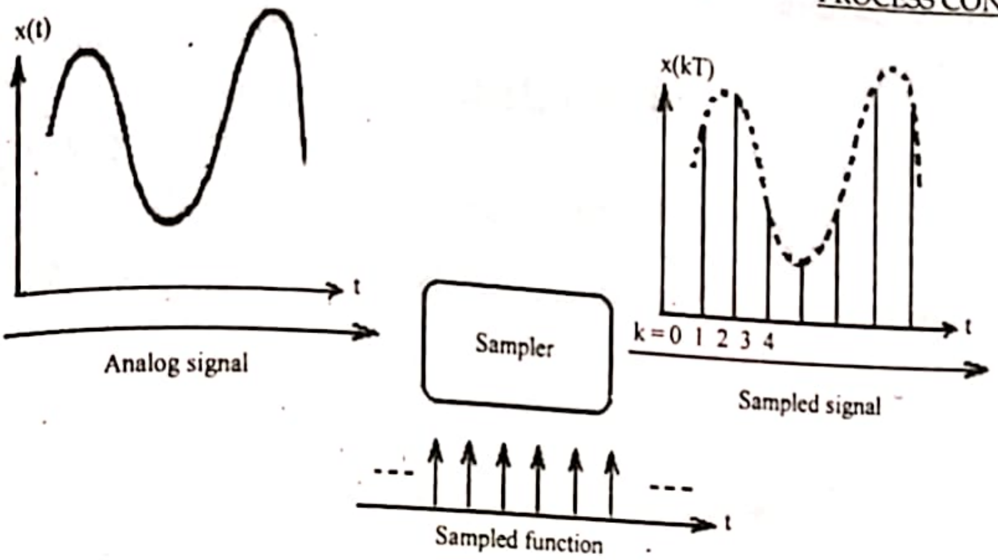



Fig. (a)

The sampled signal $x_p(t)$ is the product of the impulse train $p(t)$ and the analog signal $x_c(t)$ and mathematically written as $x_p(t) = x_c(t) \times p(t)$

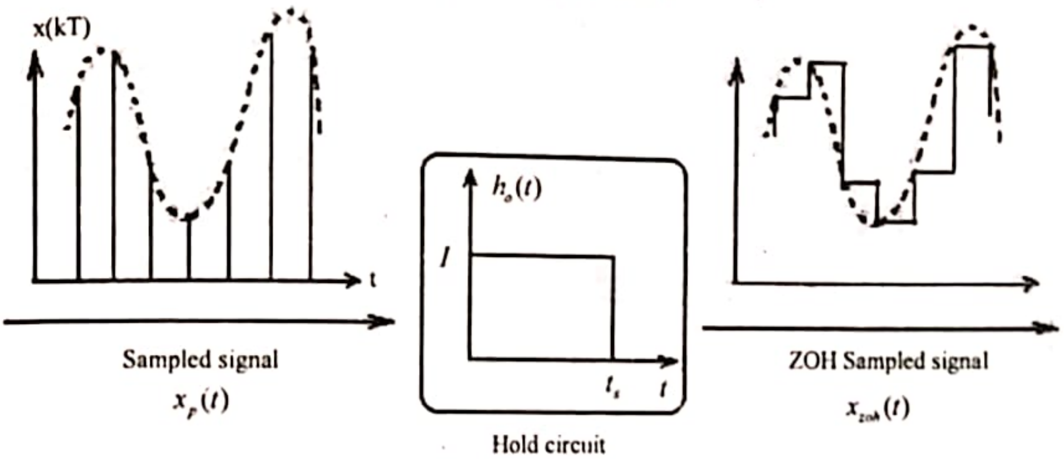



Fig. (b)

The hold circuit receives the sampled signal to output as zero order hold sampled signal