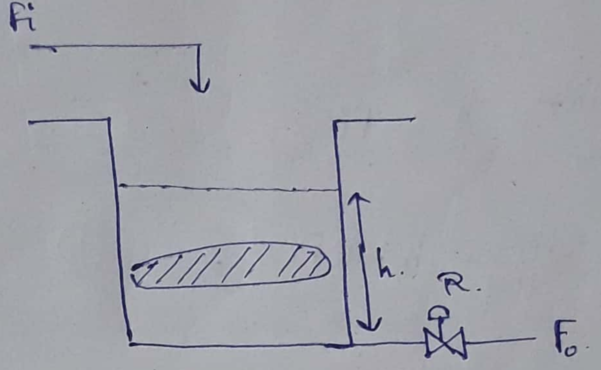
Mathematical Modeling

i) Theoritical method.
ii) Empirical method
iii) Semiemperical method.

Liquid storage tanks:-



Fi = Inlet volumetric flowrate. (m^3/sec)
Fo = Outlet volumetric flowrate.
h = liquid level.
A = Cross-sectional area.
R = Resistance to the outlet flow.

According to
the mass conservation law,

Rate of accumulation of liquid mass = inlet flowrate - outlet flowrate.

$$\frac{d(\rho v)}{dt} = \rho_i F_i - \rho_o F_o$$

$$\rho = \rho_i = \rho_o$$

$$\therefore \frac{dv}{dt} = F_i - F_o$$

Tumi Robe Nirobe !

$$\frac{d(Ah)}{dt} = F_i - F_o$$

$$F_o = \frac{h}{R}$$

$$\frac{A\,dh}{dt} = F_i - h/R.$$

$$\Rightarrow \frac{A\,dh}{dt} + \frac{h}{R} = F_i$$

$$\Rightarrow AR\frac{dh}{dt} + h = RF_i \quad \text{------} \; (1).$$

At steady state condition,

$h = h_s$, $F_i = F_{is}$.

$$0 + h_s = RF_{is} \quad \text{------} \; (2).$$

$$AR\frac{d(h-h_s)}{dt} + (h-h_s) = R(F_i - F_{is}) \qquad [(1) - (2)].$$

Let, $h' = h - h_s$ & $F_i - F_{is} = F_i'$

$$\therefore AR\frac{dh'}{dt} + h' = RF_i'$$

$$\therefore AR\,sH'(s) + H'(s) = RF_i'(s).$$

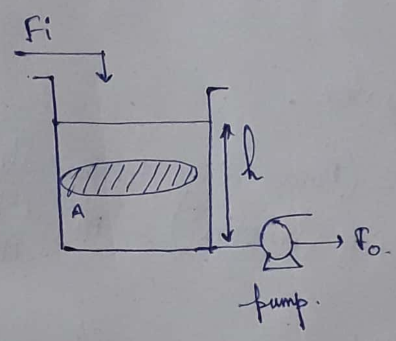
$$\therefore \frac{H'(s)}{F_i'(s)} = \frac{R}{(ARs+1)}.$$

Here,

Gain of the process$(K_p) = R$.

$\tau_p = AR$.

Liquid storage tank with constant outlet:-



$$\frac{d}{dt}(Ah) = F_i - F_o$$

$$\Rightarrow \frac{A\,dh}{dt} = F_i - F_o$$

At steady state condition,

$0 = F_{is} - F_{os}$. Here, $F_{os} = F_o$.

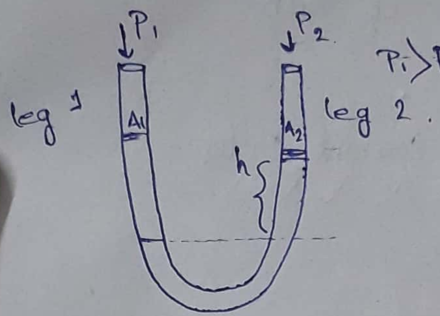
$= F_{is} - F_o$.

$$\frac{A\,dh'}{dt} = F_i'$$

$$As\,H'(s) = F_i'(s)$$

$$\Rightarrow \frac{H'(s)}{F_i'(s)} = 1/As$$

U-tube Manometer :-



$P_1 > P_2$.

$A_1 = A_2 = A$.

Force balance equation,

$$A_1P_1 - P_2A_2 - h\rho g - (\text{force due to the fluid friction}) = m\frac{d^2h}{dt^2}$$

$$A\Delta P - h\rho g - \frac{8\mu L}{R^2}A\frac{dh}{dt} = m\frac{d^2h}{dt^2}$$

$$m\frac{d^2h}{dt^2} + \frac{8\mu L}{R^2}A\frac{dh}{dt} + Ah\rho g = A\Delta P$$

$$\Rightarrow \frac{m}{A\rho g}\frac{d^2h}{dt^2} + \frac{8\mu L}{R^2\rho g}\frac{dh}{dt} + h = \frac{\Delta P}{\rho g}$$

$$\Rightarrow \frac{\rho AL}{A\rho g}\frac{d^2h}{dt^2} + \frac{8\mu L}{R^2\rho g}\frac{dh}{dt} + h = \frac{\Delta P}{\rho g} \quad [\because m = \rho AL]$$

$$\Rightarrow \frac{L}{g}\frac{d^2h}{dt^2} + \frac{8\mu L}{R^2\rho g}\frac{dh}{dt} + h = \frac{\Delta P}{\rho g}$$

$$\Rightarrow \tau^2\frac{d^2h}{dt^2} + 2\xi\tau\frac{dh}{dt} + h = \frac{\Delta P}{\rho g}.$$

$$\boxed{\tau^2 s^2 + 2\xi\tau s + 1 = 0}$$

$$\tau^2 s^2 H(s) + 2\xi\tau s H(s) + H(s) = \frac{\Delta P}{\rho g}$$

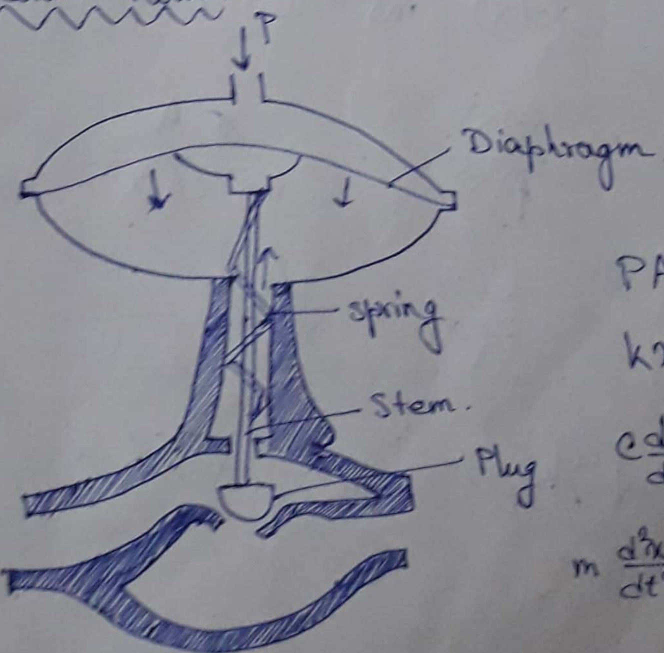
$$\tau = \sqrt{L/g} \quad ; \quad 2\xi\tau = \frac{8\mu L}{R^2\rho g}$$

$$\xi = \frac{4\mu L}{R^2 \rho g \tau}$$

$$= \frac{4\mu L}{R^2 \rho g} \cdot \sqrt{g/L}$$

$$\frac{H(s)}{\Delta P(s)} = \frac{1/\rho g}{\tau^2 s^2 + 2\xi\tau s + 1}$$

Pneumatic Control Value :-



$PA \downarrow$

$kx \uparrow$

$c\frac{dx}{dt} \uparrow$

$m\frac{d^2x}{dt^2}$

Force-balance equation,

$$PA - kx - c\frac{dx}{dt} = m\frac{d^2x}{dt^2}.$$

$$\Rightarrow \frac{PA}{k} - x - \frac{c}{k}\frac{dx}{dt} = \frac{m}{k}\frac{d^2x}{dt^2}$$

$$\Rightarrow \frac{m}{k}\frac{d^2x}{dt^2} + x + \frac{c}{k}\frac{dx}{dt} = \frac{PA}{k}.$$

$$\Rightarrow \frac{m}{k}s^2X(s) + X(s) + \frac{cs}{k}X(s) = \cancel{\frac{PA}{k}}\ \frac{Ps(A)}{k}$$

$$AP(s).$$

$$\Rightarrow ms^2X(s) + kX(s) + csX(s) = \cancel{PA(s)}$$

$$AP(s).$$

$$\Rightarrow X(s)\left\{ms^2 + k + cs\right\} = \cancel{Ps(A)}\ \cancel{PA(s)}$$

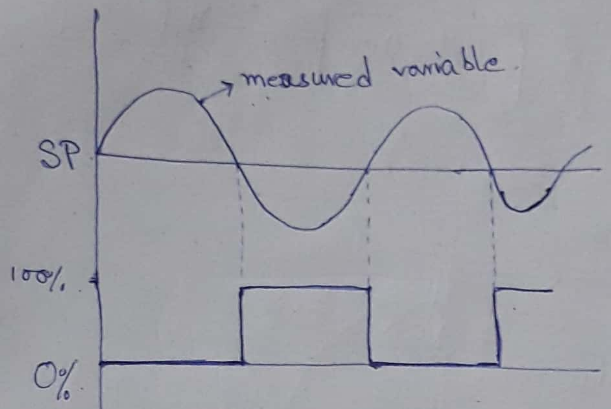
$$\Rightarrow \frac{X(s)}{P(s)} = \frac{A}{ms^2+cs+k}$$

$$\Rightarrow \frac{X(s)}{P(s)} = \frac{A/k}{m/k\, s^2 + \frac{c}{k}s + 1}$$

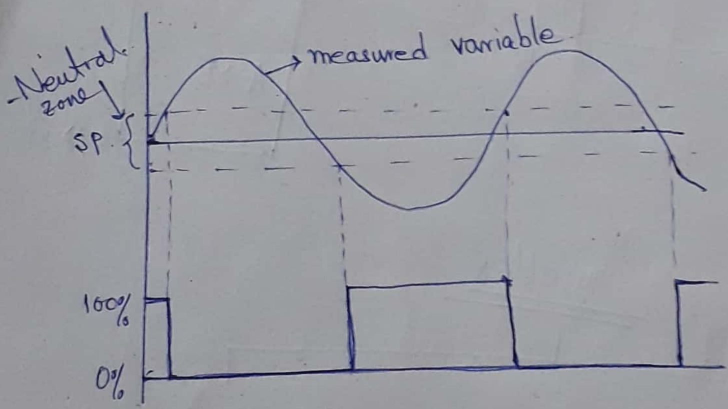
$$\tau = \sqrt{m/k}. \qquad m \ll k$$

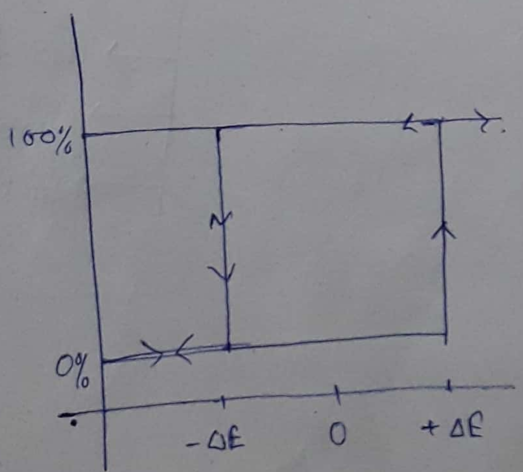
ON/OFF Controller :-

JAI SHREE RAM!



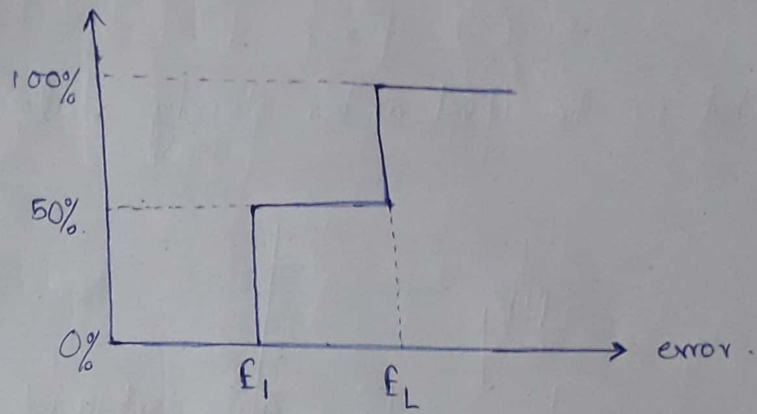
Reverse action mode





JAI SHREE RAM!

Three - position Controller:-



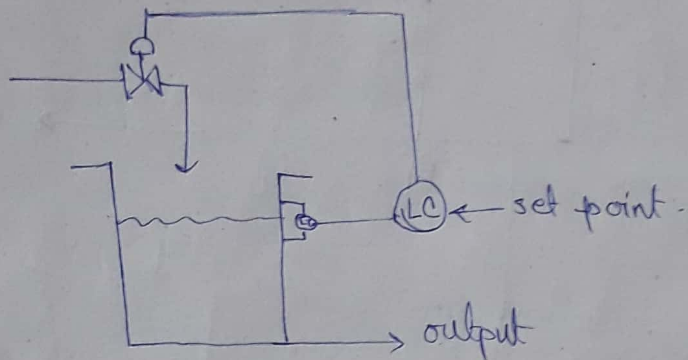
Proportional Controller:-

$$m(t) = \underbrace{K_p}_{\text{gain}} e(t)$$

Proportional band = ~~1000~~ $100\% \times 1/K_p$

$$PB = \frac{\text{percentage change in error}}{\text{percentage change in o/p}}$$

What is the level under steady state control?



given

Range of LT : 0 to 70 inch.

Level Controller : P only.

PB = 75%

Bias = 50%

Set point = 40 inches.

Load : fixed at 3.5 gpm.

Valve : Pr. drop ΔP is constant

Linear characterestics delivers 6 gpm at 100% ~~stat~~ stroke.

Soln

Inlet flow $= \frac{3.5}{6} \times 100\%$.

$m = 58.33\ \%$.

Set point $= 40/70 \times 100\%$

$= 57.14\%$.

$= r$. [koto raat ekhane thakbo]

$m = k_p\, e(t) + 50\%$

$= \frac{100}{PB} (r - c) + 50\%$.

$58.33 = \frac{100}{75} (57.14 - c) + 50$.

$\Rightarrow$ ~~57.14 - c~~ $c = 50.8925\ \%$

$c \approx 51\%$.

output Controlled variable in inches $= \frac{51}{100} \times 70$

$= 35.7$ inches

Control?

6 gpm

troke.