PID Control

Proportional, integral and derivative (PID) control is often referred to as 3-term control. P action was introduced in Chapter 22. It provides the basis for PID control, any I and/or D action is always superimposed on the P action. This chapter is concerned with the functionality of PID control and its open and closed loop behaviour. An equation for PID control is first developed in analogue form, as used in pneumatic and electronic controllers. This is then translated into a discrete form for implementation as an algorithm in a digital controller.

The principles discussed in this chapter and the equations developed are essentially the same whether the PID controller is a dedicated single loop controller, a function provided within some other control loop element such as a dp cell, or is a configurable function within a distributed control system capable of supporting multiple loops simultaneously.

23.1 Single Loop Controller

The 3-term controller, often referred to as a single loop controller (SLC), is a standard sized unit for panel mounting alongside recorders and other control room displays and switches. Multiple SLCs are typically rack mounted. It is a single unit which contains both the comparator and the controller proper. Thus it has two input signals, the measured value and the set point, and one output signal. It would also have its own power and/or air supply.

The facia of a typical SLC is depicted in Figure 23.1. Often referred to as a faceplate, it has a scale for reading values of the measured value and set point. The range of the scale normally corresponds to the calibration of the measuring element, or else is 0–100% by default. With pneumatic controllers the signals would be indicated by pointers. With electronic controllers liquid crystal displays (LCD) or light emitting diode (LED) bar displays are used whereas with digital controllers a VDU representation of the faceplate is common practice. Faceplates also show relevant alarm limits. A

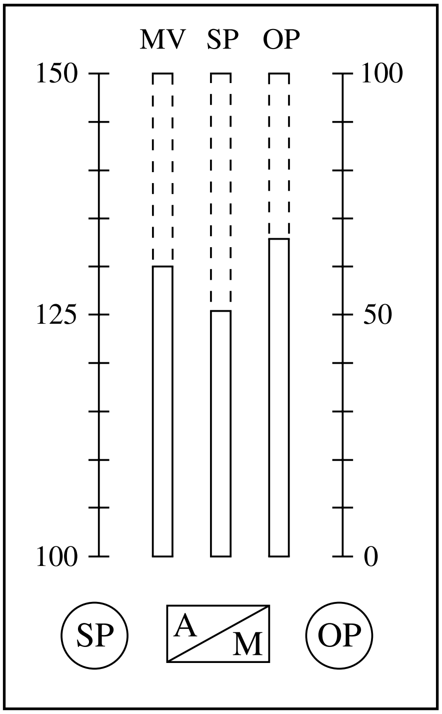

Fig. 23.1 Facia of a typical single loop controller

dial and/or dedicated pushbuttons enables the set point to be changed locally, *i.e.* by hand. Otherwise the set point is changed on a remote basis. There is often a separate scale to indicate the value of the output signal, range 0–100%, with provision for varying the output by hand when in MAN mode. A switch enables the controller to be switched between AUTO and MAN modes.

All of the above functions are operational and intended for use by plant personnel. Other functions of the controller are technical and are normally inaccessible from the faceplate. Thus, for example, the PID settings and the forward/reverse switch are typically adjusted either by dials internal to the controller or by restricted access pushbuttons. Despite its name, a modern digital SLC will provide two or more 3-term controllers, handle several discrete I/O signals, and support extensive continuous control and logic functions. Access to this functionality is restricted to engineering personnel, typically by means of a serial link.

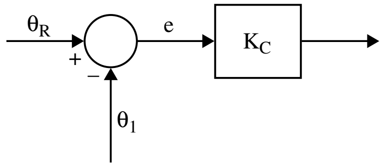

Fig. 23.2 Symbols for signals of PID controller

23.2 Proportional Action

With reference to Figure 23.2 the equations of the comparator and of a proportional controller are:

$$e = \theta_R - \theta_1 \tag{23.1}$$

$$\theta_0 = \theta_B \pm K_C.e \tag{23.2}$$

The gain K_C is the sensitivity of the controller, *i.e.* the change in output per unit change in error. Gain is traditionally expressed in terms of bandwidth, often denoted as %bw, although there is no obvious advantage from doing so. Provided the various signals all have the same range and units, gain and bandwidth are simply related as follows:

$$K_C = \frac{100}{\%\text{bw}} \tag{23.3}$$

Thus a high bandwidth corresponds to a low gain and *vice versa*. However, if the units are mixed, as is often the case with digital controllers, it is necessary to take the ranges of the signals into account. Strictly speaking, bandwidth is defined to be the range of the error signal, expressed as a percentage of the range of the measured value signal, that causes the output signal to vary over its full range.

The issue is best illustrated by means of a numerical example. Consider Figure 23.3 in which θ_1 and θ_0 have been scaled by software into engineering units of 100–150 °C and 0–100% respectively. Suppose that the controller is forward acting, $K_C = 5$, θ_R is 125°C and θ_B is 50%. What this means in practice is that signals within the range $115 < \theta_1 < 135$°C, *i.e.* an error of ±10°C, will cause the output signal to vary from 0–100%. According to the definition of bandwidth, bw $= 20/50 \times 100 =$ 40%, which is different from the value that would have been obtained by substituting $K_C = 5$ into

Equation 23.3. This is clearly a source of confusion: it is best to avoid using the term bandwidth and to consistently work with gain.

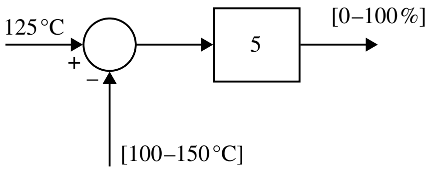

Fig. 23.3 Temperature controller for explaining bandwidth

Note that outside the range of $115 < \theta_1 < 135°C$ the controller output is constrained by the limits of the output signal range. Thus θ_0 would be either 100% for all values of $\theta_1 < 115°C$ or 0% for $\theta_1 > 135°C$. When a signal is so constrained it is said to be saturated.

In Chapter 22 the steady state response of a closed loop system was considered. In particular, the way in which increasing K_C reduces offset was analysed. Also of importance is an understanding of how varying K_C affects the dynamic response. It is convenient to consider this in relation to the same level control system of Figure 22.1. The closed loop response to step increases in set point h_R and inlet flowrate F_1 are as shown in Figures 23.4 and 23.5, respectively.

Two traces are shown on Figure 23.4. The exponential one corresponds to a low gain and depicts a so-called over-damped response. The change in set point causes a step increase in error and, because the controller is reverse acting, produces a sudden closing of the valve. Thereafter the valve slowly opens as the error reduces and the level gently rises towards the new set point with a steady state offset. Increasing K_C produces a faster asymptotic approach towards a smaller offset.

However, there is some critical value beyond which increasing K_C causes the response to become oscillatory, as depicted in the second trace. In effect, the controller is so sensitive that it has over-compensated for the increase in set point by closing the valve too much. This causes the level to rise quickly and overshoot the set point. As the level crosses the set point the error becomes negative so the controller increases its output. The resultant valve opening is more than that required to compensate for the overshoot so the level falls below the set point. And so on. Provided the value of K_C is not too high these oscillations decay away fairly quickly and the under-damped response settles out with a reduced steady state offset. Increasing K_C further causes the response to become even more oscillatory and, eventually, causes the system to become unstable.

A similar sort of analysis can be applied to Figure 23.5. The principal effects of P action are summarised in Table 23.1 on page 163.

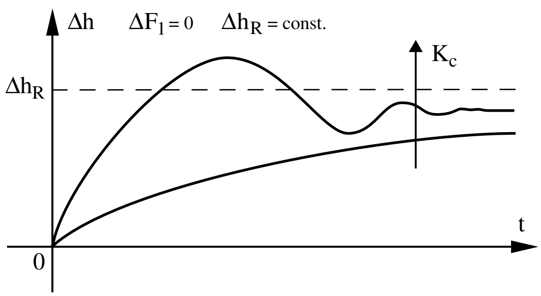

Fig. 23.4 Closed loop response to step change in set point

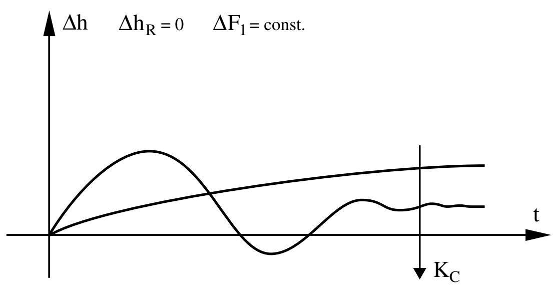

Fig. 23.5 Closed loop response to step change in inlet flow

23.3 Integral Action

The purpose of I action is to eliminate offset. This is realised by the addition of an integral term to Equation 23.2 as follows:

$$\theta_0 = \theta_B \pm K_C \left(e + \frac{1}{T_R} \int_0^t edt \right) \qquad (23.4)$$

T_R is known as the reset time and characterises the I action. Adjusting T_R varies the amount of I action. Note that, because T_R is in the denominator, to increase the effect of the I action T_R has to be reduced and *vice versa*. The I action can be turned off by setting T_R to a very large value. T_R has the dimensions of time: for process control purposes it is normal for T_R to have units of minutes.

The open loop response of a PI controller to a step change in error is depicted in Figure 23.6. Assume that the controller is forward acting, is in its AUTO mode and has a 4–20 mA output range. Suppose that the error is zero until some point in time, $t = 0$, when a step change in error of magnitude e' occurs.

Substituting $e = e'$ into Equation 23.4 and integrating gives:

$$\theta_0 = \theta_B + K_c e' + \frac{K_c}{T_R} e' t$$

The response shows an initial step change in output of magnitude $K_C e'$ due to the P action. This is followed by a ramp of slope $K_C/T_R.e'$ which is due to the I action. When $t = T_R$ then:

$$\theta_0 = \theta_B + 2K_C e'$$

This enables the definition of reset time. T_R is the time taken, in response to a step change in error, for the I action to produce the same change in output as the P action. For this reason integral action is often articulated in terms of repeats per minute:

$$\text{Repeats/min} = \frac{1}{T_R}$$

Note that the output eventually ramps up to its maximum value, 20 mA, and becomes saturated.

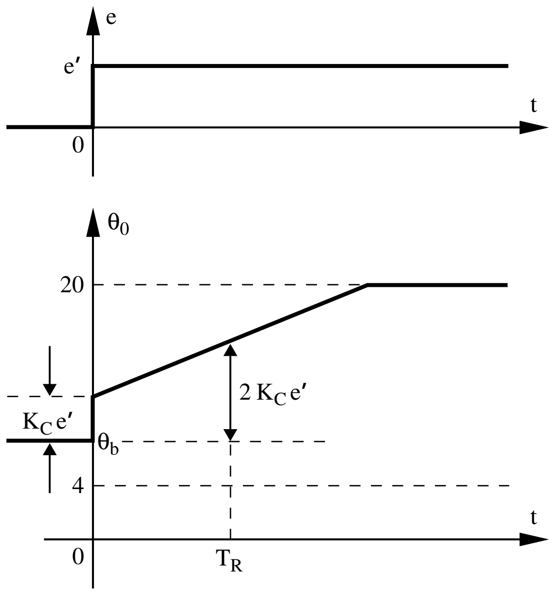

Fig. 23.6 Open loop response of PI controller to a step change in error

The closed loop response of a PI controller with appropriate settings to a step change in set point is depicted in Figure 23.7.

Again it is convenient to consider the level control system of Figure 30.1. Initially the P action dominates and the response is much as described for the under-damped case of Figure 23.4. However as the oscillations decay away, leaving an offset, the I action becomes dominant. Whilst an error persists the integral of the error increases. Thus the controller output slowly closes the valve and nudges the level towards its setpoint. As the error reduces, the contribution of the P action to the controller output decreases. Eventually, when there is zero error and the offset has been eliminated, the controller output consists of the bias term and the I action only. Note that although the error becomes zero the integral of the error is non-zero:

$$\theta_0 = \theta_B - \frac{K_C}{T_R} \int_0^t edt$$

Thus the P action is short term in effect whereas the I action is long term. It can also be seen from Figure 23.7 that the response is more oscillatory

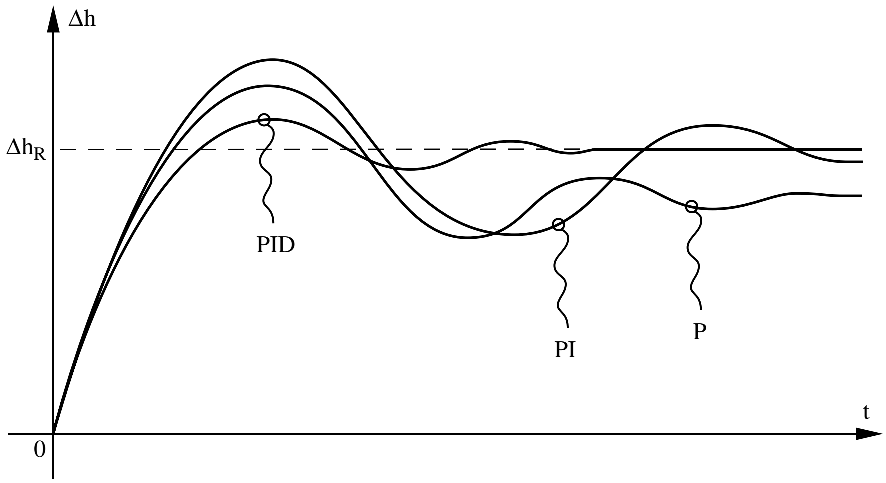

Fig. 23.7 Closed loop P, PI and PID response to a step change in set point

than for P action alone. The oscillations are larger in magnitude and have a lower frequency. In effect the I action, which is working in the same direction as the P action, accentuates any overshooting that occurs. It is evident that I action has a destabilising effect which is obviously undesirable. The principal effects of I action are also summarised in Table 23.1.

Equation 23.4 is the classical form of PI controller. This is historic, due to the feedback inherent in the design of pneumatic and electronic controllers. Note the interaction between the P and I terms: varying K_C affects the amount of integral action because K_C lies outside the bracket. An alternative non-interacting form is as follows:

$$\theta_0 = \theta_B \pm \left(K_C e + \frac{1}{T_I} \int_0^t e\mathrm{d}t \right) \qquad (23.5)$$

T_I is known as the integral time. It can be seen by inspection that $T_I = T_R/K_C$.

23.4 Derivative Action

The purpose of D action is to stabilise and speed up the response of a PI controller. This is realised by the addition of a derivative term to Equation 23.4 as follows:

$$\theta_0 = \theta_B \pm K_C \left(e + \frac{1}{T_R} \int_0^t e\mathrm{d}t + T_D \frac{\mathrm{d}e}{\mathrm{d}t} \right) \qquad (23.6)$$

T_D is known as the rate time and characterises the D action. Adjusting T_D varies the amount of D action, setting it to zero turns off the D action altogether. T_D has the dimensions of time: for process control purposes it is normal for T_D to have units of minutes.

The open loop response of a PID controller to a sawtooth change in error is depicted in Figure 23.8. Again assume that the controller is forward acting, is in its AUTO mode and has a 4–20-mA output range. Suppose that the error is zero for $t < 0$ and for $t > t'$, and that the error is a ramp of slope m for $0 < t < t'$.

Substituting $e = mt$ into Equation 23.6 gives:

$$\theta_0 = \theta_B + K_C mt + \frac{K_C}{T_R} \frac{mt^2}{2} + K_C T_D m$$

The response shows an initial step change in output of magnitude $K_C T_D m$ due to the D action. This is followed by a quadratic which is due to the P and I actions. At $t = t'$ there is another step change as the contribution to the output of the P and D

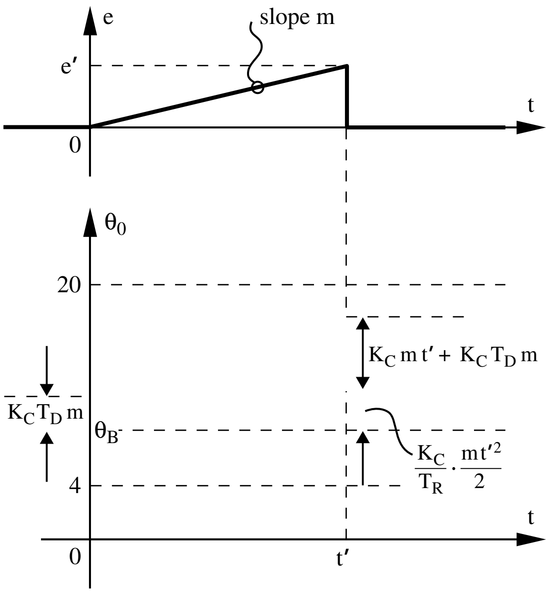

Fig. 23.8 Open loop response of PID controller to a sawtooth change in error

actions disappears. The residual constant output is due to the I action that occurred before $t = t'$.

The closed loop response of a PID controller with appropriate settings to a step change in set point is as depicted in Figure 23.7. It can be seen that the effect of the D action is to reduce the amount of overshoot and to dampen the oscillations. This particular response is reproduced in Figure 23.9 with the corresponding plots of e, Int(edt) and de/dt *vs* time.

Again, referring to the level control system of Figure 22.1, it is evident that as the level crosses the set point and rises towards the first overshoot, the sign of de/dt is negative. This boosts the controller output so that the valve opening is more than it would be due the P and I actions alone, with the obvious effect of reducing the amount of overshoot. As the level passes the first overshoot and starts to fall the sign of de/dt becomes positive and the valve is closed more than it would be otherwise. And so on. In effect the D action is anticipating overshoot and countering it.

D action depends on the slope of the error and, unlike P and I action, is independent of the magnitude of the error. If the error is constant, D action has no effect. However, if the error is changing, the D action boosts the controller output. The faster the error is changing the more the output is boosted. An important constraint on the use of D action is noise on the error signal. The spikes of a noisy signal have large slopes of alternating sign. If the rate time is too large, D action amplifies the spikes forcing the output signal to swing wildly. The principal effects of D action are summarised in Table 23.1.

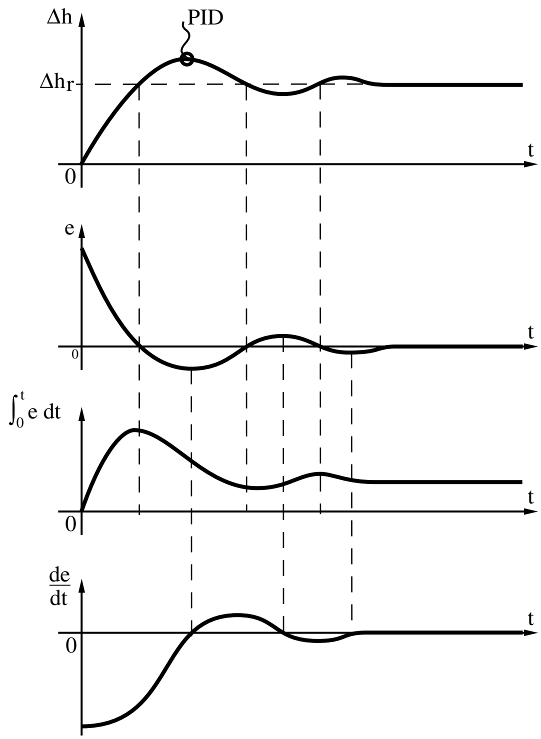

Fig. 23.9 Closed loop PID response to a step change in set point

In addition to the effects of P, I and D actions, there are a number of other operational characteristics of 3-term controllers to be considered.

23.5 Bumpless Transfer

If changes in controller output are required, it is good practice to move in a regulated rather than a sudden manner from one state to another. Process plant can suffer damage from sudden changes. For example, suddenly turning off the flow of cooling water to a jacket may cause a vessel's glass linings to crack. Suddenly applying a vacuum to a packed column will cause flashing which leads to the packings breaking up. Such sudden changes in output are invariably caused by the controller being forced to respond to step changes in error.

One source of step change in error occurs when a controller is transferred from its MAN mode of operation into AUTO. If the measured value is not at its set point, depending on the settings, the controller output may well jump to saturation. A simple way round this problem is to adjust the output in MAN mode until the measured value and set point coincide prior to switching into AUTO.

Alternatively, a bumpless transfer function may be specified. In effect, on transfer into its AUTO mode, the set point is adjusted to coincide with the measured value and hence zero error. This results in the controller being started off in AUTO mode with the wrong set point, but that may then be ramped up or down to its correct value at an appropriate rate. The integral action is also initialised by setting it to zero and whatever was the output signal in MAN mode at the time of transfer normally becomes the value of the bias in AUTO.

Thus,

$$\begin{aligned} \theta_R &= \theta_1 |_{t=0} \\ \int_{-\infty}^{0} e.dt &= 0 \\ \theta_B &= \theta_0 |_{t=0} \end{aligned}$$

Set point tracking is an alternative means of realising bumpless transfer. Thus, whilst the controller is in MAN mode, the set point is continuously adjusted to whatever is the value of the measured value. This means that when the controller is switched into AUTO mode, there is zero error and transfer is bumpless. Again, the integral action is initialised by setting it to zero.

For transfers from AUTO into MAN mode the output is normally frozen at whatever was its value in AUTO at the time of transfer.

23.6 Derivative Feedback

Another source of step change in error occurs when the controller is in its AUTO mode and the set point is suddenly changed. Of particular concern is the D action which will initially respond to a large value of de/dt and may cause the output to jump to saturation. To counter this derivative feedback may be specified, as follows:

$$\theta_0 = \theta_B \pm K_C \left(e + \frac{1}{T_R} \int_0^t e.dt - T_D \frac{d\theta_1}{dt} \right) \quad (23.7)$$

Note that the D action responds to changes in the measured value rather than to the error. Also note the minus sign which allows for the fact that the measured value moves in the opposite direction to the error signal. Whilst the set point is constant the controller behaves in exactly the same way as the classical controller of Equation 23.6. However, when the set point changes, only the P & I actions respond. This form of 3-term control is common in modern digital controllers.

23.7 Integral Windup

Another commonly encountered problem is integral windup. If the controller output saturates whilst an error exists, the I action will continue to integrate the error and, potentially, can become a very large quantity. When eventually the error reduces to zero, the controller output should be able to respond to the new situation. However, it will be unable to do so until the error has changed sign and existed long enough for the effect of the integration prior to the change of sign to be cancelled out. The output remains saturated throughout this period and the controller is effectively inoperative. A simple way round this problem is to switch the controller into its MAN mode when the saturation occurs. Switching it back into AUTO when the situation permits causes the I action to be initialised at zero.

A more satisfactory way of addressing the issue is to specify an integral desaturation facility. In effect, whilst saturation of the output occurs, the I action is suspended. This may be achieved by the I action considering the error to be zero during saturation. Alternatively, in digital controllers, the instructions used for calculating the I action may be by-passed during periods of saturation, as illustrated later on.

23.8 Worked Example

A classical 3-term controller which is forward acting has the following settings:

40% bandwidth, 5 min reset time, 1 min rate time.

Also it is known that an output of 10 mA in MANUAL is required for bumpless transfer to AUTO. This information is sufficient to deduce the parameters of Equation 23.6:

$$\theta_0 = 10 + 2.5\left(e + \frac{1}{5}\int_0^t e.dt + \frac{de}{dt}\right)$$

Suppose the controller is at steady state in AUTO when the measured value starts to decrease at the rate of 0.1 mA/min.

Thus, if $t < 0$ then $e = 0$ and $\theta_0 = 10$ mA, and if $t \geq 0$ then:

$$\frac{d\theta_1}{dt} = -0.1.$$

However, $e = \theta_R - \theta_1$. Assuming θ_R is constant then:

$$\frac{de}{dt} = -\frac{d\theta_1}{dt} = +0.1,$$

whence $e = +0.1\,t$.

Substituting for e into the PID equation gives:

$$\begin{aligned}\theta_0 &= 10 + 2.5\left(0.1\,t + \frac{1}{5}\int_0^t (0.1\,t)\,dt + \frac{d(0.1\,t)}{dt}\right)\\ &= 10.25 + 0.25\,t + 0.025\,t^2\end{aligned}$$

Inspection reveals a small jump in output at $t = 0$ of 0.25 mA followed by a quadratic increase in output with time. The controller output saturates when it reaches 20 mA, *i.e.* when:

$$20 = 10.25 + 0.25\,t + 0.025\,t^2$$

$$t^2 + 10\,t - 390 = 0$$

i.e. when $t \approx 15.4$ min. Note that this must be an open loop test: the input signal is ramping down at a constant rate and appears to be independent of the controller output.

23.9 Other Analogue Forms of PID Controller

Equation 23.6 is the classical form of PID controller and Equation 23.7 is the most common form used in digital controllers. There are, however, many variations on the theme which are supported by most modern controllers. For example, to make the controller more sensitive to large errors the P action may operate on the square of the error:

$$\theta_0 = \theta_B \pm K_C\left(\text{Sign}(e).e^2 + \frac{1}{T_R}\int_0^t e.dt - T_D\frac{d\theta_1}{dt}\right)$$

To make the controller penalise errors that persist the I action may be time weighted, with some facility for reinitialising the I action:

$$\theta_0 = \theta_B \pm K_C\left(e + \frac{1}{T_R}\int_0^t e.t.dt - T_D\frac{d\theta_1}{dt}\right)$$

It is important to be aware of exactly what form of 3-term control has been specified and what is being implemented.

23.10 Discretised Form of PID

Equation 23.7 is an analogue form of PID control and has to be translated into a discretised form for implementation in a digital controller. It may be discretised as follows:

$$\begin{aligned}\theta_{0,j} &\approx \theta_B \pm K_C\left(e_j + \frac{1}{T_R}\sum_{k=1}^{j} e_k.\Delta t - T_D\frac{(\theta_{1,j} - \theta_{1,j-1})}{\Delta t}\right)\\ &= \theta_B \pm \left(K_C.e_j + \frac{K_C.\Delta t}{T_R}\sum_{k=1}^{j} e_k - \frac{K_C.T_D}{\Delta t}(\theta_{1,j} - \theta_{1,j-1})\right)\\ &= \theta_B \pm \left(K_C.e_j + K_I.\sum_{k=1}^{j} e_k - K_D.(\theta_{1,j} - \theta_{1,j-1})\right) \qquad (23.8)\end{aligned}$$

where j represents the current instant in time and Δt is the step length for numerical integration. This