On-Off Control

On-off control, sometimes referred to as bang-bang control, is conceptually the same thing as proportional control, as described in Chapter 22, with a high controller gain. It is characterised by very small, but finite, errors causing the controller output to switch between maximum and minimum output according to the sign of the error. The response of a forward acting on-off controller to a sawtooth error is depicted in Figure 28.1.

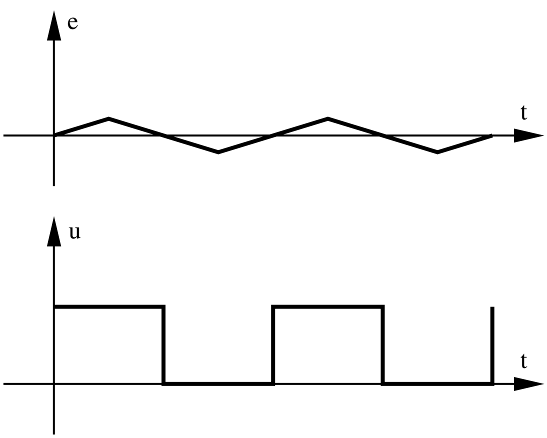

Fig. 28.1 Response of on-off controller to sawtooth error

28.1 On-Off Cycling

It is seldom that a proportional controller *per se* is used for on-off control. Much more typical is the use of amplifiers and relays in simple thermostats. On-off control is surprisingly common for simple, non-critical applications. There is more to it than meets the eye though.

Consider a tank containing a liquid whose temperature is thermostatically controlled as depicted in Figure 28.2.

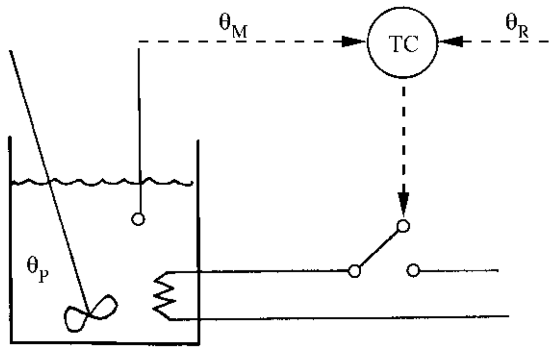

Fig. 28.2 Tank with thermostat

Depending on the temperature of the liquid in the tank, the power supply is either connected to or disconnected from the heating element. This results in the temperature cycling about the set point θ_R within a narrow band $\Delta\theta$, as depicted in Fig-

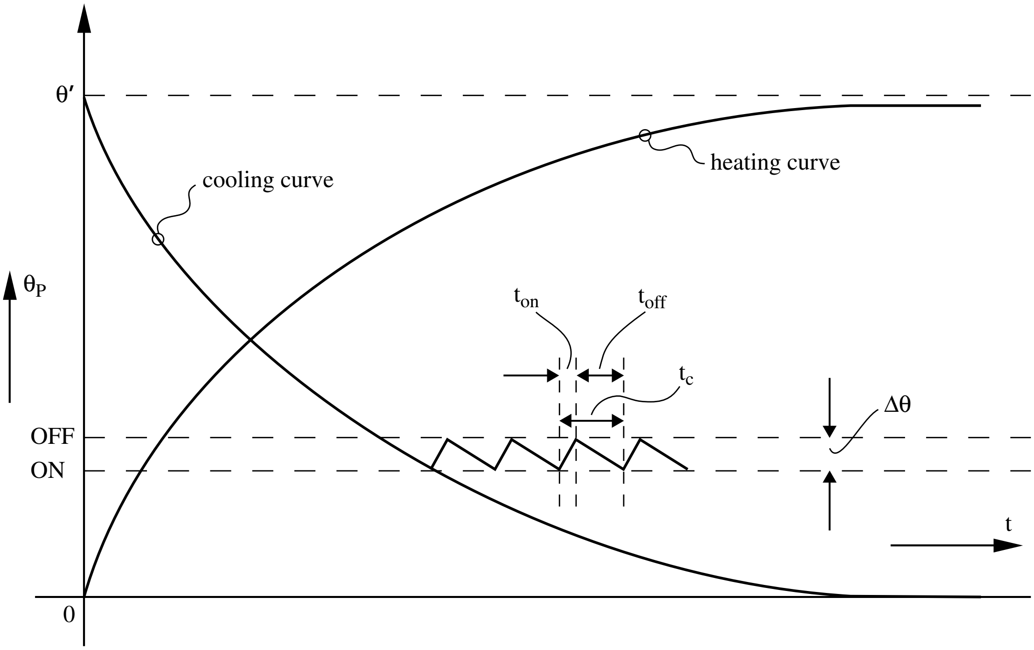

Fig. 28.3 Heating and cooling curves for tank contents

ure 28.3. Since $\Delta\theta$ is small, the sections of the heating and cooling curves may be approximated by straight lines.

28.2 On and Off Curves

Consider the heating curve. If the heater is switched on, assuming that the liquid is well mixed and that there are no heat losses due to evaporation, then an unsteady state heat balance for the contents of the vessel gives:

$$M.c_p \frac{d\theta_P}{dt} = W - U.A.\theta_P$$

where θ is measured relative to ambient temperature. This is a first order system, of the type described in Chapter 69, whose response is of the form:

$$\theta_P = \frac{W}{UA}\left(1 - e^{\frac{-UA}{Mc_p}t}\right)$$

If $t \to \infty$ then $\theta_P \to \theta' = \dfrac{W}{U.A}$. Defining the time constant $T_P = \dfrac{M.c_p}{U.A}$ yields:

$$\theta_P = \theta'.\left(1 - e^{-t/T_P}\right)$$

The slope of the heating curve is thus:

$$\frac{d\theta_P}{dt} = \frac{\theta'}{T_P}.e^{-t/T_P} = \frac{\theta' - \theta_P}{T_P}$$

Let $\overline{\theta}_P$ be the mean value of θ_P within the band $\Delta\theta$. Thus, within $\Delta\theta$:

$$\left.\frac{d\theta_P}{dt}\right|_{on} \approx \frac{\theta' - \overline{\theta}_P}{T_P} \qquad (28.1)$$

Now consider the cooling curve. If the heater is switched off, then an unsteady state heat balance for the contents of the vessel gives:

$$M.c_p \frac{d\theta_P}{dt} = -U.A.\theta_P$$

This too is a first-order system, whose response is of the form:

$$\theta_P = \theta'.e^{-t/T_P}$$

The slope of the cooling curve is thus:

$$\frac{d\theta_P}{dt} = \frac{-\theta'}{T_P}.e^{-t/T_P} = \frac{-\theta_P}{T_P}$$

So, for values of θ_P within the band $\Delta\theta$:

$$\left.\frac{d\theta_P}{dt}\right|_{off} \approx \frac{-\overline{\theta}_P}{T_P} \qquad (28.2)$$

28.3 Lag Effects

A section of the sawtooth of Figure 28.3 is reproduced in Figure 28.4.

The controller period t_C is the sum of the t_{on} and t_{off} times. Thus:

$$t_C = \frac{1}{\left.\frac{d\theta_P}{dt}\right|_{on}}.\Delta\theta + \frac{1}{\left.\frac{d\theta_P}{dt}\right|_{off}}.\Delta\theta$$

Substituting from Equations 28.1 and 28.2 gives:

$$t_C = \frac{\theta'.T_P.\Delta\theta}{\overline{\theta}_P.\left(\theta' - \overline{\theta}_P\right)} \qquad (28.3)$$

The measured temperature θ_M lags behind the tank temperature by the time constant T_M of the measuring element. Because of mechanical backlash in the switches, hysteresis, stiction, *etc.*, the heater state will not change from on to off and *vice versa* until there exists a finite error about the set point. The corresponding band of measured temperature is called the differential gap $\Delta\theta_M$.

From Figure 28.4 it can be seen that:

$$\Delta\theta = \Delta\theta_M + \left.\frac{d\theta_P}{dt}\right|_{on}.T_M + \left.\frac{d\theta_P}{dt}\right|_{off}.T_M$$

Substituting from Equations 28.1 and 28.2 gives:

$$\Delta\theta = \Delta\theta_M + \frac{T_M}{T_P}.\theta' \qquad (28.4)$$

The mean tank temperature is half way across $\Delta\theta$:

$$\overline{\theta}_P = \theta_B + \frac{\Delta\theta}{2} = \theta_B + \frac{\Delta\theta_M}{2} + \frac{T_M}{2.T_P}.\theta'$$

Assume that the backlash, *etc.*, is symmetrical about the set point θ_R:

$$\theta_R = \theta_B + \left.\frac{d\theta_P}{dt}\right|_{off}.T_M + \frac{\Delta\theta_M}{2} = \theta_B + \frac{T_M}{T_P}.\overline{\theta}_P + \frac{\Delta\theta_M}{2}$$

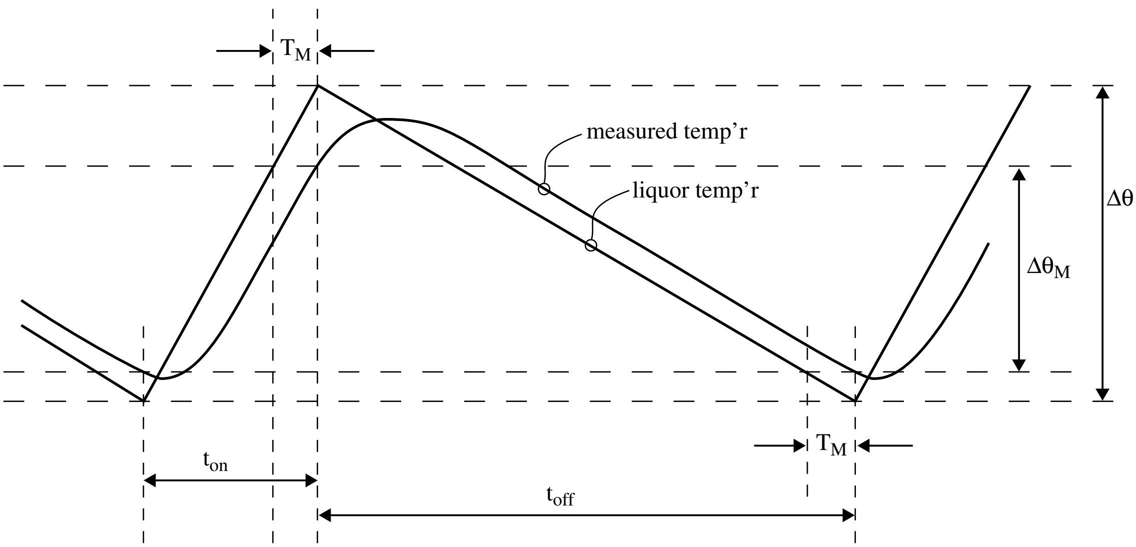

Fig. 28.4 On and off sections of sawtooth

For on-off control the offset is defined to be the difference $\overline{\theta}_P - \theta_R$, whence:

$$\text{Offset} = \overline{\theta}_P - \theta_R \equiv \frac{T_M}{T_P} \cdot \left(\frac{\theta'}{2} - \overline{\theta}_P\right) \qquad (28.5)$$

28.4 Worked Example

The temperature of the liquid in a tank is controlled by an on-off controller. The trend diagram indicates that if the set point is 65°C then the on-time is 249 s, the off-time is 150 s and the differential gap is 1.5°C. If the heater is left switched on the temperature reaches 95°C eventually. What is the temperature variation in the tank?

$$\Delta\theta_M = 1.5°\text{C}, t_{on} = 249 \text{ s and } t_{off} = 150 \text{ s.}$$

Assume an ambient temperature of 15°C:

$$\theta_R = 65 - 15 = 50°\text{C and } \theta' = 95 - 15 = 80°\text{C.}$$

From the slopes of the heating and cooling curves, Equations 28.1 and 28.2:

$$\Delta\theta = \frac{\theta' - \bar{\theta}_P}{T_P} . t_{on} = \frac{\bar{\theta}_P}{T_P} . t_{off}$$

Substituting values:

$$\left(80 - \bar{\theta}_P\right) .249 = \bar{\theta}_P .150$$

whence $\bar{\theta}_P = 49.9248$

From the definition of offset, Equation 28.5:

$$\overline{\theta}_P - \theta_R \equiv \frac{T_M}{T_P} \cdot \left(\frac{\theta'}{2} - \overline{\theta}_P\right)$$

$$-0.0752 \equiv \frac{T_M}{T_P} . - 9.925$$

whence

$$\frac{T_M}{T_P} = 0.007577$$

Substituting into Equation 28.4 gives:

$$\Delta\theta = \Delta\theta_M + \frac{T_M}{T_P} . \theta' = 1.5 + 0.007577 \times 80 = 2.106$$

Hence temperature variation in the tank is approximately 2°C.

28.5 Comments

Most on-off systems operate with a relatively large cycle time and deviation. A long cycle time gives rise to large deviations in the controlled variable, whereas a short cycle time may cause excessive wear on the relays, actuators, *etc.* Thus there is a trade off between cycle time and deviation.

Equations 28.3–28.5 give the relationships between the cycle time, width of the band and its mean, the differential gap, set point, offset and time lags. To achieve a narrow band requires minimal backlash, *etc.*, and a fast measurement, *i.e.* $T_M \ll T_P$.

There will be no offset when $\theta' = 2\,\overline{\theta}_P$ in which case:

$$\left.\frac{d\theta_P}{dt}\right|_{on} = \left.\frac{d\theta_P}{dt}\right|_{off} = \frac{\theta'}{2\,T_P}$$

The on and off times will be equal when the slopes of the on and off curves are the same.

28.6 Nomenclature

A	effective surface area	m^2
c_P	specific heat	$kJ\ kg^{-1}\ K^{-1}$
M	mass of contents of vessel	kg
t	time	s
T	time constant	s
U	overall heat transfer coefficient	$kW\ m^{-2}\ K^{-1}$
W	heater power	kW
θ	temperature	°C

Subscripts

B	ase position
C	controller
M	measurement
P	process
R	set point