

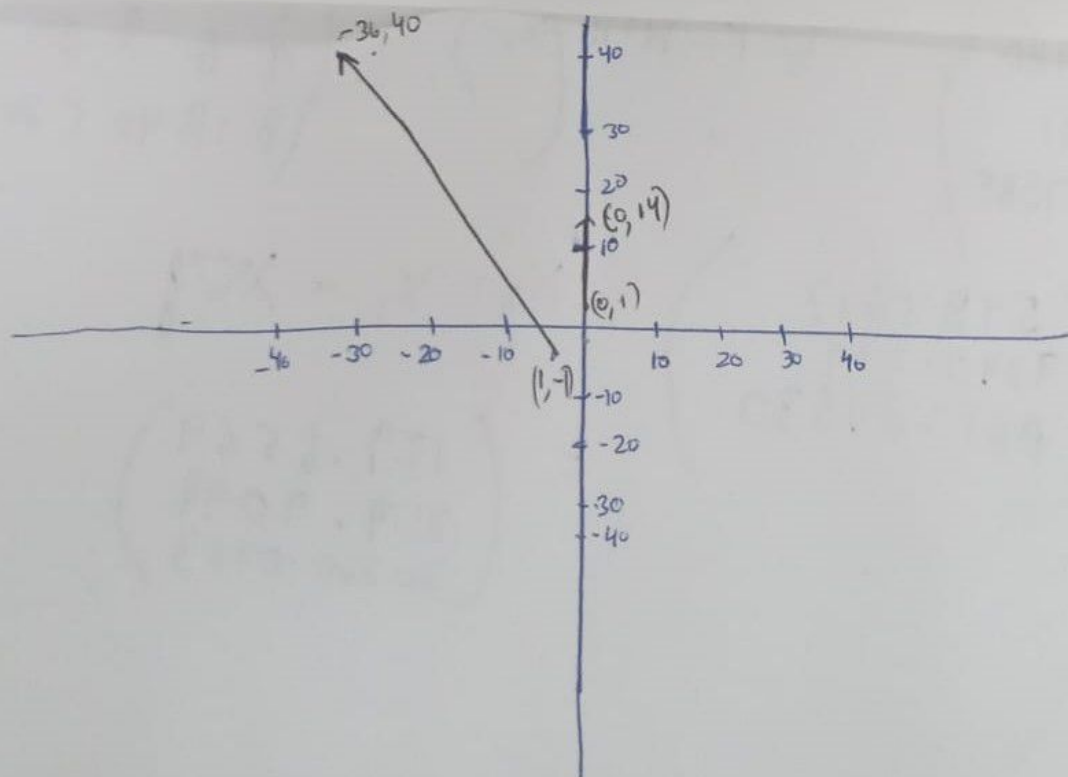
## Activity 1

a)  $f(x, y) = 18x^2y^3 + 7y^2$

$$\nabla f = (36xy^3, 54x^2y^2 + 14y)$$

b) At  $(0, 1)$  &  $(1, -1)$ ,  $\nabla f(x, y)$ :

$$= (0, 14) \text{ \& } (-36, 40)$$



# Activity 2

$$\nabla f = H^T H \cdot x - H^T z$$

$$x = 0.1, x_0 = (0.5, 0.1, 0.2)$$

$$H^T H = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 6 & 9 & 20 \\ 8 & 18 & 40.5 & 200 \end{pmatrix} \begin{pmatrix} 1 & 4 & 8 \\ 1 & 6 & 18 \\ 1 & 9 & 40.5 \\ 1 & 20 & 200 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 39 & 266.5 \\ 39 & 533 & 4504.5 \\ 266.5 & 4504.5 & 42028.25 \end{pmatrix}$$

$$H^T H \cdot \begin{pmatrix} 0.5 \\ 0.1 \\ 0.2 \end{pmatrix} - \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 6 & 9 & 20 \\ 8 & 18 & 40.5 & 200 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 11 \\ 18 \end{pmatrix} = \nabla f$$

$$\nabla f = \begin{pmatrix} 20.2 \\ 460.7 \\ 4793.85 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} 0.5 \\ 0.1 \\ 0.2 \end{pmatrix} - 0.1 \begin{pmatrix} 20.2 \\ 460.7 \\ 4793.85 \end{pmatrix} = \begin{pmatrix} 0.298 \\ -4.51 \\ -47.737 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} 0.2980 \\ -4.51 \\ -47.7385 \end{pmatrix} \quad \nabla F = H^T H (x_1) - \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 6 & 9 & 20 \\ 8 & 18 & 40.5 & 200 \end{pmatrix} z$$

$$\nabla F = \begin{pmatrix} 5484617 \\ 92425374 \\ 861323630 \end{pmatrix}$$

$$x_2 = x_1 - \lambda \nabla f$$

$$= \begin{pmatrix} 129.6569 \\ 2174.9098 \\ 20260.0963 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 129.6569 \\ 2174.9098 \\ 20260.0963 \end{pmatrix} \quad \nabla F = H^T H \begin{pmatrix} 129.6569 \\ 2174.9098 \\ 20260.0963 \end{pmatrix} - H^T z$$

~~Step~~  $x_3 = \begin{pmatrix} -54716.51 \\ -922078.83 \\ -8592976.21 \end{pmatrix}$

Aufw. 4

a) Normal equation =  $H^T \cdot H \cdot x = H^T z$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 6 & 9 & 20 \\ 8 & 18 & 40.5 & 200 \end{pmatrix} \begin{pmatrix} 1 & 4 & 8 \\ 1 & 6 & 18 \\ 1 & 9 & 40.5 \\ 1 & 20 & 200 \end{pmatrix} \begin{pmatrix} x_0 \\ v_0 \\ a \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 6 & 9 & 20 \\ 8 & 18 & 40.5 & 200 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 11 \\ 18 \end{pmatrix}$$

$$\therefore x = (H^T H)^{-1} \cdot H^T z$$

$$= \begin{pmatrix} -5.4443636 \\ 2.3927654 \\ -0.1221037 \end{pmatrix}$$

Activity 5 (a).

$$f(x, y) = (2 \cdot 12 - x)^2 + 3(y - x^2)^2 \\ = (2 \cdot 12 - x)^2 + 3(y^2 + x^4 - 2x^2y)$$

$$\text{Gradient of } f(x, y) = \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial x} = -2(2 \cdot 12 - x) + 3(0 + 4x^3 - 4xy) \\ = -4 \cdot 24 + 2x + 12x^3 - 12xy.$$

$$\frac{\partial f}{\partial y} = 0 + 3(2y + 0 - 2x^2) \\ = 6y - 6x^2.$$

$$\therefore \text{Gradient of } f(x, y) = \begin{pmatrix} 12x^3 - 12xy + 2x - 4 \cdot 24, \\ 6y - 6x^2 \end{pmatrix}.$$



Let  $C_1$  be default and  $C_2$  correspond to those who do not default.

Let's start with the home owner column, which has binary values.

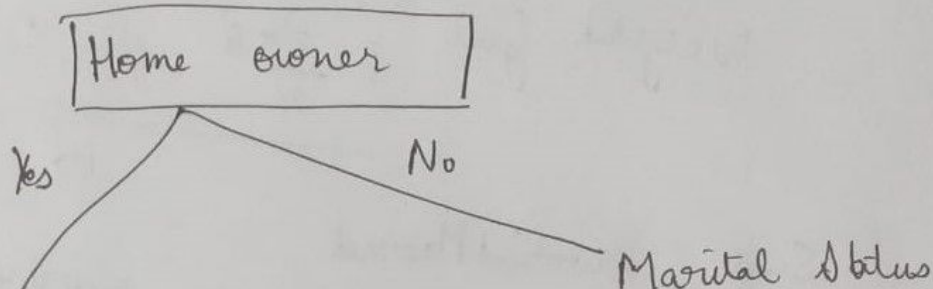
Activity 6

Homework

3.

Decision

Tree



Does not default  
Rows 3, 6, 8, 9, 10  
Default No No Yes No No.

Now we check for possible splits under marital status

1) 3 way split:

$$\frac{\frac{4 \cdot 3}{9} + \frac{0 \times 4}{16} + \frac{4 \cdot 3}{9}}{10}$$

$$= \frac{12+12}{90} = 0.266.$$

$$\text{Single} = 1 - \left(\frac{1}{3}\right)^2 - \left(\frac{2}{3}\right)^2 = \frac{4}{9} = 1 - P(C_1) - P(C_2)$$

$$\text{Married} = 1 - \left(\frac{0}{4}\right)^2 - \left(\frac{4}{4}\right)^2 = 0$$

$$\text{Divorced} = 1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{4}{9}$$

2) 2 way splits

a) Single Married & Divorced

$$\text{Single Married} = 1 - \left(\frac{1}{7}\right)^2 - \left(\frac{6}{7}\right)^2 = 0.245$$

$$\text{Divorced} = 1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = 0.444$$

$$\text{Weighted GINI} = \frac{\frac{12 \times 7}{49} + \frac{4 \times 3}{9}}{10} = 0.305$$

~~SD~~, Single Divorced<sup>(SD)</sup>, Married (M).

$$SD \text{ gini} = 1 - \left(\frac{3}{6}\right)^2 - \left(\frac{3}{6}\right)^2 = 0.5$$

$$M \text{ gini} = 1 - \left(\frac{0}{4}\right)^2 - \left(\frac{4}{4}\right)^2 = 0.$$

$$\text{Weighted Gini} = \frac{\frac{1 \times 6}{2} + 0 \times 4}{10} = 0.3.$$

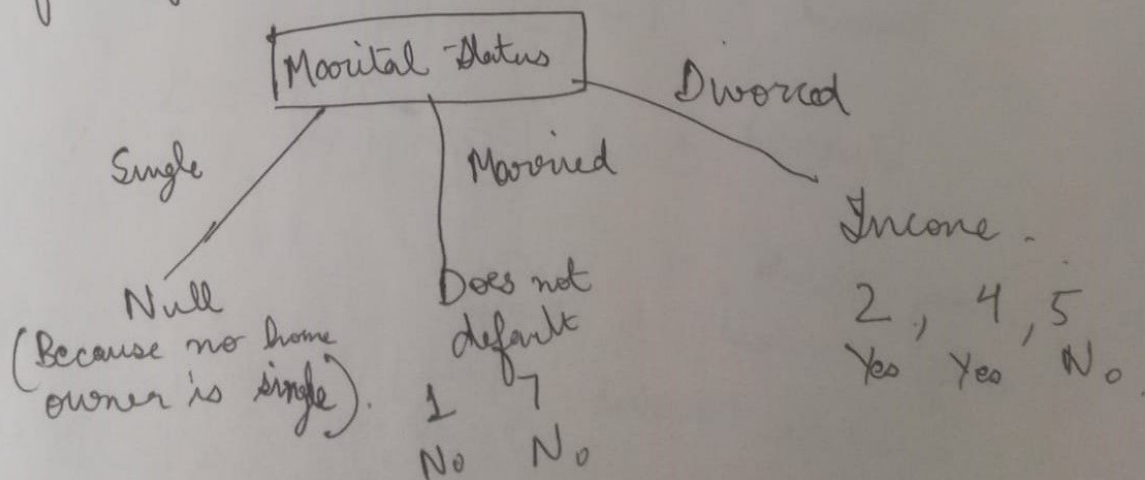
Single, Divorced Married.

$$\text{Single} : 1 - \frac{4}{9} - \frac{1}{9} = 0.444.$$

$$DM : 1 - \left(\frac{2}{7}\right)^2 - \left(\frac{5}{7}\right)^2 = \frac{20}{49}.$$

$$\text{Weighted gini} = \frac{\frac{20}{49} \times 7 + 0.444 \times 3}{10} = 0.419.$$

Therefore we opt for a 3 Way split for given it has the lowest Gini.



Income is a quantitative attribute, so we will follow the table procedure.

	Yes, 20k	Yes, 50k	No, 100k
$\leq$	>	<	<
0	2	1	
0	1	0	

Yes, 20k		Yes, 50k		No, 100k	
15k	35k	75k	115k		
Less than	More than	$\leq$	$>$	$\leq$	$>$
0	2	1	1	2	0
0	1	0	1	1	0

Gini calculation

15k

$$\leq = 0$$

$$> :- 1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{0.444}{0.444}$$

$$35k \leq :- 1 - \left(\frac{1}{1}\right)^2 - \left(\frac{0}{1}\right)^2 = 0$$

$$> :- 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = 1 - \frac{1}{2} = \frac{0.5}{0.5}$$

$$75k > :- 1 - \left(\frac{2}{2}\right)^2 - \left(\frac{0}{2}\right)^2 = 0$$

$$> :- 1 - \left(\frac{0}{1}\right)^2 - \left(\frac{1}{1}\right)^2 = \frac{0}{0}$$



115 k.

$$\leq: 1 - \left(\frac{2}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = 0.444$$

$$>: 1 - \frac{0}{0} - \frac{0}{0} = \frac{0}{0.444}$$

$\therefore$  We split at 75 k

Tree

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