Answers to questions in Lab 1: Filtering operations

Name: Rithika Harish Kumar Program: TCSCM

Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

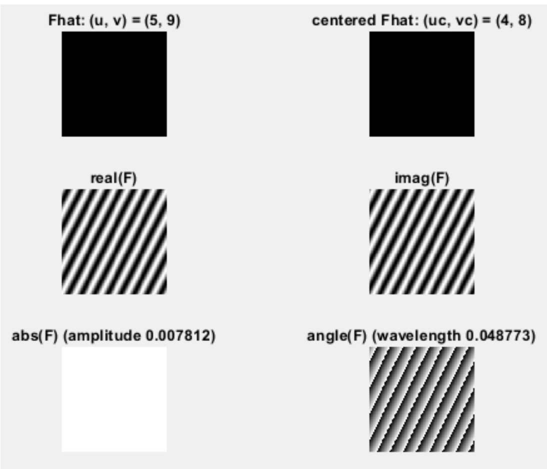
Good luck!

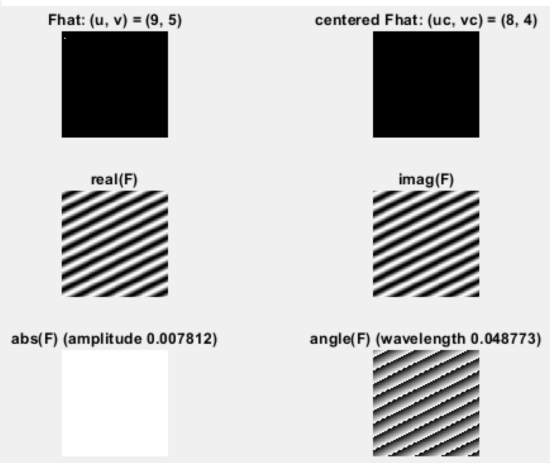
Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

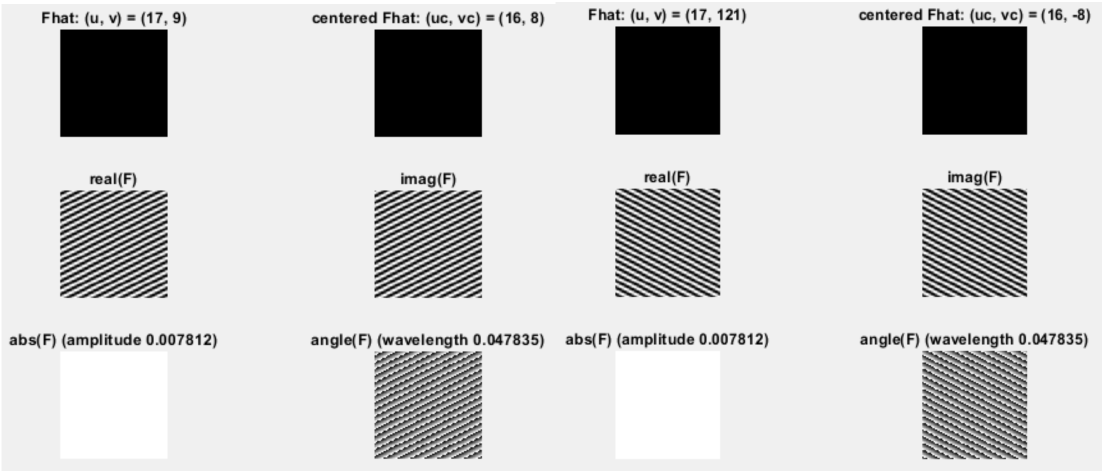
Answers:
Sine wave is created in spatial domain.

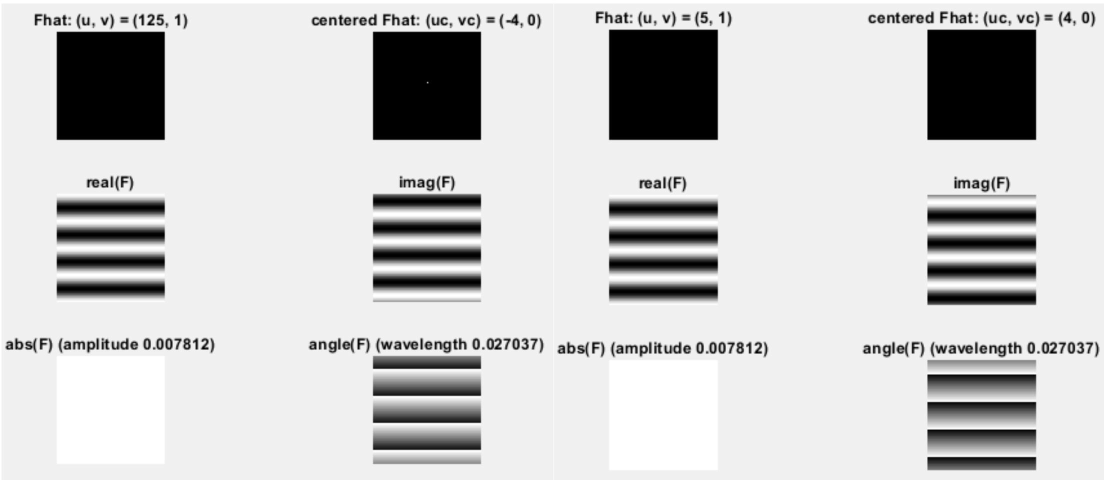
(5,9)(9,5) interchange of frequency components on x and y direction, rotation of phase, amplitude and wavelength is same.
(17,9)(17,121) inverse of frequency component on y direction, rotation of phase, amplitude and wavelength is same.
(5,1)(125,1) since y frequency component is 0, sine wave only in y direction, ($\Delta\varphi = \pi(4/-4)$) difference is the phase of the wave, amplitude and wavelength is same.

The furthur the non zero point from the origin, the larger the frequency of the wave, hence the smaller the wavelength of the spatial image. The direction of waveform is depending on the position of point relative to origin.



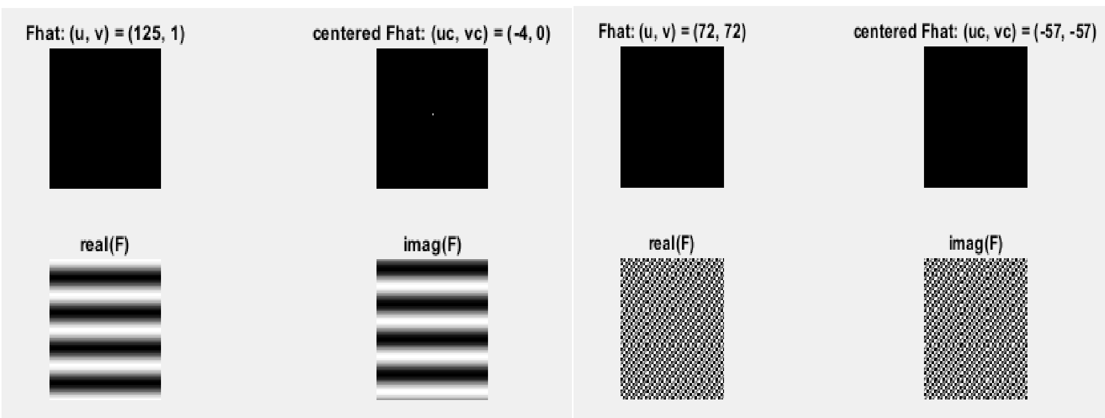






Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:
(p,q) position in fourier domain gives frequency components for creating a sine wave in x and y directions. If the point in the fourier domain is along the horizontal or vertical axes, the frequency will be zero along that direction. If the frequency is zero then the sine wave is only on one direction in the spatial domain. If the point is elsewhere then a rotation occurs on the wave and frequency is controlled by the fourier domain.



Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:
Amplitude $|f| = \sqrt{Re(f)^2 + Im(f)^2}$
Re(f) – real part
Im(f) – imaginary part
Basically, Norm of the function.
amplitude = norm(abs(F)); where F is the inverse fourier transform of the image.

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

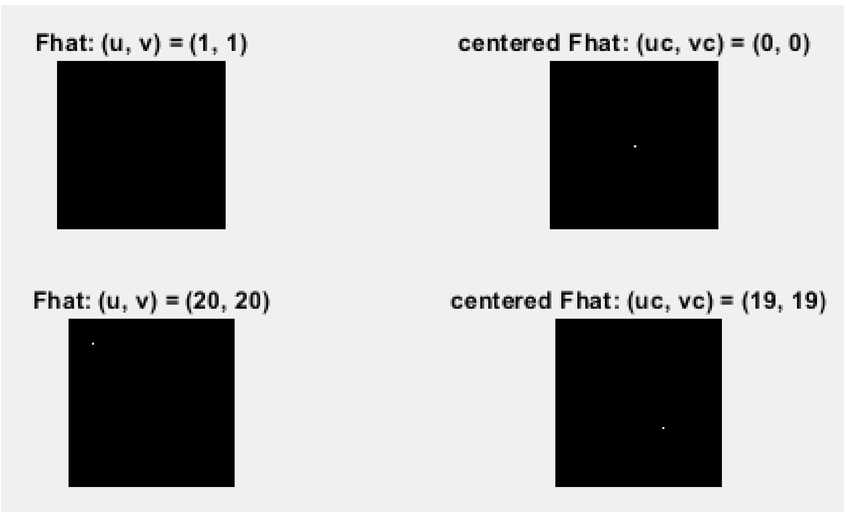
Answers:
Let $\omega = (\omega_1, \omega_2)^T$ where,
ω_1 = angular frequency in x direction
ω_2 = angular frequency in y direction.
Then, Phase of sine wave, $\varphi(\omega) = tan^{-1}(\frac{Im(\omega)}{Re(\omega)})$
Wavelength, $\lambda = \frac{2\pi}{||\omega||}$

wavelength = 2*pi./norm(angle(F)).

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:
The origin of the fourier transform is moved to the centre and the first quarter of the transform will move to the fourth. It is similar to multiplying the original function by $(-1)^{m+n}$ where m and n is image co-ords. It is called capturing. It happens because 2D fourier transform and its inverse is infinitely periodic.

Basically, the regions of the original image exchange their position diagonally.



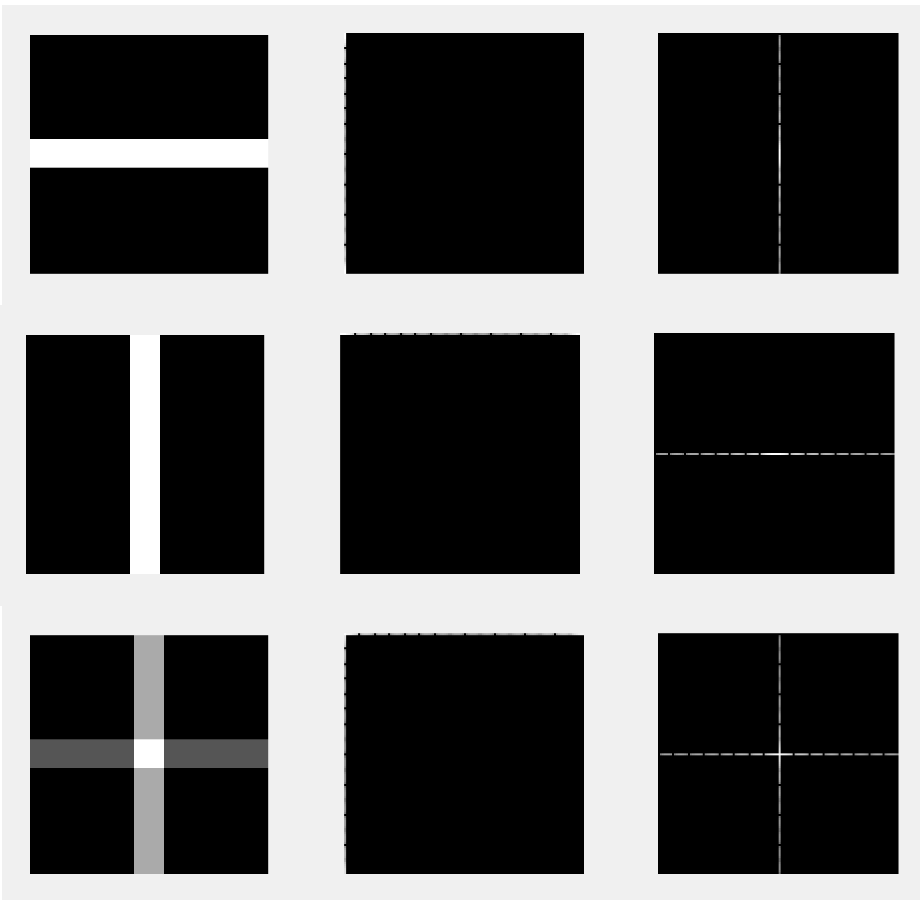
Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:
It is to visualise the mapping of angular frequency values inside the interval.

The uc and vc are the same(subtracted by 1 to go to the centre) if the pixel is in the first quarter. Otherwise it also subtracts the length of horizontal and vertical dimension from it because circshift function(shifts positions of elements circularly) is used to do the centering operation(move the centre by π or $-\pi$). (frequencies above half the length gets translated into negative frequencies)

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:
The zero point is in the top left corner. Fftshift is used to shift the zero frequency component to centre of the spectrum. The area around the origin of the transform consists of highest values. A dashed line appears in the frequency domain because the thickness of the line will be inversely proportional to the length of the object in the spatial domain and perpendicular to the direction of the biggest changes. (since one of the frequency is always zero, its concentrated on borders)



Question 8: Why is the logarithm function applied?

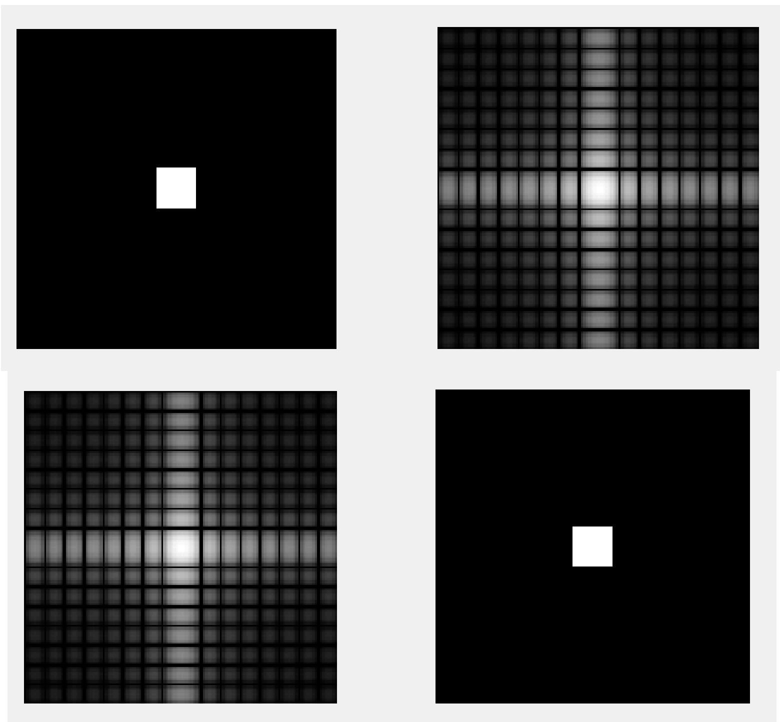
Answers:
As the dynamic range of some intensities are over powered by the DC component, to bring out the fine details logarithmic function(image enhancement) is applied. This happens due to the frequency dependent exponential decay of the energy because of the multiple reflections until capturing moment. Logarithm transformation expands the range of low pixel values, compress the range of high pixel values and redistribute the pixel values on the low range.
(enhances contrast of the image in frequency domain)

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:
The fourier transform maintains the linearity of operations in the spatial space to the frequency domain. If we add or rescale a function either before or after the fourier transform, it gives the same result.
$F[a.f_1 + b.f_2] = a.F_1 + b.F_2$

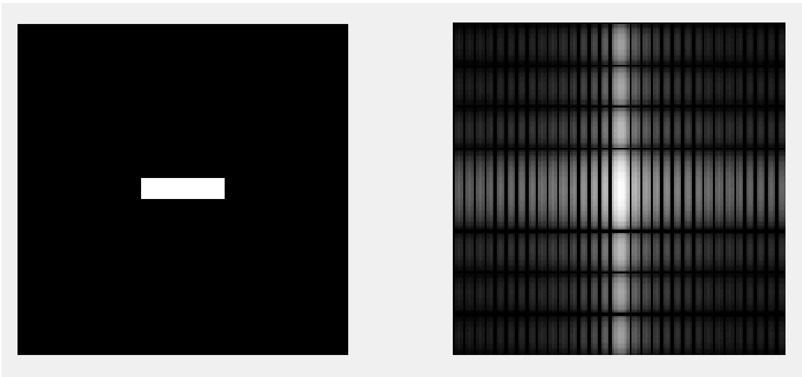
Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:
A multiplication in the fourier domain is same as the convolution in the spatial domain and a multiplication in the spatial domain is analogous to the convolution in frequency domain.
So, alternatively we can apply fourier transform seperately and then take their convolution.



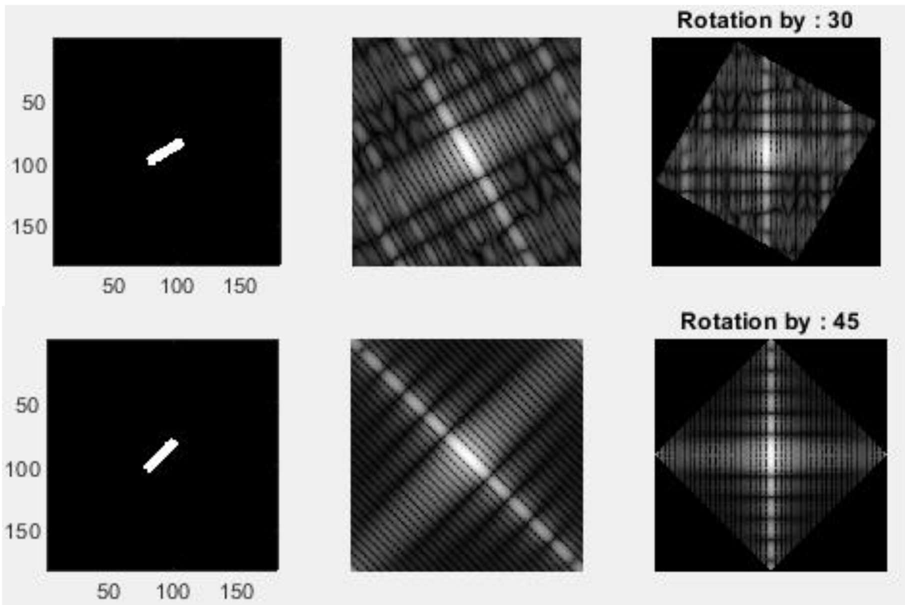
Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

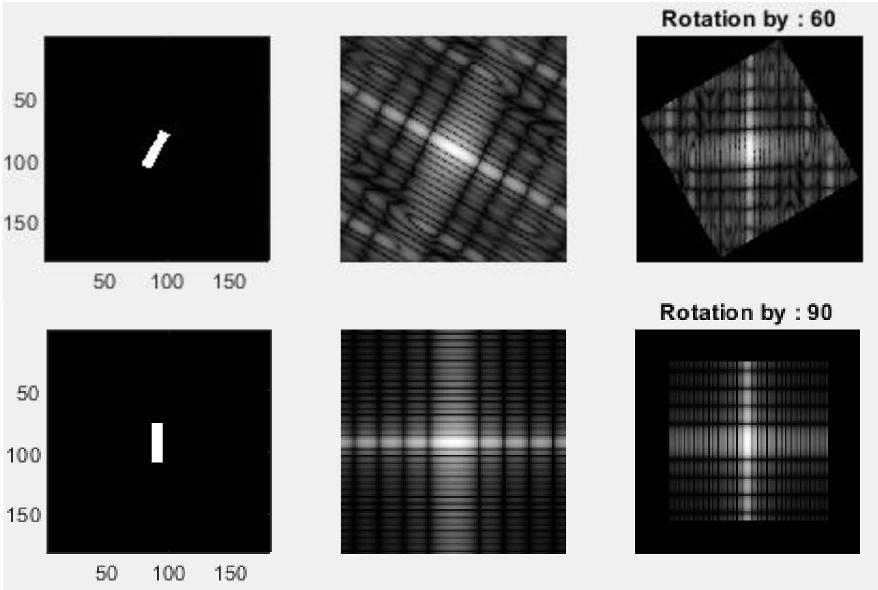
Answers:
Compression in spatial domain is same as expansion in fourier domain and vice versa.
In the previous exercise, we had a rectangular function which created a sinc response in frequency domain. Due to symmetry in spatial domain, zero crossings appeared symmetrical. But in this exercise, it is not symmetrical. This results in smaller distances in the zero crossings on frequency domain corresponding to the side with the biggest length. (lower frequency in x direction and higher frequency in y direction)



Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:
Rotation of images results in the rotation of the fourier spectrum by the same angle. Difference in orientation whereas shape is similar. Waves have different orientation but the frequency components are same. But because of rotation, original smootheness is lost. The degree of distortion depends on the angle of rotation. It can be clearly seen in images rotated with 30 and 60 degrees.

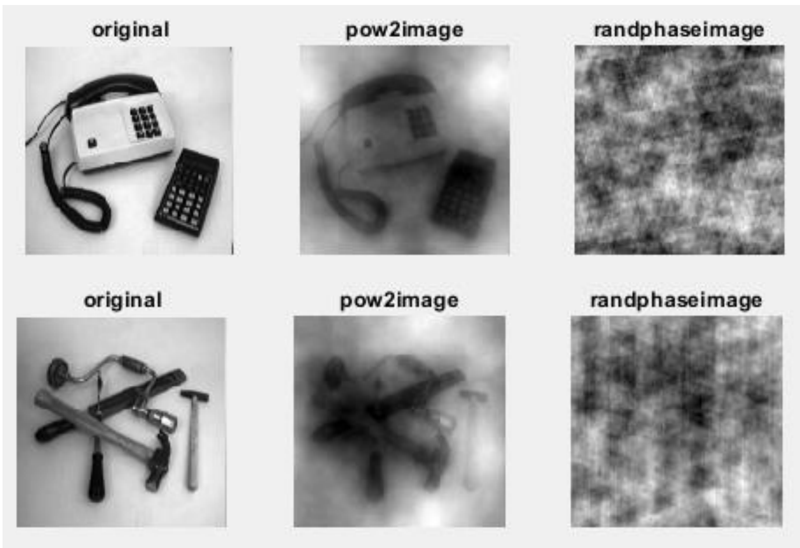




Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:
Phase unchanged; replace power spectrum – basic information is preserved. Seems like the image is smoothed.
Replace phase; power spectra unchanged – no inference can be done.
So, the phase contains main information about the image like the edges. But the spectrum visualization gives information about the direction of the biggest changes and size. But the spectra alone cannot be used to reconstruct the image.

Thus magnitude contains one-wave information, responsible for the brightness of the image and phase dictates how the delay on the superposition of them should be to reconstruct the image.



Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

t=0.1 $\begin{pmatrix} 0.0133 & 0.0 \\ 0.0 & 0.0133 \end{pmatrix}$

t=0.3 $\begin{pmatrix} 0.2811 & 0.0 \\ 0.0 & 0.2811 \end{pmatrix}$

t=1.0 $\begin{pmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{pmatrix}$

t=10.0 $\begin{pmatrix} 10.0 & 0.0 \\ 0.0 & 10.0 \end{pmatrix}$

t=100.0 $\begin{pmatrix} 100.0 & 0.0 \\ 0.0 & 100.0 \end{pmatrix}$

Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

In all cases it is = t$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

If there are even number of pixels, then gaussian distribution cannot be sampled exactly symmetrical. As the sample is taken in edges and not in centre it impacts estimated variance.

Pixel discretization(non continous spatial domain) introduces errors in frequency domain.
Bigger t(more spreading in spatial domain) – smaller errors
Value of t inversely proportional to spreading on frequency domain.

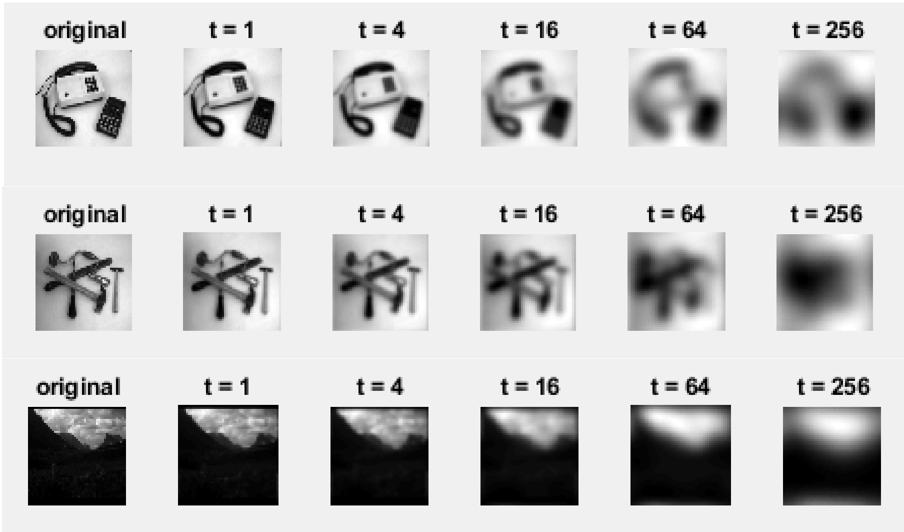
Only if the values of t are more than pi/4 they will be gaussians.

Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

Higher the t = higher the suppression of fine details and increase in blurness.
The images are smoothed.(scale space theory)

The higher the variance of t, the lower the cut-off frequency for gaussian function in frequency domain. The lower the cut off frequency, the more information is lost.



Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:
Gaussian filter : Computes a weighted average of neighborhood pixel values through a convolution mask that its coefficients gradually decrease close to zero at the edge of the boxcar. Spurious oscillations are minimized. Gaussian convolution is independent of image content. The influence that a pixel has on another depends only on their distance. Thus, image edges are blurred because pixels across discontinuities are averaged together. When the variance is large the image gets blur and integrates sap noise into image.

Median filter : It ranks all its neighbours and computes the outcome as its median. Used to remove shot noise and small regions of pixels infected by noise. Does not shift boundaries(shading), minimal degradation to edges. But when a rectangular neighbourhood is used, it creates painting like images by damaging the thin lines and corners. It is not good in removing gaussian noise. Non-linear filtering technique.

Ideal LPF : It passes without attenuation all frequencies within some radius from the origin and cuts off all the frequency outside the circle. It is useful only in the case of having all the information in the low frequency area with sudden drop of information around cutoff. It results in image blurring and also ringing(rippling near sharp edges). It performs the worst.

Gaussian Noise:

Gaussian filter – with t = 3 better results were obtained. Increase in t caused blurring effects and smoothing of sharp edges.
Median filter – with boxcar of 5 decent results were obtained. But image gets distorted.
Ideal lpf – with cut off of 0.2 cycles per pixel somewhat understandable image was obtained. Lowering the cutoff increases blurring effects.

Salt and pepper Noise:

Gaussian filter – with t =6 image was understandable. With lesser t the blurring was less but noise was more. With increase in t smoothing increased hugely.
Median filter – with boxcar = 3 better result was obtained. Although some noise is present, image was not distorted. With increase in boxcar blurring occurred which caused image distortion.
Ideal lpf – with cut off of 0.125 cycles per pixel an understandable image is obtained. Although it causes blurring and ringing effects.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

Gaussian noise : The gaussian filter works better in removing gaussian noise. The median lpf is not suitable as it generates painting like images. And ideal lpf gives blurring and ringing effects.

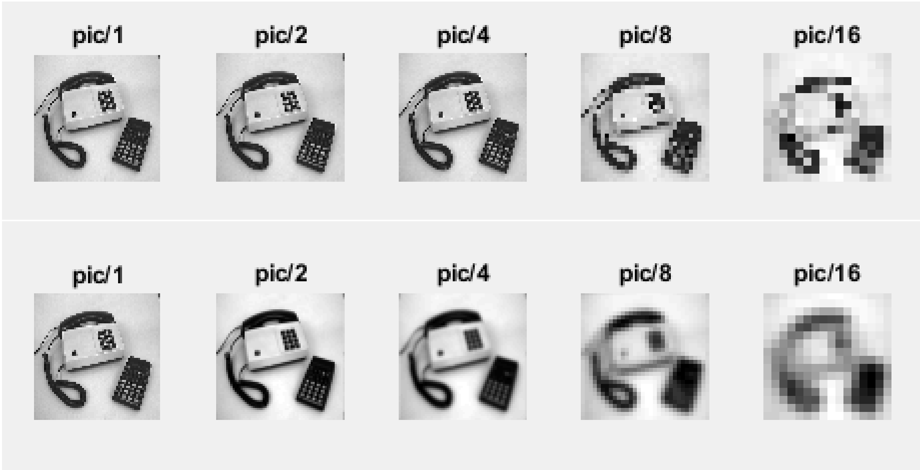
Salt and pepper noise: The median LPF works better in removing sapnoise due to its non-linearity and the less-dependency from the outliers as in the case of gaussian filter. Gaussian filter should not be used for sapnoise since the result will be just a spreading of the noise in the neighbor pixels. Ideal lpf is also not apt as it will cut completely the high frequencies causing blurring and ringing effect.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

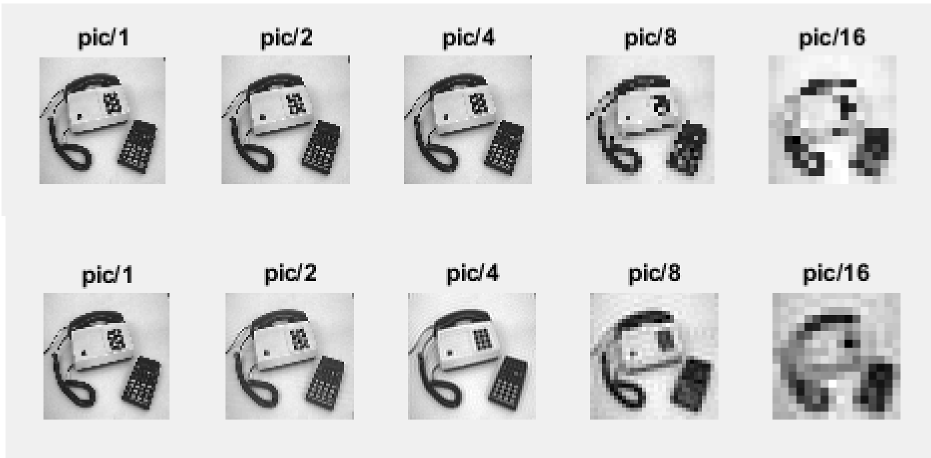
Answers:

When subsampling the images without smoothing, the edges and lines in the image appear distorted. Aliasing occurs where high frequency components of original signal masquerade as lower frequencies in sampled function. When the image is smoothed before subsampling with filters aliasing is suppressed but blurring occurs and fine details are lost. For gaussian filter t = 2 is used and for ideal lpf a cut off frequency of 0.25 is used to obtained the results below.

Gaussian filtering:



Ideal LPF:



Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
Shannon rule – a band limited function can be completely recovered if the sampling rate exceeds nyquist rate(twice the highest frequency). Aliasing occurs if the sampling rate is less than the nyquist rate.
In our case, when we use smoothing(lpf) we suppress higher frequency components and lower the nyquist rate reducing the aliasing effect. In the ideal lpf we completely eliminate the higher frequencies.
