Moving Bodies and Kinematics

Constant Acceleration Formulae

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$
- $s = \frac{1}{2}(u + v)\,t$

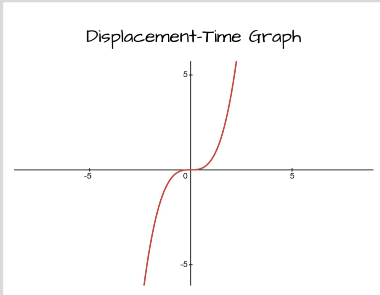
Using Calculus for Kinematics

Differentiate →

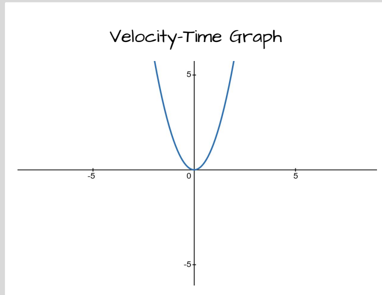
displacement | velocity | acceleration

← Integrate

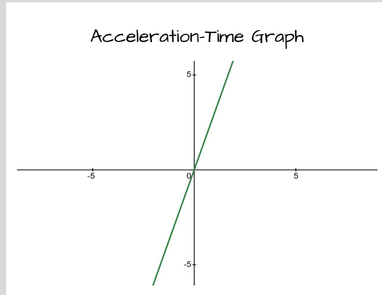
Motion Graphs



Gradient: Velocity
Area Under Curve: No Meaning



Gradient: Acceleration
Area Under Curve: Displacement



Gradient: Jolt (Not needed)
Area Under Curve: Average Velocity

The rate of change of displacement, velocity, and acceleration can be determined by their respective equations. Use the first derivative to find the time when these rates are zero-and their nature (min or max) by the second derivative.

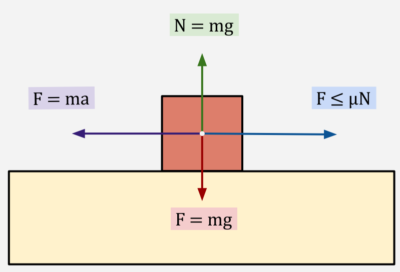
Newton's Laws of Motions

N(1): The velocity of an object will remain constant when no resultant force acts upon it.

N(2): The force acted upon an object of mass is directly proportional to the acceleration: $F = ma = mv / t$

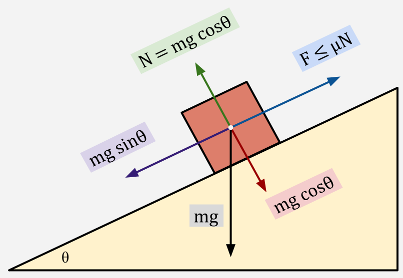
N(3): For every force acting on A by B, an equal and opposite reaction force of the same type by B on A

General Solutions for Bodies under Forces



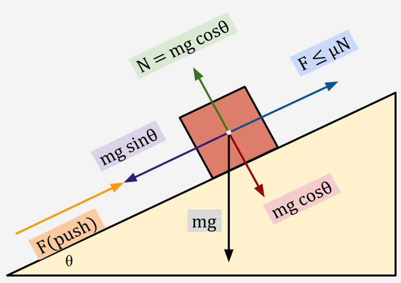
For a rough surface, μN is the maximum friction that occur. Friction will always oppose the (otherwise) resultant force.

- Static Friction is for $F \leq \mu N$
- Limiting Friction is for $F > \mu N$



The same principles applies here. The only difference is that the forces must be resolved

For a system of this nature, there is a range of force that can be applied but still maintain equilibrium.



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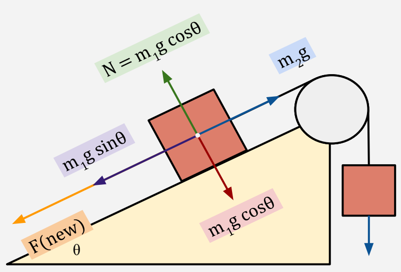
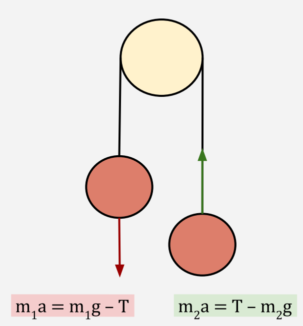


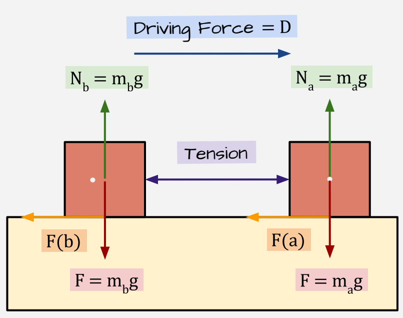
- The same principles applies here.
- The only difference is that the forces must be resolved
- For a system of this nature, there is a range of force that can be applied but still maintain equilibrium.



To calculate the Tension using

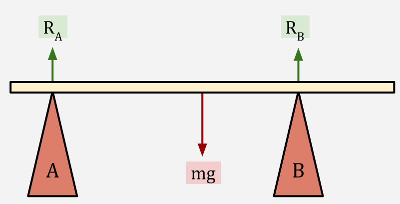
Let $m_1a = m_1g - T \rightarrow (1)$
Let $m_2a = T - m_2g \rightarrow (2)$
$(1) + (2) = a\,(m_1+m_2) = g\,(m_1-m_2)$

Use the calculated a to substitute into (1) or (2) to calculate T



This system can be modelled by the following set of equations

($\leftrightarrow$): $D - F(a) - F(b) = (m_a + m_b)\,a$
($\updownarrow$): $g\,(m_a + m_b) = N_a + N_b$
$T - F(b) = m_b\,a$
$D - T - F(b) = m_a\,a$



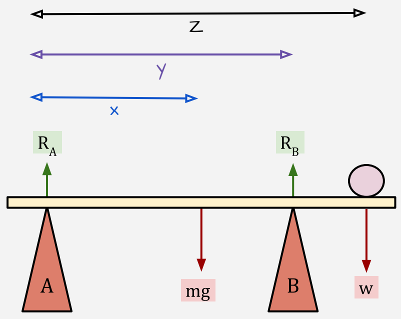
Choose one pivot to take moments about. In a uniform rod, centre of mass is at the middle of the rod

Relationship of the Forces

$R_A + R_B + \ldots = mg + \ldots$

Moments(A):

$mgx = yR_B$



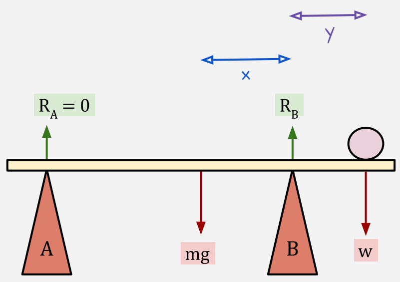
The same principles act here but with the addition of new forces.

Relationship of the Forces

$R_A + R_B + \ldots = mg + w + \ldots$

Moments(A):

$mgx + wz = yR_B$



When a rod is 'at point of tilting' about B, for example, all other reactions forces are nulled.

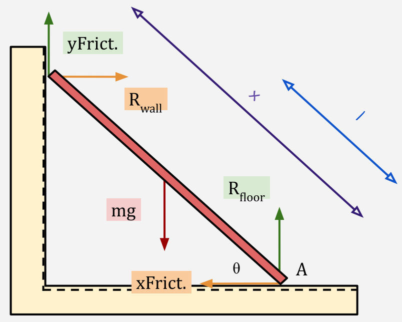
Relationship of the Forces

$R_A = 0N$

$R_B = mg + w$

Moments(B):

$mgx = yw$



A ladder leaning onto a rough wall can be modelled as below

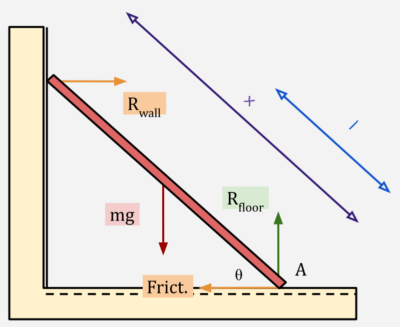
Relationship of the Forces

$R_{wall} = xFrict.$

$R_{floor} + yFrict = mg$

Moments(A):

$l\,mg\cos\theta = x\,R_{floor}\sin\theta + x\,yFric\cos\theta$



A ladder leaning onto a smooth wall can be modelled as below

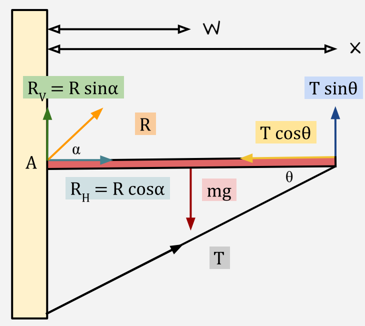
Relationship of the Forces

$R_{wall} = Frict.$

$R_{floor} = mg + \ldots$

Moments(A):

$l\,mg\cos\theta = x\,R_{floor}\sin\theta$



For a rod thrust up by another rod, the following system applies

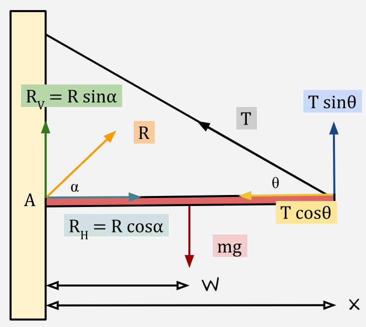
Relationship of the Forces

$R_V + T\sin\theta = mg$

$R_H = T\cos\theta$

Moments(A):

$w\,mg = x\,T\sin\theta$



For a rod hung by a string, the following system is at play

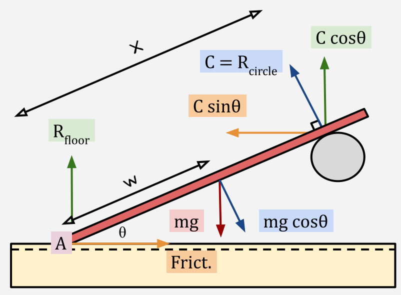
Relationship of the Forces

$R_V + T\sin\theta = mg$

$R_H = T\cos\theta$

Moments(A):

$w\,mg = x\,T\sin\theta$



For a rod on a smooth cylinder, the following system is at play

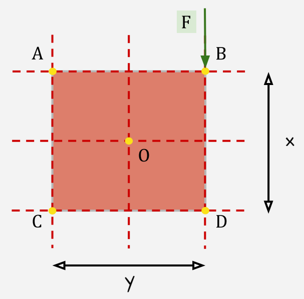
Relationship of the Forces

$C\cos\theta + R_{floor} = mg$

$C\sin\theta = Frict.$

Moments(A):

$w\,mg\cos\theta = x\,C$



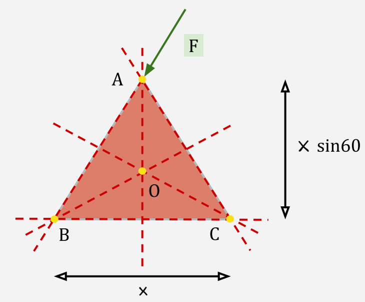
For a rectangular lamina system, the force F induces these moments about each point shown below:

Mom(A) = yF , Mom(C) = yF

Mom(B) = 0 , Mom(D) = 0

Mom(O) = ½yF

* if the line of action of the force goes through a pivot, there is no moment about that pivot



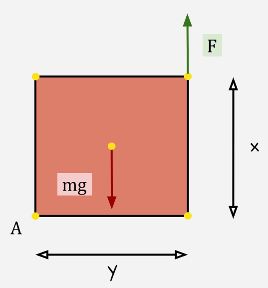
For a triangular lamina system, the force F induces these moments about each point shown below:

Mom(A) = 0

Mom(B) = ½x F cos60

Mom(C) = x sin60 F

Mom(O) = ½x sin60 F

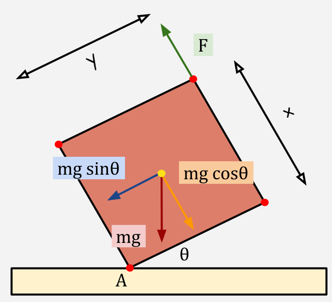


For a rectangular lamina with mass held in equilibrium the following system applies

Relationship of the Forces

$F + \ldots = mg + \ldots$

Moments(A): $\frac{1}{2}y\,mg = yF$



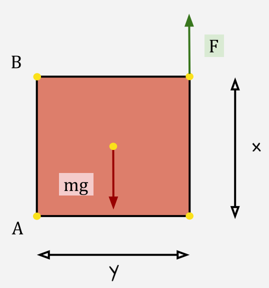
For a rectangular lamina with mass held in equilibrium at an angle the following system applies

Relationship of the Forces

$F + \ldots = mg\cos\theta + \ldots$

Moments(A):

$\frac{1}{2}y\,mg\cos\theta = xF + \frac{1}{2}x\,mg\sin\theta$



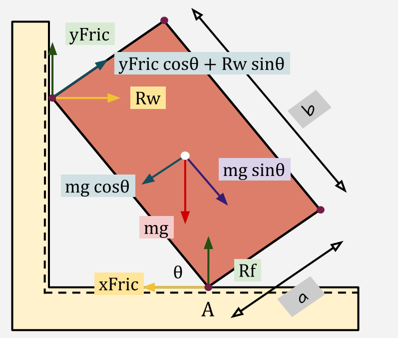
For an object with a resultant moment of a about A,

$a = \sum mom_a(\square) - \sum mom_a(\square)$

Where,

$\sum mom_a(\square) = p_1 + \ldots + mg\bar{y}$

hence, the centre of mass lies along the line $\bar{y}$



A lamina leaning onto a wall can be modelled as below

Relationship of the Forces

$R_{wall} = xFrict$ | $R_{floor} + yFrict = mg$

Moments(A | ↷):

$\frac{1}{2}a\,mg\sin\theta + b\,(yFric\cos\theta + Rw\sin\theta)$

Moments(A | ↶): $\frac{1}{2}b\,mg\cos\theta$