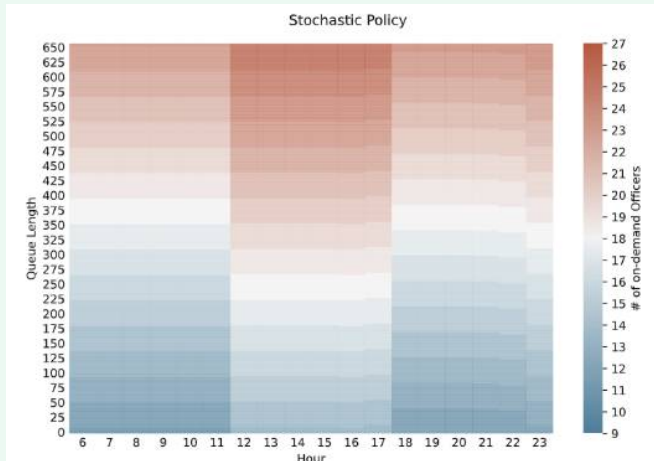


Customs Staffing Policy Interpretation



You are an expert in optimization with a PhD in Operations Research from MIT.

After implementing a Backward Induction model to find the optimal staffing policy for an airport customs booth assignment during the day given the variability of passenger arrivals, you get the following heatmap:

- x axis the time of the day
- y axis the queue length
- The color variability represents the number of on-demand officers.

Interpret the following heatmap:



The heatmap represents the **optimal stochastic policy** for staffing on-demand officers in an airport customs scenario based on queue length and time of day. Here's an interpretation:

1. X-Axis: Time of Day

- The horizontal axis spans from 6 AM to 11 PM (6 to 23).
- Key transitions occur during typical peak hours (e.g., late morning and early evening), likely corresponding to flight arrival patterns.

2. Y-Axis: Queue Length

- The vertical axis shows queue lengths from 0 to 650 passengers, divided into intervals.
- This represents the potential number of passengers waiting at customs at any given time.

3. Color Scale: Number of On-Demand Officers

- The color gradient corresponds to the number of on-demand officers allocated:
 - **Blue shades:** Fewer officers (e.g., 9 officers).
 - **White-neutral:** Medium levels of staffing.

- **Red shades:** Higher levels of staffing (e.g., up to 27 officers).

Key Insights:

1. Peak Staffing Hours:

- Red zones are visible in late morning (10-13) and early evening (16-19), indicating periods of **high passenger volume or arrival surges** that require more officers to handle the queues effectively.

2. Off-Peak Periods:

- Blue zones dominate early morning (6-9), late afternoon (14-15), and late evening (20-23). These correspond to quieter periods when fewer officers are sufficient.

3. Queue Length Patterns:

- As queue length increases, the number of officers required also increases, as seen by the upward red gradient.
- **At very high queue lengths** (e.g., >550), the model suggests maximum staffing across most time intervals.

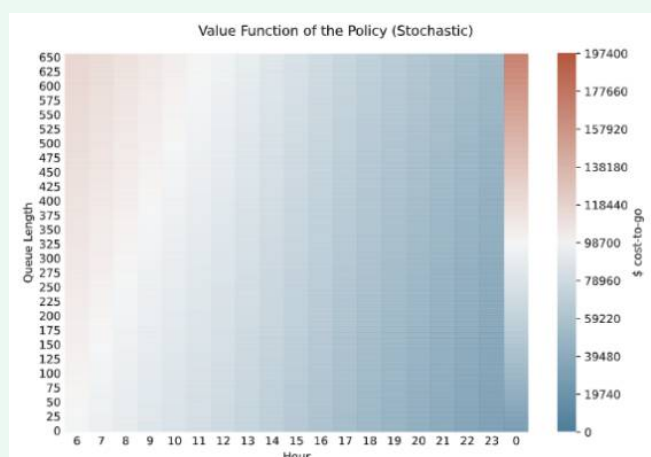
4. Sensitivity to Time and Queue:

- The staffing levels are **dynamic** and **context-dependent**, balancing between:
 - Anticipated passenger flow patterns by time of day.
 - Queue build-up thresholds requiring intervention.

Operational Takeaway:

- This policy allows for **adaptive resource allocation**, ensuring efficient use of staff:
 - During peak demand (e.g., flight arrivals in red periods), officers are added to minimize waiting time.
 - During low-demand hours, fewer officers are allocated, saving costs while maintaining service levels.

If you want, I can help refine this analysis or assist with further visualizations!



Now help me understand the respective value function of that policy. Now the only difference is that the color scale represents the cost-to-go function. This means the total cost (from the given time in the heatmap to the end of the day) of being with a given queue length at a specific hour, given that

the system will behave optimally in all subsequent periods.



The heatmap represents the **value function** (cost-to-go function) of the stochastic staffing policy. The color scale indicates the total expected cost from a specific hour and queue length to the end of the day, assuming the system operates optimally thereafter.

Key Elements:

1. X-Axis: Time of Day

- The horizontal axis spans from 6 AM to midnight (6 to 0).
- Costs decrease as the day progresses, reflecting fewer remaining hours to incur costs.

2. Y-Axis: Queue Length

- The vertical axis ranges from 0 to 650 passengers.
- Larger queues correspond to higher costs-to-go due to the greater effort required to reduce them.

3. Color Scale: Cost-to-Go

- **Blue shades (lower costs):** Found in later hours of the day (20:00 and beyond), as the time remaining to clear queues is limited.
- **Red shades (higher costs):** Found earlier in the day (morning and midday), particularly for high queue lengths, reflecting the compounding costs of managing long queues over a longer horizon.

Insights:

1. Cost-to-Go Evolution Over Time:

- Costs are higher earlier in the day because the policy anticipates the cumulative impact of long queues over the remaining hours.
- Costs taper off in the evening (blue areas) as the system approaches the end of the operational day, leaving less time for queues to accumulate further.

2. Impact of Queue Length:

- At **low queue lengths**, costs are uniformly low, as minimal effort is needed to maintain service levels.
- At **high queue lengths**, the cost is significant (red zones), especially earlier in the day, as reducing large queues over time requires substantial resources.

3. Transition Around Midday (10-14):

- The gradient in this region indicates a balancing point between ongoing costs and the need to adjust for anticipated surges or peaks.

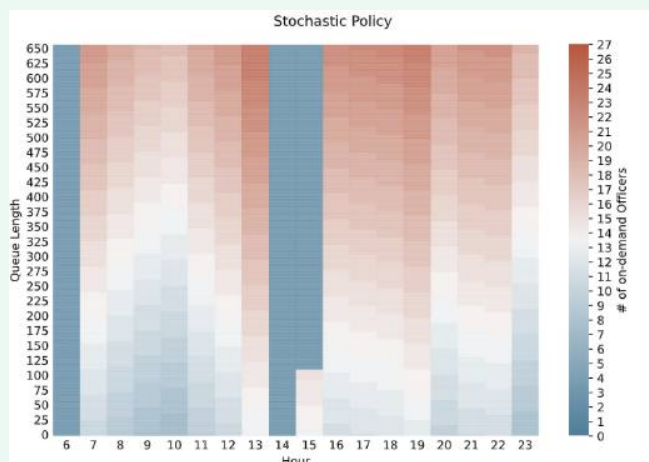
4. Policy Efficiency:

- The model demonstrates how costs are not only time-dependent but also queue-dependent, ensuring that resources are optimally allocated to minimize cumulative costs.

Operational Takeaway:

- The value function ensures proactive staffing early in the day to minimize the **long-term costs** associated with large queues.
- Toward the end of the day, the model becomes less aggressive in penalizing large queues since there's limited time for them to propagate further.
- This aligns with real-world operational strategies, where efforts are front-loaded to avoid compounding inefficiencies.

Would you like to explore further, such as sensitivity analyses or the impact of parameter changes?



Now help me analyze a different policy. The main difference is now is that the arrival rate of passengers has more variability during the day: To be specific is Arrival Distribution hourly: [154, 92, 9, 9, 229, 505, 692, 411, 322, 263, 231, 347, 417, 577, 716, 672, 460, 477, 489, 537, 378, 435, 448, 246]
The optimal policy for that arrival rate is as follow:



The new heatmap represents the **optimal stochastic policy** given a more variable arrival rate of passengers throughout the day. Here's the detailed analysis considering the updated hourly arrival distribution:

Observations on Arrival Variability:

1. Morning (6:00-11:00):

- Arrival rates start low but rise significantly around 9:00 AM (229 → 505 → 692 passengers/hour).
- The heatmap reflects this increase, with a gradual shift from **blue (low staffing)** to **red (high staffing)** as the queue length increases during this period.

2. Midday Peak (12:00-15:00):

- A peak occurs around 14:00 (577 passengers/hour) before stabilizing slightly.
- Staffing is highly dynamic, with certain queue lengths (e.g., 300-450 passengers) requiring fewer officers (evident from the blue band near 14:00). This could indicate an efficient clearing of queues before the peak demand period.

3. Evening (16:00-23:00):

- Arrival rates remain relatively high but less erratic (e.g., 672 → 460 → 489 passengers/hour).
- The heatmap reflects consistently **high staffing needs** (red zones) at larger queue lengths, ensuring passenger processing keeps up with demand.

Comparison with the Previous Policy:

1. Dynamic Adjustments:

- This policy exhibits **greater variability** in staffing decisions, reflecting the uneven arrival distribution.
- Sharp transitions between blue and red bands suggest the system anticipates and reacts to sudden changes in arrival patterns.

2. Efficiency in Staffing:

- Notably, the policy reduces the need for excess staffing at certain queue lengths during peak hours, likely due to the stochastic optimization considering system behavior across time.

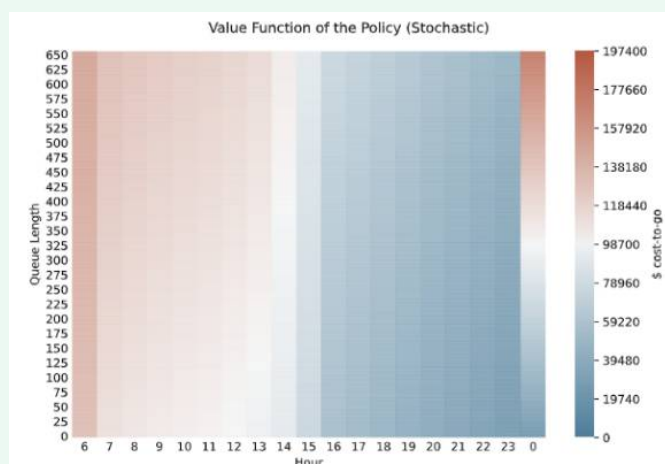
3. Proactive Resource Allocation:

- Staffing ramps up earlier in the day (e.g., 9:00 AM) and remains high during the extended peak period (12:00-18:00) compared to the previous policy.

Operational Takeaways:

- **Sensitivity to Variability:** This policy handles the more variable arrival rates effectively, dynamically adjusting staffing levels to minimize queues and costs.
- **Peak Hours Management:** High staffing levels during peak periods (e.g., 10:00-14:00 and 16:00-20:00) ensure service levels remain high despite fluctuating demand.
- **Off-Peak Efficiency:** During early morning and late evening, staffing remains minimal unless large queues are present, reducing unnecessary costs.

Would you like to compare this policy's **value function** with the previous one to analyze cost implications?



Now explain the corresponding cost to go heatmap to the previous policy



This heatmap shows the **value function (cost-to-go function)** corresponding to the revised stochastic policy. The colors represent the total expected cost from any given time and queue length to the end of the day, assuming optimal decisions are made in all subsequent periods.

Observations:

1. X-Axis (Hour of the Day):

- Early hours (6:00–14:00): **Higher costs (red zones)**.
 - These reflect the impact of high variability and surges in passenger arrivals during this period.
 - Long queues early in the day have a larger impact because there's enough time remaining for them to grow further, compounding costs.
- Later hours (15:00–23:00): **Lower costs (blue zones)**.
 - Costs taper off as the day progresses, as fewer hours remain to process passengers. Even with larger queues, there's limited time for their impact to accumulate.

2. Y-Axis (Queue Length):

- **Low Queue Lengths:**
 - Costs are uniformly low across all hours, as minimal staffing adjustments are needed, and the system remains under control.
- **High Queue Lengths:**
 - Costs increase significantly for larger queues earlier in the day (steep red gradient at the top-left).
 - Long queues late in the day (right side) still result in moderate costs, though less than in earlier hours, as there's limited time for corrective actions.

3. Dynamic Variability in Cost-to-Go:

- Compared to the previous value function, there's more distinct gradation in cost levels due to the highly variable arrival rates.
- Around **14:00**, there's a visible shift in costs (transition from red to blue). This reflects the policy's efficient handling of the peak period and a reduction in the propagation of high costs into later hours.

Insights:

1. Sensitivity to Arrival Variability:

- The policy adjusts optimally to the higher variability in passenger arrivals. While the value function shows high costs in the early hours, this reflects the realistic challenge of managing sudden surges during peak hours.

2. Queue Management Efficiency:

- The sharp drop in costs after midday (14:00–15:00) highlights the model's ability to clear queues effectively during the busiest hours.
- Late in the day, the system becomes less sensitive to large queues, as there's insufficient time for costs to escalate significantly.

3. Trade-offs Between Time and Queue Length:

- Early in the day, addressing long queues has a significant impact on reducing future costs.

- As the day progresses, even large queues contribute less to the cost-to-go because of the diminishing time horizon.
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Comparison to the Previous Policy:

- The **early hours (6:00–14:00)** have slightly **higher costs** here, as the new arrival variability introduces more uncertainty, making the system less predictable.
 - The **afternoon and evening hours** have lower costs, suggesting the revised policy more effectively clears queues by the end of the day.
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Operational Takeaway:

This value function confirms the effectiveness of the revised policy in balancing short-term actions with long-term cost minimization. It demonstrates that the system can adapt to more uncertain and variable arrival rates without losing efficiency. Would you like to explore deeper metrics, such as queue distributions or resource utilization?