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Math 430

Part I:

The differential equation

$$\frac{\partial u}{\partial t}(x, t) = \frac{\partial^2 u}{\partial x^2}(x, t) \quad 0 \le x \le \pi, \quad t \ge 0,$$

subject to the boundary conditions

$$\frac{\partial u}{\partial x}(0, t) = 0, \qquad \frac{\partial u}{\partial x}(\pi, t) = 0, \; t \ge 0,$$

and the initial condition

$$u(x, 0) = f(x), \qquad 0 \le x \le \pi$$

The funtion $f : [0, \pi]$ is differentiable $s.t \; f'(\pi) = 0$

Part II:

Express the solution of the diff eq subject to b.c and i.c as an infinite series

$$\frac{\partial u}{\partial t}(\mathrm{x}, \mathrm{t}) = \frac{\partial^2 u}{\partial x^2}(\mathrm{x},\mathrm{t})$$

$\mathrm{u(x, t) = A(x)\, B(t)}$

$\frac{d^2 A}{\mathrm{dx}^2} = -\lambda \mathrm{A} \rightarrow \mathrm{A}$ "(x) $= -\lambda \mathrm{A(x)}$, $A''(\mathrm{x}) = -\lambda \mathrm{A(x)}$, $A'(0) = 0$, $A'(\pi) = 0$

Assume case 3 : $\lambda > 0, \mu^2 = \lambda, \mu > 0$:

$\mathrm{A} = C_1\, \mu \cos(\mu \mathrm{x}) + C_2\, \mu \sin(\mu \mathrm{x})$

$A' = -C_1\, \mu \sin(\mu \mathrm{x}) + C_2\, \mu \cos(\mu \mathrm{x})$

$A'(0) = C_2\, \mu(1) = 0$

$A'(\pi) = -C_1 \cos(\mu \mathrm{x}), C_1 \in \mathbb{R} \backslash \{0\}$

$B'(\mathrm{e}) = -\lambda \mathrm{B(t)}$

$\mathrm{B(t)} = C_0\, e^{-\lambda \mathrm{t}} + C_1$

$F \sim A_0 + \sum_{\mathrm{k}=1}^{\infty} \mathrm{A_k} \cos(\mathrm{kx})\, e^{-k^2 t}$

$\mu(\mathrm{x, t}) = \cos(\mu \mathrm{x})\, e^{-\mu^2 t}$

$\mu(\mathrm{x, t}) = \mathrm{A(x)}, \mathrm{B(t)} = \cos(\mu \mathrm{x})\, e^{-\mu^2 t}$

Part III:

Write the solution of the diffusion of the dye problem if the solution of the dye is given by $f(x) = (\sin(x))^2$

The displayed formulas are an expansion of the formula provided in part II:

Solution of the diffusion of dye :

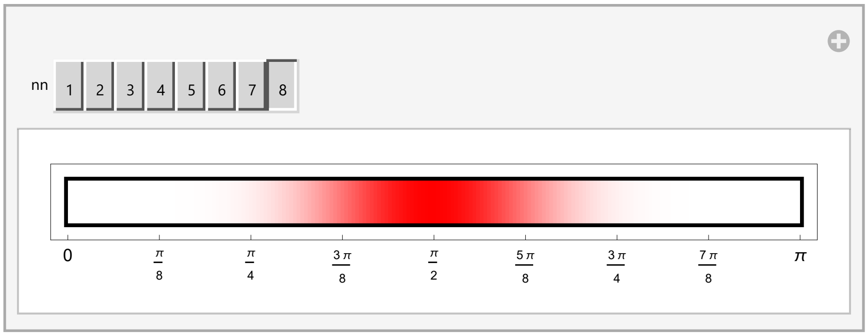
$$f(x) = (\sin(x))^8 = \frac{1}{2^8} + \frac{1}{2^7} + \sum_{k=1}^{4} (-1)^k \operatorname{binom}(8, 8-k) \cos(2\,kx)$$

$$\text{“ ”} = \frac{70}{256} + \frac{-1}{16}\cos(2\,x) + \frac{7}{32}\cos(4\,x) + \frac{-7}{16}\cos(6\,x) + \frac{-7}{16}\cos(6\,x) + \frac{35}{64}\cos(8\,x)$$

Part IV:

Illustration of the diffusion of dye when the initial distribution of dye is $f(x) = (\sin(x))^8$

```
st = Pi/200; cc = 0.002; hh = 0.2; LL = Pi;
Manipulate[Graphics[{
   {RGBColor[1, 1 - (Sin[#])^nn, 1 - (Sin[#])^nn],
      Polygon[{{# - st/2 - cc, 0}, {# + st/2 + cc, 0}, {# + st/2 + cc, hh},
        {# - st/2 - cc, hh}, {# - st/2 - cc, 0}}]} & /@
   Range[0 + st/2, LL - st/2, st], {Thickness[0.005],
   Line[{{0 - st/2, 0}, {LL + st/2, 0}, {LL + st/2, hh},
      {0 - st/2, hh}, {0 - st/2, 0}}]}}, Frame -> True,
  FrameTicks -> {{None, None}, {Range[0, Pi, Pi/8], None}},
  ImageSize -> 400],
 {nn, Range[1, 8], Setter,
  ControlPlacement -> Top}]
```




```
Manipulate[Plot[(Sin[x])^nn, {x, 0, Pi},
  PlotRange → {0, 1}, AspectRatio → Automatic, ImageSize → 400],
 {nn, Range[1, 8], Setter}, ControlPlacement → Top]
```