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Ref: Date:

Question 1

Q1 (a) Using the first 4 data points,

i	x_i	1st	2nd	3rd	4th
0	1.6	2	15	-3.3333	-3.4226
1	2	8	12	-8.8095	
2	2.5	14	1.4286		
3	3.2	15			

taking $i=0$,

$$b_1 = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$$= \frac{8-2}{2-1.6} = 15$$

$$b_2 = \frac{f[x_2,x_1] - f[x_1,x_0]}{x_2 - x_0}$$

$$= \frac{12-15}{2.5-1.6} = -3.333$$

$$b_3 = \frac{f[x_3,x_2,x_1] - f[x_2,x_1,x_0]}{x_3 - x_0}$$

$$= \frac{-8.8\overset{095}{\cancel{10}} - \cancel{3}(-3.3333)}{3.2-1.6} = -3.4\overset{22}{\cancel{23}}6$$

* $b_0 = 2$ (given in question — 1st y data point)

General Form :

$$y(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$y(3.3) = 2 + 15(3.3-1.6) + (-3.33)(3.3-1.6)(3.3-2) + (-3.423)(3.3-1.6)(3.3-2)(3.3-2.5)$$

$= \cancel{14.09}$ 14.0888 $\longrightarrow$ $\cancel{14.09}$

Q1(b) Choosing the following points shown in columns 2 and 3,

i	x_i	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
0	2	8	12	-8.8095	1.0119	0.4524
1	2.5	14	1.4286	-6.7857	2.1429	
2	3.2	15	-8.75	-2.5		
3	4	8	-12			
4	4.5	2				

For $i = 0$,

$b_0 = 8$

$b_1 = \frac{14-8}{2.5-2} = 12$

$b_2 = \frac{1.42856 \; \cancel{1.429} - 12}{3.2-2} = \cancel{-8.810} \; -8.8095$

$b_3 = \frac{-6.7857 - (-8.8095) \;\; \cancel{-6.786 - (-8.810)}}{4-2} = \cancel{1.012} \; 1.0119$

$b_4 = \frac{2.1429 - 1.0119 \;\; \cancel{2.143 - 1.012}}{4.5-2} = 0.4524$

* the points chosen was such that the value $x=3.33$ was around the centre of the range.

Substituting b_0, b_1, b_2, b_3 ~~and b_4~~ into the general form shown in part (a)

$\cancel{y(x) = 8 + 12(x-2) - 8.81(x-2)(x-2.5) + 1.01(x-2)(x-2.5)(x-3.2)}$

$\cancel{+ 0.45(x-2)(x-2.5)(x-3.2)(x-4)}$

* refer to last page for cancelled working

$y(3.3) = \cancel{14.5146} \; 14.5433$

Q1(c) Yes, there is a difference of $(14.51 - 14.09) = 0.42$ or $\frac{14.51 - 14.09}{14.51} \times 100 = 2.89\%$.

The x-value $= 3.3$ desired to be found is not in the range of values selected for part (a). ~~whereas~~ Therefore, what is actually being done is extrapolation which is known to be less accurate.

On the other hand, the range of data points selected was such that the x-value of 3.3 is situated within the range. Hence, the method performed is interpolating the y-value at the x-value desired (3.33). This yields a much more accurate answer.

Q1(d) Using the lagrange polynomial formula and expanding,

$$y(x) = \sum_{i=0}^{n=3} y_i(x) L_i(x) = y_0 L_0 + y_1 L_1 + y_2 L_2 + y_3 L_3$$

$$y(x) = y_0 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+ y_2 \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

~~$y(3.3)$ $x_0 = 1.6$, $x_1 = 2.0$, $x_2 = 2.5$, $x_3 = 3.2$~~

from part b,

$x_0 = 2$, $x_1 = 2.5$, $x_2 = 3.2$, $x_3 = 4$

$y_0 = 8$, $y_1 = \cancel{12}\ 14$, $y_2 = 15$, $y_3 = 8$

Sub → $y(x)$ and $x = 3.3$,

$$y(3.3) = \frac{28}{75} + \left(-\frac{182}{75}\right) + \left(\frac{65}{4}\right) + \left(\frac{26}{75}\right)$$

$$= 14.5433$$

* continuation on last page

Q1(e) More points mean higher order polynomial. A higher order polynomial interpolation does not mean it is more accurate. More often than not, a high order polynomial leads to 'wiggles' – it oscillates wildly. It can also result in large errors.

Question 1(b)

$y(x) = 8 + 12(x-2) - 8.8095(x-2)(x-2.5) + 1.0119(x-2)(x-2.5)(x-3.2)$

~~$+ 0.4524(x-2)(x-2.5)(x-3.2)(x-4)$~~

$y(3.3) = 14.5433$

* The table was rearranged and recalculated as a check

x_i	1st	2nd	3rd	4th	5th
3.2	15	-8.75	-7.2917	1.0119	0.4524
4	...	...	...		
2	...	...	...		
2.5	...	...			
4.5	...				

The same answer was obtained.

Question 1(d)

Both methods yield the exact same result which is $y(3.3) = 14.5433$.
This is found to be true when both methods are done to the same ~~polyno~~ polynomial order. In this case, both were done to the third polynomial