Using Taylor Series to expand $f(x,y,z)$, $g(x,y,z)$ & $h(x,y,z)$, we get in 3D :

$$f(x_{i+1}, y_{i+1}, z_{i+1}) = f(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial f_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial f_i}{\partial y}$$
$$+ (z_{i+1} - z_i)\frac{\partial f_i}{\partial z}$$

$$g(x_{i+1}, y_{i+1}, z_{i+1}) = g(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial g_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial g_i}{\partial y}$$
$$+ (z_{i+1} - z_i)\frac{\partial g_i}{\partial z}$$

$$h(x_{i+1}, y_{i+1}, z_{i+1}) = h(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial \overset{h_i}{\cancel{g_i}}}{\partial x} + (y_{i+1} - y_i)\frac{\partial \overset{h_i}{\cancel{g_i}}}{\partial y}$$
$$+ (z_{i+1} - z_i)\frac{\partial h_i}{\partial z}$$

To find (x,y,z) where the three functions intersect,

$$f(x_{i+1}, y_{i+1}, z_{i+1}) = g(x_{i+1}, y_{i+1}, z_{i+1}) = h(x_{i+1}, y_{i+1}, z_{i+1}) = 0$$

$$f(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial f_i}{\partial x} + (y_{i+1} - y)\frac{\partial f_i}{\partial y} + (z_{i+1} - z)\frac{\partial f_i}{\partial z} = 0$$

$$g(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial g_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial g_i}{\partial y} + (z_{i+1} - z)\frac{\partial g_i}{\partial z} = 0$$

$$h(x_i, y_i, z_i) + (x_{i+1} - x_i)\frac{\partial \overset{h_i}{\cancel{g_i}}}{\partial x} + (y_{i+1} - y_i)\frac{\partial \overset{h_i}{\cancel{g_i}}}{\partial y} + (z_{i+1} - z_i)\frac{\partial h_i}{\partial z} = 0$$

Rearranging and putting the above 3 equations into matrix form :

$$\begin{bmatrix} \frac{\partial f_i}{\partial x} & \frac{\partial f_i}{\partial y} & \frac{\partial f_i}{\partial z} \\ \frac{\partial g_i}{\partial x} & \frac{\partial g_i}{\partial y} & \frac{\partial g_i}{\partial z} \\ \frac{\partial h_i}{\partial x} & \frac{\partial h_i}{\partial y} & \frac{\partial h_i}{\partial z} \end{bmatrix} \begin{bmatrix} x_{i+1} \\ y_{i+1} \\ z_{i+1} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_i}{\partial x} & \frac{\partial f_i}{\partial y} & \frac{\partial f_i}{\partial z} \\ \frac{\partial g_i}{\partial x} & \frac{\partial g_i}{\partial y} & \frac{\partial g_i}{\partial z} \\ \frac{\partial h_i}{\partial x} & \frac{\partial h_i}{\partial y} & \frac{\partial \overset{h_i}{\cancel{g_i}}}{\partial z} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} - \begin{bmatrix} f_i \\ g_i \\ h_i \end{bmatrix}$$

* where f_i, g_i, and h_i are functions in terms of (x,y,z)

From 1^{st} equation and rearranging,

$$f = y^2 + z^2 - x$$

$$\frac{\partial f}{\partial x} = -1$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial f}{\partial z} = 2z$$

From 2^{nd} equation and rearranging,

$$g = x^2 + z^2 - y$$

$$\frac{\partial g}{\partial x} = 2x$$

$$\cancel{\frac{\partial h}{\partial x}} \quad \frac{\partial g}{\partial y} = -1$$

$$\frac{\partial g}{\partial z} = 2z$$

From 3^{rd} equation and rearranging,

$$h = x^2 + y^2 - z$$

$$\frac{\partial h}{\partial x} = 2x$$

$$\frac{\partial h}{\partial y} = 2y$$

$$\cancel{\partial z} \; \frac{\partial h}{\partial z} = -1$$