Ref: Date: 15/04/2020

Lab 3 Submission

Question 1

$W(y) = \frac{G}{2\mu} y(y-b)$ is the velocity in z-direction

Integrating $W(y)$ in y direction gives,

$$Q = -\frac{1}{12}\frac{Gb^3}{\mu}$$

Analytical solution ~~for~~ Q at $b = 1$ and $G/\mu = -1$,

$$\int W(y) = Q$$

$$= -\frac{1}{12}(-1)(1)^3 = \frac{1}{12} = 0.08333333$$

(a) for 1 interval : $W(0) = 0$

since $b = 1$, $W(1) = 0$ $\Big\}$ $h = \frac{1-0}{1} = 1$

$I_1 = 0.5\,(0+1)\,(W(0) + W(b))$

$I_1 = 0.5\,(1)\,(0) = 0$

for 2 intervals : $W(0) = 0$

$W(1) = 0$

$h = \frac{b-a}{n} = \frac{1-0}{2} = 0.5$

$W(0.5) = 0.1250$ is the new point

$I_2 = \cancel{0.5}\ \frac{0.5}{2}\,(0 + 0 + 2(0.1250))$

$I_2 = 0.0625$

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for 4 intervals : $W(0) = 0$

$W(1) = 0$

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$

$W(0.25) = 0.09375$
$W(0.75) = 0.09375$ } new points
$W(0.5) = 0.1250$

$$I_4 = \frac{0.25}{2}\left(0+0+2(0.09375+0.1250+0.09375)\right)$$

$$= 0.07812500$$

The error in each case is the difference between the analytical value and the trap estimate in each instance (I_1, I_2, I_4)

$\varepsilon_1 = 0.08333333 - 0 = 0.08333333$

$\varepsilon_2 = 0.08333333 - 0.0625 = 0.02083333$

$\varepsilon_4 = 0.08333333 - 0.08 = 0.00333333$

No. of Intervals	Trap. Estimate	Error
1	0	0.08333333
2	0.06250000	0.02083333
4	0.07812500	0.00333333

(b)

level	No. of Intervals	Trap Estimate	1st Richardson Extrapolation	Rhomberg Integral (2nd level R.E.)
1	1	0		
2	2	0.06250000	0.08333333	
3	4	0.07812500	0.08333333	0.08333333

lv 2, 1st Richardson Extrapolation : $\frac{4^1(0.06250000) - 0}{4^1 - 1}$

$= 0.08333333$

~~lv 2 , 2nd p~~

lv 3 , 1st Richardson Extrapolation : $\dfrac{4^1(0.07812500) - 0.06250000}{4^1 - 1}$

$= 0.08333333$

lv 3 , 2nd Richardson Extrapolation : $\dfrac{4^2(0.08333333) - 0.08333333}{4^2 - 1}$

$= 0.08333333$

∴ yes it is expected as the value obtained is the same as the analytical solution of integration of $w(y)$ which is Q

(c) For the existing limits of integrals ($a=0$, $b=1$)

$$\cancel{w(y)}\ Q = \int_a^b w(y) = \int_0^1 \frac{G}{2\mu} y(y-b)\ dy$$

for Gauss-legrende , limits should be -1 to 1

$$y = \frac{(1-0)\,y' + (1-0)}{2}$$

$$y = \frac{y'+1}{2}$$

$$w(y') = \frac{G}{2\mu}\left(\left(\frac{y'+1}{2}\right)^2 - \left(\frac{y'+1}{2}\right)b\right)$$

for 2 points Gauss-legrende , $A_1 \leftarrow c_1 = 1$ $\quad x_1 = \cancel{-0.0}\ \cancel{-0.55}\ -0.57735\,0269 \leftarrow y_1'$

$A_2 \rightarrow c_2 = 1$ $\quad x_2 = 0.57350269 \leftarrow y_2'$

new y (not derivative)

$$I = \frac{b-a}{2}\int w(y')\ dy'$$

$$= \frac{1}{2}\left[A_1 w(y_1') + A_2 w(y_2')\right]$$

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$$= -\frac{1}{2}\left(\left(\frac{-0.577350269+1}{2}\right)^2 - \left(\frac{-0.577350269+1}{2}\right), 1\right)$$

$$-\frac{1}{2}\left(\left(\frac{0.577350269+1}{2}\right)^2 - \left(\frac{0.577350269+1}{2}\right), 1\right)$$

$$= 0.08333333$$

Yes, the result is expected as it is the same as that of the value obtained analytically of the integration of w(y) which is Q