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PROGRAM 7

AIM :

To demonstrate a renewal process and find the expected waiting time until the nth renewal in case of a renewal process when renewal cycle length is distributed

a) Normally with mean μ and standard deviation σ (μ > 3σ)
b) Exponentially with parameter λ

THEORY :

A renewal process is an arrival process in which the interarrival intervals are positive, independent, and identically distributed (IID) random variables (RVs). Renewal processes (since they are arrival processes) can be specified in three standard ways: by the joint distributions of the arrival epochs S1, S2, . . . , by the joint distributions of the interarrival times X1, X2,..., and by the joint distributions of the counting RVs, N(t) for t > 0. The simplest characterization is through the interarrival times Xi since they are IID. Each arrival epoch Sn (time of nth arrival) is simply the sum X1 + X2 + · · · + Xn of n IID RVs. Hence, the total waiting time until the nth arrival is

$$S_n := \sum_{i=1}^{n} X_i.$$

a) Say the renewal cycle lengths have a normal distribution with parameters μ and σ, μ > 3σ. Since the sum of independent, normally distributed random variables is again normally distributed, where the parameters of the sum are obtained by summing up the parameters of the summands, then by the central limit theorem, Sn is approximately normally distributed for n>=20, such that

   $$Sn \approx N(n\mu, n(\sigma^2)), \text{ if } n \geq 20.$$

   Therefore, the expected waiting time until the nth renewal is given by the expectation of Sn, which is nμ.

b) Considering the renewal cycle lengths (Xi) distributed (independently and identically) exponentially with parameter λ, the renewal cycle lengths follow an Erlang distribution with parameters n and λ. Therefore, the expected waiting time until the nth renewal is the mean of En(λ) given by n/λ

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RESULT :

(a) Normally with mean μ and standard deviation σ, ($\mu > 3\sigma$)

Considering a renewal process where renewal cycle lengths are distributed normally with parameters $\mu = 10$ and $\sigma = 2$. The expected waiting time until the 30th renewal is given by:

CODE –

```
mean = 10
sd = 2 # mean and standard deviation of interarrival
time distribution
n = 30
expected_waiting_time = mean*n
print('The expected waiting time until 30th arrival in
a renewal process with normally distributed renewal
cycle lengths is: ')
print(expected_waiting_time)
```

OUTPUT :

```
The expected waiting time until 30th arrival in
a renewal process with normally distributed
renewal cycle lengths is: 300
```

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(b) Exponentially with parameter λ

Considering a renewal process where renewal cycle lengths are distributed exponentially with parameter λ = 3. The expected waiting time until the 30th renewal is given by:

CODE –

```
rate = 3 #lambda
n = 30
expected_waiting_time = n/rate
print('The expected waiting time until 30th arrival in a
renewal process with exponentially distributed renewal
cycle lengths is: ')
print(expected_waiting_time)
```

OUTPUT :

```
The expected waiting time until 30th arrival in a renewal
process with exponentially distributed renewal cycle
lengths is: 10.0
```

DISCUSSION :

The program is executed using Python. The expected waiting times until the nth renewal were computed when the interarrival times were distributed

a) Normally with mean μ and standard deviation σ (μ > 3σ)
   Expected waiting time = 300
b) Exponentially with parameter λ
   Expected waiting time = 10

If Y1, Y2,...Yn represents the renewal cycle lengths for a renewal process and are exponentially distributed with parameter λ, then the corresponding counting process { N(t), t>0 } is the homogeneous poisson process with intensity λ.