CS & IT ENGINEERING

2-3 M

Theory of Computation

Finite Automata:

Regular Expression-2

Lecture No. 3

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Regular Languages

TOPICS TO BE COVERED

- 01 Regular Expression
- 02 Operators
- 03 Basic Regular Expressions
- 04 Simplification of Reg Exps
- 05 Write Regular Exps

① Finite Automata (Regular Languages)

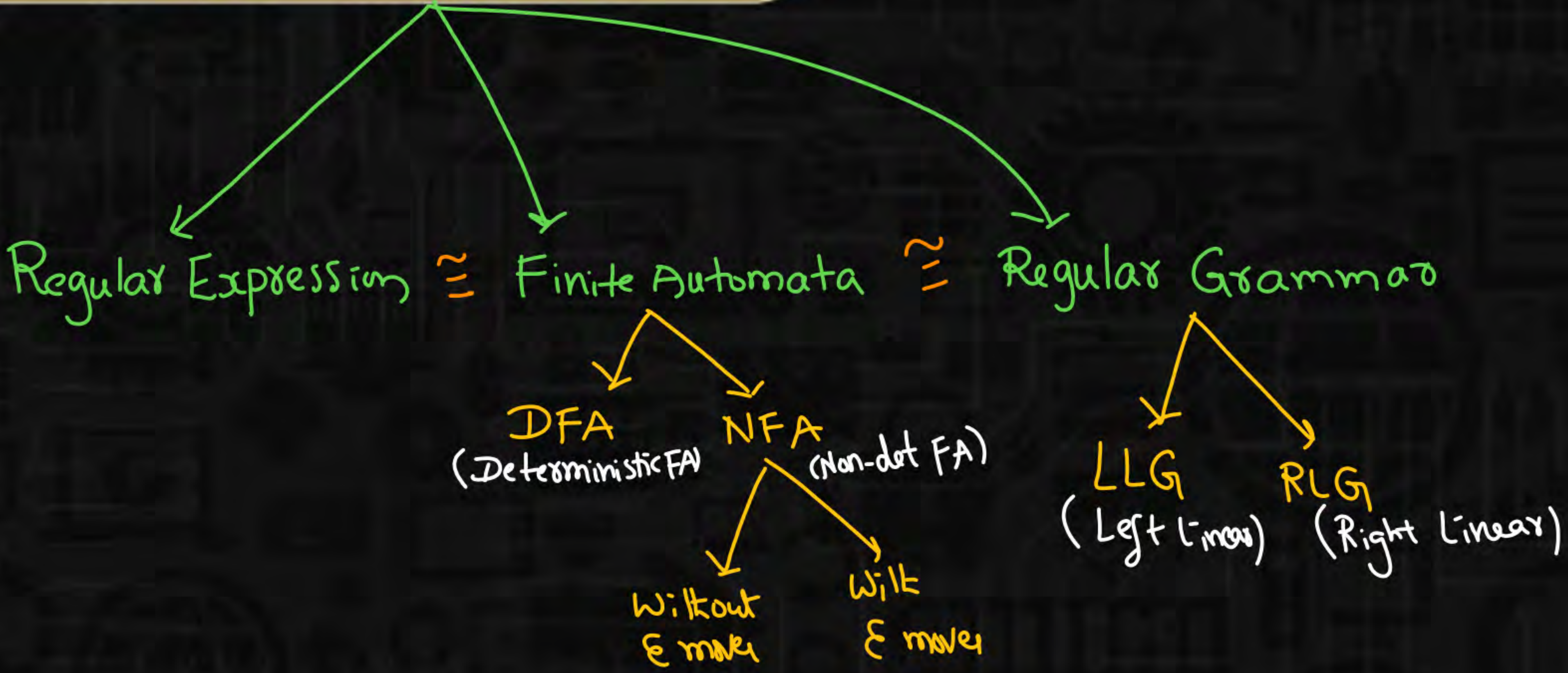
- i) Regular Expressions
- ii) Finite Automata
- iii) Regular Grammars
- iv) Regulars & Non regs
- v) Closure properties

② Pushdown Automata

③ Turing Machine

④ Undecidability

Regular Language



Regular Expression

- It generates (denotes) a regular set.
  (represents) (regular language)
- It uses 4 operators to represent

Expression: combination of operators and operands

Regular Expression:

4 operators

operands
(Regular Exps)

Digital Exp

$$\left.\begin{matrix} + \\ \cdot \\ \oplus \\ \odot \end{matrix}\right\}\text{operators}$$

Math Exp

$$\left.\begin{matrix} + \\ - \\ * \\ / \\ \vdots \end{matrix}\right\}\text{operators}$$

Relational Algebra
DBMS

$$\left.\begin{matrix} \sigma \\ \pi \\ \times \\ \bowtie \\ \vdots \end{matrix}\right\}\text{operators}$$

Discrete maths
⇓
propositional logic

$$\left.\begin{matrix} \vee \\ \wedge \\ \rightarrow \\ \leftrightarrow \\ \sim \end{matrix}\right\}\begin{matrix}\text{Operators} \\ \text{(connectives)}\end{matrix}$$

$$\text{Operators} \begin{cases} \rightarrow \text{Unary} \\ \rightarrow \text{Binary} \\ \quad \vdots \end{cases}$$

Regular Expression

→ 4 operators only used

Operators

$R_1 \cup R_2$

$R_1 \mid R_2$

Binary operators

1) → OR famous (+) (∪) (|) $\Longrightarrow R_1 + R_2$

2) → Concatenation (·) $\Longrightarrow R_1 \cdot R_2 = R_1 R_2$

Unary operators

3) → Kleene star (*) (closure) (Kleene closure) $\Longrightarrow R^*$

4) → Kleene plus (+) (positive closure) $\Longrightarrow R^+$

OR (+)

$R_1 + R_2$

R_1 or R_2

Either R_1 or R_2

$$L(\underbrace{a+b}) = \{a, b\}$$

$\Downarrow$

a ✓

b ✓

$\cancel{ab}$

$\cancel{ba}$

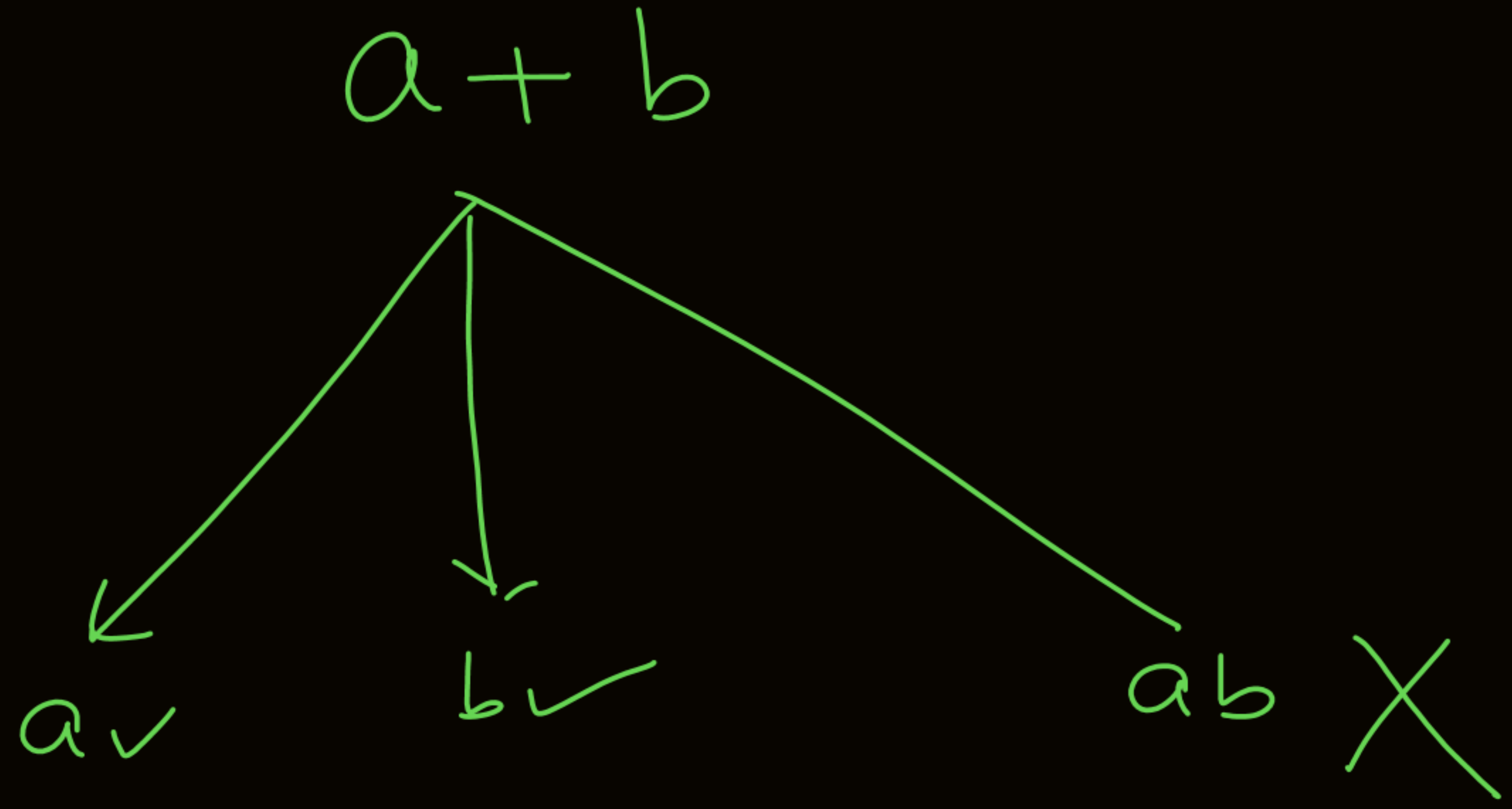
Reg Exp	Reg Language
a	$\{a\}$
ε	$\{\varepsilon\}$
ϕ empty exp	$\phi = \{\ \}$ empty set
$a+b$	$\{a, b\}$
$a+\varepsilon$	$\{a, \varepsilon\}$
$a+\phi$	$\{a\}$

$$L(R_1 + R_2) = L(R_1) \cup L(R_2)$$

$$L(a + \phi) = L(a) \cup L(\phi)$$

$$= \{a\} \cup \{\ \}$$

$$= \{a\}$$


a + b
a ✓
b ✓
ab X

$$\underbrace{abc} + \underbrace{\varepsilon} \Rightarrow \{\varepsilon, abc\}$$

$$\Downarrow$$

ε ✓

$\xcancel{a}$

$\xcancel{b}$

abc ✓

Concatenation (.)

$R_1 \cdot R_2 = R_1 R_2$

R_1 followed by R_2

- Both R_1 & R_2
- Sequence (order)
- first R_1 then R_2

$a \cdot b = \underbrace{ab}$

- a ✗
- b ✗
- ab ✓
- ~~ba~~

$$a \cdot \varepsilon = a$$

$$\underrightarrow{\varepsilon \cdot a} = a$$

$$\underrightarrow{a \cdot bc} = abc$$

$$\underrightarrow{ab \cdot c} = abc$$

$$\underrightarrow{\varepsilon \cdot abc} = abc$$

$$\underrightarrow{abc \cdot \varepsilon} = abc$$

$$\underrightarrow{\varepsilon \cdot \varepsilon} = \varepsilon$$

Kleene Star (*)

In TOC:

$2^3 = 8$ ✗

$= \underbrace{2.2.2}_{\text{3 times 2 will be repeated}}$

$2+2+2 = 6$ ✗

$\{2\} = 2$ in TOC

R^*

Kleene closure of R

$R^* = R^0 + R^1 + R^2 + R^3 + \ldots\ldots$

$a^* = a^0 + a^1 + a^2 + a^3 + \ldots\ldots$

$= \{\varepsilon, a, aa, aaa, \ldots\ldots\}$

R^* = 0 or more occurrences of R

$= R^{\geq 0}$

$$\varepsilon^{*} = \varepsilon^{0} + \varepsilon^{1} + \varepsilon^{2} + \varepsilon^{3} + \cdots$$

$$= \varepsilon + \varepsilon + \varepsilon + \varepsilon + \cdots$$

$$= \varepsilon$$

$$\boxed{\phi . \phi = \phi}$$

$$\varepsilon = \phi^{0} = \varepsilon^{0} = R^{0} = a^{0} = b^{0}$$

$$\boxed{\begin{aligned} \text{Any}^{0} &= 1X \\ &= \varepsilon \text{ in TOC} \end{aligned}}$$

$$\phi^{*} = \phi^{0} + \phi^{1} + \phi^{2} + \cdots$$

$$= \varepsilon + \underbrace{\phi + \phi + \phi + \cdots}$$

$$= \varepsilon + \phi$$

$$= \varepsilon$$

$$\varepsilon^{2} = \varepsilon . \varepsilon = \varepsilon \checkmark$$

$$\varepsilon^{3} = \varepsilon . \varepsilon . \varepsilon = \varepsilon \checkmark$$

$$\varepsilon^{4} = \varepsilon . \varepsilon . \varepsilon \varepsilon = \varepsilon$$

R^{0} = consider zero symbols in R

$= \varepsilon$

ε

empty string

non empty Exp

Exp generates empty string

It is not set

$$|\varepsilon| = 0$$

string

Length

ϕ

not string

Empty expression

Empty set

$$|\phi| = 0$$

Set

Size

$$\{\varepsilon\} \neq \{\ \}$$

$$\left|\{\varepsilon\}\right| = 1$$

size of non empty set

$$\left|\{\ \}\right| = 0$$

size of empty set

Empty string $= \varepsilon$

Empty set $= \phi = \{ \}$

$\simeq$

Empty Expression $= \phi$

$$\boxed{R^5 = RRRRR}$$

Kleene Plus ($^+$)

R^+

Positive closure of R

one or more occurrenca of R

$$R^+ = R^1 + R^2 + R^3 + \ldots$$

$$= R^{\geq 1}$$

$$a^+ \Rightarrow \{a, a^2, a^3, \ldots\}$$

$$\{a, aa, aaa, \ldots\}$$

$$R^* = R^0 + \underbrace{R^1 + R^2 + R^3 + \cdots}_{R^+}$$

✓

$$\boxed{\begin{array}{l} \varepsilon^* = \varepsilon^0 + \varepsilon^+ \\ \varepsilon = \varepsilon + \varepsilon \\ \varepsilon = \varepsilon \end{array}}$$

$$\varepsilon^* = \varepsilon$$

$$\varepsilon^+ = \varepsilon$$

$$\boxed{R^* = R^0 + R^+}$$

$$\boxed{\varepsilon^* = \varepsilon^+}$$

$R^* = R^+$ happens sometimes

why $R^+ \neq R^* - \{\varepsilon\}$?

$$\boxed{\begin{array}{l} \varepsilon^+ \neq \varepsilon^* - \{\varepsilon\} \\ \{\varepsilon\} \neq \{\varepsilon\} - \{\varepsilon\} \\ \{\varepsilon\} \neq \{\} \end{array}}$$

wff
=
well formed formula

$$R_1+R_2 = (R_1)+(R_2) = (R_1+R_2) = (R_1)+R_2 = R_1+(R_2)$$

$$R_1 \cdot R_2 = R_1R_2 = (R_1)R_2 = R_1(R_2) = (R_1R_2)$$

$$R^* = (R)^* = (R^*)$$

$$R^+ = (R)^+ = (R^+)$$

---

not wff
(not exps)

$R_1(+)R_2$	$R_1(\cdot)R_2$	$R^{(*)}$	$R^{(+)}$
✗	✗	✗	✗

Properties of $\underbrace{\overset{\text{OR}}{+} \text{ and } \overset{\text{concatenation}}{.}}_{\text{Binary}}$

- Associative
- Identity
- Commutative
- Distributive
- Annihilator (Dominator)

Associativity

$$\underbrace{(A \underset{\substack{\downarrow \\ \text{Left} \\ \text{operatr}}}{O} B) O C}_{\text{Left Associative}} = \underbrace{A O (B \underset{\substack{\downarrow \\ \text{Right operatr}}}{O} C)}_{\text{Right Associative}}$$

$$(a+b)+c = a+(b+c)$$

$$\{a, b, c\}$$

$$(a.b).c = a.(b.c)$$

$$abc$$

$$+ : \quad (A+B)+C = A+(B+C)$$

$$\boxed{+ \text{ satisfies Associative}}$$

$$\cdot : \quad \overrightarrow{(A \cdot B) \cdot C} = \overrightarrow{A \cdot (B \cdot C)}$$

$$\boxed{\bullet \text{ Satisfies Associative}}$$

Identity

$\phi + \phi = \phi$

$a + \phi = a$

$\phi + a = a$

$ab + \phi = ab$

$\varepsilon + \phi = \varepsilon$

$ab \cdot \varepsilon = ab$

$\varepsilon \cdot ab = ab$

$\varepsilon \cdot \varepsilon = \varepsilon$

$\varepsilon \cdot \phi = \phi$

$$A \bigcirc \underset{\substack{\downarrow \\ \text{Right} \\ \text{Identity}}}{I} = \underset{\substack{\downarrow \\ \text{Left} \\ \text{Identity}}}{I} \bigcirc A = A$$

$+ :$ $$A + \boxed{\phi} = \boxed{\phi} + A = A$$

$$\boxed{\phi \text{ is identity for } +}$$

$\cdot :$ $$A \cdot \boxed{\varepsilon} = \boxed{\varepsilon} \cdot A = A$$

$$\boxed{\varepsilon \text{ is identity for } \cdot}$$

Commutativity

$$\boxed{A \odot B = B \odot A}$$

$R.\epsilon = \epsilon.R$

$R = R$

$a.b \neq b.a$

$\underbrace{ab}_{} \neq \underbrace{ba}_{}$

Different

$+ \quad : \quad A + B = B + A$

+ satisfies commutative

$\cdot \quad : \quad A \cdot B \neq B \cdot A$

• not satisfies commutative

Distributivity

Left Distribution

$\underbrace{\bigcirc \text{ over } \square} : A \bigcirc (B \square C) = (A \bigcirc B) \square (A \bigcirc C)$

$\bigcirc$ is distributed over $\square$

$\underbrace{\bigcirc \text{ over } \square}_{\text{Right distribution}} : (A \square B) \bigcirc C = (A \bigcirc C) \square (B \bigcirc C)$

✓

$+$ over $\cdot$: $A+(B \cdot C) \neq (A+B) \cdot (A+C)$

$(A \cdot B)+C \neq (A+C) \cdot (B+C)$

$\boxed{+ \text{ should not be distributed over } \cdot}$

$a+bc \neq (a+b).(a+c)$

$\{a, bc\}$ $\{aa, ac, ba, bc\}$

$\cdot$ over $+$: $A \cdot (B+C) = (A \cdot B)+(A \cdot C)$

$(A+B) \cdot C = (A \cdot C)+(B \cdot C)$

✓

$a \cdot (b+c) = ab+ac$

$$\left(A + \left[(B^{+}) \cdot (C^{*})\right]\right) \Leftarrow A + B^{+} \cdot C^{*}$$

Highest

1st : Kleene star/Kleene plus

2nd : Concatenation

3rd : OR

lowest

Annihilator (Dominator)

$$A \circ \underset{\text{Dominator}}{D} = D \circ A = D$$

$a.\phi = \phi$

$\varepsilon.\phi = \phi$

$\phi.\phi = \phi$

$R.\underset{\text{identity}}{\varepsilon} = R$

$R.\underset{\text{do}}{\phi} = \phi$

ir

contains all strings

Universal set

$+ :$ $$A + \boxed{\Sigma^*} = \boxed{\Sigma^*} + A = \boxed{\Sigma^*}$$

$. :$ $$A . \boxed{\phi} = \boxed{\phi} . A = \boxed{\phi}$$

Basic Regular Expressions

	Associative	Identity	Commutative	Dominator (Annihilator)
OR	✓	ϕ	✓	Σ^*
Concatenation	✓	ϵ	✗	ϕ

Basic Regular Expressions

① $\varepsilon + \varepsilon = \varepsilon$

② $\phi + \phi = \phi$

③ $a + a = a$

④ $R + R = R$

---

⑤ $\varepsilon \cdot \varepsilon = \varepsilon$

⑥ $\phi \cdot \phi = \phi$

⑦ $a \cdot a = aa = a^2$

⑧ $R \cdot R = RR = R^2$

⑨ $\varepsilon^* = \varepsilon$

⑩ $\phi^* = \varepsilon$

⑪ $a^* = a^0 + a^1 + a^2 + a^3 + \ldots$

⑫ $R^* = R^0 + R^+$

⑬ $\varepsilon^+ = \varepsilon$

⑭ $\phi^+ = \phi$

⑮ $a^+ = a^1 + a^2 + a^3 + \ldots$

⑯ $R^+ = R^1 + R^2 + R^3 + \ldots$

$$\underset{\text{empty Exp}}{\phi} \cong \underset{\text{empty set}}{\phi}$$

$$L(\underset{\text{exp}}{\phi}) = \underset{\text{set}}{\phi}$$

$$\varepsilon \cdot \varepsilon$$

$$\begin{tikzcd} \{\boxed{\varepsilon}\} \arrow[r, bend right, "\varepsilon\varepsilon"'] & \{\boxed{\varepsilon}\} = \{\varepsilon\varepsilon\} = \{\varepsilon\} \end{tikzcd}$$

$$\phi \cdot \phi = \phi$$

$$\{\ \}\ \{\ \} = \{\ \}$$

Basic Regular Expressions

⑰ $\varepsilon + \phi = \varepsilon$

*** ⑱ $\varepsilon + a = \varepsilon + a$

⑲ $\phi + a = a$

⑳ $R + \phi = R$

*** ㉑ $R + \varepsilon = R + \varepsilon$

㉒ $a + \varepsilon = a + \varepsilon =$ ⑱

㉓ $\varepsilon \cdot \phi = \phi$

㉔ $\phi \cdot \varepsilon = \phi$

㉕ $\varepsilon \cdot a = a$

㉖ $a \cdot \varepsilon = a$

㉗ $\phi \cdot a = \phi$

㉘ $a \cdot \phi = \phi$

㉙ $R \cdot \phi = \phi$

㉚ $\phi \cdot R = \phi$

$$a + \varepsilon \Rightarrow \{a\} \cup \{\varepsilon\} = \{a, \varepsilon\} = a + \varepsilon$$

$$\underbrace{a + \phi}_{= a} \Rightarrow \{a\} \cup \{\ \} = \{a\} \Rightarrow a$$

$$\underset{\substack{\swarrow \\ \text{string}}}{\varepsilon} + \underset{\substack{\searrow \\ \text{string}}}{a} = \varepsilon + a = a + \varepsilon$$

$$\{\varepsilon\} \cup \{a\} = \{\varepsilon, a\}$$

$$= \{a, \varepsilon\}$$

Basic Regular Expressions

㉛ $(a+\varepsilon)^* =$

㉜ $(a+\varepsilon)^+ =$

㉝ $(\phi+\varepsilon)^* =$

㉞ $(\phi+a)^* =$

㉟ $(a\cdot\phi)^* =$

Q.1

Number of prefixes of "n" length string is ___
(assume all the symbols in given string are different)

A. n
[✓ B. n+1]
C. n+2
D. n-1

Q.2

$R . \overset{\text{Dominator}}{\phi} = \phi$ $(ab)^* . \phi = \phi$

$((ab)^* . \varnothing)$ is equivalent to

A. $(ab)^*$
[✓ B. $\varnothing$]
C. $(a+b)^*$
D. None

$R . \epsilon = R$

$(ab)^* . \epsilon = (ab)^*$

Q.3

$(\overset{\varepsilon}{\varnothing^*} \cup (\underset{\sim}{bb^*})) =$

A. Epsilon

B. ∅

C. b^+

[✓ D. b^*]

$b \cdot b^*$

$b \cdot \{\varepsilon, b, b^2, \ldots\}$

$b, b^2, b^3, \ldots$

b^+

$\varepsilon \cup b^+$

$\varepsilon + b^+$

$b^0 + b^+ = b^*$

$a \cdot a^* = a^+$

$a^* \cdot a = a^+$

$b \cdot b^* = b^+$

$b^* \cdot b = b^+$

Q.4

Which of the following is TRUE?

[✓ A. $(ab)^*a = a(ba)^*$]

B. $(aa)^*b = a(ab)^*$

C. $(ba)^*a = b(aa)^*$

D. All of the above

Q.5

OR operator in regular expression satisfies

A. Associative
B. Commutative
[✓ C. both A and B]
D. Neither A nor B

Q.6

Concatenation operator in regular expression satisfies

- [✓ A. Associative]
- B. Commutative
- C. both A and B
- D. Neither A nor B

Q.7

Which of the following distribution is valid in regular expressions?

A. OR over CONCATENATION

[✓ B. CONCATENATION over OR]

C. Both A and B

D. Neither A nor B

Q.8

Match the following groups over Sigma={a,b}.

Group-1:

1. OR identity
2. OR dominator
3. Concatenation identity
4. Concatenation dominator

Group-2:

- a. Epsilon
- b. Empty Expression
- c. (a+b)*
- d. (aa)*

A. 1-a, 2-b, 3-c, 4-d

[✓ B. 1-b, 2-c, 3-a, 4-b]

C. 1-d, 2-a, 3-b, 4-c

D. None

Q.9

If R1= a^*, and R2=$(aa)^*$ then R1.R2=____

A. R1
B. R2.R1
C. R1+R2
[✓ D. All of these]

Q.10

If $R1 = a(aa)^*$, and $R2 = (aa)^*$ then $R1 + R2 =$ ____

A. $R1$

B. $R2.R1$

[✓ C. $R1^*$]

D. None

Summary

OR $+$ ✓

$\cdot$ ✓

Kleene star ✓

Kleene plus ✓

ϕ^0 Take zero occurrences of ϕ

$= \varepsilon$

$R^0 = \varepsilon$

$\varepsilon^0 = \varepsilon$

$a^0 = \varepsilon$

THANK YOU GW SOLDIERS !