CS & IT ENGINEERING

Theory of Computation

Finite Automata

Regular Languages identification - 2

Lecture No. 16

By- DEVA Sir

TOPICS TO BE COVERED

Regulars & Non regulars

Languages

H.W.: DFA construction

Mode-1 3

(21) $\{a^m b^n \mid m+n = \text{even}\} = (aa)^*(bb)^* + a(aa)^* b(bb)^* \Rightarrow$ Regular

(22) $\{a^m b^n \mid m+n = \text{odd}\} = a^{\text{even}} b^{\text{odd}} + a^{\text{odd}} b^{\text{even}} = (aa)^* . b(bb)^* + a(aa)^*(bb)^* \Rightarrow$ Regular

(23) $\{a^m b^n \mid \text{If } (m=\text{even}) \text{ then } (n=\text{odd})\} = a^{\text{even}} b^{\text{odd}} + a^{\text{odd}} b^* = (aa)^* b(bb)^* + a(aa)^* b^* \Rightarrow$ Reg

(24) $\{w \mid w \in \{a,b\}^*, n_a(w) = \text{even}\} = b^*(b^* a\, b^* a b^*)^* b^* \Rightarrow$ Regular

(25) $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\} \Rightarrow$ Not reg

Mode:-1 / 4 (DFA)

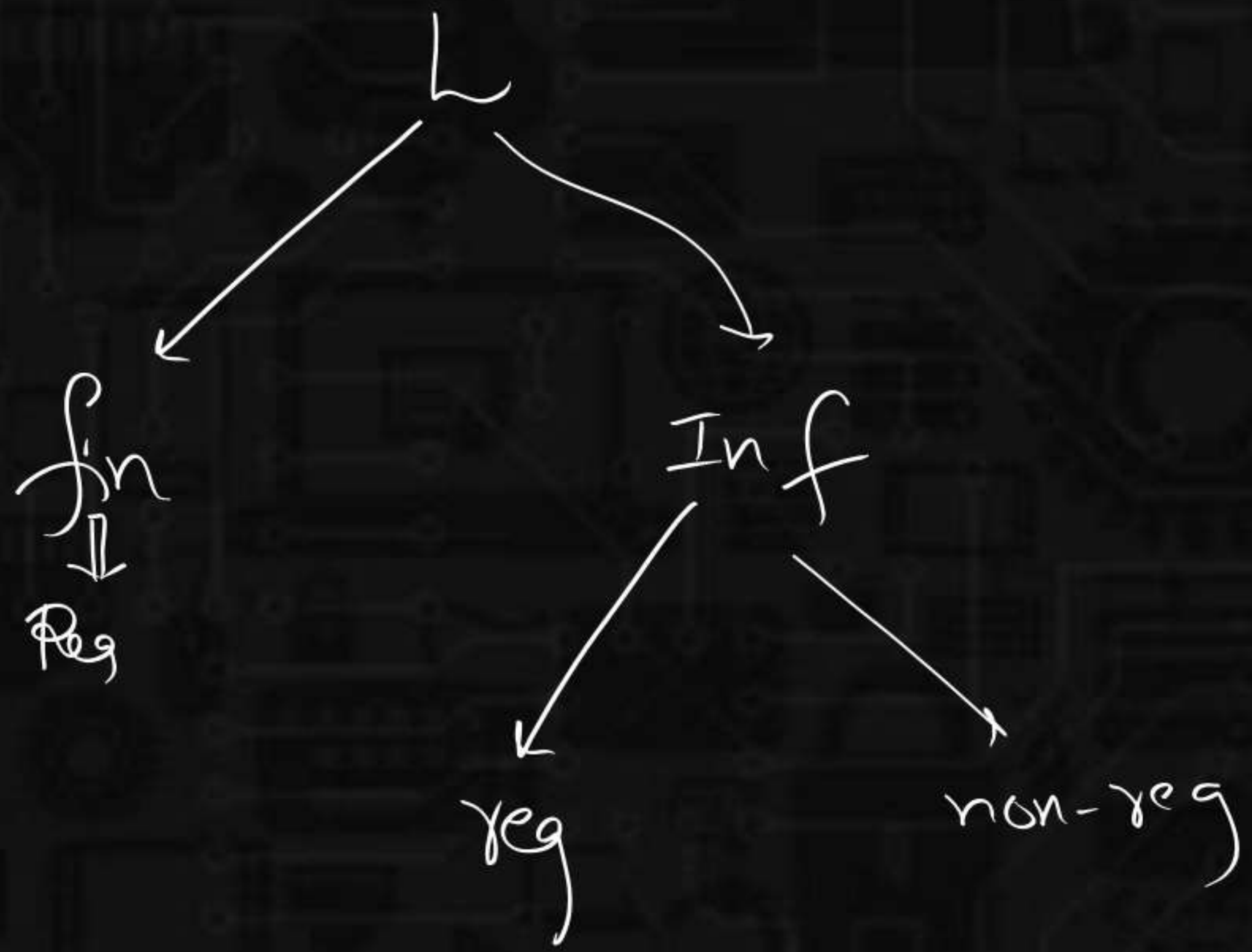
(26) $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\} \Rightarrow$ Not reg

(27) $\{w \mid w \in \{a,b\}^*, n_a(w) + n_b(w) = \text{even}\} = \{w \mid w \in \{a,b\}^*, [n_a(w)\%2=0 \,\&\, n_b(w)\%2=0] \text{ OR } [n_a(w)\%2=1 \,\&\, n_b(w)\%2=1]\}$

(28) $\{w \mid w \in \{a,b\}^*, n_a(w) + n_b(w) = \text{odd}\} \Rightarrow$ Reg $= \overline{(27)}$

(29) $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is divisible by } 100\} \Rightarrow$ Reg

(30) $\{w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } 100\} = [(a+b)^{100}]^* \Rightarrow$ Regular


L
fin
Reg
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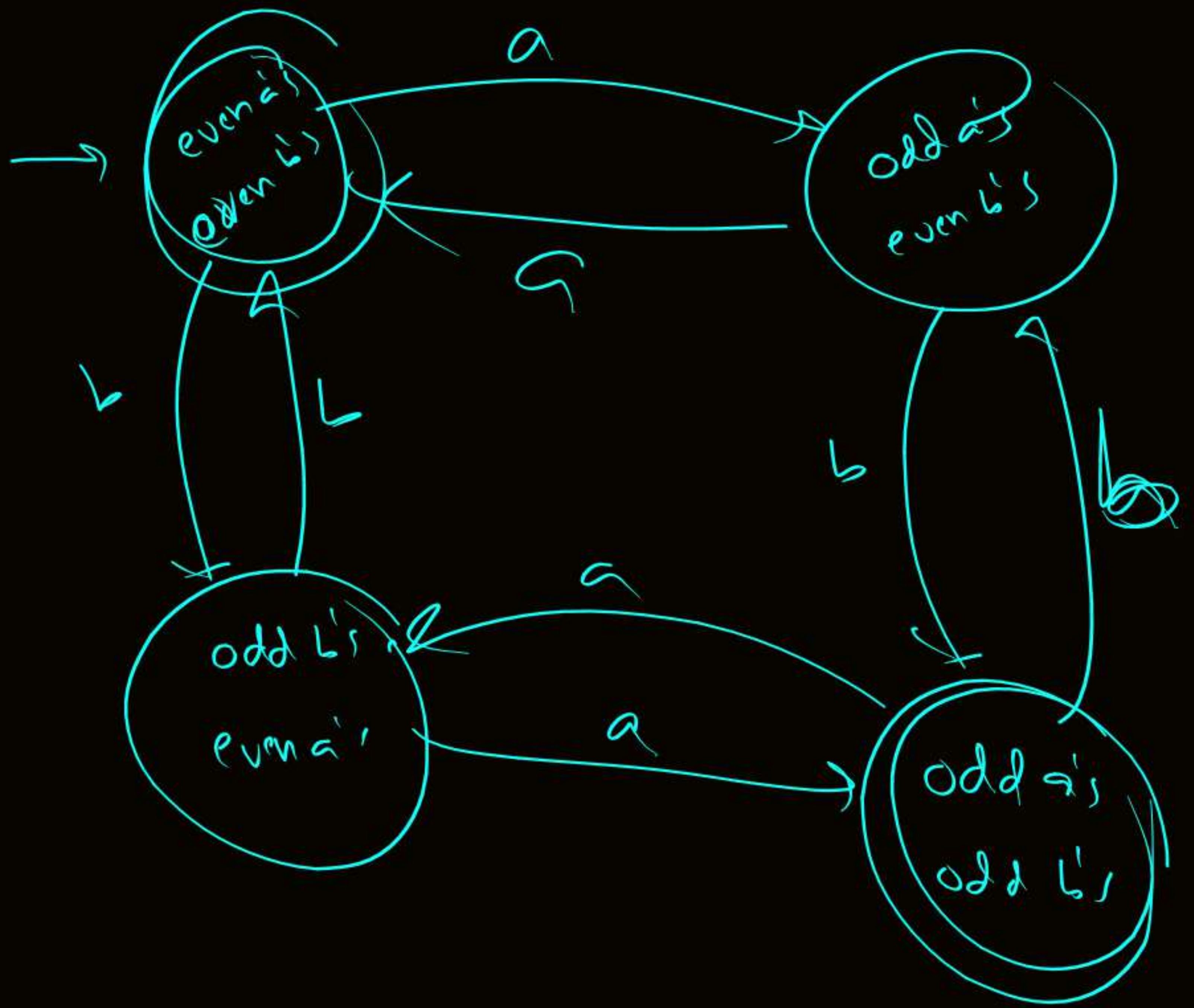
non-reg



27

#a's/2 and #b's/2

Languages

Over $\Sigma = \{a\} \rightarrow$ one symbol $\Rightarrow$ Check A.P.

(31) $\{a^n \mid n \geq 0\} = a^* = \{\underset{n=0}{\varepsilon}, \underset{n=1}{a^1}, \underset{n=2}{a^2}, \underset{n=3}{a^3}, \ldots\}$ 

(32) $\{a^{2n} \mid n \geq 0\} = (aa)^*$

(33) $\{a^{2n+100} \mid n \geq 0\} = (aa)^* a^{100}$

(34) $\{a^{\text{prime}}\} \Rightarrow$ Not regular

(35) $\{a^{200n+5}\} = (a^{200})^* a^5$

(36) $\{a^{n^2}\} \Rightarrow$

(37) $\{a^{2^n}\} \Rightarrow$

(38) $\{a^{n!}\} \Rightarrow$

(39) $\{a^{n^n}\} \Rightarrow$

Not regular (Don't form A.P.)

(40) $\{a^{m^n}\} = a^*$

non A.P. $n^n = 1^1, 2^2, 3^3, 4^4, \ldots$

A.P. $m^n = 0, 1, 2, 3, 4, 5, 6, \ldots$

$0^n \quad 1^n \quad 2^1 \quad 3^1 \quad 4^1 \ldots$

non A.P. $n! = 1, 2, 6, 24, \ldots$

Prime $= 2, 3, 5, 7, 11, 13, \ldots$

Languages

$a^* \cup \text{Any} = a^*$

$\boxed{\Sigma = \{a\}}$

(41) $\{a^n\}^* = a^*$

(42) $\{a^{2n}\}^* = a^{2n} = (aa)^*$

(43) $\{a^{n!}\}^* = a^*$

(44) $\{a^{2^n}\}^* = a^*$

(45) $\{a^{\text{Prime}}\}^* = \{\epsilon, a^2, a^3, a^4, a^5, a^6, a^7, \dots\} = \epsilon + aaa^*$

(46) $\{a^{n^n}\}^* = a^*$

(47) $\{a^{m^n}\}^* = a^*$

(48) $\{a^{n^2}\}^* = a^*$

(49) $\{a^{n^2}\}^* \{a^{100^n}\}^* = a^*$

(50) $\{a^{\text{Prime}}\}^* \{a^{n^5}\}^* = a^*$

All are regular

$\left.\begin{array}{l} w = ab \\ x = aaa \end{array}\right\}$ abababaaa $\left\{\begin{array}{l} w = \varepsilon \\ x = abababaaa \end{array}\right.$

$$\left\{ \underbrace{www\,x}_{\text{this is from}} \;\middle|\; \text{Conditions} \right\}$$

this is from
you will understand only when
you apply condition

$$\left\{ \overset{\varepsilon}{\text{string}_1},\ \overset{a}{\text{string}_2},\ \overset{b}{\text{string}_3},\ \ldots \right\} \Rightarrow$$

you will understand here

w helps whole language

$$\{w \cancel{x}\cancel{x} \mid w, x \in \{a,b\}^*\} = w = (a+b)^*$$

w can't help

$$\{w x x \mid w = ab, x \in \{a,b\}^*\} \neq w$$

Languages

Regular

(51) $\{ \overset{\varepsilon}{\cancel{w}}\,\overset{\varepsilon}{\cancel{w}}\,x \mid w, x \in \{a,b\}^* \} = (a+b)^* = x$

(52) $\{ \overset{\varepsilon}{\cancel{w}}\,x\,\overset{\varepsilon}{\cancel{w}} \mid \quad " \quad \} = x = (a+b)^*$

(53) $\{ x\,\cancel{w}\,\cancel{w} \mid \quad " \quad \} = x = (a+b)^*$

(54) $\{ \cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\,x\,\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w}\cancel{w} \mid w, x \in \{a,b\}^* \} = x = (a+b)^*$

(55) $\{ \cancel{w}\,\cancel{w}^R x \mid w, x \in \{a,b\}^* \} = x = (a+b)^*$

(56) $\{ \cancel{w}\,x\,\cancel{w}^R \mid \quad " \quad \} = x = (a+b)^*$

(57) $\{ x\,\cancel{w}\,\cancel{w}^R \mid \quad " \quad \} = x = (a+b)^*$

Not regular

(58) $\{ ww\,xx \mid w, x \in \{a,b\}^* \}$

(59) $\{ ww \mid w \in \{a,b\}^* \} \Rightarrow$

(60) $\{ ww^R \mid w \in \{a,b\}^* \}$

$$\overset{\overset{b}{a}}{w}\;\overset{\overset{b}{a}}{w}$$

$$\{ww \mid w \in \{a,b\}^*\} = \{\varepsilon\varepsilon, aa, bb, aaaa, abab, \ldots\}$$

$$\{\underset{\varepsilon}{\cancel{w_1}}w_2 \mid w_1, w_2 \in \{a,b\}^*\} = w_1 = w_2 = (a+b)^*$$

Languages

(61) $\{\cancel{w_1}^{\varepsilon} w_2 \mid w_1, w_2 \in \{a,b\}^*\} = (a+b)^* \Rightarrow$ Regular

(62) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, w_1 = w_2\} = \{ww \mid w \in \{a,b\}^*\} \Rightarrow$ Not regular

(63) $\{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = |w_2|\}$ = Set of all even length strings $= [(a+b)^2]^* \Rightarrow$ Regular

(64) $\{ww \mid w \in a^*\} = \{\varepsilon\varepsilon, aa, aaaa, \ldots\} = (aa)^* \Rightarrow$ Regular

(65) $\{w\#w \mid w \in a^*\} = \{a^n \# a^n\} \Rightarrow$ Not reg

(66) $\{ww^R \mid w \in a^*\} = (64) = ww = ww^R = w^R w = w^R w^R$ over 1 symbol $\Rightarrow$ regular

(67) $\{www \mid w \in \{a,b\}^*\} \Rightarrow$ Not regular

(68) $\{www \mid w \in a^*\} = (aaa)^* \Rightarrow$ Regular

(69) $\{w\#w \mid w \in \{a,b\}^*\} \Rightarrow$ Not reg

(70) $\{w\#w^R \mid w \in \{a,b\}^*\} \Rightarrow$ Not reg

$w\#w \Rightarrow \{\#,\ a\#a,\ b\#b,\ aa\#aa,\ \dots\}$

$w \in \{a,b\}^*$ $w=\varepsilon$ $w=a$ $w=b$ $w=aa$

Inf

$\{w\#w \mid w \in \{a,b\}^*,\ |w| \le 100\} \Rightarrow$ finite

$\Rightarrow$ reg

$\underbrace{abbaa}_{w} \# \underbrace{abbaa}_{w}$

$$\{\, \underline{w}\underline{w}\underline{w} \mid w \in \{a,b\}^* \,\}$$

$$= \{\, \varepsilon,\ aaa,\ bbb,\ aaaaaa,\ \underline{ab}\,\underline{ab}\,\underline{ab},\ \ldots \,\}$$

$w=\varepsilon \quad w=a \quad w=b \quad w=aa \quad w=ab$

63 $\{ w_1 w_2 \mid |w_1| = |w_2| \} = [(a+b)^2]^*$

$|w_1| = |w_2| = 0 \Rightarrow \varepsilon \quad \varepsilon \longrightarrow \varepsilon$

$|w_1| = |w_2| = 1 \Rightarrow a \quad a/b \longrightarrow$ 2 lengths

$= 2 \Rightarrow b \quad a/b$

aa
ab
ba
bb

aa aa/ab/ba/bb

ab aa/ab/ba/bb

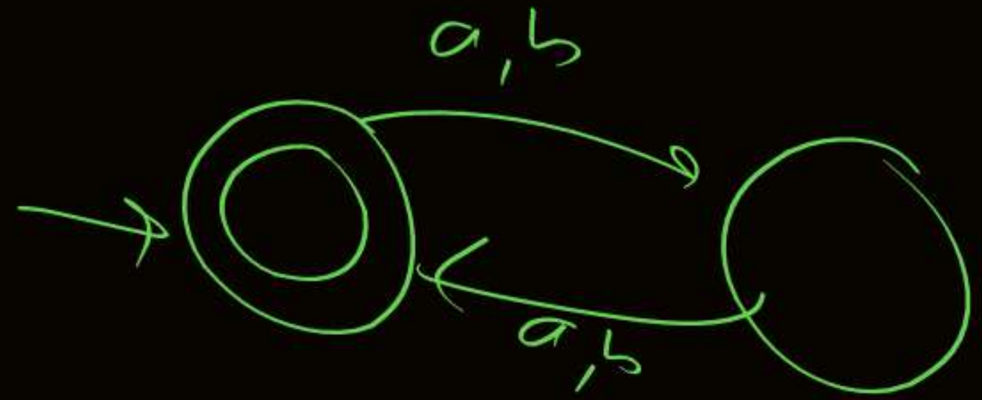
ba aa/ab/ba/bb

bb "

$\longrightarrow$ 4 lengths

$= 3 \Rightarrow$ xxx xxx $\longrightarrow$ 6 len

$= \{ w \mid |w| = \text{even}, w \in \{a,b\}^* \}$



Languages

Short cut: Put min w in given form, check resultant form covers whole language

(71) $\{wwx \mid w, x \in \{a,b\}^+\}$ $\to$ 1st: $aax + bbx$; 2nd: $aaaax + \cancel{ababx} + \cancel{babax} + bbbbx$ $\Rightarrow$ Not regular

(72) $\{wxw \mid$ " $\}$ $\to$ 1st: $axa + bxb$; 2nd: $aaxaa + \cancel{abxab} + \ldots$ $\Rightarrow$ Not regular

(73) $\{xww \mid$ " $\}$ $\to$ $xaa + xbb$; $xaaaa + \cancel{xabab} + \ldots$ $\Rightarrow$ Not regular

(74) $\{ww^Rx \mid$ " $\}$ $\to$ $aax + bbx$; $aaaax + \cancel{abbax} + \cancel{baabx} + bbbbx$ $\Rightarrow$ Not regular

*** (75) $\{wxw^R \mid$ " $\}$ $\to$ $axa + bxb$; $aaxaa + abxba + baxab + bbxbb$ $= axa + bxb \Rightarrow$ Regular

(76) $\{xww^R \mid$ " $\}$ $\to$ $xaa + xbb$; $xaaaa + \cancel{xabba} + \ldots$ $\Rightarrow$ Not regular

(77) $\{wxwy \mid w, x, y \in \{a,b\}^+\}$ $\to$ $axay + bxby$; $\boxed{a}axaay + \boxed{a}bxaby + \boxed{b}axbay + \boxed{b}bxbby$ $= axay + bxby \Rightarrow$ Regular

(78) $\{xwyw \mid w, x, y \in \{a,b\}^+\}$ $= xaya + xbyw \Rightarrow$ Regular

(79) $\{xww^Ry \mid w, x, y \in \{a,b\}^+\}$ $= xaay + xbby \Rightarrow$ Regular

(80) $\{wxwywzwpwq \mid w, x, y, z, p, q \in \{a,b\}^+\}$ $\Rightarrow$ Regular

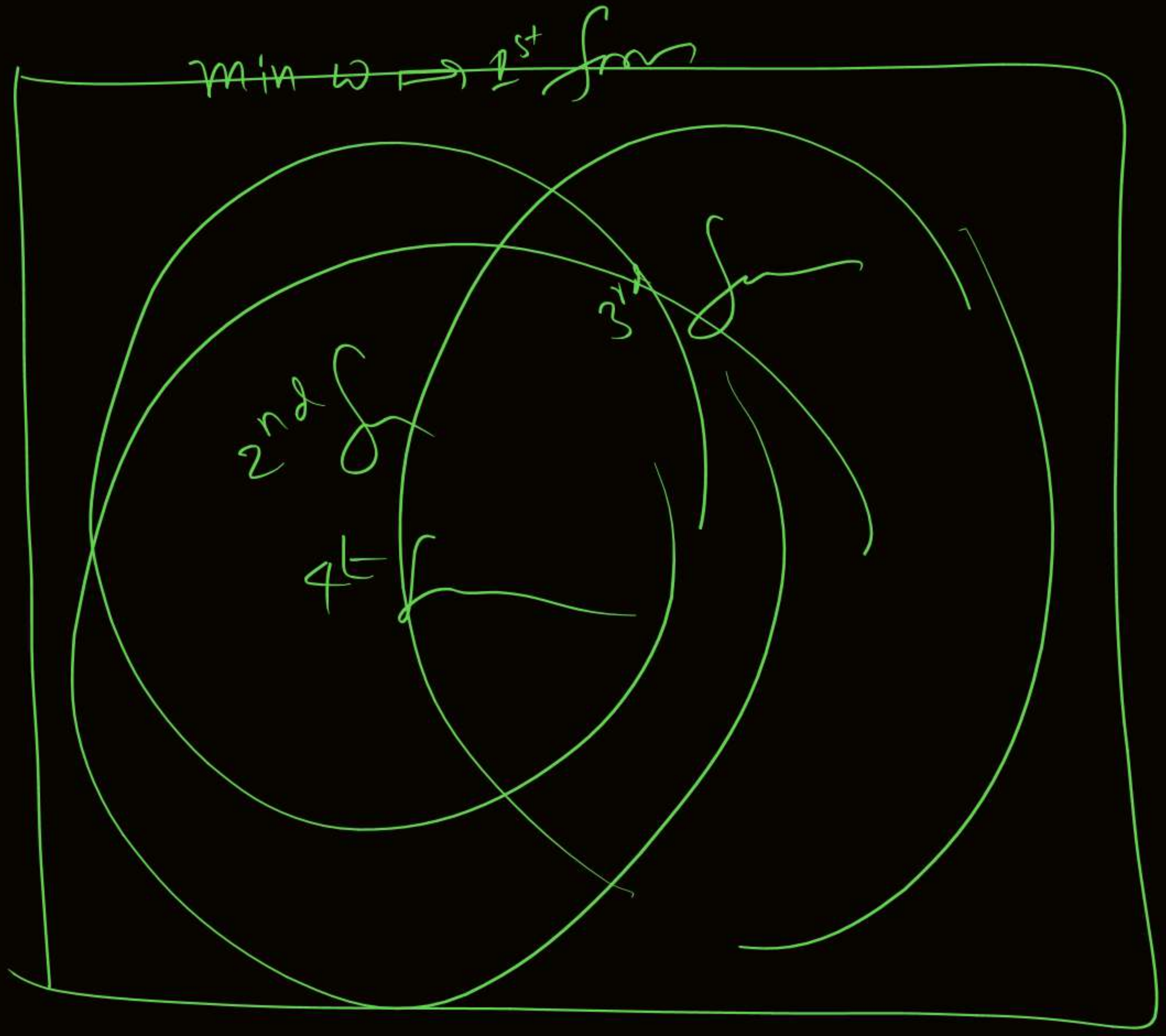
ε_n

$\cancel{\varepsilon}$

$$W_x\, x\, W_y\, y \;\Big|\; W_{x,y} \in \{a,b\}^+$$

$$a\,x\,a\,y + b\,x\,b\,y$$

$$x\,\tilde{w}^a\, y\, \tilde{w}^a = xaya + xbyb$$


min ω ⟶ 1st fn
2nd fn
3rd fn
4th fn

$$W \quad x \quad W \quad y \quad W \quad z \quad W \quad p$$

$$x \quad W \quad y \quad W \quad z \quad W \quad p \quad W$$

before befo befo bef

$$w \quad x \quad w^R \;\Big|\; w, x \in \{a,b\}^+$$

after

begin

$$x \quad w \qquad w^R \quad y$$

begin

after

$\{ \underline{w x w^R} \mid w, x \in \{a,b\}^{\boxed{+}} \} = a x a + b x b$

we can't put ϵ

Min w
$w = a/b$

$\boxed{a x a + b x b} = \boxed{a(a+b)^+ a + b(a+b)^+ b}$

1st form

$w = aa/ab/ba/bb$

If this form covered in 1st form then L is regular

2nd form

$\underbrace{aaxaa}_{\checkmark} + \underbrace{abxba} + baxab + bbxbb$

Languages

(81) $\{w \mid w \in \{0,1\}^*,\ n_0(w) = n_1(w)\}$ ⇒ Not reg

(82) $\{w \mid w \in \{0,1\}^*,\ n_{00}(w) = n_{11}(w)\}$ ⇒ Not reg

*** (83) $\{w \mid w \in \{0,1\}^*,\ n_{\boxed{01}}(w) = n_{\boxed{10}}(w)\}$ ⇒ Regular (Try DFA for this)

(84) $\{w \mid w \in \{0,1\}^*,\ n_{00}(w) = n_{01}(w)\}$ ⇒ Not regular

*** (85) $\{w \mid w \in \{0,1\}^*,\ n_{\boxed{001}}(w) = n_{\boxed{100}}(w)\}$ ⇒ Regular

*** (86) $\{w \mid w \in \{0,1\}^*,\ n_{110}(w) = n_{011}(w)\}$

(87) $\{w \mid w \in \{0,1\}^*,\ n_{000}(w) = n_{111}(w)$ ⇒ Not regular

(88) $\{w \mid w \in \{0,1\}^*,\ n_{\boxed{001}}(w) \leq n_{\boxed{100}}(w)\}$ ⇒ Regular

(89) $\{w \mid w \in \{0,1\}^*,\ n_{\boxed{001}}(w) \neq n_{\boxed{100}}(w)\}$

(90) $\{w \mid w \in \{0,1\}^*,\ n_{00}(w) \neq n_{11}(w)\}$ ⇒ Not reg

Summary

→ Reg lans, Non reg ✓


THANK YOU GW
SOLDIERS !