CS & IT ENGINEERING

Theory of Computation

Finite Automata:

NFA with epsilon moves

Lecture No. 11

By- DEVA Sir

TOPICS TO BE COVERED

01 NFA without ε-moves

02 NFA with ε-moves

03

04

05

NFA Vs DFA

→ Every DFA is NFA

→ NFA need not be DFA

→ Min DFA need not be min NFA

$$\boxed{DFA \cong NFA}$$

I) DFA ⇒ NFA (by definition)

II) NFA ⇒ DFA (subset construction)

$w \longrightarrow \boxed{\text{NFA}}$ accepts L

If $w \in L$, it has atleast one path that halts at final state.

If $w \notin L \Rightarrow$ No path exist
OR
Every path halts at nonfinal

$\delta : Q \times \Sigma \rightarrow 2^Q$

(Without ε transitions)

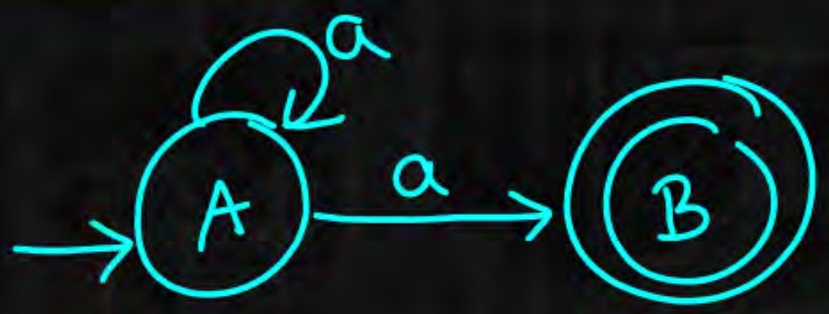
$\delta : Q \times \Sigma \cup \{\varepsilon\} \rightarrow 2^Q$

(With ε moves)

NFA

$\Sigma = \{a\}$

(1)



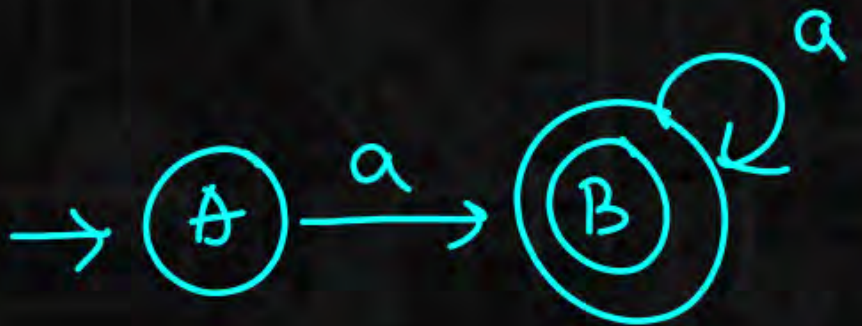
Ex

$a: A \xrightarrow{a} A \mid \underbrace{A \xrightarrow{a} B}_{\text{this path halts at final}}$

$aa:$

$aaa:$

(2)



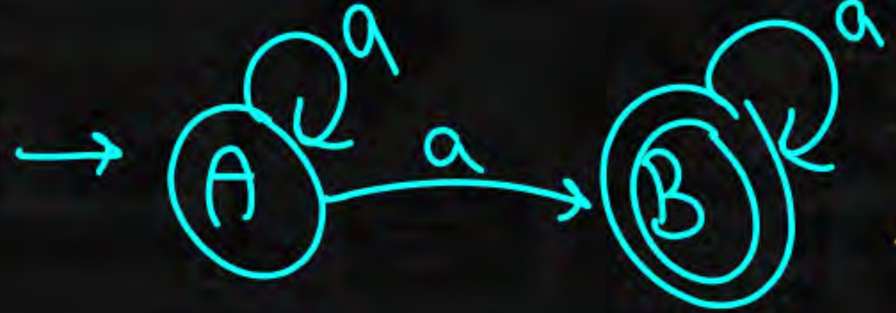
DFA

All are NFAs

$L = a^+$

accepted by all NFAs

(3)



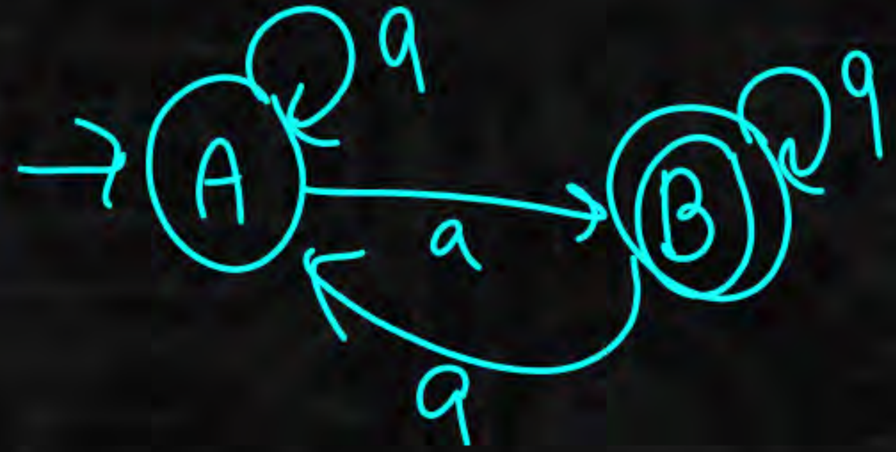
Ex

$a: A \xrightarrow{a} \underset{\text{final}}{B}$

$aa: A \xrightarrow{a} B \xrightarrow{a} \underline{B} \mid A \xrightarrow{a} A \xrightarrow{a} \underset{\text{final}}{B}$

$\vdots$

(4)



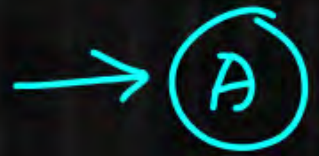
For every regular language,

i) only one min DFA exist

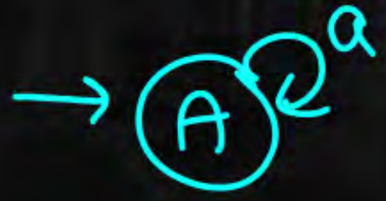
ii) $\underbrace{\text{one or more}}_{\text{not infinite}}$ min NFAs exist

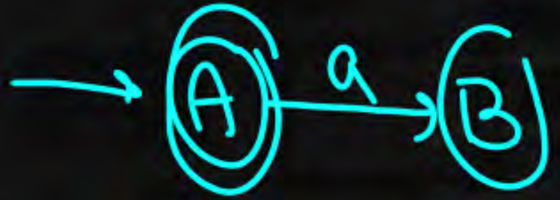
NFA

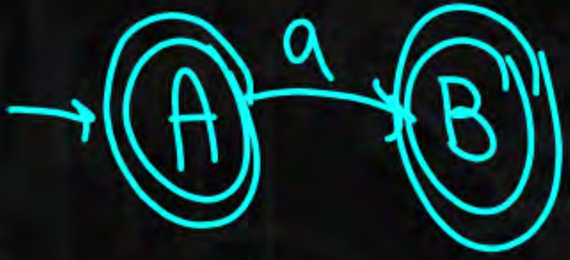
Identify language accepted by NFA. $\Sigma = \{a, b\}$

1.  $L = \phi$

2.  $L = \{\varepsilon\}$

3.  $L = \phi$

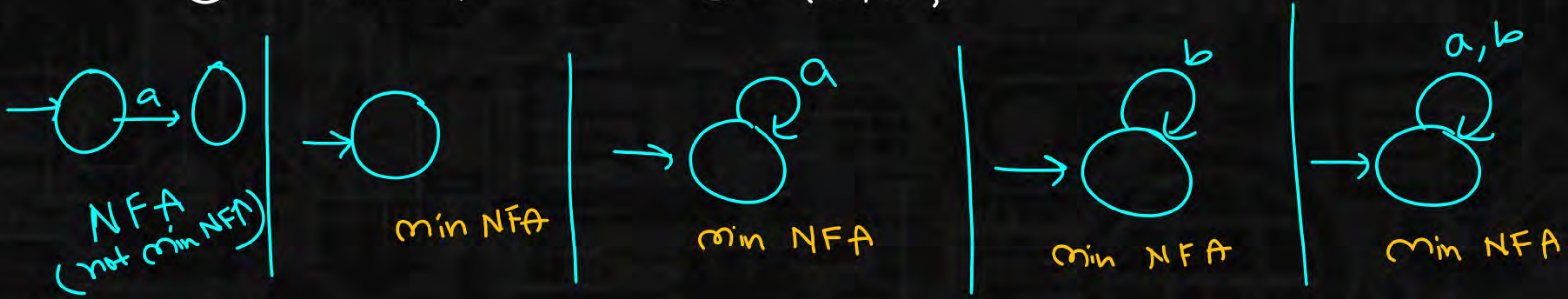
4.  $L = \{\varepsilon\}$

5.  $L = \{\varepsilon, a\}$

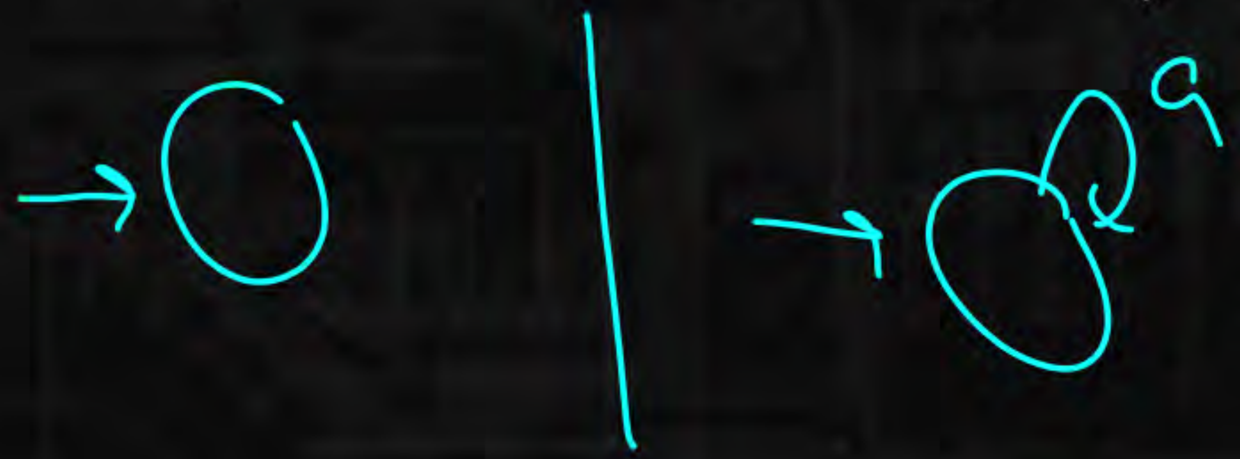
NFA

Construction

① $L = \phi$ over $\Sigma = \{a, b\}$

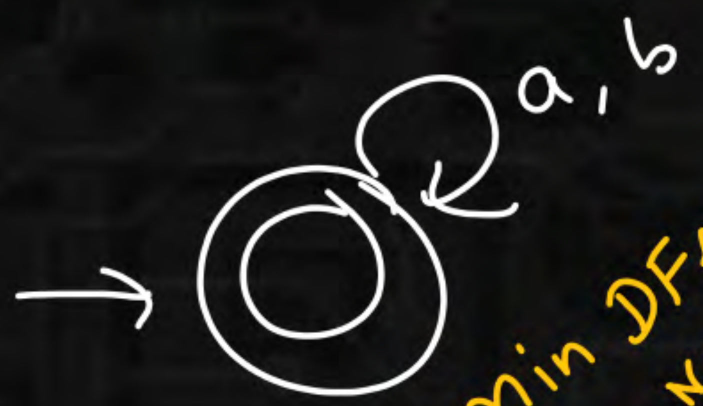


② $L = \phi$ over $\Sigma = \{a\}$



NFA

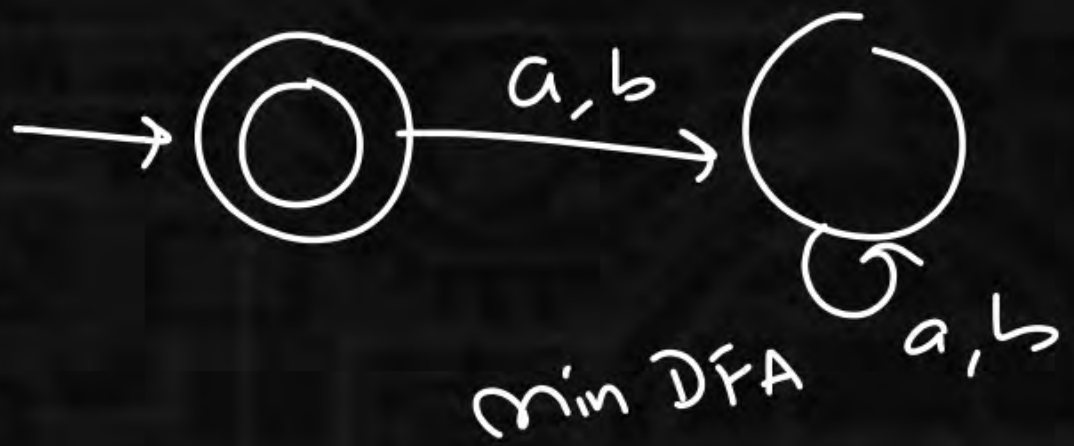
(3) $L = \Sigma^*$ over $\Sigma = \{a, b\}$



min DFA

min NFA

(4) $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

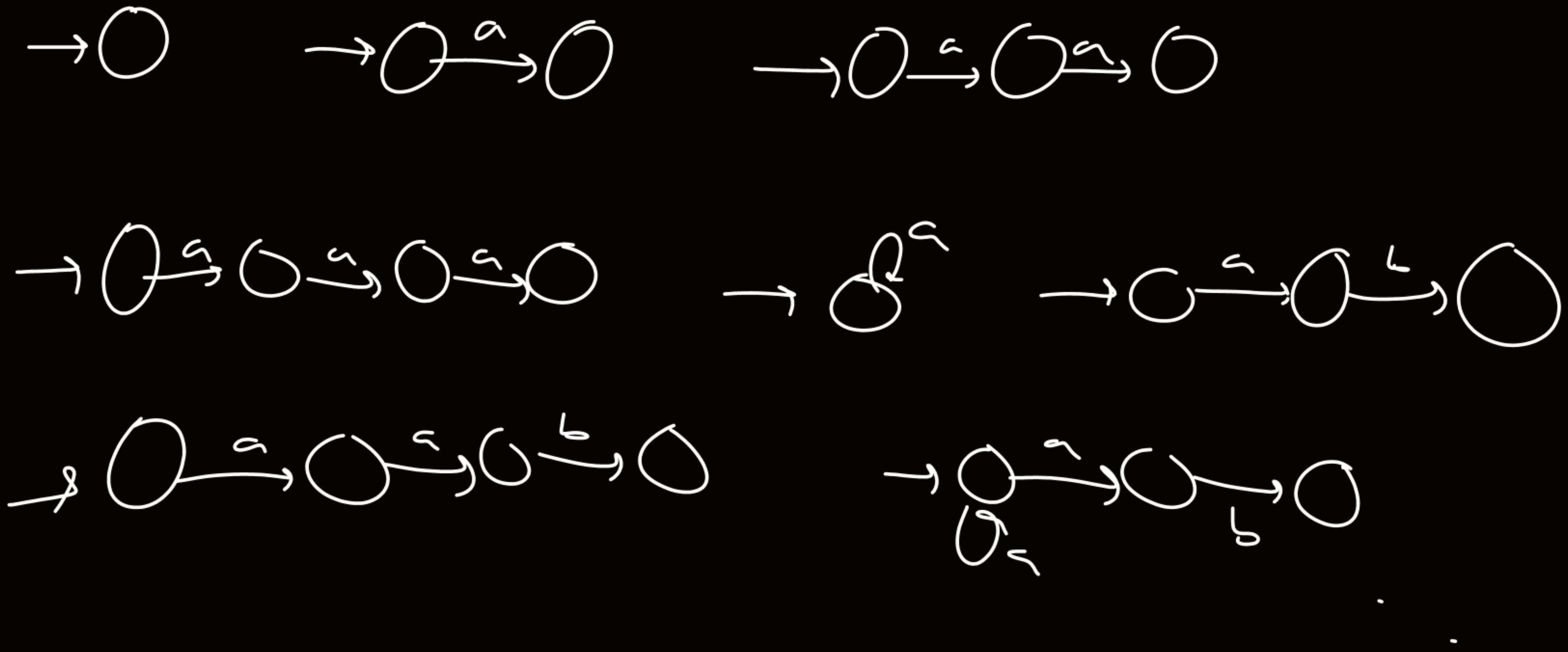


Note: Min DFA may or may not be min NFA.

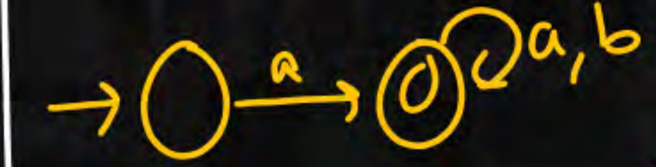
$L = \phi$ over $\Sigma = \{a, b\}$

$\Longrightarrow$ Inf NFAs | $\Rightarrow$ One min DFA

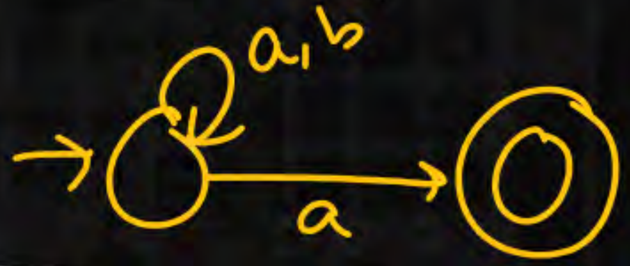
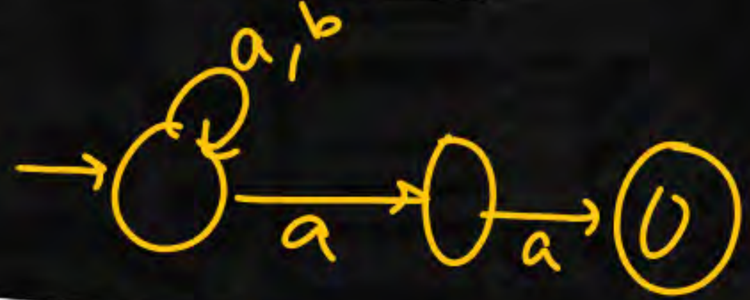
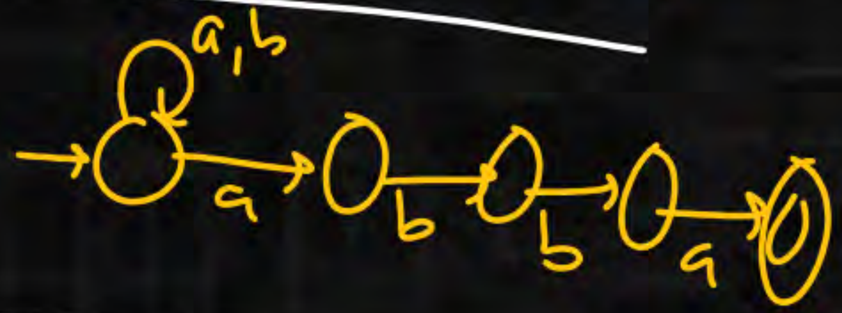
$\Longrightarrow$ Inf DFAs | $\Rightarrow$ One or more min NFAs



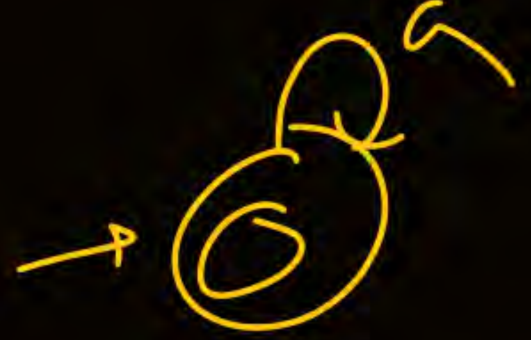
NFA

	min DFA	min NFA
(5) $L = a(a+b)^*$	3 states	2 states  Dead state not present
(6) $L = aa(a+b)^*$	4	3
(7) $L = aba(a+b)^*$	5	4

NFA

	min DFA	min NFA	
(8) $L=(a+b)^*a$	2	2	
(9) $L=(a+b)^*aa$	3	3	
(10) $L=(a+b)^*abba$	5	5	

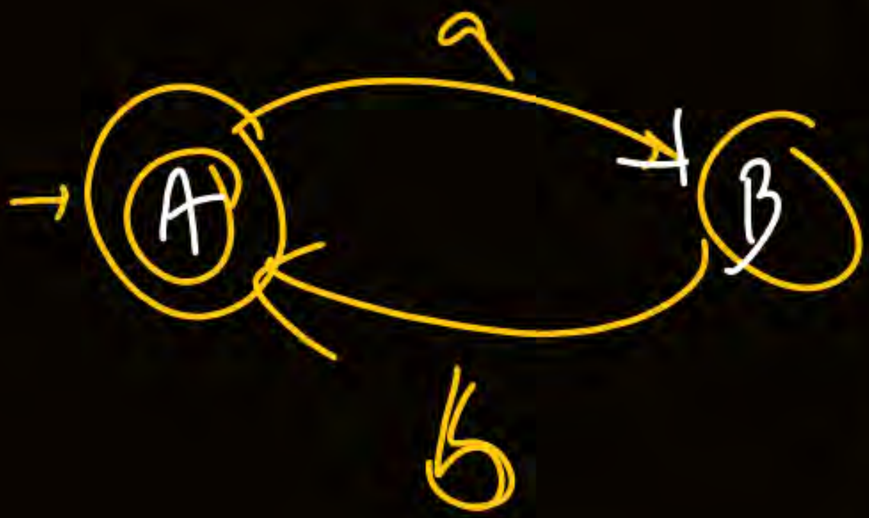
a^*



$(a+b)^*$

b^*

$(ab)^*$

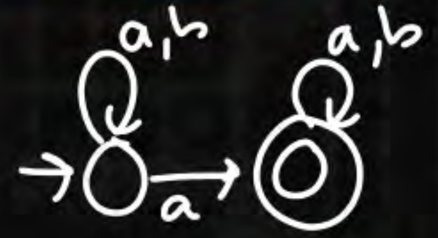
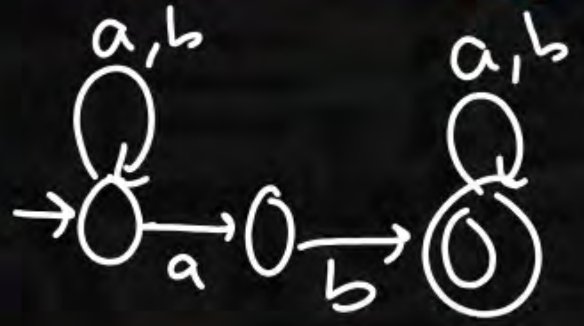


ε ✓

ab ✓ $A \xrightarrow{a} B \xrightarrow{b} A$ fin

abab ✓

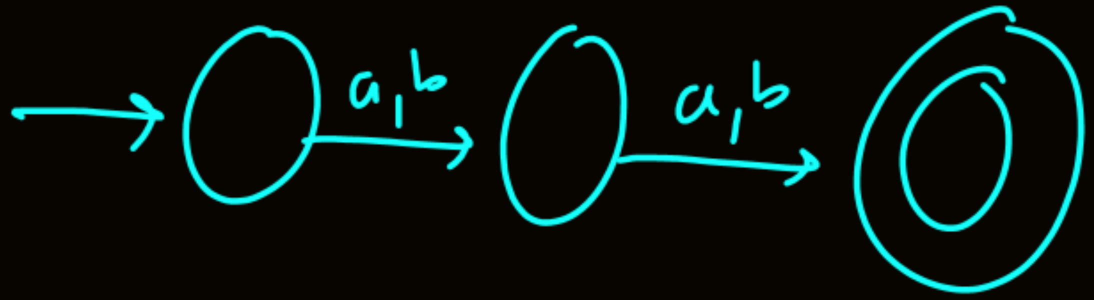
NFA

	min DFA	min NFA
(11) $L = (a+b)^* \underline{a} (a+b)^*$	2	2 
(12) $L = (a+b)^* ab (a+b)^*$	3	3 
(13) $L = (a+b)^* aaa (a+b)^*$	4	4

NFA

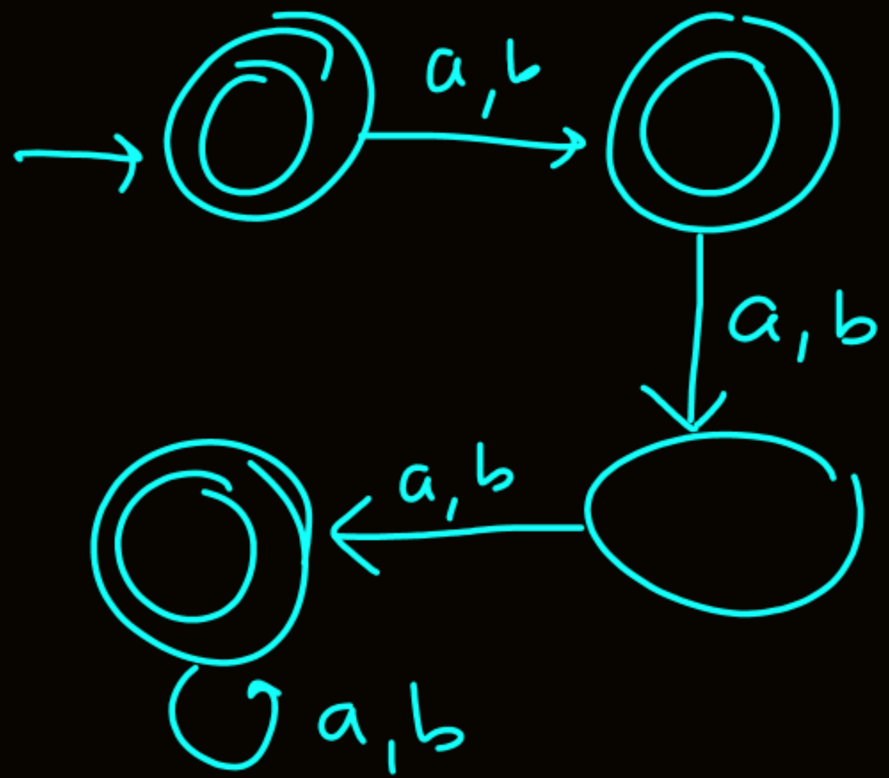
	min DFA	min NFA
(14) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert = 2\}$	4	3
(15) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert \leq 2\}$	4	3
(16) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert \geq 2\}$	3	3
(17) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert < 2\}$	3	2
(18) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert > 2\}$	4	4
*** (19) $\{w \mid w \in \{a,b\}^*, \lvert w \rvert \neq 2\}$	4	4

$|w| = 2$



3 states

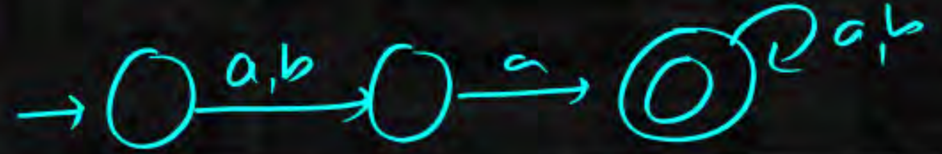
$|w| \neq 2$



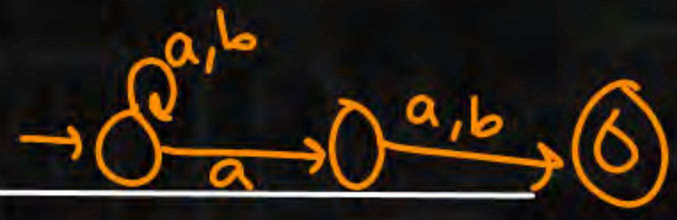
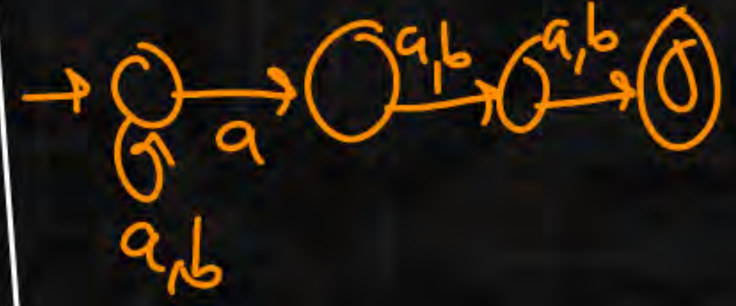
4 states

in NFA

NFA



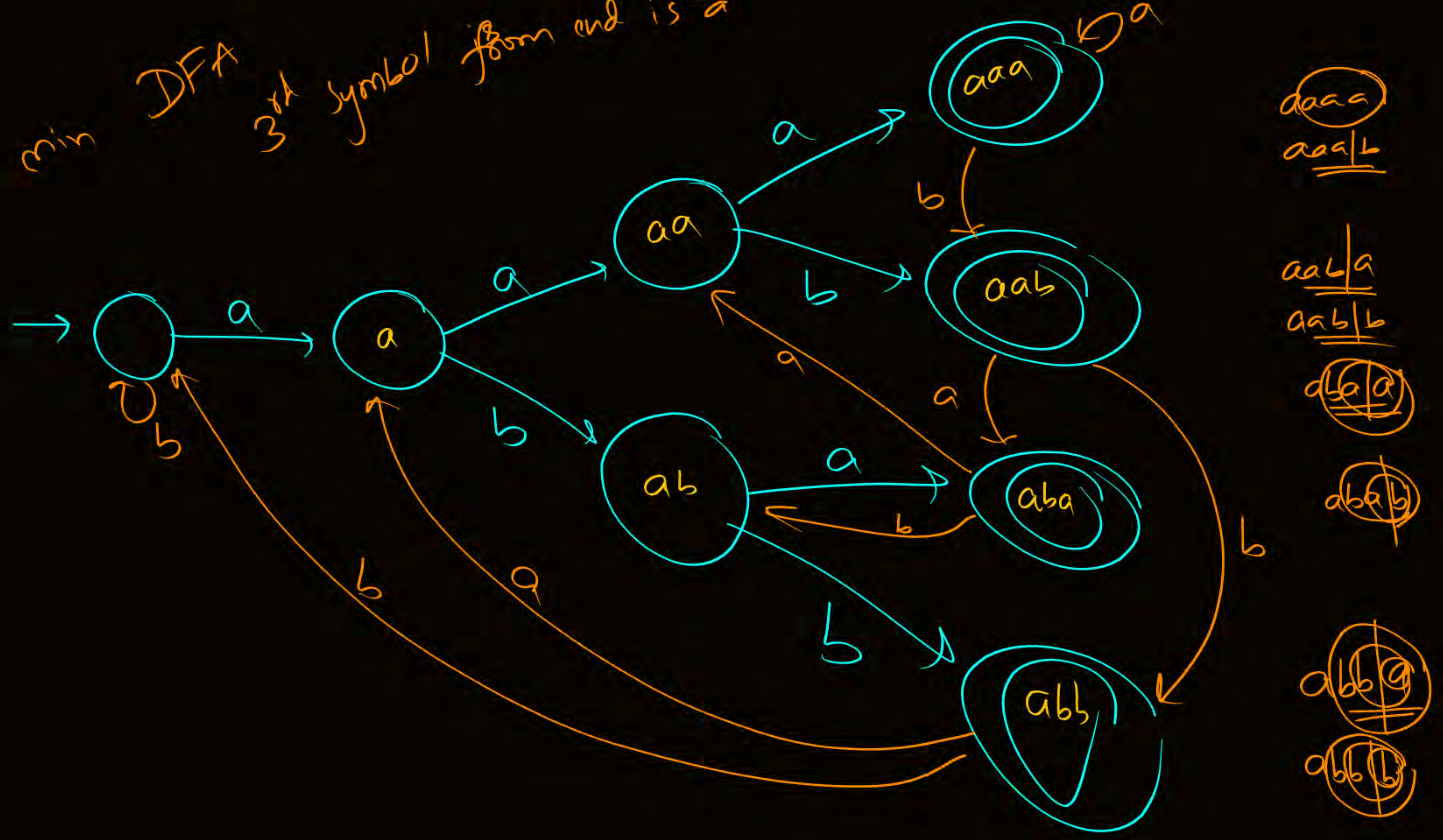
$K+2$ — K^{th} symbol

	Language	min DFA	min NFA
20	$\{xaw \mid x \in \{a,b\}, w \in \{a,b\}^*\}$ $= (a+b)a(a+b)^*$	4	3 no dead state
21	$\{xyaw \mid x,y \in \{a,b\}, w \in \{a,b\}^*\}$	5	4
22	$\{wax \mid x \in \{a,b\}, w \in \{a,b\}^*\}$ $= (a+b)^* a (a+b)$	$2^2 = 4$	3 
23	$\{waxy \mid x,y \in \{a,b\}, w \in \{a,b\}^*\}$ $(a+b)^* a (a+b)(a+b)$	$2^3 = 8$ states	4 

2^K — K^{th} symbol from end

min DFA

3rd symbol from end is 'a'



24) K^{th} symbol from begin is 'a'

For DFA : $K+2$ states

For NFA : $K+1$ states

***25) K^{th} symbol from end is 'a'

For DFA : 2^K states

For NFA : $K+1$ states ✓

NFA

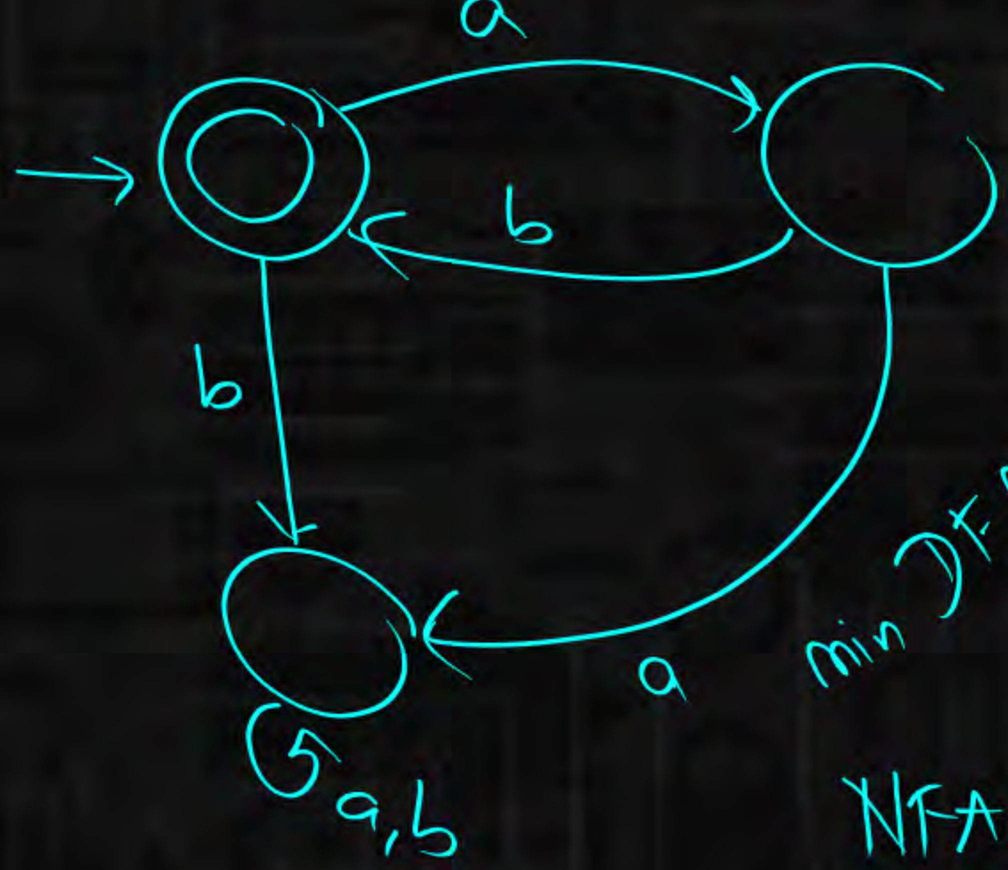
	Min DFA	Min NFA
I) $\underbrace{aaa}_{K=3}\times$	K+2	K+1
II) $\times\underbrace{aaa}_{K=3}\times$	K+1	K+1
III) $\times\underbrace{aaa}_{K=3}$	K+1	K+1
IV) $\lvert w \rvert = K$	K+2	K+1
V) $\lvert w \rvert \le K$	K+2	K+1
VI) $\lvert w \rvert \ge K$	K+1	K+1
VII) K^{th} symbol from begin is 'a'	K+2	K+1
VIII) K^{th} symbol from end is 'a'	2^K	K+1

min string length = K

NFA

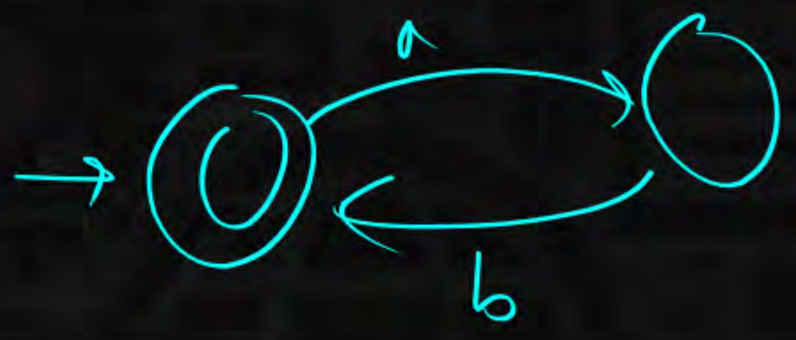
Practice:

① $L = (ab)^* = \{\varepsilon, ab, abab, \ldots\}$



min DFA

NFA but not min NFA

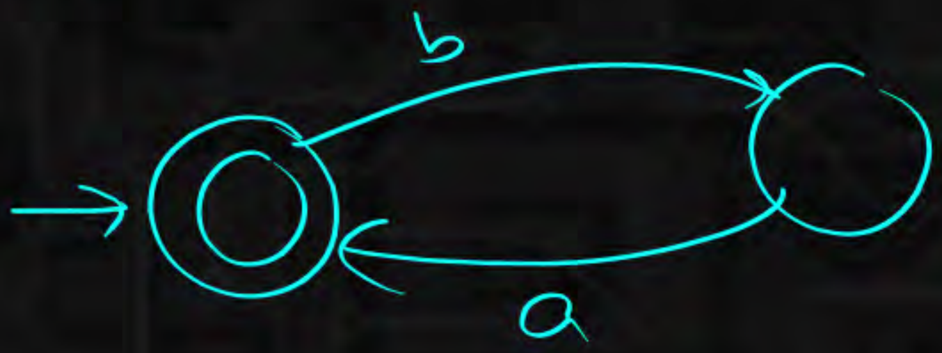


min NFA

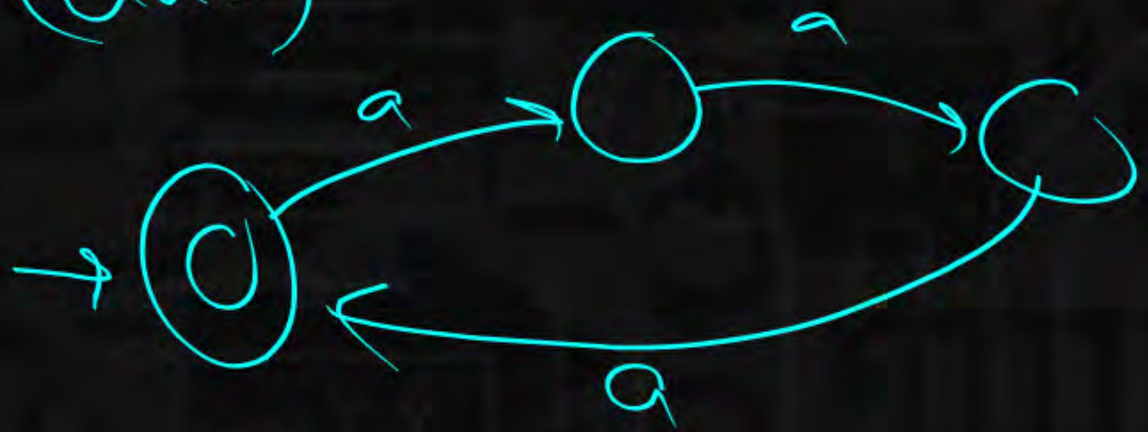
NFA

② $L = (ba)^*$

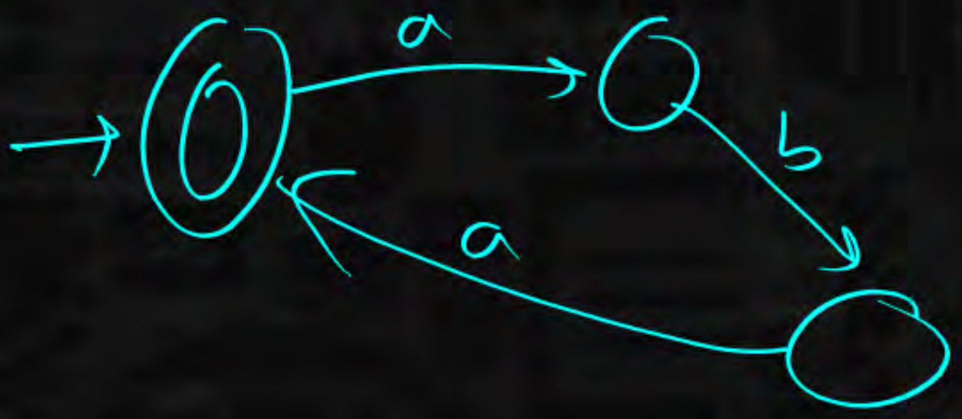
NFA



③ $L = (aaa)^*$

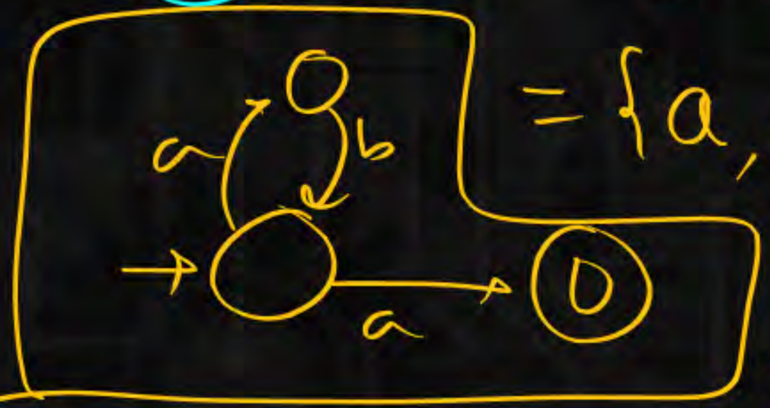


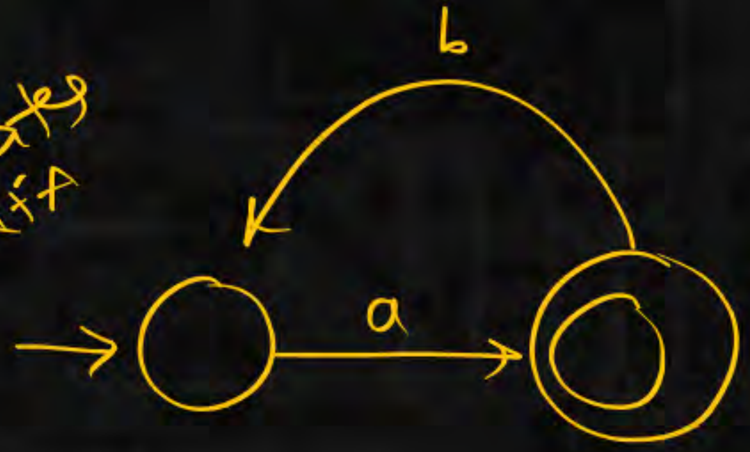
④ $L = (aba)^*$



NFA

5 $L = (ab)^*a = a(ba)^* \Rightarrow 2$ states in NFA

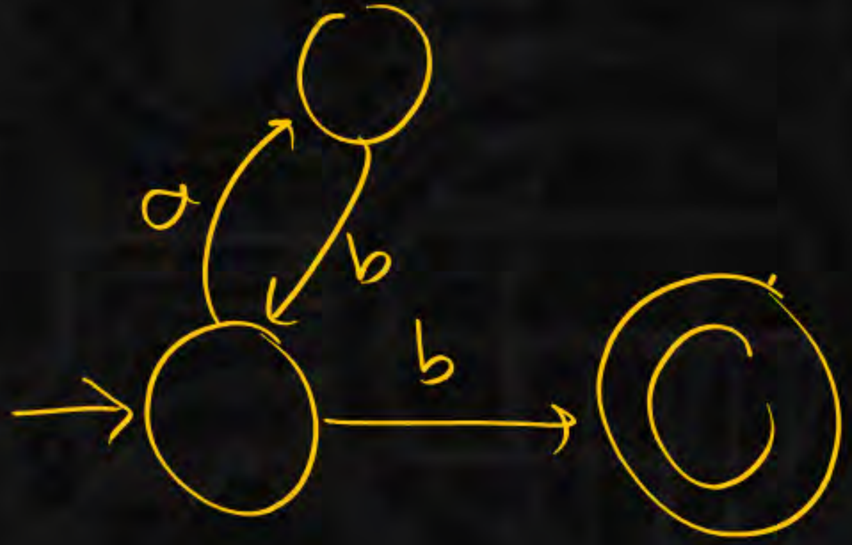




$= \{a, aba, \underbrace{(ab)^2a}_{ababa}, \quad \}$

3 ~~states~~

6 $L = (ab)^*b$



$= \{b, abb, ababb, \ldots\}$

NFA

Q = set of states in NFA

2^Q = set of states in DFA

NFA $\Longrightarrow$ DFA

n states

$\underbrace{\leq 2^n \text{ states}}$

(atmost 2^n states)

Min DFA

($\leq 2^n$ states)

$$Q = \{A, B, C\}$$

state of NFA

state of DFA

$$2^Q = P(Q) = \{\phi, \{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}$$

$$|Q| = n \Rightarrow |2^Q| = 2^n$$

NFA state

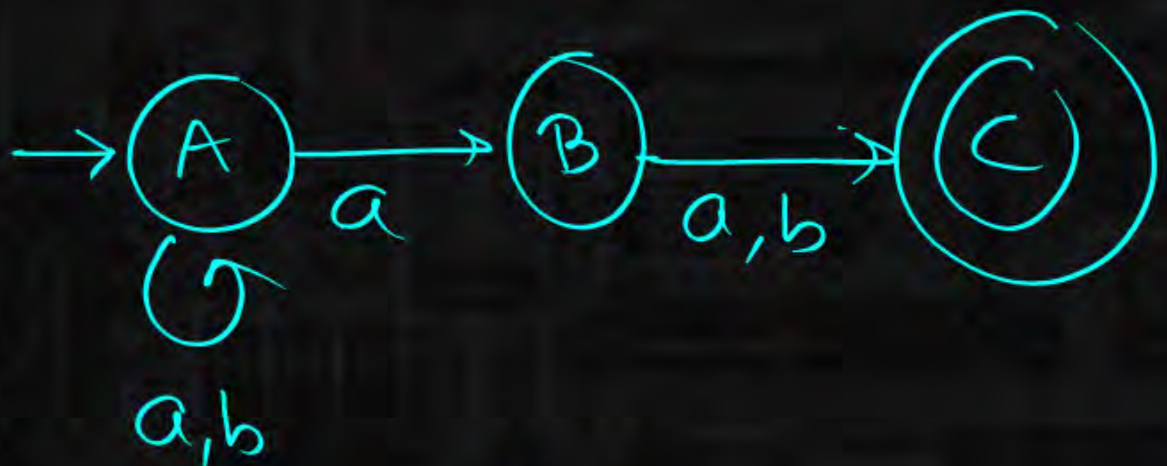
DFA states

NFA

NFA (without ε move) $\Longrightarrow$ DFA [Subset Construction]

Conversion from NFA to DFA

NFA	a	b
→A	{A,B}	{A}
B	{C}	{C}
*C	ϕ	ϕ



NFA

$$NFA = (Q, \Sigma, \delta, A, \{C\})$$

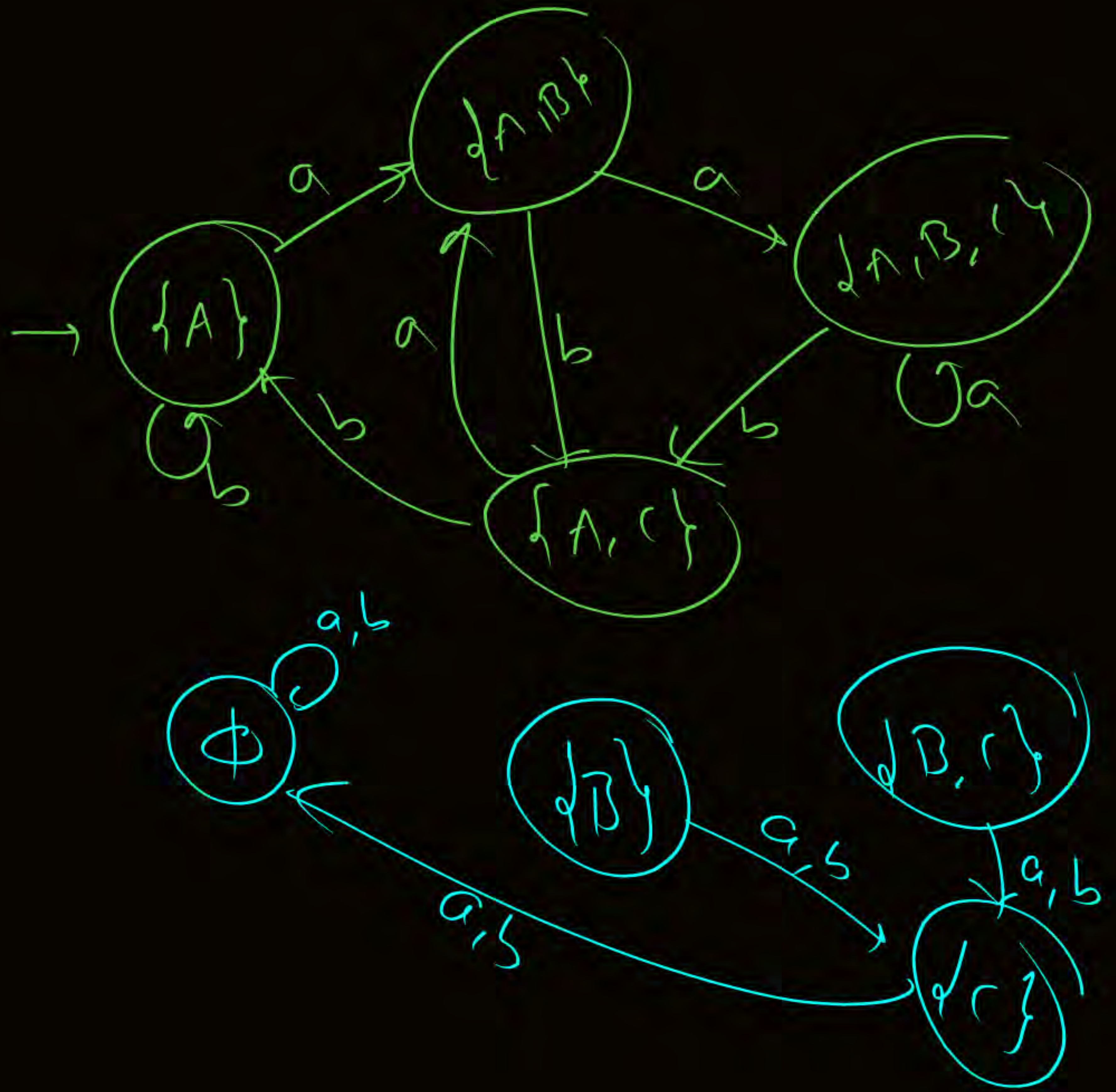
$Q \rightarrow \{A, B, C\}$

$\Sigma \rightarrow \{a, b\}$

$A \rightarrow$ Initial

$\{C\} \rightarrow$ final

	a	b
→{A}	{A,B}	{A}
{A,B}	{A,B,C}	{A,C}
*{A,B,C}	{A,B,C}	{A,C}
*{A,C}	{A,B}	{A}



{A}
{A,B}
{A,B,C}
{A,C}
a
b
a,b
Φ
{B}
{B,C}
{C}

If NFA has n states then equivalent min DFA has atmost 2^n states

If NFA has n states then equ. min DFA has max $\underbrace{\underline{2^n}}_{(\leq 2^n)}$ states

Summary

→ NFA

→ next: NFA with ε-moves


THANK YOU GW
SOLDIERS !