CS & IT ENGINEERING

Theory of Computation

Push Down Automata:

Context Free Grammar

Lecture No. 01

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TOPICS TO BE COVERED

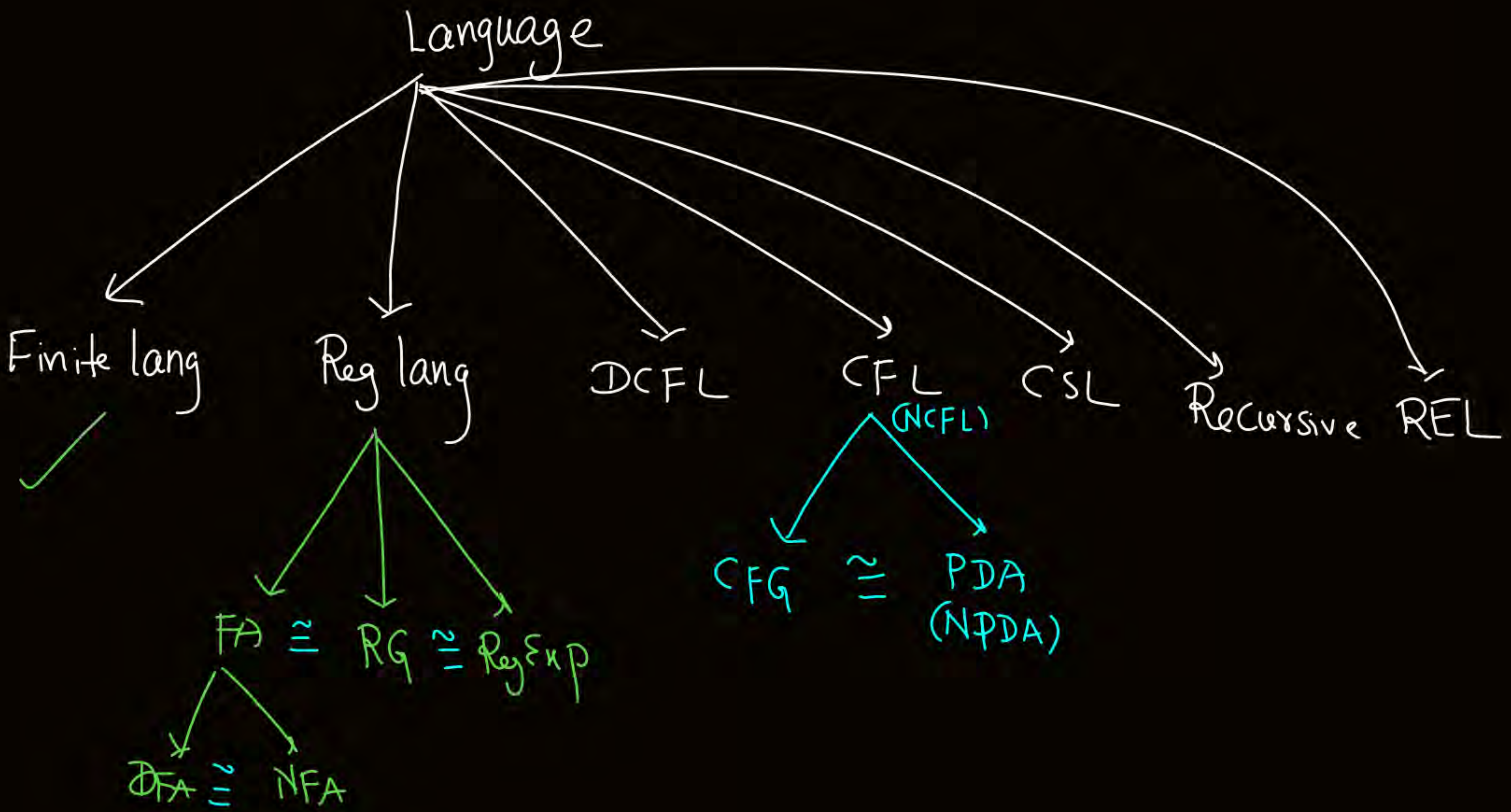
01 Context Free Grammar

02 Derivation of a String

03 Practice on CFGs

04

05


Language
Finite lang
Reg lang
DCFL
CFL
(NCFL)
CSL
Recursive
REL
FA ≅ RG ≅ RegExp
DFA ≅ NFA
CFG ≅ PDA
(NPDA)

Context Free Grammar (CFG)

Regular Grammar (RG)

LLG

$V \to VT^* \mid T^*$

RLG

$V \to T^*V \mid T^*$

Linear Grammar (LG)

$V \to T^* V T^* \mid T^*$

$S \to aS \mid b$

$S \to Sa \mid b$

$S \to aSb \mid c$

CFG

$\underbrace{V}_{\text{exactly one non-terminal}} \longrightarrow \underbrace{(V \cup T)^*}_{\text{no restriction}}$

$S \to AaBSb \mid c$

$A \to a$

$B \to b$

Every LLG is RG
Every LLG is LG
Every LLG is CFG

Every RLG is RG
Every RLG is LG
Every RLG is CFG

Every RG is LG
Every RG is CFG

Every LG is CFG $\Rightarrow$

Context Free Grammar

1. $S \to a$

- LLG ✓
- RLG ✓
- RG ✓
- LG ✓
- CFG ✓

2. $S \to Sab \mid c$

- LLG ✓
- RLG ✗
- RG ✓
- LG ✓
- CFG ✓

3. $S \to aS \mid Sb \mid c$

- LLG ✗
- RLG ✗
- RG ✗
- LG ✓
- CFG ✓

4. $S \to SS \mid a$

- RG ✗
- LG ✗
- CFG ✓

Context Free Grammar : Derivation of a String

Linear

Left Most Derivation

Right Most Derivation

Non-linear (Graph)

Parse Tree (Derivation Tree)

Context Free Grammar

Left Most Derivation

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

$\boxed{w = ab}$

$$\boxed{S} \xrightarrow{S \rightarrow AB} \boxed{AB} \xrightarrow{A \rightarrow a} \boxed{aB} \xrightarrow{B \rightarrow b} \boxed{ab}$$

Sentential form

ab (Sentence) (String)

Left Sentential form:

↳ Any sequence is derived from Start symbol 'S' using LMD

S , AB , aB , ab

No. of Substitutions = No. of steps = Length of Derivation = 3

$$\begin{tikzcd} S \arrow[r, Rightarrow] \arrow[dr, Rightarrow] \arrow[ddr, Rightarrow] & AB \arrow[d, Rightarrow] \\ & aB \arrow[d, Rightarrow] \\ & ab \end{tikzcd}$$

$$S$$

$$\Downarrow$$

$$\underbrace{\text{Any sequence}}_{\text{Sentential form}}$$

What is LMD ?

In every sentential form, $\boxed{\text{left most}}$ nonterminal is substituted to derive a string

What is RMD ?

In every sentential form, $\boxed{\text{Right most}}$ non-terminal is substituted to derive a string.

RMD [Right Most Derivation]

$S \rightarrow AB$
$A \rightarrow a$
$B \rightarrow b$

$\boxed{w = ab}$

LMD:

$$S \Downarrow AB \Downarrow aB \Downarrow ab$$

LMD

$$S \xRightarrow{S \rightarrow AB} AB \xRightarrow{B \rightarrow b} Ab \xRightarrow{A \rightarrow a} ab$$

RMD

Identify Right Sentential forms for deriving "ab" using {S→AB, A→a, B→b}

- S ✓
- AB ✓
- aB ✗
- Ab ✓
- ab ✓
- bB ✗
- Aa ✗

Context Free Grammar

Note: I) LMD and RMD need not be same to derive string.

II) $\boxed{\text{No. of steps in LMD} = \text{No. of steps in RMD}}$ if unique derivation exist for string

III) $\boxed{\text{No. of LMDs} = \text{No. of RMDs} = \text{No. of parse trees}}$

$S \to a \mid AB$

$A \to \varepsilon \mid a$

$B \to \varepsilon \mid a \mid b$

$\boxed{w = a}$ ⇒ How many derivations ? = 3 parse trees

⇒ 3 parse trees

(3 LMDs)

(3 RMDs)

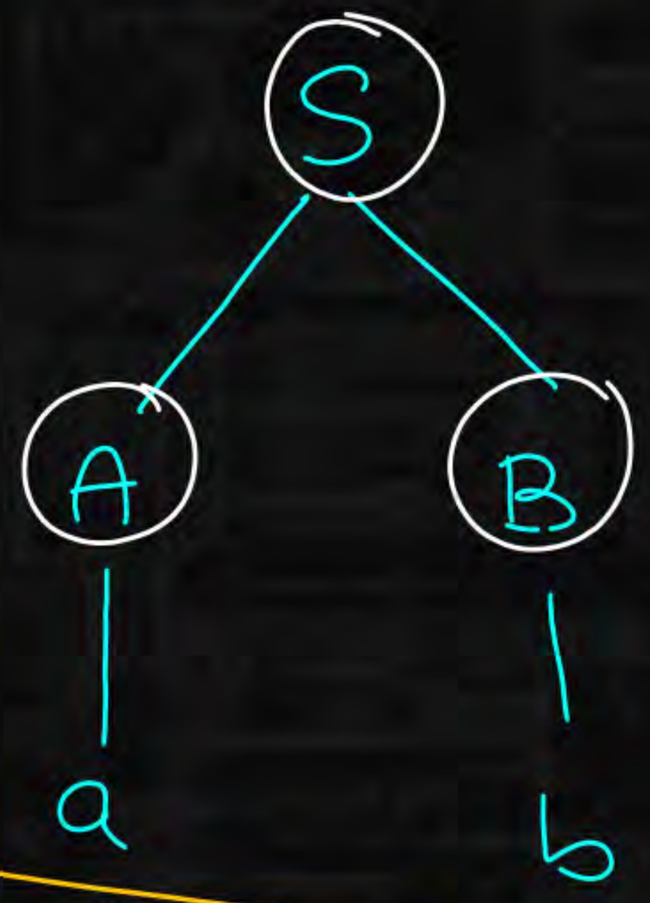
Context Free Grammar

Parse Tree

Root: S (start)

Leaf: ε or terminal

Non leaf: Non-terminal



LMD order: S, A, B

RMD order: S, B, A

$S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow b$

$\boxed{w = ab}$

= ab

Length of derivation
No. of steps in derivation
No. of steps in LMD
No. of steps in RMD
No. of Substitutions in LMD or RMD

$\Rightarrow$ No. of non leaf nodes in parse tree

I) If every string derived from given CFG has only one parse tree

then CFG is Unambiguous (not ambiguous)

$$\boxed{\forall w \in L(G)\ \ \#\text{Parse trees} = \#\text{LMDs} = \#\text{RMDs} = 1}$$

iff

G is unambiguous

II) If some string has more than one parse tree then CFG is Ambiguous

$$\boxed{\exists w \in L(G),\ \#\text{parse trees} \geq 2}$$

Context Free Grammar

Identify Ambiguous and Unamb CFGs.

In compiler, we will practice more

① $\boxed{S \to a \mid b}$

- $a \Rightarrow 1$ PT
- $b \Rightarrow 1$ PT

} Unamb CFG

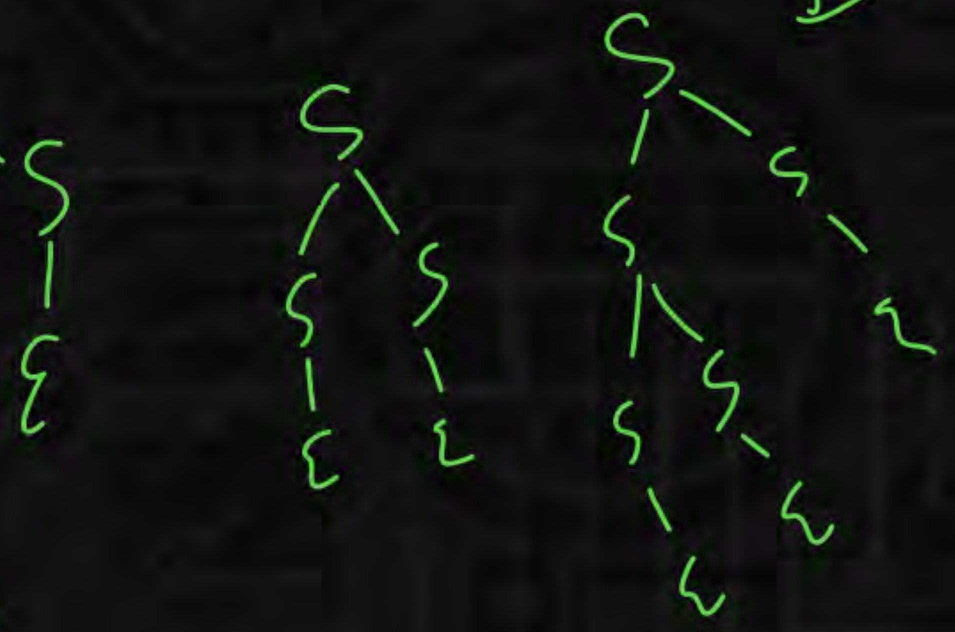
② $\boxed{S \to Sa \mid b}$

- $b \Rightarrow 1$ PT
- $ba \Rightarrow 1$ PT
- $baa \Rightarrow 1$ PT
- ⋮

} Unamb CFG

③ $\boxed{S \to SS \mid \varepsilon}$

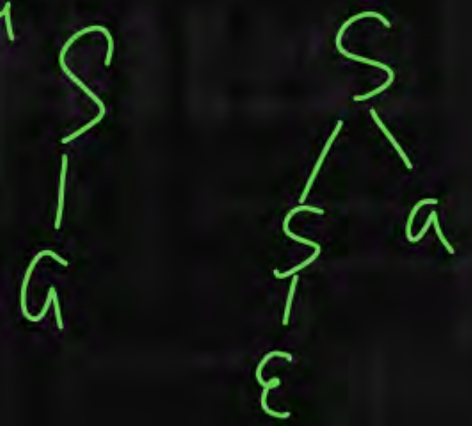
$\varepsilon \Rightarrow$ inf parse tree } Amb CFG



④ $\boxed{S \to Sa \mid a \mid \varepsilon}$

- $\varepsilon \Rightarrow 1$ PT
- $a \Rightarrow 2$ PTs
- ⋮

} Amb CFG



Context Free Grammar

Note: No Algo exist for checking CFG is Ambiguous/Unambiguous.

Amb CFG	Unamb CFG
Some string	Every string
$\Downarrow$	$\Downarrow$
>1 PT	1 PT

Context Free Grammar

Identify CFL generated by given CFG.

① $S \to Sa \mid a$ $\qquad L = aa^* = a^+$

② $S \to Sa \mid b$ $\qquad L = ba^*$

③ $S \to Sa \mid Sb \mid \varepsilon$ $\qquad L = (a+b)^*$

④ $S \to Sa \mid Sb \mid a \mid b$ $\qquad L = (a+b)(a+b)^* = (a+b)^+$

Context Free Grammar

(5) $S \to aS \mid \varepsilon$ $\quad L = a^*$

(6) $S \to aS \mid bS \mid \varepsilon$ $\quad L = (a+b)^*$

(7) $S \to aS \mid bS \mid c$ $\quad L = (a+b)^* S = (a+b)^* c$

(8) $S \to aS \mid bS \mid cS \mid dS \mid \varepsilon$ $\quad L = (a+b+c+d)^*$

(9)

$S \rightarrow Aa \Rightarrow L = A \cdot a$

$= (a+b)^+ a$

$A \rightarrow Ba \mid Bb$ $= B(a+b) = (a+b)^*(a+b) = (a+b)^+$

$B \rightarrow Ba \mid Bb \mid \varepsilon \Rightarrow (a+b)^*$

$$L = (a+b)^+ a$$

Context Free Grammar

(11) $S \to aS \mid Sb \mid c$

$$\boxed{X} \to \alpha \boxed{X} \mid \boxed{X} \beta \mid \gamma$$

$$L = \alpha^* \gamma \beta^*$$

c ✓

ac ✓

~~cb~~ ✓

acb ✓

aacb ✓

acbb

$$\boxed{a^* c b^*}$$

(12) $S \rightarrow aS \mid Sb \mid \boxed{a}$ $\qquad L = a^* a b^* = a^+ b^*$

$= \{a^m b^n \mid m \geq 1, n \geq 0\}$

(13) $S \rightarrow aS \mid Sb \mid b$ $\qquad L = a^* b^+$

*** (14) $S \rightarrow aS \mid Sb \mid a \mid b$

$L = a^*(a+b)b^*$

$= (a^*a + a^*b)b^*$

$= a^+b^* + a^*b^+$

Context Free Grammar

(15) $S \rightarrow aSb \mid c$

$$L = \{a^n c\, b^n \mid n \geq 0\}$$

c ✓

acb ✓

aacbb ✓

$a^n c\, b^n$ ✓

$$X \rightarrow \alpha X \beta \mid \gamma$$

same no-of times

$$\boxed{L = \alpha^n \gamma \beta^n}$$

Context Free Grammar

(16) $S \to aSb \mid \varepsilon$

$L = \{a^n b^n \mid n \geq 0\}$

(17) $S \to aSb \mid ab$

$L = \{a^n b^n \mid n \geq 1\}$ OR $L = \{a^{n+1} b^{n+1} \mid n \geq 0\}$

(18) $S \to aSb \mid a$

$L = \{a^{n+1} b^n \mid n \geq 0\}$

(19) $S \to aSa \mid \varepsilon$

ε ✓

aa ✓

aaaa ✓

aaaaaa ✓

$L = a^n a^n = a^{2n} = (aa)^*$

(20) $S \to aSa \mid a$

$L = a^n a a^n = \{a^{2n+1} \mid n \geq 0\}$

$= a(aa)^* = (aa)^* a$

(21) $S \to aaSb \mid \varepsilon$

$L = \{a^{2n} b^n \mid n \geq 0\}$

(22) $S \to aSbb \mid \varepsilon$

$L = \{a^n b^{2n} \mid n \geq 0\}$

(23) $S \to aaSbb \mid \varepsilon$

$L = (aa)^n (bb)^n = \{a^{2n} b^{2n} \mid n \geq 0\}$

Context Free Grammar

(24)

$S \rightarrow AB$

$A \rightarrow aA \mid \varepsilon \Rightarrow a^*$

$B \rightarrow aBb \mid \varepsilon \Rightarrow a^n b^n$

$L = a^* a^n b^n$

$= a^m a^n b^n$

$= \{a^i b^j \mid i \geq j\}$

(26)

$S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon \Rightarrow a^k b^k$

$B \rightarrow bBc \mid \varepsilon \Rightarrow b^n c^n$

$L = \{a^k \underbrace{b^k b^n} c^n\} = \{a^k b^{k+n} c^n \mid k, n \geq 0\}$

(25)

$S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon \Rightarrow a^n b^n$

$B \rightarrow bB \mid \varepsilon \Rightarrow b^*$

$L = a^n b^n b^*$

$= a^n b^n b^m$

$= \{a^i b^j \mid j \geq i\}$

(27)

$S \rightarrow AB$

$A \rightarrow aAb \mid \varepsilon$

$B \rightarrow cBd \mid \varepsilon$

$L = \{a^n b^n c^k d^k\} = \{a^m b^m c^n d^n\}$

Context Free Grammar

(28) $S \to aSb \mid A$

$\boxed{A \to cAb \mid \varepsilon} \Rightarrow c^n b^n$

$S \to aSb \mid c^n d^n$

$L = \{ w \mid w \in a^* c^* b^*, \; n_b(w) = n_a(w) + n_c(w) \}$

$L = a^k c^n b^n b^k$

$= \{ a^k c^n b^{n+k} \} = \{ a^m c^n b^{m+n} \}$

Context Free Grammar

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$S \to SA \mid \varepsilon$

$\boxed{A \to aAb \mid \varepsilon}$

$\to a^n b^n$

ε ✓

A ✓

$AA \longrightarrow ab \; a^3b^3$

AAA ✓

$\cancel{\{(a^n b^n)^* \mid n \geq 0\}}$

$n=0$

$n=1 \Rightarrow (a^1b^1)^*$

$n=2 \Rightarrow (a^2b^2)^*$

$A^* = L(A)^* = \{a^n b^n\}^*$

$= \{a^n b^n \mid n \geq 0\}^*$

i) $\{a^n b^n \mid n \geq 0\}$

ii) ✓ $\{a^n b^n \mid n \geq 0\}^*$

~~iii) $\{(a^n b^n)^* \mid n \geq 0\}$~~

iv) ✓ $\{\underbrace{a^{n_1} b^{n_1}}_{A} \underbrace{a^{n_2} b^{n_2}}_{A} \underbrace{a^{n_3} b^{n_3}}_{A} \cdots \underbrace{a^{n_k} b^{n_k}}_{A} \mid k \geq 1\}$

v) $\{a^n b^n a^n b^n \ldots k \text{ times} \mid k \geq 1\}$

not CFL $\left\{ (a^n b^n)^* \mid n \geq 0 \right\} = (a^n b^n)^0 \cup (a^n b^n)^1 \cup (a^n b^n)^2 \cup \ldots$

$n=1$

$(ab)^*$

$a^n b^n a^n b^n$

$a^2 b^2 a^2 b^2$

$a^1 b^1 aabb$ not generated

$S \rightarrow SA \mid \varepsilon$

$A \rightarrow aAb \mid \varepsilon$

$S \Rightarrow SA \Rightarrow SAA \Rightarrow AA$

$\Rightarrow aAb\,A$

$\Rightarrow ab\,A$

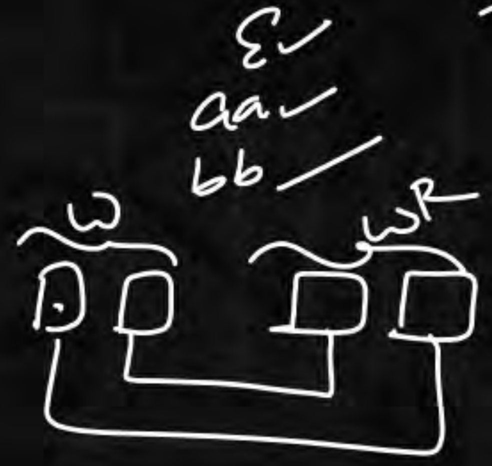
$\Rightarrow ab\,aAb$

$\Rightarrow ab\,aaAbb$

$\Rightarrow abaabb$

Context Free Grammar

(30) $S \rightarrow aSa \mid bSb \mid \varepsilon$



$L = \{ww^R \mid w \in \{a,b\}^*\}$

= set of all even length palindromes

(31) $S \rightarrow aSa \mid bSb \mid a \mid b$

$L = \{wxw^R \mid w \in \{a,b\}^*, x \in \{a,b\}\}$

= Set of all odd length palindromes

(32) $S \rightarrow aSa \mid bSb \mid a \mid b \mid \varepsilon$

L = Set of all palindromes

palindrome concept

$S = a\ b\ b\ \ldots\ \ldots\ b\ b\ a$

first — last

$$\boxed{S = S^{Rev}}$$

$X \rightarrow \boxed{\alpha}\, X\, \boxed{\alpha} \mid \boxed{\beta}\, X\, \boxed{\beta} \mid \gamma$

33 $S \to aSa \mid bSb \mid \#$

$\Sigma = \{a, b, \#\}$

$L = \{ w \# w^R \mid w \in \{a, b\}^* \}$

33 $S \to aSa \mid bSb \mid cSc \mid \varepsilon \mid a \mid b \mid c$

$L = \{ w x w^R \mid w \in \{a, b, c\}^*,\ x \in \{\varepsilon, a, b, c\} \}$

= Set of all palindromes over $\Sigma = \{a, b, c\}$

H.W.

34

$S \to ABC$

$A \to aA \mid \varepsilon$

$B \to Bb \mid \varepsilon$

$C \to aC \mid \varepsilon$

35

$S \to ABC$

$A \to aAb \mid \varepsilon$

$B \to bBd \mid \varepsilon$

$C \to dBa \mid \varepsilon$

36

$S \to AB \mid CD$

$A \to aA \mid \varepsilon$

$B \to bBc \mid \varepsilon$

$C \to aCb \mid \varepsilon$

$D \to cD \mid \varepsilon$

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$S \to aSb \mid A$

$A \to aA \mid Ab \mid a \mid b$

Summary

→ CFG ✓

Next: PDA

Every Reg lang is CFL

→ Some non regulars are CFLs
Some non regulars are not CFLs

$a^n b^n$ ✓

$a^n b^n c^{m+n}$ ✓

$a^m b^n \mid m \geq n$ ✓

$a^m b^n \mid m \leq n$ ✓

$ww^R \mid w \in \{a, b\}^*$ ✓

$w\#w^R \mid$ " ✓


THANK YOU GW
SOLDIERS !