CS & IT ENGINEERING

Theory of Computation

Finite Automata:

Closure Properties – Part 2

Lecture No. 18

By- DEVA Sir

TOPICS TO BE COVERED

01 Closure properties for regulars

02 Important Questions

03

04

05

closed $\Rightarrow$ proof by Algorithm

Not closed $\Rightarrow$ proof by Example

Closure Properites for regular languages

① Union for regular languages

Regular $Lang_1$ $\cup$ Regular $Lang_2$ $\Longrightarrow$ Regular Language

Given L_1 and L_2
$\Downarrow$ $\Downarrow$
Proof 1: $\boxed{RegExp_1 + RegExp_2}$
$\Downarrow$
$L_1 \cup L_2$

Proof 2: $\boxed{FA_1 \times FA_2}$ $\rightarrow \cup$, $\rightarrow \cap$, $\rightarrow -$
compound FA

Proof 3: L_1 $\Rightarrow$ LLG_1 (S_1); L_2 $\Rightarrow$ LLG_2 (S_2)

$\Downarrow$

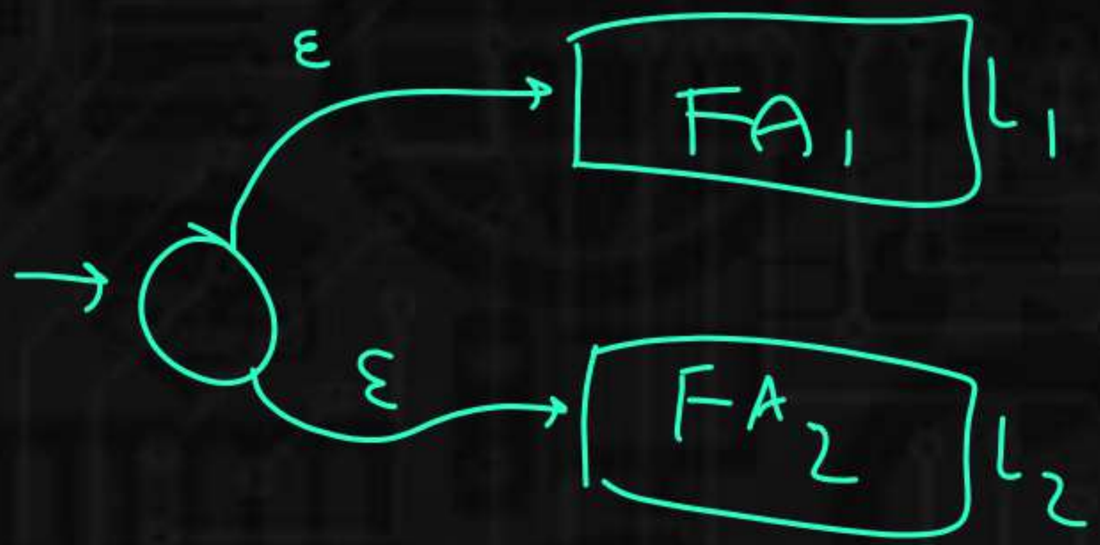
$$\boxed{\begin{array}{l} S \rightarrow S_1 \mid S_2 \\ LLG_1 \\ LLG_2 \end{array}}$$

New LLG
$\Downarrow$
$L_1 \cup L_2$

Closure Properites

Proof 4 : Use RLGs

Proof 5 : Use ε-NFA



Closure Properites

① $L_1 = a^*$, $L_2 = b^*$ $\Rightarrow L_1 \cup L_2 = a^* + b^* = \varepsilon + a^+ + b^+ = a^* + b^+ = a^+ + b^*$

$* \begin{cases} \to 0 \\ \to + \end{cases}$

$a^* b^* \Rightarrow \boxed{\varepsilon}\ \ \boxed{b^+}\ \ \boxed{a^+}\ \ \boxed{a^+ b^+}$

② $L_1 = (a+b)^*$, $L_2 = a^* b^*$ $\Rightarrow L_1 \cup L_2 = (a+b)^* = L_1$

③ $L_1 = a^+ b^+$, $L_2 = a^* b^*$ $\Rightarrow L_1 \cup L_2 = a^* b^* = L_2$

*** ④ $L_1 = a^+ b^*$, $L_2 = a^* b^+$ $\Rightarrow L_1 \cup L_2 = a^+ b^* + a^* b^+ = a^* a b^* + a^* b b^* = a^* (a+b) b^*$

Closure Properites

Note:

I) If L_1 and L_2 are regular languages then $L_1 \cup L_2$ is Regular Language

*** II) If $L_1 \cup L_2$ is Regular language then L_1 is may or may not be regular

Given $L_1 \cup L_2$ is Regular

$$\text{nonreg} \cup \Sigma^* = \Sigma^*$$

$$\text{nonreg} \quad a^n b^n \cup \Sigma^* = \Sigma^*$$

$$\text{reg} \quad a^* b^* \cup \Sigma^* = \Sigma^*$$

Closure Properites

III) If L_1 and L_2 are non regular languages then

$L_1 \cup L_2$ is May or may not be nonreg

i) $a^n b^n \cup a^n b^n \Rightarrow a^n b^n$

ii) $a^n b^n \cup \overline{a^n b^n} \Rightarrow \underset{\text{reg}}{\Sigma^*}$

$$\boxed{\text{Union is not closed for non-regular languages.}}$$

Closure Properites

IV) Reg $\cup$ Non-Reg $\Longrightarrow$ May or may not be regular

i) $\Sigma^* \cup$ nonreg $\Rightarrow$ reg (Σ^*)

ii) $\phi \cup$ nonreg $\Rightarrow$ nonreg

Closure Properites

(2) Intersection

↳ closed for regular languages

$Reg_1 \cap Reg_2 \implies$ Always Regular

Proof:

$$\begin{array}{ccc} L_1 & & L_2 \\ \text{reg} & & \text{reg} \\ \Downarrow & & \Downarrow \\ FA_1 & & FA_2 \\ & \searrow \; \times \; \swarrow & \end{array}$$

$$\underbrace{\qquad\qquad\qquad}_{FA_1 \times FA_2}$$

Compound FA

Closure Properites

① $L_1 = \phi$, $L_2 = \text{Any}$ $\Rightarrow L_1 \cap L_2 = \underline{\phi}$

② $L_1 = \Sigma^*$, $L_2 = \text{Any}$ $\Rightarrow L_1 \cap L_2 = \underline{L_2}$

③ $L_1 = a^+ b^+$, $L_2 = a^* b^*$ $\Rightarrow L_1 \cap L_2 = \underline{L_1}$

④ $L_1 = a^*$, $L_2 = b^*$ $\Rightarrow L_1 \cap L_2 = \underline{\{\varepsilon\}}$

⑤ $L_1 = ab^*$, $L_2 = a^*b$ $\Rightarrow L_1 \cap L_2 = \underline{\{ab\}}$

$ab^* = \{ab^0, ab^1, ab^2, \ldots\}$

$= \{a, ab, abb, \ldots\}$

$a^*b = \{b, ab, aab, \ldots\}$

Closure Properites

I) Reg $\cap$ Reg $\Rightarrow$ Regular language

II) Reg $\cap$ Non-reg $\Rightarrow$ May or may not be regular

III) Non Reg $\cap$ Non Reg $\Rightarrow$ may or may not be regular

$a^n b^n \cap \overline{a^n b^n} = \phi_{reg}$

$a^n b^n \cap a^n b^n \Rightarrow a^n b^n_{nonreg}$

Closure Properites

$$\left.\begin{array}{l}\phi \\ \\ \Sigma^* \end{array}\right\} \text{regs}$$

$$\left.\begin{array}{l}a^n b^n \\ \\ \overline{a^n b^n} \end{array}\right\} \text{non regs}$$

$$\phi \cap \text{Any} \Rightarrow \phi$$

$$\phi \cap a^n b^n \Rightarrow \phi$$

Closure Properites

③ Complement

↳ closed for regular languages

I) If L is Regular then $\bar{L}$ is Regular

II) If $\bar{L}$ is Regular then L is Regular

III) L is Regular iff $\bar{L}$ is Regular

IV) L is nonregular iff $\bar{L}$ is nonregular

Proof :

L is Regular

$\Downarrow$

DFA

$\Downarrow$ f↔nf

DFA

$\Downarrow$

$\bar{L}$ is Regular

$\bar{L}$ is Regular

$\Downarrow$

DFA

$\Uparrow$ f↔nf

DFA

$\Downarrow$

L is Regular

Closure Properites

GATE 2023

$L = \{w \mid w \in \{0,1\}^*$, w does not contain 3 consecutive 1's or more$\}$

no. of states required in min DFA that accepts L is ____

$\bar{L} = (0+1)^* 111 (0+1)^*$

$\bar{L} \Rightarrow$ contain 3 consecutive is 4 states

$\Downarrow$

$L \Rightarrow$ 4 states

Closure Properites

$\Sigma = \{a, b\}$

① $L = \phi \Rightarrow \bar{L} = \Sigma^*$

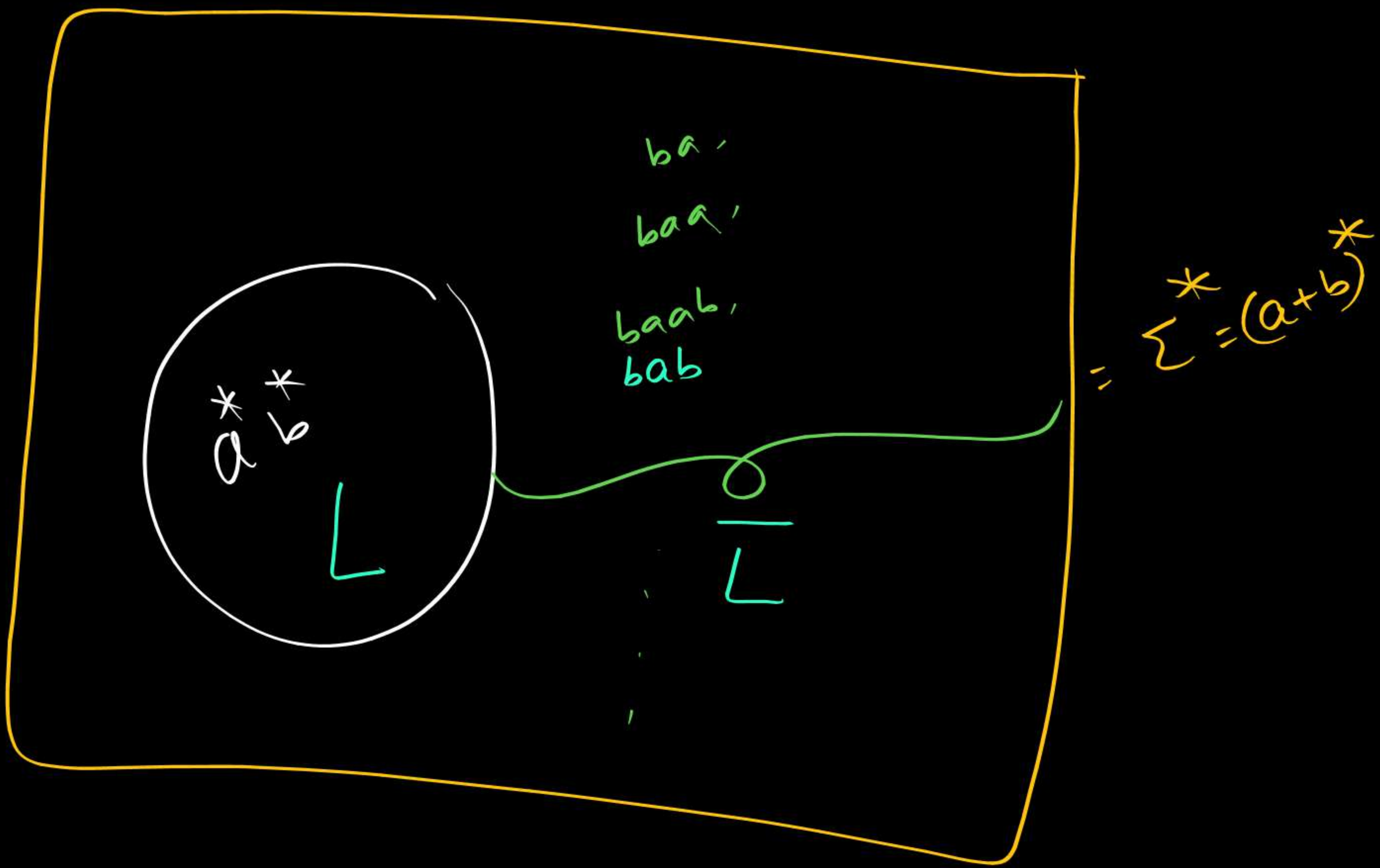
② $L = \Sigma^* \Rightarrow \bar{L} = \phi$

③ $L = a\Sigma^* \Rightarrow \bar{L} = \Sigma^* - L = \Sigma^* - a\Sigma^* = b\Sigma^* + \epsilon$

④ $L = \Sigma^* a \Rightarrow \bar{L} = \underbrace{\Sigma^* b} + \underbrace{\epsilon}$

⑤ $L = \Sigma^* a \Sigma^* \Rightarrow \bar{L} = b^*$

⑥ $L = a^* b^* \Rightarrow \bar{L} = \Sigma^* ba \Sigma^* = (a+b)^* ba (a+b)^*$



ba,
baa,
baab,
bab
$\overline{L}$
$a^* b^*$
L
$= \Sigma^* = (a+b)^*$

$$L = \underbrace{a^* b^*}$$

a's followed by b's

$$\bar{L} = \underbrace{\Sigma^* . \underbrace{ba}_{\text{must}} . \Sigma^*} \longrightarrow \text{min} = ba$$

Contains 'ba'

$$L \cup \bar{L} = \Sigma^*$$

$$\underbrace{a\Sigma^* \quad b\Sigma^* \quad \varepsilon}_{\Sigma^*}$$

Closure Properites

I) $\boxed{\overline{L} = \Sigma^* - L}$

II) $\boxed{L \cup \overline{L} = \Sigma^*}$

III) $\boxed{L \cap \overline{L} = \phi}$

$w \in L$

iff

$w \notin \overline{L}$

$w \in \overline{L}$

iff

$w \notin L$

Closure Properites

(4) Difference

↳ closed for regular languages

$Reg_1 - Reg_2 \Longrightarrow$ Always Regular

Proof:

$FA_1 \times FA_2$

Closure Properites

(1) $L_1 = a^*$, $L_2 = b^*$ $\Rightarrow$ $L_1 - L_2 = a^* - b^* = a^+$

$L_2 - L_1 = b^* - a^* = b^+$

(2) $L_1 = \phi$, $L_2 = \text{Any}$ $\Rightarrow$ $L_1 - L_2 = \phi - L_2 = \phi$

$L_2 - L_1 = L_2 - \phi = L_2$

(3) $L_1 = \Sigma^*$, $L_2 = \text{Any}$ $\Rightarrow$ $L_1 - L_2 = \Sigma^* - L_2 = \overline{L_2}$

$L_2 - L_1 = L_2 - \Sigma^* = \phi$

Note:

$L - \Sigma^* = \phi$

$\Sigma^* - L = \overline{L}$

Closure Properites

I) $Reg_1 - Reg_2 \Rightarrow$ Regular

II) Reg − NonReg $\Rightarrow$ May or may not be Regular
- i) ϕ − Nonreg $\Rightarrow$ ϕ_{reg}
- ii) Σ^* − nonreg $\Rightarrow$ nonreg

III) NonReg − Reg $\Rightarrow$ May or may not be Regular
- i) Nonreg − ϕ $\Rightarrow$ nonreg
- ii) Nonreg − Σ^* $\Rightarrow$ Reg

IV) $NonReg_1 - NonReg_2 \Rightarrow$ May or may not be regular
- i) $a^n b^n - a^n b^n \Rightarrow$ Reg
- ii) $a^n b^n - b^n a^n = \underbrace{a^n b^n \,|\, n \geq 1}_{\text{nonreg}}$

Closure Properites

(5) Concatenation

↳ closed for regular languages

Proof 1: Use Reg Exp

Proof 2: Use ε-NFA

$$Reg_1 \cdot Reg_2 \implies \text{Regular}$$

Closure Properites

$\Sigma = \{a, b\}$

1. $L_1 = a^*$, $L_2 = b^*$ $\Rightarrow$ $L_1 \cdot L_2 = a^* b^*$, $L_2 \cdot L_1 = b^* a^*$

2. $L_1 = \phi$, $L_2 = \text{Any}$ $\Rightarrow$ $L_1 \cdot L_2 = \phi$, $L_2 \cdot L_1 = \phi$

3. $L_1 = \{a\}$, $L_2 = \Sigma^*$ $\Rightarrow$ $L_1 \cdot L_2 = a\Sigma^* = a(a+b)^*$, $L_2 \cdot L_1 = \Sigma^* a = (a+b)^* a$

Closure Properites

I) $Reg_1 \cdot Reg_2 \Rightarrow$ Regular language

II) $Reg \cdot NonReg \Rightarrow$ either reg or non reg

i) $\phi \cdot Nonreg \Rightarrow reg$

ii) $\varepsilon \cdot Nonreg \Rightarrow Nonreg$

III) $NonReg \cdot Reg \Rightarrow$ either reg or not reg

*** IV) $NonReg_1 \cdot NonReg_2 \Rightarrow$ either reg or not reg

i) $\{a^n b^n\} \cdot \{a^n b^n\} \Rightarrow \{a^{n_1} b^{n_1} \cdot a^{n_2} b^{n_2}\}$

ii) $\{a^n b^n\} \cdot (\overline{\{a^n b^n\}} \cup \{\varepsilon\}) \Rightarrow \Sigma^*$

$$(L \cup \{\varepsilon\}) \cdot (\overline{L} \cup \{\varepsilon\}) \Rightarrow \Sigma^*$$

$$L \cdot \varepsilon = L$$

$$\varepsilon \cdot \overline{L} = \overline{L}$$

6 Reversal

↳ closed for regular languages

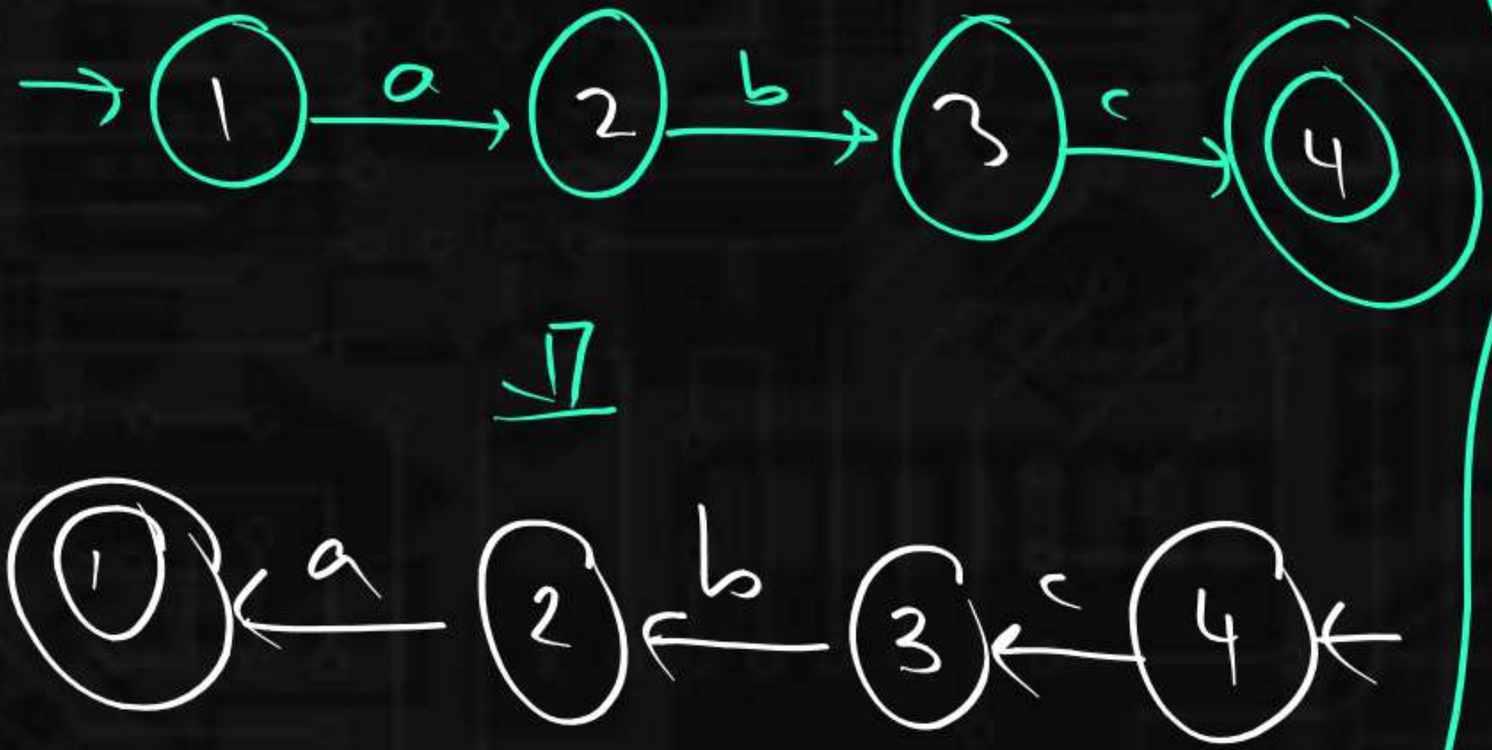
$L = ba^*$

$S \to Sa \mid b$

$\Downarrow$ Reverse every Production

$S \to aS \mid b$

$L^{Rev} = a^*b$



proof:

i) use RegExp

ii) use NFA

iii) use LLG / RLG

$L = a^* b^*$

$L^{Rev} = b^* a^*$

Closure Properites

① $L = \phi \Rightarrow L^{Rev} = \phi$

② $L = \Sigma^* \Rightarrow L^{Rev} = \Sigma^*$

③ $L = ab^* \Rightarrow L^{Rev} = b^*a$

④ $L = a\Sigma^* \Rightarrow L^{Rev} = \Sigma^*a$

⑤ $\left(L^{Rev}\right)^{Rev} = L$

$\overline{\left(\overline{L}\right)} = L$

$\left(L^{Rev}\right)^{Rev} = L$

Closure Properites

I) $(\text{Reg})^{Rev} \Rightarrow$ Regular

II) $(\text{NonReg})^{Rev} \Rightarrow$ Nonreg

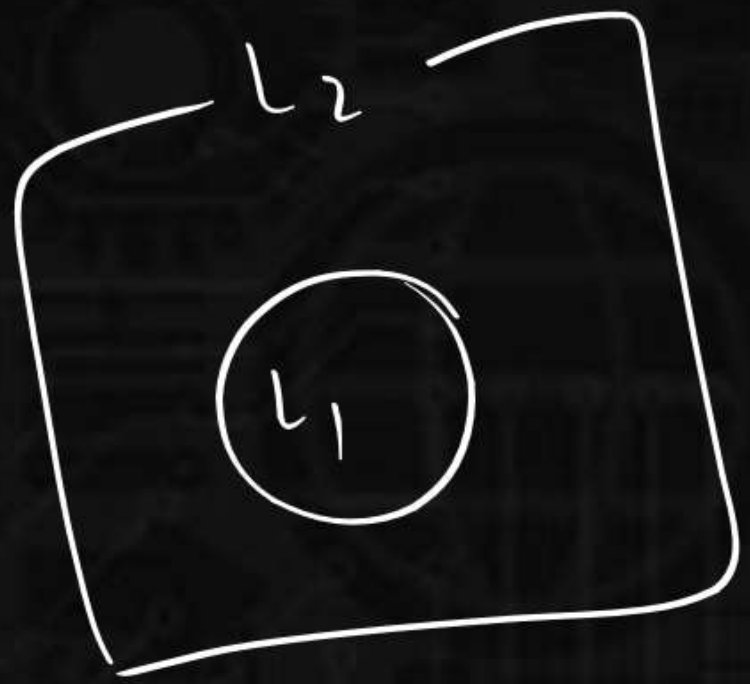
III) L is Reg iff L^{Rev} is Reg

IV) L is not reg iff L^{Rev} is not reg

Closure Properites

Summary:

- → $\cup$ ✓
- → $\cap$ ✓
- → $\bar{L}$ ✓
- → $L_1 - L_2$ ✓
- → $L_1 . L_2$ ✓
- → L^{Rev} ✓



$L_1 \cup L_2 = L_2$

$L_1 \cap L_2 = L_1$


THANK YOU GW
SOLDIERS !