CS & IT ENGINEERING

Theory of Computation

Finite Automata:

Closure Properties – Part 1

Lecture No. 17

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01 Closure properties [operations]

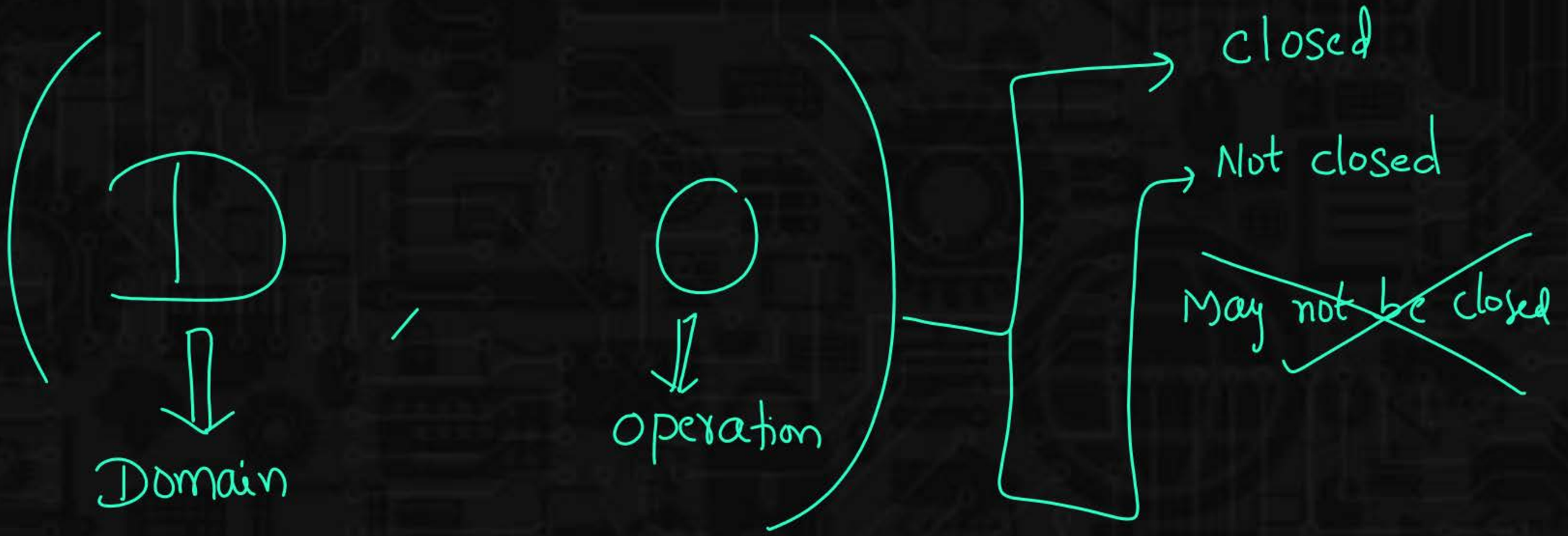
02 For Finite languages

03 For Infinite languages

04 For regular languages

05

Closure Properites



Closure Properites

(D, o) is closed

iff

$\forall x_1, x_2 \in D \Rightarrow x_1 o x_2 \in D$

Example :

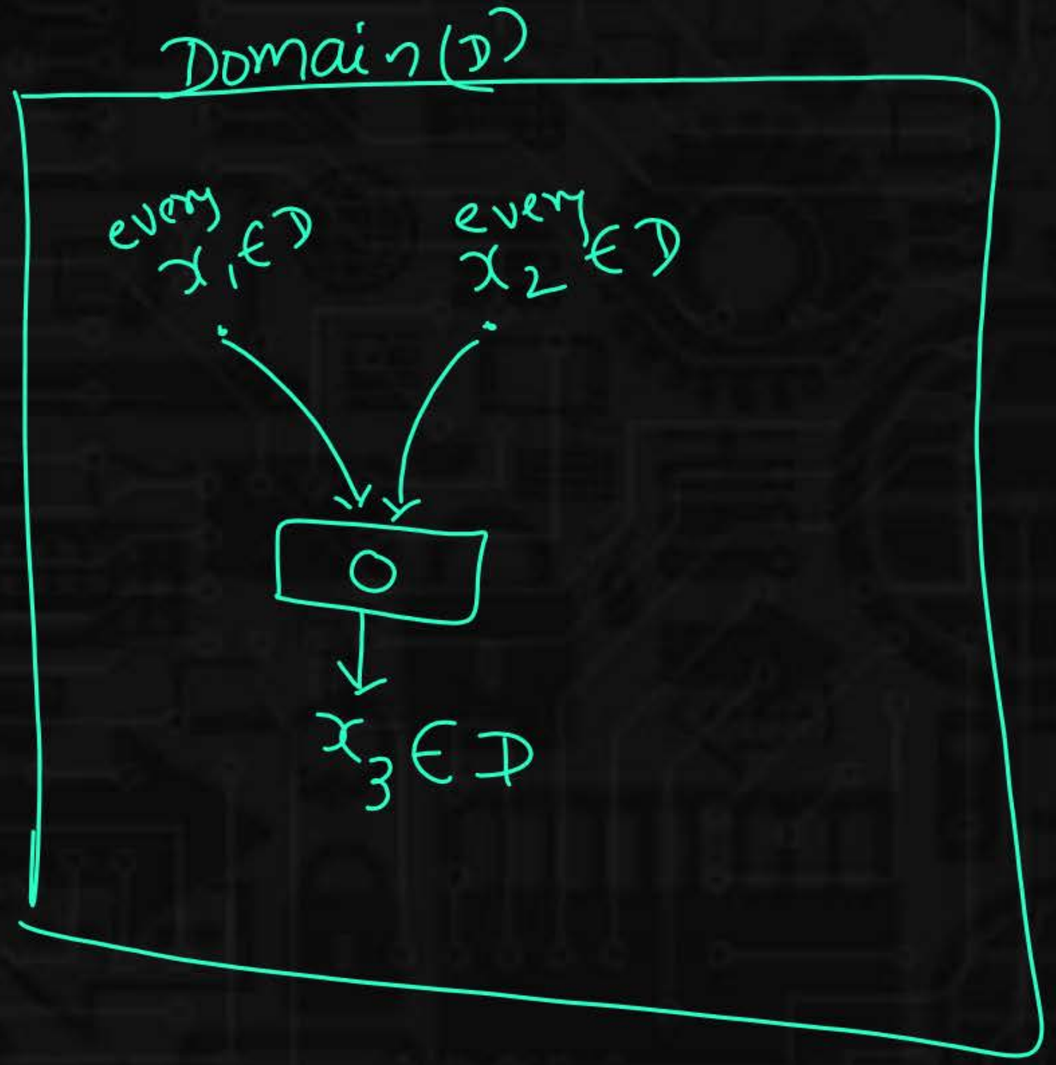
$(N, +)$ is closed

$(N, \times)$ is closed

(Set of matrices $n \times n$, $+$) is closed

(Set of languages, U) is closed

Closure Properites



Closure Properites

(D, o) is "not closed"

iff

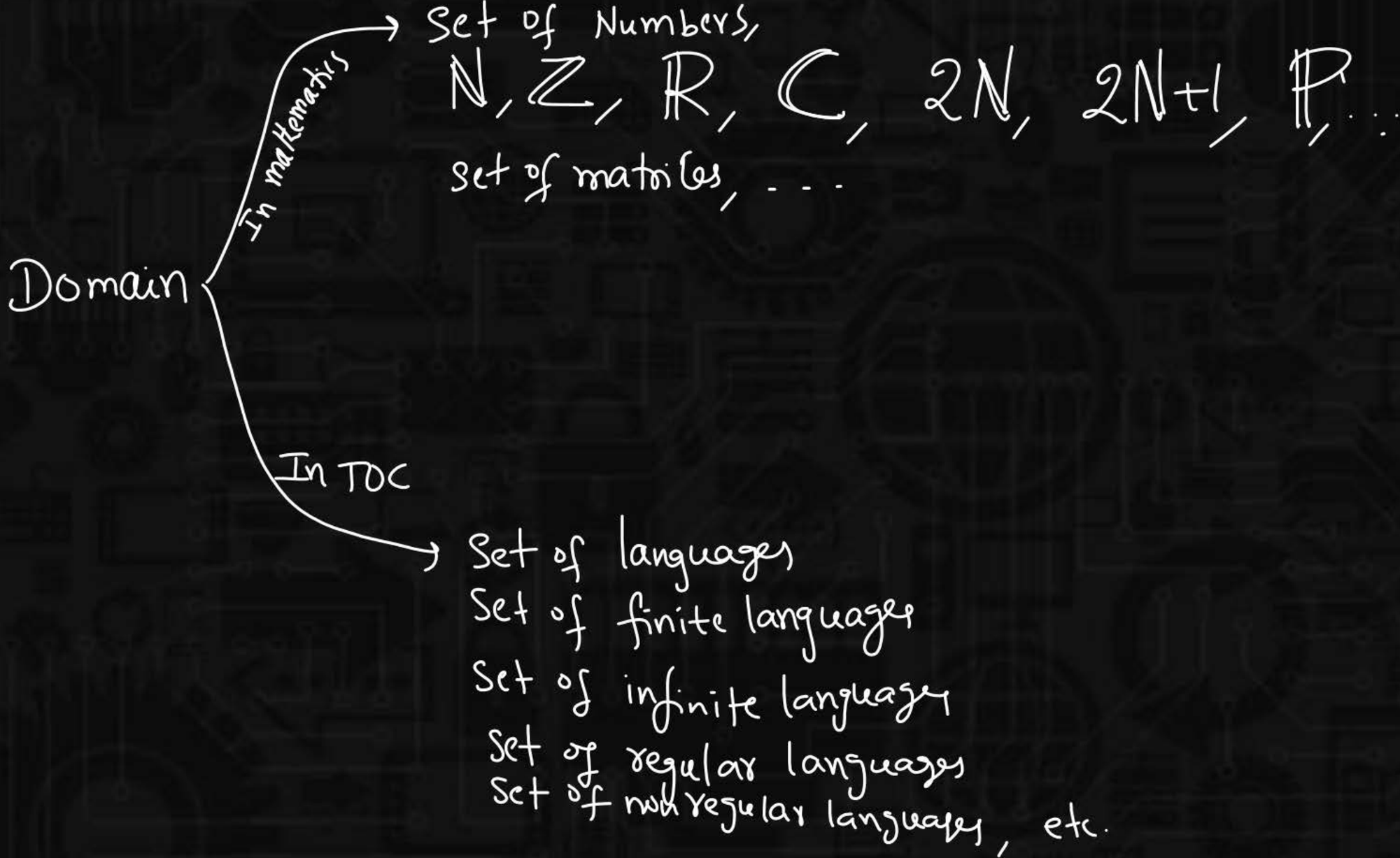
$\exists\, x_1, x_2 \in D \ni x_1 o x_2 \notin D$

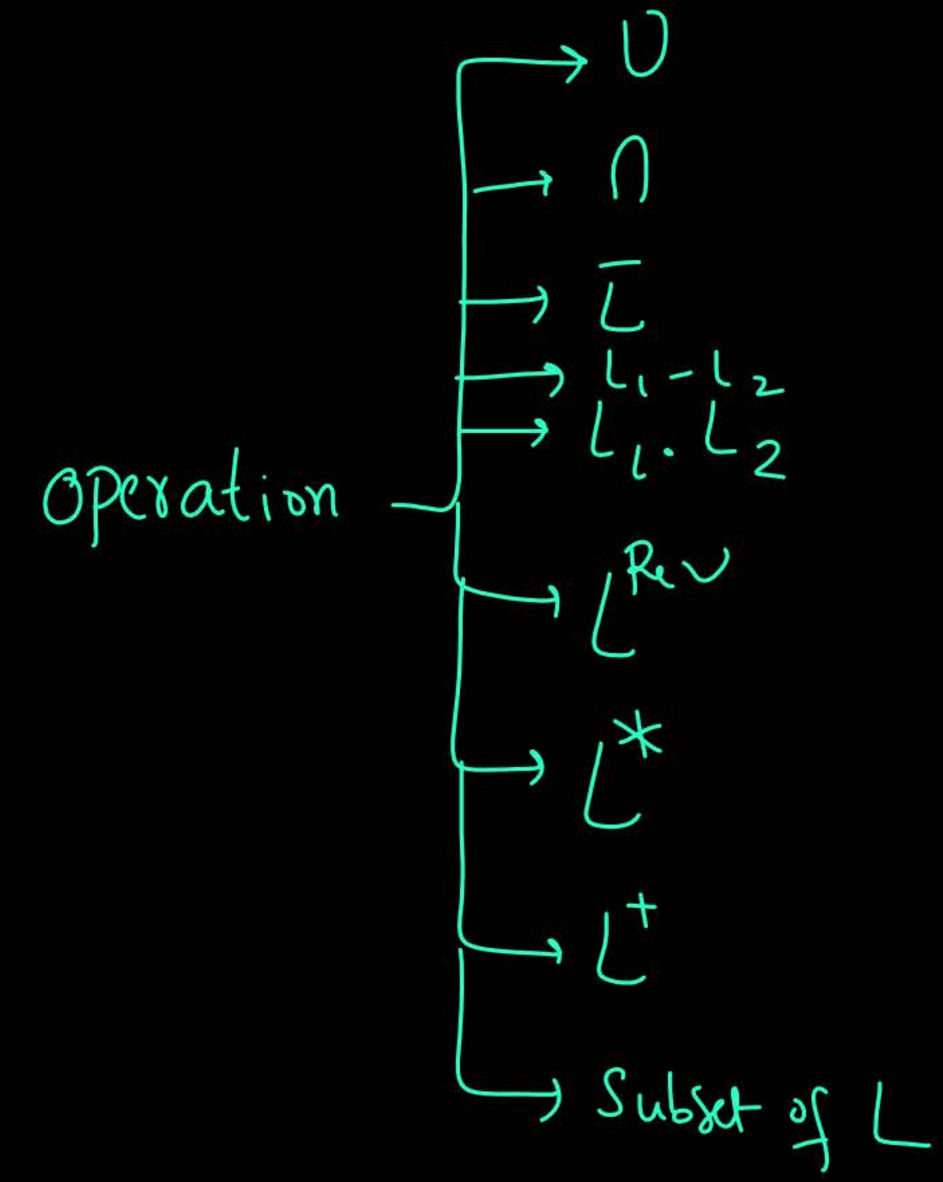
$(N, -)$ is not closed

$2 \in N$
$3 \in N$
$2-3 = -1 \notin N$

$(N, /)$ is not closed

Closure Properites





Prefix(L)

Suffix(L)

Substring(L)

$f(L)$

$h(L)$

$h^{-1}(L)$

L_1 / L_2 , - - - - - -

Closure Properites for finite languages : D = Set of finite languages

	Operation	Finite languages over Σ	
①	Union	Closed (✓)	$F_1 \cup F_2 \Rightarrow F_3$
②	Intersection	✓	$F_1 \cap F_2 \Rightarrow F_3$
③	Complement	Not closed (X)	$\overline{F} \Rightarrow$ Infinite set
④	Difference	✓	$F_1 - F_2 \Rightarrow F_3$
⑤	concatenation	✓	$F_1 . F_2 \Rightarrow F_3$
⑥	Reversal	✓	$F_1^{Rev} \Rightarrow F_2$
⑦	Kleene star	X	$F^* \Rightarrow$ may or may not finite
⑧	Kleene plus	X	$F^+ \Rightarrow$ may or may not finite
⑨	Subset	✓	Subset of (F) $\Rightarrow$ Always Finite set
⑩	Prefix	✓	Prefix (F) $\Rightarrow$ Finite set
⑪	Suffix	✓	Suffix (F) $\Rightarrow$ Finite set

Closure Properites

① Fin ∪ Fin ⟹ Always Finite

② Fin ∩ Fin ⟹ Always Finite

③ $\overline{\text{Finite language}}$ ⟹ Infinite language

$$\overline{L} = \Sigma^* - L$$

$\overline{\text{Fin}} \Rightarrow \underbrace{\Sigma^*}_{\text{Inf}} - \text{Fin} \Rightarrow$ Infinite

Closure Properites

(4) $Fin_1 - Fin_2 \Rightarrow$ (Always) Finite

(5) $Fin_1 \cdot Fin_2 \Rightarrow$ Finite

(6) $Fin^{Reversal} \Rightarrow$ Finite

(7) $Fin^{*} \Rightarrow$ Need not be Finite

$L = \phi \Rightarrow L^* = \{\varepsilon\}$

$L = \{\varepsilon\} \Rightarrow L^* = \{\varepsilon\}$

$L = \underset{fin}{\{a\}} \Rightarrow L^* = a^*$ Inf

$L = \{aa\} \Rightarrow L^* = (aa)^*$ Inf

Note

If $L \neq \phi$ and $L \neq \{\varepsilon\}$ then L^* is Infinite

Closure Properites

⑧ $(Fin)^+ \Rightarrow$ Need not be finite

⑨ Every Subset of Finite language is Finite set

$\{\varepsilon, a, aaa\}$

Finite set

$\Downarrow$

8 Subsets

every subset is finite

ϕ

$\{\varepsilon\}$

$\{a\}$

$\{aaa\}$

$\{\varepsilon, a\}$

$\vdots$

$\{\varepsilon, a, aaa\}$

Closure Properites

(10) Prefix (Finite Set) $\Rightarrow$ Finite language

$L = \{a, abb\}$

$Prefix(L) = \{\varepsilon, a, ab, abb\}$

$\cup$ { $Prefix(a) = \{\varepsilon, a\}$, $Pref(abb) = \{\varepsilon, a, ab, abb\}$

(11) Suffix (Finite Set) $\Rightarrow$ Finite language

$L = \{\varepsilon, ab, bbb\}$ Finite language

Prefix (L) $\Rightarrow$ Always finite

$Prefix(L) = Pref(\varepsilon) \cup prefix(ab) \cup Prefix(abb)$

Closure Properites for infinite languages : $\mathbb{D}$ = Set of infinite languages

	Operation	Infinite languages over Σ	
①	Union	✓	$I_1 \cup I_2 \Rightarrow I_3$
②	Intersection	✗	$I_1 \cap I_2 \Rightarrow$ Need not be Infinite
③	Complement	✗	$\overline{I} \Rightarrow$ Need not be Infinite
④	Difference	✗	$I_1 - I_2 \Rightarrow$ Need not be Infinite
⑤	concatenation	✓	$I_1 . I_2 \Rightarrow I_3$
⑥	Reversal	✓	$I^{Rev} \Rightarrow$ Infinite
⑦	Kleene star	✓	$I^* \Rightarrow$ Infinite
⑧	Kleene plus	✓	$I^+ \Rightarrow$ Infinite
⑨	Subset	✗	Subset of Inf Set $\Rightarrow$ need not be Inf
⑩	Prefix	✓	Prefix(I) $\Rightarrow$ Inf
⑪	Suffix	✓	Suffix(I) $\Rightarrow$ Inf

Closure Properites

① Inf ∪ Inf ⇒ Inf

② Inf ∩ Inf ⇏ need not be Inf

i) $a^* \cap a^* \Rightarrow a^*$

ii) $\underset{Inf}{a^*} \cap \underset{Inf}{b^*} \Rightarrow \{\varepsilon\}$

$\{\varepsilon, a, a^2 \ldots\} \cap \{\varepsilon, b, b^2 \ldots\}$

$a^+ \cap b^+ = \phi$

Closure Properites

③ $\overline{Inf} \Rightarrow$ Need not be Inf

$\Sigma = \{a, b\}$

i) $\overline{\underbrace{\Sigma^*}_{Inf}} \Rightarrow \underbrace{\phi}_{fin}$

ii) $\overline{\underbrace{a\Sigma^*}_{Inf}} \Rightarrow \underbrace{b\Sigma^* + \varepsilon}_{Inf}$

Closure Properites

(4) $Inf_1 - Inf_2 \Rightarrow$ Need not be Inf

i) $\underbrace{a^*}_{Inf} - \underbrace{a^*}_{Inf} \Rightarrow \underbrace{\phi}_{Fin}$

ii) $\underbrace{a^*}_{Inf} - \underbrace{b^*}_{Inf} \Rightarrow \underbrace{a^+}_{Inf}$

(5) $Inf_1 \cdot Inf_2 \Rightarrow Inf$

Closure Properites

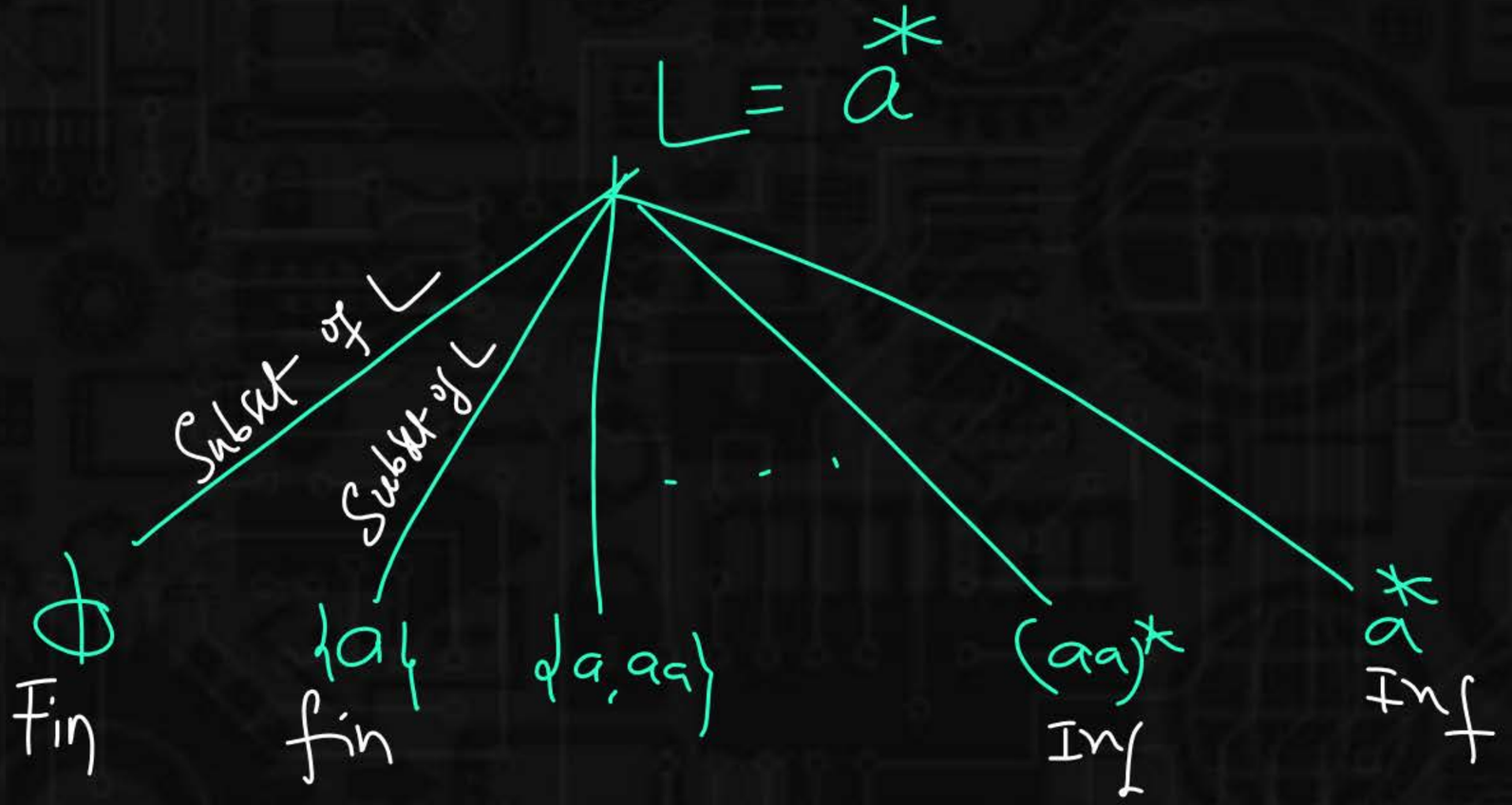
(6) $Inf^{Reversal} \Rightarrow Inf$

(7) $Inf^{*} \Rightarrow Inf$

(8) $Inf^{+} \Rightarrow Inf$

Closure Properites

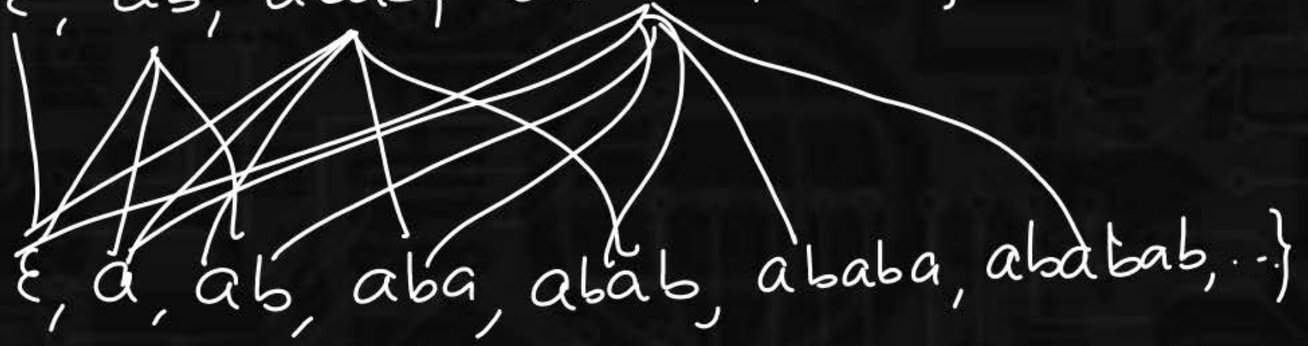
(9) (Every) Subset of Infinite set is need not be Infinite



Closure Properites

(10) Prefix (Inf lang) $\Rightarrow$ Inf lang

$L = (ab)^* = \{\varepsilon, ab, abab, ababab, \ldots\}$



$Prefix(L) = \{\varepsilon, a, ab, aba, abab, ababa, ababab, \ldots\}$

(11) Suffix (Inf) $\Rightarrow$ Infinite

Closure Properites

Same

- Union is closed for finite languages
- $(\mathcal{D}, \cup)$ is closed where $\mathcal{D}$ = set of finite languages
- (Set of finite languages, $\cup$) is closed
- Finite languages is closed under union operation
- $(\{F \mid F \text{ is Finite language}\}, \cup)$ is closed

Note :

I) Fin Set $\cap$ Any Set $\Rightarrow$ Finite Set

II) Inf set $\cup$ Any set $\Rightarrow$ Infinite Set

III) $\overline{\text{Fin set}} \Rightarrow$ Infinite Set

IV) Fin Set $-$ Any set $\Rightarrow$ Finite Set

V) $\Sigma^* \cup$ Any set $\Rightarrow \Sigma^*$

VI) $\phi \cap$ Any set $\Rightarrow \phi$

VII) ϕ . Any set $\Rightarrow \phi$

Closure Properites for regular languages

Domain = Set of regular languages

- ϕ
- $\{\varepsilon\}$
- $\{a\}$
- $\{\varepsilon, a\}$
- $\{\varepsilon, ab\}$
- a^*
- b^*
- $(a+b)^*$
- $a\Sigma^*$
- $\Sigma^* a$
- $b\Sigma^*$
- a^*b^*
- a^+b^+
- $\Sigma^* a \Sigma^*$
- $\{w \mid |w|=2, w \in \{a,b\}^*\}$

over $\Sigma = \{a, b\}$

Prime

- a
- $a^{n!}$
- a^{2^n}

← outside domain
all non regular languages

- $a^n b^n$
- $a^n b^{2n}$
- $\{ww \mid w \in \{a,b\}^*\}$

Closure Properites for regular languages:

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $\overline{L}$
4. $L_1 - L_2$
5. $L_1 \cdot L_2$
6. L^{Rev}
7. L^*
8. L^+
9. ~~Subset(L)~~

10. Prefix(L)
11. Suffix(L)
12. Substring(L)
13. Quotient(L)
14. Substitution(L)
15. Homomorphism(L)
16. ε-free Homomorphism(L)
17. $h^{-1}(L)$
18. Half(L)

19. Second Half(L)
20. one-third(L)
21. Middle $\frac{1}{3}$(L)
22. Last $\frac{1}{3}$(L)
23. Symmetric Difference(L)

24. Finite Union
25. Finite Intersection
26. Finite Difference
27. Finite Concatenation
28. Finite subset
29. Finite Substitution

30. ~~Inf U~~
31. ~~Inf ∩~~
32. ~~Inf −~~
33. ~~Inf .~~
34. ~~Inf ⊆~~
35. ~~Inf Substi~~

Closure Properites

For regular languages :

⇓

Subset is not closed

Infinite $\cup, \cap, -, \cdot, \subseteq, f$ are not closed

Closure Properites

Summary

- What is closed, not closed ?
- For finite languages
- For Infinite languages
- Next: For regular languages


THANK YOU GW
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