

CS & IT ENGINEERING

Theory of Computation

PDA : CFLs Vs DCFLs



Lecture No. 05



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TOPICS TO BE COVERED

01

CFLs V_s DCFs

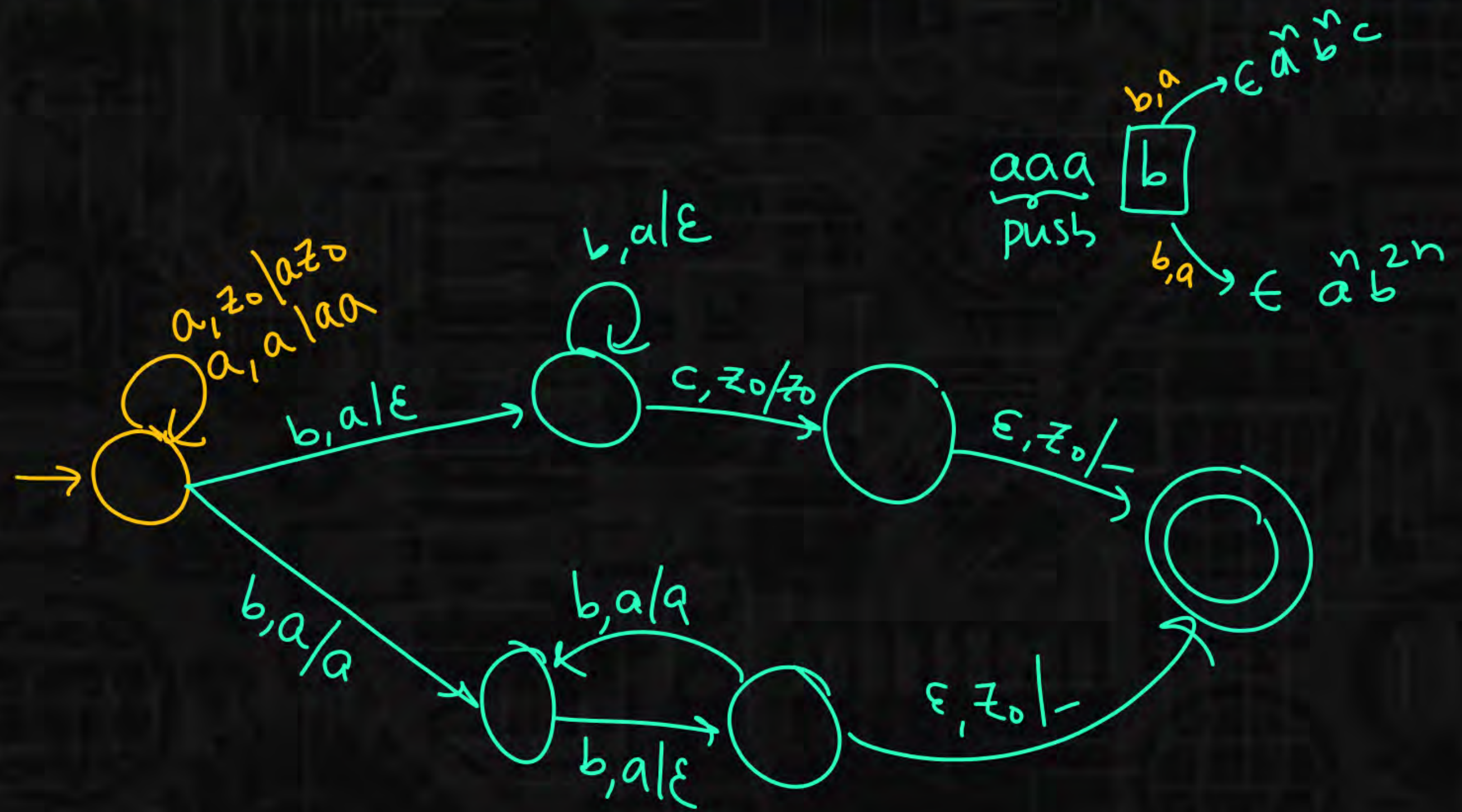
02

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04

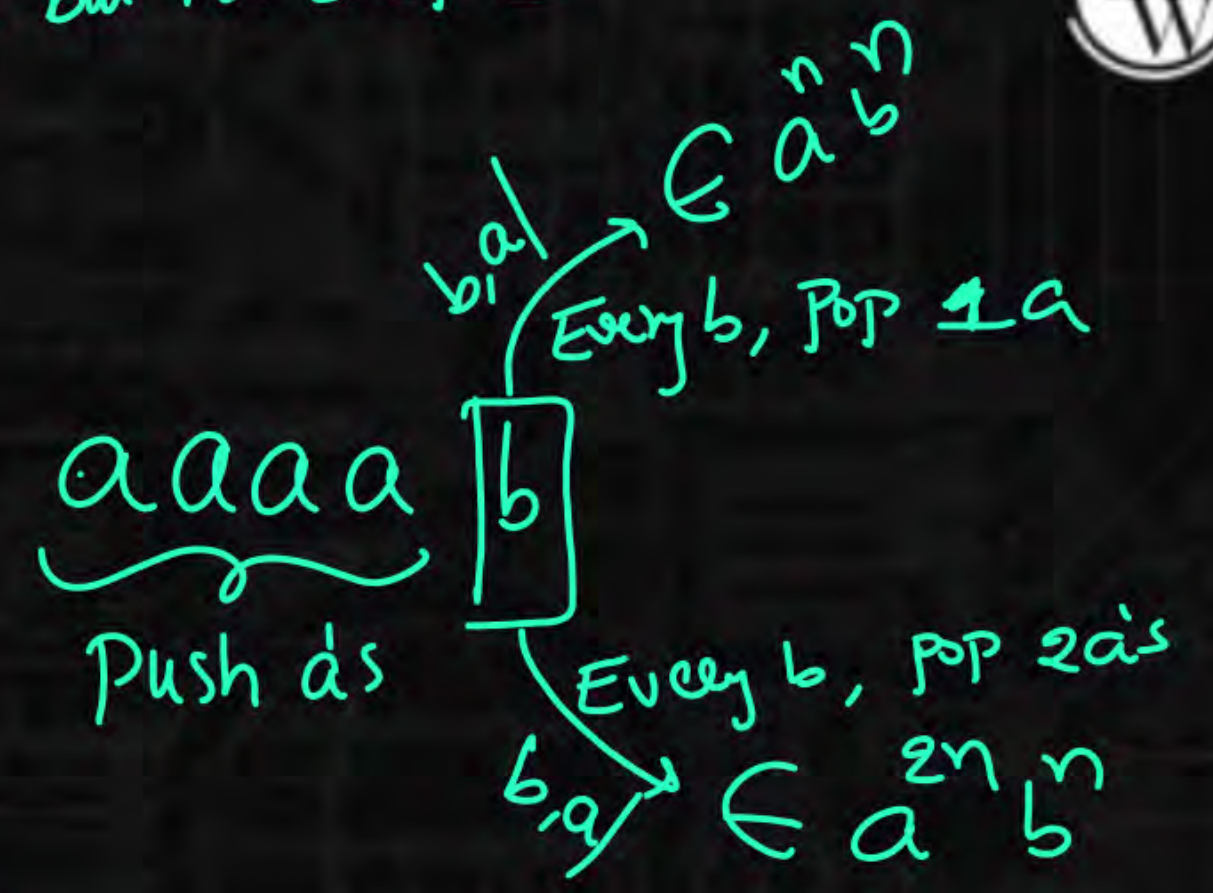
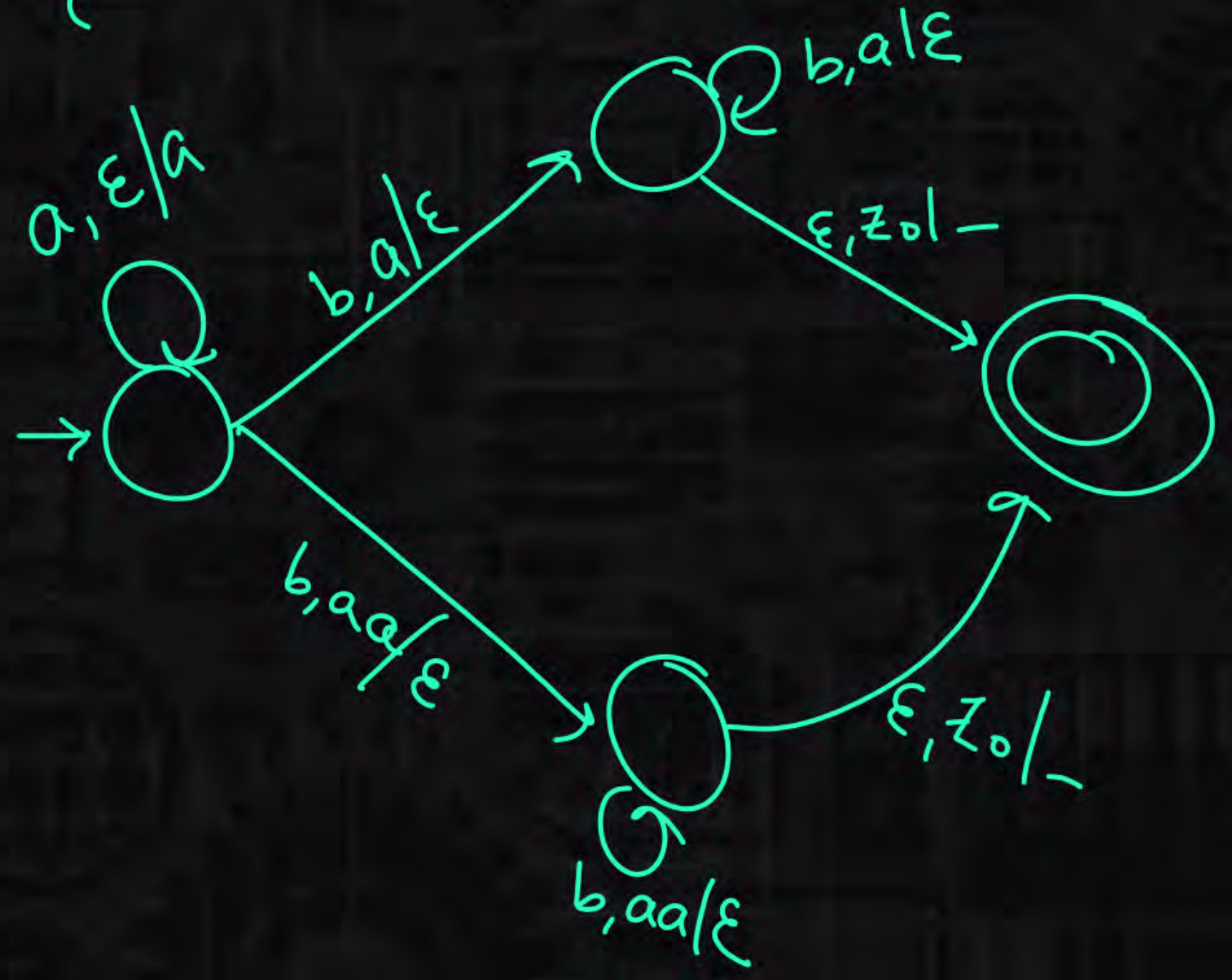
05

② $\{a^n b^n c | n \geq 1\} \cup \{a^n b^{2n} | n \geq 1\} \Rightarrow \text{CFL but not DCF}$



(29) $\{a^m b^n \mid m=n \text{ or } m=2n, m, n \geq 1\} \Rightarrow \text{CFL but not DCF L}$

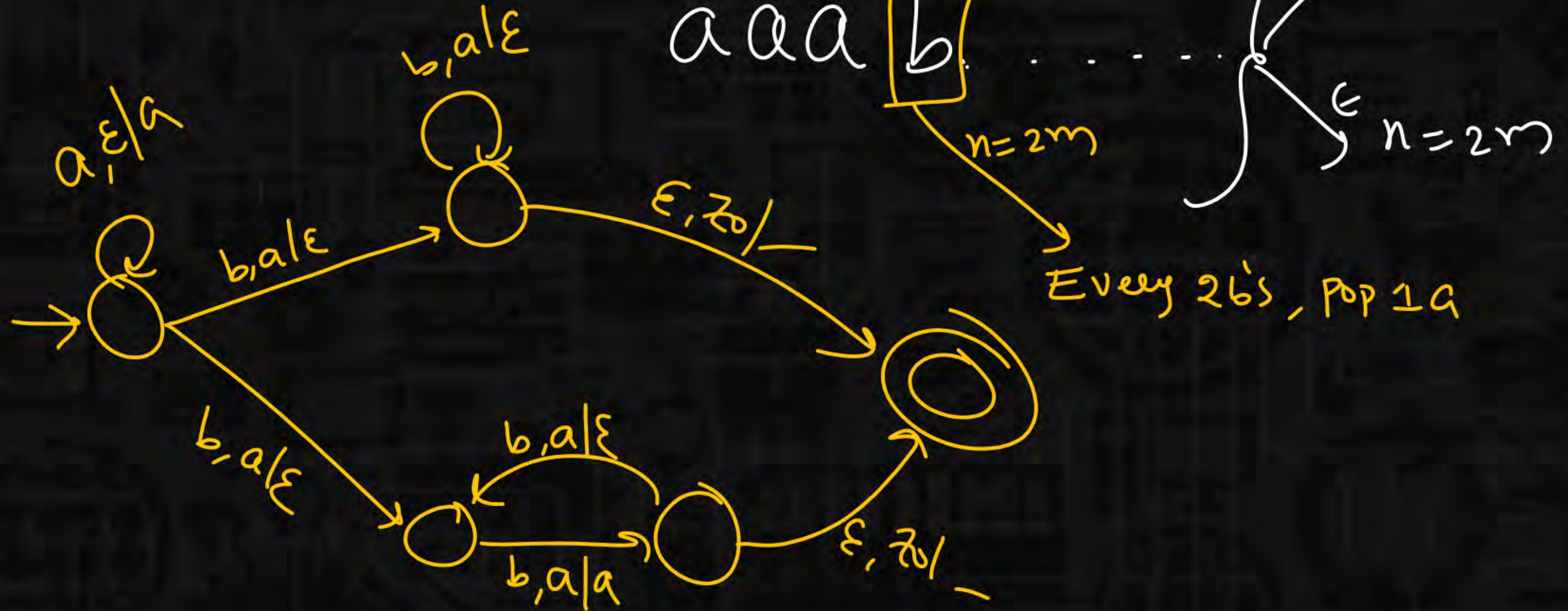
$$= \{a^n b^n \mid n \geq 1\} \cup \{a^{2n} b^n \mid n \geq 1\}$$



$b \rightarrow$
 $\epsilon \rightarrow$

23) $\{a^m b^n \mid n=m \text{ or } n=2m, n, m \geq 1\}$

$$= \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$$

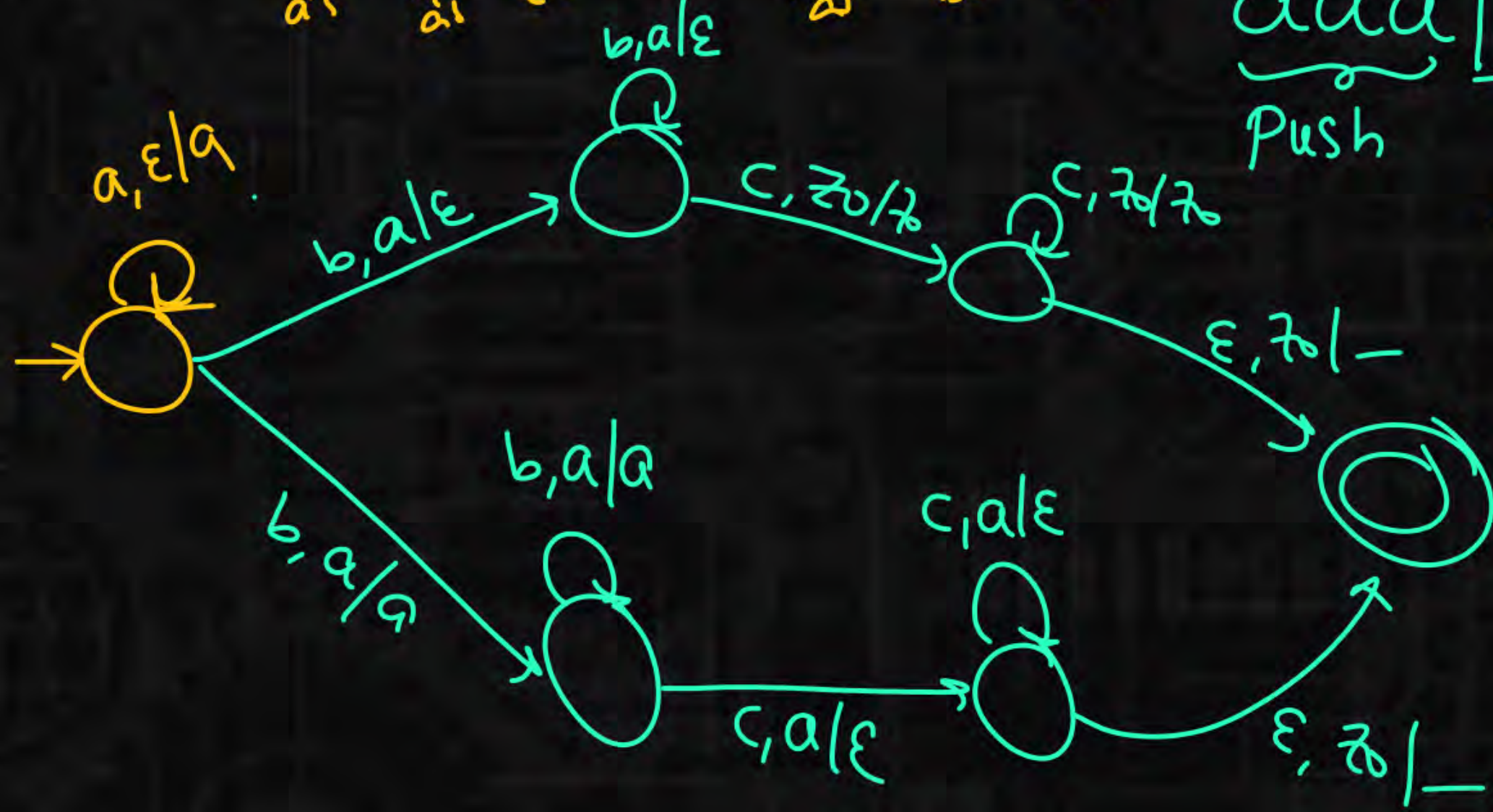
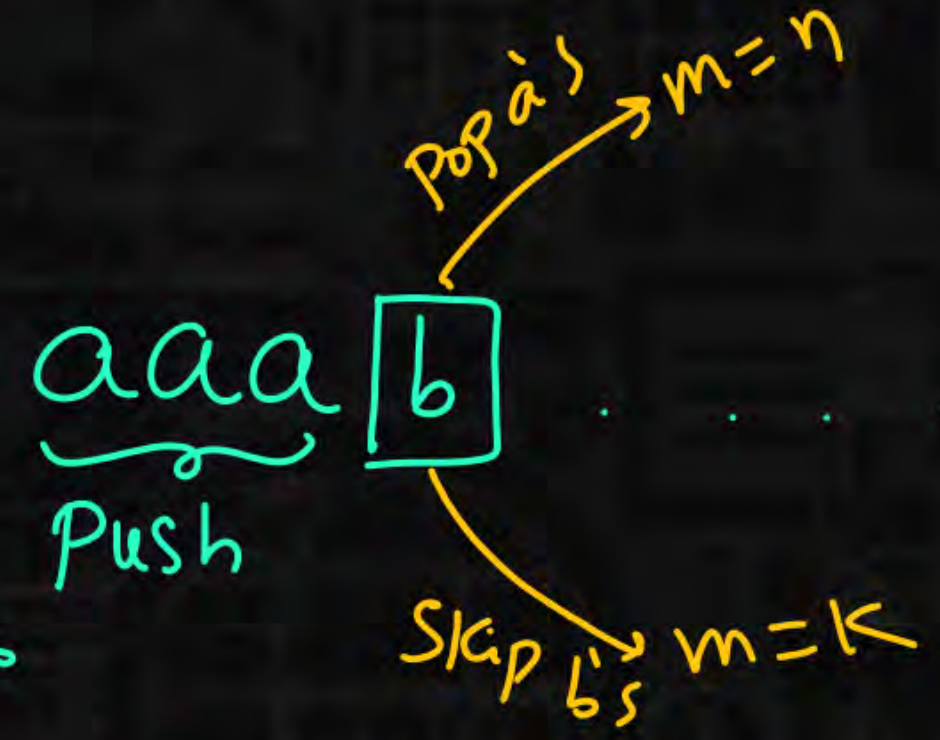


(24) $\{a^m b^n c^k \mid m=n \text{ or } m=k, m, n, k \geq 1\} \Rightarrow \text{CFL but not DCFL}$

$$= \{a^n b^n c^+ \mid n \geq 1\} \cup \{a^+ b^+ c^n \mid n \geq 1\}$$

push a 's
 pop a 's
 skip c 's

push a 's
 skip b 's
 pop a 's



(25) $\{a^m b^n c^k \mid m=n \text{ OR } n=k, m, n, k \geq 1\} \Rightarrow \text{CFL but not DCFL}$

$S \rightarrow AB/CD$

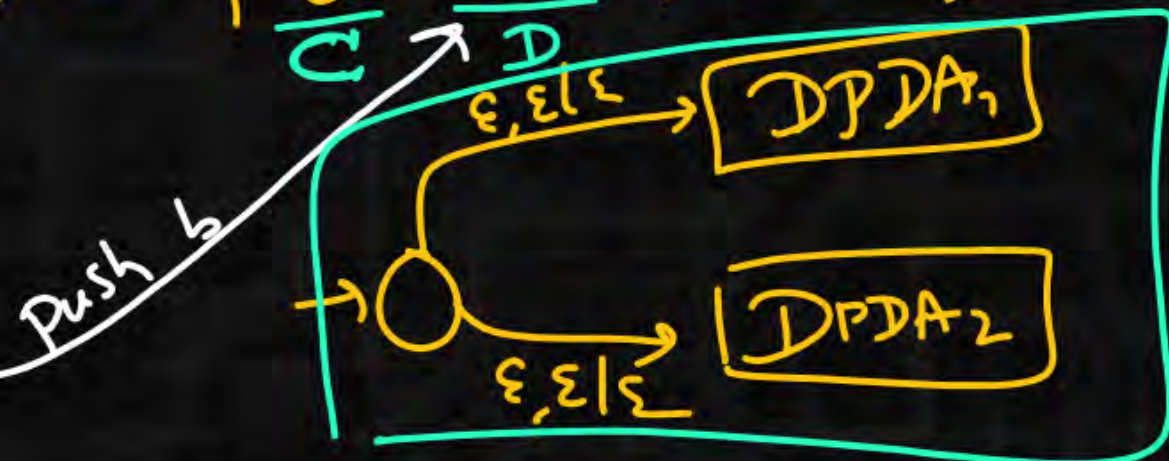
$A \rightarrow aAb/ab$

$B \rightarrow \epsilon B/c$

$C \rightarrow aC/a$

$= \{ \underbrace{a^n}_A \underbrace{b^n}_B \underbrace{c^+}_C \mid n \geq 1 \} \cup \{ \underbrace{a^+}_C \underbrace{b^n}_D \underbrace{c^n}_D \mid n \geq 1 \}$

Diagram illustrating the first part of the decomposition: $aaa \mid b$. The stack contains aaa and the input is b . An arrow labeled "push" points to the stack, and an arrow labeled "pop" points to the input.



PDA not DPDA

(26) $\{a^m b^n c^k \mid m=k \text{ OR } n=k, m, n, k \geq 1\}$

$= \{ \underbrace{a^n}_A \underbrace{b^+}_B \underbrace{c^n}_C \mid n \geq 1 \} \cup \{ \underbrace{a^+}_A \underbrace{b^n}_B \underbrace{c^n}_C \mid n \geq 1 \}$

Diagram illustrating the second part of the decomposition: $aaa \mid bbb \mid c$. The stack contains aaa and the input is $bbb \mid c$. Arrows labeled "push" point to the stack, and arrows labeled "pop" point to the input.

$$S \rightarrow X | Y$$

$$X \rightarrow aXc | aZc$$

$$Z \rightarrow bZ | b$$

$$Y \rightarrow AB$$

$$A \rightarrow aA | a$$

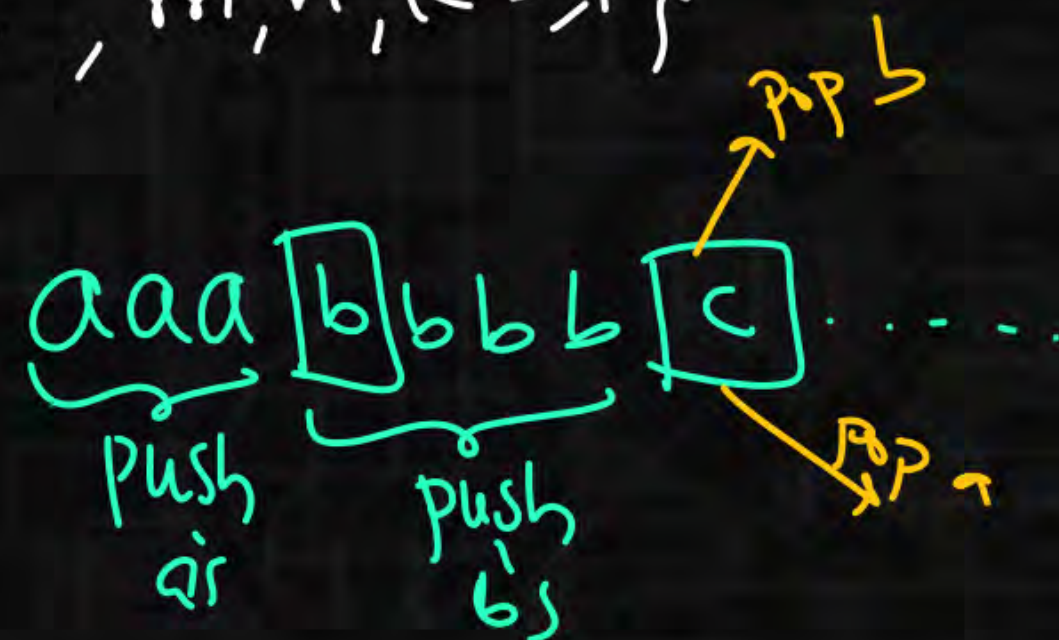
$$B \rightarrow bBc | bc$$

(26) $\{ a^m b^n c^k \mid m=k \text{ OR } n=k, m, n, k \geq 1 \}$

$= \{ a^n b^n c^n \mid n \geq 1 \} \cup \{ a^+ b^n c^n \mid n \geq 1 \}$

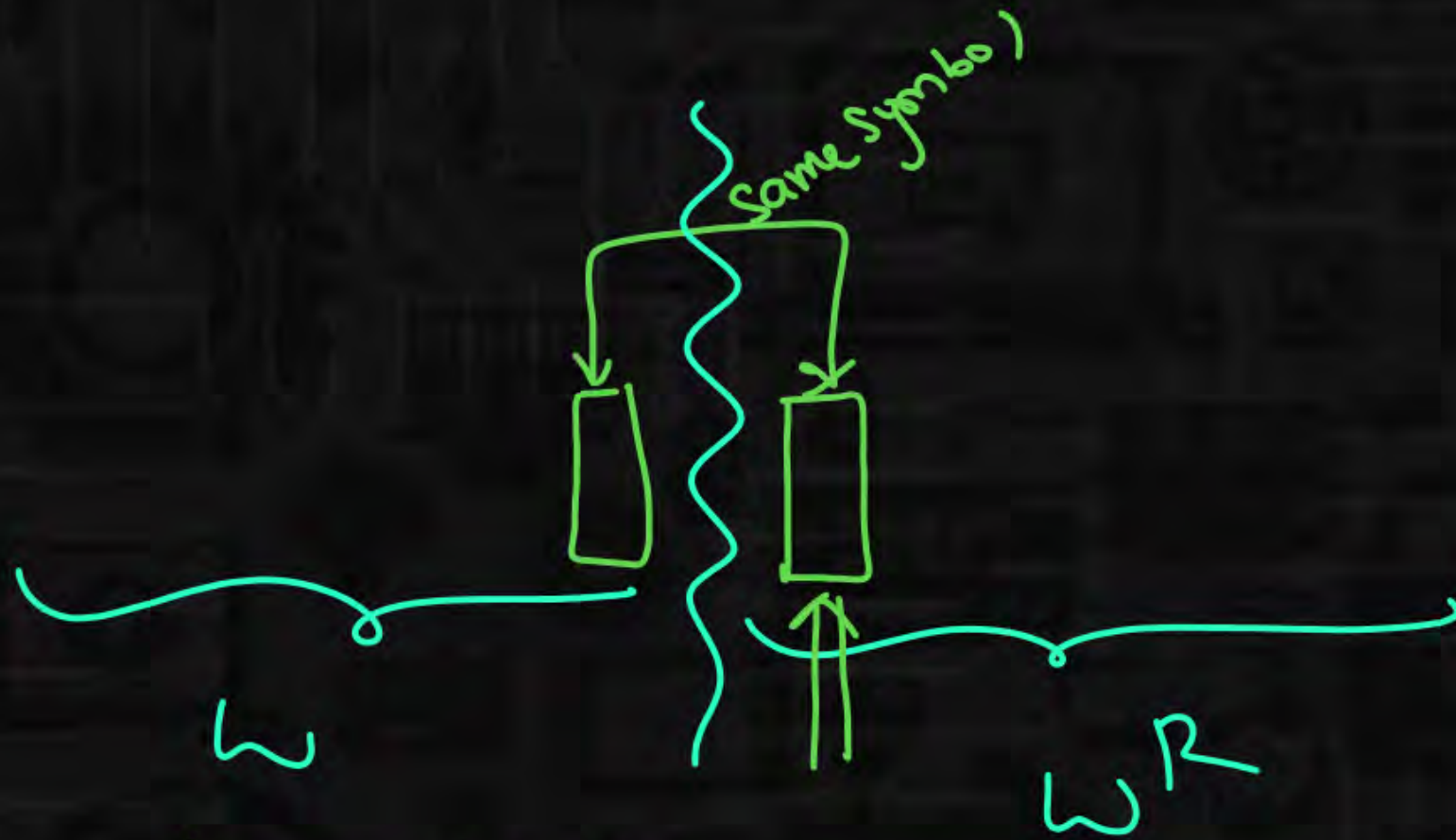
X

Y



(27) $\{ww^R \mid w \in \{a,b\}^*\} = \text{Set of even length palindromes}$

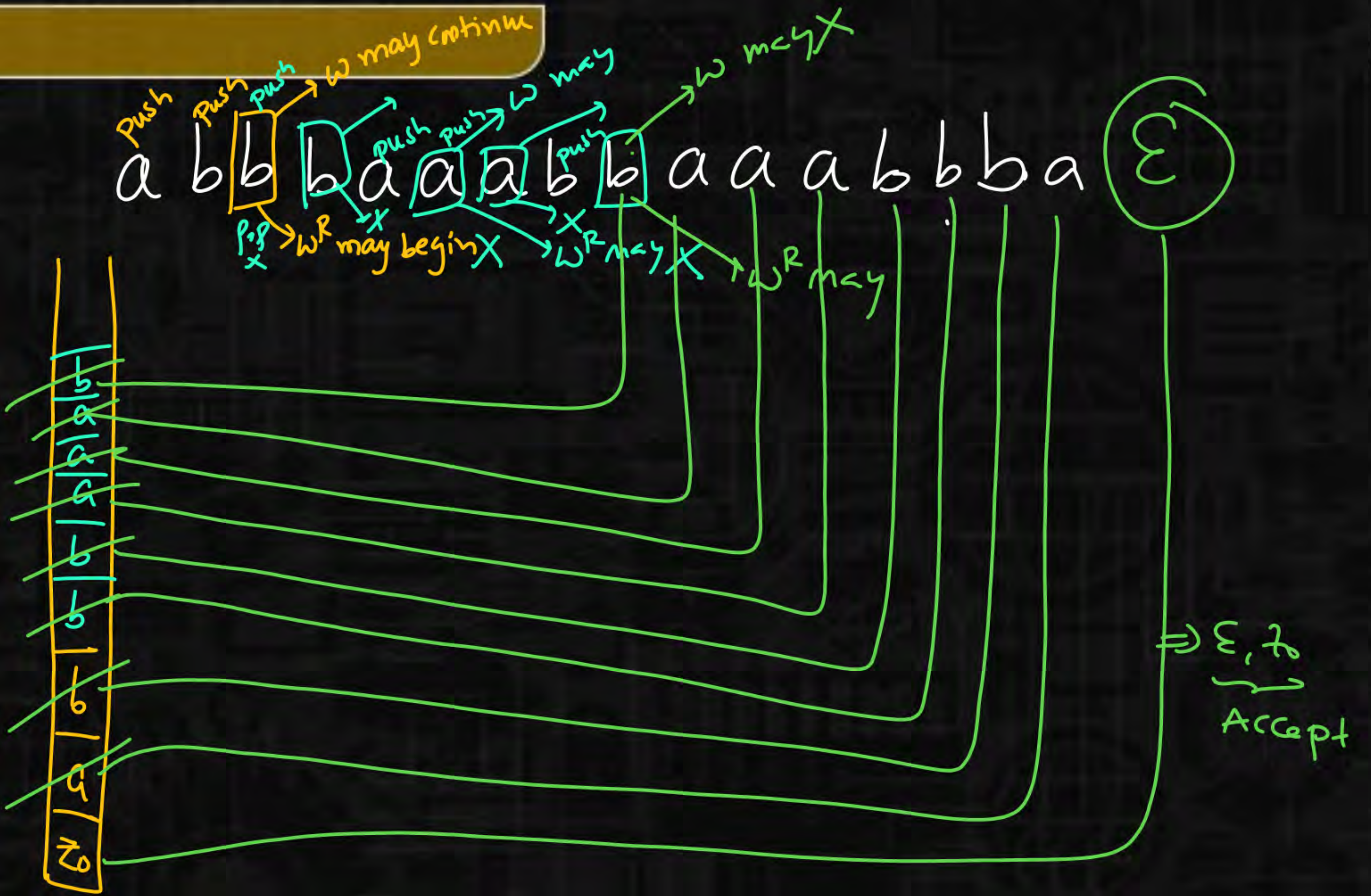
$= \{\epsilon, aa, bb, aaaa, \dots\}$ $S \rightarrow aSa \mid bSb \mid \epsilon$



last symbol of w = First symbol of w^R

I) w^R begin ?

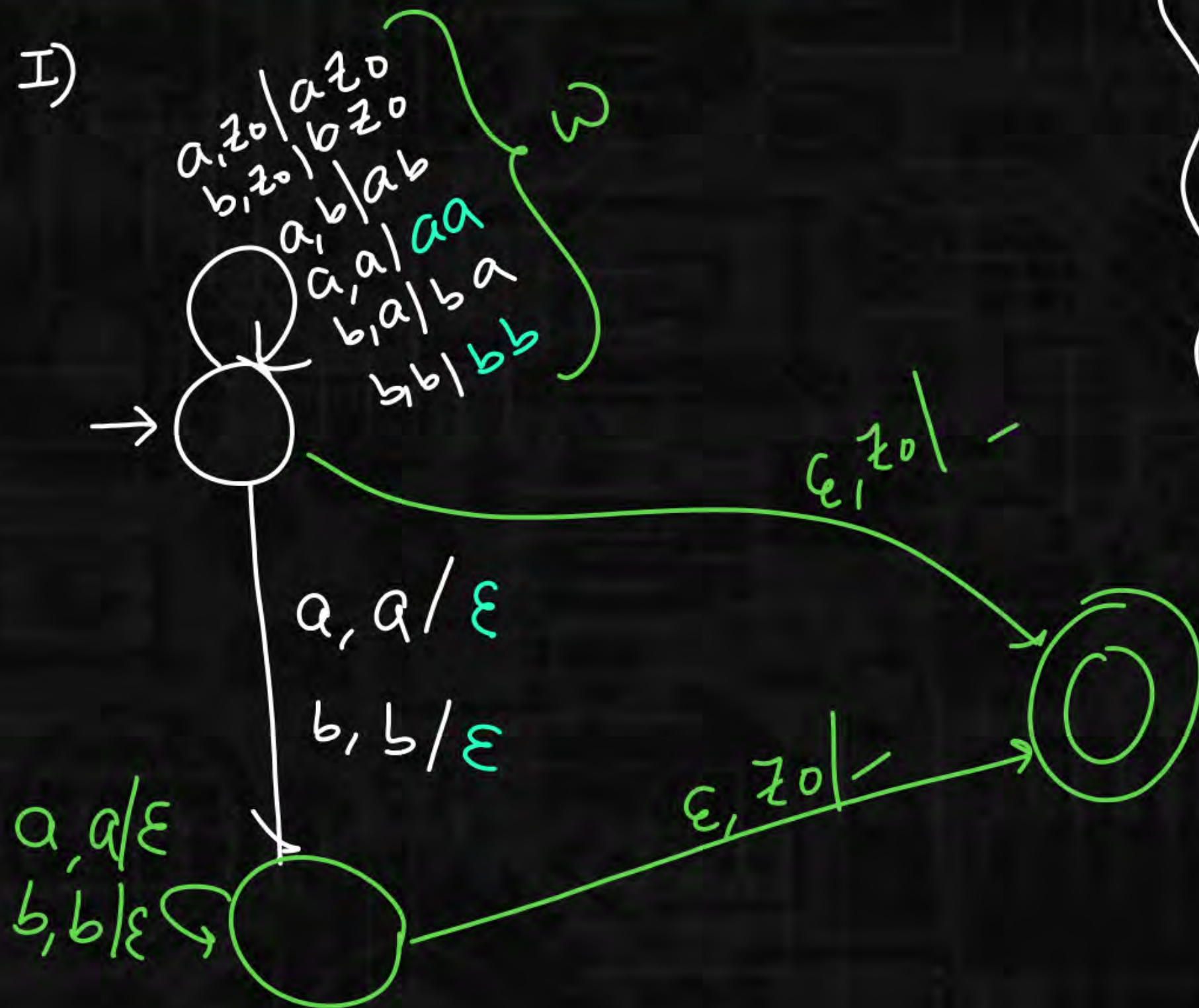
II) If w^R begins,
What is logic ?



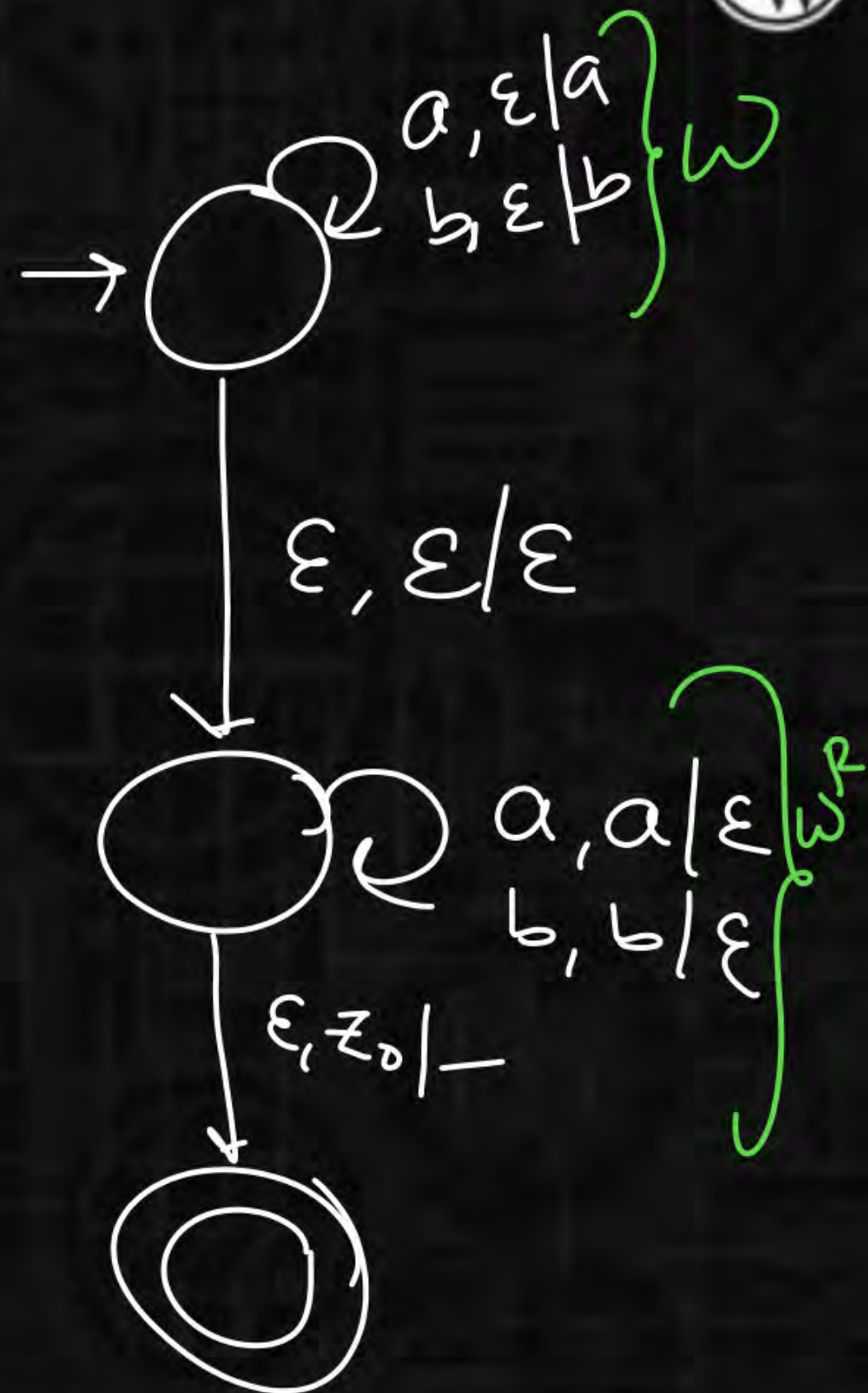
$\{ww^R \mid w \in \{a,b\}^*\}$



I)



II)

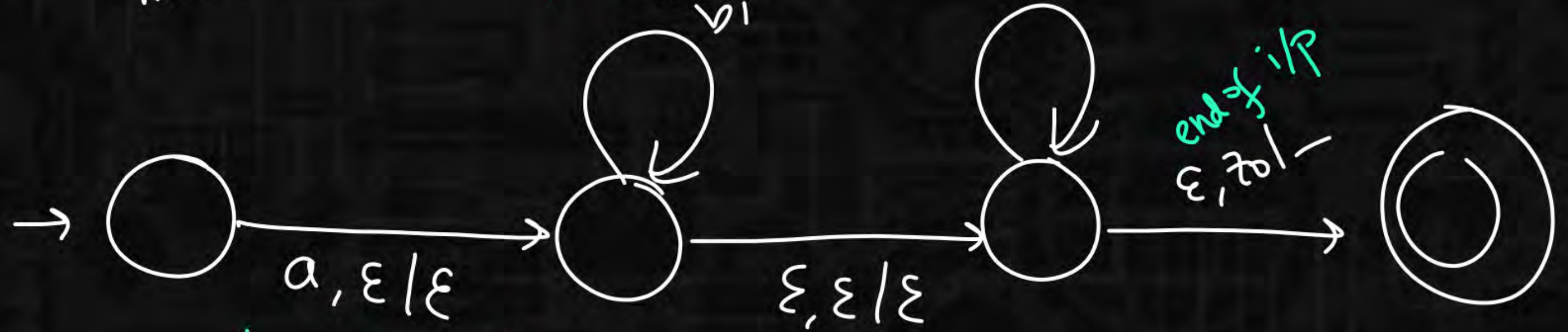


(28) $\{aww^R \mid w \in \{a,b\}^*\} \Rightarrow \text{CFL but not DCFL}$

skip
push non-det
pop non-det

$\begin{matrix} \epsilon/a \\ a/\epsilon/b \\ b/\epsilon \end{matrix}$

$\begin{matrix} b/\epsilon \\ a/\epsilon \end{matrix}$



skip

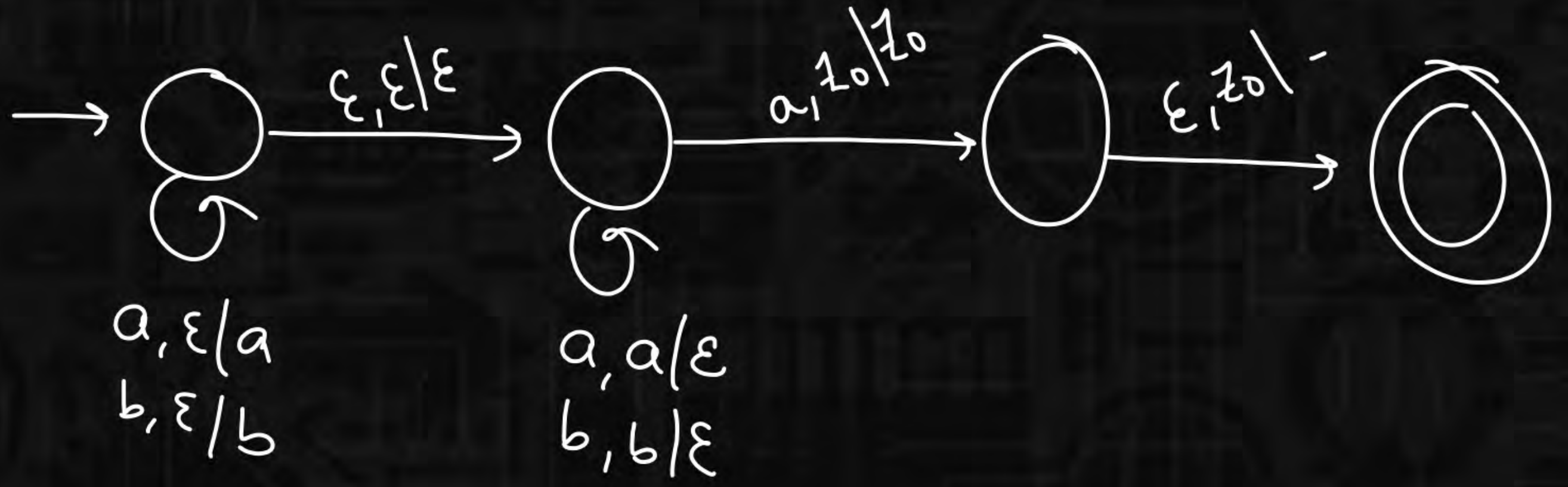
$\begin{matrix} \epsilon x \\ a \checkmark \end{matrix}$

$\begin{matrix} b x \\ a a x \\ \text{skip} \rightarrow \text{push} \end{matrix}$

end of ip
 $\epsilon, \epsilon / \epsilon, \epsilon$

(29) $\{ww^R a \mid w \in \{a, b\}^*\} \Rightarrow \text{CFL but not DFL}$

push
 pop
 skip

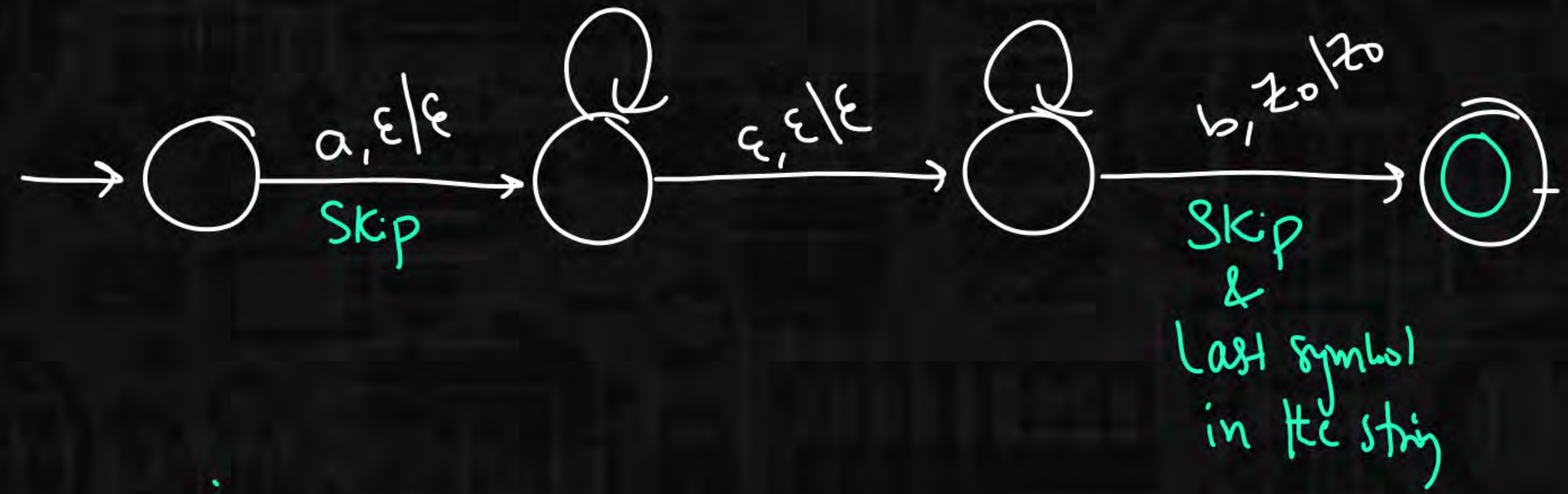


(30) $\{a w w^R b \mid w \in \{a, b\}^*\} \Rightarrow \text{CFL but not DFL}$

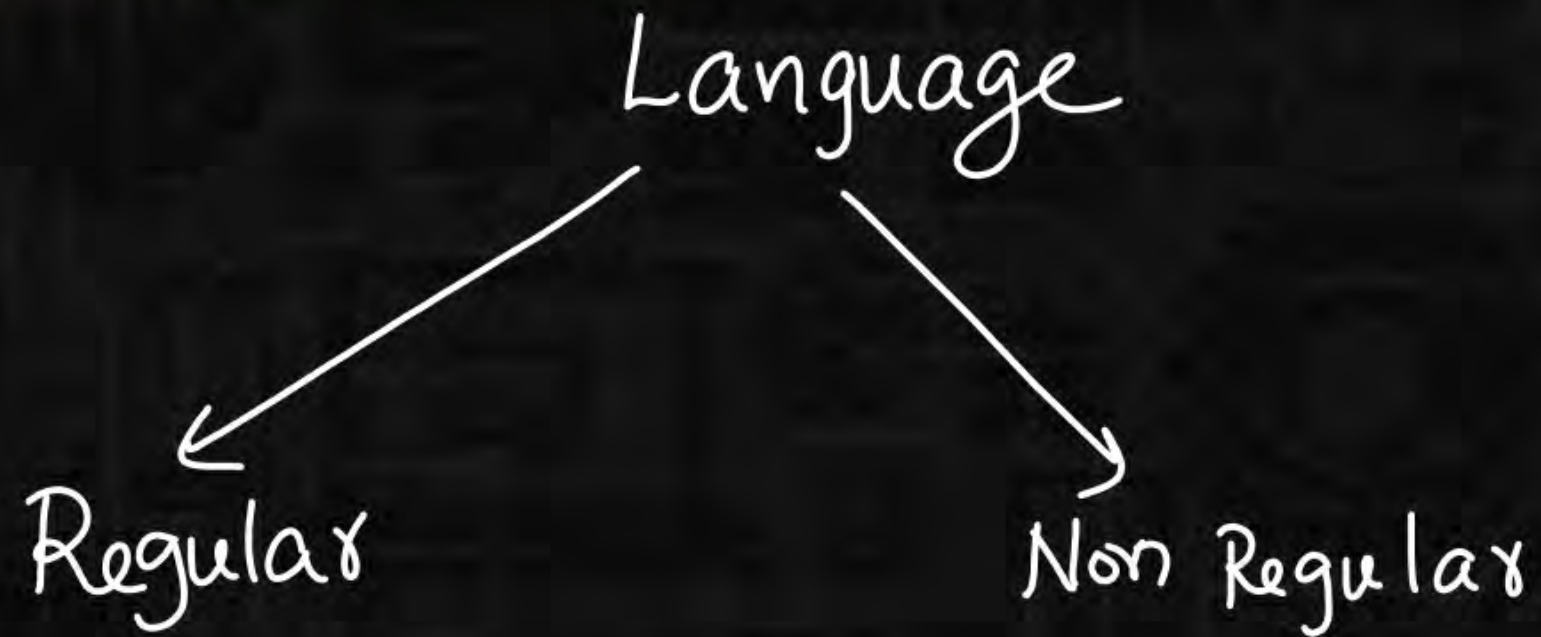
a → skip
 w → push
 w^R → pop
 b → skip

$b, \epsilon / b$
 $a, \epsilon / a$

$b, b / \epsilon$
 $a, a / \epsilon$



How to identify CFLs & DCFLs ?



I) Finite lang

II) Inf lang & over 1 symbol
that forms A.P.

III) Inf lang & over >1 symbol
that will not contain dependency
till inf and also forms A.P.

I) L over 1 symbol & do not
form A.P.

II) L over >1 symbol &
Dependency exist / do not form
A.P.

^(L) Language over Σ

Finite language

\Downarrow
 Regular
 \Downarrow
 DCFL
 \Downarrow
 CFL

$\{a^n b^n / n < 100\}$
 $\{a^n b^n / n < 100\}$

Infinite language

L over 1 symbol

L Forms A.P.

\Downarrow
 Regular a^{2n}
 \Downarrow
 DCFL
 \Downarrow
 CFL

L do not form A.P.

\Downarrow
 Not Regular $a^{n!}$

\Downarrow
Not CFL & Not DCFL

L over > 1 Symbol

No dep till inf

\Downarrow
 Regular $a^* b^*$
 \Downarrow
 DCFL
 \Downarrow
 CFL

Dep till inf or Do not form A.P.

Not Regular

$a^n b^n$
 DCFL
 \Downarrow
 CFL

Not DCFL

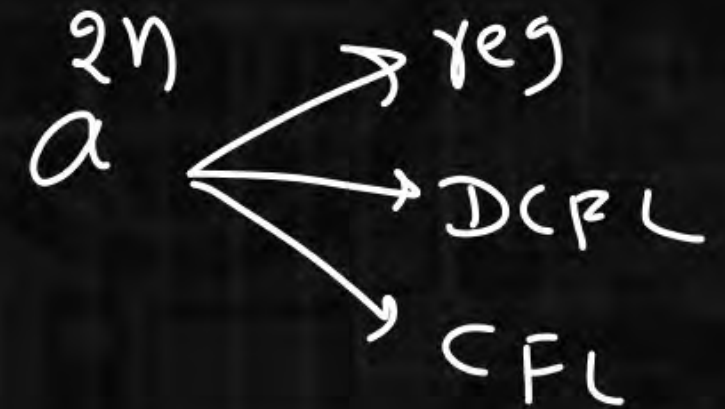
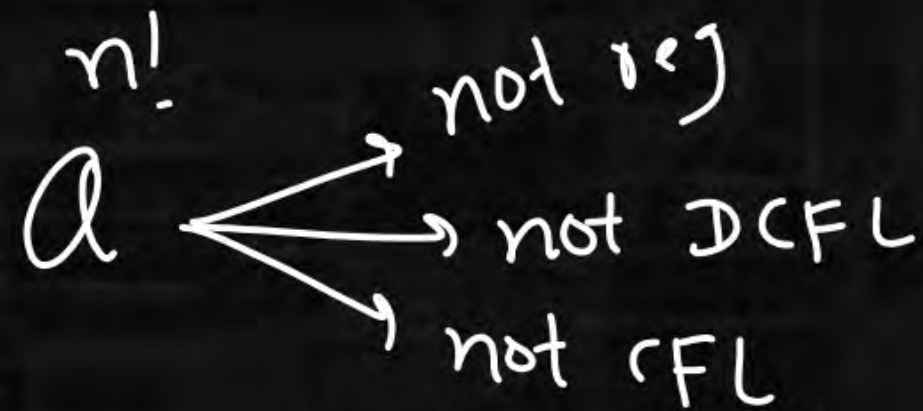
CFL ww^R

Not CFL $a^n b^n c^n$

Note: If Language is over 1 symbol then

$$FA \cong PDA \cong DPDA$$

(stack is useless in PDA because language has only 1 symbol)



Every finite language is regular

DCFL
CFL
CSL
Rec
REL

Every Regular is

DCFL
CFL
CSL
Rec
REL

Every DCFL is

CFL
CSL
Rec
REL

Identification: Finite, Reg but not fin, DCFE but not reg, CFL but not DCFE, Not CFL



① $\{a^n b^n c^n \mid n < 100\} \Rightarrow \text{Finite}$

② $\{a^n b^n c^* \mid n < 100\} \Rightarrow \text{Regular but not finite}$

③ $\{a^n! b^* c^* \mid n < 100\}$

④ $\{a^n b^n c^n \mid n \geq 1\} \Rightarrow \text{Not CFL}$

⑤ $\{a^n b^n c^* \mid n \geq 1\} \Rightarrow \text{DCFL but not reg}$
push a's, pop a's, skip

⑥ $\{a^n! b^* c^* \mid n \geq 1\} \Rightarrow \text{Not CFL}$
do not from A.P.

⑦ $\{a^n b^{n+1} c^{n+2} \mid n \geq 0\} \Rightarrow \text{Not CFL}$

⑧ $\{a^n \boxed{b^{n^2}} \mid n \geq 0\} \Rightarrow \text{Not CFL}$
do not from A.P.

⑨ $\{a^n b^{n^2} c^{n^3}\} \Rightarrow \text{Not CFL}$

(10) $\{a^{n+i} b^{n+j} \mid n, i, j \geq 0\} = a^* b^* \Rightarrow \text{Regular but not finite}$

(11) $\{a^{n+i} b^{n+j} c^{n+k} \mid i, j, k \geq 0, n \geq 0\} = a^* b^* c^* \Rightarrow \text{Regular but not finite}$

(12) $\{a^n b^{100n}\} \Rightarrow \text{DCFL but not reg}$

(13) $\{a^{50n} b^{100n}\} \Rightarrow \text{DCFL but not reg}$

(14) $\{a^n b^n c^n d^n\} \Rightarrow \text{Not CFL}$

$$\{a^{n+i} b^{n+j}\} = \underbrace{\{a^i b^j\}}_{n=0} \cup \underbrace{\{a^{1+i} b^{1+j}\}}_{n=1} \cup \underbrace{\{a^{2+i} b^{2+j}\}}_{n=2} \cup \dots$$

$$= \{a^* b^*\} \cup \{a^+ b^+\} \cup \{a a^+ b b^+\} \cup \dots$$

$$= \{a^* b^*\}$$

$$\{a^{n-1} b^{n-1}\}_{n \geq 1}$$

$$\begin{array}{l} n=0 \\ n=1 \end{array} \begin{array}{l} \cancel{a^{-1} b^{-1}} \\ a^0 b^0 \end{array}$$

$$\boxed{a^0} = \varepsilon$$

$$\{a^n b^n\}_{n \geq 0}$$

$$a^{-1} \times$$

$$\{a^{n-10} b^{n-10}\}_{n \geq 10}$$

$$a^{-2} \times$$

$$\{a^{n+2} b^{n+2}\}_{n \geq -2}$$

$$\begin{array}{l} n=-2 \\ a^0 b^0 \end{array} \checkmark$$

$$a^1 \checkmark$$

(15) $\{a^n b^n c^k d^k\} \Rightarrow \text{DCFL but not reg}$

(16) $\{a^n b^n c^k d^k e^l f^l\}$

(17) $\{a^n c^k d^k b^n\}$

(18) $\{a^m b^n c^{m+n}\}$

(19) $\{a^m b^{m+n} c^n\}$



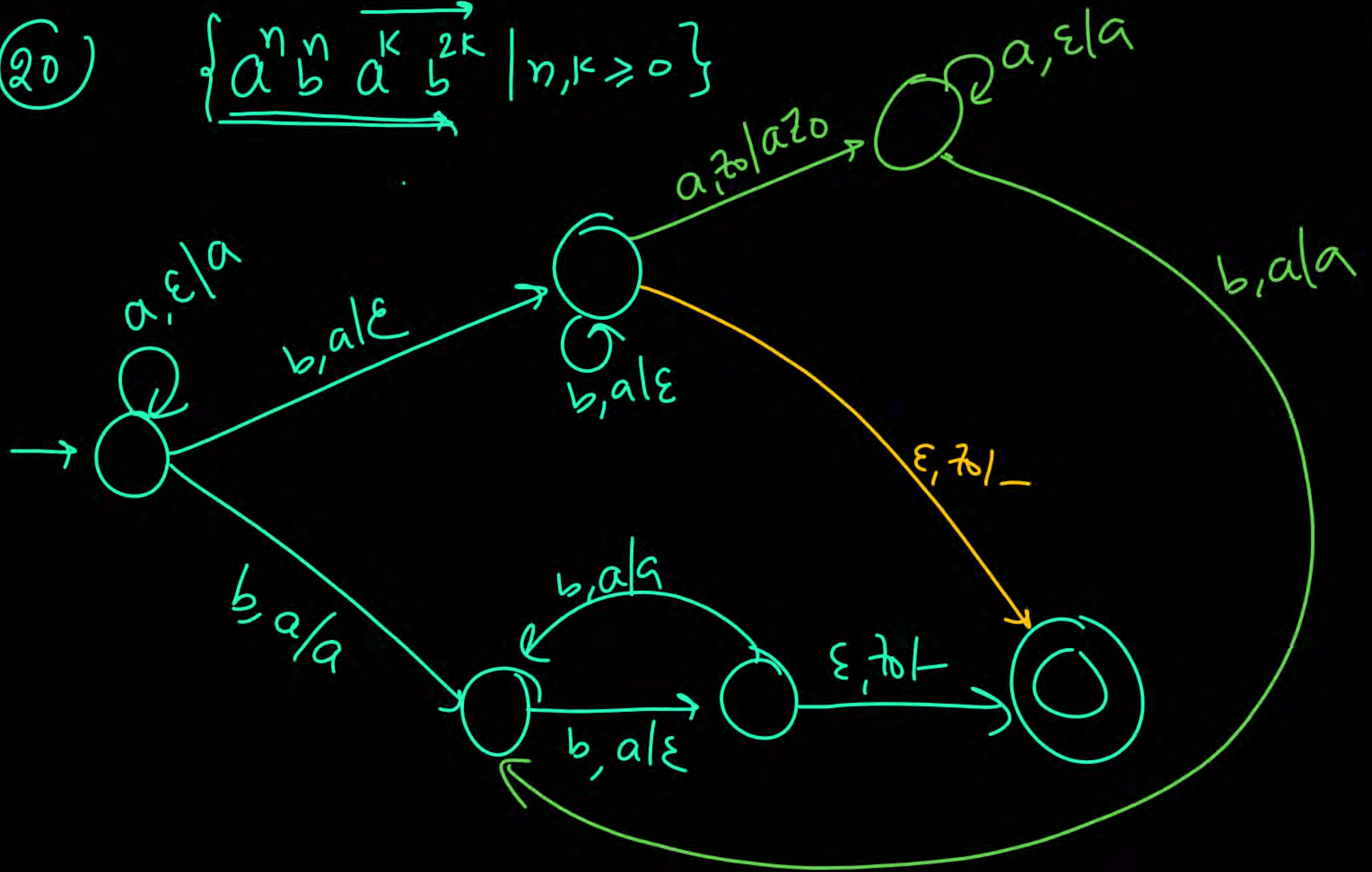
*** (20) $\{a^n b^n a^k b^{2k} \mid n, k \geq 0\}$

*** (21) $\{a^n b^n a^k b^{2k} \mid n \geq 1, k \geq 0\}$

Diagram illustrating the stack operations for (21):
 Push a's, Push b's, Pop b's, Pop a's.
 Stack state: $aaa \boxed{b} bbbbbb$

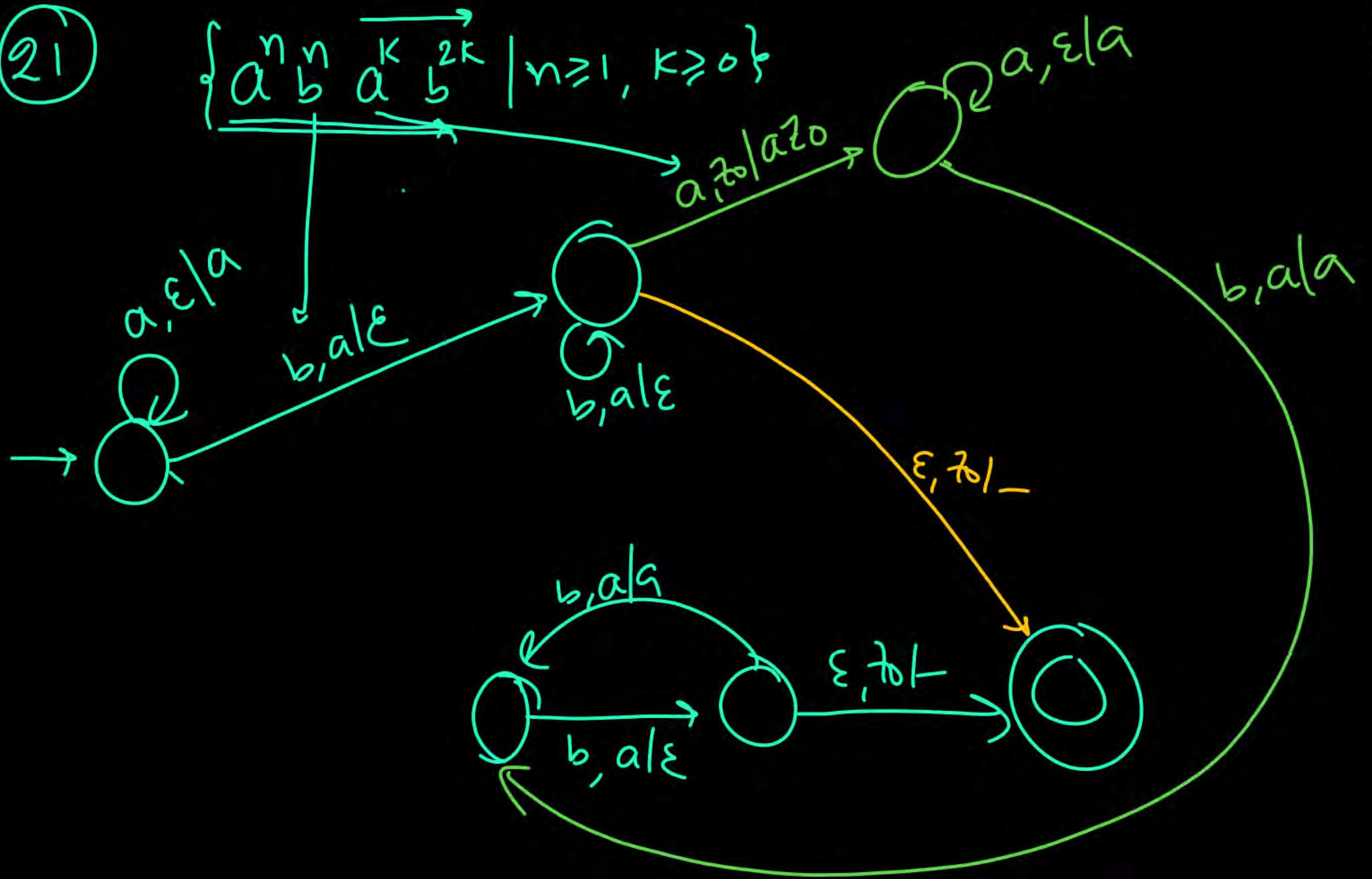
DCFL but not reg

(20) $\{ \underline{a^n b^n a^k b^{2k}} \mid n, k \geq 0 \}$



(21)

$$\{a^n b^n a^k b^{2k} \mid n \geq 1, k \geq 0\}$$



$\sim P \vee Q$
 $P \Rightarrow Q$

H.W.

(22) $\{ a^m b^n c^k \mid \text{if } (m = \text{even}) \text{ then } (n = k) \}$

(23) $\{ a^m b^n c^k \mid \text{if } (n = \text{even}) \text{ then } (m = k) \}$

(24) $\{ a^m b^n c^k \mid \text{if } (k = \text{even}) \text{ then } (m = n) \}$

(25) $\{ a^m b^n c^k \mid \boxed{\text{if } (m = n) \text{ then } (n = k)} \}$

(26) $\{ a^m b^n c^k \mid \text{if } (m \neq n) \text{ then } (n < k) \}$

DCFL but not reg

CFL but not DCFL

$(m \neq n) \vee (n = k)$

- | | | |
|------|--------------------|---------------------------------|
| (27) | $a^{n!}$ | not reg
not CFL
not DCF L |
| (28) | a^{prime} | |
| (29) | a^{n^2} | |
| (30) | a^{2^n} | |
| (31) | a^{n^n} | |

Summary



→ CFLs vs DFLs

→ Next class: \Rightarrow CFL, DFL, Not CFL,
closure properties

