


CS & IT ENGINEERING

Theory of Computation

Lecture No.- 10

A man with a beard and mustache, wearing a black polo shirt, stands with his arms crossed in front of a bookshelf. He is wearing a black watch on his left wrist.

Malleham Devasane Sir

Recap of Previous Lecture



Topic

Regular Languages

Topic

Context Free Languages

Topic

Turing Machine

Topic

Undecidability Concepts

Topics to be Covered



Topic

Regular Languages

Topic

Context Free Languages

Topic

Turing Machine

Topic

Undecidability Concepts

#Q126. Which of the following is TRUE?

Myhill-Nerode Theorem

1. L is regular if and only if \equiv_L has a finite number of equivalence classes.
2. L is not regular if and only if \equiv_L has an infinite number of equivalence classes.

A

Only 1

B

Only 2

C

Both 1 and 2

D

None of these

#Q127 . Which of the following is countable?

$\{L \mid L \text{ is finite}\}$

☒ A

Set of finite languages

☐ B

\rightarrow uncountable

Set of languages $= 2^{\Sigma^*} = P(\Sigma^*)$

☒ C

Set of regular languages

$\{L \mid L \text{ is Regular}\}$

☒ D

Set of strings $= \Sigma^*$

Set of Languages =

- 1. $\{\epsilon\}$
 - 2. $\{s\}, \{a\}, \{b\}, \{aa\}, \{ab\}, \dots$
 - 3. $\{\epsilon, a\}, \{\epsilon, b\}, \{a, b\}, \{\epsilon, aa\}, \dots$
 - 4. $\{\epsilon, a, b\}, \{\epsilon, a, aa\}, \dots$
- }

Set of strings over Σ

$$= \Sigma^*$$

$$= \{ \epsilon, a, b, aa, ab, ba, bb, \dots \}$$

$$N: \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix}$$

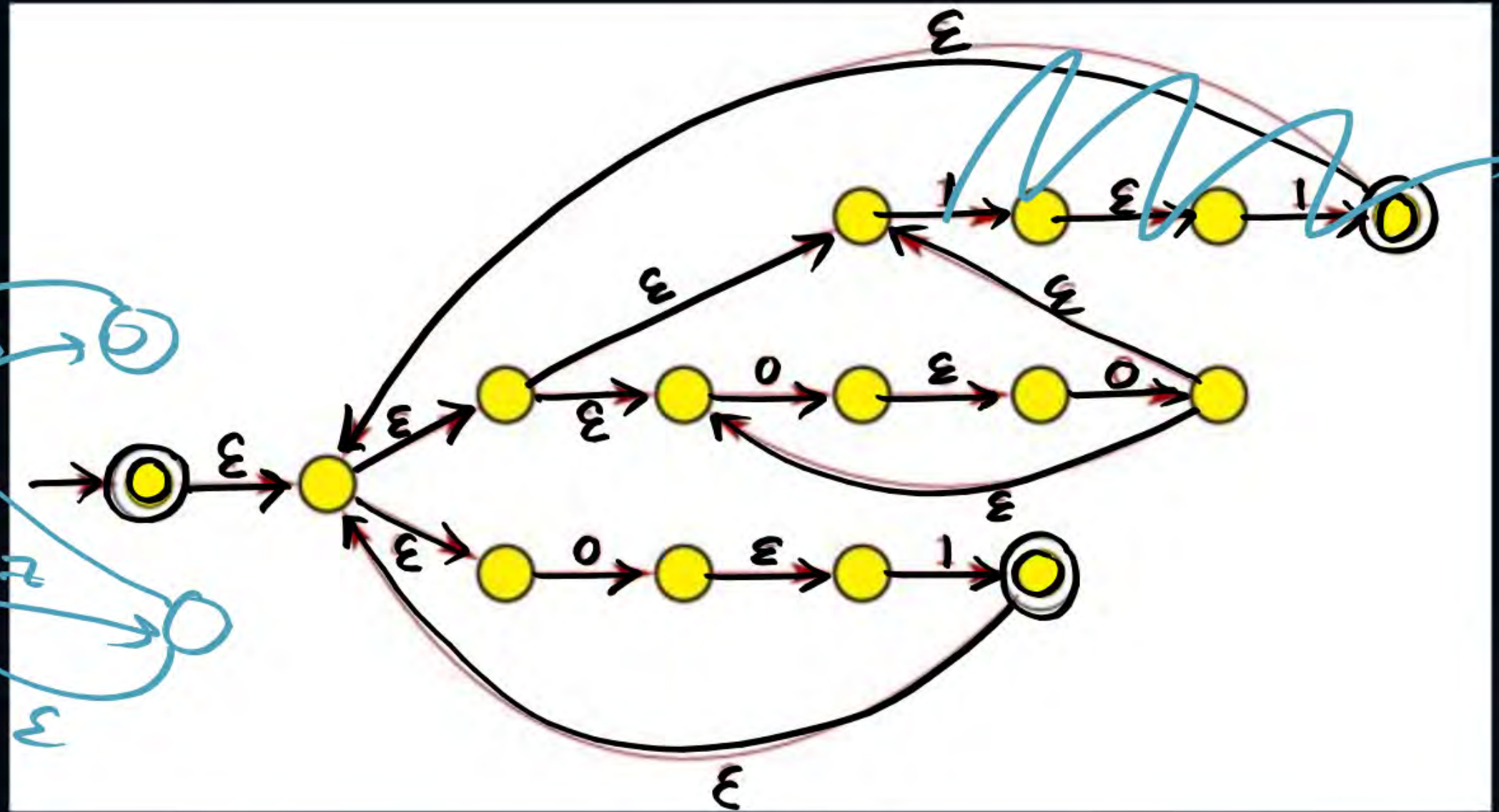
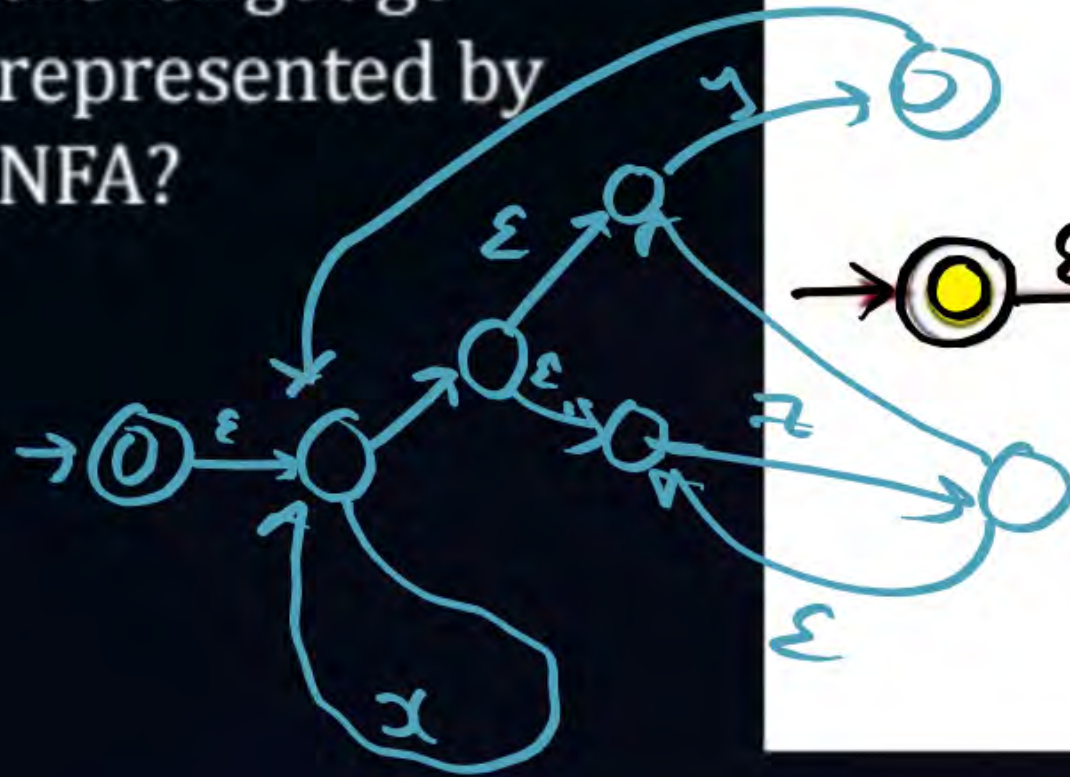
$$f: \Sigma^* \rightarrow N \text{ is Bijective}$$

X is Countable

iff

$f: X \rightarrow \text{known countable set}$ is Bijective

#Q128. What is the language represented by NFA?



A

$((00)^* + 11 + 01)^*$

C

$((00)^*(11 + 01))^*$

B

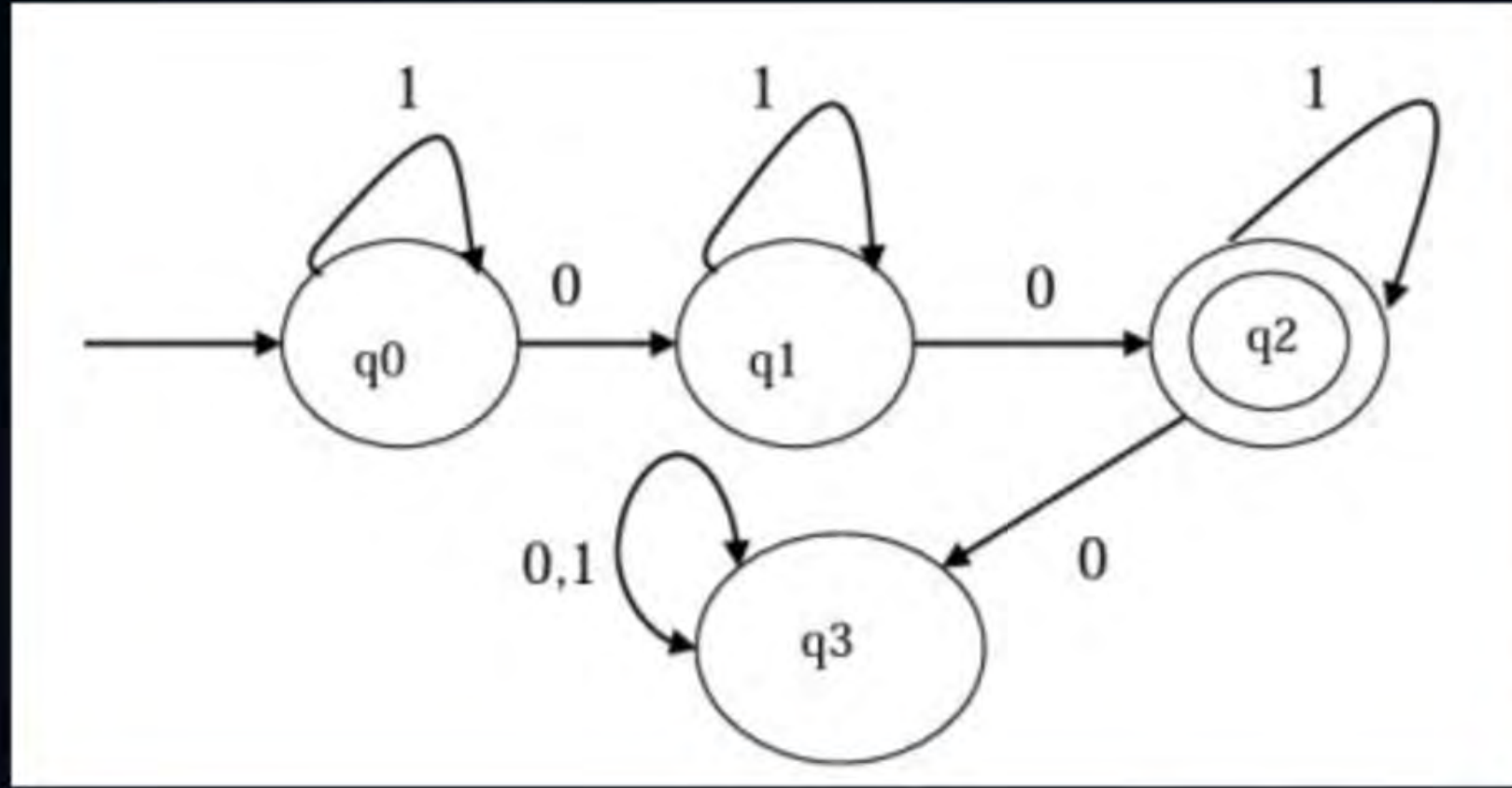
$((00)^*(11) + 01)^*$
 $(z^*y + x)^*$

D

$((00)^*(01) + 11)^*$

#Q129. What is the language accepted by FA?

$1^*01^*01^*$



A

$\{w \mid w \text{ is binary, } n_0(w) = 2\}$

B

$\{w \mid w \text{ is binary, } n_0(w) > 2\}$

C

$\{w \mid w \text{ is binary, } n_1(w) = 2\}$

D

$\{w \mid w \text{ is binary, } n_1(w) > 2\}$

#Q130. Find the language generated by the following CFG.

$$S \rightarrow AB$$

$$A \rightarrow 0A|0$$

$$B \rightarrow 1B|\epsilon$$

$$A = 0^+$$

$$B = 1^*$$

$$\left. \begin{array}{l} A = 0^+ \\ B = 1^* \end{array} \right\} L = S = AB = 0^+1^*$$

A

$$0^*1^*1 = 0^*1^+$$

C

$$0^*11^* = 0^*1^+$$

B

$$00^*1^* = 0^+1^*$$

D

$$0^*01^* = 0^+1^*$$

#Q131. Find the language generated by the following CFG.

$$S \rightarrow AC \mid CB$$

$$C \rightarrow aCb \mid \varepsilon$$

$$A \rightarrow aA \mid a$$

$$B \rightarrow bB \mid b$$

$$C = a^n b^n$$

$$A = a^+$$

$$B = b^+$$

$$S = AC + CB$$

$+ a^+ a^n b^n$ or $a^n b^+ b^n$
 $n_a > n_b$ or $n_a < n_b$
 $n_a \neq n_b$

A

$$\{a^m b^n \mid m=n\}$$

B

$$\{a^m b^n \mid m \geq n\}$$

C

$$\{a^m b^n \mid m \leq n\}$$

D

$$\{a^m b^n \mid m \neq n\}$$

#Q132. How many of the following statements are TRUE?

I. Complement of L is same as L FALSE

II. Kleene star of L is same as L FALSE

✓ III. Complement of Complement of L is same as L

IV. Kleene star of Kleene star of L is same as L FALSE

$$\overline{\overline{L}} = L$$

$$\overline{L} = \Sigma^* - L$$

$$L \neq \overline{L}$$

$$L^* = a^*$$

$$L = \{a\}$$

$$L^* = a^*$$

$$L \neq L^*$$

$$L = \{\epsilon\}$$

$$L^* = \Sigma^* = \{\epsilon\}$$

$$L = L^*$$

=

Note:

$$L^* = (L^*)^*$$

$$L^0 \rightarrow L^1 \rightarrow L^2 \rightarrow \dots$$

$$L^0 \rightarrow L^1 \rightarrow L^2 \rightarrow \dots$$

#Q133. Which of the following is correct?

A $(r + s)^* = r^* + s^*$ ✗

C $(r + s)^* = r^*s^*$ ✗

~~**B**~~ $(r + s)^* = (r^* + s^*)^*$ ✓

~~**D**~~ $(r + s)^* = (r^*s^*)^*$ ✓

#Q134. Which of the following CFGs are in CNF?

CNF

$$\begin{array}{l} V \rightarrow VV \\ \text{OR} \\ V \rightarrow T \end{array}$$

~~A~~

$S \rightarrow \underline{aa} \mid \underline{SS}$
 x ✓

~~C~~

$S \rightarrow \underline{abc} \mid \underline{SS}$
 x ✓

~~B~~

$S \rightarrow \underline{a} \mid \underline{SS}$
 ✓ ✓

~~D~~

$S \rightarrow \underline{a} \mid \underline{SSS}$
 ✓ x

#Q135. Which of the following CFGs are in GNF?

$$V \rightarrow TV^*$$

~~A~~

$$S \rightarrow \underline{a} \mid \underline{aSS}$$

~~C~~

$$S \rightarrow \underline{abc} \mid \underline{aSS}$$

~~B~~

$$S \rightarrow \underline{a} \mid \underline{bSS}$$

~~D~~

$$S \rightarrow \underline{a} \mid \underline{SSS}$$

#Q136. Union, Intersection, and Complement operations are closed for ____

~~A~~

Regular languages

$\cup, \cap, \bar{}$ ✓ ✓ ✓

~~C~~

Decidable languages

$\cup, \cap, \bar{}$ ✓ ✓ ✓

B

CFLs

$\cup, \cap, \bar{}$ ✓ ✗ ✗

D

RELs

$\cup, \cap, \bar{}$ ✓ ✓ ✗

	\cup	\cap	complement
Finite _s	✓	✓	✗
Infinite _s	✓	✗	✗
Regular _s	✓	✓	✓
DCFL _s	✗	✗	✓
CFL _s	✓	✗	✗
CSL _s	✓	✓	✓
Recursive _s	✓	✓	✓
RFL _s	✓	✓	✗

#Q137. Number of states in Min DFA that accepts all binary strings ending with 111.

= 4

Min = 111

\Downarrow

4 states



#Q138. Which of the following are CFLs but not regular?

☒ A

$\{w \mid w \text{ is binary, } n_0(w) = n_1(w)\}$

☒ B

$\{w \mid w \text{ is binary, } n_0(w) > n_1(w)\}$

☒ C

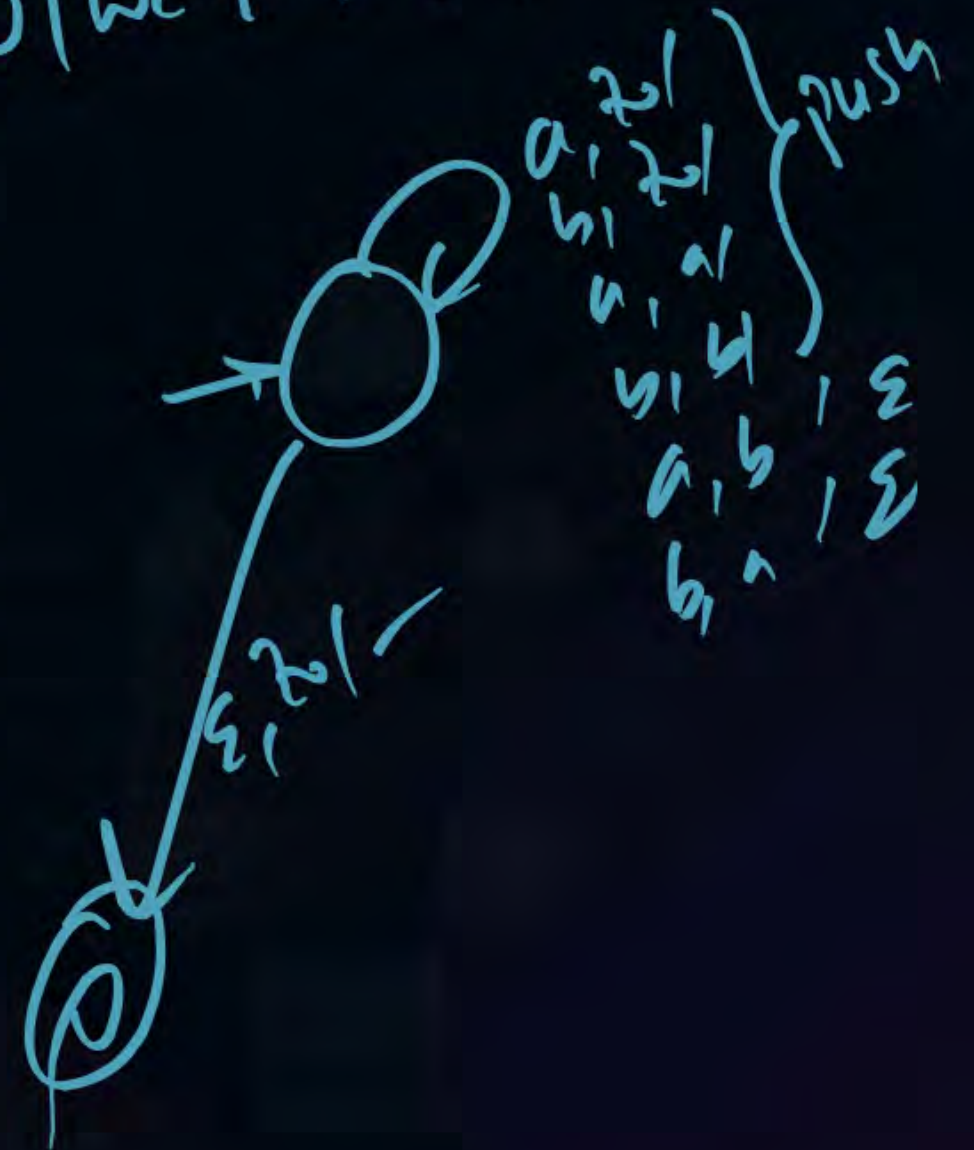
$\{w \mid w \text{ is binary, } n_0(w) < n_1(w)\}$

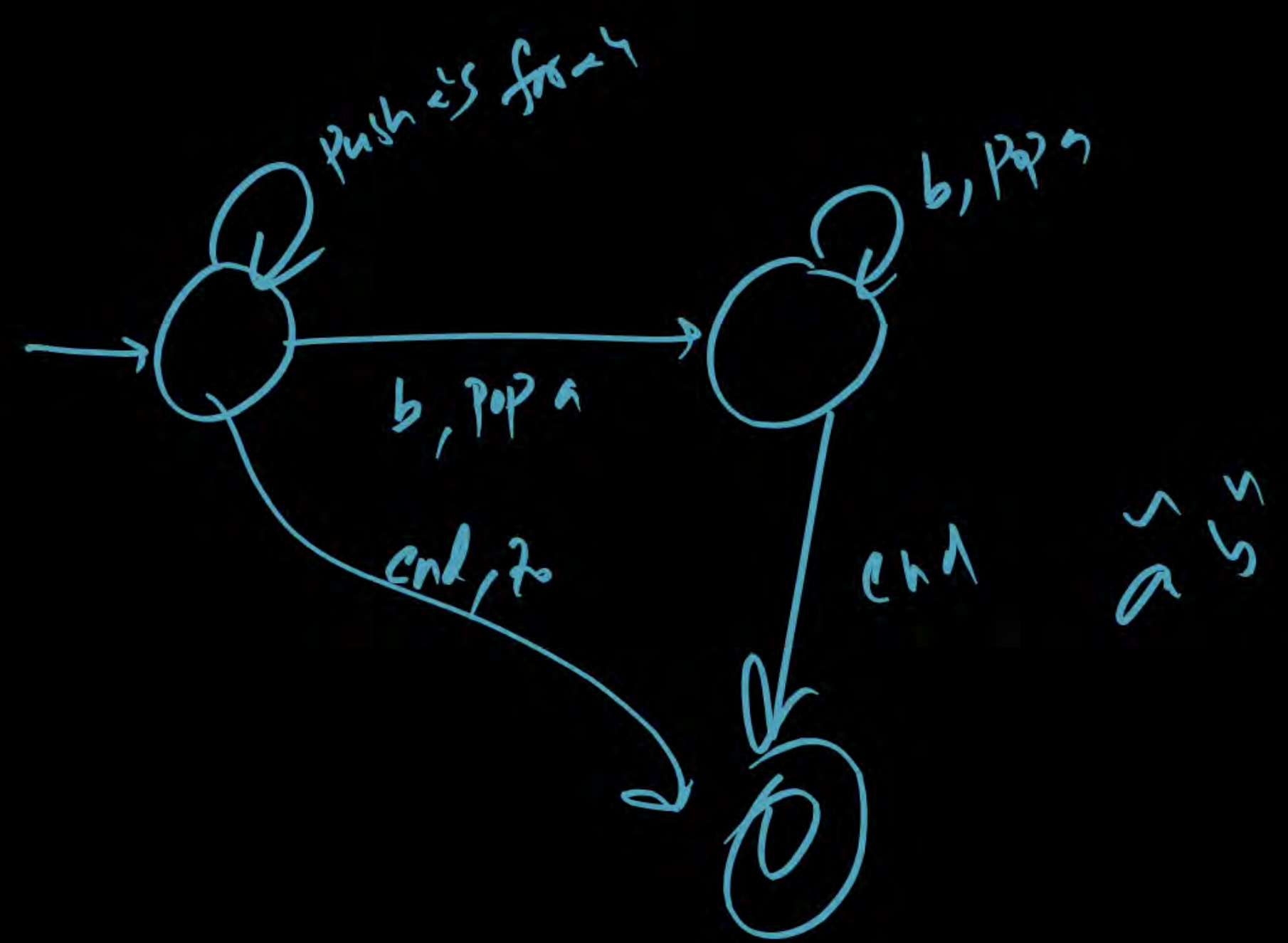
☐ D

None of these

DCFLs
not reg

$\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$





#Q139. Which of the following is CFL?

$$n=3 \Rightarrow (a^3 b^3)(a^3 b^3)(a^3 b^3)$$

$$n=2 \quad k=3$$

$$(a^2 b^2)^3 = \underbrace{a^2 b^2}_{+} \underbrace{a^2 b^2}_{+} \underbrace{a^2 b^2}_{+}$$

~~A~~

$$\{ (a^n b^n)^n \mid n \geq 0 \}$$

$$(a^2 b^3)^4$$

~~C~~

$$\{ (a^m b^n)^k \mid m, n, k \geq 0 \}$$

$$m=2 \quad n=3 \quad k=4$$

~~B~~

$$\{ (a^n b^n)^k \mid k, n \geq 0 \}$$

~~D~~

$$\{ (a^* b^*)^n \mid n \geq 0 \} = (a^* b^*)^* = (a+b)^*$$

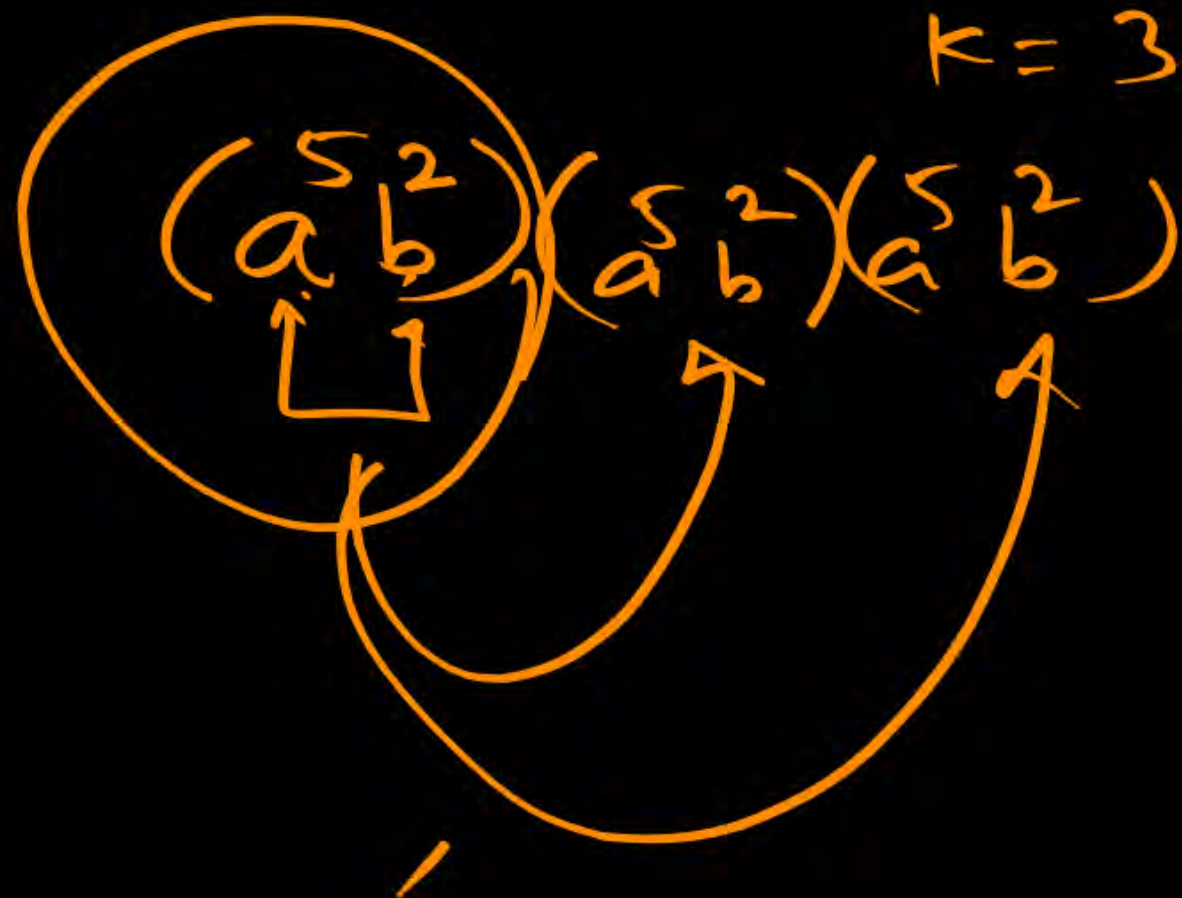
$$n=2 \Rightarrow a^* b^* a^* b^*$$

$$\left\{ \left(a^m b^n \right)^k \mid m, n, k \geq 0 \right\}$$

$$m=5$$

$$n=2$$

$$k=3$$



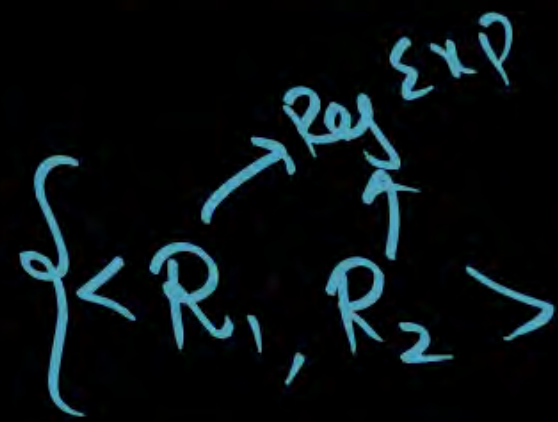
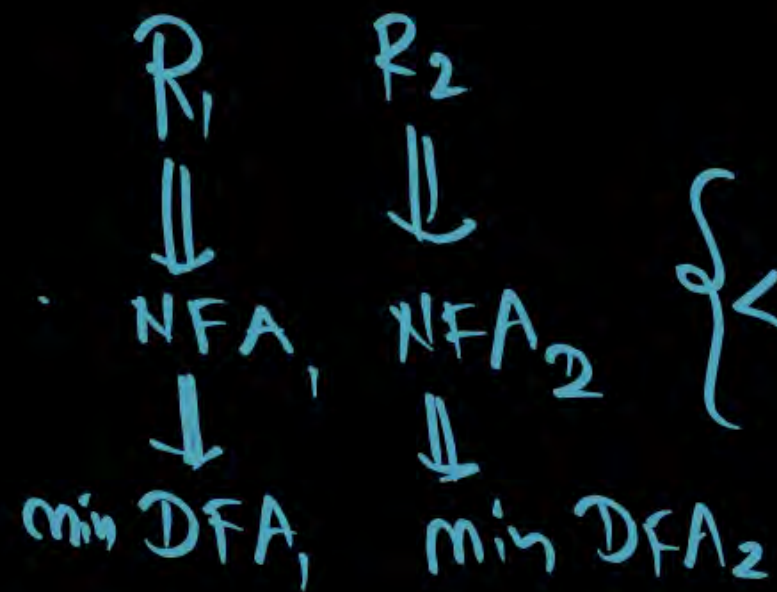
$$\left\{ \left\{ a^m b^n \mid m, n \geq 0 \right\}^k \mid k \geq 0 \right\}$$

$$\left\{ a^m b^n \right\}^2 = \left\{ a^m b^n \right\} \left\{ a^{m_2} b^{n_2} \right\}$$

$a^5 b^4 \quad a^4 b^5$

CSL Vs Recursive Set

Every CSL is Recursive



$L(R_1) = L(R_2)$
 is not CSL
 is Recursive

$a^n b^n$
 a^p
 $a^n b^n c^n$
 $wwww$
 $a^{n!}$

CSL

[MCQ]



#Q140. $\{M \mid L(M) \text{ is empty language}\} = \text{set of machines which accepts empty set}$

$$= \{M_1, M_2, M_3, \dots\}$$



Whether given TM accepts empty language

→ Yes: $L(TM) = \emptyset$ (No logic)
→ No: $L(TM) \neq \emptyset$

A

Recursive

B

REL

C

Regular

D

None of these

#Q141. $\{M \mid M \text{ accepts } abb \text{ within } 100 \text{ steps}\}$

IS ^{given} M accepts ^{given} abb within ^{given} 100 steps?

Yes: M accepts abb within 100 steps.

No: M doesn't accept abb within 100 steps.

we will check upto 100 steps

Yes: If M halts at final within 100 steps

No: If M takes 101st step, it is guaranteed M will not accept within 100 steps

☒ A

Recursive

☐ C

Regular

☒ B

REL

☐ D

None of these

#Q142. $\{M \mid M \text{ accepts } abb \text{ after } 100 \text{ steps}\}$

M not accepts abb within 100 steps

☒ A

Recursive

☐ C

Regular

☒ B

REL

☐ D

None of these

#Q143. Which of the following is decidable for PDA but not for HTM?

A

Membership

C

Equivalence

B

Emptiness

D

Finiteness

Decidable for both PDA & HTM

Undecidable for both PDA & HTM



$\frac{H}{M} \left\{ \begin{array}{l} \text{upto} \end{array} \right. \text{HTM}$

$\frac{E}{F} \left\{ \begin{array}{l} \text{upto} \end{array} \right. \text{PDA}$

$\frac{T}{E} \left\{ \begin{array}{l} \text{upto} \end{array} \right. \text{DPDA}$

$\frac{D}{S} \left\{ \begin{array}{l} \text{upto} \end{array} \right. \text{FA}$

$\text{FA} \quad \text{DPDA} \quad \text{PDA} \xrightarrow{\text{HTM}} \text{TM}$

→

→

→

#Q144. If set L is effectively enumerable by an algorithm A, and X is reducible to L then X is ____

(Recursive)
(Decidable)

Every Recursive is REL

$X \leq$ Decidable Set
may or may not be reg

$X \leq \underbrace{L}_{\text{effectively enumerable}}$

☒ A

Recursive

☒ B

REL

☐ C

Regular

☐ D

None fo these

Polynomial Time

$$\hookrightarrow O(n^p)$$

$$O(1)$$

$$\log n$$

$$n$$

$$n^2$$

$$n \log n$$

$$n^3$$

$$n^{100}$$

Deterministic Algorithms

P-Problems

P - problem: Solvable in polynomial by DTM

NP - problem: Solvable in " by NDTM
(Answer is verifiable in polynomial by DTM)

#Q145. How many of the following are Decidable Languages? = 2

- Non-decidable (Rice's Theorem)
1. $\{ \text{TM} \mid L(\text{TM}) \text{ is regular language} \}$
 2. $\{ \text{TM} \mid L(\text{TM}) \text{ is context free language} \}$
 3. $\{ \text{TM} \mid L(\text{TM}) \text{ is decidable language} \}$
 4. $\{ \text{TM} \mid L(\text{TM}) \text{ is enumerable language} \}$
 5. $\{ \text{TM} \mid L(\text{TM}) \text{ is not enumerable language} \}$
- Whether given TM accepts Regular
 " " " " CFL
 " " " " Recursive
 " " " " REL \Rightarrow Yes
 " " " " not REL \Rightarrow No

4. $L = \text{Set of all TMs}$
 $\bar{L} = \emptyset$

5. $L = \emptyset$
 $\bar{L} = \text{Set of all TMs}$

1. $L = \text{Set of some TMs}$
 $\bar{L} = \text{Set of some TMs}$ } non-decidable

Diagram 1: Mapping of integers to binary representations.

Integer	Binary Representation
0	0
-1	-1
2	10
-2	-10
3	11
4	100

Diagram 2: IEEE 754 format for 0.

Sign: 0

Exponent: 00000000

Mantissa: 000000000000000000000000

Diagram 3: IEEE 754 format for infinity.

Sign: 0

Exponent: 11111111

Mantissa: 000000000000000000000000

Set of natural numbers

Set of rational numbers

Set of integers

Set of real numbers

#Q147. Which of the following is True?

Turing Decidable
 \equiv
Recursive
 \equiv
Decidable

~~A~~

Every CFL is Turing decidable

~~B~~

Every Regular is Turing decidable

~~C~~

Every Recursive is Turing decidable

~~D~~

Every REL is Turing decidable

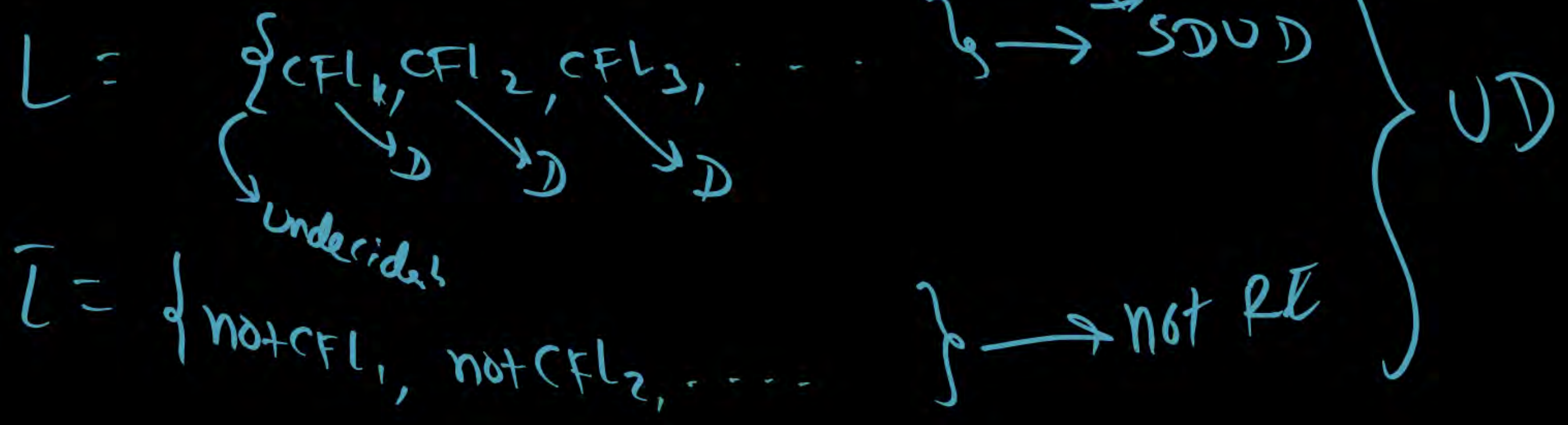
Every CFL is Decidable

$a^n b^n$ ✓

$a^* b^*$ ✓

CFL is Decidable

Set of all CFLs is undecidable



$a^n b^n = \{ \epsilon, ab, a^2 b^2, \dots \}$ is CFL
 → TM exist

$\{ a^*, \phi, b^*, a^n b^n, \dots \text{all CFLs} \}$ is Set of all CFLs
 → TM exist
 → TM not exist

#Q148. Which of the following is not decidable?

Undecidable
 If it not exist



Recursive



Complement of decidable set



Complement of not RE

Not RE or SDUD
 UD



Complement of REL

Some time Recursive or Not RE
 need not be UD



Complement of regular

Decidability
 ↑
 Regular

[MCQ]



#Q149. Kleene star is not closed for ____

$L = \{ca^n b^n\} \cup \{a^k b^{2k}\}$ is DCFL

L^* = not DCFL

Kleene star of DCFL is need not be DCFL

~~A~~

DCFLs

B

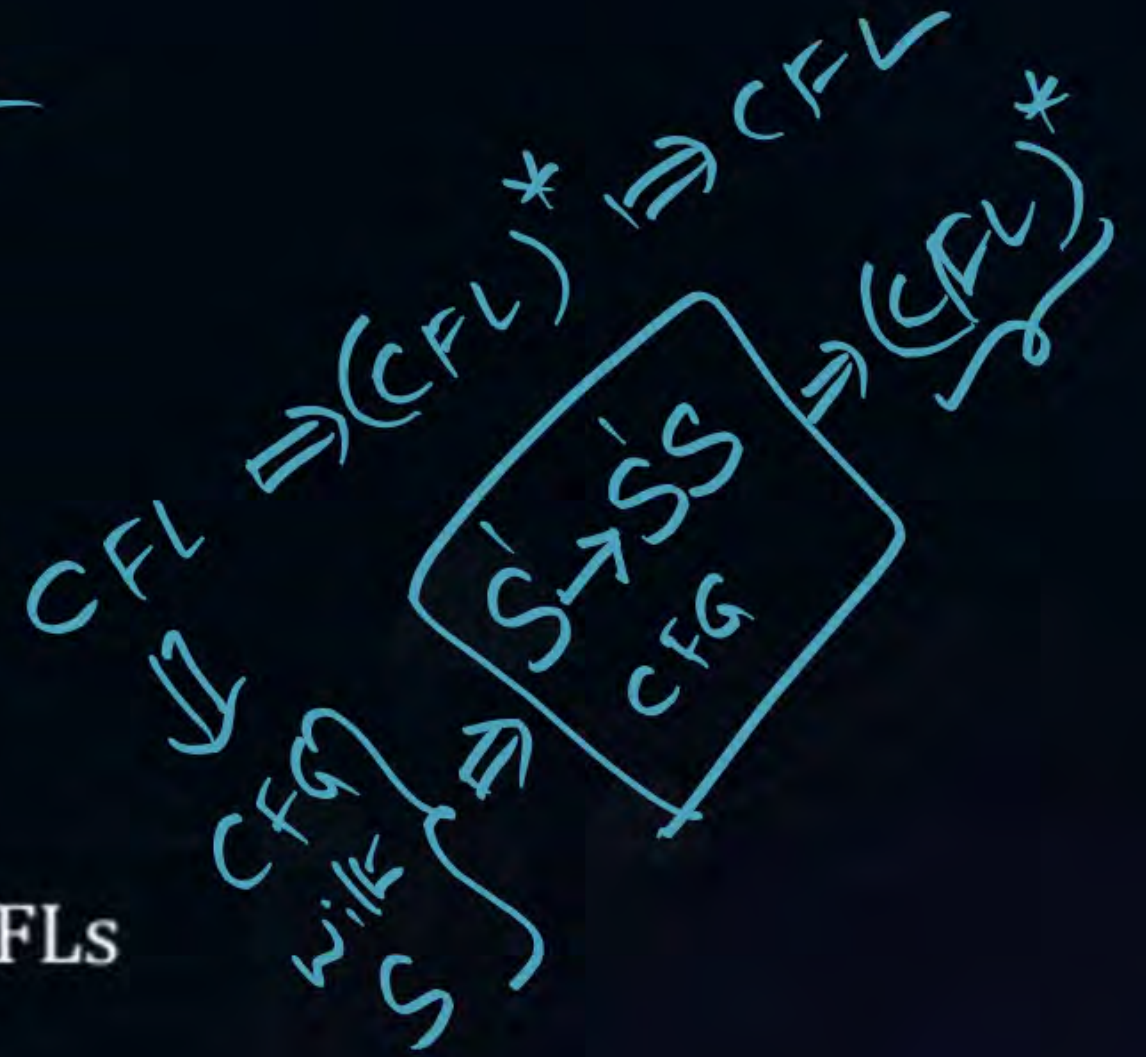
CFLs

C

Decidable languages

D

RELs



$$L = \{c a^n b^n\} \cup \{a^k b^{2k}\}$$



$$\{c\} \cup \{a^2 b^1\} \cup \{a^3 b^6\}$$

$$\subseteq \underline{caaaabbbbbb}$$

$$L^* = \left\{ \{c a^n b^n\} \cup \{a^k b^{2k}\} \right\}^*$$

Diagram illustrating the string $caaaabbb$ corresponding to the set notation $\{c a^n b^n\}$. Arrows point from the 'a' and 'b' blocks in the string to the 'a' and 'b' in the set notation.

$$c \underline{aaa} (b) bbb \checkmark$$

#Q150. Which of the following is TRUE?

~~A~~

Some sets are not REL

~~C~~

Some sets are REL

~~B~~

Some sets are countable

~~D~~

Some sets are not countable



THANK - YOU