

CS & IT ENGINEERING

Theory of Computation
Finite Automata:
Practice on DFA and NFA



Lecture No. 12



By- DEVA Sir

TOPICS TO BE COVERED

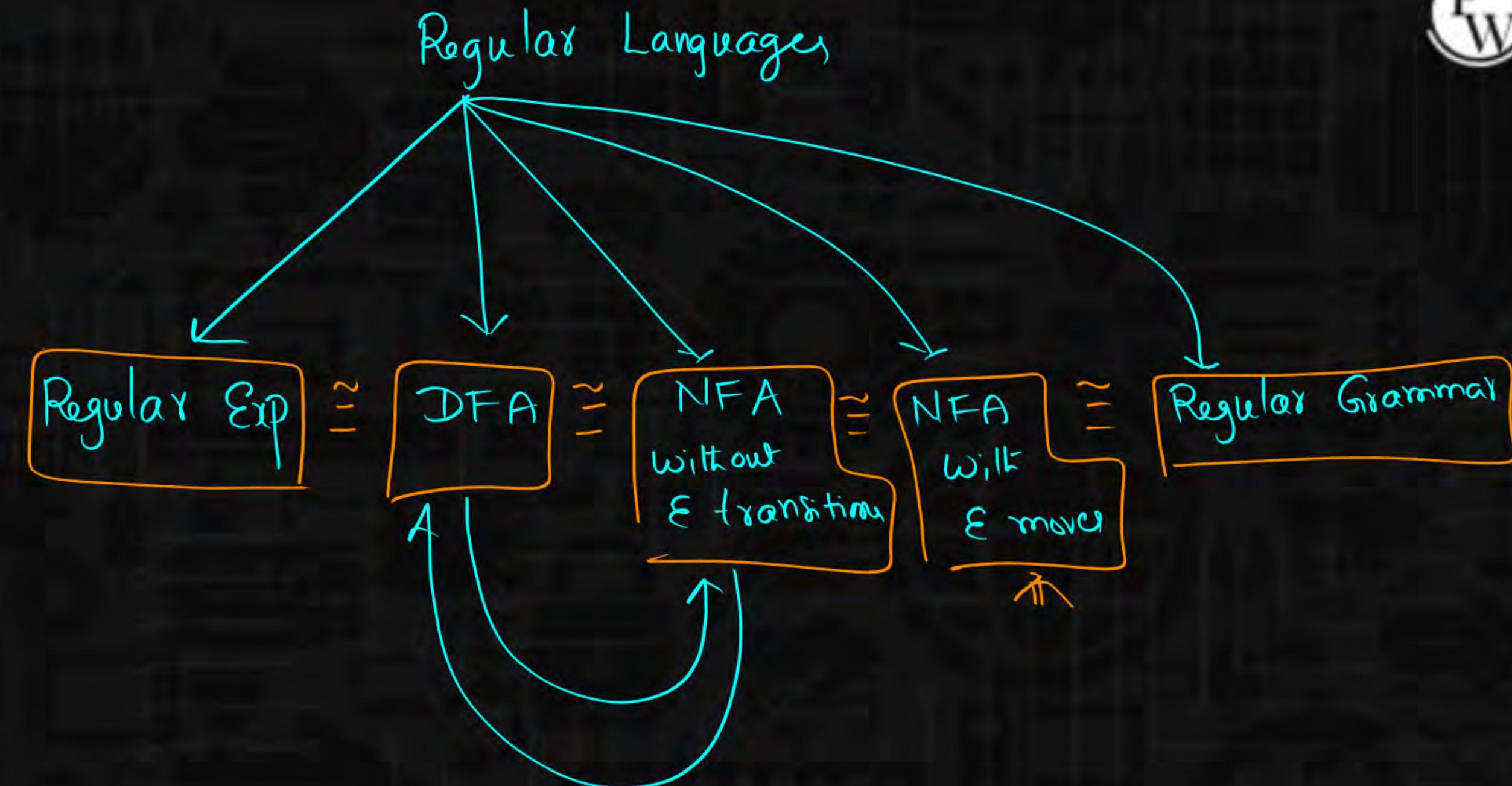
01 ϵ -NFA

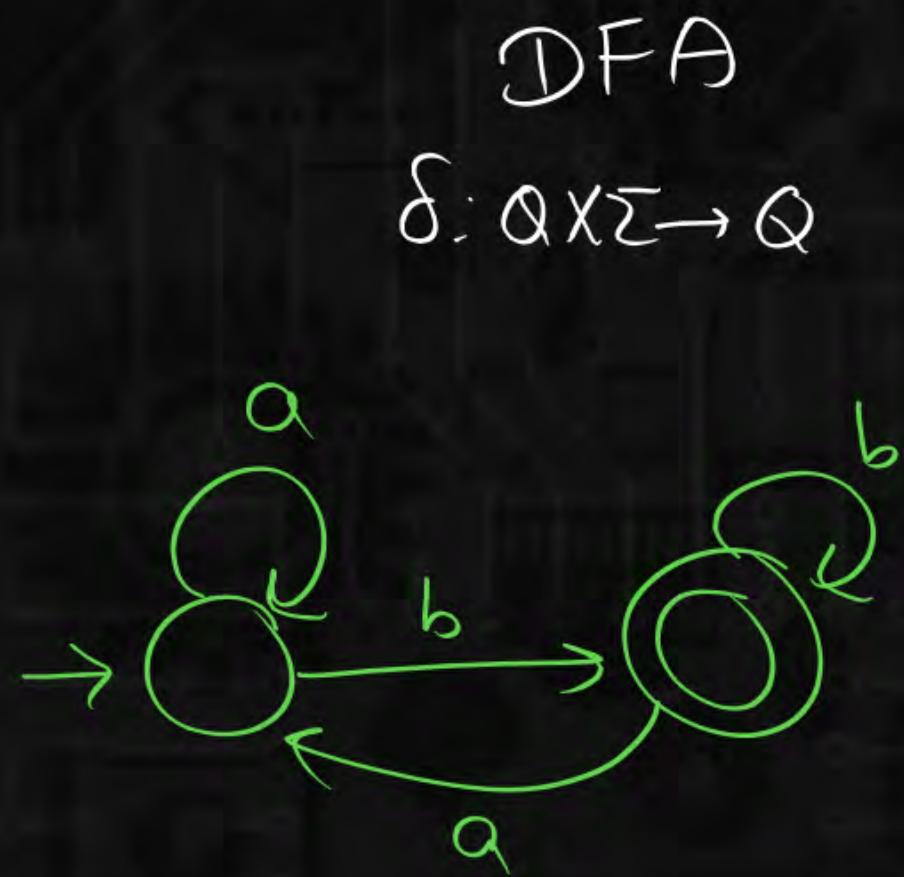
02 NFA, DFA practice

03

04

05



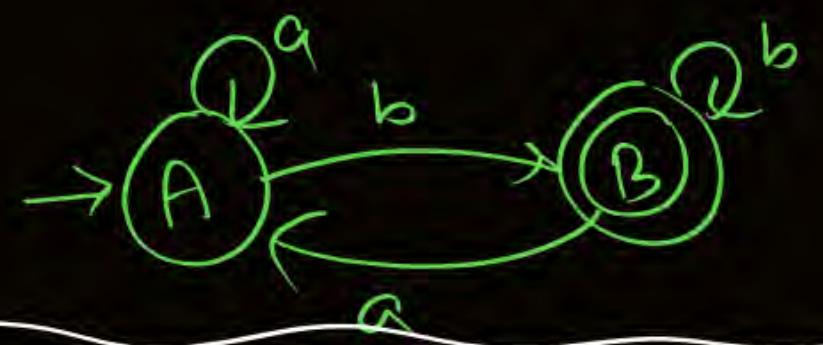


NFA
(without ϵ moves)

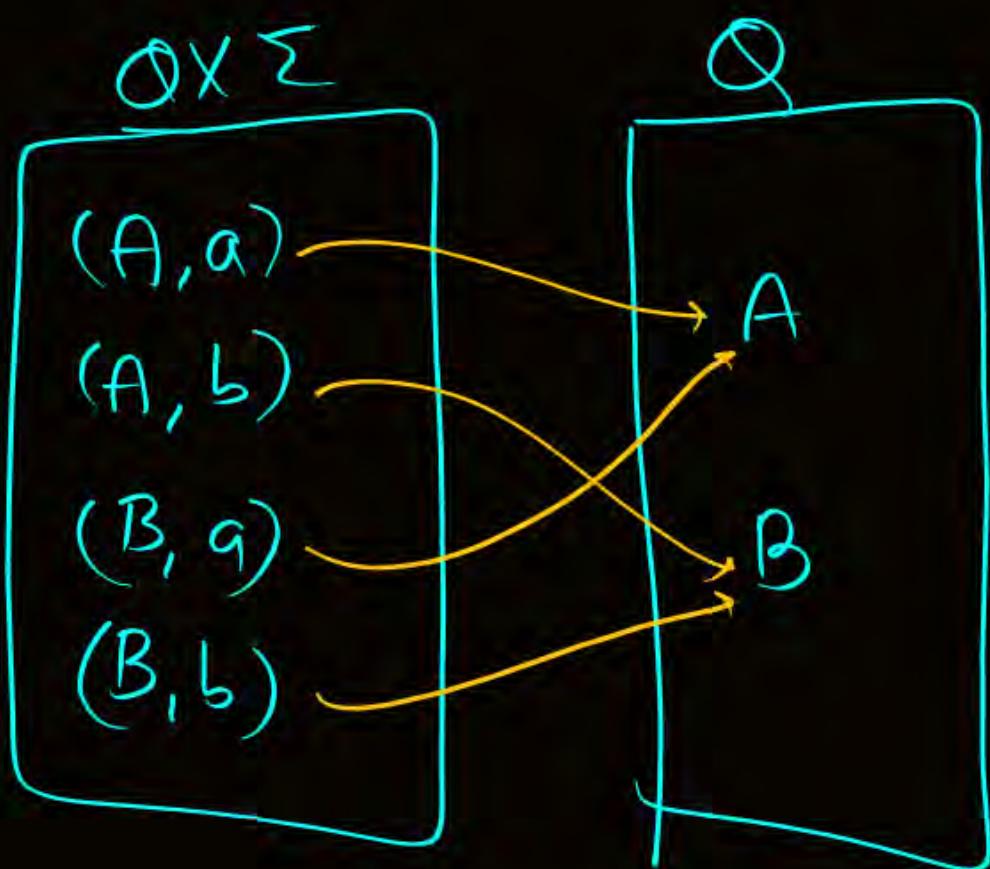
$$\delta: Q \times \Sigma \rightarrow 2^Q$$

NFA
(with ϵ -moves)

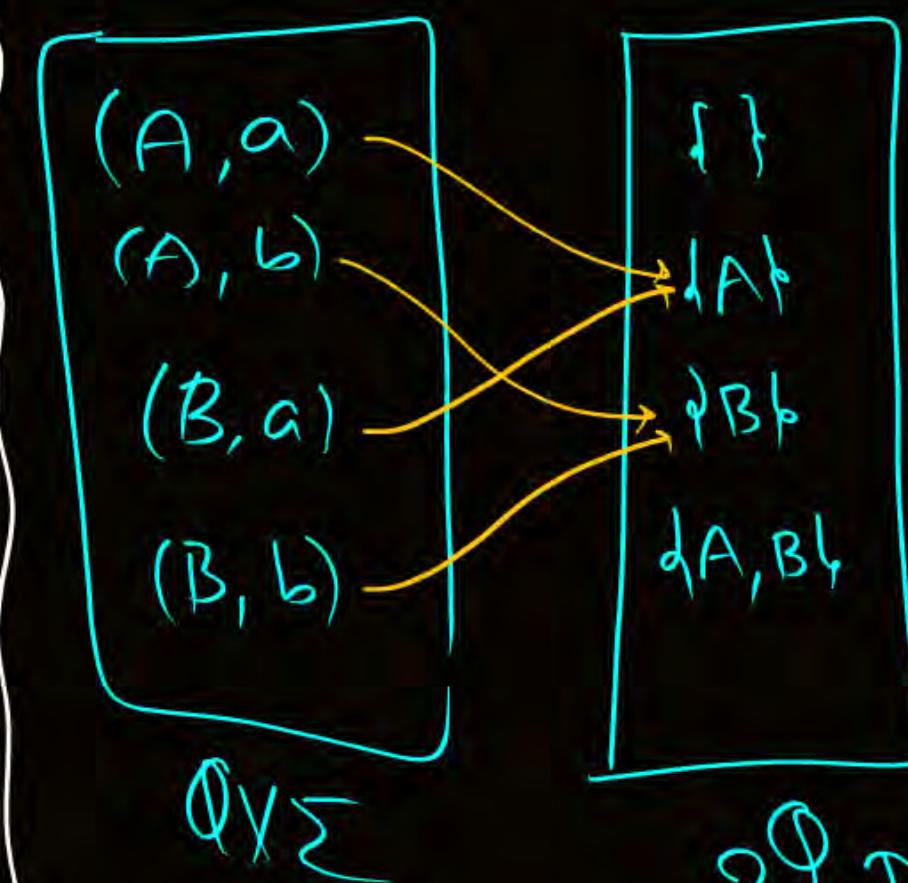
$$\delta: Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$$



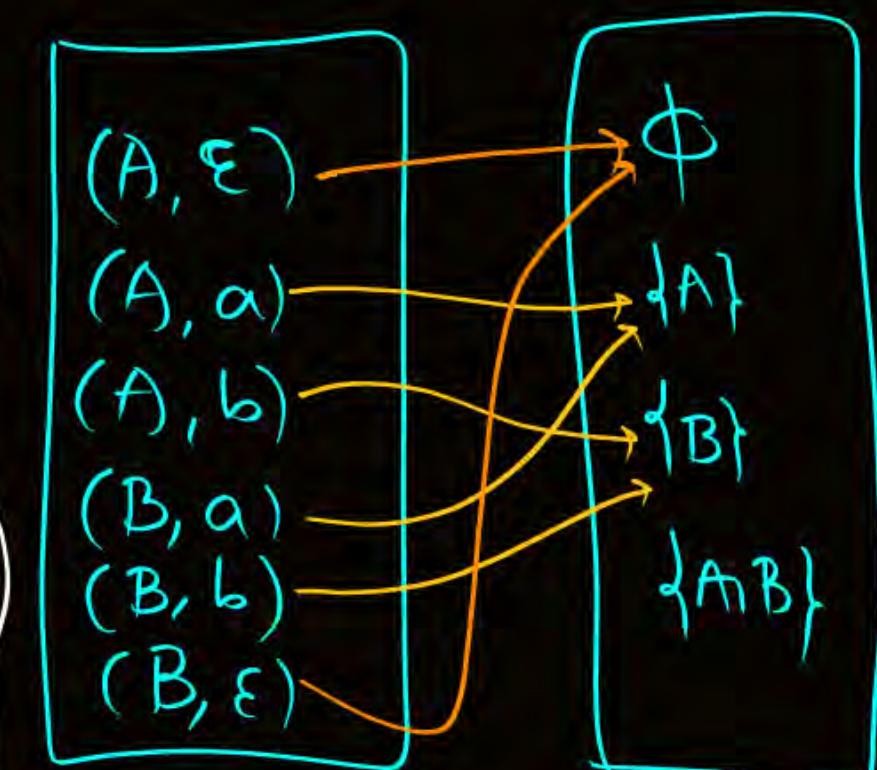
$\delta: Q \times \Sigma \rightarrow Q$

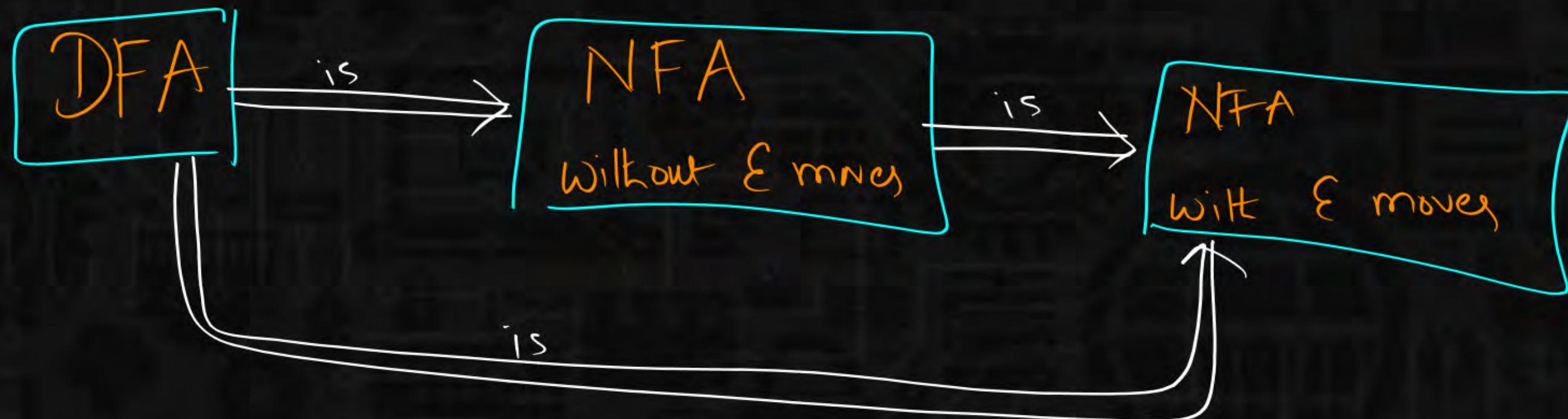


$\delta: Q \times \Sigma \rightarrow 2^Q$

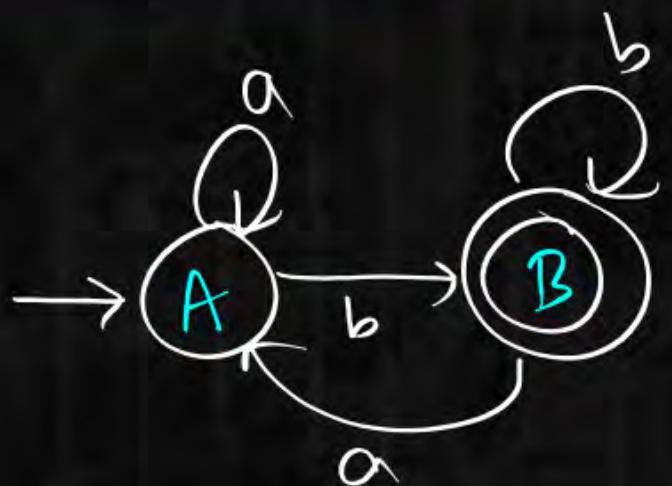


$\delta: Q \times \Sigma_E \rightarrow 2^Q$





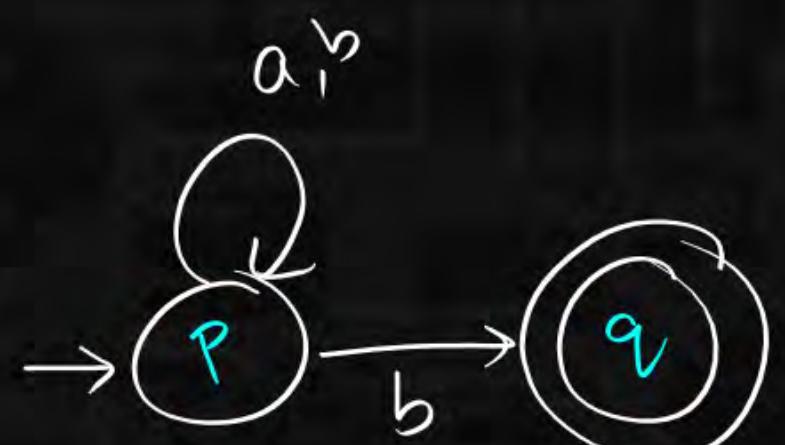
NFA



DFA ✓

NFA without ϵ ✓

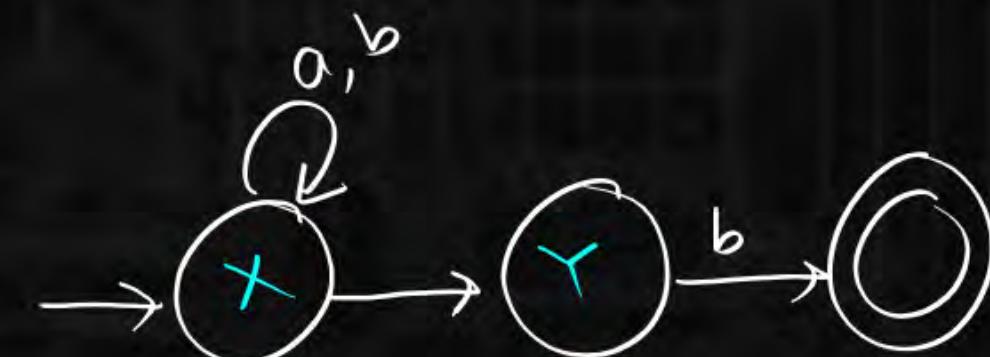
NFA with ϵ moves ✓



DFA X

NFA without ϵ ✓

NFA with ϵ ✓



DFA X

NFA without ϵ moves X

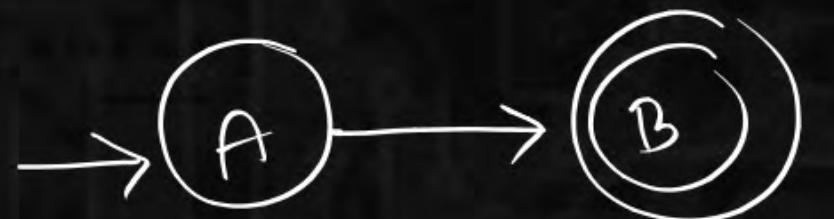
NFA with ϵ moves

$$\delta(X, \epsilon) = \{Y\}$$

.

L

①



$$L = \{\epsilon\}$$

how many paths for $w = \epsilon$?

1) A

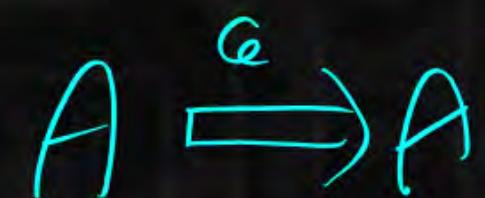
2) $A \xrightarrow[\text{Final}]{} B$



$$L = \{a\}$$

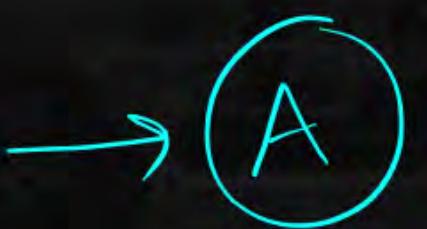
transition ① $\delta(A, \epsilon) = \begin{cases} B & \text{if } \epsilon \rightarrow \text{no trans} \\ \emptyset & \text{if } \epsilon \text{ is no trans on } \epsilon \text{ from } A \end{cases}$

patt ② $\hat{\delta}(A, \epsilon) = A$



$\hat{\delta}$ \rightarrow extended transition (patt)

③

 $L = \emptyset$

path: zero or more sequences of transitions

IS ϵ accepted?

↳ not accepted

Path: A

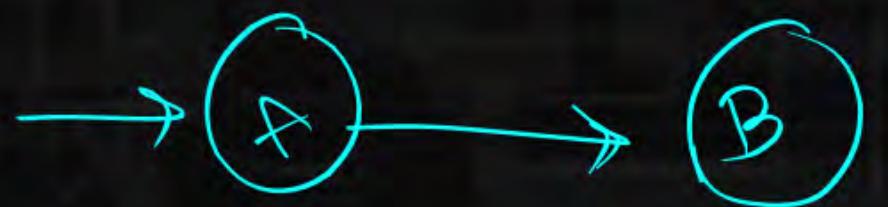
$$\hat{\delta}(A, \epsilon) = A$$

From A, by reading string ϵ , halts at A

$$\delta(A, \epsilon) = \emptyset$$

$$\hat{\delta}(A, \epsilon) = A$$

④



$$L = \emptyset$$

$$\delta(A, \varepsilon) = B$$

Pales.

$$\hat{\delta}(A, \varepsilon) = \{A, B\}$$

$$A \xrightarrow{\varepsilon} A$$

① A

Zero transitions

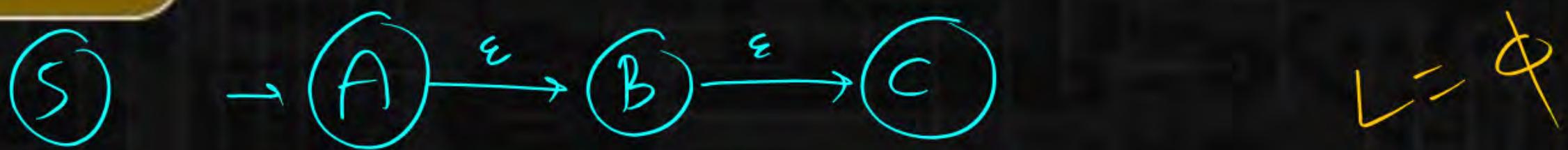
$$A \xrightarrow{\varepsilon} B$$

②

 $A \xrightarrow{\varepsilon} B$ one transition

NFA

P
W



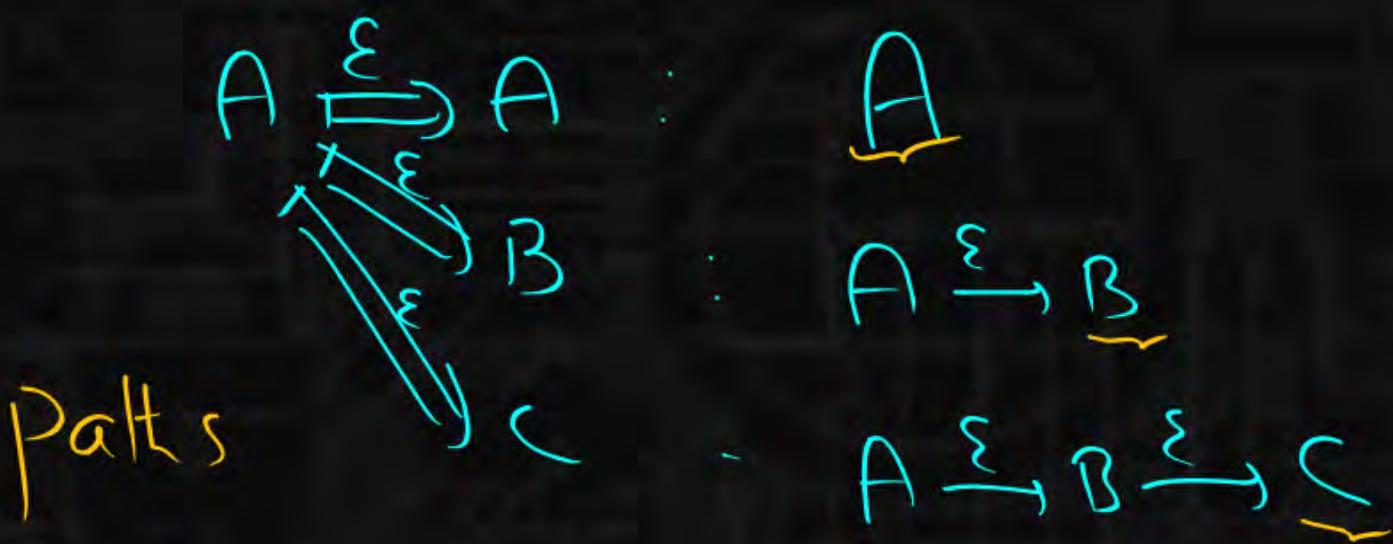
$$\delta(A, \epsilon) = B$$

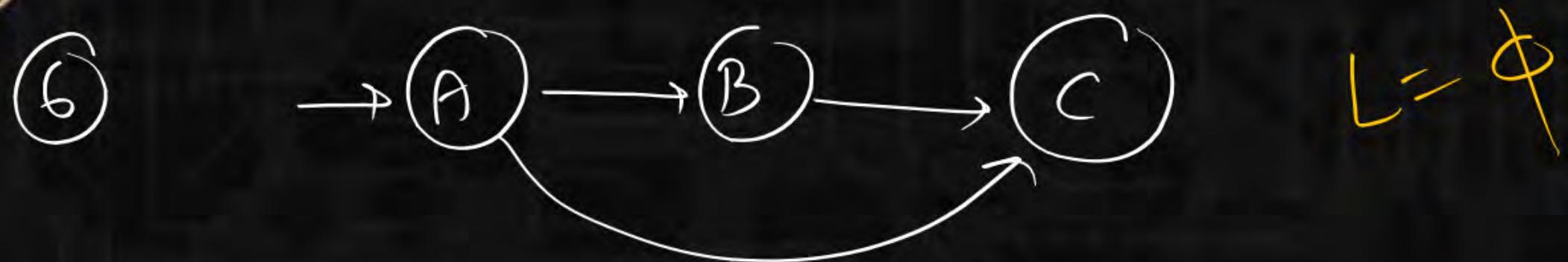
$$\delta(B, \epsilon) = C$$

$$\delta(C, \epsilon) = \emptyset$$

Transitions

$$\delta(A, \epsilon) = \{B, C\}$$





$$\begin{array}{c} A \xrightarrow{\varepsilon} B \\ A \xrightarrow{\varepsilon} C \end{array}$$

$$\delta(A, \varepsilon) = \{B, C\}$$

$$\delta(B, \varepsilon) = C$$

$$\delta(C, \varepsilon) = \emptyset$$

$$\delta(A, \varepsilon) = \{A, B, C\}$$

$$A \xrightarrow{\varepsilon} A$$

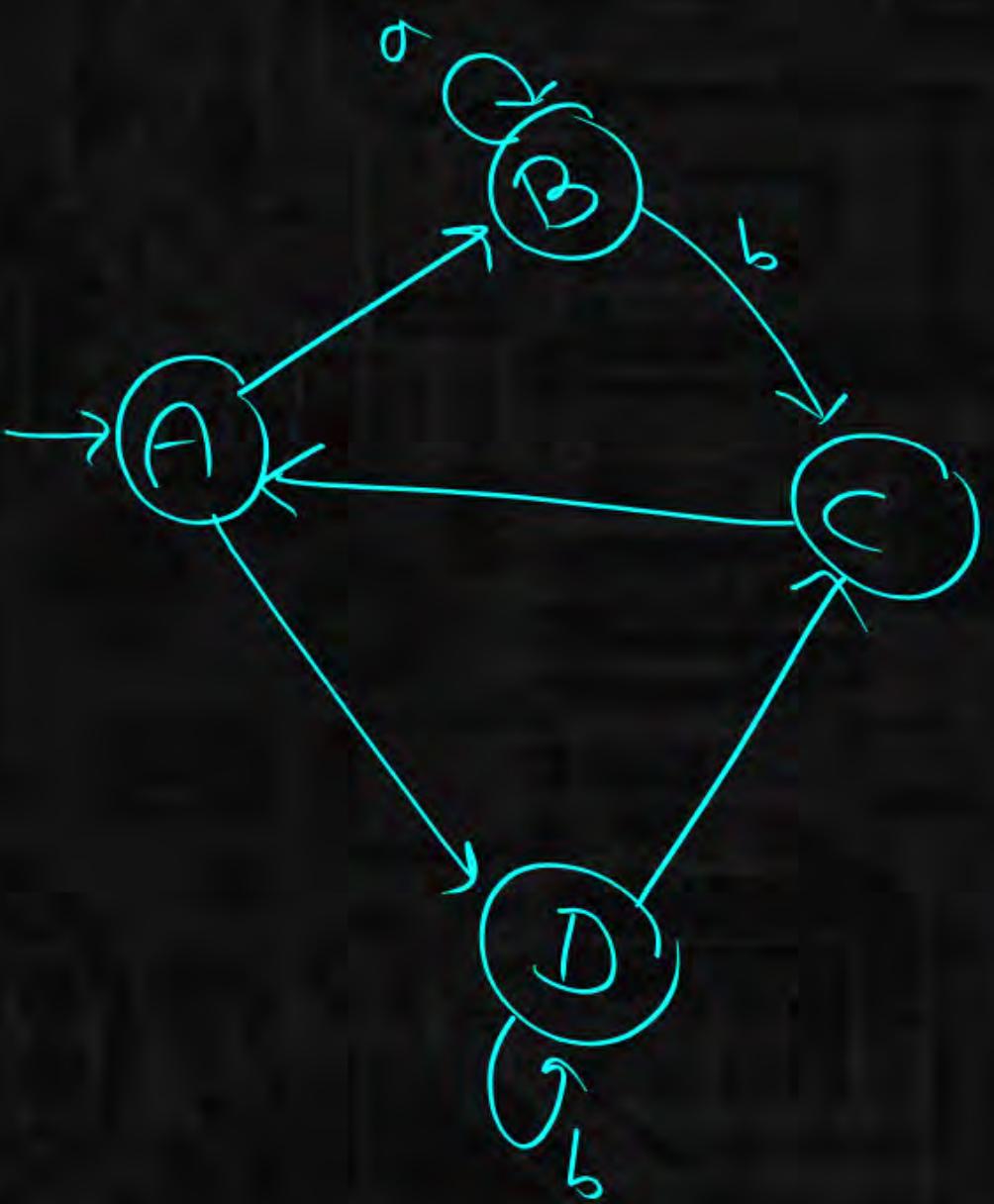
$$A \xrightarrow{\varepsilon} B$$

$$A \xrightarrow{\varepsilon} C$$

$$\begin{array}{l} A \rightarrow B \rightarrow C \\ A \rightarrow C \end{array}$$

7

ϵ -closure of state $q = \hat{\delta}(q, \epsilon)$



$$\{ \textcircled{1} \quad \epsilon\text{-closure}(A) = \hat{\delta}(A, \epsilon) = \{A, B, C, D\} \}$$

$$\textcircled{2} \quad \epsilon\text{-closure}(B) = \{B\}$$

$$\textcircled{3} \quad \epsilon\text{-closure}(C) = \{C, A, B, D\}$$

$$\textcircled{4} \quad \epsilon\text{-closure}(D) = \{D, C, A, B\}$$

$$\textcircled{5} \quad \delta(A, \epsilon) = \{B, D\}$$

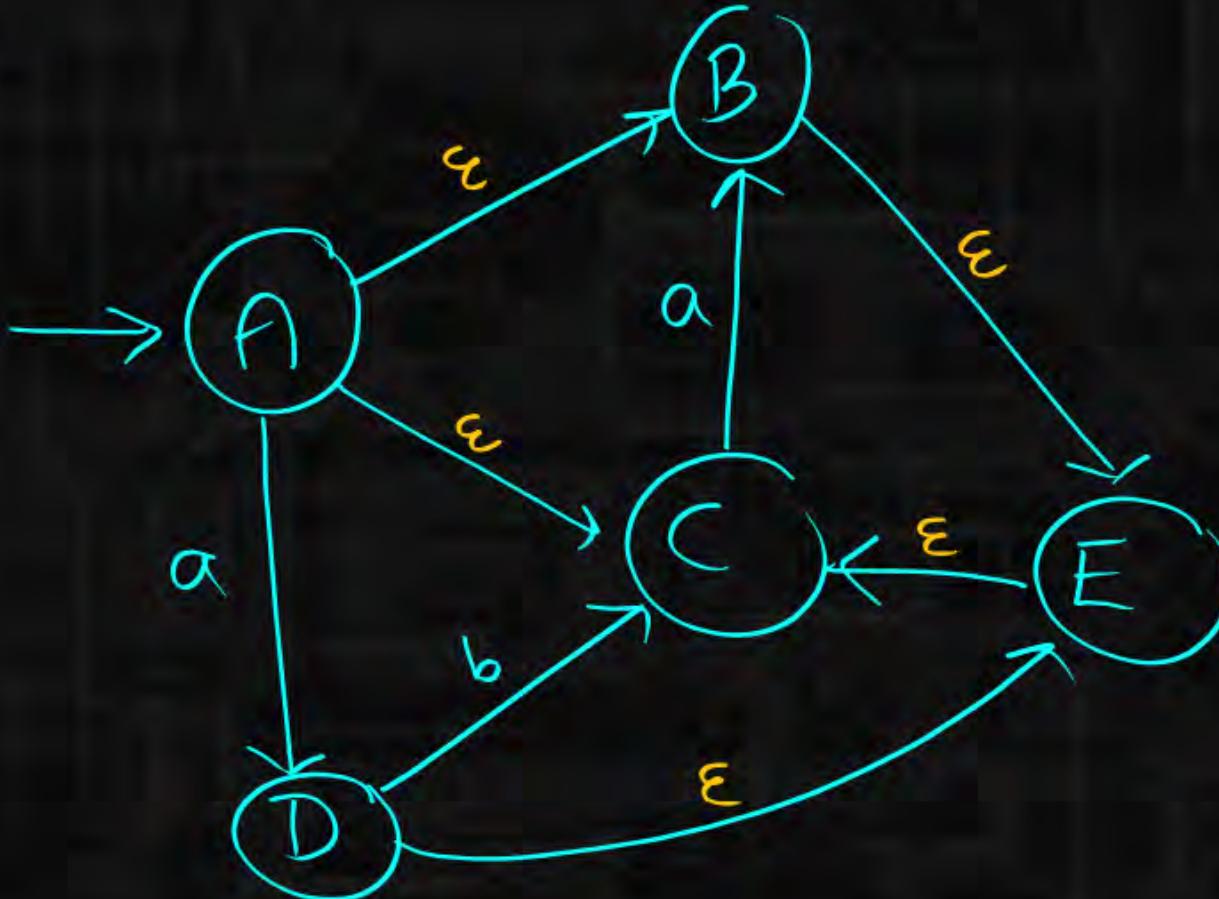
$$\textcircled{6} \quad \delta(B, \epsilon) = \emptyset$$

$$\textcircled{7} \quad \delta(C, \epsilon) = A$$

$$\textcircled{8} \quad \delta(D, \epsilon) = C$$

NFA

⑧ *



$$\begin{aligned}\mathcal{E}\text{-clo}(A) &= \{A, B, C, E\} \\ \mathcal{E}\text{-clo}(B) &= \{B, C, E\} \\ \mathcal{E}\text{-clo}(C) &= \{C\} \\ \mathcal{E}\text{-clo}(D) &= \{D, C, E\} \\ \mathcal{E}\text{-clo}(E) &= \{E, C\}\end{aligned}$$

$\mathcal{E}\text{-closure}(q)$: Set of states where each state is reachable

Without reading any string from q
 (By reading ϵ)

NFA

P
W

$$\epsilon a = a$$

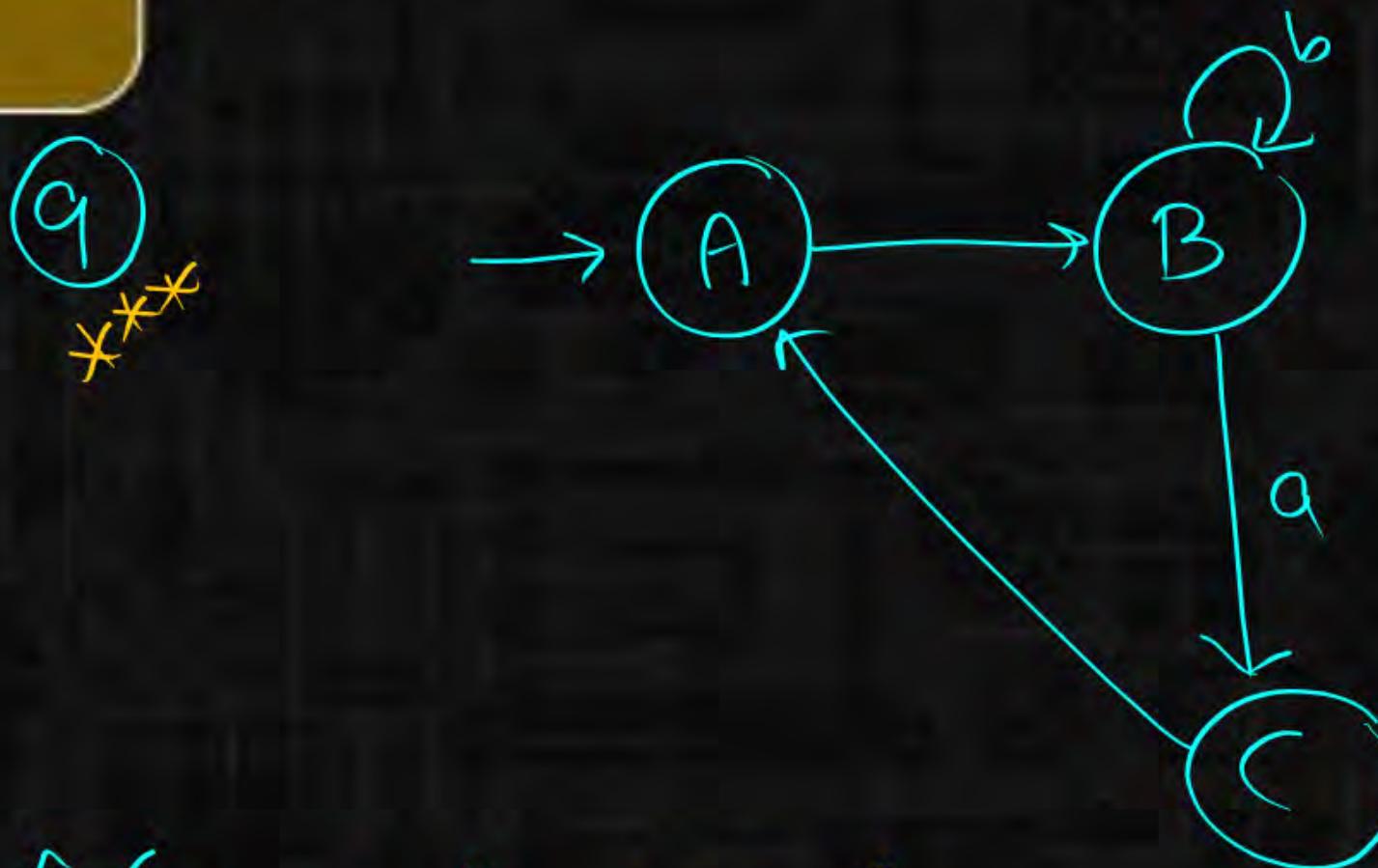
$$a\epsilon = a$$

$$a\epsilon \epsilon = a$$

$$\textcircled{1} \quad \hat{\delta}(A, aba) = \{A, B, C\}$$

$$\textcircled{2} \quad \hat{\delta}(B, ab) = \{B\}$$

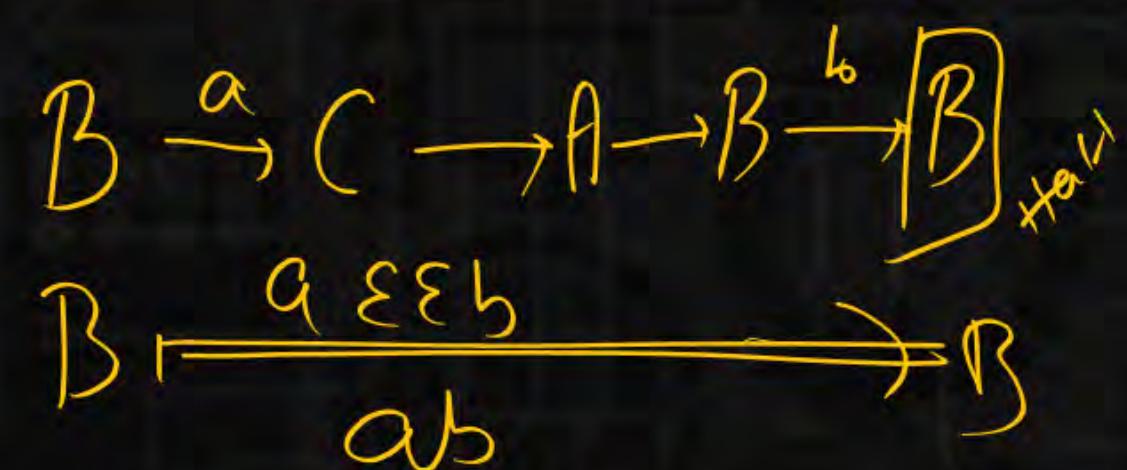
$$\textcircled{3} \quad \hat{\delta}(C, bba) = \{C, A, B\}$$



$$\hat{\delta}(A, aba) = ?$$

$$A \xrightarrow{aba} ?$$

$$A \xrightarrow{\epsilon} B \xrightarrow{a} C \xrightarrow{a} A \xrightarrow{b} B$$



$$\delta(q, \square)$$

symbol
or
 ϵ

ϵ

a

b

~~ab~~

$$\delta(q, \sim)$$

ϵ a b aa ab

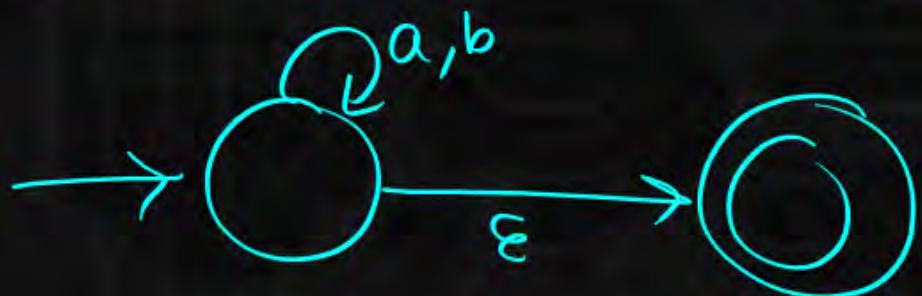
any string

NFA

Identify language accepted by NFA.

P
W

①



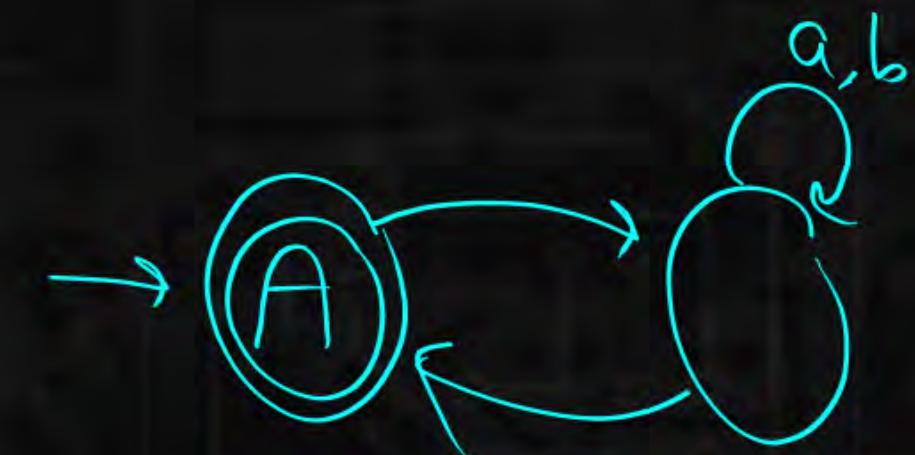
$$L = (a+b)^*$$

ε ✓
a ✓
b ✓
aa ✓
ab ✓
ba ✓
:

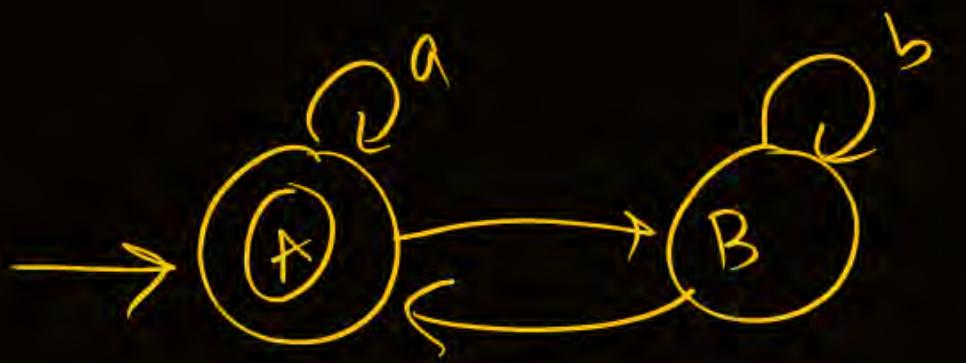


$$L = (a+b)^*$$

③



$$L = (a+b)^*$$



$\varepsilon : A$

$a : A \xrightarrow{a} A$

$b : A \xrightarrow{\varepsilon} B \xrightarrow{b} B \xrightarrow{\varepsilon} A$

$\varepsilon b \varepsilon = b$

$aa : A \xrightarrow{a} A \xrightarrow{a} A$

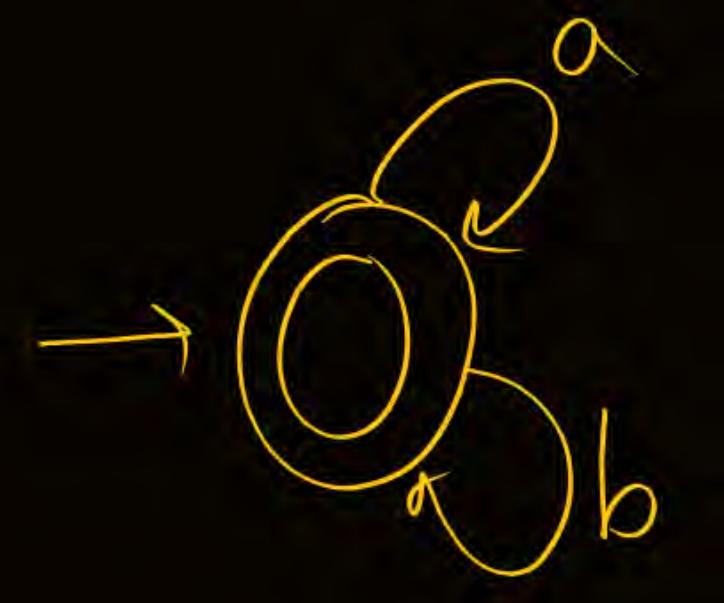
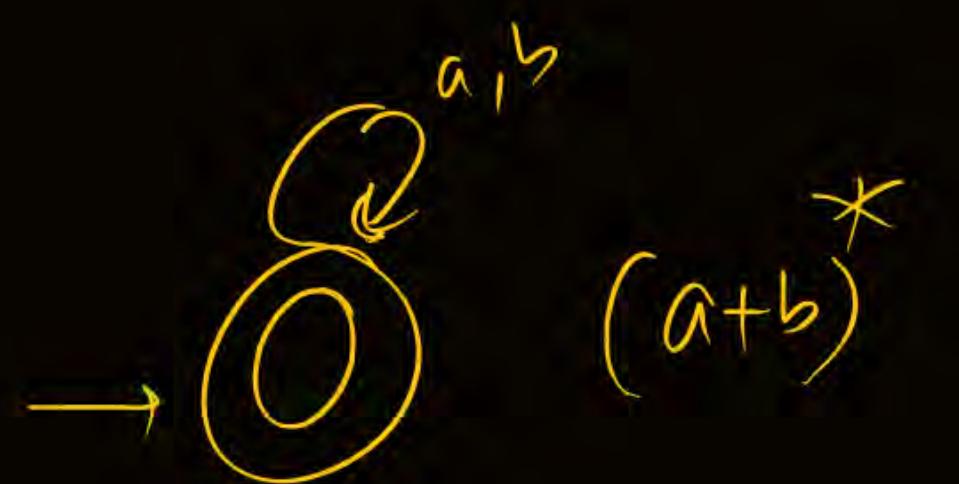
$ab : A \xrightarrow{a} A \xrightarrow{b} B \xrightarrow{b} B \xrightarrow{a} A$

$a \varepsilon b \varepsilon = ab$

$ba : A \xrightarrow{b} B \xrightarrow{a} A \xrightarrow{a} A$

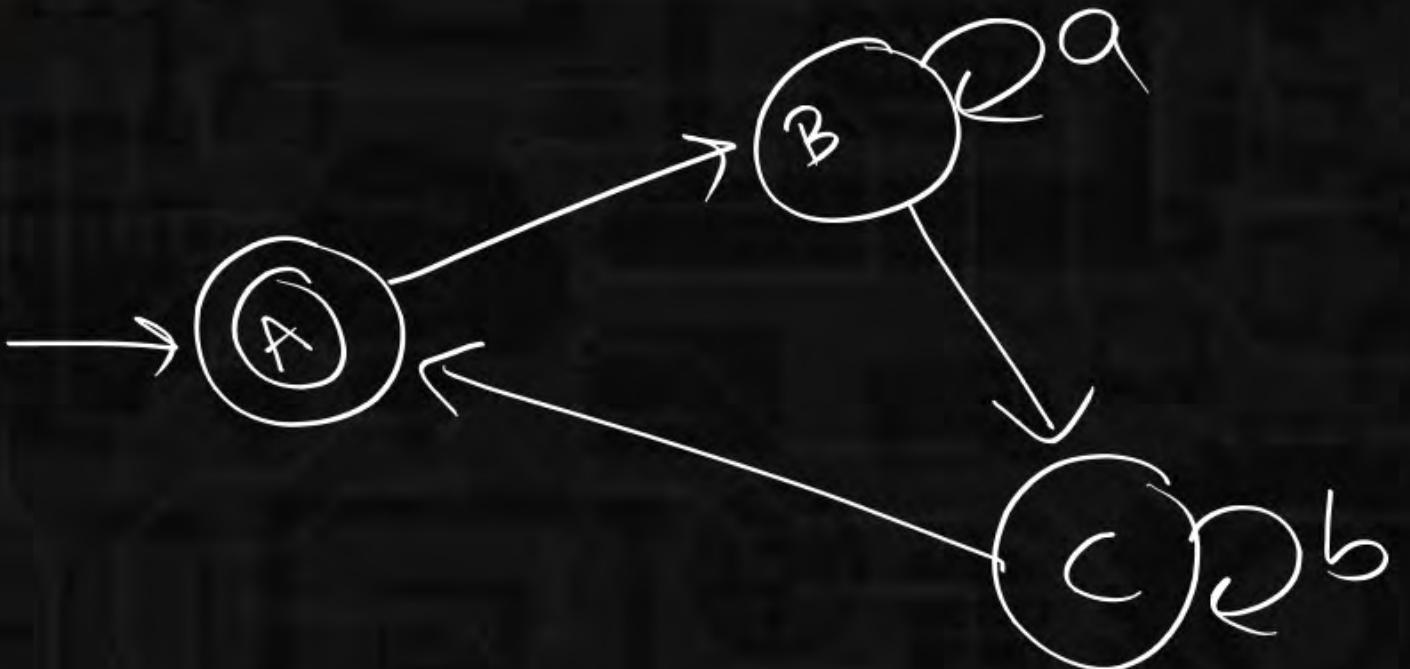
$bb : A \xrightarrow{b} B \xrightarrow{b} B \xrightarrow{b} B \xrightarrow{a} A$

$\varepsilon b \varepsilon a = ba$



NFA

④



$$= (a+b)^*$$

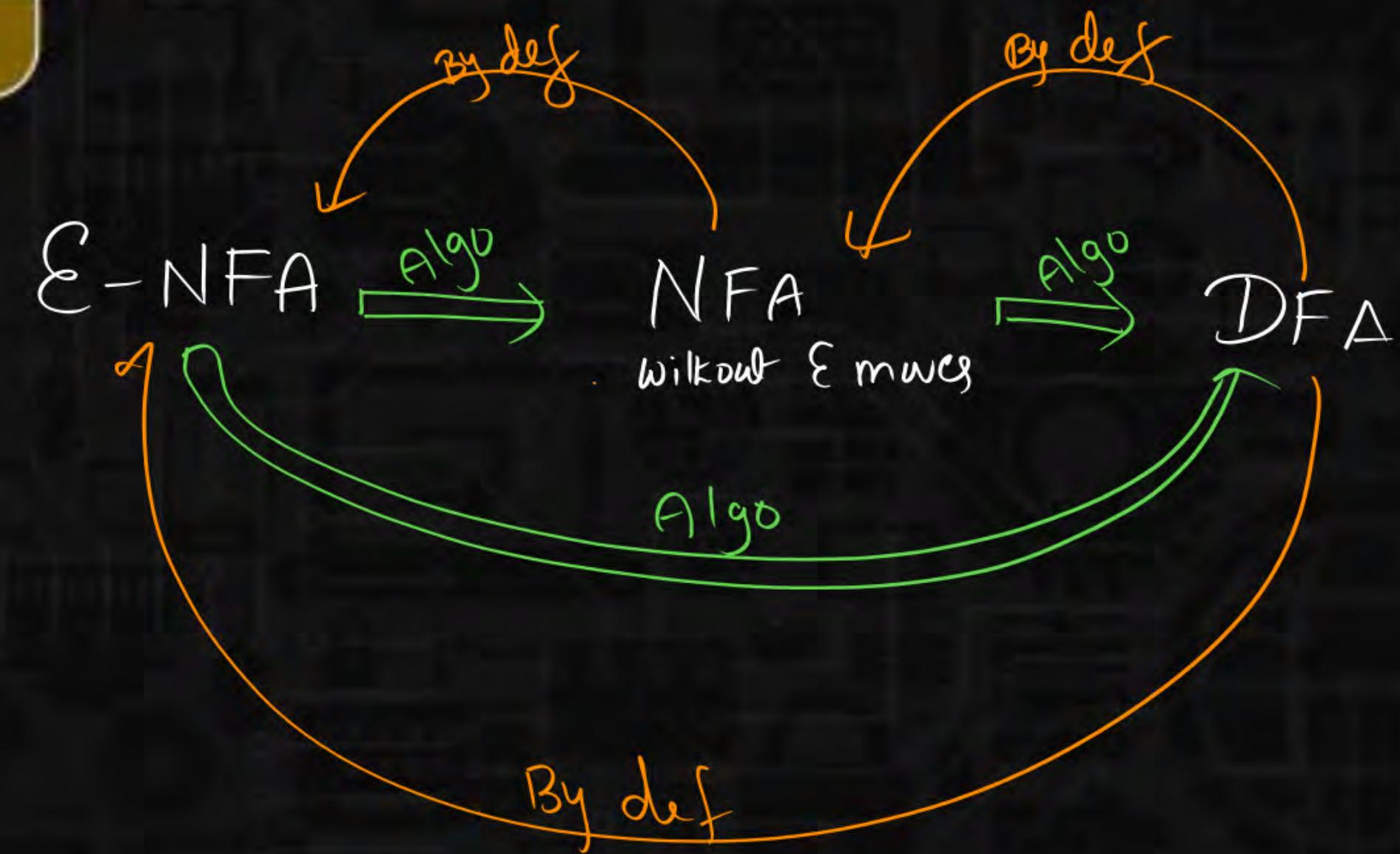
ϵ
a
b
aa
ab
ba: $\epsilon \epsilon b \epsilon \epsilon a \epsilon \epsilon$

,

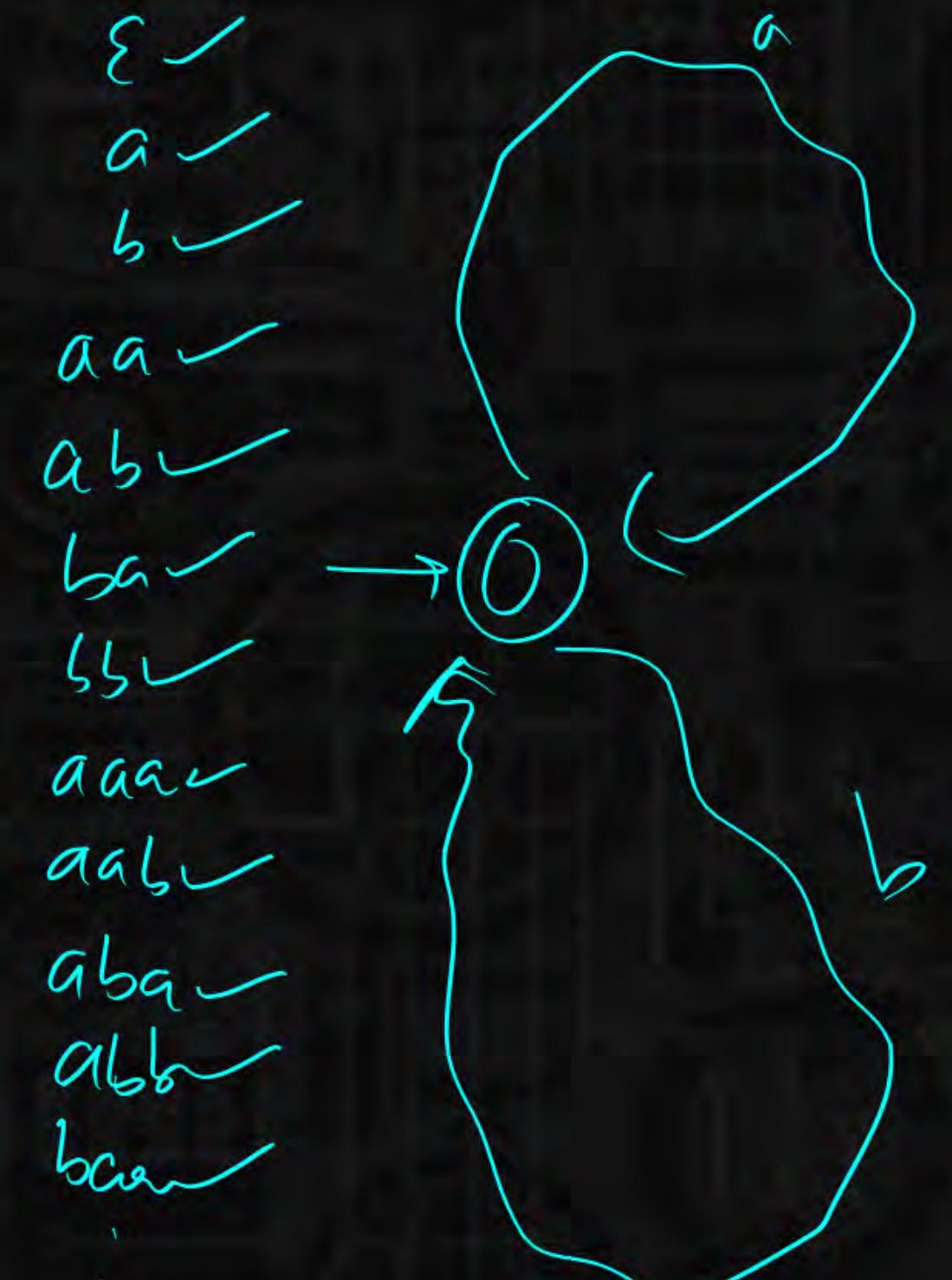
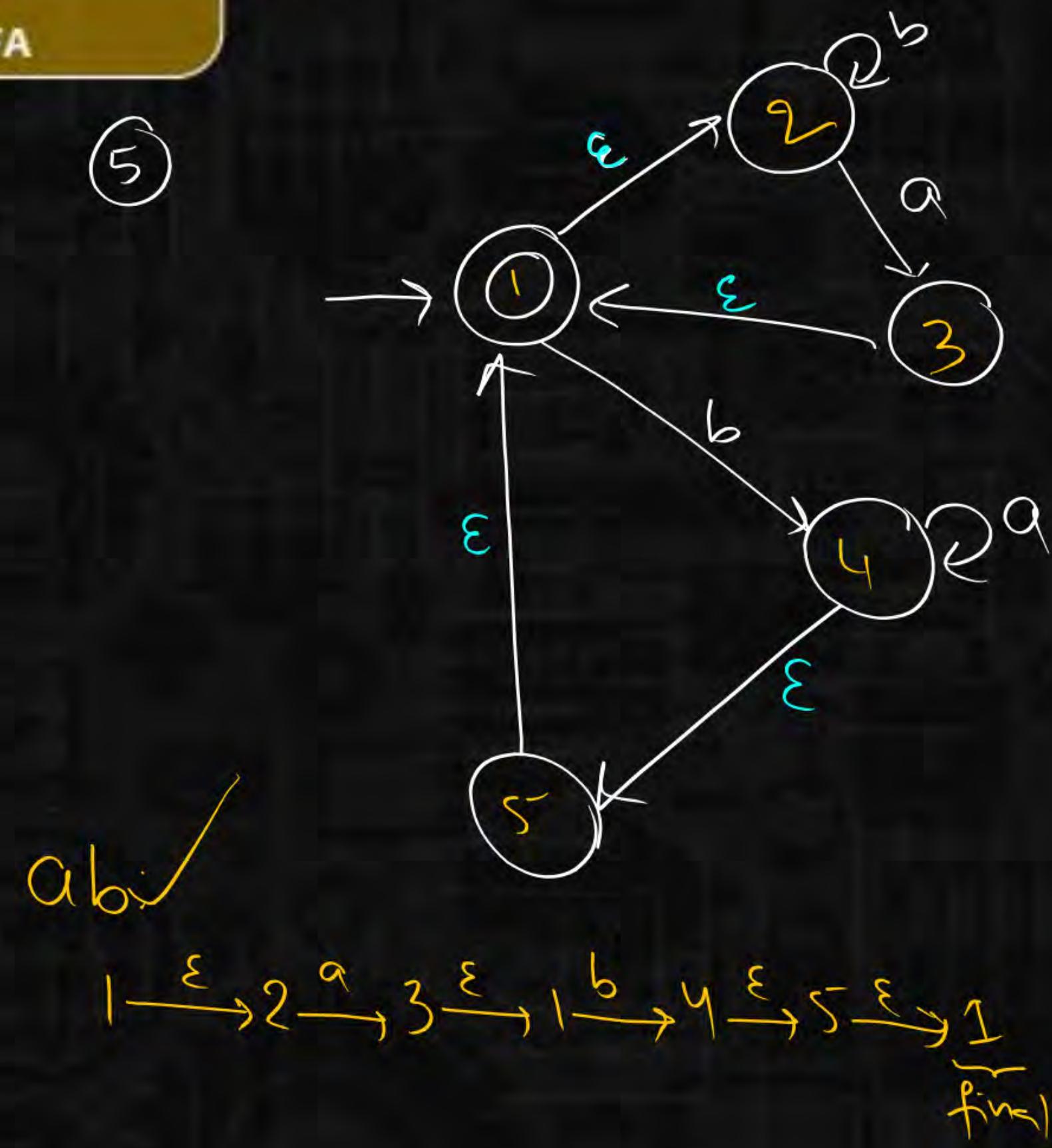
,

NFA

P
W



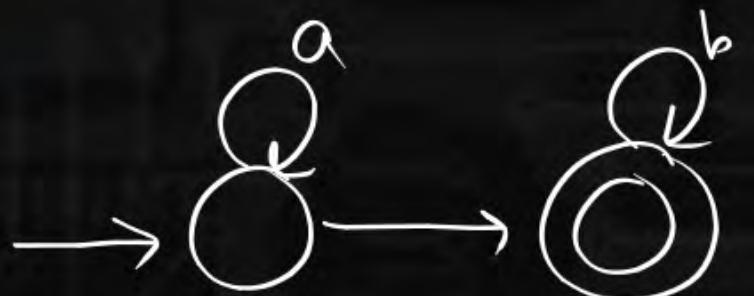
NFA



NFA

P
W

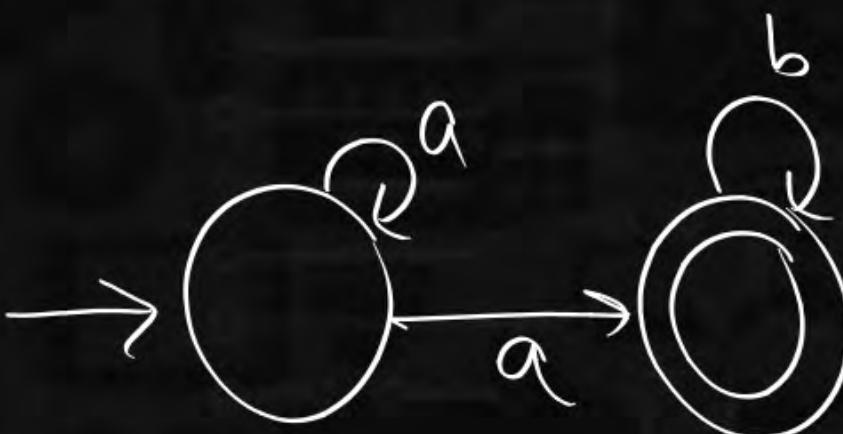
⑥



$$L = a^* b^*$$

$\epsilon \checkmark$
 $a \checkmark$
 $b \checkmark$
 $aa \checkmark$
 $ab \checkmark$
 $\boxed{ba} \times$

⑦



$$\begin{aligned}
 L &= a^* a b^* \\
 &= a^+ b^*
 \end{aligned}$$

Algo : NFA \rightarrow NFA

with ϵ moves

(n states)

NFA

without ϵ moves

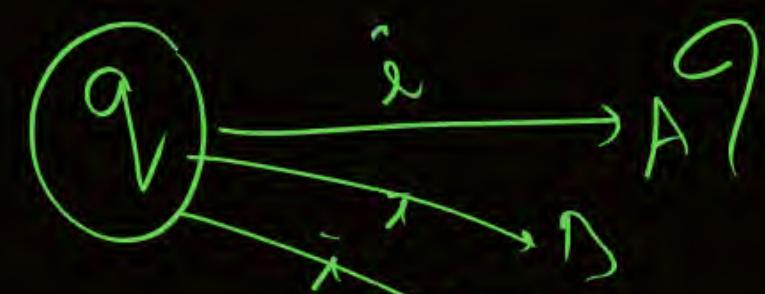
(n states)

$$(Q, \Sigma, \delta, q_0, F) \xrightarrow{\text{Given}} (Q, \Sigma, \delta', q_0, F')$$

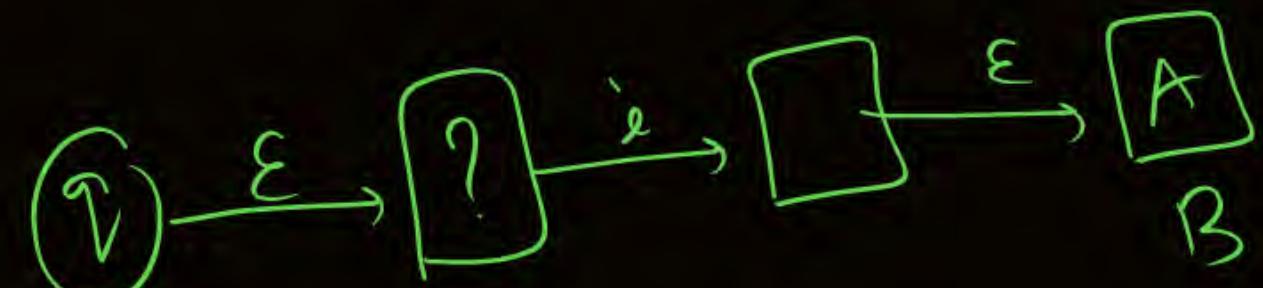
$$\delta'(q, i) = \epsilon\text{-clo} \left[\delta \left(\underbrace{\epsilon\text{-clo}(q)}_{1^{\text{st}}} \right), i \right]$$

$\forall i \in \Sigma$
 $\forall q \in Q$

3^{rd}
 2^{nd}
 1^{st}

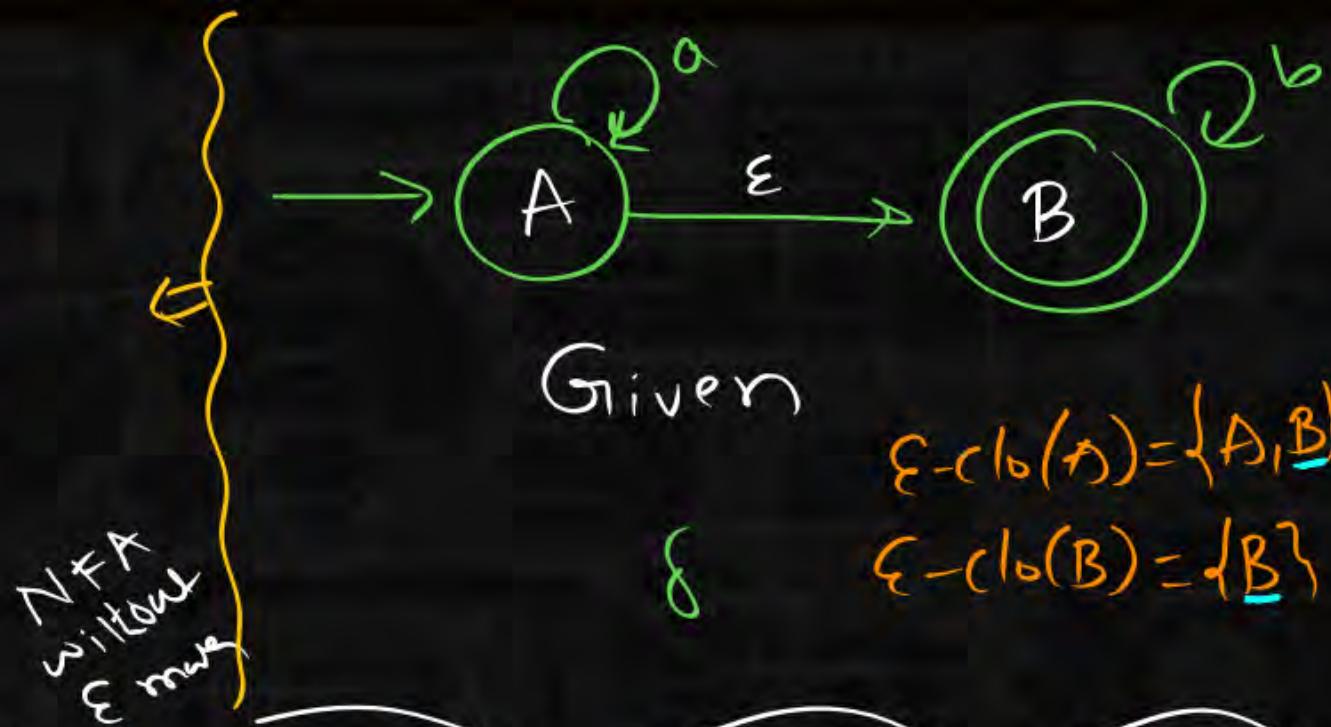
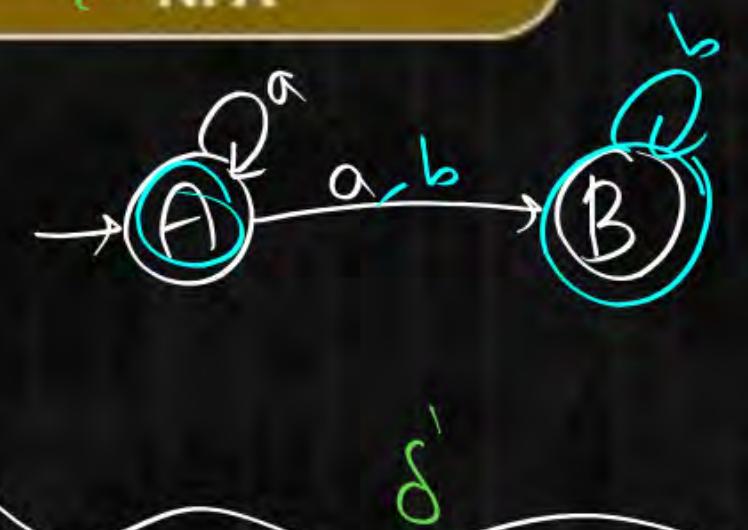


NFA
without ϵ move



using ϵ -NFA

ϵ - NFA



$$\text{I) } \delta'(A, \alpha) = \text{Eclo} \left[\delta \left(\underbrace{\text{Eclo}(A)}_{\{A, B\}}, \alpha \right) \right] = \text{Eclo} \left[\delta'(\{A, B\}, \alpha) \right]$$

$$\text{II) } \delta'(A, \beta) = \text{Eclo} \left[\delta \left(\underbrace{\text{Eclo}(A)}_{\{A, B\}}, \beta \right) \right] = \text{Eclo} \left[\underbrace{\delta'(A, \alpha)}_{\emptyset} \cup \underbrace{\delta'(B, \alpha)}_{\emptyset} \right] = \text{Eclo}(\{A, B\}) = \{A, B\}$$

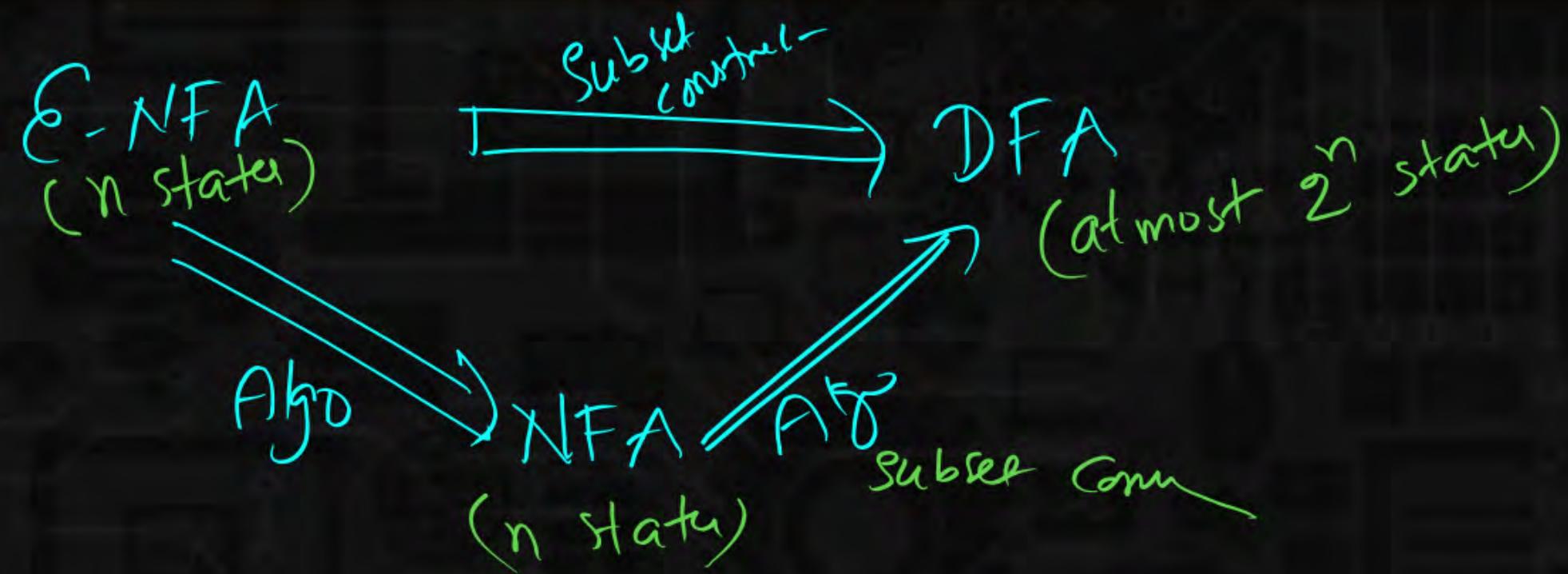
$$\text{III) } \delta'(B, \alpha) = \text{Eclo} \left[\delta \left(\underbrace{\text{Eclo}(B)}_{\{B\}}, \alpha \right) \right] = \emptyset$$

$$\text{IV) } \delta'(B, \beta) = \{B\}$$

P
W

NFA

P
W



NFA



NFA \cong DFA

Q

Let $L = \{ w \in (0+1)^* \mid w \text{ has even number of } 1s\}$, i.e., L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L ? [2010: 2 Marks]

- A $(0^*10^*)^*$
- B $0^*(10^*10^*)^*$
- C $0^*(10^*1)^*0^*$
- D $0^*1(10^*1)^*10^*$

Q

Let P be a regular language and Q be a context-free language such that $Q \subseteq P$. (For example, let P be the language represented by the regular expression $p^* q^*$ and Q be $\{p^n q^n \mid n \in \mathbb{N}\}$). Then which of the following is ALWAYS regular?

[2011: 1 Mark]

- A $P \cap Q$
- B $\Sigma^* - P$
- C $P - Q$
- D $\Sigma^* - Q$

Q

Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^* ?

- | | | | |
|----|--------------|----|---------------------------|
| 1. | abaabaaaabaa | 2. | aaaabaaaaa |
| 3. | baaaaabaaaab | 4. | baaaaabaaa [2012: 1 Mark] |

- A** 1, 2 and 3
- B** 2, 3 and 4
- C** 1, 2 and 4
- D** 1, 3 and 4

Q

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1L_2^* \cup L_1^*$?

[2013: 1 Mark]

- A $\{\epsilon\}$
- B ϕ
- C a^*
- D (ϵ, a)

Q

The length of the shortest string NOT in the language
(over $\Sigma = \{a, b\}$) of the following regular expression is

_____.

$a^*b^*(ba)^*a^*$

[2014-Set3: 1 Mark]

Q

Consider alphabet $\Sigma = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:

[2015-Set2: 2 Marks]

$$X_0 = 1X_1$$

$$X_1 = 0X_1 + 1X_2$$

$$X_2 = 0X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0 ?

- A $10(0^* + (10)^*)1$
- B $10(0^* + (10)^*)^*1$
- C $1(0 + 10)^*1$
- D $10(0 + 10)^*1 + 110(0 + 10)^*1$

Q

Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive 0s and two consecutive 1s?*

[2016-Set1: 1 Mark]

- A $(0 + 1)^*0011(0 + 1)^* + (0 + 1)^* 1100(0 + 1)^*$
- B $(0 + 1)^*(00(0 + 1)^*11 + 11(0 + 1)^* 00)(0 + 1)^*$
- C $(0 + 1)^*00(0 + 1)^* + (0 + 1)^* 11(0 + 1)^*$
- D $00(0 + 1)^*11 + 11 (0 + 1)^*00$

Q

Which one of the following regular expression represents the set of all binary strings with an odd number of 1's?

[2020: 1 Mark]

- A $(0^*10^*10^*)^*0^*1$
- B $10^*(0^*10^*10^*)^*$
- C $((0 + 1)^* 1(0 + 1)^*1)^*10^*$
- D $(0^*10^*10^*)^*10^*$

Q

Which of the following regular expressions represents(s) the set of all binary numbers that are divisible by three? Assume that the strings ϵ is divisible by three.

[2021-Set2-MSQ: 2
Marks]

- A $(0^*(1(01^*0)^*1))^*$
- B $(0 + 1(01^*0)^* 1)^*$
- C $(0 + 11 + 10(1 + 00)^*01)^*$
- D $(0 + 11 + 11(1 + 00)^*00)^*$

Q

Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w . What is the minimum number of states in a non-deterministic finite automaton that accepts L ?

[2010: 2 Marks]

- A** $n-1$
- B** n
- C** $n+1$
- D** 2^{n-1}

Q

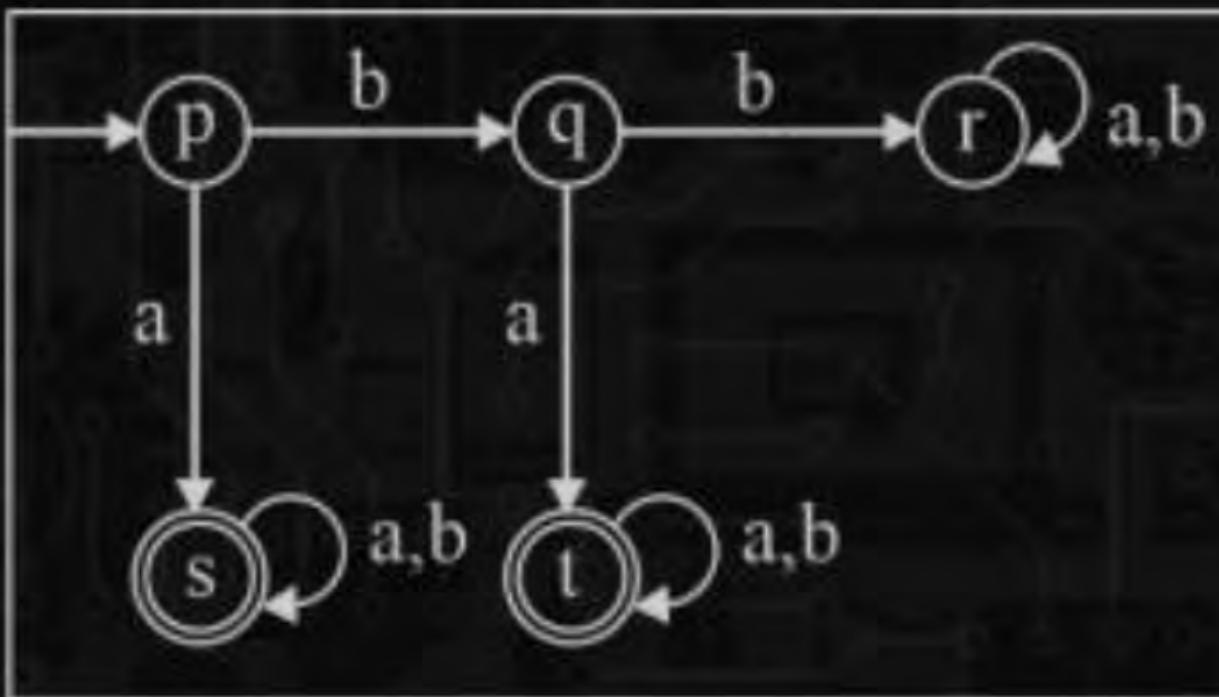
The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

- A Finite state automata
- B Deterministic pushdown automata
- C Non-deterministic pushdown automata
- D Turing machine

[2011: 1 Marks]

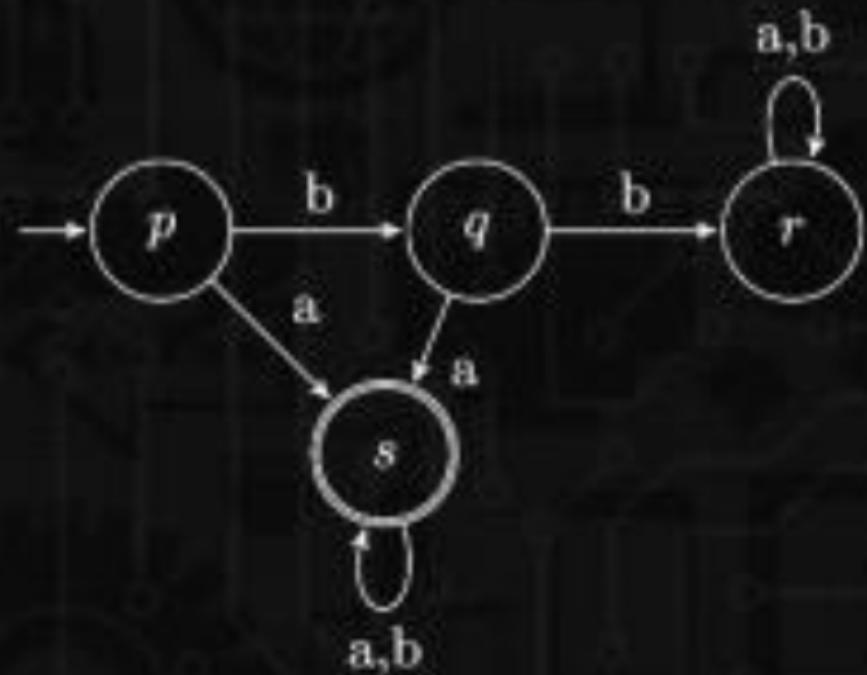
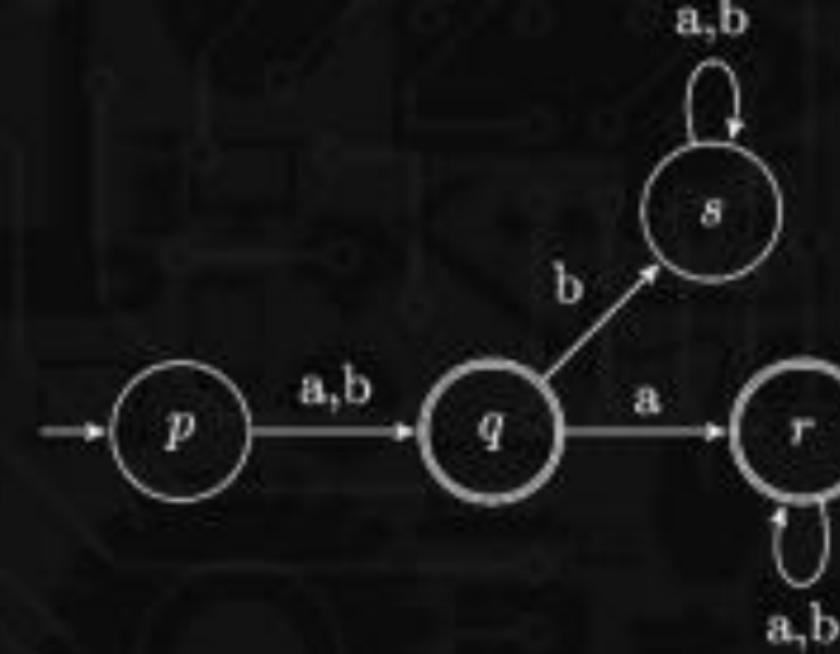
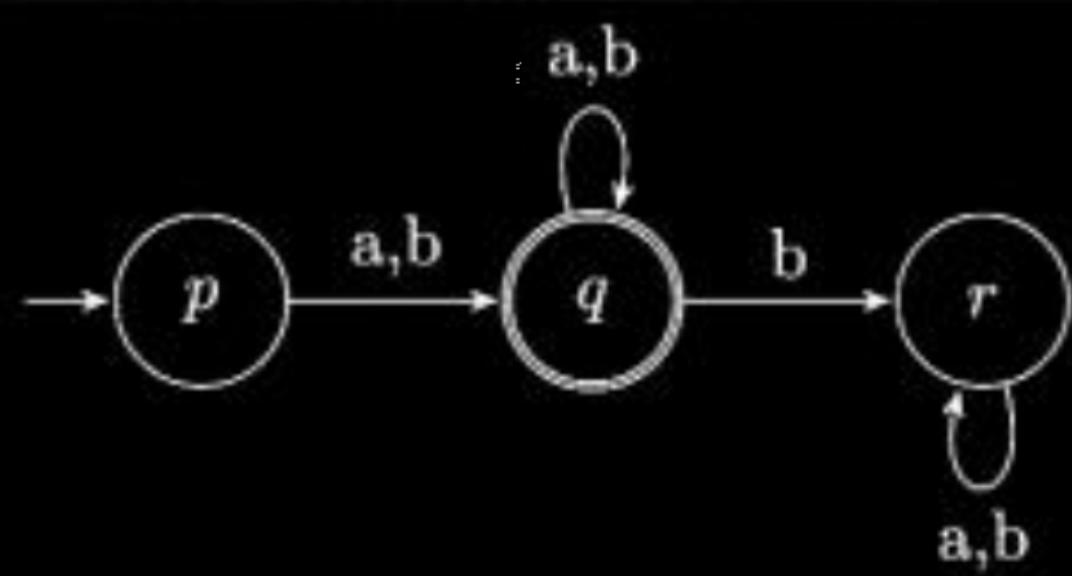
Q

A deterministic finite automaton (DFA) D with alphabet $\Sigma = \{a, b\}$ is given below:



Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

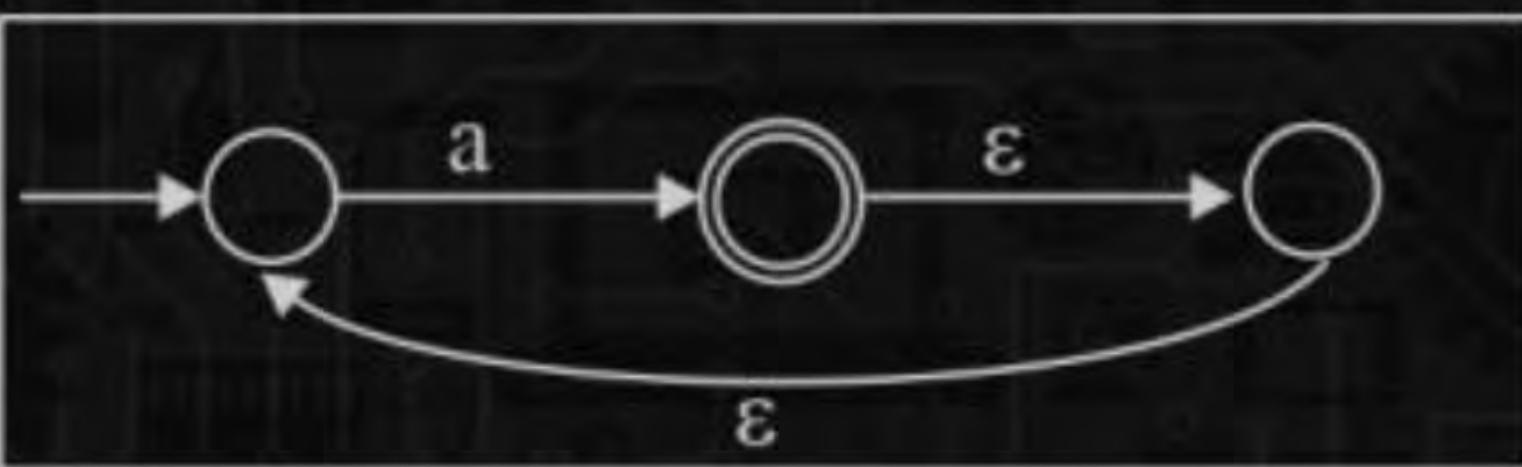
[2011:2 Mark]

A**B****C****D**

Q

What is the complement of the language accepted by the NFA shown below? Assume $\Sigma = \{a\}$ and ϵ is the empty string.

[2012: 1 Mark]



- A \emptyset
- B $\{\epsilon\}$
- C a^*
- D $\{a, \epsilon\}$

Q

Consider the set of strings on $\{0, 1\}$ in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially complete DFA that accepts this language is shown below.



The missing arcs in the DFA
are? [2012: 2 Marks]

A

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11		0			

B

	00	01	10	11	q
00		0			1
01			1		
10				0	
11		0			

C

	00	01	10	11	q
00		1		0	
01		1			
10			0		
11		0			

D

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

Q

Consider the DFA A given below:



- Which of the following are FALSE?
1. Complement of $L(A)$ is context-free.
 2. $L(A) = L((11^*0 + 0)(0 + 1)^*0^*1^*)$
 3. For the language accepted by A, A is the minimal DFA.
 4. A accepts all strings over $\{0, 1\}$ of length at least 2.

[2013: 1 Mark]

- A 1 and 3 only
- B 2 and 4 only
- C 2 and 3 only
- D 3 and 4 only

Q

Which one of the following is TRUE?

[2014-Set1: 1 Mark]

- A The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.
- B The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.
- C The language $L = \{w \mid w \text{ has } 3k + 1 \text{ b's for some } k \in \mathbb{N} \text{ with } \Sigma = \{a, b\}\}$ is regular.
- D The language $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular.

Q

Which of the regular expressions given below represent the following DFA?



- I. $0^*1(1 + 00^*1)^*$
- II. $0^*1^*1 + 11^*0^*1^*$
- III. $(0 + 1)^*1$

[2014-Set1: 2 Mark]

- A I and II only
- B I and III only
- C II and III only
- D I, II, and III

Q

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

- I. $L_1 \cdot L_2$ is a regular language
- II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is CORRECT?

- A** Only I
- B** Only II
- C** Both I and II
- D** Neither I nor II

[2014-Set2: 1 Mark]

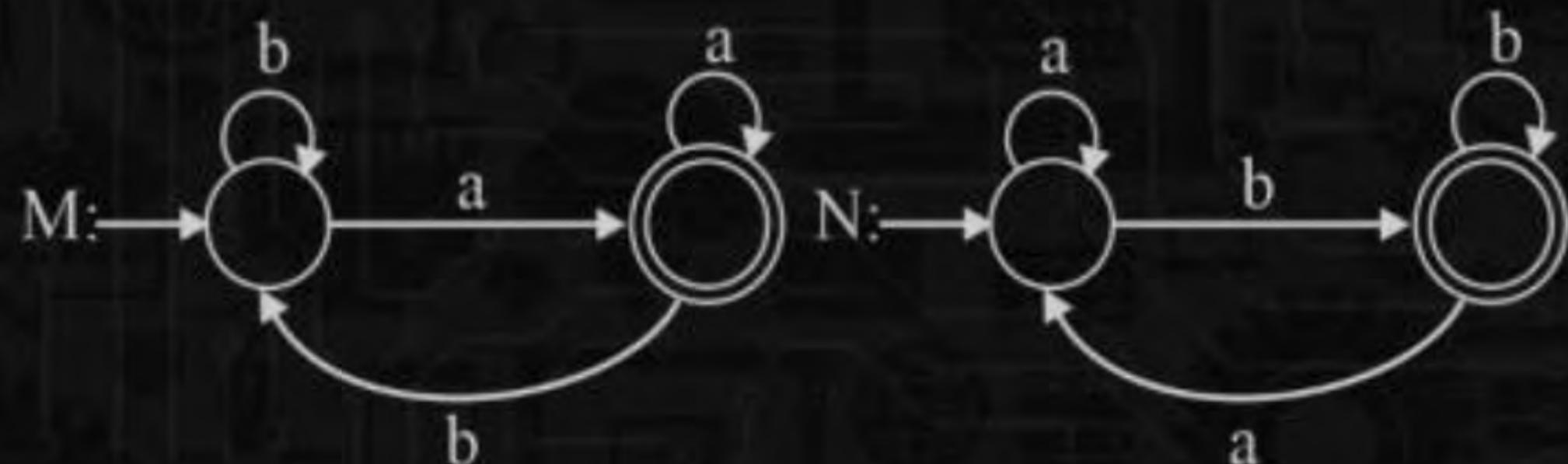
Q

Let $L_1 = \{\omega \in \{0,1\}^* \mid \omega \text{ has at least as many occurrences of } (110) \text{'s as } (011)\text{'s}\}$. Let $L_2 = \{\omega \in \{0,1\}^* \mid \omega, \text{ has at least as many occurrences of } (000)\text{'s as } (111)\text{'s}\}$. Which one of the following is TRUE?

[2014-Set2: 2 Marks]

- A** L_1 is regular but not L_2
- B** L_2 is regular but not L_1
- C** Both L_1 and L_2 are regular
- D** Neither L_1 nor L_2 are regular

Q



Consider the DFAs M and N given above. The number of states in a minimal DFA that accepts the language $L(M) \cap L(N)$ is

[2015-Set1: 2 Marks]

Q

The number of states in the minimal deterministic finite automaton corresponding the regular expression $(0 + 1)^*(10)$ is ____.

[2015-Set2: 2 Marks]

Q

Let L be the language represented by the regular expression
 $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0, 1\}$.

What is the minimum number of states in a DFA that
recognizes L' (complement of L)?

[2015-Set3: 1 Mark]

A

4

B

5

C

6

D

8

Q

Which of the following languages is/are regular?

$L_1: \{wxw^R \mid w, x \in \{a, b\}^* \text{ and } |w|, |x| > 0\}$ w^R is the reverse of string $w\}$

$L_2: \{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$

$L_3: \{a^p b^q c^r \mid p, q, r \geq 0\}$

[2015-Set2: 2 Marks]

- A** L_1 and L_3 only
- B** L_2 only
- C** L_2 and L_3 only
- D** L_3 only

Q

The number of states in the minimum sized DFA that accepts the language defined by the regular expression

$(0 + 1)^* (0 + 1) (0 + 1)^*$ is ____.

[2016-Set2: 1 Mark]

Q

Consider the following two statements:

- I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^* .
- II. There exists a regular language A such that for all language B, $A \cap B$ is regular.

[2016-Set2: 2 Marks]

Which one of the following is CORRECT

- A Only I is true
- B Only II is true
- C Both I and II are true
- D Both I and II are false

Q

Consider the language L given by the regular expression $(a + b)^*b(a + b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is ____.

[2017-Set1: 2 Marks]

Q

The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1 aw_2 \mid w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ____.

[2017-Set2: 1 Mark]

Q

Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ϵ -NFA whose transition table is given below:

[2017-Set2: 2 Mark]

δ	ϵ	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

The $\hat{\delta}(q_2, aba)$ is

- A ϕ
- B $\{q_0, q_1, q_3\}$
- C $\{q_0, q_1, q_2\}$
- D $\{q_0, q_2, q_3\}$

Q

Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N . Which one of the following is necessarily true?

[2018: 1 Mark]

- A $k \geq 2^n$
- B $k \geq n$
- C $k \leq n^2$
- D $k \leq 2^n$

Q

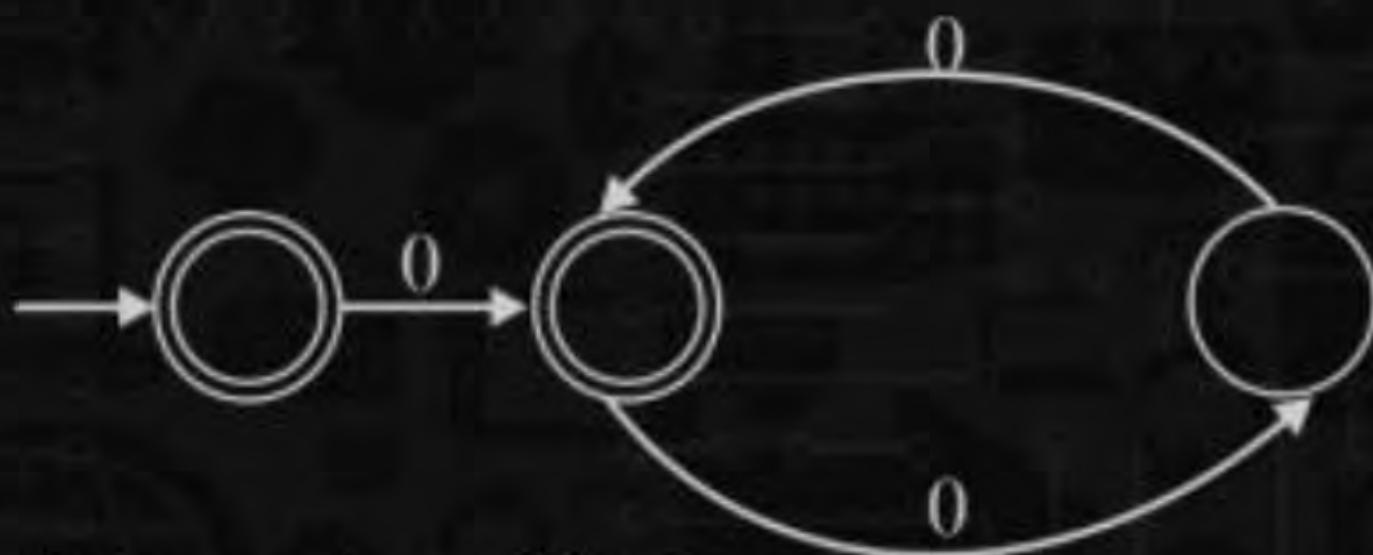
Given a language L , define L^i as follows:

$$L^0 = \{\epsilon\}$$

$$L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton



The order of L_1 is ____.

[2018: 2 Marks]

Q

For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L ?

[2019: 1 Mark]

- A** 9
- B** 24
- C** 3
- D** 5

Q

Let Σ be the set of all bijections from $\{1, \dots, 5\}$ to $\{1, \dots, 5\}$, where id denotes the identity function, i.e. $id(j) = j, \forall j$.

Let \circ denote composition on functions.

For a string $x = x_1 x_2 \dots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \dots \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$.

The minimum number of states in any DFA accepting L is ____.

[2019: 2 Marks]

Q

Consider the following language:

$L = \{x \in \{a, b\}^* \mid \text{number of } a's \text{ in } x \text{ is divisible by 2 but not divisible by 3}\}$

The minimum number of states in a DFA that accepts L is ____.

[2020: 2 Marks]

Q

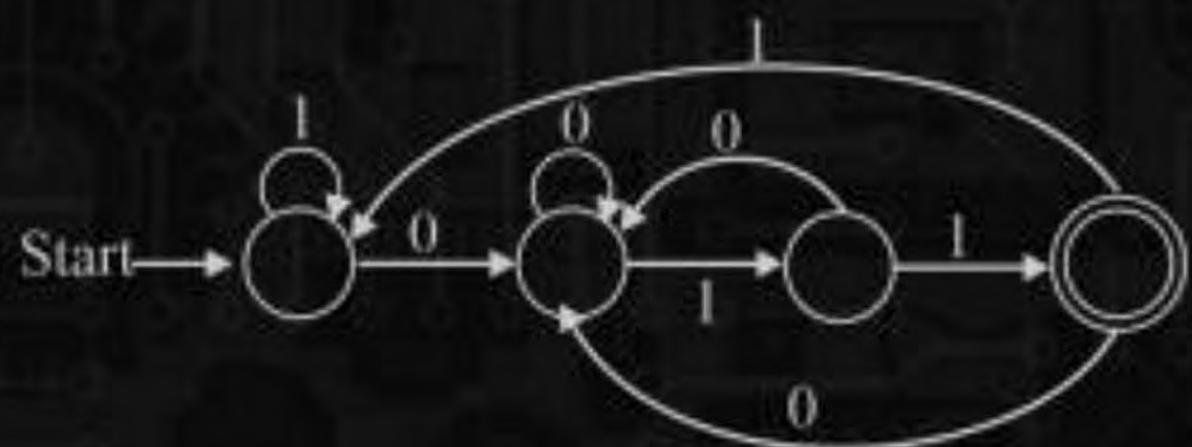
Consider the following language:

$$L = \{w \in \{0, 1\}^* \mid w \text{ ends with the substring } 011\}.$$

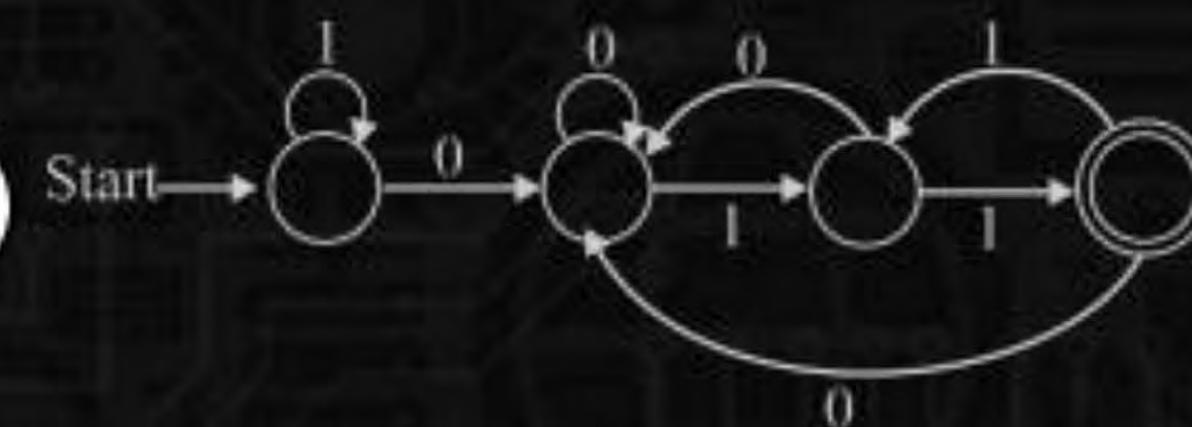
Which one of the following deterministic finite automata
accepts L?

[2021-Set1: 2 Marks]

A



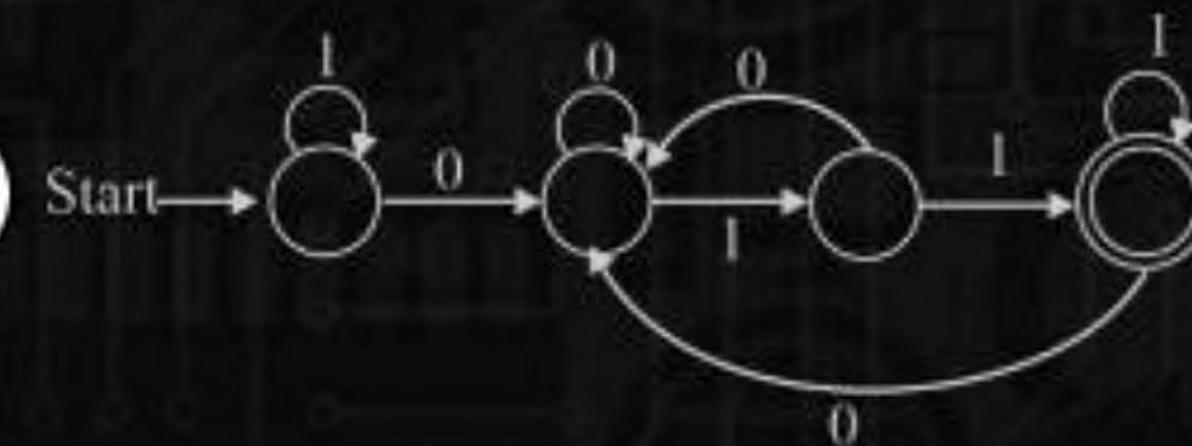
B



C



D



Q

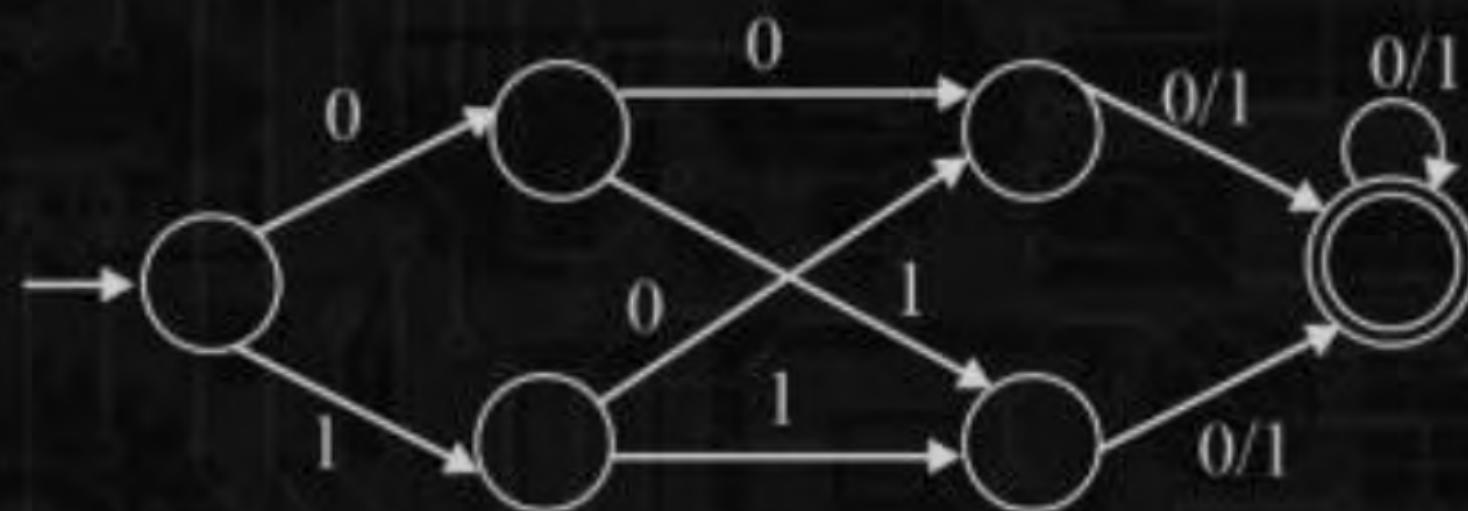
Let $L \subseteq \{0, 1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

[2021-Set2: 1 Marks]

- A $\{0, 1\}^* - L$
- B $L \cup \{01\}$
- C $L \cdot L$
- D $L - \{01\}$

Q

Consider the following deterministic finite automaton (DFA).



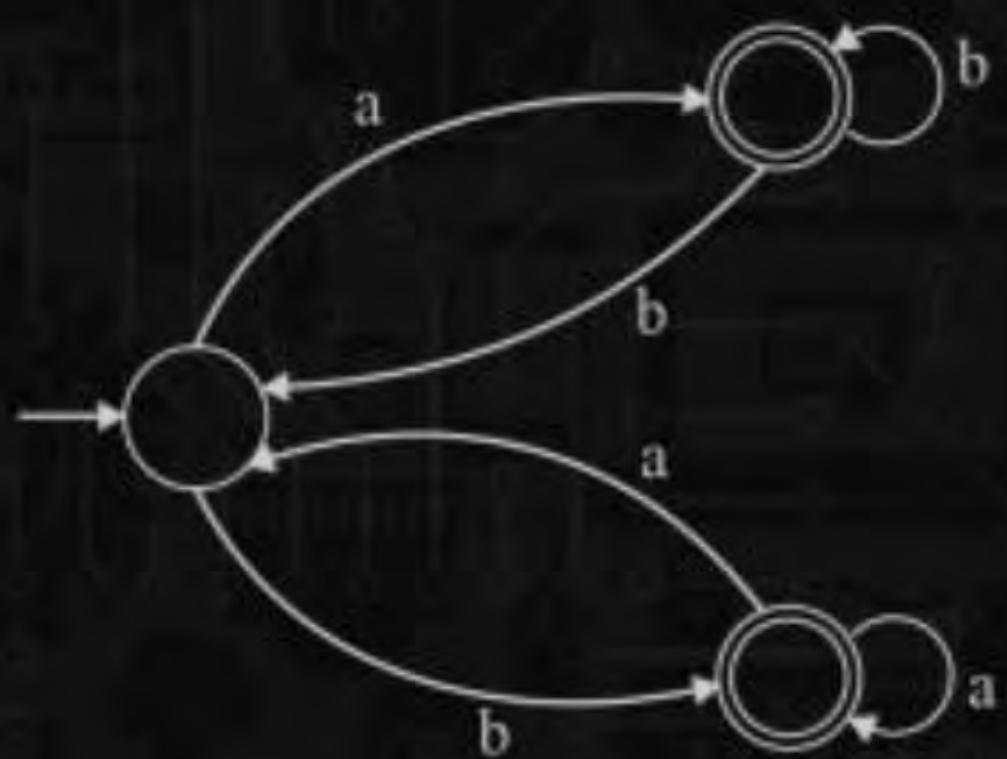
The number of strings of length 8 accepted by the above automaton is ____.

[2021-Set2: 1 Marks]

Q

Which one of the following regular expressions correctly represents the language of the finite automaton given below?

[2022: 1 Mark]



- A $ab^*bab^* + ba^*aba^*$
- B $(ab^*b)^*ab + (ba^*a)^*ba^*$
- C $(ab^*b + ba^*a)^*(a^* + b^*)$
- D $(ba^*a + ab^*b)^*(ab^* + ba^*)$

Summary

↳ NFA Vs DFA ✓

NFA with ϵ -moves ✓

