CS & IT ENGINEERING

Theory of Computation

Turing Machine : Recursively Enumerable

Turing Machine (Part 3)

Lecture No. 3

By- DEVA Sir

01 Closure Properties

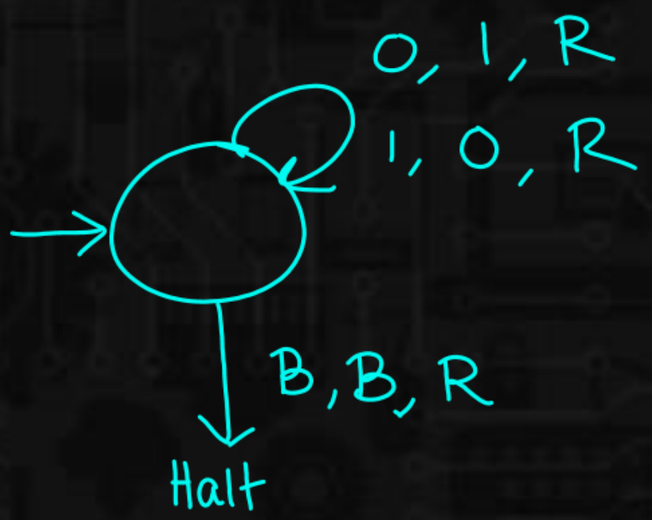
02 Termnology: Decidable

03 Terminology: Undecidable

04

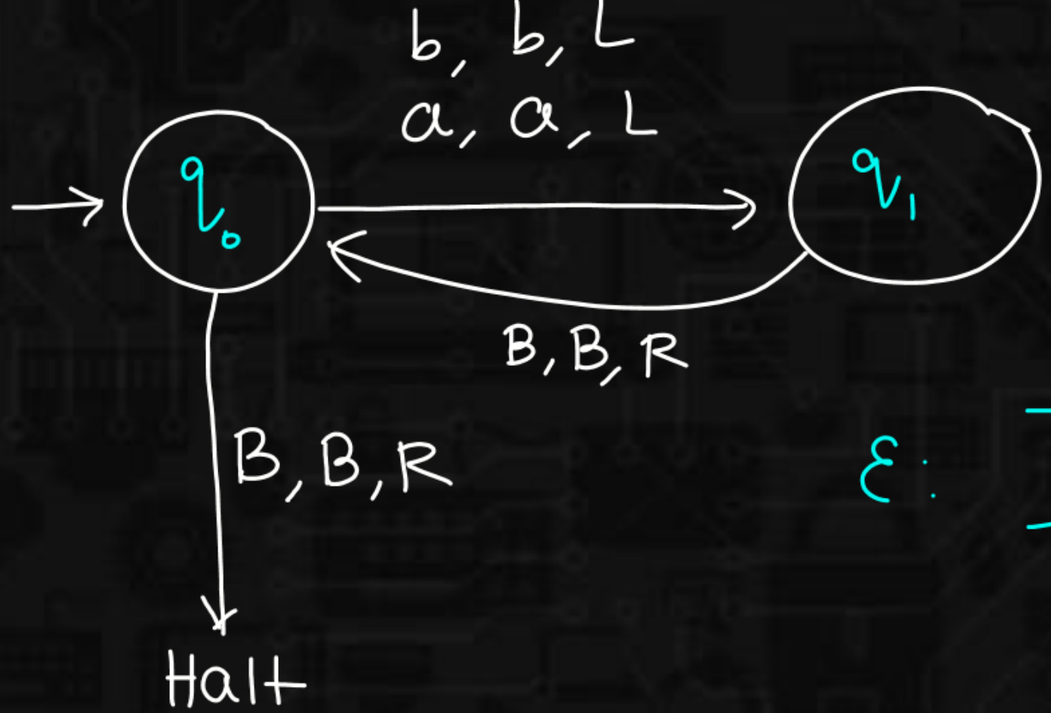
Practice

I)



1's complement

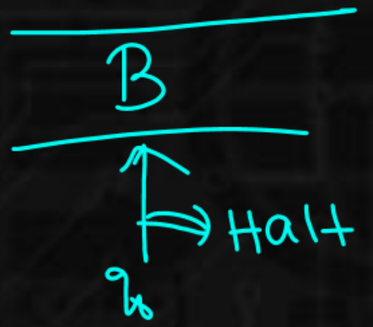
II)



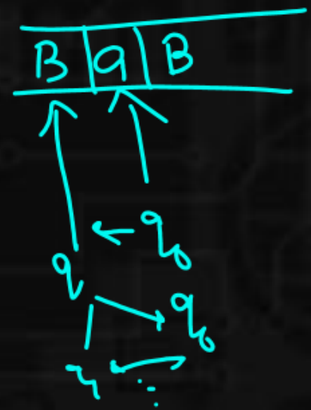
$(a+b)^+$ doesn't Halt

ε halts

ε :



a :



a ↺

b ↺

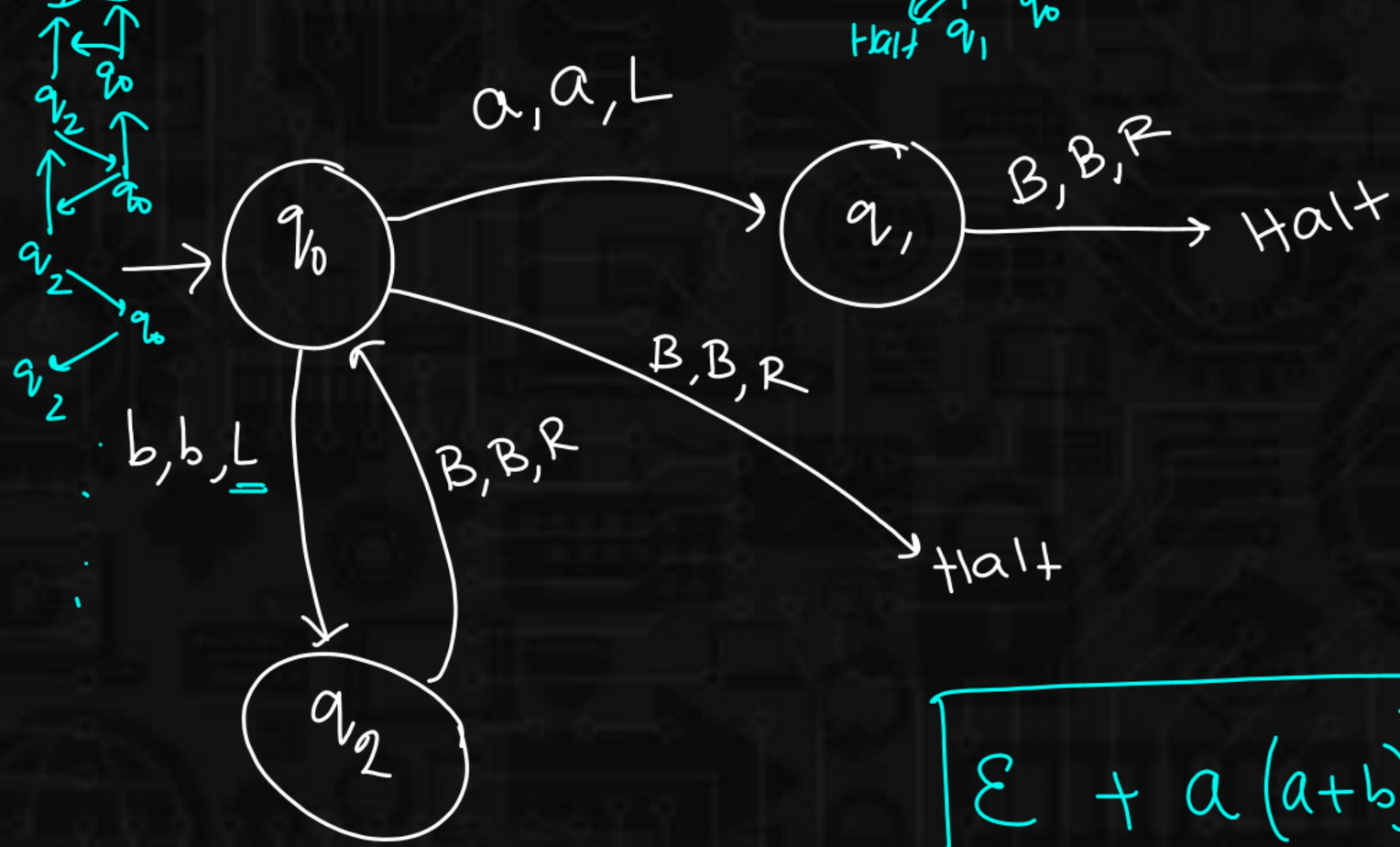
aa ↺

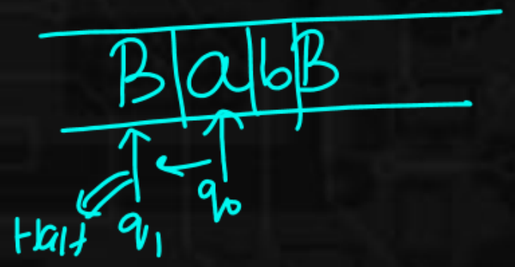
ab ↺

ba ↺

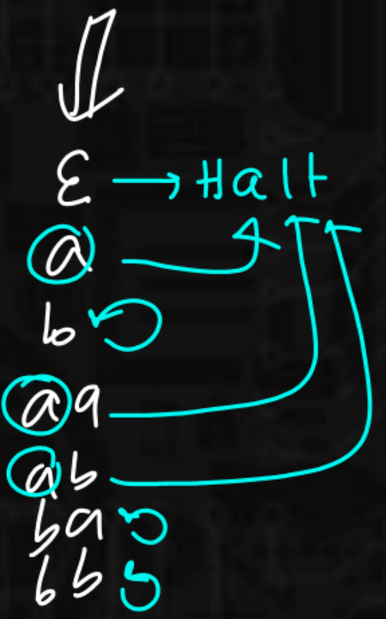
bb ↺

III) Bbab B





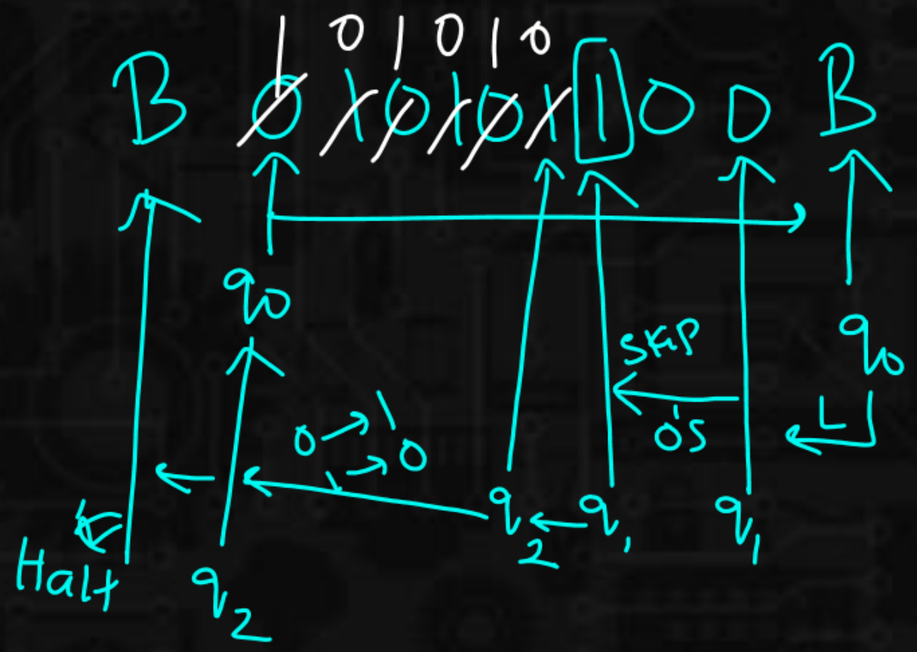
$(a+b)^*$

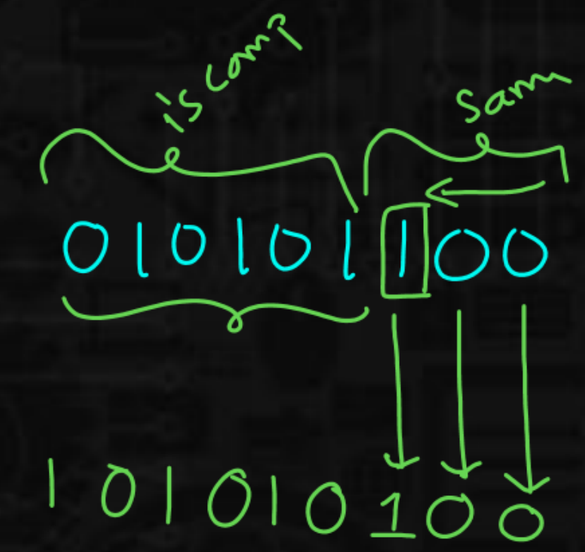


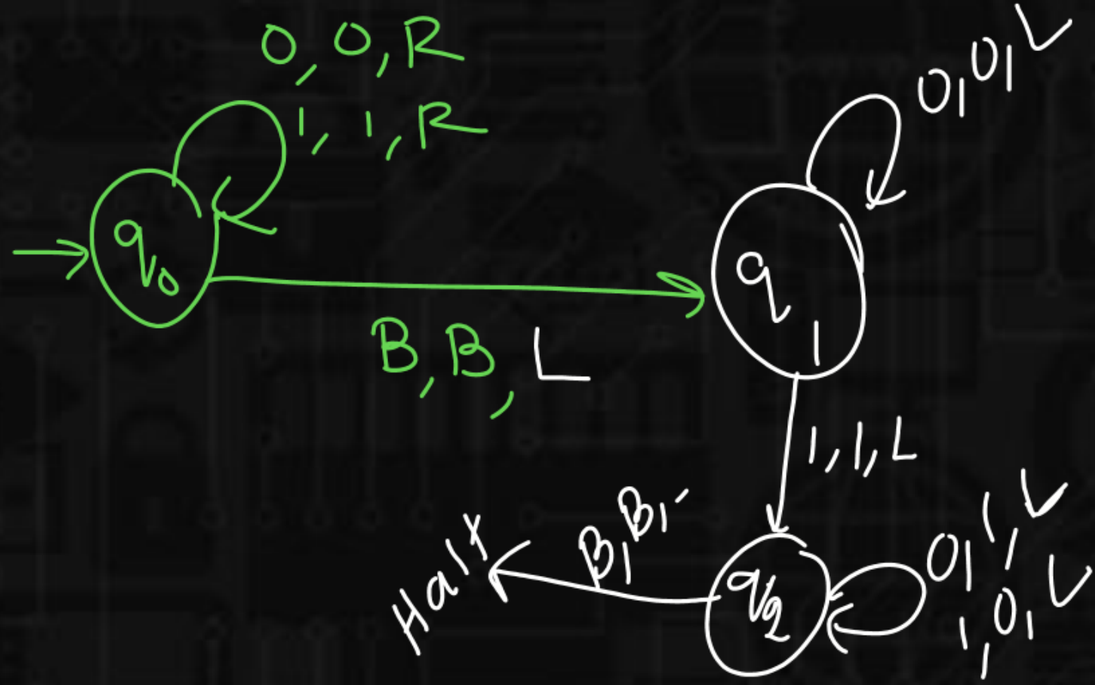
$\varepsilon + a(a+b)^* \Rightarrow$ Halt

$b(a+b)^* \Rightarrow$ Never halts

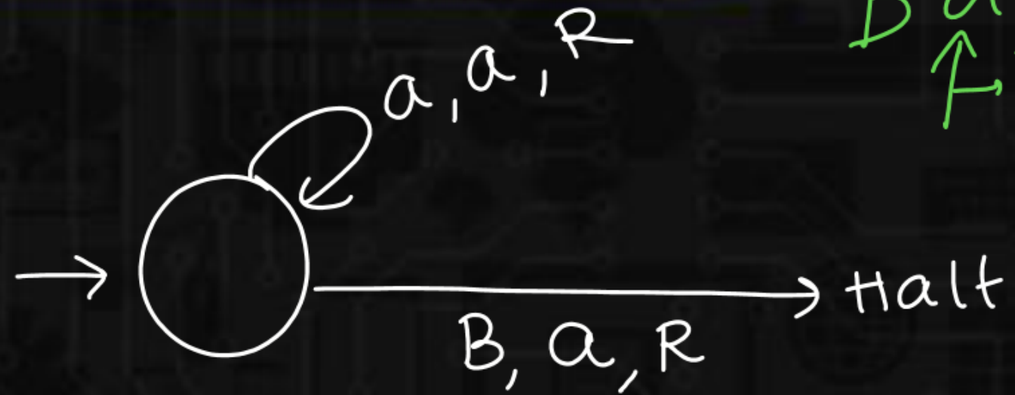
IV) 2's complement







V)



B a a a B B B

Unary Increment:

$$f(x) = x+1$$

n	n+1
ε	a
a	aa
aa	aaa

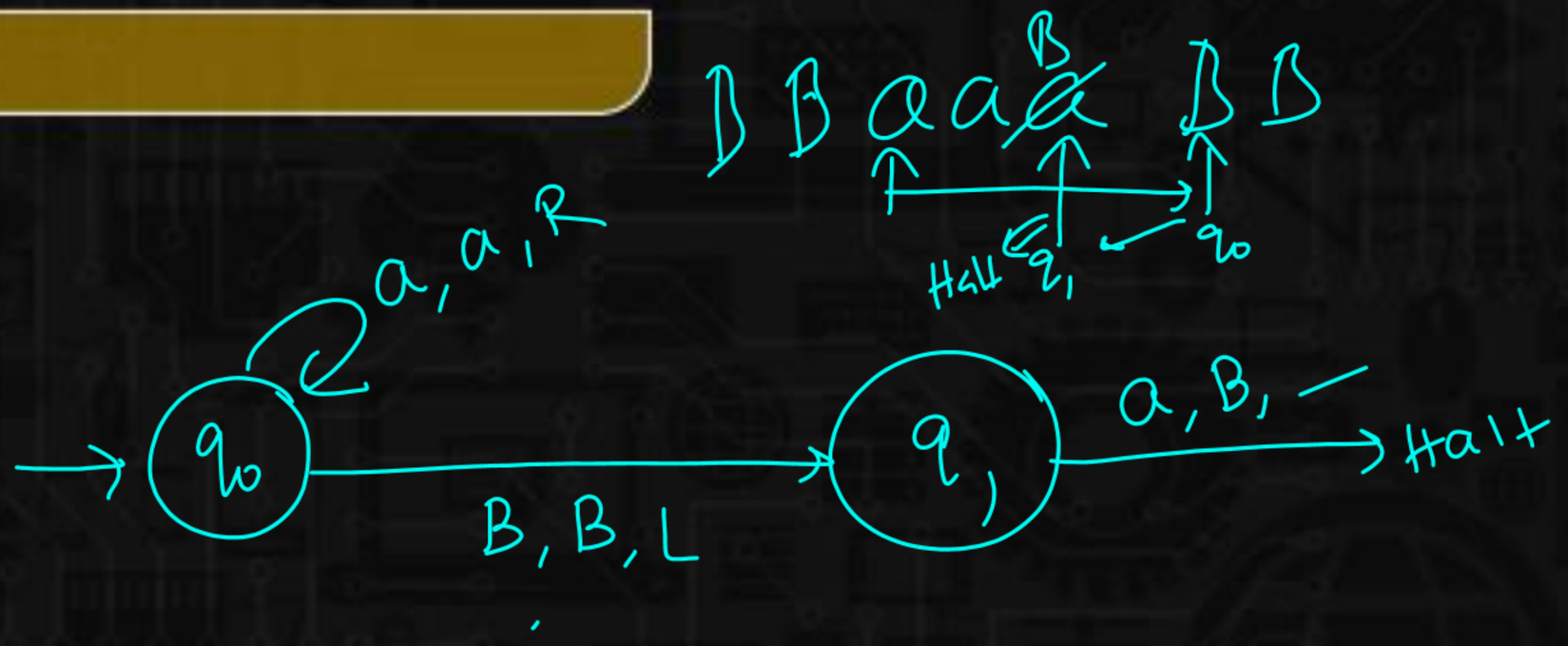
A) Increment

B) Decrement

C) Accepts a^*

D) None

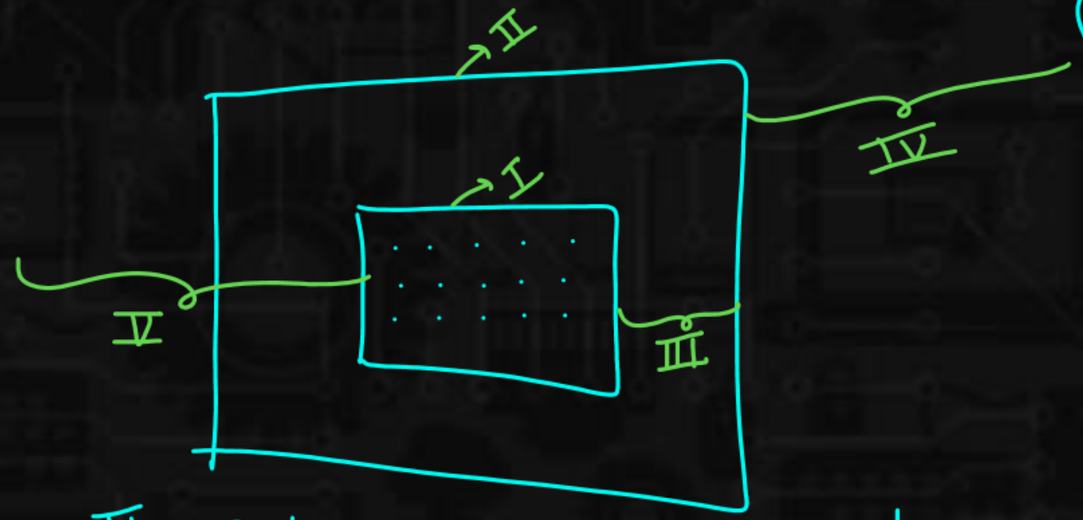
VII)



Decrement in unary

$$f(x) = x - 1 \quad ; x > 0$$

δ	a	B
$\rightarrow q_0$	(q_0, a, R)	(q_1, B, L)
q_1	$(Halt, B, -)$	—



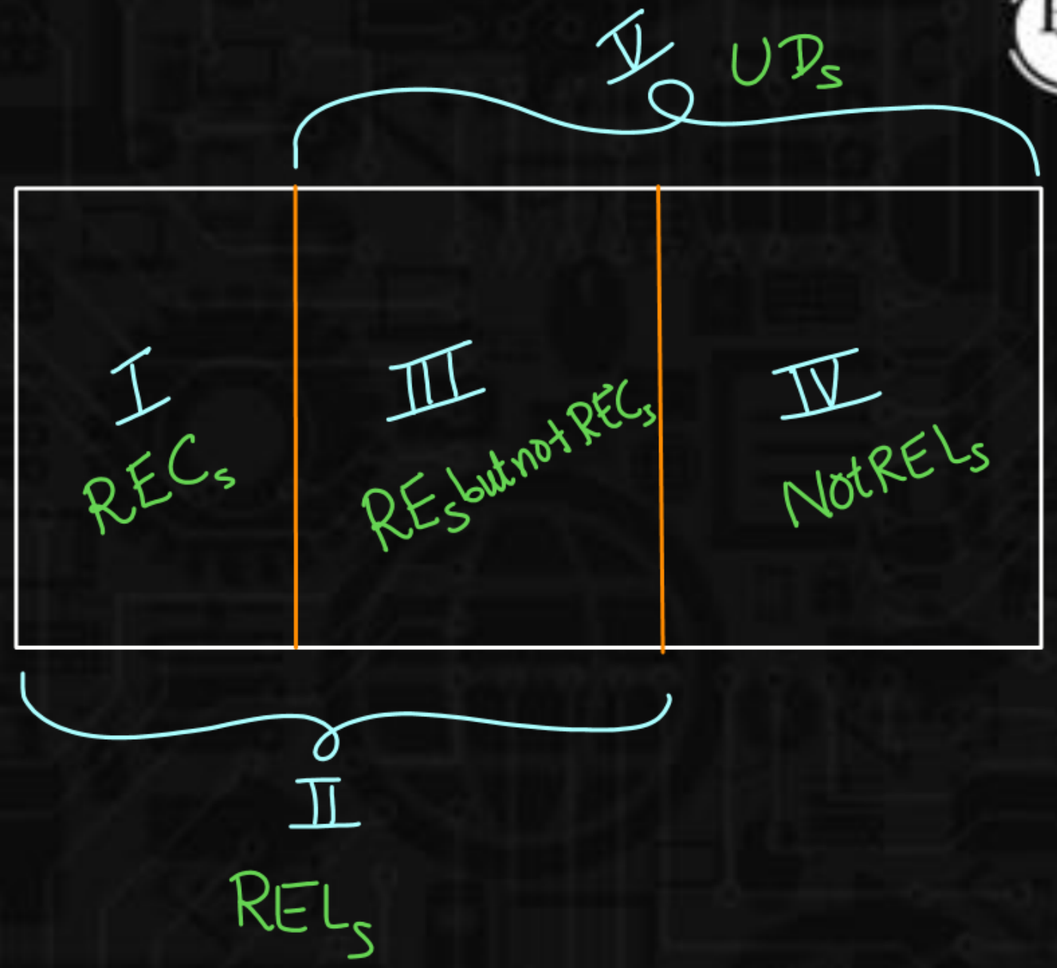
V : Set of all not recursive languages

I : Set of all recursive languages

II : I ∪ III = Set of all REL_s

III : II − I = Set of all REL_s but not recursive

IV : Set of all not REL_s



a^*

$(a+b)^*$

$a^* b^*$

$a^n b^n$

$a^n b^n c^n$

$ww^R \mid w \in \{a,b\}^*$

$w\#w \mid$ ..

CSLs

$a^{n!}$

a^{2^n}

a^{prime}

a^{n^n}

$a^n \# a^n \# a^n$

CSLs

Language

↳ Set of all strings over Σ

Examples:

- Set of symbols
- Set of grammars
- Set of languages
  - Set of regs
  - Set of DCFLs
  - Set of finite languages
- Set of DFAs
- Set of TMs
- Set of objects


L =
Symbol / String / Language / Reg Exp / Grammer / DFA / NFA / PDA / DPDA / TM / . . .

TM $\cong$ Program

HTM $\cong$ Halting Program $\cong$ Algorithm

"Closure properties" for RECs

[Not closed: $\subseteq$, f, h, f_{finite}, $Inf(\cup, \cap, -, \cdot, \subseteq, f)$]

1. Union
2. Intersection
3. Complement
4. Difference
5. Concatenation
6. Reversal
7. Kleene Star
8. Kleene plus

~~9~~ Subset

10. prefix
11. suffix
12. substring

*** ~~13~~ $f(L)$

~~14~~ $h(L)$

15. ε-free $h(L)$
16. $h^{-1}(L)$

17, 18, 19, 20, 21, ~~22~~ Finite $(\cup, \cap, -, \cdot, \subseteq, f)$

Finite substitution

~~23~~, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~ Infinite $(\cup, \cap, -, \cdot, \subseteq, f)$

"Closure properties" for RELs

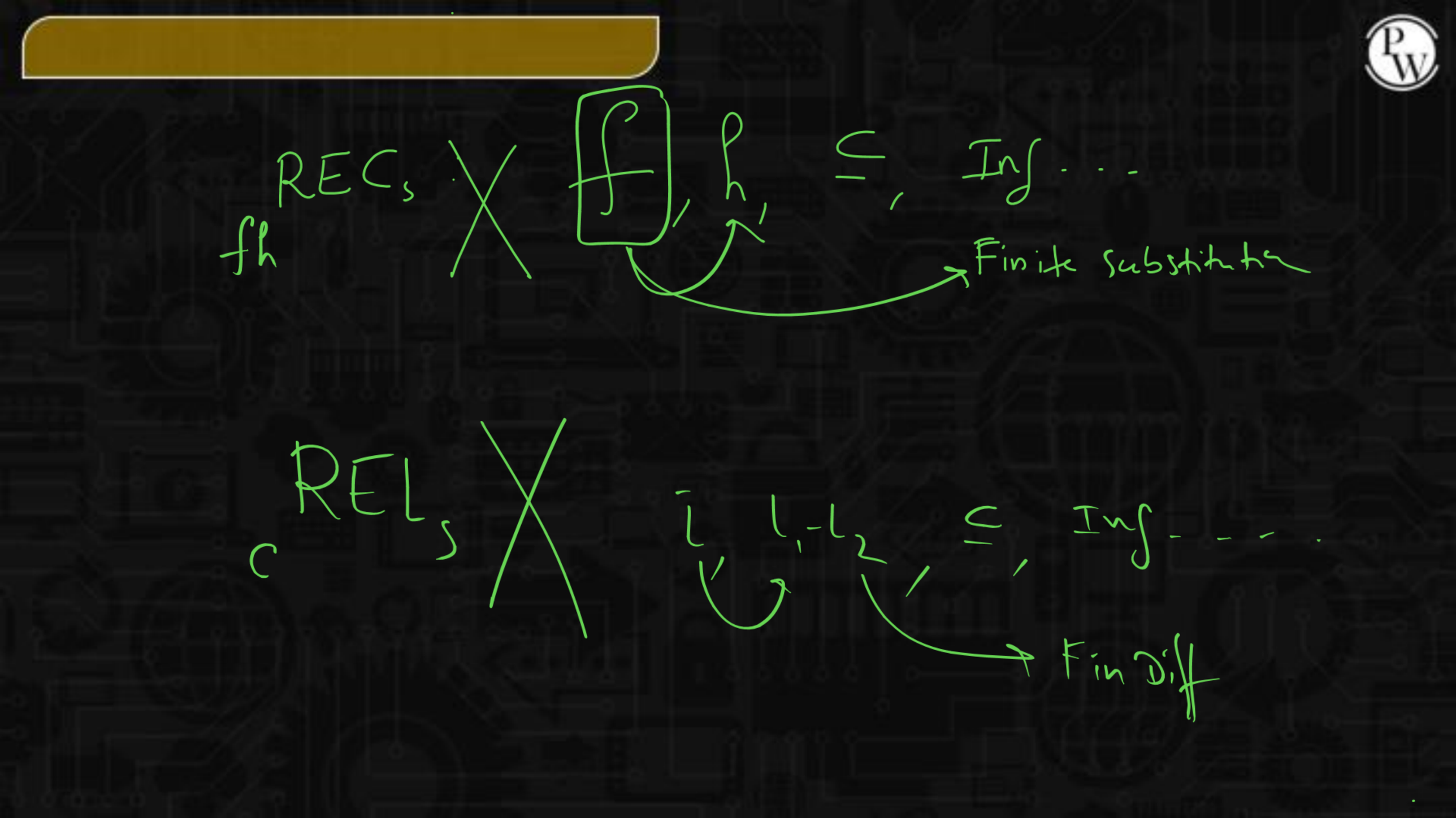
Not closed :
$\bar{L}$, $L_1 - L_2$, $\subseteq$, FinDiff, Inf($\cup, \cap, -, ;, \subseteq, f$)

1. Union
2. Intersection
3. ~~Complement~~
4. ~~Difference~~
5. Concatenation
6. Reversal
7. Kleene Star
8. Kleene plus
9. ~~Subset~~
10. prefix
11. suffix
12. substring
13. $f(L)$
14. $h(L)$
15. ε-free $h(L)$
16. $h^{-1}(L)$
17.
18.
19. ~~Finite~~
20.
21.
22.

Finite($\cup, \cap, -, ;, \subseteq, f$)

23. ~~Infinite~~
24. ~~Infinite~~
25. ~~Infinite~~
26. ~~Infinite~~
27. ~~Infinite~~
28. ~~Infinite~~

Infinite($\cup, \cap, -, ;, \subseteq, f$)


fh
RECs
f, h
Finite substitution
c
RELs
i, l1-l2
FinDiff

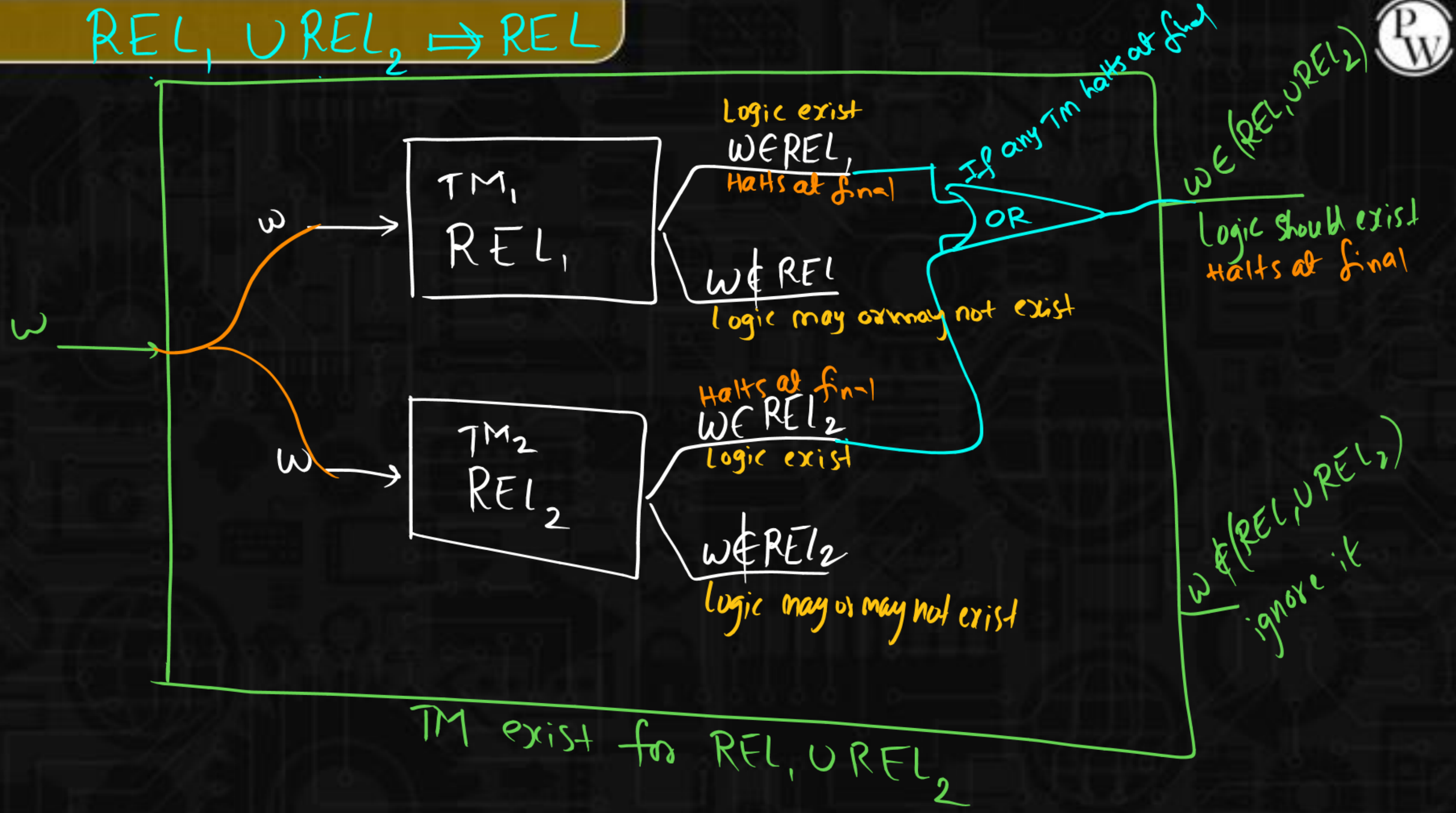
① Union

- closed for RELs
- closed for Recursive languages (RECs)

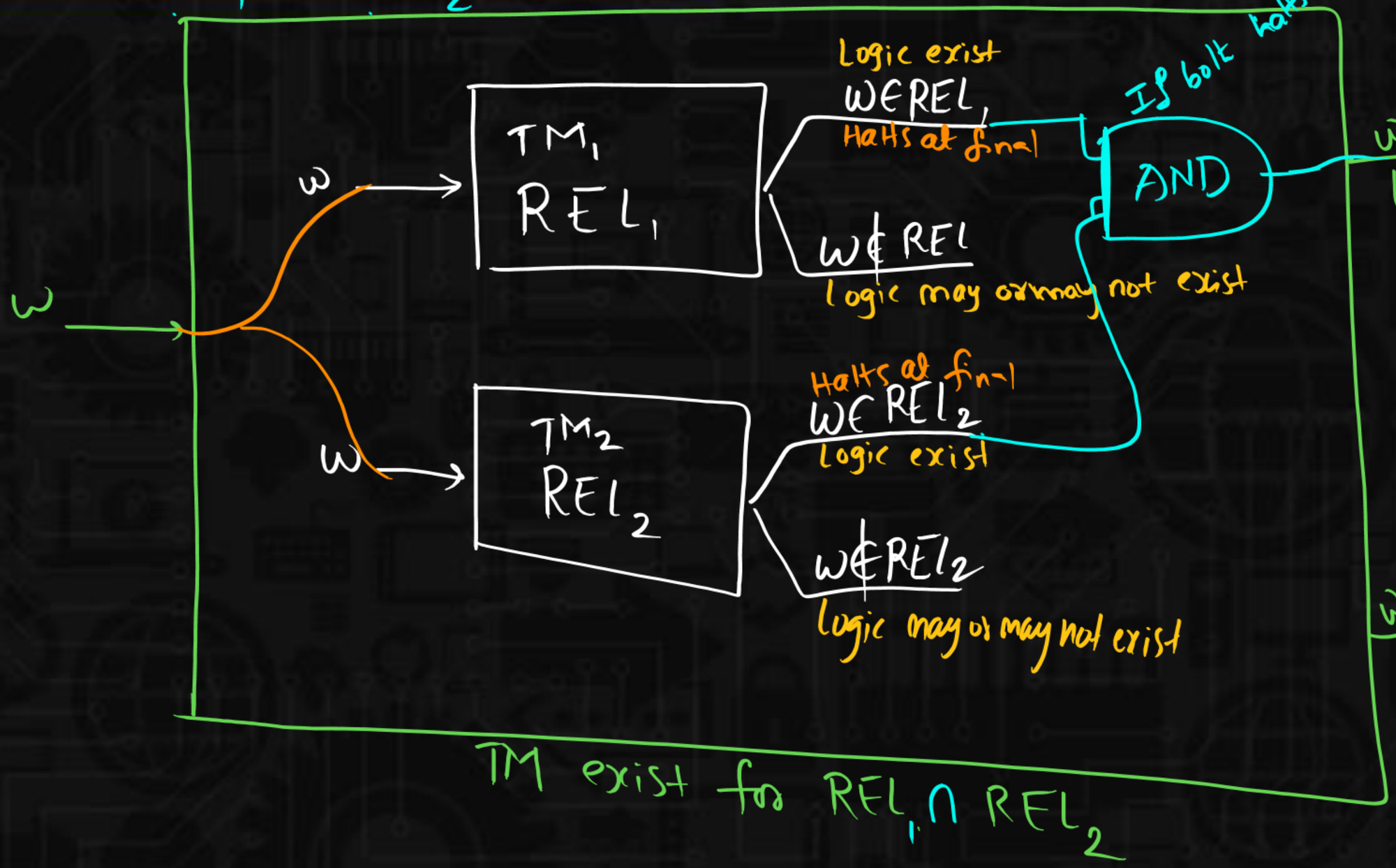
$$REL_1 \cup REL_2 \Rightarrow REL$$

$$REC_1 \cup REC_2 \Rightarrow REC$$

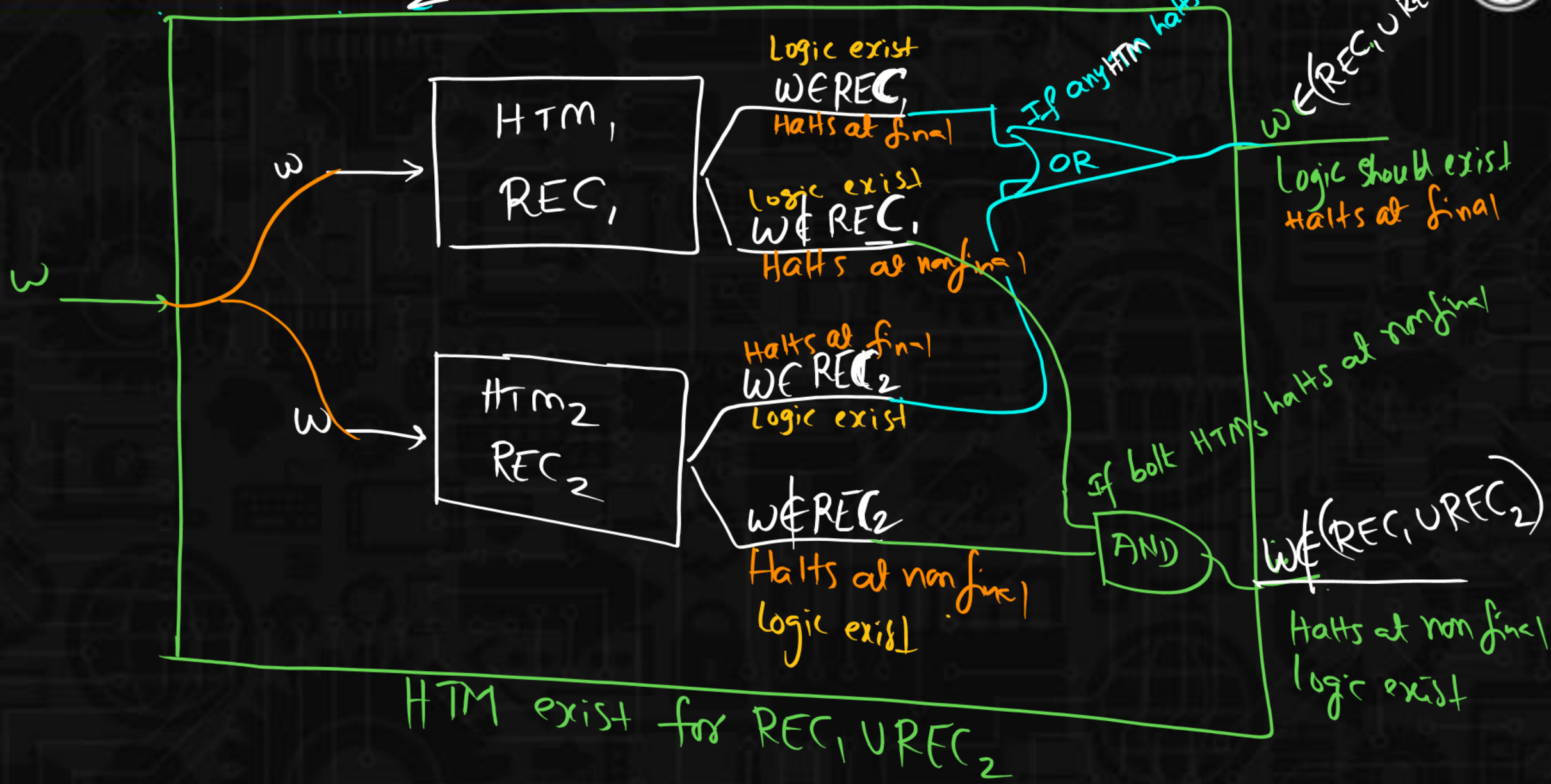
$REL_1 \cup REL_2 \Rightarrow REL$



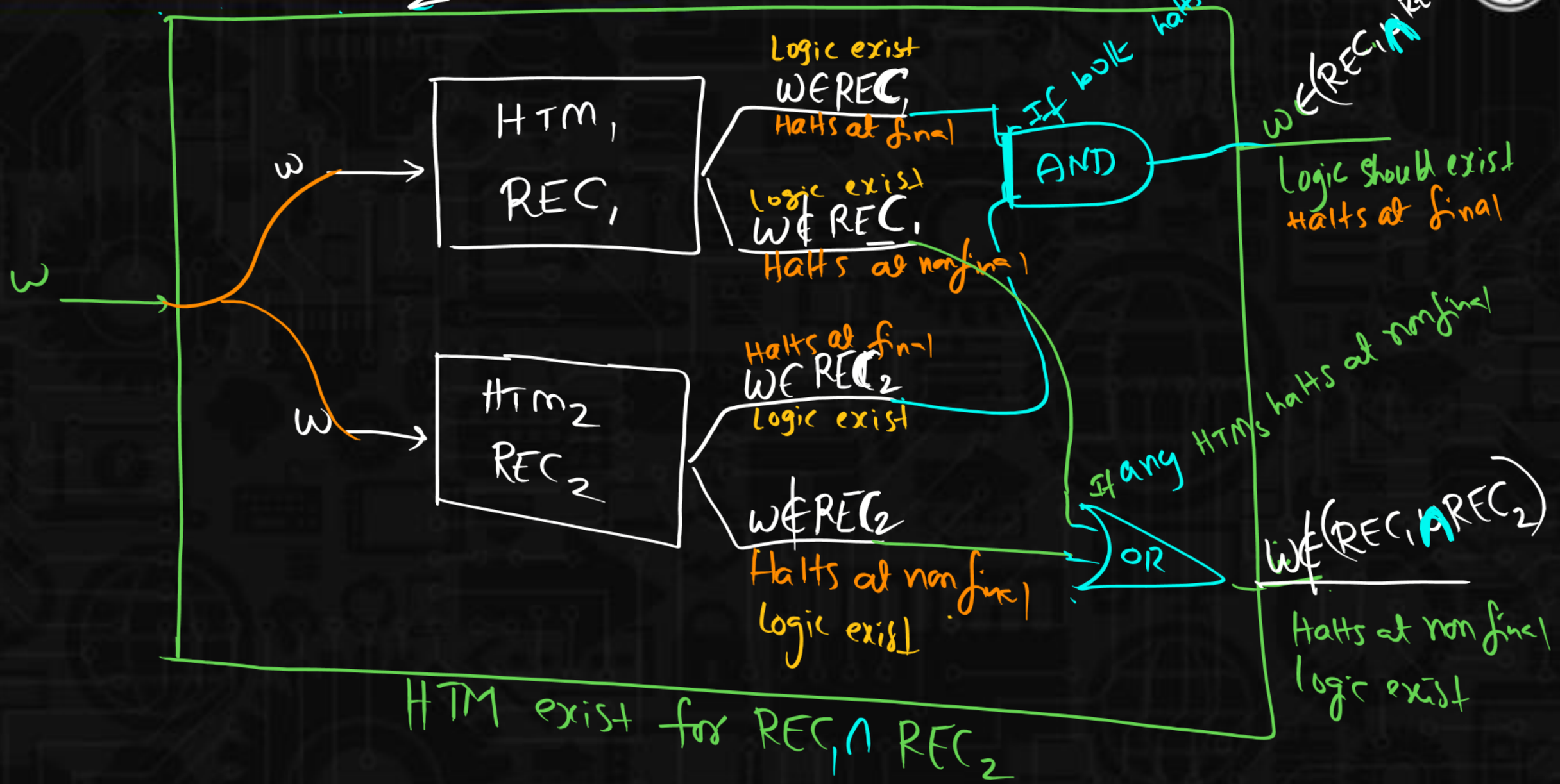
$REL_1 \cap REL_2 \Rightarrow REL$



$REC_1 \cup REC_2 \Rightarrow REC$



$REC_1 \cap REC_2 \Rightarrow REC$



① Union

↳ closed for RELs & RECs

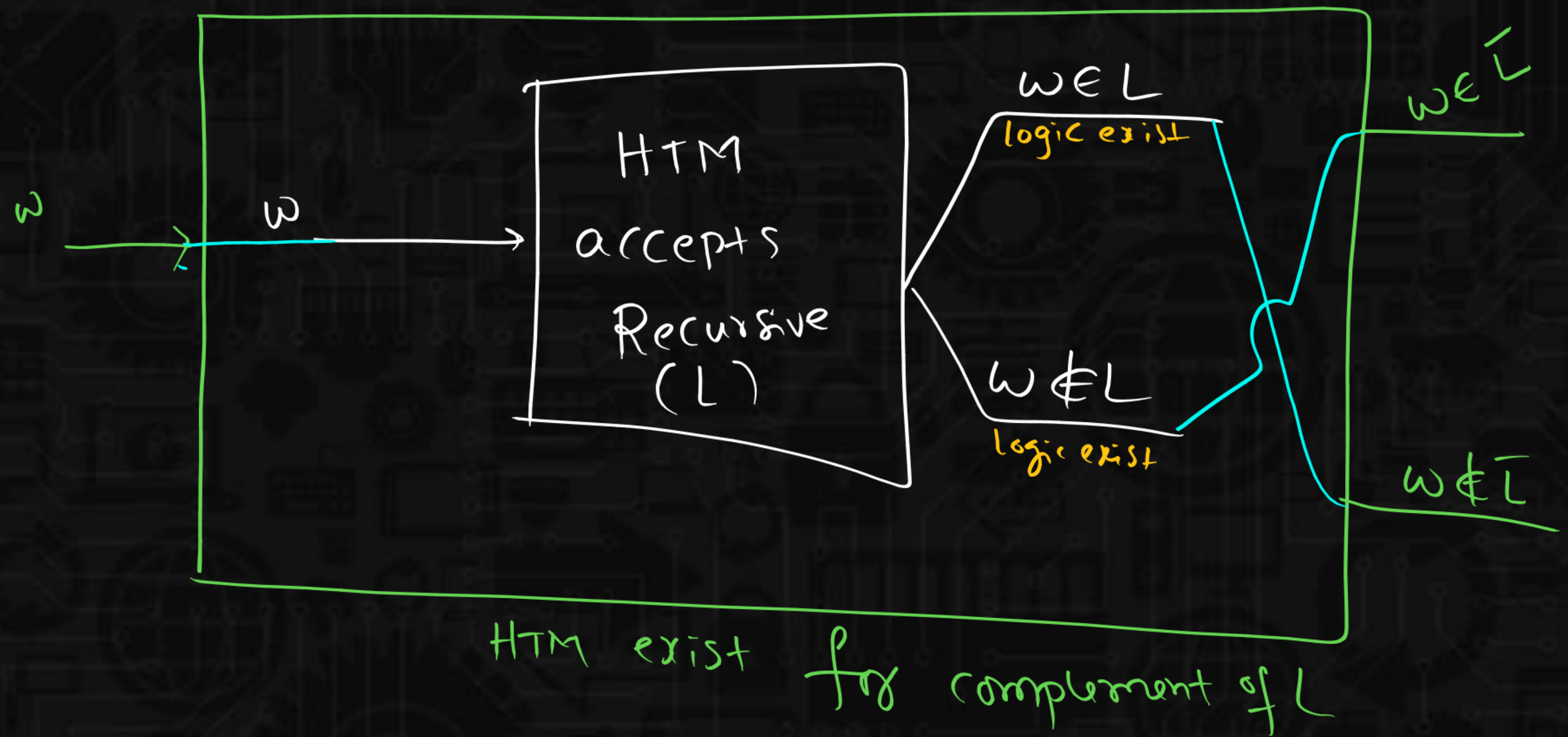
② Intersection

***③ complement

↳ closed for RECs

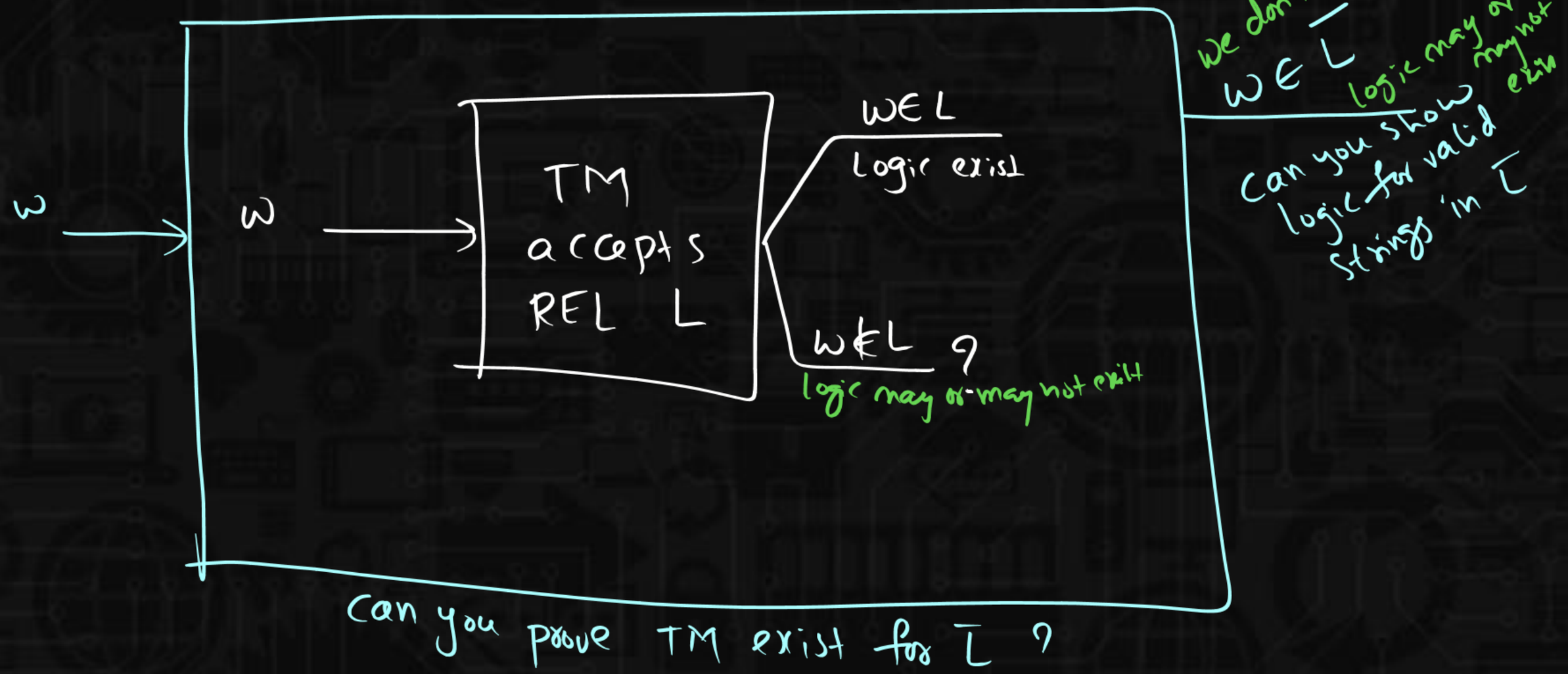
↳ Not closed for RELs

Complement of Recursive language is Recursive



Complement of REL is

may or may not be REL
(either Recursive or Not REL)



REL ⇨ either "Recursive" or "Not REL"

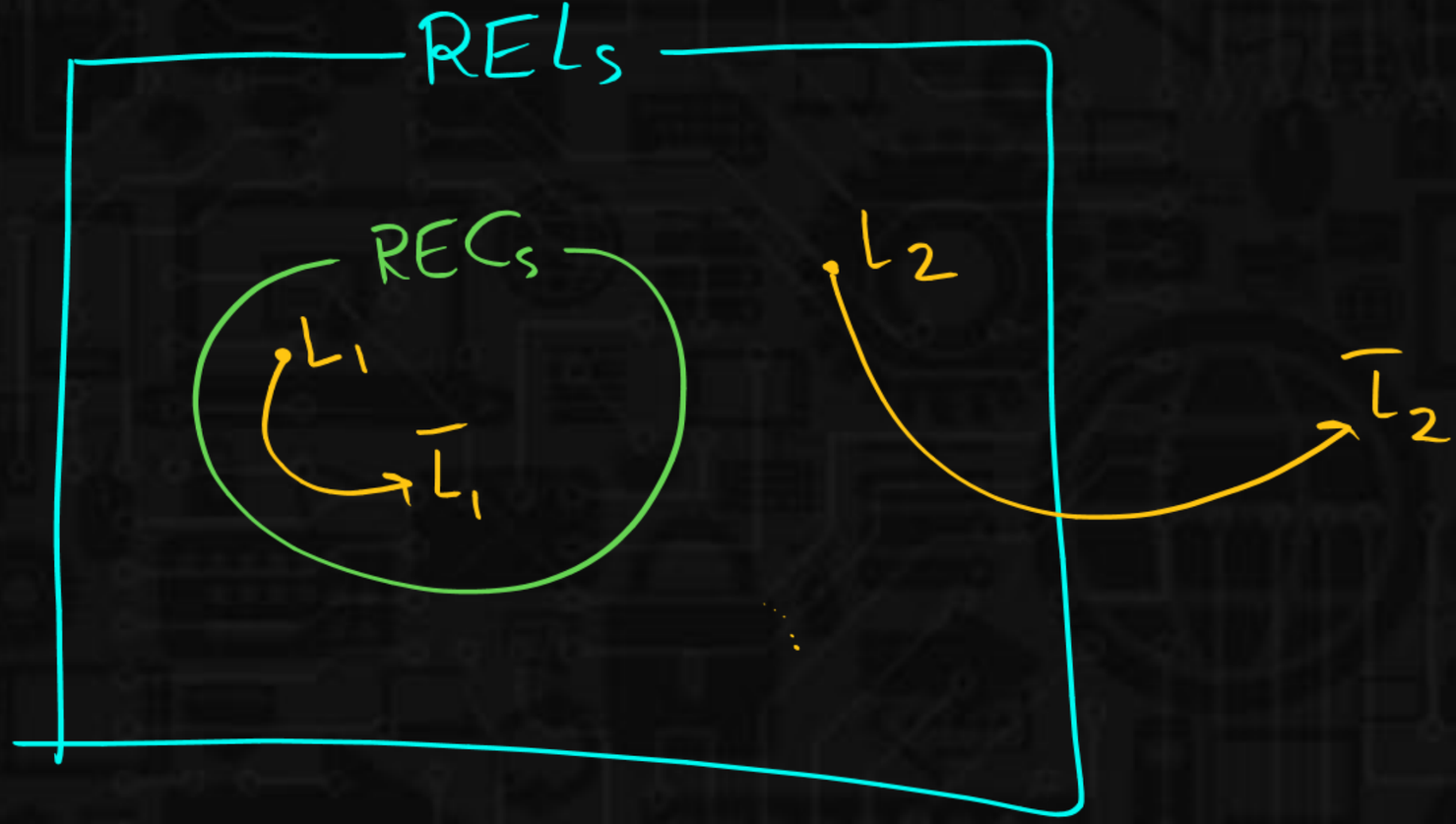
If L is Recursive then $\bar{L}$ is Recursive

If L is "REL but not recursive"
then $\bar{L}$ is Not REL

I) $\overline{REL}$ $\Rightarrow$ may or may not be REL

II) $\overline{\overline{REL}}$ $\Rightarrow$ either "Recursive" OR "Not REL"

III) $\overline{\overline{REL}}$ $\Rightarrow$ Never be "REL but not be Recursive"


RELs
RECs
L_1
$\bar{L}_1$
L_2
$\bar{L}_2$

① $REL_1 \cup REL_2 \Rightarrow REL$

② $REC_1 \cup REC_2 \Rightarrow REC$

③ $REC \cup REL \Rightarrow REL$

④ $REC \cup Reg \Rightarrow REC$

⑤ $REC \cup CFL \Rightarrow REC$

⑥ $REC \cup CSL \Rightarrow REC$

⑦ $REL \cup Reg \Rightarrow REL$

⑧ $REL \cup CFL \Rightarrow REL$

⑨ $REL \cup CSL \Rightarrow REL$

*** ⑩ $REL_1 \cap REL_2 \Rightarrow REL$

⑪ $REC_1 \cap REC_2 \Rightarrow REC$

⑫ $REC \cap REL \Rightarrow REL$

⑬ $REC \cap Reg \Rightarrow REC$

⑭ $REC \cap CFL \Rightarrow REC$

⑮ $REC \cap CSL \Rightarrow REC$

⑯ $REL \cap Reg \Rightarrow REL$

⑰ $REL \cap CFL \Rightarrow REL$

⑱ $REL \cap CSL \Rightarrow REL$

***

DCFL $\cap$ CFL $\Rightarrow$ CSL

CFL $\cap$ CSL $\Rightarrow$ CSL

DCFL $\cap$ DCFL $\Rightarrow$ CSL

CFL $\cap$ CFL $\Rightarrow$ CSL

$\overline{Rec}$ $\Rightarrow$ Rec

$\overline{REL}$ $\Rightarrow$ either Recursive OR Not REL

$\overline{\text{REL but not Rec}}$ $\Rightarrow$ Not REL

Summary

- closure properties
  - complement
    - Important
  - 5 classes


THANK YOU GW
SOLDIERS !