CS & IT ENGINEERING

Theory of Computation

Finite Automata:

DFA-5

Lecture No. 10 (no DPP)

By- DEVA Sir

01 Model- X

02 XI

03 XII

04

05

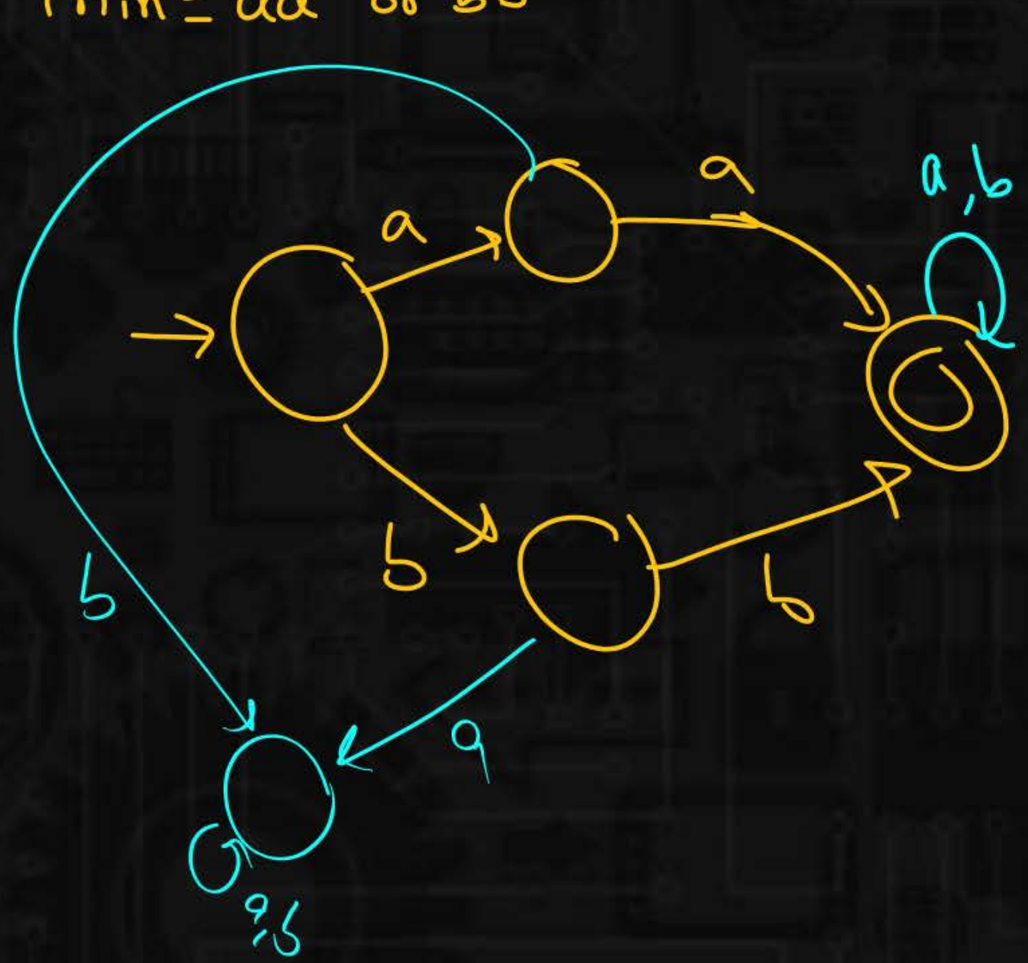
DFA Construction

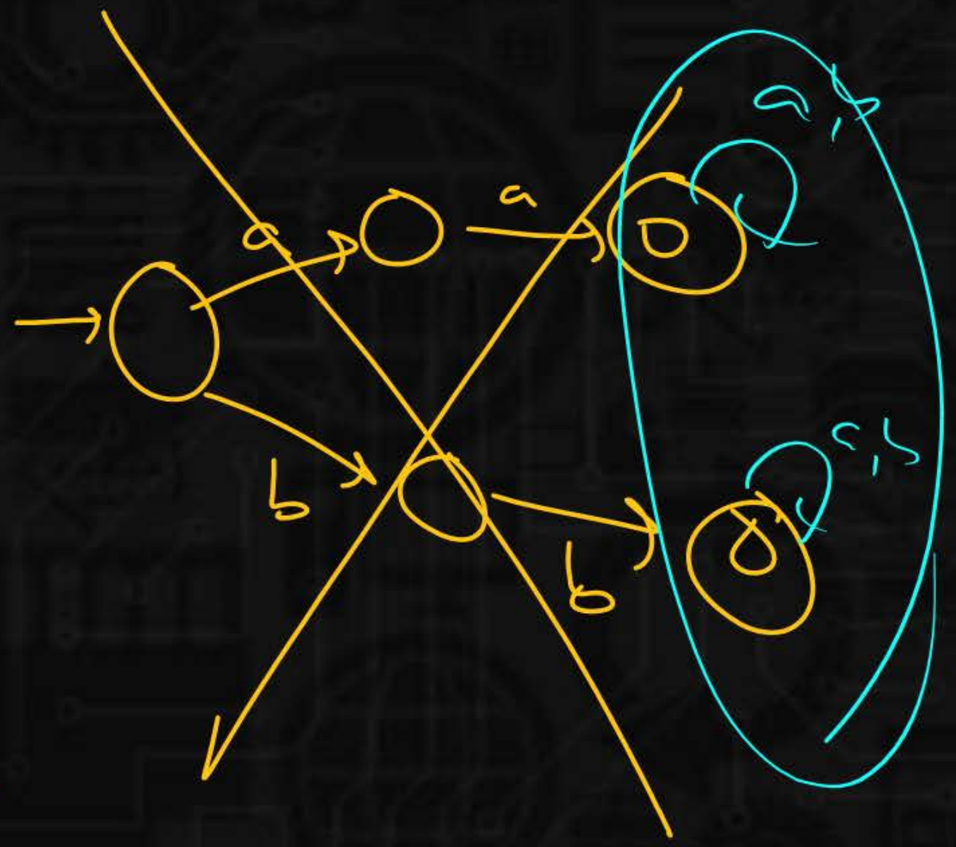
Module-X

(86) $(aa+bb)$ ✗

5 states ✓

min = aa or bb





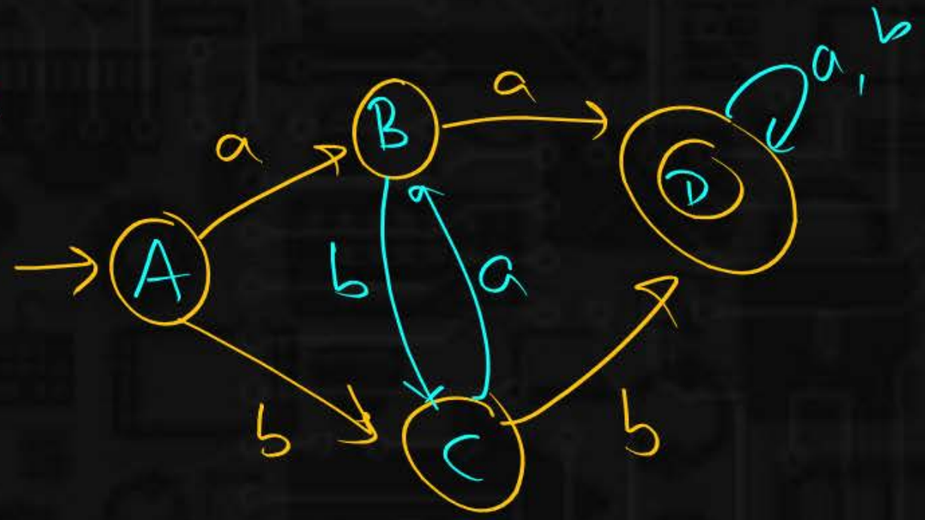
DFA Construction

(87) $X(aa+bb)X$

min = aa or bb

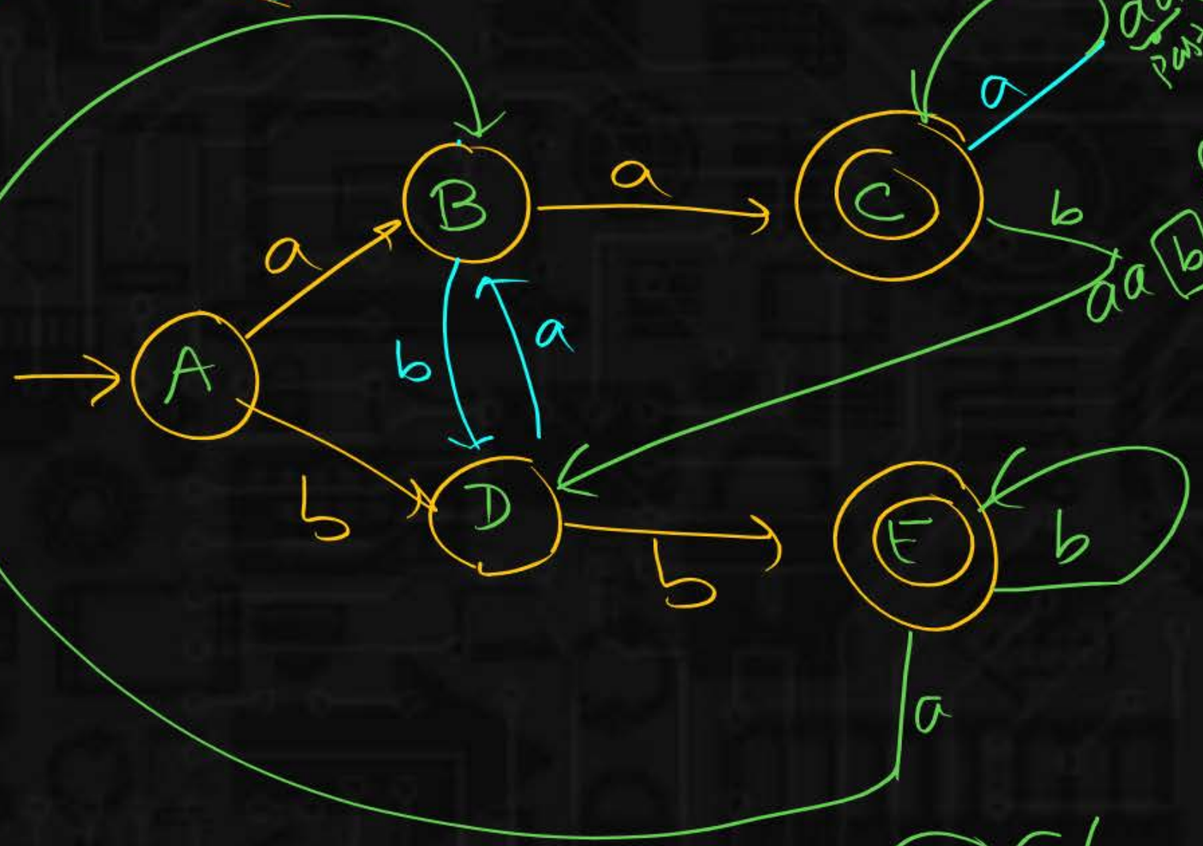
= 4 States

A : waits for aa
B : waits for a
C : waits for b
D : final



DFA Construction

*** (88) $X(aa+bb)$



A: Waiting for aa or bb

B: Waiting for a

C: final
(ends with aa)

D: Waits for b

E: final
(ends with bb)

aa a ∈ L
aa part power

aa b b ∈ L
future

bb a a ∈ L

= 5 states

Are you clear why C & E are needed as different finals ?

I) Past present $\in L$

↳ goto final

II) If Past present $\notin L$ then

i) past . present . (future) $\in L$

some minimum

goto particular state waiting for future

ii) Past present . future $\notin L$

goto Dead every

(89) $a \times b$

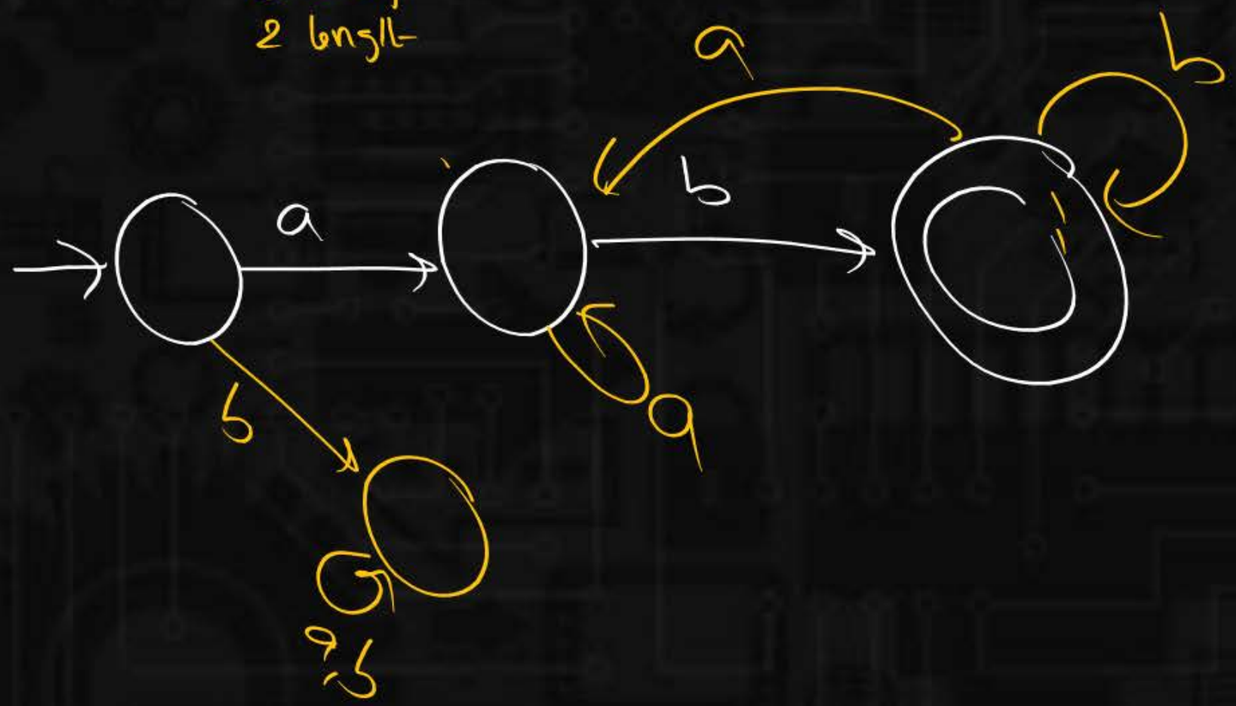
4 states

$min = ab$

$|ab| + 1 + 1 = 4$ states

$|ab| + 1$: to accept 2 length

1: Dead



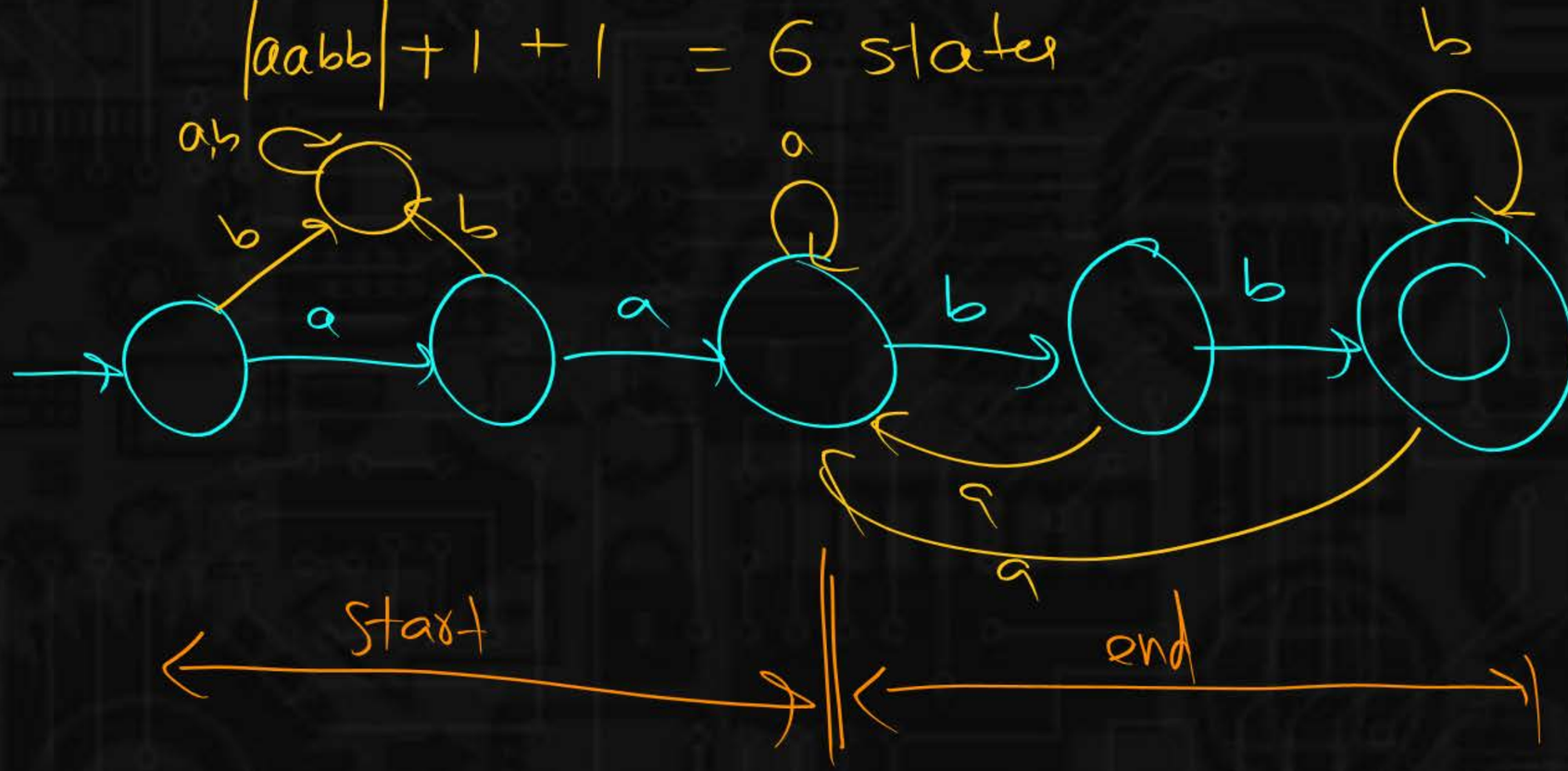
DFA Construction

(90) $aa(a+b)^*bb$

= 6 states

min = aabb

|aabb| + 1 + 1 = 6 states



(91) starts with 'a' and contains 'a'

= starts with a

= a X ⇒ 3 states

aX and XaX = aX

DFA Construction

(92) ends with 'a' and contains 'a'

$$X a \cap X a X = X a$$

↳ 2 states

DFA Construction

(93) aaa ✗ bbb

↳ |aaabbb| + 1 + 1 = 8 states

Home Work ***

(94) Starts with aaa and Ends with aab

aaa ✗ aab

aaaaab not min string

6 + 1 + 1 = 8 stat

|aaab| + 1 + 2 + 1 ⇒ 8 states

min string; extra states (common symbols); Dead

Min = aaab

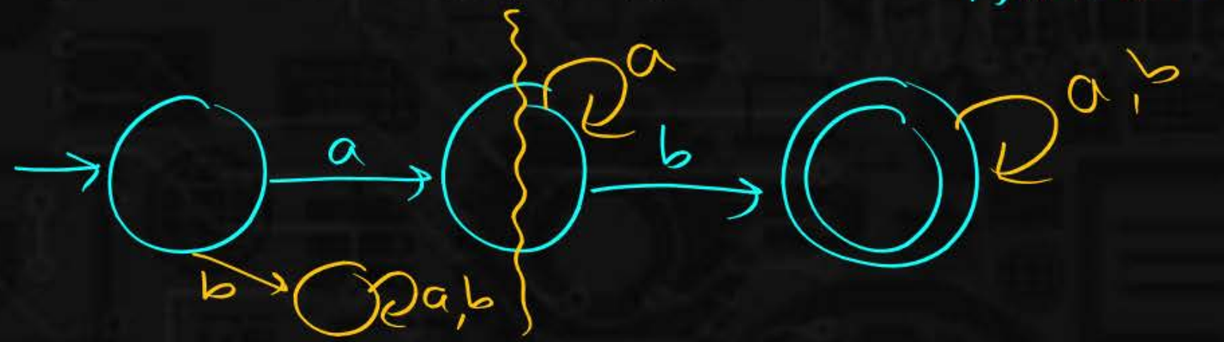
start; end

DFA Construction

(95) Starts with 'a' and contains 'b'

$R = aXbX$

4 states

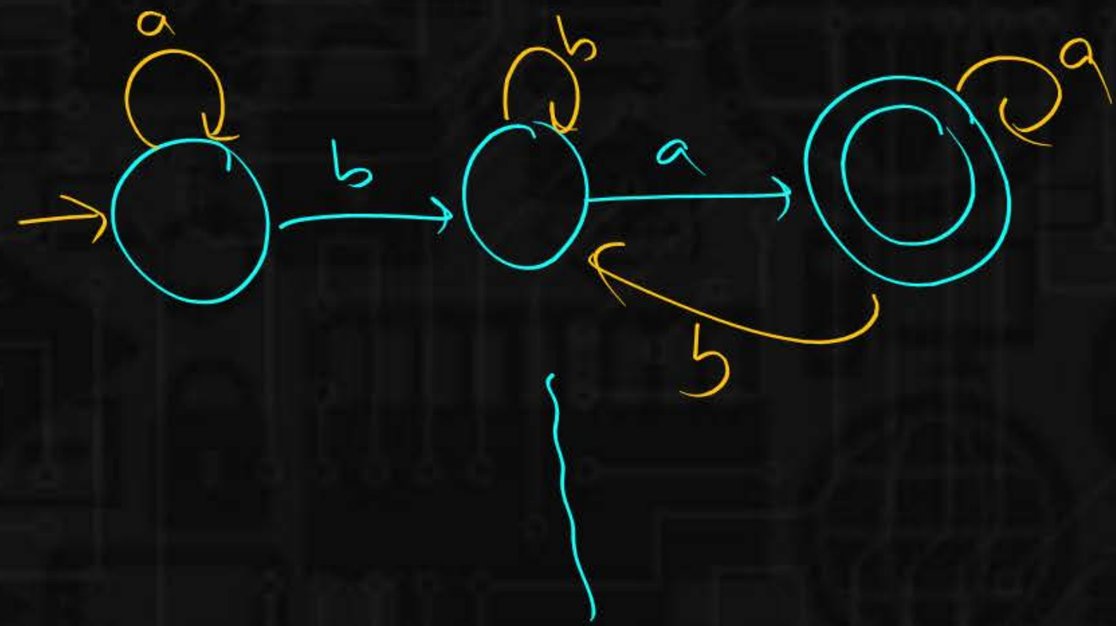


Min = ab

(96) ends with 'a' and contains 'b'

$R = XbXa$

3 states



Min = ba

DFA Construction

Model-XI [Composition based] FA, XFA2

compound FA

4 states finals may differ

(97) $\{w \mid w \in \{a,b\}^*, n_a(w) = \text{even}, n_b(w) = \text{even}\}$

(98) $\{w \mid$ " $, n_a(w) = \text{even}, n_b(w) = \text{odd}\}$

(99) $\{w \mid$ " $, n_a(w) = \text{odd}, n_b(w) = \text{even}\}$

(100) $\{w \mid$ " $, n_a(w) = \text{odd}, n_b(w) = \text{odd}\}$

(101) $\{w \mid$ " $, n_a(w) = \text{even}$ OR $n_b(w) = \text{even}\}$

(102) $\{w \mid$ " $, n_a(w) = \text{even}$ but not $n_b(w) = \text{even}\}$

DFA Construction

L_1

$n_a(w) = \text{even}$

Div by 2

L_2

$n_b(w) = \text{even}$

Divisible by 2

$Q_1 = \{A, B\}$

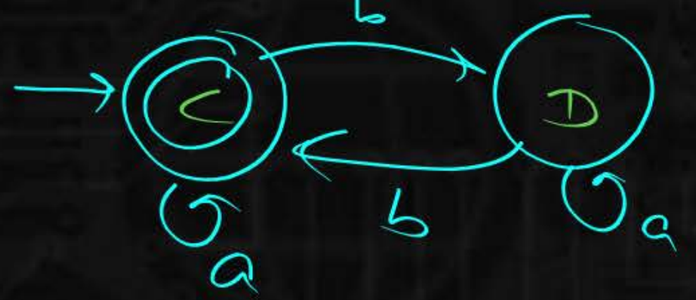
$Q_2 = \{C, D\}$

$Q_1 \times Q_2 = \{(A,C), (A,D), (B,C), (B,D)\}$

FA$_1$

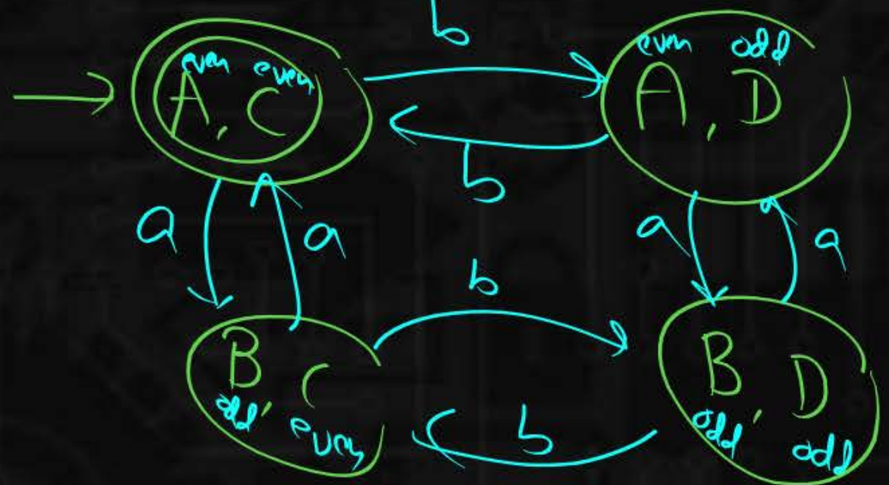
$\times$

FA$_2$



$\delta(\{A,C\}, a) = \{B, C\}$

$\delta(A, a)$ $\quad$ $\delta(C, a)$



FA$_1$ X FA$_2$

DFA Construction

103 $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is div by } 2, n_b(w) \text{ is div by } 3\}$

$2 \times 3 = 6$ states

104 $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is div by } 10, n_b(w) \text{ is div by } 20\}$

$10 \times 20 = 200$ states

DFA Construction

Models :

Special	I	$\Phi \to 1$ $\quad \Sigma^* \to 1$ $\quad \{\varepsilon\} \to 2$ $\quad \Sigma^+ \to 2$
Length	II	$\lvert w\rvert = K \to K+2$ $\quad \lvert w\rvert \le K \to K+2$ $\quad \lvert w\rvert \ge K \to K+1$ $\quad \lvert w\rvert < K \to K+1$ $\quad \lvert w\rvert > K \to K+2$ $\quad \lvert w\rvert \ne K \to K+2$
no. of a's	III	$n_a(w) = K$ $\quad n_a(w) \le K$ $\quad n_a(w) \ge K$ $\quad n_a(w) < K$ $\quad n_a(w) > K$ $\quad n_a(w) \ne K$
over 1 symbol	IV	$a^* \to 1$ $\quad (aa)^* \to 2$ $\quad a(aa)^* \to 2$ $\quad \{a^{K_1 n + K_2} \mid n \ge 0\}$ $K_1 > K_2 \Rightarrow K_1$ states, $K_1 \le K_2 \Rightarrow K_2+1$ states $\quad (aa+aaa)^* \to 3$ $\quad (a^3+a^4)^* \to 7$
Sequence	V	$a^*b^* \to 3$ $\quad a^+b^* \to 4$ $\quad a^*b^+ \to 3$ $\quad a^+b^+ \to 4$
Remainder: Length	VI	$\lvert w\rvert$ is div by $K \to K$ states $\quad \lvert w\rvert = K_1 n + K_2$ If $K_1 > K_2 \Rightarrow K_1$ states, If $K_1 \le K_2 \Rightarrow K_2+1$ states
Remainder: no. of a's	VII	$n_a(w)$ is div by $K \to K$ states $\quad n_a(w) = K_1 n + K_2$
Start/contain/end	VIII	$aX \to 3$ $\quad abX \to 4$ $\quad aaaX \to 5$ $\quad XaX \to 2$ $\quad XaaaX \to 4$ $\quad Xa \to 2$ $\quad Xab \to 3$ $\quad Xaba \to 4$ $\quad Xaaaab \to 6$
Position based	IX	K^{th} symbol from begin is 'a' $\to K+2$ $\quad K^{th}$ symbol from end is 'a' $\to 2^K$
Multiple conditions	X	$(aa+bb)X$ $\quad X(aa+bb)X$ $\quad X(aa+bb)$ $\quad aXb$ $\quad aaXbb$
composition	XI	$n_a(w)$ is div by K_1 and/or but not $n_b(w)$ is div by $K_2 \Rightarrow K_1 \times K_2$

DFA Construction

Model-XII (Many things but one thing) ← Tricky

105 $\{w \mid w \in \{a,b\}^*, w \text{ starts with 'a'}, w \text{ starts with 'b'}\} = \phi$

$\hookrightarrow L = \phi \rightarrow$ 1 state

106 $\{w \mid w \in \{a,b\}^*, w \text{ ends with 'a'}, w \text{ ends with 'b'}\}$

$L = (a+b)^+ \rightarrow$ 2 state

107 $\{w \mid w \in \{a,b\}^*, w \text{ contains 'a' OR } w \text{ contains 'b'}\} = (a+b)^+$

108 $\{w \mid w \in \{a,b\}^*, w \text{ starts with 'a' or 'b'}\} = (a+b)^+$

109 $\{w \mid w \in \{a,b\}^*, w \text{ ends with 'a' or 'b'}\} = (a+b)^+$

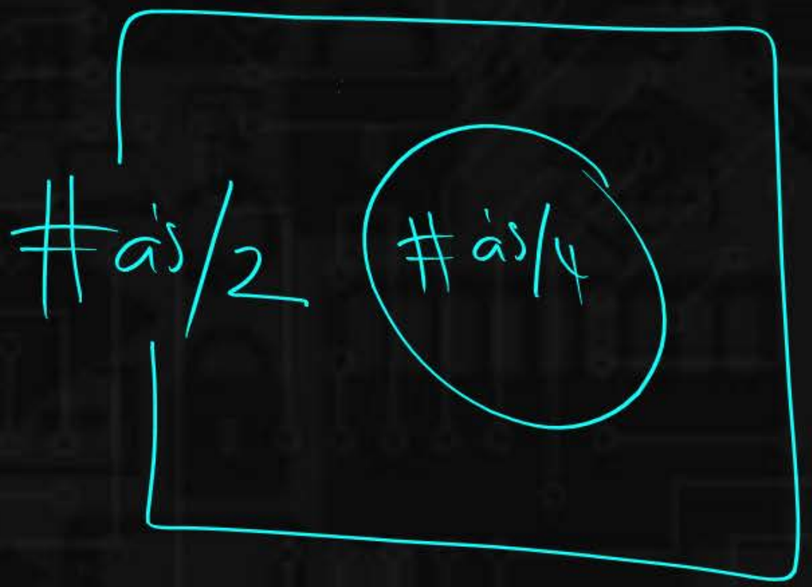
DFA Construction

(110) $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is div by 2}, n_a(w) \text{ is div by 4}\}$

$\#a's/2 \cap \#a's/4 = \#a's/4 \Rightarrow$ 4 states

(111) $\{w \mid w \in \{a,b\}^*, n_a(w) \text{ is div by 2 OR } n_a(w) \text{ is div by 4}\}$

$\#a's/2 \cup \#a's/4 \Rightarrow \#a's/2 \Rightarrow$ 2 states

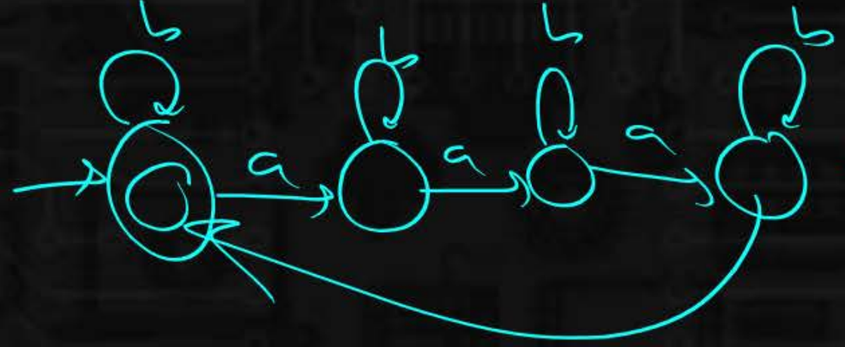


DFA Construction

#a's/2 and #a's/4

#a's = 0, 2, 4, 6, 8 ... ∩ #a's = 0, 4, 8, 12, 16, 20, ...

{ε, aa, aab, aba, baa, ...} ∩ {ε, aaaa, aaaab, aabaa, ...}



= #a's/4

↓

4 States

$L = ab(a+b)^* \Longrightarrow \Sigma = \{a, b\}$

$L = (aa)^* \Longrightarrow \Sigma = \{a\}$

DFA Construction

Model - XIII [no. of a's, no. of b's, <, <=, >=]

H.W.

(112) $\{w \mid w \in \{a,b\}^*, n_a(w) = 1, n_b(w) = 2\}$

(113) $\{w \mid$ " , $n_a(w) \leq 1, n_b(w) = 2\}$

(114) $\{w \mid$ " , $n_a(w) \geq 1, n_b(w) = 2\}$

(115) $\{w \mid$ " , $n_a(w) = 1, n_b(w) \leq 2\}$

(116) $\{w \mid$ " , $n_a(w) \leq 1, n_b(w) \leq 2\}$

(117) $\{w \mid$ " , $n_a(w) \geq 1, n_b(w) \leq 2\}$

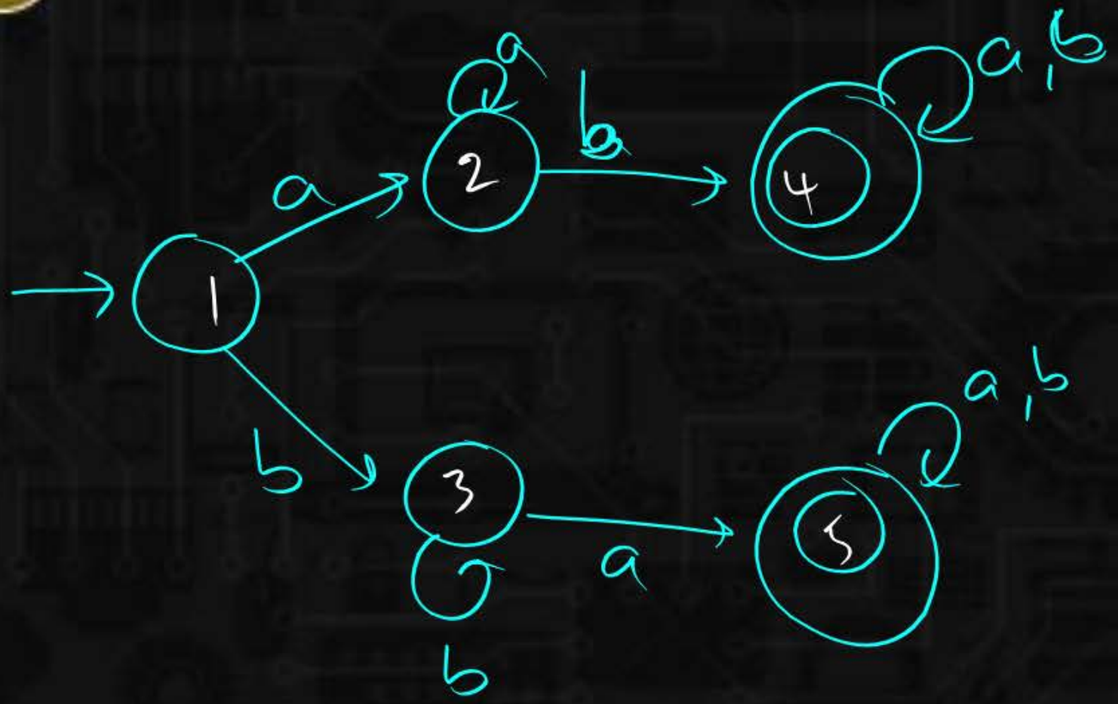
(118) $\{w \mid$ " , $n_a(w) = 1, n_b(w) \geq 2\}$

(119) $\{w \mid$ " , $n_a(w) \leq 1, n_b(w) \geq 2\}$

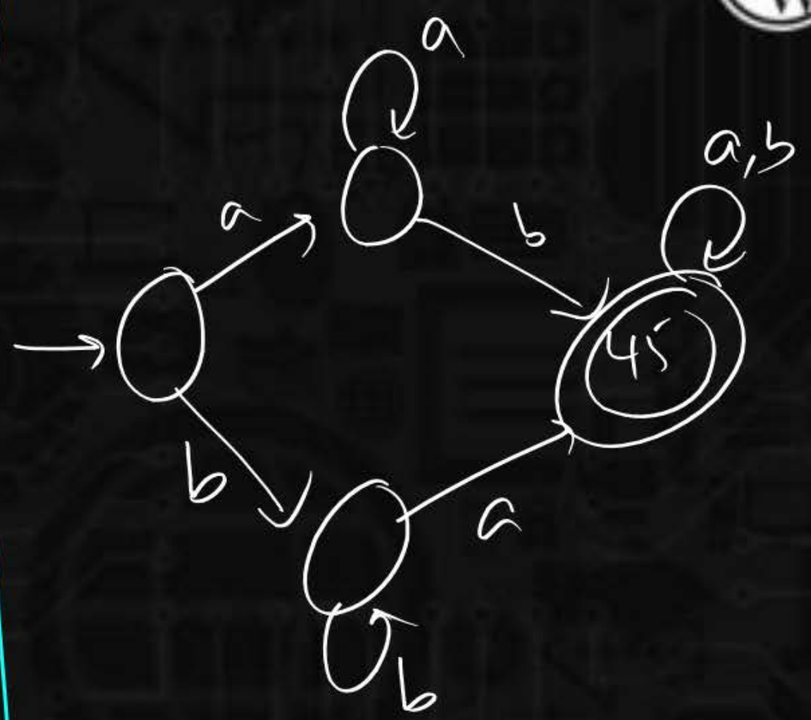
(120) $\{w \mid w \in \{a,b\}^*, n_a(w) \geq 1, n_b(w) \geq 2\}$

DFA Construction

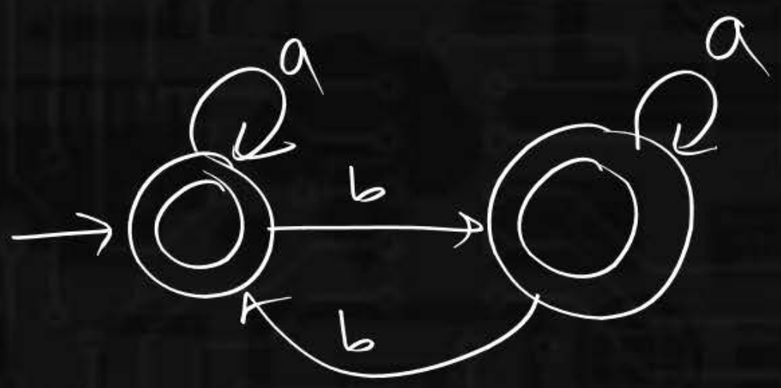
DFA Minimization



$4 \cong 5$



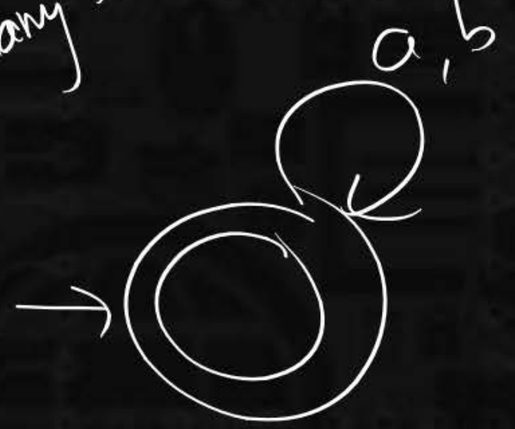
DFA Construction



DFA

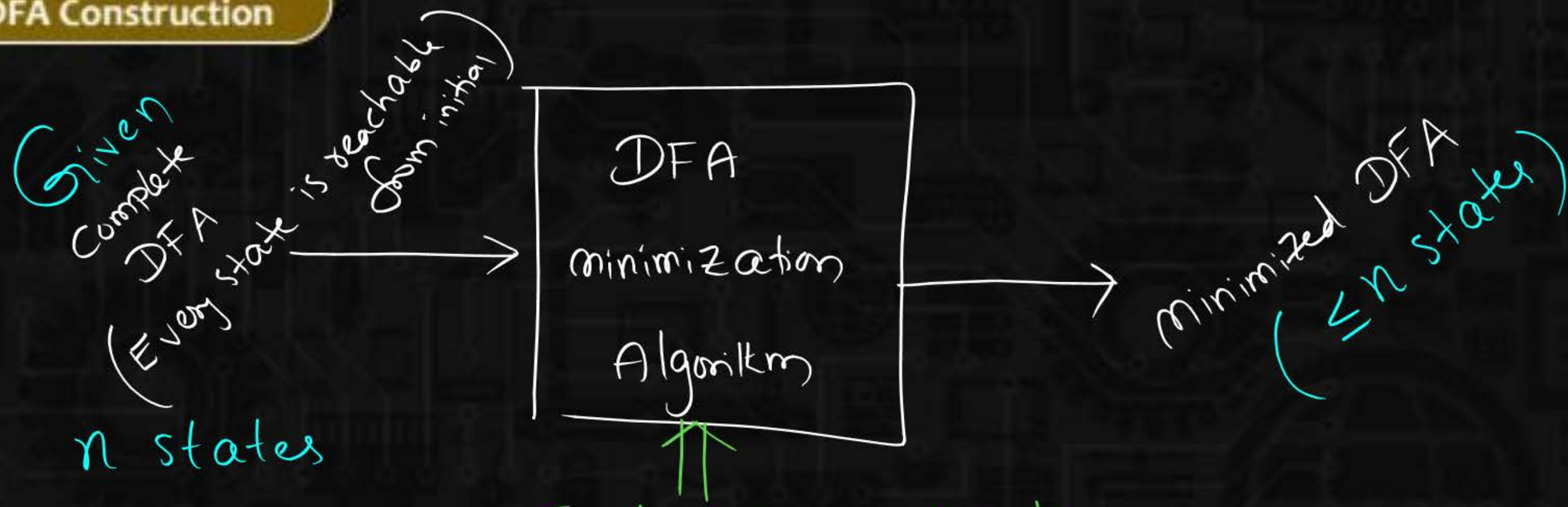
$\Longrightarrow$

How many states in min DFA?



$= 1$

DFA Construction



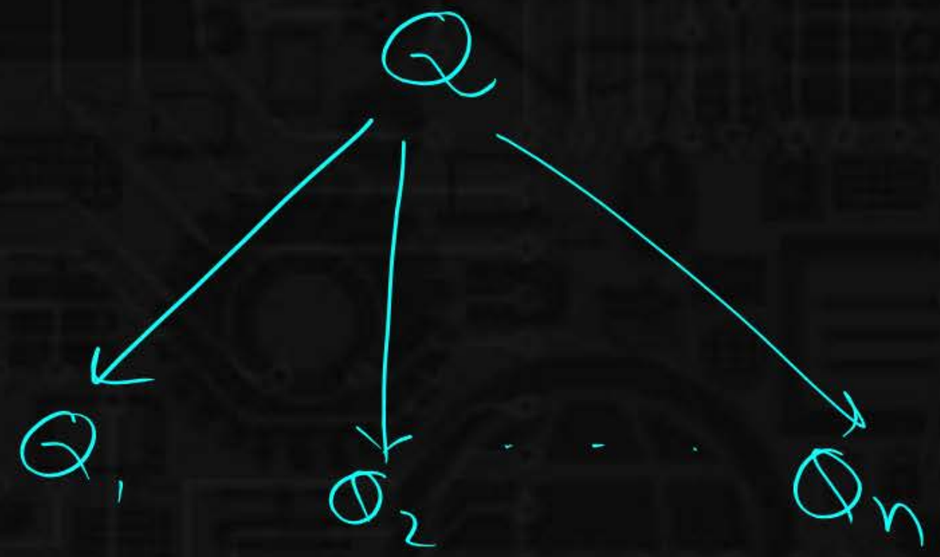
Minimization Algorithm (Partition Algorithm)

- Table Approach
- * Set Approach

} (Partition based Algorithms)

DFA Construction

$\{Q_1, Q_2, \ldots, Q_n\}$ is a partition on Q



I) Every Q_i is subset of Q (non empty)

II) $Q_1 \cup Q_2 \cup \ldots \cup Q_n = Q$

III) Every Q_i, Q_j $\Rightarrow$ $Q_i \cap Q_j = \phi$ $i \neq j$

DFA Construction

$Q = \{q_0, q_1, q_2\}$

I) $\{\{q_0\}, \{q_1\}, \{q_2\}\}$ ⇒ partition with 3 subsets is a partition on Q

Partition with 2 subsets

II) $\{\{q_0, q_1\}, \{q_2\}\}$,,

III) $\{\{q_0, q_2\}, \{q_1\}\}$,,

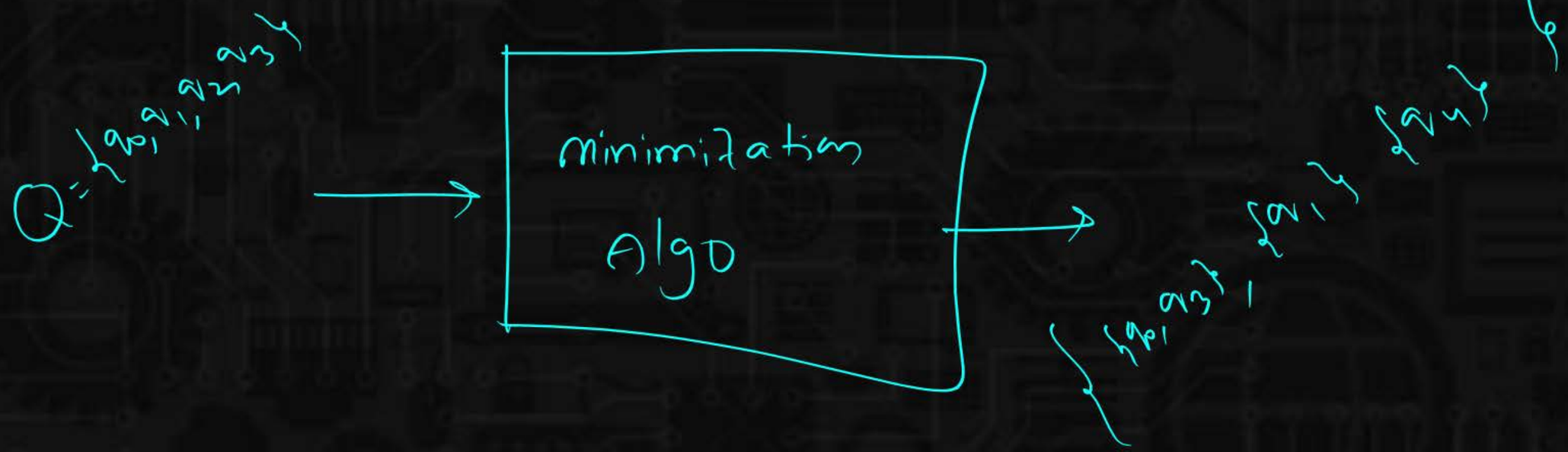
IV) $\{\{q_1, q_2\}, \{q_0\}\}$,,

V) $\{\{q_0, q_1, q_2\}\}$,,

↳ Partition with 1 subset

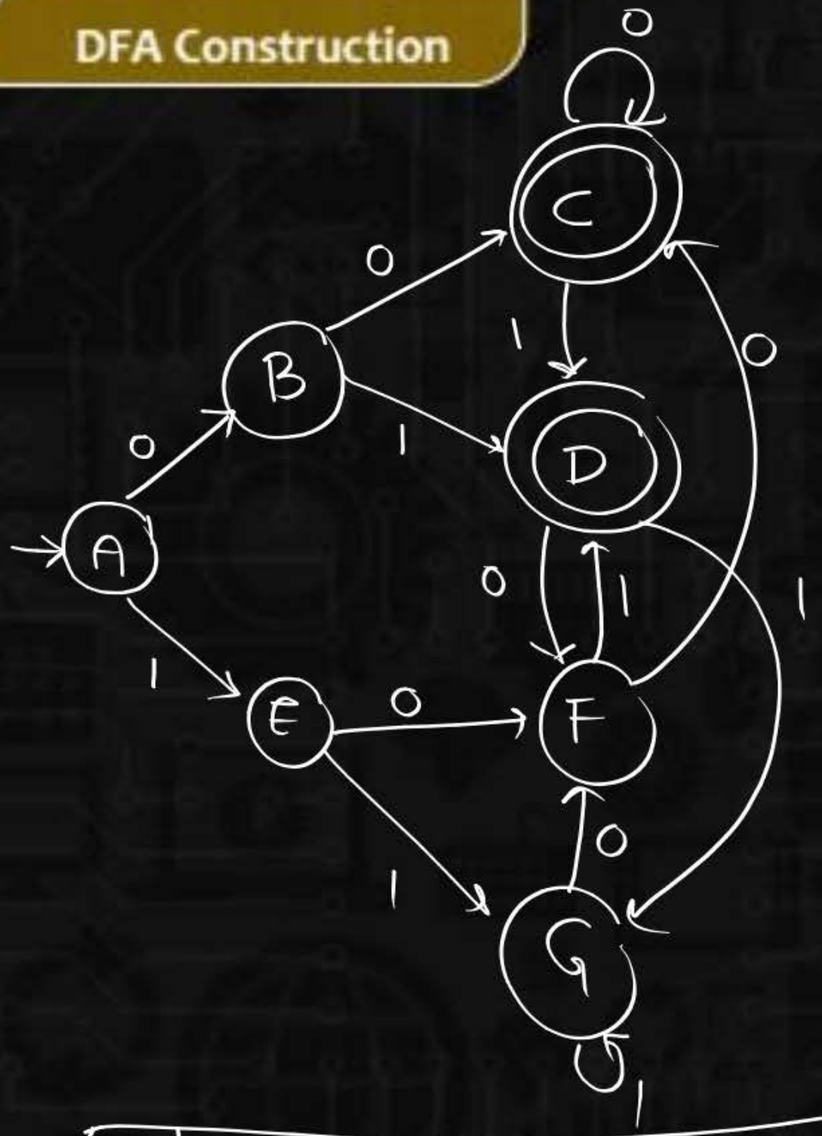
Total No. of partitions on set Q with 3 elements = 5

DFA Construction



$q_0 \cong q_3$

DFA Construction

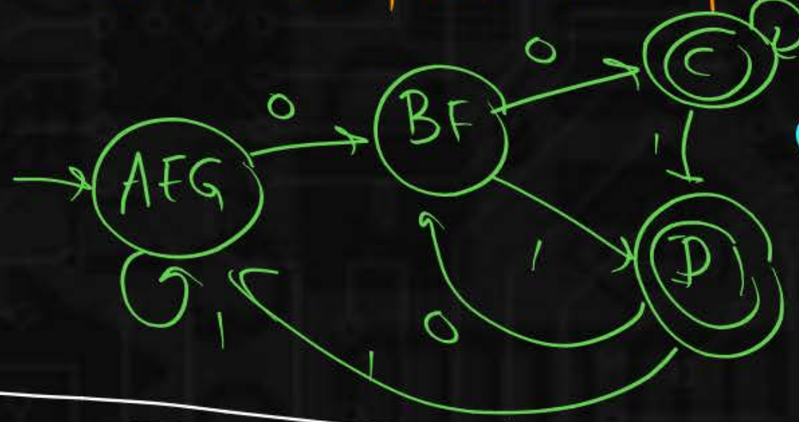


2nd Symbol from end is '0'

Step 1: {A, B, E, F, G} {C, D}
Non finals finals

Step 2: {A, E, G} {B, F} {C} {D}

Step 3: {A, E, G} {B, F} {C} {D}
Same as previous step



B, F → 0: C, C; 1: D, D

C, D → 0: C, F (in diff sets) ✗

Final State ≠ Non-final State

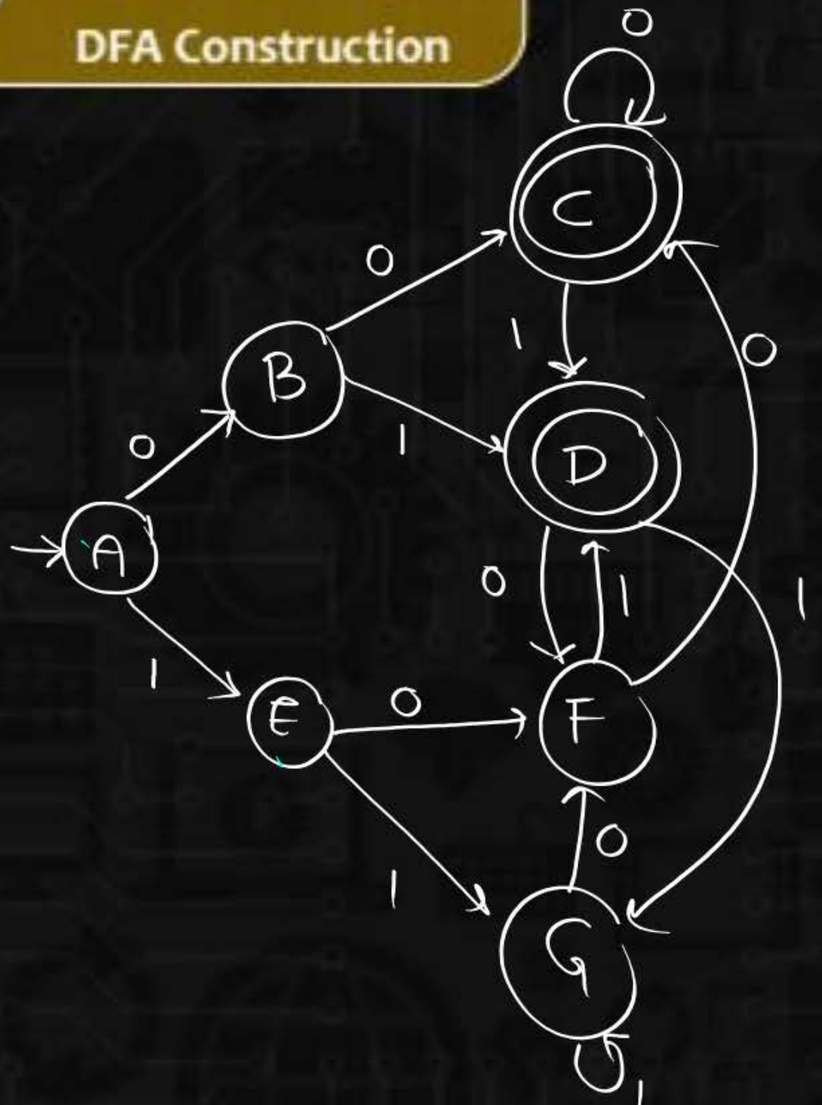
✗ A, B → 0: B, C (in different sets) ⇒ B ≠ C ⇒ A ≠ B

A, E → 0: B, F (in same set); 1: E, G (in same set)

A, F → 0: B, C (in diff sets) ⇒ B ≠ C ⇒ A ≠ F ✗

A, G → 0: B, F (in same set); 1: E, G (in same set)

DFA Construction



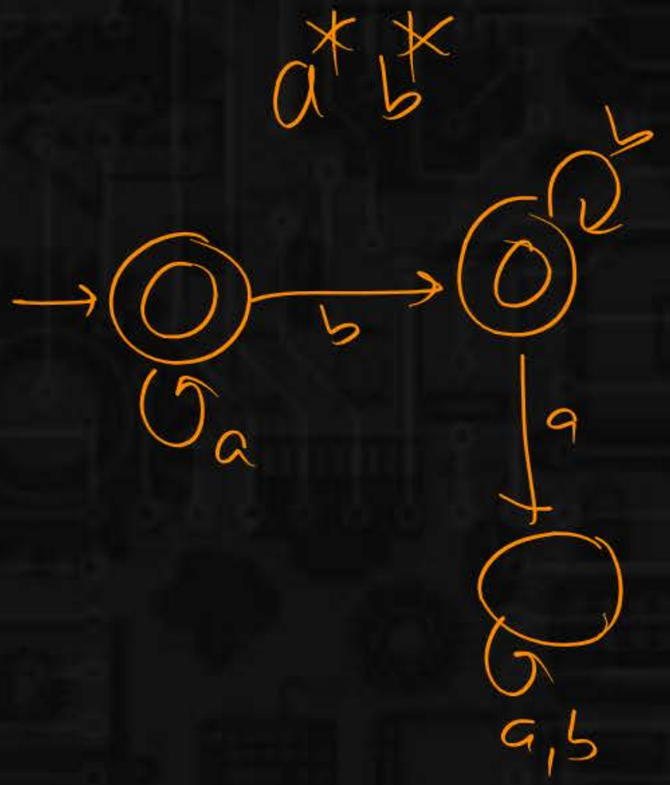
2nd Symbol from end is '0'

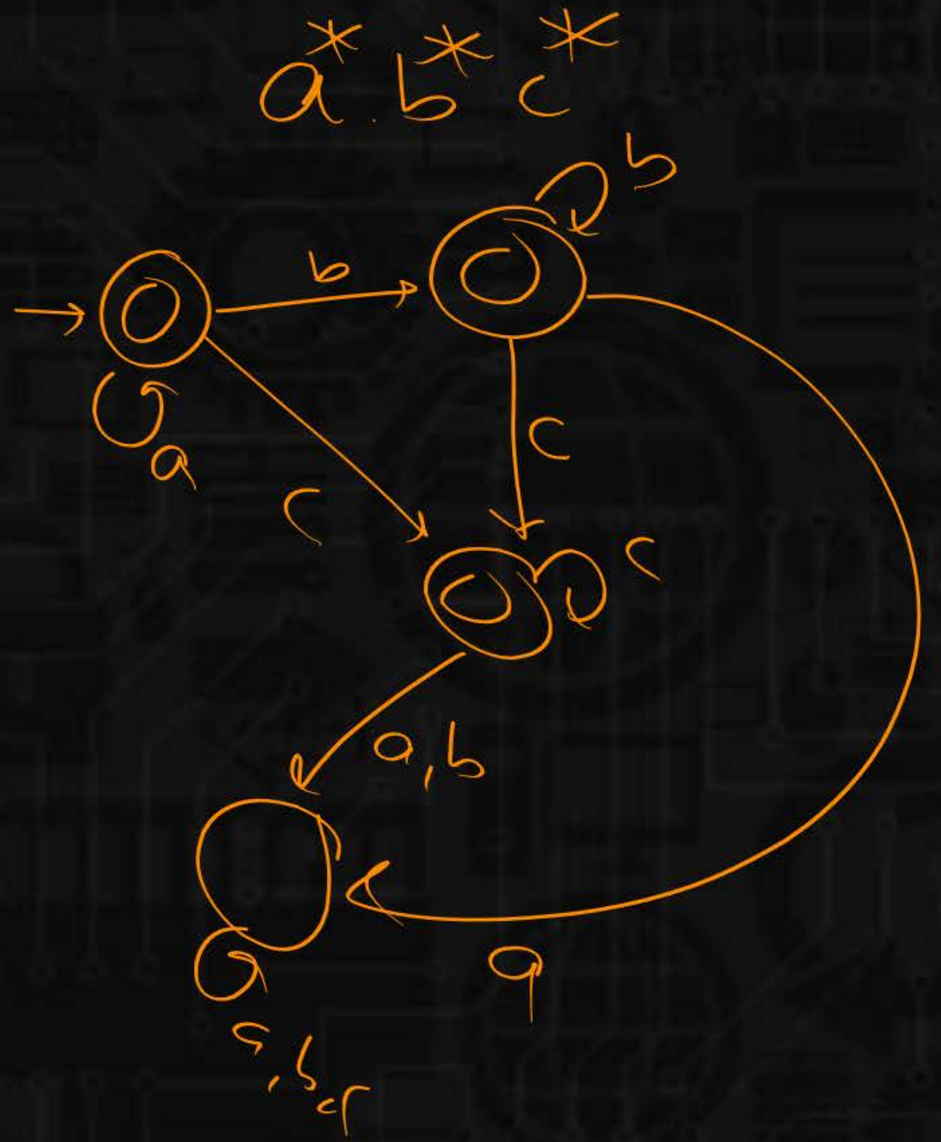
ABEFG | C D

AEG | BF | C | D

AEG | BF | C | D

DFA Construction





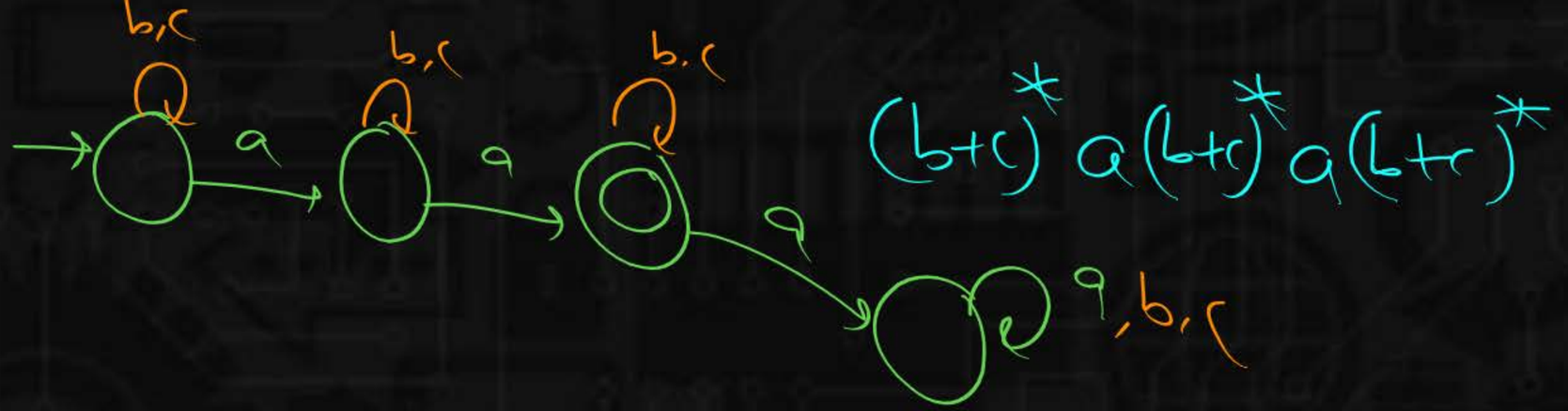
DFA Construction

#a's = 2 over $\Sigma = \{a, b\}$

$b^* a b^* a b^*$

#a's = 2 over $\Sigma = \{a, b, c\}$

$(b+c)^* a (b+c)^* a (b+c)^*$



Summary

- models of DFA
- minimization of DFA
- Next Session: NFA


THANK YOU GW
SOLDIERS !