

CS & IT ENGINEERING

Theory of Computation

Turing Machine : Recursively Enumerable

Turing Machine (Part 2)



Lecture No. 2



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TOPICS TO BE COVERED

01 Recursive Language Vs REL

02 LBA Vs HTM Vs TM

03 TM Construction

04 Closure Properties of RECs and RELs

Undecidable

not decidable

not recursive

no Hm

→ Tm may exist
or

Tm may not exist

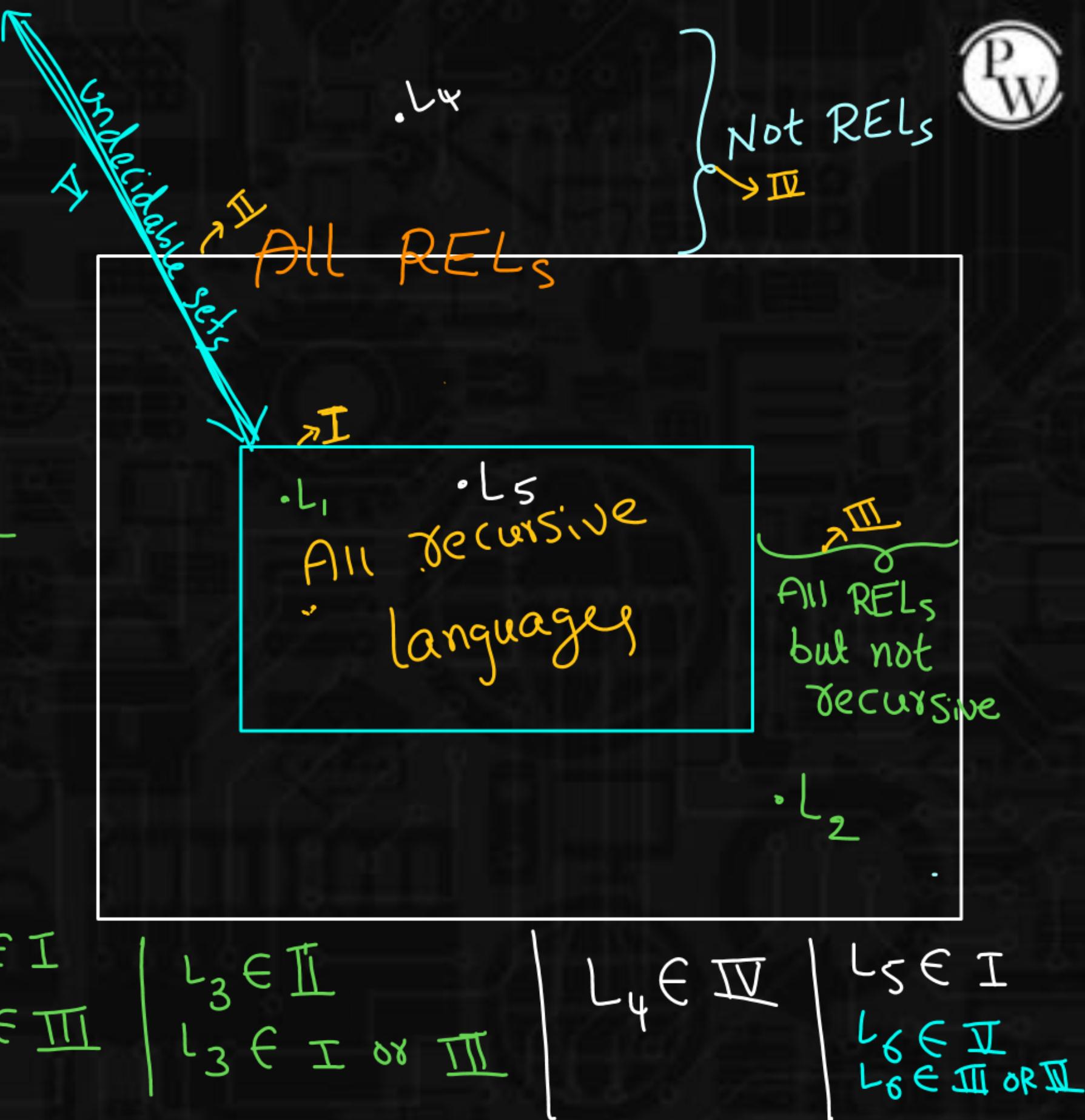
REC_s Vs REL_s

(Recursive Sets)
(Decidable sets)

(Recursively Enumerable Sets)

HTM exist TM exist

- ✓ L₆ is Undecidable set
- ✓ L₅ is Decidable Set
- ✓ L₁ is Recursive Set
- ✓ L₂ is REL but not recursive
- ✓ L₃ is REL
- ✓ L₄ is not REL



LBA V_s HTM V_s JM

- It represents REL_s
- It represents REC_s
- It represents CSL_s

LBA V_s HTM V_s TM

P
W

It halts for valid strings

It is a TM that always halts

$w \rightarrow \boxed{\begin{array}{l} (\text{Total TM}) \\ \text{HTM} \\ (\text{TM that always halts}) \\ \hline \text{accepts } L \end{array}}$

If $w \in L$, Halts at final
If $w \notin L$, Halts at nonfinal

It is HTM but tape is linearly bounded

Recursive Set Vs RE Set

(REC)

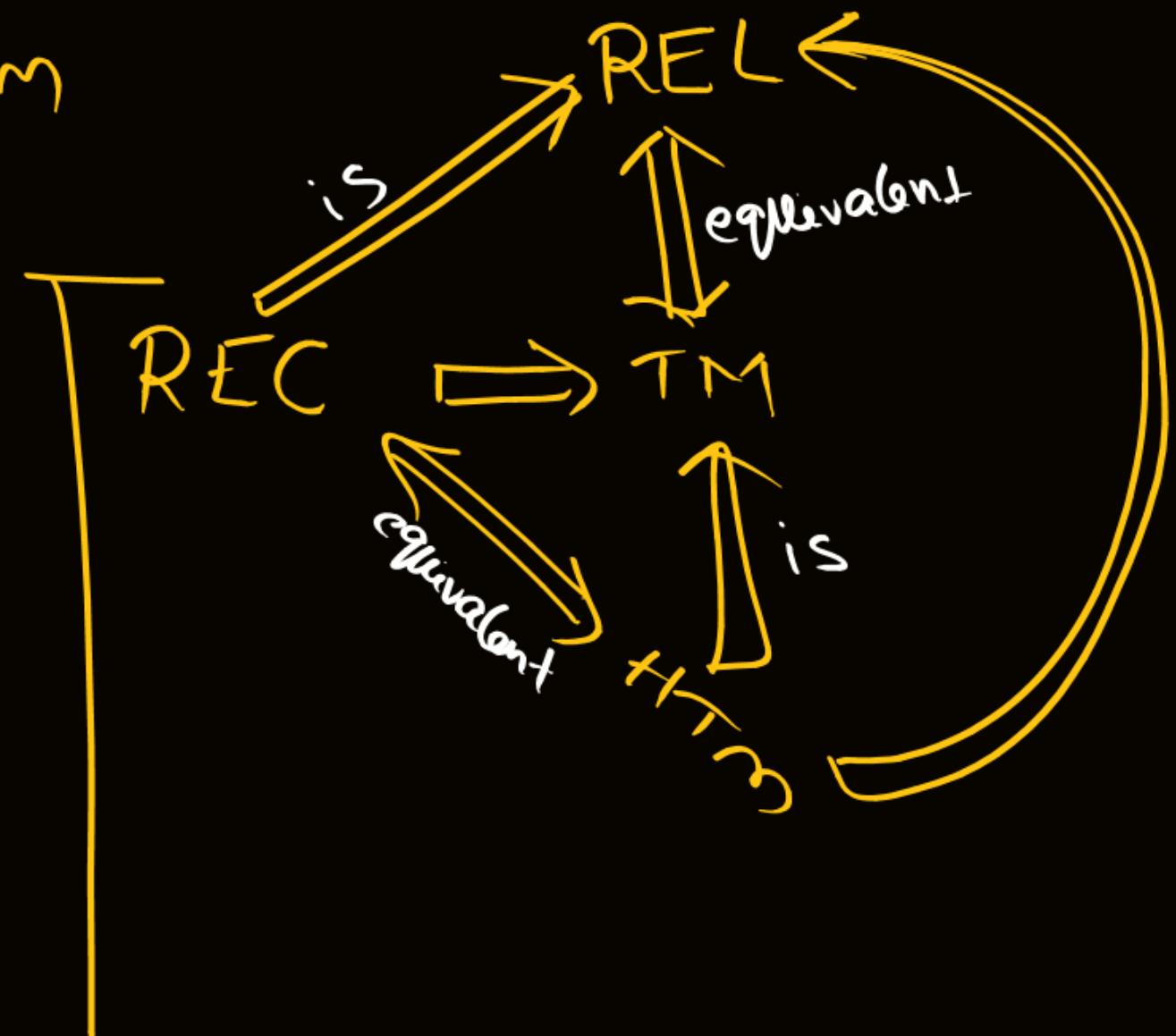
(REL)

- TM acceptable set (TM recognizable set)
- It has TM
 - (but HTM may or may not exist)
- It has HTM
(It also has TM)
- TM decidable set
- TM enumerates in lexicographical order

Note :
I) Every REC is REL
II) REL need not be REC

$\text{REC} \equiv \text{HTM} \equiv \text{Halting program} \equiv \text{Algorithm}$

$\text{REL} \equiv \text{TM} \equiv \text{program}$



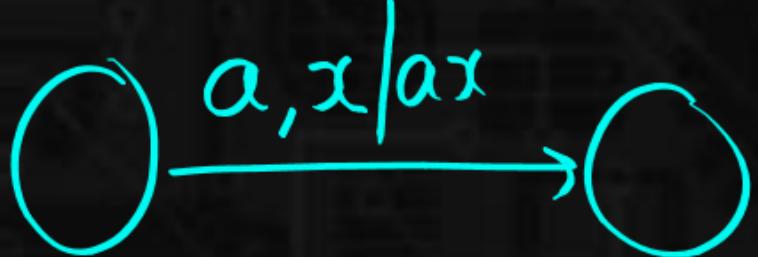
Construction of Turing M/c :

FA

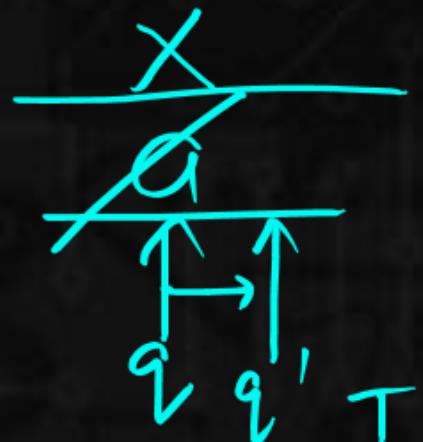


DFA: $Q \times \Sigma \rightarrow Q$

PDA



DPDA: $Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$



$q \xrightarrow{a} q'$ TM

From q , by reading a , move to q' by writing x , moved right



DTM: $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$

$$\delta(q, a) = (q', X, R)$$

Present state
 next state
 with
 read

How to remember 'a' ?

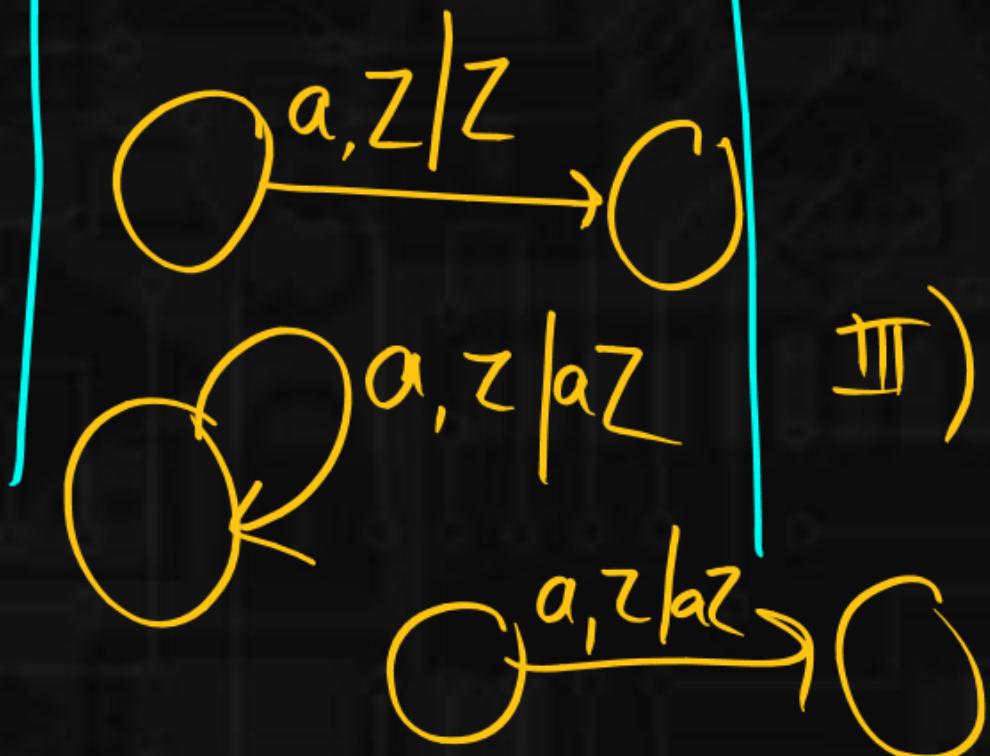
FA



change the state

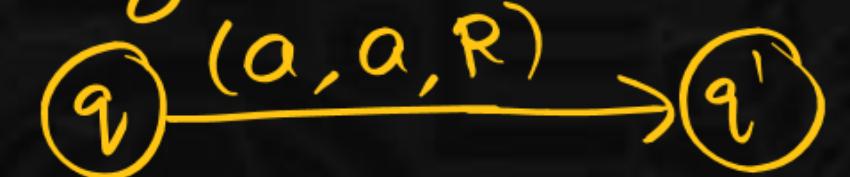
PDA

- I) change state
- II) push onto stack
- III) I & II

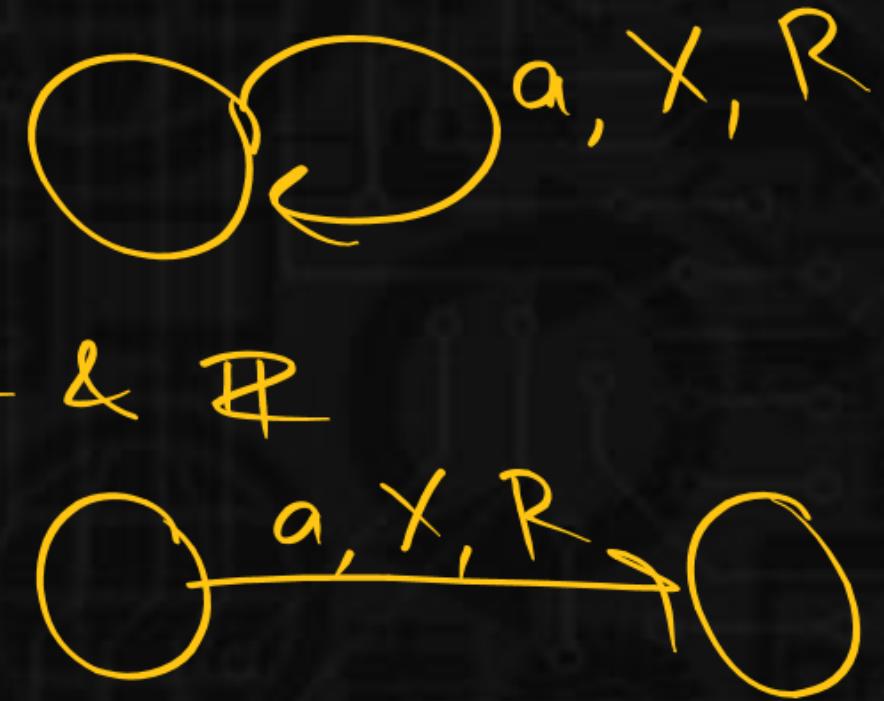


TM

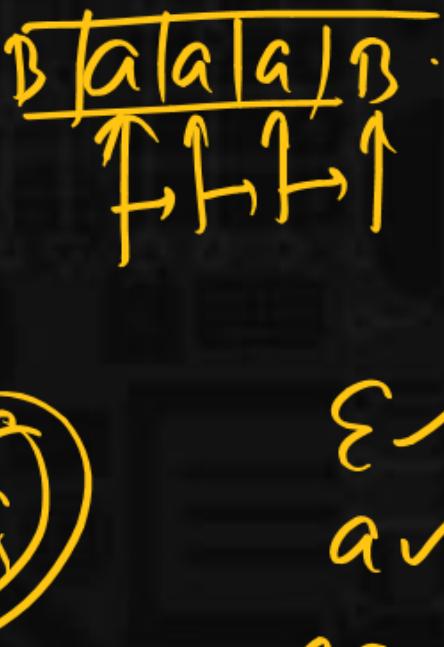
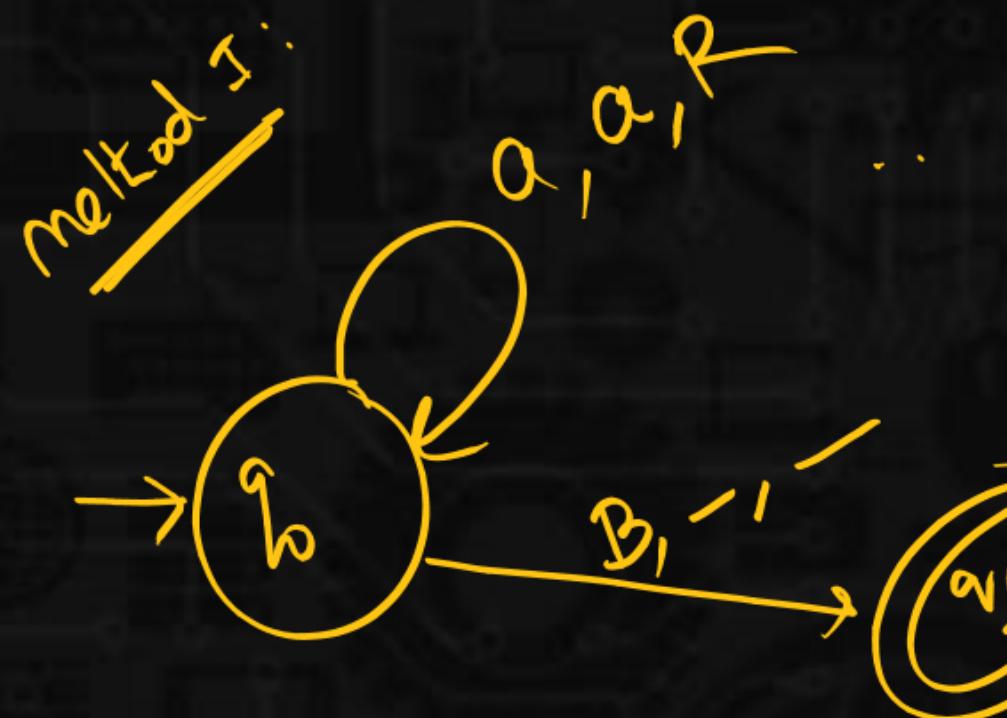
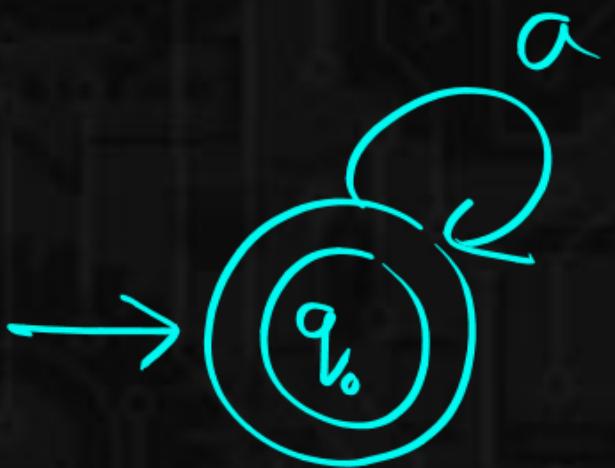
- I) change state



- II) Write with new symbol

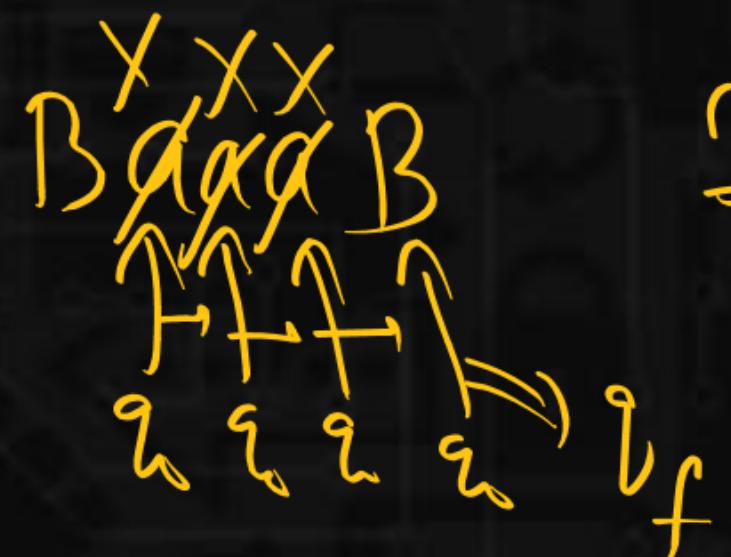


① $L = a^*$

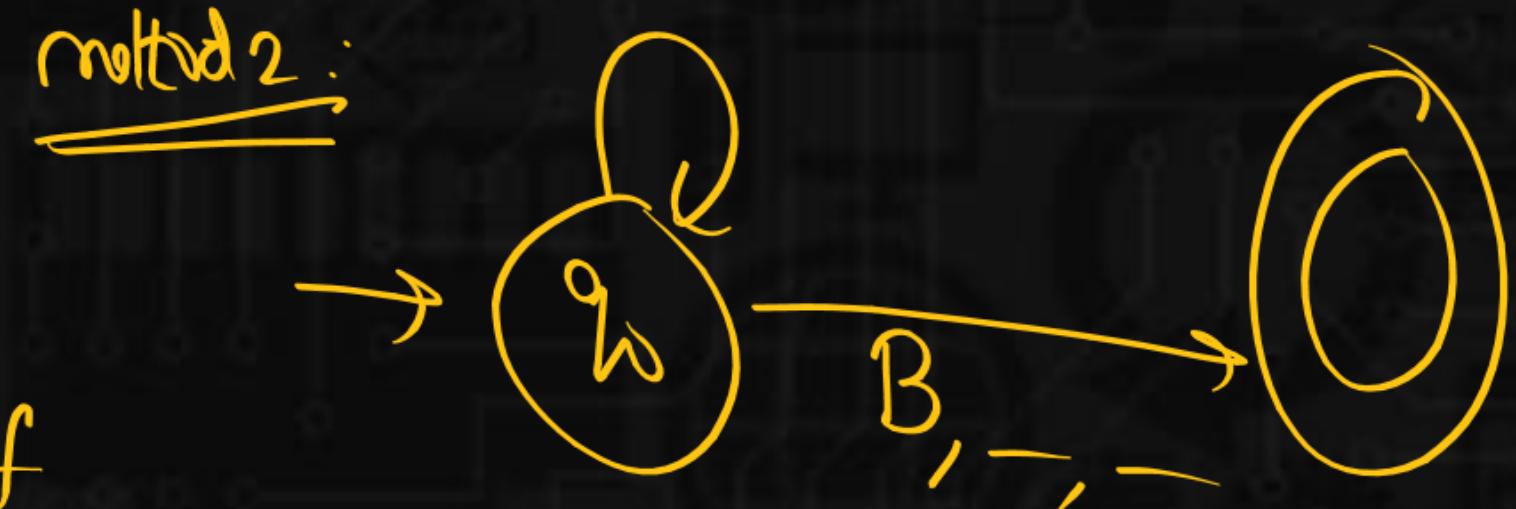


ε ✓
a ✓
aa ✓
aaa ✓
⋮

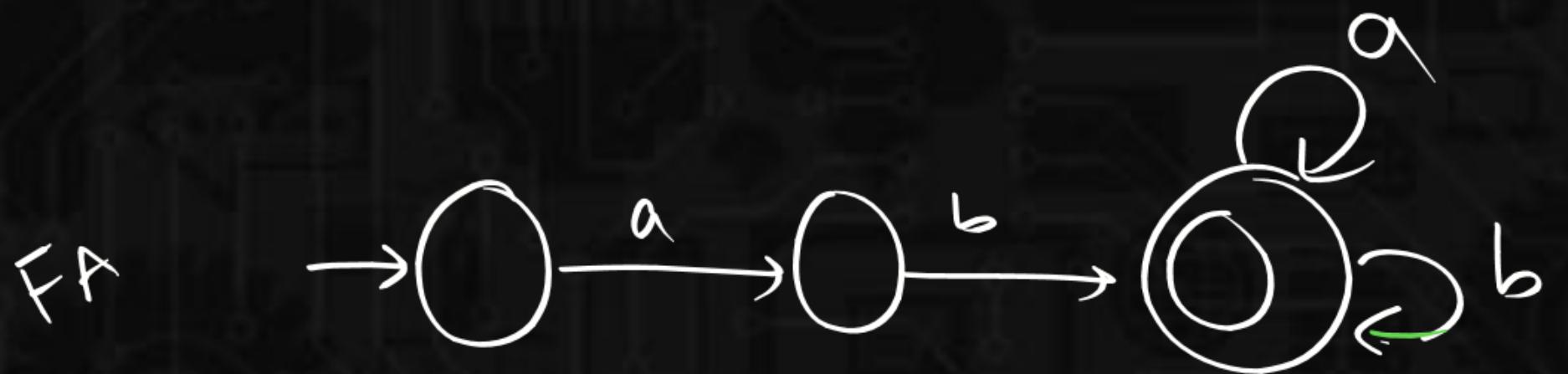
FA $\xrightarrow{\quad}$ TM



a, X, R



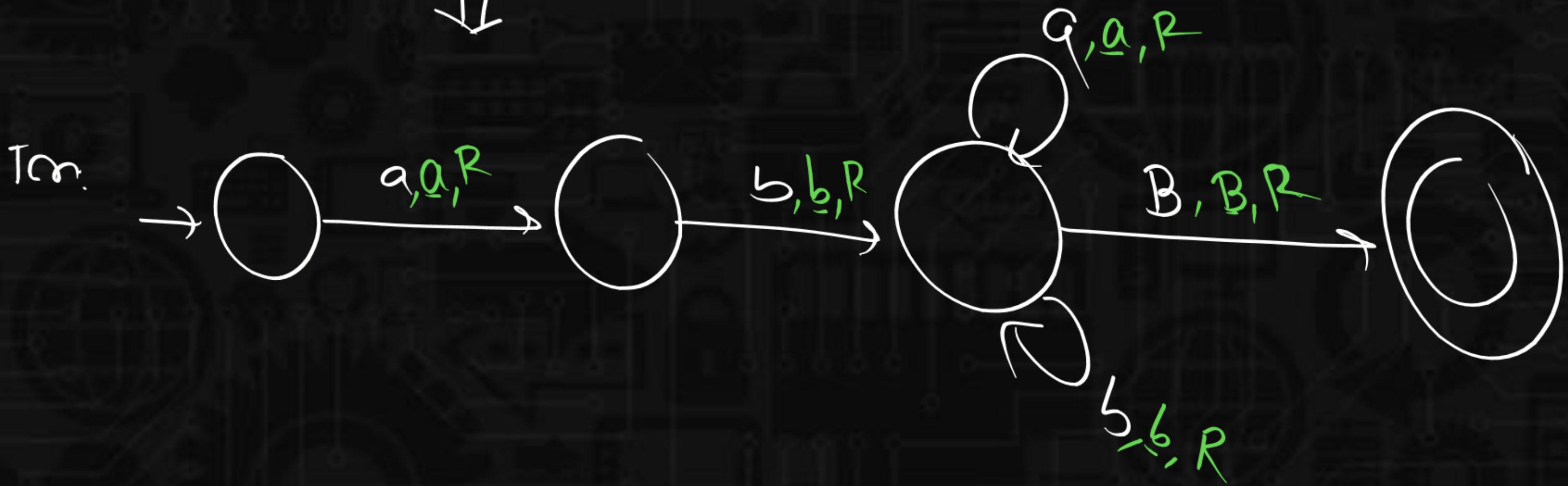
② $L = ab(a+b)^*$



$$\Sigma = \{a, b\}$$



↓



Note I) Every FA is convertible to TM

** II) If TM always moves right direction in configuration (every transition)

then $TM \cong FA$

$L(TM)$ is Regular

*** III) If TM tape is read only then $TM \cong FA$

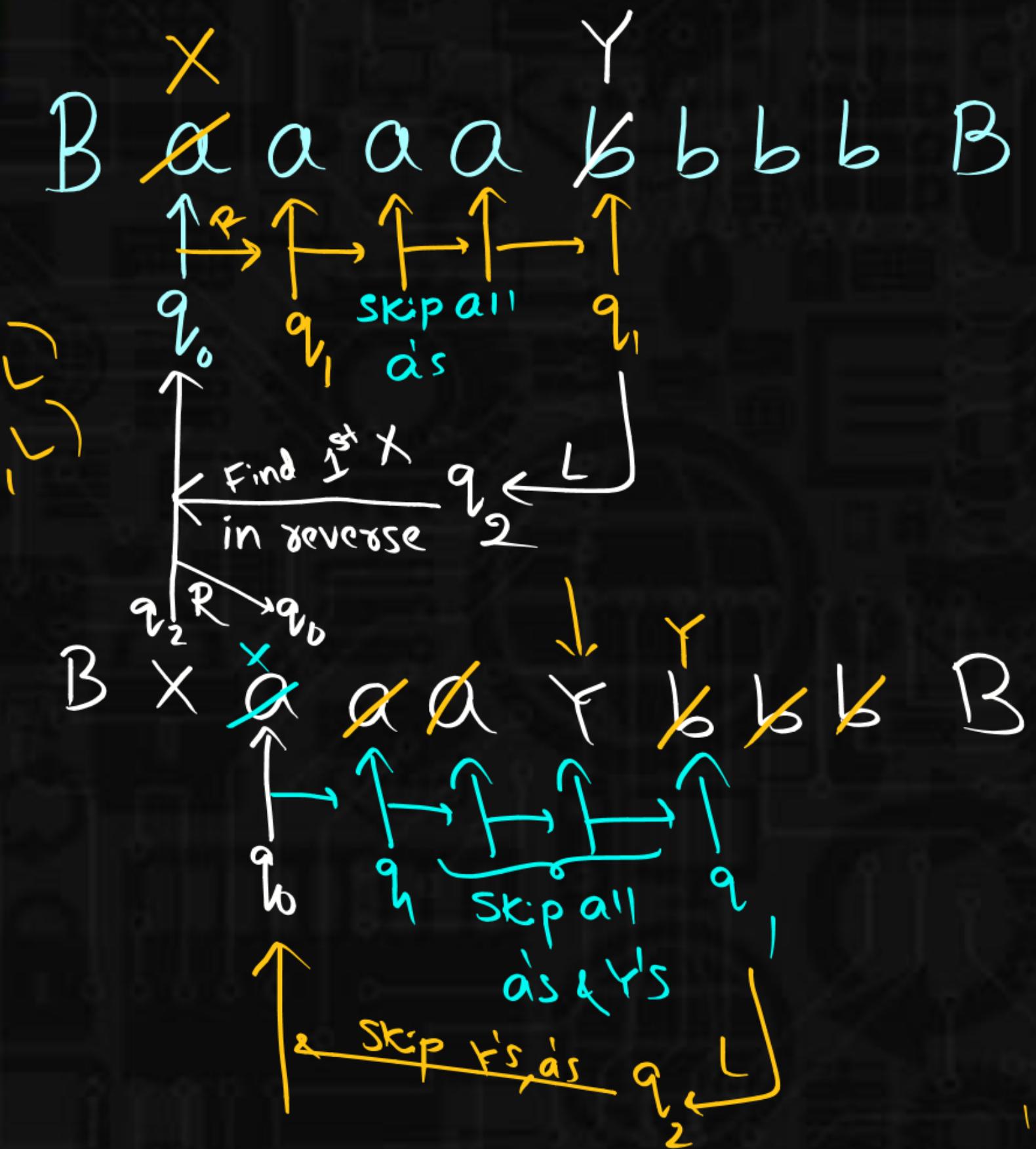
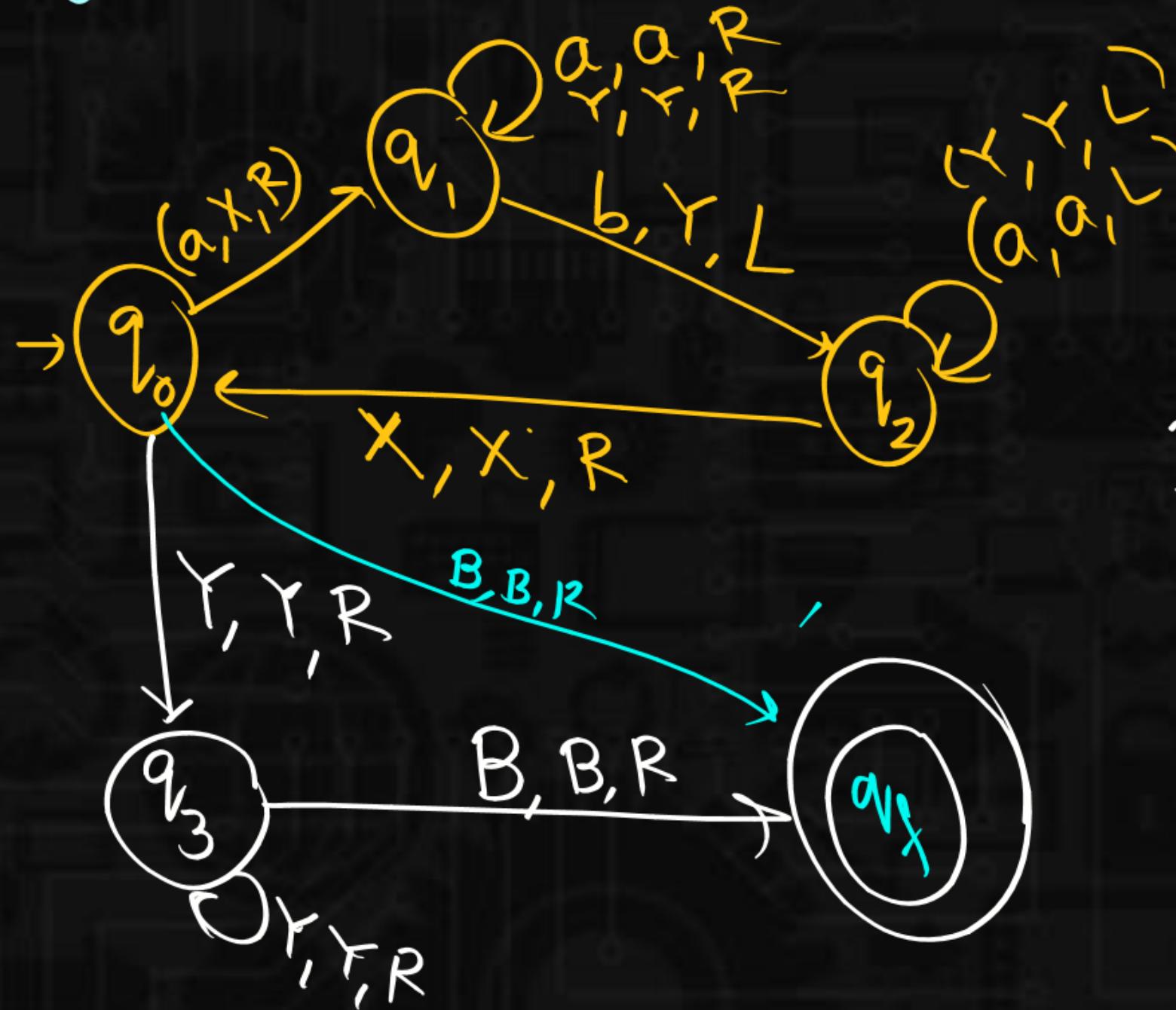
**** IV) If TM tape is read only and head moves unidirectional then $TM \cong FA$

$$③ L = \{a^n b^n \mid n \geq 0\}$$

P
W

q_1 : Find b , replace with Y

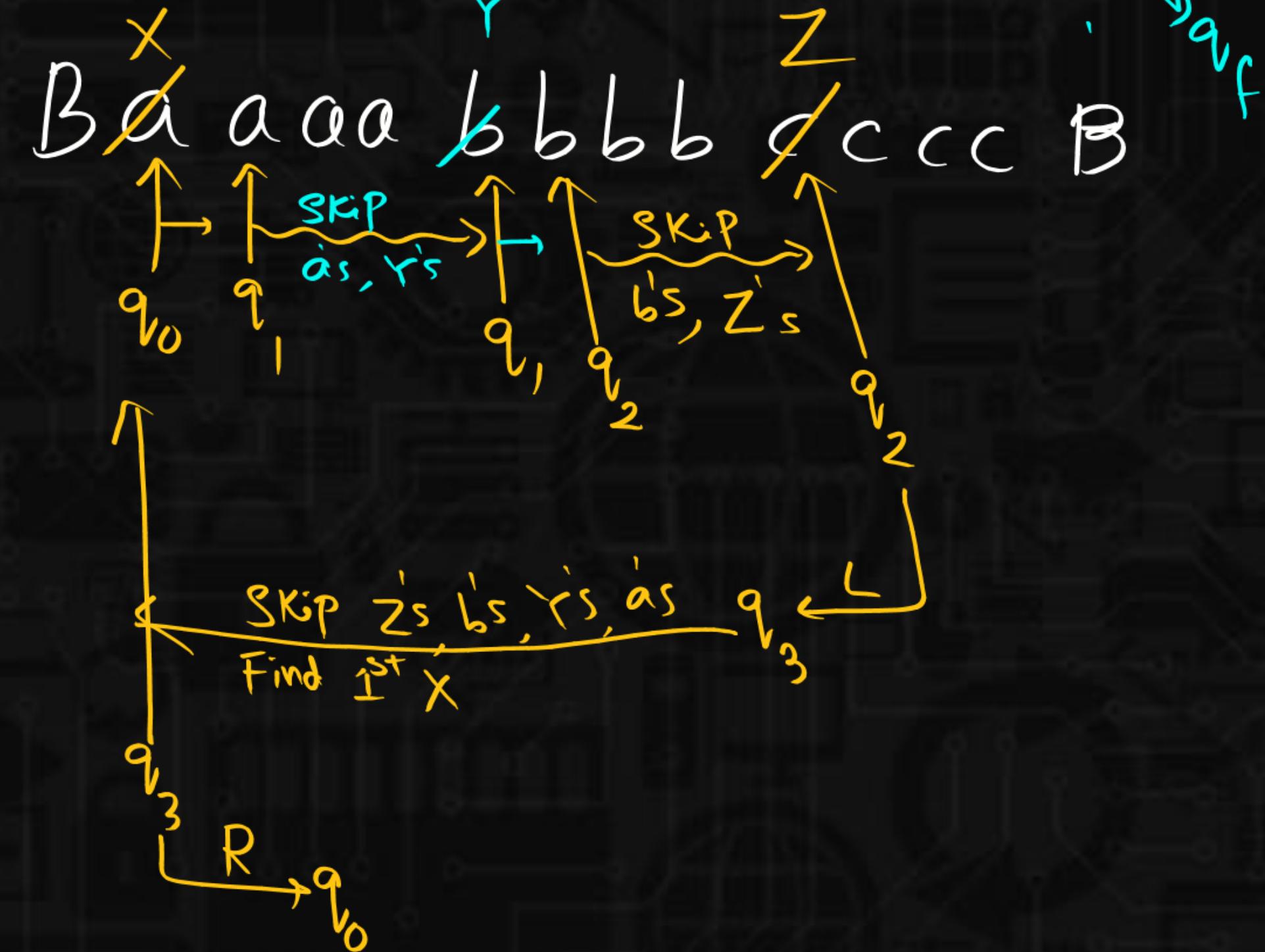
q_0 : Replace a with X





④ $\{a^n b^n c^n \mid n \geq 0\}$

H.V.



⑤ $\{ww^R \mid w \in \{a,b\}^*\}$



⑥ $\{w\#w^R \mid w \in \{a,b\}^*\}$

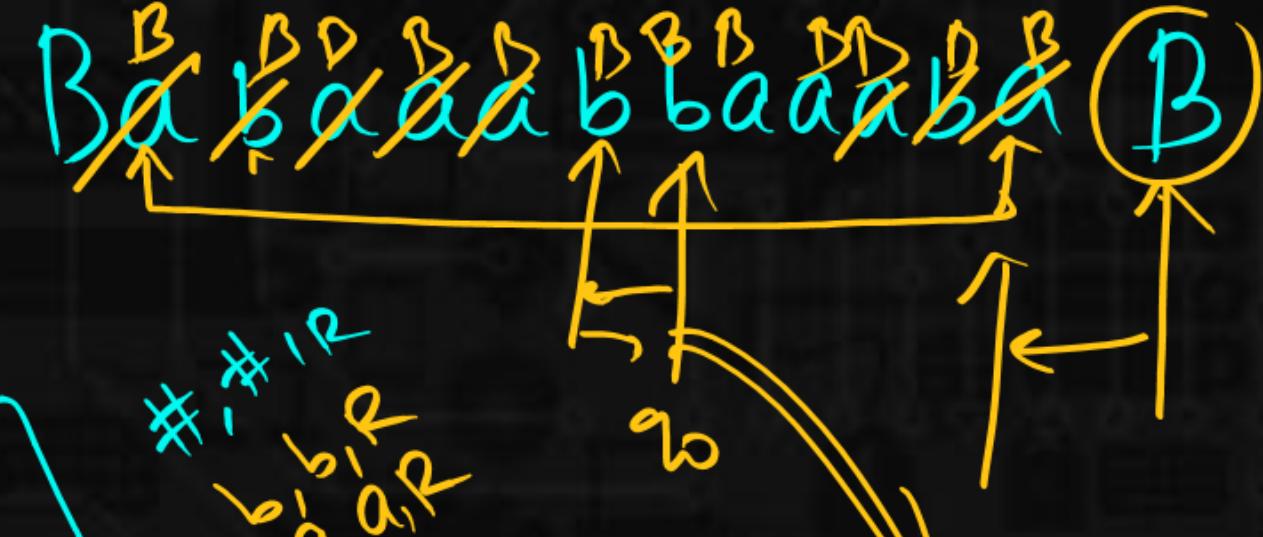
⑦ $\{w\#w \mid w \in \{a,b\}^*\}$

⑧ $\{a^n b^n c^n \mid n \geq 1\}$

⑨ $\{a^n b^n c^n \mid n \geq 1\}$



⑤ $\{ww^R \mid w \in \{a,b\}^*\}$



⑥ $\{w\#w^R \mid w \in \{a,b\}^*\}$

6

$\#\#^R$
 $\#b^R$
 $b^R a^R$

⑦ $\{w\#w \mid w \in \{a,b\}^*\}$

$a^R b^R \# a^R b^R$

⑧ $\{a^n b^n c^n \mid n \geq 1\}$

⑨ $\{a^n b^n c^n \mid n \geq 1\}$



P
W

Summary



TM Construction

Acceptance ✓

Halting → next

Functionality ↗

