CS & IT ENGINEERING

Theory of Computation

Finite Automata:

DFA-1

Lecture No. 5

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TOPICS TO BE COVERED

01 Regular Expressions

02 Finite Automata

03 Deterministic FA

Important Regular Exps

1) $(a+b)^*$

2) $a(a+b)^* = \boxed{a(a+b+\varepsilon)^+ = a(a^*b^*)^* = a(b^*a^*)^*} = (ab^*)^+$

3) $b(a+b)^* = (ba^*)^+$

4) $(a+b)^*a = (b^*a)^+$

5) $(a+b)^*b = (a^*b)^+$

6) Even no. of a's

7). odd no. of a's $= (b^*ab^*ab^*)^*b^*ab^*$

8) #a's divisible by 3 =

9) $[(a+b)^3]^*$

#a's is divisible by 3

$$(b^* a b^* a b^* a b^*)^* + b^*$$

Even no. of a's over $\Sigma = \{a, b\}$

$$= b^*(\boxed{b^*} a b^* a b^*)^* . b^* = (b^* a b^* a b^*)^* + b^*$$

$$= (b^* a b^* a \boxed{b^*})^* b^* = (b^* a b^* a)^* b^*$$

$$= b^*(\boxed{b^*} a b^* a b^*)^*$$

$$= b^*(a b^* a b^*)^*$$

$L = \{\varepsilon, b, aa, bb, aab, aba, baa, bbb, \ldots\}$

zero a's
2 a's
4 a's

even no. of a's

no. of b's is any

Even no. of a's but exclude zero no. of a's

a's a's a's

$\boxed{\text{even} = 2, 4, 6, \ldots}$

Ex

b ✗ zero a's

bb ✗ invalid

⋮

$$(b^* a b^* a b^*)^+$$

$$\underbrace{\underset{\text{must}}{\boxed{a}}\underbrace{(a^*b^*)^*}} = a\underbrace{(b^*a^*)^*}$$

$$= a(b^1a^0)^1$$

$$= a\ b$$

$$ab = \underbrace{a}\underbrace{(a^0b^1)^1}$$

$$= a\ b$$

$$aX = \{\underset{\text{Min}}{a},\ a\underset{a/b}{\square},\ a\underset{a/b}{\square}\underset{a/b}{\square},\ a\underset{a/b}{\square}\underset{a/b}{\square}\underset{a/b}{\square},\ \ldots\}$$

$$= \{a, aa, ab, aaa, aab, aba, bbb, \ldots\}$$

$+$, $\cup$, $|$ } or

$a(a \cup b)^*$

$a(a+b)^* = (ab^*)^+$

a . . . . ✓

a . . . . ✓

$a(\ )^0 = a$ $\qquad$ $(ab^0)^1 = a$

$a(a\ )^1 = aa$ $\qquad$ $(ab^0)^1(ab^0)^1 = aa$

$a(b)^1 = ab$ $\qquad$ $(ab^1)^1 = ab$

$\vdots$ $\qquad$ $\vdots$

Important Regular Exps

1. $(a+b)^*$

$= 2.\ (a+b)^+ + \varepsilon$

$= 3.\ (a+b+\varepsilon)^+$

$= 4.\ (a^* b^*)^*$

$= 5.\ (b^* a^*)^*$

$= 6.\ (a^* + b^*)^*$

$= 7.\ (a^* + b)^*$

$= 8.\ (a + b^*)^*$

$= 9.\ (a+b)^+ + a^*$

$= 10.\ (a+b)^+ + (ab)^*$

$= 11.\ (a^* b^*)^+$

$= 12.\ (b^* a^*)^+$

$= 13.\ (a^* + b^*)^+$

$= 14.\ (a^+ + b^+ + \varepsilon)^+$

$a \quad b$

$= (a^* b^* a^*)^*$

$= (a^* b^* a^* b^*)^+$

$= (a^* b^* + a b^+)^+$

$= (a^* b^*)^+ + (a^* b^+)^+$

$$(a+b+\varepsilon)^{+} = (a^{+}+b^{+}+\varepsilon)^{+}$$

$$= (\boxed{a}+\underbrace{aa+aaa+}_{\text{no need}}\cdots+\boxed{b}+\underbrace{bb+bbb+\cdots}_{\text{no need}}+\boxed{\varepsilon})^{+}$$

$$(a^* b^*)^*$$

$$(\underbrace{a+b}_{\to ab})^* \to (a)^1 (b)^1$$

1st 2nd

$$\underbrace{a+b}_{\to ab \times}$$

$$\to \varepsilon = (\quad)^0 = (\underbrace{a^0 b^0}_{\varepsilon^1})^1 = (\underbrace{a^0 b^0}_{\varepsilon^2})^2 = (\underbrace{a^0 b^0}_{\varepsilon^3})^3 = \cdots$$

$$\to a = (a)^1 = (a)^1 (\varepsilon)^1 = (a)^1 (\varepsilon)^1 (\varepsilon)^1$$

```
for - - - .
{
  for - - - . { - - - }
  for - - . { - - }
}
```

$$\underbrace{\text{Any Exp}}_{\text{All strings generated?}} = (a+b)^*$$

$$(\underbrace{ab^*}_{a} + \underbrace{ba^*}_{b})^*$$

$$a^* + b^* = \varepsilon + a^+ + b^+$$

$$\left.\begin{matrix} a^* \\ b^* \end{matrix}\right\} \begin{matrix} \varepsilon, a, aa, aaa, \ldots, b, bb, bbb, \ldots \\ \cancel{ab}, \cancel{ba}, \ldots \end{matrix}$$

3 types: ε

a^+

b^+

$$(a+b)^* = (a^*b^*)^*$$

$(a+b)^*$	$(a^*b^*)^*$
ε ✓	ε ✓
a ✓	a ✓
b ✓	b ✓
aa ✓	aa ✓
ab ✓	ab ✓
⋮	⋮

Every string generated by both

$$\underbrace{(a+b)^*}_{\text{All}} + (ab)^* = (a+b)^*$$

$$\text{All} \cup \text{Any} = \text{All}$$

$$(a^+ + b^+)^* = \underbrace{(a+b)^*}_{\text{All}}$$

$$(a + \underbrace{aa}_{} + \cdots + b + \underbrace{bb}_{} + \cdots)^*$$

$$aa \rightarrow (a^2)^1$$

$$(a^1)^2$$

ba
11

$a^+b^+ \neq b^+a^+$

$(a^+b^+)^* \neq (b^+a^+)^*$

$(a^+b^+)^*$	$(b^+a^+)^*$
a ✗	a ✗
b ✗	b ✗
ε ✓	ε ✓
ab ✓	ba ✓
~~ba~~	~~ab~~

$\overset{ba\ \times}{\boxed{a^*b^*}} \neq \overset{ab\ \times}{\boxed{b^*a^*}}$

$(a^*b^*)^* = (b^*a^*)^* = (a+b)^*$

$a^*b^* \neq a^*+b^* \neq a+b$

$(a^*b^*)^* = (a^*+b^*)^* = (a+b)^*$

every w satisfies condition

atmost 3 a's

$L = \{w \in \{a, b\}^* : \underbrace{\#_a(w) \leq 3}\}. = \{\varepsilon, a, b, aa, ab, ba, bb, \ldots, \cancel{aaaa}, \ldots\}$

min

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \rightarrow$ min $= \varepsilon$

B. $b^* \boxed{(a)} b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \rightarrow$ min $= a$

C. $b^* \boxed{(a)} b^* \boxed{(a)} b^* (a \cup \varepsilon) b^* \rightarrow$ min $= aa$

D. $b^* (a \cup \varepsilon) b^* \boxed{(a)} b^* \boxed{(a)} b^* \rightarrow$ min $= aa$

ε not generated

$$\underbrace{\#a(\omega)}_{\text{No. of } a\text{'s in } \omega}$$

$$n_a(\omega)$$

$$L = \{ x \mid \underbrace{x \in \Sigma^*}_{x \text{ is any string}} \} = \Sigma^*$$

$$= \{ x : x \in \Sigma^* \}$$

$$= \{ x \in \Sigma^* \}$$

$$\boxed{\begin{array}{c}\{a, b\}^* \\[2em] (a+b)^*\end{array}}$$

$$(a \cup b)^*$$

$$(\{a\} \cup \{b\})^*$$

atleast 3a's

$L = \{w \in \{a, b\}^* : \underbrace{\#_a(w) >= 3}\}$.

A. $b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \longrightarrow \varepsilon$ min

B. $(a \cup b)^* (a) b^* (a \cup \varepsilon) b^* (a \cup \varepsilon) b^* \longrightarrow$ min = a

C. $b^* (a) b^* (a) b^* (a \cup \varepsilon) b^* \longrightarrow$ min = aa

D. $(a \cup b)^* \boxed{a} (a \cup b)^* \boxed{a} (a \cup b)^* \boxed{a} (a \cup b)^* \longrightarrow$ min = aaa

X a X a X a X

$L1 = a^*b^*$

$L2 = a^+b^+$

Find L2-L1.

A. a^*

B. b^*

C. $a^* + b^*$

D. None

$\{a^m b^n \mid m,n \geq 1\}$

$L_2 = \underbrace{a^+ b^+}$ $\Rightarrow \{ab, aab, abb, aaab, aabb, abbb, \ldots\}$

$L_1 = a^* b^*$ → ε, a^+, b^+, a^+b^+

$\{a^m b^n \mid m \geq 0, n \geq 0\}$

$L_2 - L_1$

$\cancel{a^+ b^+} - \{\varepsilon, a^+, b^+, a^+b^+\}$

ϕ

$L_1 - L_2 = \varepsilon + a^+ + b^+$

$= a^* + b^*$

$= a^* + b^+$

$= a^+ + b^*$

L1 = a*+b* and L2 = a*b*.
Which of the following is TRUE?

A. ~~L1 = L2~~
B. ~~L1 U L2 =(a+b)*~~
C. L1* = L2*
D. None

$$a^* + b^* \neq a^* b^*$$

$$L_1 \cup L_2 = \underbrace{a^* + b^*}_{L_1} + \underbrace{a^* b^*}_{L_2}$$

$\cancel{ba}$

L1 = a*+b* and L2 = a*b*.
Which of the following is TRUE?

A. L1 is subset of L2
B. ~~L2 is subset of L1~~
C. ~~L1 U L2 = L1~~
D. None

$L_1 = a^* + b^*$

$L_2 = a^* b^*$

$$\boxed{L_1 \subset L_2}$$

$L_1 \subset L_2$

$L_1 \Rightarrow a^*, b^*$

$L_2 \Rightarrow a^*, b^*, a^+b^+$

If $X \subset Y$ then

$$\boxed{X \cup Y = Y}$$

$$\boxed{X \cap Y = X}$$

$L1 = a^+b^+$ and $L2 = a^*b^*$.

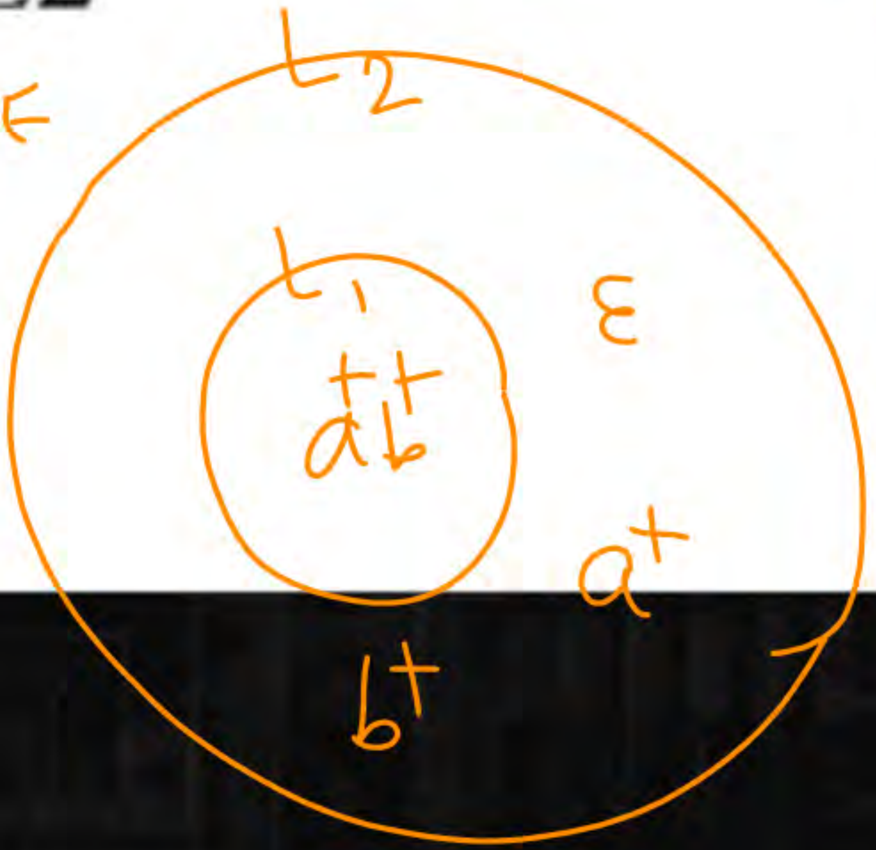
Which of the following is $\boxed{\text{FALSE}}$?

TRUE A. L1 is subset of L2

B. $L1^* = L2^*$ FALSE

TRUE C. L1 U L2 = L2

D. None



$$\underline{(a^+b^+)^*} \neq \underbrace{(a^*b^*)^*}_{\text{every string possible}}$$

a ×

b ×

$L1 = a^+ + b^+$ and $L2 = a^* + b^*$.

Which of the following is $\boxed{\text{TRUE?}}$

A. $\cancel{L1 = L2}$

B. $\cancel{L1^+ = L2^+}$

C. $L1 \cup L2 = L2$

D. None

$L_1 \subset L_2$

$\boxed{L_1 \cup \{\varepsilon\} = L_2}$

$\underbrace{a^+ + b^+}_{\varepsilon \text{ is missing}} \neq a^* + b^*$

$\underbrace{(a^+ + b^+)^+}_{\varepsilon \text{ is not generated}} \neq \underbrace{(a^* + b^*)^+}_{\text{every string generated}}$

$L1 = a^+$ and $L2 = a^*$

Which of the following is TRUE?

A. $L1+ = L2^*$

B. $L1^+ = L2^+$

C. $L1^* = L2^+ = L_2 = L_2^*$

D. None

$L_1 = a^+ \Rightarrow L_1^+ = (a^+)^+ = a^+$

$\Rightarrow L_1^* = (a^+)^* = a^*$

$L_2 = a^* \Rightarrow L_2^* = (a^*)^* = a^*$

$\Rightarrow L_2^+ = (a^*)^+ = a^*$

$A = a^*$ and $B = b^*$

$AB = ?$

same

A. $\{ a^n b^n \mid n >= 0 \}$

B. $\{ a^m b^n \mid m, n >= 0 \}$

C. $(a+b)^*$ $\supset AB$

D. None

$AB = a^* \cdot b^* = a^* b^*$

$L = \{ a^m b^n \mid m, n \geq 0 \}$

$= a^* b^*$

$A = aa^* \text{ and } B = bb^*$

$(AUB)^* = ?$

A. $\{ a^n b^n \mid n >= 0 \}$

B. $\{ a^m b^n \mid m, n >= 0 \}$

C. $(a+b)^*$

D. None

$A = aa^* = a^+$

$B = bb^* = b^+$

$(A \cup B)^* = (a^+ + b^+)^*$

$= (a + b)^*$

$(AB)^* = (a^+ b^+)^* \neq (a+b)^*$

Given the language L = {ab, aa, baa},
which of the following strings are not in $\boxed{L^*}$?

1) abaabaaabaa $\in L^*$
2) aaaabaaaa $\in L^*$
3) baaaaabaaaab $\notin L^*$

A. 1 only
B. 2 only
C. 3 only
D. None

$L = ab + aa + baa$

$L^* = (ab + aa + baa)^*$

b

The length of the shortest string NOT in the language (over $\Sigma = \{a, b\}$) of the following regular expression is ____________________.

$a^*(ba)^*a^*$

A. 1

B. 2

C. 3

D. 4

ε ✓

a ✓

$\boxed{b}$ is not

aa

ab

ba

bb

order

$L = \underbrace{a(a+b)^*}_{\text{Start with 'a'}}$ is equivalent to ________

A. $(ab^*)^+$

B. $(a^+b^*)^+$

C. $a^*(ab^*)^+$

D. All of the above

$(ab^*)^+ = (a^+b^*)^+$

$\boxed{(ab^*)^+}$

$(aab^*)^+$

$(aaab^*)^+$

$L = \underbrace{(a+b)^*b}$ is equivalent to ________

A. $(ab^*)^+$ → min = a

B. $(a^+b^*)^+$

C. $b^*(ab^*)^*\boxed{b}$ must

D. None

b

$\boxed{a}$ b

$\boxed{b}$ b

$\boxed{a}$ $\boxed{a}$ b

a b b

b a b

b b b

$$(a+b)^* = b^*(ab^*)^* = a^*(ba^*)^* = (b^*a)^* b^* = (a^*b)^* a^*$$

$$(ab^*)^+ \leftarrow aX$$

$$(ba^*)^+ \leftarrow bX$$

$$(b^*a)^+ \leftarrow Xa$$

$$(a^*b)^+ \leftarrow Xb$$

$\underbrace{(b + ba)}_{b(\varepsilon+a)}$ $(b + a)^*$ $\underbrace{(ab + b)}_{(a+\varepsilon)\,b}$

$b \quad (a+b)^* \quad b$

$= b\,(a+b)^*\,b$

A. $(a+b)^*$

B. $a(a+b)^*a$

C. $b(a+b)^*b$

D. None

$(\varepsilon+a)(a+b)^*(\varepsilon+a)$

$= (a+b)^*$

$\Rightarrow$ multiple of 3

$\{w \in \{a, b\}^* : \#_a(w) \equiv_3 0\}$.

#a's is divisible by 3

A. $(b^*\boxed{a}b^*\boxed{a}b^*\boxed{a})^*\underline{b}^*$

B. $(b^*ab^*ab^*\underline{a})^*$ $\leftarrow$ b ✗

C. $(ab^*ab^*a)^*$ $\longleftrightarrow$ b ✗

D. $(ab^*ab^*a)^*b^*$ $\leftarrow$ baaa ✗

$\underbrace{(a \cup b)^* \, (a \cup \varepsilon)} \, b^* = (a+b)^* \boxed{b^*} = (a+b)^*$

A. $\underbrace{(a+b)}_{R} \cdot \underbrace{(a+b)^*}_{R^*} = (a+b)^+$

B. $(a+b)^*$

C. $(aa+b)^*$

D. None

$L = \{w \in \{a, b\}^* \mid w \text{ has bba as a substring}\}$
Which of the following describes L ?

A. $(a \cup b)^* \boxed{bba} (a \cup b)^*$
B. $(a \cup b)^{+} \boxed{bba} (a \cup b)^*$
C. $(a \cup b)^{+} \boxed{bba} (a \cup b)^{+}$
D. $(a \cup b)^* \boxed{bba} (a \cup b)^{+}$

$L = \{w \in \{a, b\}^*\}$ = $(a+b)^*$

1. $(a + b)^*$
2. $(a + b + \text{epsilon})^+$
3. Epsilon + $(a + b)^+$
4. $(a^*b^*)^*$
5. $(b^*a^*)^*$
6. $(a^+b^+)^*$ → b ✗ → a ✗

How many of above are equivalent to given L ?

A. 4 B. 5 C. 6 D. 3

$\Sigma = \{0\}$

Which Two of the following four regular expressions are equivalent?

(i) $\underbrace{(00)^*(\varepsilon + 0)}_{\text{All}} = 0^*$ (ii) $\underbrace{(00)^*}_{\text{even 0's}}$ (iii) $\underbrace{0^*}_{\text{All}}$ (iv) $\underbrace{0\,(00)^*}_{\text{odd 0's}}$

(a) (i) and (ii)
(b) (ii) and (iii)
~~(c)~~ (i) and (iii)
(d) (iii) and (iv)

(GATE - 96)

$$(00)^*(\varepsilon + 0)$$

$$\underbrace{(00)^*}_{\text{even}} + \underbrace{(00)^*0}_{\text{odd}} = 0^*$$

$$\underbrace{\underbrace{(00)^*}_{\text{even}} \cup \underbrace{0(00)^*}_{\text{odd}}}_{\text{All}} = 0^*$$

If the regular set A is represented by $A = (01+1)^*$ and the regular set 'B' is represented by $B = ((01)^* 1^*)^*$, which of the following is true? **(GATE - 98)**

(a) $A \subset B$

(b) $B \subset A$

(c) A and B are incomparable

(d) $A = B$

$A = (x+y)^*$

01 1

$B = (x^* y^*)^*$

$A \subseteq B$ ✓

$A \subset B$ ✗

$A \supseteq B$ ✓

$A \supset B$ ✗

$A = B$ ✓

The string $\boxed{1101}$ does not belong to the set represented by **(GATE - 98)**

$1101 \in$ (a) $110^* (0+1)$

$1101 \in$ (b) $1(0+1)^*101$

$1101 \in$ (c) $(10)^* (01)^* (00+11)^*$

(d) $(00+(11)^* 0)^*$

$1101 \notin$

$110110 \in$

this is diff string

Let S and T be languages over $\sum = \{a, b\}$ represented by the regular expressions $\boxed{(a + b^*)^*}$ and $\boxed{(a + b)^*}$, respectively. Which of the following is true? **(GATE - 2000)**

(a) $S \subset T$

(b) $T \subset S$

(c) $S = T$

(d) $S \cap T = \phi$

$S \cap T = S$

$= T$

H.W.

Consider the set $\sum^*$ of all strings over the alphabet $\sum = \{0, 1\}$. $\sum^*$ with the concatenation operator for strings **(GATE - 03)**

Group Theory

(a) Does not form a group

(b) Forms a non-commutative group

(c) Does not have a right identity element

(d) Forms a group if the empty string is removed from $\sum^*$

Discrete maths:

&

TOC

The regular expression $\underbrace{0^*(10^*)^*}$ $= (0+1)^*$ denotes the same set as **(GATE - 03)**

(a) $(1^*0)^*1^* = (0+1)^*$

(b) $0+(0+10)^* \rightarrow 1 \times$

(c) $\underbrace{(0+1)^*10(0+1)^*} \rightarrow \varepsilon \times$

(d) None of the above

Which one of the following languages over the alphabet $\{0, 1\}$ is described by the regular expression $(0+1)^*\boxed{0}(0+1)^*\boxed{0}(0+1)^*$ **(GATE - 09)**

at least 2 0's

(a) The set of all strings containing the substring 00

(b) The set of all strings containing at most two 0's

(c) The set of all strings containing at least two 0's

(d) The set of all strings that begin and end with either 0 or 1

Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1 L_2^* \cup L_1^*$? **(GATE - 13)**

(a) $\{\varepsilon\}$ (b) ϕ

(c) a^* (d) $\{\varepsilon, a\}$

$$L_1 L_2^* \cup L_1^* = \boxed{\phi \cdot a^*} \cup \boxed{\phi^*}$$

$$= \phi \cup \{\varepsilon\}$$

$$= \{\varepsilon\}$$

The length of the shortest string NOT in the language (over Σ = {a, b}) of the following regular expression is ___=3___.

(GATE – 14-SET3)

$$a^*b^*(ba)^*a^*$$

ϵ ✓
a ✓
b ✓
aa ✓
ab ✓
ba ✓
bb ✓
aaa ✓
aab ✓
aba ✓
abb ✓
baa ✓
$\boxed{bab}$
bba
bbb

~~bab~~

$\underbrace{bab}$

3 len

Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0's and two consecutive 1's? (GATE – 16 – SET1)

00

11

containing 00 and 11 as substring

(a) $(0+1)^* \boxed{0011}(0+1)^* + (0+1)^* \boxed{1100}(0+1)^*$

001011 ×

↳ contains 0011 or 1100

(b) $(0+1)^* (00(0+1)^* 11 + 11(0+1)^* 00)(0+1)^*$

(c) $(0+1)^* \boxed{00}(0+1)^* + (0+1)^* \boxed{11}(0+1)^*$

(d) $00(0+1)^* 11 + 11(0+1)^* 00$

00 11 + 11 00

Let $r = 1(1+0)^*$, $s = 11^*0$ and $t = 1^*0$ be three regular expressions. Which one of the following is true? **(GATE - 91)**

(a) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$

(b) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$

(c) $L(s) \subseteq L(t)$ and $L(s) \subseteq L(r)$

(d) $L(t) \subseteq L(s)$ and $L(s) \subseteq L(r)$.

Which of the following regular expression identities are true?

(GATE - 92)

(a) $r(^*) = r^*$ not wff

(b) $(r^*s^*)^* = (r+s)^*$

(c) $(r+s)^* = r^* + s^*$

(d) $r^*s^* = r^* + s^*$

Which one of the following regular expressions represents the set of all binary strings with an odd number of $1'$s?

A. $(\boxed{(0+1)^*}1\boxed{(0+1)^*}1)^*10^*$ ← 01 ×

B. $(0^*10^*10^*)^*0^*1$ → 10 ×

C. $10^*(0^*10^*10^*)^*$ → 01 ×

D. $(0^*10^*10^*)^*10^*$ → 01 ×

no answer

GATE 2020

Which one of the following regular expressions over {0, 1} denotes the set of all strings $\boxed{\textbf{not} \text{ containing } 100}$ as a substring?

(GATE - 97)

(a) $0^*(1^+0)^*$ → 1 ✗

(b) 0^*1010^* → ε ✗

(c) $0^*1^*01^*$ → ε ✗

(d) $0^*(10+1)^*$ ✓

valid : ε, 0, 1, 00, 01, 10, 11, ... } valid

000 ✓
001 ✓
010 ✓
011 ✓
100 ✗
101 ✓
110 ✓
111 ✓

1000 ✗
0100 ✗
0011 ✓
⋮

Model GATE Questions

- Identify correct regular expression for given Language.
- Find the expression that can generate given String.
- Find the string that can be generated by given expression.
- Find shortest length string generated by given expression.
- Identify the equivalent expression for given expression.
- Identify equivalent expressions from given expressions.
- In FA *** Find number of equivalence classes for the language generated by given expression.
- Find the language generated by expression is finite or infinite.

Summary

→ Reg Exp Revision done

Next class : FA


THANK YOU GW
SOLDIERS !