CS & IT ENGINEERING

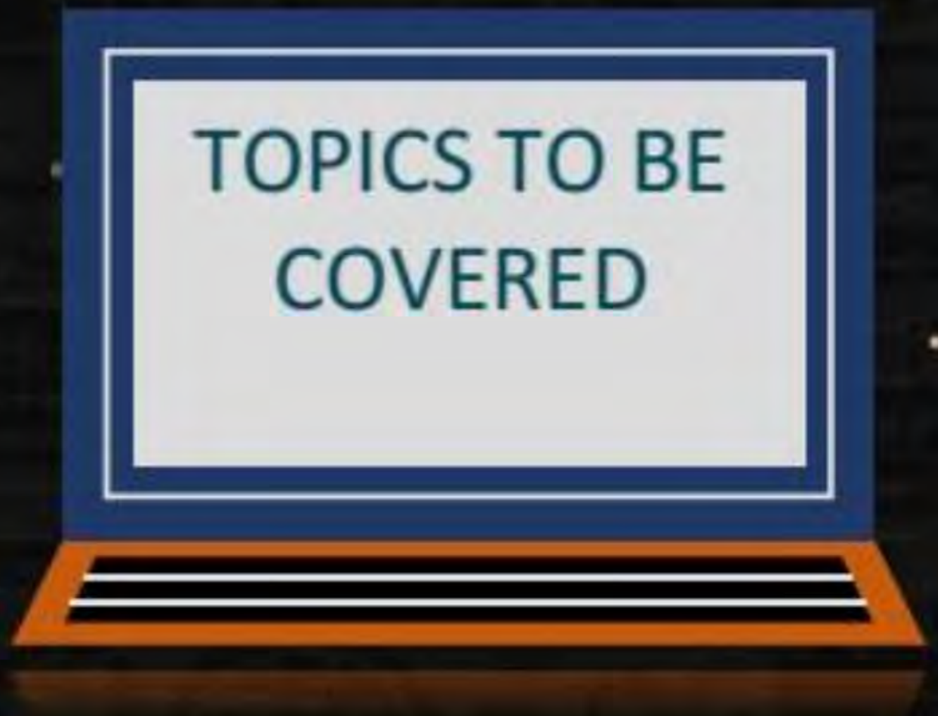
Theory of Computation

Finite Automata:

DFA-3

Lecture No. 7

By- DEVA Sir



Construction of DFA

Model- I

Model- II

Model - III

Model - IV

Model - V

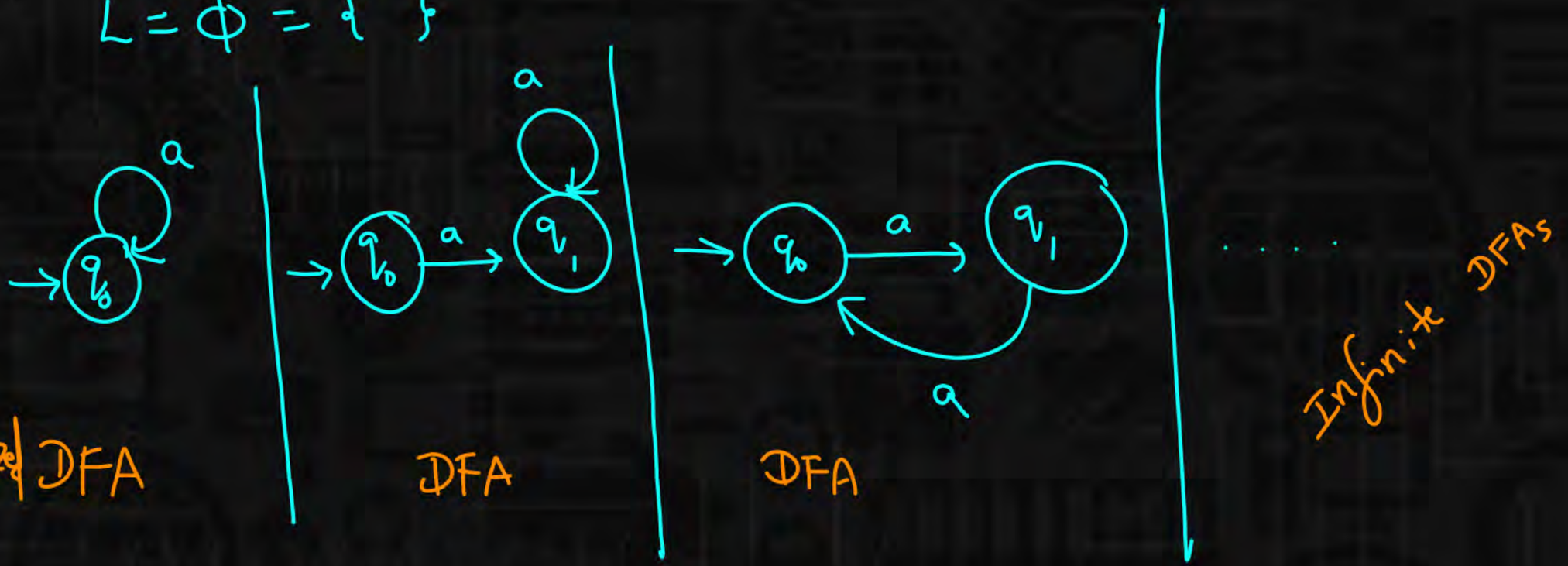
Model - VI

DFA Construction

Mode-I [Special problems : ϕ, Σ^*, $\{\varepsilon\}$, Σ^+]

① $L = \phi$ over $\Sigma = \{a\}$

$L = \phi = \{\ \}$



For every regular language, unique (only one) min DFA exist

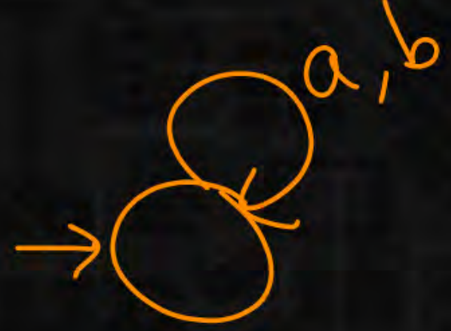
DFA Construction

① $L = \phi$ over $\Sigma = \{a\}$

1 state

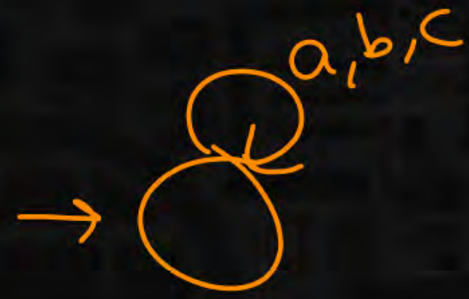
εx
ax
aax
$aaax$
⋮
$\cdots$

② $L = \phi$ over $\Sigma = \{a, b\}$



1 state

③ $L = \phi$ over $\Sigma = \{a, b, c\}$



1 state

DFA Construction

(4) $L = \Sigma^*$ over $\Sigma = \{a\}$

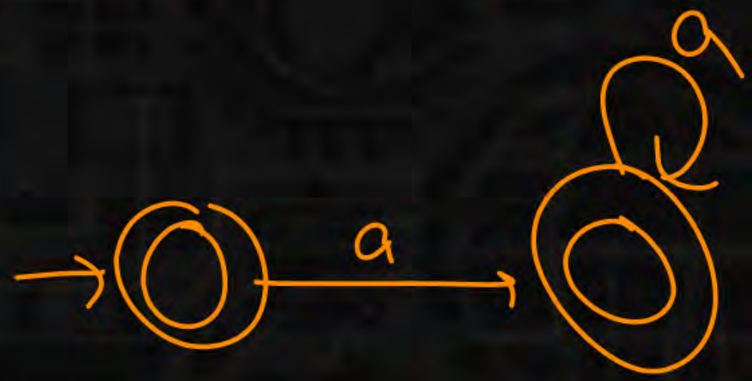
$a^2 = a.a = aa$

$L = \{\boxed{\epsilon}, a, a^2, a^3, a^4, \ldots\}$

min

ϵ ✓
a ✓
aa ✓
aaa ✓
⋮

Min DFA

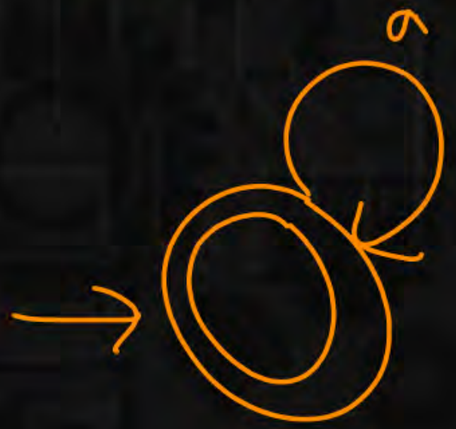


ϵ ✓
a ✓
aa ✓
aaa ✓
⋮

DFA Construction

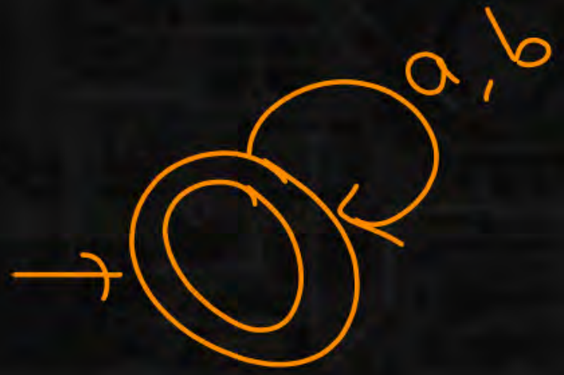
④ $L = \Sigma^*$ over $\Sigma = \{a\}$

$= a^*$

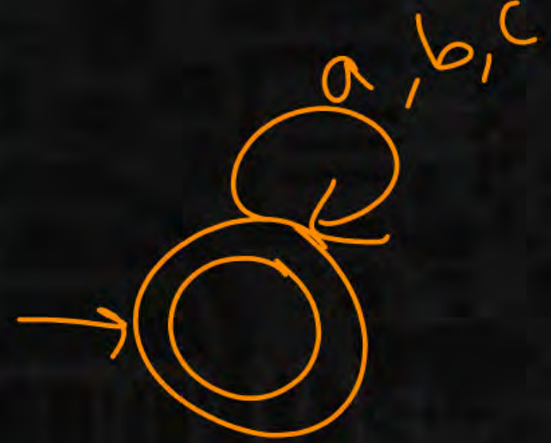


⑤ $L = \Sigma^*$ over $\Sigma = \{a, b\}$

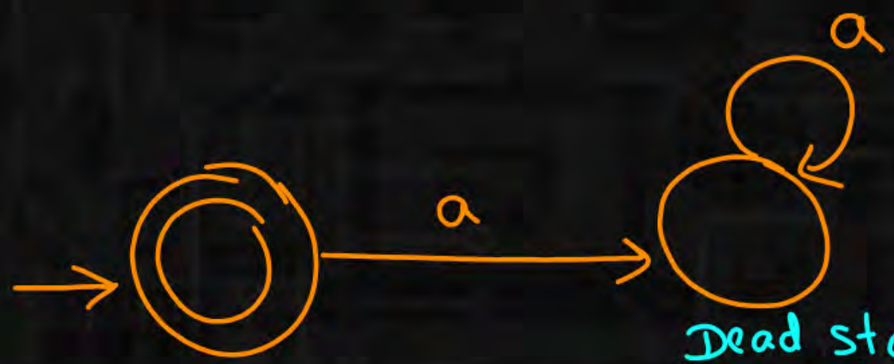
$= (a+b)^*$



⑥ $L = \Sigma^*$ over $\Sigma = \{a, b, c\}$



(7) $L = \{\varepsilon\}$ over $\Sigma = \{a\}$



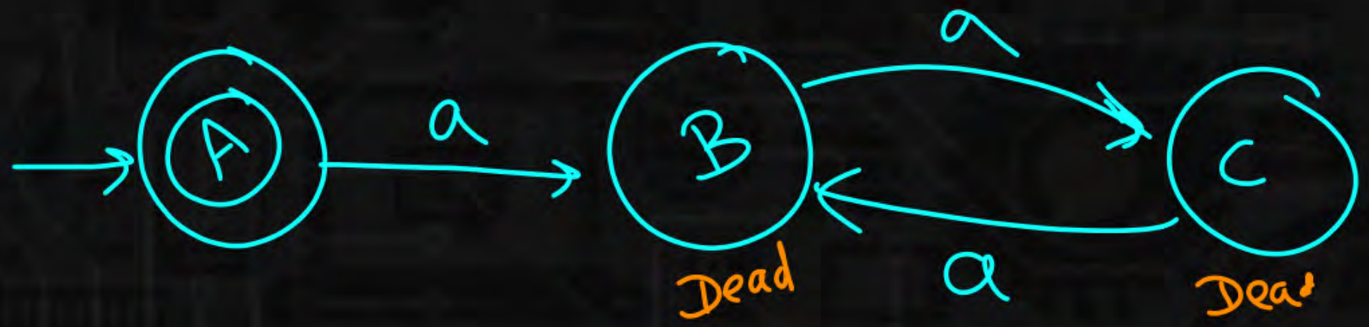
Dead state (Trap state)
(special nonfinal)

There is no path from this nonfinal to final

Min DFA

ε ✓
a ✗
aa ✗
aaa ✗
a^4 ✗
a^5 ✗

DFA Construction



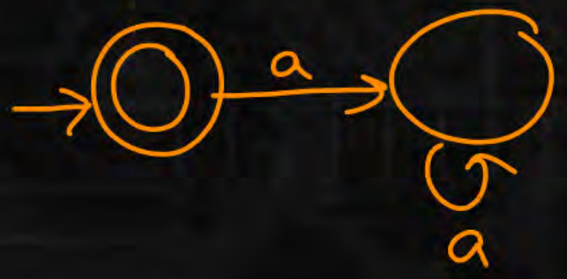
DFA ✓

Min DFA ✗

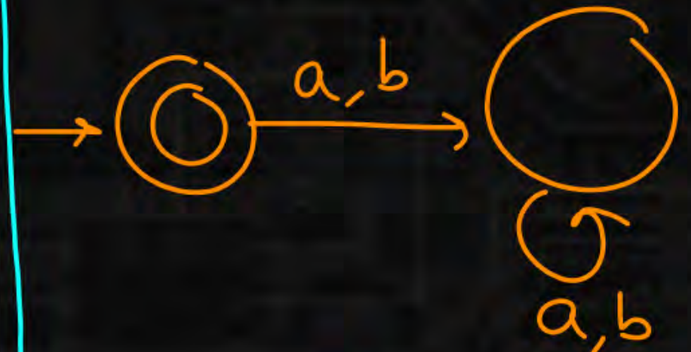
$L = \{\varepsilon\}$

DFA Construction

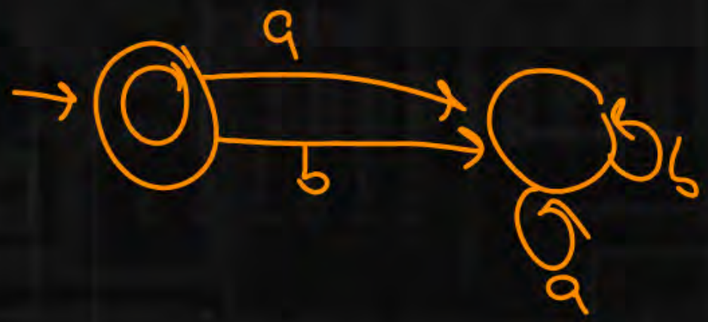
(7) $L = \{\varepsilon\}$ over $\Sigma = \{a\}$



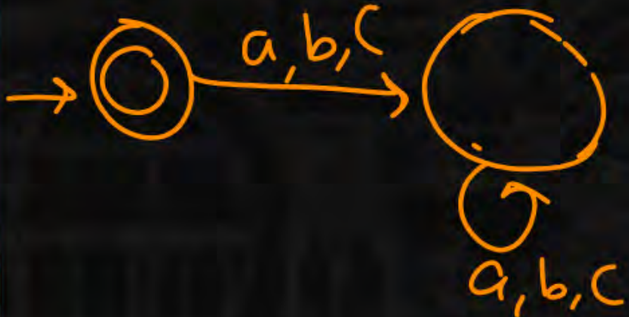
(8) $L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$



OR



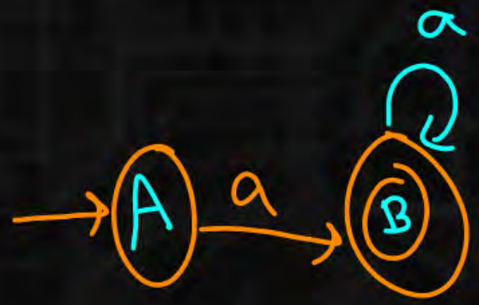
(9) $L = \{\varepsilon\}$ over $\Sigma = \{a, b, c\}$



DFA Construction

(10) $L = \Sigma^+$ over $\Sigma = \{a\}$

$L = a^+ = \{a, aa, aaa, \ldots\}$ (a: min)

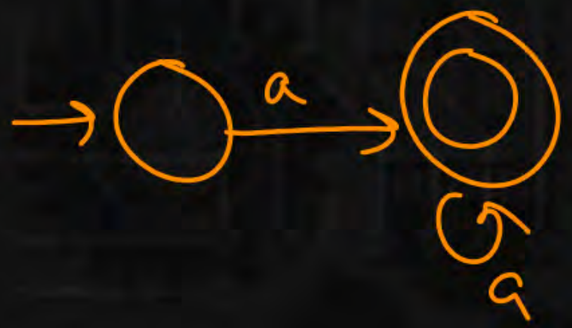


Ex
a ✓
aa ✓
a^3 ✓
a^4 ✓
⋮

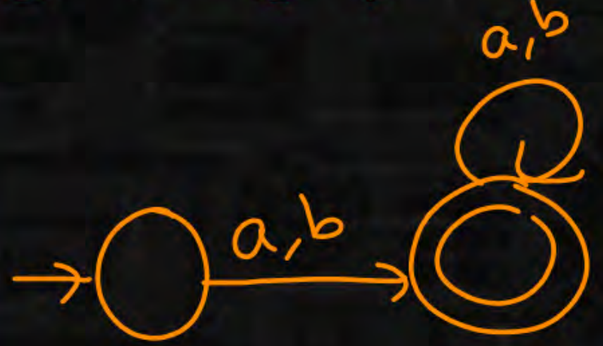
Note: Dead state is always nonfinal.
But nonfinal may or may not dead state.

DFA Construction

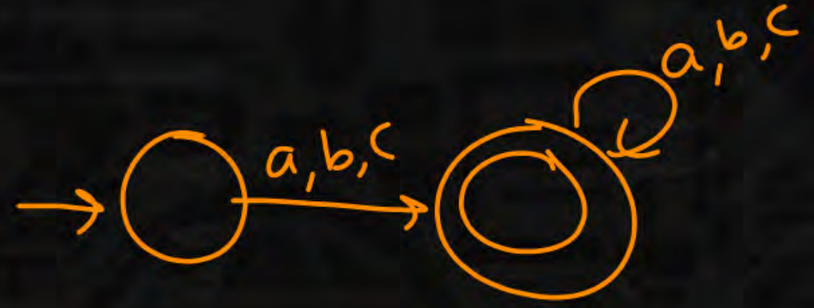
(10) $L = a^+$



(11) $L = (a+b)^+$



(12) $L = (a+b+c)^+$



DFA Construction

$L_1 = \phi$ | $L_2 = \Sigma^*$ | $L_3 = \{\varepsilon\}$ | $L_4 = \Sigma^+$

$\overline{L_1} = L_2$

$\overline{L_2} = L_1$

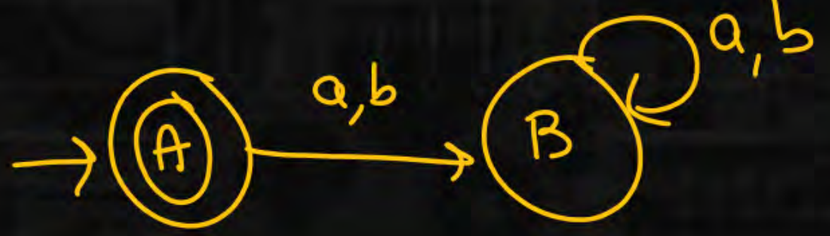
$\overline{L_3} = L_4$

$\overline{L_4} = L_3$

DFA Construction

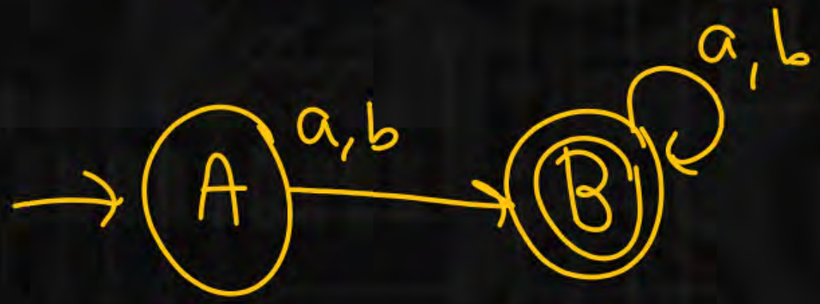
$L = \{\varepsilon\}$ over $\Sigma = \{a, b\}$

$\bar{L} = \Sigma^+$ over $\Sigma = \{a, b\}$



L

$\bar{L}$: Interchange finals and nonfinals in DFA that accepts L

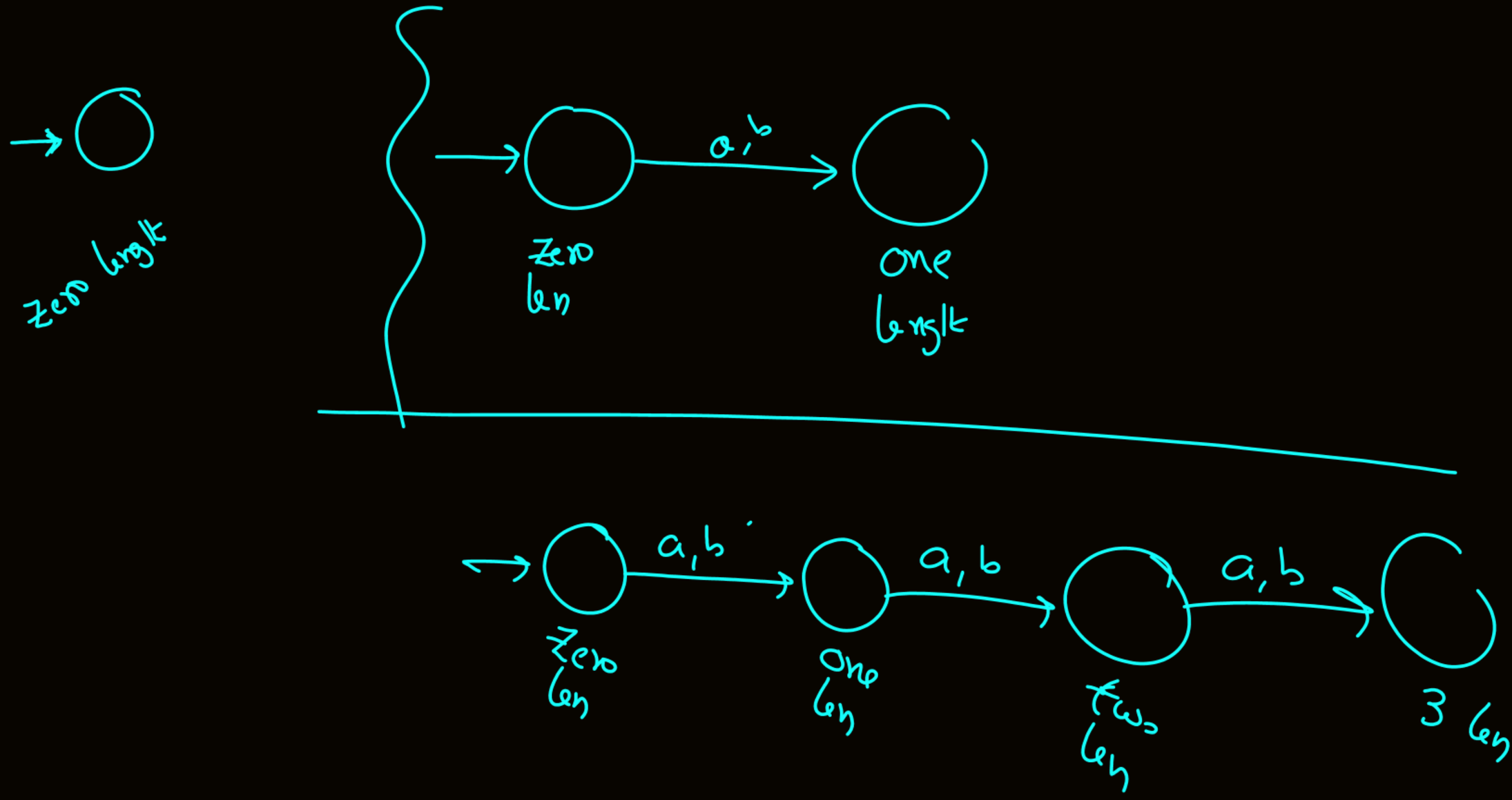


DFA Construction

If L is regular and min DFA that accepts L has n states then

no. of states in min DFA that accepts $\bar{L}$ $= n$

If n_1 finals and n_2 non finals in min DFA that accepts L then n_2 finals and n_1 non finals in min DFA that accepts $\bar{L}$.


Zero length
Zero len
a, b
one length
a, b
a, b
a, b
Zero len
one len
two len
3 len

DFA Construction

Model-II [length based problems: $=, <, >, \leq, \geq$]

(13) $L = \{w \mid w \in \{a,b\}^*, |w| = 2\}$ $\Rightarrow$ 4 States in Min DFA

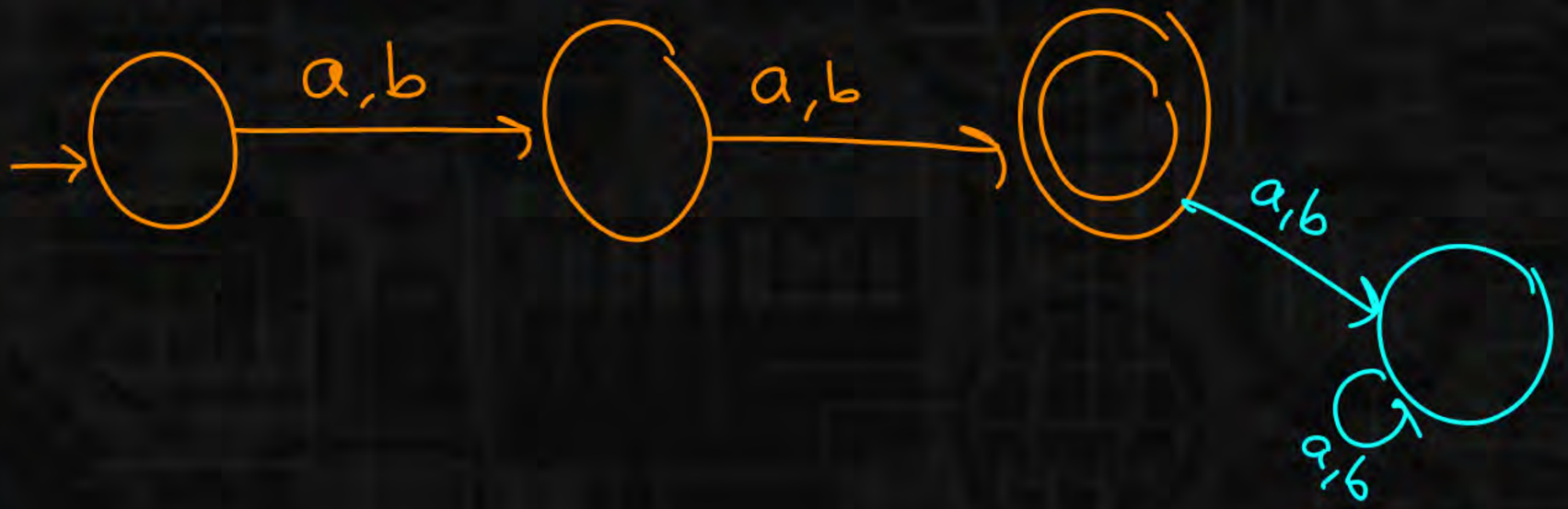
$=$ Set of all 2 length strings over $\Sigma = \{a,b\}$

$= \{aa, ab, ba, bb\}$

Note:

If $|w| = K$

then $(K+2)$ states in Min DFA



DFA Construction

(13) $\{w \mid w \in \{a,b\}^*, |w| = 2\}$ ⟹ 4 states in min DFA

(14) $\{w \mid w \in \{a,b\}^*, |w| = 100\}$ ⟹ 102 states

*** (15) $\{w \mid w \in \{a,b\}^*, |w| = \underbrace{K}_{\text{constant}}\}$ ⟹ K+2 states

1 final, K+1 non final

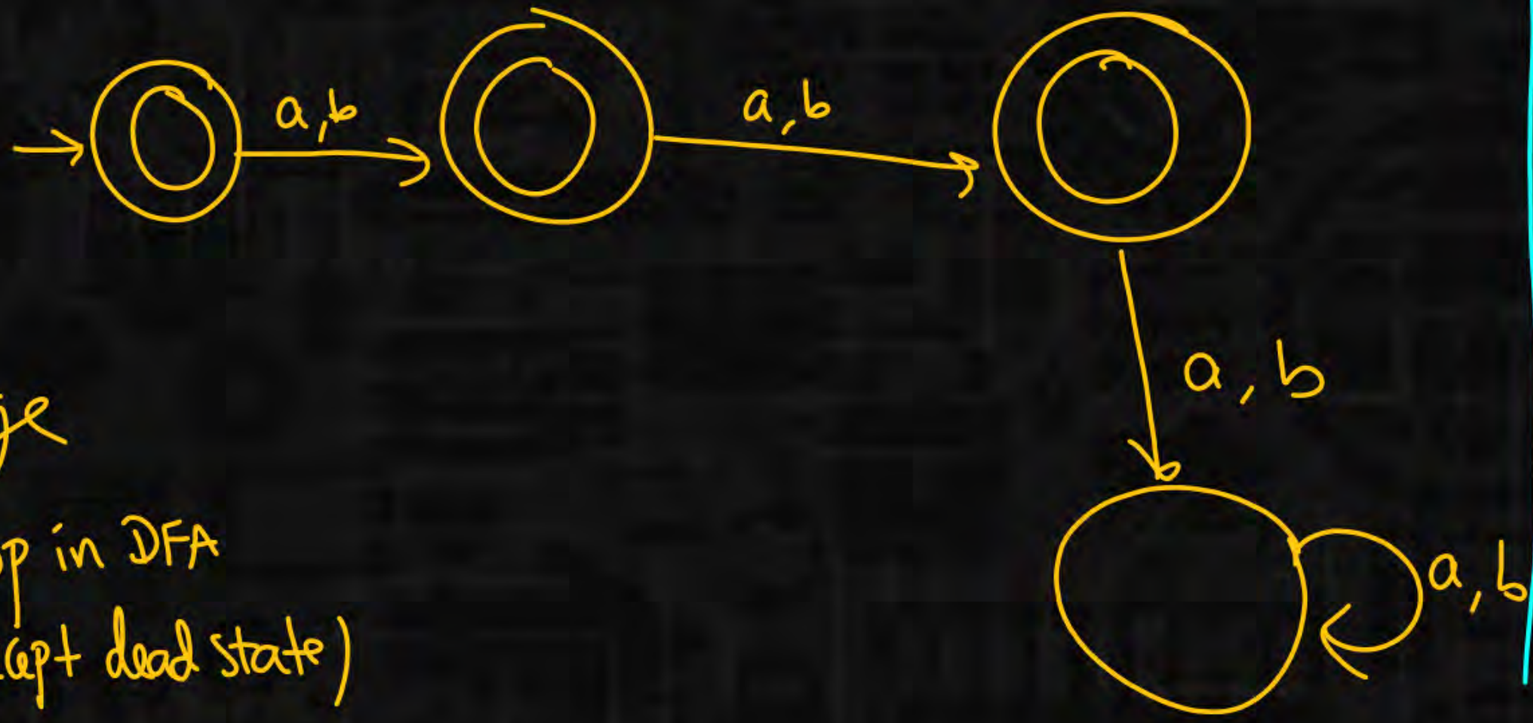
GATE: $\underbrace{(a+b).(a+b)\ldots n \text{ times}}_{\text{exactly } n \text{ length string}}$

$= (a+b)^n$ ⟹ n+2 states in DFA

fixed n

DFA Construction

(16) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \leq 2}_{\text{Length of } w \text{ is atmost 2}}\}$



Finite language
↳ no loop in DFA
(except dead state)

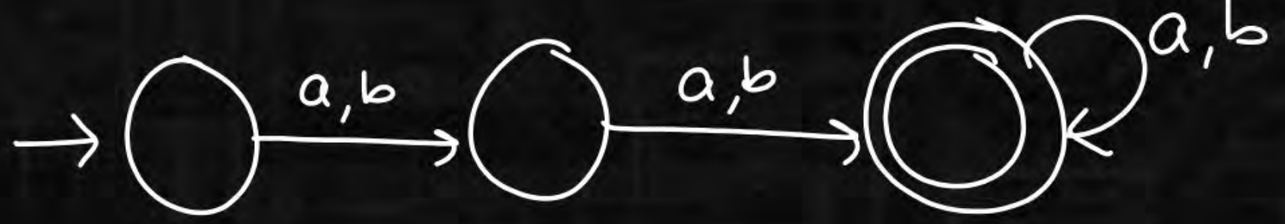
$(\varepsilon + a + b)^2$

$\varepsilon + a + b + aa + ab + ba + bb$

Note:
If $|w| \leq K$
then $K+2$ states
in DFA

DFA Construction

(17) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| \geq 2}_{\text{Atleast 2 length}}\}$



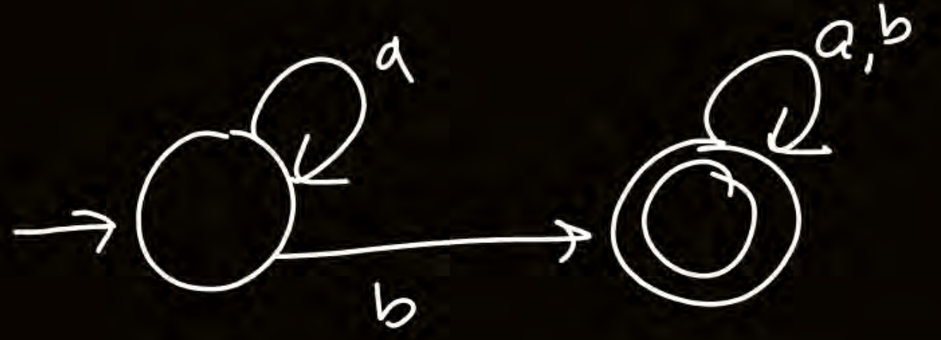
$(a+b)^2 (a+b)^*$

If $|w| \geq k$ then $k+1$ states in DFA.

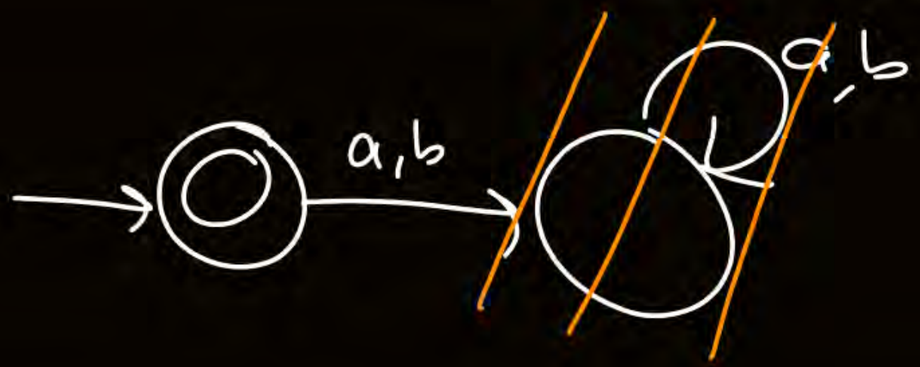
Infinite language
↳ loop exist in DFA (except dead state)

Check loop

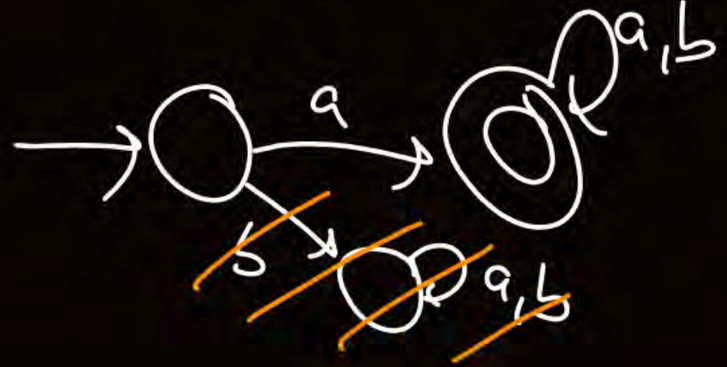
Inf lang



Finite



Infinit



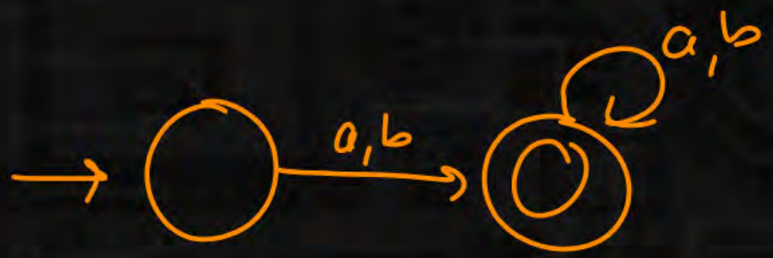
DFA Construction

IS $L = \phi$?

$L = \phi$

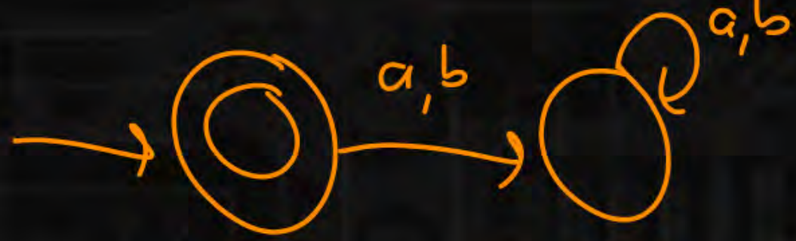
No final state

$L \neq \phi$



Final exist

$L \neq \phi$

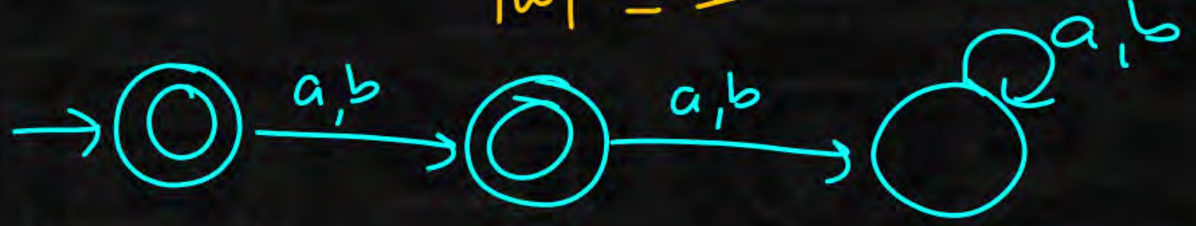


Final exist

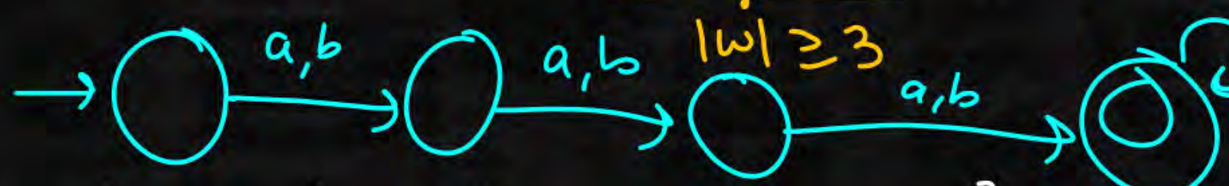
Language is empty $\Rightarrow L = \phi$

String is empty $\Rightarrow \omega = \varepsilon$

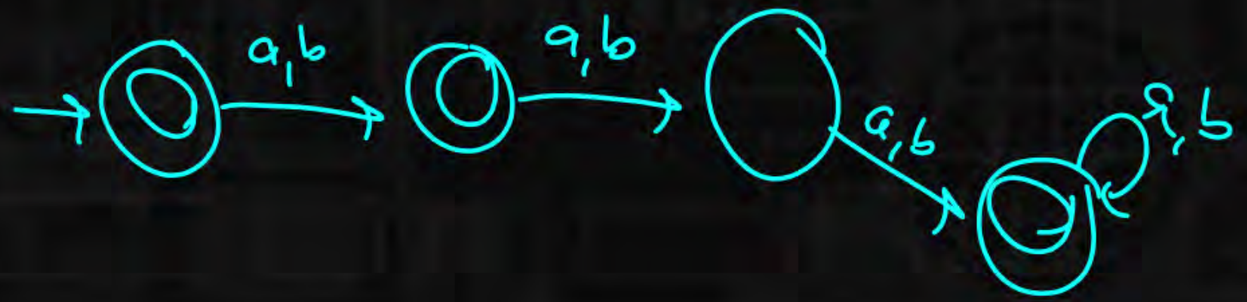
(18) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| < 2}_{|w| \leq 1}\}$ ⟹ 3 states



(19) $\{w \mid w \in \{a,b\}^*, \underbrace{|w| > 2}_{|w| \geq 3}\}$ ⇒ 4 states



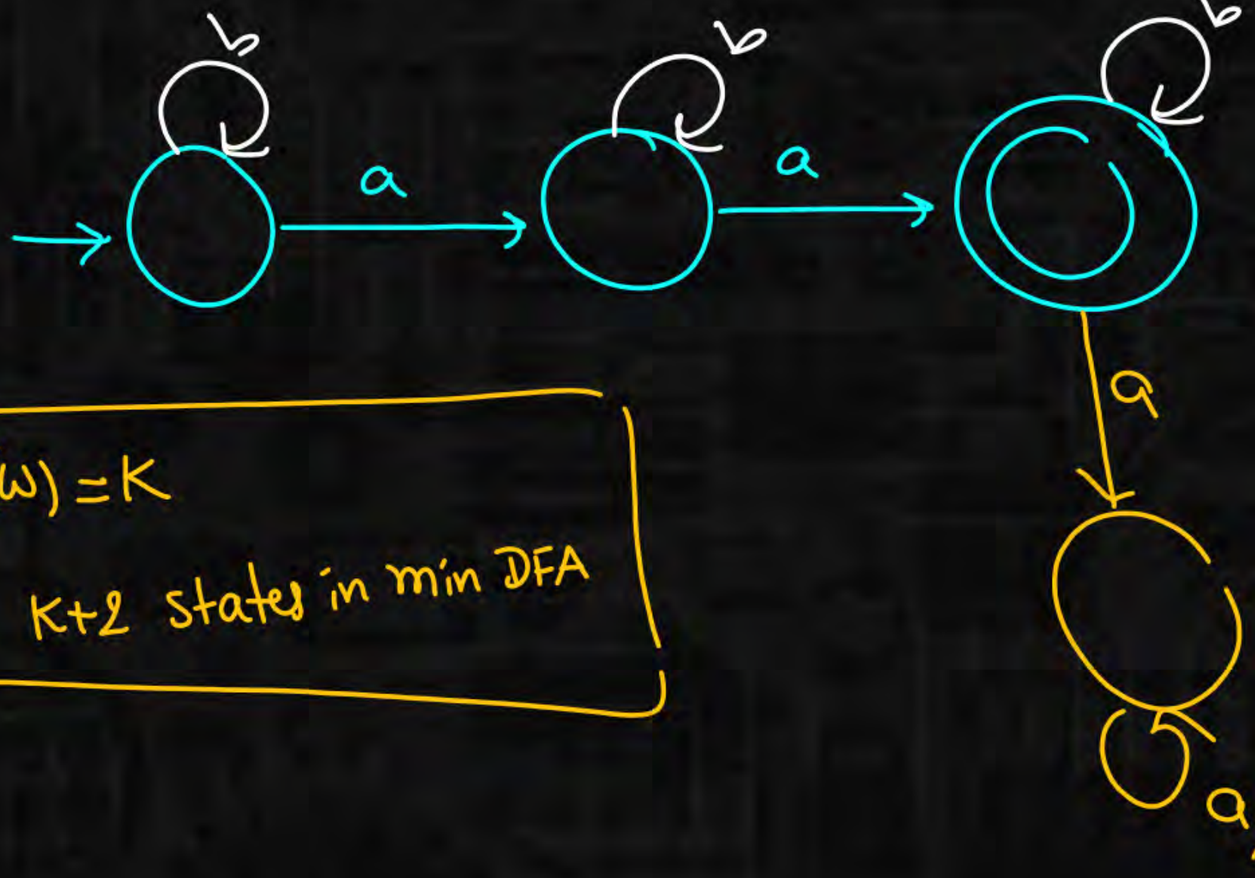
(20) $\{w \mid w \in \{a,b\}^*, |w| \neq 2\}$ ⇒ 4 states



DFA Construction

Model-III [No. of a's : $=, \leq, \geq, <, >, \neq$]

(21) $\{w \mid w \in \{a,b\}^*, n_a(w) = 2\}$



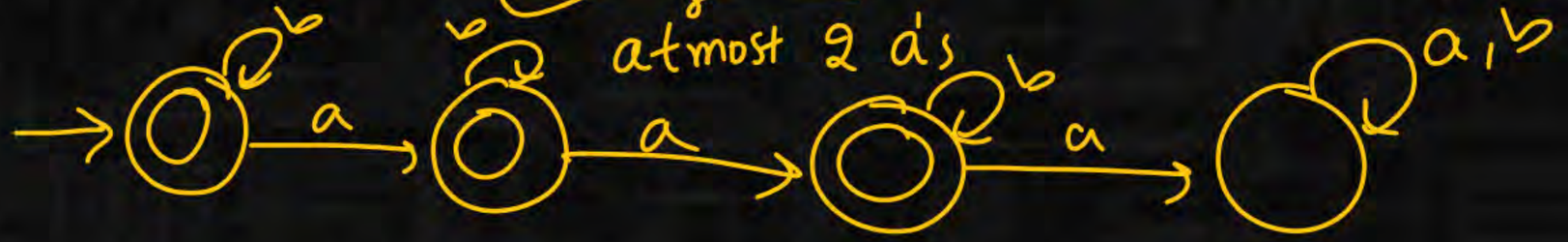
If $\#_a(w) = K$

then K+2 states in min DFA

$\lvert w \rvert = 2$	$(a+b)^2$
$\#_a(w) = 2$	$b^* a b^* a b^*$

DFA Construction

(22) $\{w \mid w \in \{a,b\}^*, \#a(w) \leq 2\}$ ⇒ 4 states



(23) $\{w \mid w \in \{a,b\}^*, \#a(w) \geq 2\}$ ⇒ 3 states

(24) $\{w \mid w \in \{a,b\}^*, \#a(w) < 2\}$ ⇒ 3 states

(25) $\{w \mid w \in \{a,b\}^*, \#a(w) > 2\}$ ⇒ 4 states

(26) $\{w \mid w \in \{a,b\}^*, \#a(w) \neq 2\}$ ⇒ 4 states

DFA Construction

$w \in \{a,b\}^*$ | No. of states in min DFA

①	$\|w\| = K$	$K+2$
②	$\|w\| \leq K$	$K+2$
③	$\|w\| \geq K$	$K+1$
④	$\|w\| < K$	$K+1$
⑤	$\|w\| > K$	$K+2$
⑥	$\|w\| \neq K$	$K+2$

No. of states in min DFA

⑦	$n_a(w) = K$	$K+2$
⑧	$n_a(w) \leq K$	$K+2$
⑨	$n_a(w) \geq K$	$K+1$
⑩	$n_a(w) < K$	$K+1$
⑪	$n_a(w) > K$	$K+2$
⑫	$n_a(w) \neq K$	$K+2$

DFA Construction

Model-IV [language over 1 symbol, forms Arithmetic progression]

(27) $a^* = \{a^n \mid n \geq 0\}$

(28) $(aa)^* = \{a^{2n} \mid n \geq 0\}$

(29) $a(aa)^* = \{a^{2n+1} \mid n \geq 0\}$

(30) $\{a^{3n+2} \mid n \geq 0\}$

(31) $\{a^{3n+5} \mid n \geq 0\}$

A.P.

0, 1, 2, 3, . . .

0, 2, 4, 6, 8, . . .

2 2 2

Summary

Model-I: ϕ, Σ^*, $\{\varepsilon\}$, Σ^+

ϕ → 1 state; Σ^* → 1 state; $\{\varepsilon\}$ → 2 states; Σ^+ → 2 states

Model-II: Length, $=, \neq, <, \leq, \geq, >$

Model-III: no. of a's: $=, \neq, <, \leq, \geq, \geq$

⇨ Model-IV: over 1 symbol, forms A.P.


THANK YOU GW
SOLDIERS !