

# CS & IT ENGINEERING

Theory of Computation

Push Down Automata.  
Part-3



Lecture No. 04



By- DEVA Sir



# TOPICS TO BE COVERED

01

PDA construction

Using Final State mechanism

02

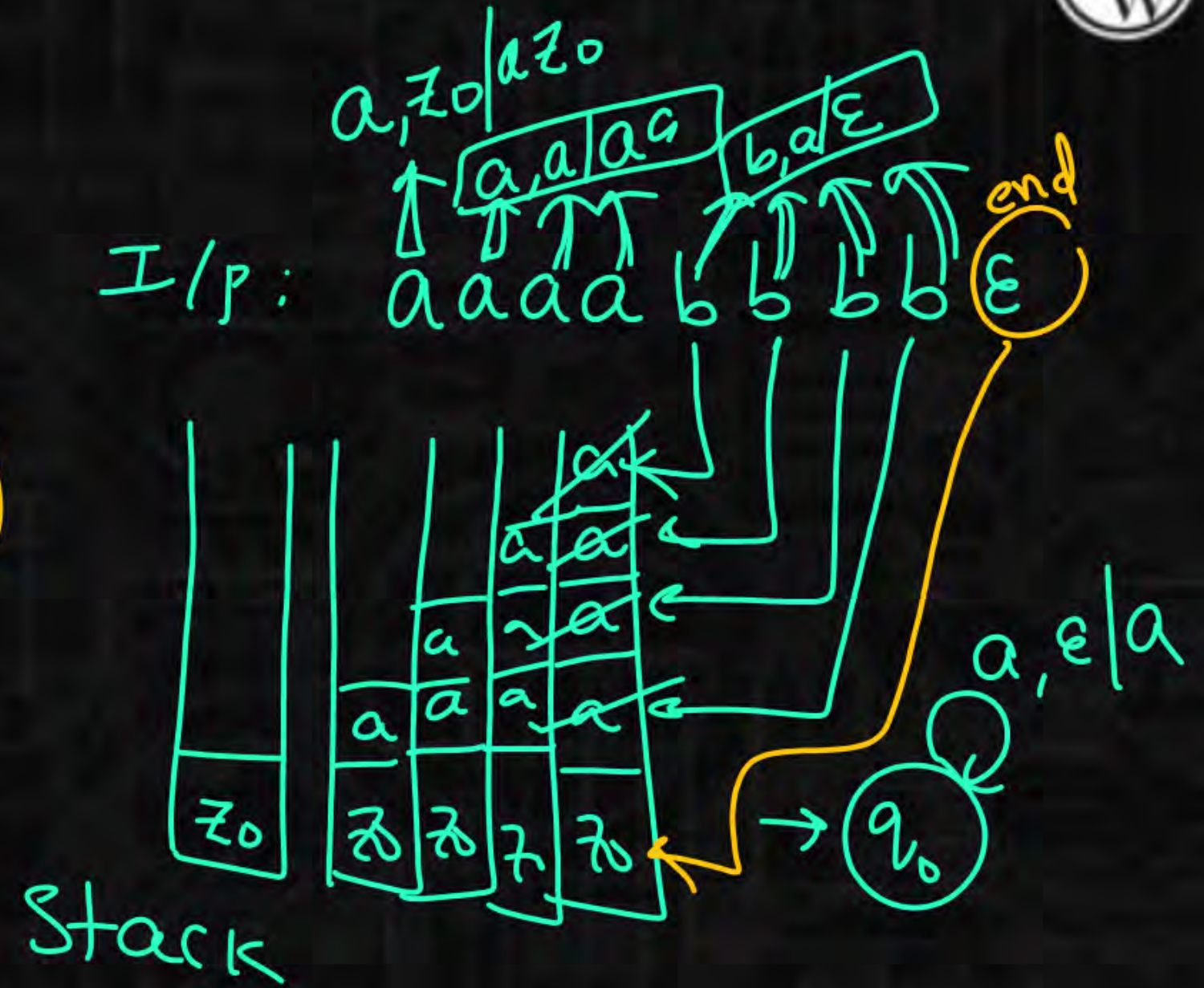
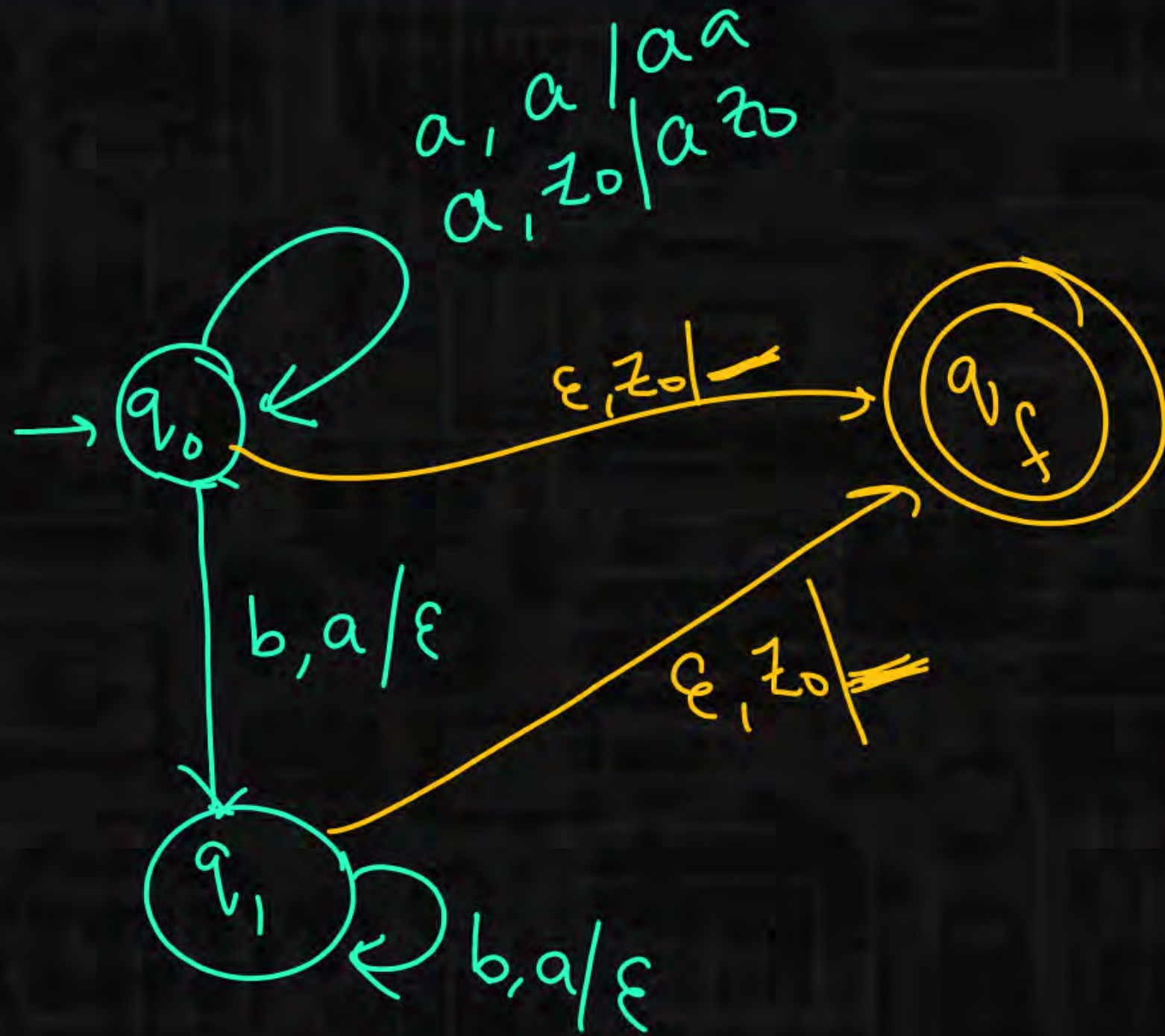
03

04

05

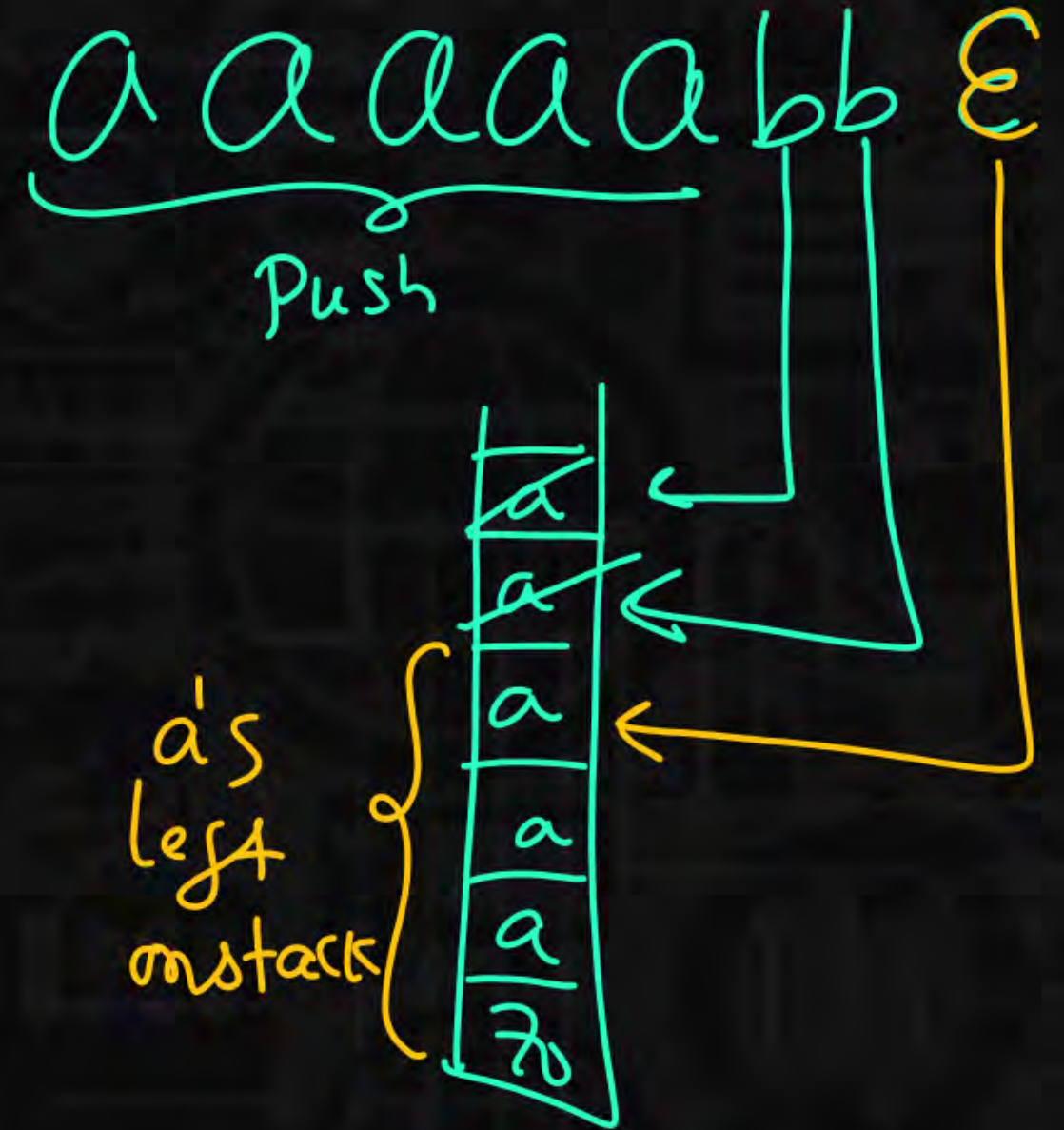
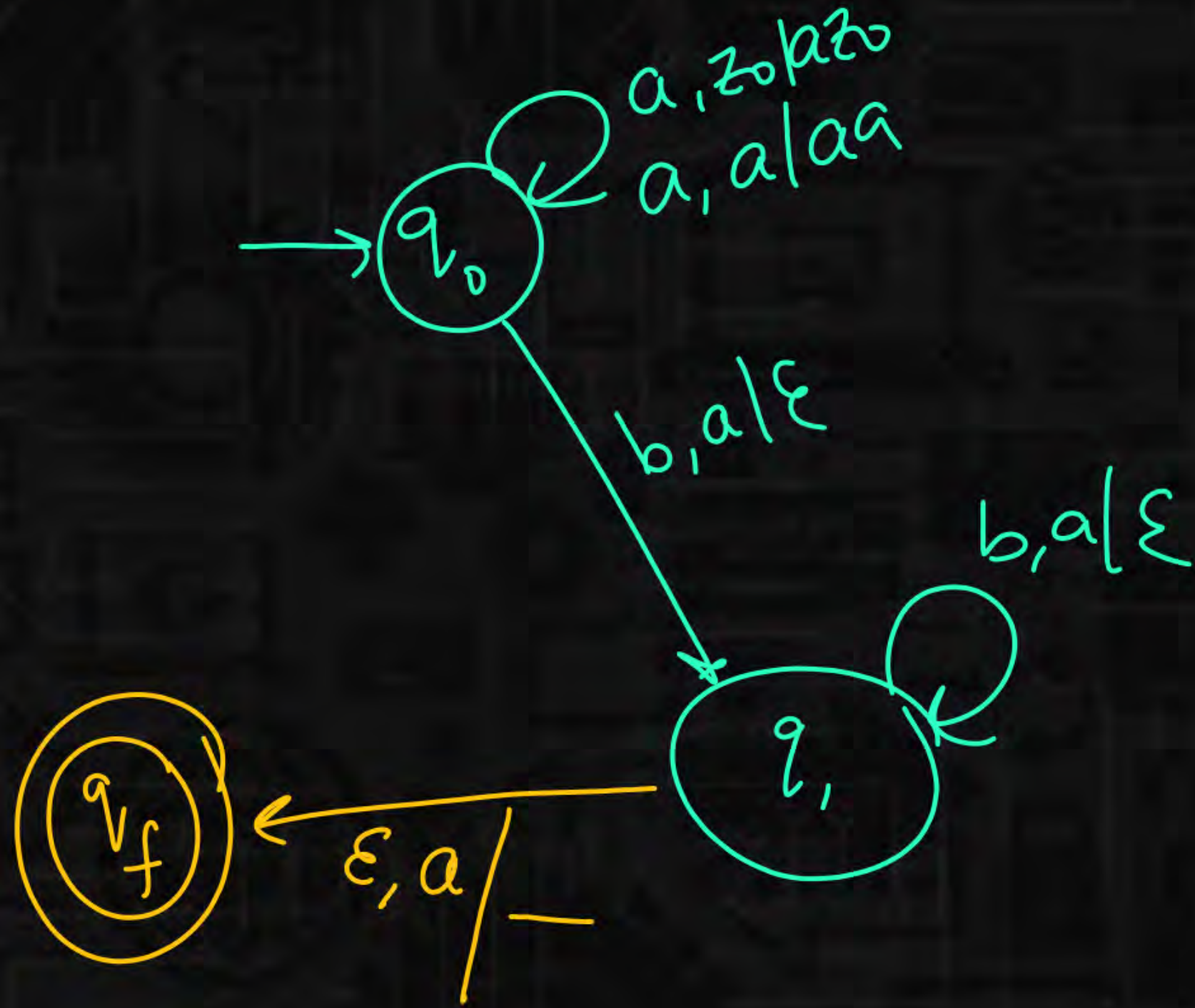


①  $\{a^n b^n \mid n \geq 0\}$



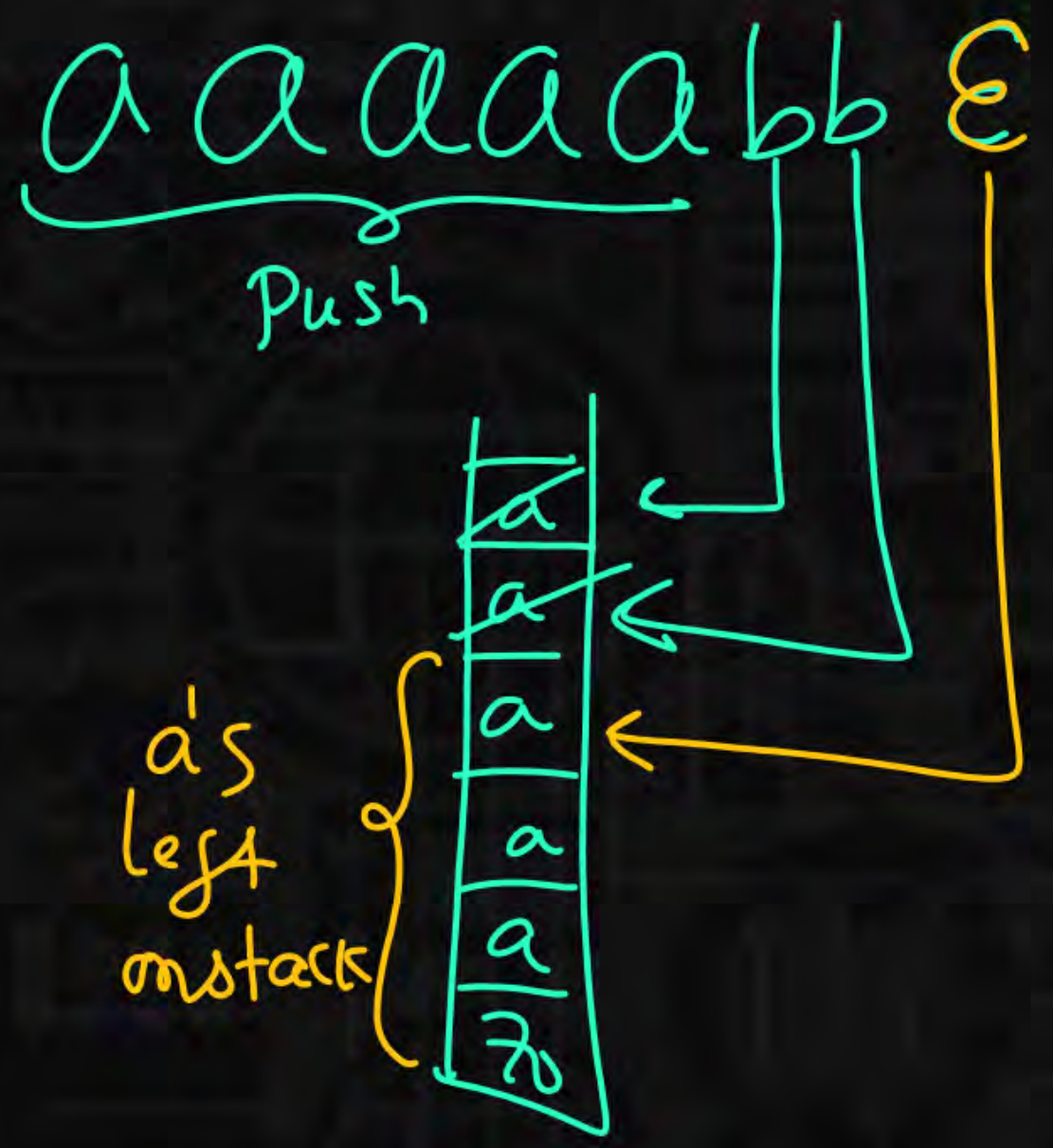
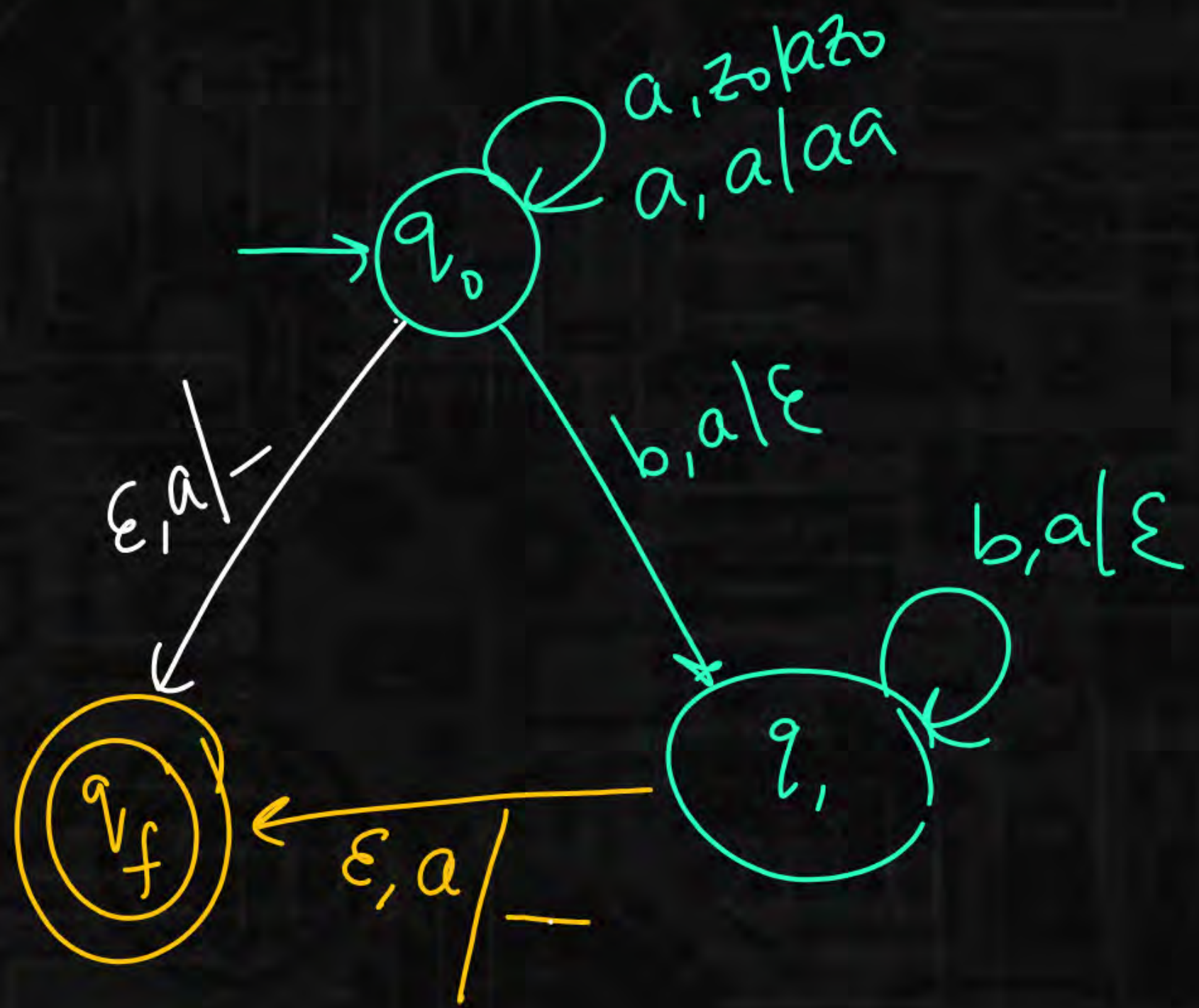


②  $\{a^m b^n \mid m > n, m, n \geq 1\}$





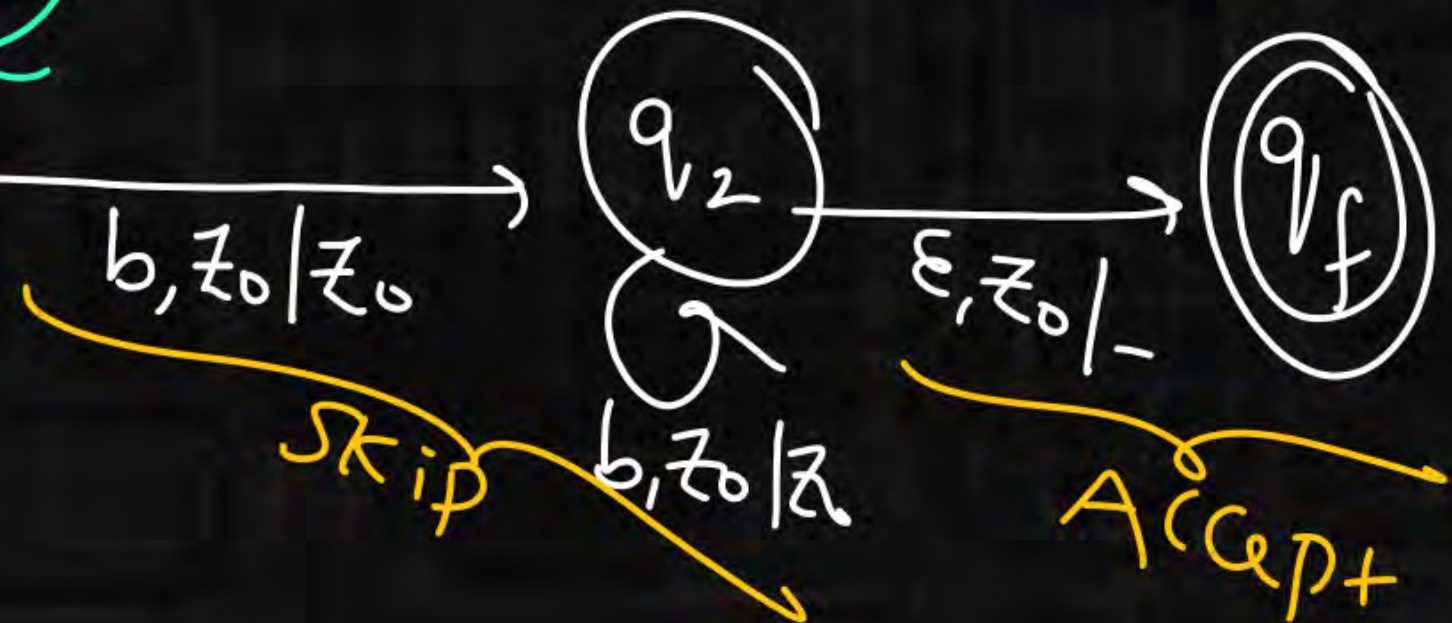
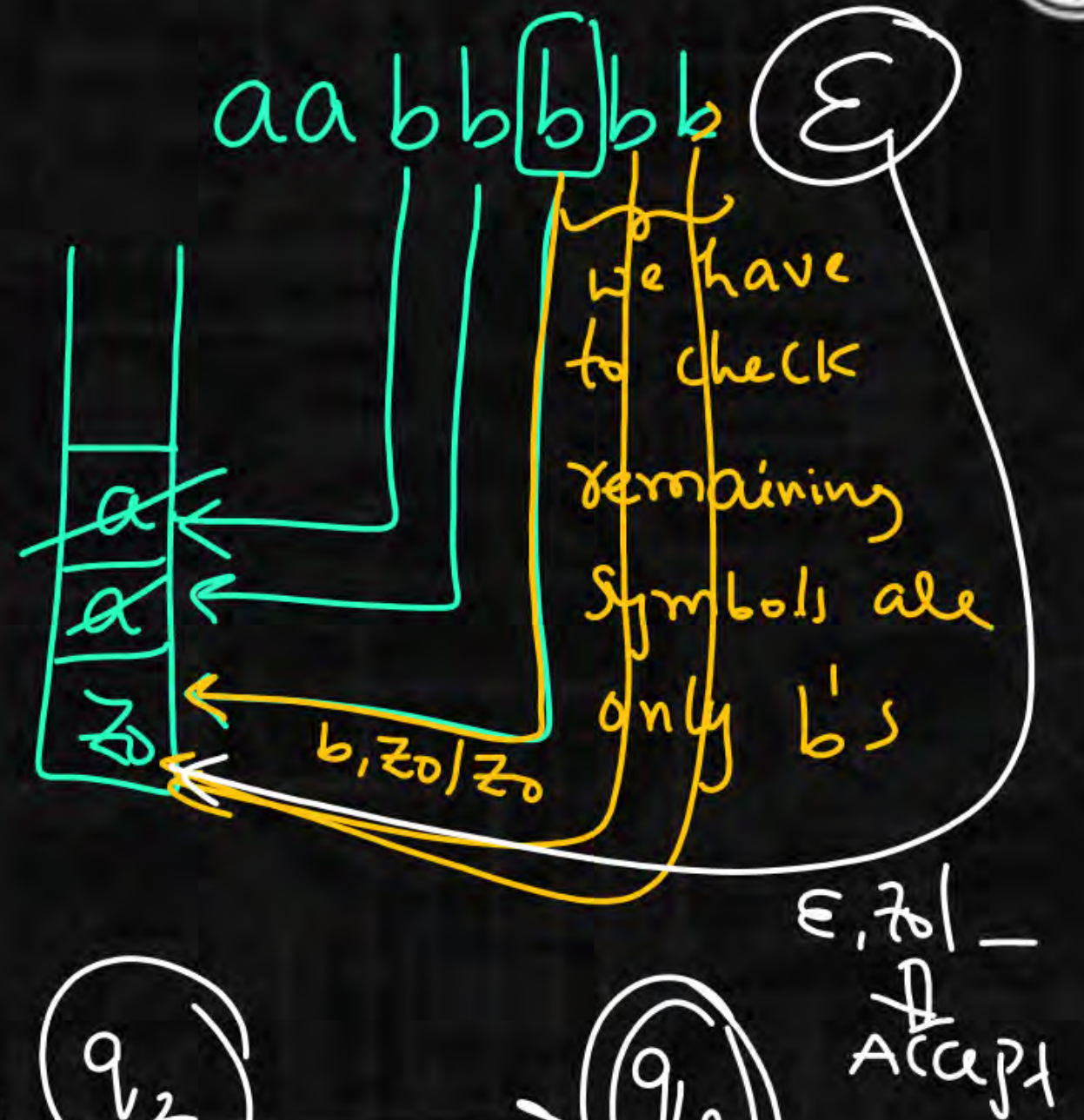
③  $\{a^m b^n \mid m > n, m, n \geq 0\} = \textcircled{2} \cup a^+$



Every CFL can be accepted by PDA with 1 state.

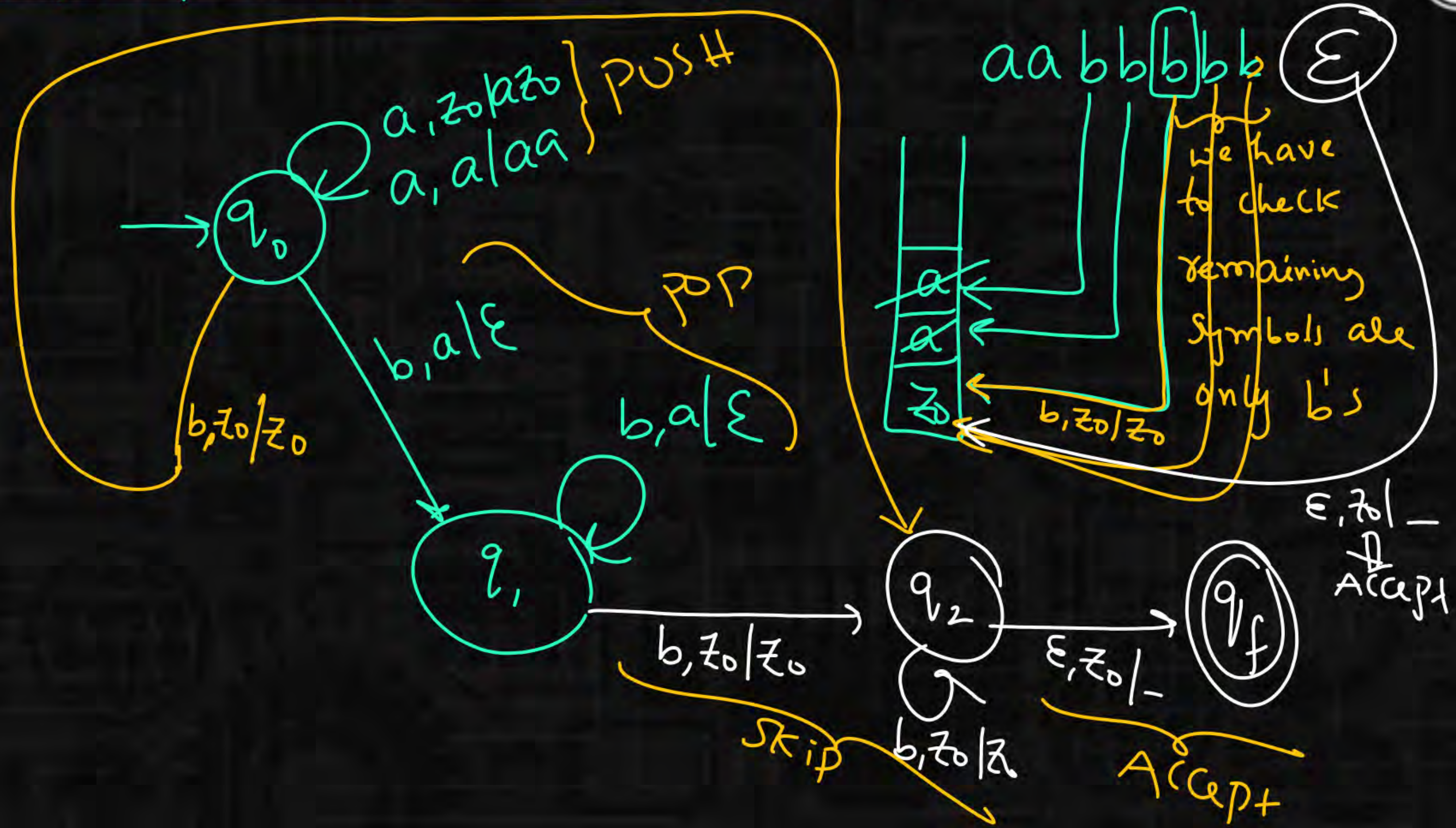
CFG  $\Rightarrow$  PDA  
Algorithm





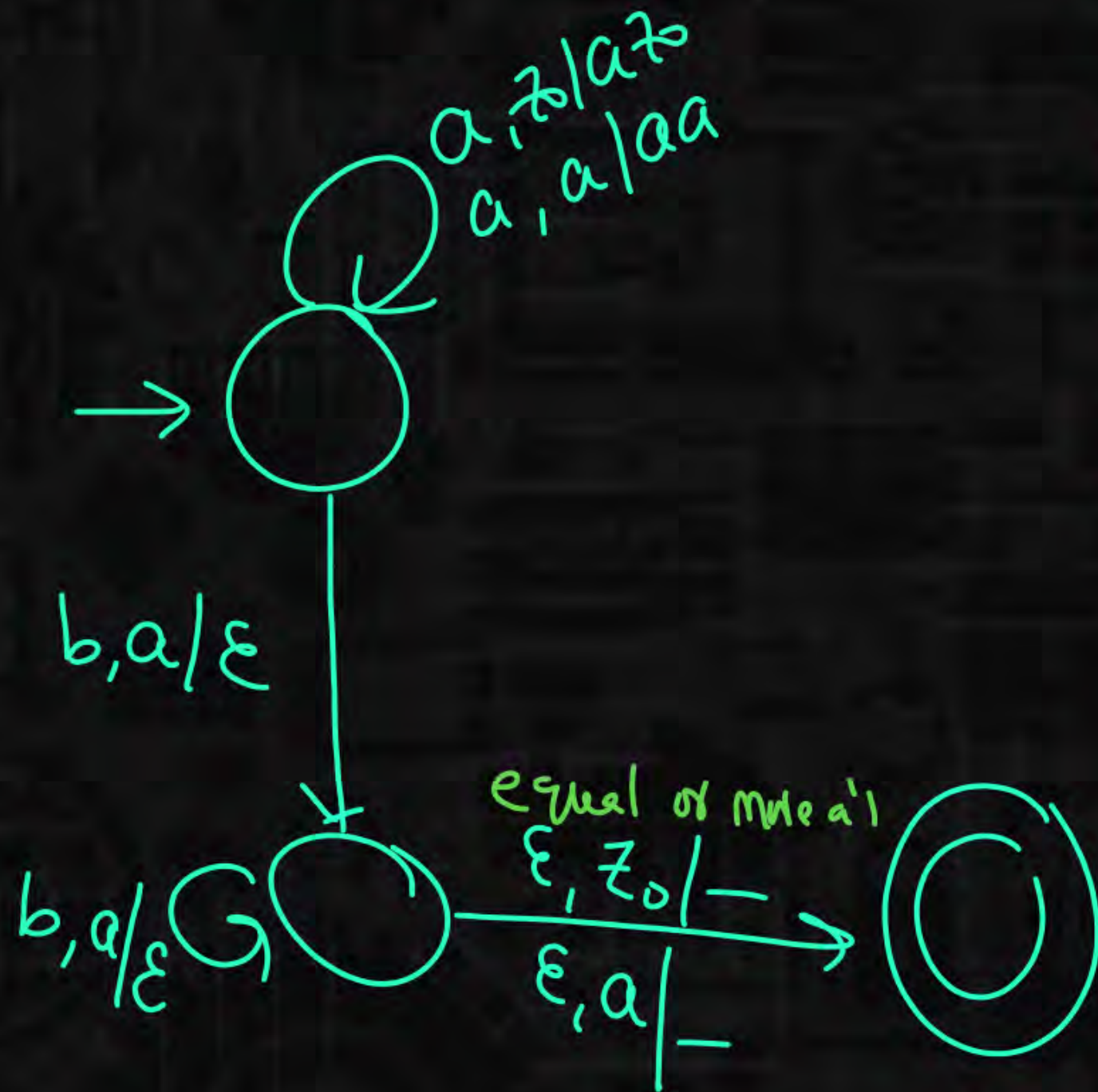


⑤  $\{a^m b^n \mid m < n, m, n \geq 0\} = \textcircled{4} \cup b^+$

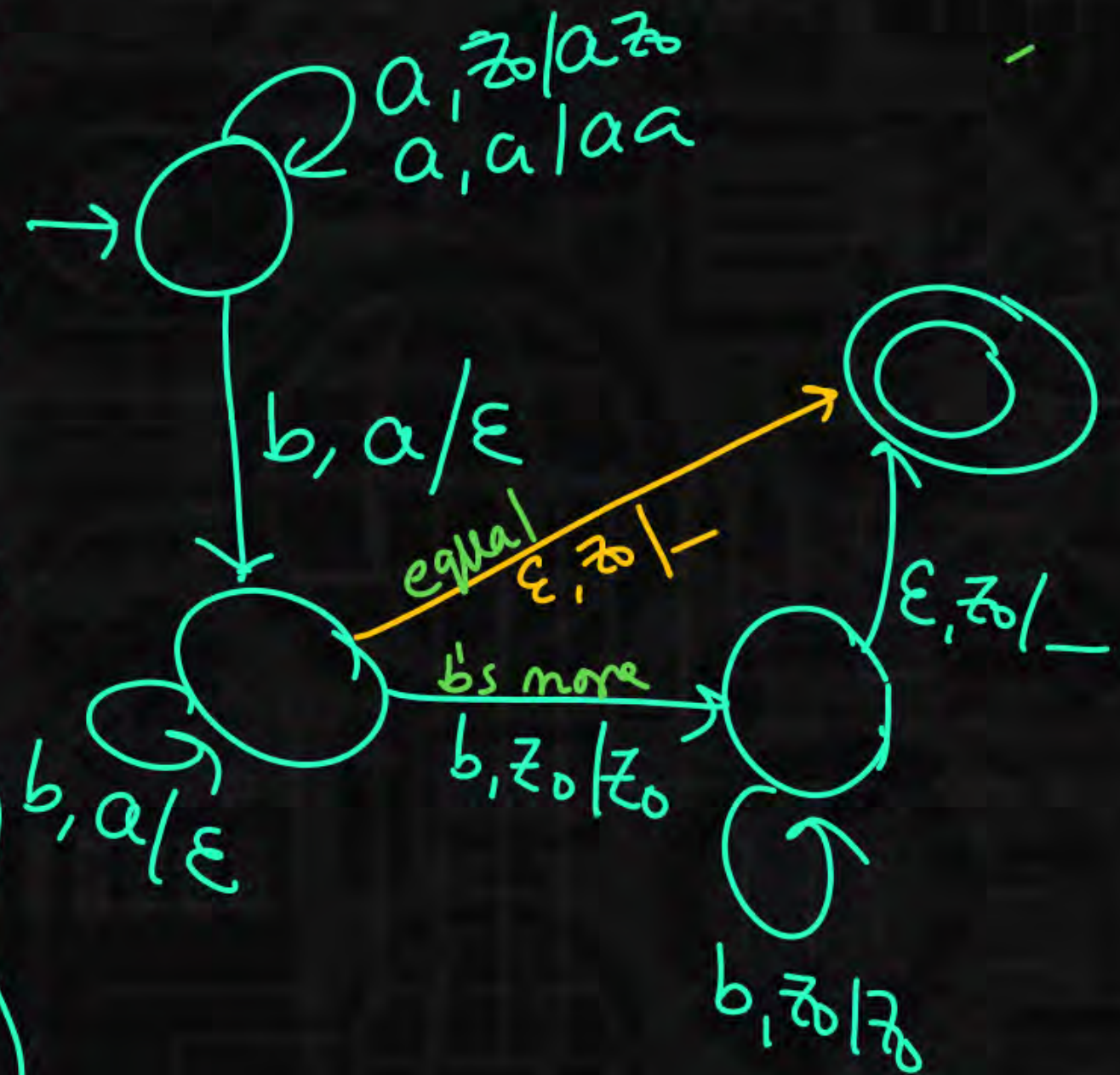




⑥  $\{a^m b^n \mid m \geq n, m, n \geq 1\}$   
 $m > n$  or  $m = n$

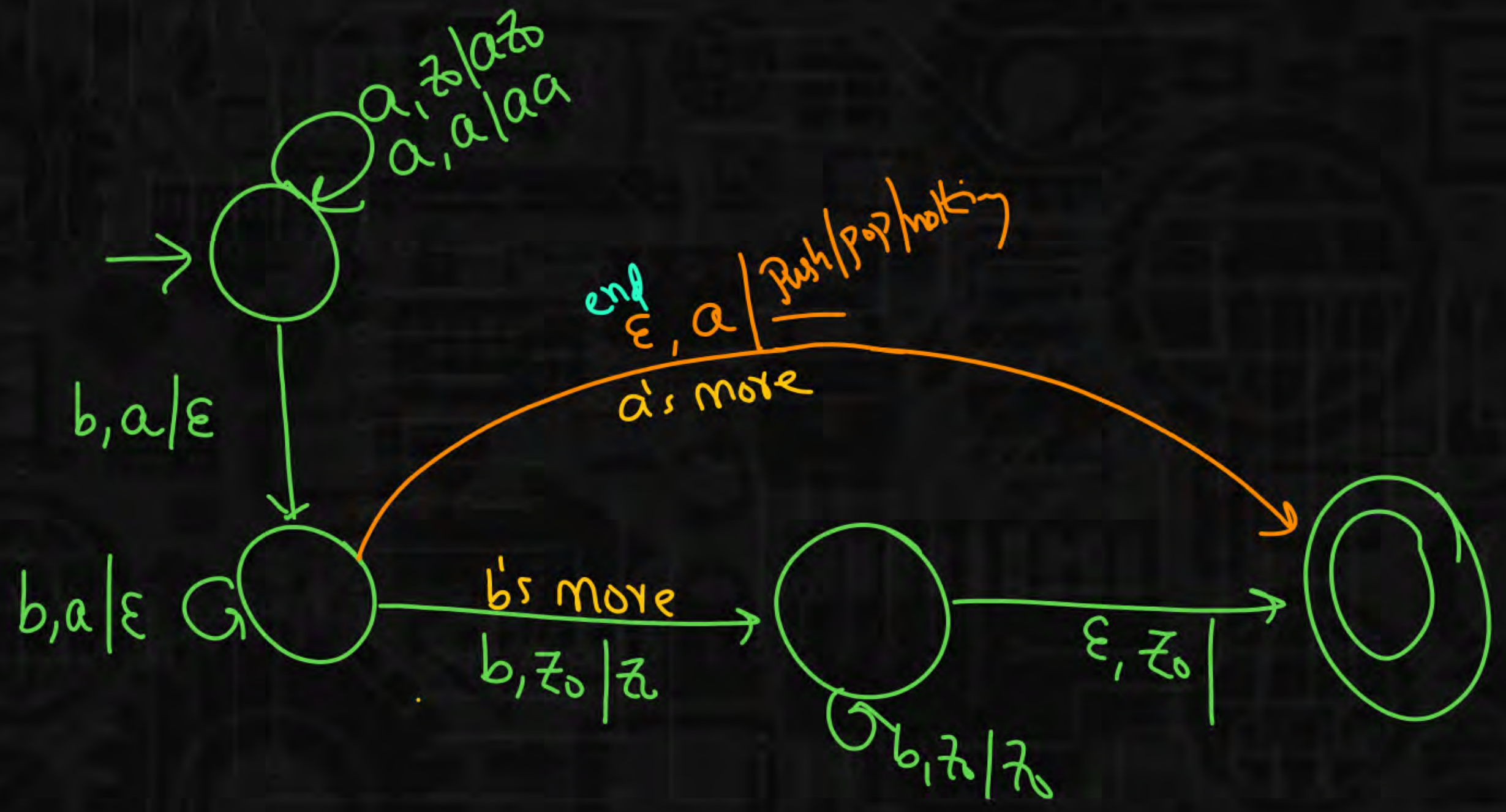


⑦  $\{a^m b^n \mid m \leq n, m, n \geq 1\}$   
 $m < n$  or  $m = n$



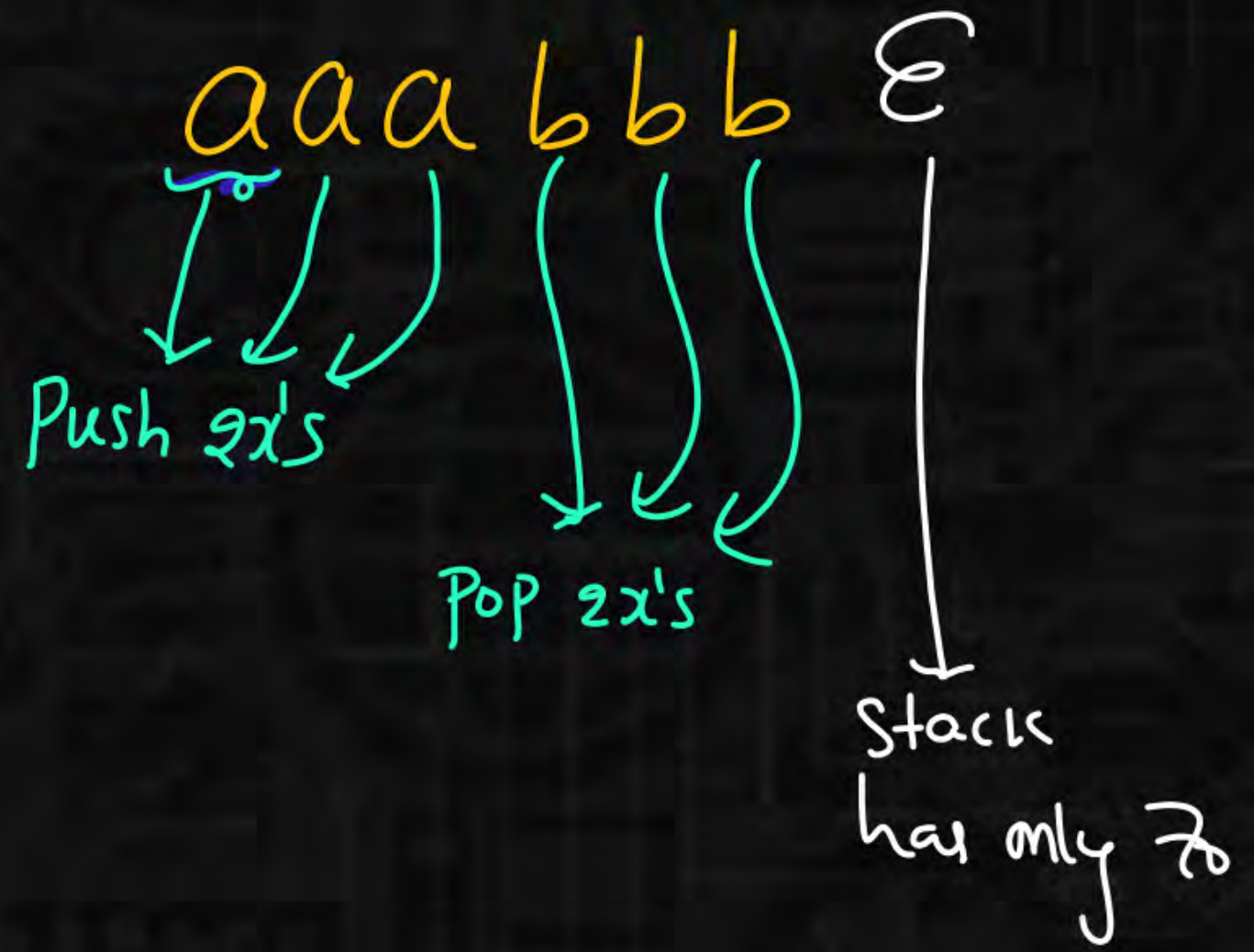
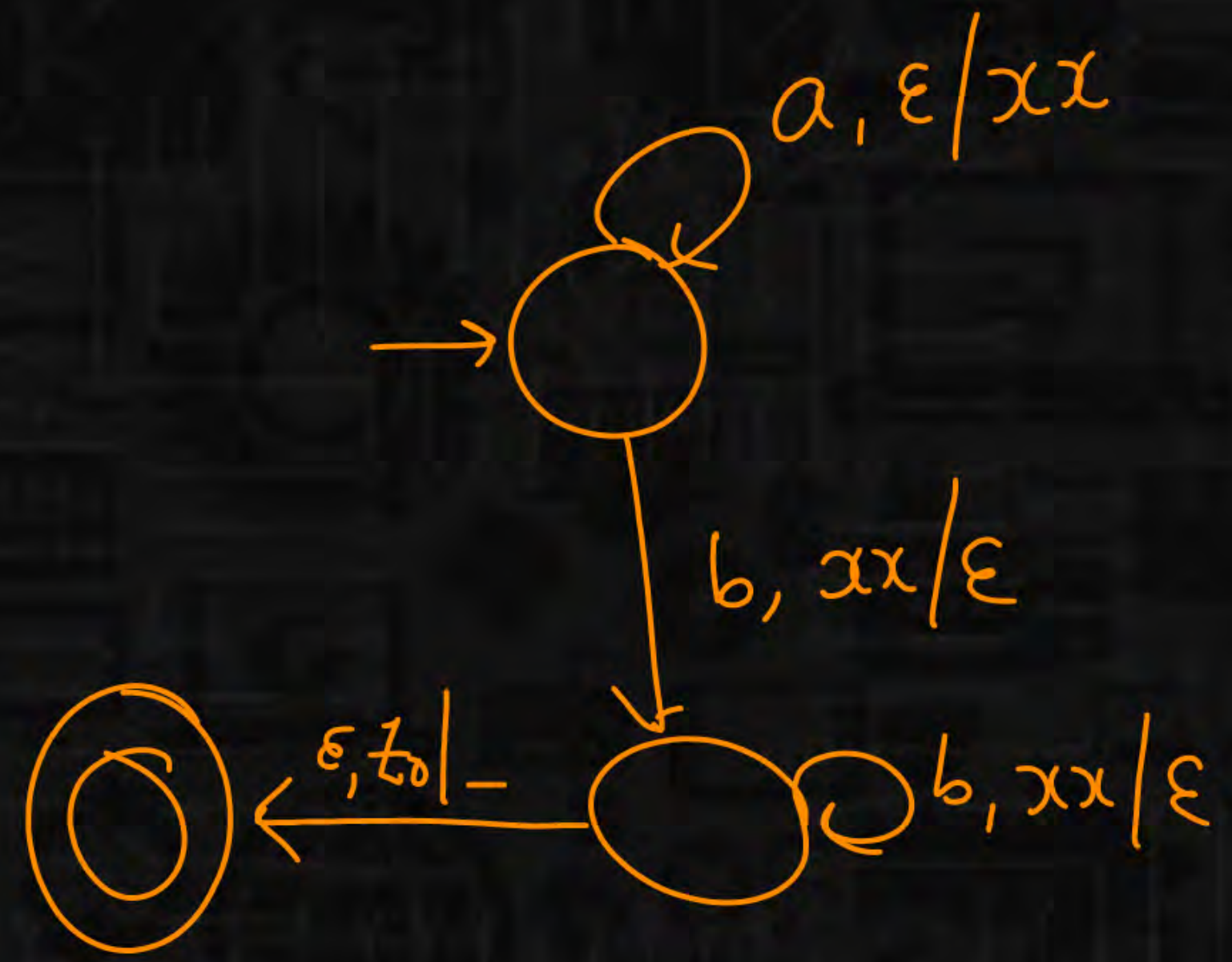


⑧  $\{a^m b^n \mid m \neq n, m, n \geq 1\}$   
 $m > n$  or  $m < n$





⑨  $\{a^n b^n \mid n \geq 1\}$

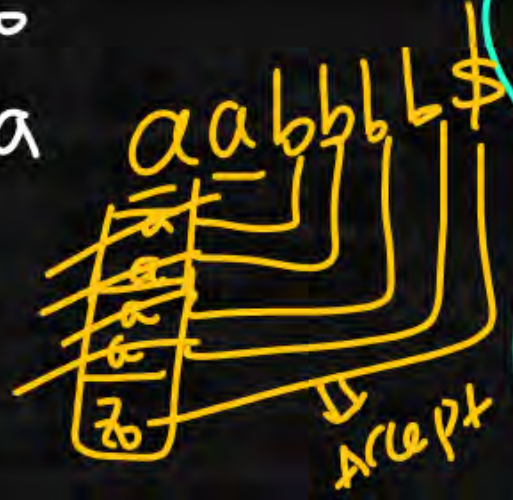
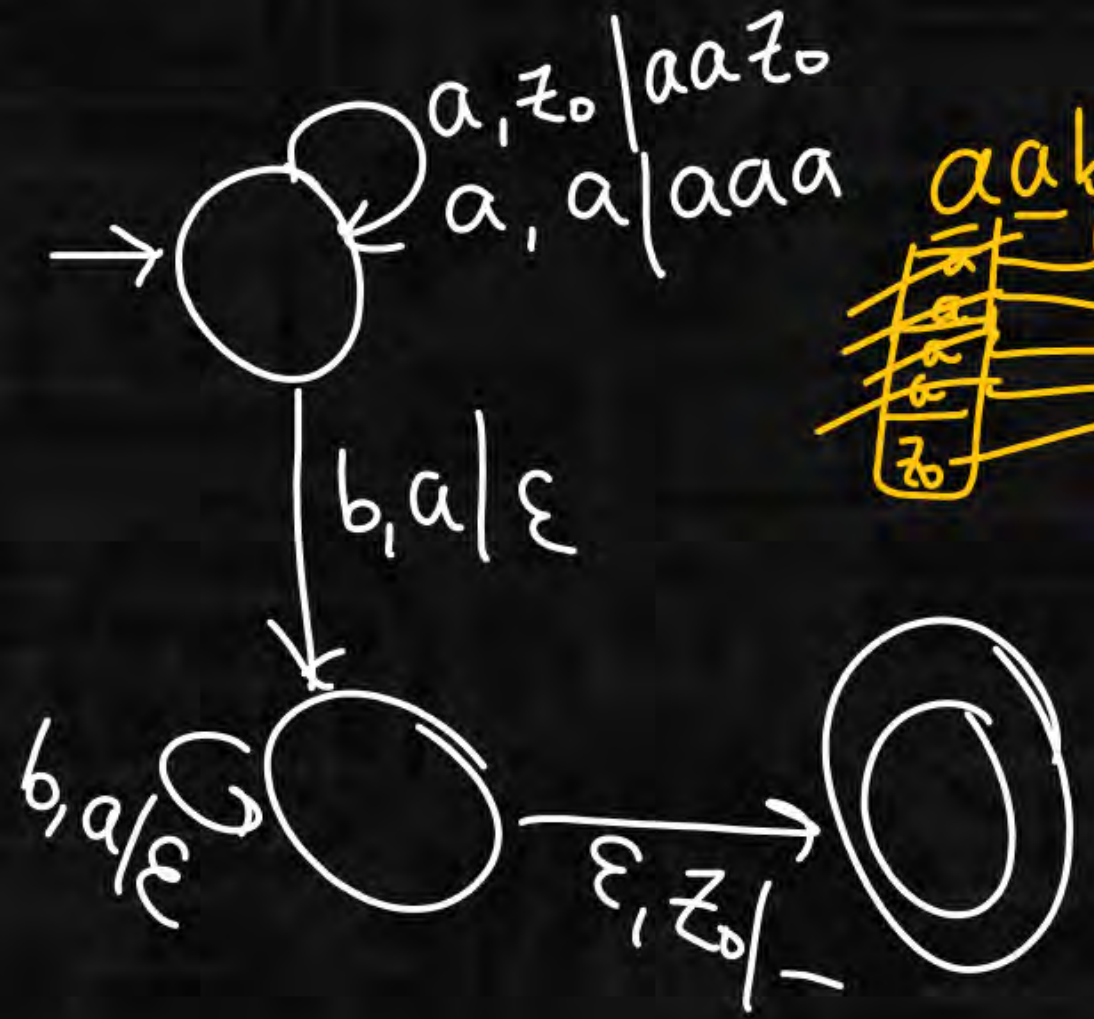




⑩  $\{a^n b^{2n} \mid n \geq 1\}$

Each a,  
Push 2a's

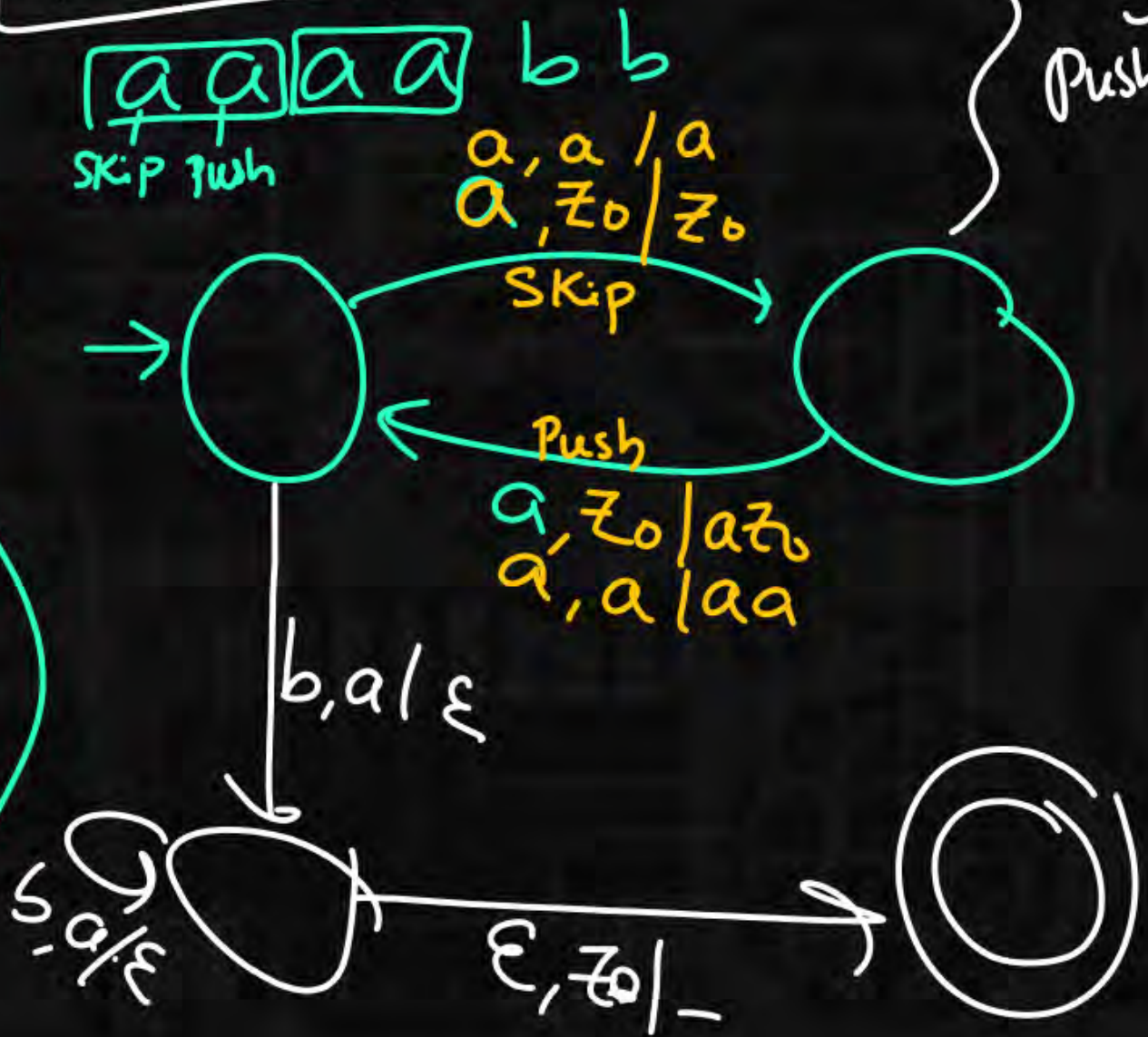
Each b,  
Pop 1b



⑪  $\{a^{2n} b^n \mid n \geq 1\}$

Every 2a's  
Push 1a

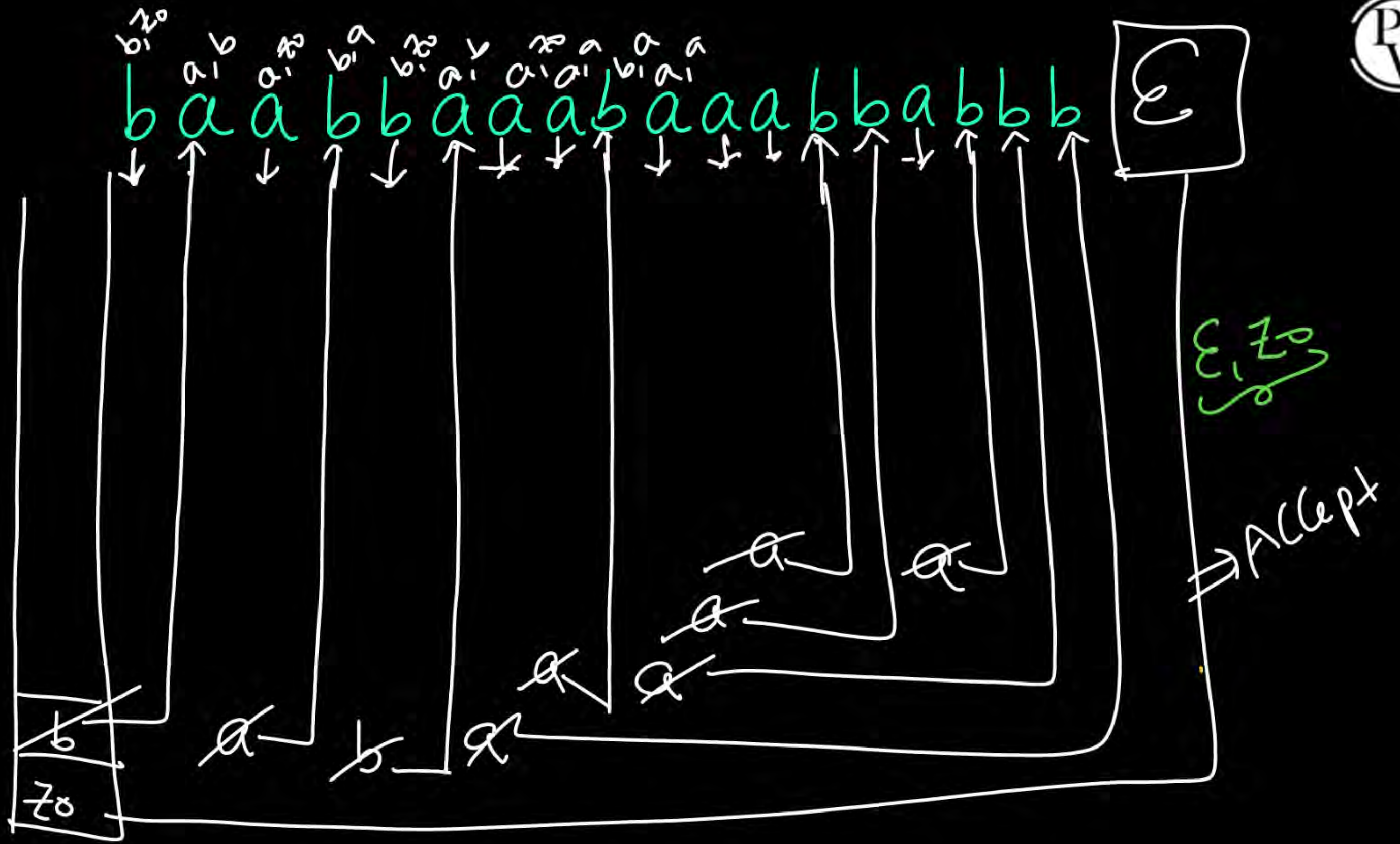
Every b,  
Pop 1a



Other logic:  
 $a^{2n} b^n$   
 $\downarrow$   
 Every a, Push 1a  
 $\downarrow$   
 Every b, Pop 2a's

aaaa bb  
 skip push  
 $a, a / a$   
 $a, z_0 / z_0$   
 skip







baabbaaabaabbabb

I



I/p = a or b

Push

II



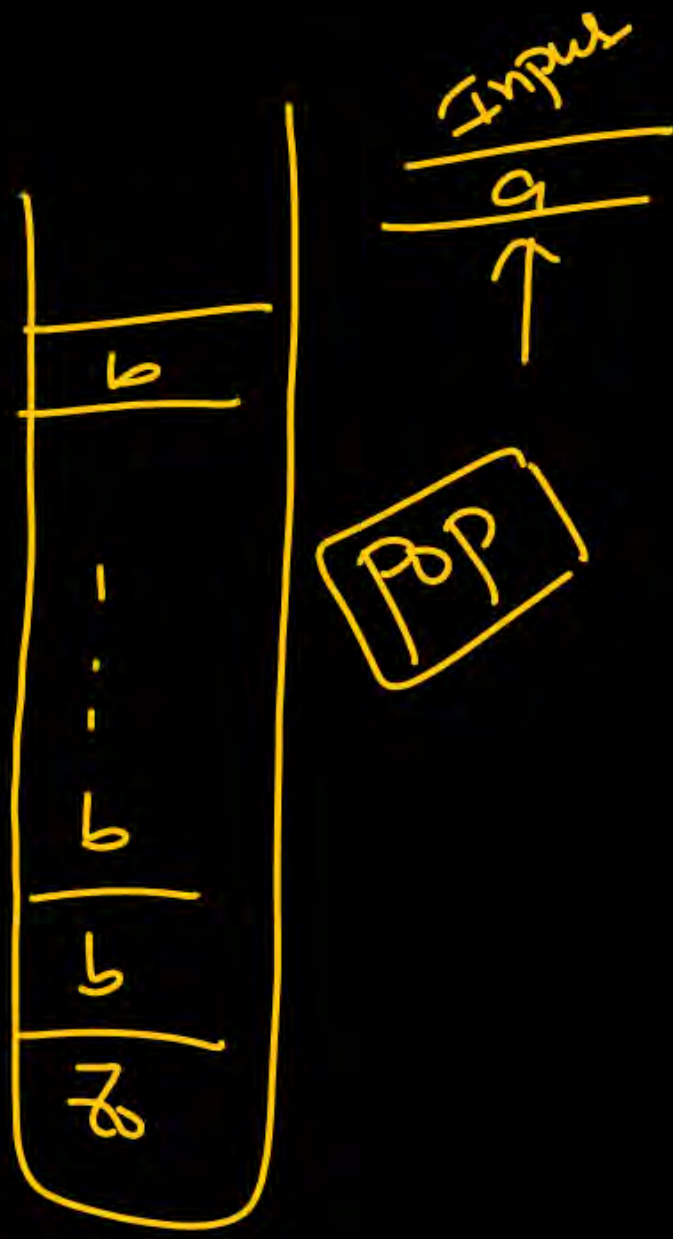
Input  
a  
↑  
Push

(OR)

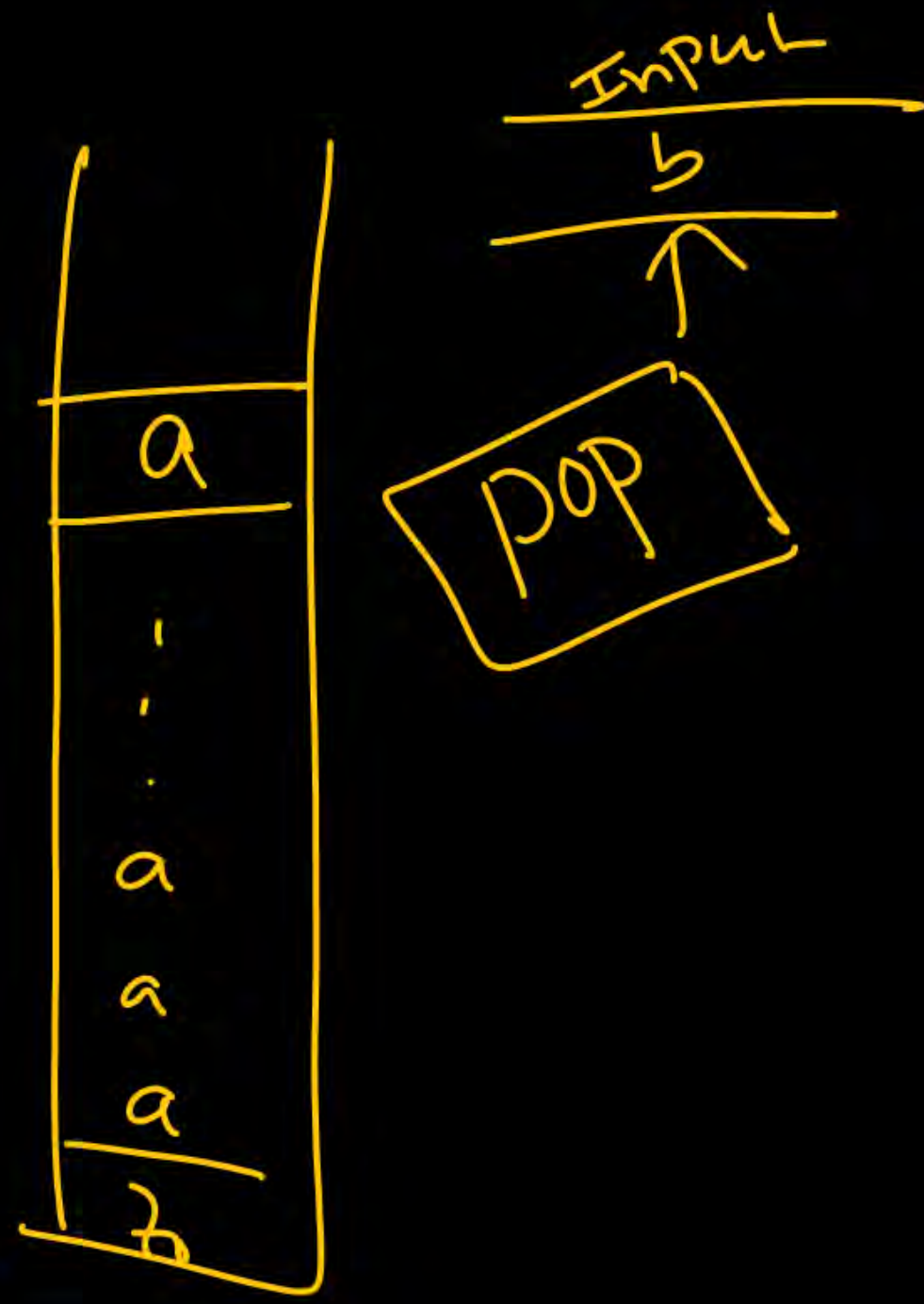


Input  
b  
↑  
Push





OR





12

12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

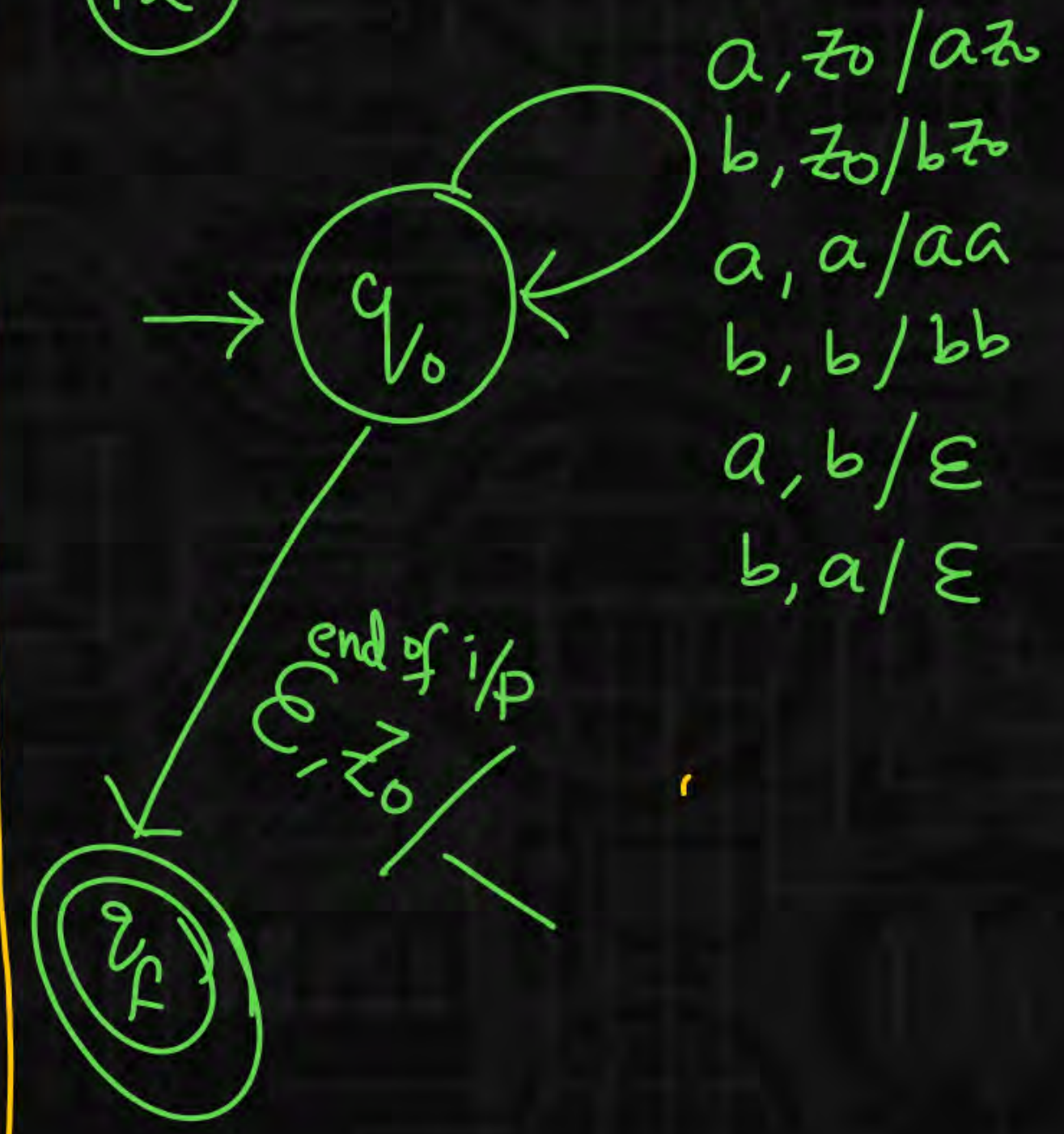
13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$

14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$

15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$

16)  $\{w \mid \text{ " }, n_a(w) \leq n_b(w)\}$

17)  $\{w \mid \text{ " }, n_a(w) \geq n_b(w)\}$





(12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

(13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$

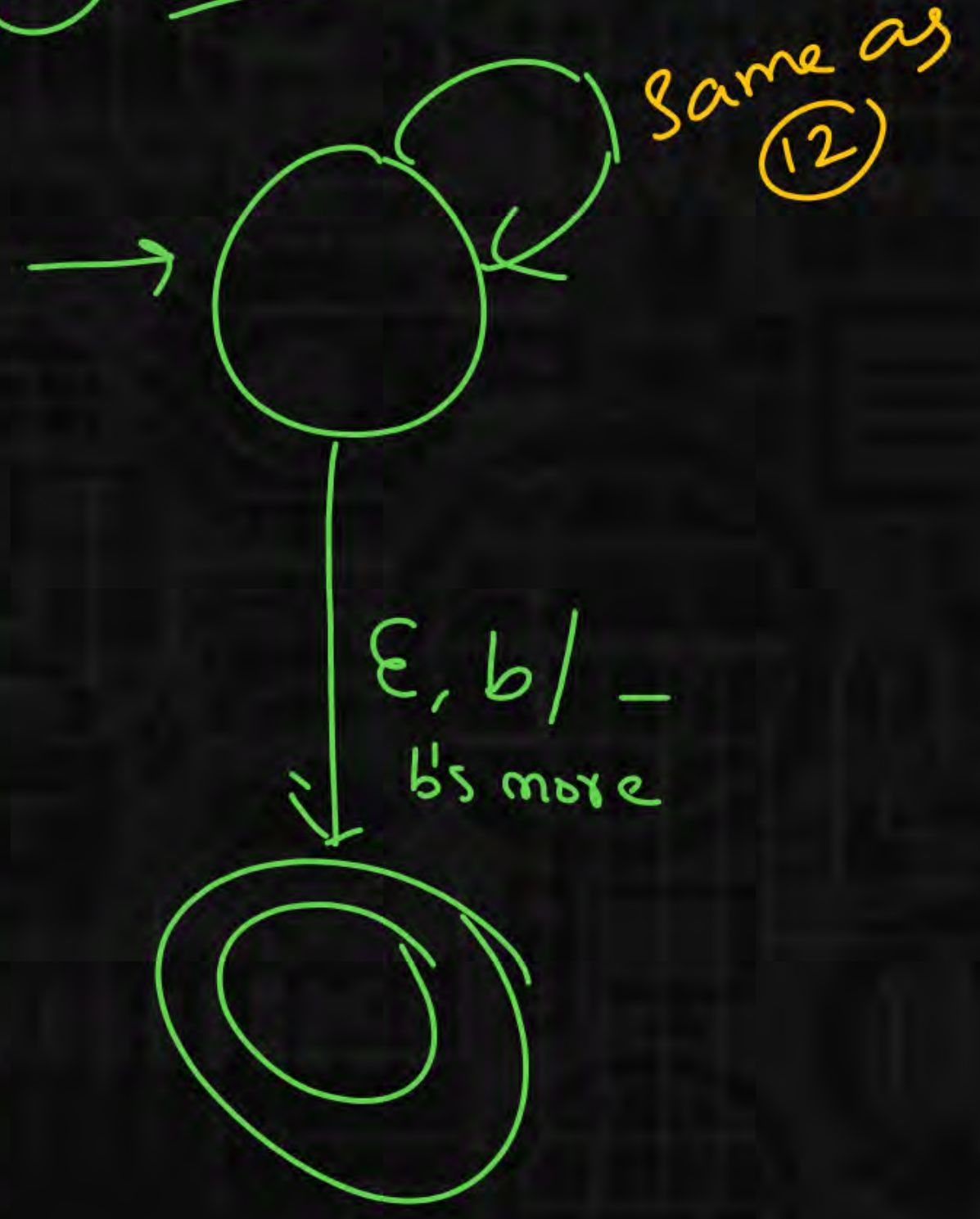
(14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$

(15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$

(16)  $\{w \mid \text{"}, n_a(w) \leq n_b(w)\}$

(17)  $\{w \mid \text{"}, n_a(w) \geq n_b(w)\}$

(13) b's more

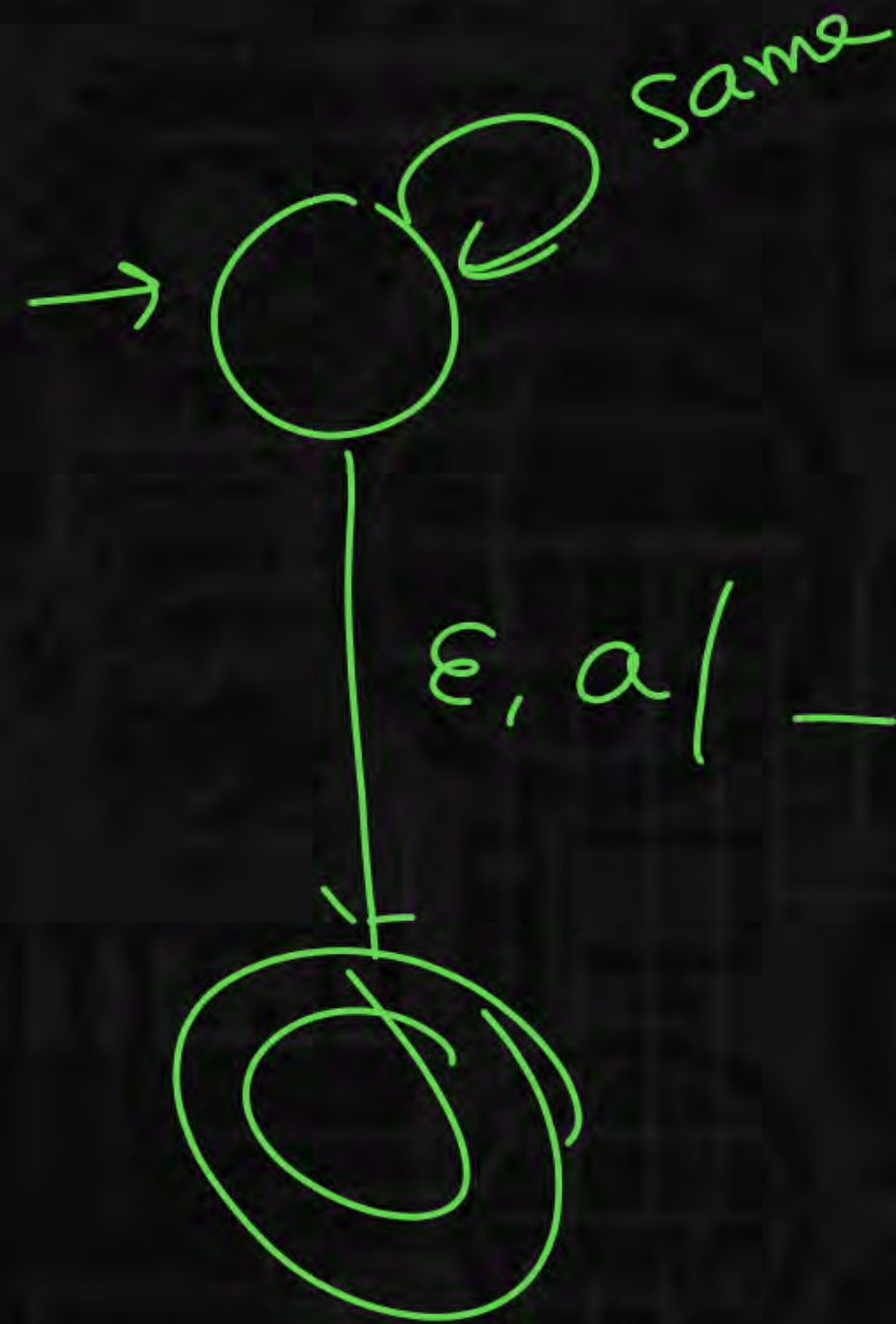




- (12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$
- (13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$
- (14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$
- (15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$
- (16)  $\{w \mid \text{"}, n_a(w) \leq n_b(w)\}$
- (17)  $\{w \mid \text{"}, n_a(w) \geq n_b(w)\}$

(14)

a's more





(12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

(13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$

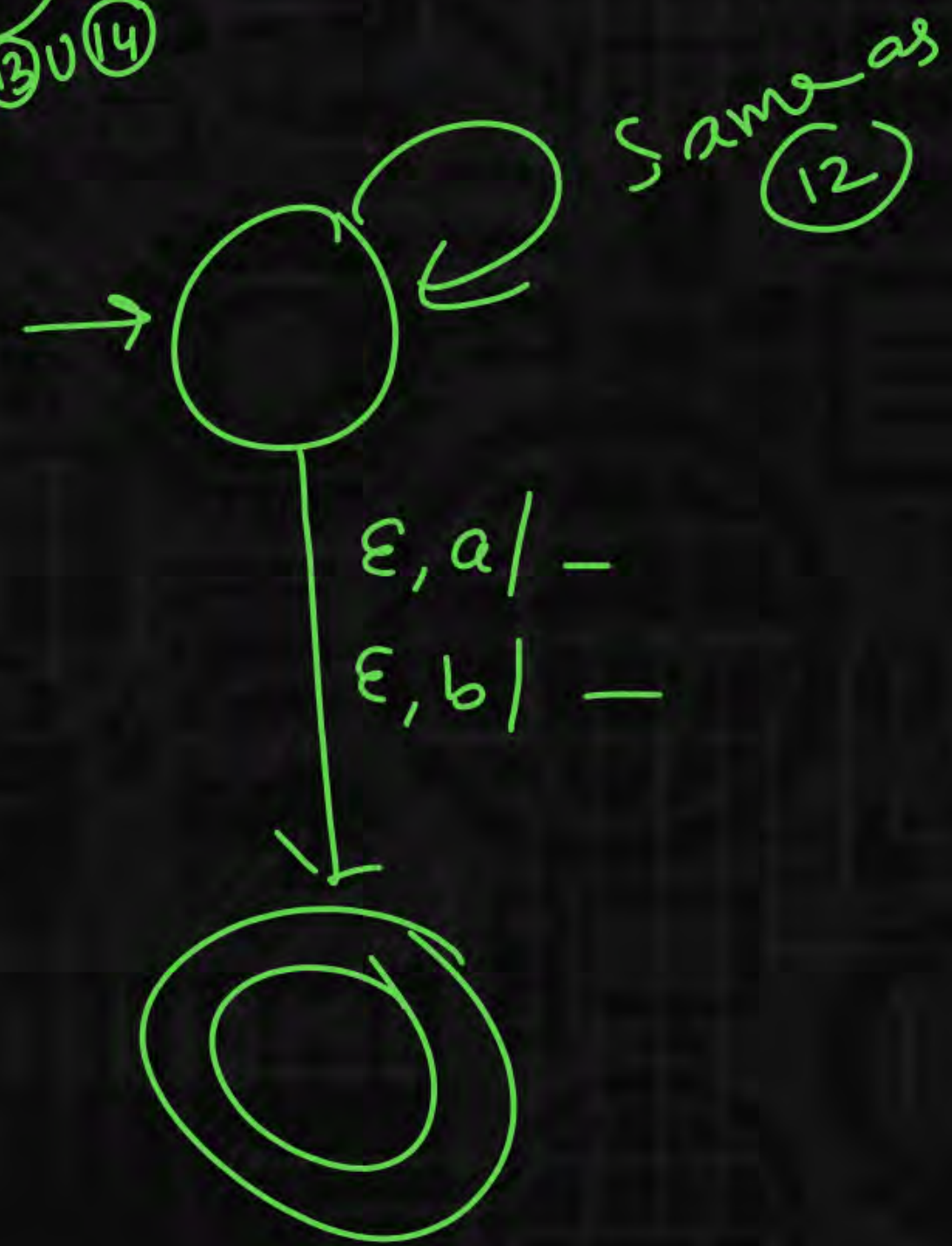
(14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$

(15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$

(16)  $\{w \mid \text{"}, n_a(w) \leq n_b(w)\}$

(17)  $\{w \mid \text{"}, n_a(w) \geq n_b(w)\}$

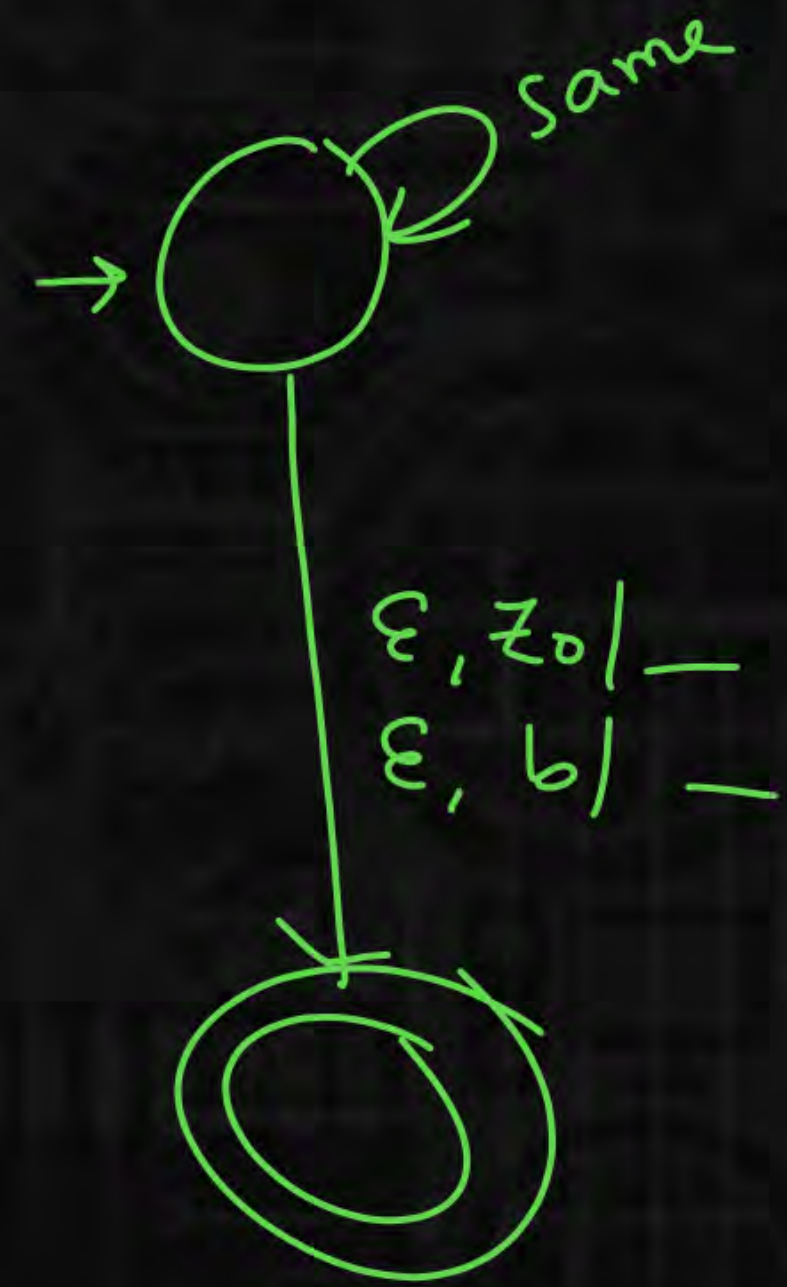
(15) Either a's more or b's more  
 $= (13) \cup (14)$





- (12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$
- (13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$
- (14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$
- (15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$
- (16)  $\{w \mid \text{"}, n_a(w) \leq n_b(w)\}$
- (17)  $\{w \mid \text{"}, n_a(w) \geq n_b(w)\}$

(16)  $n_a \leq n_b$





(12)  $\{w \mid w \in \{a,b\}^*, n_a(w) = n_b(w)\}$

(13)  $\{w \mid w \in \{a,b\}^*, n_a(w) < n_b(w)\}$

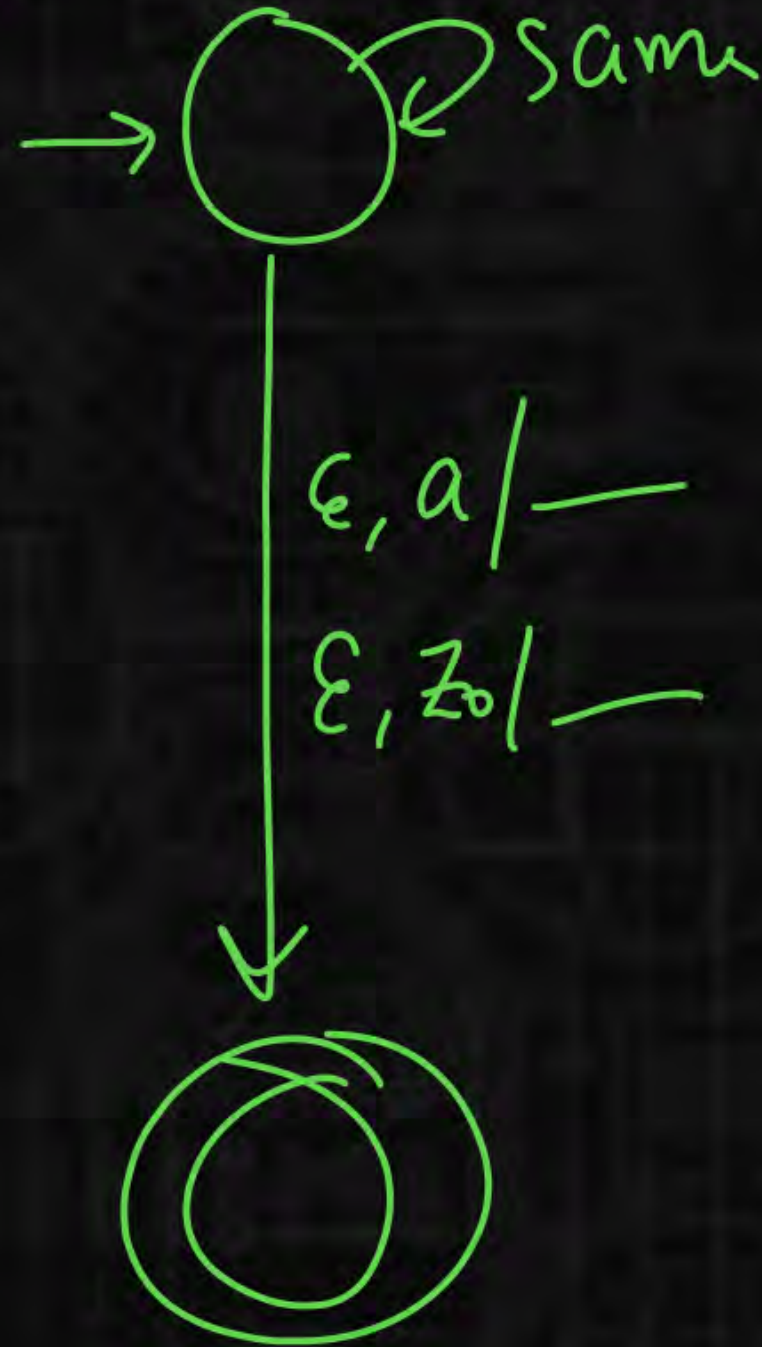
(14)  $\{w \mid w \in \{a,b\}^*, n_a(w) > n_b(w)\}$

(15)  $\{w \mid w \in \{a,b\}^*, n_a(w) \neq n_b(w)\}$

(16)  $\{w \mid \text{"}, n_a(w) \leq n_b(w)\}$   
 → (12)  $\cup$  (13)

(17)  $\{w \mid \text{"}, n_a(w) \geq n_b(w)\}$   
 = (12)  $\cup$  (14)

(17)





18  $\{w \# w^R \mid w \in \{a,b\}^*\}$

push

skip

pop

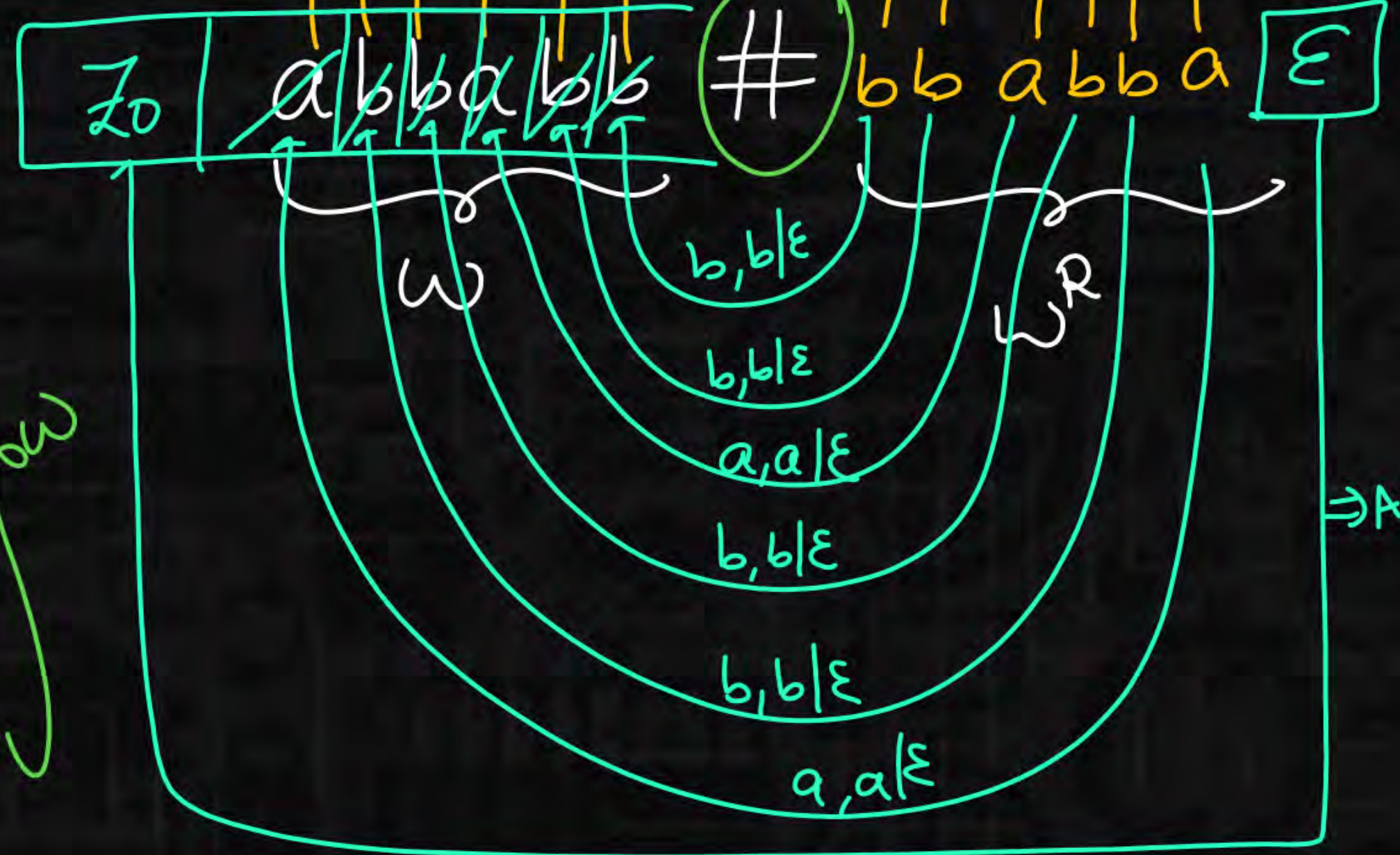


a, a/epsilon  
b, b/epsilon

a, z0 / a z0  
b, z0 / b z0  
a, a / a a  
a, b / a b  
b, a / b a  
b, b / b b

#, a / a  
#, b / b  
#, z0 / z0

b, b / epsilon  
a, a / epsilon



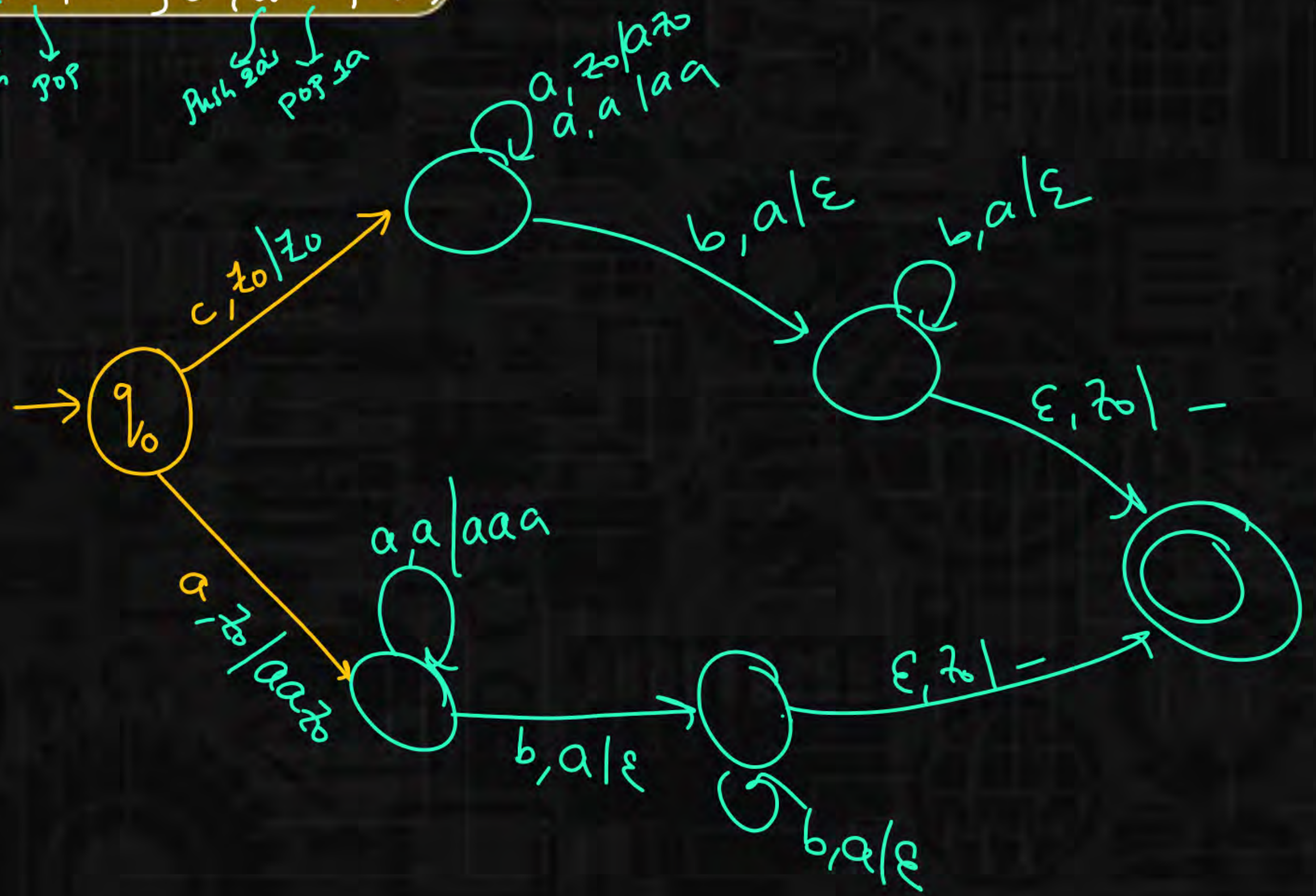
Accept



19  $\{c a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$

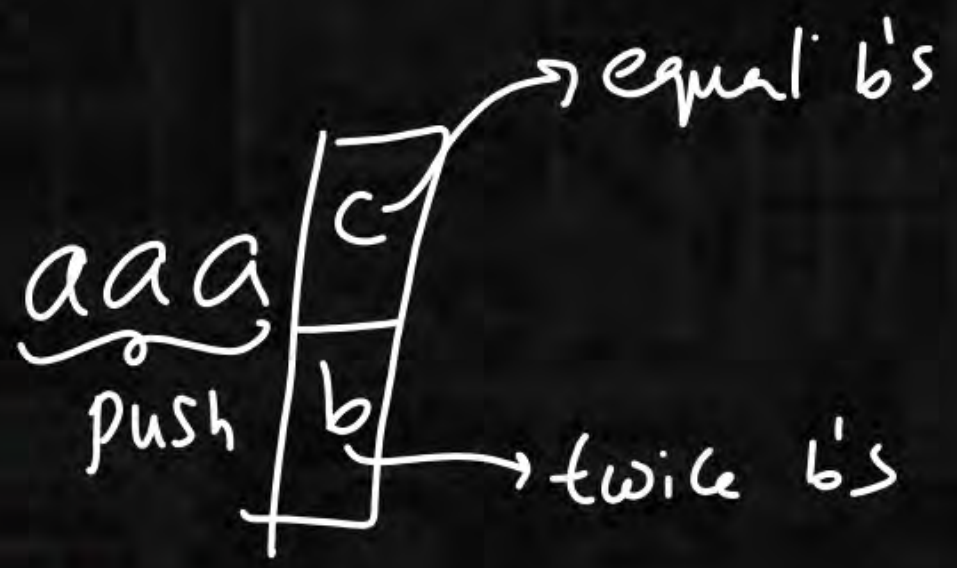
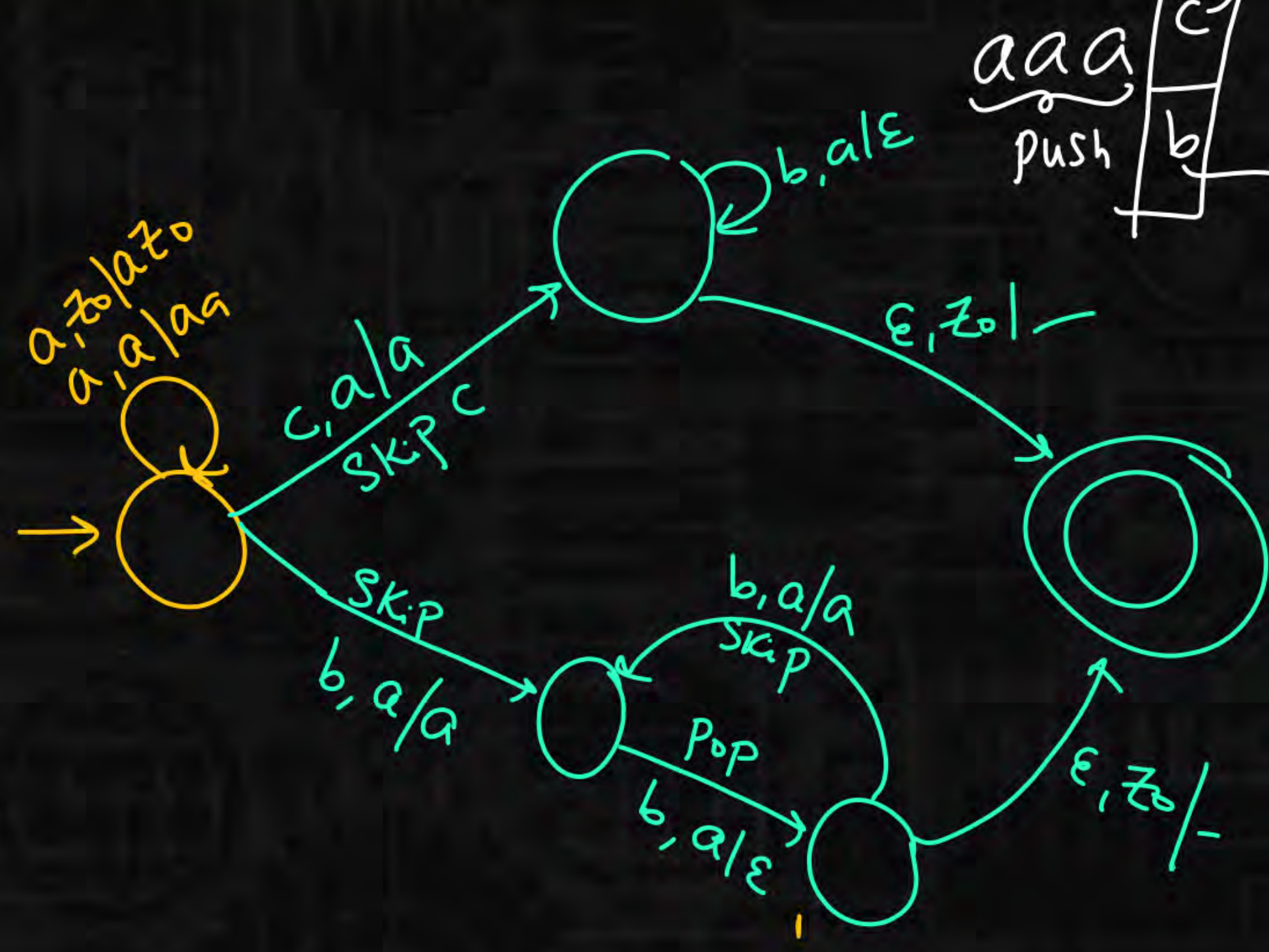
skip push pop

push  $2a$  pop  $1a$





20  $\{a^n c b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$





First 20 languages are DCFLs (so, CFLs)

① to ②①



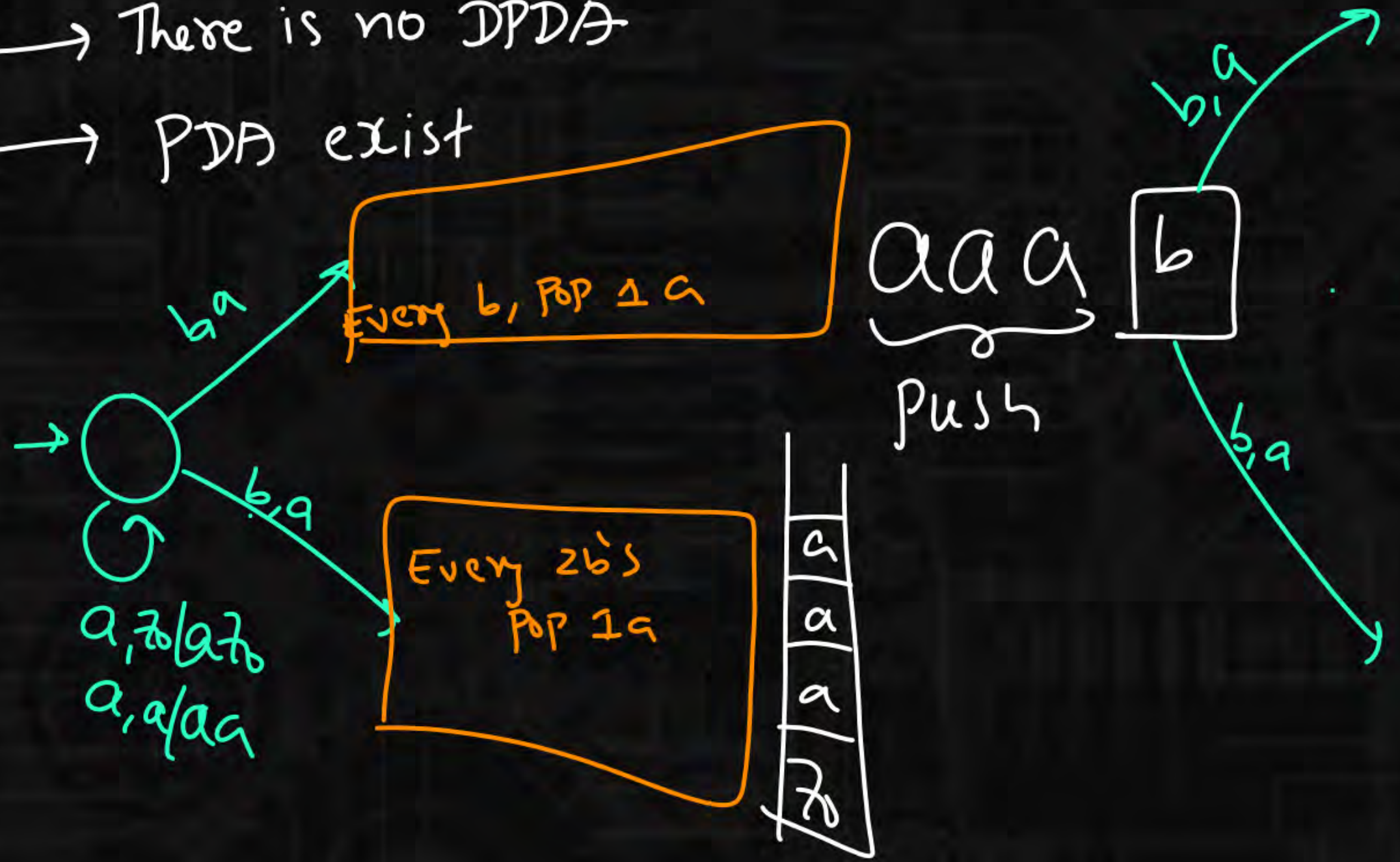
Every DCFL is CFL

CFL need not be DCFL



②  $\{a^n b^n c \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$  is not DCF

- CFL but not DCF
- There is no DPDA
- PDA exist



$Q \times \Sigma \times \Gamma \rightarrow$



8PM-10PM

(21)

$$\{a^n b^n c \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$$

(22)

$$\{a^m b^n \mid m=n \text{ OR } m=2n, m, n \geq 1\}$$

(23)

$$\{a^m b^n \mid n=m \text{ OR } n=2m, m, n \geq 1\}$$

(24)

$$\{a^m \boxed{b^n} c^k \mid m=n \text{ OR } m=k, m, n, k \geq 1\}$$

(25)

$$\{a^m \boxed{b^n} c^k \mid m=n \text{ OR } n=k, m, n, k \geq 1\}$$

(26)

$$\{a^m \boxed{b^n} c^k \mid m=k \text{ OR } n=k, m, n, k \geq 1\}$$

(27)

$$\{ww^R \mid w \in \{a, b\}^*\}$$

PDA exist  
CFs but not CFL

No PDA



$$S \rightarrow \underline{b} S \mid \underline{a b^* a} S \mid \epsilon$$

$$(b + a b^* a)^*$$

min DFA

no. of states = no. of eqv classes

$$S \rightarrow b S \mid a A \mid \epsilon$$

$$A \rightarrow \underline{a} S \mid \underline{b} A$$

$$A \equiv b^* a S$$



# Summary



→ PDA & DPDA construction

→ Next: CFLs, Vs, DCFLs

↘ CFL but not DCFL



