CS & IT ENGINEERING

Theory of Computation

Finite Automata:

NFA / DFA-4

Lecture No. 8

By- DEVA Sir

TOPICS TO BE COVERED

01 Remaining DFA Models

I, II, III ✓

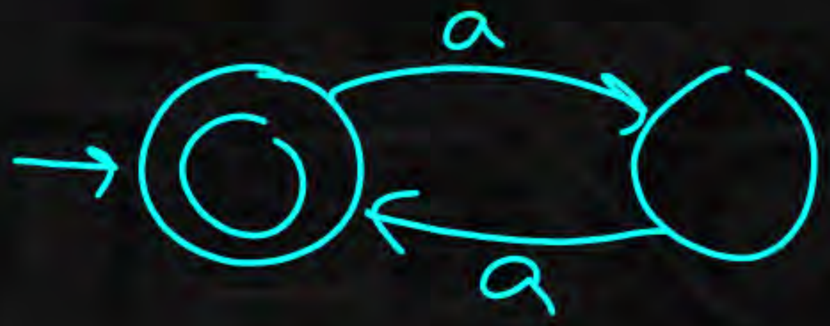
IV

↓

DFA Construction

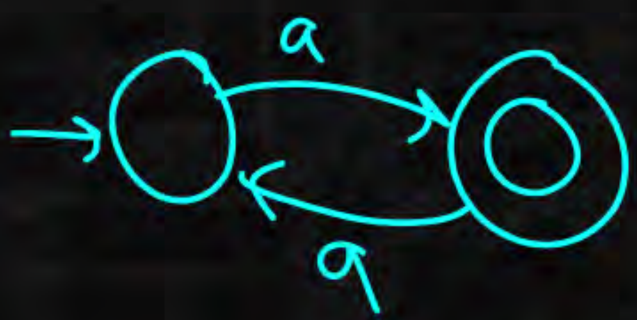
Model-IV [language over 1 symbol, forms Arithmetic progression]

(27) $a^* = \{a^n \mid n \geq 0\}$ → 

$L =$ (28) $(aa)^* = \{a^{2n} \mid n \geq 0\}$ → 

$= \{\varepsilon, a^2, a^4, a^6, \ldots\}$

$\bar{L} =$ (29) $a(aa)^* = \{a^{2n+1} \mid n \geq 0\}$

$= \{a, a^3, a^5, a^7, \ldots\}$ → 

(30) $\{a^{3n+2} \mid n \geq 0\}$

(31) $\{a^{3n+5} \mid n \geq 0\}$

A.P.

$0, 1, 2, 3, \ldots$

$0, 2, 4, 6, 8, \ldots$

$2 \quad 2 \quad 2$

$a^* \Rightarrow \boxed{\varepsilon}, a^1, a^2, a^3, a^4 \ldots$

$0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots$

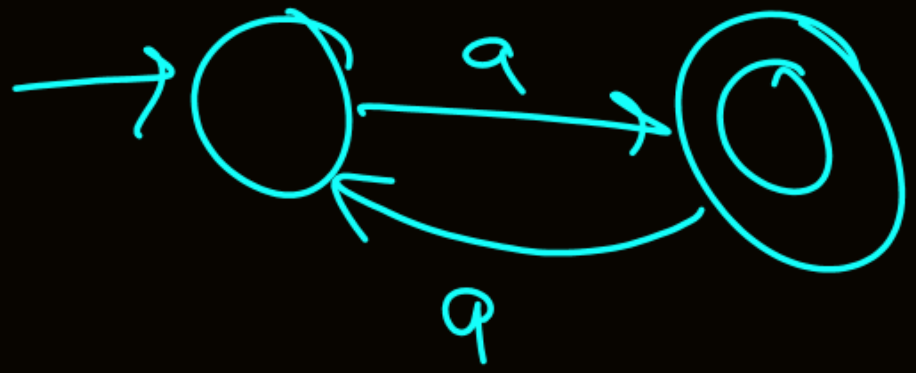
$$a(aa)^* \Longrightarrow a(aa)^0 = a$$

$$a(aa)^1 = a^3$$

$$a(aa)^2 = a(aa)^1(aa)^1$$

$$= a^5$$

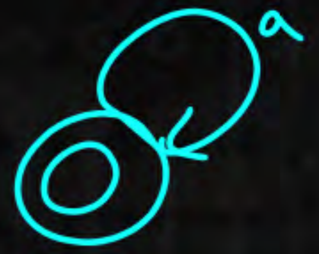
$$L = \{\boxed{a^1}, a^3, a^5, a^7, \ldots\}$$



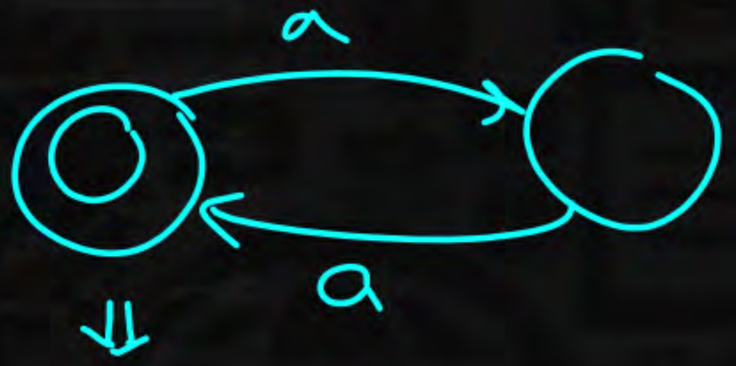
$$\underbrace{(aa)^*}_{\text{even a's}} + \underbrace{a(aa)^*}_{\text{odd a's}} = \underbrace{a^*}_{\text{All a's}}$$

DFA Construction

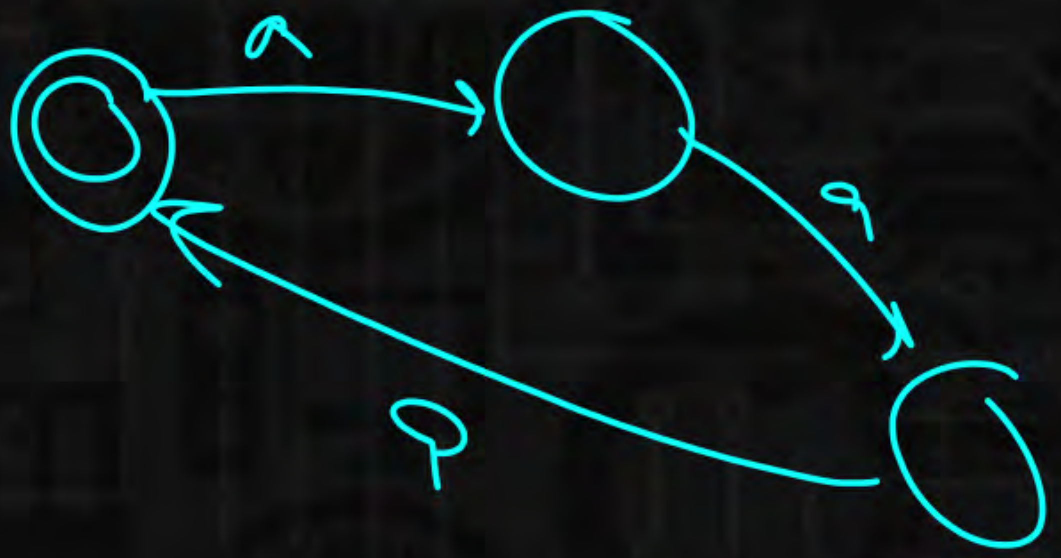
1



2



3



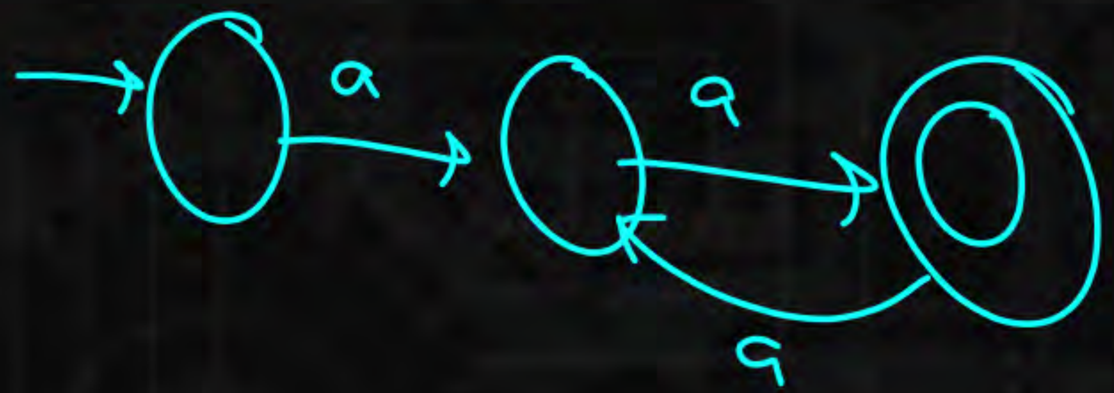
DFA Construction

30. $\left\{ a^{3n+2} \mid n \geq 0 \right\} = \left\{ a^{2}, a^{5}, a^{8}, a^{11}, a^{14}, \dots \right\}$

Min

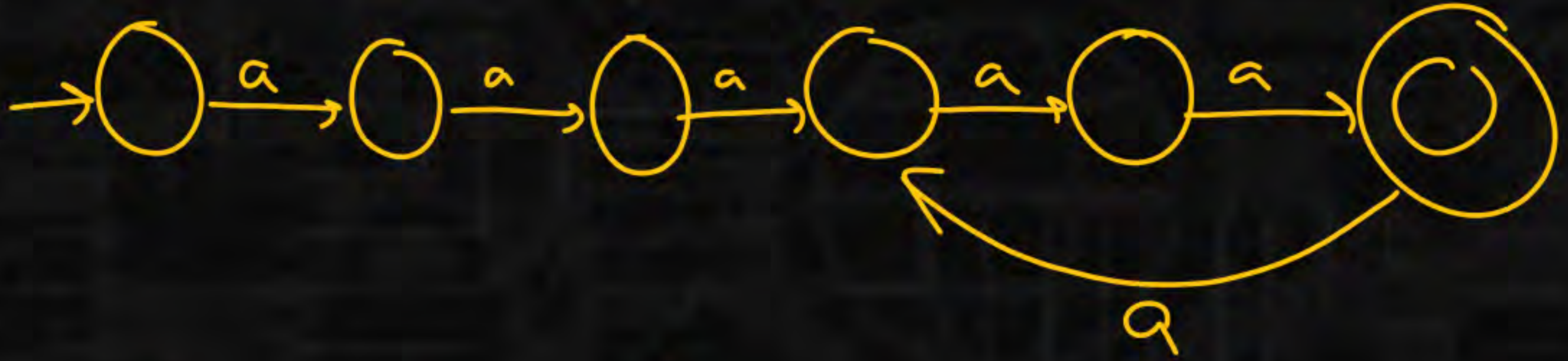
$n=0$ $n=1$ $n=2$

32. $\left\{ a^{2n+2} \mid n \geq 0 \right\}$



31 $\{a^{3n+5} \mid n \geq 0\} = \{\underbrace{a^5, a^8}_{3\text{ a's}}, \underbrace{a^8, a^{11}}_{3\text{ a's}}, \underbrace{a^{11}, a^{14}}_{3\text{ a's}}, \ldots\}$

$$\boxed{a^{3n+5} = a^{3n} \cdot a^5 = (aaa)^* a^5}$$



ε a a^2 a^3 a^4 a^5

a^6 a^7 a^8

a^9 a^{10} a^{11}

$a^{3n+3} \mid n \geq 0$ a^{3n+4} a^{3n+5}

DFA Construction

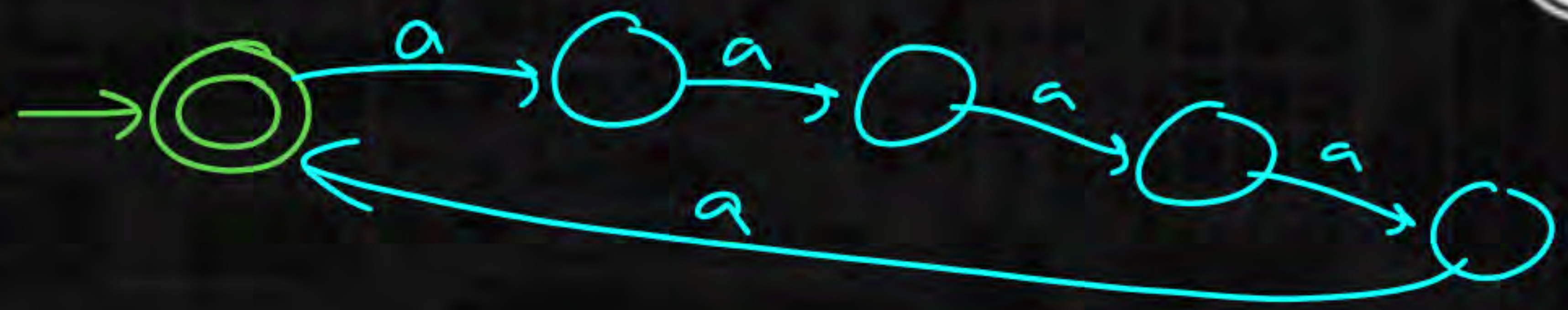
$5n = 5n + \boxed{0}$

$a^{k_1 n + k_2}$

$\boxed{k_1 > k_2}$

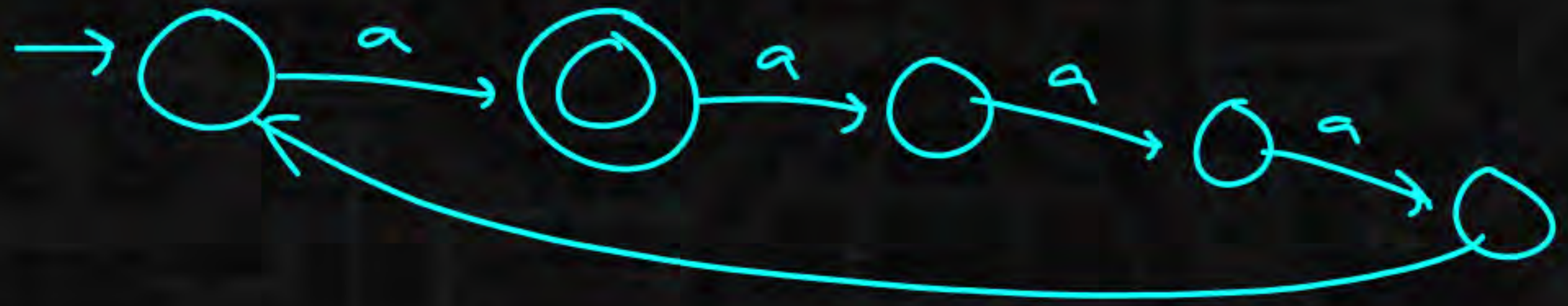
32 $\{a^{\boxed{5}n} \mid n \geq 0\} \Rightarrow$

5 states



33 $\{a^{\boxed{5}n+\boxed{1}} \mid n \geq 0\} \Rightarrow$

5 States



34 $\{a^{\boxed{5}n+\boxed{4}} \mid n \geq 0\}$

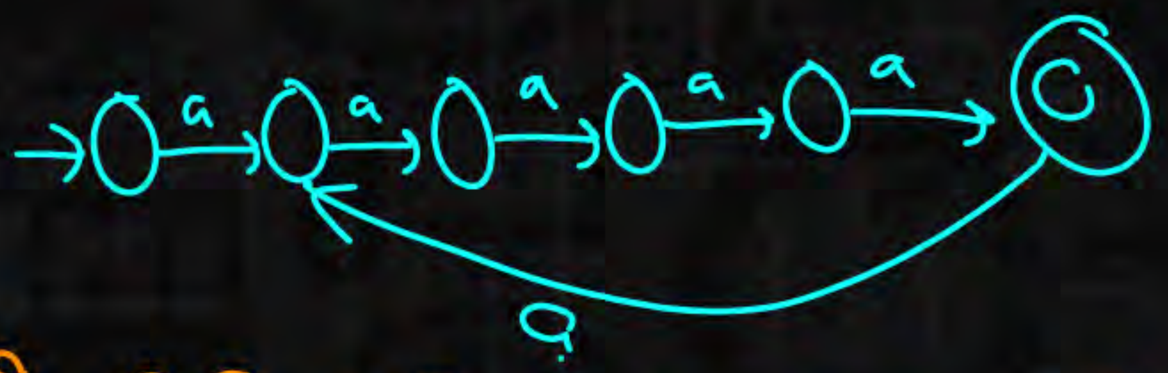
5 states

$a^{k_1 n + k_2}$

$\boxed{k_1 \leq k_2}$

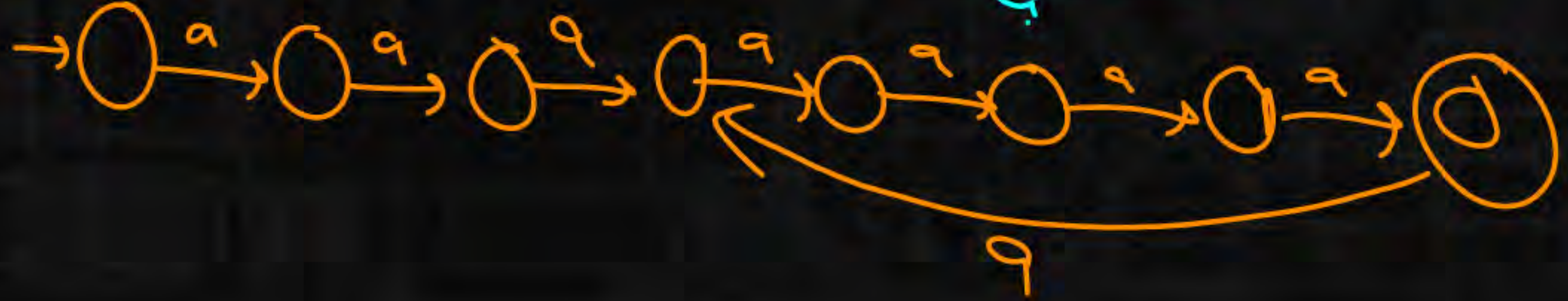
35 $\{a^{5n+\boxed{5}} \mid n \geq 0\} = \{a^5, a^{10}, a^{15}, \dots\}$

6 states



36 $\{a^{5n+\boxed{7}} \mid n \geq 0\}$

8 States



(37) $\{a^{2n} \mid n \geq 1\} = \{\boxed{a^2}, a^4, \ldots\} = \{a^{2n+2} \mid n \geq 0\} \Rightarrow$ 3 states

(38) $\{a^{3n+2} \mid n \geq 1\} = \{a^{3n+\boxed{5}} \mid n \geq 0\} \Rightarrow$ 6 states

(39) $\{a^{2n+1} \mid n \geq 1\} = \{a^{2n+\boxed{3}} \mid n \geq 0\} \Rightarrow$ 4 states

$$\left\{ a^{\boxed{K_1}n+\boxed{K_2}} \mid n \geq 0 \right\}$$

If $K_1 > K_2$

K_2 acts as remainder

$\Downarrow$

K_1 states in min DFA

If $K_1 \leq K_2$

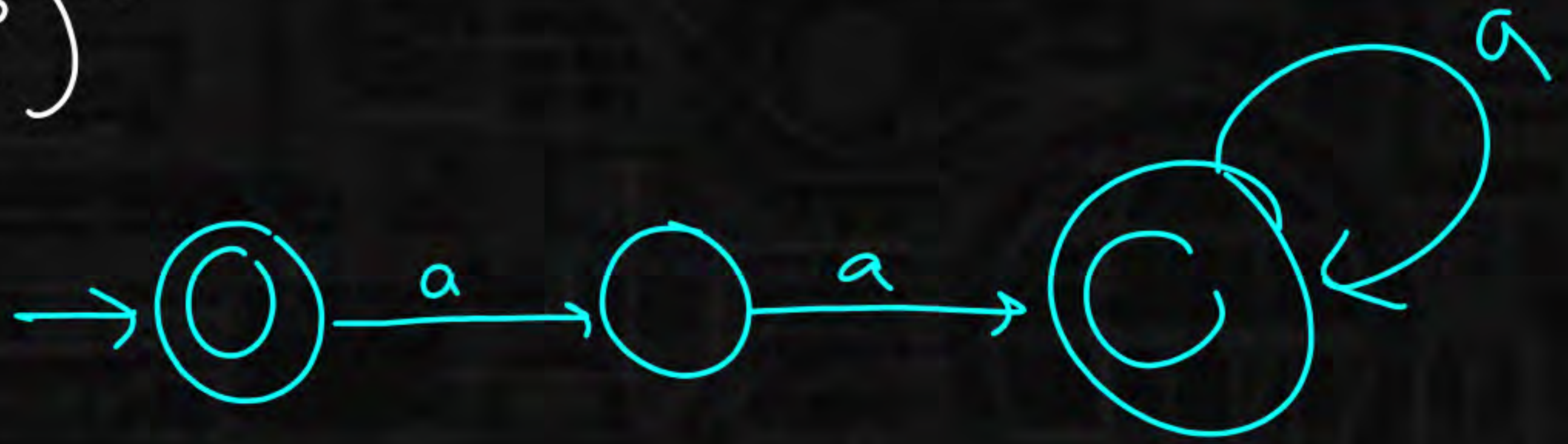
$\Downarrow$ (K_2 will not be remainder)

K_2+1 States

40 $(aa+aaa)^* = \{\boxed{\varepsilon}, a^2, a^3, a^4, a^5, \dots\}$

A.P. begins here

$(a^2+a^3)^*$



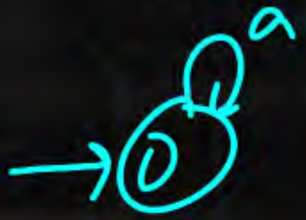
$L = \{a^n \mid n \neq 1\}$

$L = \{a^n \mid n=0 \text{ OR } n \geq 2\}$

$= \varepsilon + aaa^*$

$= \varepsilon + aa^+$

DFA Construction

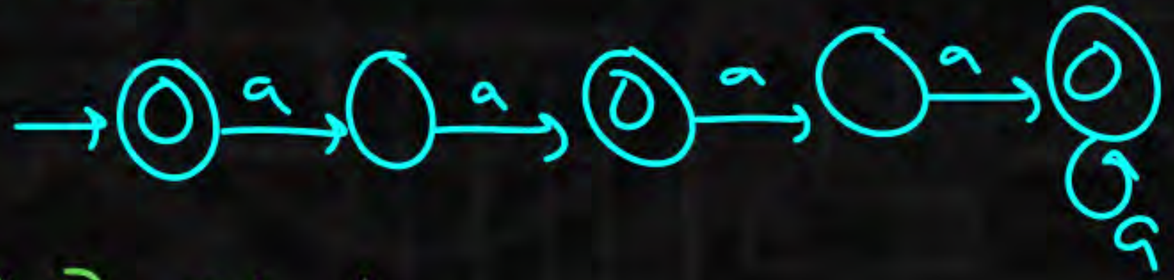
(41) $(a+aaa)^* = a^*$  $\Rightarrow$ 1 state

(42) $(aa+aaaa)^* = (aa)^* \Rightarrow$ 2 states 

A.P. begins

(43) $(aa+aaaaa)^* = \{\varepsilon, a^2, \boxed{a^4}, a^5, a^6, a^7, a^8, a^9, \dots\} \Rightarrow$ 5 states

(44) $(aaa+aaaa)^*$



$= \{\varepsilon, a^3, a^4, \boxed{a^6}, a^7, a^8, a^9, \dots\} \Rightarrow$ 7 States

$$(aaa+aaaa)^* = (a^3+a^4)^*$$

$$= (a^3+a^4)^0 + (a^3+a^4)^1 + \underbrace{(a^3+a^4)^2}_{} + \dots$$

$a^3 . a^3 = a^6$

$a^3 . a^4 = a^7$

$a^4 . a^3 = a^7$

$a^4 . a^4 = a^8$

✓ $\varepsilon \leftarrow (\quad)^0$

× $a \leftarrow$ ×

× $a^2 \leftarrow$ ×

✓ $a^3 \leftarrow (a^3+a^4)^1$

✓ $a^4 \leftarrow (a^3+a^4)^1$

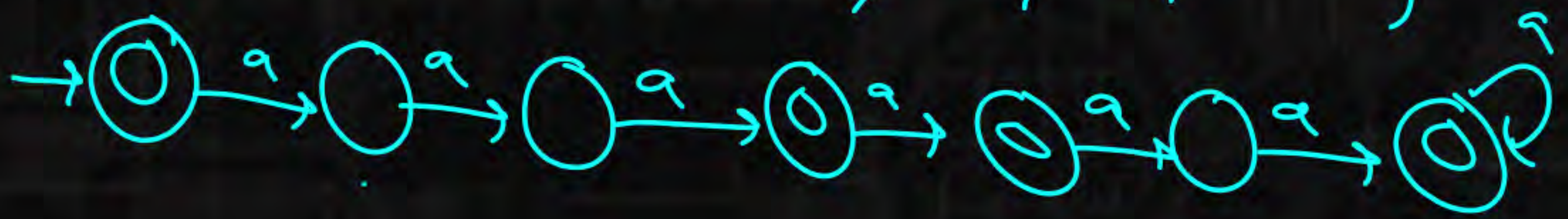
× $a^5 \leftarrow$ ×

✓ $\boxed{a^6}$ ✓

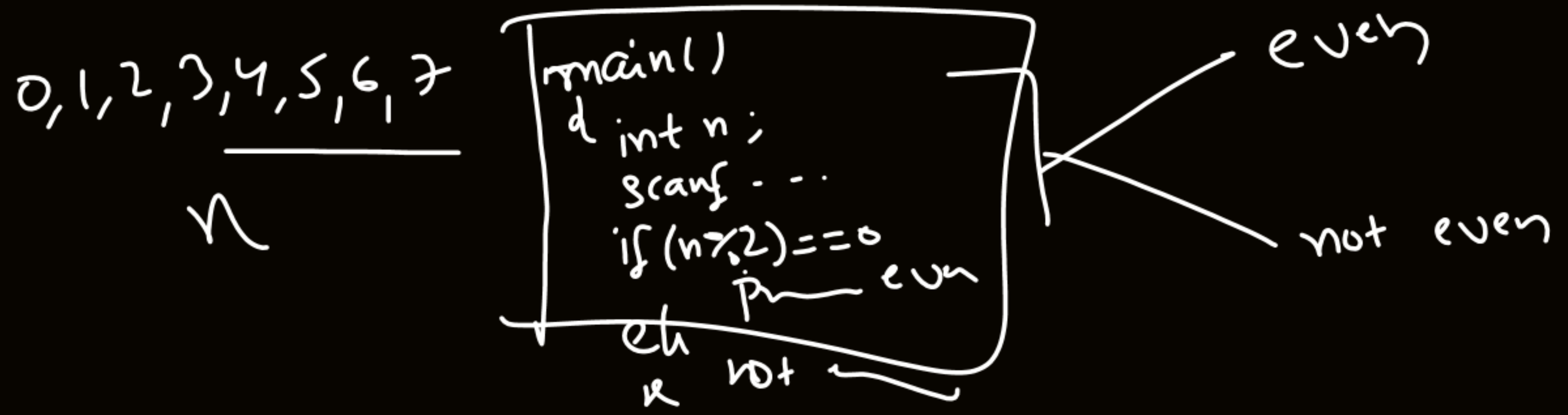
✓ a^7 ✓

$\vdots$

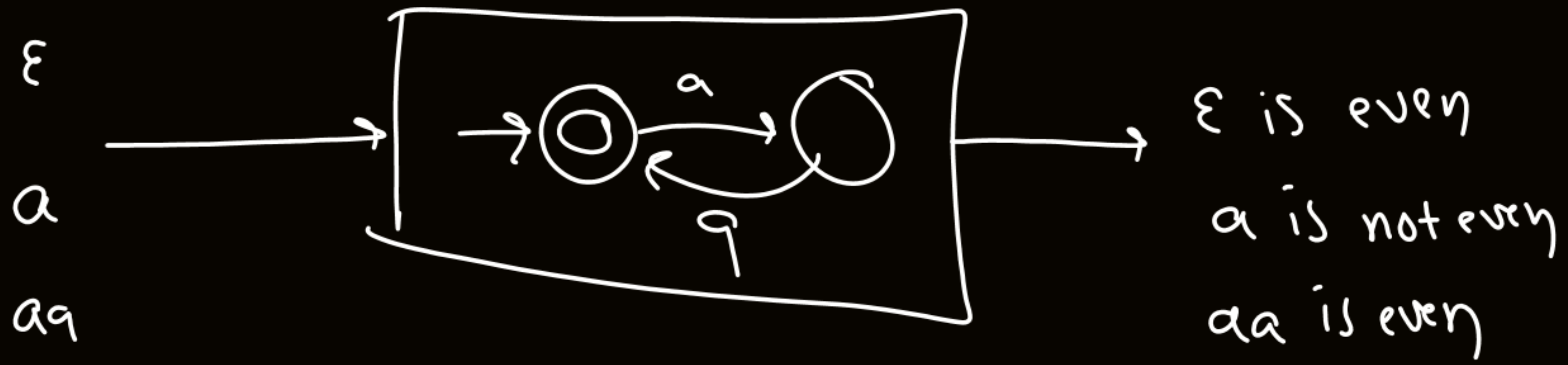
$$= \{\varepsilon, a^3, a^4, a^6, a^7, a^8, a^9, a^{10}, \dots\}$$



check given number is even or not.



$L = (aa)^*$ over $\Sigma = \{a\}$



DFA Construction

Model-V [Sequence based]

45 $L = \{ a^m b^n \mid m, n \geq 0 \}$

$\boxed{\text{Any no. of a's}}$ followed by $\boxed{\text{Any no. of b's}}$

$= a^* b^*$

$P = a^*$

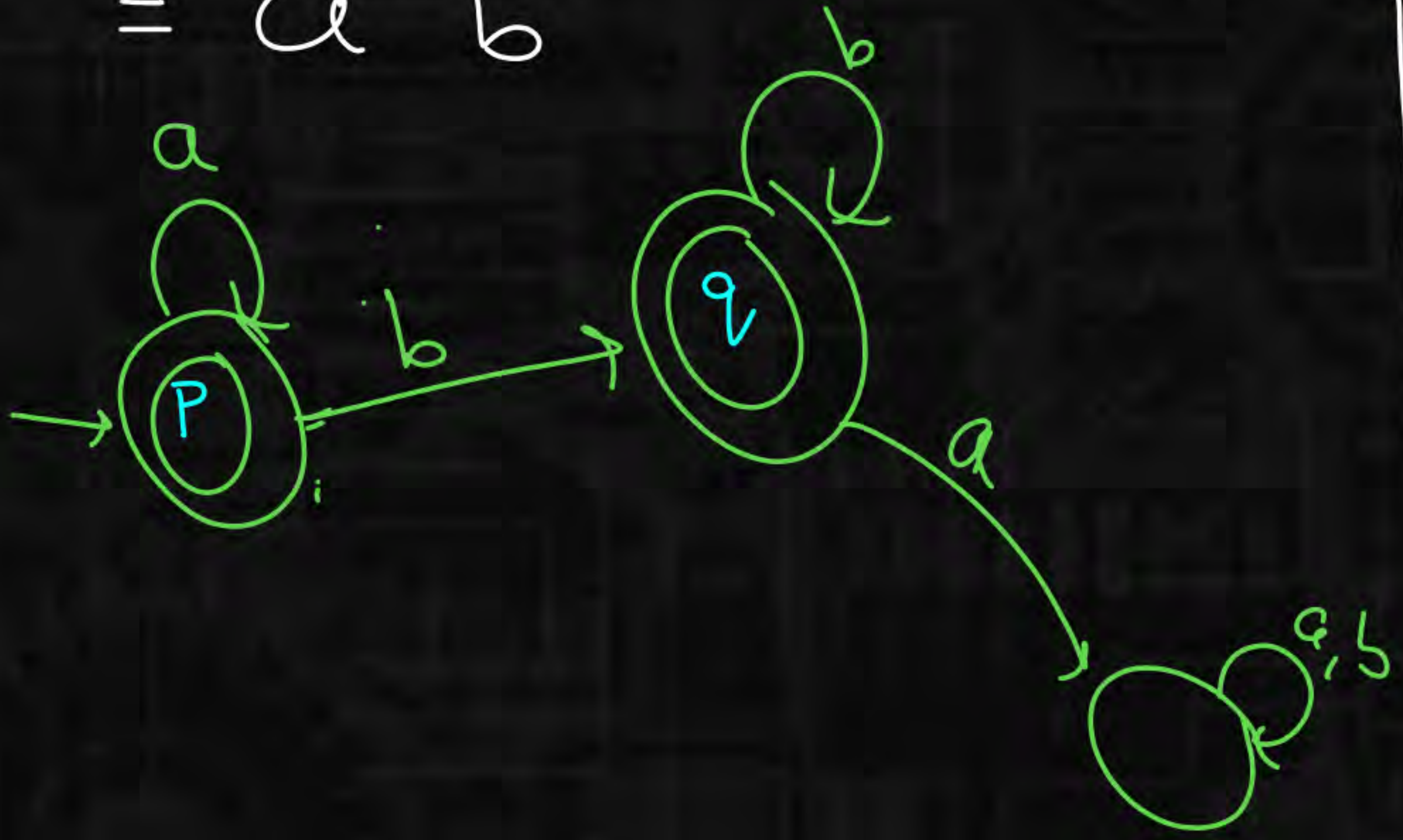
$q = a^* b b^*$

$P + q$

$= a^* + a^* b^+$

$= a^* (\epsilon + b^+)$

$= a^* b^*$



$a^* b^*$:
- ϵ
- a^+
- b^+
- $a^+ b^+$

b never comes before a

a never comes after b

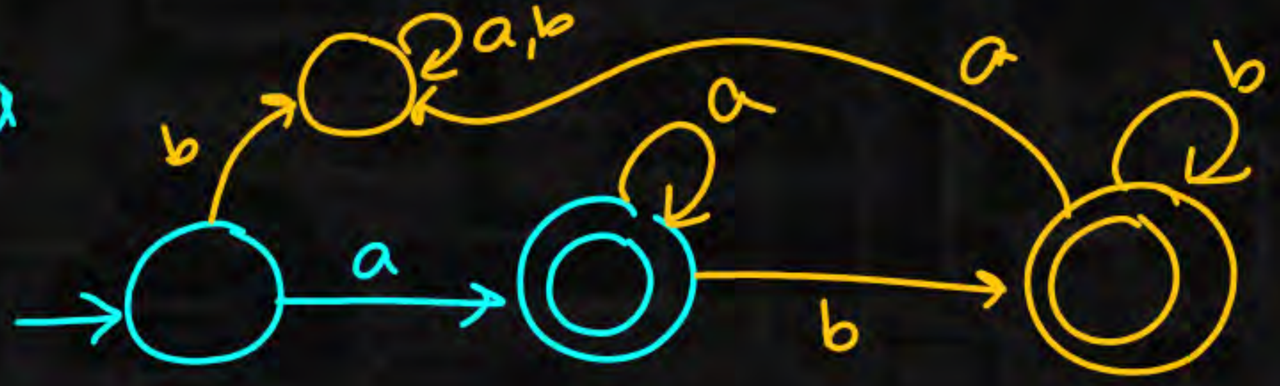
$a^m b^n$

$a^m b^n$	ε	$m = 0$ $n = 0$
	a^+	$m \neq 0$ $n = 0$
	b^+	$m = 0$ $n \neq 0$
	$\underline{a}^+ b^+$	$m \neq 0$ $n \neq 0$

DFA Construction

(46) a^+b^*

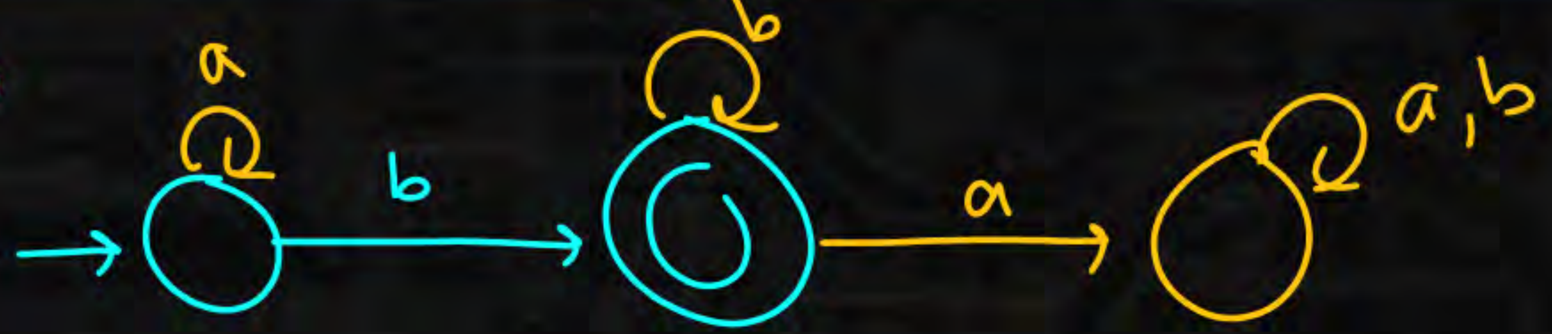
min = a



(47) a^*b^+

$= a^*bb^*$

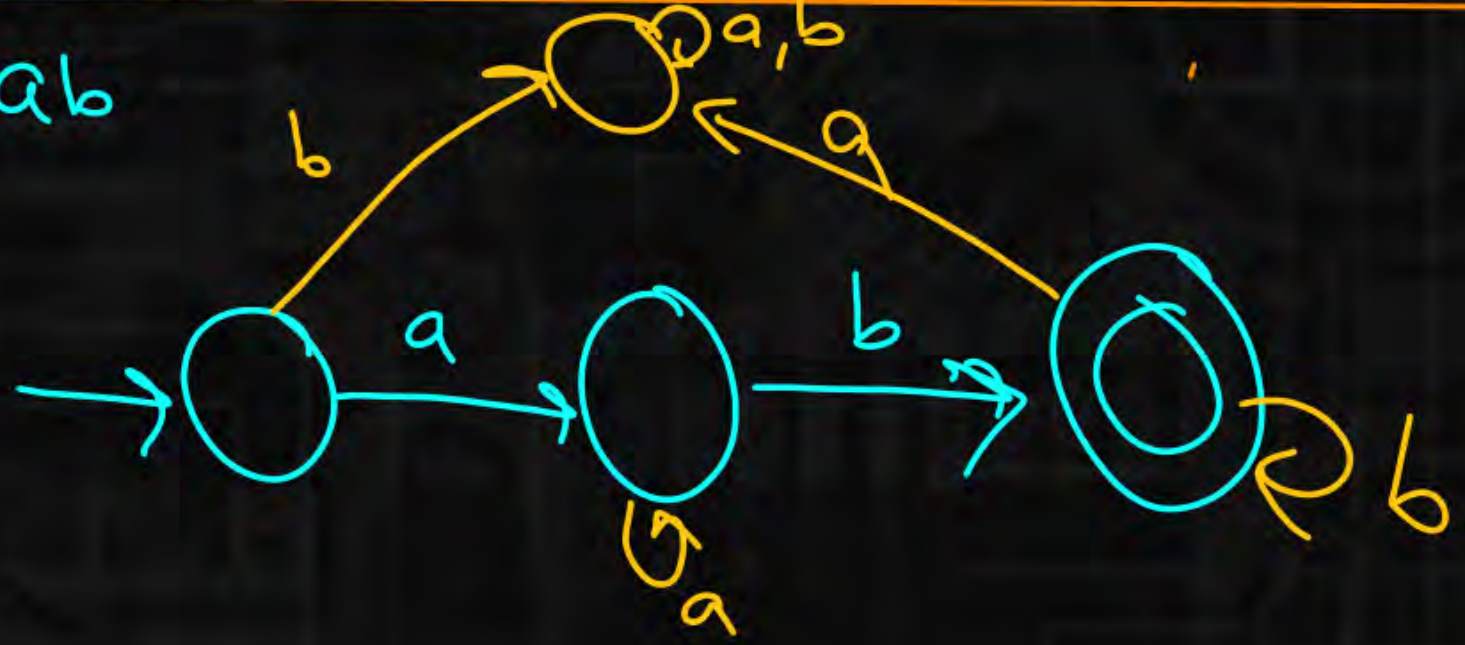
min = b



(48) a^+b^+

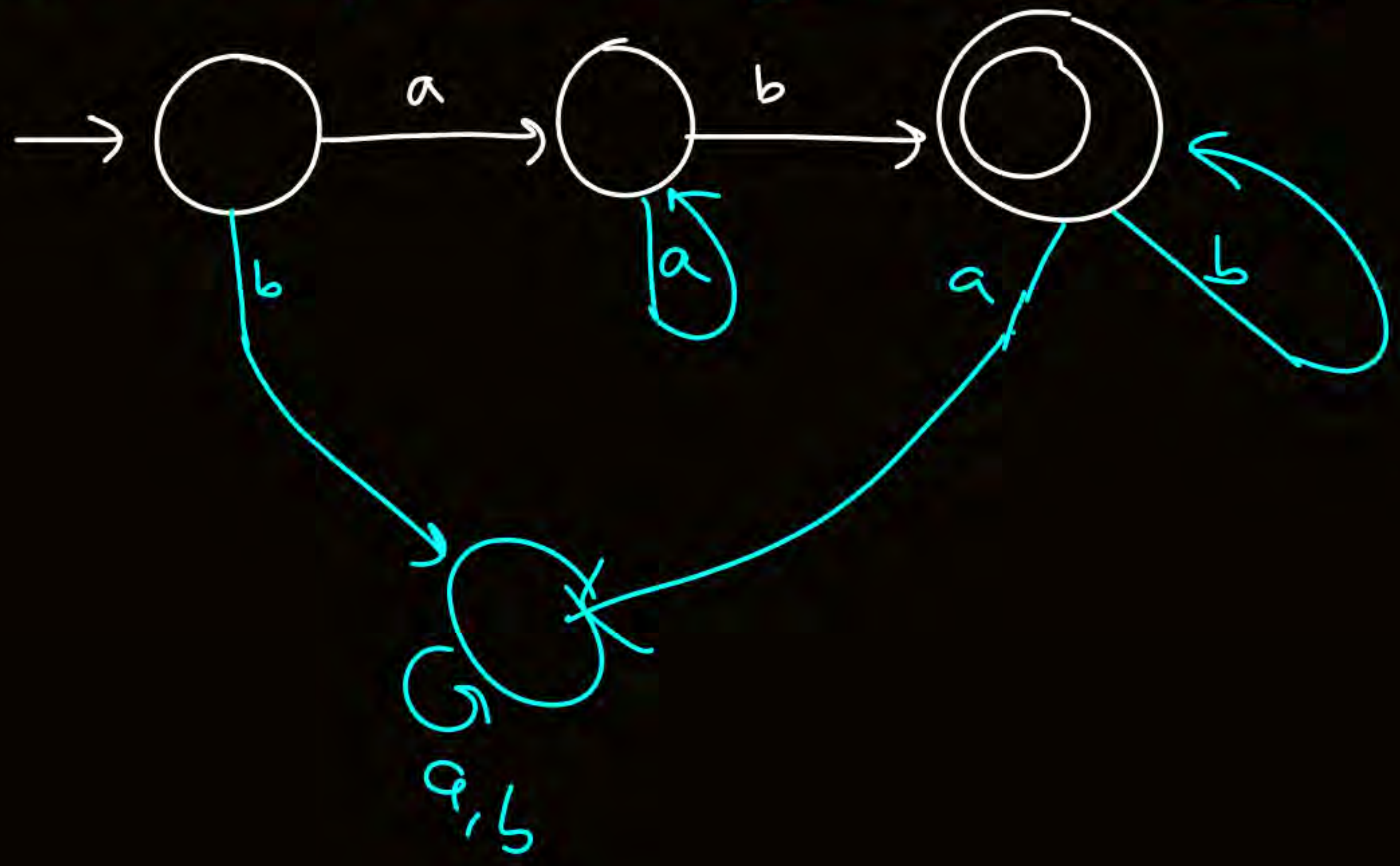
$= aa^*bb^*$

min = ab



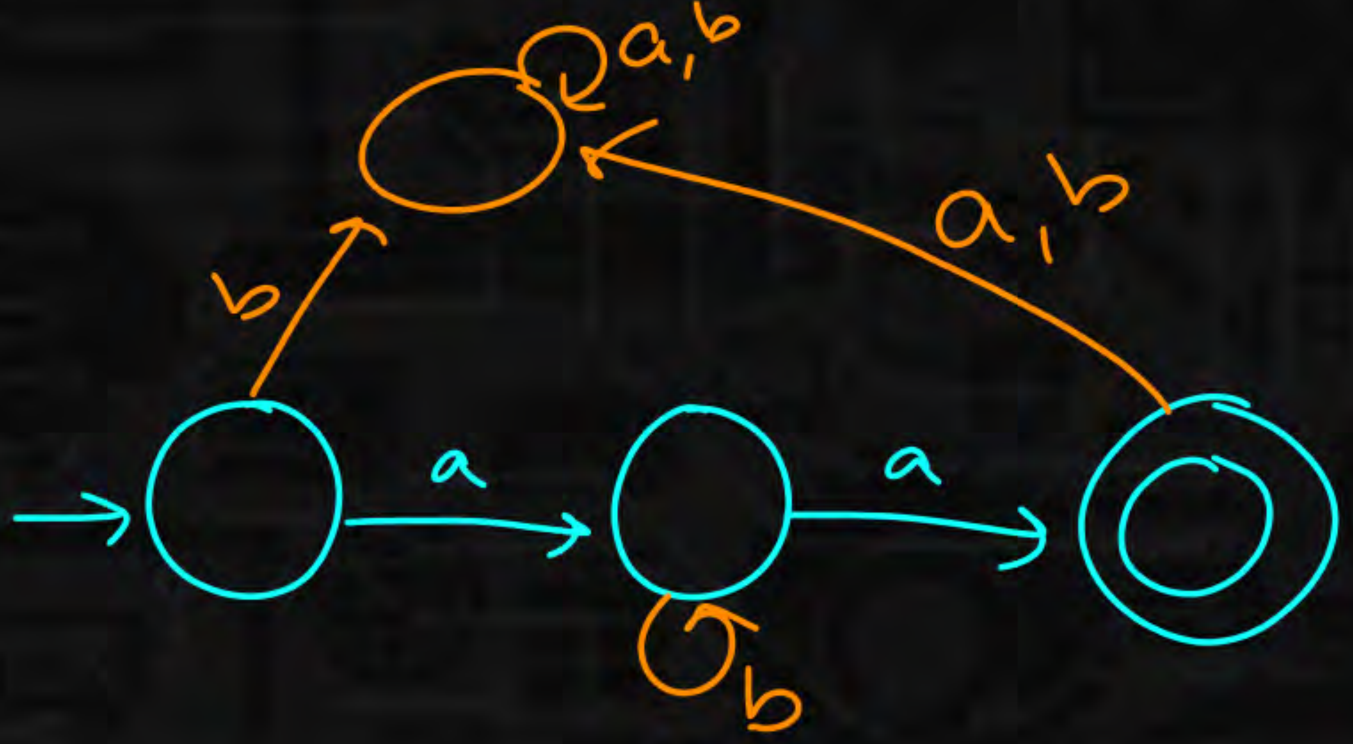
$$a^+ b^+ = \{\underbrace{\boxed{ab}}_{\text{min}}, aab, abb, aabb, \ldots\}$$

Atleast one a followed atleast one b



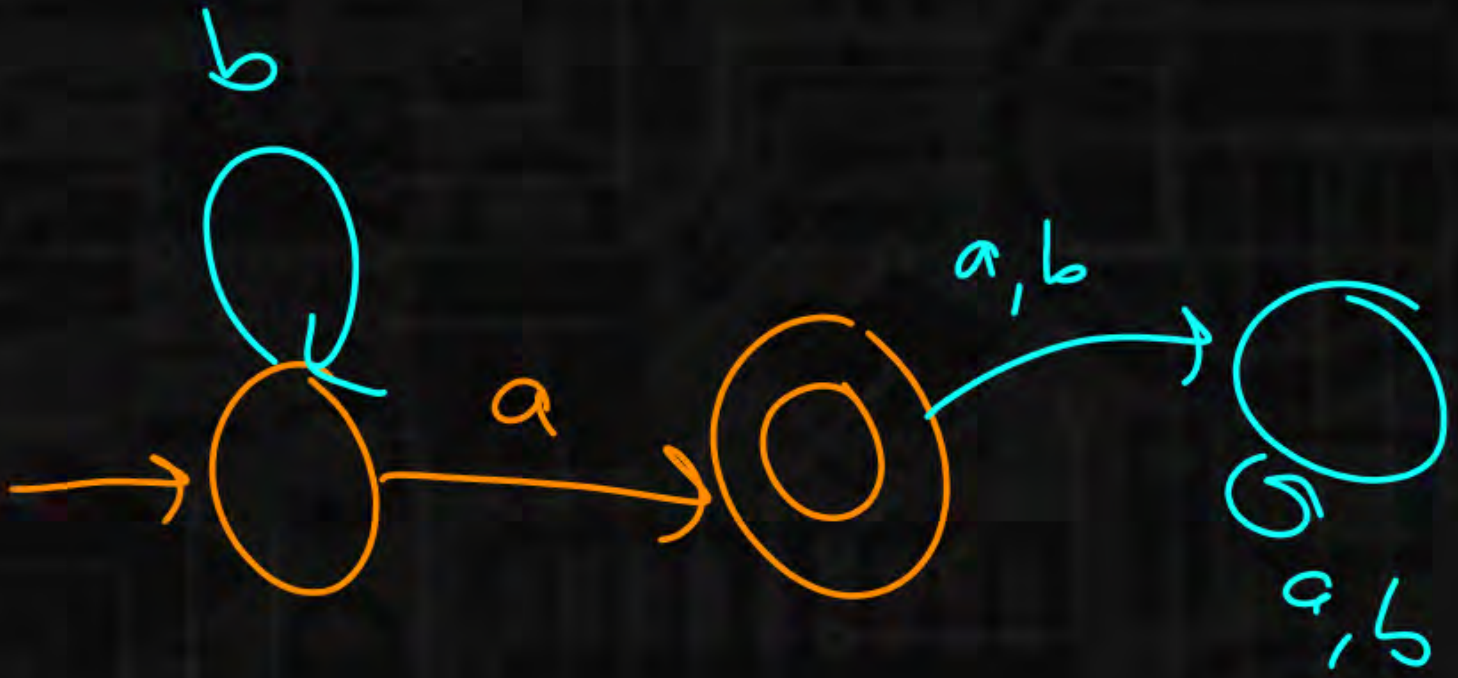
49 ab^*a

min = aa



50 b^*a

min = a



DFA Construction

H.W.:

51. b^*a^*
52. b^+a^*
53. b^*a^+
54. b^+a^+

$\Sigma = \{a, b\}$

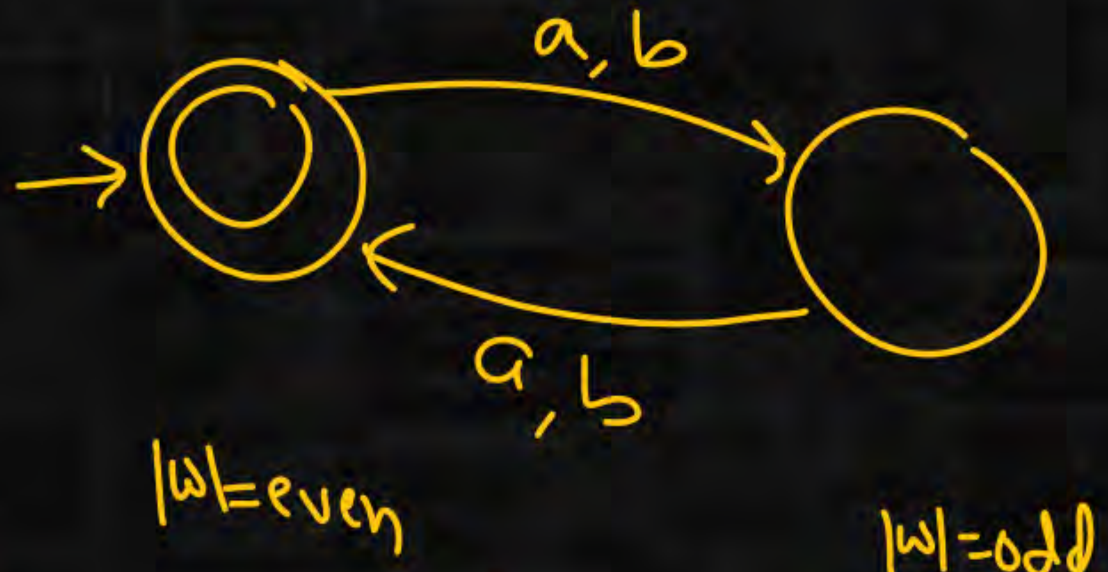
55. $a^*b^*c^* = \{a^m b^n c^k \mid m, n, k \geq 0\}$
56. $a^*b^+c^*$
57. $a^*b^*c^+$
58. $a^+b^*c^*$
59. $a^+b^+c^*$
60. $a^+b^+c^+$

DFA Construction

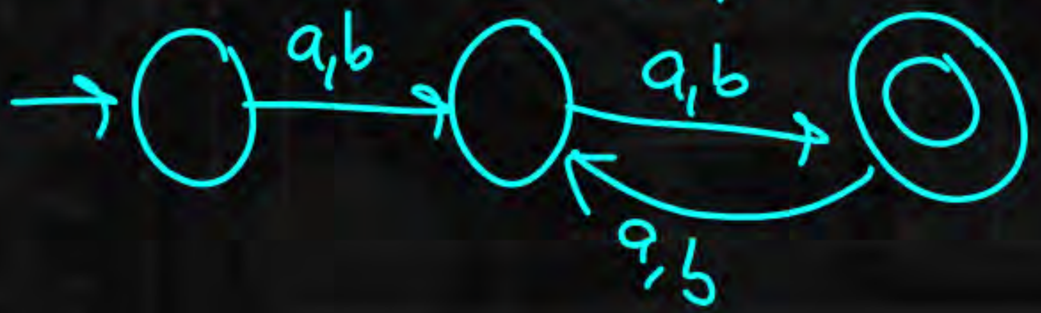
Model-VI : [Remainder & Length based]

61. $\{w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } 2\}$

$$= [(a+b)^2]^*$$



62. $\{w \mid w \in \{a,b\}^+, |w| \text{ is divisible by } 2\}$



$$= [(a+b)^2]^+$$

I : special

II : $|w| = , \leq , \geq , < , > , \neq \; K$

III : $n_a(w) = , \leq , \geq , < , > , \neq \; K$

IV : L over 1 symbol but forms A.P.

V : sequence

DFA Construction

$|w|$ is divisible by 2

$|w|$ is even

$|w| \% 2 = 0$

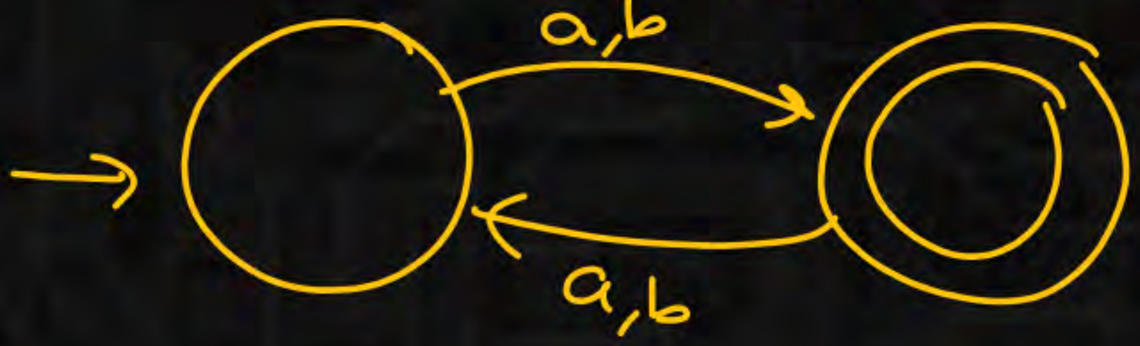
$\text{rem}\left(|w|/2\right) = 0$

$|w| \cong_2 0$

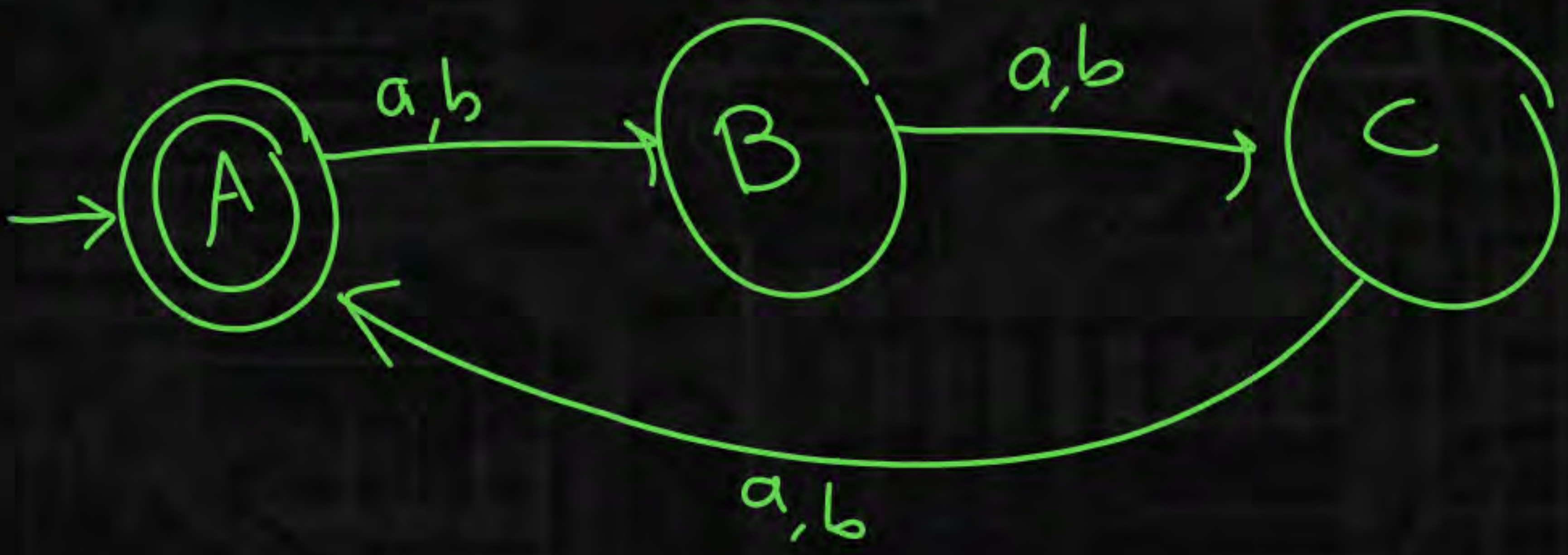
$|w| = 0 \bmod 2$

$|w| = 2n, \; n \geq 0$

63 $\{w \mid w \in \{a,b\}^*, \underbrace{|w| = odd}_{|w| \% 2 = 1}\} = \overline{61}$



64 $\{w \mid w \in \{a,b\}^*, |w| \text{ is divisible by } 3\}$

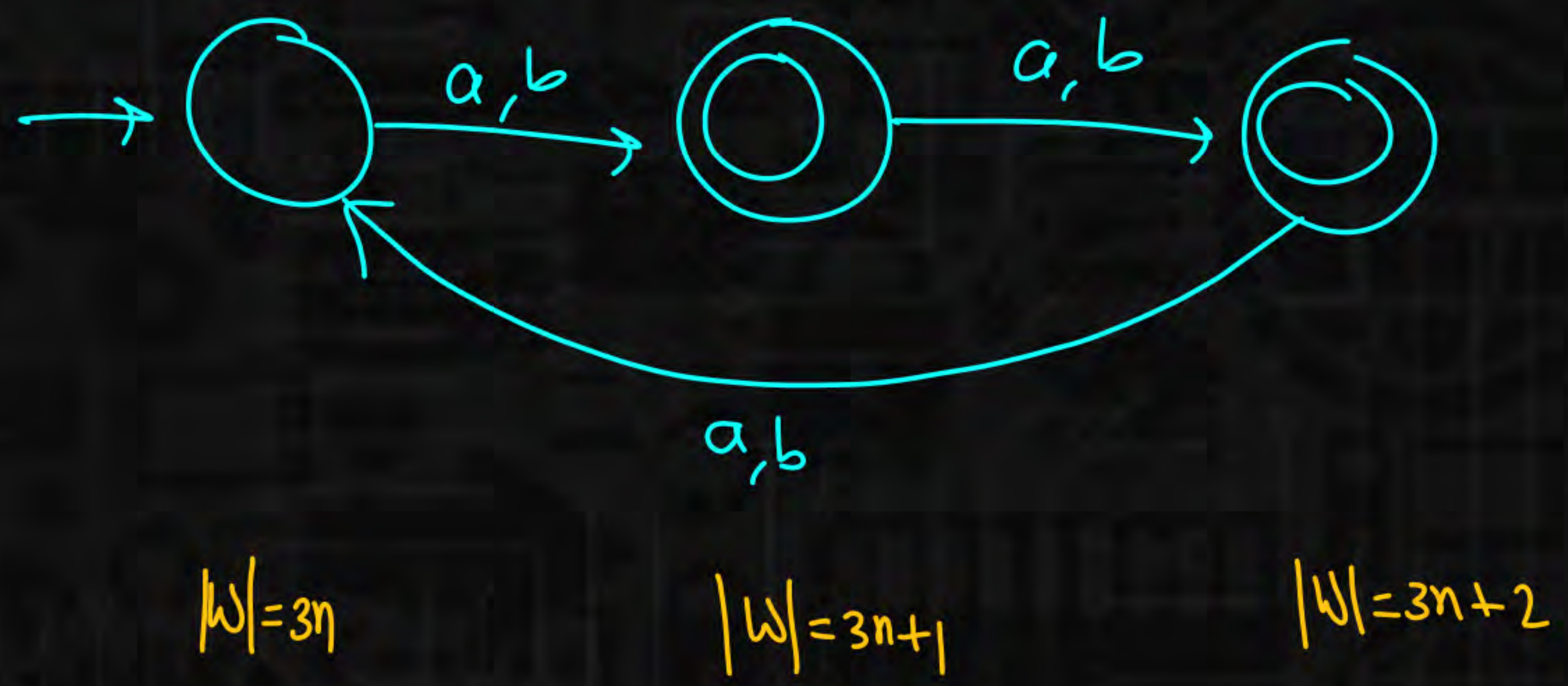


rem

$|w| \% 3$ — $\boxed{0}$, 1, 2

(65) $\{ w \mid w \in \{a,b\}^*, |w| \text{ is not divisible by } 3 \}$

$(|w| \% 3 \neq 0)$



66 $\{w \mid w \in \{a,b\}^*,\ |w| = \underbrace{100n+5}_{K_1 > K_2},\ n \geq 0\}$

$K_1 n + K_2$ → remainder

→ K_1 states

$\boxed{100 \text{ states}}$

67 $\{w \mid w \in \{a,b\}^*,\ |w| = \underbrace{100n+123}_{K_1 \leq K_2},\ n \geq 0\}$

$K_1 n + K_2$

→ 124 states

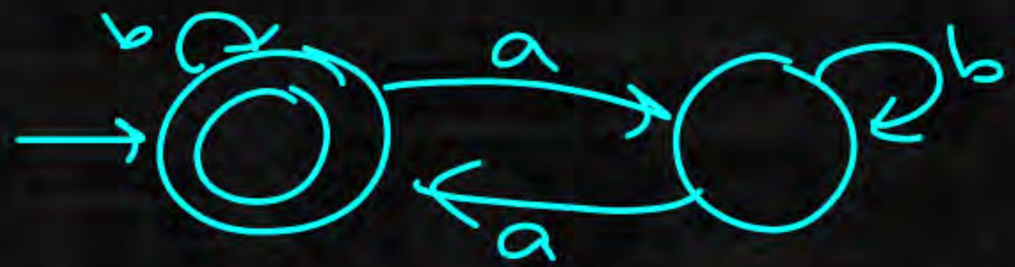
(K_2+1)

DFA Construction

Model-VII [Remainder based & no. of a's]

(68) $\{ w \mid w \in \{a,b\}^*, \; n_a(w) \text{ is divisible by } 2 \}$

$\rightarrow$ 2 States



(69) $\{ w \mid w \in \{a,b\}^*, \; n_a(w) \text{ is divisible by } 3 \}$

$\rightarrow$ 3 States

In general

(70) $\{ w \mid w \in \{a,b\}^*, \; n_a(w) = k_1 n + k_2, \; n \geq 0 \}$

Case I : If $k_1 > k_2$ then k_1 states

Case II : If $k_1 \leq k_2$ then $k_2 + 1$ States

DFA Construction

Home Work

Model-VIII [Start / contain / end]

$\Sigma = \{a, b\}$

min=a, 3 states (71) $L = \underline{a}(a+b)^* \Rightarrow \{w \mid w \in \{a,b\}^*, w \text{ starts with 'a'}\}$

min=aa, 4 states (72) $L = \underline{a}\underline{a}(a+b)^* \Rightarrow \{w \mid w \in \{a,b\}^*, w \text{ starts with 'aa'}\}$

|min string| + 2

min=a, 2 states (73) $L = (a+b)^* a \Rightarrow$ Set of all strings ending with 'a'

min=aa, 3 states (74) $L = (a+b)^* aa \Rightarrow$ Set of all strings ending with 'aa'

min=a, 2 states (75) $L = (a+b)^* a (a+b)^* \Rightarrow \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*\}$

min=aa, 3 states (76) $L = (a+b)^* aa (a+b)^* \Rightarrow \{w \mid w \in \{a,b\}^*, w \text{ contains 'aa' as substring}\}$

|min string| + 1

exactly ⇒ Dead state required

At least ⇒ Dead state is not required

At most

Start

end

contain

Dead state is not required

$$\Sigma = \{\varepsilon, a\}$$

$$= \{x, y\}$$

$$(\varepsilon + a)^* = (x+y)^*$$

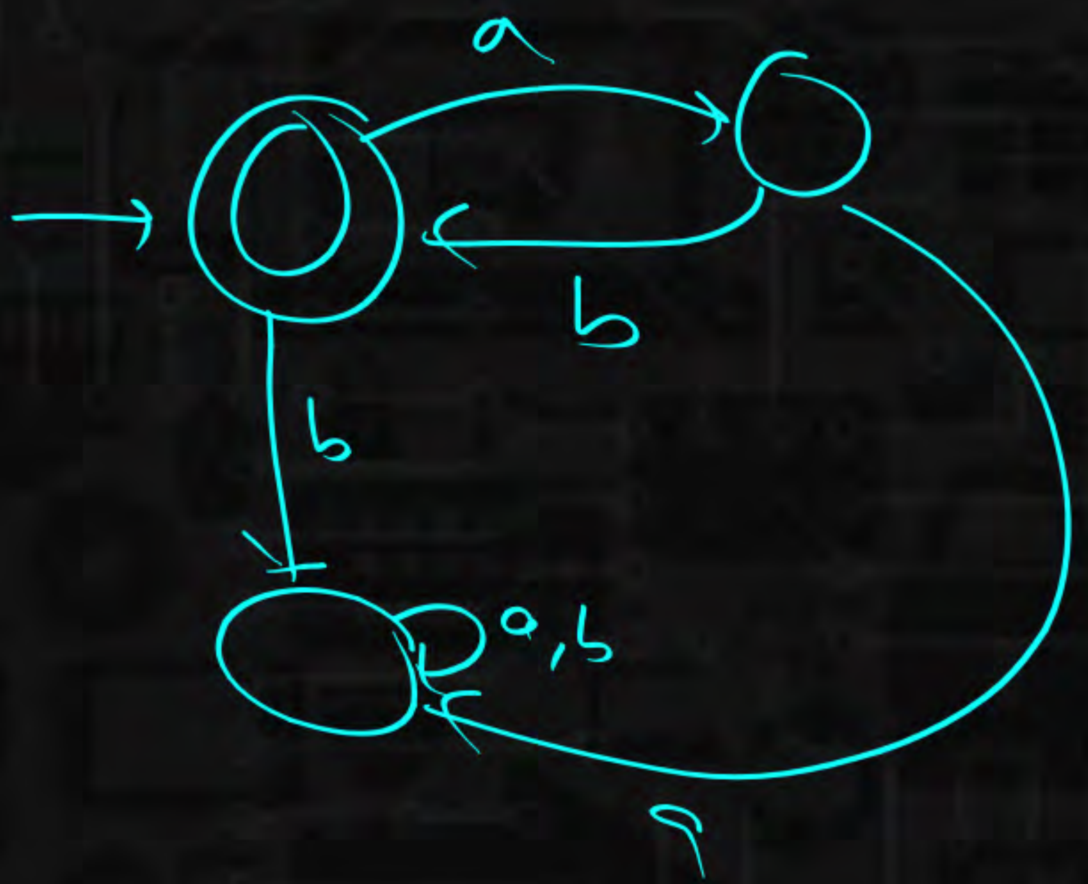
In this question, it is i/p symbol

$$(\varepsilon a)^* \neq a^*$$

$$(xy)^*$$

DFA Construction

$$L = (ab)^*$$



Summary

model - I
⋮
model - VII

⇒ easy
DFA is easy
takes less time to design

model - VIII

⇒ Answer might be easy
DFA construction takes time


THANK YOU GW
SOLDIERS !