CS & IT ENGINEERING

Theory of Computation

Undecidability & Decidability: Languages

Lecture No. 2

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01 Decision Properties Table

Problems & Languages

Problem Vs Language

Problem: Yes / NO

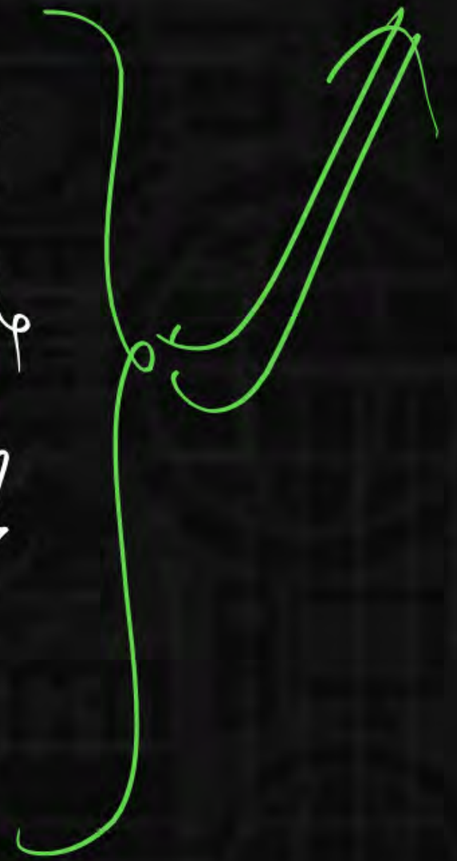
Language: Valid / Invalid

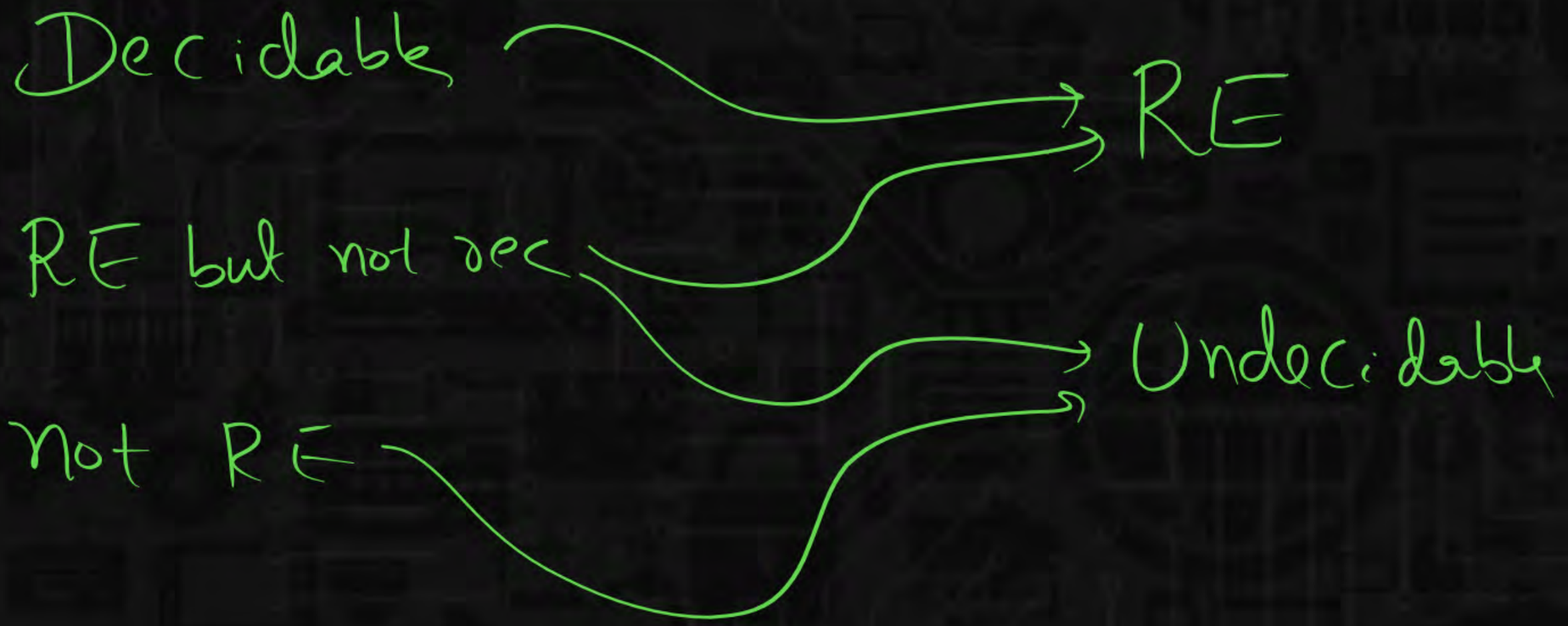
Problem: Whether given TM halts on w $\Longleftrightarrow$ RE but not rec.

Language: $\{TM \mid TM \text{ halts on } w\}$

$\{\langle TM, w\rangle \mid TM \text{ halts on } w\}$

$\{\langle TM, \#\rangle \mid \text{"} \}$




Decidable
RE
RE but not dec
Undecidable
not RE

Problem is RE but not rec

Language is RE but not rec

① Halting & Non-Halting

By definition FA, DPDA, PDA, LDA and HTM halts always

I) IS FA halts on w ?
(FA: given, w: given)

II) IS DPDA halts on w ?

III) IS PDA " " w ?

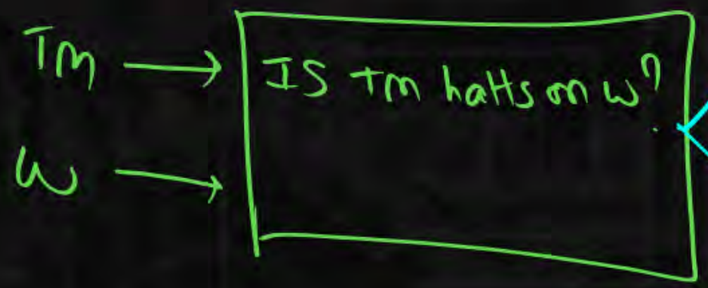
IV) IS HTM " " w ?

⇒ Decidable

Halting problem is decidable for FA/DPDA/PDA/LDA/HTM

Note: Non-halting also decidable for FA/DPDA/PDA/LDA/HTM

V) IS TM " " w ?



⇒ Undecidable [RE but not Recursive]

Note: Non-halting is not RE for TM

Languages depends on Haltings.

① $L_1 = \{ \text{DFA} \mid \text{DFA halts on } \varepsilon \}$ = set of all DFAs

② $L_2 = \{ \text{PDA} \mid \text{PDA halts on ab} \}$

③ $L_3 = \{ \text{HTM} \mid \text{HTM halts on } \underset{\text{any}}{w} \}$

④ $L_4 = \{ \langle \text{DPDA}, 011 \rangle \mid \text{DPDA halts on ab} \}$

⑤ $L_5 = \{ \text{TM} \mid \text{TM halts on } \varepsilon \}$ ⇒ RE but not Rec

⑥ $\overline{L_1} = \{ \text{DFA} \mid \text{DFA does not halt on } \varepsilon \} = \{\ \}$

Decidable

(2) Membership

CYK Algorithm

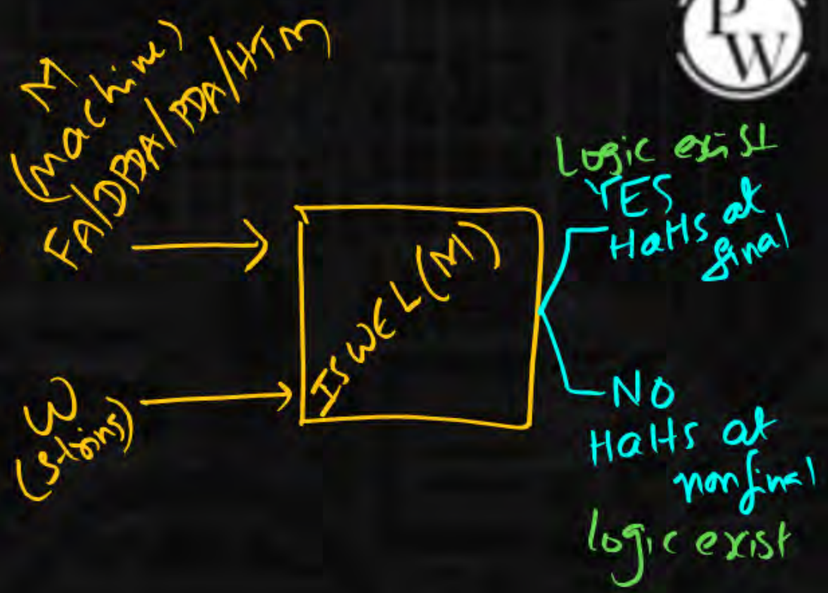
I) IS $w \in L(FA)$?

II) IS $w \in L(DPDA)$?

III) IS $w \in L(PDA)$?

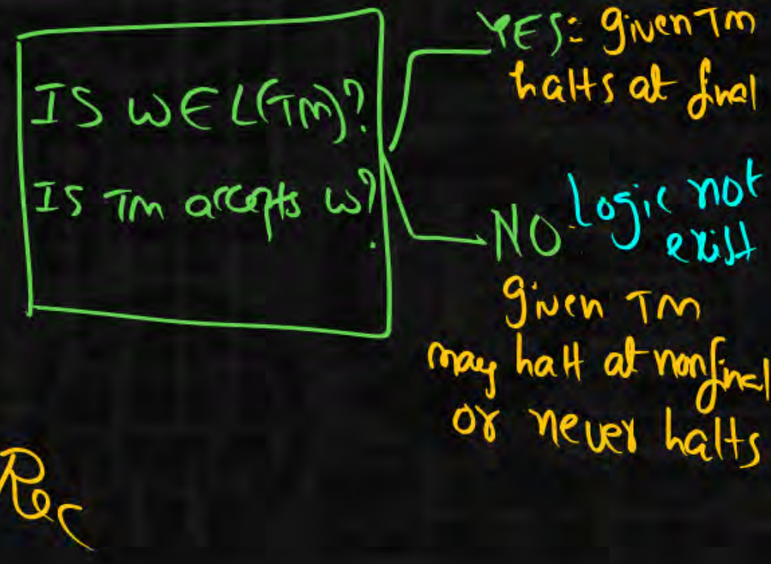
IV) IS $w \in L(HTM)$?

⇒ Decidable



V) IS $w \in L(TM)$?

⇒ RE but not Rec



CYK Algorithm / membership Algo

- Bottom up parsing
- $O(|w|)^3 = O(n^3)$
- Dynamic programming

FA / DPDA / PDA $\Rightarrow$ CFG $\Rightarrow$ CNF CFG (G)

Not REC $\Rightarrow$ either not REC or RE but not rec

(2)$_b$ Non-membership

I) Is $w \notin L$(FA)?
II) Is $w \notin L$(DPDA)?
III) Is $w \notin L$(PDA)?
IV) Is $w \notin L$(TM)?

$\Rightarrow$ Decidable

V) Is $w \notin L$(TM)?

- Is TM doesn't accepts w?
- Is TM doesn't accept ε?
- Is TM " " a?
- Is TM " " abb?

$\Rightarrow$ Not RE

① $L_1 = \{ M \mid M \text{ is DFA}, M \text{ accepts } \epsilon \}$

② $L_2 = \{ \langle DPDA, 0 \rangle \mid DPDA \text{ accepts } abb \}$

③ $L_3 = \{ \langle PDA, \#, gate \rangle \mid PDA \text{ accepts } aaa \}$

④ $L_4 = \{ \langle HTM, \#, @ \rangle \mid HTM \text{ accepts } ab \}$

⇒ Decidable

⑤ $L_5 = \{ Tm \mid Tm \text{ accepts } \epsilon \}$ ⇒ RE but not rec

⑥ $L_6 = \{ \langle TM, 0, 1 \rangle \mid TM \text{ doesn't accept } abb \}$ ⇒ Not RE

③ Emptiness

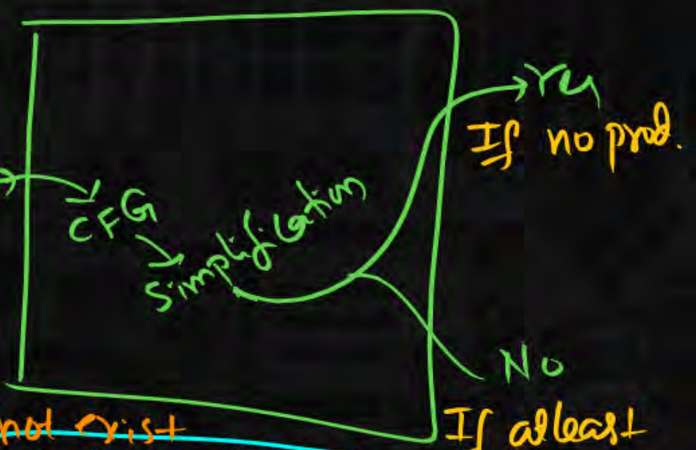
I) Is $L(\underbrace{FA}_{\text{given}}) = \phi$?

II) Is $L(DPDA) = \phi$?

III) Is $L(PDA) = \phi$?

⇒ We can apply Simplification Algo

⇒ Decidable



IV) Is $L(HTM) = \phi$?

HTM → ISL(HTM) = ϕ ?

⇒ Not RE

Logic not exist

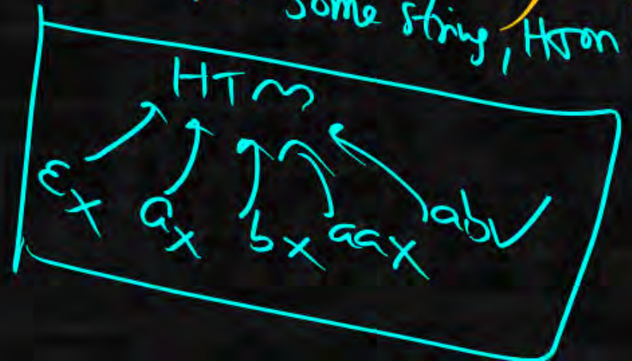
Infinite no. of string halts at non final on every string

Yes HTM does not accept every string

No HTM accepts atleast one string

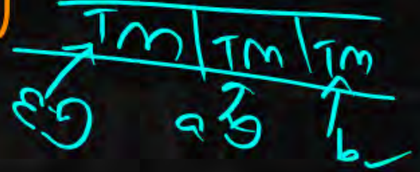
Logic exist

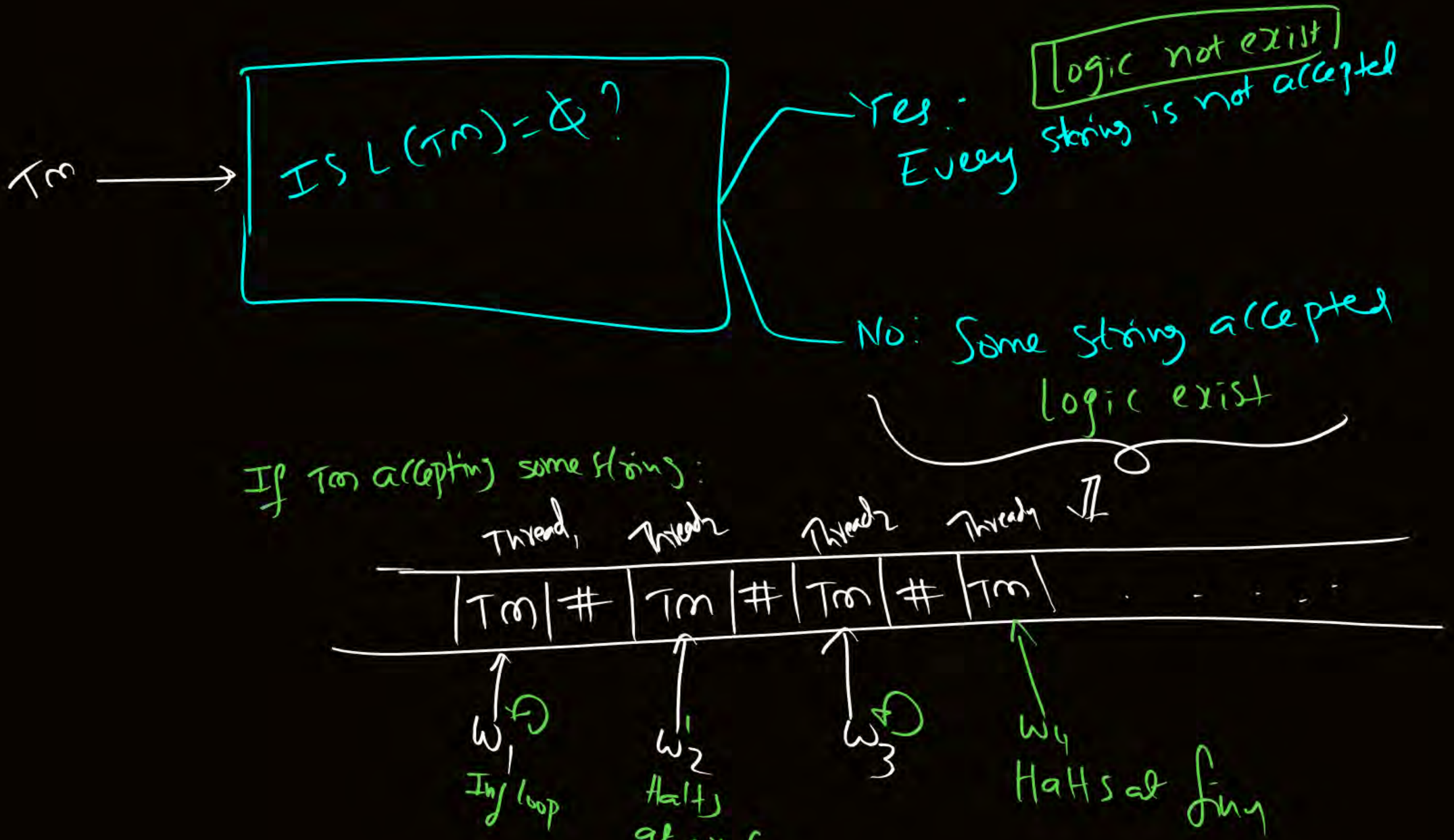
For some string, HTM halts at final



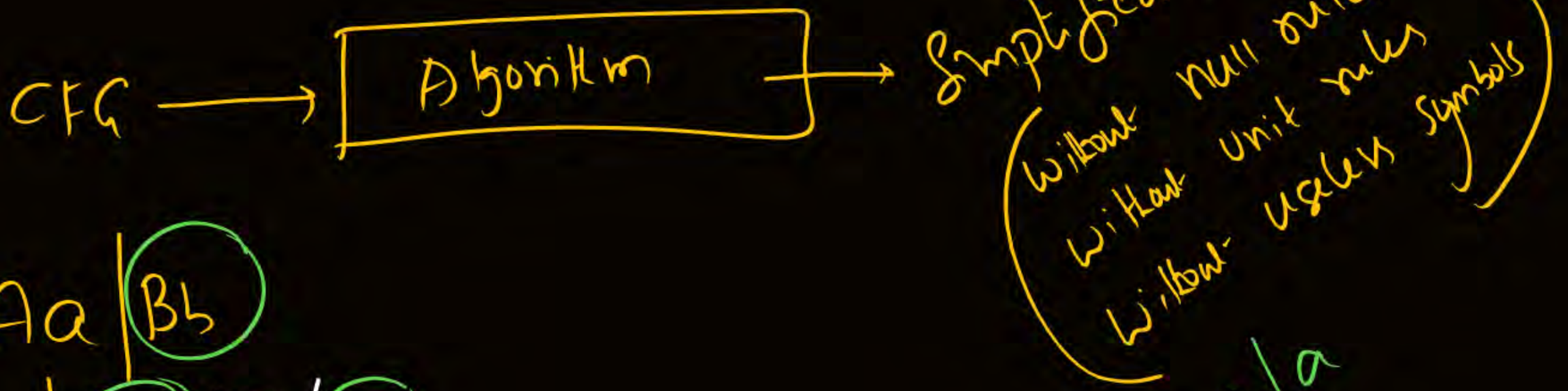
V) Is $L(TM) = \phi$?

⇒ Not RE




Tm
IS L(Tm) = ϕ ?
Yes: Every string is not accepted
logic not exist
No: Some string accepted
logic exist
If Tm accepting some string:
Thread1
Thread2
Thread3
Thread4
Tm
#
Tm
#
Tm
#
Tm
w1
Inf loop
w2
Halts at non fin
w3
w4
Halts at fin

Simplification of CFG :



$S \rightarrow Aa \mid Bb$

$A \rightarrow b \mid De \mid \varepsilon \mid B$

$B \rightarrow Be$

$E \rightarrow f$

$S \rightarrow Aa \mid a$

$A \rightarrow b$

(3) Non-emptiness

I) Is $L(\underbrace{FA}_{\text{given}}) \neq \phi$?

II) Is $L(DPDA) \neq \phi$?

III) Is $L(PDA) \neq \phi$?

$\Rightarrow$ Decidable

IV) Is $L(HTM) \neq \phi$?

Yes. Logic exist

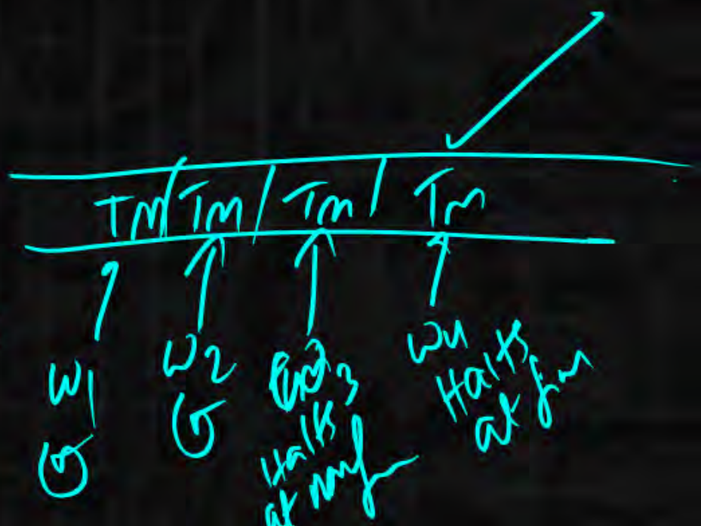
$\Rightarrow$ RE but not rec

No

V) Is $L(TM) \neq \phi$?

Is TM accepts some string?

$\Rightarrow$ RE but not rec



(4) Finiteness

Dependency Graph

I) IS L(FA) = Finite ?

II) IS L(DPDA) = Finite ?

III) IS L(PDA) = Finite ?

⇒ Decidable

Undecidable

IV) IS L(HTM) = " ? ⇒

V) IS L(TM) = " ? ⇒

⇒ Not RE

HTM → IS HTM accepting finite lang?

Yes: L(HTM) = fin — we need to check all strings ⇒ logic not exist

No: L(HTM) = Inf — we need to check all strings ⇒ logic not exist

FA / DPDA / PDA

⇓

CFG

$S \rightarrow AB$

$A \rightarrow aA \mid b$

$B \rightarrow b$

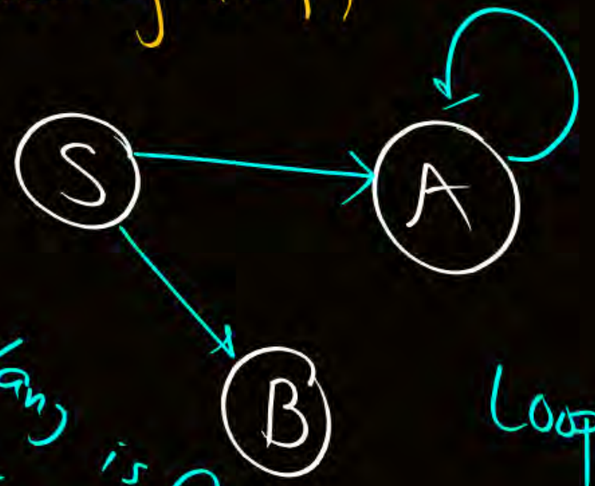
⇓

Construct Dependency Graph

⇓

If loop not exist ⇒ lang is finite

If loop exist ⇒ lang is infinite



Loop exist

(4)b Infiniteness (non finiteness)

(1) For FA
(2) For DPDA
(3) For PDA
} Decidable

(4) For HTM
(5) For TM
} $\Rightarrow$ Not RE

In general:

not RE $\Rightarrow$ either not RE OR (RE but not REC)

Summary

- H ✓
- M ✓
- E ✓
- F ✓

- T
- E
- D
- S


THANK YOU GW
SOLDIERS !