CS & IT ENGINEERING

Theory of Computation

Lecture No.- 04

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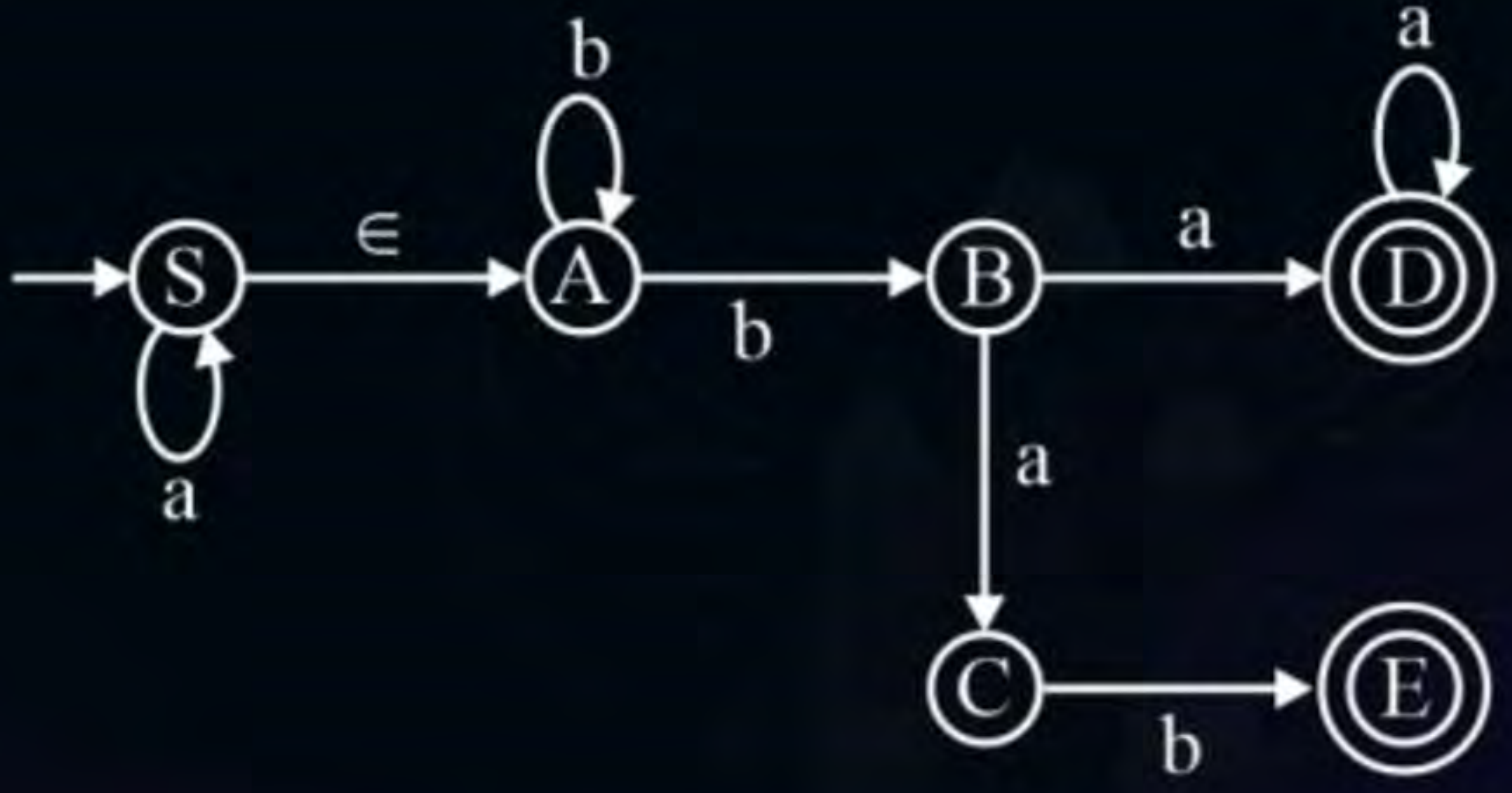
Topics to be Covered

- **Topic** Regular Expression
- **Topic** Finite Automata
- **Topic** Regular Grammar
- **Topic** Closure Properties

Regular Languages : MSQ

Q43. Consider the following $\in$-NFA:
Which of the following strings are accepted?



A abab ✓ a ε b a b

B baab

C bbaa ✓ ε b b a a

D abaa ✓ a ε b a a

Regular Languages : MCQ

L = Set of all languages generated by LLGs

[MCQ]

Q44. let L be the set of all the languages accepted by all grammars where every production is in the form of $\boxed{V \to VT^* \text{ or } V \to T^*}$.

LLG

Let Q be the set of all languages accepted by all grammars where every production of grammar is in the form of $\boxed{V \to T^*V \text{ or } V \to T^*}$

RLG

Which of the following is correct?

(Note: T is terminals and V is non-terminals)

Q = Set of all languages generated by RLGs

A. $L \geq Q$

B. $L \leq Q$

C. $L = Q$

D. $L \neq Q$

$L_s(LLG_s) = L$

$L_s(RLG_s) = Q$

$\boxed{L = Q}$

$L = Q$ = Set of all regular languages

Regular Languages : MSQ

Q45. Consider the following grammar G:

G: $S \rightarrow aS \mid bS \mid a\cancel{a}S \mid b\cancel{b}S \mid a$ $(a+b)^* a$

Which of the following is correct regular expression for above grammar G?

A. $(a + b)^* a$

B. $(a + b + aa + bb)^* a$ $= (a+b)^* a$

C. $(a + b + aa + bb + ba)^* a$ $= (a+b)^* a$

D. None of these

Regular Languages : MCQ

Q46. Consider the following deterministic finite automaton (DFA).

000
001
⋮
111
8 strings

The number of strings of length 3 accepted by the above automaton is____.

A 2

B 4

C 6

D 8

Regular Languages : MCQ

Q47. Consider the following DFA.

Which one of the following language L accepted by above DFA?

A) $L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 011\}$ ✗

B) $L = \{w \in \{0, 1\}^* \mid w \text{ starts with } 011\}$ ✗

C) $L = \{w \in \{0, 1\}^* \mid w \text{ has substring } 011\}$

D) $L = \{w \in \{0, 1\}^* \mid w \text{ ends with } 11\}$

Regular Languages : MSQ

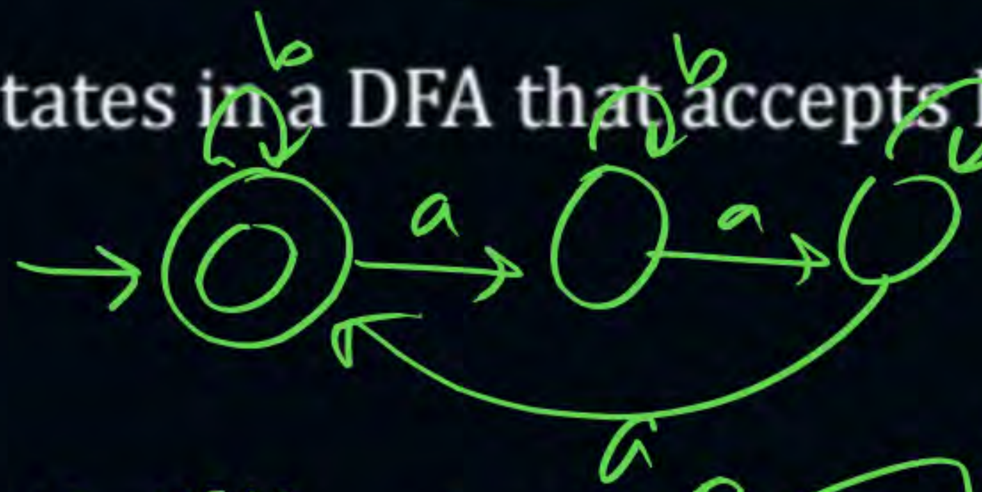
Q48. Consider the following language:

÷3 //

div by 3

$L = \{x \mid x \in \{a, b\}^*$, number of a's in x is $\boxed{\text{divisible by 3 or divisible by 6}}\}$

The minimum number of states in a DFA that accepts L is ___.

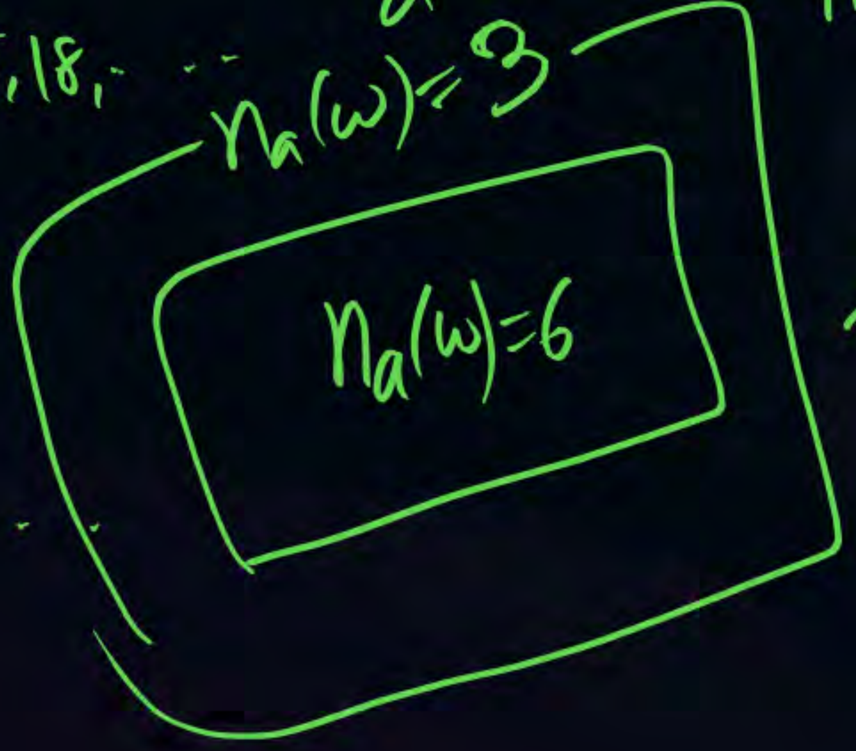


$\underbrace{\text{\# a's is div by 3}}$

\# a's = 0, 3, 6, 9, 12, 15, 18, ...

$\underbrace{\text{\# a's is div by 6}}$

\# a's = 0, 6, 12, 18, ...



\# a's is div by 3
OR
\# b's is div by 6

$3 \times 6 = 18$

Regular Languages : NAT

Q49. Consider the following statements:

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular. → F
II. If $L_1 \cap L_2$ is regular, then both L_1 and L_2 must be regular. → F
III. If Complement of L is regular, then L must be regular. → True
IV. If $L_1 - L_2$ is regular, then both L_1 and L_2 must be regular. → False
V. If L^* is regular, then L must be regular. → False

How many of above statements are FALSE?

= 4 //

L^* is Reg

$\not\Downarrow$

L need not be Reg

$\{a^{\text{Prime}}\}^*$ is Reg

$\not\Downarrow$

a^{prime} is not reg

$L_1 - L_2$ is Reg

$\Downarrow\!\!\!\!/$

L_1 and L_2 need not be Regular

$$a^n b^n - a^n b^n \Rightarrow \phi$$

$L_1 \cap L_2$ = Regular

$\Downarrow$

L_1 and L_2 need not be Regular

$a^n b^n \quad \bigcap \quad b^n a^n \Rightarrow$ Reg
$\{\epsilon\}$

If $\underbrace{L_1 \cup L_2}_{?}$ is Regular then L_1 need not be reg
L_2 need not be reg

$\{a^n b^n\} \cup \overline{\{a^n b^n\}} \Rightarrow$ Regular
$(a+b)^*$

$$\boxed{L \cup \bar{L} = \Sigma^*}$$

$\Sigma = \{f_1, f_2\}$

Q50. Let Σ be the set of all bijections from {1, 2} to {1, 2}, where *id* denotes the identity function, i.e. $id(j) = j, \forall j$.
Let $\circ$ denote composition on functions.
For a string $x = x_1 x_2 \ldots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \ldots \circ x_n$.
Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$.
The minimum number of states in any DFA accepting L is _____.

Identity Function ✓
Bijective Function. ✓
Composition.

$$L = \{ x \mid \pi(x) = id \}$$

Bijective Function: [one to one correspondence]

↳ It is $\underbrace{\text{Injective}}_{\text{one to one func}}$ & $\underbrace{\text{Surjective}}_{\text{onto func}}$

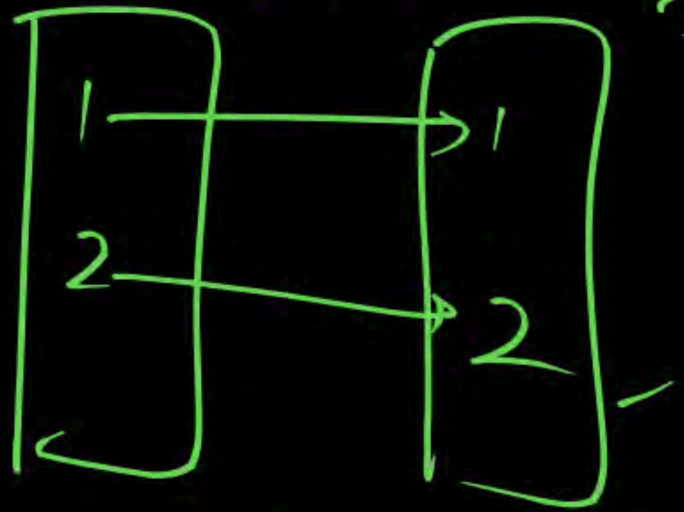
Set
↓
function
↙ ↘
$\underbrace{\text{Inj} \qquad \text{Surj}}_{\text{Bij}}$

$$\Sigma = \{\overset{id}{\boxed{f_1}}, f_2\} = \{a, b\}$$

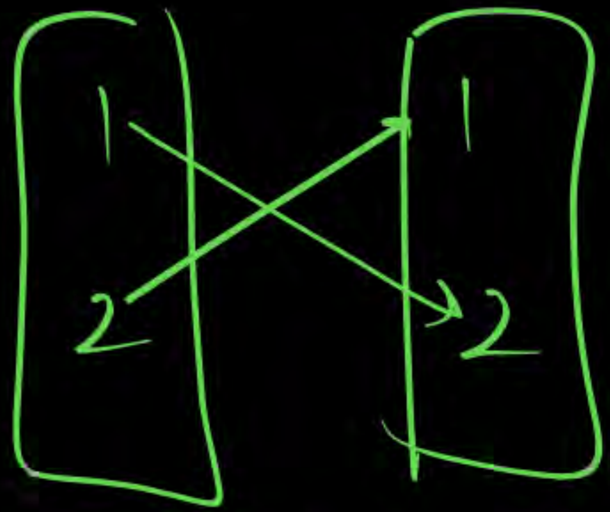
$$\Sigma^* = \{\varepsilon, \underset{a}{f_1}, \underset{b}{f_2}, \underset{aa}{f_1f_1}, \underset{ab}{f_1f_2}, \underset{ba}{f_2f_1}, \underset{bb}{f_2f_2}, \ldots\}$$

Id

$f_1 = id$

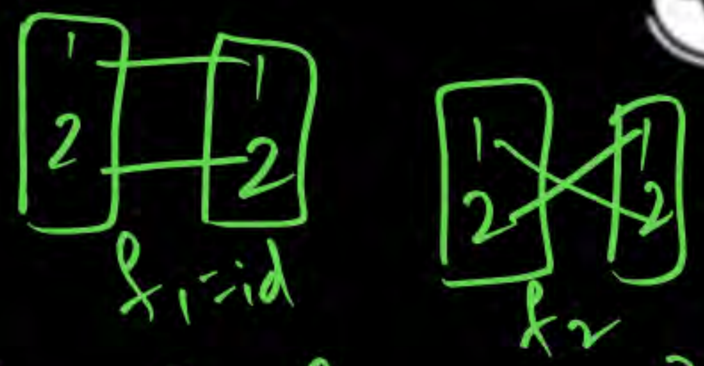


f_1



f_2

$$\Sigma = \{ f_1, f_2 \} = \{a, b\}$$



$$\Sigma^* = \{ \underset{\checkmark}{\varepsilon},\ \underset{\checkmark}{\underset{a}{f_1}},\ \underset{\times}{\underset{b}{f_2}},\ \underset{\checkmark}{\underset{aa}{f_1 f_1}},\ \underset{\times}{\underset{ab}{f_1 f_2}},\ \underset{\times}{\underset{ba}{f_2 f_1}},\ \underset{\checkmark}{\underset{bb}{f_2 f_2}},\ \dots \}$$

$$L = \{ \underbrace{\varepsilon}_{\text{choice}},\ f_1,\ f_1 f_1,\ f_2 f_2,\ \dots \}$$

$$\Downarrow$$

$$\pi(f_1) = id$$

Regular Languages : MSQ

Q51. If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is TRUE?

$y \in L^R$

A $\{xy \mid x \in L, y^R \in L\}$ is Regular $L \cdot L^R$ is Reg

B $\{y \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $x\boxed{y} \in L\}$ is Regular Suffix(L)

C $\{x \in \Sigma^* \mid \exists y \in \Sigma^*$ such that $\boxed{x}y \in L\}$ is Regular prefix(L)

D $\{ww^R \mid w \in L\}$ is Regular is FALSE

$$\text{Suffix}(L) = \{ y \mid xy \in L, \ x \in \Sigma^* \}$$

$$= \{ y \in \Sigma^* \mid \exists x \in \Sigma^* \ni xy \in L \}$$

$$\text{Prefix}(L) = \{ x \mid xy \in L, \ y \in \Sigma^* \}$$

$\{xy \mid x \in L,\ y^R \in L\} = \{xy \in L L^R\}$

$\{xy \mid x \in L,\ y \in L^R\}$

$L \cdot L^R$

Given L is Reg
$\Downarrow$
L^R is also Reg

$\Bigg\}$ $L \cdot L^R$ is Reg

Q52. For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{5+3k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

A) 5

B) 3

C) 7 ✓

D) 4

$\underbrace{a^{3K+5}}$

$\Downarrow$

6 states in min DFA

$\boxed{\text{Constant} \geq 6}$

Regular Languages : NAT

TOC & Algo

Q53. Given a language L, define L^i as follows:

$$L^0 = \{\epsilon\}$$
$$L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k-1}$.

Consider the language L_1 (over alphabet 0) accepted by the following FA.

L^0

$\times\; L^1 = L^0$ $\quad K=1$

$\times\; L^2 = L^1$ $\quad K=2$

$\times\; L^3 = L^2$ $\quad K=3$

$L^0 = \{\varepsilon\}$

$L^1 = L^0 \cdot L = L = \varepsilon + 0(00)^*$

$L^2 = L^1 \cdot L = (\varepsilon + 0(00)^*) \cdot (\varepsilon + 0(00)^*) = 0^*$

$L^3 = L^2 \cdot L = 0^* \cdot L = 0^*$

The order of L_1 is ____.

$\boxed{= 3}$

$\boxed{L^3 = L^2}$ $\quad L^{\boxed{K}} = L^{K-1}$ for small K

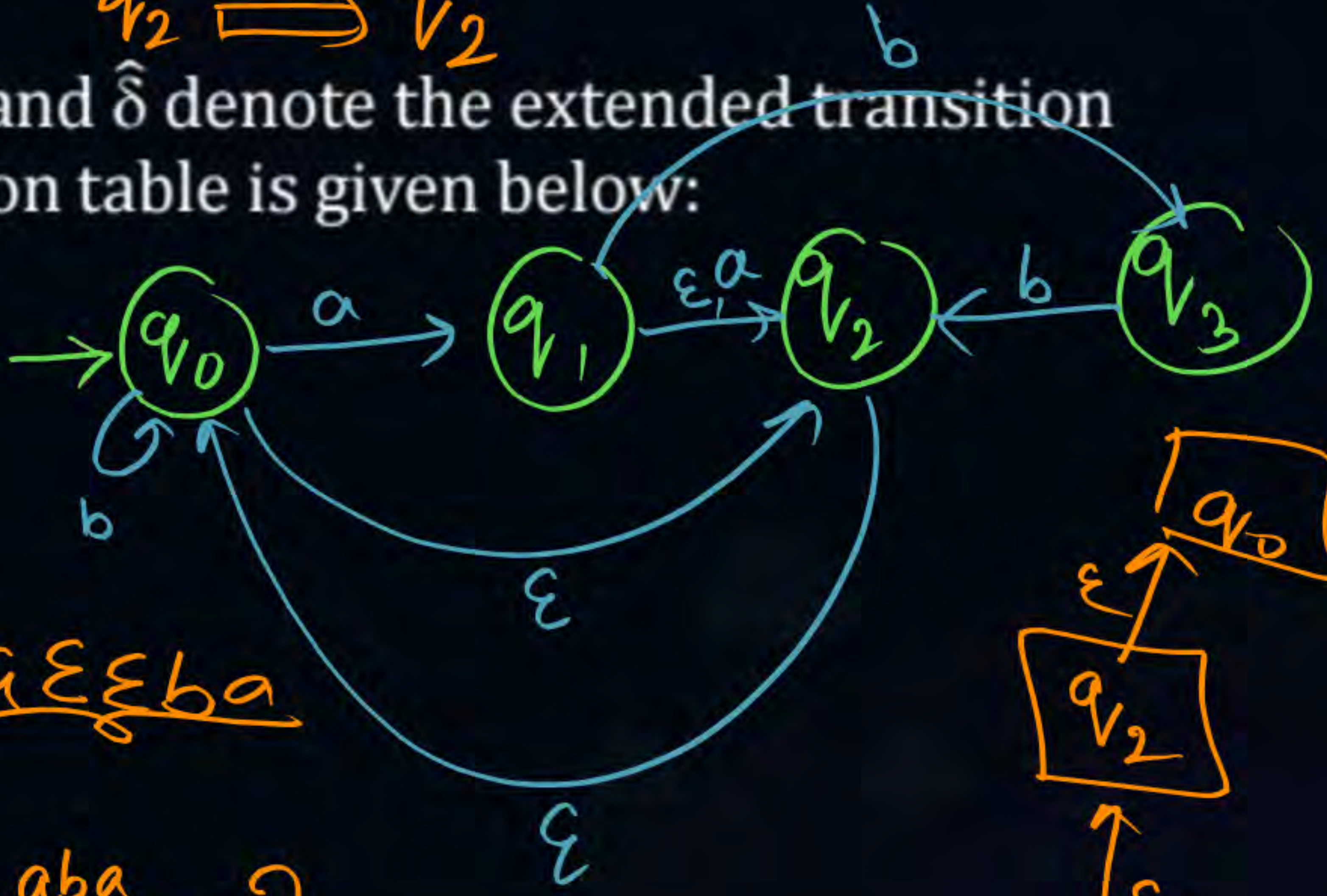
$L = \varepsilon + 0(00)^*$

$q_2 \xrightarrow{aba} q_1$ $\quad q_2 \xrightarrow{aba} q_0$

$q_2 \xrightarrow{aba} q_2$

Q54. Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the ε-NFA whose transition table is given below:

δ	$\in$	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$



εaεεba

Then $|\hat{\delta}(q_2, aba)|$ is ______

$q_2 \xrightarrow{aba} ?$

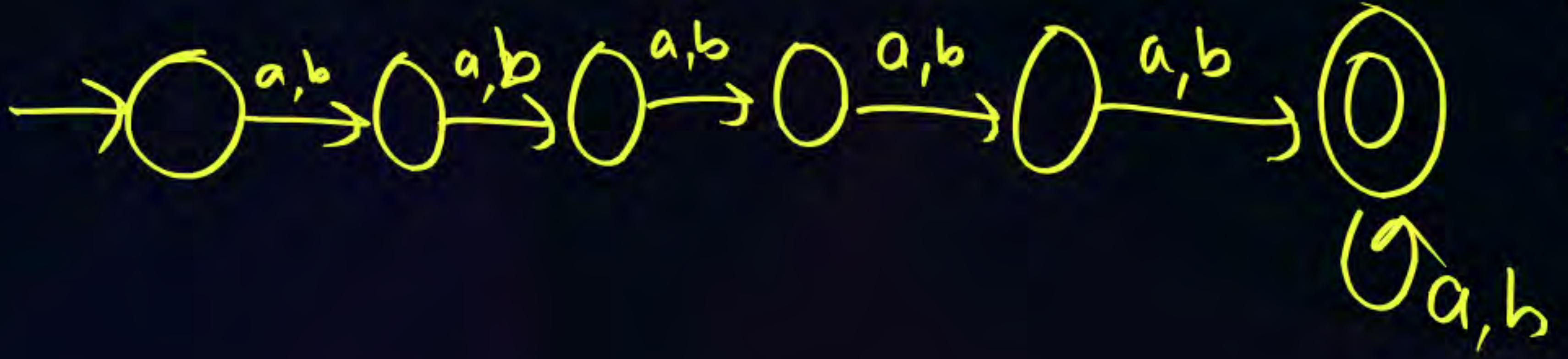
$q_2 \xrightarrow{\varepsilon} q_0 \xrightarrow{a} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{\varepsilon} q_0 \xrightarrow{b} q_0 \xrightarrow{a} \boxed{q_1} \xrightarrow{\varepsilon} \boxed{q_2} \xrightarrow{\varepsilon} \boxed{q_0}$

$|\{q_1, q_2, q_0\}| = 3$

Q55. Find the minimum possible number of states of a DFA that accepts the regular language $L = \{\underbrace{\underbrace{w_1}_{\geq 2}\ \underbrace{w_2}_{\geq 3}}_{\geq 5} \mid w_1, w_2 \in \{a, b\}^*, |w_1| \geq 2, |w_2| \geq 3\}$ is ____.

$\{w \mid w \in \{a,b\}^*, |w| \geq 5\}$

$\Downarrow$



$5+1 = 6$ states

Regular Languages : NAT

Q56. If G is a grammar with productions

$$S \rightarrow Sa \mid Sb \mid Saa$$

where S is the start variable, then which one of the following strings is not generated by G?

A abab

B aaab

C abbaa

D babba

Q57. How many of the following languages are regular?

$L_1 = \{wxw^R \mid w, x \in \{a, b\}^*, w^R \text{ is the reverse of string } w\}$
$L_2 = \{a^n b^m \mid m, n \geq 0\}$
$L_3 = \{a^p b^q c^r \mid p, q, r \geq 0\}$
$L_4 = \{\omega \mid \omega \in \{0,1\}^*, \omega \text{ has equal number of } (00)\text{'s and } (11)\text{'s}\}$.

Q58. If $L_1 = \{a^n \mid n \geq 0\}$and $L_2 = \{b^n \mid n \geq 0\}$, then how many of the following statements are TRUE?

I. $L_1 \cdot L_2$ is a regular language
II. L_1 / L_2 is a regular language
III. $L_1 U L_2$ is a regular language

Regular Languages : MCQ

Q59. Which one of the following is TRUE?

A. Kleene closure of $\{a^n b^n \mid n \geq 0\}$ is regular.

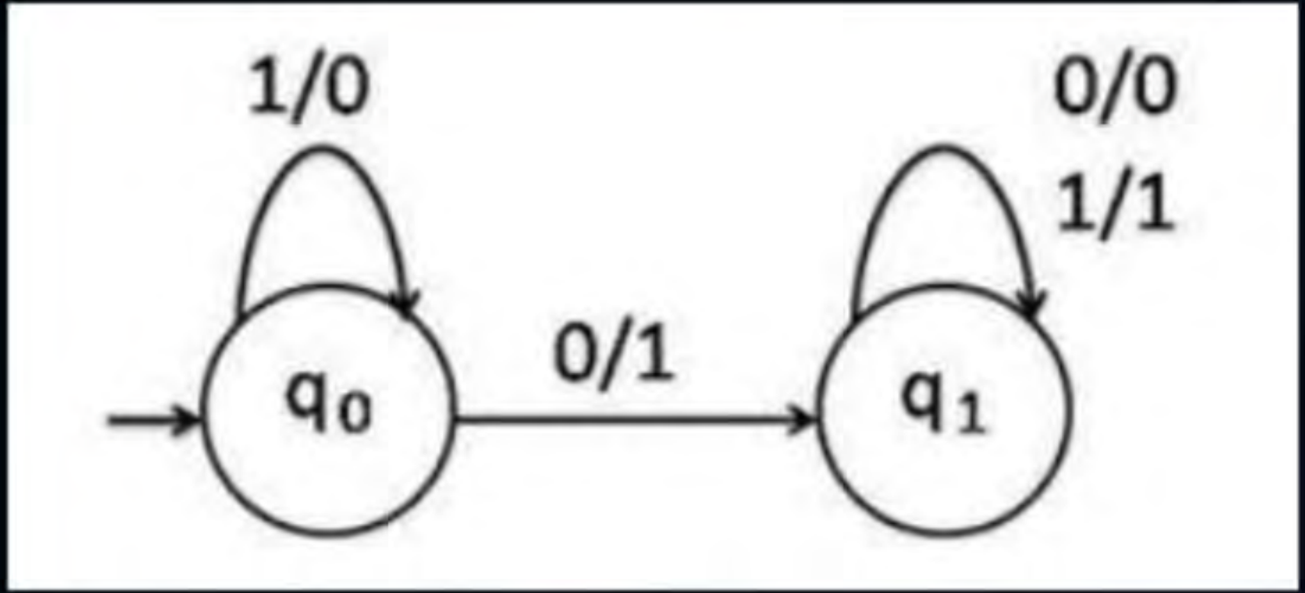
B. Kleene closure of $\{a^n \mid n \text{ is prime}\}$ is regular.

C. Kleene closure of $\{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular.

D. Kleene closure of $\{wxw \mid w, x \in \Sigma^* \text{ with } \Sigma = \{0, 1\}\}$ is regular.

Regular Languages : NAT

Q60. Consider the following FSM with output. It takes binary input in reverse order of actual binary number and produces binary output. To see actual output, produced output should be considered in reverse. Identify TRUE statement.



A. It increments given input

B. It decrements given input

C. It left shifts given input

D. It right shifts given input

$$\underbrace{a^n b^n}_{\text{DCFL}} \subset \underbrace{a^* b^*}_{\text{reg}}$$

$$\underbrace{\text{Set of regs} \subset \text{Set of DCFLs}}$$

$$\underset{\text{DCFL}}{a^n b^n} \supset \underset{\text{reg}}{\{ab\}}$$

$$\underset{\text{DCFL}}{a^*} = \underset{\text{Reg}}{a^*}$$

THANK - YOU