Branch : CSE & IT   Batch : Hinglish

Subject : THEORY OF COMPUTATION

Chapter : Finite Automata

Topic : Closure Properties of Regular Languages - 2

DPP 14

[MCQ]

1. Consider a regular language L.
If $L^* = \{a^{prime}\}^*$ is regular, then which of the following is true?
(a) $L = \{a^{prime}\}$ is regular
(b) $L = \{a^{prime}\}$ is not regular
(c) $L = \{a^{prime}\}$ is regular and finite.
(d) None of these.

[MSQ]

2. Consider a regular language L, which of the following statements are true regarding L.
(a) Prefix(L) = $\{w \mid ww_1 \in L, w_1 \in \Sigma^*\}$ is regular.
(b) Suffix(L) = $\{w \mid w_1w \in L, w_1 \in \Sigma^*\}$ is regular.
(c) Half(L) = $\{w \mid ww_1 \in L, |w| = |w_1|\}$ is regular.
(d) L is closed under infinite intersection.

[MCQ]

3. Let's consider L_1 and L_2 are two regular sets defined over (Σ = a, b), then
(a) $L_1 \cap L_2$ is irregular
(b) $L_1 \cup \overline{L_2}$ is not regular
(c) L_1^* is not regular
(d) $\Sigma^* - L_1$ is regular

[MCQ]

4. Let's suppose the languages $L_1 = \{a\}$ & $L_2 = \{\phi\}$. Then $L_2L_1^* \cup L_2^*$?
(a) $\{\phi\}$ (b) $\{\in\}$
(c) $\{a^*\}$ (c) $\{a\}$

[MCQ]

5. Consider a regular language L over the alphabet $\Sigma = \{a, b\}$. L is defined as x = (a + b*) (bab*).
If homomorphism h is defined over T = {c, d, e} and
h(a) = cd
h(b) = cddec
Then the regular language h(L) is given as
(a) (cd + cddec) (cddec cd cddec)
(b) (cddec) (cd + cddec*)
(c) (cd + (cddec)*) ((cddec) (cd) (cddec)*)
(d) None of these

[MCQ]

6. Consider the following statements:
S_1: if $L_1 \cup L_2$ is regular, then both L_1 and L_2 are regular.
S_2: Regular language is closed under infinite union.
(a) S_1 is true.
(b) S_2 is true
(c) Both S_1 and S_2 are true
(d) Both S_1 and S_2 are false.

[MSQ]

7. Regular language is closed under
(a) Subset
(b) Complement
(c) Finite union
(d) Infinite Intersection

Answer Key

1. **(b)**
2. **(a, b, c)**
3. **(d)**
4. **(b)**
5. **(c)**
6. **(d)**
7. **(b, c)**

Hints & Solutions

1. (b)
If L^* is regular then L is need not to be regular.
Hence, If $L^* = \{a^{prime}\}*$ is regular, then $L = \{a^{prime}\}$ is not regular. Hence, option (b) is correct.

2. (a, b, c)
Regular language is closed under Prefix, Suffix and half of the language. But regular language are not closed under infinite intersection.
So, a, b, c are correct.

3. (d)
(a) Regular language is closed under intersection, So
option (a) is false.
(b) Regular language is closed under complementation and union. Therefore, option (b) is false.
(c) Regular language is closed under kleene closure.
So, option (c) is false.
(d) $\Sigma^* - L_1 = \Sigma^* \cap \overline{L_1}$, Regular language is closed under intersection and complementation. So, option (d) is correct.

4. (b)
$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \ldots L^k \cup L^{k-1} \ldots$
we know, $L^0 = \in$.
ϕ acts as 0 in multiplication. So, concatenation of ϕ with any other language will result in ϕ.
Given,

$L_1 = \{a\}$

$L_2 = \phi$

$L_2 L_1^* \cup L_2^*$

$L_1^* = \{a\}^0 \cup \{a\}^1 \cup \{a\}^2 \ldots$
$= \in \cup a \cup aa \ldots$
$= a^*$

$L_2^* = \{\phi\}^0 \cup \{\phi\}^1 \cup \{\phi\}^2 \ldots$
$= \in \cup \phi \cup \phi \ldots$
$= \{\in\}$

$\therefore L_2 L_1^* \cup L_2^* = \phi \cdot a^* \cup \{\in\}$
$= \phi \cup \{\in\}$
$= \{\in\}$

So, option (b) is correct answer.

5. (c)
Homomorphism is a function from strings to string which is based on concatenation.
for any a and b $\in \Sigma^*, h(a,b) = h(a)h(b)$
L is defined as
$x = (a+b)^* (bab^*)$
then,
$h(L) = (h(a) + h(b)^* (h(b)h(a)h(b)^*)$
$= (cd + (cddec)^*)((cddec)(cd)(cddec)^*)$.

6. (d)
S_1: If $L_1 \cup L_2$ is regular, then L_1 and L_2 may be regular.
Consider $L_1 = \{a^n b^n, n \geq 0\}$ and consider L_2 be the complement of L1.
So, $L_1 \cup L_2 = \{a^n b^n\} \cup \{a^n b^n\}^c$
$= (a+b)^*$
this is regular but L_1 and L_2 are DCFL.
S_2: Regular language is not closed under infinite union.

7. (b, c)
Regular language are closed under complement and finite union.