# CS & IT ENGINEERING

## Theory of Computation

### Push Down Automata:

PDA – Part 2

**Lecture No. 03**

By- DEVA Sir

01 Push Down Automata

02

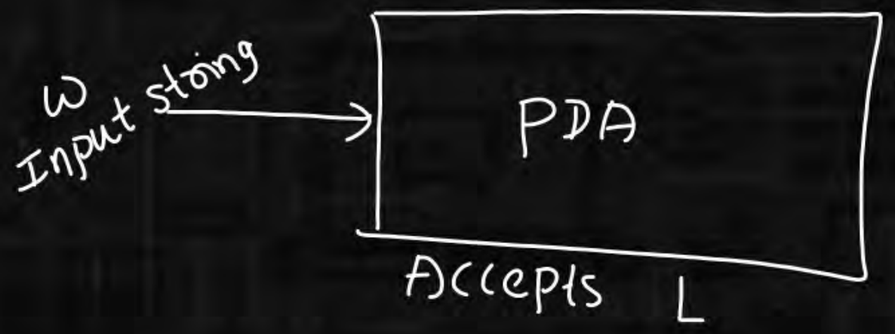
03

04

05

# Push Down Automata

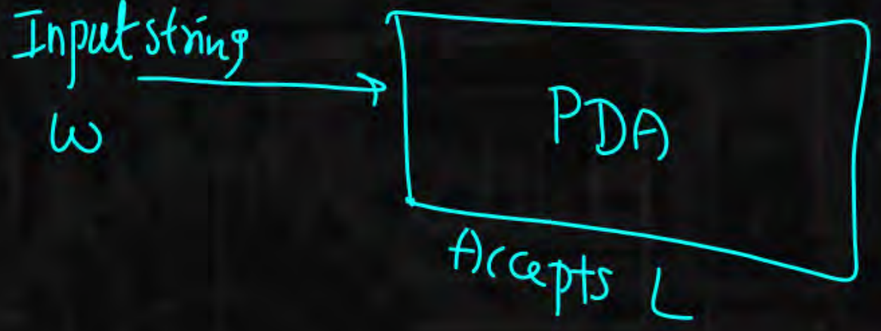
## I) Acceptance by Final State




After reading whole string

If $w \in L$, it halts at final state

## II) Acceptance by Empty stack




After reading whole string

If $w \in L$, stack will be empty

# Final State Acceptance

Assume stack initially has $Z_0$

$\Sigma = \{a, b\}$

1.

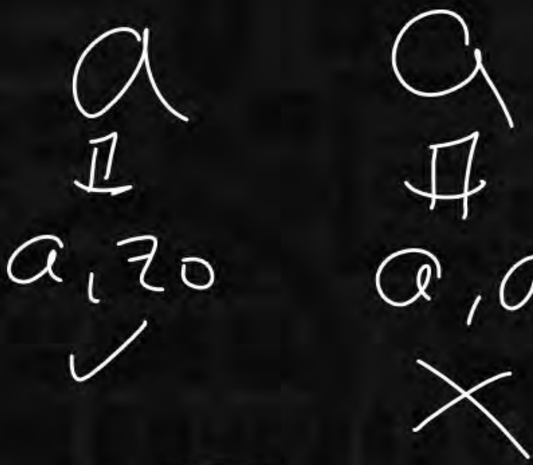
$L = \phi = \{\ \}$

2.

$L = \{\varepsilon\}$

3.


$L = \{\varepsilon, a\}$




$\varepsilon$ : $A_\varepsilon$

a : $A \xrightarrow{a} A$

aa X $A \xrightarrow{a} A \xrightarrow{a}$

(4)

$a, z_0 | z_0$

$L = a^*$

$a, \varepsilon | \varepsilon$

$\varepsilon : A$

$a : A \xrightarrow{a} A$

OR

$a, X | X$

$aa : A \xrightarrow{a} A \xrightarrow{a} A$

$aaa : A \xrightarrow{a} A \xrightarrow{a} A \xrightarrow{a} A$

(5)

$a, z_0 | a z_0$ [push 1st a]

$a, a | aa$ [Push remaining a's]

$L = a^*$

✓ $\varepsilon : A$

✓ $a : A \xrightarrow{a} A$

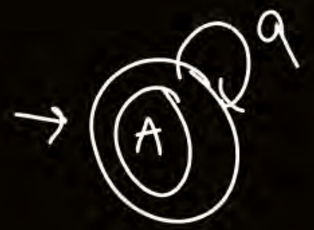
✓ $aa : A \xrightarrow{a} A \xrightarrow{a} \underbrace{A}_{\text{final}}$

✓ aaa

✓ aaaa

⋮

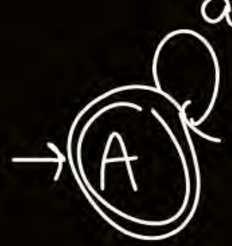
FA $\Longrightarrow$ PDA




$a$

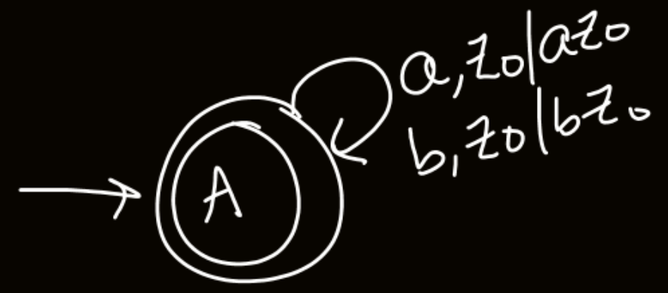
add $z_0 | z_0$

$\Longrightarrow$




$a, z_0 | z_0$

6




$L = \{\varepsilon, a, b\}$

✓ $\varepsilon$ : A

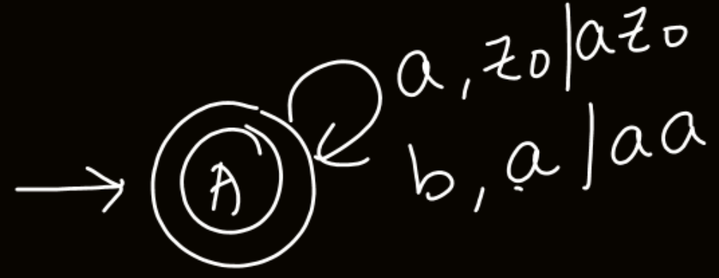
✓ a : $A \xrightarrow{a} A$

X aa : $A \xrightarrow{a} A \xrightarrow{a}$

✓ b :

$A \xrightarrow{b} A$

7




$\varepsilon$ ✓ A

a ✓

$$L = \varepsilon + ab^*$$

aa ✗

ab ✓

abb ✓

abbb ✓

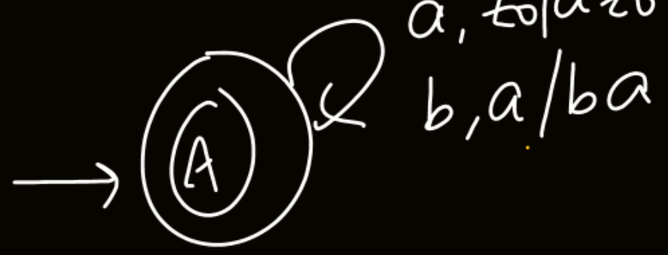
b ✗

ab ✓

$A \xrightarrow{b, z_0 |} \text{no trans}$

$A \xrightarrow[\text{Push } a]{a, z_0} A \xrightarrow[\text{Push } a]{b, a} A$

(8)




$L = \{\varepsilon, a, ab\}$

$\varepsilon$ ✓

a ✓

bX

ab ✓

abbX

9

$L = \{\varepsilon\}$

$/\varepsilon$ : $A$

$Xa$ : $A \xrightarrow{a, z_0}$ no pulls

$Xaa$

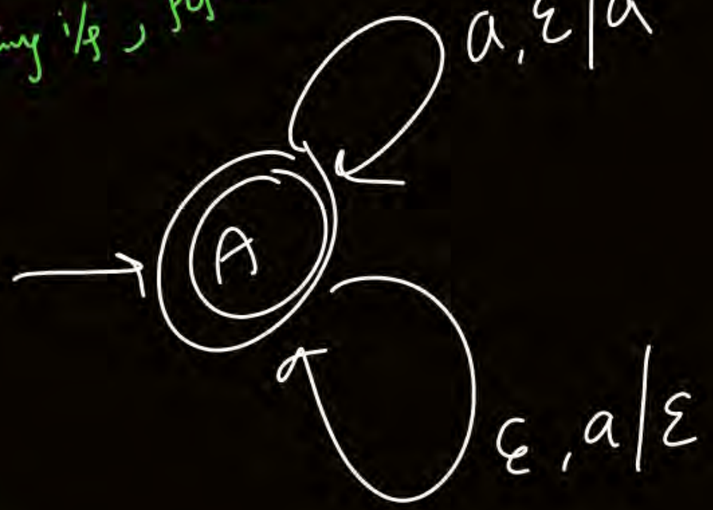
⋮

(10)

a, ε | a  Push 'a' for i/p 'a'

ε, a | ε  without reading i/p, pop 'a'

OR




$L = a^*$

ε ✓

a ✓

aa ✓

⋮

(11)

Read i/p 'a', Don't look at stack, Do nothing on stack

Read i/p 'b', " " A

$L = (a+b)^*$

a ✓ A —a→ A

b ✓ A —b→ A

(12)

Push 'b' without looking at tos

Push 'a' "

$L = (a+b)^*$ ε ✓

a ✓

b ✓

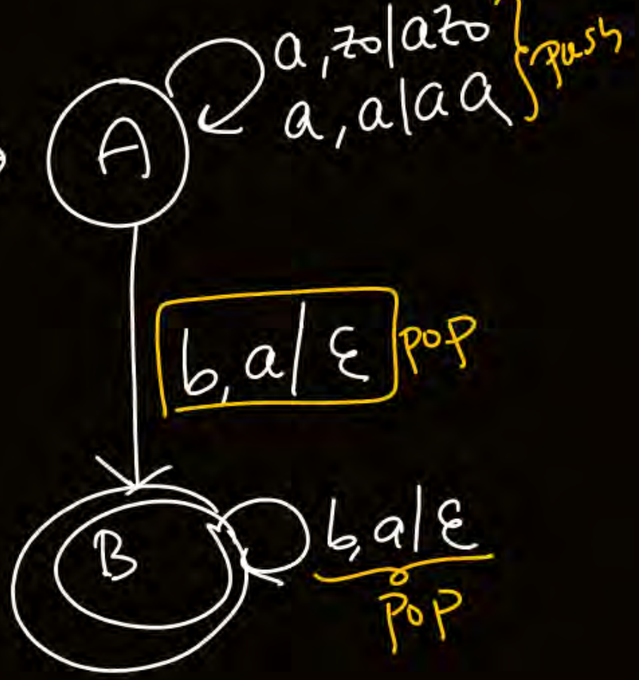
aa

A —a→ A —a→ A

aba ✓

(13)

$L = \{ a^m b^n \mid m \geq n \}$, $m, n \geq 1$




Check following

1) ε ✗ : A

2) a ✗ A $\xrightarrow{a, z_0 / a z_0}$ A (not final)

3) ab ✓ A $\xrightarrow{a, z_0 / a z_0}$ A $\xrightarrow{b, a / \varepsilon}$ B (final)

4) aaabb ✓

A $\xrightarrow{a, z_0 / a z_0}$ A $\xrightarrow{a, a / a a}$ A $\xrightarrow{a, a / a a}$ A $\xrightarrow{b, a / \varepsilon}$ B $\xrightarrow{b, a / \varepsilon}$ B (final)

5) aabbb ✗ (no path)

Using Final state mechanism

//

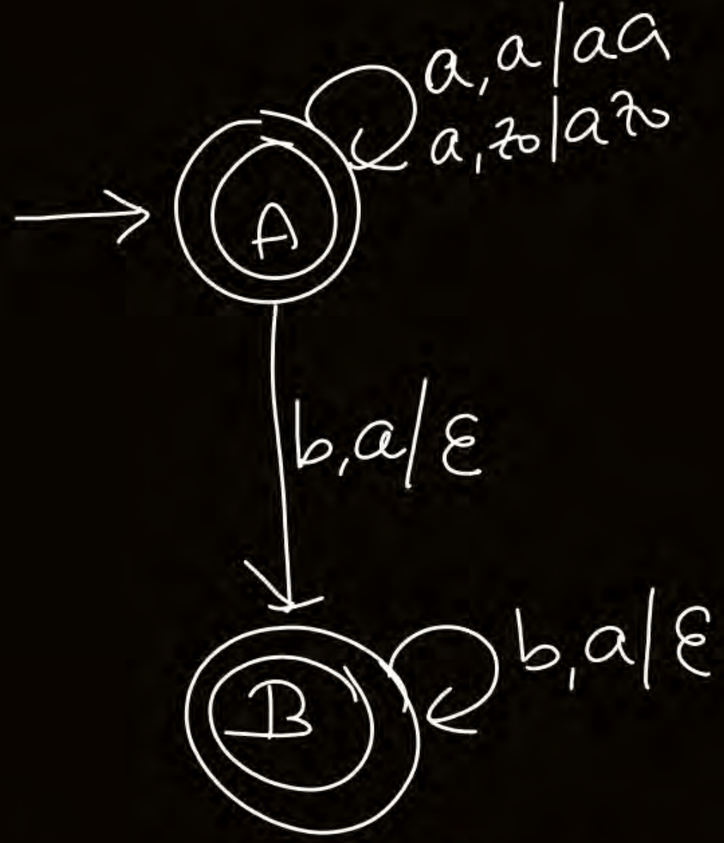
At the end of input, only state is matter

Where it halts

Stack is not required
(may be empty or not empty)

***14

GATE PYQ




$L = \underbrace{a^*}_{n=0} \cup \{a^m b^n \mid m \geq n,\ m,n \geq 1\}$

$= \{a^m b^n \mid m \geq n,\ m,n \geq 0\}$

A : $a^*$

B : $\{a^m b^n \mid m >= n \quad m,n \geq 1\}$

PDA

Acceptance Mechanism By

Empty stack

At the end of i/p

aaba

Is stack empty ?

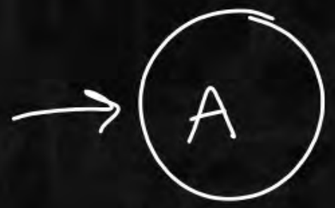
Try to make empty stack

# Push Down Automata

Using only Empty stack ! (Assume initially $Z_0$ is on stack)

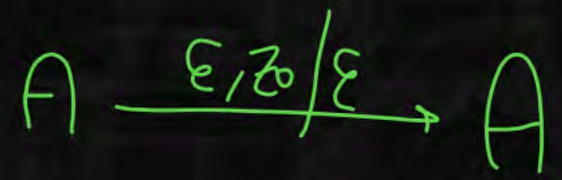
(there is no meaning to final state)

1.



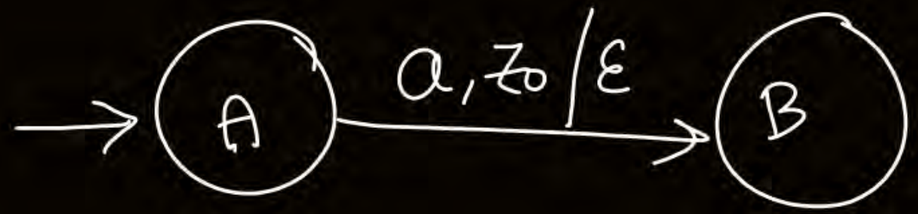

$L = \phi$

2.



$L = \{\varepsilon\}$

(3)




$L = \{a\}$

| Complete the input |
| --- |
| check Stack |

ε A

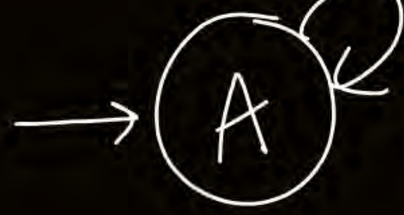
stack is not empty

a A → a B

aa A → a B → a

no pdl

(4)




$L = a^*$

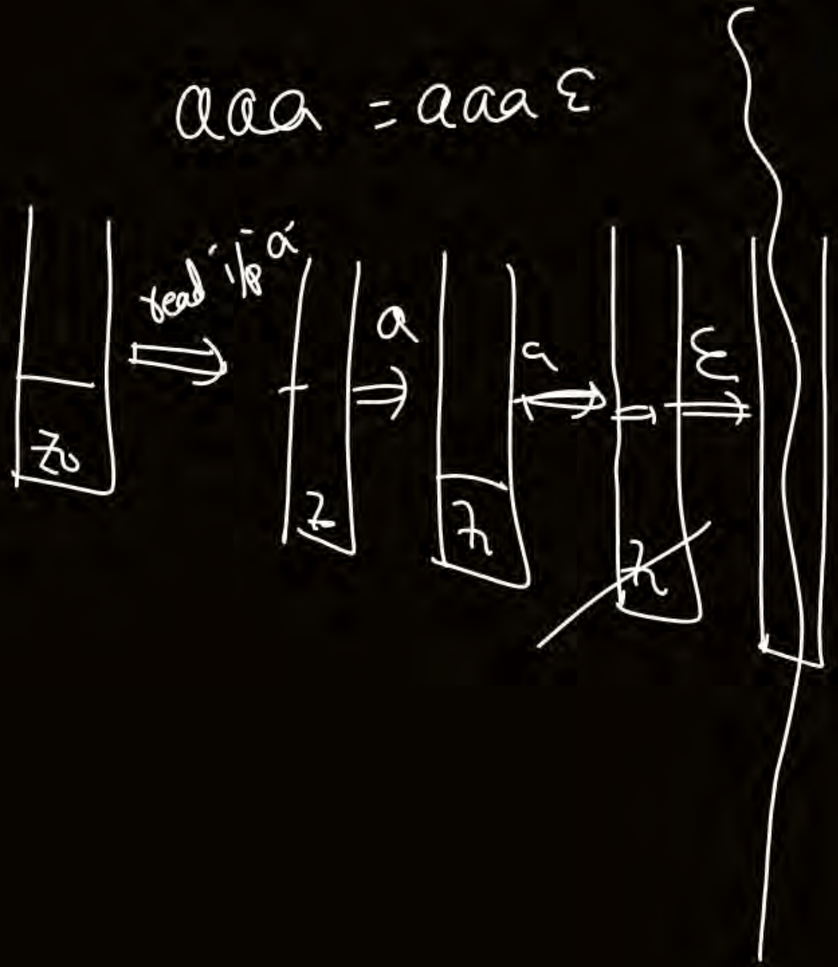
ε ✓ we can make stack empty

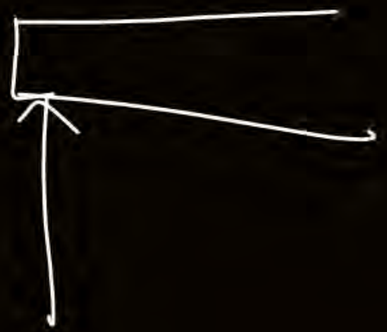
a ✓ "

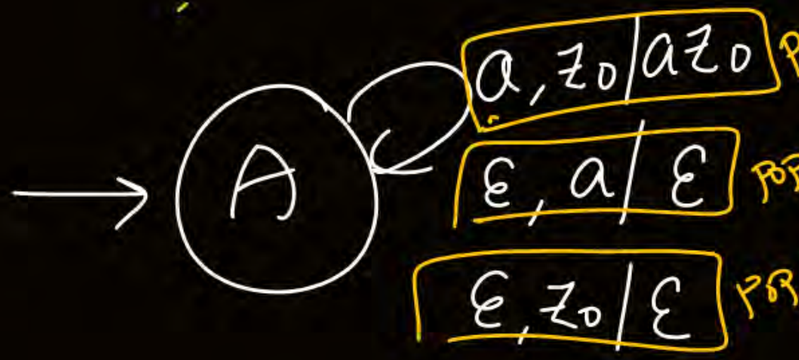
aa ✓ "

aaa ✓ "

aaa = aaaε





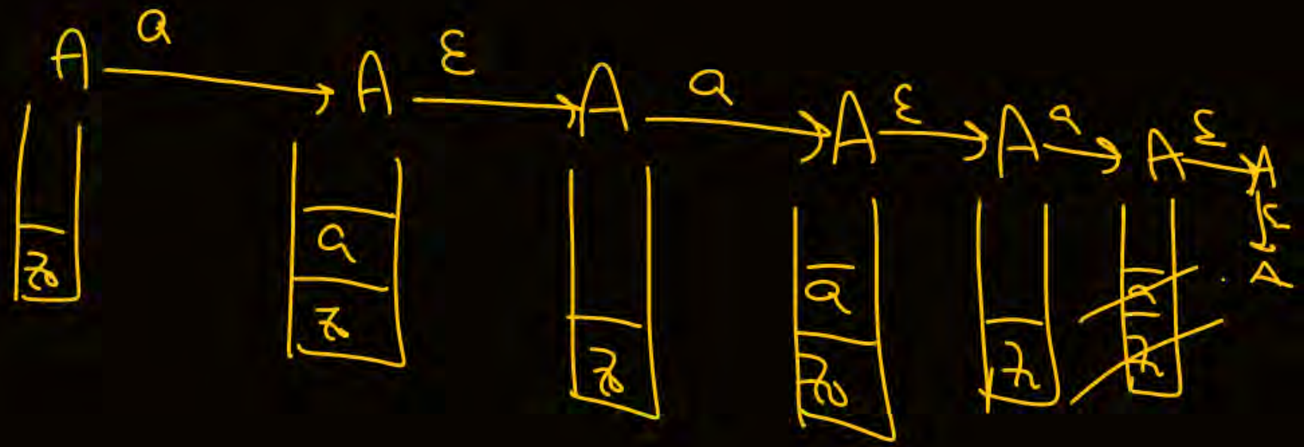



$L = a^*$

ε: A $\xrightarrow{\varepsilon, Z_0 | \varepsilon}$ A

a: A $\xrightarrow{a, Z_0 | aZ_0}$ A $\xrightarrow{\varepsilon, a | \varepsilon}$ A $\xrightarrow{\varepsilon, Z_0 | \varepsilon}$ A

aaa:

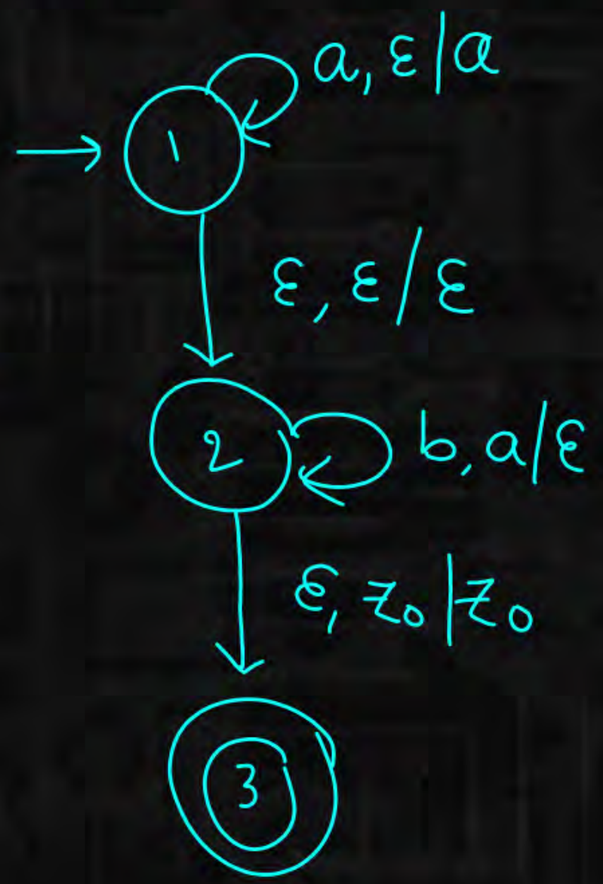


# Push Down Automata

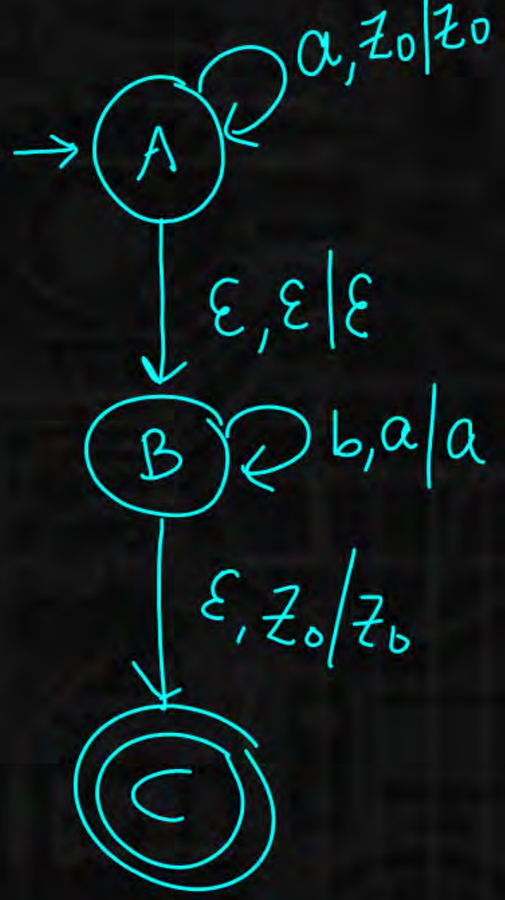
Home Work :

Using Final state

① $L = \{a^n b^n \mid n \geq 0\}$




②



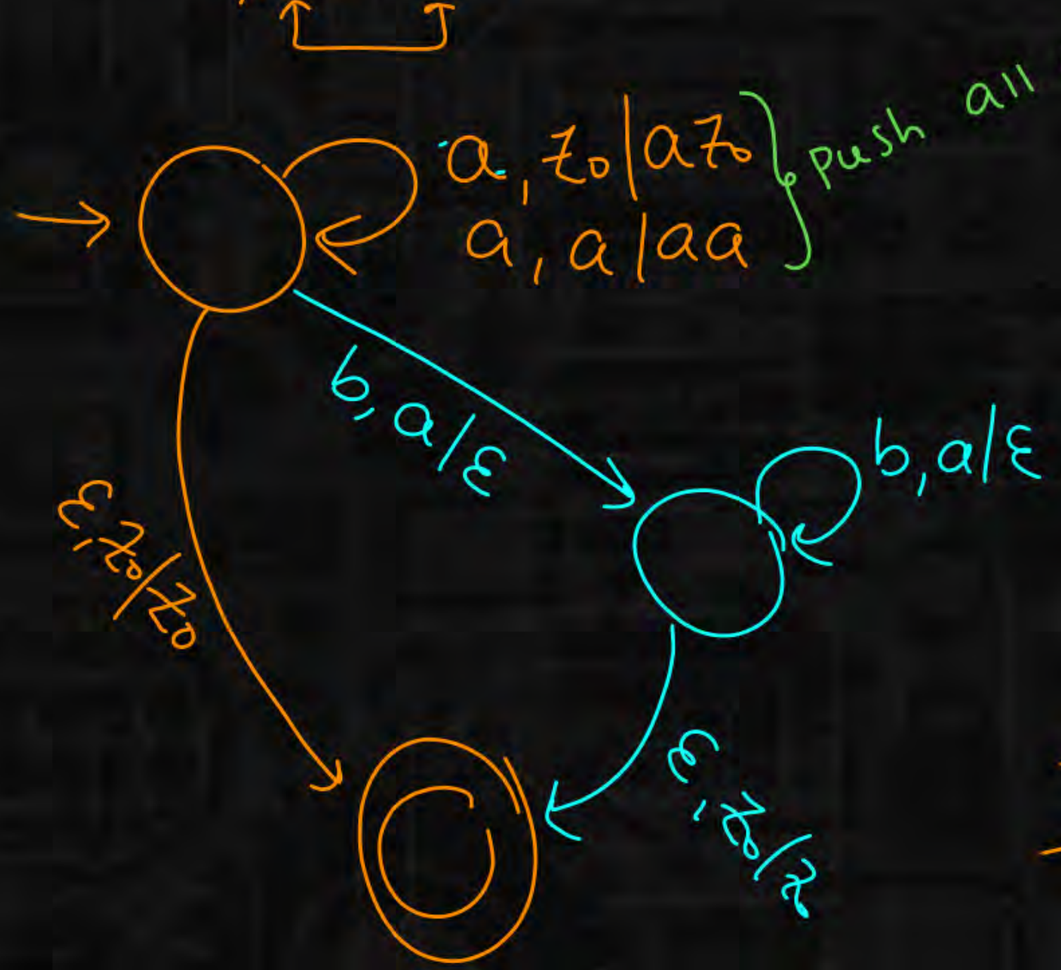

$L = a^*$

# Push Down Automata

construction of PDA

① $\{a^n b^n \mid n \geq 0\}$

Push a's  Pop a's

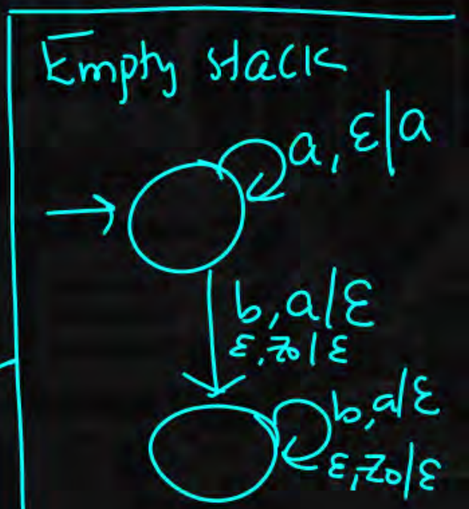


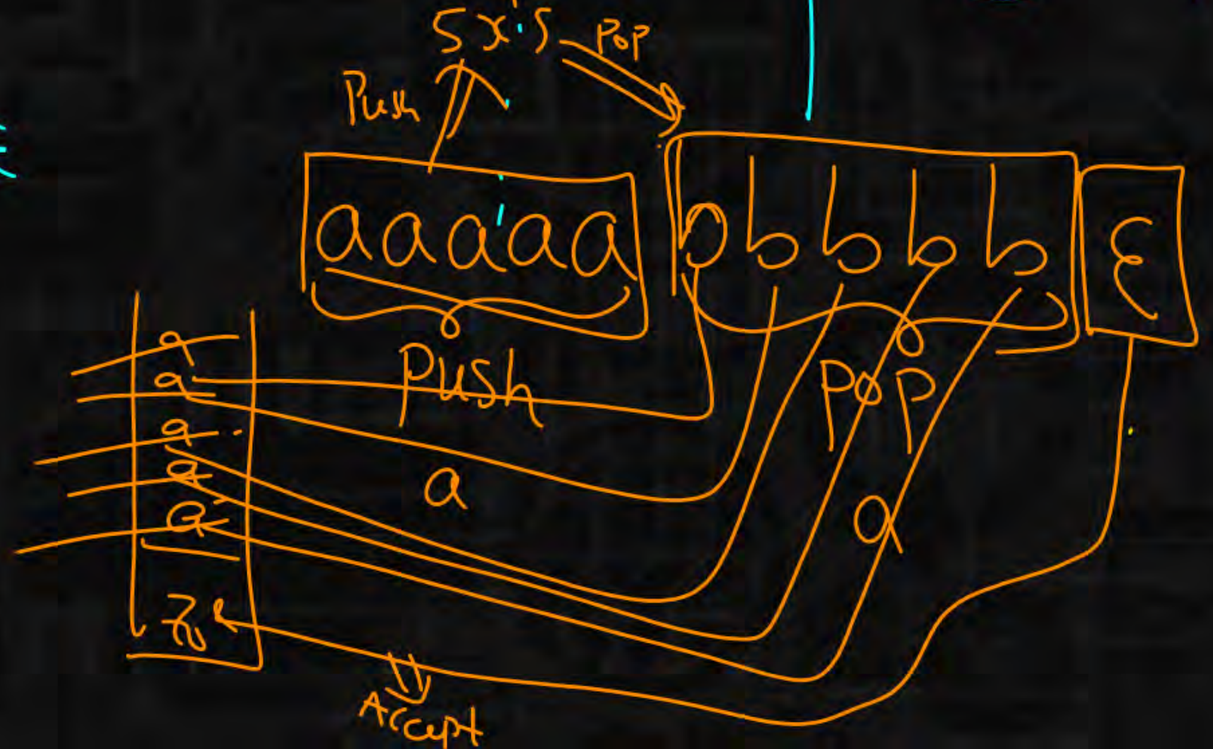

ε ✓

ab ✓

aabb ✓

aaabbb ✓



# Push Down Automata

Construct PDA with Final state Acceptance

1. $\{a^n b^n \mid n \geq 0\}$
2. $\{a^m b^n \mid m > n,\ m, n \geq 1\}$
3. $\{a^m b^n \mid m < n,\ m, n \geq 1\}$
4. $\{a^m b^n \mid m \neq n,\ m, n \geq 1\}$
5. $\{a^m b^n \mid m \leq n,\ m, n \geq 1\}$
6. $\{a^m b^n \mid m \geq n,\ m, n \geq 1\}$

## Summary

→ What is PDA ?

Next: CFLs & DCFLs ⇨ How to construct PDA & DPDA

THANK YOU GW
SOLDIERS !