CS & IT ENGINEERING

Theory of Computation

Undecidability & Decidability : Decision Properties Table

Lecture No. 1

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TOPICS TO BE COVERED

01 Decision Properties Table

Terminology

I) Set of all Recursive languages

II) Set of all RELs

III) Set of all RELs but not Recursive

IV) Set of all Not RELs

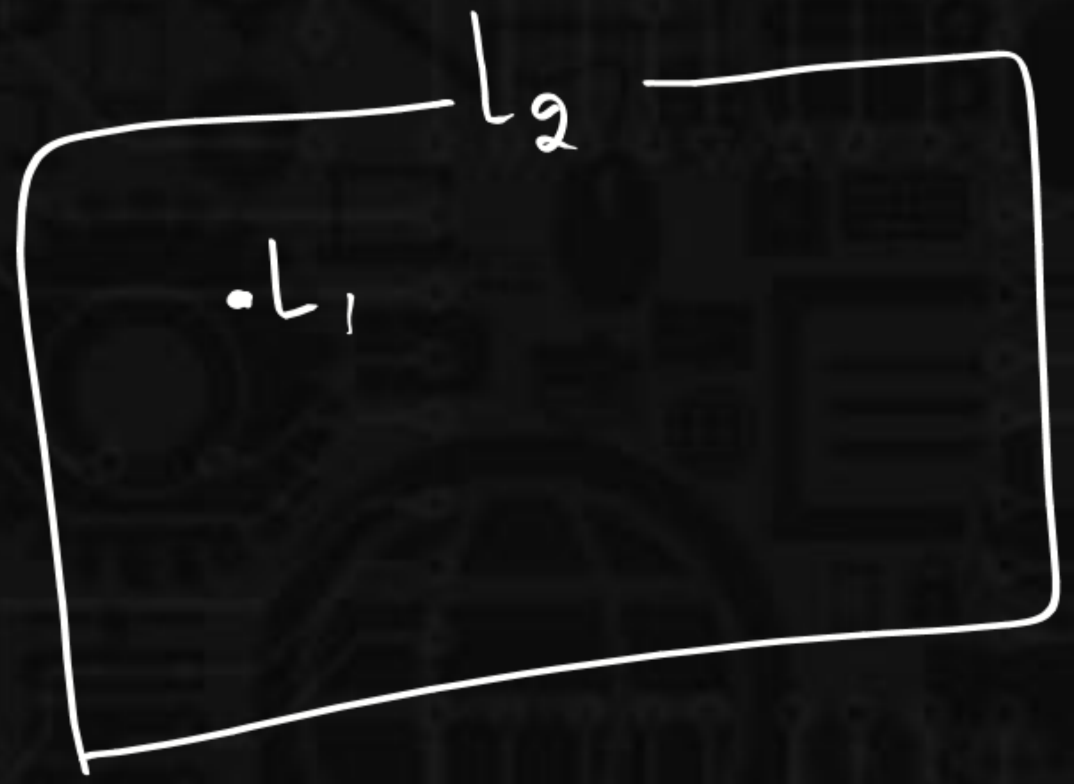
V) Set of all undecidable languages

$L_1 = \text{REL}$

$L_2 = \text{Set of all RELs}$

$= \{L_1, \ldots\}$

$L_1 \in L_2$



Recursive language (L)

$\cong$ $\Updownarrow$

Decidable Language

$\cong$ $\Updownarrow$

$\boxed{L \text{ is REL and } \bar{L} \text{ is REL}}$

$\cong$ $\Updownarrow$

$\boxed{L \text{ has TM and } \bar{L} \text{ has TM}}$

$\cong$ $\Updownarrow$

$\boxed{L \text{ has HTM}}$

$\cong$ $\Updownarrow$

$\boxed{\text{Logic exist for valid \& Invalid}}$

Recursive language is ______

I) REL

II) Decidable language

III) Turing decidable

IV) Turing Acceptable

V) Enumerable language

VI) Semi-decidable language

L is Recursive

iff

L is REL and $\bar{L}$ is REL

HTM for L

$w \in L \Rightarrow$ logic exist

$w \notin L \Rightarrow$ logic exist

Recursively Enumerable language (L) [REL]

$\cong$

$\boxed{\text{Semi-decidable language [SD]}}$ (Partially decidable)

$\cong$

$\boxed{\text{(Turing) Recognizable Set (Acceptable)}}$

$\cong$

$\boxed{\text{(TM) Enumerable set}}$

$\cong$

$\boxed{\text{L has TM}}$

$\cong$

$\boxed{\text{Logic exist for valid strings}}$

REL
SD } same

REL
↳ may be "Recursive"
or
"REL but not recursive"

I) $\underbrace{\text{Recursive language}}_{\text{Turing Decidable}}$ is $\underbrace{\text{REL}}_{\text{Turing recognitable}}$ $\Longrightarrow$

II) Recursive is Recognitable

$\Longrightarrow$

$\cancel{\Longleftarrow}$

III) Decidable is Recognitable

Acceptable
↳ valid has logic

Decidable
↳ valid & Invalid } logic exist

"REL but not Recursive"

(L)

$\cong$

Logic exist only for valid

$\cong$

L has TM but no HTM

$\cong$

Undecidable but REL

$\cong$

Undecidable and semidecidable

$\cong$

Semidecidable but undecidable

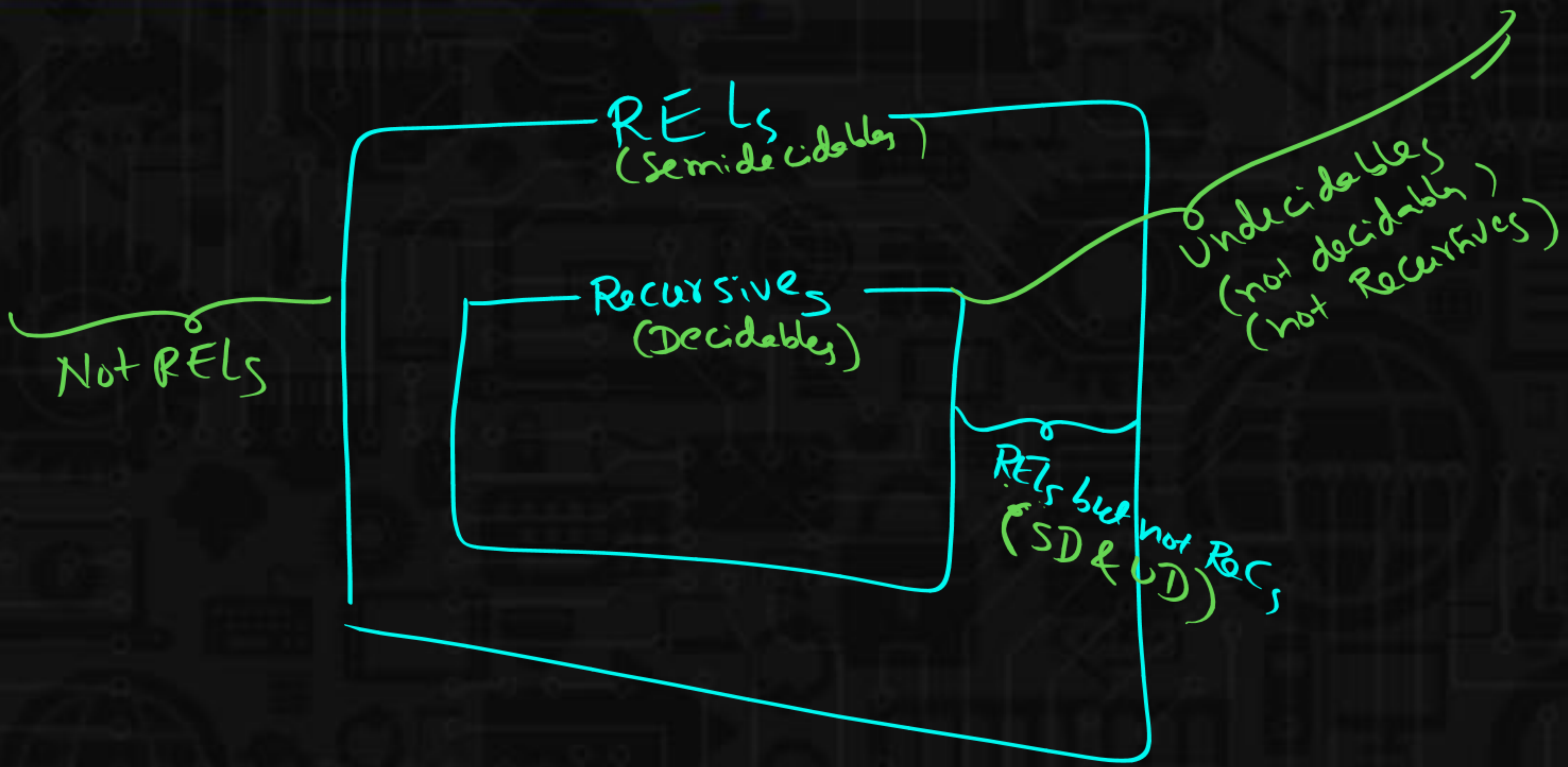
REL but not Recursive is ____

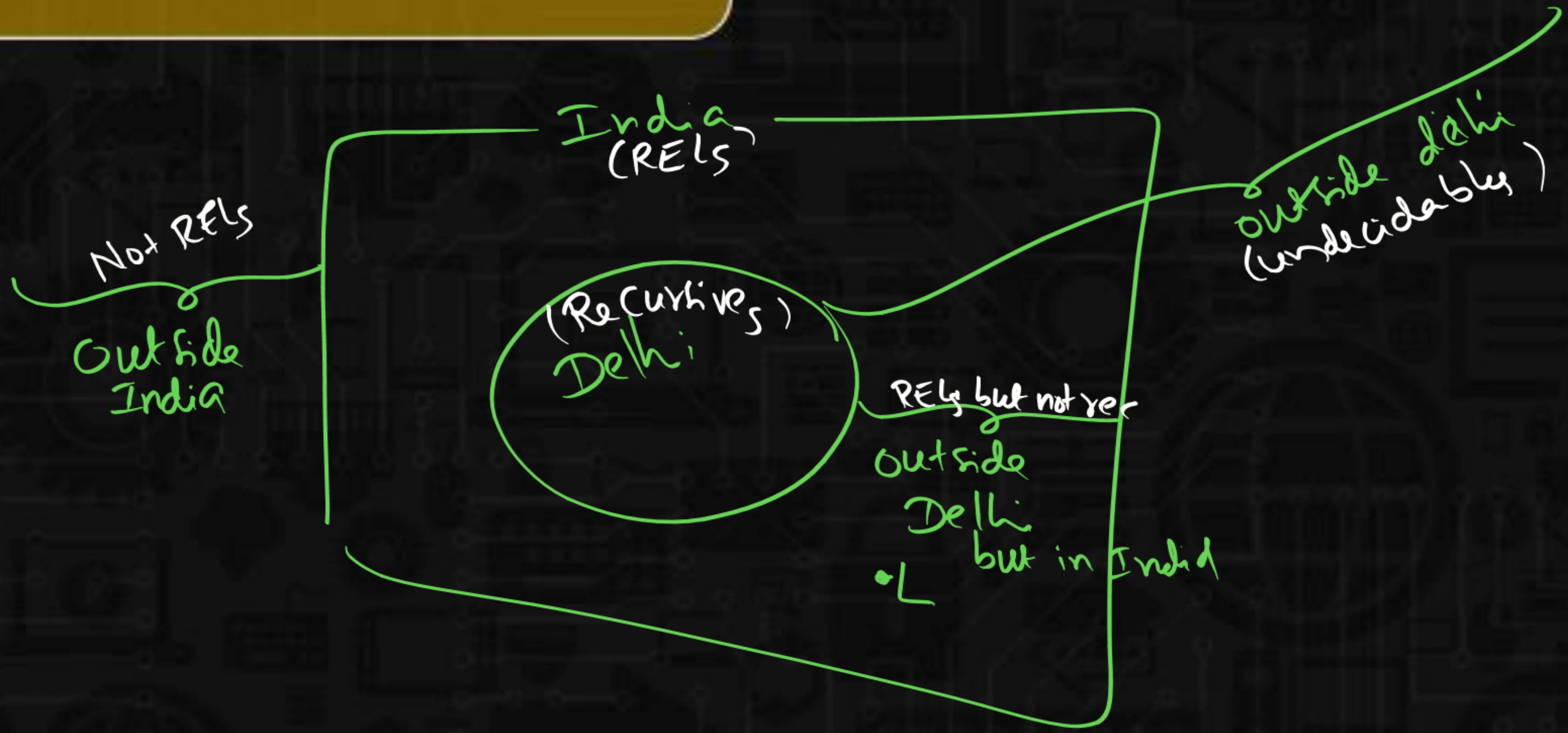
I) REL
II) Undecidable
III) not recursive
IV) Semidecidable (RE)
V) not decidable

X is Y $(X \Rightarrow Y)$

$X \cong Y$ $(X \Rightarrow Y$ & $Y \Rightarrow X)$

$X \Leftrightarrow Y$


RELs
(Semidecidables)
Recursives
(Decidables)
Not RELs
Undecidables
(not decidables)
(not Recursives)
RELs but not RecS
(SD & UD)


India
(RELs)
Not RELs
Outside India
(Recursives)
Delhi
RELs but not rec
outside Delhi but in India
L
outside delhi
(undecidables)

Not REL

(L)

$\cong$

undecidable but not semidecidable

$\cong$

$\boxed{\text{not semidecidable}}$

$\cong$

Logic not exist for valid

$\cong$

L has no TM

Not REL is ____

I) undecidable

II) not recursive

III) not semidecidable

IV) not "REL but not rec"

V) not decidable

Undecidable Language (L)

$\cong$

not decidable

$\cong$

not recursive

$\cong$

L has no HTM

$\cong$

Either "$\underbrace{\text{REL but not rec}}$" OR "$\underbrace{\text{Not REL}}$"

Undecidable lang is ___

1) not decidable
2) not recursive
3) not regular
4) not CFL
5) not CSL
6) REL or not REL
7) REL but not rec or not REL

* Recursive Language

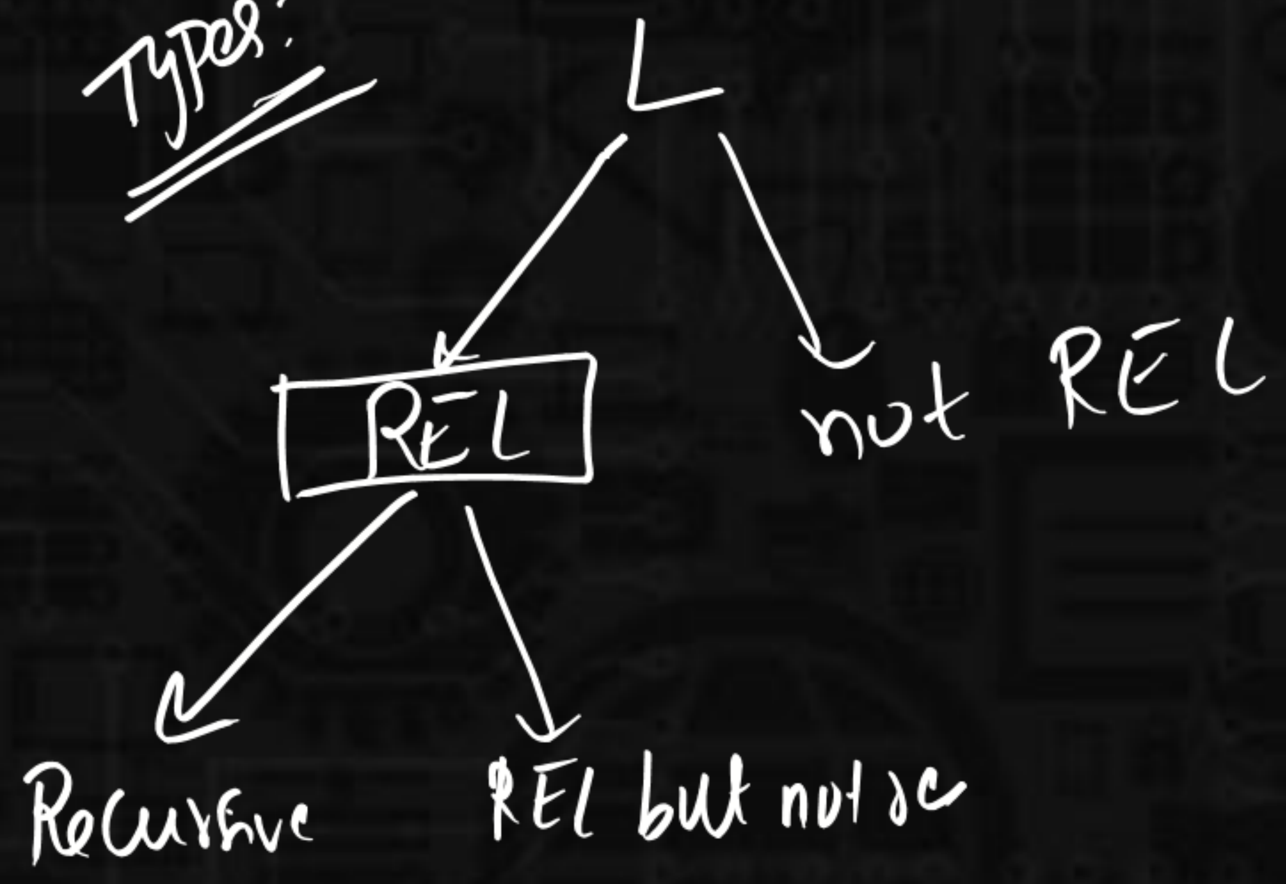
REL

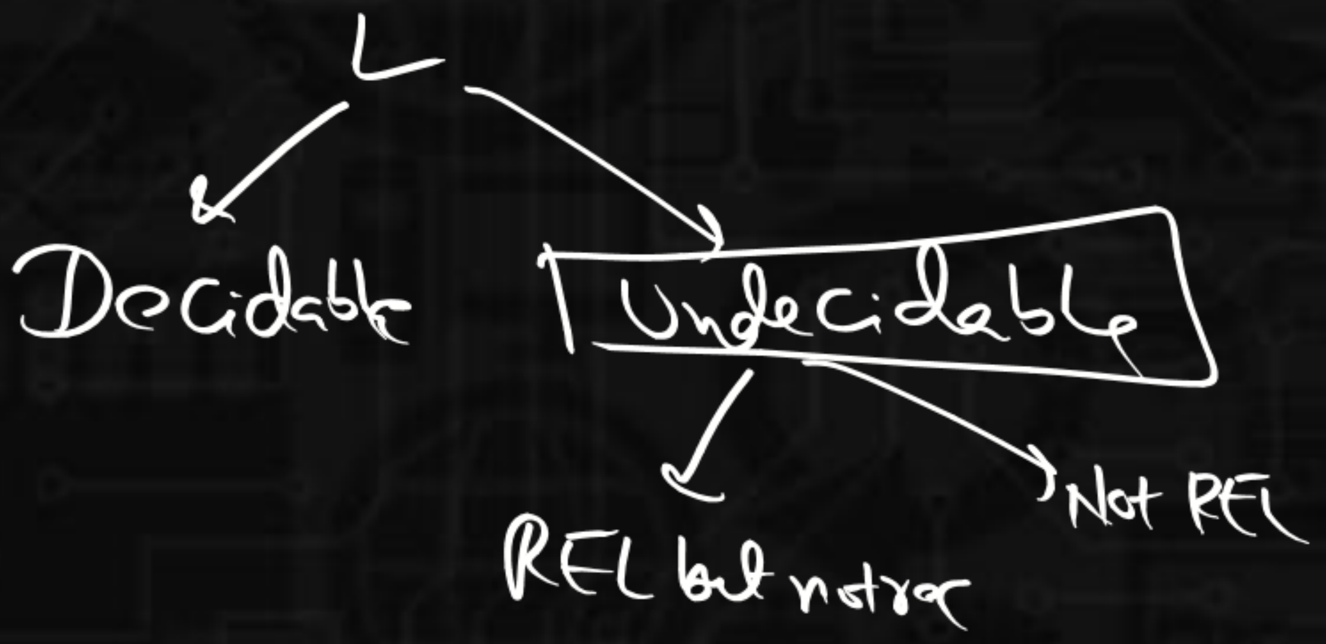
* RE but not recursive

* Not REL

Undecidable language

Types:





L is Recursive $\Longrightarrow$ Decidable, REL, Semidecidable

logic exist for valid & Invalid

L is REL but not rec $\Longrightarrow$ SD & UD, SD, UD, REL, not rec

logic exist only for valid

L is not REL $\Longrightarrow$ not SD, UD, not rec, ...

logic not exist for valid

I) $\overline{\text{Recursive Language}}$ is REC

II) $\overline{\text{REL}}$ is need not be REL (REC or not REL) (Never be REL but not rec)

III) $\overline{\text{REL but not rec}}$ is Not REL

IV) $\overline{\text{Not REL}}$ is either "not REL" OR "REL but not rec" (Undecidable)

V) $\overline{\text{Undecidable language}}$ is Undecidable

L is Not REL
(logic not exit for valid)
(logic may exist for Invalid)

$\bar{L}$ is not REL
valid of $\bar{L}$ has no logic

$\bar{L}$ is REL but not rec
logic exist for valid of $\bar{L}$

$\bar{L}$ is undecidable

Undecidable
("Not REL" or "REL but not rec")

$\underbrace{\text{Not REL}}$ $\xrightarrow{\text{comp}}$ UD

$\underbrace{\text{REL but not rec}}$ $\xrightarrow{\text{comp}}$ Not REL

$\underbrace{\text{UD} \quad \text{Not REL}}_{\text{UD}}$

Decision properties Table

	problems	FA/Reg/RegExp	DPDA/DCFLs	PDA/CFLs	LBA/CSLs HTM/RECs	TM/RELs
H	① Halting & Non Halting	D	D	D	D	UD
M	② Membership & Non membership	D	D	D	D	UD
E	③ Emptiness & Non emptiness	D	D	D	UD	UD
F	④ Finiteness & Non Finiteness	D	D	D	UD	UD
T	⑤ Totality & Non totality	D	D	UD	UD	UD
E	⑥ Equivalence & Non equivalence	D	D	UD	UD	UD
D	⑦ Disjoint & Non disjoint	D	UD	UD	UD	UD
S	⑧ Set containment & Non set containment	D	UD	UD	UD	UD

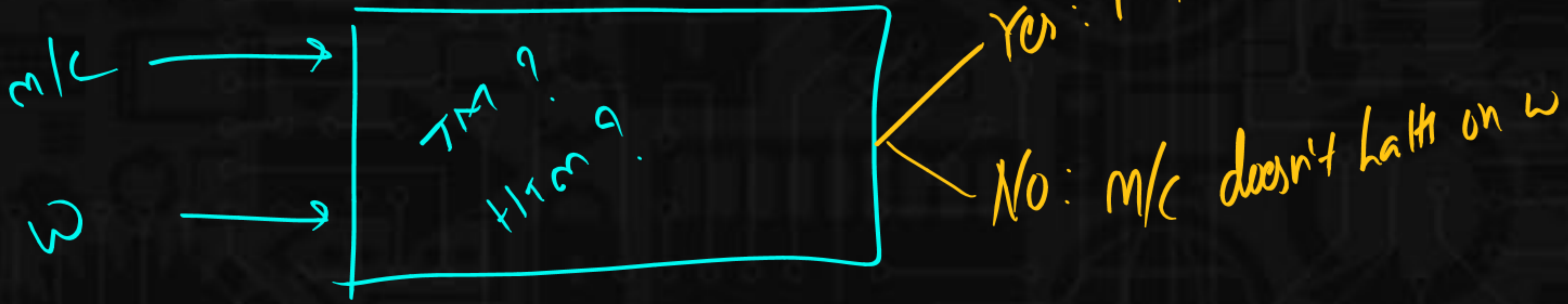
	DPDA	PDA	HTM
HM	D	D	D
EF	D	D	
TE	D		

① Halting problem

Is $\underbrace{\text{given M/c}}$ halts on $\underbrace{\text{given string}}$?

Whether M/c halts on w.

M/c halts on w



Halting problem is decidable for $\underbrace{\text{FA / DPDA / PDA / LBA / HTM}}_{\text{always halts}}$

may be at final or non final

Halting problem is "$\underbrace{\text{undecidable}}_{\text{RE but not rec}}$" for $\underbrace{\text{TM}}_{\text{may halt or Never halt}}$

TM →
w →

Is TM halts on w

Yes: $\underbrace{\text{TM halts on w}}_{\text{Logic exist}}$

No: $\underbrace{\text{TM never halts}}_{\text{logic not exist}}$

(1) Non-Halting

IS give M/C doesn't halt on w ?

Whelter M/C doesn't halt on w.

M/C doesn't halt on w

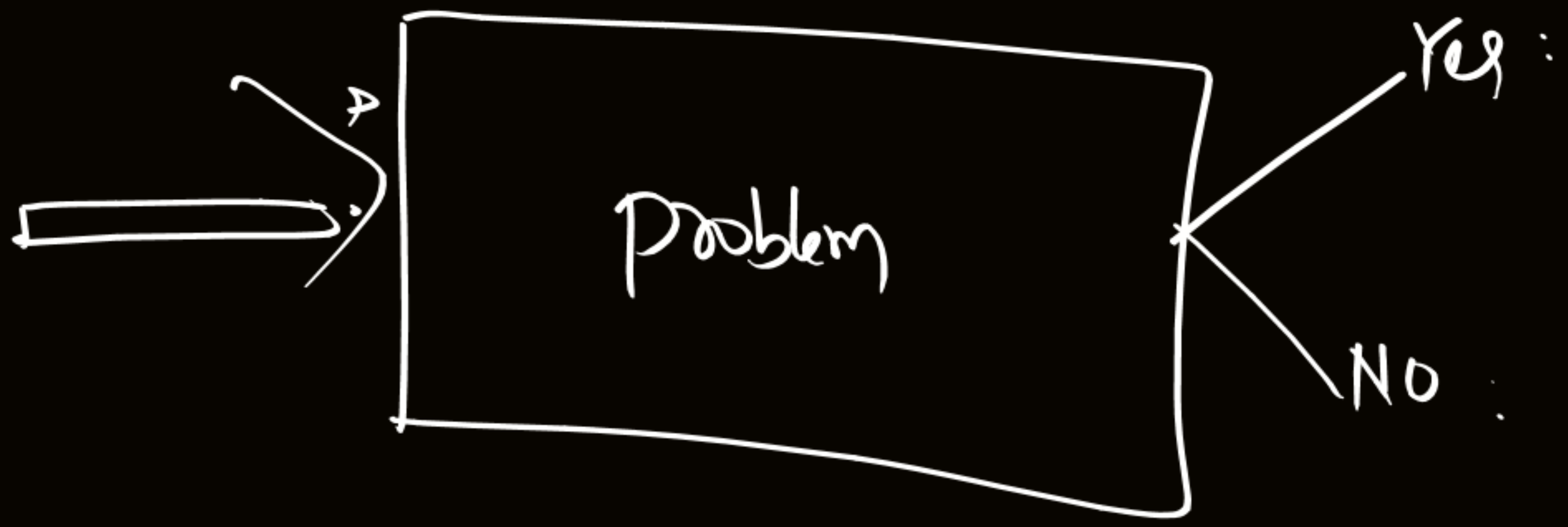
(2)a Membership

- → IS given M/C $\underbrace{\text{accepts}}$ on $\overset{\text{given}}{\text{string}}$ ω ?
- → IS $\omega \in L$(given M/C) ?
- → IS M accepts ω ?
- → M accepts ω

(2)b Non-membership

- → Is M doesn't accept ω ?

IS $\omega \notin L(M)$?



I) Both Yes & No have logic

↳ problem is Decidable

Undecidable
II) only Yes has logic

↳ RE but not Rec

III) Yes has no logic

↳ Not RE

③ₐ Emptiness

- → Is given m/c accepts nothing ?
- → Is $L(M) = \{\}$?
- → Is $L(M) = \phi$?

③ᵦ Non-emptiness

- → Is given m/c accepts some string ?

  Is $L(M) \neq \phi$?

(4)a Finiteness

- Is given m/c accepts finite language?
- Is $L(M) = $ finite?
  - M: given m/c

(4)b Non-finiteness (Infiniteness)

- Is $L(M) = $ Infiniteness?

5a Totality

- Is $L(M) = \Sigma^*$?
  M → given
- Is given m/c accepts every string (everything) ?

$$\overline{\Sigma^*} = \phi$$

Is $L(M) = \Sigma^*$? = Is $L(M) \neq \Sigma^*$?

5b Not totality

- Is $L(M) \neq \Sigma^*$?
- Is M/c not accepting some string ?

⑥$_a$ Equivalence

↳ IS $\overset{\text{given}}{M_1}$ equivalent to $\overset{\text{given}}{M_2}$?

IS $L(M_1) = L(M_2)$?

IS $M_1 \cong M_2$?

⑥$_b$ Non-equivalence

↳ IS $L(M_1) \neq L(M_2)$?

7 a Disjoint

$\hookrightarrow$ IS $L(\overset{\text{given}}{M_1}) \cap L(\overset{\text{given}}{M_2}) = \phi$?

7 b Not Disjoint

$\hookrightarrow$ IS $L(M_1) \cap L(M_2) \neq \phi$?

⑧a Set containment [Subset checking]

↳ IS $L(M_1) \subseteq L(M_2)$?

IS $L_1 \subseteq L_2$?

IS $L_1 \cap \overline{L_2} = \phi$?

⑧b Not Set Containment

↳ Is $L_1 \not\subseteq L_2$?

IS $L_1 \cap \overline{L_2} \neq \phi$?

closure property

Subset of $(\underset{\text{given}}{L}) \Longrightarrow$ result language ?

Decision property

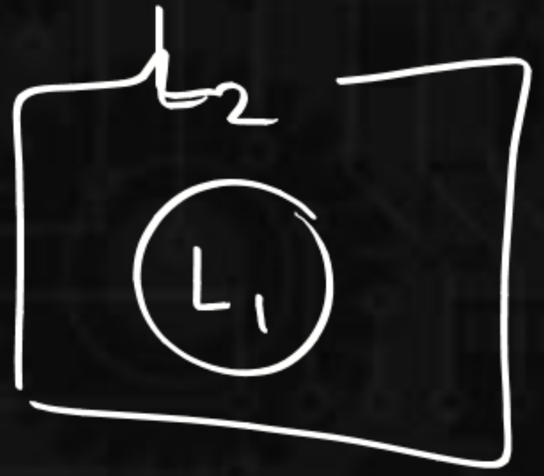
Is $\underset{\text{given}}{L_1} \subseteq \underset{\text{given}}{L_2}$?

→ Yes

→ NO

Summary

→ Decision properties



$\boxed{L_1 \subseteq L_2}$

same

$\boxed{L_1 \cap \overline{L_2} = \phi}$

H
M
E
F
T
E
D
S


THANK YOU GW
SOLDIERS !