Branch : CSE & IT Batch : Hinglish

Theory of Computation

Push Down Automata

Context Free Languages: Closure properties

DPP-05

[MCQ]

1. The intersection of CFL and a regular language will be

(a) Always regular
(b) Always CFL
(c) Always not regular
(d) None of these

[MCQ]

2. Consider the following grammars G_1, G_2 and G_3:

G_1 : $S \to P\ Q$
$P \to 0\ P\ 1 | \in$
$Q \to 1\ Q\ 2 | \in$

G_2 : $S \to 0\ S1 | Q$
$P \to 1Q\ 2 | \in$

G_3 : $S \to P\ Q\ |\ Q$
$P \to 0\ P\ 1 |\ 01$
$Q \to 1\ Q\ 2 | \in$

Here, {S,P,Q} are variables where S is start symbol. {0,1,2} are terminals.
Which of the following is true?

(a) G_1 and G_2 are equivalent.
(b) G_1 and G_3 are equivalent.
(c) G_2 and G_3 are equivalent.
(d) None of these.

[MSQ]

3. Consider the following regular expressions P, Q and R over $\sum = \{a, b\}$:
P = ab + aQ + bR
Q = baQ + bR
R = Raba + a
Which of the following regular expression will produce all the strings accepted by above regular expression?

(a) $ab + ba(aba)^* [\in + a(ba)^*]$
(b) $ab + [\in + a(ba)^*]\ ba(aba)^*$
(c) $ab + a(ba)^+\ ba(aba)^*$
(d) $ab + a(ba)^+\ (aba)^* + ba(aba)^*$

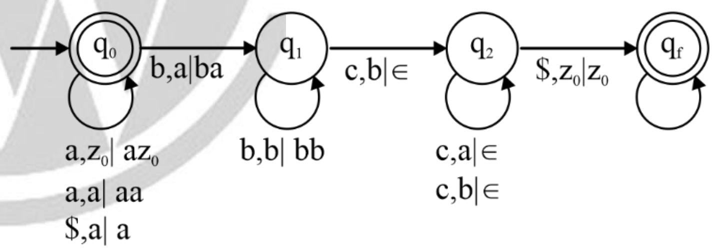
[MCQ]

4. Consider the following language.
L_1 = Context free language.
L_2 = Deterministic context free language.
L_3 = Context sensitive language.
L_4 = Regular
Which of the following is incorrect?

(a) $L_2 . L_4$ is always DCFL.
(b) $L_1 \cap L_3$ is CSL.
(c) $\Sigma^* - L_3$ is CSL.
(d) None of the above.

[MCQ]

5. Consider the following push down automata.
PDA = $\{Q, \Sigma, \delta, \Gamma, q_0, Z_0, q_f\}$



$\$\ Z_0\ |\ Z_0$

Which of the following language is accepted by above PDA?

(a) $L = \{a^*\} \cup \{a^p\ b^q\ c^r \mid p, q, r \geq 1, p + q = r\}$
(b) $L = \{a^{p+q}\ b^{q+r} \mid p, q, r \geq 0\}$
(c) $L = \{a^p\ b^q\ c^r \mid p, q, r \geq 1\}$
(d) None of the above

[MSQ]

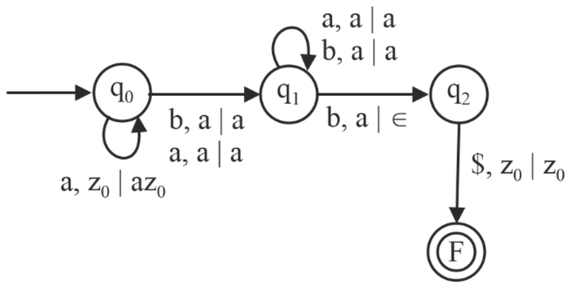
6. Consider the following language:
$L_1 = \{ab^n\ a^{2n} \mid n \geq 1\}$
$L_2 = \{aab^n\ a^{3n} \mid n \geq 1\}$
Which of the following is correct?

(a) $L_1 \cup L_2$ is DCFL but not regular.
(b) $L_1 \cup L_2$ is CFL but not DCFL.
(c) $L_1 \cup L_2$ is CSL but not CFL.

(d) $L_1 \cup L_2$ is DCFL and also CFL.

[MCQ]

7. Consider the following PDA:



Here q_0 is a starting state and F is a final state. Then the language accepted by above PDA is?

(a) Regular but finite

(b) Regular but infinite

(c) CFL but not regular

(d) None of these

[MSQ]

8. Suppose, L is any CFL language on alphabet $\Sigma = \{a, b\}$, and the following language:

$L_1 = L - \{w\, x\, w^R \mid w, x \in \{a,b\}^*\}$

$L_2 = L_1 \cdot L$

$L_3 = \overline{L}_1 \cup L$

Which of the following is/are correct?

(a) L_1 is finite.

(b) L_2 is CFL.

(c) L_3 is regular.

(d) None of these.

Answer Key

1. **(b)**
2. **(b)**
3. **(b, d)**
4. **(a)**
5. **(a)**
6. **(a, d)**
7. **(b)**
8. **(a, b, c)**

Hints and Solutions

1. (b)

- CFL ∩ Regular
- Always CFL

Hence, option (b) is correct.

2. (b)

$L(G_1) = \{0^n 1^n 1^m 2^m | m, n \geq 0\}$
$= \{0^n 1^{m+n} 2^m | m, n \geq 0\}$

$L(G_2) = \{0^m 1^n 2^n 1^m | m, n \geq 0\}$

$L(G_3) = \{0^n 1^{m+n} 2^m | m, n \geq 0\}$

Hence, option (b) is correct.

3. (b, d)

P = ab + aQ + bR

Q = baQ + bR

R = Raba + a

Apply Arden's Theorem:

$R = a(aba)^*$

$Q = (ba)^* bR$

$Q = (ba)^* ba(aba)^*$

P = ab + aQ + bR

P = aQ | bR | ab

$= a[(ba)^*ba(aba)^*] + ba(aba)^* + ab$

$r^* r = r^+$

$(ba)^*ba = (ba)^+$

$P = a(ba)+ (aba)^* + ba(aba)^* + ab$

Exactly match with option (d)

$P = a[(ba)^* \underline{ba(aba)^*}] + \underline{ba(aba)^*} + ab$

$P = [a(ba)^* + \in]ba(aba)^* + ab$

$= ab + [\in + a(ba)^*]ba(aba)^*$

Exactly match with opetion (b)

Hence, option (b, d) are correct.

4. (a)

(a) DCFL·Regular ↑

DCFL· DCFL

CFL **(False)**

(b) CFL ∩ CSL

CSL ∩ CSL

CSL **(True)**

(c) Σ* – CSL

$\Sigma^* \cap \overline{CSL}$

CSL

Hence, option (a) is correct.

5. (a)

- State q_0 will accept all the a's i.e. a*

At state q_f

Number of C = number of a's + number of b's

So, $L = \{a^*\} \cup \{a^p b^q c^r | p + q = r, p, q, r \geq 1\}$

Hence, option (a) is correct.

6. (a, d)

- $L_1 = \{ab^n a^{2n} | n \geq 1\}$ is DCFL
  $L_2 = \{aab^n a^{3n} | n \geq 1\}$ is DCFL
- $L_1 \cup L_2$ will be DCFL for L_1 skip first a and for L_2 skip 2 a's. Push and pop are clear so $L_1 \cup L_2$ will be DCFL but not regular
- Every DCFL is CFL also.

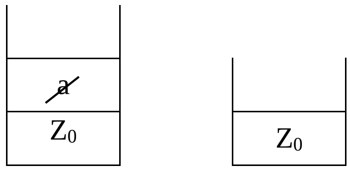
Hence, option (a, d) is correct

7. (b)

- In given PDA first a will be pushed into stack

a
Z_0

- After first 'a' it will skip all the a's and b's
- And it will be pop 'a' on last input b.



empty stack

Regular expression = $a(a+b)^* b$ (regular but infinite)

Hence, option (b) is correct.

8. (a, b, c)

$L_1 = CFL - (a+b)^*$

$= CFL \cap [(a+b)^*]^C$

$= \phi$

$L_2 = \phi \,.\, CFL$

$= \phi$

$L_3 = \overline{\phi} \cup CFL$

$= (a+b)^* \cup CFL$

$= (a+b)^*$

(a) L_1 = finite true

$L_1 = \phi$

(b) L_2 is CFL

$L_2 = \phi$ is regular and every regular is CFL.

(c) L_3 is regular

$L_3 = (a+b)^*$

Hence, (a, b, c) are correct option