CS & IT ENGINEERING

Theory of Computation

Finite Automata
Regular Languages identification - 1

Lecture No.15

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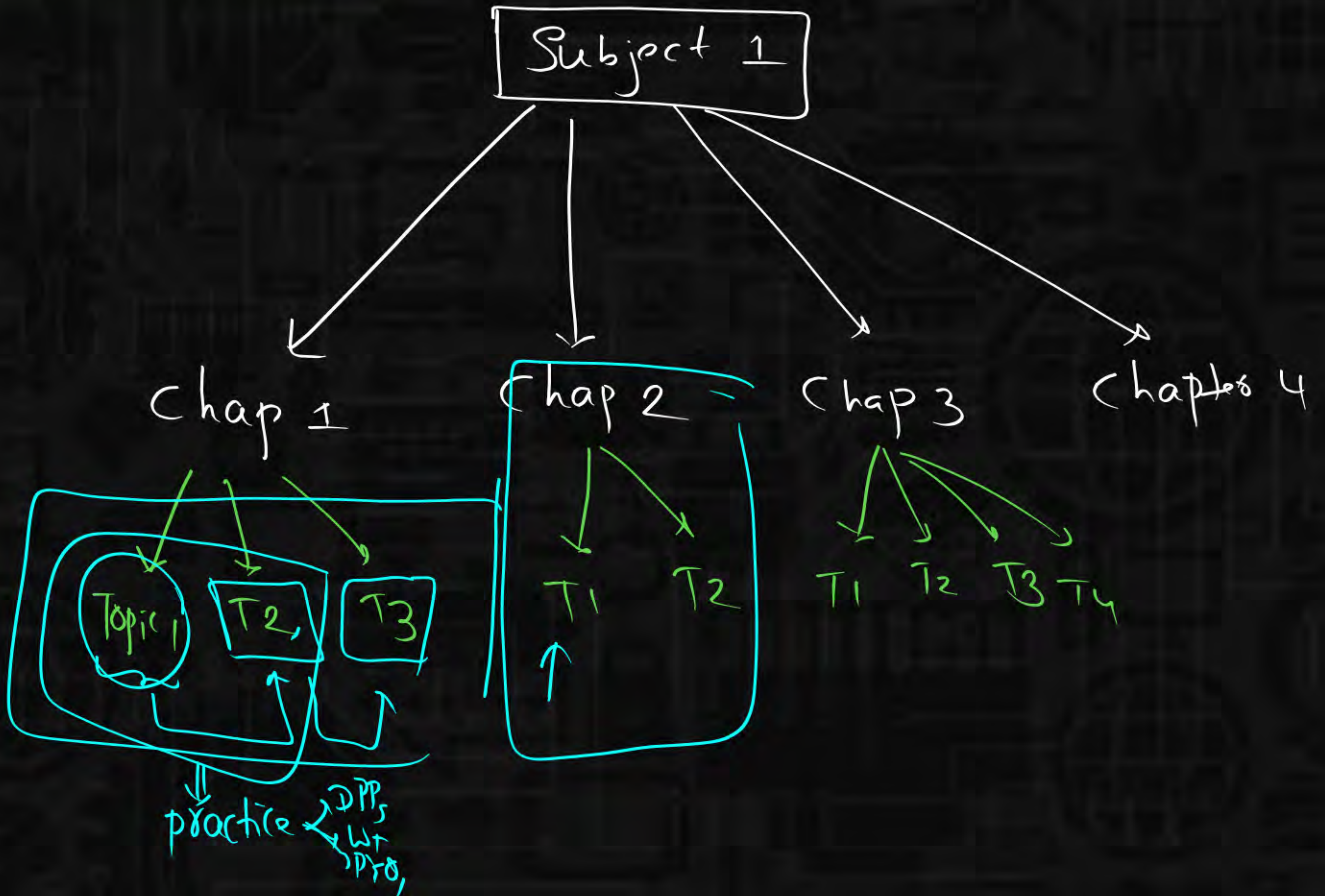
TOPICS TO BE COVERED

Regular Grammar

Regulars & Non regulars

Revision Strategy



Regular Grammar

(30) $S \rightarrow Sab \mid Scd \mid e$ $\qquad L = e(ab+cd)^*$

(31) $S \rightarrow Saa \mid \varepsilon$ $\qquad L = (aa)^*$

(32) $S \rightarrow Saa \mid b$ $\qquad L = b(aa)^*$

(33) $S \rightarrow Sa \mid A$

$\boxed{A \rightarrow Ab \mid \varepsilon}$

b^*

$S \rightarrow Sa \mid b^*$ $\qquad \boxed{L = b^* a^*}$

RG

(34) $S \to aS \mid A$

$A \to bA \mid \varepsilon$

$\underbrace{A \to bA \mid \varepsilon}_{b^*}$

$S \to aS \mid b^*$

$$\boxed{L = a^* b^*}$$

(35)

$S \to aA \mid bA$

$A \to aB \mid bB$

$B \to aC$

$C \to aC \mid bC \mid \varepsilon$

$L = L(S) = (a+b).A = (a+b).(a+b)\,a\,(a+b)^*$

1st, 2nd, 3rd symbol

$L(A) = (a+b)\,L(B) = (a+b)\,a\,(a+b)^*$

$L(B) = a\,(a+b)^*$

$L(C) = (a+b)^*$

RG

(36)

$S \rightarrow Aa \mid Ab$

$A \rightarrow Ba \mid Bb$

$B \rightarrow Ca$

$C \rightarrow Ca \mid Cb \mid \epsilon$

$\rightarrow (a+b)^*$

$\rightarrow (a+b)^* a$

$A = B(a+b)$

$\rightarrow (a+b)^* a (a+b)$

$A(a+b) = (a+b)^* a (a+b)(a+b)$

3rd symbol from end is 'a'

Home work

(37)
$S \rightarrow aA \mid bA$
$A \rightarrow aB \mid bB$
$B \rightarrow \varepsilon$

(38)
$S \rightarrow aA \mid bA$
$A \rightarrow aB \mid bB$
$B \rightarrow aB \mid bB \mid \varepsilon$

(39)
$S \rightarrow aA \mid bA \mid \varepsilon$
$A \rightarrow aB \mid bB \mid \varepsilon$
$B \rightarrow \varepsilon$

(40)
$S \rightarrow bS \mid A$
$A \rightarrow aB$
$B \rightarrow bB \mid C$
$C \rightarrow aD$
$D \rightarrow bD \mid \varepsilon$

(41)
$S \rightarrow bS \mid A$
$A \rightarrow aB \mid B$
$B \rightarrow bB \mid C$
$C \rightarrow aD \mid D$
$D \rightarrow bD \mid \varepsilon$

(42)
$S \rightarrow aS \mid bS \mid A$
$A \rightarrow aB$
$B \rightarrow aB \mid bB \mid C$
$C \rightarrow aD$
$D \rightarrow aD \mid bD \mid \varepsilon$

(43) $S \rightarrow aS \mid bS \mid cS \mid \varepsilon$

(44) $S \rightarrow Sa \mid Sb \mid Sc \mid \varepsilon$

H.W.

(45) $S \rightarrow aS \mid A \Rightarrow a^*b^*c^*$

$A \rightarrow bA \mid B = b^*c^*$

$B \rightarrow cB \mid \varepsilon$ c^*

(46) $S \rightarrow aS \mid bS \mid cS \mid d$

(47) $S \rightarrow aA \mid \varepsilon \Rightarrow S \rightarrow abS \mid \varepsilon$

$A \rightarrow bS$

$L(A) = bS$

$L = (ab)^*$

(48) $S \rightarrow aA$

$A \rightarrow bS \mid \varepsilon$

(49) $S \rightarrow aA \mid aB \mid aD$

$A \rightarrow bA \mid \varepsilon$

$B \rightarrow cB \mid \varepsilon$

$D \rightarrow dD \mid \varepsilon$

(50) $S \rightarrow aS \mid A$

$A \rightarrow \# B$

$B \rightarrow aB \mid \varepsilon$

Identification of Regulars & Non-regulars :

Regular language may be finite or infinite.

Non regular is always infinite

Language (Set) over Σ

Finite set

$\Downarrow$

It is always Regular

$\{a^n b^n \mid n \leq 3\}$

$= \{\underset{n=0}{\epsilon}, \underset{n=1}{ab}, \underset{n=2}{aabb}, \underset{n=3}{aaabbb}\}$

$\epsilon + ab + aabb + a^3b^3$

Infinite set

Regular

$a^* b^*$

Non-regular

$\{a^n b^n \mid n \geq 0\}$

I. Every Finite set is regular.

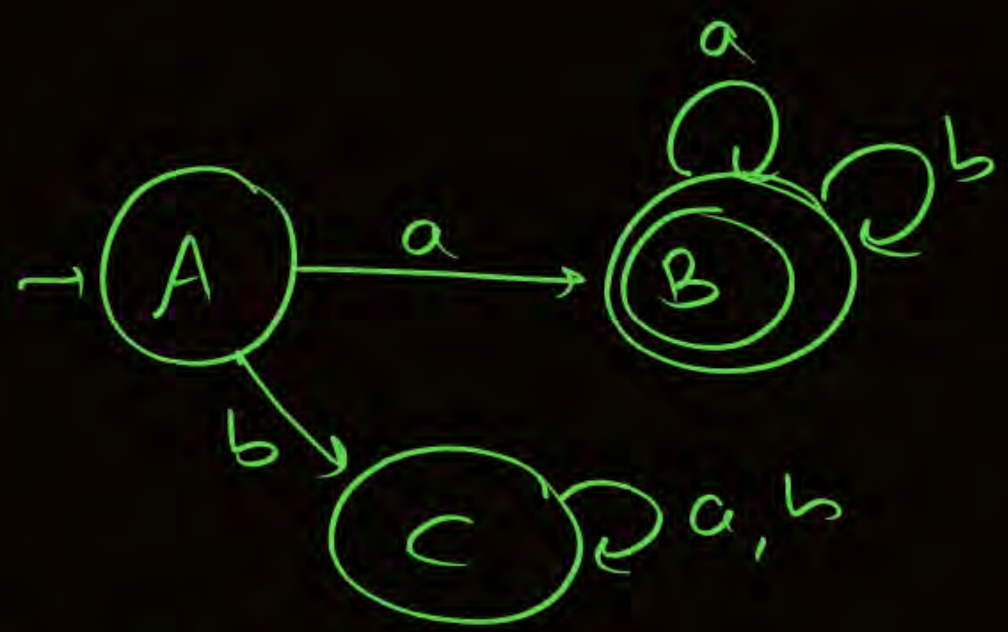
II. Some infinite languages are regulars and Some infinite languages are non-regulars

$$\underbrace{L = a(a+b)^*}$$

Regular

$\Downarrow$

O(1) space, we can solve




```
void C( )
{
  ch = getchar();
  if(ch=='a' || ch=='b')
      C();
  if (ch == '\n')
      Reject
}
```

```
char ch;   1 Byte
void main()
{
  ch = getchar()
  if (ch == 'a')  B( )
  if (ch == 'b')  C( )
  if (ch == '\n')  Reject
}
```

```
void B( )
{
  ch = getchar( );
  if (ch == 'a' || ch == 'b')  B( )
  if (ch == '\n')  ACCEPT
}
```

L is Regular Language

- → FA exist (DFA, NFA)
- → Reg Exp exist
- → Reg Grammar exist (LLG, RLG)
- → Finite no. of equivalence classes (no. of states in min DFA) [Each state in min DFA represents one equivalence class.]

Σ^*

[A] [B] [C]

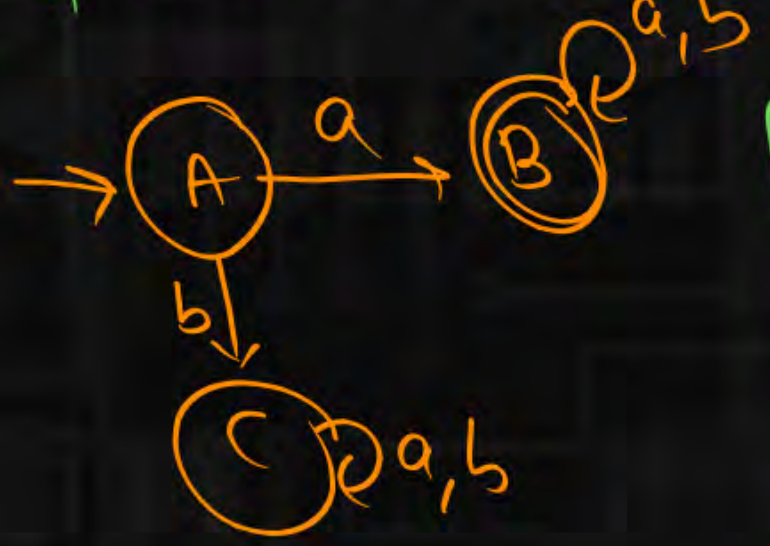
$\boxed{\varepsilon \cup a(a+b)^* \cup b(a+b)^*}$

$= \Sigma^*$

$\underbrace{L = a(a+b)^*}_{\text{Regular}}$

⇓

3 equ classes

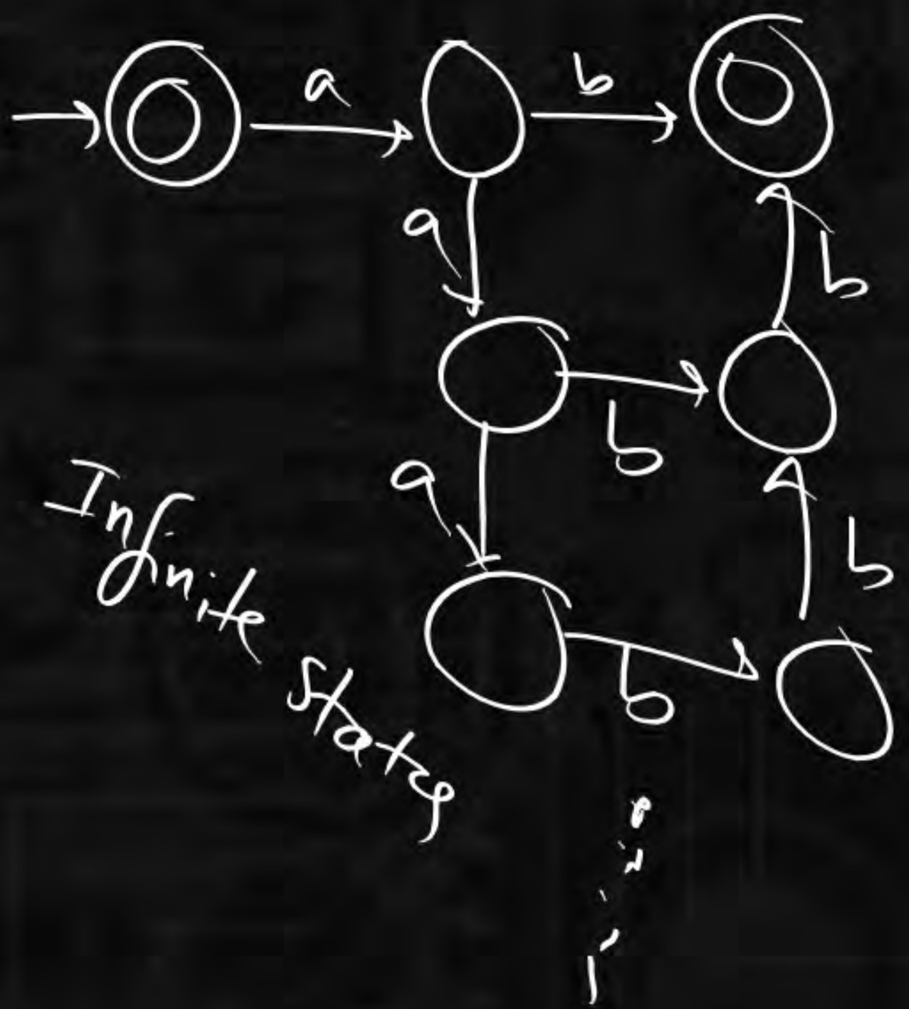


$[A] = \varepsilon$

$[B] = a(a+b)^*$

$[C] = b(a+b)^*$

$L = a^n b^n$ $\Rightarrow$ FA not exist



L is non regular language

- FA never exist
- Reg Exp not exist
- Reg Grammar not exist
- Infinite no. of equivalence classes present

Language (L)

- (I) Finite language
  - ⇓ $\boxed{\text{Regular language}}$
- Infinite Language
  - over 1 symbol
    - (II) L Forms A.P.
      - ⇓ $\boxed{\text{Regular language}}$
      - a^*, a^{2n}, a^{3n+5}
    - (III) Not forms A.P.
      - ⇓ $\boxed{\text{Not regular}}$
      - $a^{n!}$
      - a^{prime}
  - over more than 1 symbol
    - (IV) No Dependency till infinite
      - ⇓ $\boxed{\text{Regular}}$
      - $a^* b^*$
    - (V) Dependency till infinite OR non A.P. Series
      - ⇓ $\boxed{\text{Not Regular}}$
      - $a^{n!} b^*$, $a^n b^n$

$\{x \mid x \geq 10, x \in N\}$

$\{10, 11, 12, \dots\}$

$$\{ \underbrace{\alpha}_{} \mid \underbrace{\beta}_{} \boxed{,} \underbrace{\gamma}_{} \}$$

String form — condition — AND — condition

$\Sigma = \{\#, a, b\}$

$$\{ \underbrace{\#w}_{} \mid w \in (ab)^*, |w| \geq 1 \}$$

$w \in \{\cancel{\#}, ab, abab, \dots\}$

$= \boxed{\{\#ab, \#abab, \dots\}}$

every #w Satisfies given Conditions

$\{1, 2, 3\}$

, → OR

$$\gcd(\overset{a^m}{m}, \overset{b^n}{n}) = 1$$

m	n
1	2
	3
	4
	5
	⋮
2	1
	3
	5
	7
	9
	⋮
3	1
	2
	4
	5
	7
	8

L

$a^m \qquad b^n$

#a's   #b's

Depen

Languages

1. $L_1 = \{a^m b^n \mid m, n \geq 0\} = a^* b^*$ Inf Regular
2. $L_2 = \{a^m b^n \mid m > n\}$ Inf Non regular
3. $L_3 = \{a^m b^n \mid m \geq n\}$ Inf Not regular
4. $L_4 = \{a^m b^n \mid m < n\}$ Inf Not regular
5. $L_5 = \{a^m b^n \mid m < n \text{ or } m = n\}$ Not reg
6. $L_6 = \{a^m b^n \mid m < n \text{ or } m > n\}$ Not reg
7. $L_7 = \{a^m b^n \mid m \neq n\} = (6)$ Not reg
8. $L_8 = \{a^m b^n \mid m = n \text{ or } m \neq n\} = a^* b^* = (1)$ Regular
9. $L_9 = \{a^m b^n \mid m \leq n \text{ or } m \geq n\} = (8)$ Regular
10. $L_{10} = \{a^m b^n \mid m < n < 100\} \Rightarrow$ Finite language, Regular

Languages

(11) $L_{11} = \{a^m b^n \mid m=n,\ m\neq n\} = \{\ \} = \phi$ Regular

(12) $L_{12} = \{a^m b^n \mid m<n,\ m>n\} = (11) \Rightarrow$ Regular

(13) $L_{13} = \{a^m b^n \mid m<n,\ m=n\} = (11) \Rightarrow$ Regular

ϕ

(14) $L_{14} = \{a^m b^n \mid m=n=2\} = \{a^2 b^2\} \Rightarrow$ Finite, Regular

(15) $L_{15} = \{a^m b^n \mid m<n<0\} = \phi = (11) = (12) = (13) \Rightarrow$ Regular

(16) $L_{16} = \{a^m b^n \mid m<n<100\} \Rightarrow$ Finite, Regular

(17) $L_{17} = \{a^m b^n \mid m>n>100\} \Rightarrow$ Not regular

(18) $L_{18} = \{a^m b^n \mid \gcd(m,n)=1\} \Rightarrow$ Not regular

(19) $L_{19} = \{a^m b^n \mid \mathrm{LCM}(m,n)=1\} = \{a^1 b^1\} \Rightarrow$ finite, Regular

(20) $L_{20} = \{a^m b^n \mid m\%2=0,\ n\%3=0\} = \{(aa)^*(bbb)^*\} \Rightarrow$ Regular

Model-XII $\{a^m b^n \mid m\%x, n\%y\}$

① $\{a^m b^n \mid m\%2=0, n\%3=0\}$

② $\{a^m b^n \mid m\%2\neq 0, n\%3=0\}$

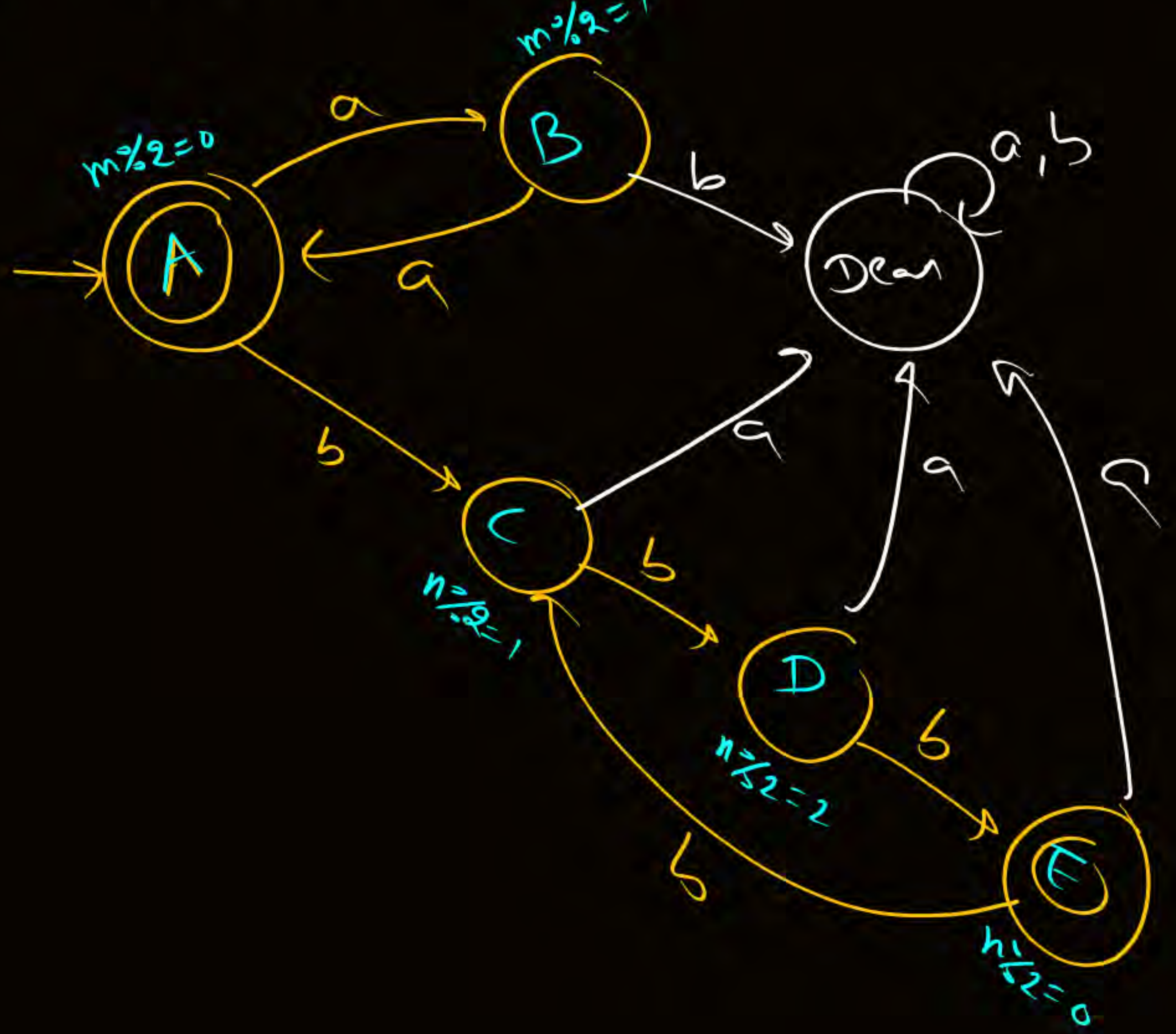
③ $\{a^m b^n \mid m\%2\neq 0, n\%3\neq 1\}$

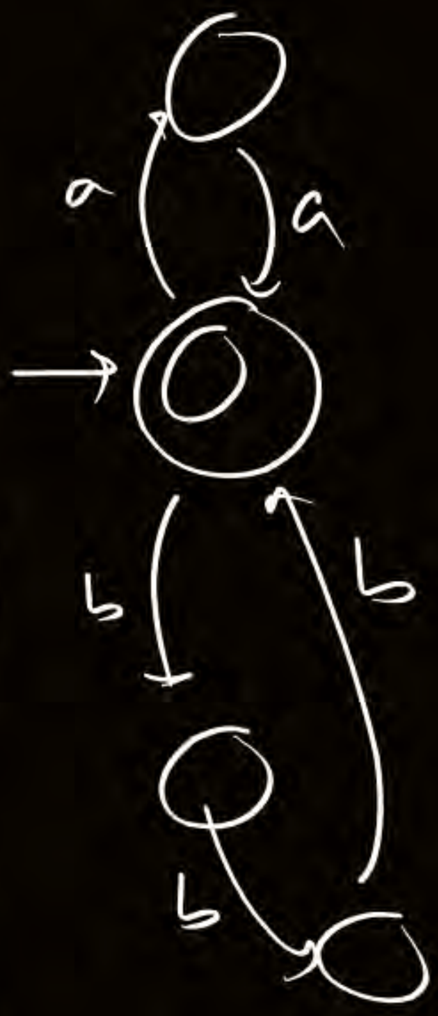
④ $\{a^m b^n \mid m\%2=1, n\%3=2\}$

⑤ $\{a^m b^n \mid m\%2=1, n\%3\neq 2\}$

DFA Construction

$$a^m b^n \quad m\%2=0 \quad n\%3=0 = (aa)^*(bbb)^*$$





$$=(aa+bbb)^*$$

Summary

$\rightarrow$ Languages

$\quad\rightarrow$ Regular

$\quad\rightarrow$ Nonregular


THANK YOU GW
SOLDIERS !