


CS & IT ENGINEERING

Algorithm

Miscellaneous Topics

Lecture No. 12



By- Dr. Khaleel Khan
Sir

Topics to be Covered



Topic

Backtracking and Branch & Bound





Represents one of the most general techniques. Many problems which deal with searching for a set of solutions or which ask for an optimal solution satisfying some constraints can be solved using the backtracking formulation.

The name backtrack was first coined by D. H. Lehmer in the 1950's. Early workers who studied the process were R. J. Walker who gave an algorithmic account of it in 1960 and Golomb and Baumert who presented a very general description of backtracking coupled with a variety of applications. (See the references for further details).

In order to apply the backtrack method, the desired solution must be expressible as an n-tuple (x_1, x) where the x_i are chosen from some finite set S_i . Often the problem to be solved calls for finding one vector which maximizes (or minimizes or satisfies) a criterion function $P(x_1, \dots, x_n)$. Sometimes it seeks all such vectors which satisfy P .

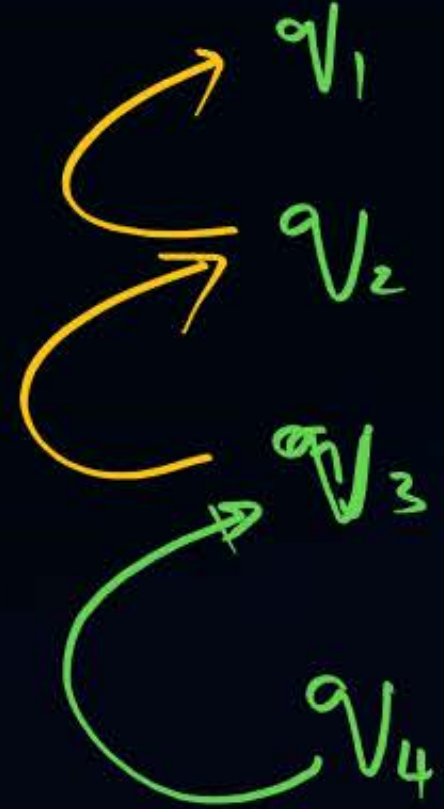
① n-Queens: n-Queens (q_1, \dots, q_n); $n \times n$ C/H/RP

$n=4$ $\langle q_1, \dots, q_4 \rangle$

$\times [1 \dots n] = \langle x_1, x_2, x_3, x_4 \rangle$

$$1 \leq x_i \leq n$$

x_i = Position of q_i i.e. placed in row 'i';



x	x	x	
x	x		

$x: \langle 2, 4, 1, 3 \rangle \langle 3, 1, 4, 2 \rangle$

Solution Space

↓ Backtracking
(State-Space)

4!
 $\langle 1, 2, 3, 4 \rangle$
 $\langle 1, 2, 4, 3 \rangle$
 $\langle 1, 3, 2, 4 \rangle$
 $\langle 1, 3, 4, 2 \rangle$
 \vdots



Backtracking

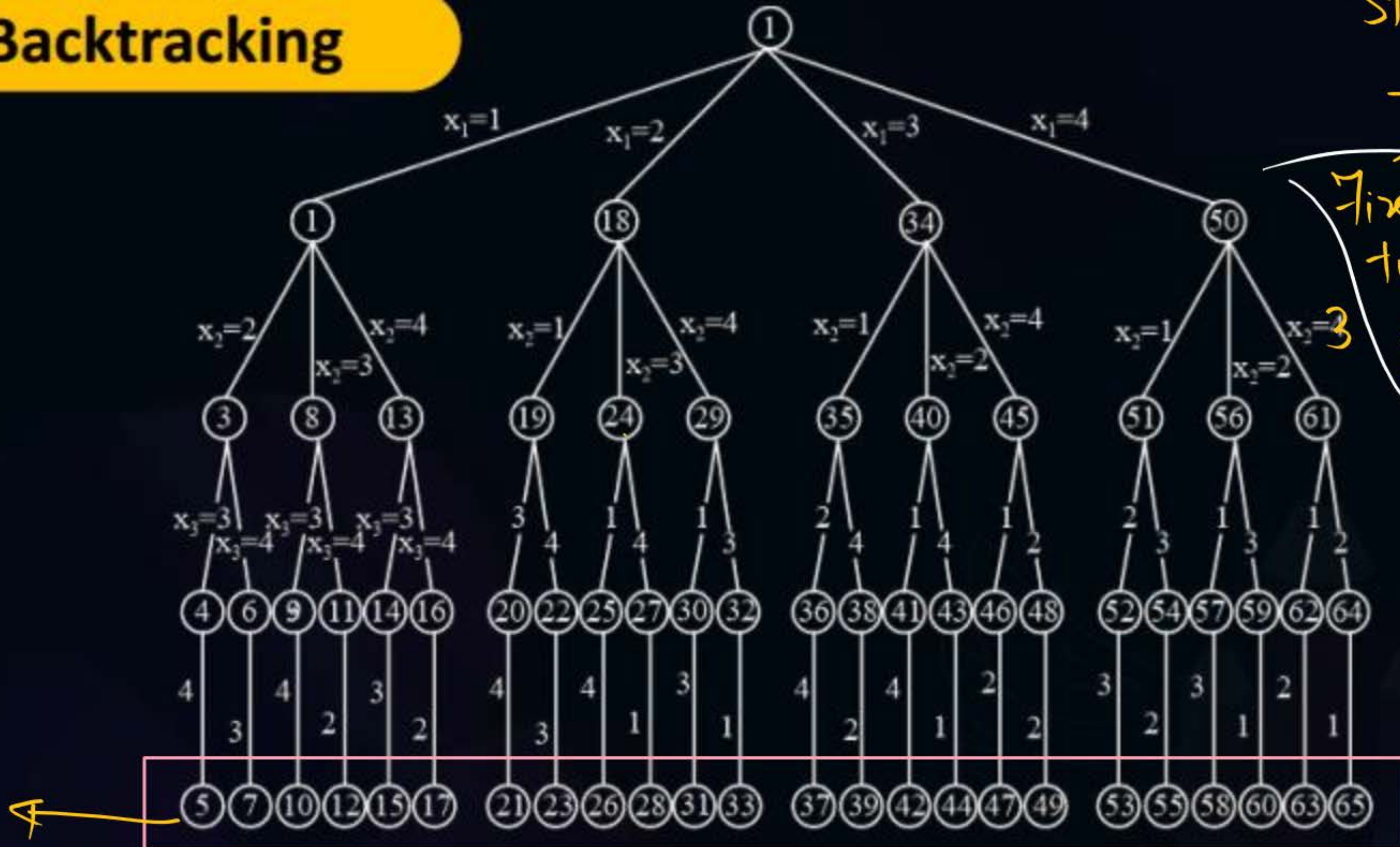


State-Space Tree

Fixed tuple size

Variable tuple size

$\langle 1, 2, 3, 4 \rangle$



Actual Solution

Tree organization of the 4-queens solution space. Nodes are numbered as in depth first search

STATE Space Tree



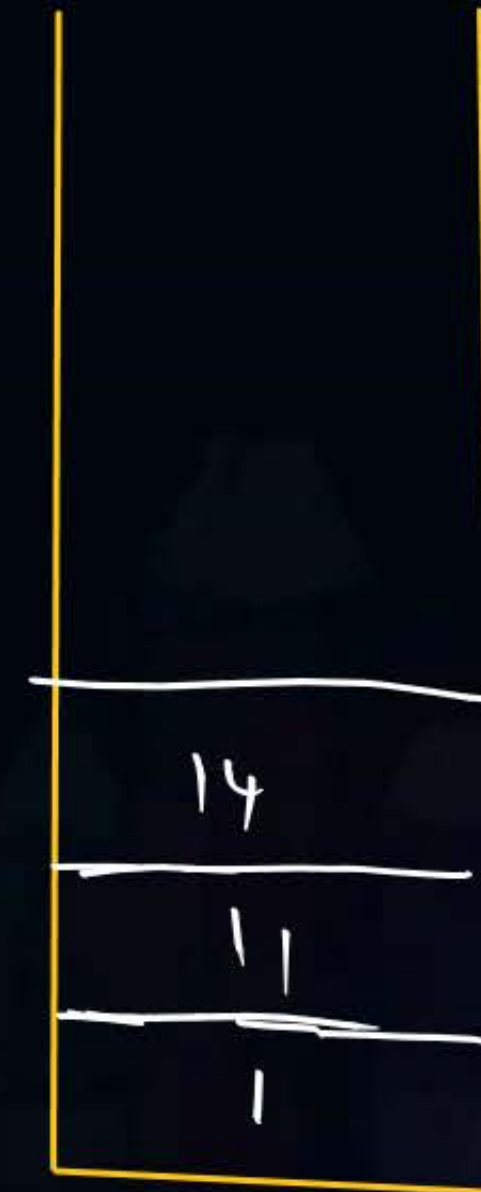
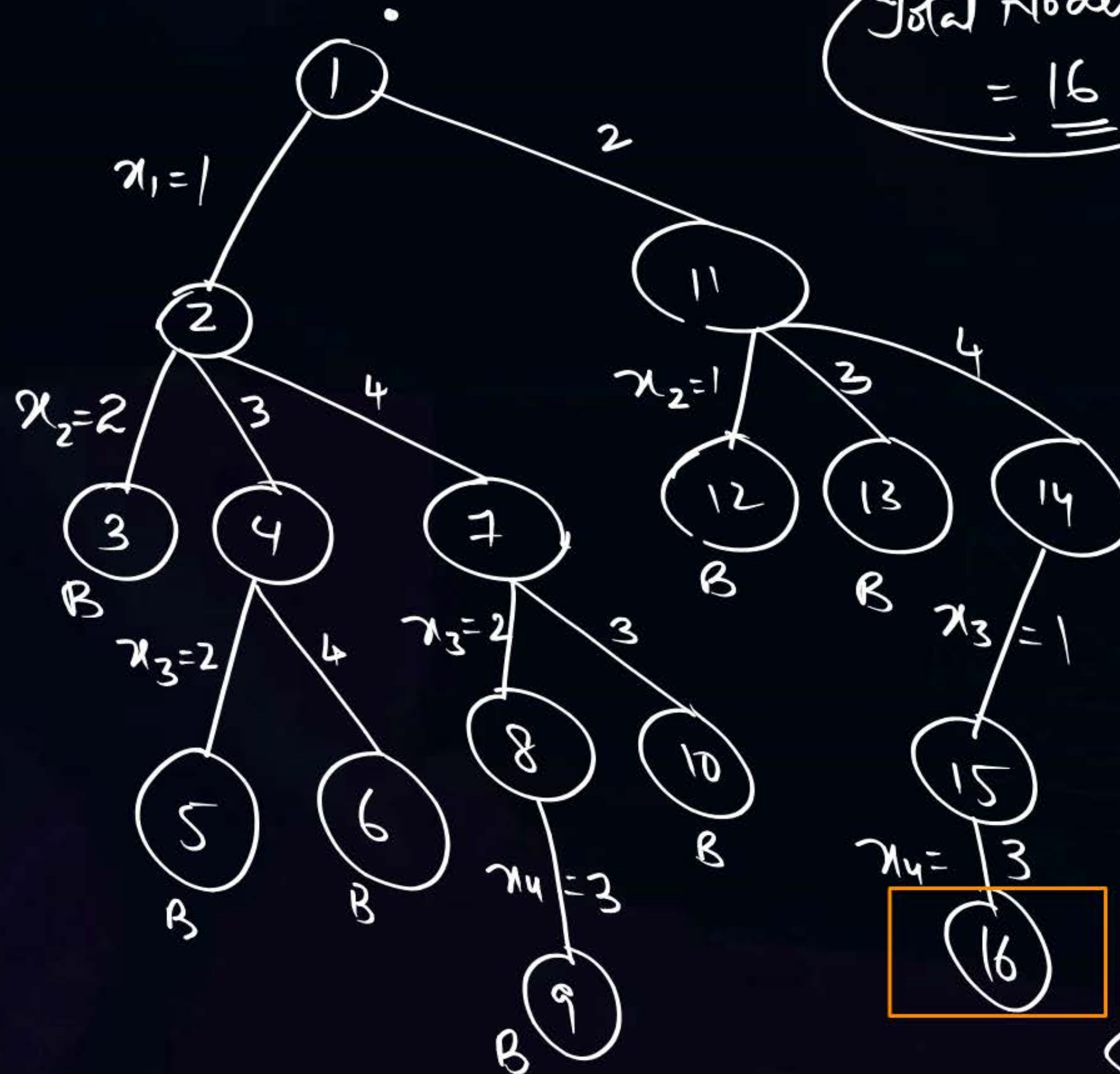
⇒ Backtracking uses D.F.S to search for Feasible
in the Solution Space, & it also applies Optimal Solns
Bounding Fns to Kill/Bound those Nodes,
Criterion Fns that are Not feasible/optimal

Backtracking = D.F.S + Bounding fns

Portion of State-Space Tree generated due to Backtracking Process for 4-Q's



Total Nodes = 16



STACK

$\langle 2, 4, 1, 3 \rangle$

Variable tuple Size Formulation of State-Space Tree



Sum of Subsets: $\langle S \circ S \rangle$

n -elements; M

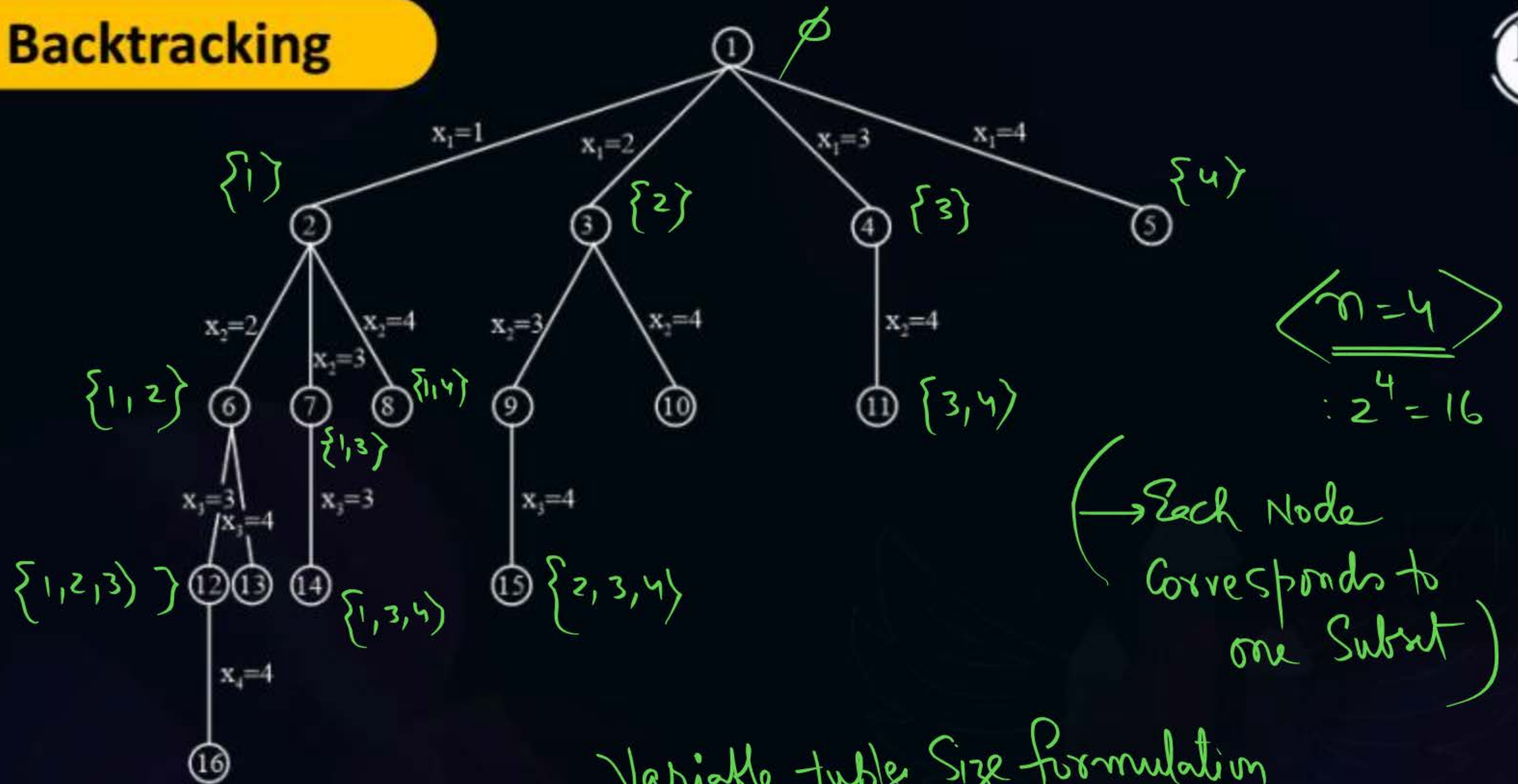
$A: \langle 1 \dots n \rangle$

$n=5$; $M=50$

$A: \langle 10, 20, 30, 40, 50 \rangle \} \{2^n\}$

$\{ \underline{20, 30} \}$ $\{ \underline{50} \}$
 $\{ \underline{10, 40} \}$

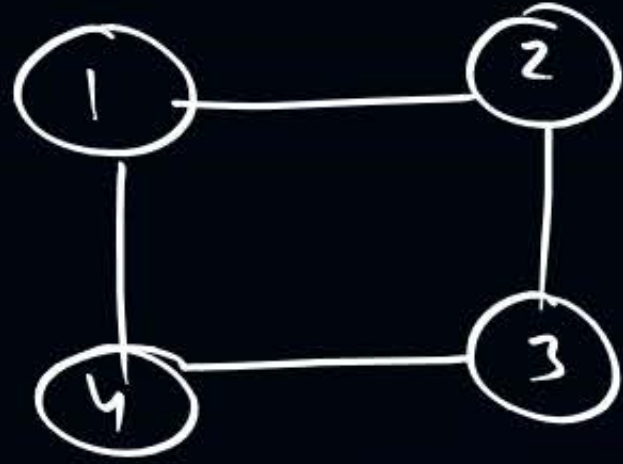
Backtracking



Variable tuple Size formulation
for Sum of Subsets

3) Graph Coloring: It is reqd to paint the graph in such a way that no two adjacent vertices should have the same color;

$\langle x_1, x_2, x_3, x_4 \rangle$

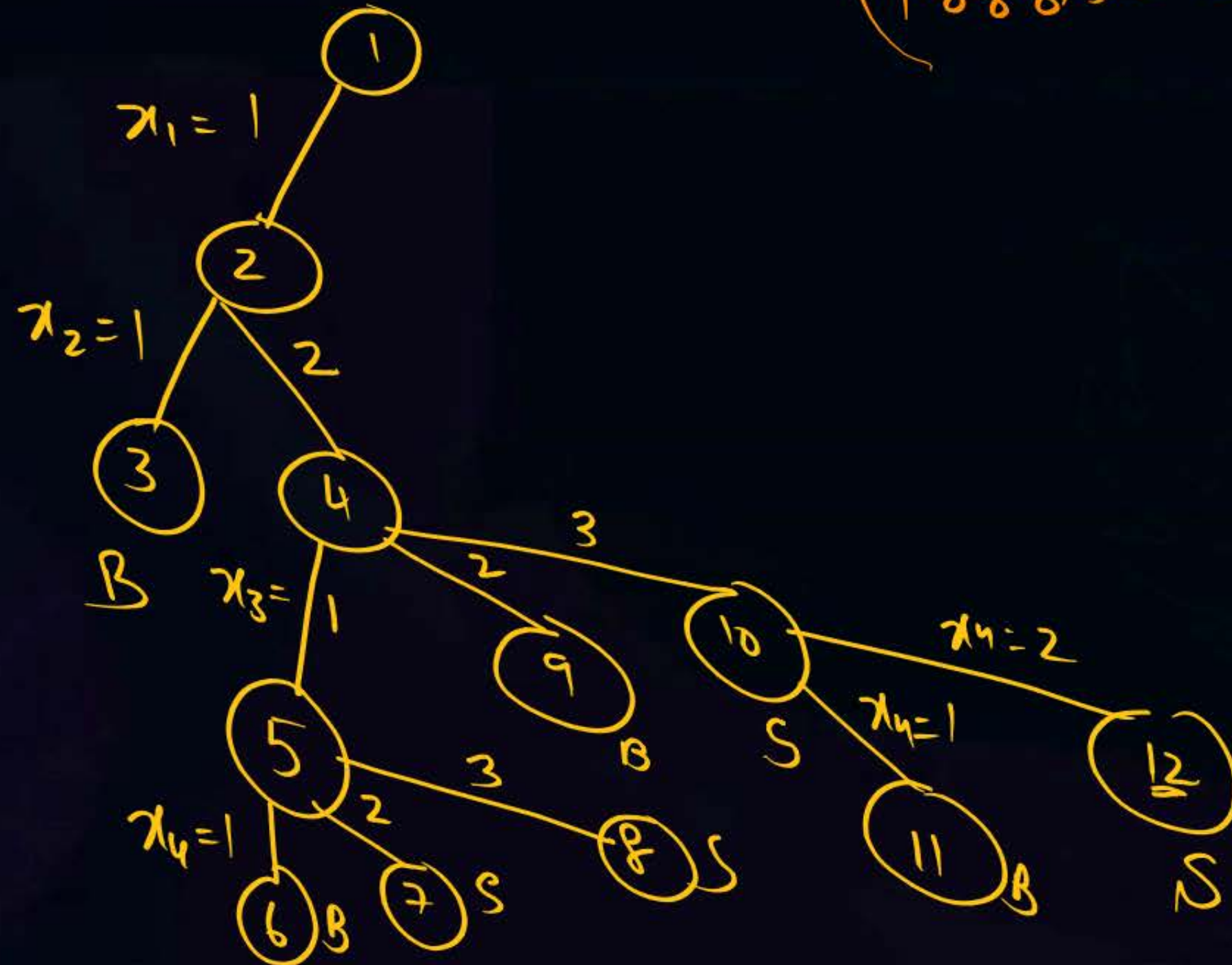


no. of colors $m = 3$

(Possible orderings)

<chromatic no.>

(Painting of Atlas)



Popular Problems Solved by Backtracking

- (i) N-Queens
- (ii) Sum of Subsets
- (iii) Graph Coloring
- (iv) Hamiltonian Cycle
- (v) 0/1 KNAPSACK

Branch & Bound: → Stack

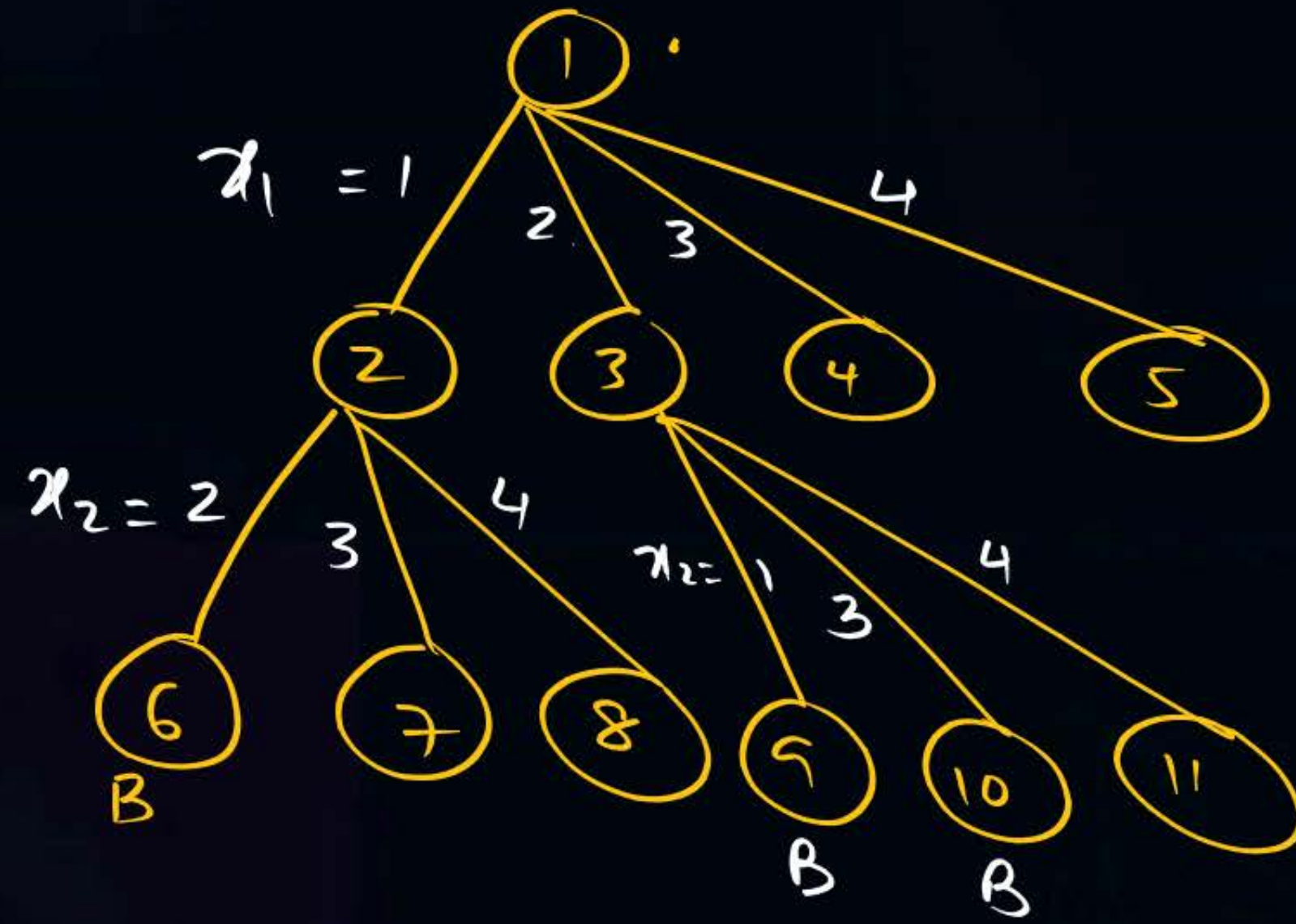
1) Backtracking = DFS + Bounding fns

2) Branch & Bound = BFS + "

→ Queue:
 FIFO ✓
 LIFO ✓

FIFO-BB
 LIFO-BB

Solving 4-Queens Problem using FIFO-BB



$Q(FIFO)$

2	3	4	5	7	8	11
--------------	--------------	--------------	---	---	---	----

No. of Nodes

Generated =

31

Branch & Bound

- 1) n-Queens
- 2) 15-Puzzle Problem
- 3) J.S.D
- 4) T.S.P
- 5) 0/1 KNAP

8	3	12	15
4	9	5	
6	2	1	14
7	10	11	13

1) Complete the Syllabus

↳ (notes)

2) Practice Problems (class)

3) P. 4. Q's < 25-30 yrs >

4) Test - Series < Time Mgmt
Coverage of Syllabus

5) No Peer Comparison

THANK - YOU