

CS & IT ENGINEERING

Algorithms

Miscellaneous Topics

Lecture No.- 05

By- Dr. Khaleel Khan sir



Recap of Previous Lecture

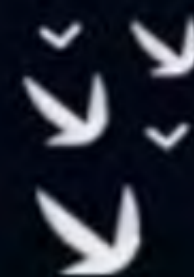


Topic

Minimum Cost Spanning Trees



Topics to be Covered



Topic

Minimum Cost Spanning Trees

Shortest Paths

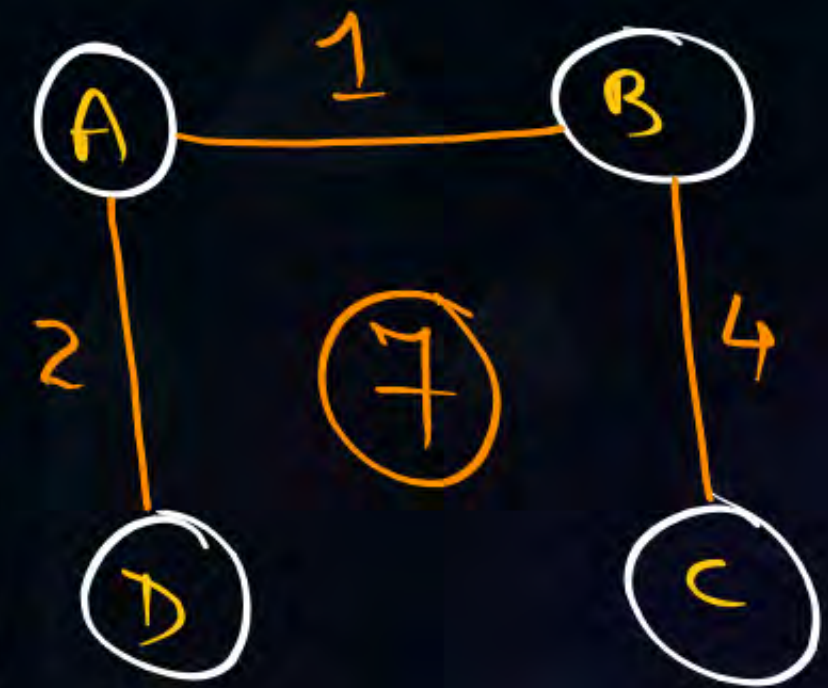
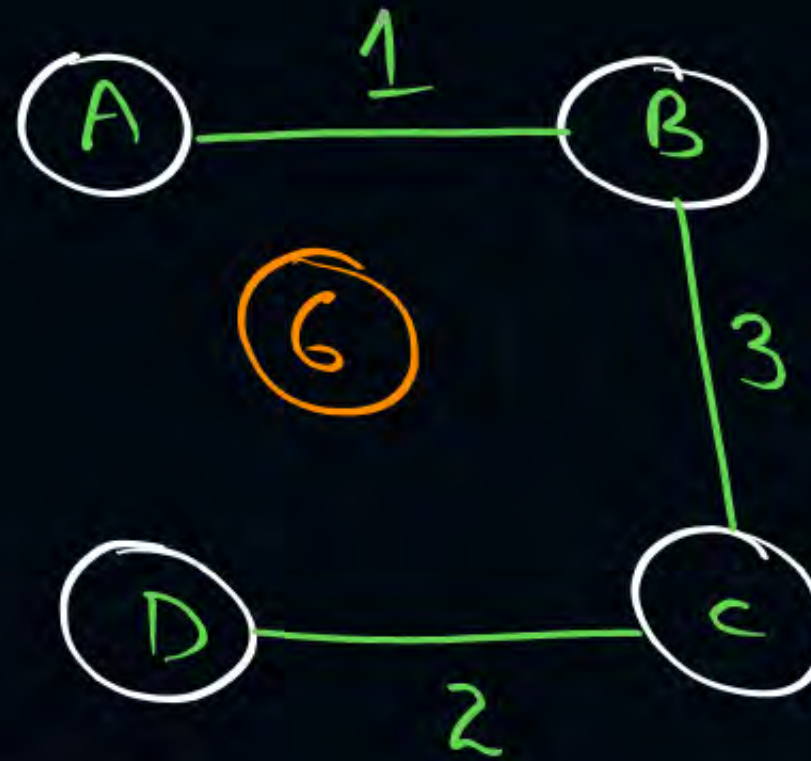




Topic : III. Greedy Method

- Q. Let G be a complete undirected graph with 4 vertices and edge weights are $\{1, 2, 3, 4, 5, 6\}$. The maximum possible weight that a minimum weight Spanning Tree can have is 7. ✓

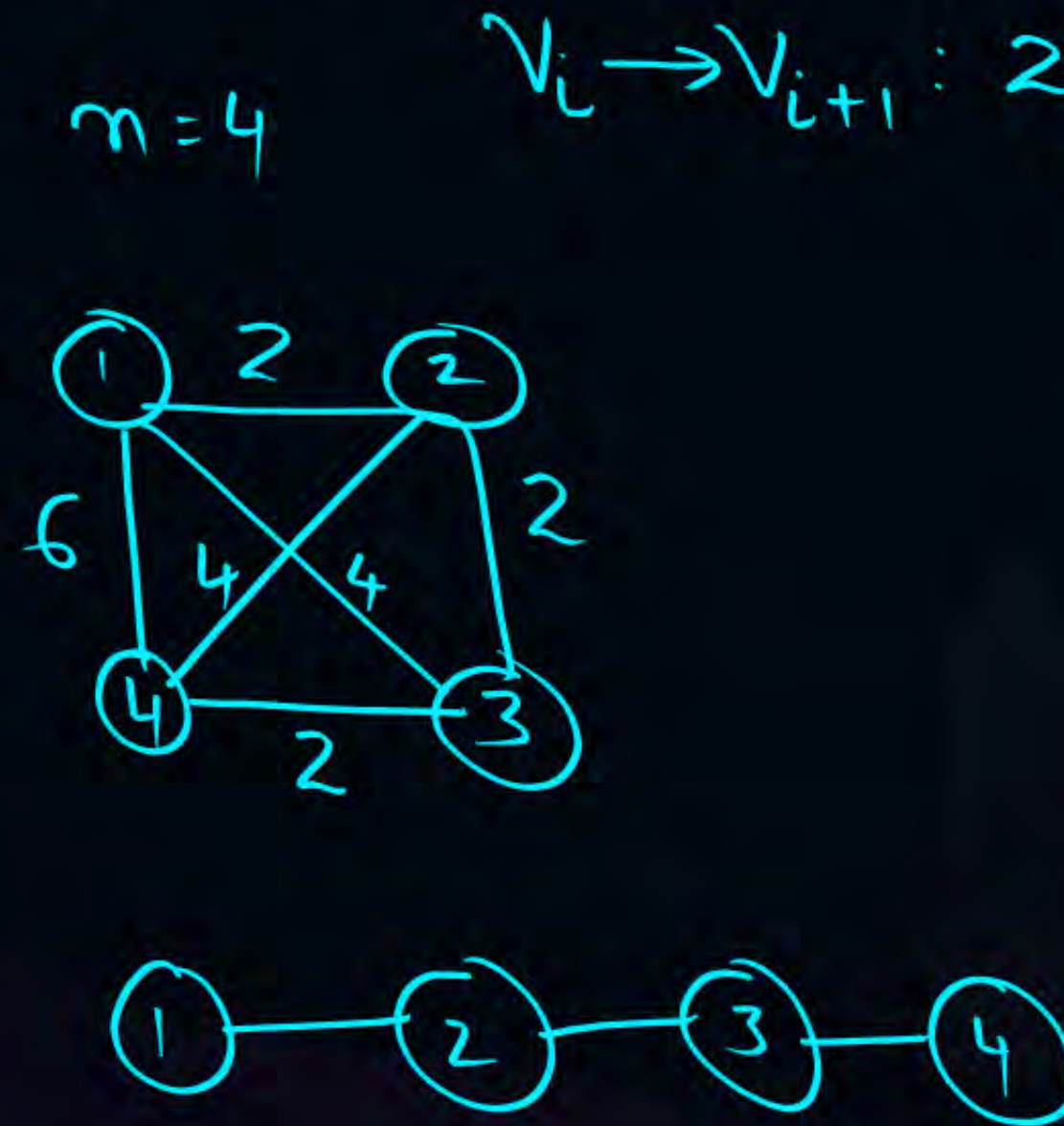
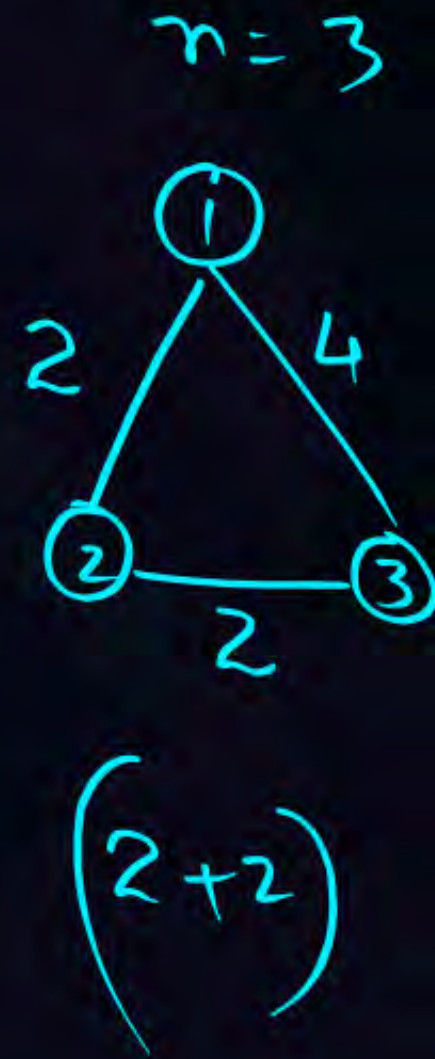
MCST





Topic : III. Greedy Method

- Q. Consider a complete weighted Graph with n vertices numbered V_1 to V_n . Two vertices V_i & V_j having edge between them has a cost value of $2|i - j|$. The weight of minimum cost Spanning Tree of such a graph is _____.



$$\begin{aligned} &= 2(n-1) \\ &\Rightarrow (2n-2) \checkmark \end{aligned}$$



Topic : III. Greedy Method

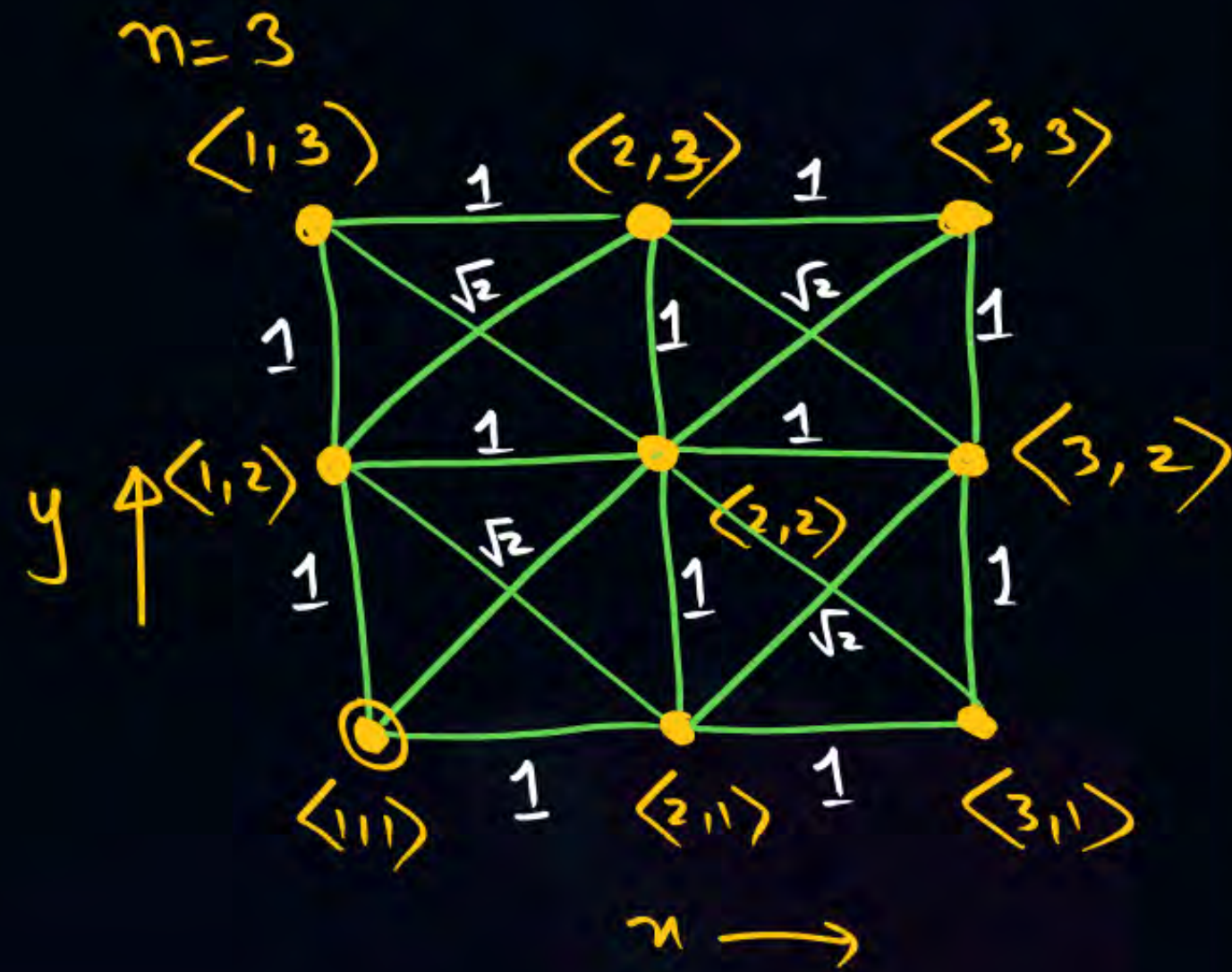
Q. Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes 995. ✓ ₊₅

$$\Rightarrow 500 + 99 \times 5 = 995 \checkmark$$

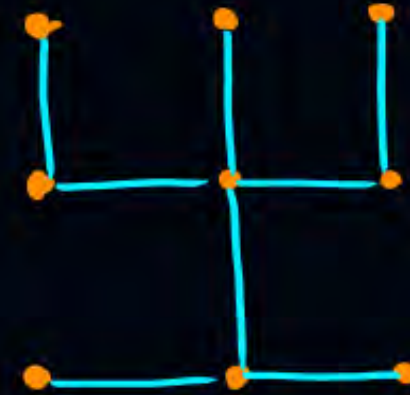


Topic : III. Greedy Method

- Q. Consider a Graph whose vertices are points in a plane with integer coordinates (x, y) where $1 \leq x \leq n$, $1 \leq y \leq n$, $n > 2$ is an integer. 2 vertices $\langle x_1, y_1 \rangle$ & $\langle x_2, y_2 \rangle$ are adjacent iff $|x_1 - x_2| \leq 1$ & $|y_1 - y_2| \leq 1$. The cost of such an edge is given by the distance between them. Compute the weight of min cost Spanning Tree of such graph for a given value of n .



$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$



$n=3$
 $\text{Cost} = 8$
 $(3^2 - 1) \times 1$

For a value of 'n'
 No. of vertices of G' : (n^2)
 No. of edges of mcs: $(n^2 - 1) \cdot 1$


$$: (n^2 - 2)\sqrt{2} + 1$$



Q2) What is the cost of Max-Cost Sp-Tree,



Topic : III. Greedy Method

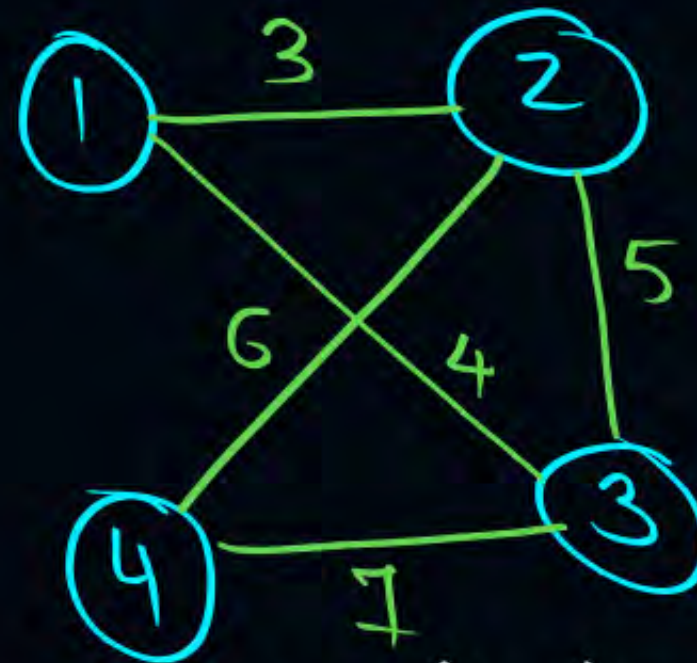
Q.  Consider a graph with 'n' vertices $n > 2$. The vertices are numbered V_1 to V_n . Two vertices V_i & V_j are adjacent iff $0 < |i - j| \leq 2$. The weight of such an edge is $i + j$. The weight of minimum cost Spanning Tree of such a graph for a value of n is _____.

$n=3$



$$\therefore \begin{pmatrix} 3+4 \\ (3+2 \times 2) \end{pmatrix}$$

$n=4$



$$\therefore \begin{pmatrix} 3+2(2+3) \\ (3+4+6) \end{pmatrix}$$

n

$$\therefore 3+2(2+3+4+\dots+n-1)$$

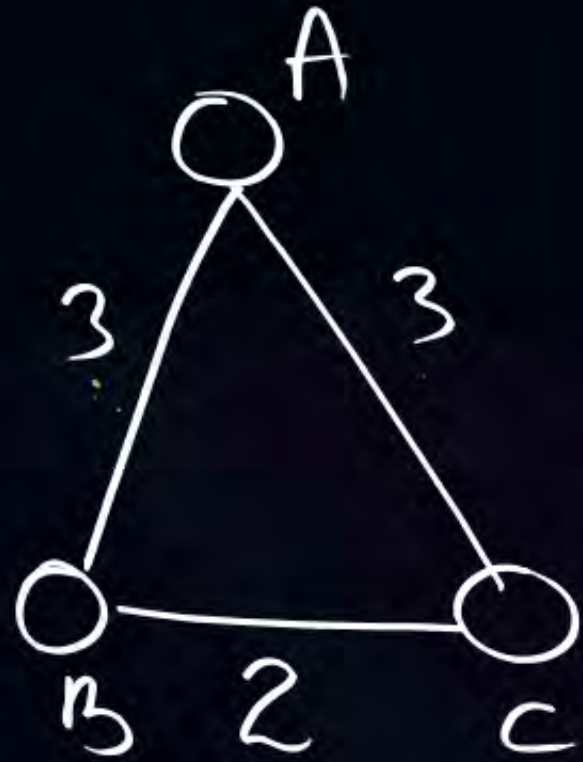
$$\therefore 3+2\left[\frac{n(n-1)}{2}-1\right]$$

$$\therefore 3+\cancel{2}\left[\frac{n^2-n-2}{\cancel{2}}\right]$$

$$\boxed{n^2-n+1}$$

(nc_r)

(1)

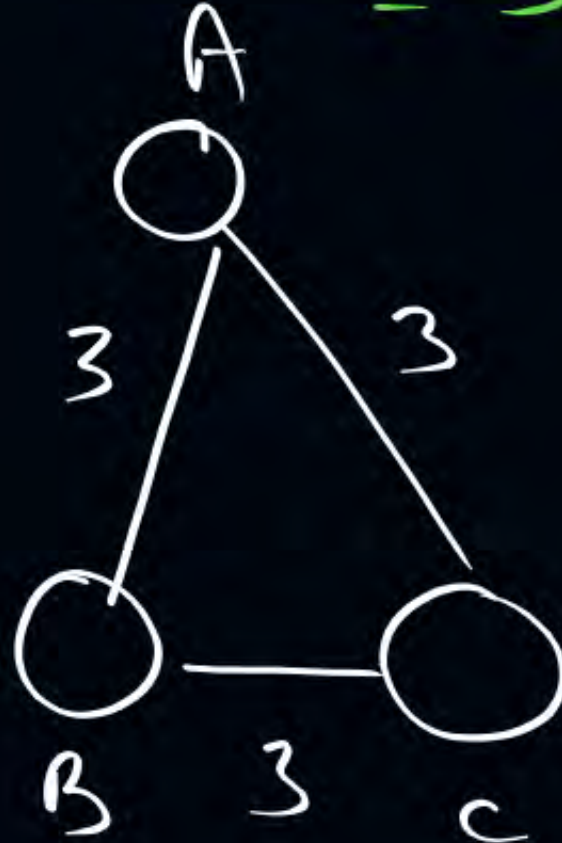


No. of mcsT's: (2c₁)
0

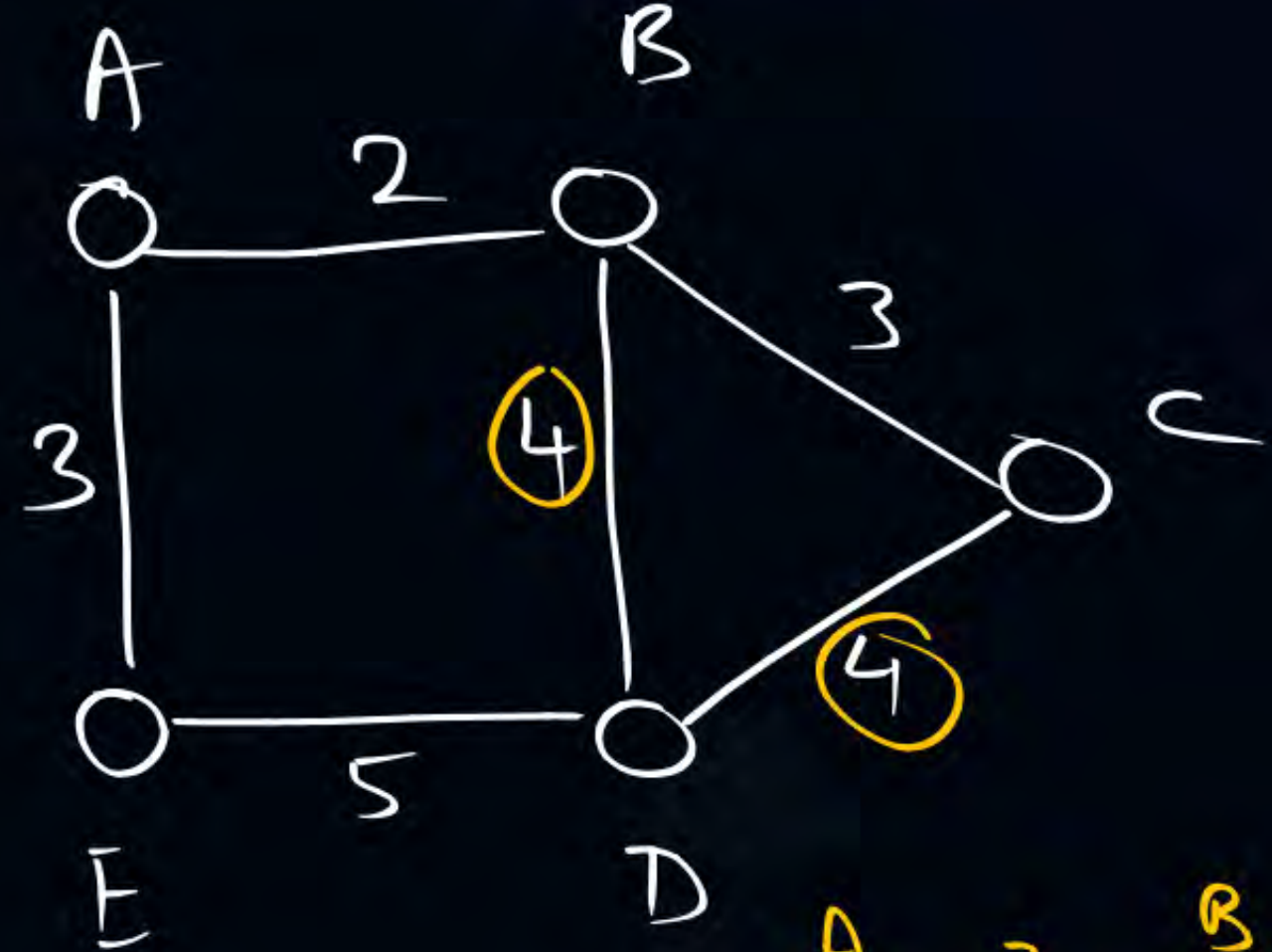


(3c₂)

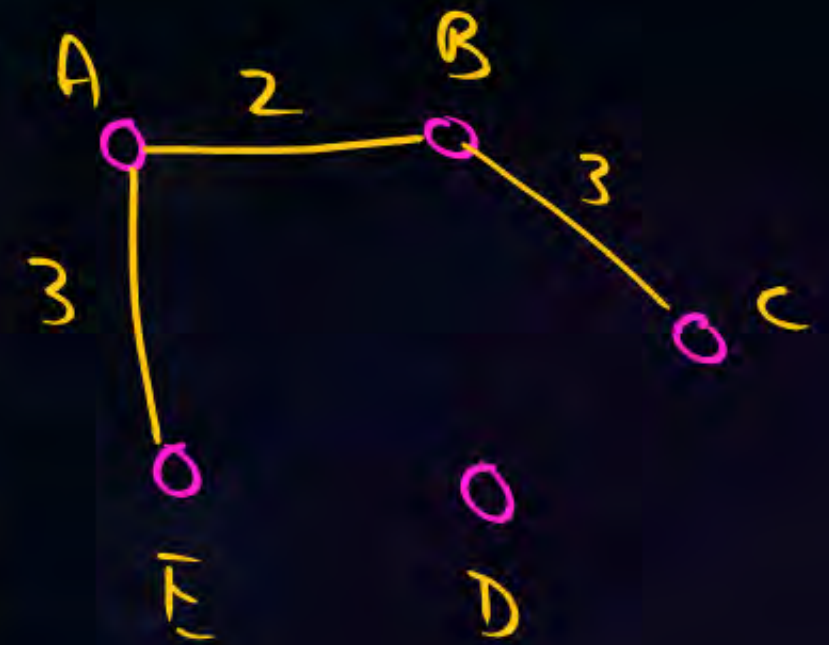
= 3 ✓



(2c₁)



No. of mcsT's:
(2)

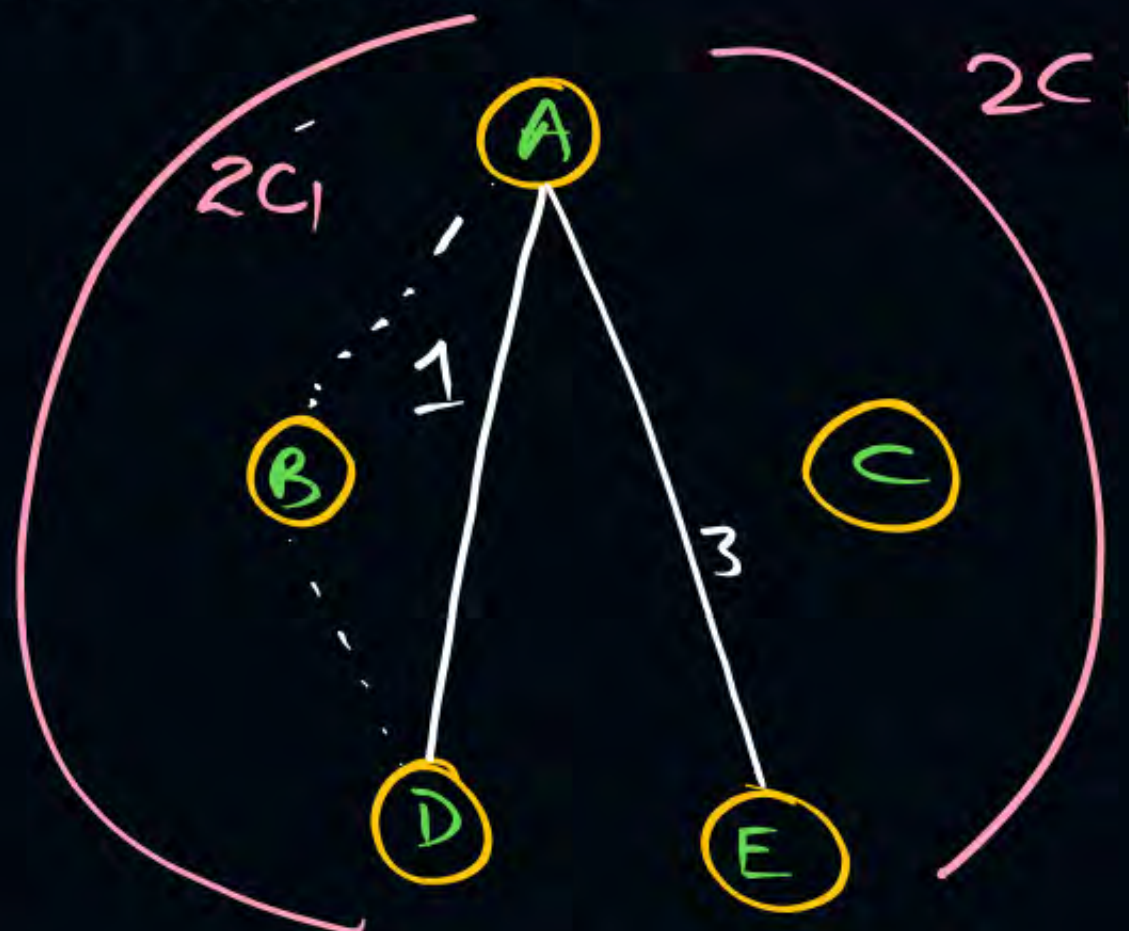




Topic : III. Greedy Method



Q. Consider the following undirected graph G:



(i) If $x > 5$ then $N = 2$

(ii) If $x < 5$ $N = 2$

(iii) If $x = 5$

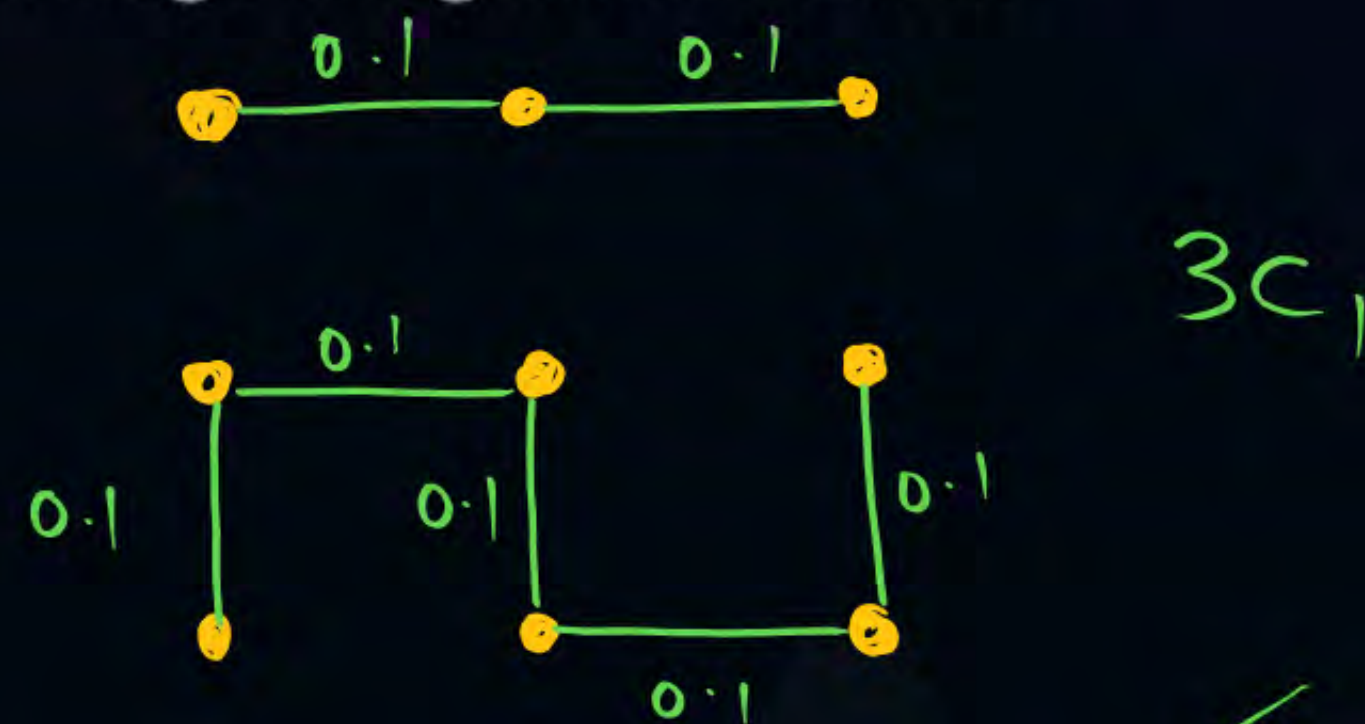
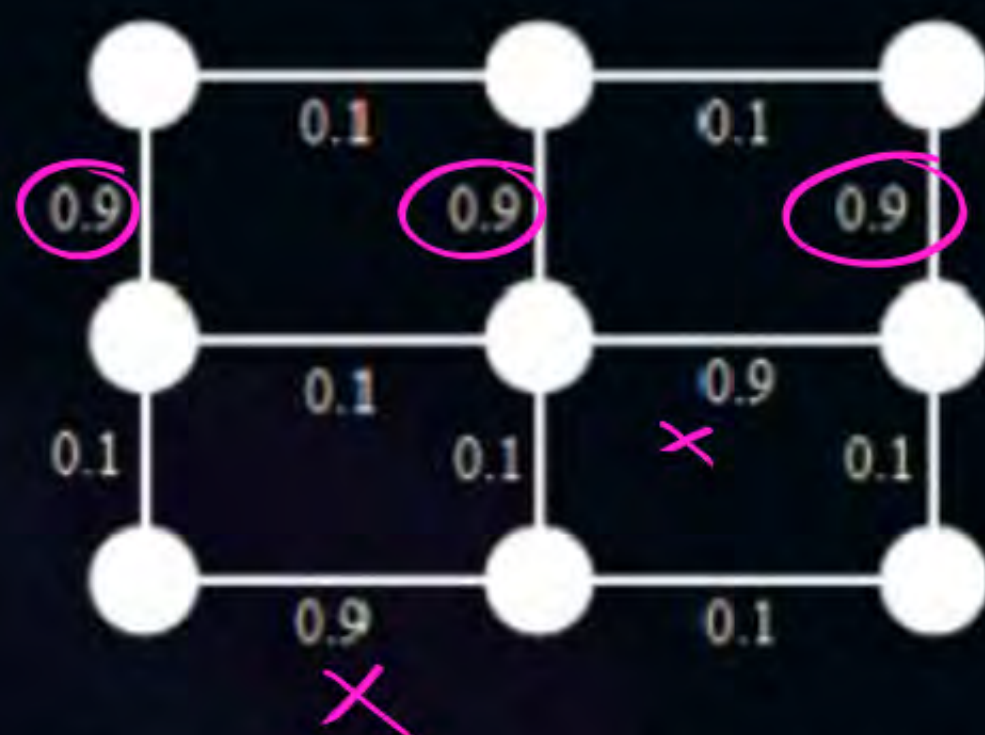
Total No. of
MCSTs = $2C_1 * 2C_1$
= 4

Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G . The number of MWSTs of G for this value of x is 4.



Topic : III. Greedy Method

Q. Consider the following undirected graph with edge weights as shown



The number of minimum-weight spanning trees of the graph is 3 ✓



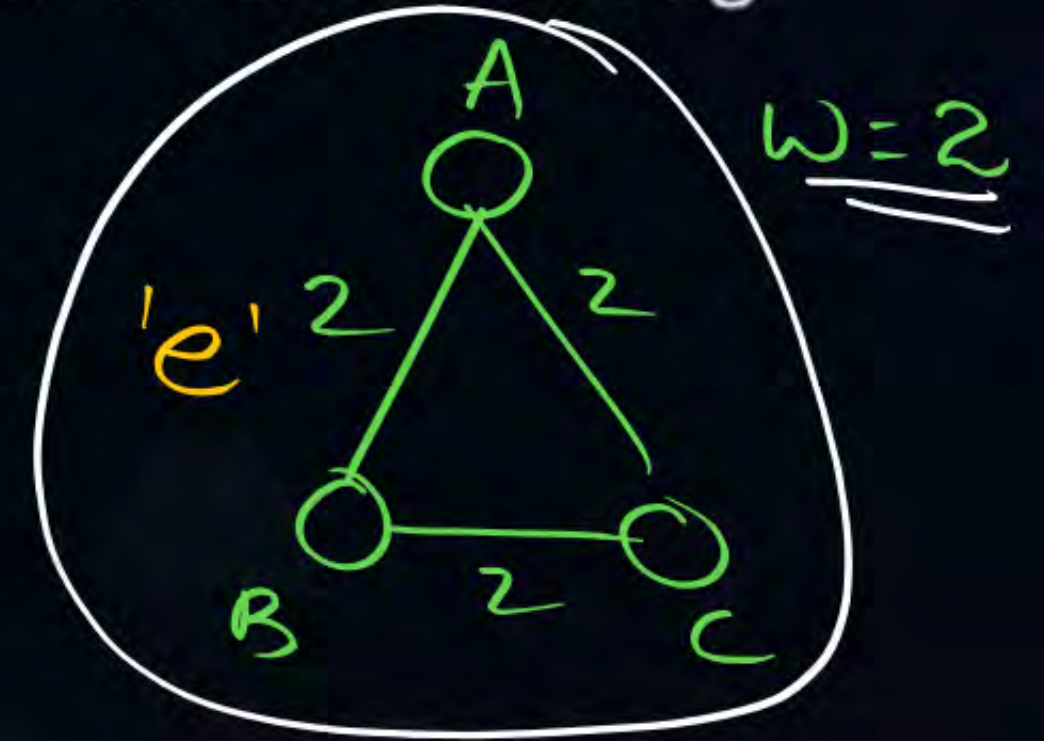
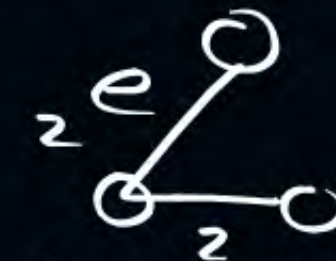
Topic : III. Greedy Method

Q. Let w be the minimum weight among all edge weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is False?

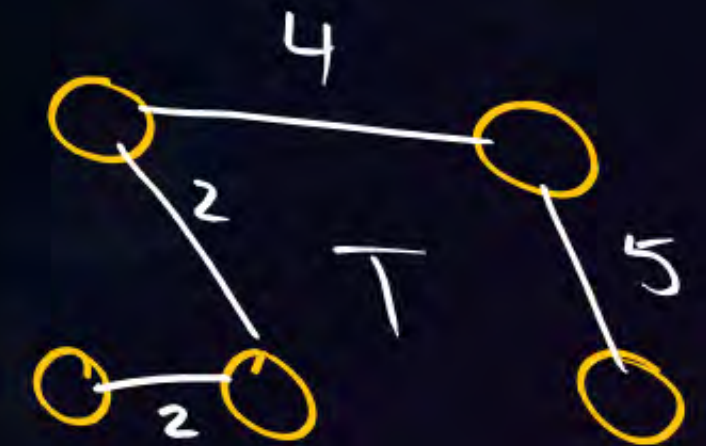
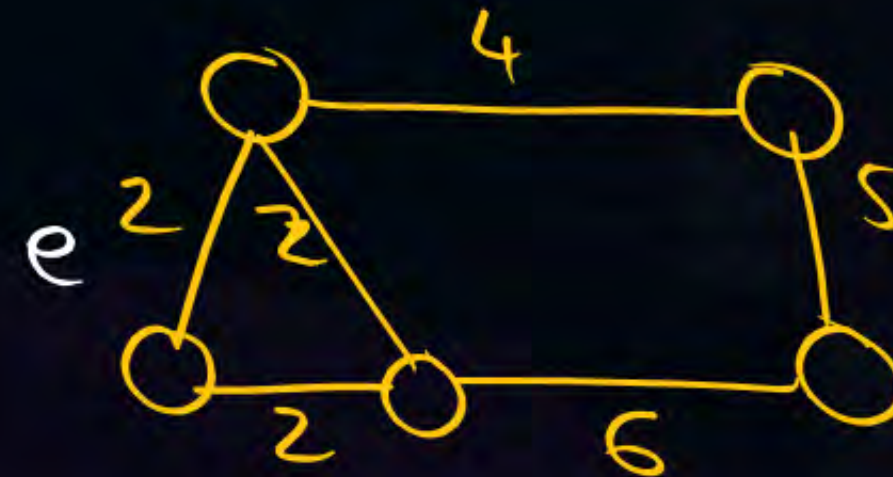
one
T i. There is a minimum Spanning Tree containing 'e' always.

T ii. Every minimum Spanning Tree has an edge of weight 'w'.

F iii. 'e' is present in every minimum Spanning Tree.



T iv. If 'e' is not present in a minimum Spanning Tree named 'T' then there will be a cycle formed by adding 'e' to T.





Topic : III. Greedy Method

Q. $G = (V, E)$ is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G . Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G , then every MST of G includes e

II. If e is the heaviest edge of some cycle in G , then every MST of G excludes e

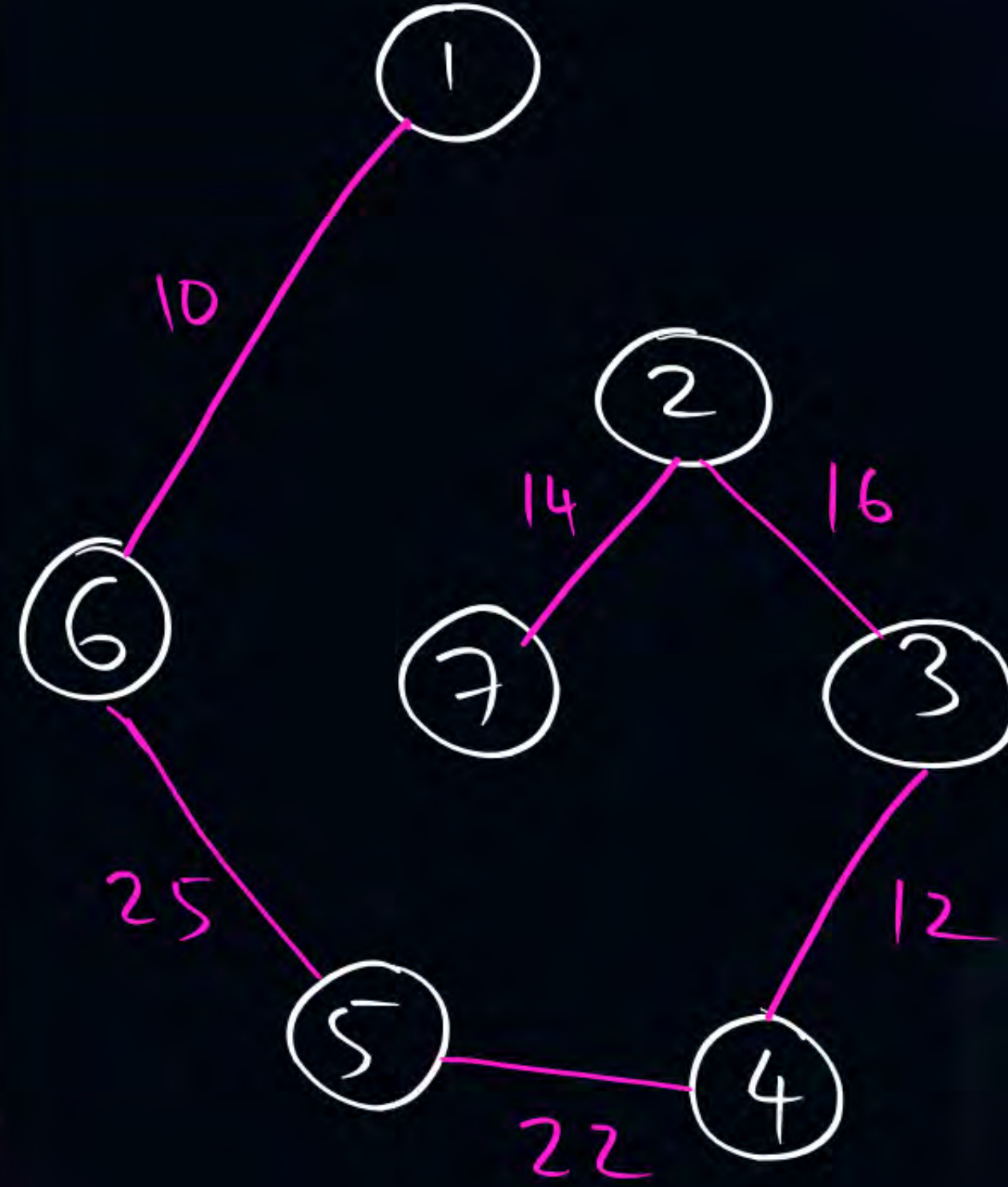
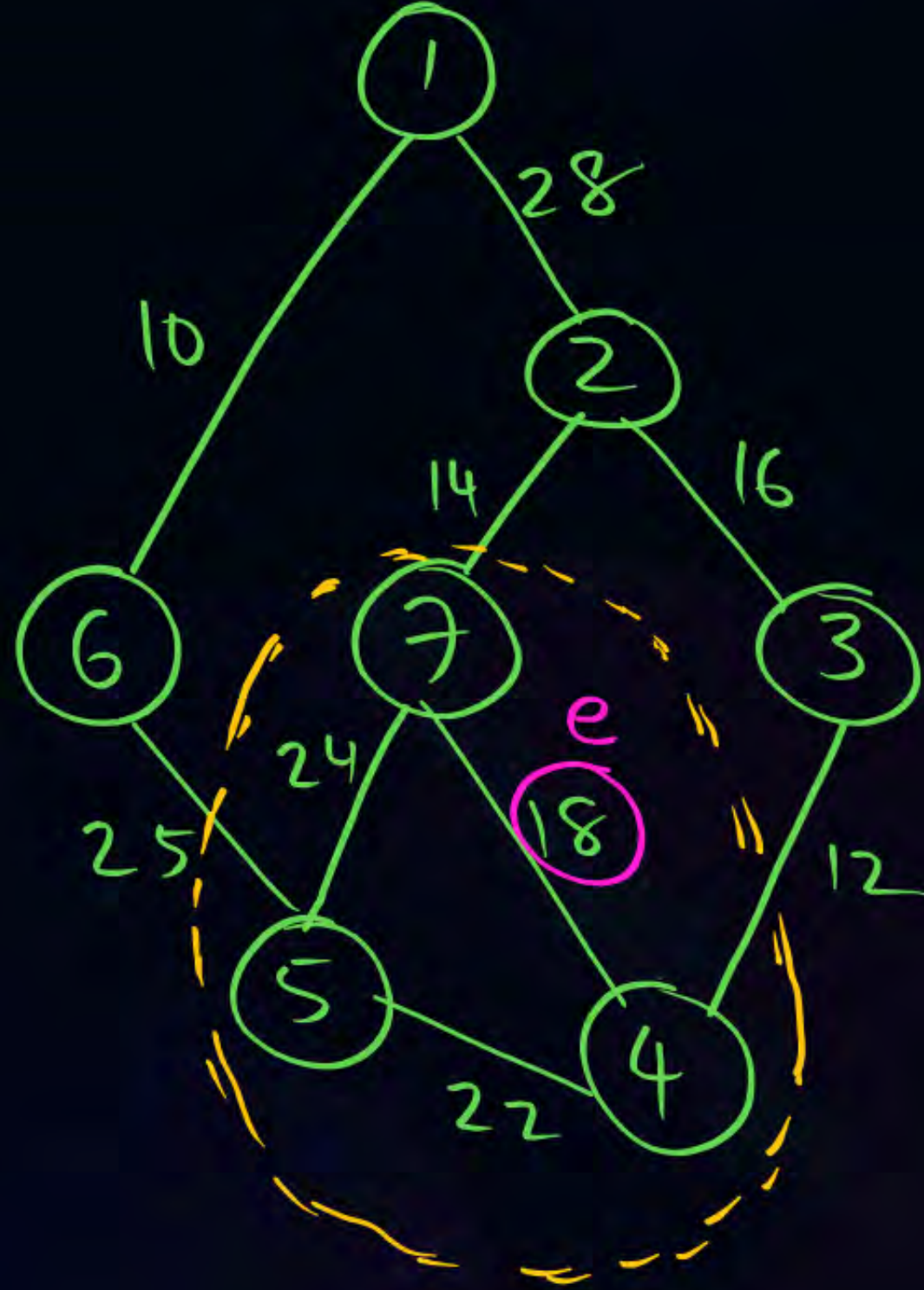
(a) I only

✓ (b) II only

(c) both I and II

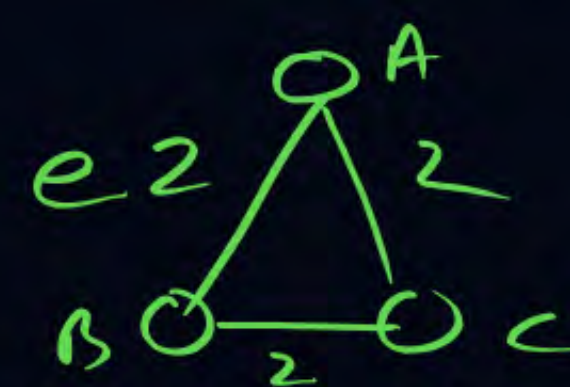
(d) neither I nor II

← Digkstra's Algo. property →





Topic : III. Greedy Method



Q. Let G be a connected undirected weighted graph. Consider the following two statements.

✚ S1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G

└ S2: If every edge in G has distinct weight, then G has a unique minimum spanning tree.

Which one of the following options is correct?

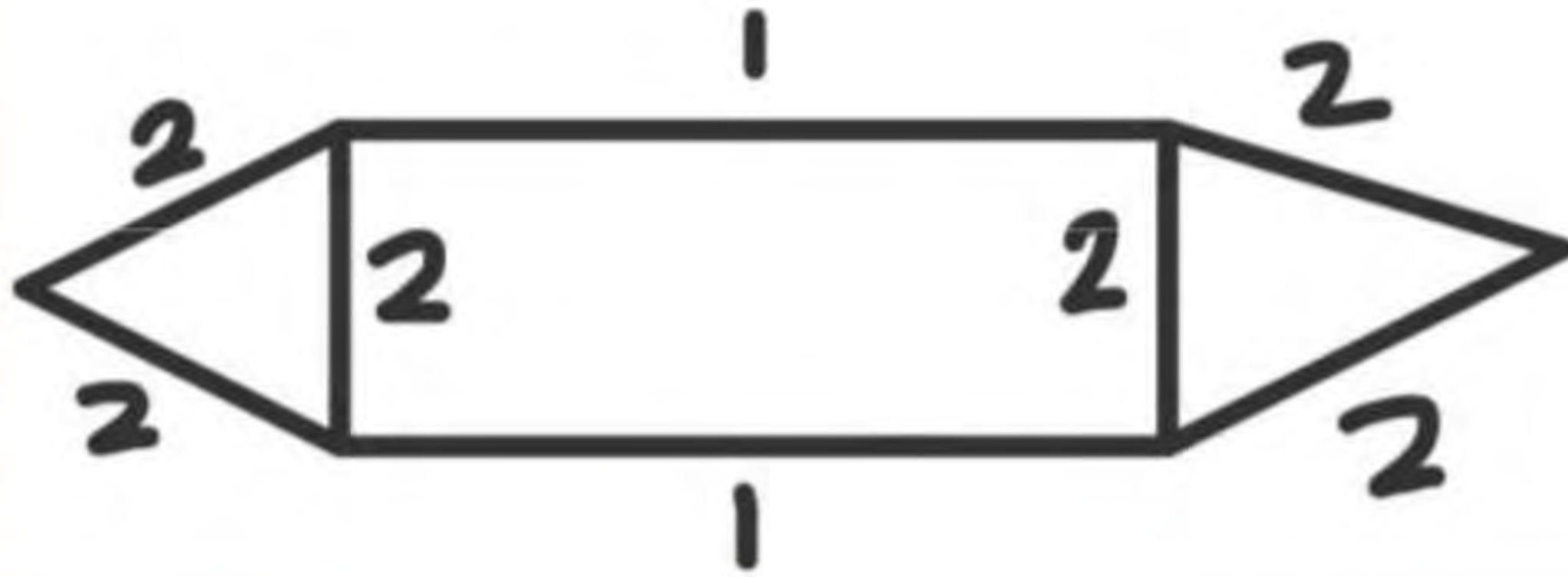
(a) Both S1 and S2 are true

(b) S1 is true and S2 is false

✓ (c) S1 is false and S2 is true

(d) Both S1 and S2 are false.

No of minimal spanning trees



H/w



6) Single Source Shortest Paths (SSSP)

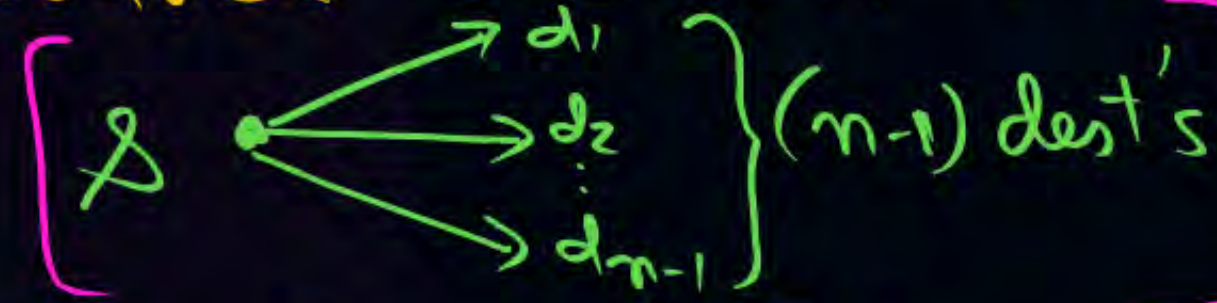


< Dijkstra's Algorithm >

(i) Single pair Shortest Path



(ii) Single Source Shortest Paths



(iii) All-Pairs Shortest Paths

< Floyd-warshall's Algo. > [D.P]
($\langle i, j \rangle$ every pair Sh-Path)

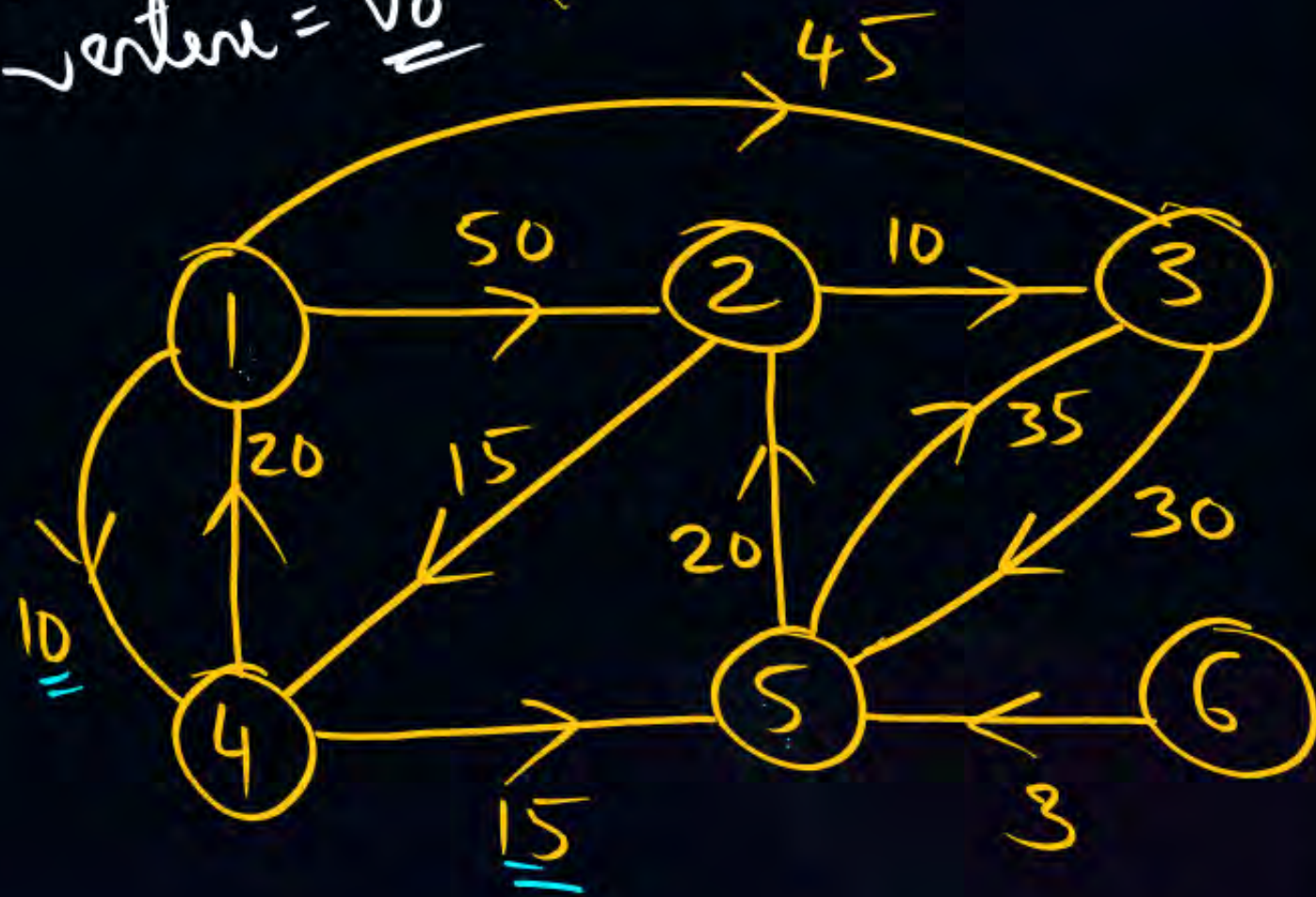
May NOT work with -ve wt. edges

Dijkstra's Algo (Greedy)

Bellman-Ford (D.P)

(Can work with -ve wt. edges)

Source vertex = v_0 (Dijkstra's Greedy Algo)



C	1	2	3	4	5	6
1	0	50	45			
2						
3						
4						
5						
6						

$c(i,j)$ = edge cost

a) Matrix-method : $v_0 = 1$

vertex	Selected	d-values					
		1	2	3	4	5	6
Step 1	1	-	50	45	10	∞	∞
Step 2	{1, 4}	-	50	45	10	25	∞
Step 3	{1, 4, 5}	-	45	45	10	25	∞
Step 4	{1, 2, 4, 5}	-	45	45	10	25	∞
Step 5	{1, 2, 3, 4, 5}	-	45	45	10	25	∞



$v_0 = 6$

$v_0 = 6$

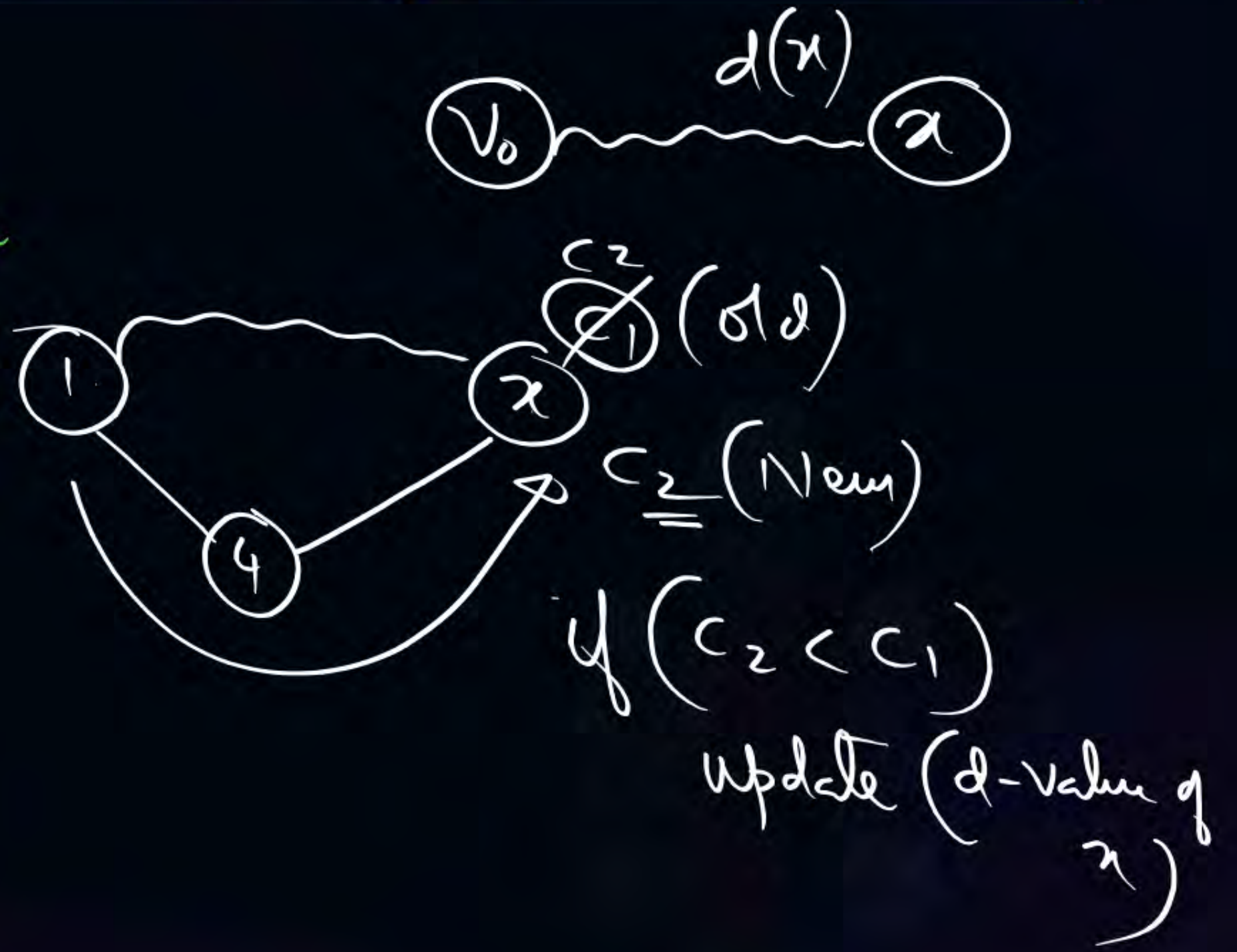
Vertex
Selected

d-values

		1	2	3	4	5	6
Step 1	{6}	∞	∞	∞	∞	3	-
Step 2	{6, 5}	∞	23	38	∞	3	-
Step 3	{6, 2, 5}	∞	23	33	38	3	-
Step 4	{6, 2, 3, 5}	∞	23	33	38	3	-
Step 5	{6, 2, 3, 5, 4}	58	23	33	38	3	- ✓


$d(x)$ = distance from source (v_0) to vertex
' x ' known so far

(Relaxation
Probers)



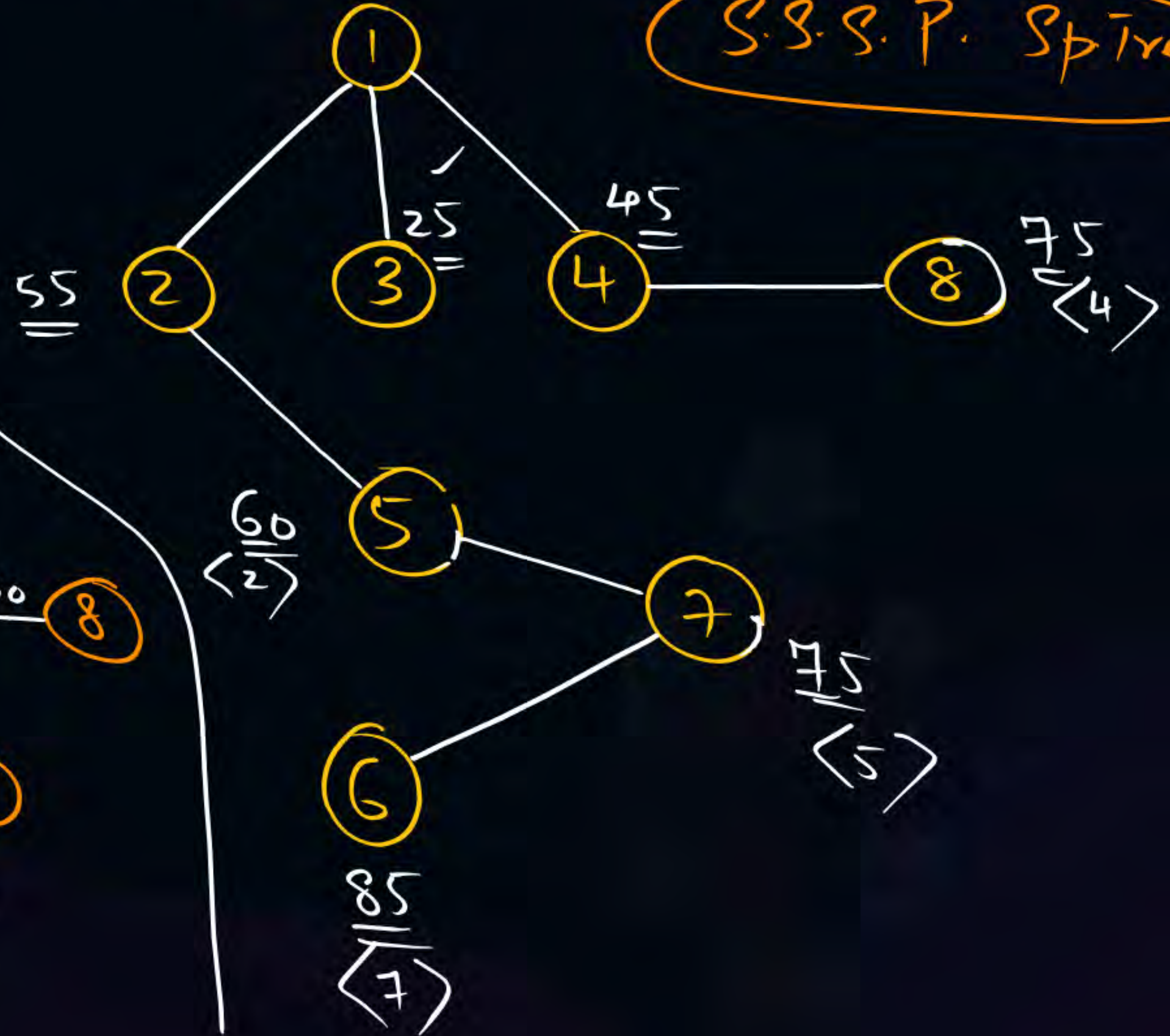
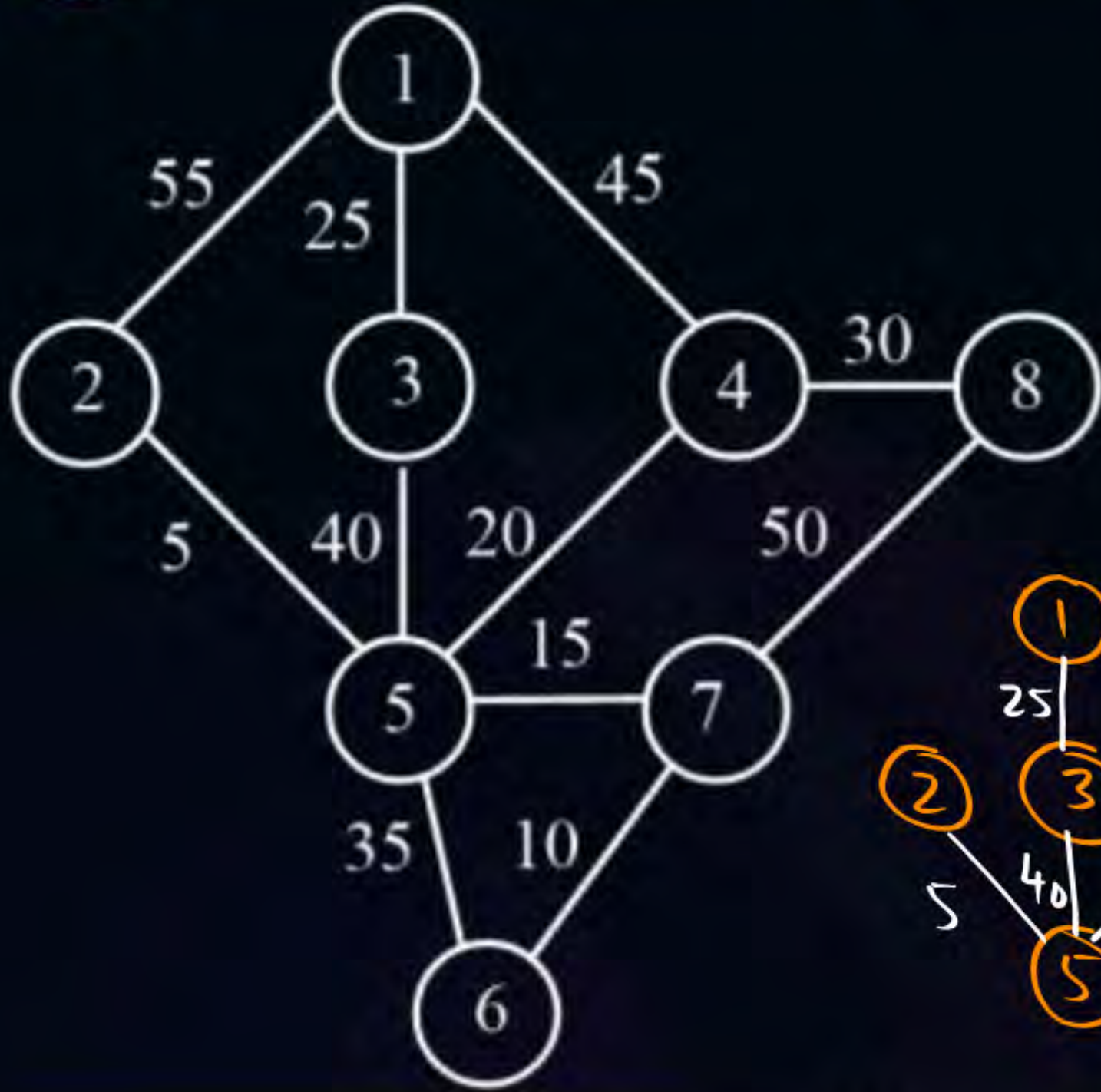


Topic : Greedy Method

* Dijkstra's Algo [Spanning Tree] 

$v_0 = 1$

S.S.P. SpTree



THANK - YOU