COMPUTER SCIENCE



Computer Organization and Architecture

Floating Point Representation



Lecture_01

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Floating Point Representation





- 10 Introduction of COA.
- (2) Machine Instruction & Addressing Modele.
- (3) Floating Point Representation



Number System

Floating Point Representation.

Number System:

Sign [-ve].

MSB [LSB]

Magnitude Format
[Signed & Un signed]
(tre 4-re) (only tre)

@ Complement Format [18 Complement, 2's Complement) magnitude Format

unsigned (0102-1)

Signed 1's Conflement 2's conflement (2-1) to +(2-1) (2-1) (2-1) (2-1) (2-1) (2-1)

Complement

n bit number. (I) Magnitude Format

1) O Unsigned Range = 0 to 2 -1 0000 >0

(B) 4 bit unsigned Range = 0 to 2 -1 > 0 to 15 1111 -> 15

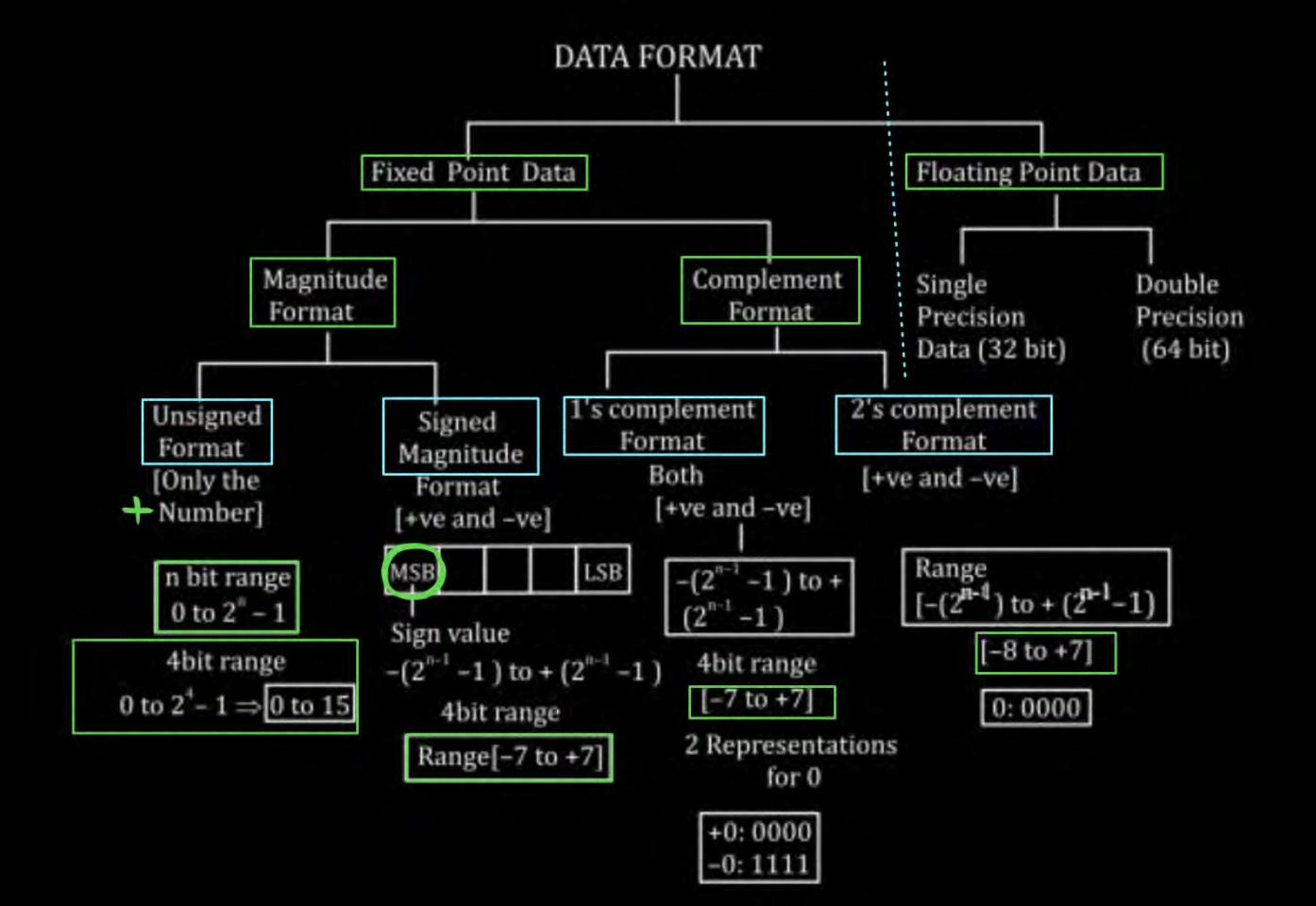
(2) Signed Range = $-(2^{n-1})$ to $+(2^{n-1})$ 4 bit Signed Range = -2^{n-1} to $+2^{n-1}$ = -7 to +7

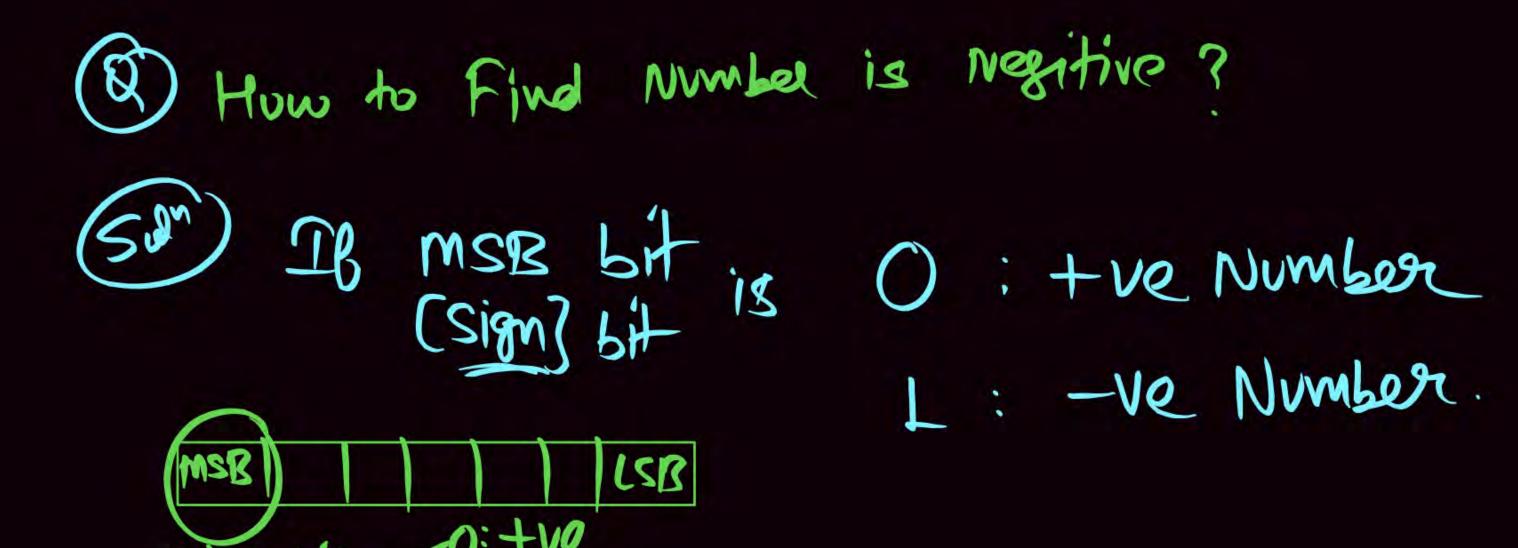
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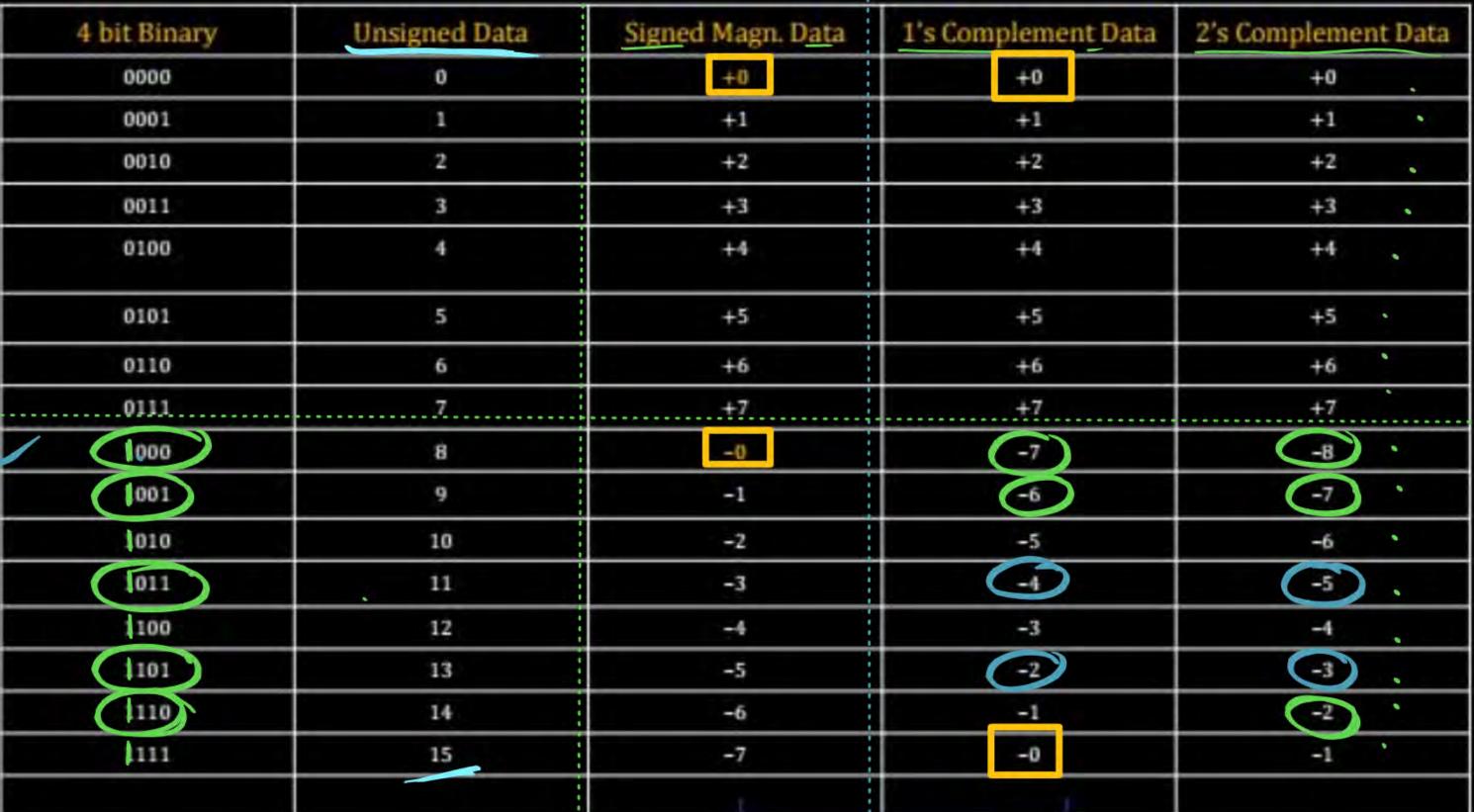
er u bit number (II). Complement bonnat.

② 2 Complement Range =
$$-(2^{n-1})$$
 to $+(2^{n-1})$

.









tow to Find 1's com	blement 1 2's	Compleme	mt:
1's Comp	lement:	2'S COW	plement
6 Gonverted into		1's Comple	ment
	1	+1_	
1			
	1's Complement		Conflement
(a) T000 =>	0111 [-73		1 = 1000 = [-8
(a) 100T 3	0110 [-6]	0110	=> 0111 [-7].
(e) 1101 =)	0010 [-2]	00100111	⇒ 0011 (-3].
	0100 (-4)	010/00011	⇒ 0011 [-3]. ⇒ 0101 [-5].

(B) In Computer System WHY 2's complement one Used to Represent Negitive Number, even we have signed 4 1's complement also?

(SE)

Signed Magnitude.

+0:0000

-0:1000

18 complement

+0:0000

-0:1111

Redundant Representation of 0 in signed & 1's Complement 30 2's complement are used in computer system.

n bit 2's complement farge =
$$-(2^{n-1})$$
 to $+(2^{n-1})$

(B) 4 5it 2's complement =
$$-(4-1)$$
 to $+(2-1)$
Range = -8 to $+7$.

.

Floating Point Representation:

```
WHY Floating point Representation?
        To Represent very-very large Number= (986512247...)
       To Represent very - very small Number = (00......)

Nearly: (...0)

16 bit Number Range = -(2^{16-1}) to +(2^{16-1}-1) \Rightarrow -2 to +2-1
                                  =) - 32k to +(32k-1)
we want to Represent 51.000 its Not bossible With 16 bit Data.
```

Floating-Point Representation



Principles

- With a fixed-point notation it is possible to represent a range of positive and negative integers centered on or near 0.
- By assuming a fixed binary or radix point, this format allows the representation of numbers with a fractional component as well
- Limitations:
 - Very large numbers cannot be represented nor can very small fractions
 - The fractional part of the quotient in a division of two large numbers could be lost

Floating-Point Representation



16 bit fixed point data format then

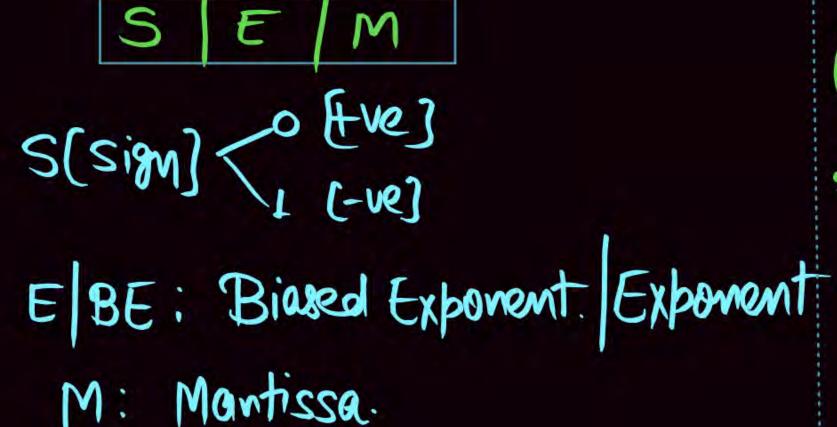
Range =
$$-2^{16-1}$$
 to $+(2^{16-1}-1)$

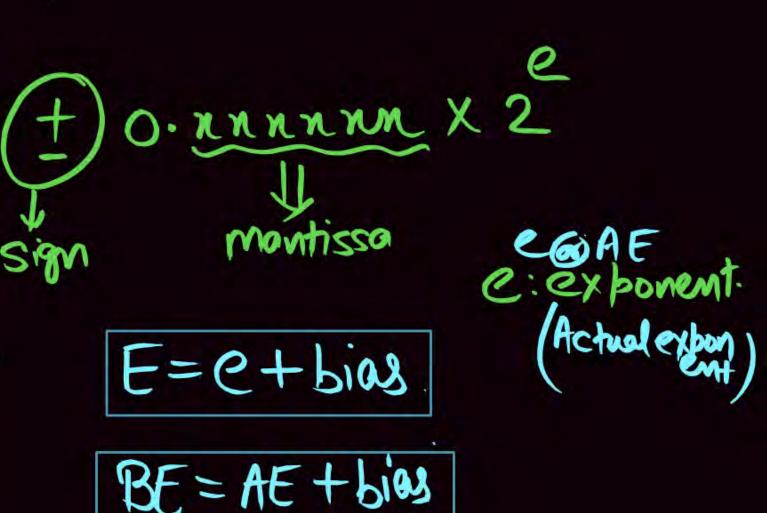
$$\Rightarrow$$
 -(2¹⁵) to + (2¹⁵ - 1)

If we want to store 61,000 then we cannot store

So floating point representation is to represent very large data and very small fraction and consume less memory

Floating Point Representation





Floating-Point Representation

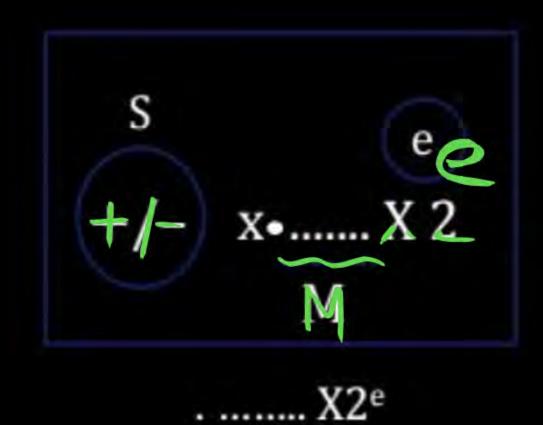


S	Е	М
5	L	1

S: sign bit
$$\frac{0}{1}$$
 +ve

E: exponent

M: Mantissa



6.5 in Binary $\Rightarrow 110.1$

Number System

16 8 4 2 1 . (.5) (.25) (.125) (0.8625)

- 3 11.5 => 1011·1
- (4) 13.25 => 1101·01
- (5) 16.625 => 10000·101
- (6) $6.37 \Rightarrow 110.011$
- (F) 19.25 => 10011.01

(8) $4.75 \Rightarrow 100.11$ (9) $11.625 \Rightarrow 1011.101$

(6) $37 \Rightarrow 100101$

(i) ·37 = 0.011.

(12) 13.75 => 1101.1

(13) $29.625 \Rightarrow 1101.101$

·625 => 0·101

100.5005

+(6.5)

110·L

2 2 2 2 2 . 2 2 2 8 4 2 1 . (5) (.25)

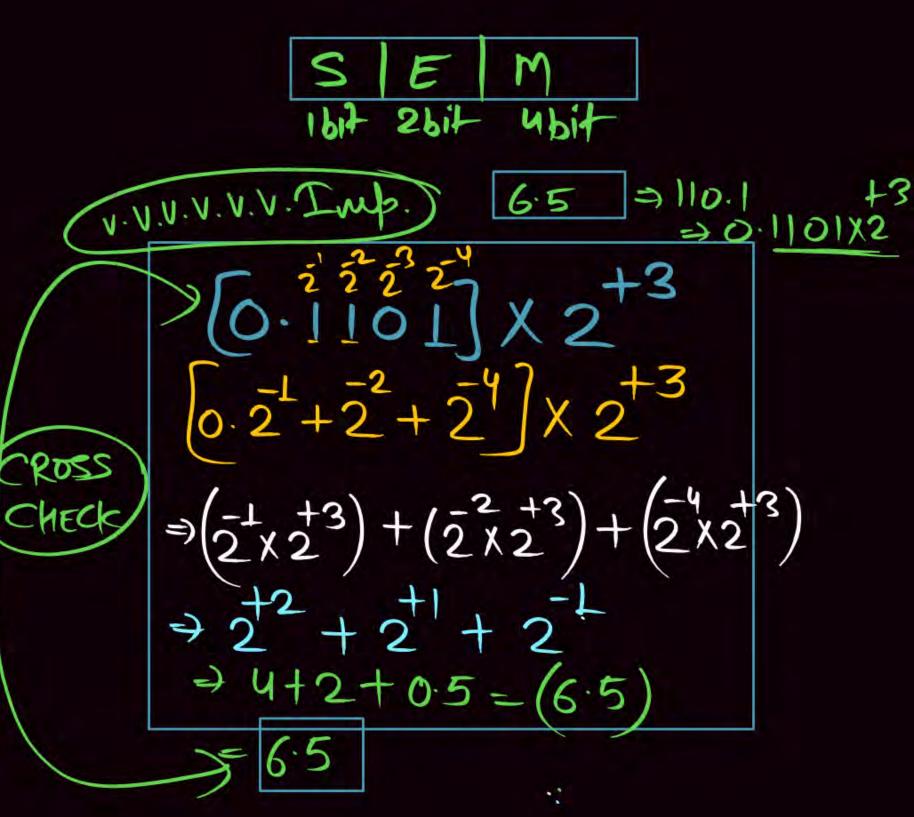
Another Method
to chack 2

to chack 2

2

Techning Databits side
Right Alignment: 2-12.

Left Alignment: 2-12.



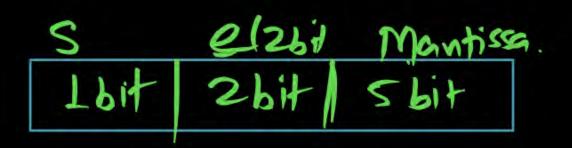
(a)
$$+(6.5)$$

 $+110.1$
 $+0.1101 \times 2$
Mantissa
Sign = 0
 $M = 1101$
 $C = +3 \Rightarrow [11]$

$$(4.5)$$
 + (4.5) + (100.1)

.

$$\begin{array}{l}
(8.3) + (4.75) \\
+ (100.11) \\
=) + 0.10011 \times 2^{+3} \\
S = 0
\end{array}$$





5m(16h) (26it) Montissa (5bit)
0 | 1 | 10011

$$+6.5$$

$$6.5 = (110.1)_2$$

$$\frac{0.1101}{\text{S}} \times \frac{2^3}{2^e}$$

$$S = 0 (+)$$

$$M = 1101$$

$$e = 3 = (11)_2$$

S	e	M
0	11	1101



Very. Imp

$$6.5 = 110.1$$

$$=.1101 \times 2^{3}$$

$$= [.2^{-1} + 2^{-2} + 2^{-4}] \times 2^{3}$$

$$= [2^2 + 2^1 + 2^{-1}]$$

$$= 6.5$$



100.1

 0.1001×2^{3}

S = 0 (+ve)

M = 1001

e = 3[11]

S e M

0 11 1101



100.11

$$.10011 \times 2^{3}$$

$$S = 0$$

M: 10011

$$e = 3 \Rightarrow (11)_2$$

S	e	M
0	11	10011

NOTE:



Mantissa alignment process is used to adjust the decimal point; in this process right alignment increments the exponent and left alignment decrements the exponent.

Right Alignment

6.5
110•1
⇒.1101 ×
$$(2^3)^{2^{+3}}$$

$$\Rightarrow [.2^{-1} + 2^{-2} + 2^{-4}] \times 2^{3}$$

$$\Rightarrow 2^{2} + 2^{1} + 2^{-1}$$

$$\Rightarrow 4 + 2 + 0.5$$

$$\Rightarrow 6.5 \text{ Ans}$$

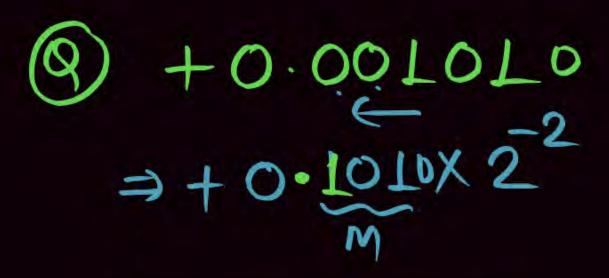
Left Alignment

Data:
$$0.00000000101 \times 2^{+5}$$
 $1.01 \times 2^{-8} \times 2^{+5}$
 $[1.01 \times 2^{+5-8}]$
 $+1.01 \times 2^{-3}$

(Align to use upto 8 times)

(a) WHy 'E' Required ?

E': Bias Exponent



S: 0 [+ke]

M: 1010

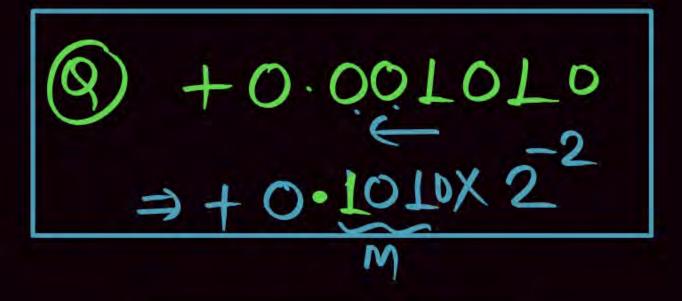
> No Provision to tell-that

exponent is Nepitive.

Number is positive [: S=0] but exponent is Ngritive. Signific How to Deal With this Ngritive exponent.

is exponent is negitive.

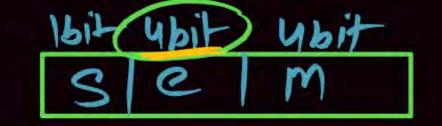
then 2's complement taking

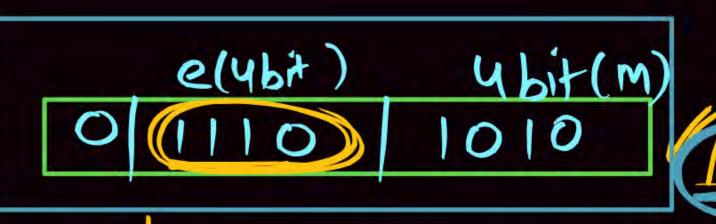


W: 1010



's complements Hone 'e' create Confusion

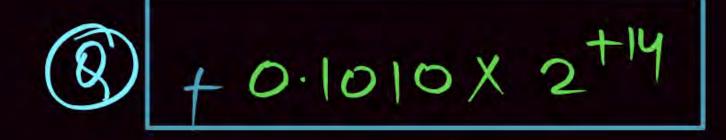




5 (8) But My Question is

2's complement. How we can say

if another example with having (2=14)

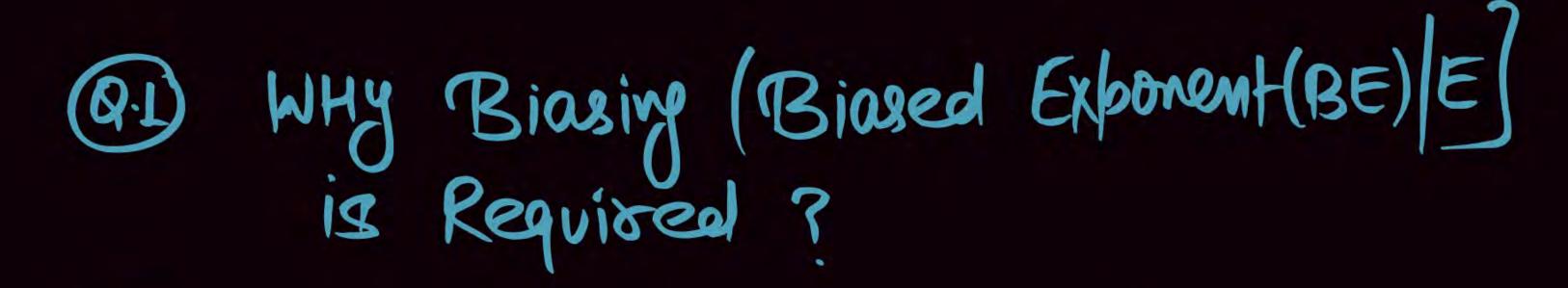




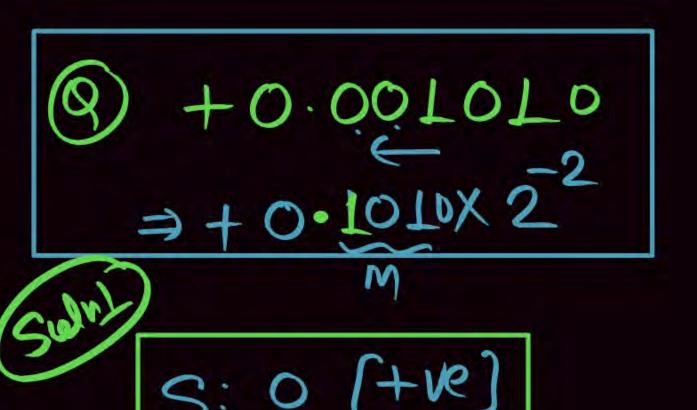
M: 1010

e=+14 => 1110

2=414

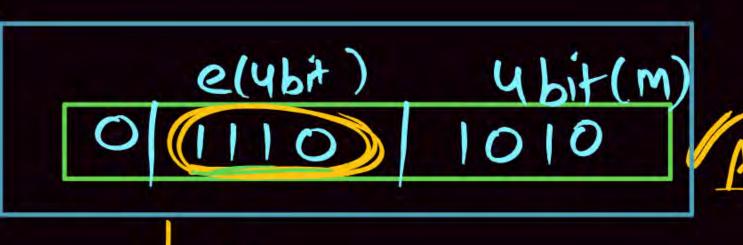


(0.2) How bias is decided ? value selected.



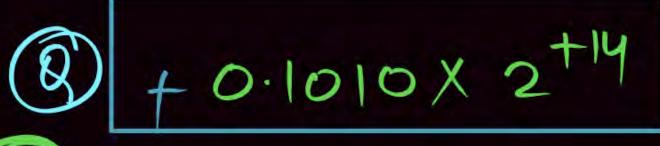
M: 1010

Sel M

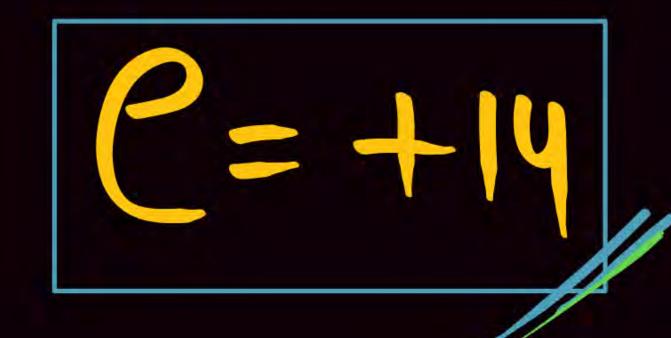


5 (8) But My Question is

1110









(Sel I) It we take Actual exponent [e] then there is No Provision to Represent the Negitive exponent. BC2 Sign bit is telling Number + ve(6) @ -ve(ib S=1].

if we take 2's complement of vepitive exponent then it Creates Ambiguity Like in Previous 2 example
-2: 1110 } create Ambiguity.
+14: 1110 } create Ambiguity.

So the Sulution is instead of writing in 2's complement

Used Bios exponent [E BE]



BE = AE + bing



(50 M2)

biasing: To Convert a Number into 'O' 60 + ve Number.

How bios value Select: ?

N bit 2's Complement Range = $-(2^{n-1})$ to $+(2^{n-1})$ 4bit 2's Complement = $-(2^{n-1})$ to $+(2^{n-1})$ = [-8 to +7] How bias is select?

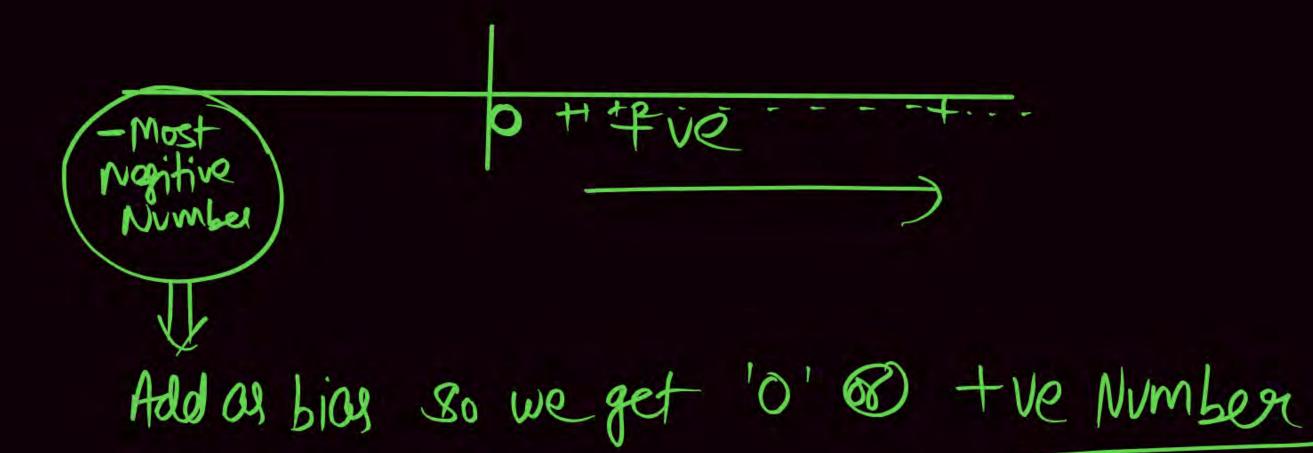
16 Exponent = K-1

bias = 2

its exponent is kbit them bios = 2^{k-1}

is exponent k bit then 2.8 compounent = (-2^{k-1}) to $(+2^{k-1})$ exponent 4 bit 2's complement Range = -2^{k-1} to $+2^{k-1}$ = (-8) to +7

(Note) In order to Convert All Number (Nepitive 4 Positive) Into Positive Number take Most Highest Nepitive Number 2 ADD 040 bing.

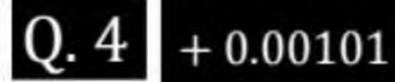


Exponent =
$$Kbit$$

Exponent = $4bit$

= -8 +0+7

AE(e)	E) BE (e+bios
<u>-8</u>	-8+8=67 $-7+8=1$
-6	=2
-S	= 3 = u
Ö	0+8 =8
)	1+8 =9
(+7)	7+8 +115)





$$0.101 \times 2^{-2}$$

$$M = 101$$

$$E = -2$$

$$S = 0$$

S	E(4bit)	M(5 bit)
0	1110	10100
	E	M

$$E = -2 = (1110)_2$$
 2's complement

Biasing: is method in which we convert the negative number into

the positive number

Bit	Bit	Bit
S	Е	М



S = Sign

E/BE = Exponent or

BE = bias exponent

M = Mantissa

$$E = e + bias$$

Bias = 2^{K-1}

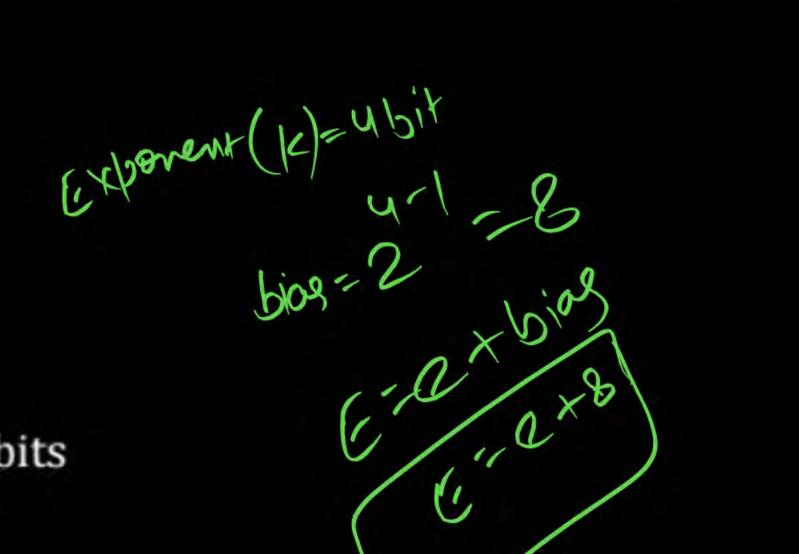
where K is exponent bits

Example

If K = 4 bits

Exponent = 4 bit then

bias =
$$2^{K-1} = 2^{4-1} = 8$$



Bias =
$$2^{K-1}$$
 = 2^{4-1}
bias = 8

$$E = e + bias$$

$$E = e + 8$$

$$E = 4 bit$$

or

Excess 8 code

$$2^{K-1} = 8$$

$$2^{K-1} = 23$$

$$K - 1 = 3$$

$$K = 4$$

$$E = 4 bit$$

			ŀ	İ	ŀ	T

Excess 8 code

bigg = 8

e [origin	al exponent]	Stored exponent [BE] E F=セナルルショーと
-8	-8+B	- 0
- 7	-7+8	1
- 6	-6+8	2
- 5	-5+6	3
-4	-418	4
- 3	-3+8	5
-2	-2+8	6
-1	-1+8	7
0	0+8	8
1	1+8	9
2	2+8	10
3	3+8	11
4	3+8	12
5	5+8	13
6	8+8	14
7	7+8	15



Exponent = Kbit

$$bios = 2$$

Excess-8: bios=8

Excess- 16 : bios = 16

Excess - 32 : bias = 32

Excass-64: bios=64.

Excess: 8

$$bios = 2$$

$$Excess: 16$$
 $2^{k-1} = 2^{4}$
 $2^{k-1} = 2^{4}$

$$K-1=4$$
 $K=5$ Exponent =4 bit

bias =
$$2$$

From previous question

0.00101

$$0.101 \times 2^{-2}$$

$$M = 101$$

Bias =
$$2^{5-1}$$

$$Bias = 16$$

$$e = -2$$

$$E = e + bias$$

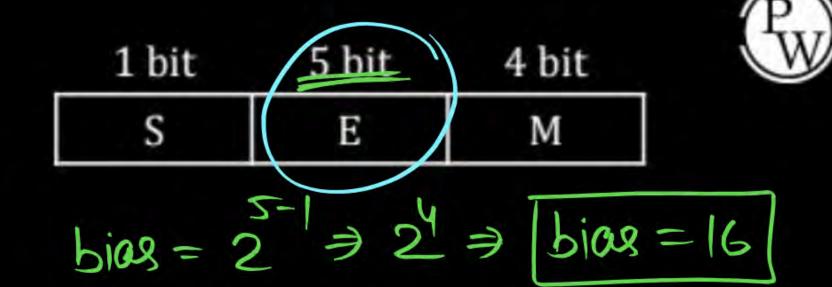
$$E = -2 + 16$$

$$E = 14$$

$$E = (01110)_2$$

Formula: $(-1)^S \times 0.M \times 2^e$

$$(-1)^0 \times 0.101 \times 2^{E-bias}$$



1 bit	5 bit	4 bit
0	01110	1010

Ans

$$0.101 \times 2^{14-16} = 0.101 \times 2^{-2}$$





Explicit Normalized

Syntax

Formula to get number [value formula] $(-1)^s \times 0.M \times 2^e$

$$(-1)^{s} \times 0.M \times 2^{E-bias}$$

Implicit Normalized

Syntax

Formula to get number [value formula]

$$(-1)^s \times 1.M \times 2^e$$

$$(-1)^S \times 1.M \times 2^{E-bias}$$

Explicit

Implicit



0.1 After the point,

Immediate first bit should be 1

Example

(101.11)

 0.10111×2^{3}

M = 10111,

e = 3

E = e + bias

Before the point 1 means 1.

Example

(101.11)

 1.0111×2^{2}

M = 0111,

e = 2

E = e + bias

+(4.875) format



Then do explicit and implicit normalization

Explicit

(+4.875)

100.111

 0.100111×2^{3}

M = 100111

e = 3, bias = 2^{4-1}

E = 3 + 8

E = 11

E = 1011

1 bit	4 bit	5 bit
0	1011	10011

Value Formula: $(-1)^S \times 0.M \times 2^e$

 $(-1)^0 \times 0.10011 \times 2^{11-8}$

 0.10011×2^{3}

100.11

4.75

(Not getting very accurate)

Implicit

$$(+4.875)$$

100.111

$$1.00111 \times 2^{2}$$

$$M = 00111$$

$$e = 2$$
, bias $= 2^{4-1}$

$$E = 2 + 8$$

$$E = 10$$

$$E = 1010$$

1 bit	4 bit	5 bit
0	1010	00111



Value Formula: $(-1)^S \times 1.M \times 2^e$

$$(-1)^0 \times 1.00111 \times 2^{10-8}$$

$$1.00111 \times 2^{2}$$

(Getting very accurate)



Consider a 16 bit register used to store floating point number. Mantissa is Explicit normalized signed fraction number. Exponent is in Excess-32 form then what is 16-bit for –(29.75)₁₀ in the register?



Solution

1 bit	6 bit	9 bit
S	E	М

-29.75

-11101.11

 0.11101111×2^{5}

M: 1110111

e = 5

bias = 2^{6-1}

bias = 32

 $E = 5 + 32 = 37 = (100101)_2$

S(1 bit) E(6 bit) 1 100101

111011100

M(9 bit)



Q. +21.75 1 bit 7 bit Implicit? S E

Pw

10101.11

$$1.0101111 \times 2^4$$

$$M = 010111$$

$$e = 4$$
, bias = 2^{7-1}

$$E = 4 + 64$$

$$E = 68 = (1000100)_2$$

Value Formula:

8 bit

M

$$(-1)^S \times 1.M \times 2^e$$

$$(-1)^0 \times 1.010111 \times 2^{68-64}$$

$$1.0101111 \times 2^{4}$$

$$10101.11 = (21.75)_{10}$$

Ans

S(1bit)	E(7bit)	M(8 bit)
0	1000100	01011100

Hexadecimal = $(445C)_{16}$

