COMPUTER SCIENCE

Database Management System

FD's & Normalization

Lecture_08

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01 Finding Super keys

02 Lossy and Lossless Join

1. RDBMS Concept
2. FD Concept & its type
3. Attribute closure
4. SUPER KEY
5. Candidate key
6. Finding Multiple C.K
7. Membership set
8. Equality b/w 2 FD set
9. Minimal Cover
10. Closure of FD Set

Note

Any Super Set of key (Candidate key) is also Super key.

Finding Number of super keys

Consider a Relational Schema R with 'n' Attribute ($A_1, A_2, A_3, A_4, A_5 \ldots\ldots A_n$)

How many super keys are possible ?

(i) With Only Candidate key A_1 ?

(ii) With Only Candidate key A_1, A_2 ?

(iii) With Only Candidate key $A_1 A_2, A_3 A_4$?

(iv) With Only Candidate key $A_1 A_2, A_2 A_3$?

(v) With Only Candidate key A_1, A_2, A_3 ?

Soln 1 With Only Candidate key A_1

A_1

$A_1\ A_2$

$A_1\ A_2\ A_3$

A_i $A_2\ A_3\ A_4\ A_5 \ldots\ldots A_n$ ($n-1$)

$$\#\text{Superkey} = 2^{n-1}$$ Ans

Ex R(ABCD) C.k : A

$$\#\text{Superkeys} = 2^{n-1}$$

$$\Rightarrow 2^{4-1} \Rightarrow 2^3$$

$= 8$ Superkeys

(Q) R(ABCD) with C.k : A then How many #Super keys ?

Soln R(ABCD)

A BCD

$2^3 = 8$ Super key.

A, AB, AC, AD,

ABC, ACD, ABD, ABCD.

n = 4 Attribute

R(ABCD) C.k : A

$\#\text{Super keys} = 2^{n-1}$

$= 2^{4-1} + 2^3$

$\Rightarrow$ 8 Super keys

R(ABCD) with Ck : A

A	B	C	D	
0	0	0	0	→ A
0	0	1		→ AD
0	1	0		→ AC
0	1	1		→ ACD
1	0	0		→ AB
1	0	1		→ ABD
1	1	0		→ ABC
1	1	1		→ ABCD

8 Super keys Ans

Soln 2 With Only Candidate keys A_1, A_2 ?

A_1

$A_1\ A_2$

$A_1\ A_2\ A_3$

$A_1\ A_2\ A_3\ A_4$

$A_1\ A_2\ A_3\ A_4\ A_5$

$A_1\ \underbrace{A_2 A_3 A_4 A_5 \ldots A_n}_{(n-1)}$

A_2

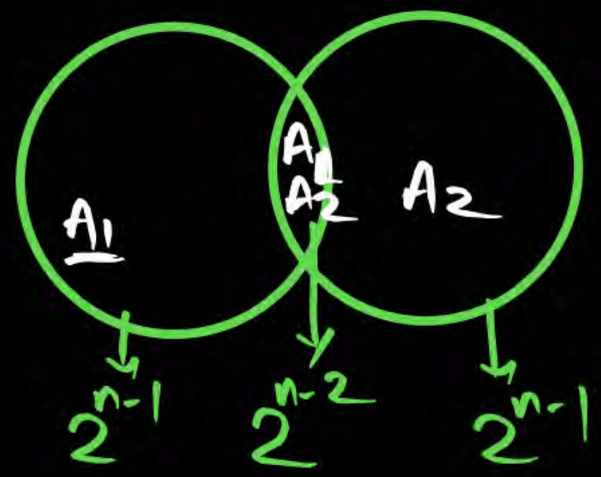
$A_2\ A_1$

$A_2\ A_1\ A_3$

$A_2\ A_1\ A_3\ A_4$

$A_2\ A_1\ A_3\ A_4\ A_5$

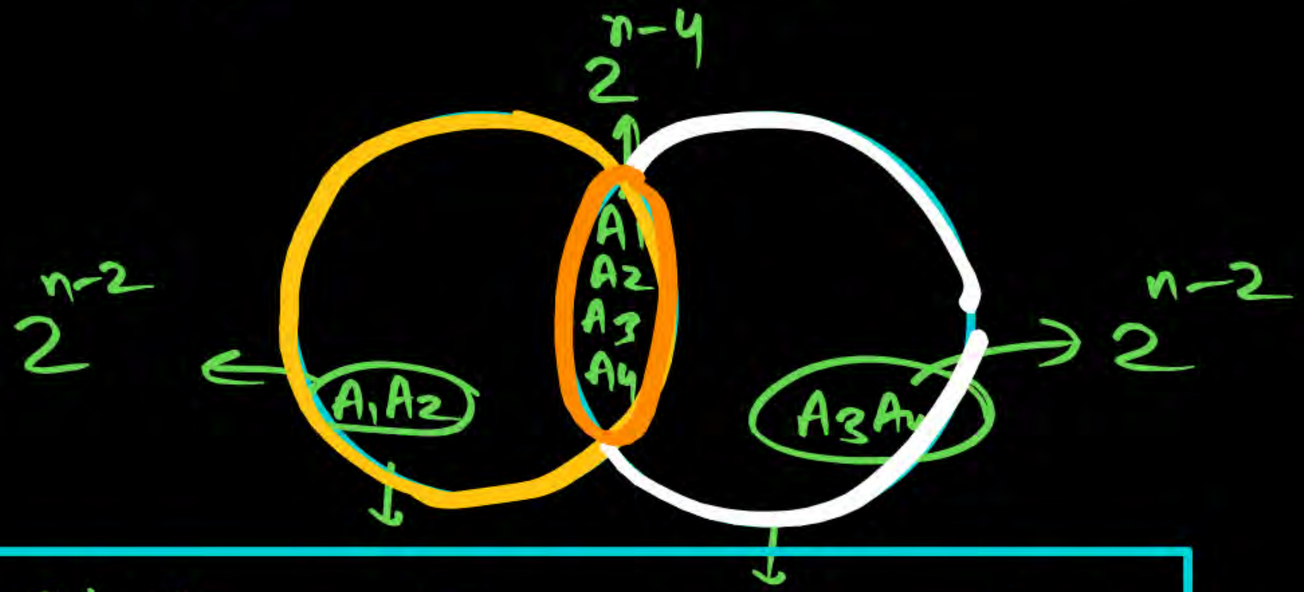
$A_2\ \underbrace{A_1 A_3 A_4 A_5 \ldots A_n}_{n-1}$



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

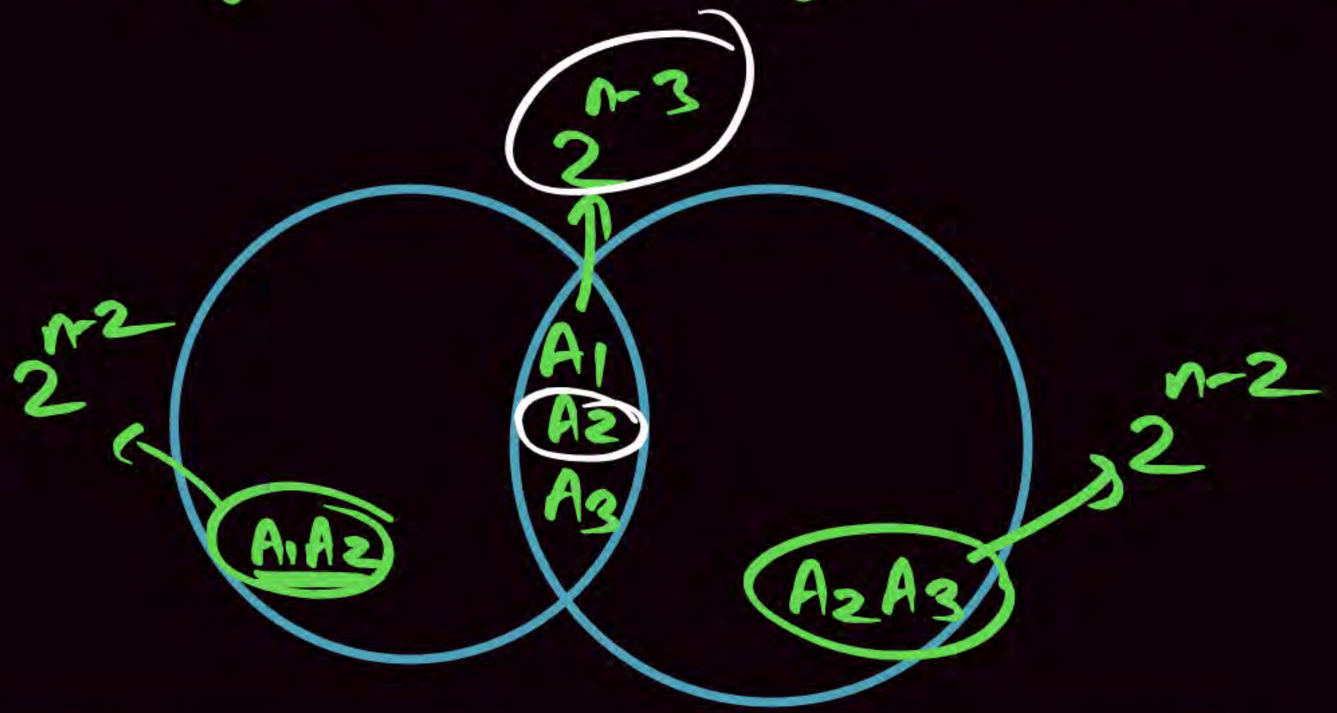
$$\#\text{Superkey} = 2^{n-1} + 2^{n-1} - 2^{n-2}$$ Ans

Soln 2 With only Candidate key A_1A_2, A_3A_4 ?



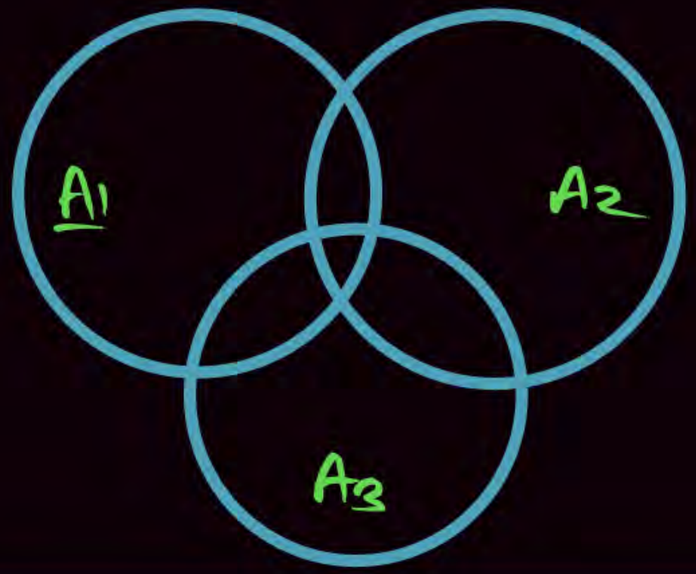
$$\#\text{Super keys} = 2^{n-2} + 2^{n-2} - 2^{n-4}$$

Sorry With Only Candidate key A_1A_2, A_2A_3 ?



$$\#\text{ super keys} = 2^{n-2} + 2^{n-2} - 2^{n-3}$$

Soln's With Only Candidate key A_1, A_2, A_3 ?



$$\#\text{Super keys} = 2^{n-1} + 2^{n-1} + 2^{n-1} - 2^{n-2} - 2^{n-2} - 2^{n-2} + 2^{n-3}$$

Finding Number of super keys

Q.1 R(ABCDE) With Ck : A ?

Q.2 R(ABCDE) with Ck A, B ?

Q.3 R(ABCDE) with Ck AB, CD ?

Q.4 R(ABCDE) with Ck AB, BC ?

Q.5 R(ABCDE) with Ck A, B, C ?

Q.6 R(ABCDE) with Ck : ABC, DE

Q.7 R(ABCDE) with Ck A, BCD ?

R(ABCDE)

Soln 1 R(ABCDE) C.k : A

subset keys $= 2^{5-1} = 2^4$

$= 16$ Ans

Soln 2 C.k A, B

$2^{5-1} + 2^{5-1} - 2^{5-2}$

$\Rightarrow 2^4 + 2^4 - 2^3$

$= 16 + 16 - 8$

$= 24$ Ans

Soln 3 C.k : AB, CD

$2^{5-2} + 2^{5-2} - 2^{5-4}$

$2^3 + 2^3 - 2^1$

$8 + 8 - 2$

$= 14$ Ans

Soln 4 AB, BC

$2^{5-2} + 2^{5-2} - 2^{5-3}$

$2^3 + 2^3 - 2^2$

$8 + 8 - 4$

$= 12$ Ans

Soln 5 Ck [A, B, C]

$2^{5-1} + 2^{5-1} + 2^{5-1} - 2^{5-2} - 2^{5-2} - 2^{5-2} + 2^{5-3}$

$\Rightarrow 2^4 + 2^4 + 2^4 - 2^3 - 2^3 - 2^3 + 2^2$

$\Rightarrow 16 + 16 + 16 - 8 - 8 - 8 + 4$

$= 32 - 4$

$= 28$ Ans

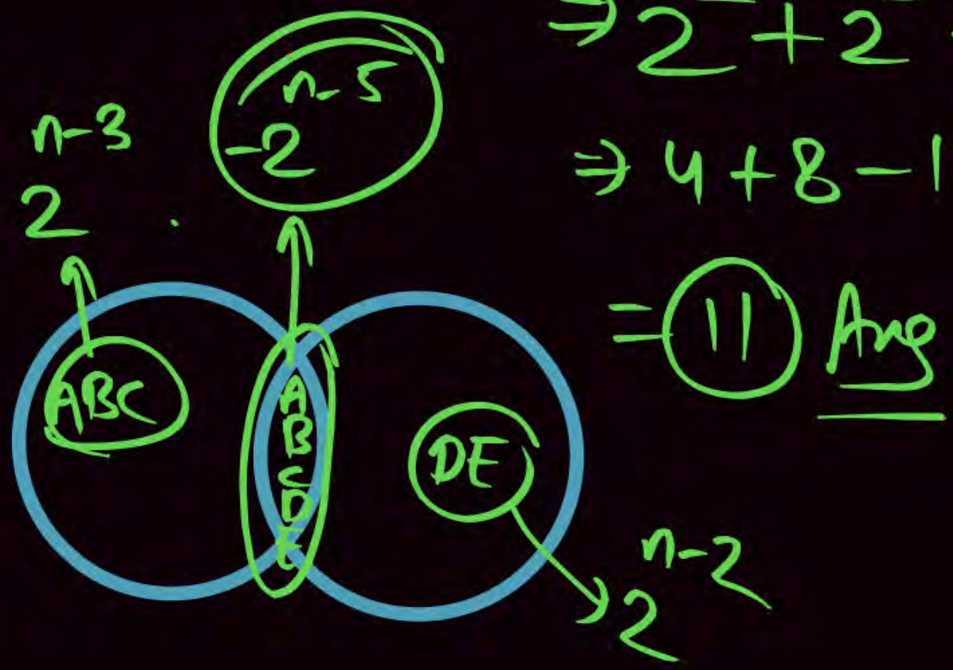
Soln6 CK : ABC . DE

$$\#\text{Subsequences} = 2^{5-3} + 2^{5-2} - 2^{5-5}$$

$$\Rightarrow 2^2 + 2^3 - 2^0$$

$$\Rightarrow 4 + 8 - 1$$

$$= 11 \text{ Ans}$$



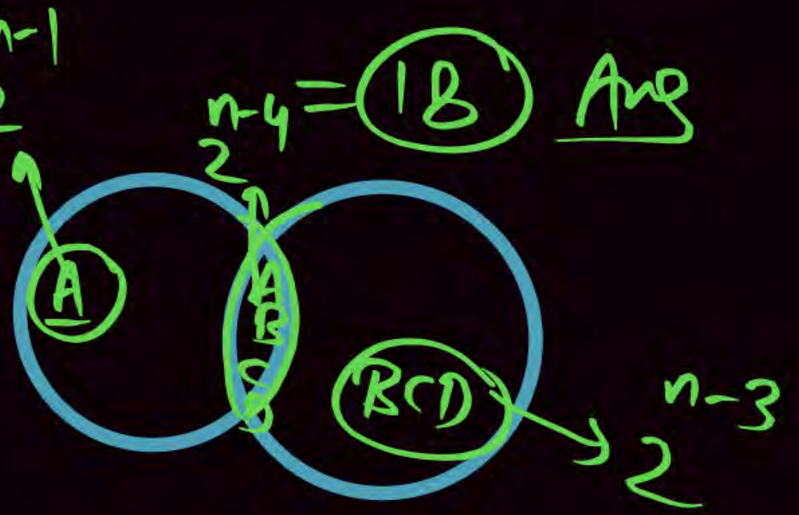
Soln7 A, BCD.

$$2^{5-1} + 2^{5-3} - 2^{5-4}$$

$$2^4 + 2^2 - 2^1$$

$$16 + 4 - 2$$

$$n-4 = 18 \text{ Ans}$$



GATE 2013 1 R(EFGH) with C.k E ?

PSUQ R(ABCDE) with C.k : A, CD ?

Soln

$$\#\text{Subset keys} = 2^{5-1} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^4 + 2^3 - 2^2$$

$$= 16 + 8 - 4$$

$$= 20$$ Ans

Soln R(EFGH) C.k : E

$$\#\text{Super keys} = 2^{4-1}$$

$$= 2^3$$

$$= 8 \text{ Super key}$$

R(EFGH)

E FGH

$\Downarrow$

2^3

8 Super keys.

E, EF, EG, EH

EFG, EGH, EFH, EFGH

(E) F G H

0 0 0 → E

0 0 1 → EH

0 1 0 → EG

0 1 1 → EGH

1 0 0 → EF

1 0 1 → EFH

1 1 0 → EFG

1 1 1 → EFGH.

8 Super key

GATE 2022

R(ABCDE) with Candidate key AB

Super keys

Soln

$$\#\text{Super keys} = 2^{5-2}$$

$$= 2^3$$

$$= 8 \text{ Super keys}$$

R(AB CDE)

$\#S.K = 2^3 = 8$ Super keys

AB, ABC, ABD, ABE,

ABCD, ABDE, ABCE, ABCDE.

Q.1 R(ABCDE) C.k : A, BC

$$\#\ \text{super keys} = 2^{5-1} + 2^{5-2} - 2^{5-3}$$

$$\Rightarrow 2^4 + 2^3 - 2^2$$

$$\Rightarrow 16 + 8 - 4$$

$$= \text{(20 super keys) Ans}$$

Q1 R(ABCDE) C.k : A, BC.

II^nd Method :

~~A~~BCDE = 2^4 = 16

BCDE = 2^2 = 4

BC, BCD, BCE, BCDE

20 subset keys.

(Q.2) R(ABCDE) Ck : [A, B, CD]

~~A~~ B C D E : $2^4 = 16$

~~B~~ C D E : $2^3 = 8$

CD E $2^1 = 2$

26 Super key

R(ABCD) C.k : [A, B, C, D]

Super keys ?

Total # S.k (Max.)

$2^n - 1$ (n:4)

$2^4 - 1$

(15) Ans

~~A~~ B C D $\Rightarrow 2^3 = 8$ [A, AB, AC, AD, ABC, ACD, ABD, ABCD]

~~B~~ C D $= 2^2 = 4$ [B, BC, BD, BCD]

~~C~~ D $= 2^1 = 2$ [C, CD]

D $= 2^0 = 1$ [D]

15 (15) Ans

(Q) R(ABCDE) C.k [A, B, C, D, E]

then # Subset keys ?

~~A~~ BCDE	$= 2^4$	16
~~B~~ CDE	$= 2^3$	8
~~C~~ DE	$= 2^2$	4
D E	2^1	2
E	2^0	1
		31

Here n : 5

Total max. # S.k

$= 2^n - 1$

$\Rightarrow 2^5 - 1$

$= 31$ Subset keys

Ans

Note

Total Maximum Number of Super keys $= 2^n - 1$

n: # of Attribute

(under the Assumption if every Single Attribute is Candidate key)

Note

Maximum Number of Candidate key $= {}^{n}C_{\lfloor \frac{n}{2} \rfloor}$

n: # of Attribute

Q) 7 Attribute then max # CK.

Soln $^{n}C_{\lfloor \frac{n}{2} \rfloor}$

$^{7}C_{\lfloor \frac{7}{2} \rfloor} \Rightarrow {}^{7}C_{3}$ $\frac{7 \times 6 \times 5}{3!}$

$= 35$ Ans

$\lfloor 2.1 \rfloor = 2$

floor

$\lfloor 2.9 \rfloor = 2$

$\lfloor 2.8 \rfloor = 2$

Ceiling

$\lceil 2.1 \rceil = 3$

$\lceil 2.2 \rceil = 3$

R(ABCDEF) then find Maximum # C.K ? (n: 6 Attribute)

(i) If every Single Attribute form a key : $6C_1$: 6

(ii) If every 2 Attribute form a key : $6C_2$: 15

(iii) 3 Attribute " " $6C_3$: 20

(iv) 4 Attribute " " $6C_4$: 15

(v) 5 Attribute " " $6C_5$: 6

(vi) 6 Attribute " " $6C_6$: 1

$nC_{\lfloor \frac{n}{2} \rfloor}$

$6C_{\lfloor \frac{6}{2} \rfloor}$

$\Rightarrow 6C_3$

$= 20$ Ans

R(ABCDEF)

(i) $^6C_1$: A, B, C, D, E, F.

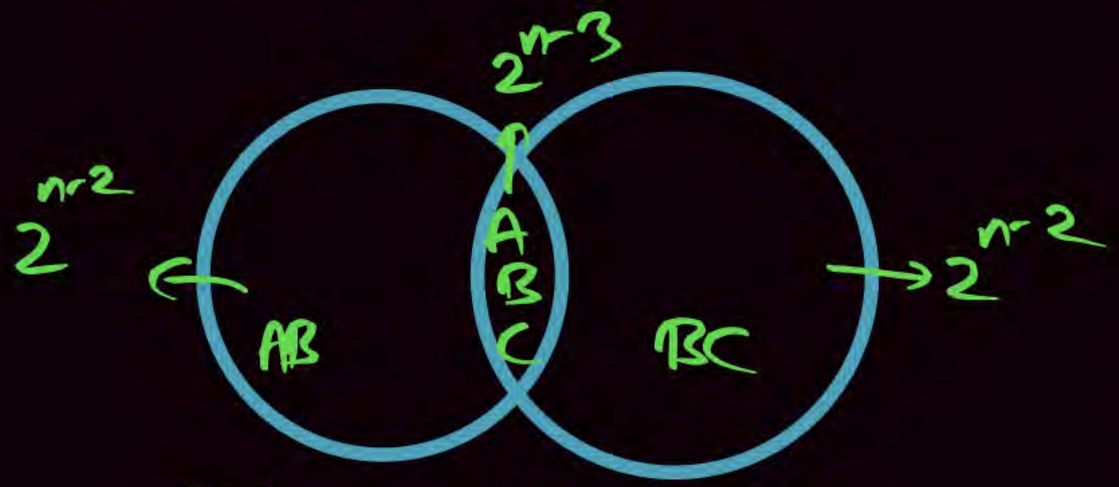
(ii) $^6C_2$: AB, AC, AD, AE, AF, BC, BD, BE, BF,
CD, CE, CF, DE, DF, EF = (15)

$^6C_3$: ABC, ACD, ADE, AEF.....

(iii) every 6 Attribute form a CK : ABCDEF = (1)

(Q.1) R(ABCDE) C.k[AB, BC]

Ck : [AB, BC)



$2^{5-2} + 2^{5-2} - 2^{5-3}$

$2^3 + 2^3 - 2^2$

$\Rightarrow 8+8-4$

$= 12$ Ans

II nd Method

R(ABCDF)

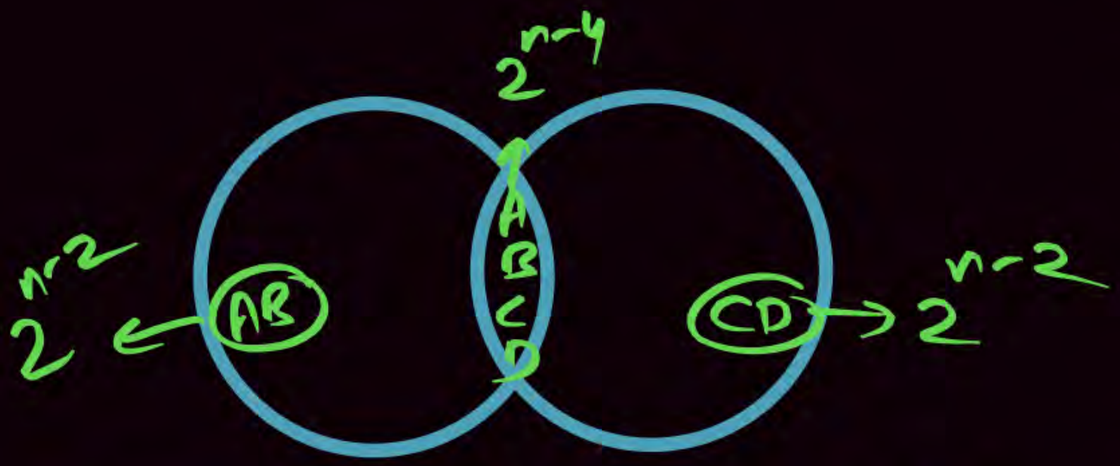
~~AB~~ CDE $= 2^3 = 8$

BC DE $= 2^2 = 4$

12 Super keys

Q. R(ABCDE) C.k : [AB, CD]

Venn Diagram [AB, CD]



$$\#\text{Subel keys} = 2^{5-2} + 2^{5-2} - 2^{5-4}$$

$$\Rightarrow 2^3 + 2^3 - 2^1$$

$$= 8 + 8 - 2$$

= 14 Subel keys.

R(ABCDE) Ck [AB, CD]

AB CDE = $2^3 = 8$

CD E = $2^1 = 2$

~~10 subel key~~

14 ✓

AB	CD
ABC	CDE
ABD	CDA
ABE	CDB
ABCD	CDAE
ABDE	CDBE
ABCE	
ABCDE	

= 14 Ans

Other Approach.

A Relation R with n Attributes, if every (n-1) attributes form a Candidate key. then Total Number of Super key.

Soln

(n-1) Attribute ⇒ Super key = $^nC_{n-1}$ + Any Super set of $^nC_{n-1}$

$^nC_{n-1}$ ^nC_n

Super key = $^nC_{n-1} + {}^nC_n$

= n+1 Ans

Eg R(ABC)

3C.K

AB
BC
AC

n: 3 Attribute

Super keys
AB
BC
AC
ABC

(4) Ans

Eg R(ABCD)

n = 4

every (n-1) ⇒ every 3 Attribut

(4-1) = 3

C.k.

$^nC_{n-1}$ ^nC_n

S.k = $^4C_3$ + $^4C_4$

$^4C_3$ ⇒ 4 C.k

4C.k

ABC
ACD
ABD
BCD

5S.k

ABC
ACD
ABD
BCD
ABCD

n+1

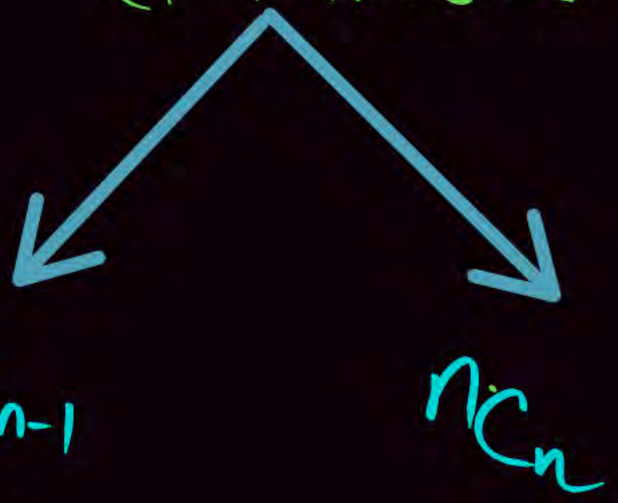
4+1 ⇒ 5 S.k Ans

CASE 1 Other Approach.

(1) A Relation R with n Attributes, if every (n-1) attributes form a Candidate key. then Total Number of Super key.

Soln

(n-1) Attribute ⇒ Super key = $^{n}C_{n-1}$ + Any Super set of $^{n}C_{n-1}$



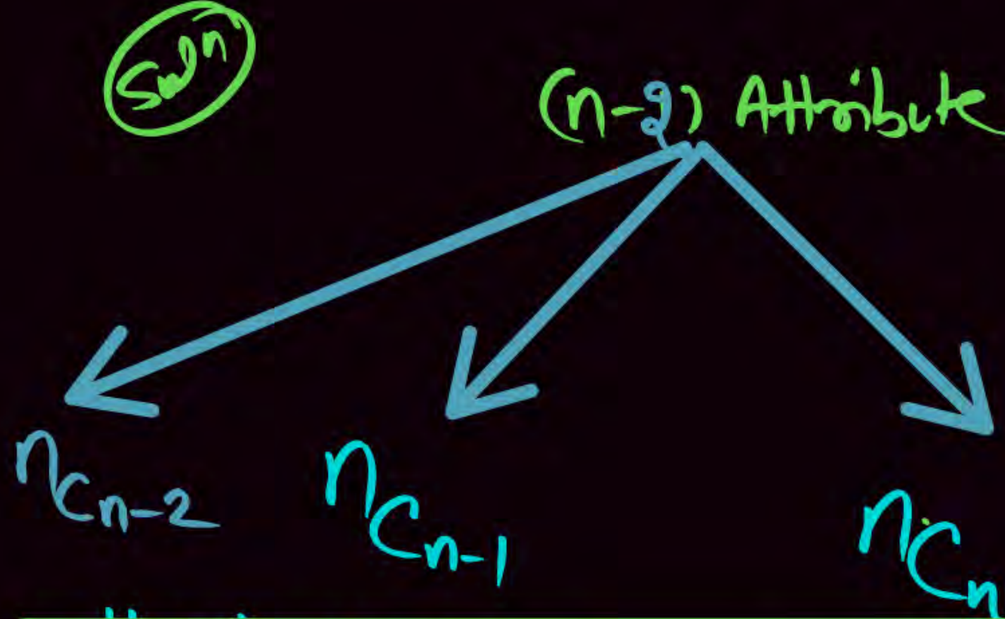
$^{n}C_{n-1}$ $^{n}C_{n}$

Super key

CASE II Other Approach.

(2) A Relation R with n Attributes, if every (n-2) attributes form a Candidate key. then Total Number of Super key.

Soln



$$\#\text{Super key} = {}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n$$

Ans

$n=5$ $(n-2)$

(eg) R(ABCDE) If every 3 Attribute form a C.k then # super keys?

Soln

$$\#\text{super keys} = {}^5C_3 + {}^5C_4 + {}^5C_5$$

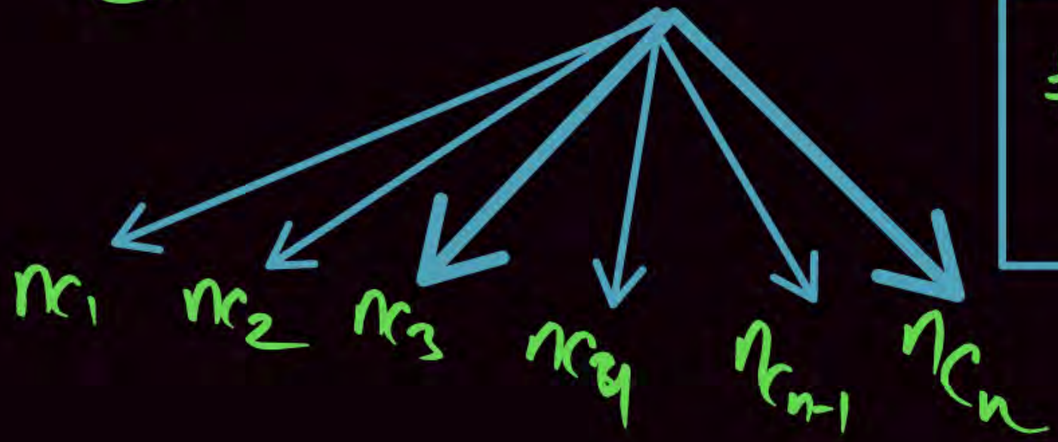
$$= 10 + 5 + 1$$

$$= 16$$

CASE III Other Approach.

(3) A Relation R with n Attributes, if every Single attributes form a Candidate key. then Total Number of Super key.

Soln



$$\#\text{Super keys} = {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots {}^nC_n$$

(Eg) R(ABCD) C.k [A, B, C, D].

$${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$$

$$\Rightarrow 4 + 6 + 4 + 1$$

$$= (15) \text{ Ans}$$

$$2^4 - 1 = (15) \text{ Ans}$$

(Q) R(PQRST) C.k : [P, T, QR, RS]

Q
R
S
QS

Total Maximum # Super key = $2^n - 1 \Rightarrow 2^5 - 1$

= 31 Super keys

Not a key	Q
(Never, key)	R
	S
	QS

31 − 4 = 27 Ans

Candidate key: Minimal

QR, RS CK.

Q
R
S
QS

Never Candidate key

THANK YOU!