

CS & IT ENGINEERING



Data structure &
Programming

Arrays

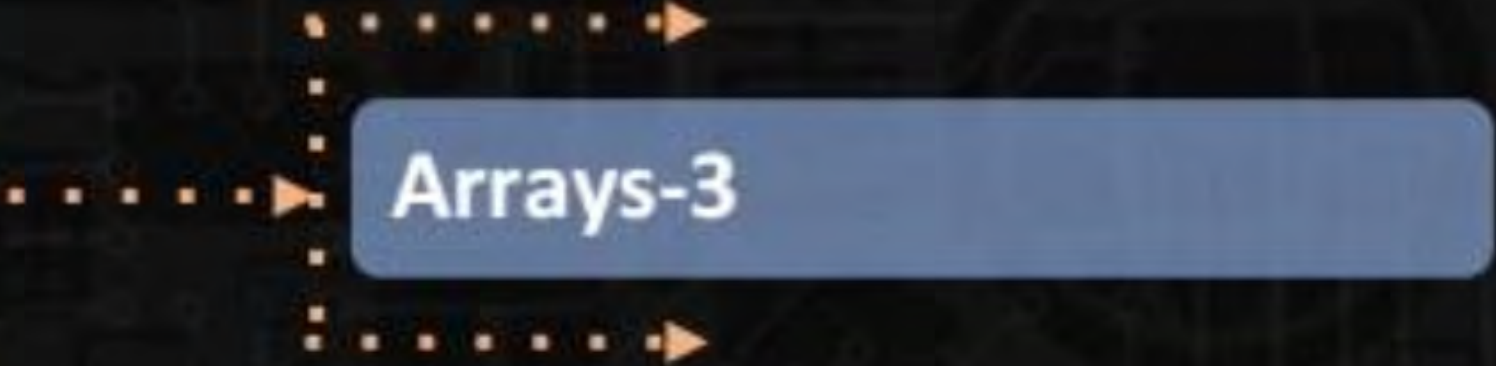
Lec- 03



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TOPICS TO BE
COVERED



Arrays-3

CMD
LTM

$\omega = 1 \text{ byte}$
 $BA = 1000$

7×7
 $A[-3..3][-5..1]$
 $\text{add}(A[3][-1])$

already filled

as per
formula

$$3 - (-1) \\ = 4$$

$i - j$

$$4 - 2$$

$= 2 \text{ elements}$

$$\text{Total} = 22 + 2 \\ = 24 \text{ elements}$$

col.
already
filled

$$= -5 \text{ to } -2 \\ = -2 - (-5) + 1 \\ = -2 + 6 \\ = 4$$

$$7 + 6 + 5 + 4 \\ \Rightarrow 22 \text{ element}$$

$$-3 - (-5) = 2$$

	-5	-4	-3	-2	-1	0	1
-3	<u>X</u>	0	0	0	0	0	0
-2	X	X	0	0	0	0	0
-1	X	X	X	0	0	0	0
0	X	X	X	X	0	0	0
1	X	X	X	X	<u>X</u>	0	0
2	X	X	X	X	<u>X</u>	X	0
3	X	X	X	X	X	X	X

Memory = 24 byte

$$\text{add}(q_3, -1) = 1000 + 24 \\ = 1024$$

Upper triangular Matrix

Square matrix

$$a_{21} = 0$$

$$a_{31} = 0$$

$$a_{32} = 0$$

$$a_{41} = 0$$

$$a_{42} = 0$$

$$a_{43} = 0$$

$$a_{ij} = 0 \quad i > j$$

$$\begin{bmatrix} & 1 & 2 & 3 & 4 \\ 1 & a_{11} & a_{12} & a_{13} & a_{14} \\ 2 & 0 & a_{22} & a_{23} & a_{24} \\ 3 & 0 & 0 & a_{33} & a_{34} \\ 4 & 0 & 0 & 0 & a_{44} \end{bmatrix}$$

all entries are zero

RMO

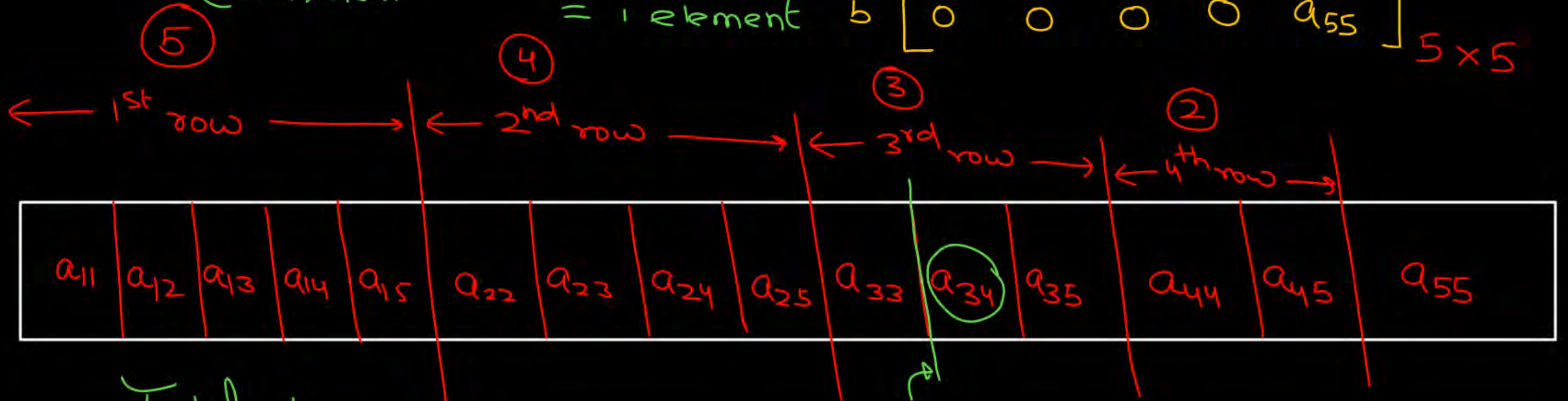
add (a_{34})

Rows already filled
= 1 to 2
= 2 rows
(5+4) elem

within row whose index is 3, elem already filled before a_{34}
= 4-3
= 1 element

	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	0	a_{22}	a_{23}	a_{24}	a_{25}
3	0	0	a_{33}	a_{34}	a_{35}
4	0	0	0	a_{44}	a_{45}
5	0	0	0	0	a_{55}

5x5



Total ele = 10 elem.

UTM in RMO

add(a_{ij})

Rows already filled

= index 1, 2, 3, ..., (i-1)

1st $\Rightarrow N$
 2nd $\Rightarrow N-1$
 3rd $\Rightarrow N-2$
 ...
 i-1 $\Rightarrow N-(i-2)$

$$N + (N-1) + \dots + (N-i+2)$$

$$\Rightarrow \frac{(i-1)}{2} [N + (N-i+2)]$$

Rows already filled

	1	2	3	...	j-1	j	...	N
1	a_{11}	a_{12}	a_{13}	...				a_{1N}
2	0	a_{22}	a_{23}	...				a_{2N}
3	0	0	a_{33}	...				a_{3N}
...								
i-1								
i						a_{ij}		
...								
N	0	0	0	...				a_{NN}

UTM in RMO

add(a_{ij})

Rows already filled

= index 1, 2, 3, ..., (i-1)

1st $\Rightarrow N$
 2nd $\Rightarrow N-1$
 3rd $\Rightarrow N-2$
 ...
 i-1 $\Rightarrow N-(i-2)$

$N + (N-1) + \dots + (N-i+2)$

$$\Rightarrow \frac{(i-1)}{2} [N + (N-i+2)]$$

Rows already filled

within i-th row
 ele. already filled before $\Rightarrow a_{ij}$
 $= (j-1)$

	1	2	3	...	j-1	j	...	N
1	a_{11}	a_{12}	a_{13}	...				a_{1N}
2	0	a_{22}	a_{23}	...				a_{2N}
3	0	0	a_{33}	...				a_{3N}
...								
i-1								
i						a_{ij}		
...								
N	0	0	0	...				a_{NN}

$$\text{Total element already filled} = \frac{i-1}{2} [2N - (i-2)] + (j-i)$$

$$= (i-1) \cdot N - \frac{(i-1)(i-2)}{2} + (j-i)$$

$$\text{memory already filled} = \left[(i-1)N - \frac{(i-1)(i-2)}{2} + (j-i) \right] * w$$

No need

$$\text{add}(a_{ij}) = BA + \left[(i-1)N - \frac{(i-1)(i-2)}{2} + (j-i) \right] * w$$

formula ✓

$A[-12..12][-12..12]$

$$12 - (-12) + 1 = 25$$

$w = 2$ byte

BA = 1000

add($A_{0,3}$)

$$\text{add}(A_{0,3}) = 1000 + 474 = 1474$$

UTM
RMO

rows filled

$$= -12 \text{ to } -1$$

$$= -1 - (-12) + 1$$

$$= 12 \text{ rows}$$

1st row → 25

2nd row → 24

3rd row → 23

...

12th row → 14

$$\Rightarrow \frac{12}{2} [25 + 14]$$

$$= 6 \times 39$$

$$= 234$$

elem filled within row index 0
before $A_{0,3}$

$$= 3 - 0 = 3$$

$$\text{Total ele} = 234 + 3 = 237$$

$$\text{Memory filled} = 237 \times 2 = 474 \text{ bytes}$$



$a[100][100]$

block
transfer

$\Rightarrow 1\text{KB}$

4 byte

$$\Rightarrow 10000 \times 4 = 40000$$

Row-
wise

```
for(i=0; i<99; i++)  
{
```

```
  for(j=0; j<99; j++)  
  {
```

```
    pf("%d", a[i][j],  
3
```

```
}
```

$a_{00} a_{01} a_{02} \dots$

$a[100][100]$

4 byte

$$\Rightarrow 10000 \times 4 = 40000$$

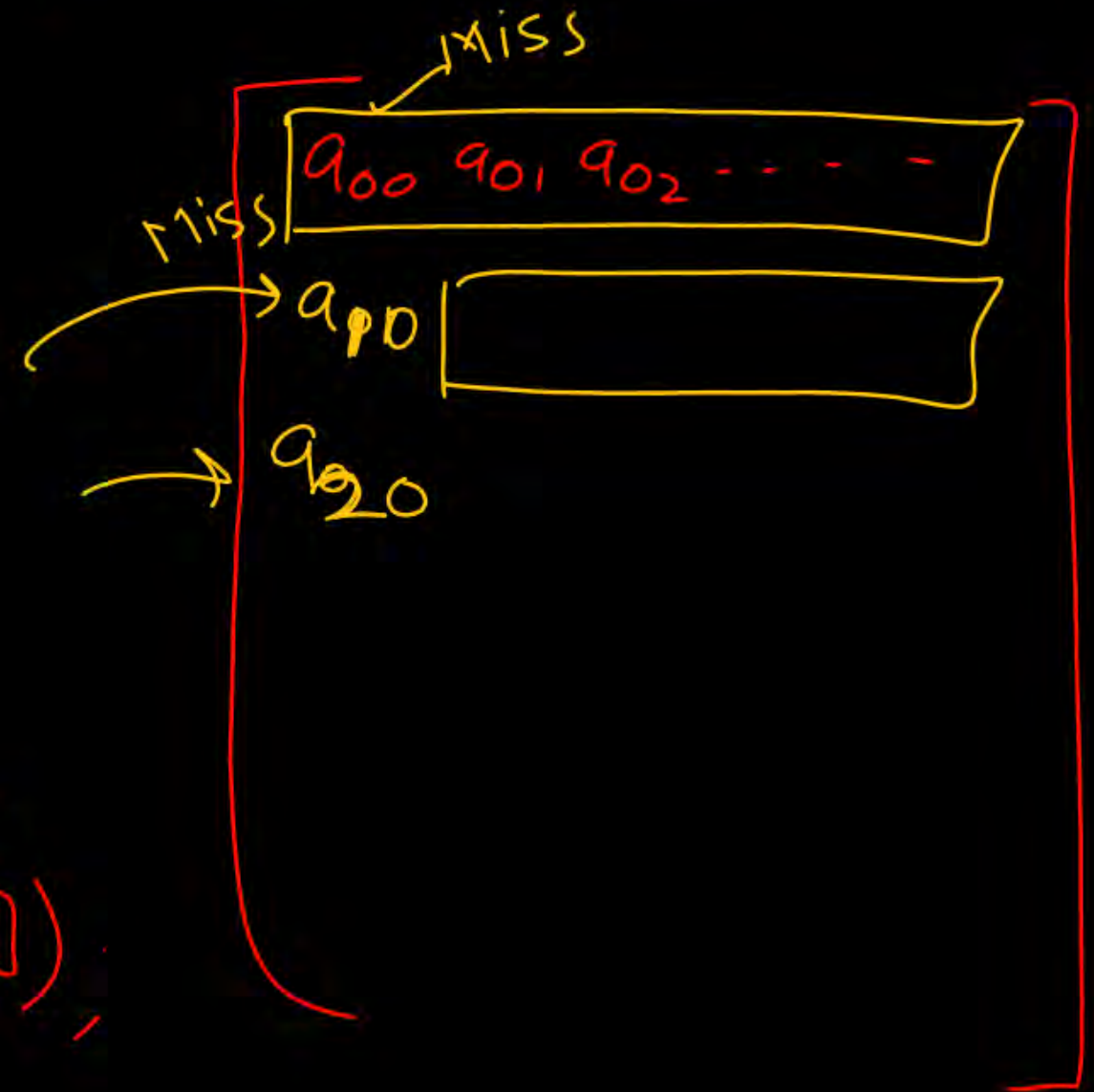
Row-wise

```
for(i=0; i<99; i++)  
{
```

```
  for(j=0; j<99; j++)  
  {
```

```
    pf("%d", a[i][j],  
    )
```

```
  }
```



UTM in CMO

$\text{add}(a_{45})$
 within col 5
 ele. already
 filled before
 a_{45}
 = row index
 1 to 3
 = $3-1+1$
 = 3 elem
 col. already
 filled
 index
 = 1, 2, 3, 4
 = 4
 1 + 2 + 3 + 4
 = 10 elem.

	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	0	a_{22}	a_{23}	a_{24}	a_{25}
3	0	0	a_{33}	a_{34}	a_{35}
4	0	0	0	a_{44}	a_{45}
5	0	0	0	0	a_{55}

a_{11}	a_{12}	a_{22}	a_{13}	a_{23}	a_{33}	a_{14}	a_{24}	a_{34}	a_{44}	a_{15}	a_{25}	a_{35}	a_{45}	a_{55}
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Tri-diagonal Matrix

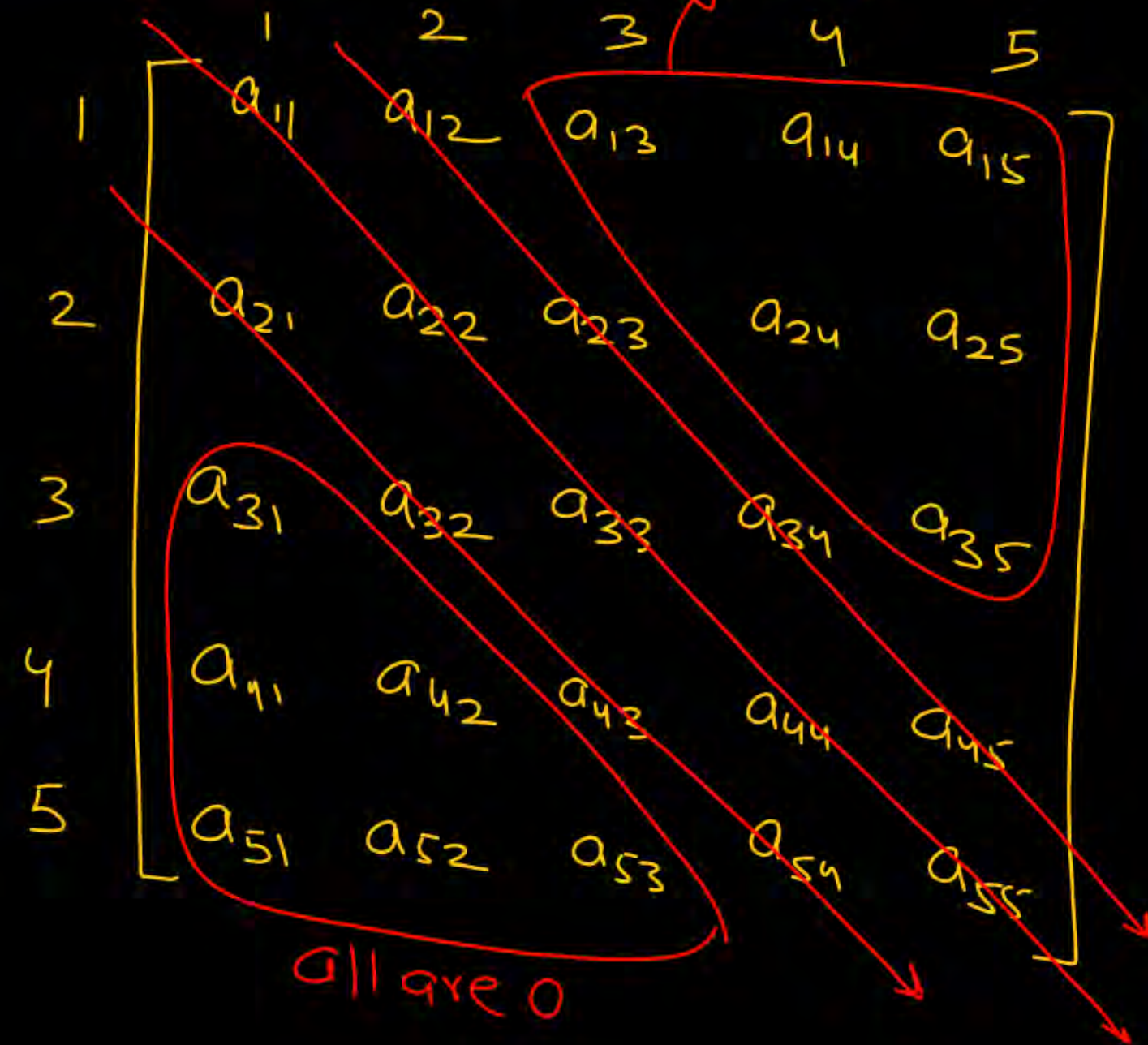
It is a square matrix

⇒ Main diagonal

⇒ diagonal just above main diagonal

⇒ diagonal just below main diagonal

except them
all entries are 0



of
element
first Row $\Rightarrow 2$

last Row $\Rightarrow 2$

rem. rows $\Rightarrow 3$

first row \Rightarrow

last
row \Rightarrow

$$\begin{array}{c} 1 \quad 2 \quad 3 \quad 4 \quad 5 \\ 1 \left[\begin{array}{ccccc} a_{11} & a_{12} & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & a_{23} & 0 & 0 \\ 3 & 0 & a_{32} & a_{33} & a_{34} & 0 \\ 4 & 0 & 0 & a_{43} & a_{44} & a_{45} \\ 5 & 0 & 0 & 0 & a_{54} & a_{55} \end{array} \right] \end{array}$$

$n \times n$ tridiagonal matrix

Total rows = n

1st $\Rightarrow 2$

last row $\Rightarrow 2$

How many

remaining rows

$$= (n-2)$$

Element in row except 1st, last = 3

Total elem

$$= 2 + 2 + (n-2) \cdot 3$$

1st row last row

$$= 2 + 2 + 3n - 6$$

$$= \boxed{3n-2}$$

RMO

add(a_{45})

rows already
filled
= 3

1st $\Rightarrow 2$

2nd $\Rightarrow 3$

3rd $\Rightarrow 3$

8 elements

within 4th row
ele before

a_{45}

$$= (5 - 4) + 1$$

= 2 elements

3 rows

	1	2	3	4	5
1	a_{11}	a_{12}	0	0	0
2	a_{21}	a_{22}	a_{23}	0	0
3	0	a_{32}	a_{33}	a_{34}	0
4	0	0	a_{43}	a_{44}	a_{45}
5	0	0	0	a_{54}	a_{55}

a_{11}	a_{12}	a_{21}	a_{22}	a_{23}	a_{32}	a_{33}	a_{34}	a_{43}	a_{44}	a_{45}	a_{54}	a_{55}
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Generalize

$N \times N$

— tridiagonal

RMO

add(a_{ij})



within i th
row, elements
already filled

before $a_{ij} = (j - i + 1)$

Total ele before a_{ij}

$$= 3i - 4 + j - i + 1$$

$$= 2i + j - 3$$

२२-११

Rows
already filled

$$= 1 \text{ to } i-1$$

$$= (i-1) \text{ rows}$$

out of these $(i-1)$ rows

1st row $\Rightarrow 2$

$$\Rightarrow 2 + 3(i-2)$$

rem. $(i-2)$ rows में $\Rightarrow 3$ elem.

$$= 2 + 3i - 6$$

$$= \boxed{3i - 4}$$

new

Z matrix

- ① Sparseband
- ② Toeplitz



	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	0	0	0	a_{24}	0
3	0	0	a_{33}	0	0
4	0	a_{41}	0	0	0
5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

Grate

→ Add calculation

→ indirect way

Total ele $\Rightarrow 11$

↓
index $+2\pi$ & π



A

feedback &
review

$A_{ii} \rightarrow 0$

$A_{i+1,i}$

$A_{i+2,i}$

$A_{i+3,i}$

