CS & IT ENGINEERING

Data Structure

Trees-1

DPP 01 Discussion Notes

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01 Question

02 Discussion

Q.1 A binary tree has 1024 leaves. The number of nodes in the tree having two children is ___1023___.

[NAT]

Ex1

leaf nodes = 1

no. of nodes with 2 child = 0

Ex2.

leaf. node = 1

nodes with 2 - child = 0

Ex2.

leaf node = 2

nodes with 2-childs = 1

$\boxed{\#\text{ nodes with 2-childs} = \#\text{ leaf nodes} - 1}$

$1024 - 1$

$= 1023$

Q.2 The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 9 are-

[MCQ]

A. 1024, 9

B. 1023, 10

C. 511, 9

D. 512, 10

$h = 9$

$n_{min} = h + 1$

$n_{max} = 2^{h+1} - 1$

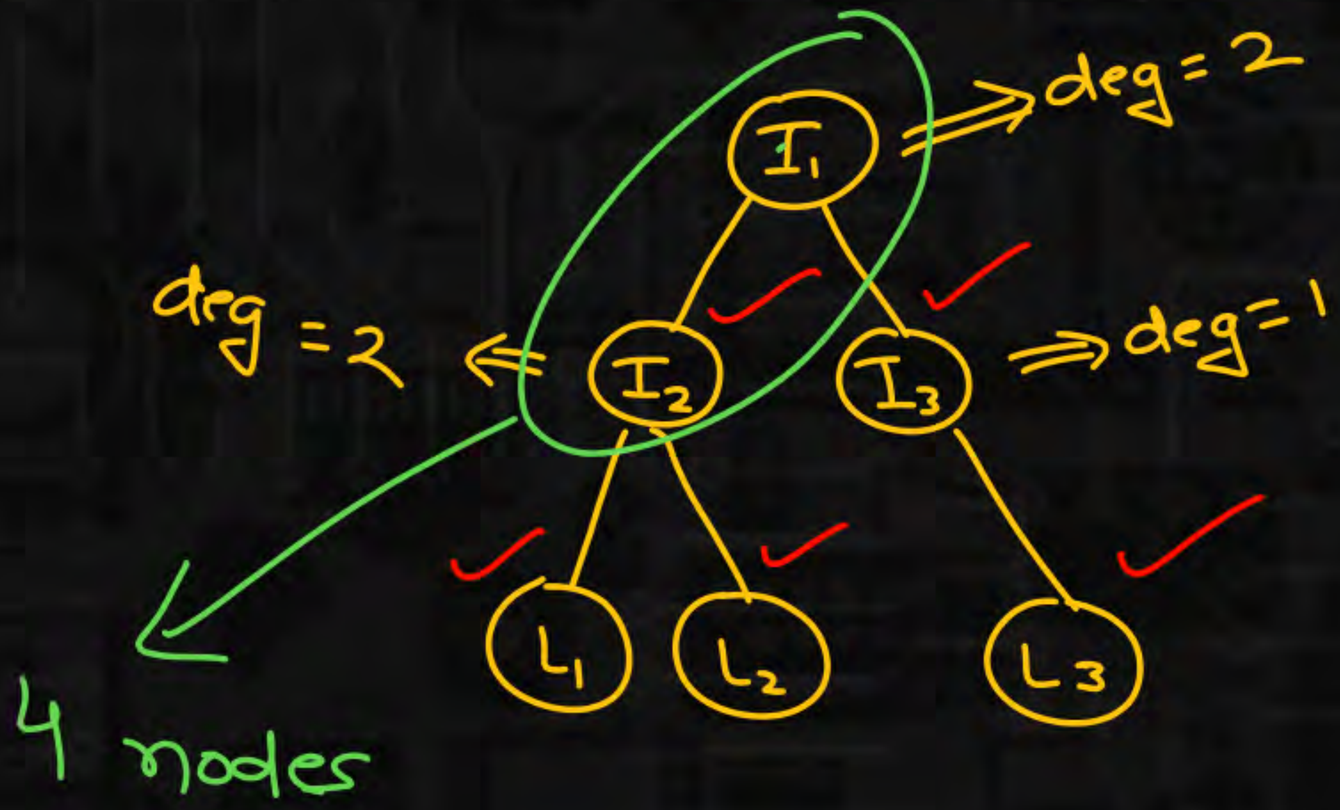
$n_{min} = 9 + 1 = 10$

$n_{max} = 2^{9+1} - 1 = 2^{10} - 1 = 1024 - 1 = 1023$

$\boxed{1023, 10}$

Q.3 In a binary tree, the number of internal nodes of degree 1 is 6, and the number of internal nodes of degree 2 is 12. The number of leaf nodes in the binary tree is ______.

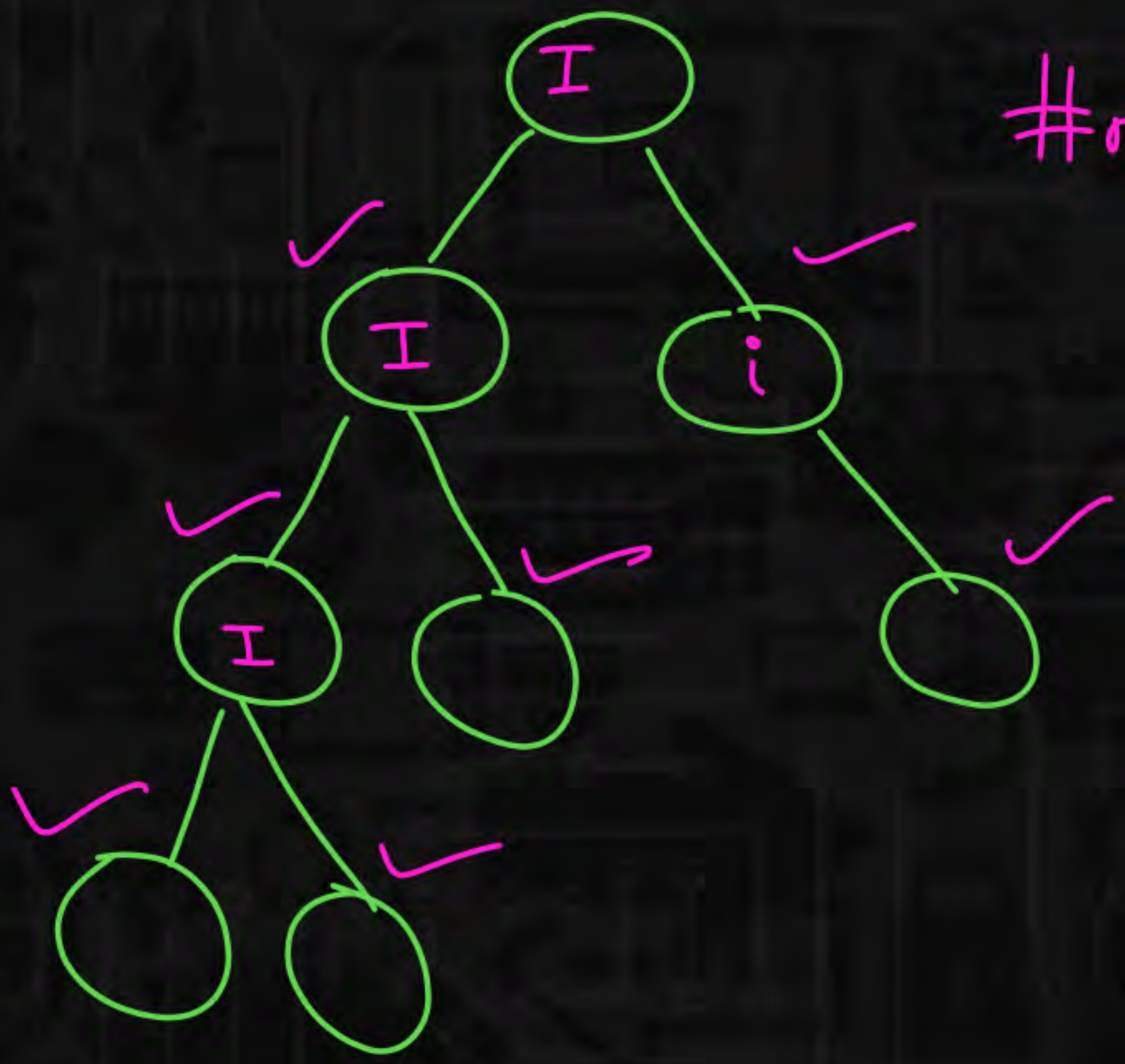
[NAT]



$2 + 2 + 1$

$$\# \text{ nodes} = 2 + 2 + 1 + \underset{\text{Root}}{1}$$

Q.3 In a binary tree, the number of internal nodes of degree 1 is 6, and the number of internal nodes of degree 2 is 12. The number of leaf nodes in the binary tree is ______. **[NAT]**



no. of nodes of deg 2

$\# \text{nodes} = 3 \times 2 + 1 \times 1 + 1$

no. of nodes of deg 1

$3 + 1 + 4$

Q.3 In a binary tree, the number of internal nodes of degree 1 is 6, and the number of internal nodes of degree 2 is 12. The number of leaf nodes in the binary tree is 13.

[NAT]

Total nodes $= 12 \times 2 + 6 \times 1 + 1$

$= 24 + 6 + 1$

Total nodes $= 31$

$12 + 6 +$ Leaf nodes $= 31$

leaf nodes $= 31 - 18$

$= 13$

Q.4

A strict k-ary tree T is a tree that contains exactly 0 or k children. The number of leaf nodes in tree T if there are exactly 'p' internal nodes is- [MCQ]

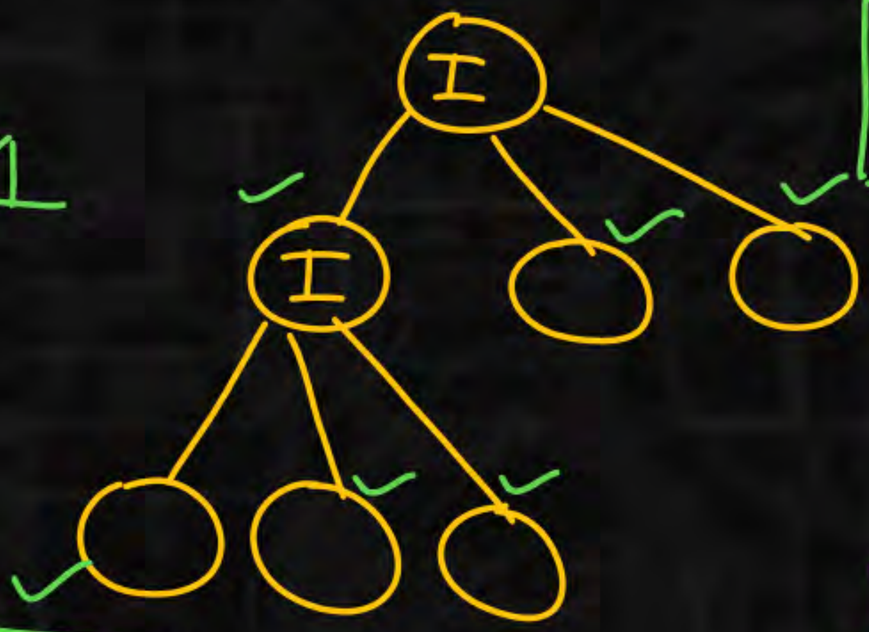
Strict 3-ary tree : → 0 child (leaf) → 3 child (Internal)

A. $(k-1)p + 1$

B. $pk + 1$ # nodes = $2 \times 3 + 1$

C. $pk + 1 + p$

D. None



$\boxed{\#\text{nodes} = p \times k + 1}$

$\boxed{\#\text{nodes} = p \times 3 + 1}$

$L + P = PK + 1$

$L = PK + 1 - P$

$\boxed{L = P(K-1) + 1}$

Q.5 A linked list is used to store a binary tree with $\boxed{\text{1024 nodes}}$. The number of null pointers present is <u>1025</u>.

[NAT]

ROOT

Left		Right
X	10	X

nodes = 1

Null pointer = 2

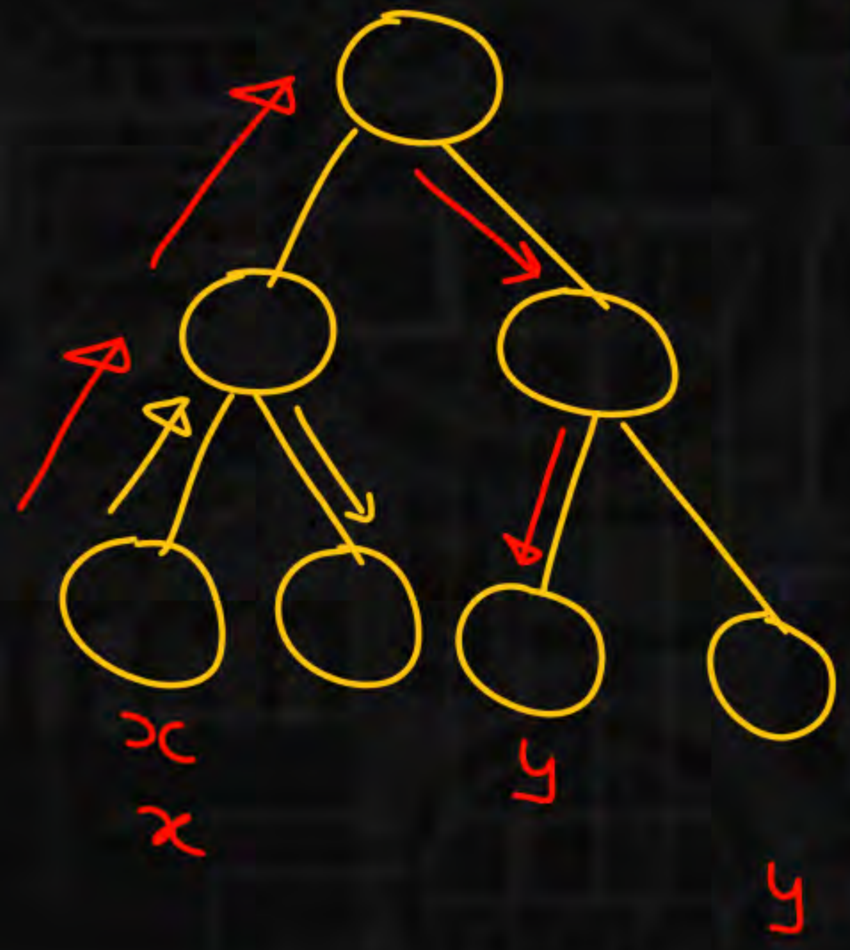
$$\text{Null Pointers} = n+1$$
$$= 1024+1$$
$$= 1025$$

Q.6 Let T be a full binary tree with 4 leaves. (A full binary tree has every level full). Suppose two leaves x and y of T are chosen uniformly and independently at random. The expected value of the distance between x and y in T (i.e., the number of edges in the unique path between x and y) is (rounded off to 2 decimal places) ____.

[NAT]

$4 \times 4 = 16$

Path length	ways	P(i)
0	4	4/16
2	4	4/16
4	8	8/16



Q.6

Let T be a full binary tree with 4 leaves. (A full binary tree has every level full). Suppose two leaves x and y of T are chosen uniformly and independently at random. The $\boxed{\text{expected value}}$ of the distance between x and y in T (i.e., the number of edges in the unique path between x and y) is (rounded off to 2 decimal places)

2.50

[NAT]

$4 \times 4 = 16$

Path length	ways	P(i)
0	4	4/16
2	4	4/16
4	8	8/16

$$E(i) = \sum i \times P(i)$$

$$= 0 \times \frac{4}{16} + 2 \times \frac{4}{16} + 4 \times \frac{8}{16}$$

$$= \frac{8}{16} + \frac{32}{16} = 2.5$$

Q.7 The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 2 children is-

Total

[MCQ]

A. $\frac{n+1}{2}$

B. $\frac{n-1}{2}$

C. $\frac{n}{2}$

D. $n-1$

Every internal node = 2 childs

I : # of internal nodes
L : # of leaf nodes

Total no. of nodes = $I \times 2 + 1$

$$\boxed{L + I = 2I + 1}$$

$$\boxed{n = L + I}$$
$$\Downarrow$$
$$\boxed{I = n - L}$$

$$L = I + 1$$
$$= n - L + 1$$
$$L = n - L + 1$$
$$2L = n + 1 \Rightarrow L = \frac{(n+1)}{2}$$


THANK YOU GW
SOLDIERS !