Q.1

Consider a hash table H with 512 slots. If 128 keys are to be stored in H, the load factor of H is.

[NAT]

$$\lambda = \frac{n}{m}$$

$$\lambda = \frac{128}{512} = \left(\frac{1}{4}\right)$$

Q.2 Consider a hash function that distributes keys (uniformly.) The hash table size is 2024. After hashing of how many keys will the probability that any new key hashed collides with an existing one exceed 0.75? _____

$$x$$

$$\frac{1}{2024} \times x = \frac{3}{4}$$

$$x = \frac{3 \times \overset{506}{\cancel{2024}}}{\cancel{4}} = 1518$$

Q.3 Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h1 and h2. Further suppose our hashing schemes uses h1 for the even keys and h2 for the odd keys. What is the expected number of keys in a slot?

[MCQ]

A. $\dfrac{n}{m}$

B. $\dfrac{m}{n}$

C. $\dfrac{2n}{m}$

D. $\dfrac{m}{2n}$

Q.4 A hash table contains 9 buckets and uses linear probing to resolve collisions. The key values are integers and the hash function used is key % 9. If the values 41, 157, 72, 76, 31 are inserted in the table, in what location would the last key be inserted?

7
_____.

[NAT]

$$h(41) = 41 \% 9 = 5$$
$$h(157) = 157 \% 9 = 4$$
$$h(72) = 72 \% 9 = 0$$
$$h(76) = 76 \% 9 = \textcircled{4}$$
$$h(31) = 31 \% 9 = \textcircled{4}$$

Index	Value
0	72
1	
2	
3	
4	157
5	41
6	76
7	(31)
8	

Q.5 Which one of the following hash functions on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 for i ranging from 0 to 2024?

[MCQ]

(A.) $h(i) = (12 * i) \bmod 10$

(B.) $h(i) = (11 * i^2) \bmod 10$

(C.) $h(i) = i^3 \bmod 10$

(D.) $h(i) = i^2 \bmod 10$

Even

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64$

$9^2 = 81$

$100^2 = 100$

1, 4, 9, 6, 5

6, 9, 4, 1, 0

2, 3, 7, 8

Which one of the following hash functions on integers will distribute keys **most uniformly** over **10 buckets numbered 0 to 9** for i ranging from 0 to 2024?

[MCQ]

A. $h(i) = (12 * i) \bmod 10$

B. $h(i) = (11 * i^2) \bmod 10$

C. $h(i) = i^3 \bmod 10$

D. $h(i) = i^2 \bmod 10$

Ev

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$
$8^3 = 512$
$9^3 = 729$
$10^3 = 1000$

1,8,7,4,5,6,
3,2,9,0

Q.6 Consider a double hashing scheme in which the primary hash function is $h_1(k) = k \bmod 17$, and the secondary hash function is $h_2(k) = 1 + (k \bmod 13)$. Assume that the table size is 17. Then the address returned by probe 2 in the probe sequence (assume that the probe sequence begins at probe 0) for key value $k = 127$ is __13__.

$$m = 17$$

$$H(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod m$$

$$H(127, 2) = \left(127 \bmod 17 + 2 \cdot \left(1 + 127 \bmod 13\right)\right) \bmod$$

$$(8 + 2 \cdot 11) \bmod 17$$

$$30 \bmod 17 = \boxed{13}$$

Q.7 Consider a hash table with 11 slots. The hash function is $h(k) = k$ mod 11. The collisions are resolved by chaining. The following 11 keys are inserted in the order: 28, 19, 15, 20, 33, 30, 42, 63, 60, 32, 43. The maximum, minimum, and average chain lengths in the hash table, respectively, are-

[NAT]

A. 3, 0, 1

B. 3, 3, 3

C. 3, 0, 2

D. 4, 0, 1

$h(28) = 28 \bmod 11 = 6$

$h(19) = 19 \bmod 11 = 8$

$h(15) = 15 \bmod 11 = 4$

$h(20) = 20 \bmod 11 = 9$

$h(33) = 33 \bmod 11 = 0$

$h(30) = 30 \bmod 11 = 8$

$h(42) = 42 \bmod 11 = 9$

$h(63) = 63 \bmod 11 = 8$

$h(60) = 60 \bmod 11 = 5$

$h(32) = 32 \bmod 11 = 10$

$h(43) = 43 \bmod 11 = 10$

Q.7 Consider a hash table with 11 slots. The hash function is h(k) = k mod 11. The collisions are resolved by chaining. The following 11 keys are inserted in the order: 28, 19, 15, 20, 33, 30, 42, 63, 60, 32, 43. The maximum, minimum, and average chain lengths in the hash table, respectively, are-

A. 3, 0, 1

B. 3, 3, 3

C. 3, 0, 2

D. 4, 0, 1

$0 \to 33$
$1 \to \times$
$2 \to \times$
$3 \to \times$
$4 \to 15$
$5 \to 60$
$6 \to 28$
$7 \to \times$
$8 \to 19 \to 63 \to 30$
$9 \to 20 \to 42$
$10 \to 32 \to 43$

$\dfrac{1+0+0+0+1+1+1+0+3+2+2}{11} \Rightarrow 1$

$h(28) = 28 \bmod 11 = 6$ ✓
$h(19) = 19 \bmod 11 = 8$ ✓
$h(15) = 15 \bmod 11 = 4$ ✓
$h(20) = 20 \bmod 11 = 9$ ✓
$h(33) = 33 \bmod 11 = 0$ ✓
$h(30) = 30 \bmod 11 = 8$ ✓
$h(42) = 42 \bmod 11 = 9$ ✓
$h(63) = 63 \bmod 11 = 8$ ✓
$h(60) = 60 \bmod 11 = 5$ ✓
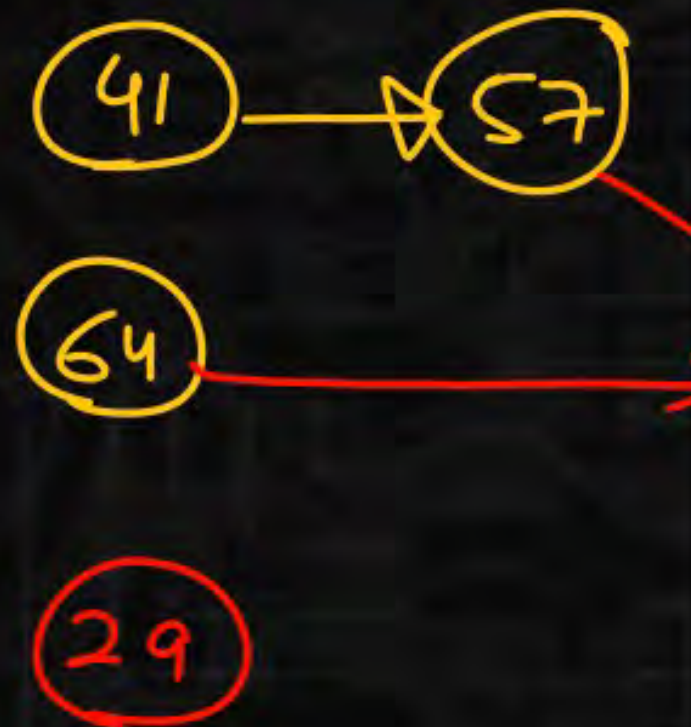$h(32) = 32 \bmod 11 = 10$
$h(43) = 43 \bmod 11 = 10$

Q.8 A hash table of length 8 uses open addressing with hash function h(k)= 2 + k mod 8, and linear probing. After inserting 5 values into an empty hash table, the table is as shown below.

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

[MCQ]

0	
1	
2	64
3	41
4	57
5	72
6	
7	29

41 ⟶ 57

64

29

A. 10

B. 9

C. 15

D. 8

$h(64) = 2 + 64 \mod 8 = 2$
$h(41) = 2 + 41 \mod 8 = 3$
$h(57) = 2 + 57 \mod 8 = 3$
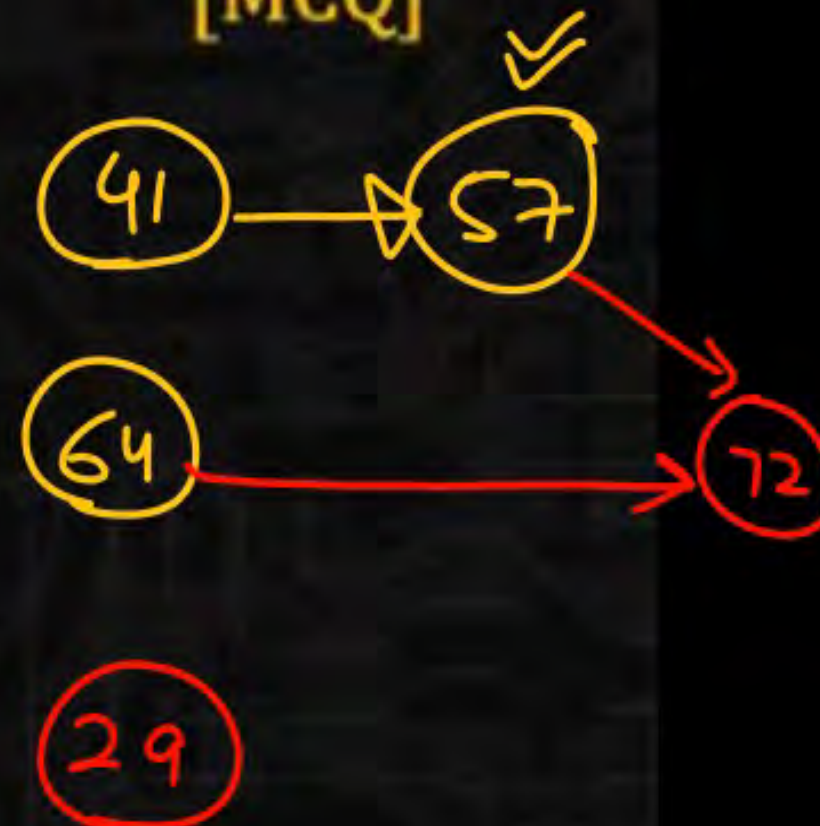$h(72) = 2 + 72 \mod 8 = 2$
$h(29) = 2 + 29 \mod 8 = 2$

A hash table of length 8 uses open addressing with hash function h(k)= 2 + k mod 8, and linear probing. After inserting 5 values into an empty hash table, the table is as shown below.

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

[MCQ]

0	
1	
2	64
3	41
4	57
5	72
6	
7	29

(i) 41, 64   (ii) 64, 41

A. 10

B. 9

(i) 41 57 64

C. 15

D. 8

(ii) 64 41 57

(iii) 41 64 57

41 → 57

64 → 72

29

Q.8 A hash table of length 8 uses open addressing with hash function h(k)= 2 + k mod 8, and linear probing. After inserting 5 values into an empty hash table, the table is as shown below.

How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

0	
1	
2	64
3	41
4	57
5	72
6	
7	29

A. 10

B. 9

C. 15

D. 8

(handwritten annotations)

41 → 57

64 → 72

29

(i) 41 57 64 72

(ii) 64 41 57 72

(iii) 41 64 57 72

3 ways × 5 = 15