TOPICS TO BE COVERED

Arrays-2

rmo Vs cmo

2·D array - C MO

int a[2][3]

$$0 \quad 1 \qquad 0 \quad 1 \quad 2$$

$$\begin{array}{cc} & 0 \qquad 1 \qquad 2 \\ 0 & \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \\ 1 & \end{array}$$

← col with index=0 → ← col with index=1 → ← col with index=2 →

| a_{00} | a_{10} | a_{01} | a_{11} | a_{02} | a_{12} |

int $a[3][4]$

$$\begin{array}{c c c c c}
 & 0 & 1 & 2 & 3 \\
0 & a_{00} & a_{01} & a_{02} & a_{03} \\
1 & a_{10} & a_{11} & a_{12} & a_{13} \\
2 & a_{20} & a_{21} & a_{22} & a_{23}
\end{array}$$

add (a_{23})

← col. with index = 0 → ← col. with index = 1 → ← col with index = 2 → ← col index = 3 →

| a_{00} | a_{10} | a_{20} | a_{01} | a_{11} | a_{21} | a_{03} | a_{12} | a_{22} | a_{03} | a_{13} | a_{23} |

1000

int $a[3][4]$
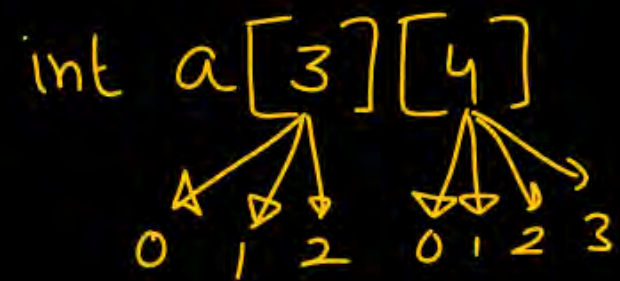
$\underset{0\ 1\ 2}{\underbrace{\qquad}}\quad \underset{0\ 1\ 2\ 3}{\underbrace{\qquad}}$
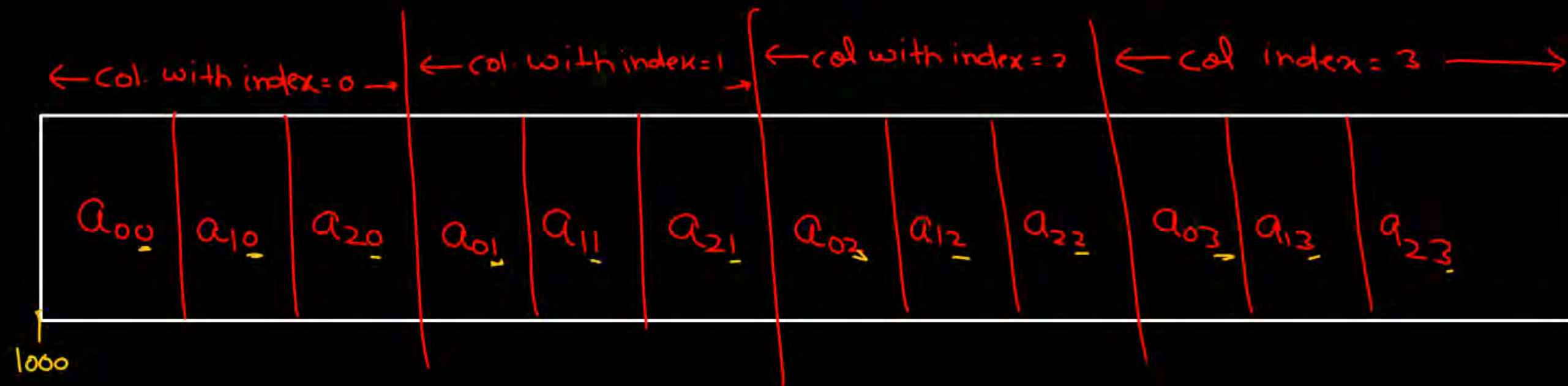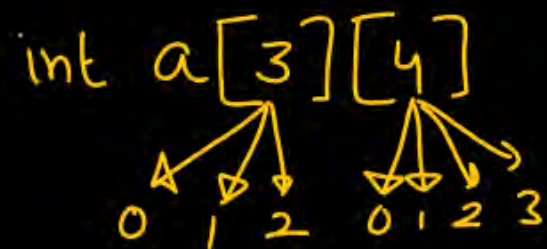
add (a_{23})

within col index 3
elem. already filled
before a_{23}

= row with index
0 to 1

= 1 - 0 + 1 = ②

How many col. already
filled before col. with
index 3

= ③ (0 to 2)

2 - 0 + 1 = 3

$$
\begin{array}{c}
\quad\ 0\quad\ \ 1\quad\ \ 2\quad\quad\ 3 \\
0\ \left[\begin{array}{cccc} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \end{array}\right]
\end{array}
$$

1

2

← col. with index=0 →   ← col. with index=1   ← col with index =2   ← col index = 3 →

| a_{00} | a_{10} | a_{20} | a_{01} | a_{11} | a_{21} | a_{02} | a_{12} | a_{22} | a_{03} | a_{13} | a_{23} |

1000   1044
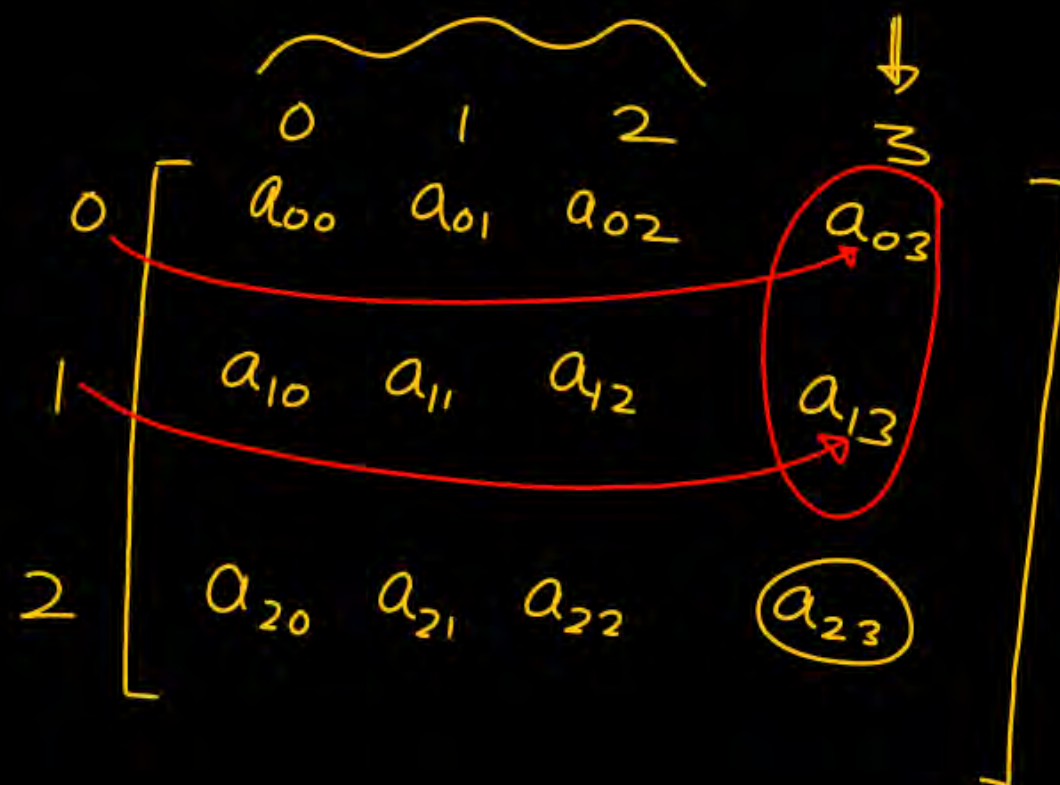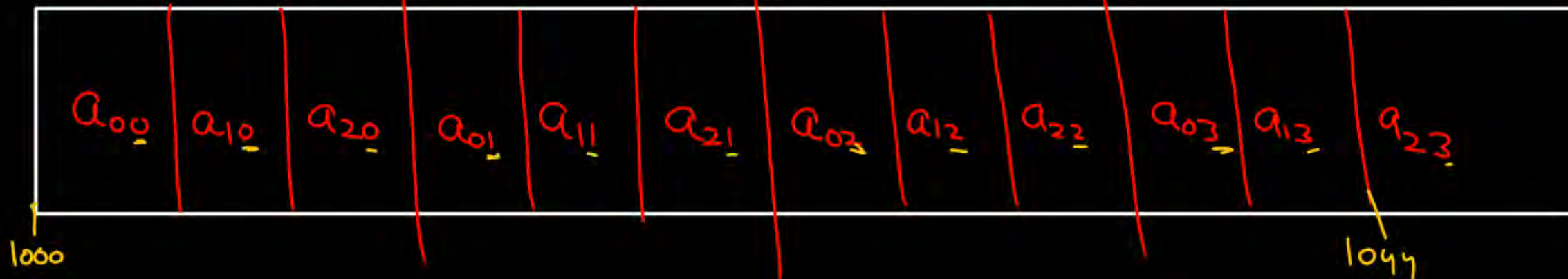
After 3 col. & 2 elements a_{23} is stored
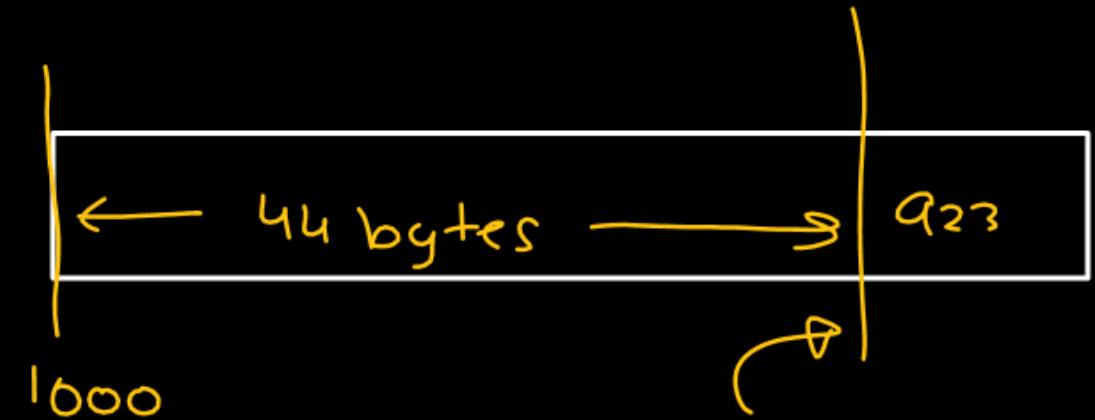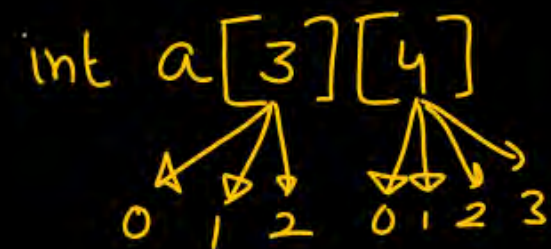
$a[3][4]$ → Every index = 3 element

Every col ⇒

Total elements = $3 \times 3 + 2$

= $9 + 2$

= 11 elements

Memory already filled = $11 \times 4 = 44$ bytes

← 44 bytes → | a_{23}

1000

$add(a_{23}) = 1000 + 44$

$= 1044$

int a[3][4]

0 1 2   0 1 2 3

add(a₂₃)

400 relative → age के हिसाब
से

1 2 - - - 10
11 12 - - - -20

$$int\ a[3][4]$$

3 col

	0	1	2	3
0	a_{00}	a_{01}	a_{02}	a_{03}
1	a_{10}	a_{11}	a_{12}	a_{13}
2	a_{20}	a_{21}	a_{22}	a_{23}
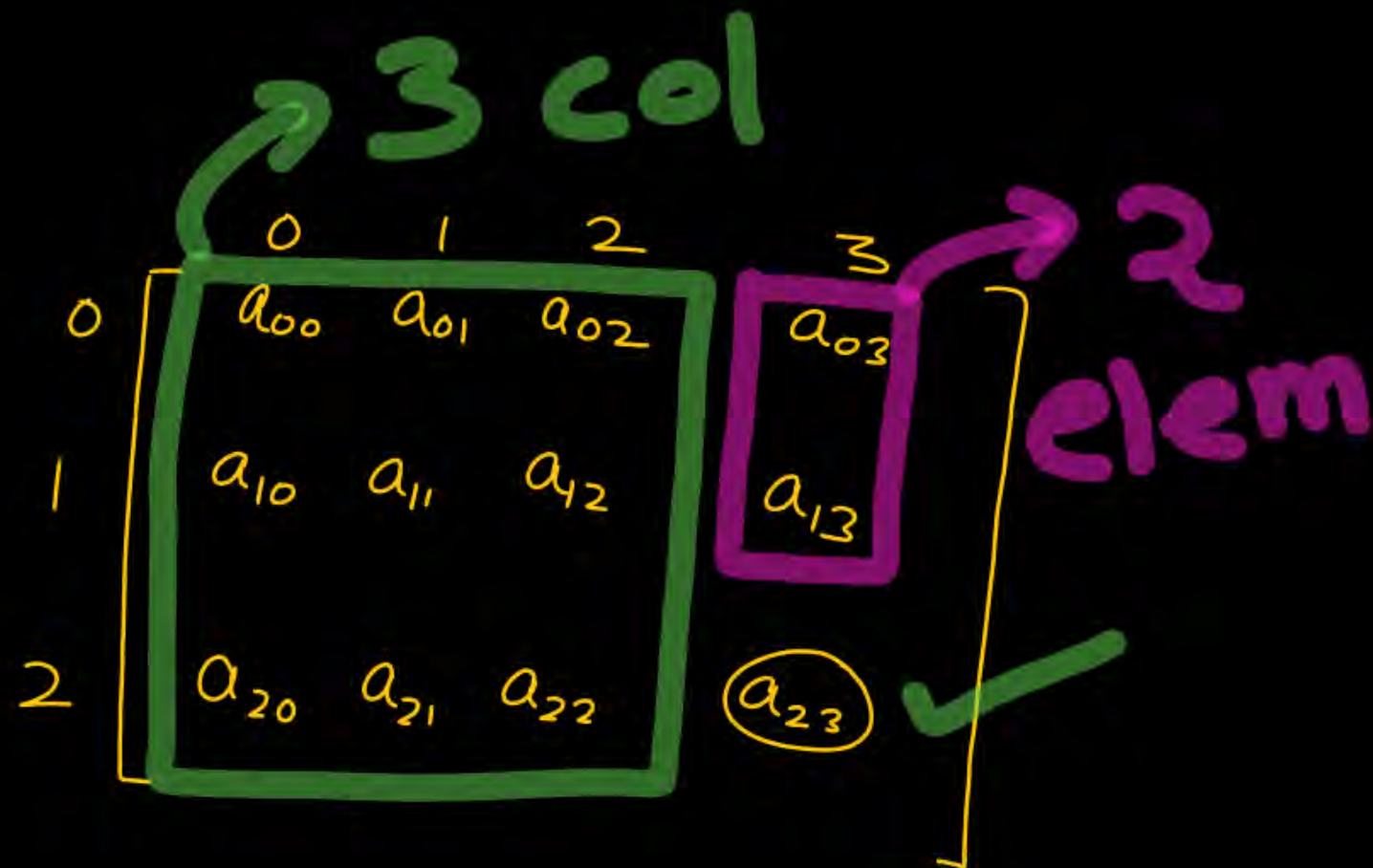
2 elem

CMO

$w = 2\,bytes$

$BA = 1000$

$$6-(-5)+1 \quad 3-(-3)+1$$

$$a\,[\,-5..6\,]\,[\,-3..3\,]$$

Every index in this dim = 12 ele.

$add(a_{0,0})$

index already filled
$= -5\ to\ -1$
$= -1-(-5)+1$
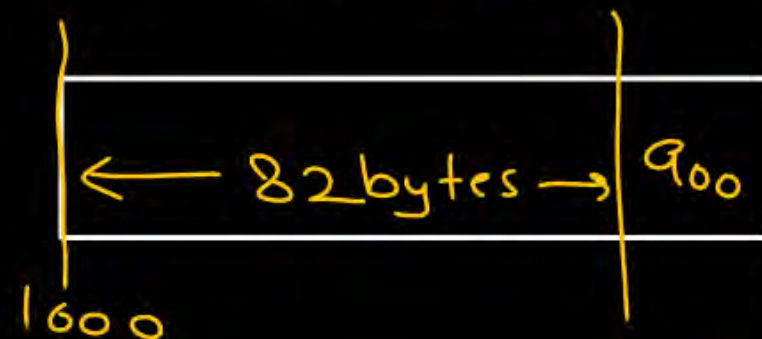$= 5$

index already filled before index 0
$= -3\ to\ -1$
$= -1-(-3)+1$
$= 3$

3×12 element

Total ele already filled $= 3 \times 12 + 5$
$= 41\ elements$

Memory already filled
$= 41 \times 2$
$= 82\ bytes$

$\leftarrow 82\,bytes \rightarrow$ | a_{00}

1600

$add(a_{00}) = 1000 + 82$
$= 1082$
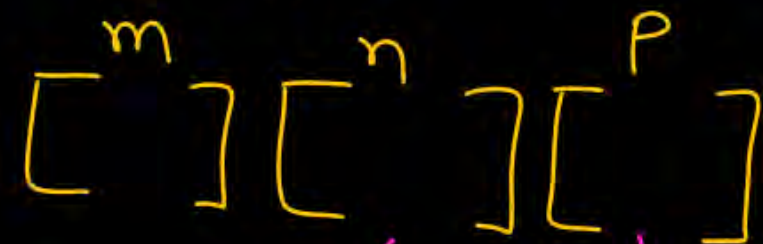
Sparse matrix

(i) Lower triangular matrix

(ii) Upper triangular matrix

(iii) tri-diagonal matrix

Lower triangular matrix

$\checkmark$ A LTM is a square matrix

all entries $= 0$

$$
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
\end{array}
$$

$$
\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
\end{array}
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44} \\
\end{bmatrix}
$$
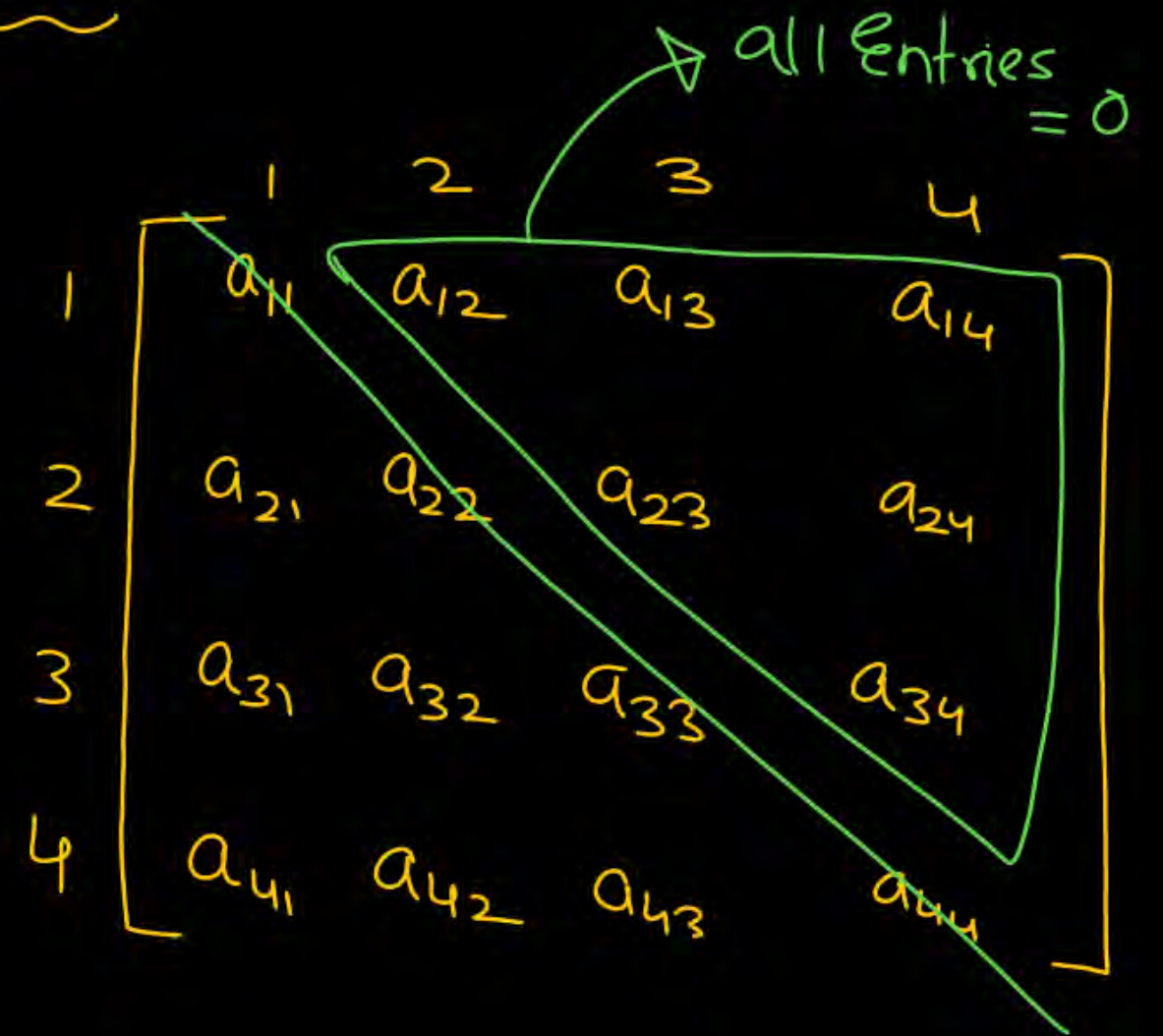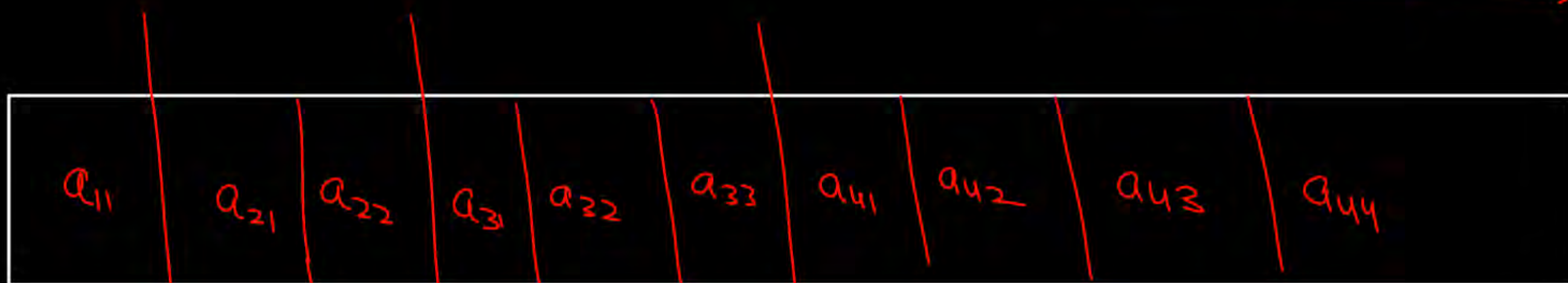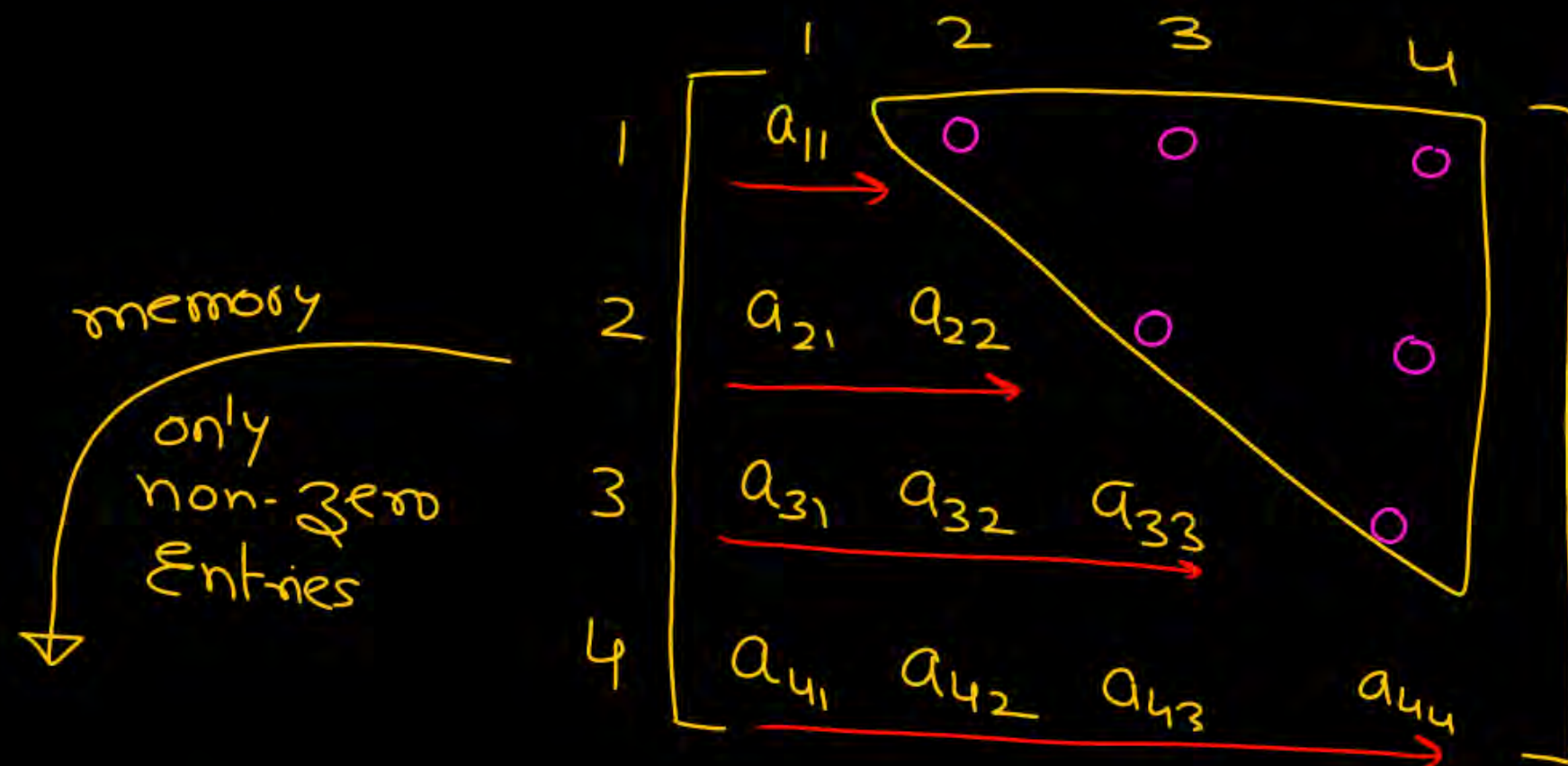
Lower triangular matrix

↗A LTM is a square matrix

$\boxed{A_{ij} = 0, \ i < j}$

$a_{12} = 0$
$a_{13} = 0$
$a_{14} = 0$
$a_{23} = 0$
$a_{24} = 0$
$a_{34} = 0$

row < col

all entries = 0

$$\begin{array}{c c c c}
 & 1 & 2 & 3 & 4 \\
1 & a_{11} & a_{12} & a_{13} & a_{14} \\
2 & a_{21} & a_{22} & a_{23} & a_{24} \\
3 & a_{31} & a_{32} & a_{33} & a_{34} \\
4 & a_{41} & a_{42} & a_{43} & a_{44}
\end{array}$$

RMO

Lower triangular matrix

$$
\begin{array}{c c c c c}
 & 1 & 2 & 3 & 4 \\
1 & a_{11} & \circ & \circ & \circ \\
2 & a_{21} & a_{22} & \circ & \circ \\
3 & a_{31} & a_{32} & a_{33} & \circ \\
4 & a_{41} & a_{42} & a_{43} & a_{44}
\end{array}
$$

memory

only
non-zero
Entries

| a_{11} | a_{21} | a_{22} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} | a_{44} |

add(a_{43})

$a[4][5]$ X
$a[5][5]$ ✓

$$\begin{array}{c} & 1 & 2 & 3 & 4 & 5 \\ 1 & \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & 0 & 0 & 0 \\ 3 & a_{31} & a_{32} & a_{33} & 0 & 0 \\ 4 & a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ 5 & a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \end{array}$$

RMO

| a_{11} | a_{21} | a_{22} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} | a_{44} | a_{51} | a_{52} | a_{53} | a_{54} | a_{55} |

1600   1004
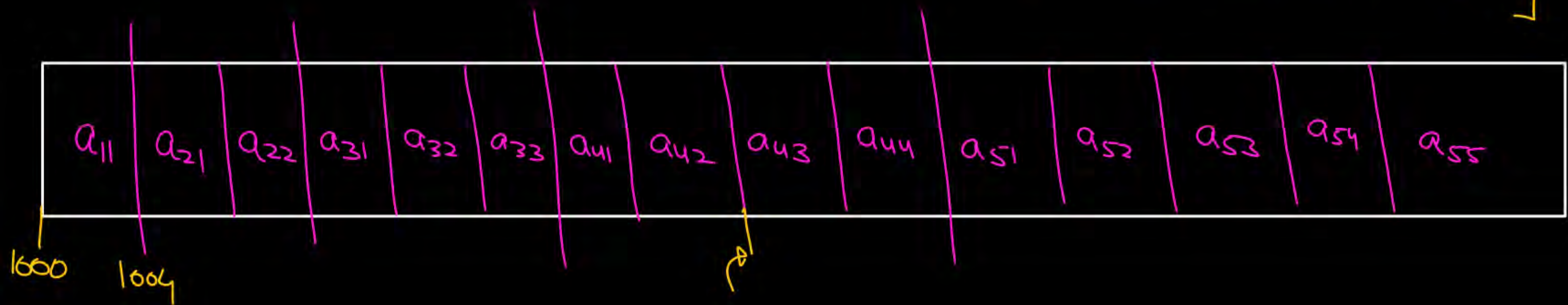
$a[4][5]$ X
$a[5][5]$ ✓

add(a_{43})

How many rows
already filled before
row with index 4

= 1 to 3

= 3 rows

= 1,2,3 index

✓ ✓ ✓
1+2+3 = 6 element

within row
with index 4
elements filled
before a_{43}

= col with index
1 to 2

= 2-1+1 = 2

3 rows
already
filled

	1	2	3	4	5
1	a_{11}	0	0	0	0
2	a_{21}	a_{22}	0	0	0
3	a_{31}	a_{32}	a_{33}	0	0
4	a_{41}	a_{42}	a_{43}	a_{44}	0
5	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}

| a_{11} | a_{21} | a_{22} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} | a_{44} | a_{51} | a_{52} | a_{53} | a_{54} | a_{55} |

1000 1004

Total 8 elements already filled

$$LTM \Rightarrow N \times N$$

add(A_{ij})

Rows already filled

within i^{th} row elem. already filled before A_{ij}

Rows already filled

Rows already filled = index 1 to $i-1$

= index $1, 2, 3, \cdots i-1$

$1 + 2 + 3 \cdots + (i-1)$

$= \dfrac{(i-1)(i)}{2}$ elements

Total elem already filled = $\left[\dfrac{i(i-1)}{2} + (j-1) \right]$

= col with index 1 to $j-1$

= $j-1-1+1$

= $(j-1)$ elements

$$\text{Size} = \omega \text{ bytes}$$

Memory already filled before A_{ij} = $\left[\dfrac{i(i-1)}{2} + (j-1) \right] \omega$ bytes

$$\text{add}(A_{ij}) = BA + \left[\dfrac{i(i-1)}{2} + (j-1) \right] \omega$$

22 ना नहीं है

formula निश्चय देगा

Q/

RMO

$w = 2$ byte

$BA = 1000$

$A[-5..5][-7..3]$

$add(A_{1,-2})$

Rows
already filled

-5 to 0

$= 0 - (-5) + 1$

$= 6$ rows

$1 + 2 + 3 + 4 + 5 + 6$

$= \dfrac{6 \times 1}{2} = 21$ elements

within row index 1

Col. already filled/
Elem. already filled

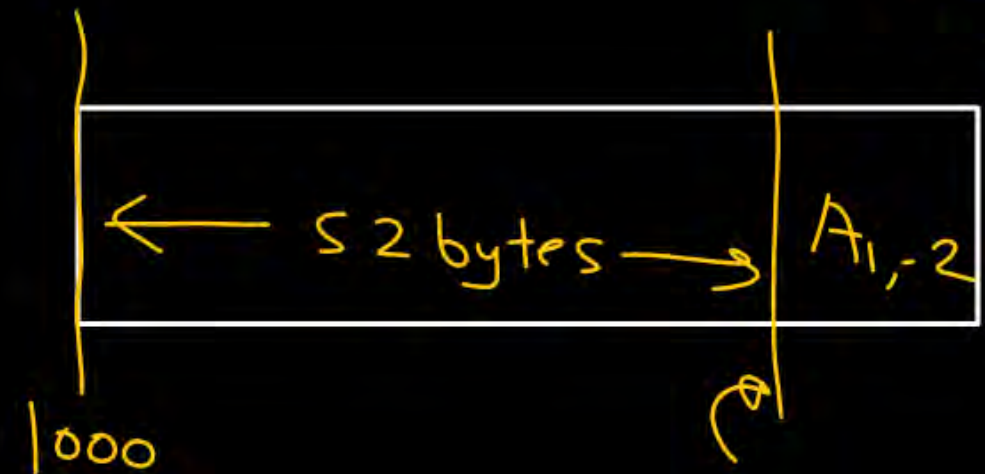$= -7$ to -3

$= -3 - (-7) + 1$

$= -3 + 8$

$= 5$ elements

Total elem. already filled

$= 26$

Memory already filled

$= 26 \times 2$

$= 52$ bytes

← 52 bytes → $A_{1,-2}$

1000

$add(A_{1,-2}) = 1000 + 52$

$= 1052$

LTM in CMO

add($\overset{\leftarrow}{a_{53}}$)

within intended COL(3^r col),
ele. already filled
before a_{53}
$= (5 \cdot 3)$
$= 2$ elements

col. already
filled
$=$ index 1 to 2
$=$ index 1, 2
$5 + 4$
$= 9$ elements

$$\begin{array}{c} & 1 & 2 & 3 & 4 & 5 \\ 1 & a_{11} & 0 & 0 & 0 & 0 \\ 2 & a_{21} & a_{22} & 0 & 0 & 0 \\ 3 & a_{31} & a_{32} & a_{33} & 0 & 0 \\ 4 & a_{41} & a_{42} & a_{43} & a_{44} & 0 \\ 5 & a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{array}$$

← 1st col → | ← 2nd col → | ← 3rd col → | 4th col | ← 5th col →

| a_{11} | a_{21} | a_{31} | a_{41} | a_{51} | a_{22} | a_{32} | a_{42} | a_{52} | a_{33} | a_{43} | a_{53} | a_{44} | a_{54} | a_{55} |

$$N \times N$$

$$\text{add}(a_{ij})$$

col. already filled
index $1, 2, 3, \cdots j-1$

index $1 \rightarrow N-(1-1)$

index $2 \rightarrow N-(2-1)$

index $3 \rightarrow N-(3-1)$

index $j-1 \Rightarrow N-(j-2)$

$$\left[N + (N-1) + (N-2) + \cdots (N-j+2) \right]$$

$$= \frac{j-1}{2} \left[N + (N-j+2) \right]$$

	1	2	3	$\cdots$	$j-1$	j	$\cdots$	$\cdots$	N
1	a_{11}	0	0		0	0			0
2	a_{21}	a_{22}	0		0	0	—		0
3	a_{31}	a_{32}	a_{33}		0	0	—	—	0

a_{ij}

$a_{N,1} \quad a_{N,2} \cdots \cdots \cdots \cdots \cdots a_{N,N}$

$N \times N$

add (a_{ij})

within col.
index

elem. already filled
before a_{ij}

$(i-j)$

col. already filled
index $1, 2, 3, \cdots j-1$

index 1 $\longrightarrow$ $N-(1-1)$

index 2 $\longrightarrow$ $N-(2-1)$

index 3 $\longrightarrow$ $N-(3-1)$

index $j-1$ $\Longrightarrow$ $N-(j-2)$

$$\left[N + (N-1) + (N-2) + \cdots (N-j+2) \right]$$

$$= \frac{j-1}{2} \left[N + (N-j+2) \right]$$

	1	2	3	...	j-1	j	...	N
1	a_{11}	0	0		0	0		0
2	a_{21}	a_{22}	0		0	0	— —	0
3	a_{31}	a_{32}	a_{33}		0	0	— —	0

$i-1$

i

a_{ij}

| N | $a_{N,1}$ | $a_{N,2}$ | | | | | — | $a_{N,N}$ |

$$\text{Total ele.} = \frac{j-1}{2}\left[N + (N - (j-2))\right] + (i-j)$$

$$= \frac{(j-1)}{2}\left[2N - (j-2)\right] + (i-j)$$

$$= N(j-1) + \frac{(j-1)(j-2)}{2} + (i-j)$$

रटना नहीं है

Memory already filled $= \left[N(j-1) + \frac{(j-1)(j-2)}{2} + (i-j)\right]\omega$

$add(a_{ij}) = BA + \left[N(j-1) + \frac{(j-1)(j-2)}{2} + (i-j)\right]\omega$

LTM
CMO

$$20-(-20)+1$$
$$A[-20..20][-20..20]$$
$$41 \times 41$$

$$\omega = 1 \text{ byte}$$
$$BA = 1000$$

$$\text{add} \left(A[-2][-6] \right)$$

← 487 → $a_{-2,-6}$

1000

$$\Rightarrow 1000 + 487$$
$$= 1487$$

within col index -6
elem already filled
before $a_{-2,-6}$

col. already filled
$$= \boxed{-20, -19, \ldots -8, -7} -6$$
$$= -20 \text{ to } -7$$
$$= -7 - (-20) + 1$$
$$= -7 + 20 + 1$$
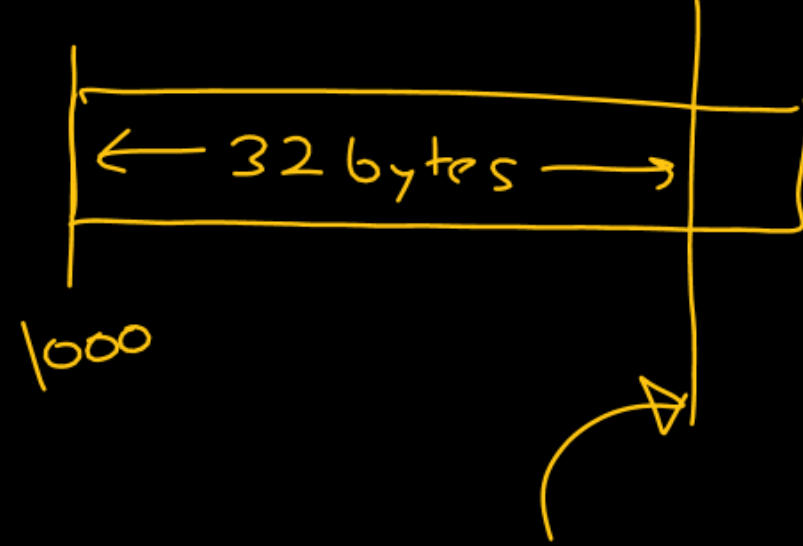$$= 14 \text{ cols}.$$

$$\Rightarrow -2 - (-6)$$
$$= -2 + 6$$
$$= 4 \text{ element}$$

$$41 + 40 + \cdots + 28$$

$$\Rightarrow \frac{14}{2} [41 + 28]$$

$$= 7 \times 69$$
$$= 483 \text{ elem}.$$

1st → 41
2nd → 40
3rd → 39
14th → 28

Total ele $= 483 + 4 = 487$
Memory $= 487 \times 1 = 487$ byte.

LTM
CMO

$A[-3..3][-5..1]$

$w = 1\,byte$ add $(A[3][1])$

$BA = 1000$

* * *

formula wale

~~लिखेंगे~~

Memory already filled = 8×4 = 32 bytes

$\leftarrow$ 32 bytes $\longrightarrow$

1000

1000 + 32

= 1032