

CS & IT ENGINEERING

Data Structure & Programming

Stack and Queues

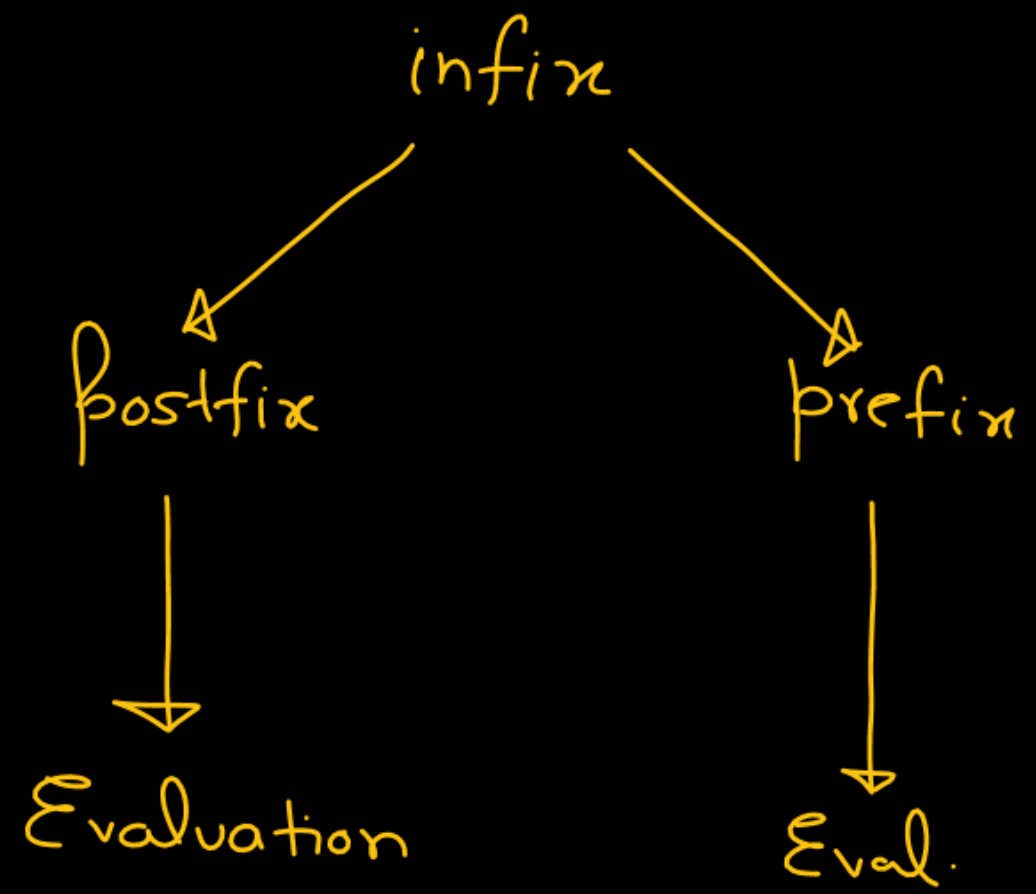
Lec- 03



By- Pankaj Sharma sir

TOPICS TO BE
COVERED

Stack-III



Postfix Evaluation

Infix: $2 + 3 \times 5$

Postfix: $2 \ 3 \ 5 \ \times \ +$

using stack

Operator $\Rightarrow (*)$

a) Pop 1st elem $\Rightarrow A$

b) Pop 2nd elem $\Rightarrow B$

$B \circledast A$

push result onto stack

5
3
2

$$\begin{array}{r} \xrightarrow{\hspace{2cm}} \\ 5, 3 \\ \hline 3 \times 5 \\ = 15 \end{array}$$

2 3 5 X +

X

+

15, 2

$$2 + 15 = 17$$

5	
3	15
2	17

5, 3

$$3 \times 5 = 15$$

Ex 2:

infix : $2 + 3 \times 4 - 6 / 2$

Postfix : $2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$
 ↑

4
3 12
2

(i) \times

4, 3
←
 3×4
push 12

Ex 2:

infix: $2 + 3 \times 4 - 6 / 2$

Postfix: $2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$
 ↑ ↑

(ii) +

→
12, 2

←

$2 + 12$

14
push 14

4
3 12
2 14

(i) ×

4, 3

←

3×4

push 12

Ex 2:

infix: $2 + 3 \times 4 - 6/2$

Postfix: $2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$
 $\uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow$

(iii) /

$\xrightarrow{\quad}$
 $2, 6$
 $\xleftarrow{\quad}$
 $6/2$
 $= 3$
push 3

(ii) +

$\xrightarrow{\quad}$
 $12, 2$
 $\xleftarrow{\quad}$
 $2 + 12$
 $= 14$
push 14

4	2
3	3 12 6
2	14

(i) \times

4, 3
 $\xleftarrow{\quad}$
 3×4
push 12

Ex 2:

infix: $2 + 3 \times 4 - 6 / 2$

Postfix: $2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$
 $\uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow$

-

3

14

→
3, 14

↓

14 - 3

⑪ ✓

	4		2
3	3	12	6
11	2		14

Ex 2:

infix: $2 + 3 \times 4 - 6 / 2$

Postfix: $2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$

$2\ 3\ 4\ \times\ +\ 6\ 2\ /\ -$
Scan from
L to R

$2\ 12\ +\ 6\ 2\ /\ -$

$14\ 6\ 2\ /\ -$

$14\ 3\ -$

$$14 - 3 = 11$$

A function f defined on stacks of integers satisfies the following properties.

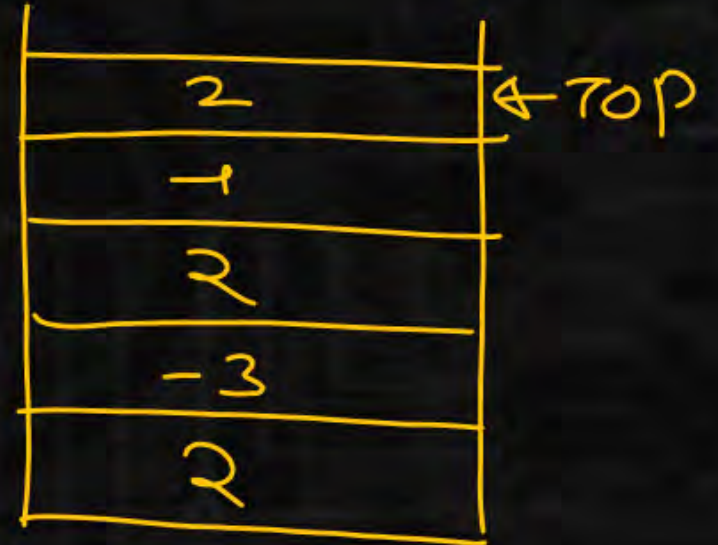
$f(\emptyset) = 0$ and $f(\text{push}(S, i)) = \max(f(S), 0) + i$ for all stacks S and integer i .

If a stack S contains the integers 2, -3, 2, -1, 2 in order from bottom to top, what is $f(S)$?

- A** 6
- B** 4
- ☒ **C** 3
- D** 2

$$f(\text{stack is empty}) = 0$$

$$\begin{aligned} f(\text{Push}(S, 2)) &= \max(f(S), 0) + 2 \\ &= \max(0, 0) + 2 = 2 \end{aligned}$$



$$f(s) = 2$$

$$\begin{aligned} f(\text{Push}(s, -3)) &= \max(f(s), 0) + (-3) \\ &= \max(2, 0) + (-3) = -1 \end{aligned}$$

$$\begin{aligned} f(\text{Push}(s, 2)) &= \max(f(s), 0) + 2 \\ &= \max(-1, 0) + 2 = 2 \end{aligned}$$

$$f(\text{Push}(s, -1)) = \max(f(s), 0) + (-1) = \max(2, 0) + (-1) = 1$$

$$\begin{aligned} f(\text{Push}(s, 2)) &= \max(f(s), 0) + 2 \\ &= \max(1, 0) + 2 = 1 + 2 = \boxed{3} \end{aligned}$$

-1
2
-3
2
s

The result of evaluating the postfix expression
 $10\ 5 + 60\ 6 / * 8 -$ is

direct ✓
using stack X



A 284

B 213

C 142

D 71

$10\ 5 + 60\ 6 / * 8 -$

$15\ 60\ 6 / * 8 -$

$15\ 10 * 8 -$

$150 - 8 -$

$150 - 8$
 $= 142$

The best data structure to check whether an arithmetic expression has balanced parentheses is-

- A** queue
- B** stack
- C** tree
- D** list

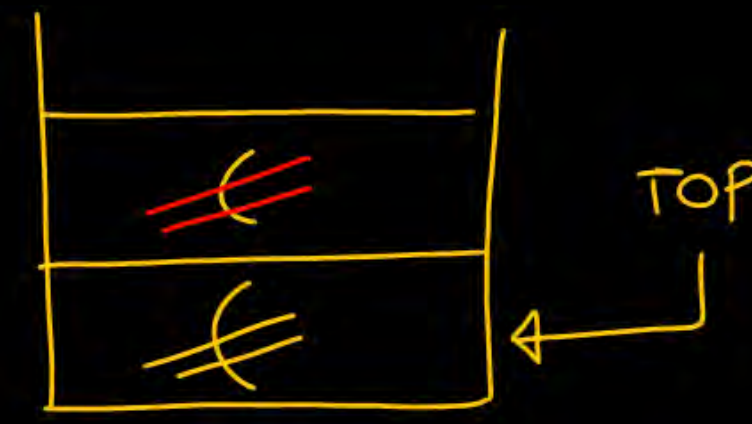
()

() ()

(())

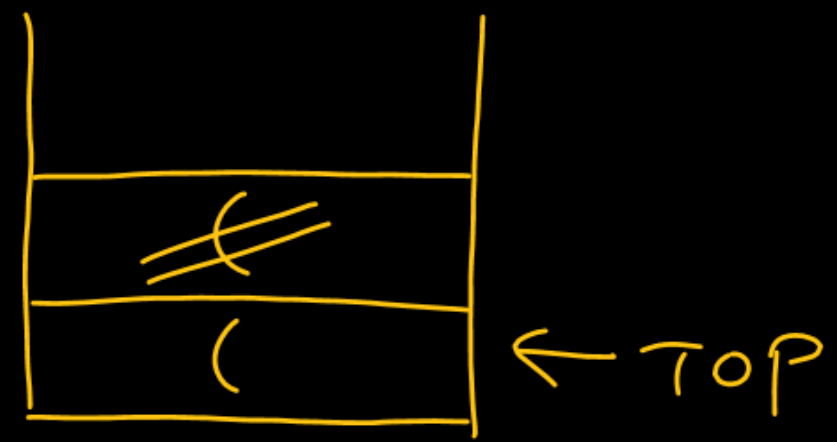
(()) () ()

/P :

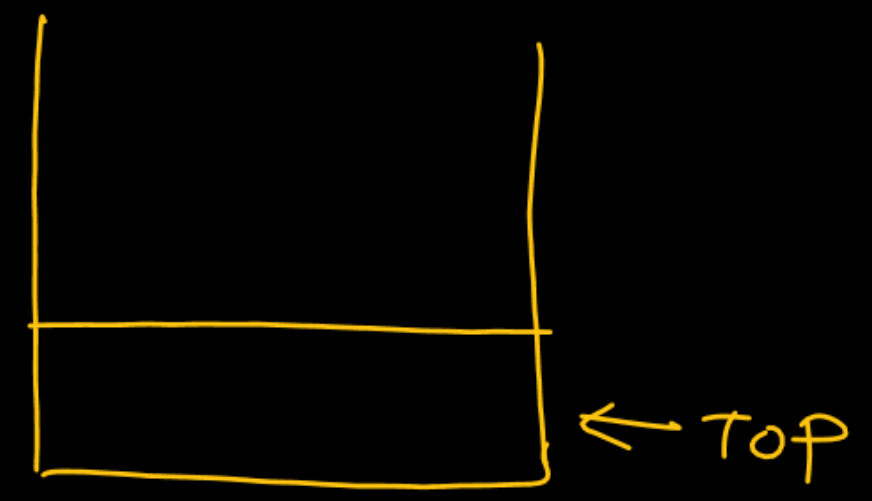


most recently
occured paranthesis

i/p : (()



i/p :) ((



Prefix Evaluation

infix: $2 + 3 \times 5$

Prefix: $+ 2 \times 3 5$

Reverse Prefix: $5 3 \times 2 +$ [↑]

$$\begin{array}{r} \times \quad \longrightarrow \\ 3, 5 \\ 3 \times 5 \\ 15 \end{array}$$

3 2
17 5 15

$$\begin{array}{r} + \\ 2, 15 \\ \hline 2 + 15 \\ = 17 \end{array}$$

Stack

Push →

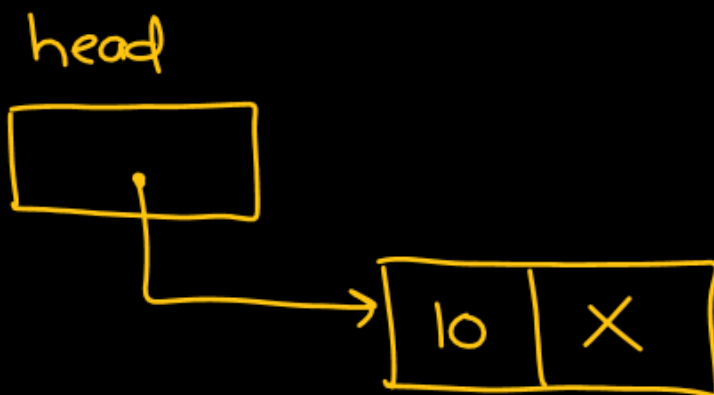
Pop →

using l.l.

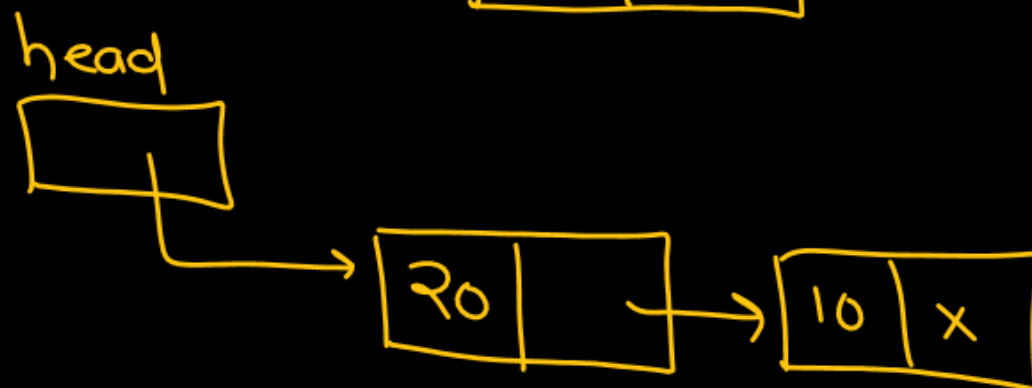
10, 20, 30

Push(10)

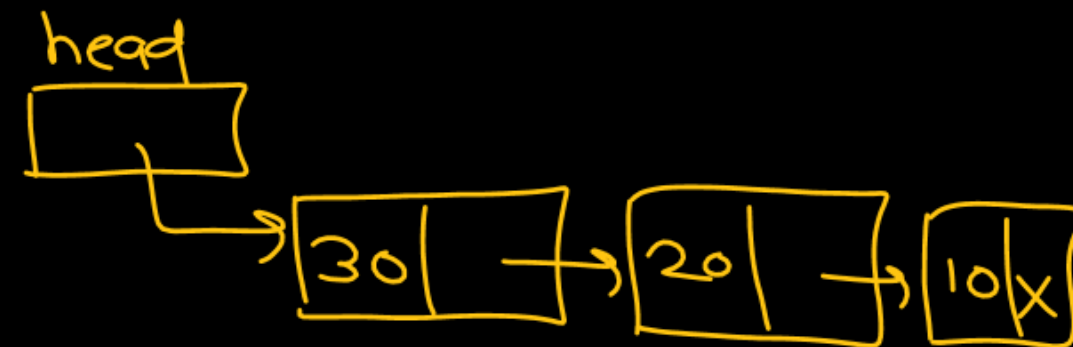
(i)



Push(20) (ii)



(iii) Push(30)



Pop() ⇒ deletion from
beg.

Push() ⇒ Insert at
begin

Recursion

Tower of Hanoi Problem:

Given 3 pegs and n disc each of distinct size.

$n=3$

(i) we can not put a large size plate above smaller size plate.

(ii) move only 1 disk at a time



A
Source



B
Aux



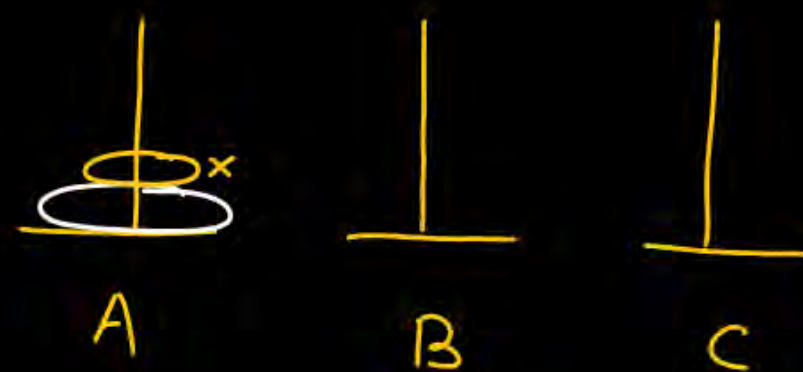
C
Dest

Target:
Move all disc from Source peg to Dest. peg.

I/P:



$n=2$

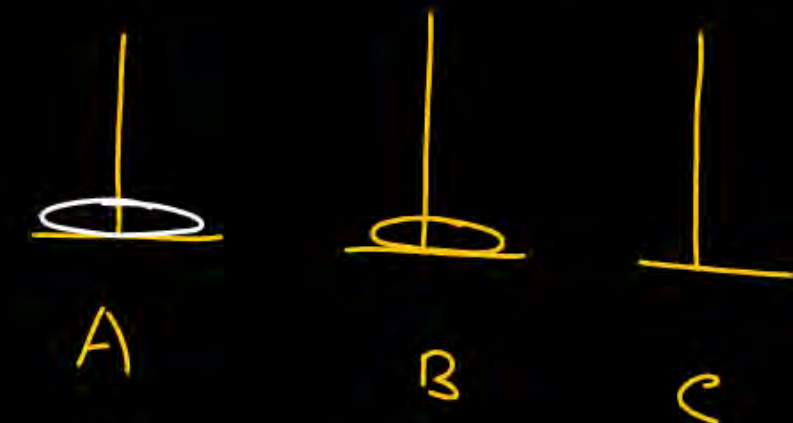


$A \rightarrow B$

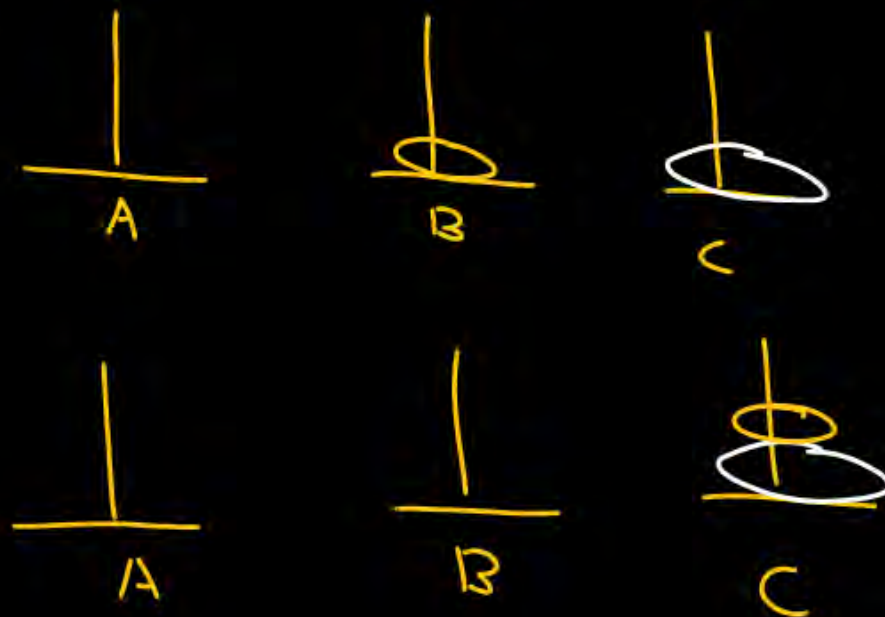
O/P:



$A \rightarrow C$



$B \rightarrow C$

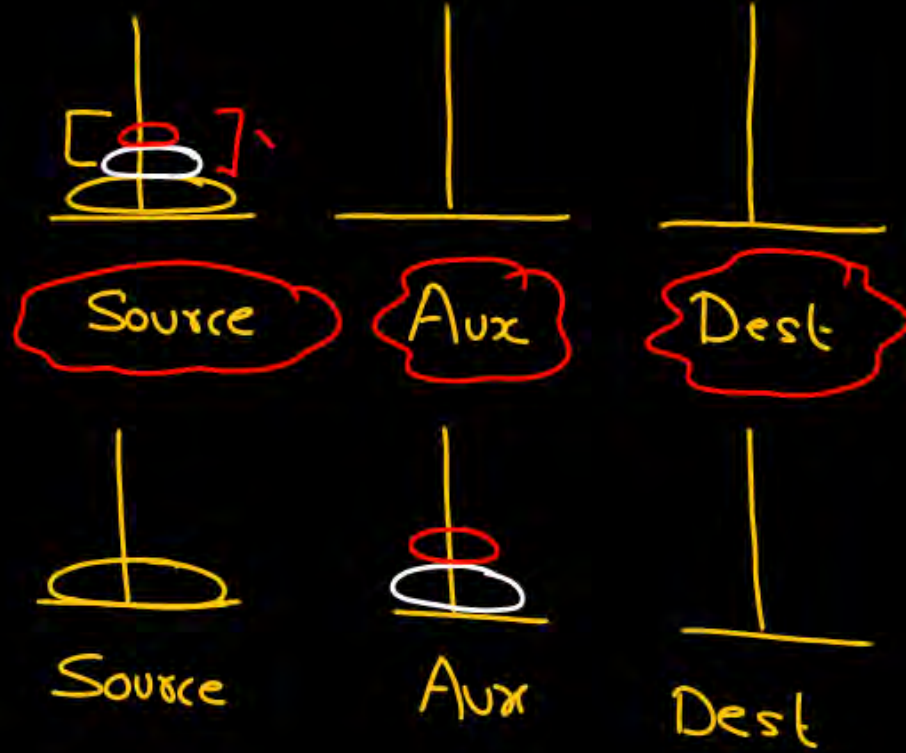


TOH(n , Source, Dest, Aux)

no. of
plate

$n=3$

(i)



(ii)



(iii)

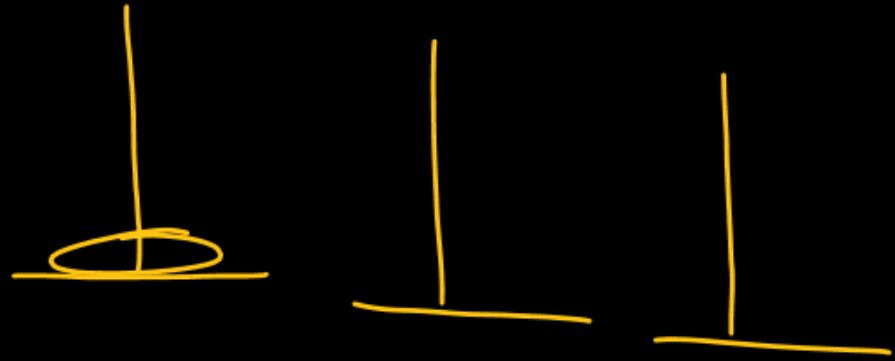


TOH(n , Source, Dest, Aux)

- 1.) TOH($n-1$, Source, Aux, Dest)
- 2.) Move Source \rightarrow Dest
- 3.) TOH($n-1$, Aux, Dest, Source)

TOH

$n \Rightarrow \text{small}$



```
if ( n is small )  
{
```

Easy case

No rec. is req.

Can be answered directly
}

```
else {
```

n is large

Not Easy

Rec. is needed

```
}
```

```
void TOH(int n, char Source, char Dest, char Aux)
```

```
{  
    if (n == 1)  
    {  
        printf("%c → %c", Source, Dest);  
        return;  
    }
```

```
    TOH(n-1, Source, Aux, Dest);
```

```
    printf("%c → %c", Source, Dest);
```

```
    TOH(n-1, Aux, Dest, Source);  
}
```

```
void main() {
```

```
    int n;
```

```
    printf("Enter the value of n");  
    scanf("%d", &n);
```

```
    TOH(n, 'A', 'C', 'B');
```

```
}
```



```
void TOH(int n, char Source, char Dest, char Aux)
```

```
{
  if(n == 1)
  {
    printf("%c → %c", Source, Dest);
    return;
  }
```

```
  TOH(n-1, Source, Aux, Dest);
```

```
  printf("%c → %c", Source, Dest);
```

```
  TOH(n-1, Aux, Dest, Source);
}
```

TOH(2, 'A', 'C', 'B')

^{S D A}
TOH(1, 'A', 'B', 'C')

A → B

A → C

^{S D A}
TOH(1, 'B', 'C', 'A')

B → C

~~A~~

A

~~A~~

A

~~B~~

B

~~B~~

B

~~C~~

C

~~C~~

C

~~A~~

A

~~A~~

A

~~B~~

B

~~B~~

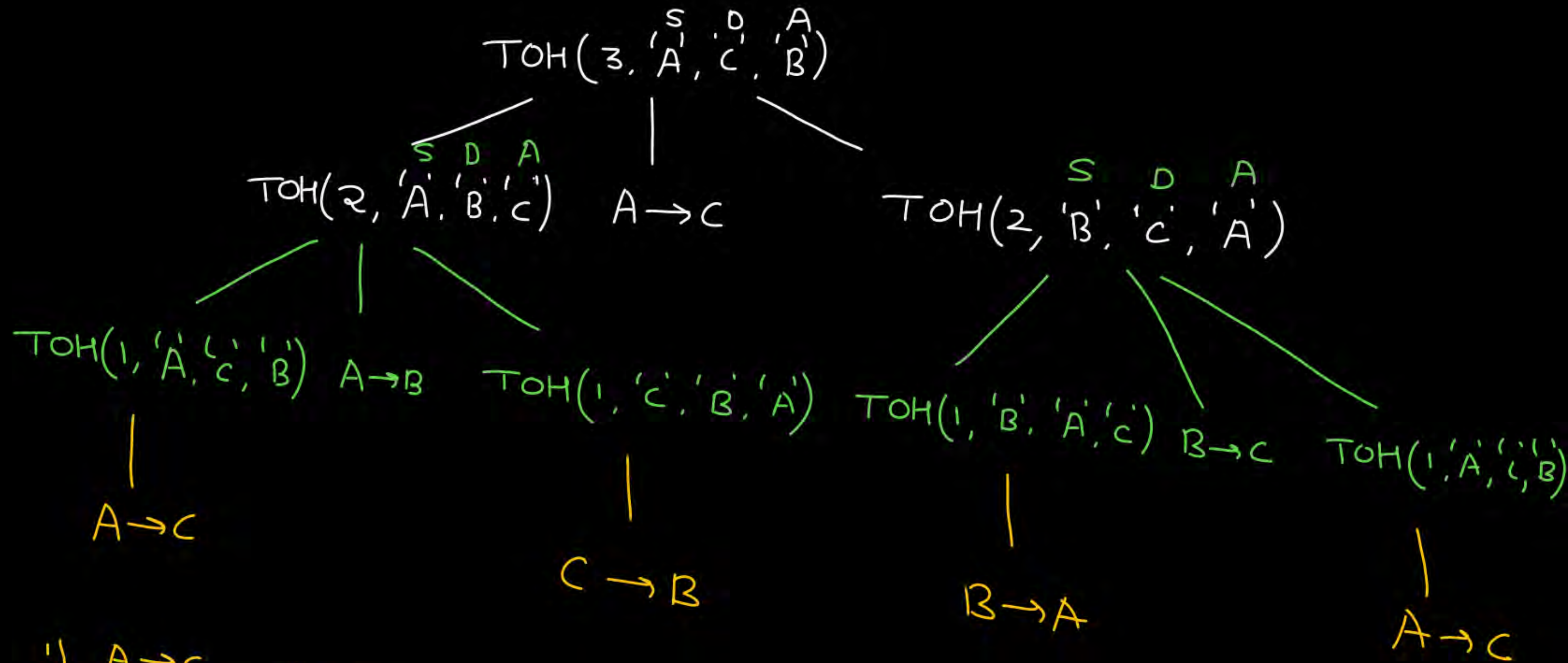
B

~~C~~

C

~~C~~

C



1) $A \rightarrow C$
 2) $A \rightarrow B$
 3) $C \rightarrow B$
 4) $A \rightarrow C$
 5) $B \rightarrow A$

6) $B \rightarrow C$
 7) $A \rightarrow C$



A



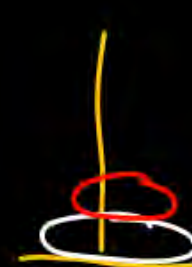
B



C



A



B



C



A



B



C



A



B



C



A



B



C



A



B



C



A



B



C



A



B



C

- 1) $A \rightarrow C$
- 2) $A \rightarrow B$
- 3) $C \rightarrow B$
- 4) $A \rightarrow C$
- 5) $B \rightarrow A$

- 6) $B \rightarrow C$
- 7) $A \rightarrow C$

{
TOH
Fibonacci
Queue
}

PYQs

Stack → recursion

Which of the following is essential to convert an infix expression to post-fix expression efficiently?

- ☒ A An operator stack
- ☐ B An operand stack
- ☐ C An operator and an operand stack
- ☐ D A parse tree

The following sequence of operations is performed on stack:
 PUSH(10), PUSH(20), POP, PUSH(10), PUSH(20), POP, POP,
 POP, PUSH(20), POP. The sequence of the value popped out is-

- A** 20 10 20 10 20
- ☒ **B** 20 20 10 10 20
- C** 10 20 20 10 20
- D** 20 20 10 20 10

10 20	20 10	20	
--------------------------------	--------------------------------	---------------	--

20, 20, 10, 10, 20

Which of the following permutations can be obtained in the output (in the same order) using a stack assuming that the input sequence is 1, 2, 3, 4, 5 in that order?

A

3, 4, 5, 1, 2 ✗

B

3, 4, 5, 2, 1 ✓✓

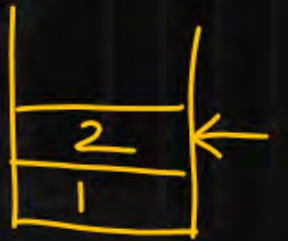
C

1, 5, 2, 3, 4

D

5, 4, 3, 1, 2

Push(1), Push(2), Push(3), Pop(), Push(4), Pop(), Push(5), Pop()



A program attempts to generate as many permutations as possible of the string "abcd" by pushing the character a, b, c, d in the same order onto a stack but it may pop off the top character at any time. Which one of the following strings

CANNOT be generated using this program?

A abcd ✓

B dcba ✓

C cbad ✓

D cabd

Push(a), Push(b), Push(c), Pop(), Pop(), Pop(), Push(d), Pop()

Which of the following is essential to convert an infix expression to post-fix expression efficiently?

- A** An operator stack
- B** An operand stack
- C** An operator and an operand stack
- D** A parse tree

The postfix expression corresponding to the infix expression $a+b*c-d^e^f$ is-

\wedge
 \leftarrow
 R to L

$d^{\wedge} \underbrace{e^{\wedge} f}_{\textcircled{1}}$

$$a + b * c - d^{\wedge} [e f^{\wedge}]$$

$$a + b * c - [d e f^{\wedge \wedge}]$$

$$= a + [b c x] - [d e f^{\wedge \wedge}]$$

$$[a b c x +] - [d e f^{\wedge \wedge}]$$

$$= a b c x + d e f^{\wedge \wedge} _$$

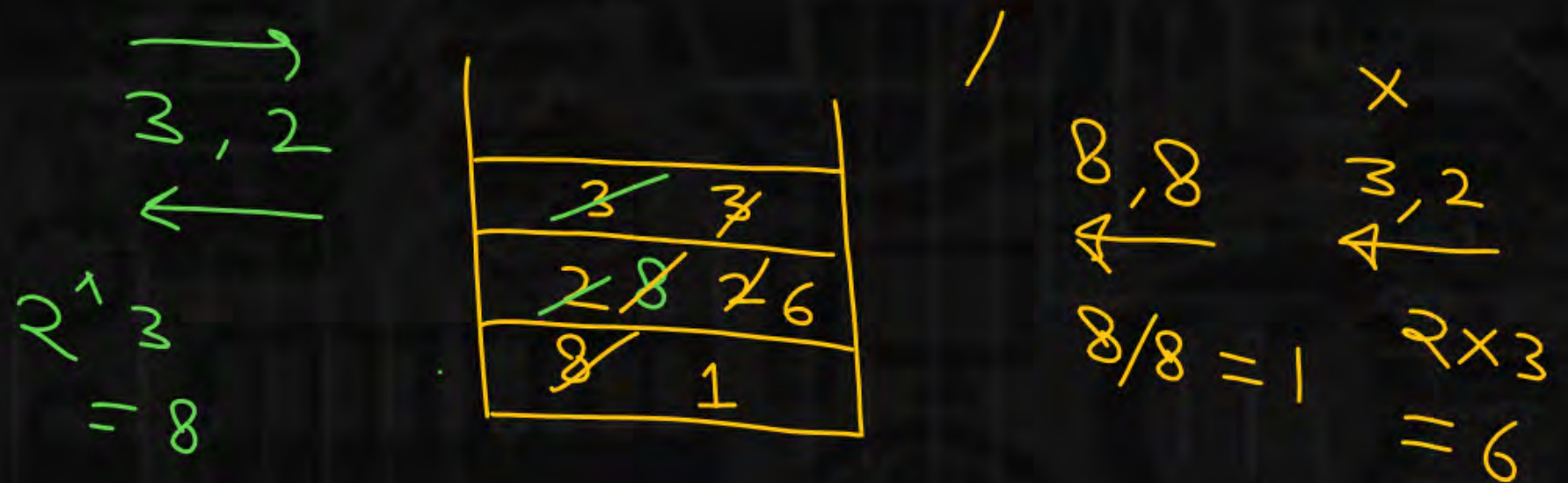
The following postfix expression with single digit operand is evaluated using a stack:

$$8\ 2\ 3\ ^\wedge\ /\ 2\ 3\ *\ +\ 5\ 1\ *\ -$$

Note that $^\wedge$ is the exponentiation operator. The top two elements of the stack after the first $*$ is evaluated are:

$$8\ 2\ 3\ ^\wedge\ /\ 2\ 3\ \times\ +\ 5\ 1\ \times\ -$$

- ☒ A 6, 1
- ☐ B 5, 7
- ☐ C 3, 2
- ☐ D 1, 5



Consider the following C program:

```
#include <stdio.h>
```

```
#define EOF -1
```

```
void push (int);
```

```
/*push the argument on the stack*/
```

```
int pop (void); /*pop the top of the stack */
```

```
void flagError();
```

```
int main()
```

```
{
```

```
    int c, m, n, r,
```

```
    while((c = getchar()) != EOF){
```

```
        if (isdigit(c) push(c);
```

```
        else if ((c == '+') || (c == '*')){
```

```
            m = pop();
```

```
            n = pop();
```

```
            r = (c == '+') ? n + m : n*m;
```

```
            push(r);
```

```
        }else if (c != ' ')
```

```
            flagError( );
```

```
        }
```

```
        printf(" %c ", pop());
```

```
}
```

What is the output of the program for the following?

5 2* 3 3 2 + * +

✗ ✗

A

15

~~B~~

25

C

30

D

150

5 2 ✗ 3 3 2 + ✗ +

10 3 3 2 + ✗ +

10 3 5 ✗ +

10 15 +

25

Let S be a stack of size $n \geq 1$. Starting with the empty stack, suppose we push the first n natural numbers in sequence, and then perform n pop operations. Assume that PUSH and POP operations take X secs each and Y seconds elapse between the end of one such stack operation and the start of the next operation. For $m \geq 1$, define the stack life-time of m as the time elapsed from the end of PUSH(m) to the start of POP operation that removes m from S . The average stack-life of an element is-

17. W

- A** $n(X+Y)$
C $3Y+2X$

- B** $n(X+Y)-X$
D $Y+2X$

The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
-	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression $2 - 5 + 1 - 7 * 3$ in this language is 9.

$$\begin{aligned} & [2 - (-1)] \times 3 \\ & \quad 3 \times 3 \\ & \quad 9 \end{aligned}$$

$$\begin{aligned} & \left[2 - \left(\overset{\checkmark}{5 + 1} - 7 \right) \right] \times 3 \\ & \quad (2 - (6 - 7)) \times 3 \end{aligned}$$

