CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No. 06

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

All mothers are female.

All of x — x is mother — x is female.

$\forall x$ — $MT(x)$ — $F(x)$

$$\forall x\,[MT(x) \quad F(x)]$$

Case 1:

$$\forall x\,[MT(x) \wedge F(x)]$$

Case 2:

$$\forall x\,[MT(x) \rightarrow F(x)]$$

All mothers are female

X.

$\forall x\,[MT(x) \wedge F(x)]$

F. $\wedge$	F
$\wedge$	
F $\wedge$	F
$\wedge$	
T $\wedge$ T	T

false

- BABA
- female.
- mother/female.

True.

$\forall x\,[MT(x) \rightarrow F(x)]$

ALL $(\rightarrow)$

F $\rightarrow$	True.
$\wedge$	
F $\rightarrow$	True
$\wedge$	
T $\rightarrow$ T	T.

Some cats are black.

Some of x, x is cat x is black. True

- maggie/white.
- Alia/bl
- cow

Domain: Animal.

$\exists x \qquad CT(x) \qquad Bl(x)$

$\exists x\,[\,CT(x) \wedge bl(x)\,]$

True

T $\wedge$ F	F
$\vee$	
T $\wedge$ T	T
$\vee$	
$\wedge$	

$\exists x\,[\,CT(x) \rightarrow bl(x)\,]$

True

T $\rightarrow$ F	F
$\vee$	
T $\rightarrow$ T	T
$\vee$	
$\rightarrow$	

False.

Some cats are black.

Some of x, x is cat x is black.

$\exists x$ $CT(x)$ $Bl(x)$

- Alia/W
- cow
- kat/W

X.

Some	$\rightarrow$
T	T
F	T.

Some	$\wedge$
T	T
F	F

$\exists x[CT(x) \wedge bl(x)]$

False

$\boxed{T \wedge F}$ F

$\boxed{F \wedge}$ F

$\boxed{T \wedge F}$ F

$\exists x[CT(x) \rightarrow bl(x)]$

True

$\boxed{T \rightarrow F}$ F.

$\boxed{F \longrightarrow}$ T.

Same behaviour :

	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$
some(T)	T	T	T	T.
Some(F)				

Some real no. are rationals (GATE)

A)

B) $\exists x\,[\text{Realno}(x) \wedge \text{Rational}(x)]$

C)

D

WARNING:

English → logical

we just convert

we dont find the truth value.

All graphs are connected.

$$\forall x \left[Graph(x) \rightarrow Connected(x) \right]$$

1. All graphs are connected.

$$\forall x\,[G(x) \rightarrow C(x)]$$

3. not (all graphs are connected)

$$\neg(\forall x\,(G(x) \rightarrow C(x)))$$

2. All graphs are not connected $\equiv$ 4. no graphs are connected.

$$\forall x\,[G(x) \rightarrow \neg C(x)] \qquad \forall x\,[G(x) \rightarrow \neg C(x)]$$

not all $\quad \neg \forall x[\rightarrow]$

no/none $\quad \forall x[\quad \rightarrow \neg \quad]$

all of my friends are perfect

not all of my friends are perfect

$\neg \forall x [\quad \rightarrow \quad]$

all of my friends are not perfect $\equiv$ none of my friends are perfect.

$\forall x [\quad \rightarrow \neg \quad]$

$p(x): x > 0$

$q(x): x$ is even

$r(x): x$ is perfect square.

$s(x): x$ is $\div$ by 4.

$t(x): x$ is $\div$ by 5

$$\forall x\, [q(x) \wedge r(x) \to s(x)]$$

$$\exists x\, [p(x) \wedge q(x)]$$

1) There exist a positive int that is even.

2) if x is even then x is not divisible by 5.

$$\forall x\, [q(x) \to \neg t(x)]$$

$2 = 3$

3) no even int is $\div$ by 5

4) There exist even int $\div$ by 5

5) if x is even and x is perfect square then x is $\div$ by 4.

not all

$\neg \forall x [\quad \rightarrow \quad]$

no/none.

$\forall x [\quad \rightarrow \neg \quad]$

no graphs are connected.

$\forall x [G(x) \rightarrow \neg C(x)]$

OR

$\forall x [\neg G(x) \vee \neg C(x)]$

$\forall x [\neg [G(x) \wedge C(x)]]$

$\boxed{\exists x [G(x) \wedge C(x)]}$

Gold and Silver ornaments are precious

(GATE)

A) $\forall x\,[G(x) \lor S(x) \to P(x)]$

B) $\forall x\,[\boxed{G(x) \land S(x)} \to P(x)]$

x

English
mathematics

$$\forall x \forall y \left[(x > y) \rightarrow (x - y > 0) \right]$$

$$\forall x \forall y \left[(x < y) \rightarrow \exists z (x < z < y) \right]$$

$$\forall x \forall y \left[|x| = |y| \rightarrow (y = \pm x) \right]$$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

Establish the validity of the following arguments.

a) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

b)
$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \neg r \\ \hline \therefore \neg(p \vee r) \end{array}$$

c)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) *P* and *R* only

(c) *P* and *S* only

(d) P, Q, R and S

d)

$$p \wedge q$$
$$p \rightarrow (r \wedge q)$$
$$r \rightarrow (s \vee t)$$
$$\neg s$$

$$\therefore t$$

e)

$$p \rightarrow (q \rightarrow r)$$
$$p \vee s$$
$$t \rightarrow q$$
$$\neg s$$

$$\therefore \neg r \rightarrow \neg t$$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

Let p, q, r and s be four primitive statements. Consider the following arguments:

$P: [(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

$Q: [(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

$R: [[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

$S: [p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

S_1: $\{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

S_2: $\{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

The statement formula $\{(a \rightarrow c) \wedge (b \rightarrow d) \wedge (c \rightarrow \sim d)\} \rightarrow (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

The statement formula

$\{((\sim p \vee q) \to r) \wedge (r \to (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \to \sim t)\} \to p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !