CS & IT ENGINEERING

Discrete Mathematics

GRAPH THEORY

Lecture No. 2

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TOPICS TO BE COVERED

01 Degree Sequence

02 Graphical sequence

03 Havell-Hakimi Thm

04 Inequalities Thm

05 Theorem no . 7

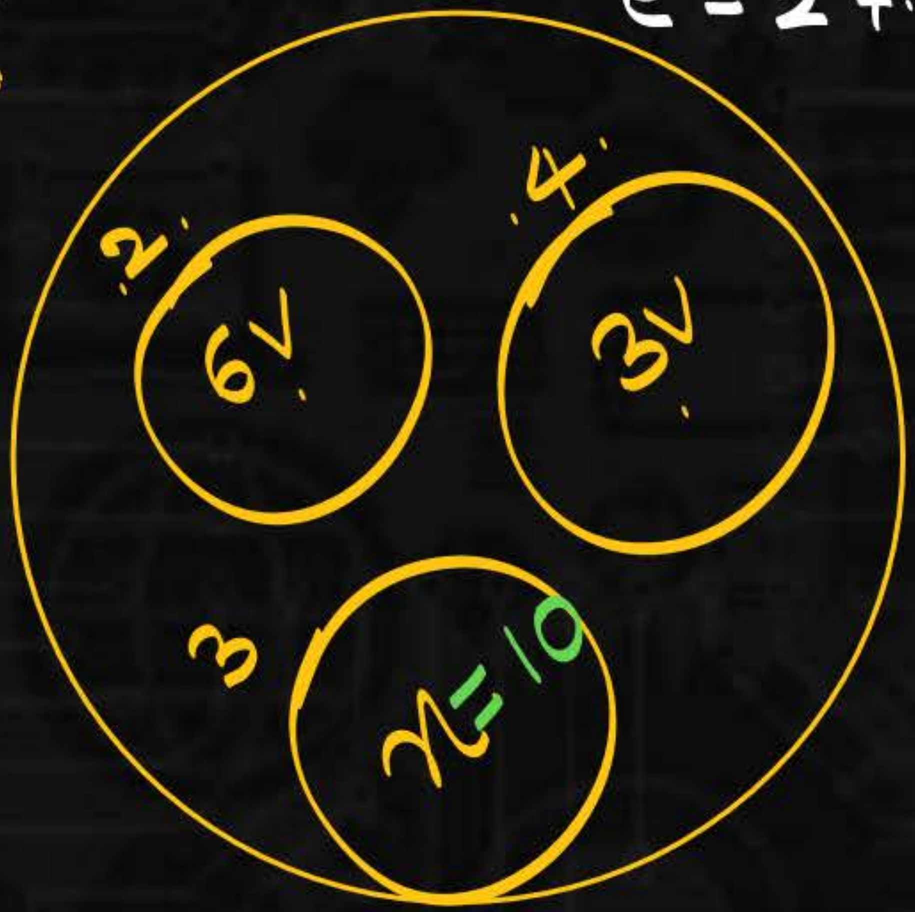
Degree Sequence

Ans: 19 (GATE)

Consider a Graph having 27 edges. 6 vertices which is having degree 2, 3 vertices which is having degree 4. and remaining vertices is having degree 3, Total vertices ?

Total vertices
$= 6+3+10$
$= 19$

$e = 27.$



x = remaining vertices.

$\sum d(v_i) = 2e$

$6\times 2 + 3\times 4 + x\times 3 = 2\cdot 27$

$12 + 12 + 3x = 54.$

$3x = 54 - 24$

$3x = 30$

$x = 10$

Degree Sequence

G has order 14 (vertices), size 27 (edge). Degree of each vertex of G is 3, 4 or 5. There 6v of degree 4 how many vertices will have degree 3 ?

let x be total vertices of degree 3.

$\Sigma d(v_i) = 2e$

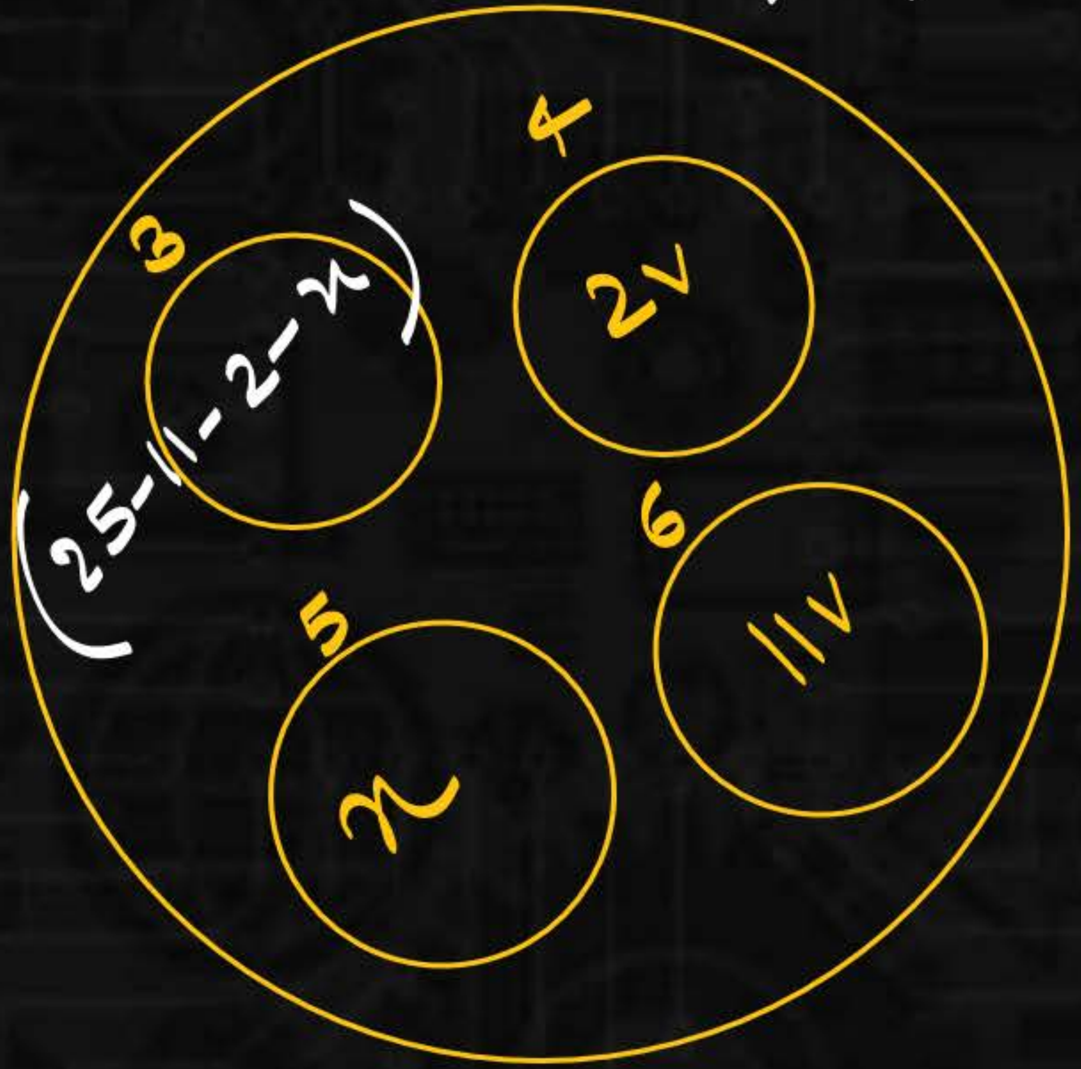
$3 \cdot x + 6 \cdot 4 + 5(8 - x) = 2 \cdot 27$

$\boxed{x = 5}$

Degree Sequence

$n = 25 \quad e = 62.$

Degree of every vertex of Graph of order 25, size 62 is 3, 4, 5 or 6. There 2 vertices of degree 4. 11 vertices of degree 6 how many vertices will have degree 5?



$x = 7$

Degree Sequence

$\delta(G) = 3$.

Graph $e = 35$, degree of each vertex is atleast 3.

maximum no. of vertices ?

$\delta(G) = 3$ $\quad e = 35$

$\delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1$.

$\delta(G) \le \frac{2e}{n}$

$3 \le \frac{2 \cdot 35}{n}$

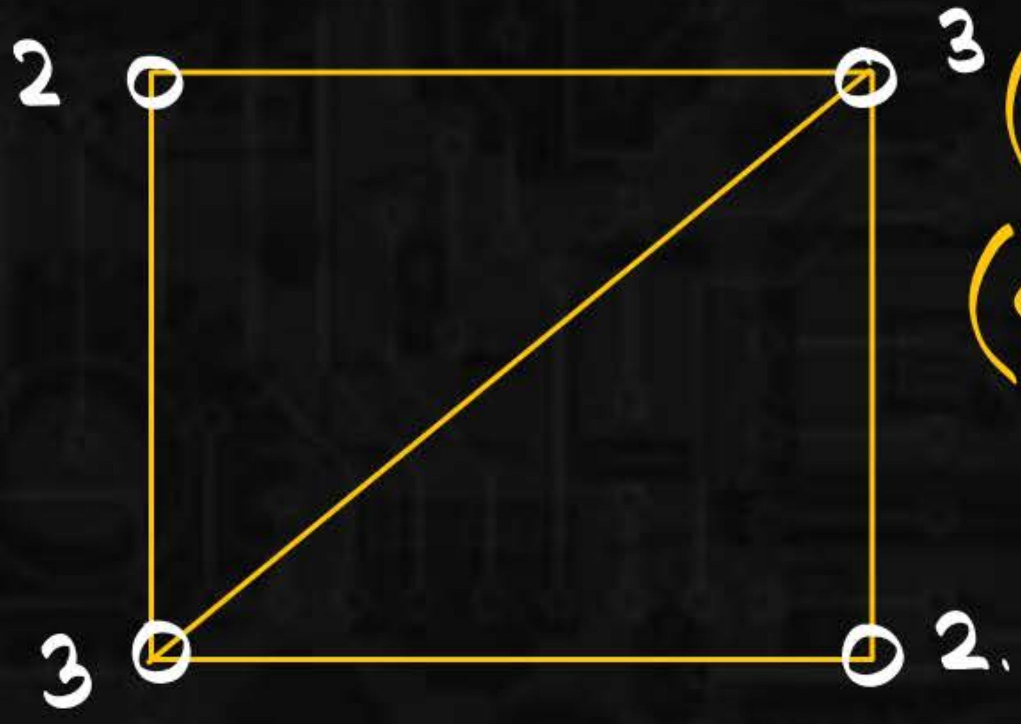
$n \le \frac{70}{3}$

$n \le 23.3$

$n = 23$

Degree Sequence

$\delta(G)$ $\Delta(G)$



($\delta(G)$) minimum degree = 2.

($\Delta(G)$) maximum degree = 3

$$\frac{2e}{n} = \frac{3+3+2+2}{4} = \frac{10}{4} =$$

$$\boxed{\delta(G) < \frac{2e}{n} < \Delta(G)} \quad - \text{(I)}$$

Degree Sequence

$$\boxed{\delta(G) = \frac{2e}{n} = \Delta(G)}$$

$\delta(G) = 2.$

$\Delta(G) = 2.$

$$\text{avg. degree} = \frac{(2+2+2+2)}{(\text{Total vertices})} = \frac{8}{4} = 2.$$

$$= \frac{\Sigma d(v_i)}{n} \qquad \Sigma d(v_i) = 2e.$$

$$= \frac{2e}{n}.$$

Degree Sequence

Thm: $\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$

$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1.$

atleast.

atmost degree.

Thm 3:

In simple Graph

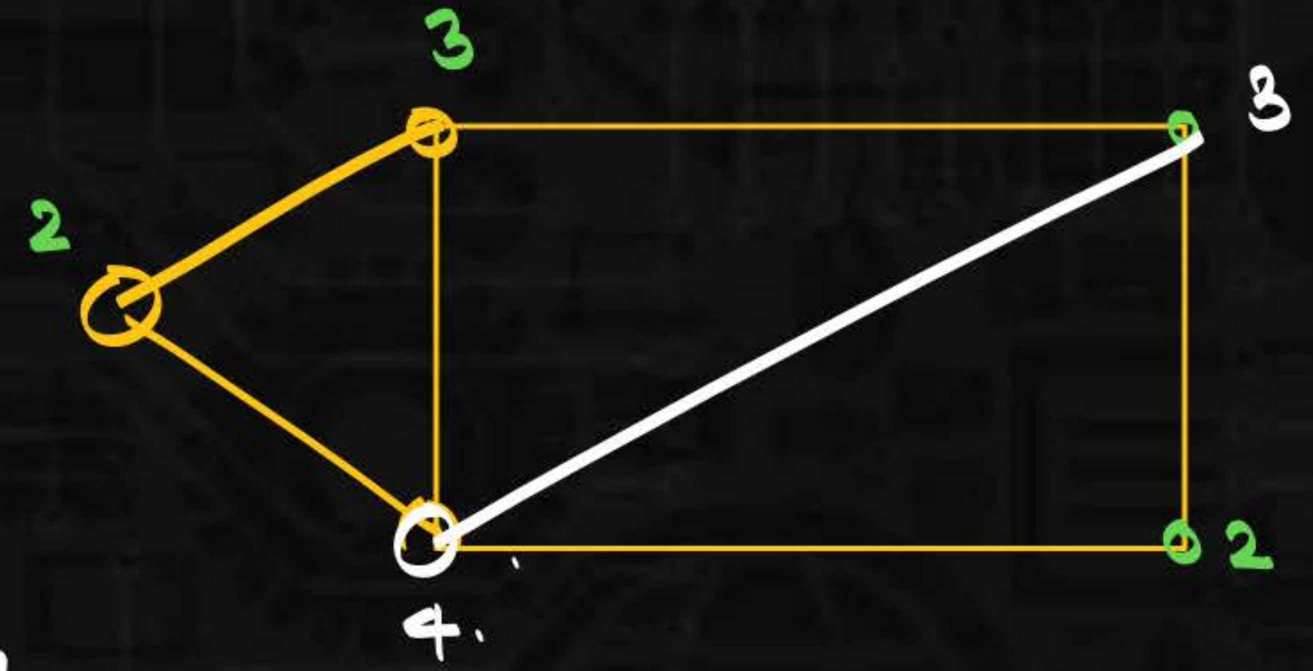
maximum degree $\leq n-1$.

$\Delta(G) \leq n-1.$

Degree Sequence

Degree sequence:

writing degrees of all vertices either in increasing or decreasing order.



$\{4, 3, 3, 2, 2.$

OR.

$\{2, 2, 3, 3, 4.$

Degree Sequence

5, 2, 2, 2, 2, 1. what will be edges in Graph?

m1:

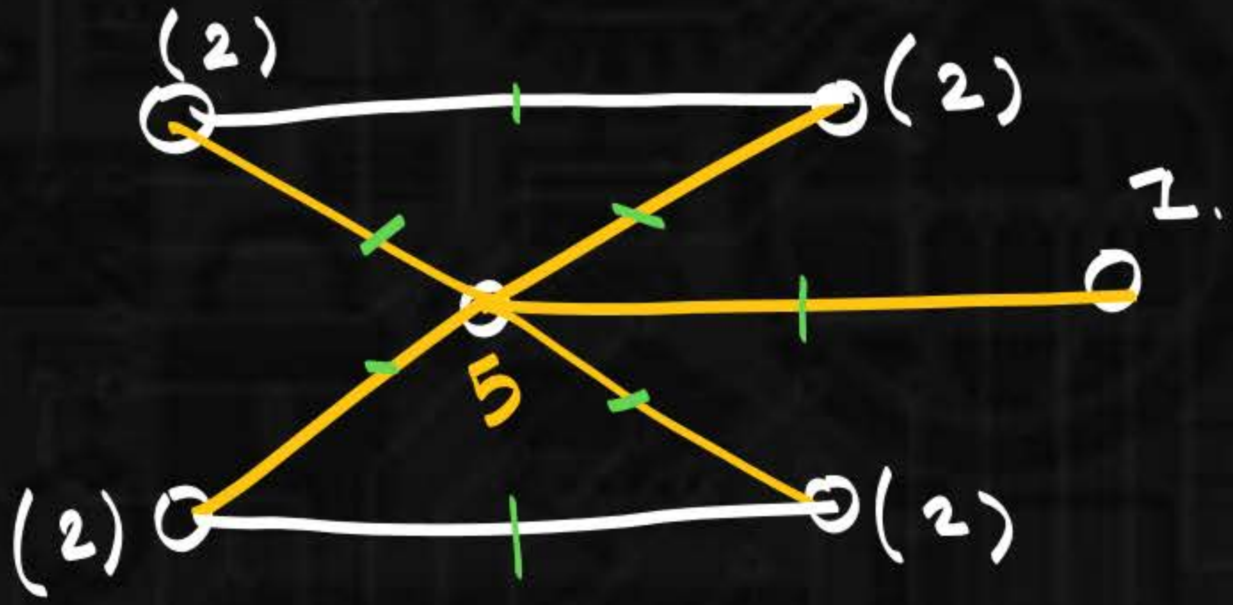
$$\sum d(v_i) = 2e.$$

$$5+2+2+2+2+1 = 2e.$$

$$14 = 2e$$

$$e = 7$$

m2: 5, 2, 2, 2, 2, 1. Total vertices = 6.



$e = 7$

Degree Sequence

3, 3, 3, 1, what will be edges in G?

$\Sigma d(v_i) = 2e$

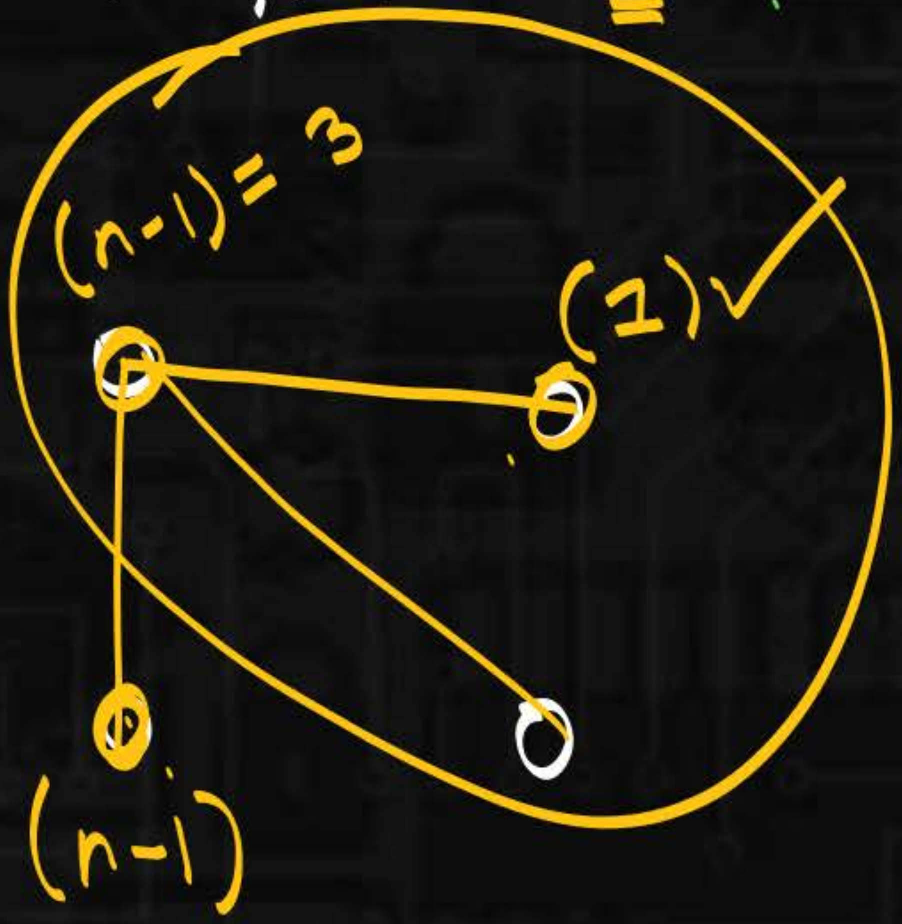
$3+3+3+1 = 2e$

$10 = 2e$

$e = 5$

3, 3, 3, 1 Totalvertices = 4

(n-1), (n-1) . . . 1



no simple Graph.

Degree Sequence

$\boxed{\text{Degree sequence} \rightarrow \text{simple Graph.}}$ = Graphical sequence

Degree sequence $\not\rightarrow$ simple Graph

5, 2, 2, 2, 2, 1. → Graphical sequence.

3, 3, 3, 1 → not Graphical sequence.

Degree Sequence

Graphical ?

A) 5, 4, 3, 2, 1. ✗

B) 4, 4, 3, 2, 1. ✗

C) 3, 3, 3, 3, 2. ✓

d) 2, 2, 2, 2. ✓

e) 1, 1, 1, 1, 1. ✗

not Graphical.

A) 5, 4, 3, 2, 1.

$n = 5$ $\quad \Delta(G) \leq n-1$

$\Delta(G) \leq 4$

Reason 1: Thm 2 violate.

Reason 2: Thm 3. $\Delta(G) \leq n-1$.

Reason 3:

Degree Sequence

b) 4, 4, 3, 2, 1.

3, 3, 3, 1.

[n-1, n-1 . . . . 1] → not Graphical

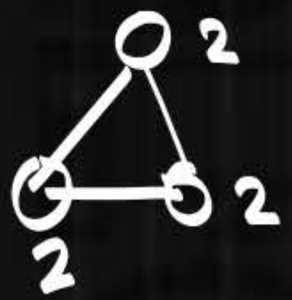
Total vertices = n.

n-1, n-1 . . . .

n = 3

n-1, n-1, n-1

2, 2, 2

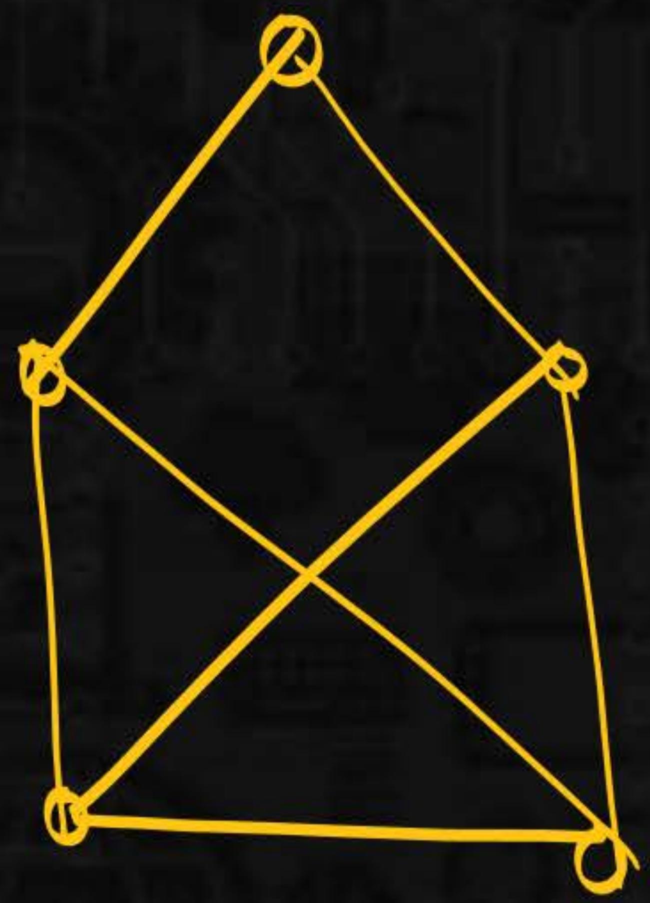


n-1, n-1 1

→ not Graphical.

Degree Sequence

c) 3, 3, 3, 3, 2.



d) 2, 2, 2, 2.

E) 1, 1, 1, 1, 1.

Thm 2 violates.

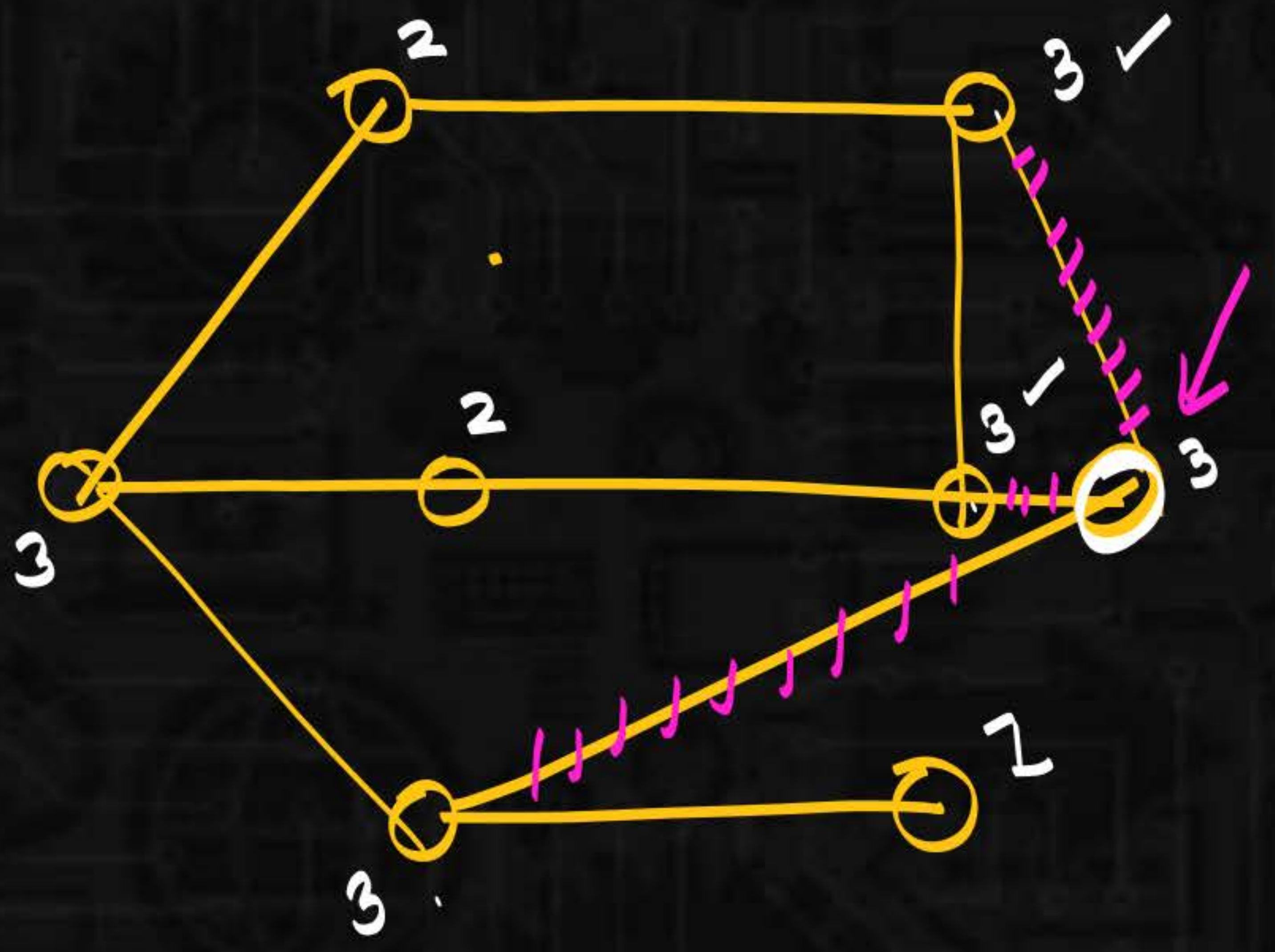
no. of odd degree vertices should be even.

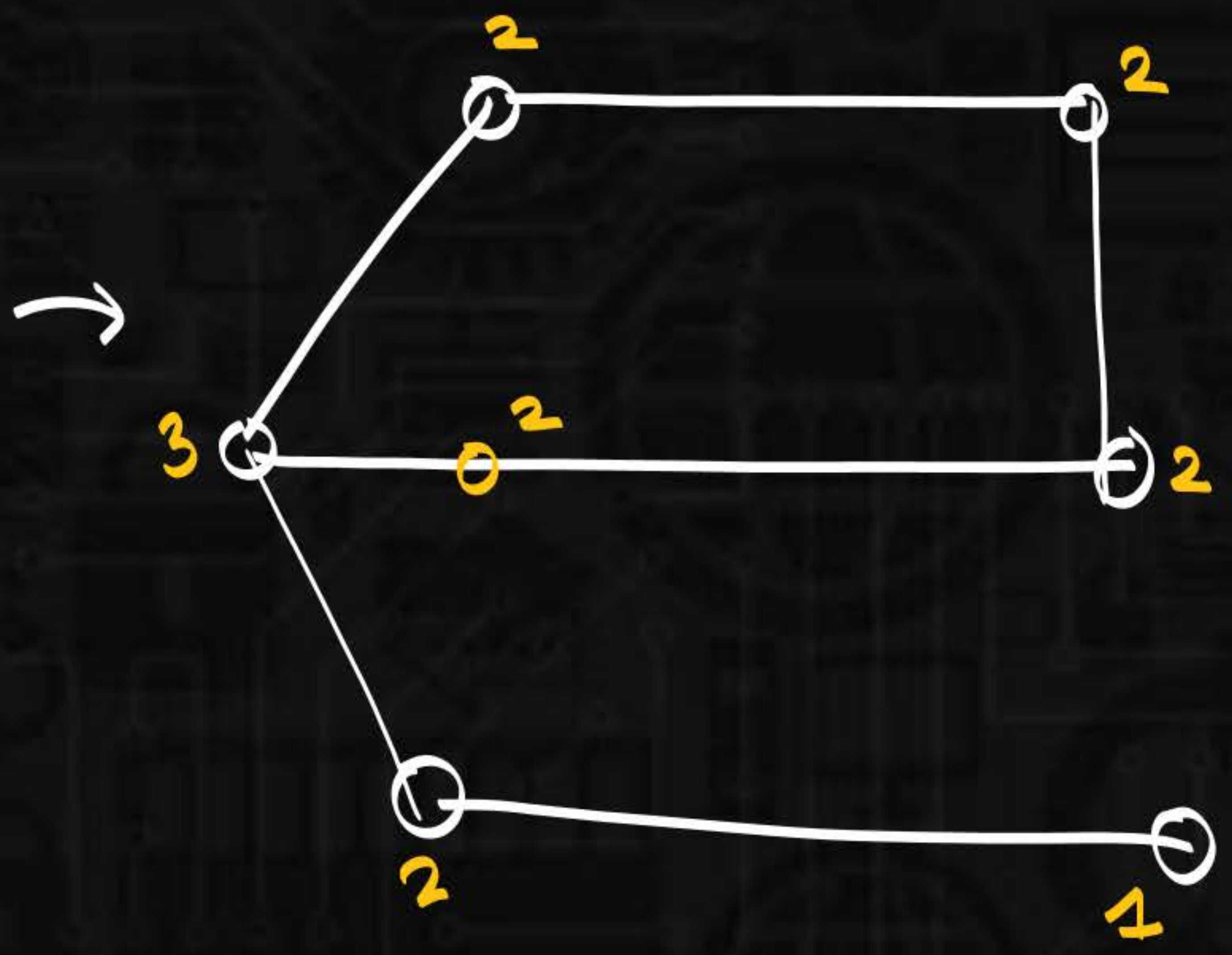
Degree Sequence

~~3~~, 3, 3, 3, 3, 2, 2, 1

2 2 2 3 2 2 1

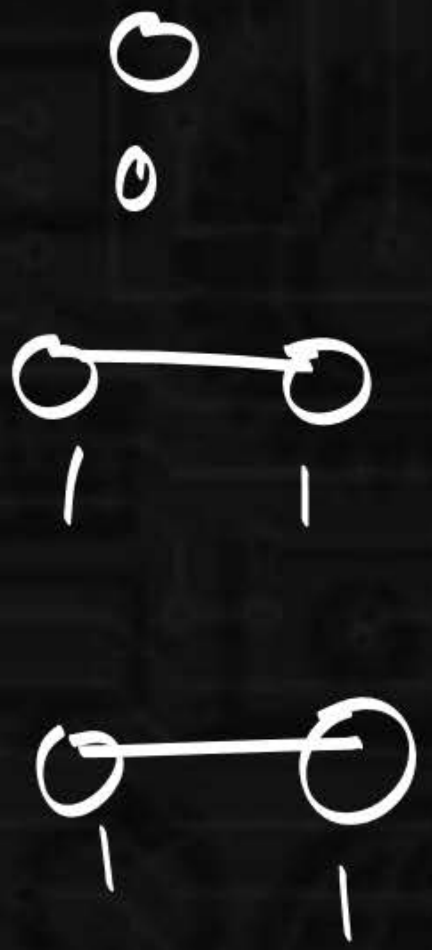
3 2 2 2 2 2 1.





Degree Sequence

11110



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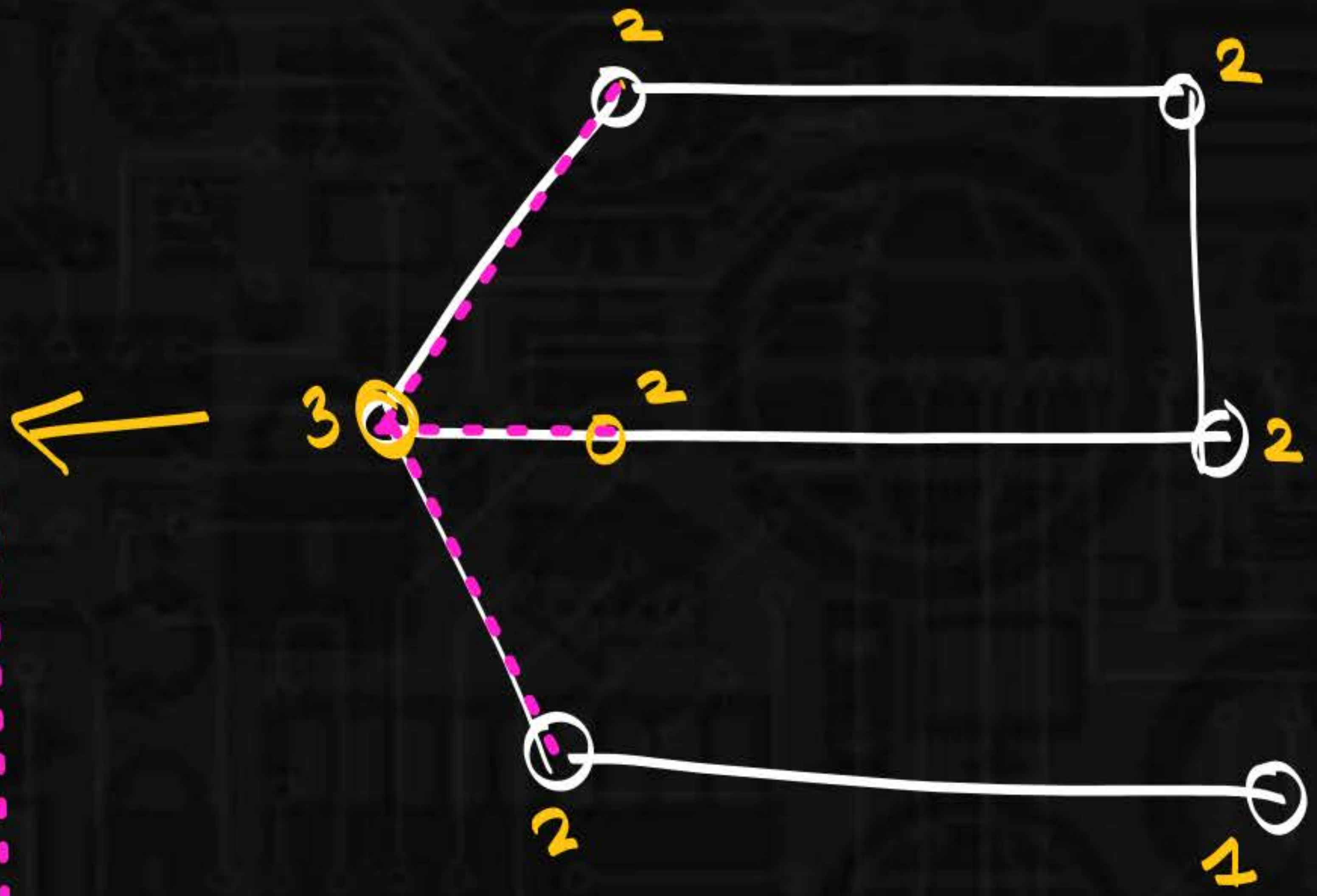

Degree Sequence

~~3~~, 3, 3, 3, 3, 2, 2, 1.

2, 2, 2, 3, 2, 2, 1

→ ~~3~~, 2, 2, 2, 2, 2, 1 (ordering)

1, 1, 1, 2, 2, 1

~~2~~, 2, 1, 1, 1, 1

1, 0, 1, 1, 1

~~1~~ 1 1 1 0 (ordering)

0 1 1 0 → 1 1 0 0 (ordering)

Graphical?

cut, count, mark
dlt 1.
ordering.

Degree Sequence

Graphical ?

A) 5, 4, 3, 2, 1. ✗

B) 4, 4, 3, 2, 1. ✗

C) 3, 3, 3, 3, 2. ✓

d) 2, 2, 2, 2. ✓

e) 1, 1, 1, 1, 1. ✗

~~5~~, 4, 3, 2, 1.

not Graphical

~~4~~, 4, 3, 2, 1.

~~3~~, 2, 1, 0

1, 0, -1.

not graphical.

Degree Sequence

Graphical ?

A) 7, 6, 5, 4, 4, 3, 2, 1. ✓

B) 6, 6, 6, 6, 3, 3, 2, 2. ✗

C) 7, 6, 6, 4, 4, 3, 2, 2. ✓

D) 8, 7, 7, 6, 4, 2, 1, 1. ✗

cut, count, mark.

dlt (-1)

ordering.

Degree Sequence

$\cancel{7}, 6, 6, 4, 4, 3, 2, 2.$

$\cancel{5}, 5, 3, 3, 2, 1, 1.$

$4, 2, 2, 1, 0, 1$

(ordering)

$\cancel{4}, 2, 2, 1, 1, 0.$

$1, 1, 0, 0, 0.$

Degree Sequence

steps for @egree sequence Q.

1) check Thm 2.

2) check Thm 3.

3) n-1, n-1 .... 1.

4) if all degrees are distinct (simple Graph is not possible)

eg: 5, 4, 3, 2, 1.

5) Havell- Hakimi


THANK YOU GW
SOLDIERS !