CS & IT ENGINEERING

Discrete maths

Graph theory

Lecture No. 3

By- SATISH YADAV SIR

01 Complete Graph

02 Cycle graph

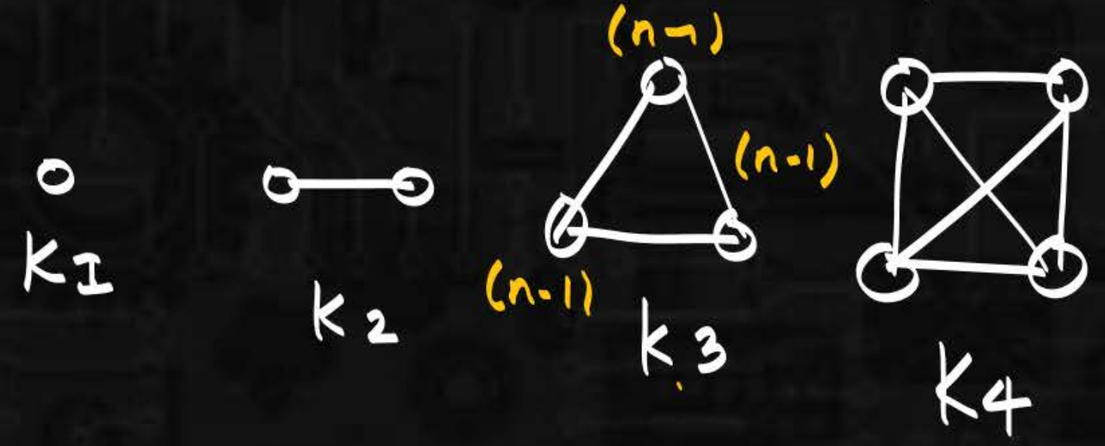
03 Wheel graph

04 Bipartite graph

05 Line graph

Types of graph

Complete Graph $(K_n)(n \geq 1)$



1. D.S: $n-1, n-1, n-1 \ldots n-1$ (n times)

$\sum d(v_i) = 2e$.

$n.(n-1) = 2e$.

2. $e = \frac{n(n-1)}{2}$.

3. $\delta(G) = \frac{2e}{n} = \Delta(G) = n-1$.

Types of graph

Regular Graph: $\left(\delta(G) = \frac{2e}{n} = \Delta(G) \right)$

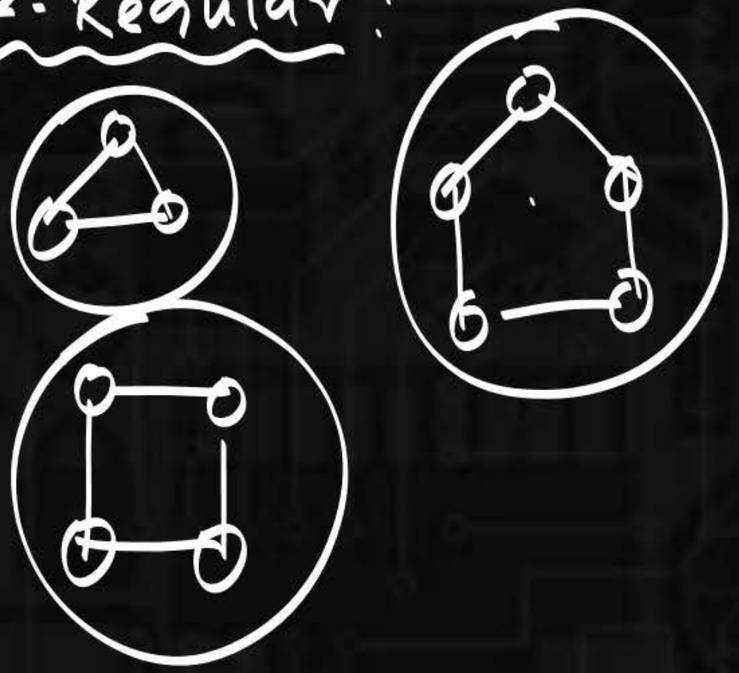
Degrees of all vertices are same.

Degrees of all vertices are k.

k-regular Graph.

1-Regular

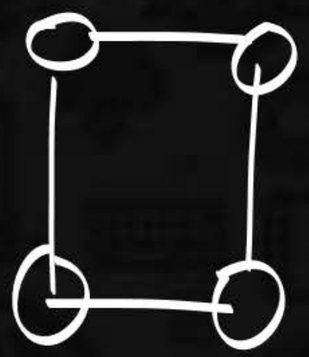
2-Regular:



Types of graph

All K_n are regular Graph.

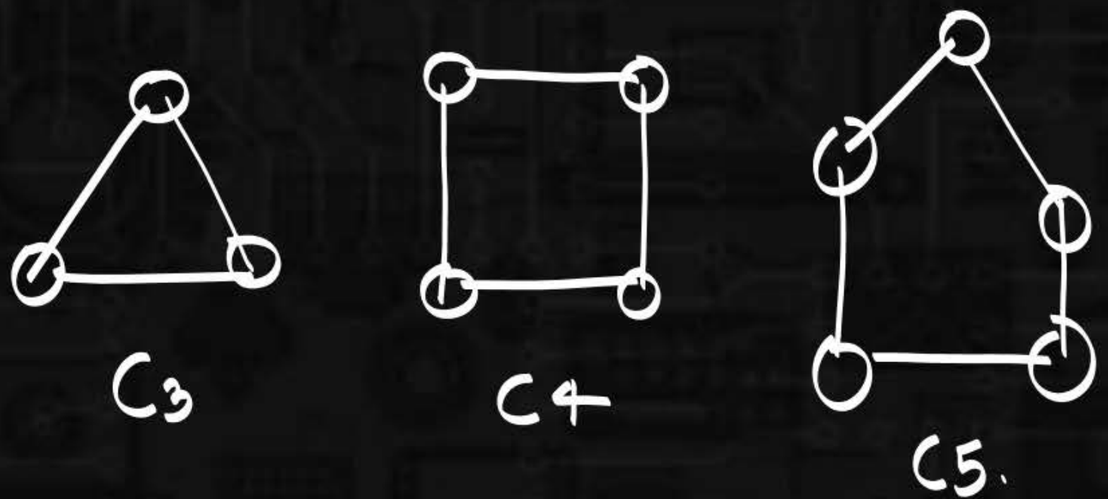
→ all regular Graphs are K_n (false)



2- Regular

Types of graph $\{\delta(G) = \frac{2e}{n} = \Delta(G) = 2\}$

Cycle Graph $(C_n)(n \geq 3)$



1. Degrees of all vertices in C_n is 2.

2. $\Sigma d(v_i) = 2e$.

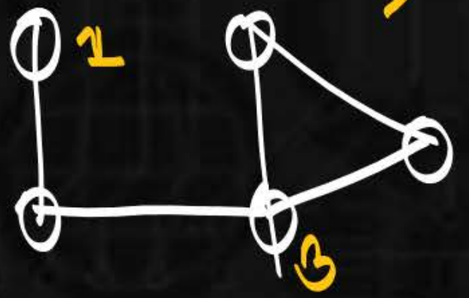
$n \cdot 2 = 2e$

$n = e$

3. $C_n \rightarrow n = e$ (T)

$n = e \rightarrow C_n$ (False)

$n = 5$ $e = 5$



4. all C_n are regular Graph.

all regular Graphs are not C_n.

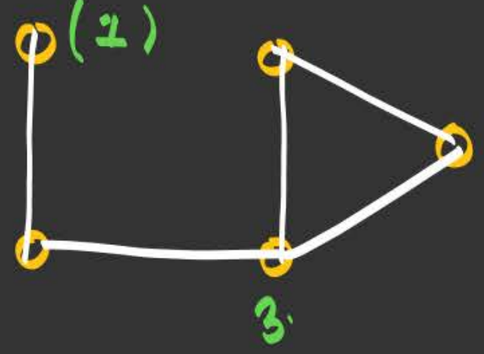
$C_n \longrightarrow n = e.$
(True)

(false)

if Graph is having $n = e$ then it is cycle Graph.

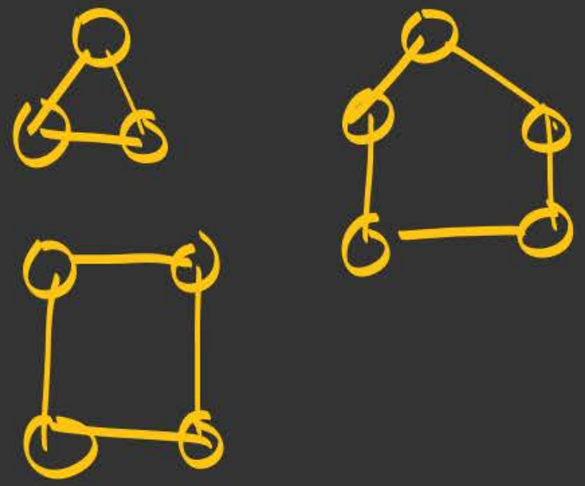
Case 1:

$n = 5$ $e = 5$



. . . . . C5

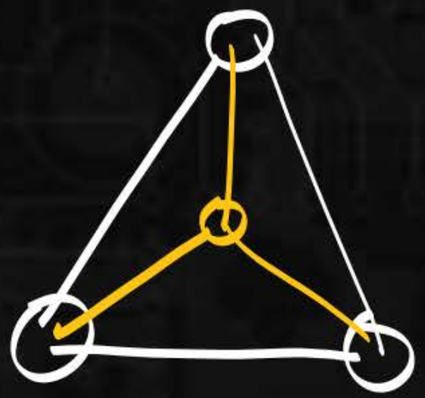
All C_n are Regular Graph (T)



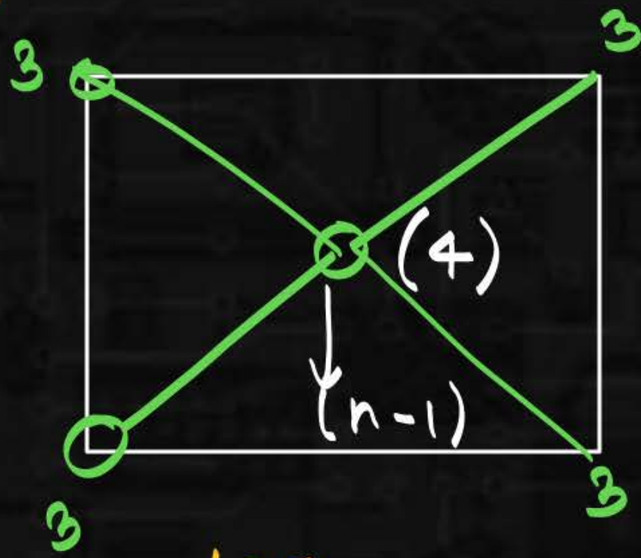
all regular Graphs are cycle Graph.

Types of graph

Wheel Graph (W_n) $(n \geq 4)$



W_4



W_5

$n-1$ connect corner

C_4 $(n-1)$

$n=5$ W_5 $e=4$ $(n-1)$ $e=4$

W_n $n-1 + n-1$

$e(W_n) = 2n - 2$

$= 2(n-1)$

DS: $n-1$, (3, 3, 3, 3, 3 .... 3)

← n-1 times →

W_{100}: 99, (3, 3, 3 ..... 3)

... 99 times ...

Types of graph

if Graph is (W_n) then $e = 2(n-1)$. (True)

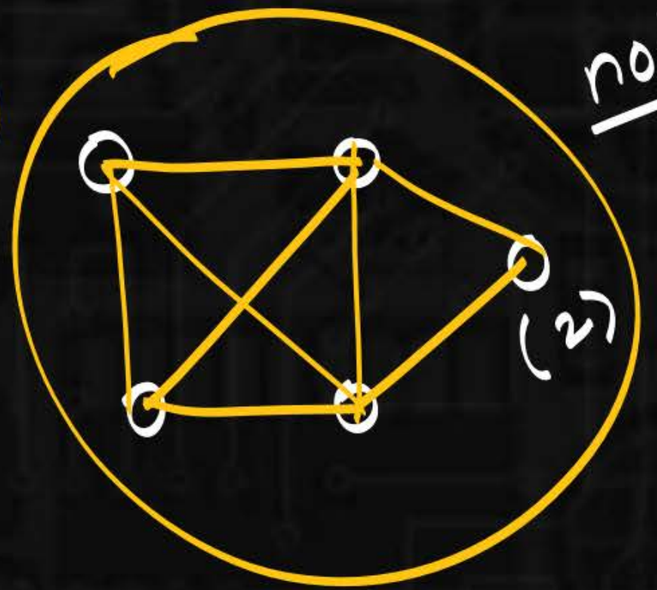
if G is having $e = 2(n-1)$ then G is W_n. (false)

not wheel Graph.

$n = 5 \quad e = 2(n-1)$

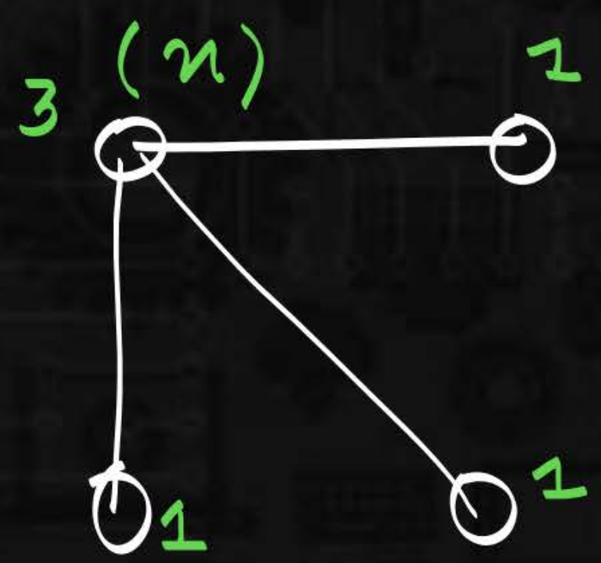
$= 2(4)$

$= 8$

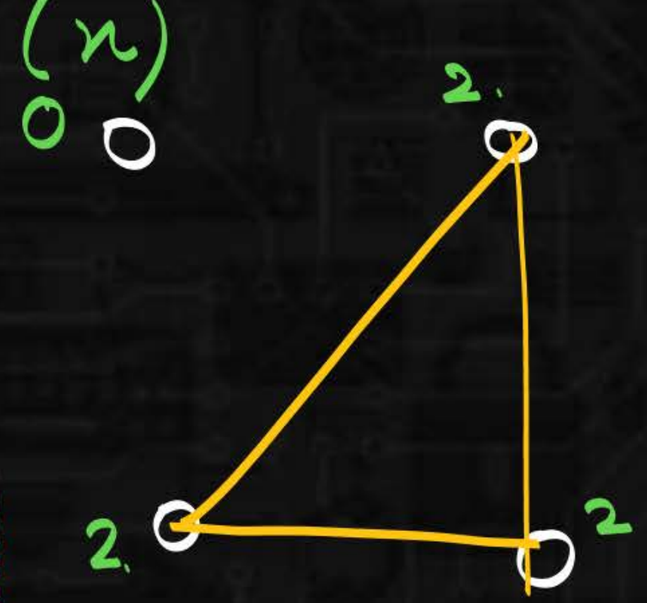


Types of graph

Complement Graph : ($\bar{G}$)



edges → absent



edges → present

$$G + \bar{G} = K_n$$

$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

$$e(\bar{G}) = \frac{n(n-1)}{2} - e(G)$$

K_4 3 3 3 3

$G \rightarrow$ 3, 1, 1, 1. Total vertices = 4.

$\rightarrow \{\bar{G}$ 0, 2, 2, 2.$\}$

1) $e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$.

2)

K_n $\quad n-1, n-1, n-1, \ldots n-1$

$G \rightarrow d_1, d_2, d_3, \ldots d_n$

$\bar{G}$ $\quad n-1-d_1, n-1-d_2, n-1-d_3, \ldots n-1-d_n$.

Types of graph

if $n = 10$ $e(G) = 20$ $e(\bar{G}) = ?$

$e(\bar{G}) = 25$

$n(n-1) = 272$

$n = 17$

→ $e(G) = 56$ $e(\bar{G}) = 80$ $n = ?$

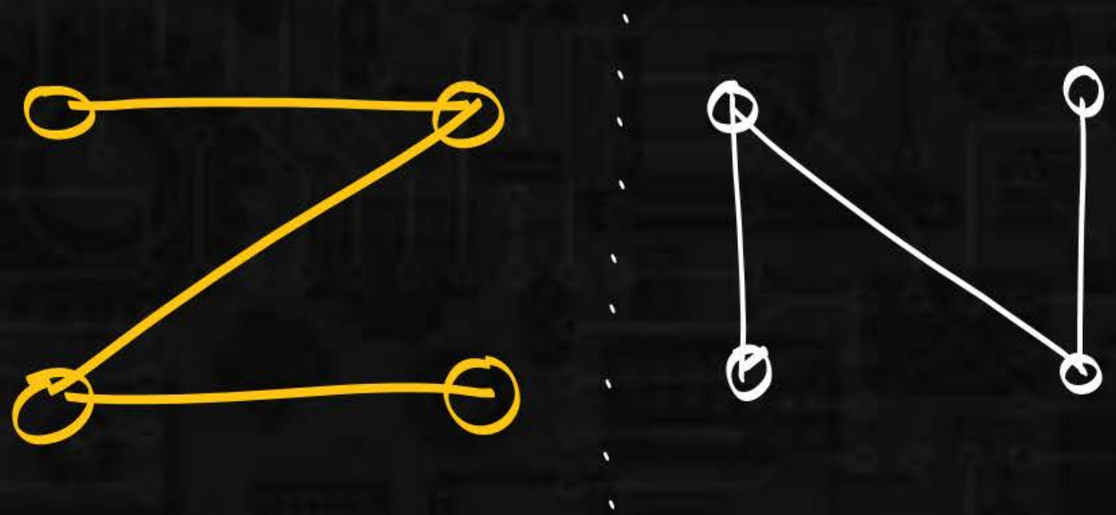
$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

$$56 + 80 = \frac{n(n-1)}{2}$$

$$2(136) = n(n-1)$$

Types of graph

Self - complement ($G \equiv \bar{G}$)



$$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$$

$$e + e = \frac{n(n-1)}{2}$$

$$2e = \frac{n(n-1)}{2}$$

$$\boxed{e = \frac{n(n-1)}{4}}$$

$n = 4 \quad e = \frac{n(n-1)}{4} = \frac{4 \cdot 3}{4} = 3$

$\frac{n}{4}$ OR $\frac{n-1}{4}$

$n = 5 \quad e = \frac{5 \cdot 4}{4} = 5$

$n = 6 \quad e = \frac{6 \cdot 5}{4} = \frac{15}{2} = 7.5$ X

$n = 7 \quad e = \frac{7 \cdot 6}{4}$

$\left\{ \frac{n}{4} \text{ OR } \frac{n-1}{4} \right.$

$\frac{n-0}{4}$ OR $\frac{n-1}{4}$

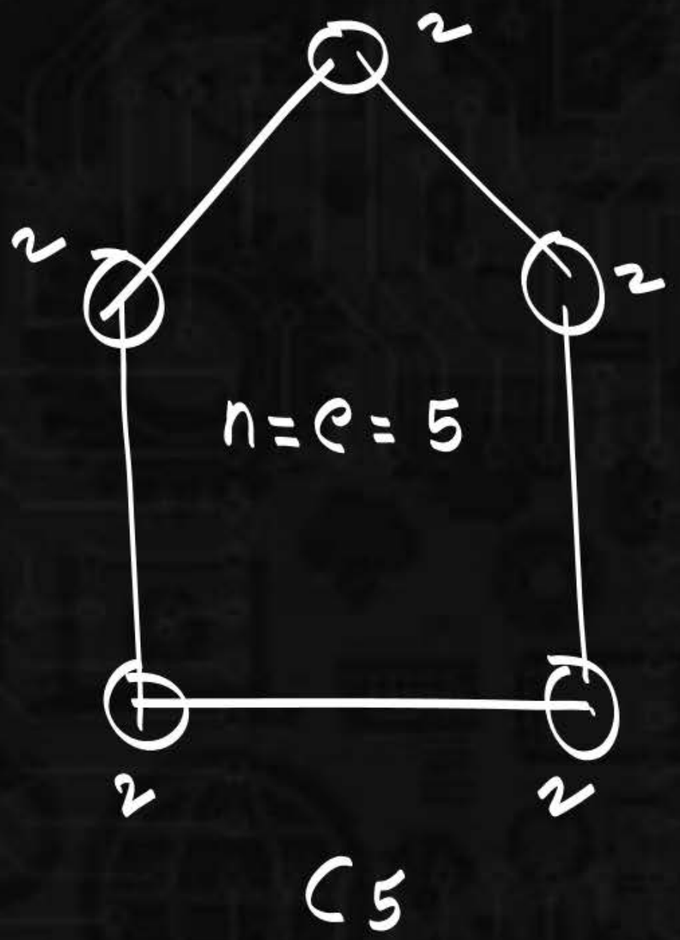
$n \equiv 0 \pmod 4$ OR $n \equiv 1 \pmod 4$

$n \equiv 0$ OR $1 \pmod 4$

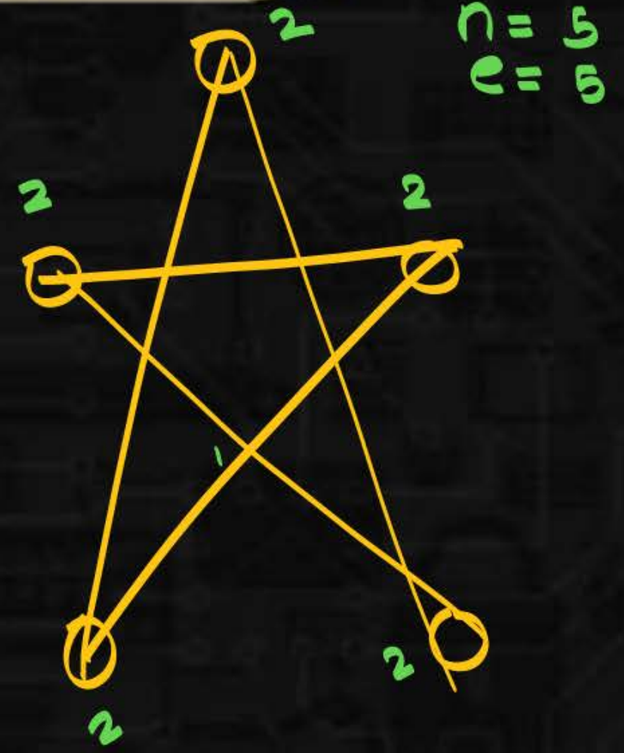
1. a, b are having same remainder when divides by n.
2. $a \equiv b \pmod n$ $\quad 0 \equiv 4 \pmod 4$ $\quad 1 \equiv 5 \pmod 4$
3. $\frac{a-b}{n} \in z$. $\quad \frac{1-5}{4} = -\frac{4}{4} = -1 \in z$.

$1 \equiv 2 \equiv 3$

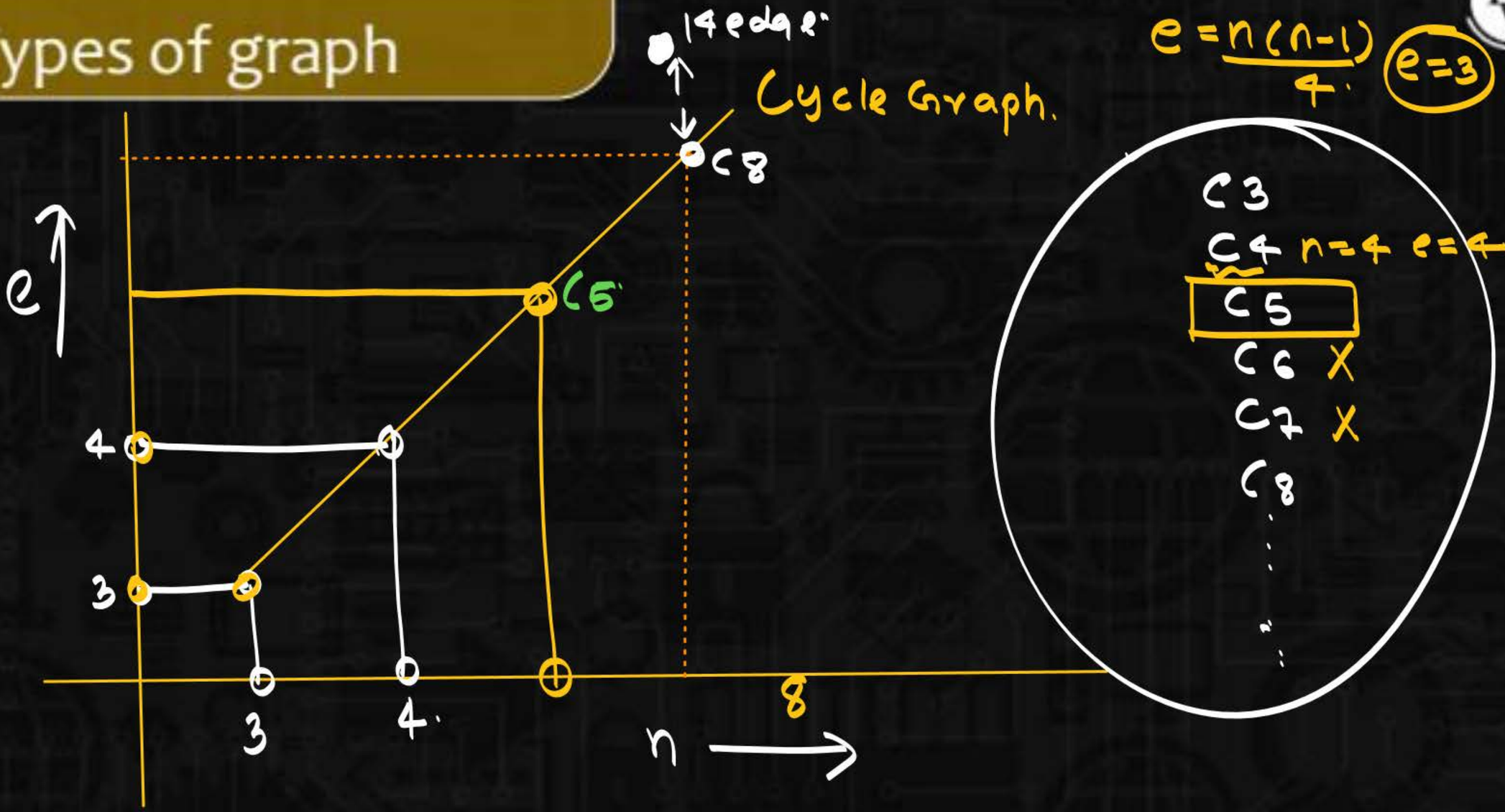
Types of graph



C_5

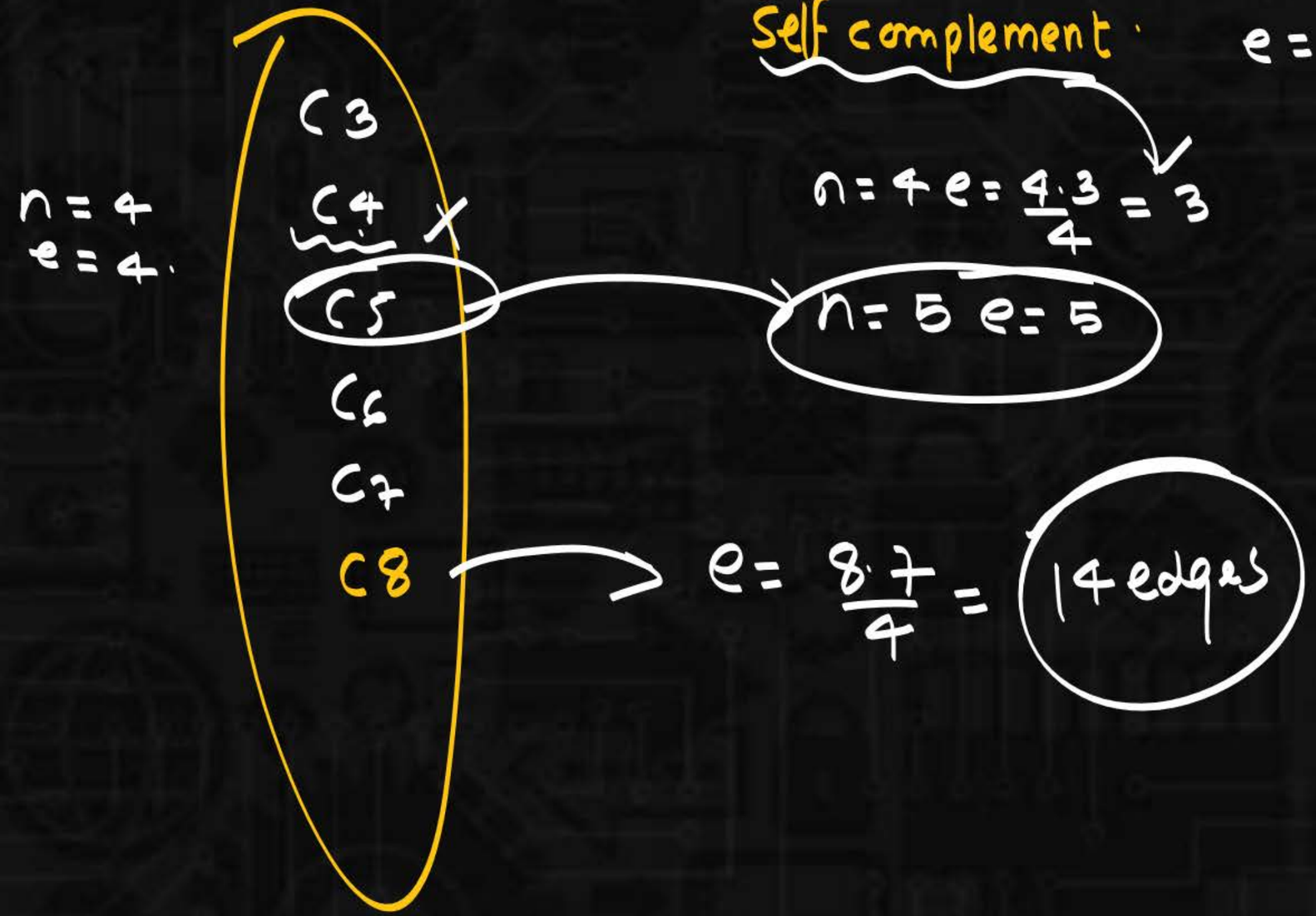


Types of graph



$e = \frac{n(n-1)}{4}$ $e=3$

Types of graph




THANK YOU GW
SOLDIERS !