CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 04

By- SATISH YADAV SIR

01 sum rule

02 Product rule

03 Practice

COMBINATORICS

$$\frac{1}{1-ax} = 1 + ax + (ax)^2 + (ax)^3 + (ax)^4 + \cdots$$

$a = 1$

$$\frac{1}{1-x} = (1 + x + x^2) + x^3 + x^4 + \cdots \infty$$

$a = 2$

$$\frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + \cdots$$

$$\frac{1}{1+ax} = 1 - ax + (ax)^2 - (ax)^3 + (ax)^4 - \cdots$$

$a = 1$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$$

$a = 2$

$$\frac{1}{1+2x} = 1 - 2x + (2x)^2 - (2x)^3 + \cdots$$

COMBINATORICS

How many ways to select 6 students in a class of 10?

$(x^1 + x^0) \times (x^1 + x^0)$

$(x+1) \times (x+1) \ldots\ldots$

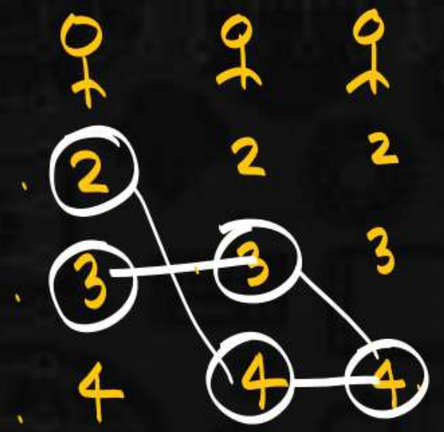
$10C_6$.

$(1+x)^{10}$

coefficient of x^6.

COMBINATORICS

How many ways we can distribute 10 coins among 3 children such that each child gets at least 2 coins and atmost 4 coins?



Ans:

2, 4, 4
4, 2, 4
4, 4, 2
3, 3, 4
4, 3, 3
3, 4, 3

Giving 2 coins is same as writing x^2.

$x^2 \times x^4 \times x^4 = x^{10}$ 3 coins — " — x^3.

COMBINATORICS

$(x^2+x^3+x^4)\times(x^2+x^3+x^4)\times(x^2+x^3+x^4)$

Coefficient of x^{10} : $x^2 \cdot x^4 \cdot x^4$: x^{10}

$x^2 \cdot x^4 \cdot x^4 = x^{10}$

$x^{e_1+e_2+e_3} = x^{10}$ $x^{e_1} \times x^{e_2} \times x^{e_3} = x^{10}$

$e_1+e_2+e_3=10$ atleast attempt.

COMBINATORICS

$(x^2+x^3+x^4) \times (x^2+x^3+x^4) \times (x^2+x^3+x^4)$

$(x^2+x^3+x^4)^3$ coefficient of x^{10}

take x^2 common.

$(x^2)^3 (1+x+x^2)^3$

$x^6 (1+x+x^2)^3 = x^6 \left(\frac{1-x^3}{1-x}\right)^3 = x^6 (1-x^3)^3 (1-x)^{-3}$

COMBINATORICS

$x^6(1-x^3)^3(1-x)^{-3}$

$x^6(1-3x^3+3x^6-x^9)\times(1-x)^{-3}$

$(x^6-3x^9+3x^{12}-x^{15})(1-x)^{-3}$

$(x^6-3x^9+3x^{12}-x^{15})(-3C_0\,1^{-3-0}(-x)^0+-3C_1\,1^{-3-1}(-x)^1$

no use no use

$+-3C_2\ldots\ldots$

$x^6\times-3C_4(1)^{-}(-x)^4-3x^9\times-3C_1\,1(-x)\ldots-3C_4\,1^{-3-4}(-x)^4$

coefficient of x^{10}

when x^6 will multiply by x^4 it will give x^{10}

COMBINATORICS

$$(x^6 - 3x^9 + \dots)(1-x)^{-3}$$

$$x^6 \times \left({}^{-3}C_4\, 1\,(-x)^4\right) - 3x^9\left({}^{-3}C_1\, 1\,(-x)^7\right)$$

$$\left(x^6\, \frac{(-1)^4}{1}\, {}^{3+4-1}C_4\, \frac{(-1)^4 x^4}{1}\right) - 3x^9\left((-1)\, {}^{3+1-1}C_1\, (-1)\times x\right)$$

$$x^{10}\, {}^{6}C_4 - 3x^9 x^1\, {}^{3}C_1 = x^{10}\left({}^{6}C_4 - 3\times 3\right)$$

$$= x^{10}(15-9) \neq 6x^{10}$$

COMBINATORICS

$$(x^2+x^3+x^4)^3$$

$$(x^2)^3(1+x+x^2)^3$$

$$x^6\left(\frac{1-x^3}{1-x}\right)^3$$

$$x^6(1-x^3)^3(1-x)^{-3}$$

$$x^6(1-3x^3+3x^6-x^9)(1-x)^{-3}$$

$$x^6\left(-3_{C_1}(1)(-x)^4\right)-3x^9\left(-3_{C_1}(1)(-x)\right)$$

-ve. -ve

$$\left(6_{C_4}x^{10}-3\times 3_{C_1}x^9\cdot x\right)$$

$$(15-9)x^{10}=6x^{10}$$

COMBINATORICS

(GATE)

Coefficient of x^{12} in $(x^3+x^4+x^5\ldots)^3$

$(x^3)^3(1+x+x^2\ldots)^3$

$x^9\left(\frac{1}{1-x}\right)^3$

$x^9(1-x)^{-3}$

$x^9 \times \left(\frac{-3 \cdot -4 \cdot -5}{-x \cdot -x \cdot -x}\right)$

$x^9\left((-1)^3\ {}^{3+3-1}C_3\ 1\cdot(-1)^3\cdot x^3\right)$

${}^5C_2\, x^{12}$

COMBINATORICS

How many ways to distribute 15 similar coins among 4 children, such that each child gets atleast 3 & atmost 6.

$(x^3+x^4+x^5+x^6)^4$

$(x^3)^4(1+x+x^2+x^3)^4$

$x^{12}\left(\frac{1-x^4}{1-x}\right)^4$

$x^{12}(1-x^4)^4(1-x)^{-4}$

coefficient of x^{12}.

$x^{12}(1-4x^4\ldots)(1-x)^{-4}$

$x^{12}\left(-{}^4C_3\,1\,(-x)^3\right) = {}^6C_3\,x^{15}$

COMBINATORICS

coefficient of x^7 in $(1+x+x^2+x^3\ldots)^{15}$

Ans: $^{21}C_7$

coefficient of x^{50} in $(x^7+x^8+x^9\ldots)^6$

$(x^7)^6\left(1+x+x^2+\ldots\right)^6$

$x^{42}\left(1+x+x^2\ldots\right)^6$

$x^{42}\left(\frac{1}{1-x}\right)^6 = x^{42}(1-x)^{-6}$

$-6\ C_8\ (-x)^8$

$^{13}C_8$

COMBINATORICS

24.

In how many ways two dozen identical robots be assigned to 4 assembly line a) atleast 3 robots to each line? b) atleast 3 atmost 9.

$24-12=12$

CBR

$(x^3+x^4+x^5\ldots)^4$

$(x^3)^4(1+x+x^2\ldots)^4$

$x^{12}\left(\frac{1}{1-x}\right)^4$

$x^{12}(1-x)^{-4}$

$x^{12}\left(-{}^{-4}C_{12}(-x)^{12}\right)$

${}^{15}C_{12}={}^{15}C_3$

$\square\,|\,\square\,|\,\square\,|\,\square$

$12+3C_3$

${}^{15}C_3$

COMBINATORICS

24.

In how many ways two dozen identical robots be assigned to 4 assembly line a) atleast 3 robots to each line? b) atleast 3 atmost 9.

$\left(15C_3 - 4 \times 8C_3\right)$

COMBINATORICS

$(x^3 + x^4 \ldots x^9)^4$ 24

$x^{12}(1 + x + x^2 \ldots x^6)^4$

$x^{12}\left(\frac{1-x^7}{1-x}\right)^4$

$x^{12}(1-x^7)^4(1-x)^{-4}$

$x^{12}(1 - 4x^7 + 6x^{14} - 4x^{21} + 1x^{28})(1-x)^{-4}$

$-{}^4C_{12} - 4 \times {}^{-4}C_5$

$\left({}^{15}C_{12} - 4 \times {}^8C_5\right) = \left({}^{15}C_3 - 4 \times {}^8C_3\right)$


THANK YOU GW
SOLDIERS !