CS & IT ENGINEERING

Discrete Mathematics

GRAPH THEORY

Lecture No.9

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TOPICS TO BE COVERED

01 Properly coloring

02 Chromatic number

03 Chromatic Number in Graphs

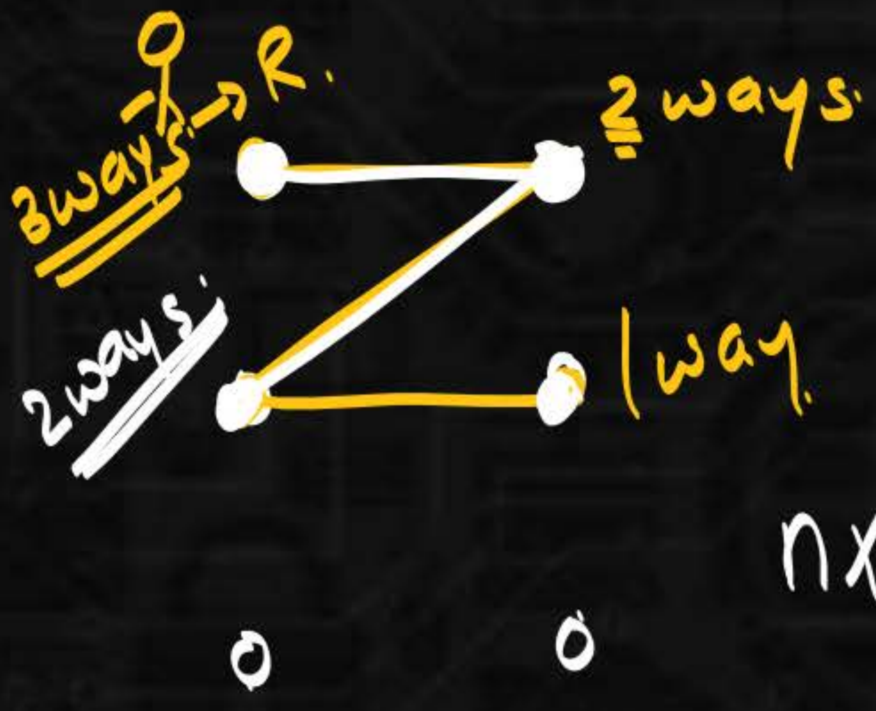
04 Subgraphs

05 Graph operations

$K_{n,n}$ $(n \geq 2)$

$n!(n-1)!/2.$

$K_{3,3}$



$n = 3$ to 90 to 90

R	L.
$3 \times 2 \cdot 1$	2×1

$3! \cdot 2!/2.$

R	L.
$n \times n-1 \times n-2$	$n-1 \times n-2 \ldots$

$n!(n-1)!/2.$

$X(G) = 2.$

$X(G)$: min + properly coloring.

1. diff color.

2. min no. of color.

properly coloring:

paint all vertices with diff colors such that adjacent should not have same clr.

Chromatic no ($X(G)$)

min no. of colors, such that adjacent should not have same color.

1)  $\chi(G) = 1$.

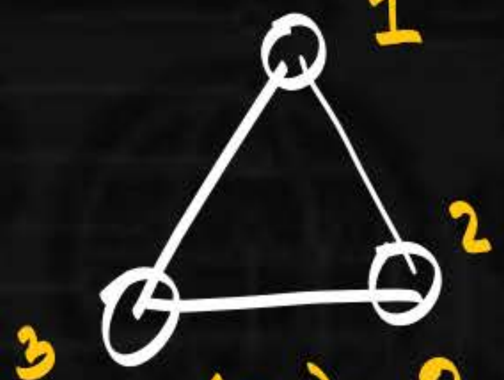
2) <u>Tree</u>: $\chi(\text{Tree}) = 2$

$\begin{cases} \chi(G) = K. \\ K\text{-colorable.} \end{cases}$

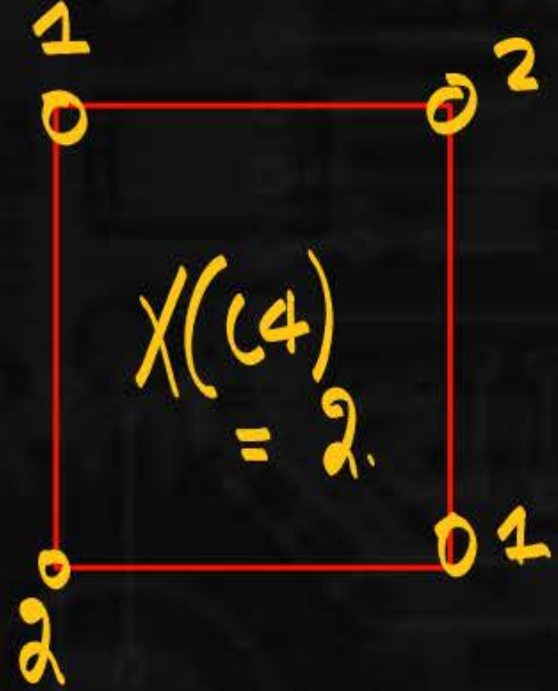
Every Tree is 2-colorable.

$\chi(C_n) = 2$ — n is even. → Even length cycle

$\chi(C_n) = 3$ — n is odd. → odd length. cycle.

3) 

$\chi(C_3) = 3$



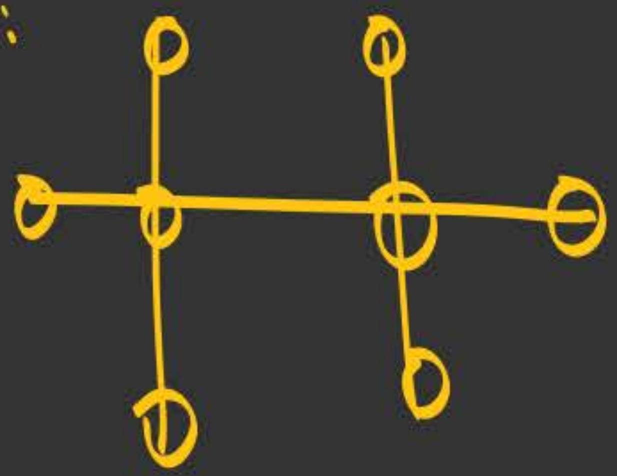
Every Tree is 2-colorable. (True)

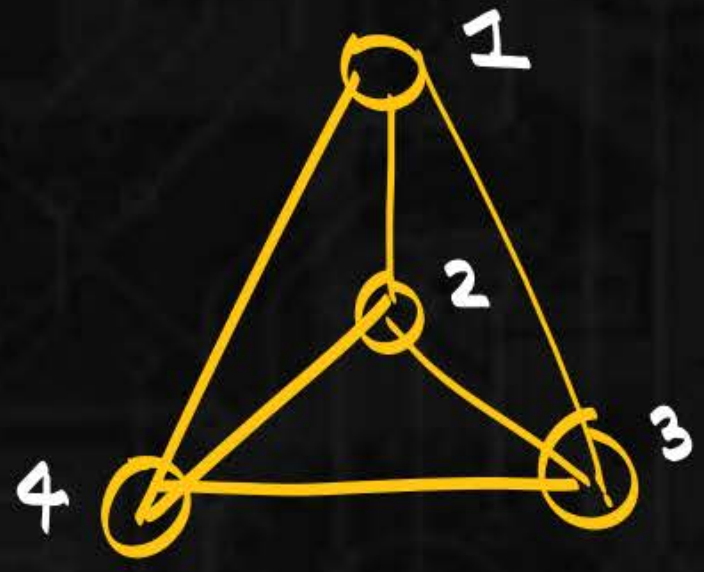
Every 2-colorable Graph is Tree (false)

→ Even length cycle is 2-colorable (True)

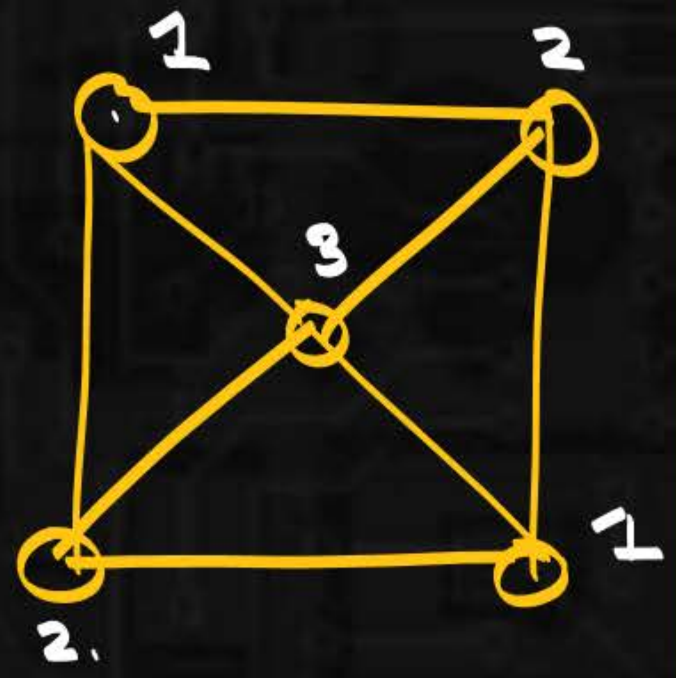
→ Every 2 colorable is Even length cycle. (false)

eg:





$\chi(W_4) = 4.$



$\chi(W_5) = 3.$

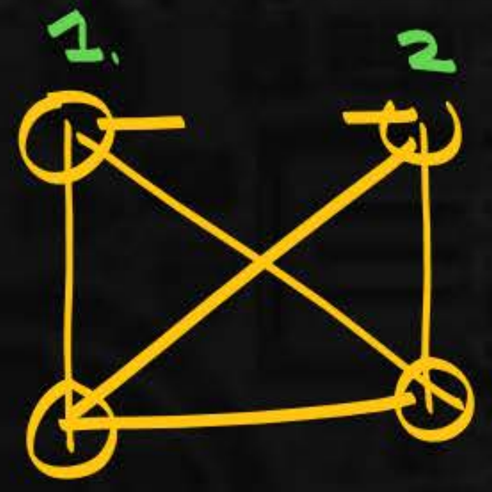
$$\chi(W_n) = 3 \qquad n \to \text{odd}$$
$$\chi(W_n) = 4. \qquad n \to \text{even}$$

$\chi(K_n) = n.$

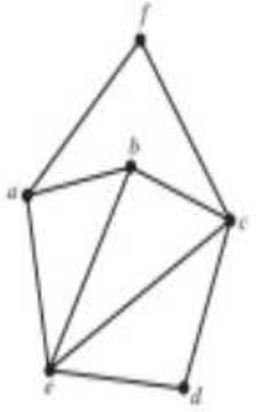
$\chi(K_1) = 1.$

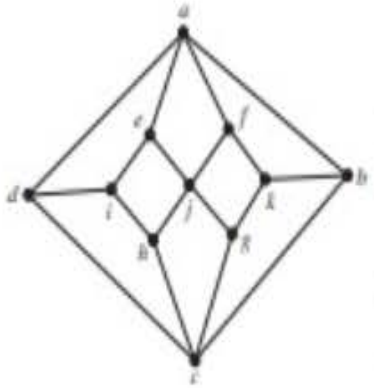
$\chi(K_2) = 2.$

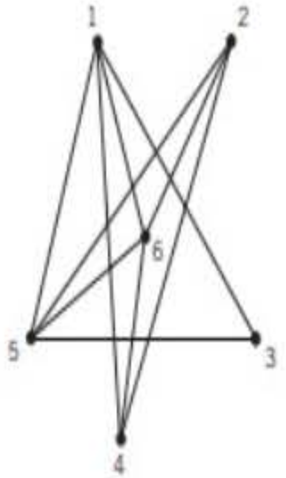
$\chi(K_3) = 3.$

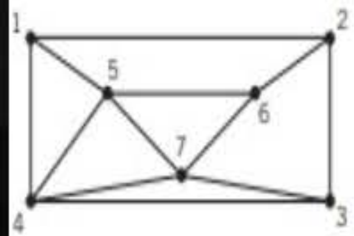


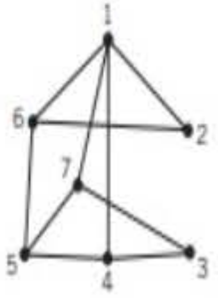
$\chi(K_n) = n.$



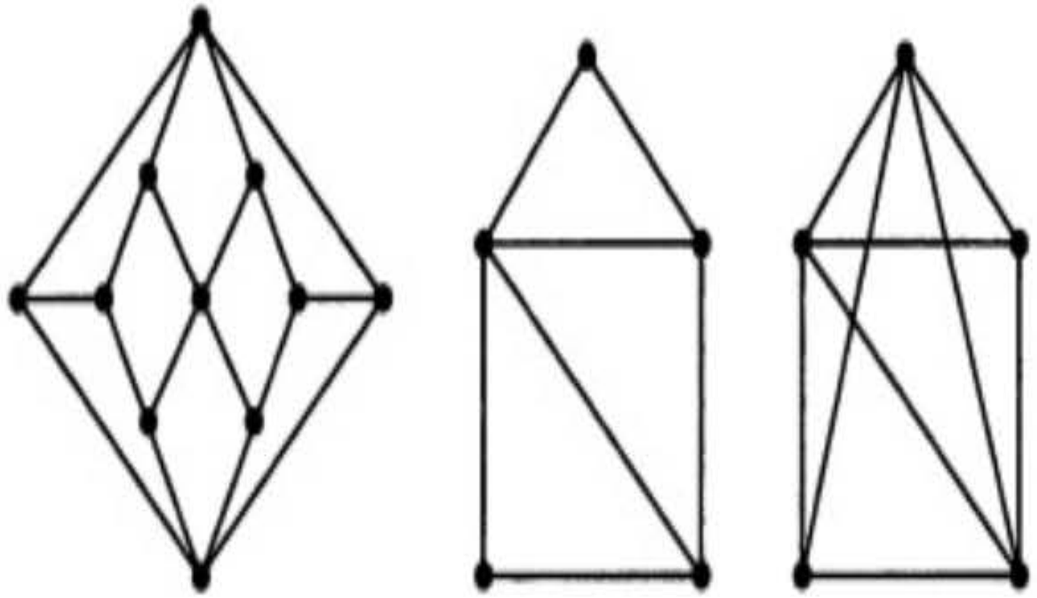








5. Determine the chromatic numbers of the following graphs:

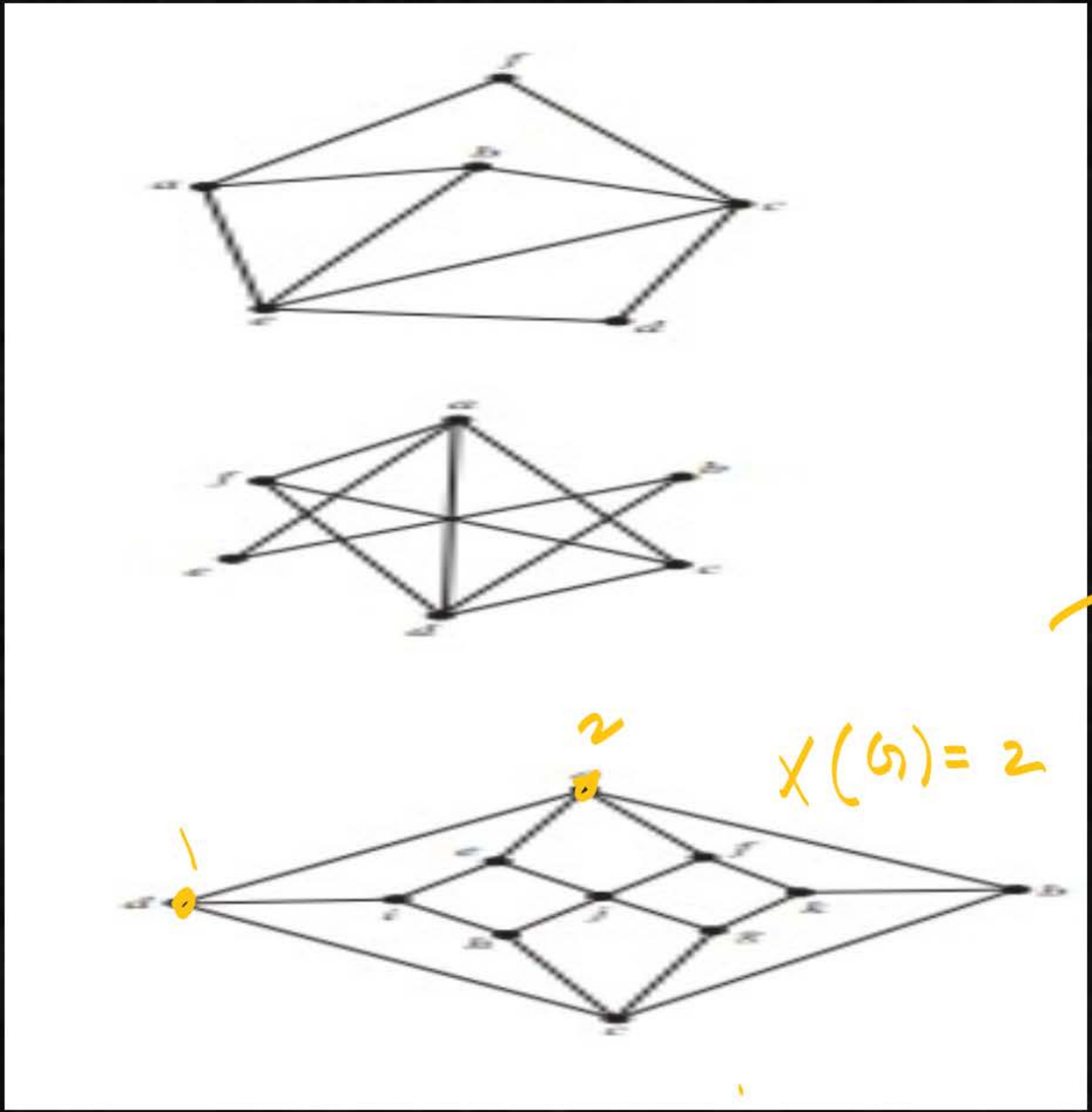


$\beta(G)$

$\chi(G)$

10. What is the chromatic number of the graph obtained from K_n by removing one edge?

The *Petersen graph* $\mathcal{P}$ is the graph whose vertices are the ten 2-subsets of $\{1,2,3,4,5\}$ in which two vertices are joined by an edge if and only if their 2-subsetss are disjoint.



$\chi(G) = 2.$

Bipartite Graph:

2 or more isolated vertices:

B.P.G.

$\chi(G)=1$.

→ Bipartite Graph does not contains odd length cycle

→ Tree $\chi(G)=2$

→ Even length cycle. $\chi(G)=2$.

{ Every B.PG is 2-colorable.

& viceversa is also True }

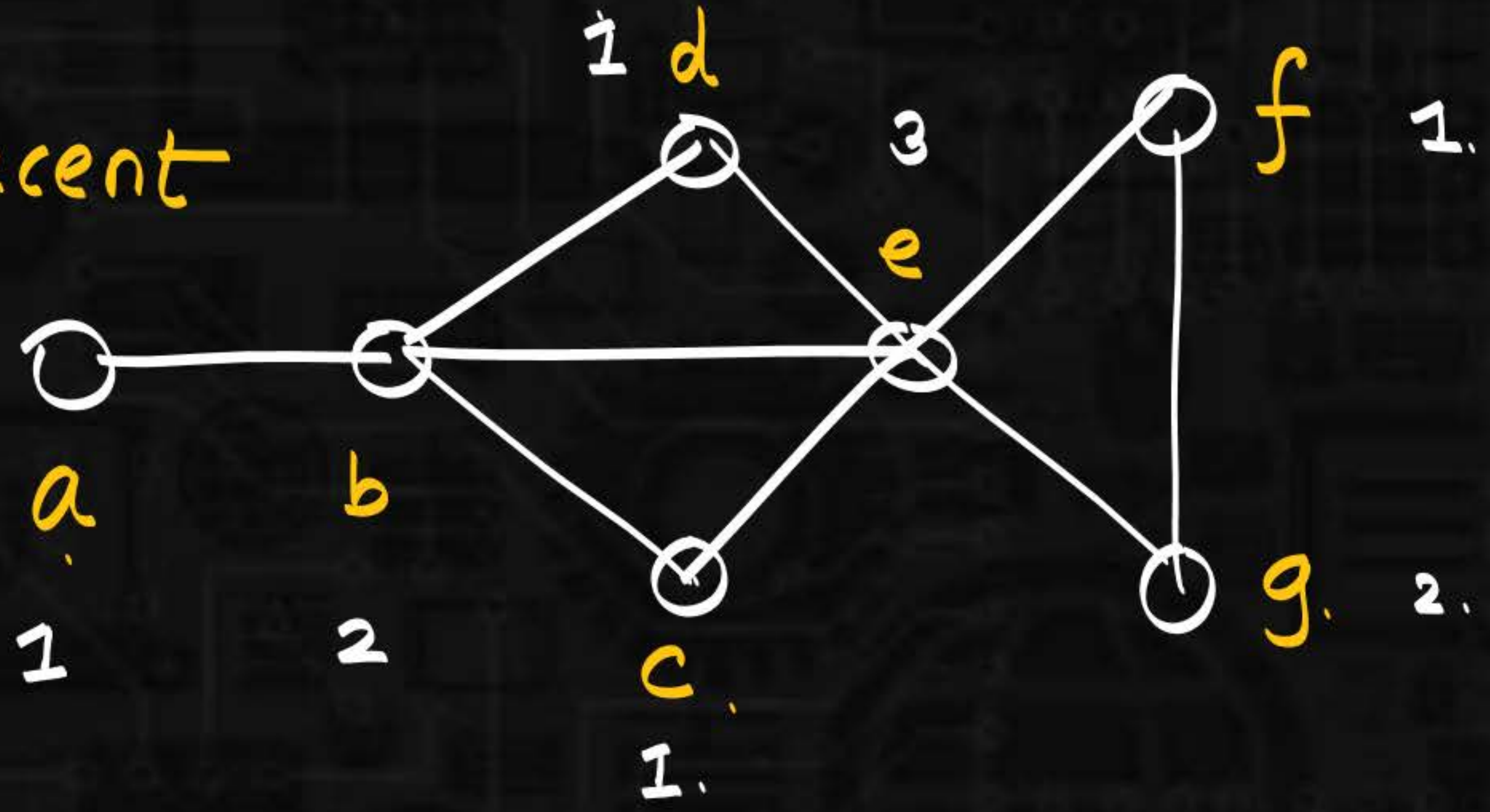
$\chi(G) = 3.$

non adjacent

$1 \rightarrow \{a, c, d, f\}$

$2 \rightarrow \{b, g\}$

$3 \rightarrow \{e\}$



chromatic partioning $= \chi(G) = 3.$

Independent set:

set of non adjacent vertices.

{a} – IS.

{a, b} → not IS.

{a, c} → IS.

{a, c, d} → IS.

{a, c, d, f} → IS.

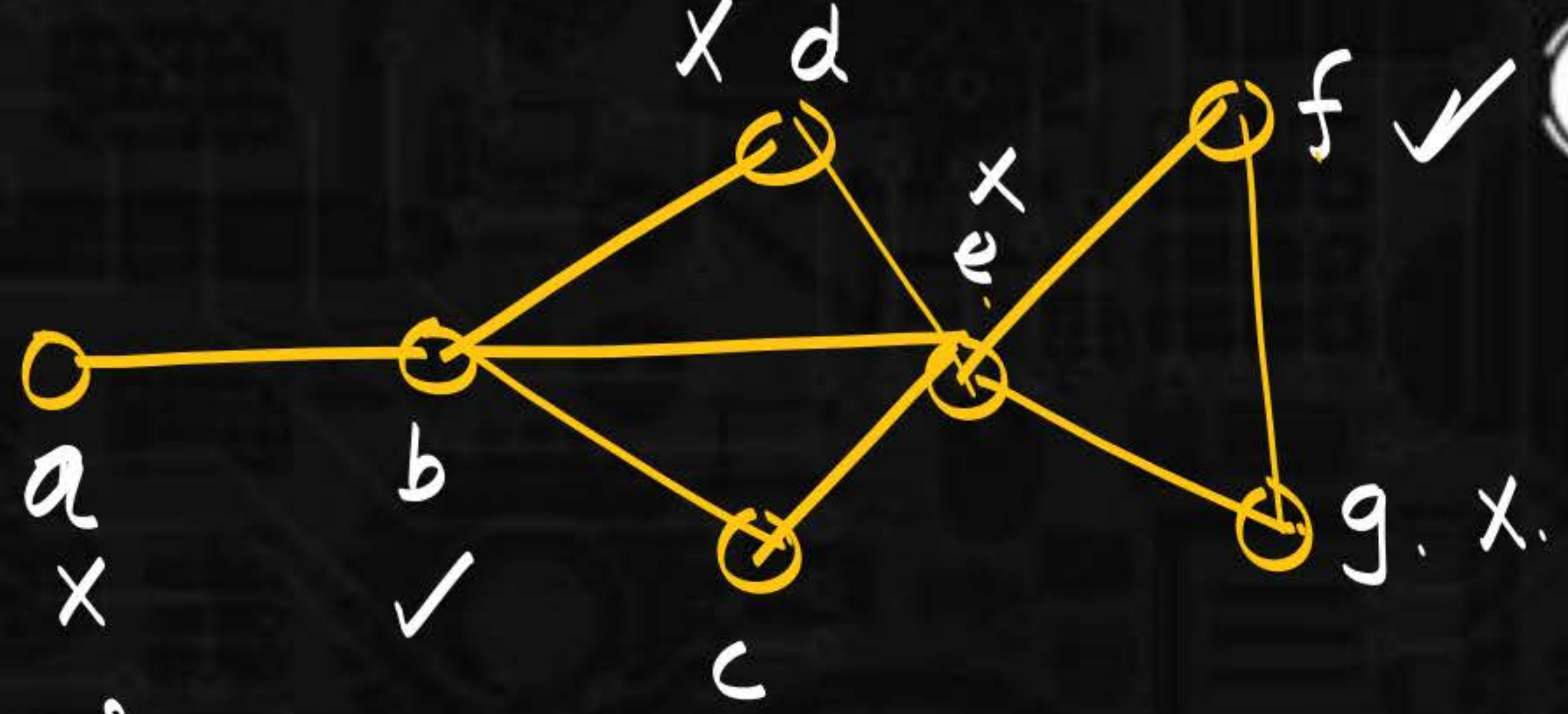
(maximal IS)

can we add new vertex into this.

NO

Independent set:

set of non adjacent vertices.



{b} → IS.

{b, f} → IS.

(MIS)

{b} - IS.

{b, g} → IS

(MIS)

{a, e} → MIS.

Independent set: set of nonadjacent vertices.

maximal Independent set: Independent set, such that we can not add new vertex into this.

$\{a, c, d, f\}$
$\{b, g\}$
$\{b, f\}$
$\{a, e\}$

Independencenos ($\beta(G)$)

no. of vertices present in largest " "

$\beta(G) = 4$

maximal → not related to size. but property


THANK YOU GW
SOLDIERS !