Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics

Mathematical Logic

DPP-02

[MSQ]

1. Which of the following is/are logical equivalence?

I. $\sim(p \to q)$

II. $\sim(p \to q) \wedge (q \to r)$

III. $p \wedge \sim q$

IV. $(p \vee q) \to r$

(a) I and II (b) I and III

(c) II and IV (d) II and III

[MSQ]

2. Consider the following statement

S_1: $(p \to q) \wedge (p \to r)$

S_2: $p \to (q \wedge r)$

Which of the following is True?

(a) S_1 is tautology

(b) S_1 is contingency

(c) S_1 is logically equivalence to S_2

(d) None of these

[MSQ]

3. Which of the following is logically equivalence?

(a) $(p \to r) \vee (q \to r)$

(b) $(p \leftrightarrow q) \vee (q \to r)$

(c) $(p \wedge q) \wedge r$

(d) $(p \leftrightarrow r) \wedge (q \leftrightarrow r)$

[MCQ]

4. Consider the following statement

S_1: $\sim (p \leftrightarrow q)$

S_2: $p \leftrightarrow \sim q$

Which of the following is correct?

(a) S_1 is tautology

(b) S_2 is contradiction

(c) S_1 is equivalence to S_2

(d) None of these

[MCQ]

5. Consider the following statement

S_1: $\sim (p \vee (\sim p \wedge q))$

S_2: $\sim p \wedge \sim q$

Which of the following is correct?

(a) S_1 is tautology

(b) S_2 is contradiction

(c) S_1 is equivalence to S_2

(d) S_1 is not equivalence to S_2

Answer Key

1. **(b, c)**
2. **(b, c)**
3. **(a, b, c)**
4. **(c)**
5. **(c)**

Hints and solutions

1. **(b, c,)**

Two statements forms are logical equivalent if and only if their resulting truth values are identical for each variation of statement variables.

I. $\sim(p \to q)$

$= \sim(\sim p \vee q)$

$= p \wedge \sim q$

Hence, I is logically equivalent to III.

II. $(p \to r) \wedge (q \to r)$

$= (\overline{p} + r) \wedge (\overline{q} + r)$

$= \overline{p}\,\overline{q} + \overline{p}r + \overline{q}r + r$

$= \overline{p}\,\overline{q} \mid \overline{p}r + r$

$= \overline{p}\,\overline{q} + r$

$= \overline{(p \vee q)} + r \equiv (p \vee q) \to r$

Hence, II and IV are logically equivalence.

2. **(b, c)**

Statement S_1: $(p \to r) \wedge (p \to r)$

$= (\overline{p} + q) \wedge (\overline{p} + r)$

$= \overline{p} + \overline{p}r + \overline{p}q + qr$

$= \overline{p} + \overline{p}q + qr$

$= \overline{p} + qr$

$= p \to (q \wedge r) \neq 41$

Hence, S_1 is not tautology and S_1 is logically equivalent to S_2.

Statement S_2: $p \to (q \wedge r)$

$= \overline{p} + (q \wedge r)$

$= \overline{p} + qr \neq 1$ or 0

Hence, statement S_2 is contingency.

3. **(a, b, c)**

Option A: $(p \to r) \vee (q \to r)$

$= (\overline{p} + r) \vee (\overline{q} + r)$

$= \overline{p} + r + \overline{q} + r$

$= \overline{p} + \overline{q} + r$

$= \overline{pq} + r \equiv \overline{(p \wedge q)} + r$

$\equiv (p \wedge q) \to r$

So, option A is logically equvalence to option C.

Option B: $(p \leftrightarrow r) \vee (q \to r)$

$= \overline{p}\,\overline{r} + pr + \overline{q} + r$

$= \overline{p}\,\overline{r} + \overline{q} + pr + r$

$= \overline{p}\,\overline{r} + \overline{q} + r$

$= \overline{p}\,\overline{r} + r + \overline{q}$

$= \overline{p} + r + \overline{q}$

$= \overline{p} + \overline{q} + r$

$= \overline{(p \wedge q)} + r = (p \wedge q) \to r$

So, option B is also logically equvalence to option A.

4. **(c)**

Statement S_1: $\sim(p \leftrightarrow q)$

$= \sim(\overline{p}\,\overline{q} + pq)$

$= (p + q)(\overline{p} + \overline{q})$

$= p\overline{q} + q\overline{p}$

Statement S_2: $p \leftrightarrow \sim q$

$= \overline{p}q + p\overline{q}$

Hence, S_1 and S_2 are equivalence to each other.

5. **(c)**

Statement S_1: $\sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge [\sim(\sim p \wedge q)]$

$= \sim p \wedge [\sim(\sim p \vee \sim q)]$

$= \sim p \wedge [p \vee \sim q)]$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

$= \sim p \wedge \sim q)$

Hence, S_1 is equivalence to S_2.