CS & IT ENGINEERING

Discrete maths
GRAPH THEORY

Lecture No. 1

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Definition of Graph

02 Handshaking Lemma

03 Types of Graphs

04 No of Graphs

05 Simple Graphs theorem

Basics of Graph

(10 – 18)

- Graph Theory (4 - 6)
- logic (2 - 4)
- set theory (2 - 4)
- Combinatorics. (2 - 4)

<u>Ref</u>:

Keneth. H. Rosen.

@SatishsirP

Basics of Graph

$G = (\{\ldots\ldots\}, \{\ldots\ldots\})$

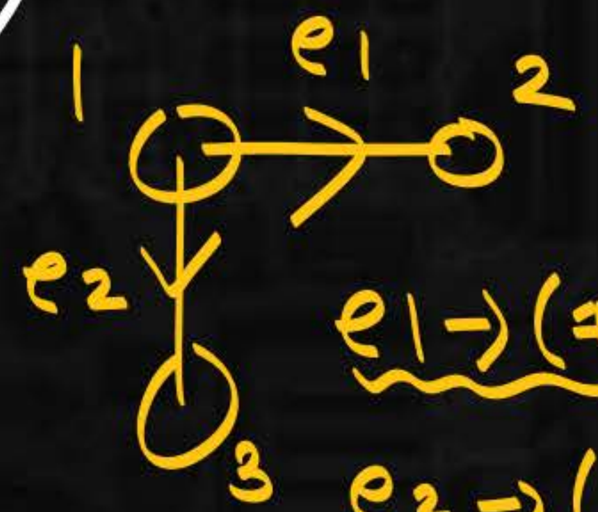
joint / point → vertex / vertices.

line / branch → edge / edges.

Graph $G = (V, E)$

E → set of edges.

V → set of vertices



$e_1 \to (1, 2)$

$e_2 \to (1, 3)$

each edge must be associated with unorder (pair of vertices.)

Basics of Graph

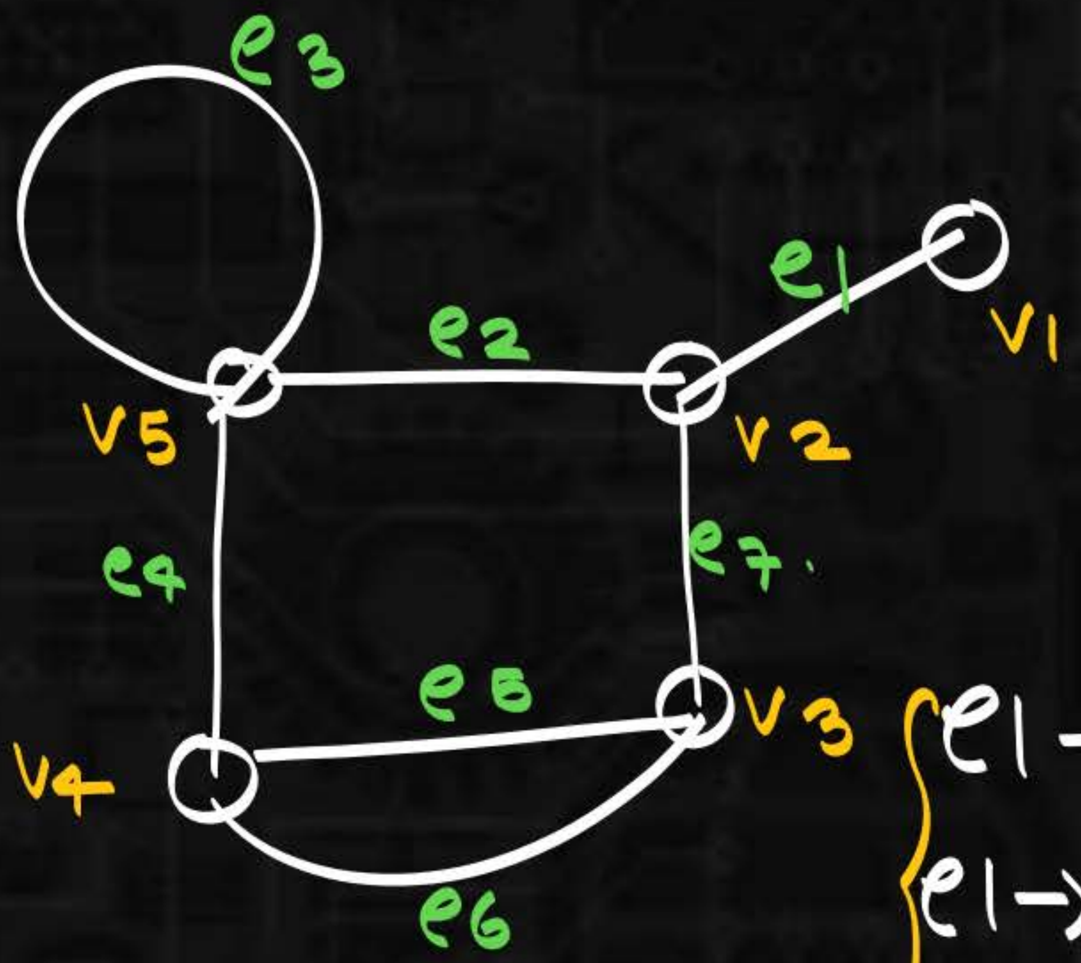
$G = (V, E)$

V = set of vertices.

$V = \{v_1, v_2, v_3, v_4, v_5\}$

E = set of edges.

$E = \{e_1, e_2 \ldots\ldots e_7\}$.



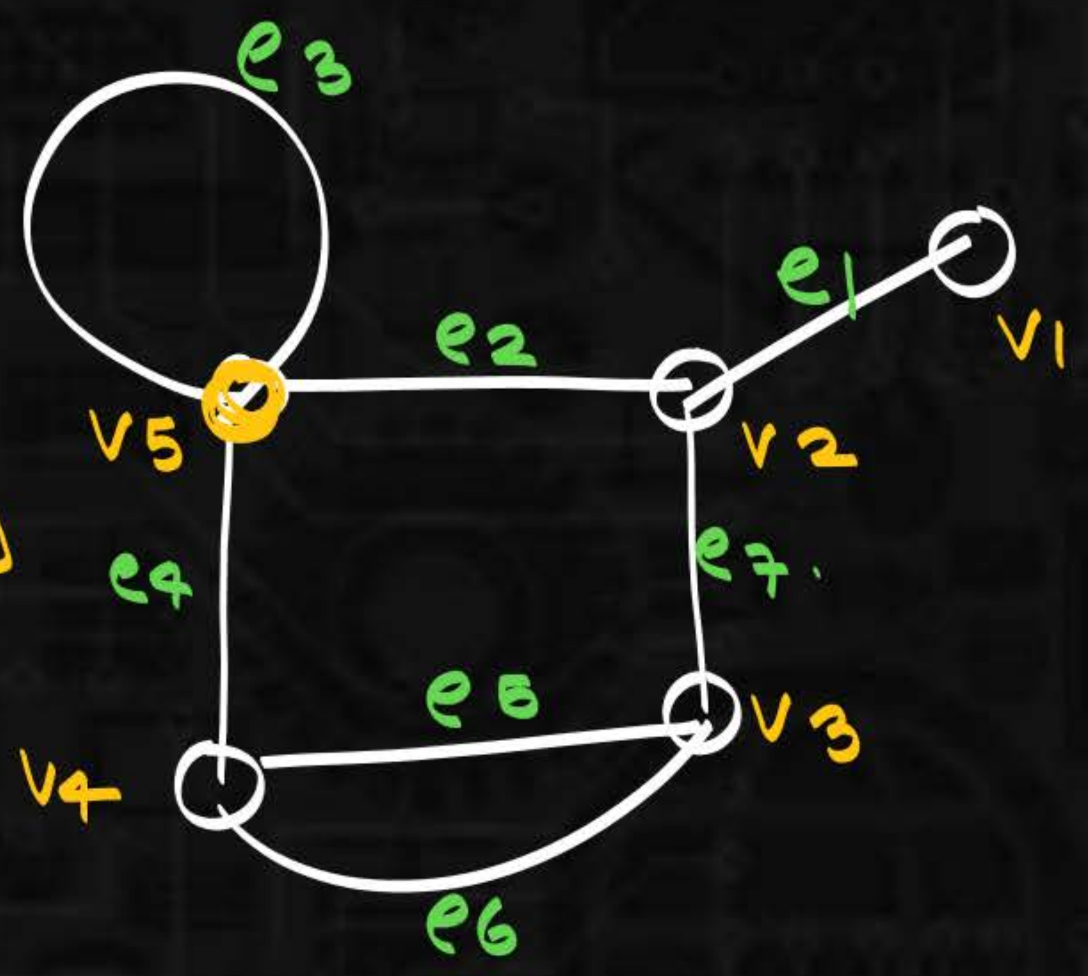
$e_1 \rightarrow (v_1, v_2)$ ✓

$e_1 \rightarrow (v_2, v_1)$ ✓

Basics of Graph

end vertices :

unorder pair of vertices are called endvertices.



$e3 \rightarrow (v5, v5)$

Self-loop/loop → when endvertices are same

edge → loop.

Basics of Graph

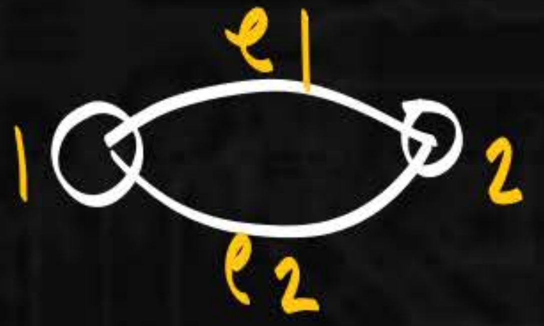
∥ edges :

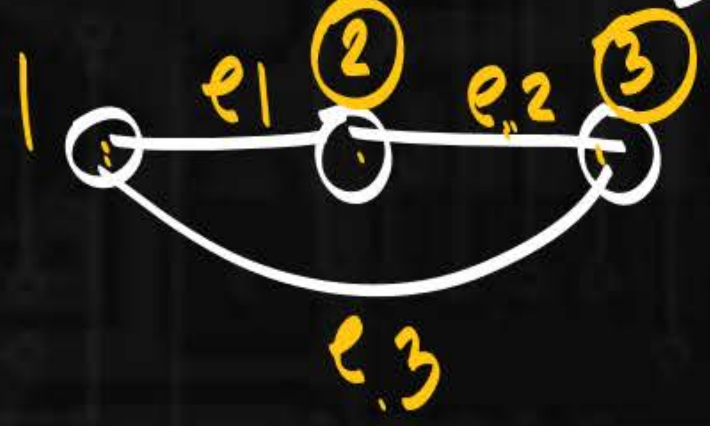
$e5 \rightarrow (v3, v4)$

$e6 \rightarrow (v3, v4)$

2 or more edges associated with same end vertices.

e_2, e_3 are not ∥.





$e1 \rightarrow (1, 2)$

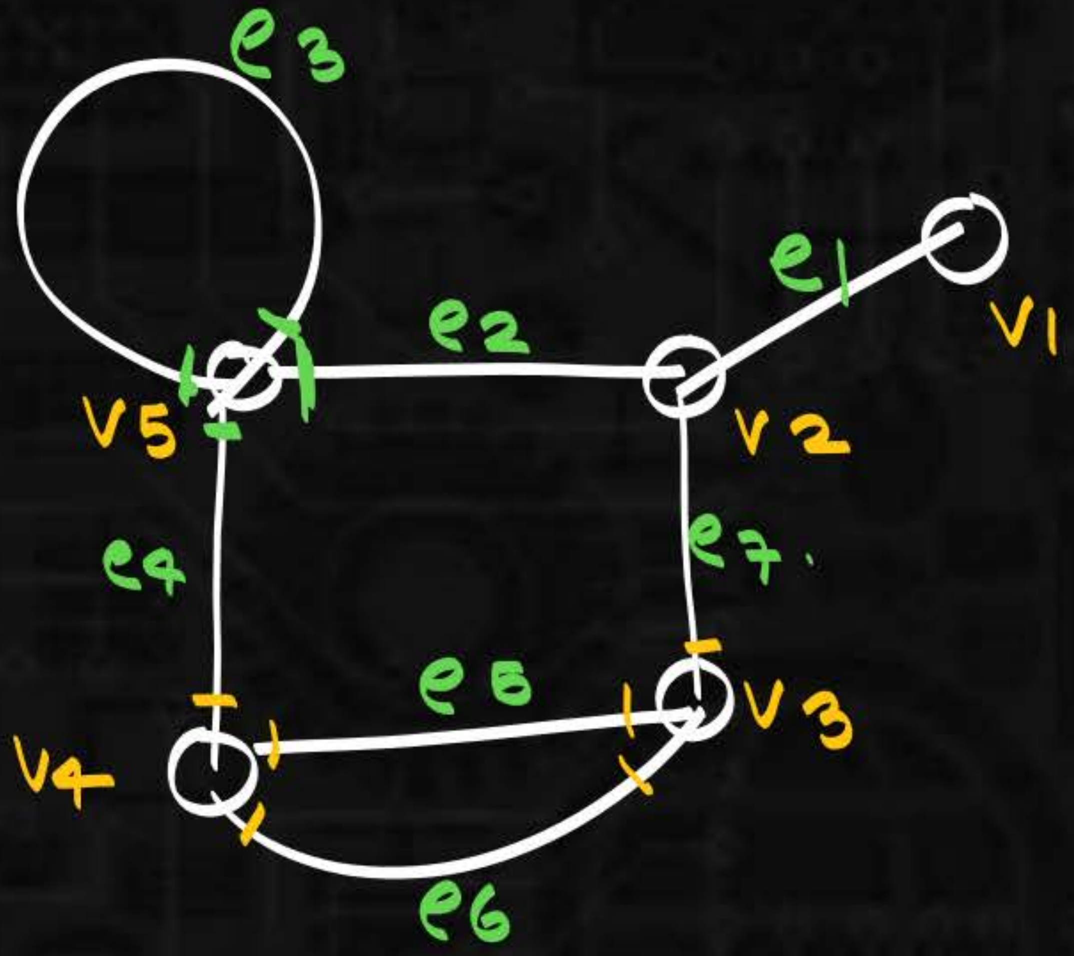
$e3 \rightarrow (1, 3)$

they are not ∥ edges.

Basics of Graph

incident point:

meeting point of vertex & edge



Degree/valency $(d(v_i))$

no. of incident point.

$d(v_4) = 3$ $\quad$ $d(v_1) = 1.$

$d(v_3) = 3$ $\quad$ $d(v_5) = 4.$

Basics of Graph

pendant vertex:

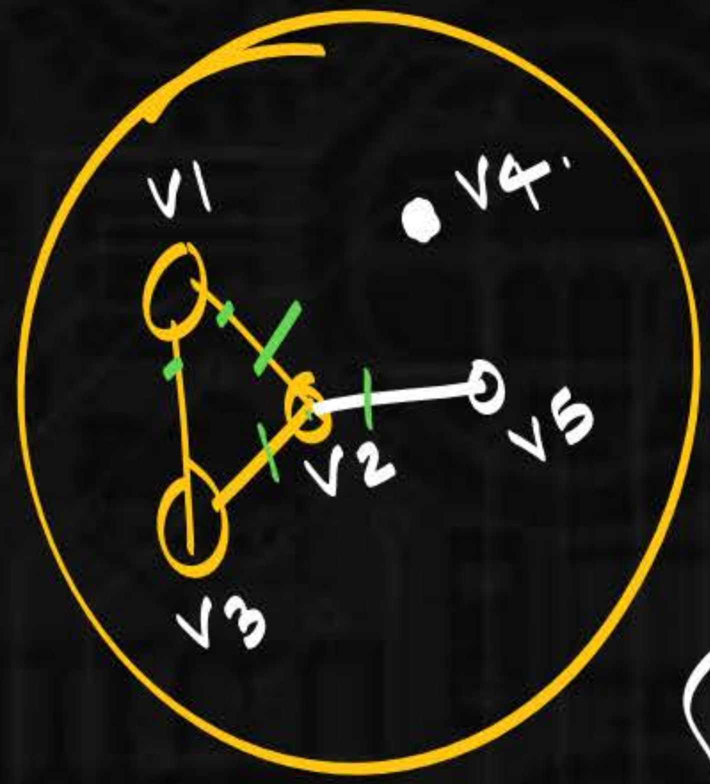
degree 1 vertex

null graph:

Set of isolated vertices

Isolated vertex:

degree 0 vertex



$d(v_1) = 2$

$d(v_3) = 2$

$d(v_2) = 3$

$d(v_5) = 1$

pendant vertex.

$d(v_4) = 0$

isolated vertex

$$G = (\underbrace{V}_{\text{set}}, \overbrace{E}^{\text{set}})$$

$$\begin{cases} V \neq \phi \\ E = \phi \end{cases}$$

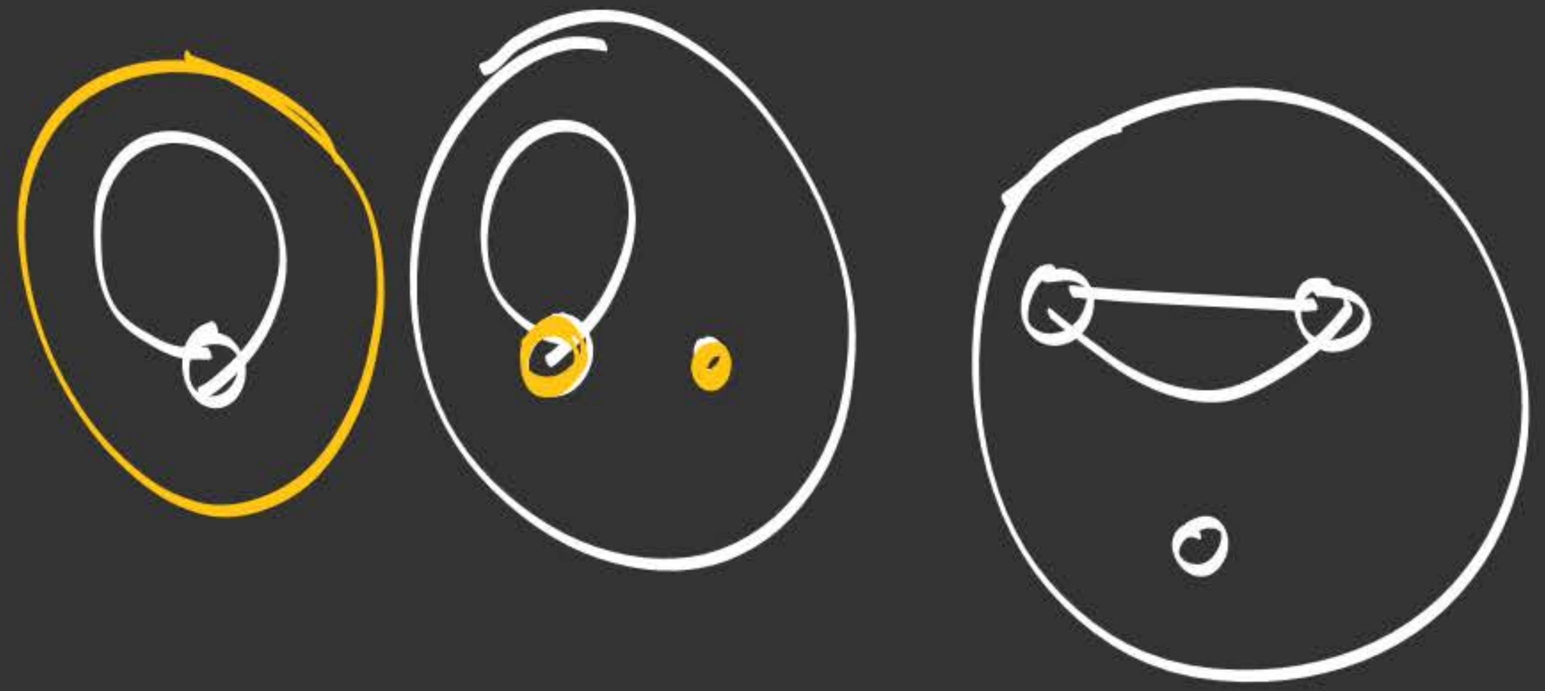
Graph

Trivial Graph :

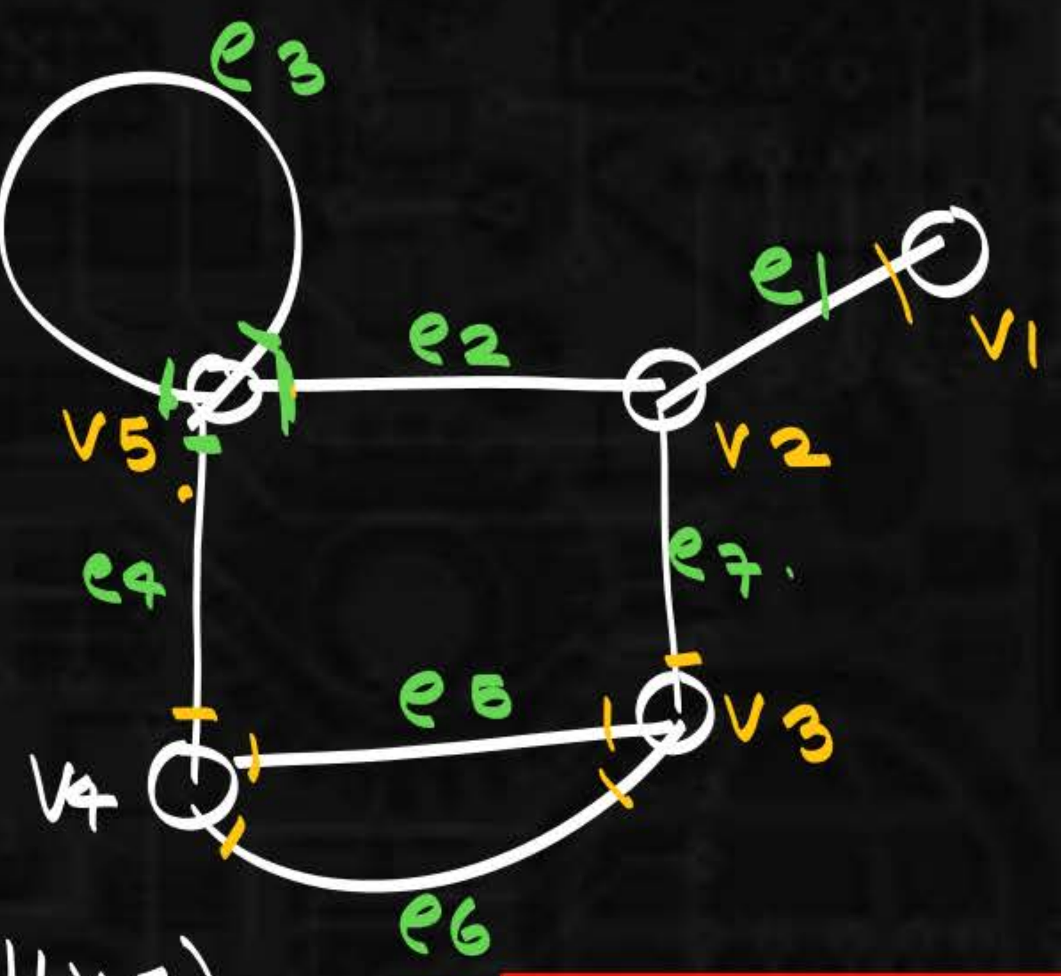
$G = (V, E)$

$V = \{v_1\}$

$E = \phi$



Basics of Graph



$d(v_1) = 1 \quad d(v_2) = 3$

$d(v_5) = 4 \quad d(v_4) = 3$

$d(v_3) = 3.$

$$d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$

$$= 1 + \underbrace{3 + 3 + 3} + 4$$

$$= 14 = 2 \times 7 \rightarrow \text{no. of edges.}$$

$$\boxed{\sum d(v_i) = 2e.}$$

Basics of Graph

$\boxed{L.H.S = R.H.S}$

degree		edge
2	$=$	$2(1)$
$2+2$	$=$	$2(1+1)$
$2+2+2$	$=$	$2(1+1+1)$

$$\sum d(v_i) = 2e$$

Thm1:

Sum of degrees of all vertices is equals to twice the no. of edges.

$$\boxed{\sum d(v_i) = 2e.}$$

Basics of Graph

$\sum d(v_i) = 2e$ → even / odd

$\sum d(v_i) = \text{even}$

$\sum d(v_i) = 17$ ✗

$\sum d(v_i) = 7$ ✗

7 no. of edges Graph is possible.

$\sum d(v_i) = \text{at least } 5$

$= 6, 8, 10 \ldots$

Basics of Graph

$\sum d(v_i) = \text{even}$

$d(v_1) + d(v_2) + \cdots d(v_n) = \text{even}$

odd degree.. even degree vertex

$\boxed{1, 3, 5} + \boxed{e, e} = \text{even}$

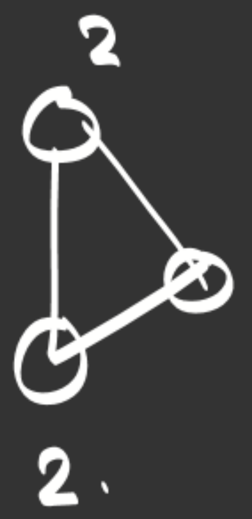
$\text{odd} + \text{even} = \text{even}$

$\text{odd} = \text{even}$

it has to be even.

Thm 2:

no. of odd degree vertices will always be even.



(even degree vertex)

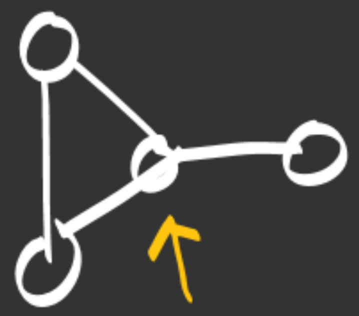
\# odd degree vertex = 0

(odd ...

odd degree vertex

\# odd degree vertex = 2.

even....



odd...

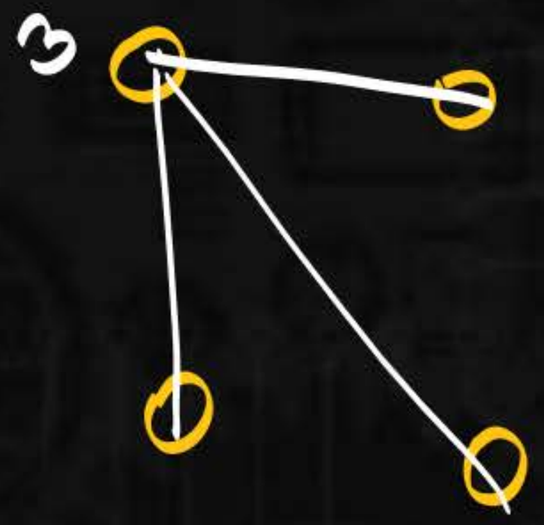
Basics of Graph

	ll edges	loop.
Simple	✗	✗
multigraph	✓	✗.
Pseudograph	✓	✓

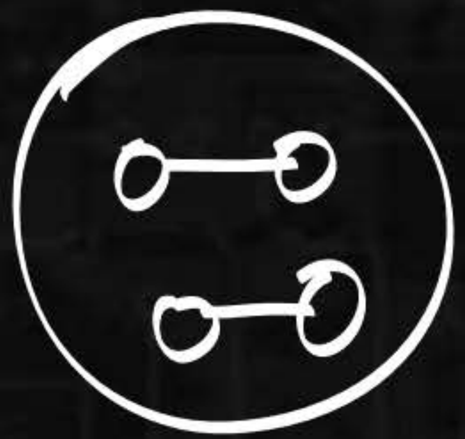
Basics of Graph

Thm3: In Simple Graph maximum degree $\leqslant n-1$.

$n = 4$



$n = 4$



$n = 4$

Basics of Graph

Thm 4: In Simple Graph maximum no. of edges $\leq \frac{n(n-1)}{2}$.

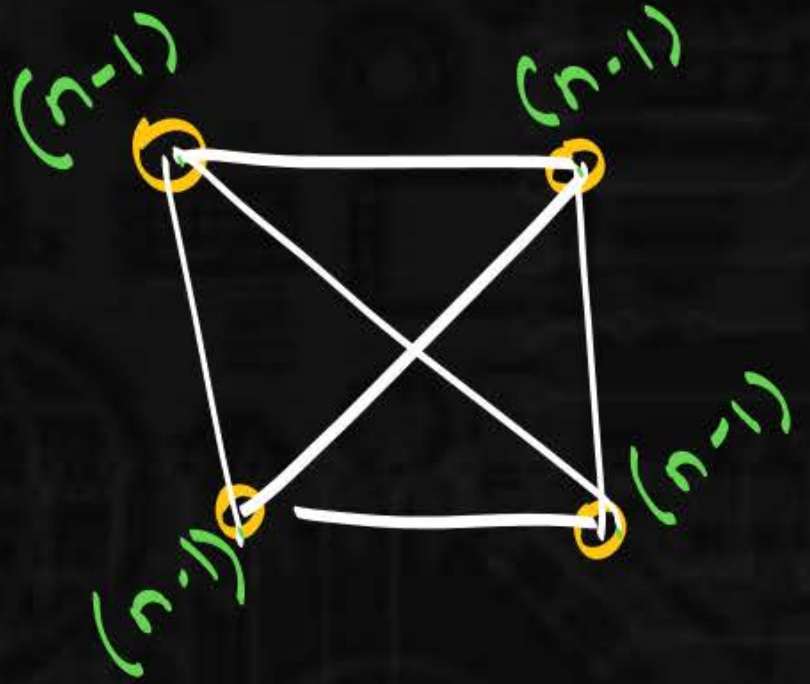
n = Total vertices

$n = 4$ (vertices)

$\sum d(v_i) = 2e.$

$n \cdot (n-1) = 2e.$ → Total vertices = n. Degree of each vertex is $n-1$.

$e = \frac{n(n-1)}{2}.$



max no. of edges = 6.

Basics of Graph

Total no. of graphs $n = 4$.

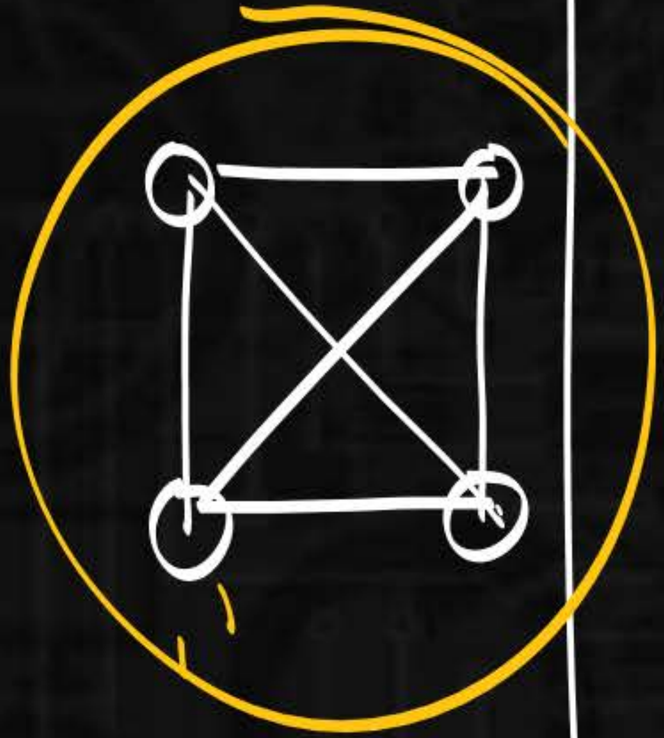
Total vertices $= 4$.

$n = 4$.

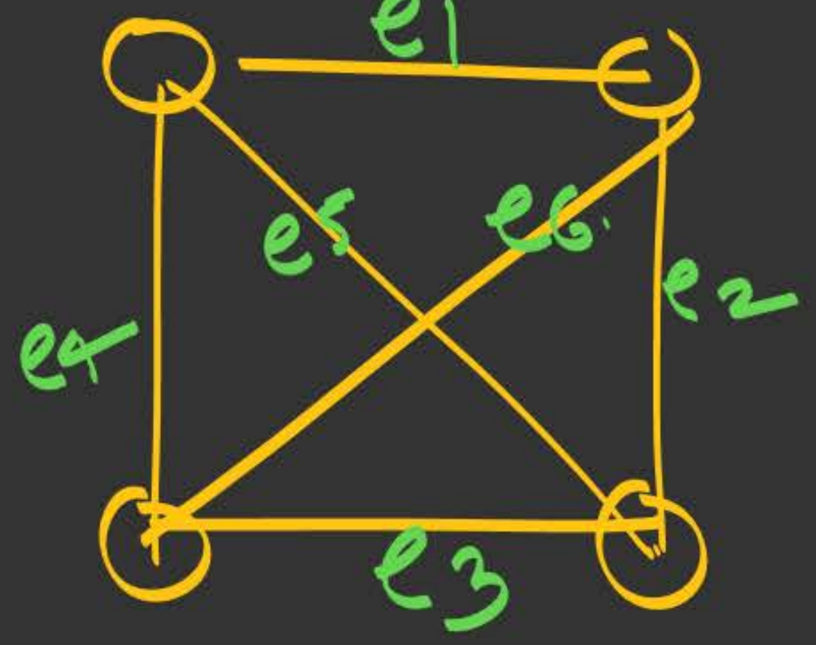
max no. of edges $= 6$.

$n = 4$
$e = 0$

$n = 4$
$e = 1$



2^6

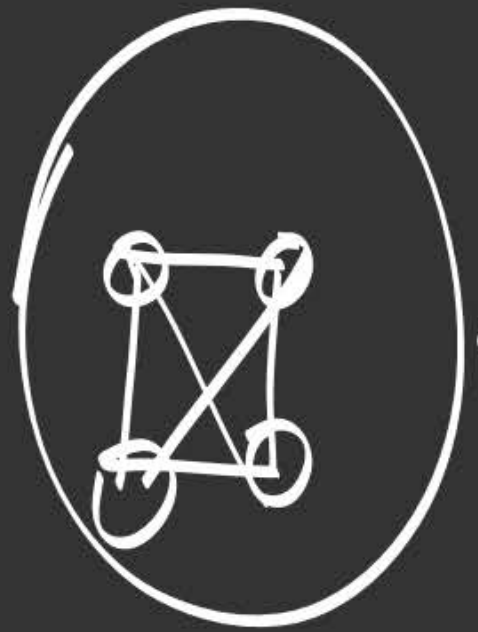


2^6

e1	e2	e3	e4	e5	e6.
0	0	0	0	0	0
1	0	0	0	0	0
0	1	0	0	0	0
1	1	1	1	1	1

→ $^6C_1$.

2 edges → $^6C_2$.



$$^6C_0 + {^6C_1} + {^6C_2} + {^6C_3} + {^6C_4} \cdots {^6C_6} = 2^6.$$

1 edge.

2 edges

Basics of Graph

n = Total vertices.

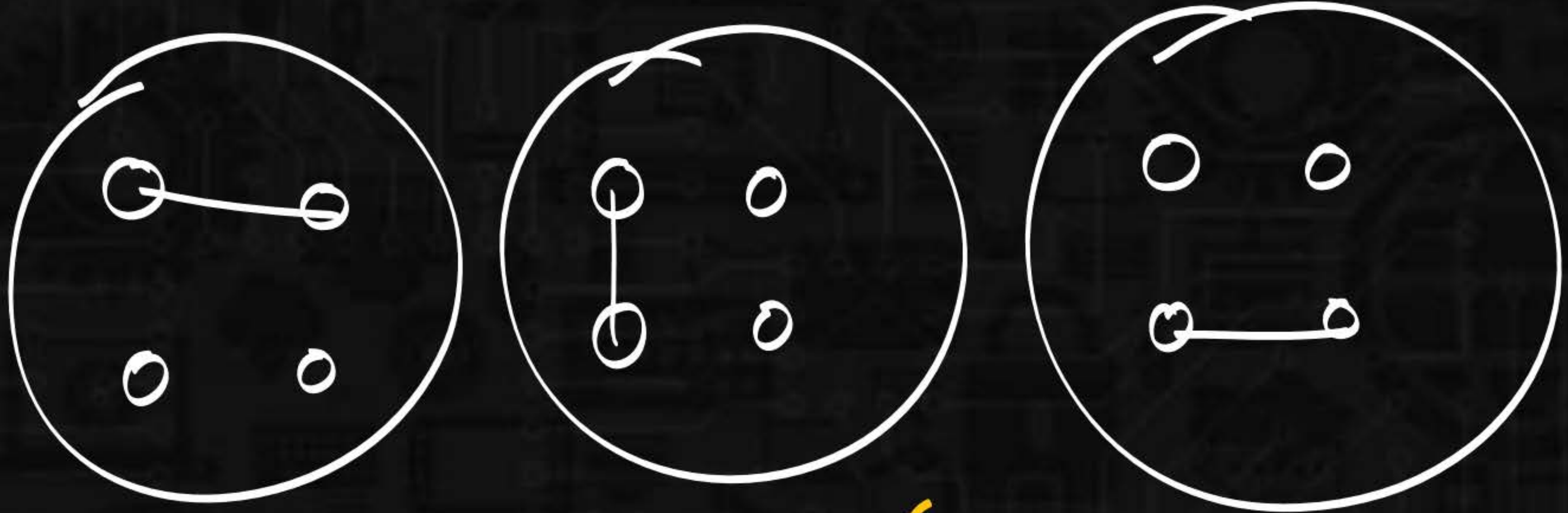
Total no. of graphs $= 2^{\frac{n(n-1)}{2}}$

$n \rightarrow$ Total vertices.

no. of graphs with e edges $^{\frac{n(n-1)}{2}}C_e$.

Basics of Graph

how many graphs are possible with 4 vertices & 1 edge.



$\{e_1, e_2, e_3 \cdots e_6\}$

$\{e_1\}$

$^6C_1$ $\{e_2\}$

$\{e_3\}$

$\vdots$

$\{e_6\}$

$^6C_1$.


THANK YOU GW
SOLDIERS !