CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 01

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TOPICS TO BE COVERED

01 sum rule

02 Product rule

03 Practice

COMBINATORICS

Sum/product Rule.
Combination with Rept^n
Inclusion - Exclusion
pigeonhole principle.
Derangement.
Euler - ϕ - function.
Generating function
Recurrence Relation.

count + property.

COMBINATORICS

product Rule.:

Task T → T_1 → n_1 ways to do task T_1.

Task T → T_2. → n_2 ways to task T_2.

Total ways $= n_1 \times n_2$.

* for each of n_1 ways, we can perform n_2
* both tasks are happening simultaneously

COMBINATORICS

$k = 0$

$\boxed{\text{for } i = 1 \text{ to } n_1}$ T_1

$\boxed{\text{for } j = 1 \text{ to } n_2}$ T_2

$k = k + 1$

$i = 1 \quad j = 1 \ldots\ldots n_2$

$i = 2 \quad j = 1 \ldots\ldots n_2$

$\vdots$

$i = n_1 \quad j = 1 \ldots\ldots n_2$

$k = n_1 \times n_2$

COMBINATORICS

Total signals it can generate if we have 2-bits.

T.

T_1 → n_1 ways. $n_1 = 2$. (0/1)

T_2 → n_2. $n_2 = 2$. (0/1)

$0 \to 0 \to 00$

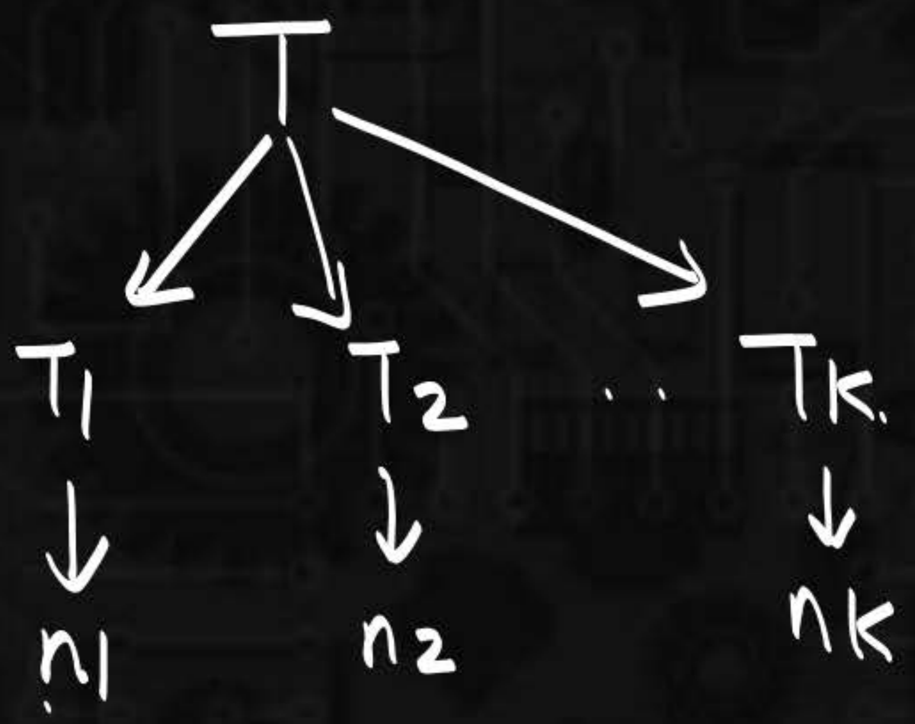
$0 \to 1 \to 01$

$1 \to 0$

$1 \to 1$

$$\text{Total ways} = n_1 \cdot n_2 = 2 \cdot 2 = 2^2$$

COMBINATORICS



Simultaneously

Total ways = $n_1 \times n_2 \cdots n_k$.

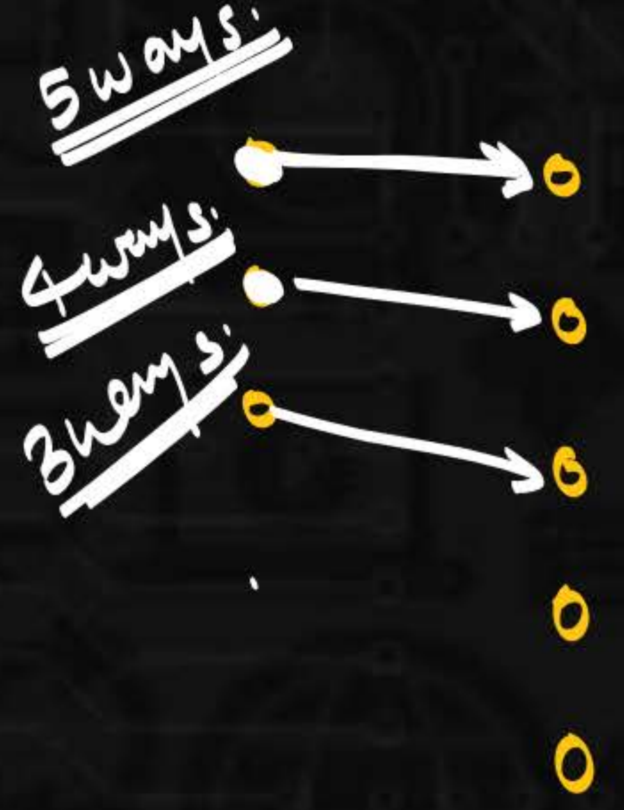
```
k = 0
for x1 = 1 to n1.        T1
  for x2 = 1 to n2.      T2
    for x3 = 1 to n3     T3
      :
        for xk = 1 to nk.    Tk
          k = k + 1.
```

$\boxed{K = n_1 \cdot n_2 \cdot n_3 \cdots n_k}$

COMBINATORICS

$f: A \to B$ (1:1) Total ways

$|A| = 3 \quad |B| = 5$ $\qquad = 5 \cdot 4 \cdot 3$



How many ways we can generate number plates. Such that 4 characters followed by 2 digit ?

Ans: $26^4 \cdot 10^2$.

$$\underset{26}{\underline{A}} \times \underset{26}{\underline{B}} \times \underset{26}{\underline{A}} \times \underset{26}{\underline{C}} \;\Big|\; \underset{10}{\underline{0}} \times \underset{10}{\underline{2}}$$

Sum Rule :

Task T → T_1 → n_1 ways to do task T_1.

Task T → T_2. → n_2 ways to task T_2.

Total ways $= n_1 + n_2$.

both the tasks are not happening simultaneously.

COMBINATORICS

Sum Rule. :

```
k = 0
for i = 1 to n1.
    k = k+1.
for j = 1 to n2.
    k = k+1.
```

Product Rule.

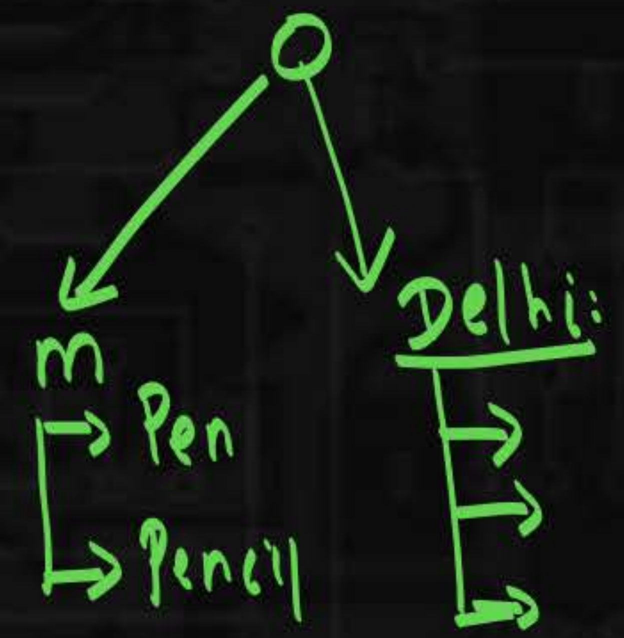
```
k = 0
for i = 1 to n1.
for j = 1 to n2.
    k = k+1.
```

COMBINATORICS

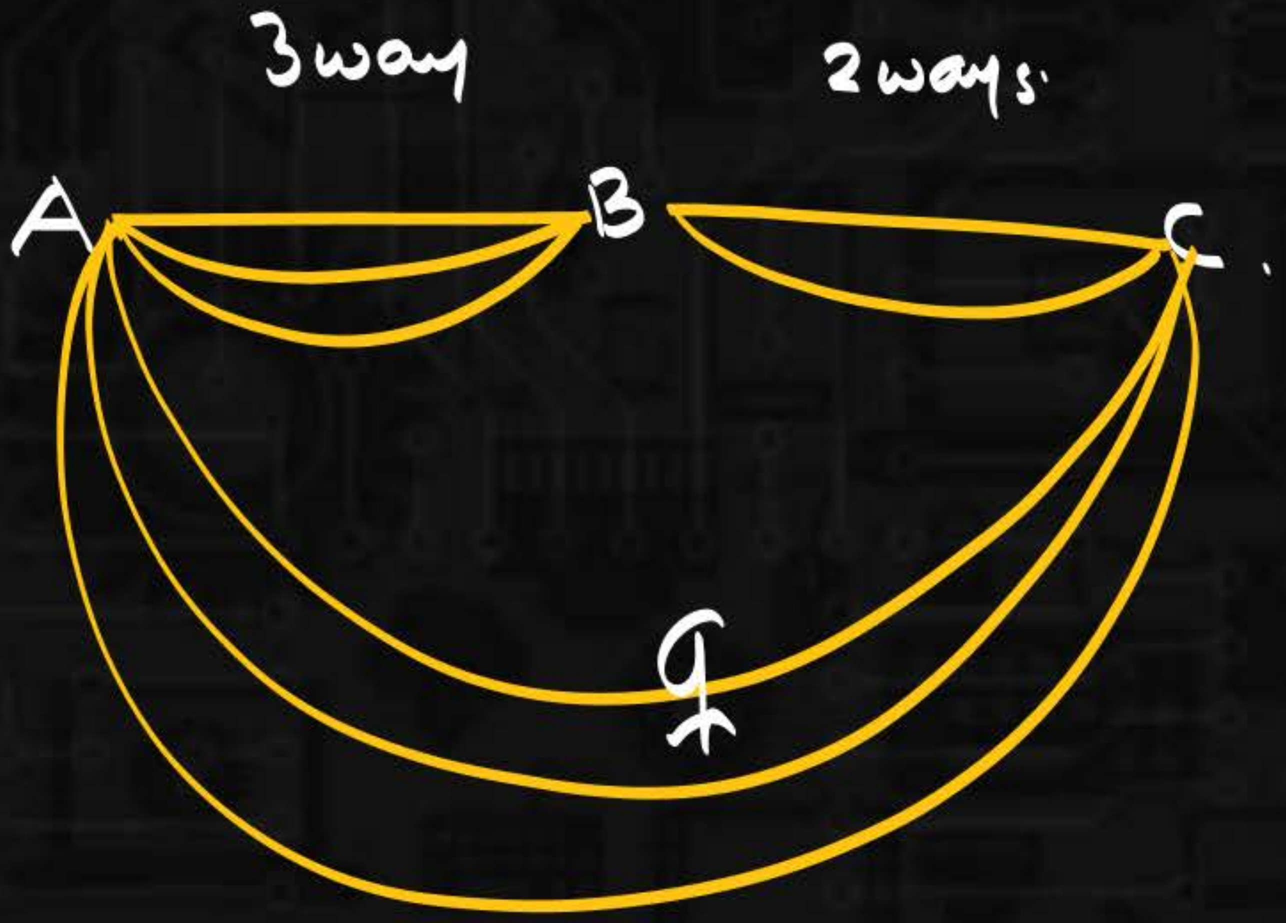
Babita ji wants to go for shopping,
3 ways to do shopping in Delhi, (chair, table, bed)
2 ways to do shopping in Mumbai (pen, pencil)

how many ways she can do shopping?

Total ways = 3+2.



COMBINATORICS



1) A — B — C (via B)

Total ways = 3×2.

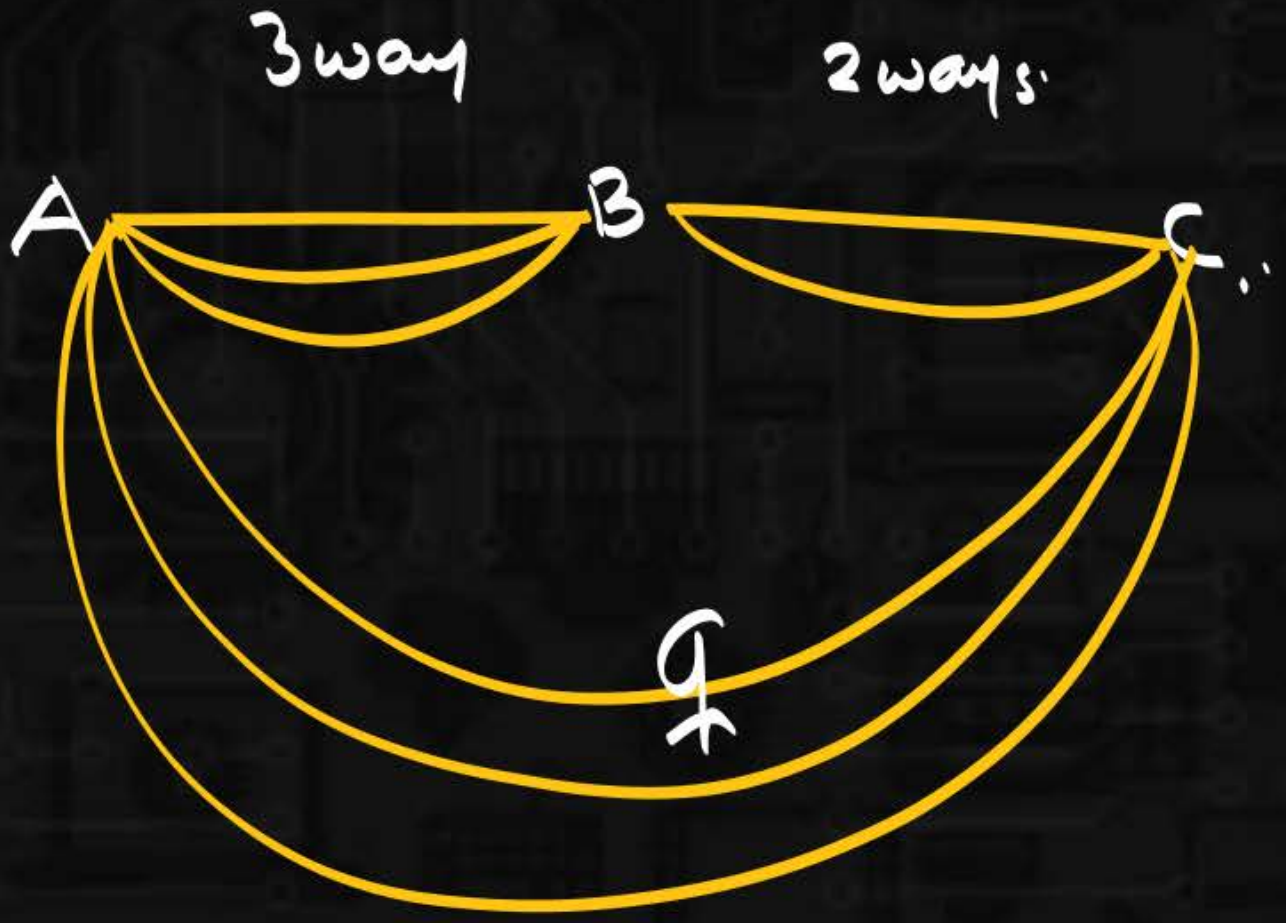
2) A — B — C

Direct / via B.

$3 + 3 \times 2$

$3 + 3 \times 2$
$= 3 + 6 = 9$ ways.

COMBINATORICS



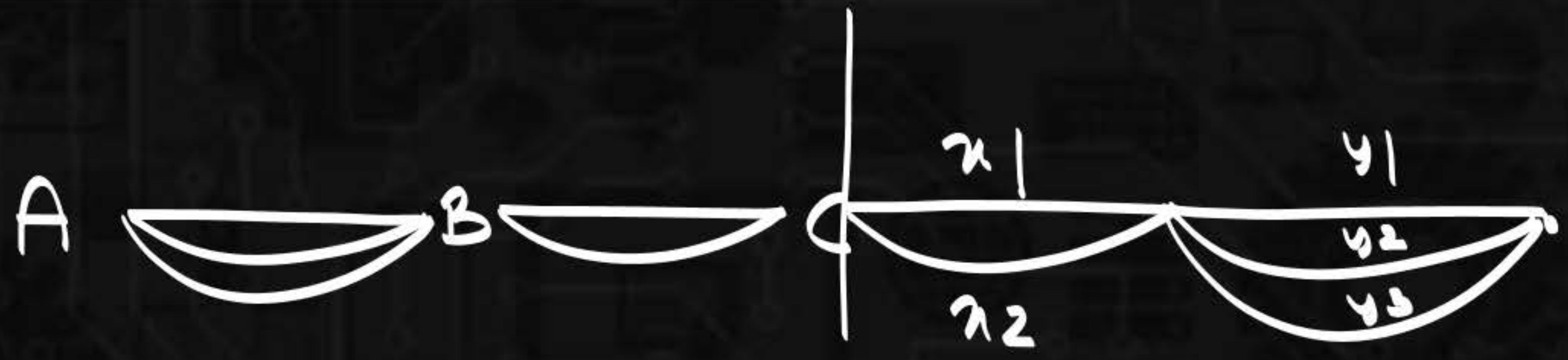
3) $A \underset{\text{via } B}{\text{———}} C \underset{\text{via } B}{\text{———}} A$

$$= 3 \cdot 2 \cdot 2 \cdot 3$$
$$= 6 \times 6$$
$$= 36.$$

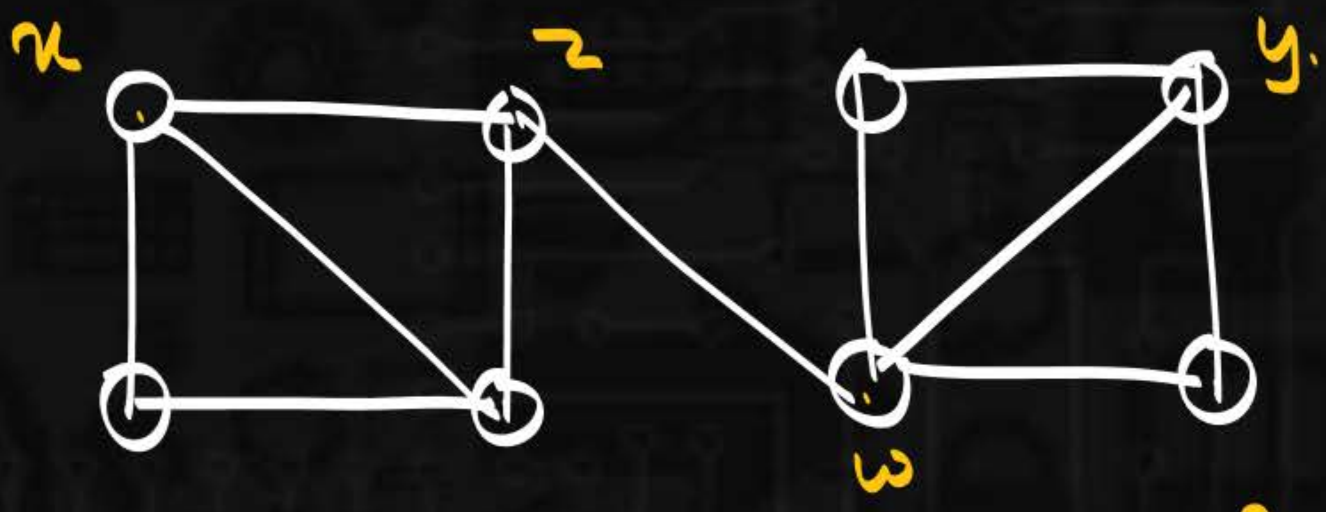
4) A ——— C ——— A.

$$(6+3) \times (3+6) = 9 \cdot 9 = 81 \text{ ways.}$$

COMBINATORICS



$3 \times 2.$ $\qquad\qquad$ 2×3



how many paths are there from x to y ?

$= 3 \cdot 3 = \underline{9 \text{ paths}}.$

x — z — w — y

$3 \times 1 \cdot\cdot 3$

COMBINATORICS

How many no plates we can generate if 4 characters followed by 1 or 2 or 3 or 4 digit?

$$\{\_\ \_\ \_\ \_\ |\ \_\} + \{\_\ \_\ \_\ \_\ |\ \_\ \_\} + \{\_\ \_\ \_\ \_\ |\ \_\ \_\ \_\} + \{\_\ \_\ \_\ \_\ |\ \_\ \_\ \_\ \_\}$$

$$26^4 \cdot 10 \quad + \quad 26^4 \cdot 10^2 \quad + \quad 26^4 \cdot 10^3 \quad + \quad 26^4 \cdot 10^4$$

$$26^4 \cdot 10 + 26^4 \cdot 10^2 + 26^4 \cdot 10^3 + 26^4 \cdot 10^4$$

$$= 26^4 (10 + 10^2 + 10^3 + 10^4)$$

COMBINATORICS

How many no plates we can generate if 1 or 2 or 3 or 4 characters followed by 1 or 2 or 3 or 4 digit?

Ans: $(26+26^2+26^3+26^4)(10+10^2+10^3+10^4)$

COMBINATORICS

How many ways we can select 2 diff language movies.

if we have 8 English movies.

10 Hindi movies.

20 Telugu movies.

8×20

$80 + 200 + 160$

$= 440$

$\boxed{8E}$ $\boxed{10H}$ $\boxed{20T}$

$8 \times 10 + 10 \times 20 + 8 \times 20$

COMBINATORICS

How many ways we can distribute 100 prizes among 3 students?

100

$$100 \times 99 \times 98$$

Q2: 100 prizes → 5 students.

$$100 \times 99 \times 98 \times 97 \times 96.$$

COMBINATORICS

n objects $\rightarrow r$ places

100 pr $\rightarrow$ 3 plac.

$(100)(100-1)(100-2)$

R place

$\underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ } \ldots \underline{\ \ }$

n ways $\cdot (n-1) \cdot (n-2) \qquad (n-r+1)$

$100 \cdots (100-3+1)$

$$n \cdot (n-1) \cdot (n-2) \cdots (n-R+1) \left(\frac{(n-R)!}{(n-R)!}\right) = \frac{n!}{(n-R)!} = {}^{n}P_{R}$$

14. There are five different roads from City A to City B, three different roads from City B to City C, and three different roads that go directly from A to C.

(a) How many different ways are there to go from A to C via B?

(b) How many different ways are there from A to C altogether?

(c) How many different ways are there from A to C and then back to A?

(d) How many different trips are there from A to C and back again to A that visit B both going and coming?

(e) How many different trips are there that go from A to C via B and return directly from C to A?

(f) How many different trips are there that go directly from A to C and return to A via B?

(g) How many different trips are there from A to C and back to A that visit B at least once?

(h) Suppose that once a road is used it is closed and cannot be used again. Then how many different trips are there from A to C via B and back to A again via B?

(i) Using the assumption in (h) how many different trips are there from A to C and back to A again?

23. A tire store carries 10 different sizes of tires, each in both tube and tubeless variety, each with either nylon, rayon cord, or steel-belted, and each with white sidewalls or plain black. How many different kinds of tires does the store have?


THANK YOU GW
SOLDIERS !