Branch : CSE/IT Batch : Hinglish

DISCRETE MATHEMATICS
Mathematical Logic

DPP-05

[MCQ]

1. Let p(x) and q(x) denote the following open statements.
p(x): $x^2 > 0$
q(x): x is odd
for the universe of all integers, determine the truth or falsity of each of the statement.
S_1: $\forall x\ [p(x)\rightarrow q(x)]$
S_2: $\exists x\ [p(x)\rightarrow q(x)]$
which of the following is true?
(a) S_1 only (b) S_2 only
(c) Both S_1 and S_2 (d) Neither S_1 nor S_2

[MCQ]

2. Consider following two First Order Logic Statements:
S_1: $[\forall x(\sim P(x)\vee Q(x))] \rightarrow [\forall x\ P(x)] \rightarrow [\forall x\ Q(x)]$
S_2: $[\exists x\ P(x)] \rightarrow [\exists x\ Q(x)] \rightarrow [\exists x\ (P(x)\rightarrow Q(x))]$
Which of the following is valid?
(a) S1 only (b) S2 only
(c) Both S1 and S2 (d) None of these

[MSQ]

3. $P(y) = \sqrt{y}$ is real in the domain of Z^+, then which of the following is / are correct?
(a) $\forall y\ P(y)$ (b) $\exists y\ P(y)$
(c) $\forall y \sim P(y)$ (d) $\exists y \sim P(y)$

[MCQ]

4. Which of the following is not valid logical expression?
(a) $\forall x\ [P(x) \rightarrow Q(x)] \rightarrow [\forall x\ P(x)] \rightarrow [\forall x\ Q(x)]$
(b) $\forall x\ [P(x) \vee Q(x)] \rightarrow [\forall x\ P(x)] \vee [\forall x\ Q(x)]$
(c) $\exists x\ [P(x) \wedge Q(x)] \rightarrow [\exists x\ P(x)] \wedge [\exists x\ Q(x)]$
(d) $\forall x\ [P(x) \leftrightarrow Q(x)] \rightarrow [\forall x\ P(x)] \leftrightarrow [\forall x\ Q(x)]$

[MCQ]

5. Consider following logical expressions:
I: $\forall y[P(y) \rightarrow Q] \leftrightarrow [\forall y\ P(y)] \rightarrow Q$
II: $\exists y[P(y) \rightarrow Q] \rightarrow [\exists y\ P(y)] \rightarrow Q$
which of the following logical expression is valid?
(a) I only (b) II only
(c) Both I and II (d) None of these

Answer Key

1. **(b)**
2. **(c)**
3. **(a, b)**
4. **(b)**
5. **(d)**

Hints and Solutions

1. **(b)**

Statement S_1: $\forall x[p(x) \rightarrow q(x)]$

As we know the $\forall x$ connected through '$\wedge$' operator.

So, check the statement for x = 3

$\therefore$ $[p(3) \rightarrow q(3)] = [(3^2 > 0) \rightarrow (3 \text{ is odd})]$

$\equiv$ [True $\rightarrow$ True] $\equiv$ True

Now, check the statement for x = 2

$\therefore$ $[p(2) \rightarrow q(2)] = [(2^2 > 0) \rightarrow (2 \text{ is odd})]$

$\equiv$ [True $\rightarrow$ False] $\equiv$ False

Here S_1 is false.

Statement S_2: True

If $\exists x$ is true for one value then the overall the truth value of the statement will be true.

So, Check the statement for x = 3

$\therefore$ $[p(3) \rightarrow q(3)] = [(3^2 > 0) \rightarrow (3 \text{ is odd})]$

$\equiv$ [True $\rightarrow$ True] $\equiv$ True

Hence, S_2 is True.

2. **(c)**

S_1: $[\forall x\,(P(x) \rightarrow Q(x)] \rightarrow [\forall x\, P(x)] \rightarrow [\forall x\, Q(x)]$

(Property of Predicate Logic)

$[\forall x\,(\sim P(x) \vee Q(x)] \rightarrow [\forall x\, P(x)] \rightarrow [\forall x\, Q(x)]$

S_2: $[\exists x\, P(x)] \rightarrow [\exists x\, Q(x)] \rightarrow [\exists x\,(P(x) \rightarrow Q(x)]$

Proof:

$$(P_1 \vee P_2) \rightarrow (Q_1 \vee Q_2) \rightarrow [(P_1 \rightarrow Q_1) \vee (P_2 \rightarrow Q_2)]$$

$$(P_1'P_2' + Q_1 \vee Q_2) \rightarrow [(P_1' + Q_1) + (P_2' + Q_2)]$$

$$(P_1 + P_2) \cdot (Q_1'\ Q_2') + P_1' + Q_1 + P_2' + Q_2$$

$$P_1Q_1'Q_2' + P_2Q_1'Q_2' + P_1' + Q_1 + P_2' + Q_2$$

$$\boxed{A'B + A = A + B}$$

$$P_1 + P_2 + P_1' + Q_1 + P_2' + Q_2$$

$$P_1 + P_1' = 1 \text{ and } 1 + \text{anyting} = 1$$

$$1 + P_1 + P_2 + Q_1 + Q_2 + P_2'$$

1 True

Hence both are valid.

3. **(a, b)**

$P(y) = \sqrt{y}$ is real

domain = positive integers (z^+)

(a) $\forall y\, P(y)$ True

For every values of y, $\sqrt{y}$ is real because domain is positive integer

(b) $\exists y\, P(y)$ True

For some values of y, $\sqrt{y}$ is real

(c) $\forall y \sim P(y)$ False

(d) $\exists y \sim P(y)$ False

4. **(b)**

$(P_1 \vee Q_1) \wedge (P_2 \vee Q_2) \rightarrow (P_1 \wedge P_2) \vee (Q_1 \wedge Q_2)$

$(P_1 + Q_1) \wedge (P_2 + Q_2) \rightarrow P_1P_2 + Q_1Q_2$

$P_1'\, Q_1' + P_2'\, Q_2' + P_1P_2 + Q_1Q_2$

$P_1P_2 + Q_1Q_2 + P_1'\, Q_1' + P_2'\, Q_2'$ (Invalid)

Remaining all are valid.

5. **(d)**

I: $\forall y[P(y) \rightarrow Q] \leftrightarrow [\forall y\, P(y)] \rightarrow Q$

$(P_1 \rightarrow Q) \wedge (P_2 \rightarrow Q) \equiv (P_1 \wedge P_2) \rightarrow Q$

$(P_1' + Q) \wedge (P_2' + Q) \equiv P_1' + P_2' + Q$

$P_1'\, P_2' + P_1'Q + P_2'Q + Q \equiv P_1' + P_2' + Q$

$P_1'\, P_2' + Q \not\equiv P_1' + P_2' + Q$ (invalid)

II: $\exists y\, [P(y) \rightarrow Q] \rightarrow [\exists y\, P(y)] \rightarrow Q$

$(P_1 \rightarrow Q) \vee (P_2 \rightarrow Q) \rightarrow (P_1 \vee P_2) \rightarrow Q$

$P_1' + P_2' + Q \rightarrow P_1'\, P_2' + Q$

$P_1\, P_2\, Q' + P_1'\, P_2' + Q$

$P_1P_2 + P_1'\, P_2' + Q$ (Invalid)

Hence, option (d) is correct