CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 06

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TOPICS TO BE COVERED

- 01 onto Functions
- 02 1:1 correspondance Functions
- 03 Number of Functions
- 04 Types of Functions
- 05 Various Examples in Functions

Partial order relation :

(RAT) $\begin{cases} \text{Reflexive} \\ \text{Antisymmetric.} \\ \text{Transitive} \end{cases}$

$R : \{ (a,b) \mid a \mid b \}$ ✓ Set: Z^+ (POR)

Ref: $aRa \quad a \mid a$. ✓

Anti: $aRb \wedge bRa \rightarrow a = b$. OR $a \mid b \wedge b \mid a \rightarrow a = b$.

$\underbrace{3 \mid 6}_{T} \wedge \underbrace{6 \mid 3}_{F}$

$\underbrace{T \wedge F}_{F}$

$\underbrace{F \rightarrow}_{T}$

Transitive:

$a \mid b \wedge b \mid c \rightarrow a \mid c$.

$R_2 : \{(a, b) \mid a \le b\}$. set: Z

✓ $\underline{R}$: aRa. $a \le a$ (T)

✓ $\underline{Anti}$: $aRb \wedge bRa \rightarrow a = b$.

$a \le b \wedge b \le a \rightarrow a = b$.

✓ $\underline{T}$: $a \le b \wedge b \le c \rightarrow a \le c$.

$$\underbrace{\underbrace{\underbrace{2 \le 3}_{T} \wedge \underbrace{3 \le 2}_{F}}_{F} \longrightarrow}_{T}$$

Set : A

set

A x A

partition

RST

EQR

Result : { ..... }

Set : A

A x A

RAT

POR

Result : { ..... }

Relation

Set : { ... a, b ... }

Relation : R.

* aRb OR bRa.

a, b are comparable.

Set : Z^+ : { 1, 2, 3 .... ∞ }.
(Partial order set)

Relation : 1. (Division)
(POR)

w.r.t this Relation only partial elements are comparable

Choose any a, b :

$a = 3 \quad b = 6$.

aRb OR bRa.

$a|b$ OR $b|a$.

$\underset{T}{3|6}$ OR $\underset{F}{6|3}$.

3, 6 are comparable

$a = 3 \quad b = 5$.

aRb OR bRa.

$a|b$ OR $b|a$.

$\underset{F}{3|5}$ OR $\underset{F}{5|3}$.

3, 5 are not comparable

Set : z : $\{ \cdots 0, \cdots \}$. (Total order set)

Relation : $\leq$. (Total order Relation)

Choose any 2 elements.

$a = 10^2 \quad b = 10^4$

aRb OR bRa.

$a \leq b$ OR $b \leq a$.

$\underline{10^2 \leq 10^4}_{T}$ OR $10^4 \leq 10^2$.

any 2 elements are always comparable.

→ all elements are comparable.

Total elements are comparable.

Set:

Relation → RAT → Yes → POR → check all elements are comparable

RAT ↓ NO

check all elements are comparable ↑ NO

check all elements are comparable ↓ Yes ↓

(Total order Relation

linear order relation)

Set: { 1, 2, 3, 4, 6, 12 }.

$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,12)$

$(2,2)\ (2,4)\ (2,6)\ (2,12)$

$(3,3)\ (3,6)\ (3,12)$

$(4,4)\ (4,12)$

$(6,6)\ (6,12)$

$(12,12) \}$.

POR

(poset)

Partial order set.

(POR)

(Set, Relation) (poset)

$\rightarrow \big(\{1, 2, 3, 4, 6, 12\}, \mid \big)$

poset.

$(D_{12}, \mid)$

(set, Relation)

$(Z^+, |)$ – poset

OR

$(Z, \leq)$ –

toset

lower ← → upper

$(a, b) \in R$

OR

$a R b$

or

b
$\uparrow$
a

same level :

$a \to b$ ✗

$a \searrow b$ ✗

Set: $\{1, 2, 3, 4, 6, 12\}$.

$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,12)$

$(2,2)\ (2,4)\ (2,6)\ (2,12)$

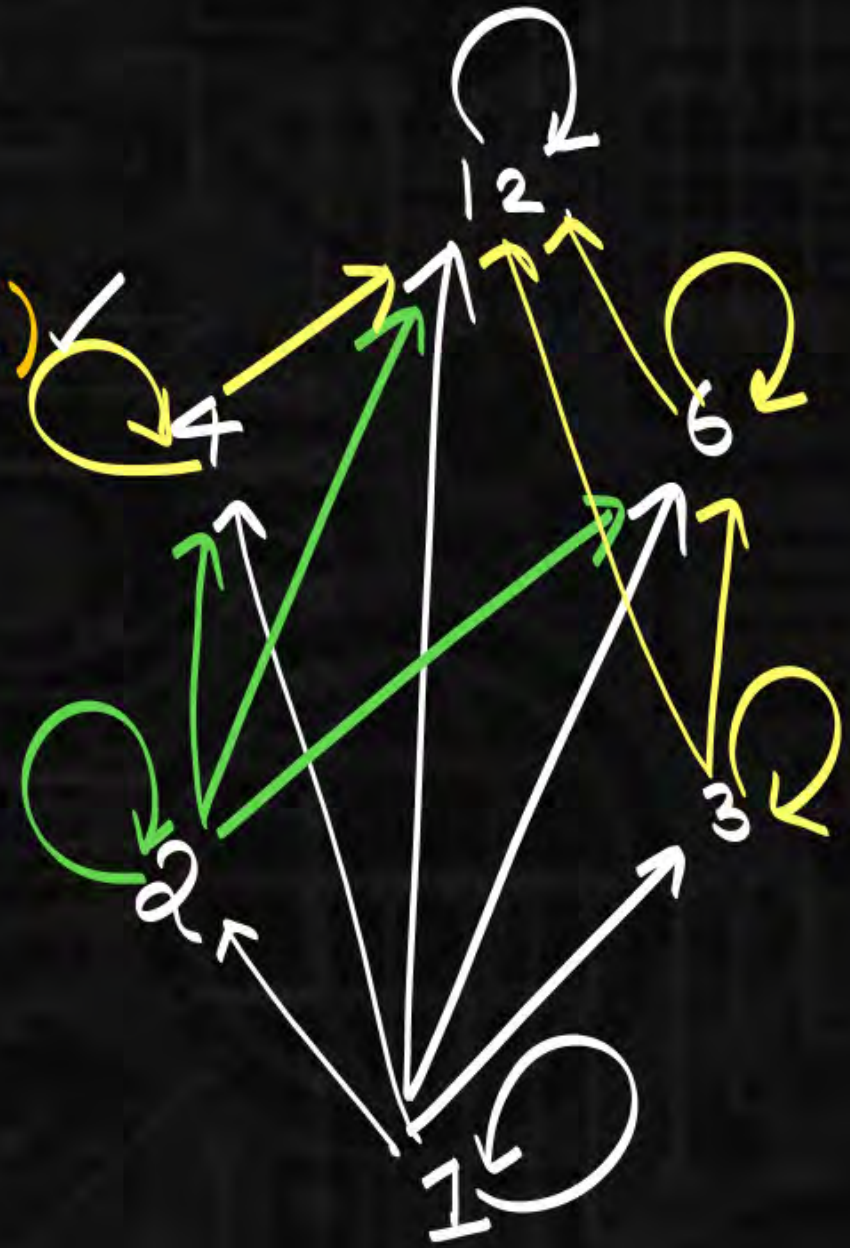
$(3,3)\ (3,6)\ (3,12)$

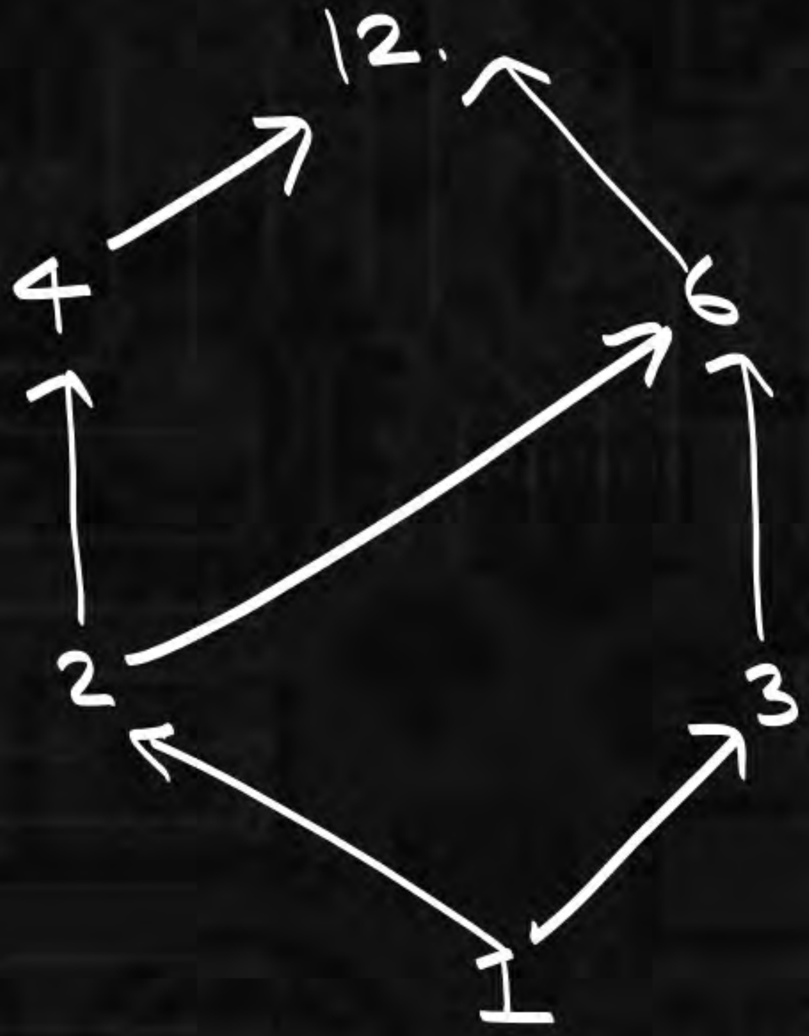
$(4,4)\ (4,12)$

$(6,6)\ (6,12)$

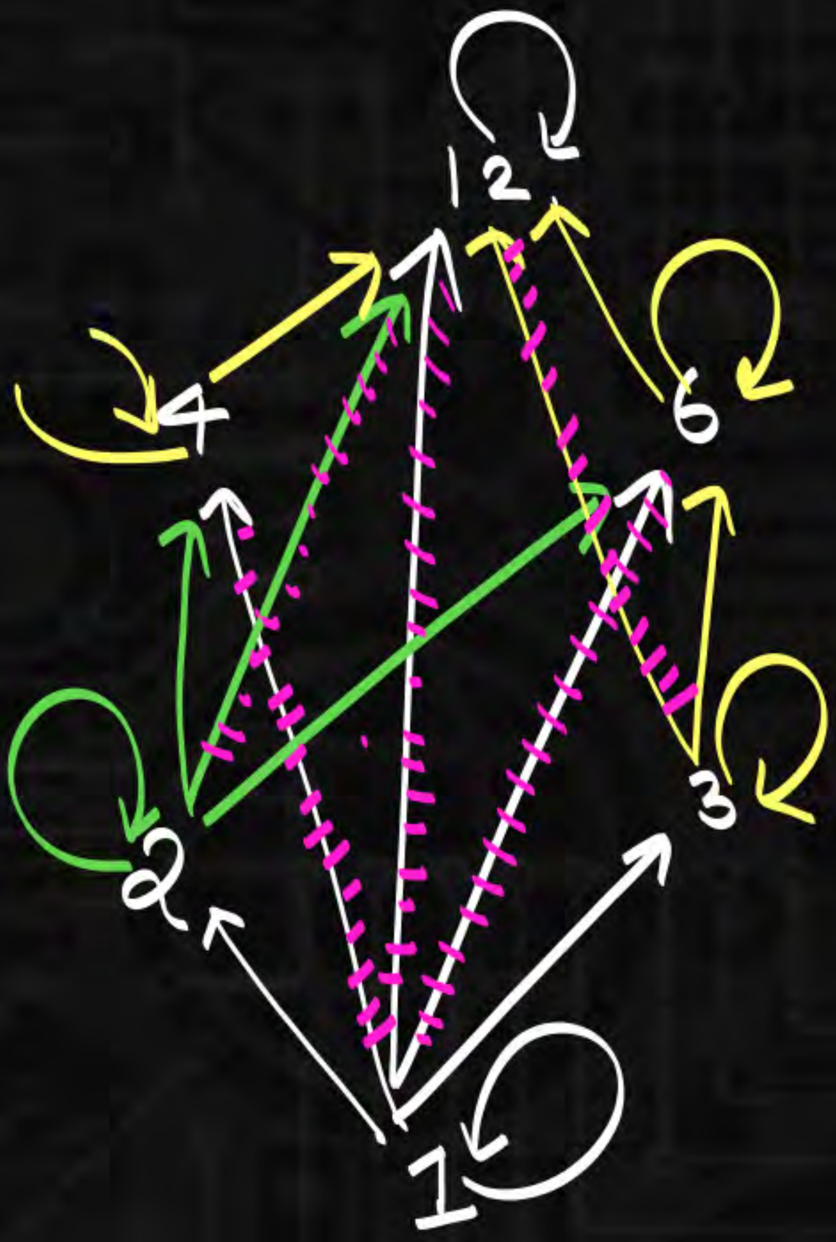
$(12,12) \}$.

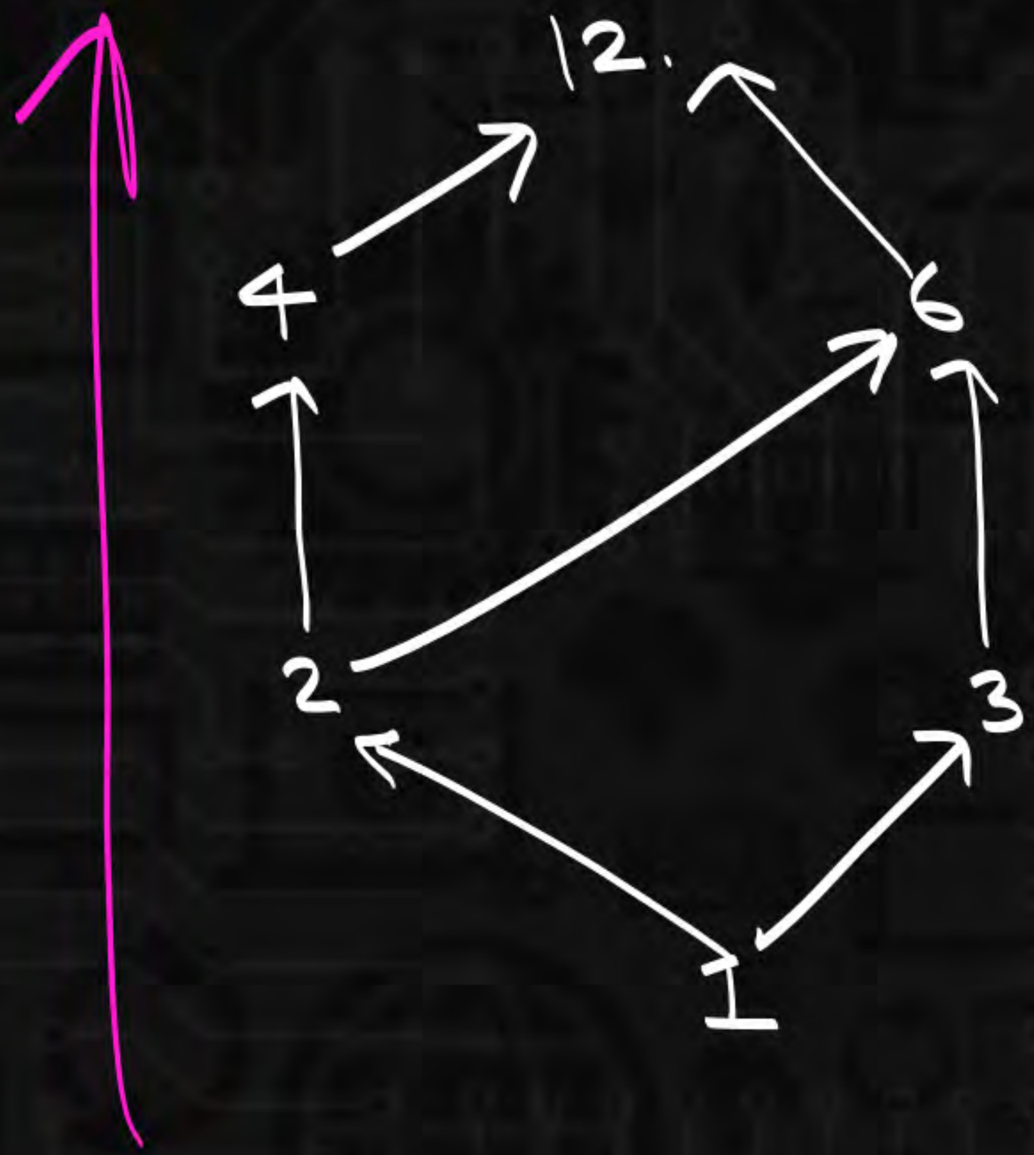
POR





- Remove selfloop.
- Remove transitive arrows.





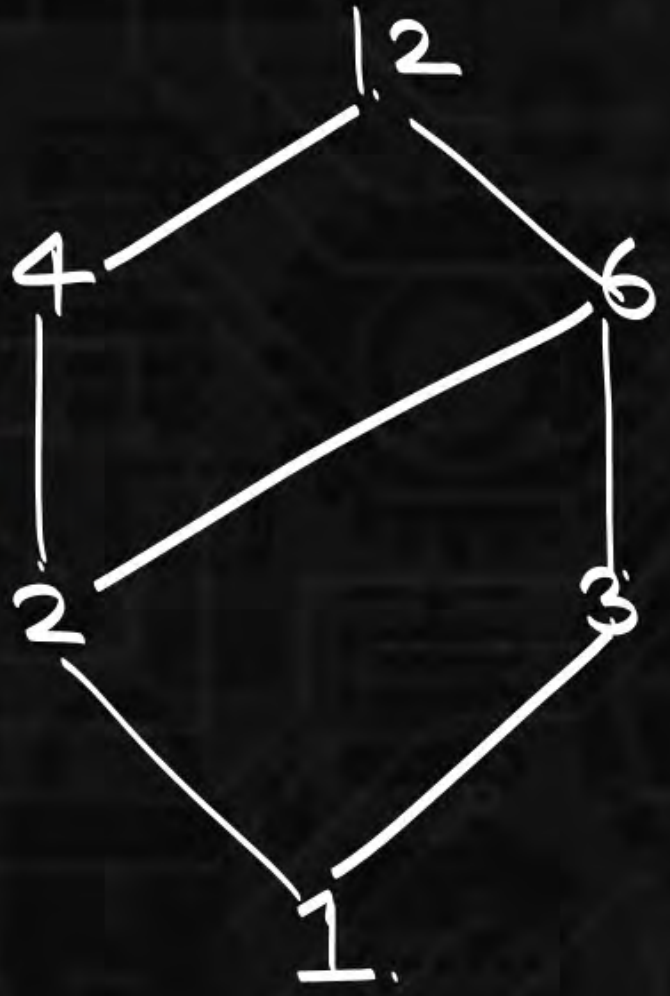
R: { ...

→ ... }

OR

$(D_{12}, |)$ OR

poset



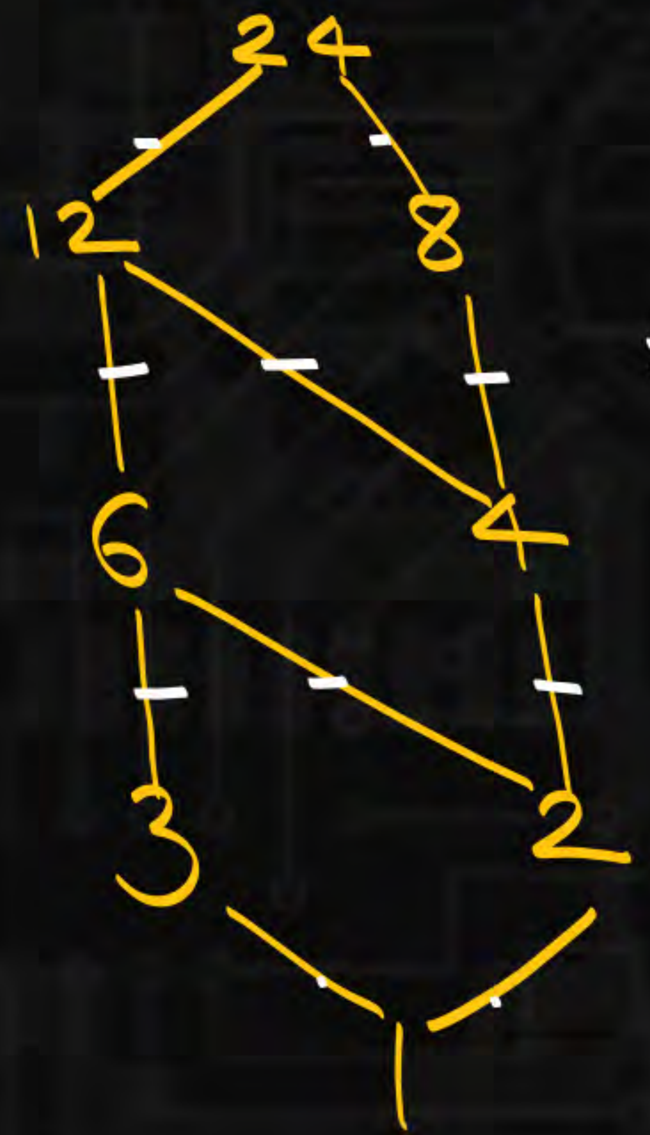
Hasse diagram

$(D_{24}, |)$

or

$(D_{36}, |)$

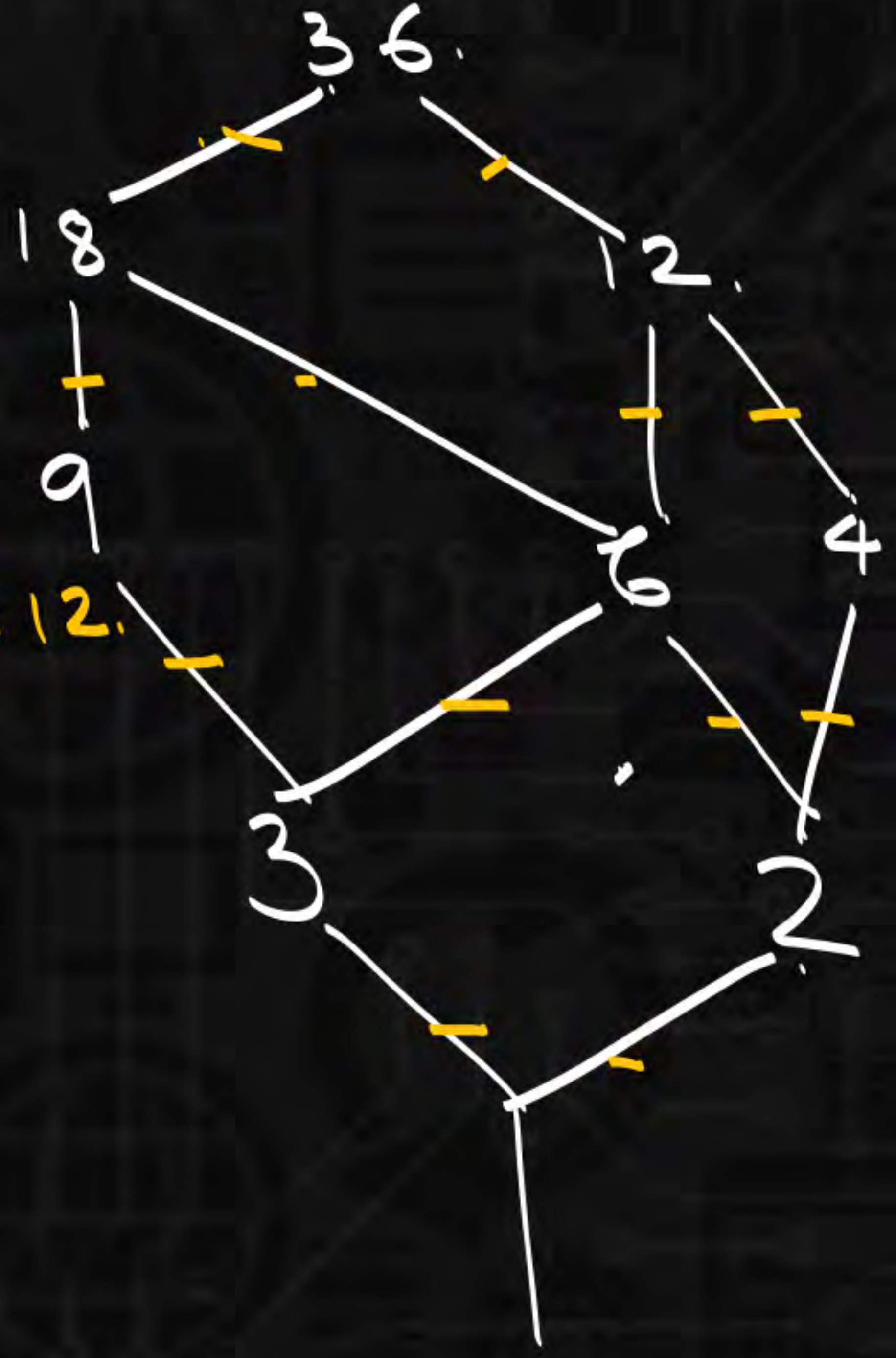
$(\{1, 2, 3, 4, 6, 8, 12, 24\}, |)$



# of edges = 10

$(D_{36}, |)$

$(\{1, 2, 3, \cancel{4}, \cancel{6}, \cancel{9}, \cancel{12}, 18, \cancel{36}\}, |)$



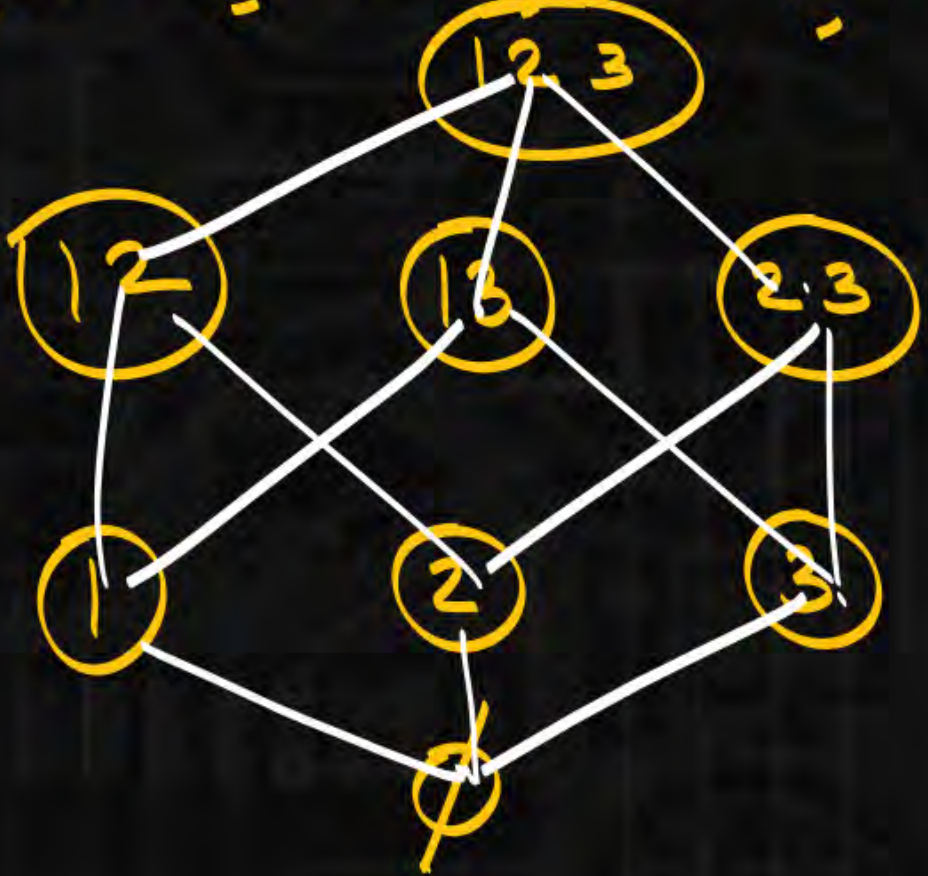
# of edges = 12.

$A = \{1, 2, 3\}$ (Set, Relation)

set. R.

$(P(A), \subseteq)$

$(\{\phi, \{1\}, \{2\}, \{3\}, \{12\}, \{13\}, \{23\}, \{123\}\}, \subseteq)$



R: $ARA \quad A \subseteq A.$

Anti: $ARB \wedge BRA \rightarrow A = B.$

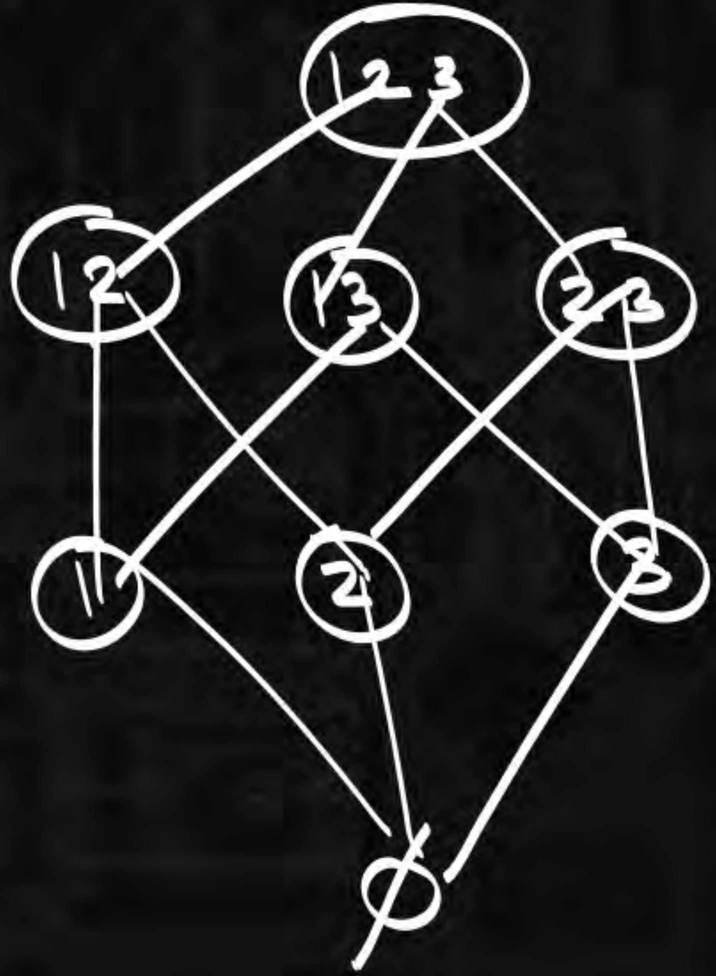
$A \subseteq B \wedge B \subseteq A \rightarrow A = B.$

T: $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C.$

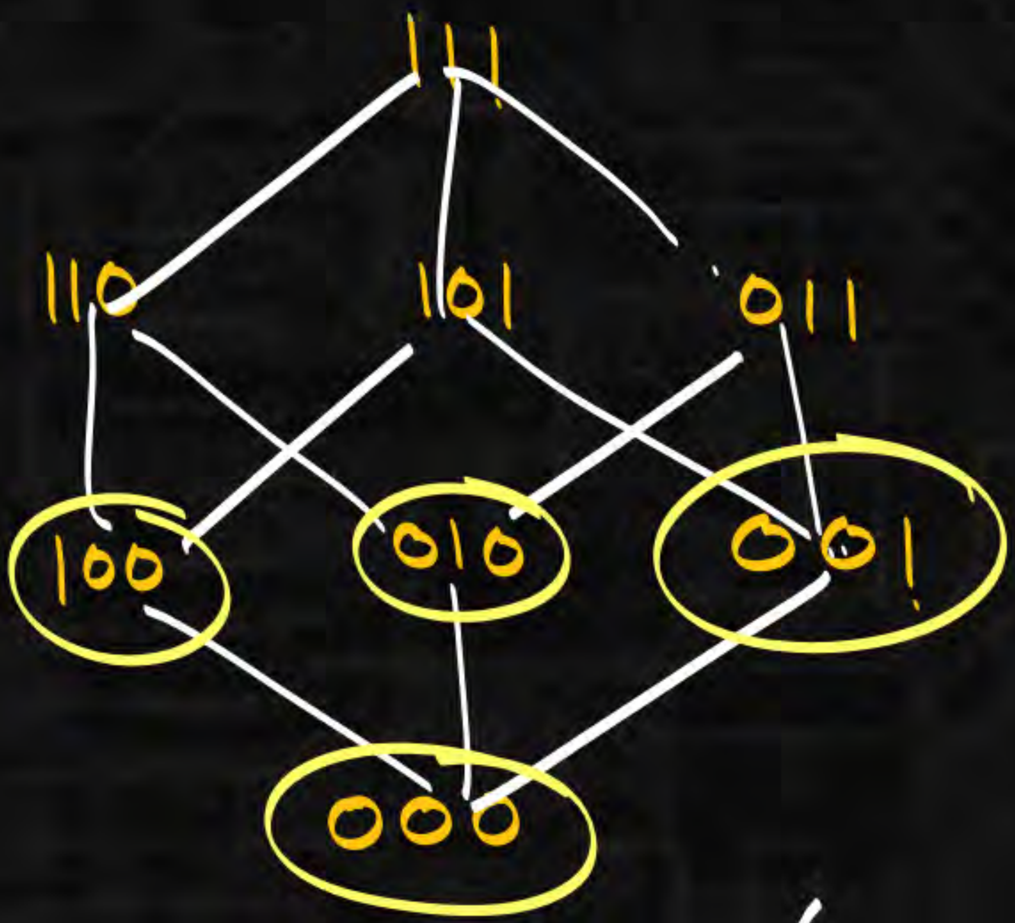
$A = \{1, 2, 3, 4\}$

$(P(A), \subseteq)$

no. of edges = ?



$$\frac{1\ 2\ 3}{1\ 0\ 0}$$



Hypercube (Q_3)

$A = \{1, 2, 3, 4\}$

$(P(A), \subseteq)$

# of edges =

$\Sigma\, d(v_i) = 2e$

$n \cdot 2^n = 2e$

$\boxed{e = n \cdot 2^{n-1}}$

$n = 4 \quad e = 4 \cdot 2^{4-1}$

$= 4 \cdot 2^3 = 4 \cdot 8$

$= 32$


THANK YOU GW
SOLDIERS !