CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 02

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TOPICS TO BE COVERED

01 onto Functions

02 1:1 correspondance Functions

03 Number of Functions

04 Types of Functions

05 Various Examples in Functions

Functions

$f: A \to B$

1:1 correspondance. (bijective = injective + surjective)

1:1 $\wedge$ onto

$A \le B \wedge A \ge B \longrightarrow A = B$

$A = B$

Functions

$f : G_1 \rightarrow G_2 \ (1:1 \ c)$

$f : V_1 \rightarrow V_2 \qquad |V_1| = |V_2|$

$f : E_1 \rightarrow E_2 \qquad |E_1| = |E_2|$

Functions

$\{a, b, c\}$

subset.

1 : 1 c

$|A| = |B|$

2^3

		$a\ b\ c$		
$\emptyset$	$\rightarrow$	0 0 0	F F F	
$\{a\}$	$\rightarrow$	1 0 0		
$\{b\}$	$\rightarrow$	0 1 0		
$\{c\}$	$\rightarrow$	0 0 1.	2^3.	2^3.
$\{ab\}$		.		
$\{bc\}$		.		
$\{ac\}$		.		
$\{abc\}$		1 1 1	T T T	

$a\ b\ c$

Represent

$\{a\} \rightarrow$ present

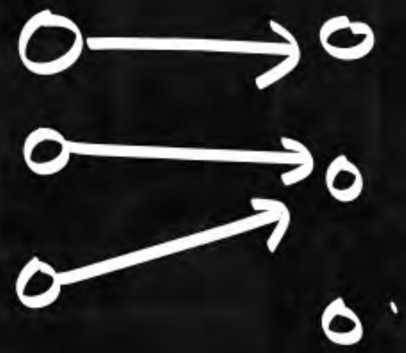
b, c absent.

a	b	c
1	0	0

$1 \rightarrow T$

$0 \rightarrow F$.

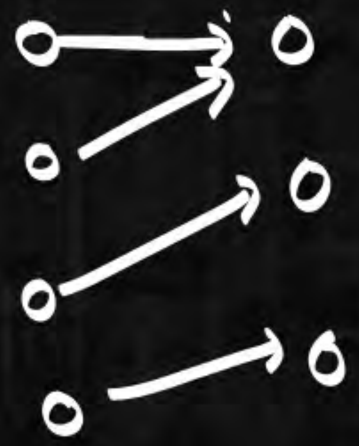
Functions



1:1 ✗

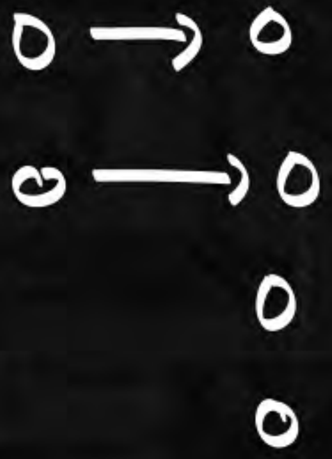
onto ✗

Funct^n ✓



1:1 ✗

onto ✓



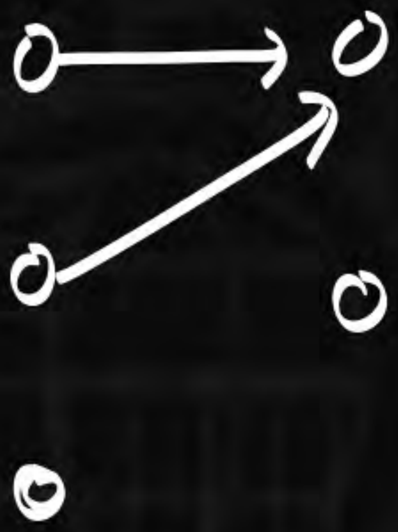
1:1 ✓

onto ✗

1:1 ✓

onto ✓

1:1c ✓



not a function.

Functions

identity Function $i_A : A \to A$.

$f : A \to A$

$$a \to a$$
$$b \to b$$
$$c \to c.$$

$f(x) = x.$

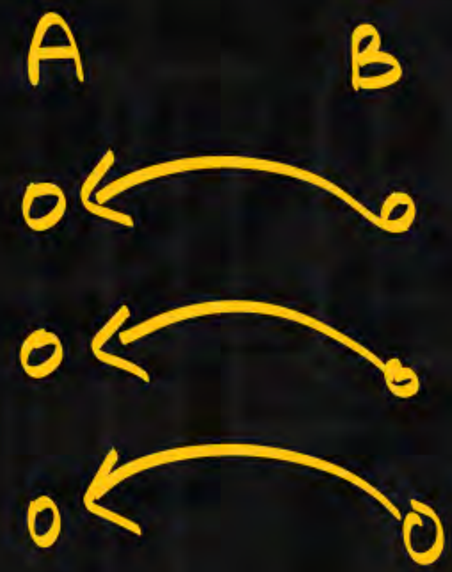
Inverse function : f^{-1}

Invertible Function

$f^{-1} \neq \frac{1}{f}.$

$f : A \to B$

$f^{-1} : B \to A$



Functions

$a \longrightarrow x$

$b \longrightarrow y$

$c \longrightarrow z$.

$f(a) = x$.

$f(b) = y$

$f(c) = z$.

$a \longleftarrow x$

$b \longleftarrow y$

$c \longleftarrow z$.

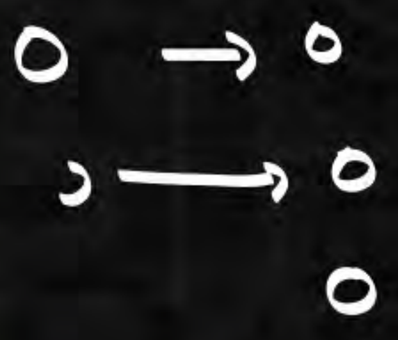
$f^{-1}(x) = a$.

$f^{-1}(y) = b$

$f^{-1}(z) = c$

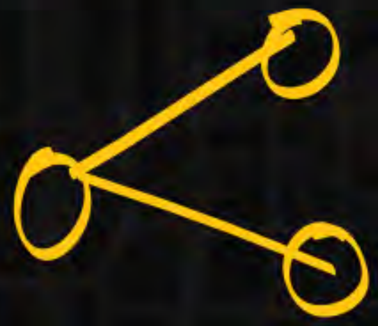
not 1:1 c

1:1



onto

not a function



not a function

Functions

$f: Z \to Z.$

$f(x) = x+3$

$\vdots$

$0 \longrightarrow 3$

$1 \longrightarrow 4$

$f(x) = x+3$

$f(1) = 1+3$

1:1 ✓

onto ✓

3 ways:	a	1
2 ways:	b	2
1 way	c	3

$f: A \to B$ (1:1 c)

$|A| = |B| = n$

what will be no of 1:1 c.

Functions $= n!$.

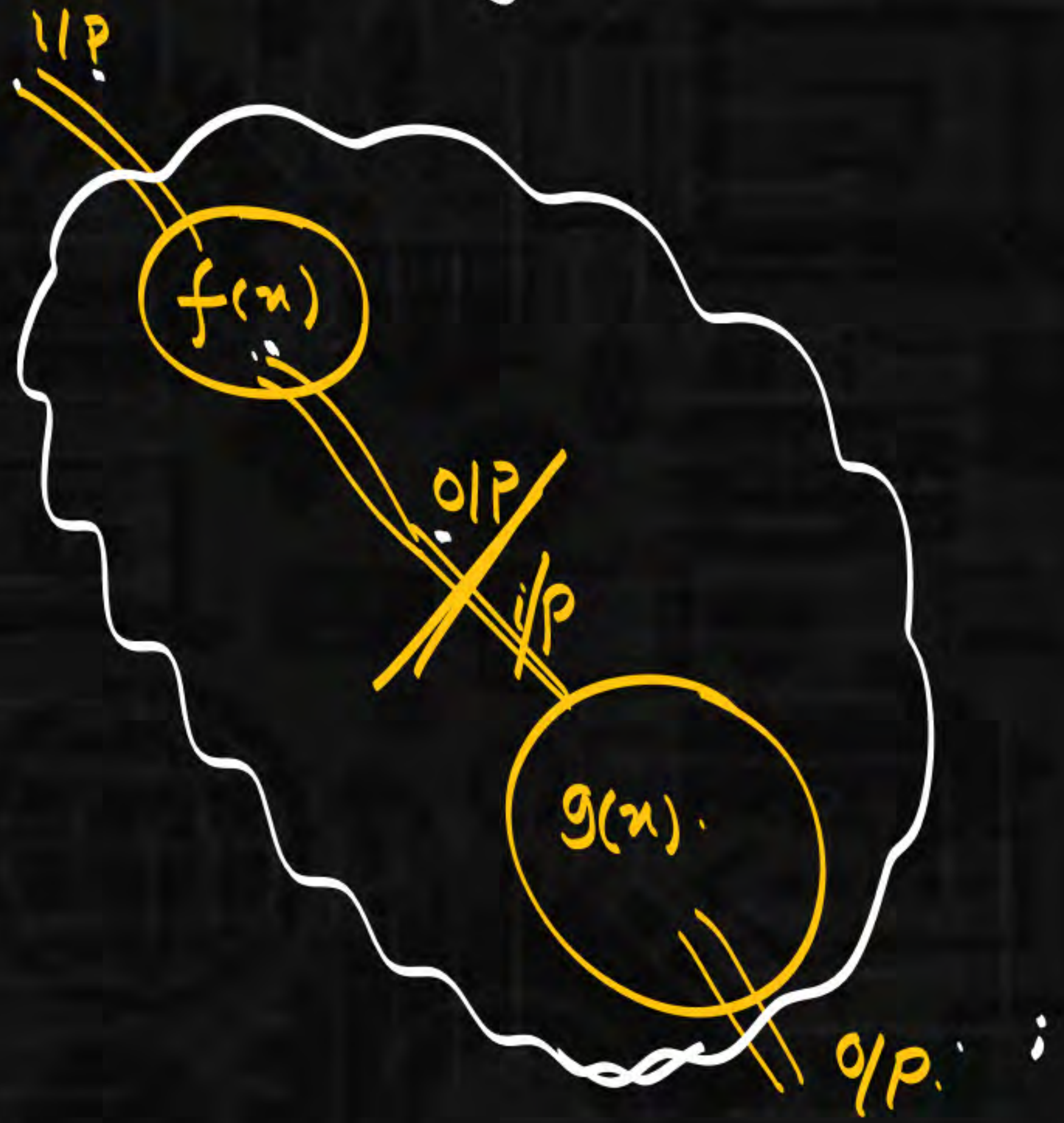
$3 \cdot 2 \cdot 1$

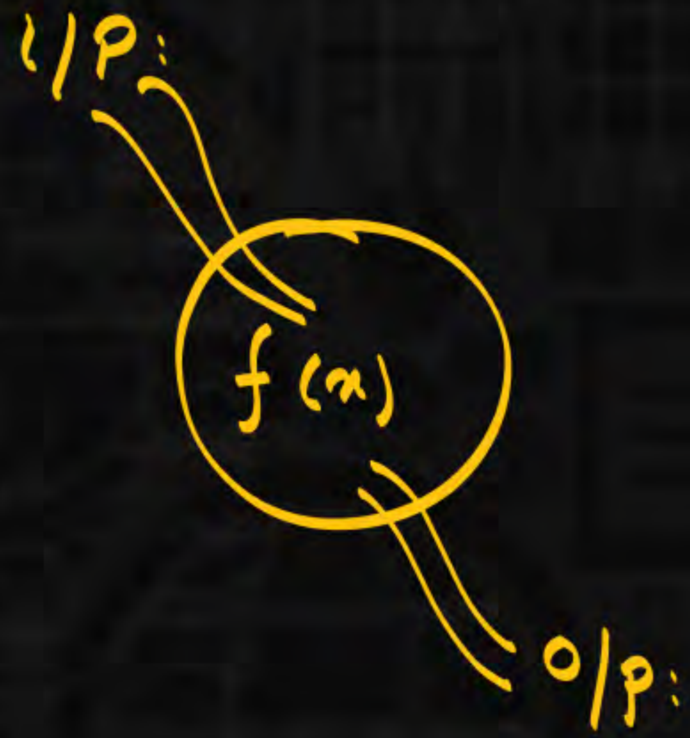
$n \cdot (n-1) \cdots 1$

$= n!$.

Functions

Compositions of functions :





Functions

$f : A \to B \quad g : B \to C$

$g(f(\,)) \Rightarrow \boxed{g \circ f : A \to C}$

$a \longrightarrow x \qquad x \longrightarrow 0$

$b \longrightarrow y \qquad y \longrightarrow 0$

$c \longrightarrow z \qquad z \longrightarrow 1$

$\qquad\quad w \qquad w \longrightarrow 2$

$f(a) = x \qquad g(x) = 0$

$f(b) = y \qquad g(y) = 0$

$f(c) = z \qquad g(z) = 1$

$\qquad\qquad\quad g(w) = 2$

$\boxed{f(a) = x}$

$g(x) = 0$

$\downarrow$

$\boxed{g(f(a)) = 0}$

$g(f(b)) = 0$

$g(f(c)) = 1$

Functions

$f(x) = x+1 \quad g(x) : 2x+3$

$gof = g(f(\,)) = g(f(x))$

$= g(x+1)$

$= 2(x+1)+3$

$= 2x+5$

$fog = f(g(x))$

$= f(\boxed{2x+3})$

$= (2x+3)+1$

$= 2x+4$

Functions

$f, g: R \to R$

$f(x) = x^2 + x - 1 \quad g(x): 3x + 2$

$fog(x) - gof(x) = 6(x+1)^2.$

Q.2: $g(x) = 1 - x + x^2 \quad f(x) = ax + b$

(NAT)

$gof(x) = 9x^2 - 9x + 3$

what will be $a+b = ?$

0.

a b   a, b

3, −1   −3 +2

Functions

$f, g : R \to R$

$gof(x) = g(f(x))$

$-a + 2ab = -9.$

$= g(ax+b)$

$a = 3$

$a = -3$

$g(ax+b) = 1-(ax+b)+(ax+b)^2.$

$= 1-(ax+b)+(ax+b)^2.$

$g(a) = 1-a+a^2.$

Q.2: $g(\underline{x}) = 1-\underline{x}+\underline{x}^2 \quad f(x) = ax+b$

$= 1-(ax+b)+(a^2x^2+2axb+b^2)$

$a^2 = 9$

$\boxed{a = \pm 3.}$

$gof(x) = 9x^2-9x+3$

$= (1-b+b^2) + (\boxed{-ax+2axb}) + a^2x^2$

$b^2-b+1 = 3$

$b^2-b-2 = 0$

$b^2-2b+b-2 = 0$

$b = 2, -1$

$\underline{9}x^2-9x+3 = \underline{a^2}x^2 + (-a+2ab)x + \boxed{1-b+b^2}$

Functions

$f : A \to B$

$(1:1c)$ ✓

$\downarrow$

Inverse exist ✓

$\downarrow$

$f^{-1} : B \to A$

2nd. 1st.

$f \circ \underline{f^{-1}} = i_B$

$f^{-1} \circ f = i_A$

A B

$a \longrightarrow 1$

$b \longrightarrow 2$

$c \longrightarrow 3$

Functions

$f: A \to B \quad g: B \to C$

if f, g are $1:1$ then $g \circ f$ is $1:1$ (True)

f ($1:1$) $\qquad$ g ($1:1$)

$a_1 \xrightarrow{f} b_1 \xrightarrow{g} x_1$

$a_2 \xrightarrow{f} b_2 \xrightarrow{g} x_2$

$\circ \longrightarrow x_3$

x_4

$a_1 \longrightarrow x_1$

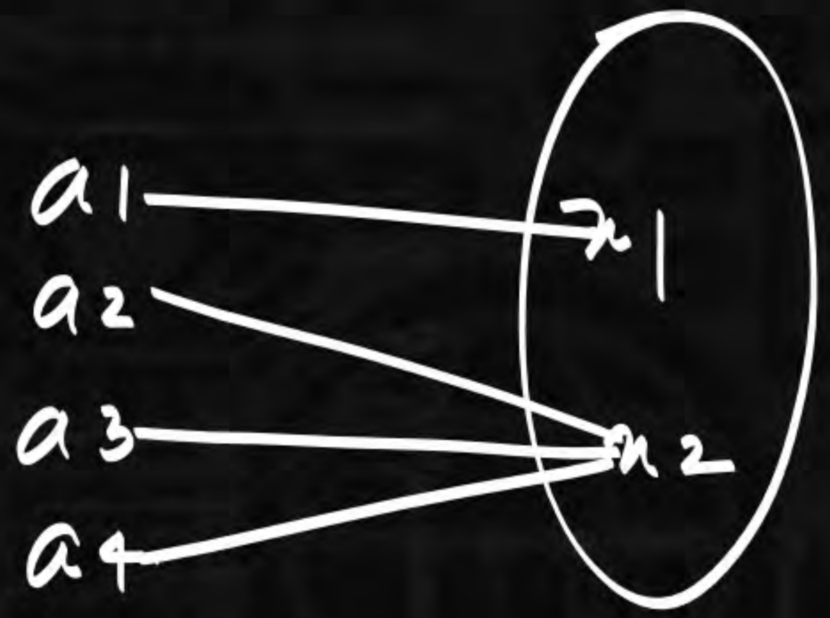
$a_2 \longrightarrow x_2$

x_3

x_4

Functions

$f: A \to B \quad g: B \to C$



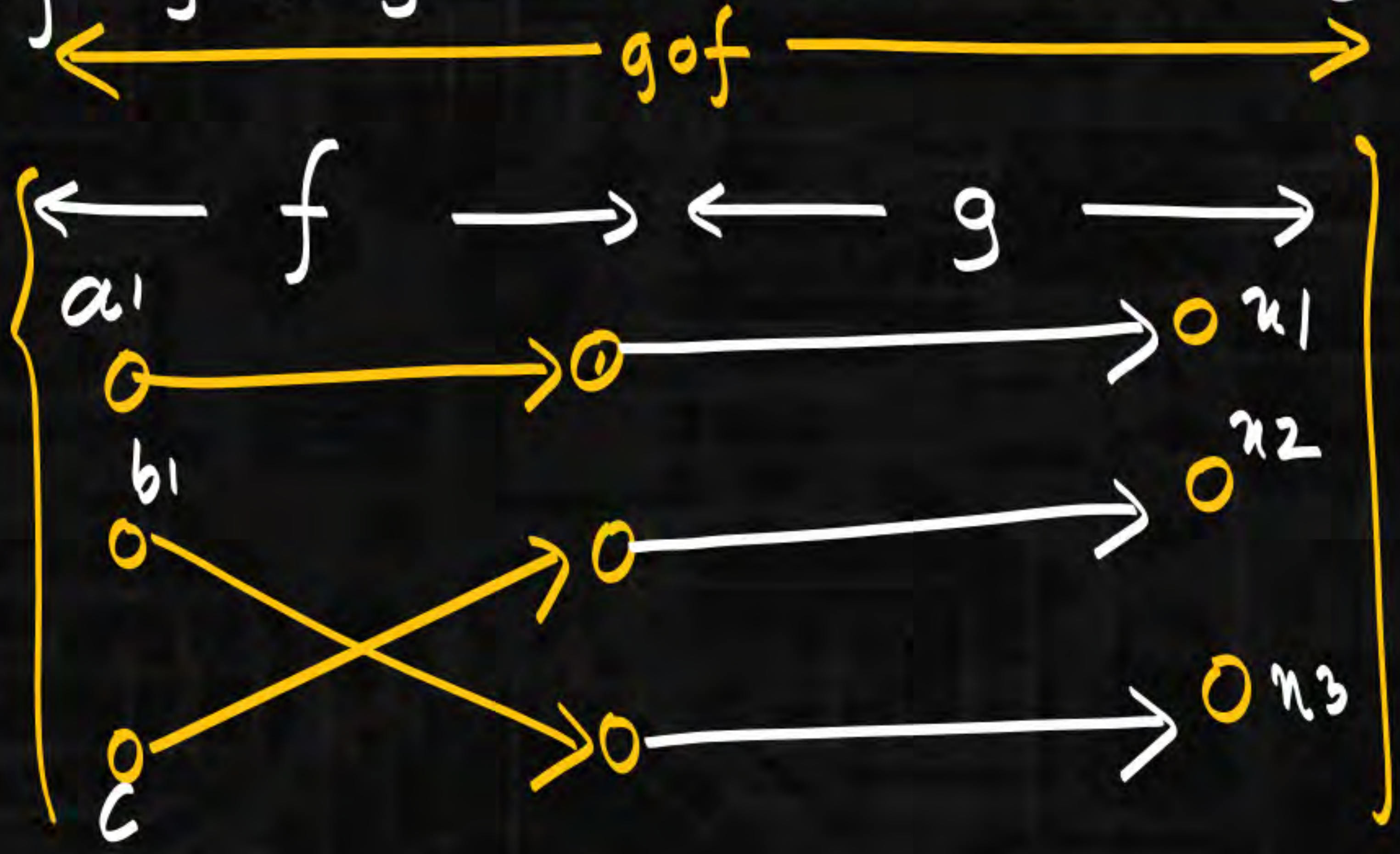
3) if f, g are onto then $g \circ f$ is onto (True)

Functions

$f: A \to B \quad g: B \to C$

True. check.

1) if f & g are 1:1c then $g \circ f$ is 1:1c.

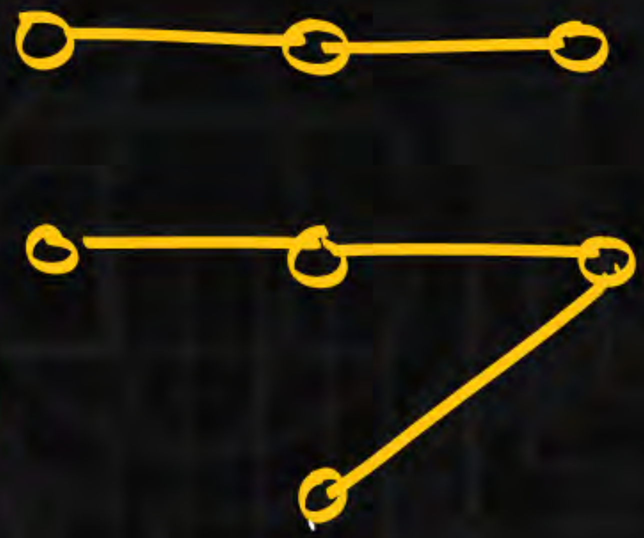


$g \circ f$ — (1:1c ✓

$$\left\{ \begin{array}{l} a_1 \to x_1 \\ b_1 \to x_3 \\ c \to x_2 \end{array} \right\}$$

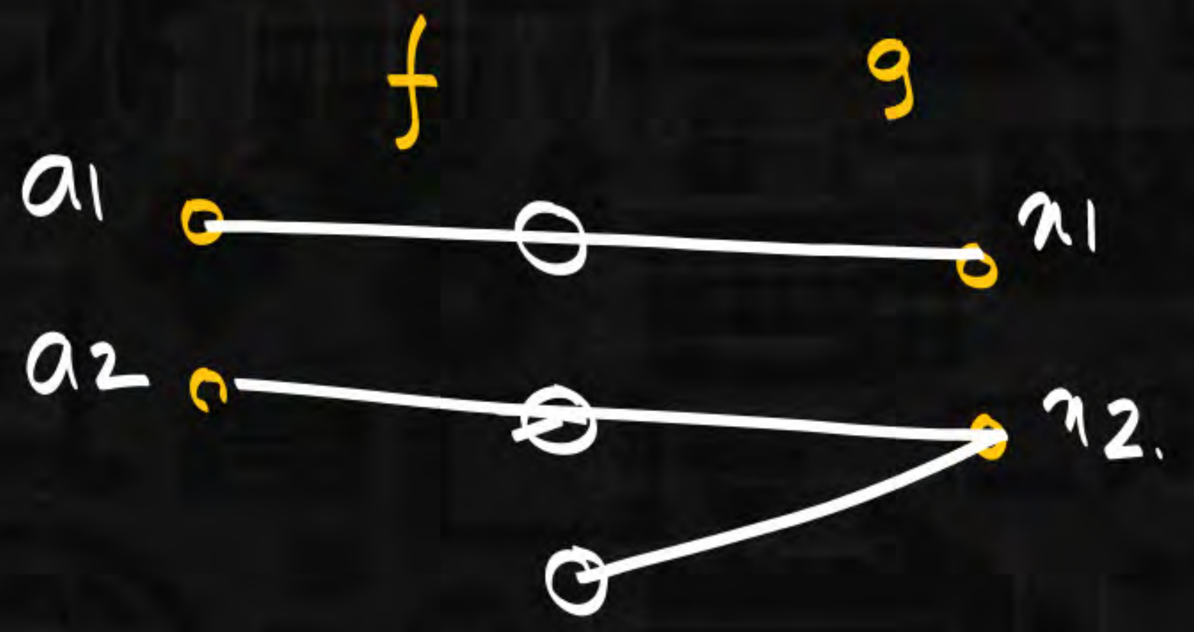
Functions

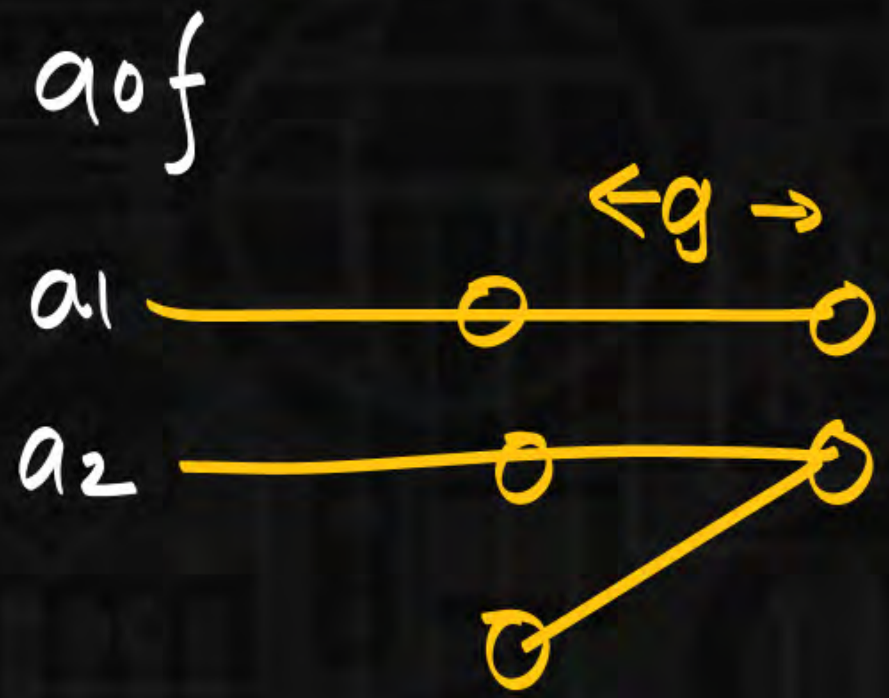
1) if $g \circ f$ is onto then f is onto (F)
2) if $g \circ f$ is onto then g is onto (T)
3) if $g \circ f$ is 1:1 then f is 1:1 (T)
4) if $g \circ f$ is 1:1 then g is 1:1. (F)



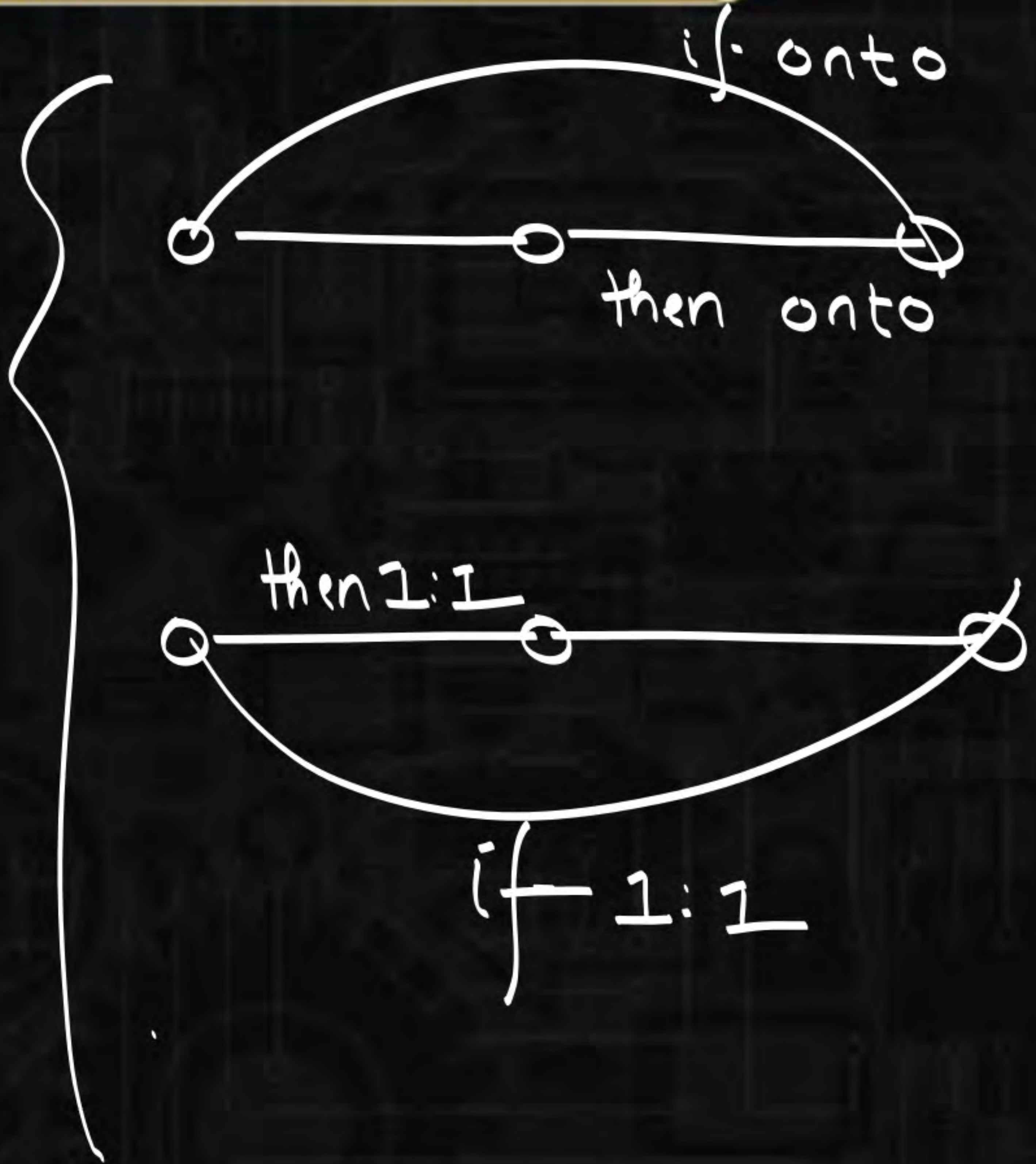
Functions

1) if gof is onto then f is onto (false)





Functions




THANK YOU GW
SOLDIERS !