CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No. 04

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

$$\begin{array}{l} P \to Q \\ P \\ \hline \therefore Q \end{array}$$

OR

$$(P \wedge (P \wedge Q)) \to Q \quad 1.$$

$$\begin{array}{ll} P_1: & (a \wedge b) \to (c \wedge d) \\ P_2: & (a \wedge b) \\ \hline & (c \wedge d) \end{array}$$

$$\Big(\big((a \wedge b) \to (c \wedge d) \big) \wedge (a \wedge b) \Big) \to (c \wedge d)$$

$$\begin{array}{l} \square \to \bigcirc \\ \square \\ \hline \therefore \bigcirc \end{array}$$

tautology

$$\begin{array}{l} P \to Q \\ Q \\ \hline \therefore P \end{array} \quad (\text{fallacy})$$

$$\text{if } \overline{\underline{A} \cup B = \underline{A} \cup C}^{\,T} \text{ then } \overline{B = C}^{\,\text{Check}} \quad \times$$

$\{1\} \cup \{2\,3\} \quad \{1\} \cup \{1\,2\,3\}$ false.

$\overline{\{1\,2\,3\}} \qquad \overline{\{1\,2\,3\}}$

$$\frac{\begin{array}{c} a \lor b \\ \neg a \end{array}}{b} \quad \text{OR} \quad \frac{\begin{array}{c} \neg a \lor b \\ a \end{array}}{b} \quad \text{OR} \quad \frac{\begin{array}{c} a \lor b \\ \neg b \end{array}}{a}$$

$$\frac{\begin{array}{c} \square \lor \bigcirc \\ \neg \square \end{array}}{\bigcirc}$$

predicate ~~logic~~ → T → F

(predicate variable)

$P(x)$: $\cancel{x}$ is even no. (open statement) → stmt, where we can not define truth value but after input, it changes to simple stmt → Define

$x = 0$

$P(0)$: $\cancel{0}$ is even no. (T)

$x = 1$

$P(1)$: $\cancel{1}$ is even no. (F)

Domain / universe / Domain of discourse / universe of discourse.

input

$x = 0$

open stmt:

x is even no.

0 is even no (simple)

S_2

Domain: set of possible choices for open stmt

1.(F)
2.(T)

x is even no

→ ALL.

→ some

$1 \cdots 100$

quantity:

↓

Tool:

↓

quantifier:

Tool → quantity

↓

Truth value

ALL → universal quantifier: $(\forall x)$

↳ check all stmts are True or not.

$\forall x\, p(x)$:

for every value of x, x will satisfy $p(x)$

for all of x, such that $p(x)$

$\forall x\, p(x) \rightarrow$ True

→ all elements are True

$\forall x\, p(x) \rightarrow$ False

→ atleast 1 false.

D: $\{0, 1, 2\}$.

$P(x): x^2 \leq 9$.

$\forall x(P(x))$

True

$P(0): 0^2 \leq 9$.

$P(1): 1^2 \leq 9$

$P(2): 2^2 \leq 9$.

D: Z.

$P(x): x^2 \geq 0$

$\forall x P(x)$

True

D: Z.

$P(x): x^2 + 3 = 9$.

$\forall x P(x) \to$ false

Some :
atleast 1.

Existential quantifier ($\exists x P(x)$)

$\exists x P(x)$:

there exist x , which satisfy $P(x)$

some value of x , which satisfy $P(x)$

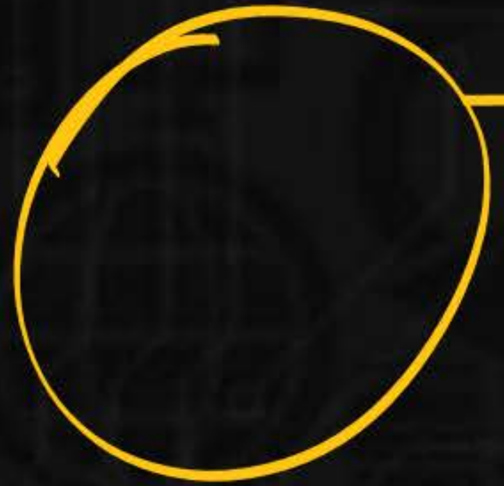
there is atleast 1 value of x. which satisfy $P(x)$

there exist x, for $P(x)$

$\exists x P(x) \to$ True

→ atleast 1 element is True

$\exists x P(x) \to$ False



→ all elements are false

$D: \{1, 2, 3\}.$

$p(x): x^2 = 4.$

$\exists x\, p(x)$ → True

$D: \{1, 3, 5\}.$

$p(x): x^2 = 4.$

$\exists x\, p(x)$ → false.

$D: \mathbb{Z}.$

$p(x): x^2 \geq 0$

$\exists x\, p(x)$ → True

$D: \mathbb{Z}.$

$p(x): x^2 \geq 0$

$\forall x\, p(x)$ → True

$D: \mathbb{Z}$

$p(x): x^2 \geq 0$

$D: \mathbb{Z}.$

$p(x)\ x^2 \geq 0$

$\underbrace{\forall x\, p(x)}_{\text{True}} \rightarrow \exists x\, p(x)$ (valid)

$\forall x\, p(x) \rightarrow \exists x\, p(x)$

$\forall x\, P(x) \rightarrow$ True.

$\downarrow$

ALL.

$D: \{1, 2, 3\}$

$\forall x\, P(x) \equiv P(1) \land P(2) \land P(3)$

$\exists x\, P(x) \rightarrow$ True.

$\hookrightarrow$ atleast 1 True

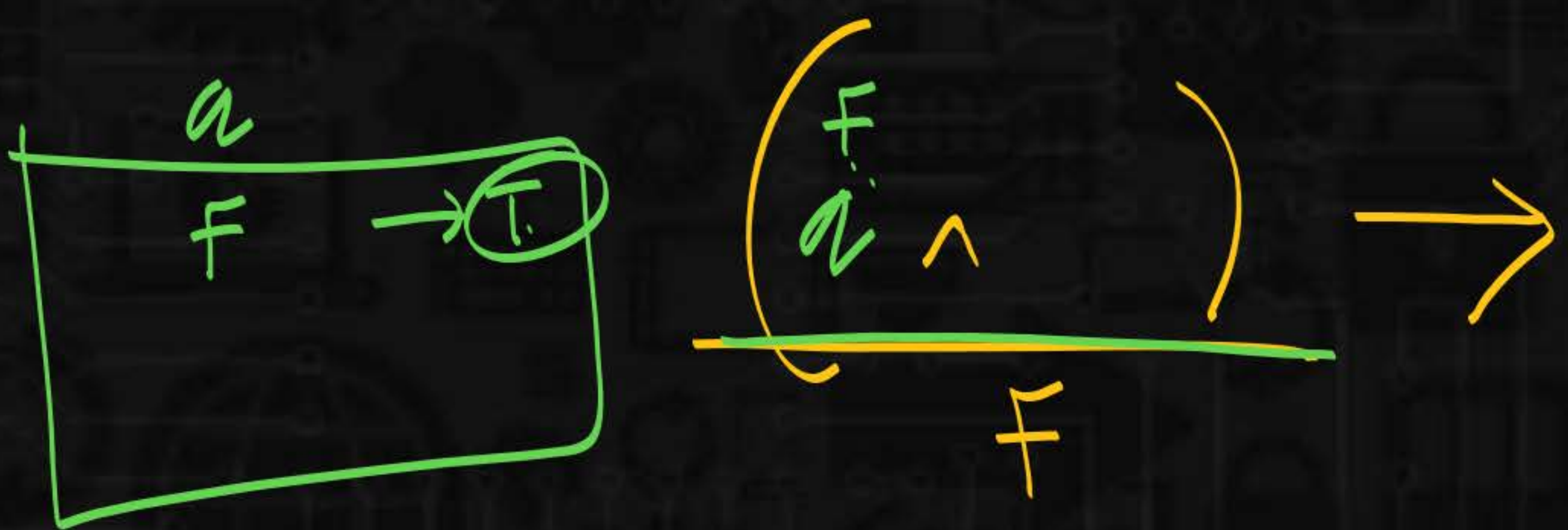
$D: \{1, 2, 3\}$

$\exists x\, P(x) \equiv P(1) \lor P(2) \lor P(3)$

negation of quantifier:

$$\neg\{\forall x\, p(x)\} \equiv \exists x\, \neg p(x)$$

$$\neg \exists x\, p(x) \equiv \forall x\, \neg p(x)$$



D: $\{1, 2, 3\}$.

$$\neg\{\forall x\, p(x)\} \equiv \neg\{p(1) \wedge p(2) \wedge p(3)\}$$

negate both side

$$\equiv \neg p(1) \vee \neg p(2) \vee \neg p(3)$$

$$\equiv \exists x\, \neg p(x)$$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

Establish the validity of the following arguments.

a) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$ ✓

b)

$$\begin{array}{l} p \rightarrow q \\ \wedge \\ \neg q \\ \wedge \\ \neg r \\ \hline \therefore \neg(p \vee r) \end{array}$$

✓

$\equiv \neg p \vee q$

$\neg q$

$\vdash \neg p$

$\wedge$

$\neg R$

$\neg p \wedge \neg R \equiv \neg(p \vee R)$

c)

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

$\equiv \neg p \vee q$

$\neg r \vee \neg q$

$\vdash \neg p \vee \neg R.$

$$\begin{array}{c} R \\ \hline \neg p. \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$ –

Which of the above arguments are valid?(GATE - 04)

Type-3

$\downarrow$

cmos

Type-1

(a) P and Q only

(b) *P* and *R* only

(c) *P* and *S* only

(d) P, Q, R and S

d) $p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

e) $p \rightarrow (q \rightarrow r)$

$p \vee s$

$t \rightarrow q$

$\neg s$

$\therefore \neg r \rightarrow \neg t$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \lor (p \land q) \lor (p \land q \land \sim r)) \land ((p \land r \land t) \lor t)$ is

(a) $p \land t$

(b) $q \land t$

(c) $p \land r$

(d) $p \land q$

$S_1: \{(\sim p \to (q \to \sim W)) \wedge (\sim S \to q) \wedge \sim t \wedge (\sim p \vee t)\} \to (w \to s)$

$S_2: \{(q \to t) \wedge (s \to r) \wedge (\sim q \to s)\} \to (\sim t \to r)$

S_2:

$q \to t$

$s \to R$

$\neg q \to s$

$\neg t \to R$

$q \to t \equiv \neg t \to \neg q$

$\neg q \to R$ $\quad \neg q \to R$

$\neg t \to R$

Resolution:

$\neg q \vee t$

$\neg s \vee R$

$q \vee s$

$t \vee R$

$\neg q \vee t$

$q \vee R$

$t \vee R.$

$S_1: \{(\sim p \to (q \to \sim W)) \wedge (\sim S \to q) \wedge \sim t \wedge (\sim p \vee t)\} \to (w \to s)$

$S_2: \{(q \to t) \wedge (s \to r) \wedge (\sim q \to s)\} \to (\sim t \to r)$

$p \vee (\quad)$

$\underline{S_1}$: $\neg p \to (q \to \neg w) \equiv \neg(\neg p) \vee (q \to \neg w) \equiv p \vee \neg q \vee \neg w$

$$\begin{array}{l} \neg p \to (q \to \neg w) \\ \neg s \to q \\ \neg t \\ \neg p \vee t \\ \hline w \to s \end{array}$$

$$\begin{array}{l} \underline{\underline{s \vee q}} \\ \neg t \\ \neg p \vee t \\ \hline \neg w \vee s \end{array} \to \begin{array}{l} p \vee (\neg q \vee \neg w) \\ \neg p \\ \hline \neg q \vee \neg w \\ s \vee q \\ \hline \neg w \vee s \end{array}$$

The statement formula $\{(a\to c)\wedge(b\to d)\wedge(c\to\sim d)\}\to(\sim a\vee\sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

$P_1 \wedge P_2 \wedge P_3$

$P_2 \wedge P_1 \wedge P_3$.

Step 1: Conclusion.

$a \to c$ $\quad \neg a \vee c$

$b \to d$ $\quad \neg b \vee d$ $\quad a \to \neg d$

$c \to \neg d$. $\quad \neg c \vee \neg d$. Resolution.

$\neg a \vee \neg b$. $\quad \neg a \vee \neg b$

OR

$a \to \neg b$.

$\neg a \vee c$

$\neg b \vee \neg c$.

$\neg a \vee \neg b$

(valid)

Hypothetical syllogism:

$((a \to b)\wedge(b \to c)) \to (a \to c)$

$(\underbrace{(a\to c)}_{1st} \wedge \underbrace{(c\to\neg d)}_{3rd}) \to (a \to \neg d)$

2nd premises contra. $\neg d \to \neg b \longrightarrow (a \to \neg b)$

$$\left.\begin{array}{l} a \to b \\ b \to c \end{array}\right\} \to \quad a \to c$$

$$\begin{array}{c} a \to c \\ \neg a \\ \hline c \\ \neg c \vee x \\ \hline x \end{array}$$

$$\begin{array}{c} a \to b \\ b \to c \\ \neg a \\ \neg c \vee x \\ \hline x \end{array}$$

The statement formula

$\{((\sim p \vee q) \rightarrow r) \wedge (r \rightarrow (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \rightarrow \sim t)\} \rightarrow p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !