CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No. 03

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

premises ⟶ conclusion.

$\underbrace{\text{premises}}_{\text{True}} \longrightarrow \underbrace{\text{conclusion.}}_{\text{Check.}}$

Check. → True / false

Inference Rule :

Considering premises as true check the conclusion.

if it's true it comes under I.R.

premises | conclusion | → propositional stmt.

premises $\rightarrow$ conclusion OR $\dfrac{\text{premises}}{\therefore \text{ conclusion}}$

$$\left(\overline{P_1 \wedge P_2 \wedge P_3 \wedge \ldots P_n}^{\text{True}}\right) \rightarrow \overline{Q}^{\text{Check}} \quad \text{OR} \quad \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \vdots \\ \hline Q \end{array}$$

P_1: mobile left OR Right pocket.

P_2: mobile is not in left pocket.

Q: Right pocket

valid

$$((L \vee R) \wedge \neg L) \to R.$$

P_1: $L \vee R \to T$

$\wedge$

P_2: $\neg L \to T$

$\therefore R \to T$

L	R	$L \vee R$		$\neg L$	
T	T	T	$\wedge$	F	X
T	F	T		F	X
F	T	T		T	
F	F	F		T	

P_1: if Graph is planar then $e \leq 3n-6$. (T)

P_2: Graph is planar

$e \leq 3n-6$.

$$\begin{array}{l} P \rightarrow Q \\ P \\ \hline \therefore Q \ (e \leq 3n-6) \end{array}$$

$$\underbrace{((P \rightarrow Q) \wedge P)}_{T} \rightarrow Q$$

valid.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$$\overset{T}{\overline{((P \to Q) \wedge \overset{r}{P})}} \to Q \qquad \overset{F}{}$$

$$\underline{\underline{(T \to F)}_{F}}_{F} \longrightarrow$$

$Q = F$

P_1: if perfect matching exist then no. of vertices will be even.

P_2: no. of vertises even.

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perfect matching.

P: perfect matching.

Q: no. of vertices.

$$\begin{array}{l} P \to Q \ (T) \\ \underline{Q} \\ \therefore P \end{array}$$

$$\left(\overline{(P \to Q) \land Q}^{\,T}\right) \to P \quad \underline{\text{Check}}$$

	P	Q	$P \to Q$
may	T	T	T
	T	F	F
mayor not	F	T	T
	F	F	T

1. $P \to Q$
P
$\therefore Q$

$[(P \to Q) \wedge P] \to Q$

(modus ponens)

2. $P \to Q$
$\neg Q$
$\therefore \neg P$

OR $\neg Q \to \neg P$
$\neg Q$
$\neg P$

$((P \to Q) \wedge \neg Q) \to \neg P$

(modus tollens)

3. $P \vee Q$
$\neg P$
$\therefore Q$

OR $P \vee Q$
$\neg Q$
$\therefore P$

OR $\neg P \vee Q$
P
Q

OR $P \vee \neg Q$
Q
P

(Disjunctive syllogism)

4. $a \to b$
$b \to c$
$a \to c$

$((a \to b) \wedge (b \to c)) \to (a \to c)$

(Hypothetical Syllogism)

5. $\dfrac{T}{p \wedge q \to p}$

OR

$p \wedge q \to q$

(Simplification)

$p \to p \vee q$ OR $\dfrac{p}{\therefore p \vee q}$ (addition)

$$\begin{array}{c} P \vee \cancel{Q} \\ \cancel{\neg Q} \vee R \\ \hline P \vee R \end{array} \qquad \left(\overline{(P \vee Q) \wedge (\neg Q \vee R)}^{\,T}\right) \to \overline{P \vee R} \quad \text{(Resolution)}$$

Question:

$$\left(a \wedge (a \to b) \wedge (\neg b \vee c)\right) \to c.$$

valid ?

valid

Gi a

Gi $a \to b$ (modus ponens)

$$\frac{\begin{array}{l} a \\ a \to b \end{array}}{b}$$

Given $\neg b \vee c$.

$$\frac{\begin{array}{l} b \\ \neg b \vee c \end{array}}{c}$$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$(\boxed{p \wedge} q) \vee (\boxed{p \wedge} \neg q) \vee (\neg p \wedge \neg q)$

$p \wedge (q \vee \neg q)$

$\frac{p \wedge T}{p}$ $\vee (\neg p \wedge \neg q)$

$p \vee (\neg p \wedge \neg q)$

$(p \vee \neg p) \wedge (p \vee \neg q)$

$T \wedge (p \vee \neg q)$

$(p \vee \neg q)$

P and Q are two propositions. Which of the following logical expressions are equivalent?

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III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

$p \wedge (q \vee \neg q)$

$p \wedge T$

$p \quad \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q) = (p \vee \neg p) \wedge (p \vee q)$

$T \wedge (p \vee q) \equiv p \vee q$

Establish the validity of the following arguments.

a) $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$

b)
$$\begin{array}{l} p \to q \\ \neg q \\ \underline{\neg r} \\ \therefore \neg(p \vee r) \end{array}$$

c)
$$\begin{array}{l} p \to q \\ r \to \neg q \\ \underline{r} \\ \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P, Q, R and S

d)

$$\begin{array}{l} p \land q \\ p \to (r \land q) \\ r \to (s \lor t) \\ \neg s \\ \hline \therefore t \end{array}$$

e)

$$\begin{array}{l} p \to (q \to r) \\ p \lor s \\ t \to q \\ \neg s \\ \hline \therefore \neg r \to \neg t \end{array}$$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p\wedge(\sim r \vee q \vee \sim q))\vee((r \vee t \vee \sim r)\wedge \sim q)$ is

(a) $p\wedge\sim q$

(b) $p\vee\sim q$ ✓

(c) t

(d) $(p\rightarrow\sim q)$

$$\left(p\wedge(\neg r \vee \boxed{q\vee\neg q})\right)\vee\left((r\vee t\vee\neg R)\wedge\neg q\right)$$

$$\underbrace{(\neg r \vee T)}_{T}$$

$$p\wedge T$$

$$p$$

$$\boxed{r\vee\neg R}\vee t$$

$$\boxed{T\vee t}$$

$$\downarrow$$

$$(T\wedge\neg q)$$

$$\neg q$$

Ans: $p\vee\neg q$ ✓

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

Arrangement:

(a) $p \wedge t$ ✓

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$\boxed{p \vee (p \wedge q)}$

$\underbrace{t \vee (t \wedge p \wedge r)}_{t}$

$p \vee (p \wedge \boxed{q \wedge \neg R})$

p

$\mathbf{S_1}: \{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

$\mathbf{S_2}: \{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

The statement formula $\{(a \rightarrow c) \wedge (b \rightarrow d) \wedge (c \rightarrow \sim d)\} \rightarrow (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

The statement formula

$\{((\sim p \vee q) \rightarrow r) \wedge (r \rightarrow (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \rightarrow \sim t)\} \rightarrow p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !