Branch : CSE & IT **Batch : Hinglish**

DISCRETE MATHEMATICS
Mathematical Logic

DPP-06

[MCQ]
1. Consider a function, $P(x, y, z) = x + y + z = 15$ and domain = z, then which of the following is correct?
(a) $\forall x \exists y \exists z\, P(x, y, z)$ (b) $\exists z \forall x \forall y\, P(x, y, z)$
(c) $\forall x \exists z \forall y\, P(x, y, z)$ (d) $\exists z\, \exists y \forall x\, P(x, y, z)$

[MCQ]
2. Consider an asymmetric function $P(x, y) = x^2 + y^2 = 10.0$ on domain integer, then which of the following is correct?
(a) $\exists x \exists y\, P(x, y)$ (b) $\forall x \exists y\, P(x, y)$
(c) $\forall y \exists x\, P(x, y)$ (d) None of these

[MSQ]
3. Which of the following is/ are negation of $[\forall x \exists y \forall z (P(x, y, z) \oplus Q(x, y, z))]$
(a) $\exists x \forall y \exists z\, (\sim P(x, y, z) \oplus \sim Q(x, y, z))$
(b) $\exists x \forall y \exists z\, (P(x, y, z) \Rightarrow \sim Q(x, y, z))$
(c) $\exists x \forall y \exists z\, (P(x, y, z) \Leftrightarrow Q(x, y, z))$
(d) $\exists x \forall y \exists z\, (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))$

[NAT]
4. Consider the following logical expressions
(a) $\forall x \forall y\, P(x, y) \leftrightarrow \exists y \forall x\, P(x, y)$
(b) $[\forall x\, P(x)] \vee Q \leftrightarrow \forall x[P(x) \vee Q]$
(c) $\forall x[P(x) \wedge Q] \leftrightarrow [\forall x\, P(x)] \wedge Q$
(d) $\exists x[P(x) \vee Q] \leftrightarrow [\exists x\, P(x)] \wedge Q$
Total invalid expressions are ___?

[MCQ]
5. Consider the following statements
S_1: There is someone who is loved by everyone.
S_2: Every real number has its corresponding negative.
Here L(x, y) denotes "x loves y"
P(x, y) denotes "$x + y = 0$"
Which of the following represent the correct predicate logic of the given statement?
(a) $S_1{:}\exists x \forall y\, L(x, y)$, S_2: $\exists y \forall x\, p(x, y)$
(b) $S_1{:}\forall x \exists y\, L(x, y)$, S_2: $\forall x \forall y\, p(x, y)$
(c) S_1: $\exists y \forall x\, L(x, y)$, S_2: $\forall x \exists y\, p(x, y)$
(d) None of these.

Answer Key

1. (a)
2. (d)
3. (c, d)
4. (2)
5. (c)

Hints and Solutions

1. (a)

(a) $\forall x\exists y\exists z\, P(x, y, z)$

$z + y = 15 - x$

$15 -$ integer $=$ integer **True**

(b) $z = 15 - x - y$ **False**

z must be independent, here z depends on x and y.

(c) $z = 15 - x - y$ **False**

z should not depend on y.

(d) $y + z = 15 - x$ **False**

the value of $(y + z)$ is depending on x, $(y + z)$ must be independent, so this expression is also **False.**

2. (d)

(a) $\forall x\exists y\, P(x, y)$ **False**

$x^2 + y^2 = 10.0$

$F(1, 3) = 1 + 9 = 10$

Here, 10 is integer but output must be 10.0, it will never come because 10.0 is not an integer.

(b) $\forall x\exists y\, P(x, y)$ **False**

10.0 will never come.

(c) $\forall y\exists x\, P(x, y)$ **False**

Hence, option (d) is correct

3. (c, d)

Negation of XOR operator is biconditional.

p	q	$p \oplus q$	$p \leftrightarrow q$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

(c) $\sim [\forall x\exists y\forall z(P(x, y, z)\oplus Q(x, y, z))]$

$[\exists x\forall y\exists z \sim (P(x, y, z) \oplus Q(x, y, z))]$

$[\exists x\forall y\exists z\, (P(x, y, z) \Leftrightarrow Q(x, y, z))]$ True

(d) Property:

$P \leftrightarrow Q \equiv \sim P \leftrightarrow \sim Q$

$P'Q' + PQ \equiv P'Q' + PQ$

$\sim [\forall x\exists y\forall z(P(x, y, z)\oplus Q(x, y, z))]$

$[\exists x\forall y\exists z\, (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))]$ True

Hence, option (c, d) are correct.

4. (2)

(a): Invalid

$\forall x\forall y P(x, y) \rightarrow \exists y\forall x\, P(x, y)$ (One way true)

$\forall y\forall x P(x, y) \rightarrow \exists y\forall x\, P(x, y)$

(b): $[\forall x P(x)] \vee Q \leftrightarrow [\forall x P(x) \vee Q]$

$(P_1 \wedge P_2) + Q \equiv (P_1 \vee Q) \wedge (P_2 \vee Q)$

$P_1P_2 + Q \equiv P_1P_2 + P_1Q + P_2Q + Q$

$P_1P_2 + Q \equiv P_1P_2 + Q$ (valid)

(c): $\forall x[P(x) \wedge Q] \leftrightarrow [\forall x\, P(x)] \wedge Q$

$(P_1 \wedge Q) \wedge (P_2 \wedge Q) \equiv (P_1 \wedge P_2) \wedge Q$

$P_1QP_2\, Q \equiv P_1\, P_2\, Q$

$P_1\, P_2Q \equiv P_1\, P_2Q$

Valid

(d): $\exists x(P(x) \vee Q) \leftrightarrow [\exists x\, P(x)] \wedge Q$

$(P_1 \vee Q) \vee (P_2 \vee Q) \equiv (P_1 \vee P_2) \wedge Q$

$P_1 + Q + P_2 + Q \equiv (P_1 + P_2)\, Q$

$P_1 + P_2 + Q \not\equiv (P_1 + P_2)\, Q$ Invalid

Total 2 expressions are invalid

5. (c)

Statement S_1: There is someone who is loved by everyone.

- Assume, variables x and y denote people
- A predicate $L(x, y)$: denotes "x loves y"

$\therefore \exists y\forall x\, L(x, y)$ there is someone who is loved by everyone.

Statement S_2: Every real number has its corresponding negative.

- Assume, a real number is denoted as x and its negative as y.
- A predicate $p(x, y)$ denotes "$x + y = 0$"

$\therefore\ \forall x\exists y\, p(x, y)$

Hence, option c is correct answer.