CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No. 07

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

nested ~~/~~ quantifier :

↓

Tool

↓

Quantity.

↓

Truth value.

D: $\{1, 2, 3\}$.

$\forall x \forall y (x + y = 10)$

$(1 + y$

$D: \{1, 2, 3\}$.

$P(x, y): (x \times y \leq 9)$

$\forall x \forall y$

for all of x, all of y, P(x, y)

for every value of x, every value of y, P(x, y)

$\rightarrow$ $D: \{1, 2, 3\}$.

$\forall x \forall y (x \times y \leq 9)$

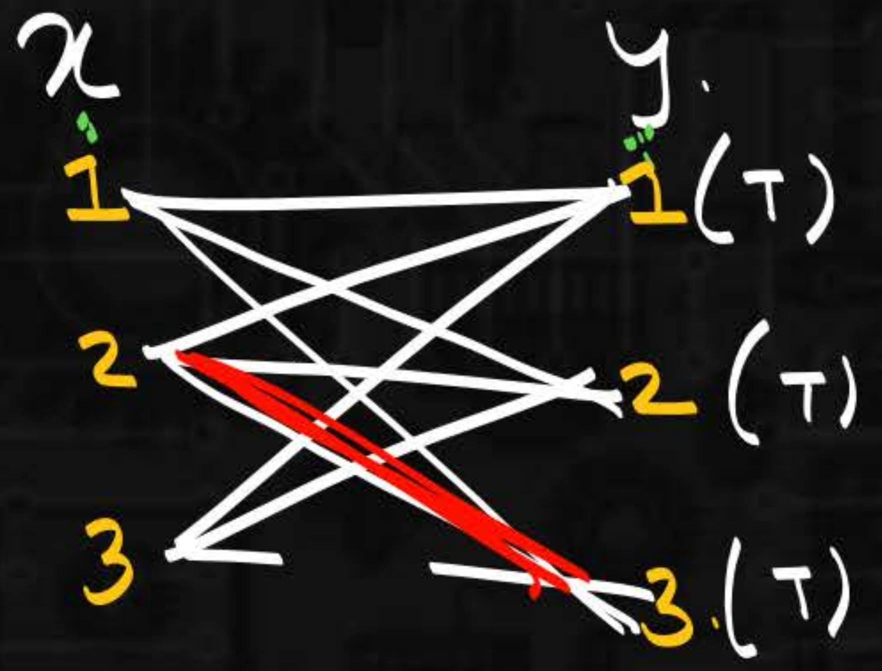
$x=1 \quad y=1 \quad 1 \times 1 \leq 9 \ (T)$

$x=1 \quad y=2 \quad 1 \times 2 \leq 9 \ (T)$

$x=1 \quad y=3 \quad 1 \times 3 \leq 9 \ (T)$

$x=2 \quad y=1 \quad 2 \times 1 \leq 9 \ (T)$

$D: \{1, 2, 3\}$. $x \times y \le 9$.



$x=1 \quad y=1$

$1 \times 1 \le 9$.

edge → pair (x, y)

$x=1 \quad y=2$

$1 \times 2 \le 9$.

$x=1 \quad y=3$

$1 \times 3 \le 9$.

$\forall x \forall y \rightarrow$ True.

→ True → all pairs.

→ it is true when all edges are True

$\forall x \forall y \rightarrow$ False

atleast 1 edge is false.

$D: \{1, 2, 3\}$

$P(x, y): x \times y = 4.$

false.

$\forall x \forall y \, (x \times y = 4)$

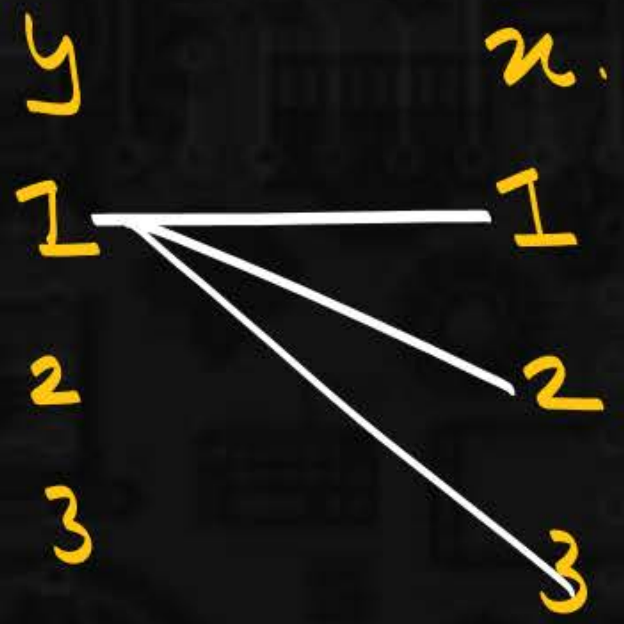
$x = 1 \quad y = 1$

$1 \times 1 = 4 \; (F)$

x	y	
1	1	(False)
2	2	
3	3.	

$D: \{1, 2, 3\}.$

$\rightarrow \forall y \forall x (x \times y \le 9)$



$D: \{1, 2, 3\}.$

$P(x, y): x \times y \le 9.$

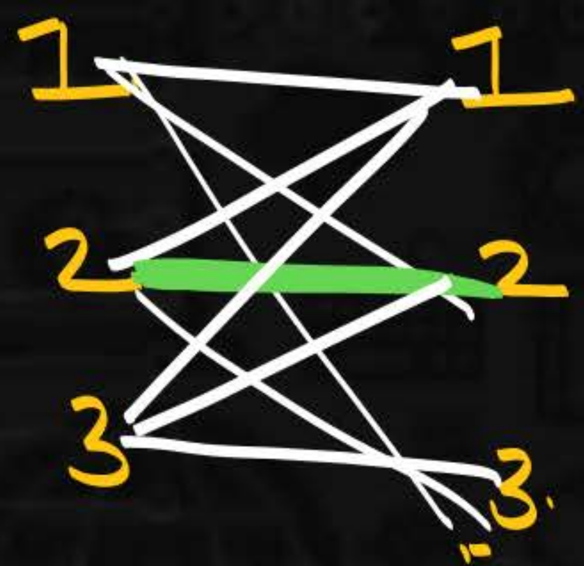
$\forall x \forall y (x \times y \le 9) \quad \forall y \forall x (x \times y \le 9)$

$$\boxed{\forall x \forall y \equiv \forall y \forall x.}$$

$D: \{1, 2, 3\}$

$P(x, y): x \times y = 4$

x y



$(F)\,/\,F$

$(F)\,/\,(T)$

(F)

$x = 1$ $y = 1$

$|\alpha| = 4$

$\exists x \, \exists y \, (x \times y = 4)$

There exist value of x, exist value of y.
such that $P(x, y)$

atleast 1 value of x, atleast 1 value of y.

Some of x, some of y $(P(x, y))$

$D: \{1, 2, 3\}$.

$\exists x \exists y \, (x \times y = 4)$

$\exists x \exists y \rightarrow$ True.
when it sees
atleast 1 edge is True.

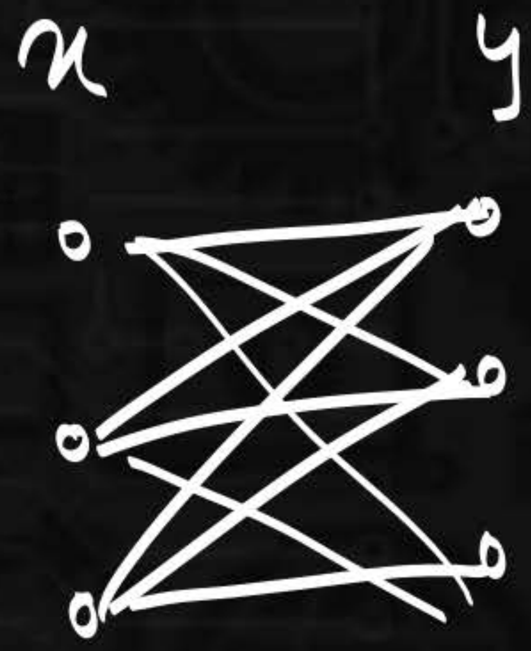
$\exists x \exists y \longrightarrow$ false.
when all edges
are false

y	x
1	1
2	2
3	3

$\exists x \exists y \equiv \exists y \exists x$

D: O.S

$\forall x \forall y$



D: O.S

$\forall y \forall x$

D: O.S

$\exists x \exists y$

D: O.S

$$\begin{cases} \forall x \forall y \equiv \forall y \forall x \\ \exists x \exists y \equiv \exists y \exists x \end{cases}$$

$$\begin{cases} \forall x \forall y \rightarrow \exists x \exists y \\ \forall y \forall x \rightarrow \exists x \exists y \\ \forall x \forall y \rightarrow \exists y \exists x \\ \forall y \forall x \rightarrow \exists y \exists x \end{cases}$$

$D: \mathbb{Z}$.

$P(x,y): x+y=10$.

$\forall x\, \exists y\, (x+y=10) \rightarrow$ True.

x		y
-3	$\longrightarrow$	13
$\vdots$		
1	$\longrightarrow$	9
2	$\longrightarrow$	8
$\vdots$		

$x+y=10$

$y=10-x$.

$x=1 \quad y=10-1=9$.

$x=2 \quad y=10-2=8$

$x=a \quad y=10-a$.

$a\in \mathbb{Z} \quad y=10-a \in \mathbb{Z}$.

$\forall x\, \exists y$:

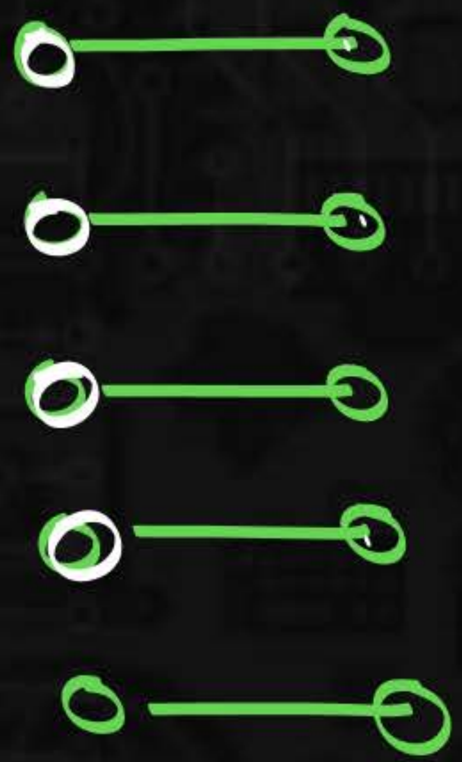
for all of x,

there exist y.

for all value of x.

Some value of y.

$\forall x \; \exists y \rightarrow$ True.



all boys, there exist gf

D: Z.

$\forall x \exists y\ (x+y=10)$

↳ True.

$x+y=10$

$\underset{\text{True}}{\underline{\forall x \exists y}} \not\Rightarrow \underline{\exists y \forall x}$

$y = 10 - x.$

$y = 10 - 10$

D: Z. false.

$\exists y \forall x\ (x+y=10)$

$y \qquad x.$

Expect

there exists.

for all of x.

Forcing y to be constant.

$\exists y \forall x \longrightarrow \forall x \exists y$

True. True.

y x

True

x y

(True)

$\forall x \exists y \not\rightarrow \exists y \forall x$

$\exists y \forall x \rightarrow \forall x \exists y$

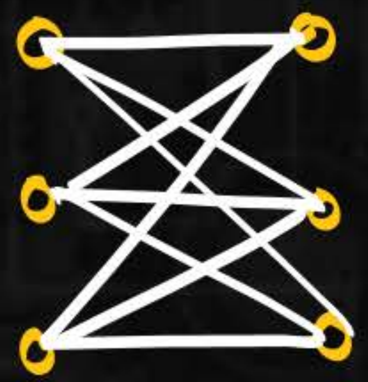
$P(x,y): \left(\frac{x}{y} \in \mathbb{Z}\right)$

$\exists y \, \forall x \left(\frac{x}{y} \in \mathbb{Z}\right) \longrightarrow \forall x \, \exists y \left(\frac{x}{y} \in \mathbb{Z}\right)$

True

1.

$\forall x \forall y$



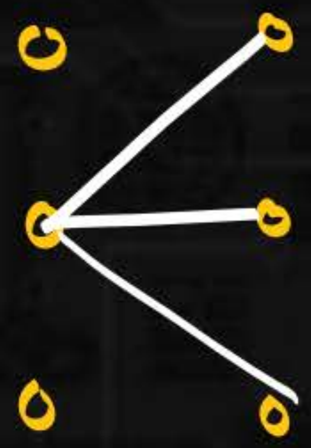
2.

$\forall x \exists y$

OR

3.

$\exists y \forall x$



4.

$\exists x \exists y$

$\forall x \exists y \not\rightarrow \exists y \forall x$

$\exists y \forall x \rightarrow \forall x \exists y$

$1 \rightarrow \text{all}$

$\boxed{\text{all} \rightarrow 4}$

$1 \rightarrow 2$

qedy.

1. $\forall x \forall y \equiv \forall y \forall x$ 5

3. $\exists y \forall x$

2. $\forall x \exists y$

4. $\exists x \exists y \equiv \exists y \exists x$ 8

$\exists x \forall y$ 6

$\forall y \exists x$ 7

1 → all.

5 → all.

all → 4.

all → 8.

D: Z.

True $x=1$ $y=1$.

1. $\exists x \exists y\, [x \cdot y = 1]$

2. $\exists x \exists y\, [(2x + y = 5) \wedge (x - 3y = -8)]$ → True. : $x = 1$ $y = 3$.

3. $\exists x \exists y\, [(3x - y = 7) \wedge (2x + 4y = 3)]$ → $x =$ __ $y =$ __

it is false

$x, y \notin Z$.

D: Z.

S1: $\forall n\, \exists m\, [n + m = 5] \rightarrow T$

S2: $\exists n\, \forall m\, [n \cdot m = m] \rightarrow T$

S3: $\forall m\, \exists n\, [m \cdot n = 1] \rightarrow F$

S4: $\exists m\, \forall n\, [m + n = 0] \rightarrow F$

$m \cdot n = 1$

$m = \frac{1}{n}$

m n

$\exists m \forall n \rightarrow$ forcing m to be constant

m n

$m + n = 0$

eqt^n is changing m every time hence it is false

For every integer n bigger than 1, there is prime strictly between n and 2n.

A) $\forall n \left[(n>1) \rightarrow \exists x \left[p(x) \wedge (n<x<2n) \right] \right]$ — A

B) $\forall n \left[(n>1) \wedge \exists x \left[p(x) \rightarrow (n<x<2n) \right] \right]$

C) $\forall n \left[(n>1) \rightarrow \exists x \left[p(x) \rightarrow (n<x<2n) \right] \right]$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

Establish the validity of the following arguments.

a) $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$

b)
$$\begin{array}{l} p \to q \\ \neg q \\ \neg r \\ \hline \therefore \neg(p \vee r) \end{array}$$

c)
$$\begin{array}{l} p \to q \\ r \to \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P, Q, R and S

d) $p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

e) $p \rightarrow (q \rightarrow r)$

$p \vee s$

$t \rightarrow q$

$\neg s$

$\therefore \neg r \rightarrow \neg t$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

Let p, q, r and s be four primitive statements. Consider the following arguments:

$P: [(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

$Q: [(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

$R: [[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

$S: [p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$S_1: \{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

$S_2: \{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

The statement formula $\{(a \to c) \wedge (b \to d) \wedge (c \to \sim d)\} \to (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

The statement formula

$\{((\sim p \lor q) \to r) \land (r \to (s \lor t)) \land (\sim s \land \sim u) \land (\sim u \to \sim t)\} \to p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !