CS & IT ENGINEERING

Discrete Mathematics
Graph Theory

Lecture No.12

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TOPICS TO BE COVERED

01 covering set

02 Covering number

03 Planar Graph

04 Euler's Formula In planarity

05 Sum of Degrees in Region

Graph

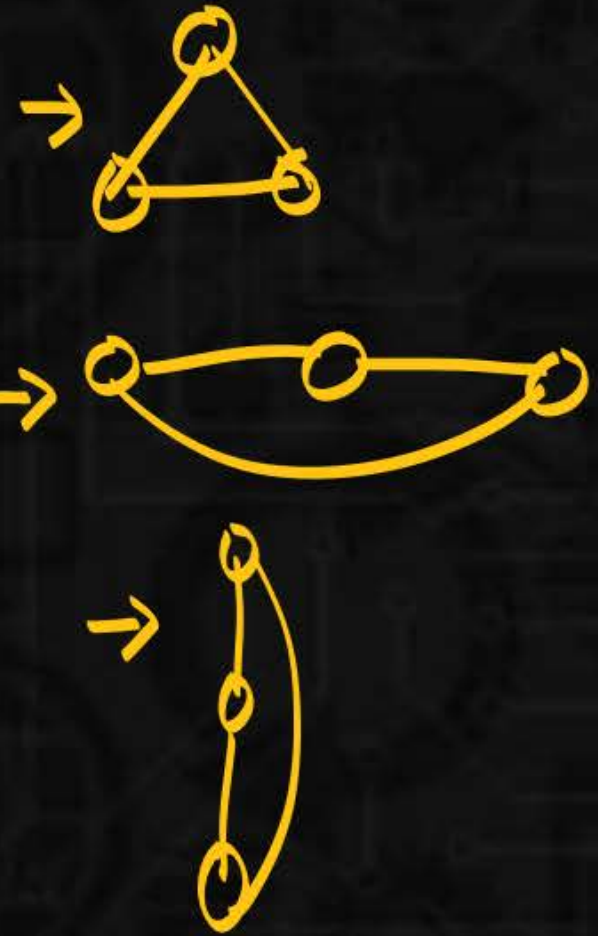
planar

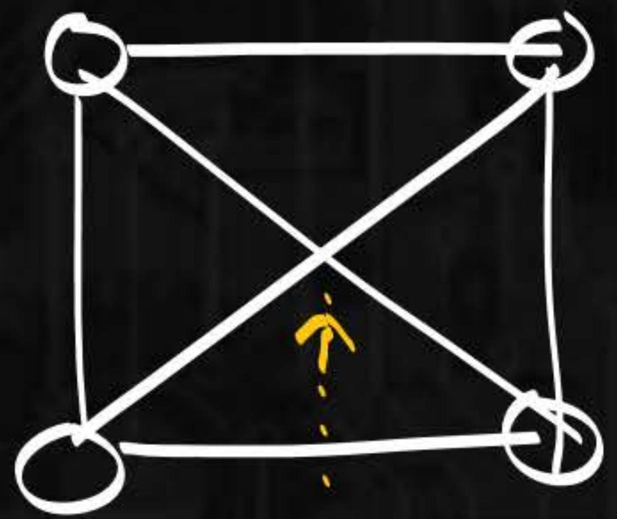
if we can draw a graph on a plane without intersection of it's edges.

non planar

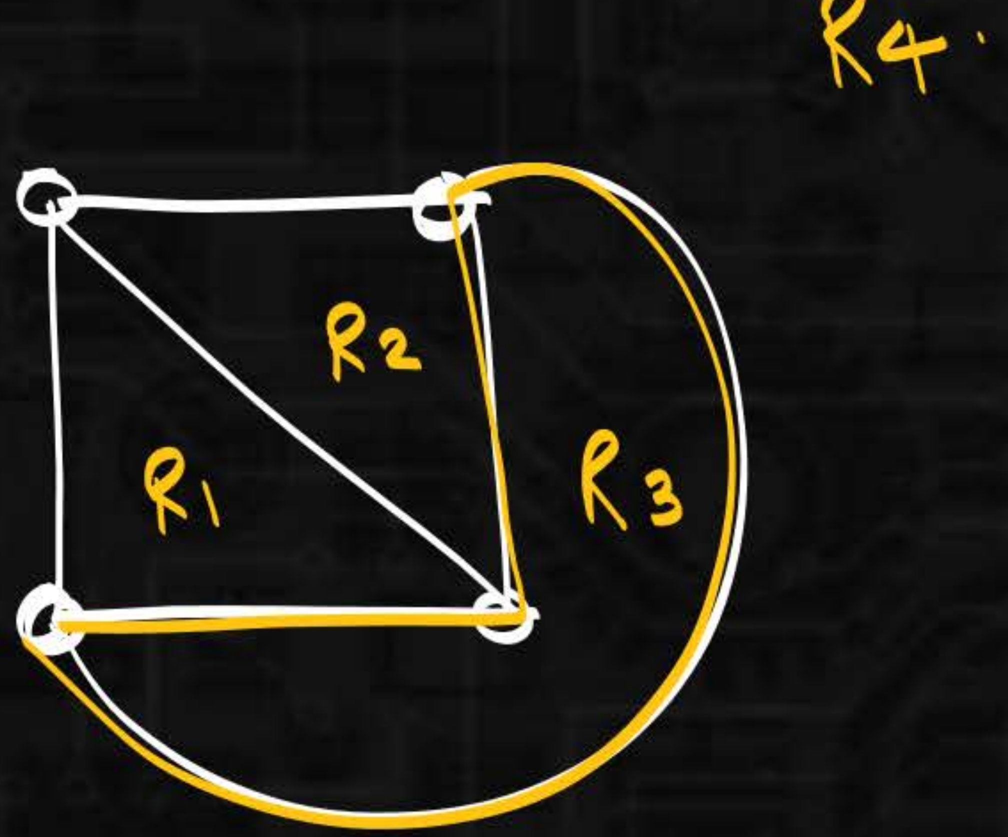
Otherwise it is nonplanar.

it does not matter how we draw graph.





intersection of edges.

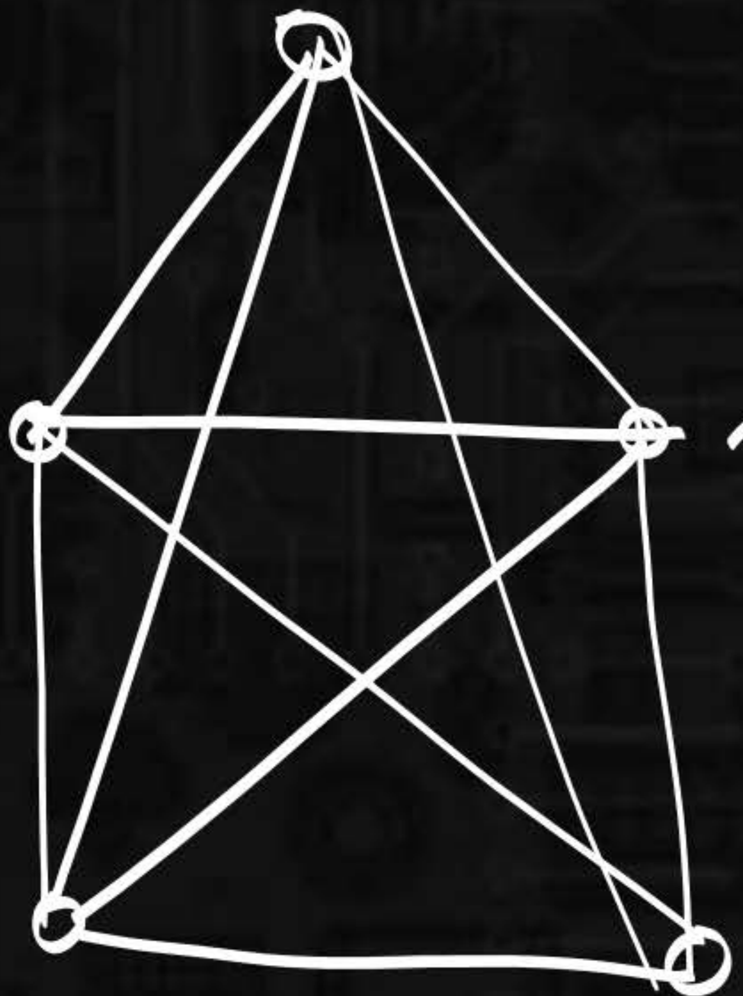


planar Graph.

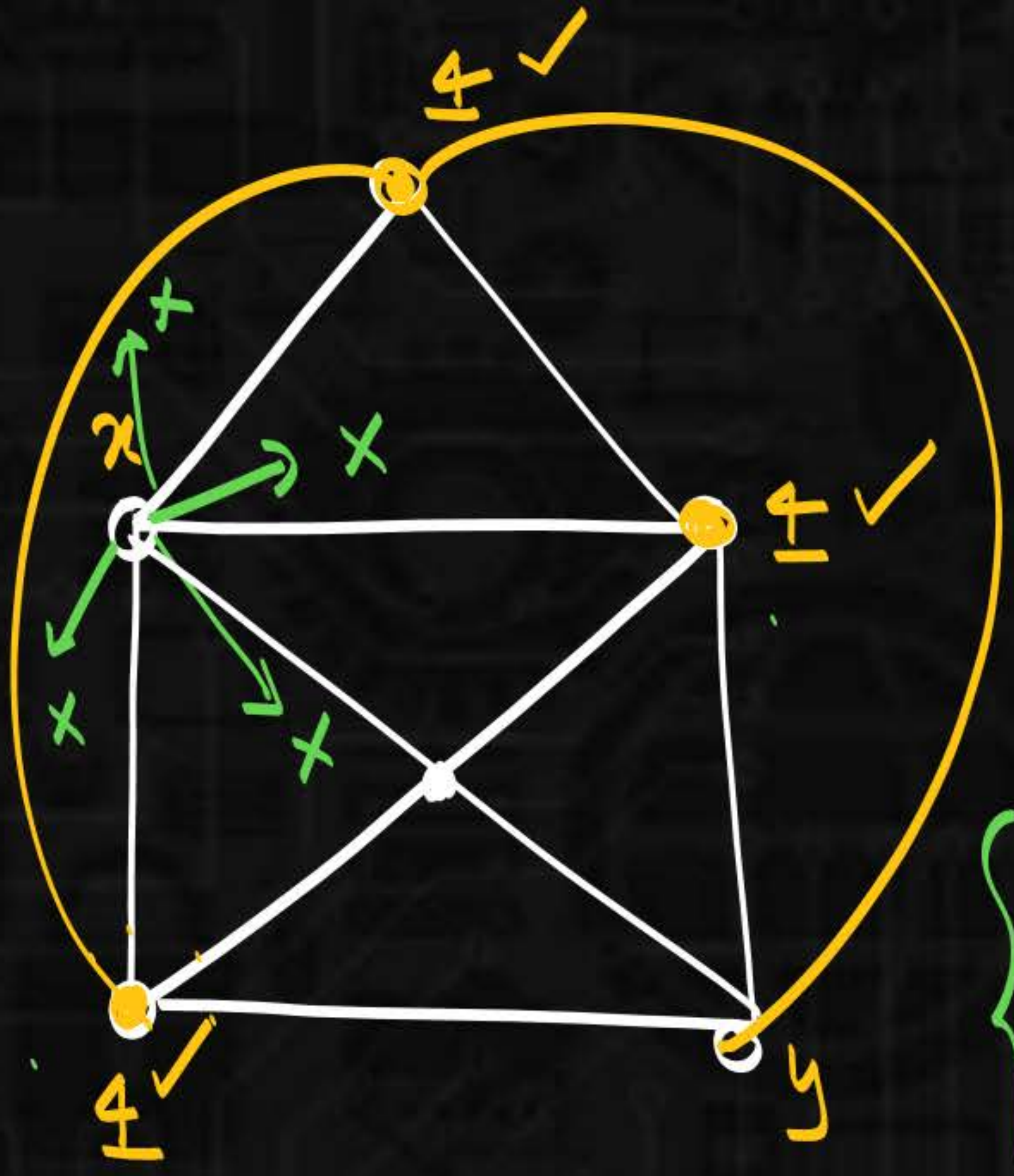
K_5

$n = 5$

$e = 10$



K_5



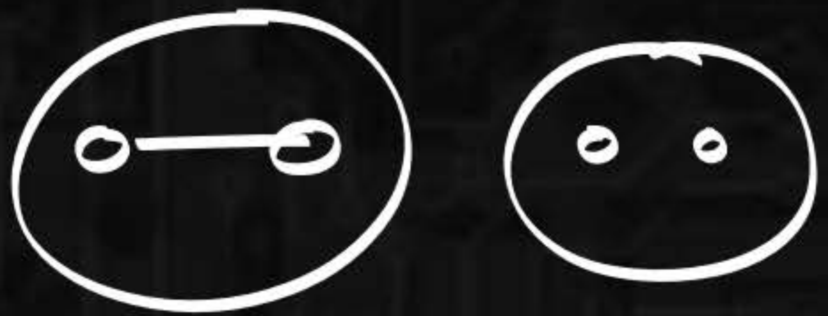
$\left\{ \begin{array}{l} n = 5 \\ e = 9 \end{array} \right.$

Planar

$\boxed{(K_5 - e)}$

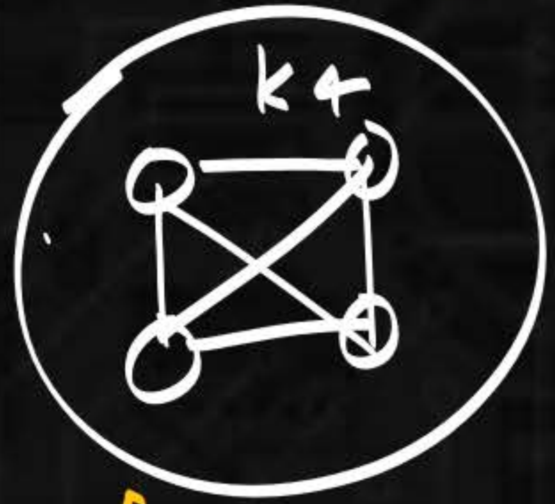
K_5 is non planar

$n = 1$ 

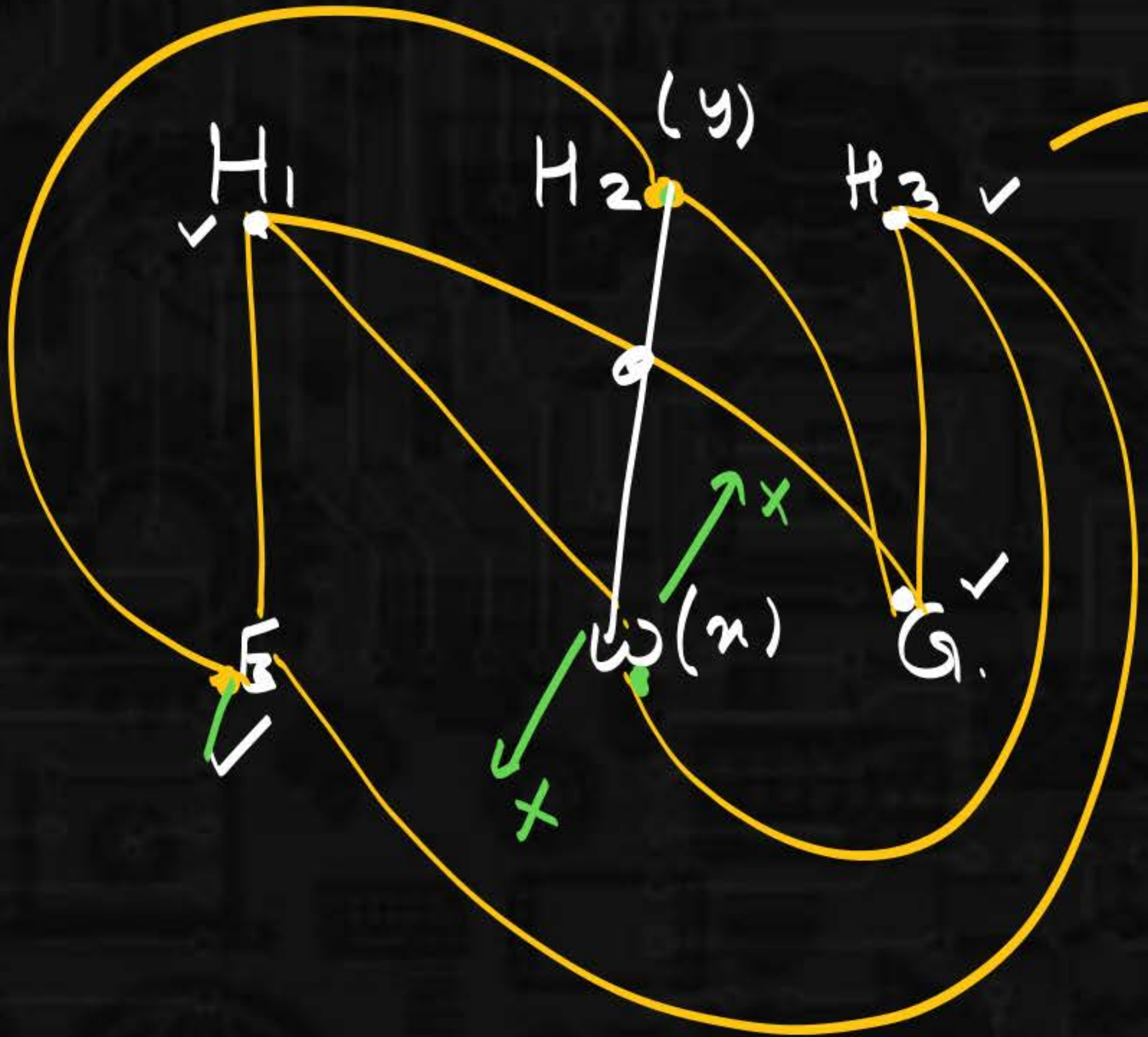
$n = 2$ 

$n = 3$

planar

$n = 4$ 

$n = 5$ K5 nonplanar



(K3, 3 - e) → planar Graph.

K3, 3 is nonplanar.

Obs:

→ both graphs are regular Graph.

→ Removal of single edge from both the graphs will make both graphs as a planar Graph.

⇒ K5 is nonplanar Graph with min no. of vertices.
K3,3 is nonplanar Graph with min no. of edges.

min. no. of vertices

nonplanar K_5 | $n = 5$ | $e = 10$

nonplanar $K_{3,3}$ | $n = 6$ | $e = 9$

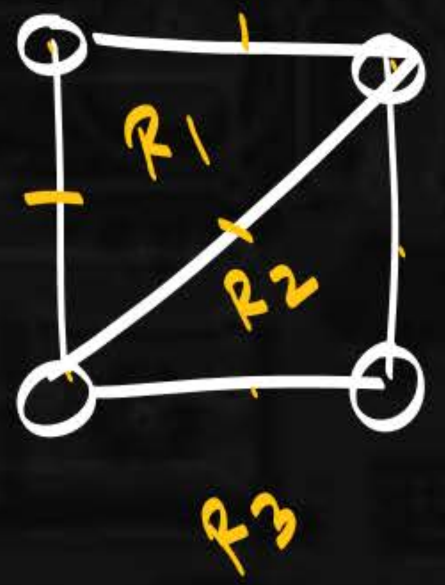
min no. of edges

Euler's formula.:

n = vertices e = edges f = faces/Region

(PLANAR GRAPH)

$$n - e + f = 2$$



$n = 4.$

$e = 5$

$$n - e + f = 2$$

$$4 - 5 + f = 2$$

$$f = 3.$$

$n - e + f = 2.$

Case 1:

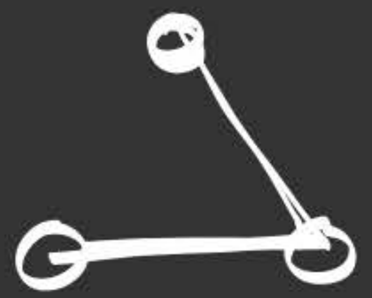
$\boxed{n\uparrow \quad e\uparrow}$

$n = 2 \quad e = 1$

$n - e + f = 2.$

$2 - 1 + f = 2$

$f = 1.$



$n\uparrow \quad e\uparrow$

$n = 3 \quad e = 2$

$n - e + f = 2.$

$3 - 2 + f = 2$

$f = 1$

Case 2:

$e\uparrow \quad f\uparrow$

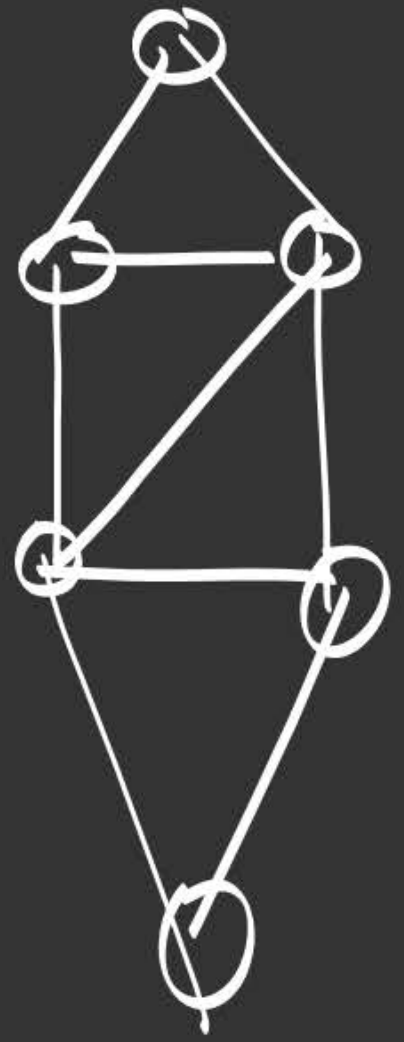
$n = 3 \quad e = 3$

$n - e + f = 2$

$3 - 3 + f = 2$

$f = 2$

$n - e + f = 2$



Consider a Planar Graph with $n = 10$ $e = 15$
what will be no of closed faces ?

$$n - e + f = 2$$

$$10 - 15 + f = 2.$$

$$-5 + f = 2.$$

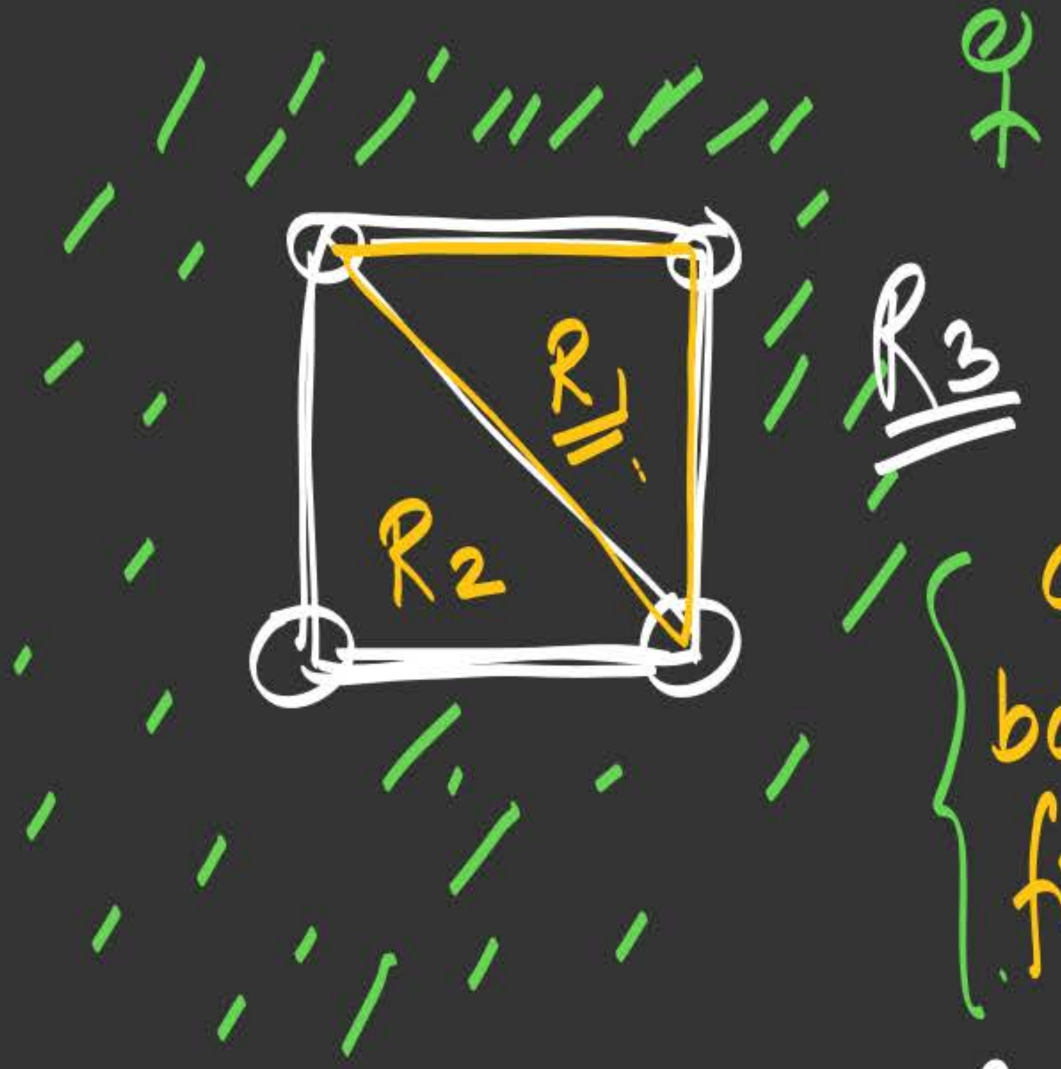
$$f = 7$$

$$\text{Closed} = \text{Total} - 1.$$

$$= 7 - 1.$$

$$= 6.$$

Region/face: (R/f)

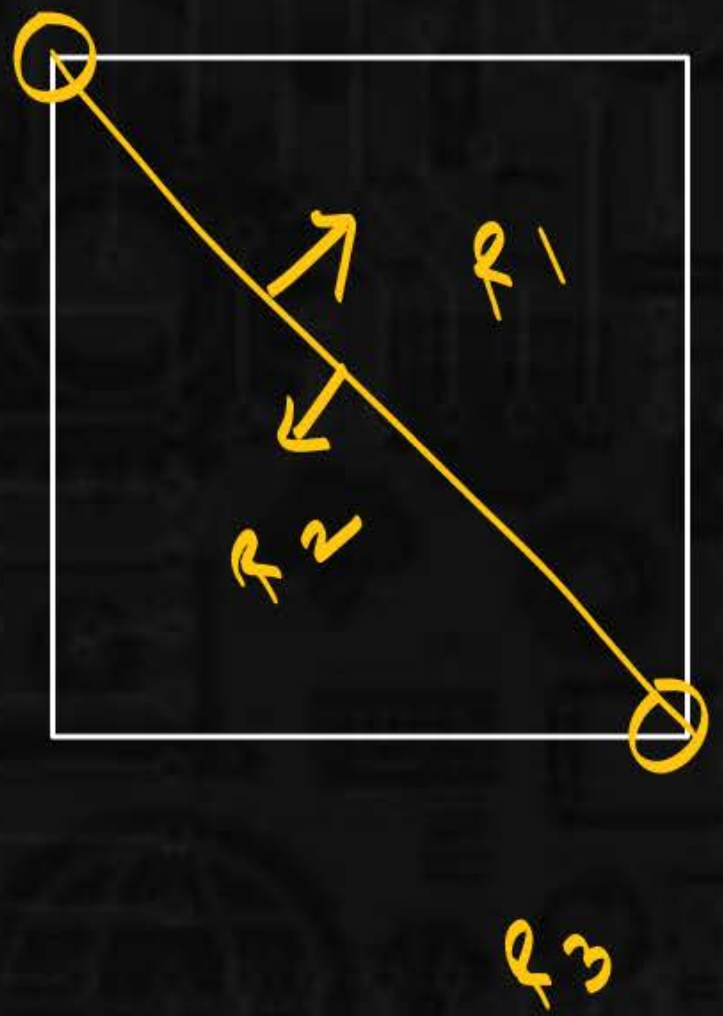


faces/Region

- closed, bounded, finite — R_1, R_2
- open, unbounded, Infinite, (Alien) — R_3

embedding: Planar Graph → Draw → plane → it creates Region/face

Degree of regions : no. of edges involved in regions.



$deg(R_1) = 3$ $\quad$ $deg(R_3) = 4$.

$deg(R_2) = 3$

$$deg(R_1) + deg(R_2) + deg(R_3)$$

$$= 3 + 3 + 4 = 10 = 2 \cdot 5$$

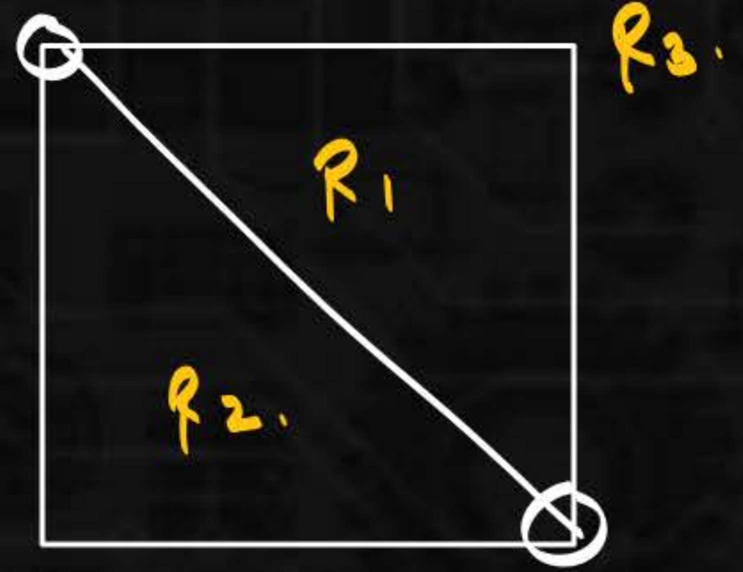
5 → no. of edges

$$\boxed{\sum deg(R_i) = 2e.}$$

$\deg(R_1) \geq 3$

$\deg(R_2) \geq 3$

$\deg(R_3) \geq 3.$



Graph.

(Regions are made by atleast 3 edges)

$$\underbrace{\deg(R_1) + \deg(R_2) + \deg(R_3)} \geq 3 + 3 + 3$$

$$\sum d(R_i) \geq 3 \cdot 3 \longrightarrow \text{faces}$$

$$\boxed{2e \geq 3f}$$

Planar:

$2e \geq 3f$ $\quad$ $n - e + f = 2$

$f = 2 + e - n$

(Region are made by atleast 3 edges)

$2e \geq 3(2 + e - n)$

$2e \geq 6 + 3e - 3n$

$3n - 6 \geq 3e - 2e$

$3n - 6 \geq e$

Thm: if G is planar then $e \leq 3n - 6$.

(viceversa is not true)

Thm:

if G is planar then $e \leq 3n-6$.

Contrapositive:

if $\neg(e \leq 3n-6)$ then $\neg$(G is planar)

~~Thm~~ ~~if~~ $e > 3n-6$ then G is nonplanar.

viceversa is not true

if $e \leq 3n-6$ then G is planar (false)

$K_{3,3}$ $e=9$ $n=6$.

$e \leq 3n-6$

$9 \leq 3(6)-6$

$9 \leq 18-6$

$9 \leq 12$ (True)

$\underline{p \to q}$ Inverse $= \neg p \to \neg q$.

Converse $= q \to p$ (viceversa)

Contrapositive: $\neg q \to \neg p$

$$\boxed{p \to q \equiv \neg q \to \neg p.}$$

$n = 11$

then $G / \bar{G}$ will be nonplanar.

if $e > 3n - 6$ then G is nonplanar

$\boxed{n = 11}$

$e > 3(11) - 6$

if $e > 27$ then G is nonplanar

$e(G) + e(\bar{G}) = e(K_{11})$

$e(G) + e(\bar{G}) = \frac{11 \cdot 10}{2}$

$e(G) + e(\bar{G}) = 55$

27 : 28

→ 28 : 27

20 : 35

26 : 29

$e(G) + e(\bar{G}) = 55$

$\deg(R_1) \geq 4$

$\deg(R_2) \geq 4.$

$\deg(R_3) \geq 4$

Thm:

if G is planar then $e \leq 2n-4.$

(Regions are made by atleast 4. edges)

$n - e + f = 2$

$f = 2 + e - n.$

$\cancel{2}e \geq \cancel{4}^{2}(2 + e - n)$

$e \geq 2(2 + e - n)$

$e \geq 4 + 2e - 2n$

$2n - 4 \geq 2e - e.$

$2n - 4 \geq e.$

$\deg(R_1) + \deg(R_2) + \deg(R_3) \geq 4 + 4 + 4.$

$\geq 4 \cdot 3$

$2e \geq 4 \cdot f$

(Regions are made by atleast 5 edges)

$2e \geq 5f$ $\qquad$ $n - e + f = 2$

$e \leq \frac{5}{3}n - \frac{10}{3}$

(atleast 3)	if G is planar then $e \leq 3n-6$
(at - 4)	if G is planar then $e \leq 2n-4$
(at - 5)	if G is planar then $e \leq \frac{5}{3}n - \frac{10}{3}$

A planar Graph has vertices only of degree 5 and 7
if there are 10 vertices of degree 7. Ans: 22

prove that atleast — vertices of degree 5.

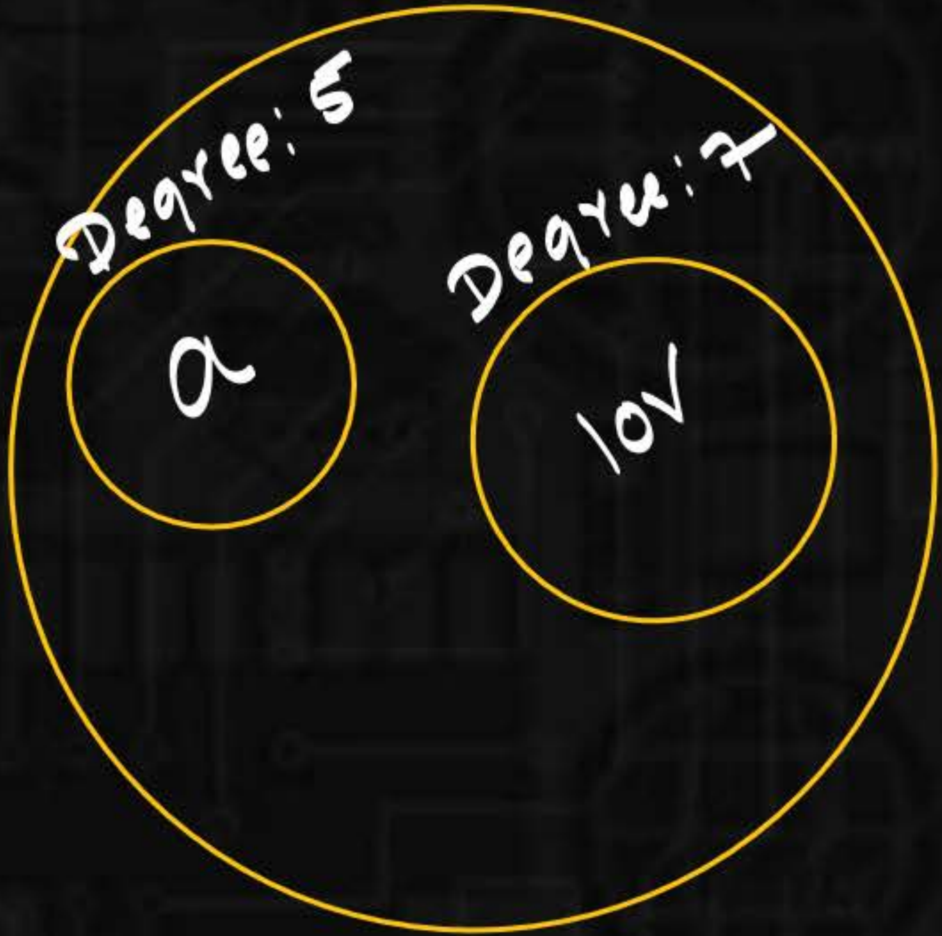
$$\Sigma\, d(v_i) = 2e$$

$$5 \cdot a + 10 \cdot 7 = 2e.$$

$$e = \tfrac{1}{2}(5a + 70)$$

if G is planar then $\boxed{e \le 3n - 6}$

$$\tfrac{1}{2}(5a + 70) \le 3(10 + a) - 6.$$



Total vertices = 10 + a.

$e \leq 3n-6$.

$5 \cdot a + 10 \cdot 7 = 2e$.

$\frac{1}{2}(5a+70) \leq 3(10+a)-6$.

$e = \frac{1}{2}(5 \cdot a + 70)$

$5a+70 \leq 6(10+a)-12$

$5a+70 \leq 60+6a-12$.

$\underline{70-60}+12 \leq 6a-5a$.

$10+12 \leq a$.

$22 \leq a$.


THANK YOU GW
SOLDIERS !