CS & IT ENGINEERING

Discrete Mathematics
Graph Theory

Lecture No. 08

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TOPICS TO BE COVERED

01 Annalysis In Connectivity

02 Various definition in Connectivity

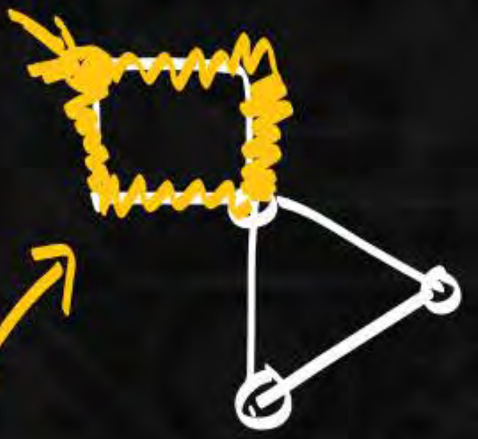
03 Edge Connectivity

04 Vertex Connectivity

05 Largest inequality theorem

Connectivity in Graphs

Trail : R.V / R.~~E~~.



Closed Trail : Trail + starting = ending vertex

Euler circuit : Closed Trail + covers all edges exactly once.

Euler ckt

$G = (V, E)$

Connectivity in Graphs

Euler CKt → Euler Graph.

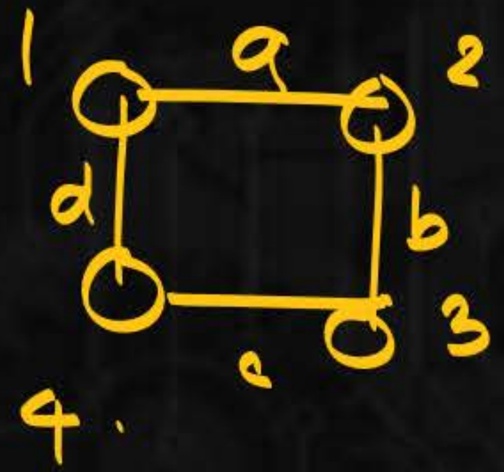
→ A

→ all bridges.

closed.

all edges.

Connectivity in Graphs

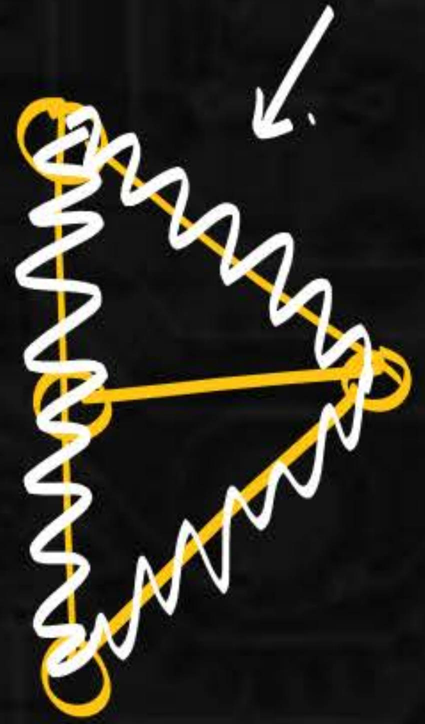


1 – a – 2 – b – 3 – c – 4 – d – 1

Euler/ckt

G = (V, E) → Euler Graph

1) ✓

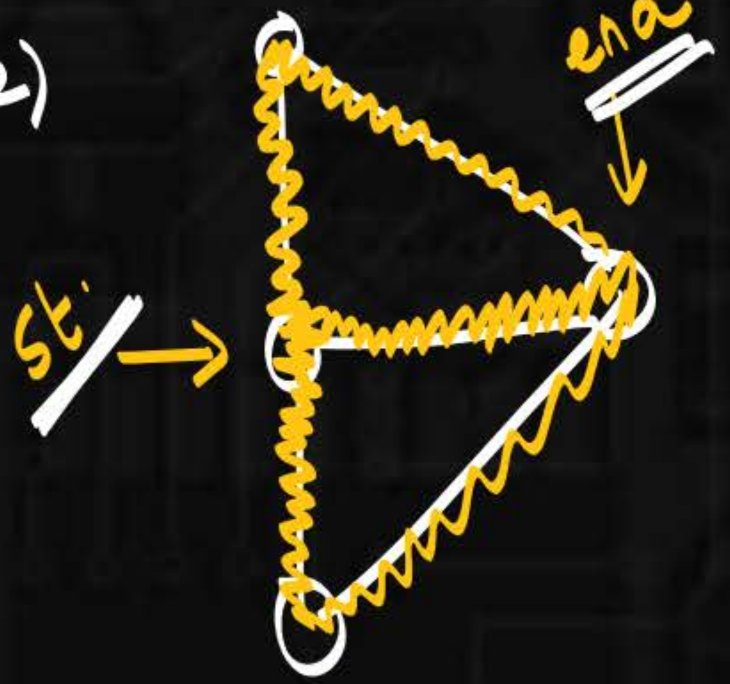


(any vertex)

A ✓

all edges ✗

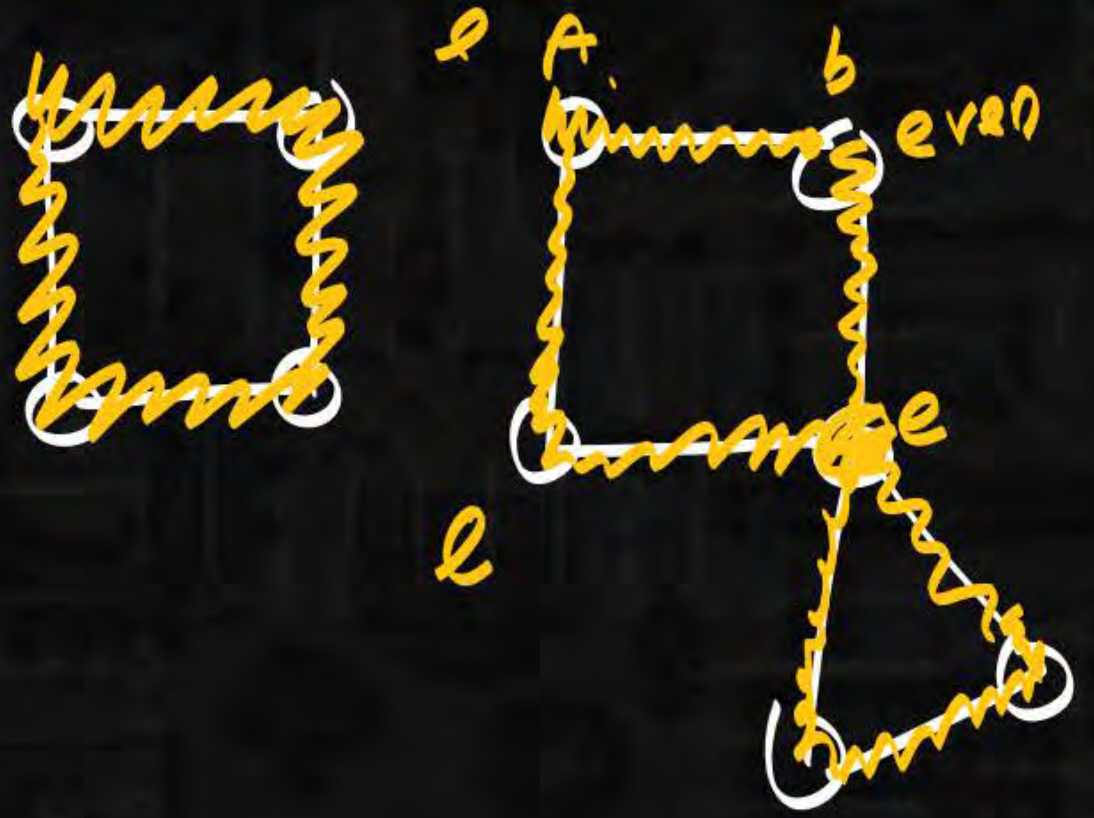
2)

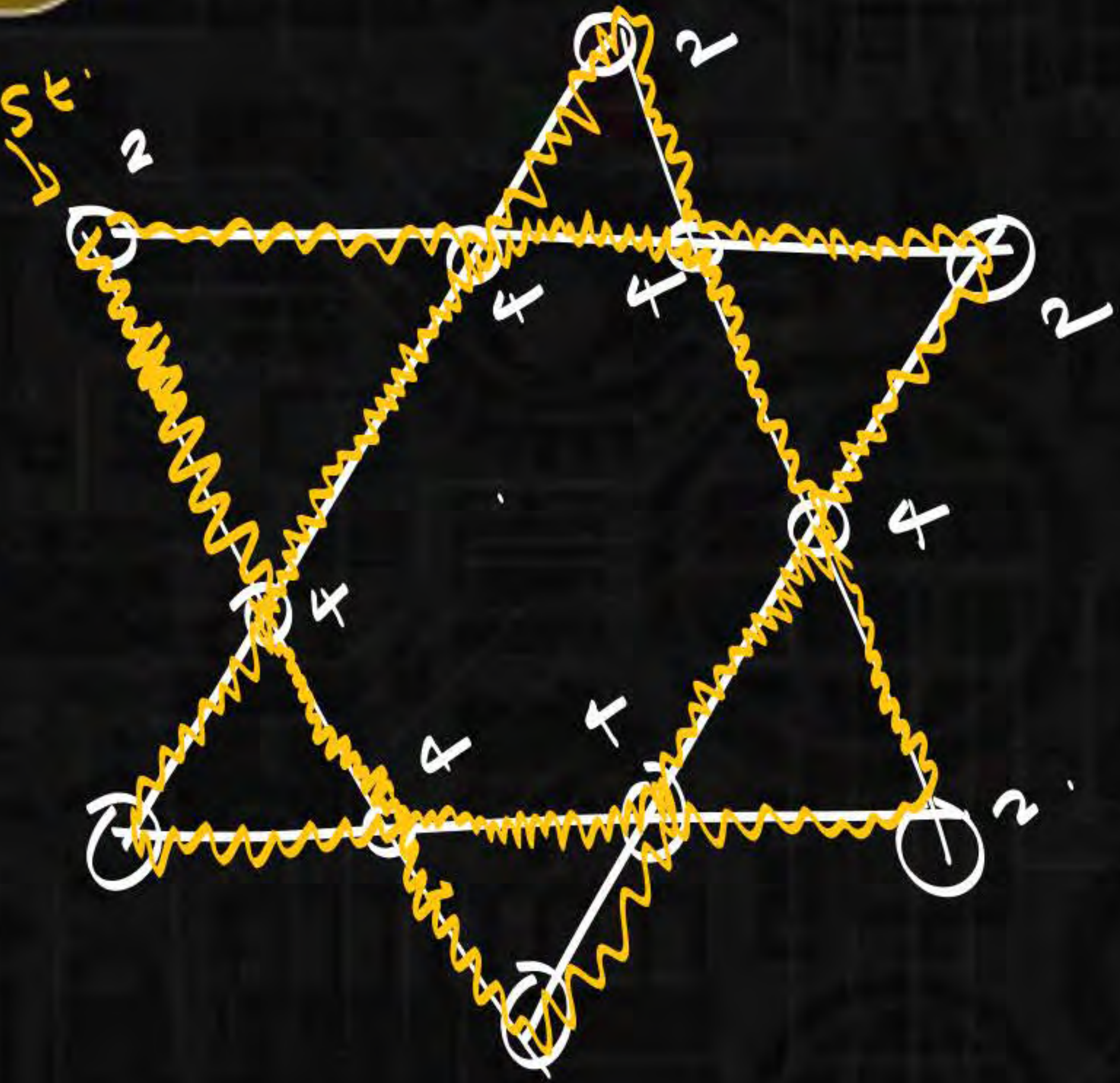


A ✗

all edges ✓

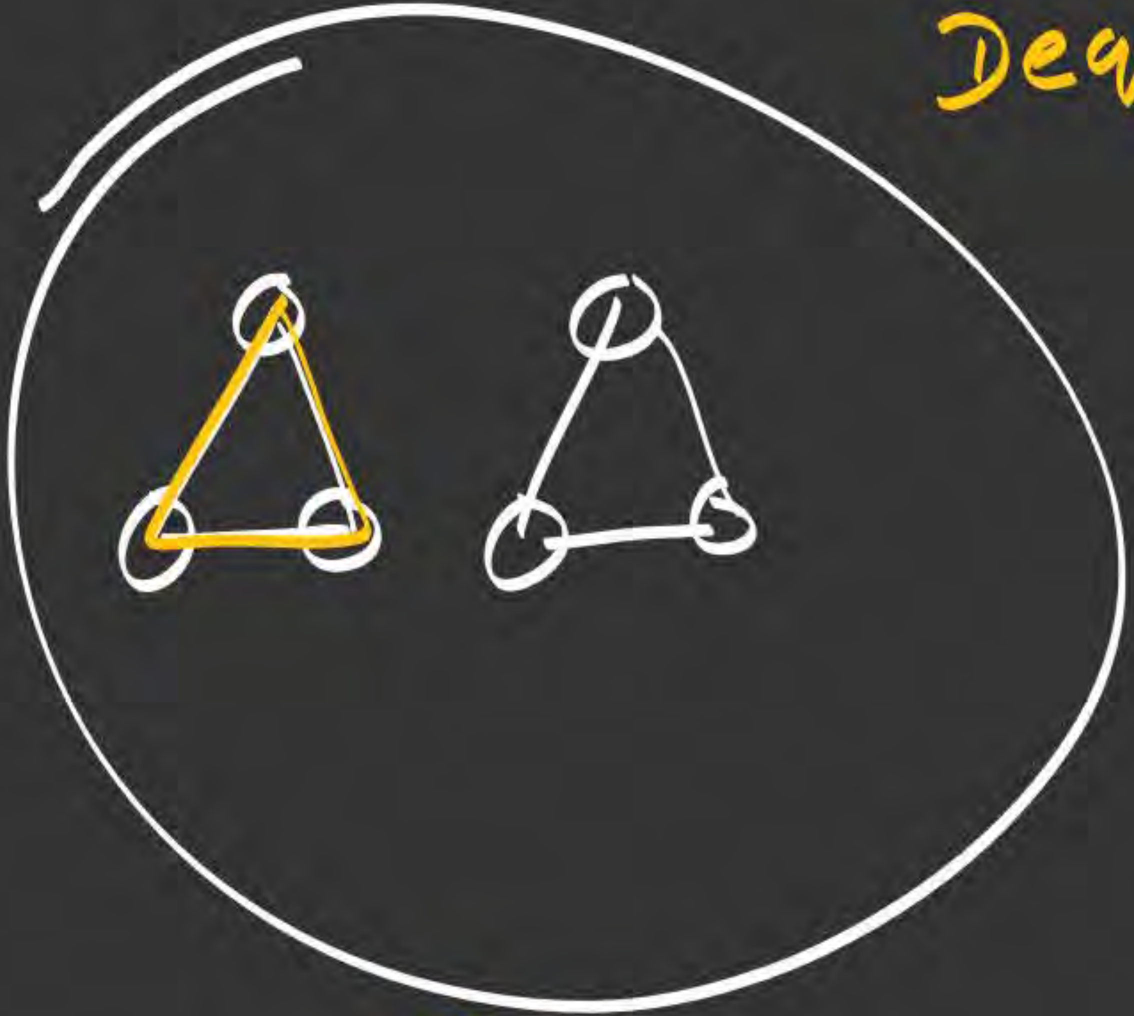
Connectivity in Graphs





D.C.G.

Degrees of all are 2.



all edges

Trivial Graph

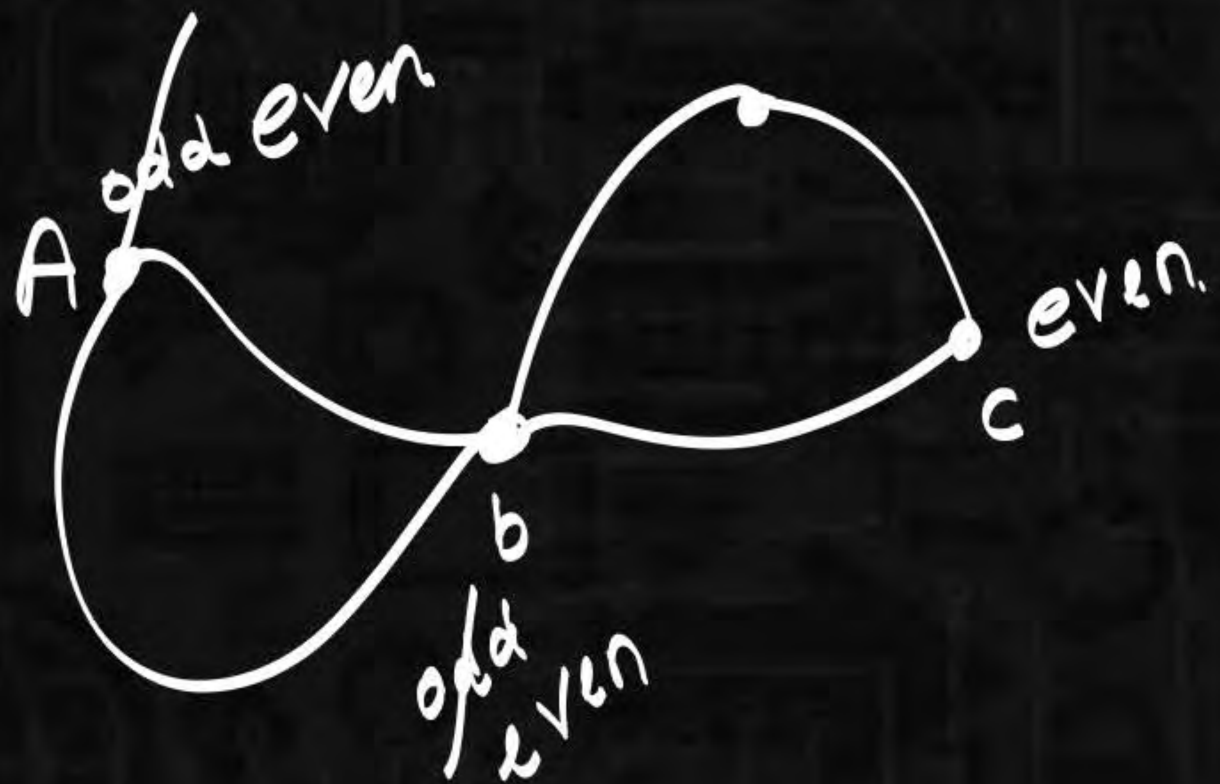
(K_1)

isolated vertex.

Connectivity in Graphs

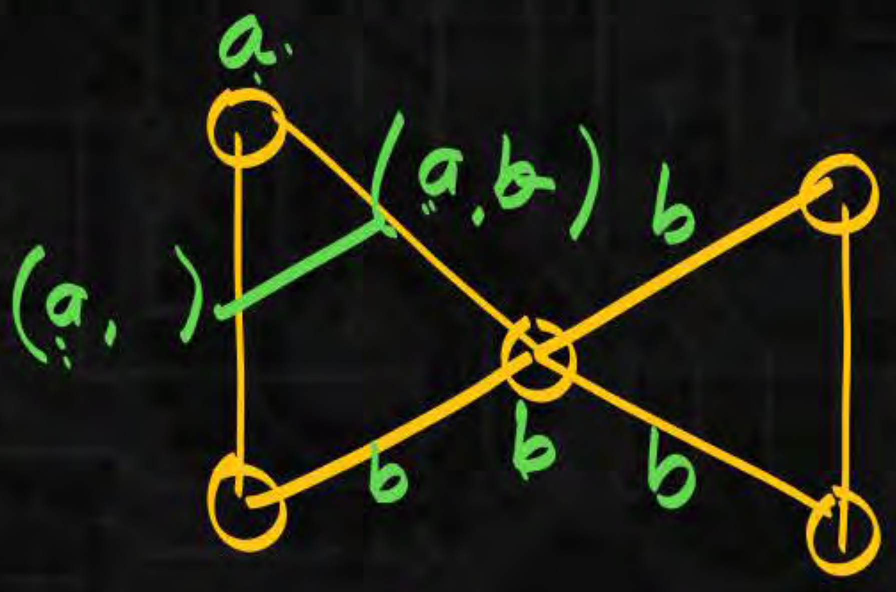
Thm: Graph is Euler Graph iff degrees of all vertices are even.

(non Trivial connected Graph)

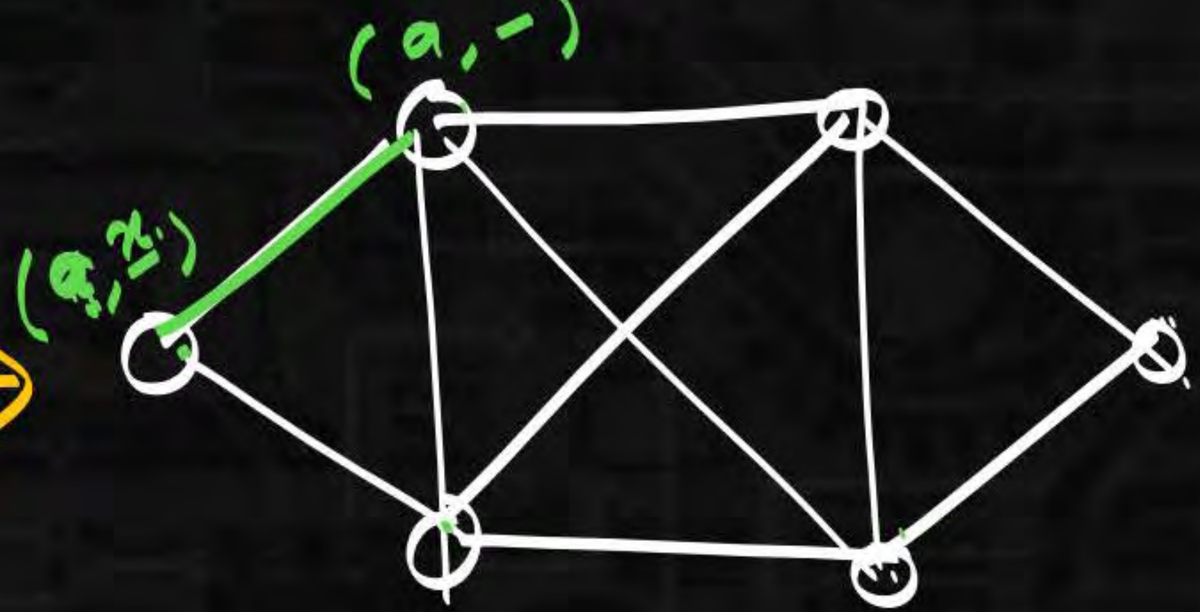


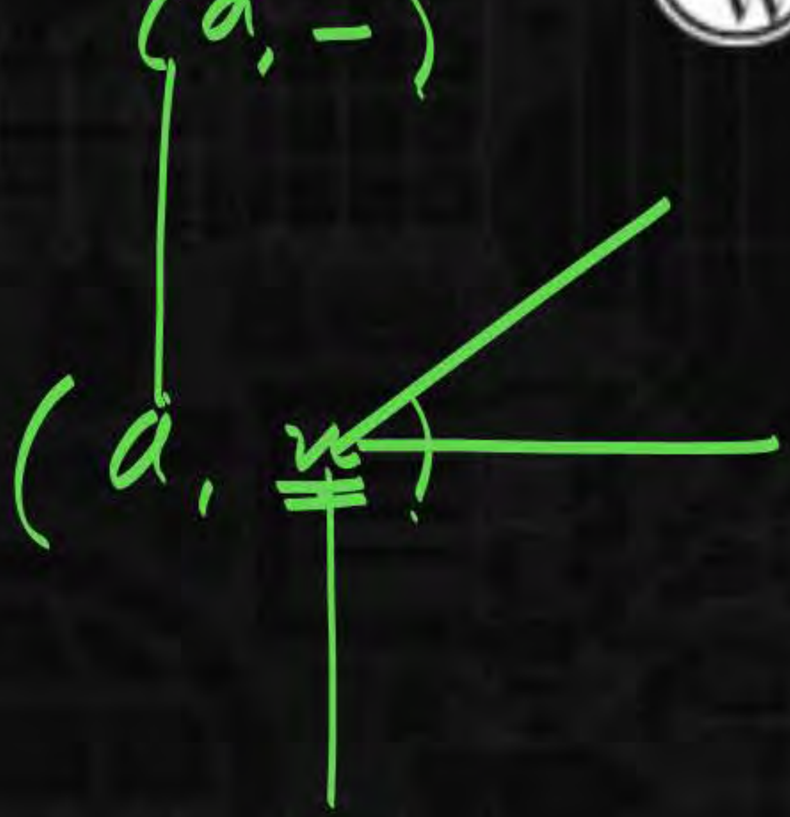
A
all edges cover

Connectivity in Graphs



$\rightarrow$



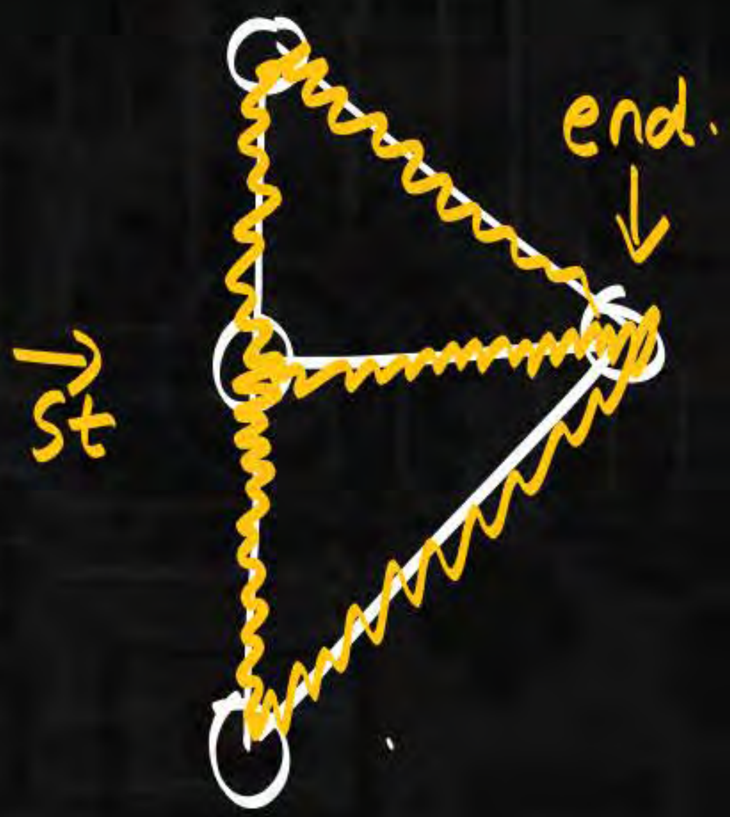


Euler Graph:

Degrees of all vertices are even $\rightarrow 2, 4, 6 \ldots$

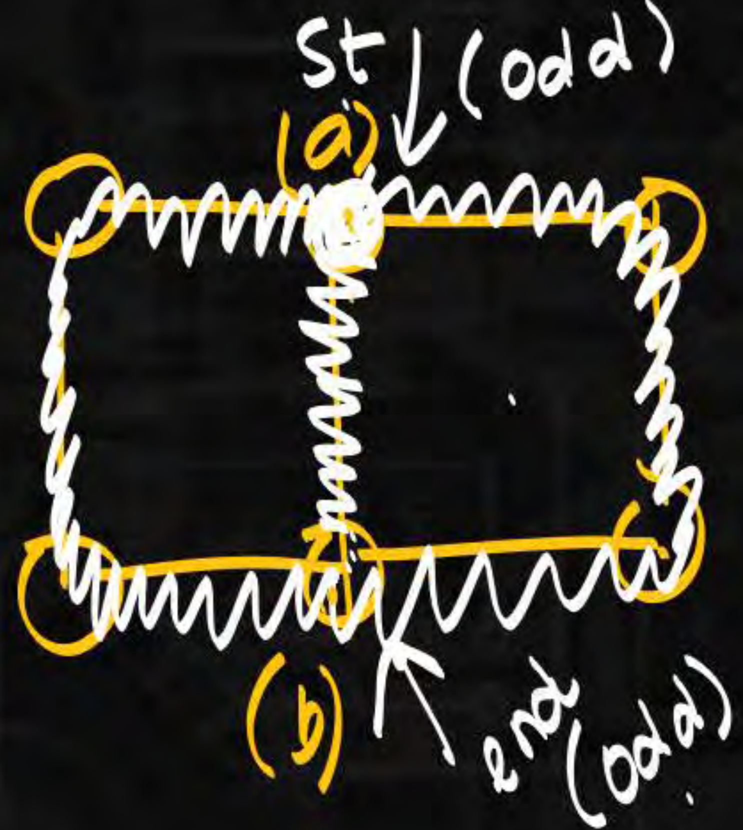
note: Line Graph of euler Graph will always be Euler Graph.

Connectivity in Graphs



{ $A^{\circlearrowleft}$

→ all edges }



Trail,

open Trail:

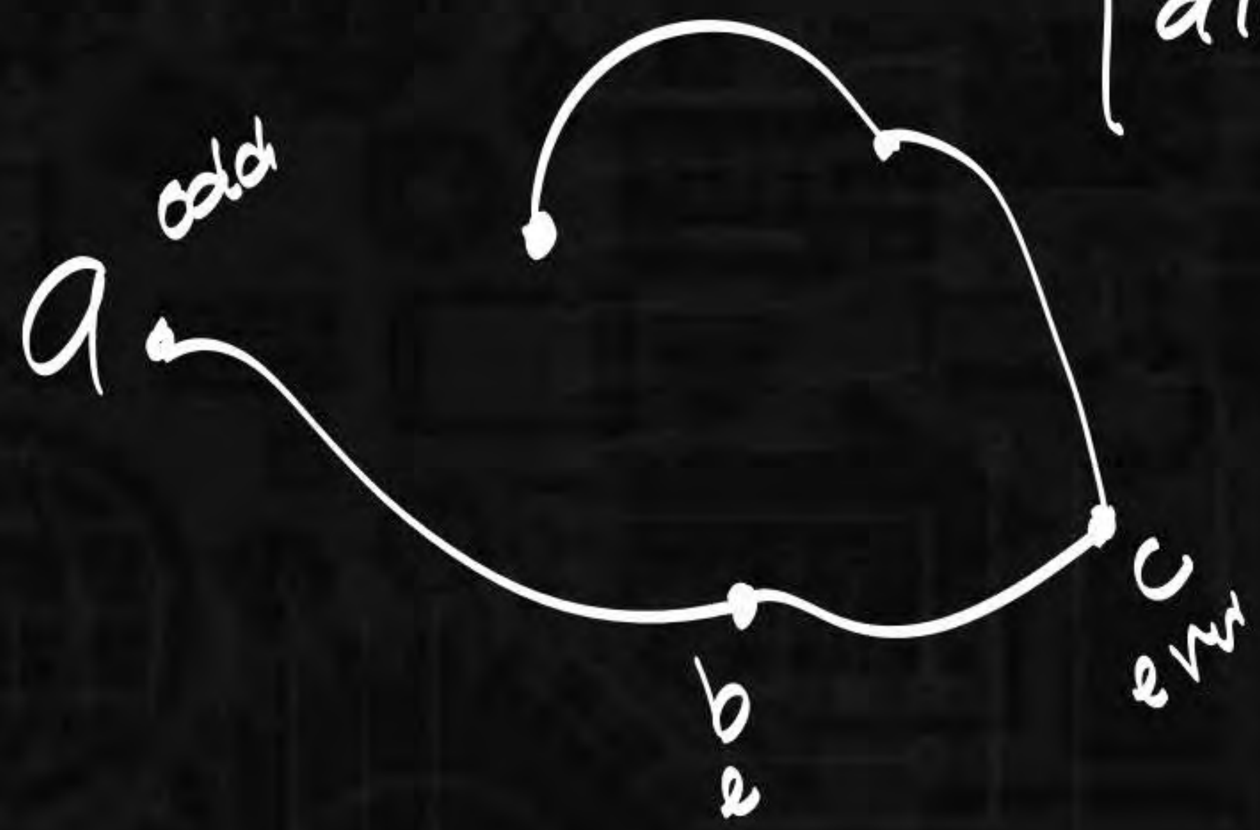
Euler line.
Euler path.
unicursal line.

⇒ { open Trail
+
covers all edges exactly once.

Connectivity in Graphs

Thm: Graph contains Euler path iff it has exactly 2 odd degree vertex.

{ st ≠ ending
all edges }



Connectivity in Graphs

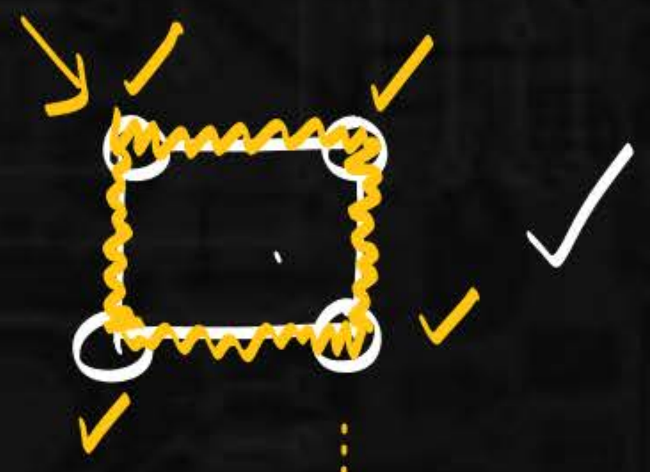
$\boxed{\text{Graph} \rightarrow \text{Hamiltonian ckt}}$

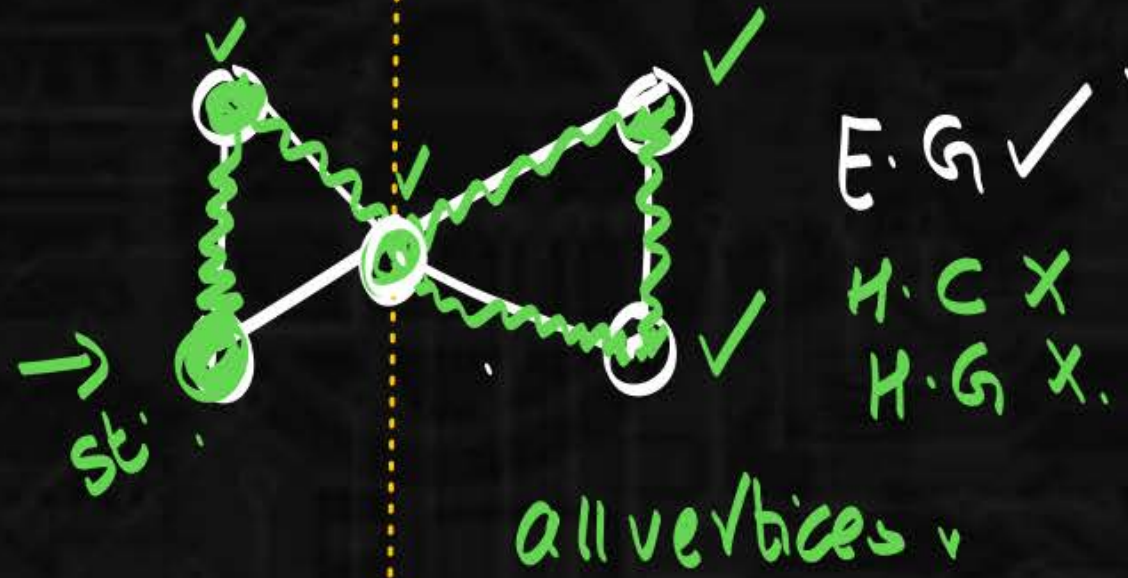
Hamiltonian Graph.

Path :

Closed Path: Path + st = ending vertex.

Hamiltonian ckt : { Closed Path. + cover all vertices exactly once.





Connectivity in Graphs

all C_n — E.G ✓ H.G ✓

W_n — E.G ✗ H.G ✓

$K_{m,n}$

Euler Graph ($m = n = \text{even}$ $m, n \geq 2$).
Hamiltonian Graph.

Connectivity in Graphs

$K_{m,n}$

$K_{m,n}$ $(m = n = \text{even})$ $(m\,n \geq 2)$

$K_{1,1}$

E.G X

$K_{2,2}$ (E G)

$K_{2,3}$

3 (odd)

Connectivity in Graphs

$K_{m,n}$

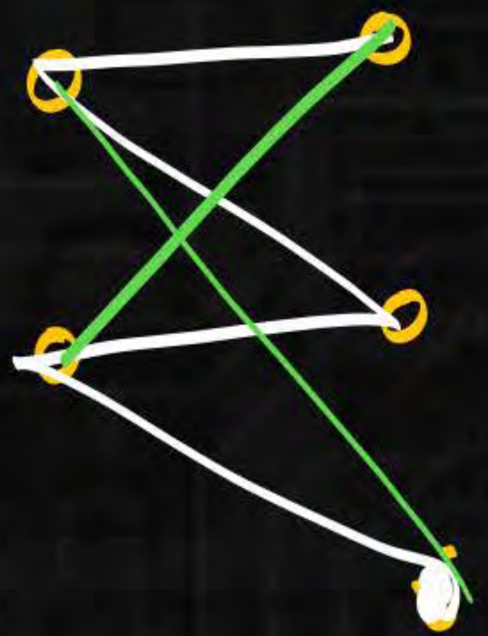
$\left(\begin{array}{c} m = n \\ m, n \geq 2 \end{array} \right)$

$K_{1,1}$ X

$m \neq n$

$K_{1,2}$

$K_{2,3}$



$K_{3,3}$

Connectivity in Graphs

K_n $(n \geq 3)$

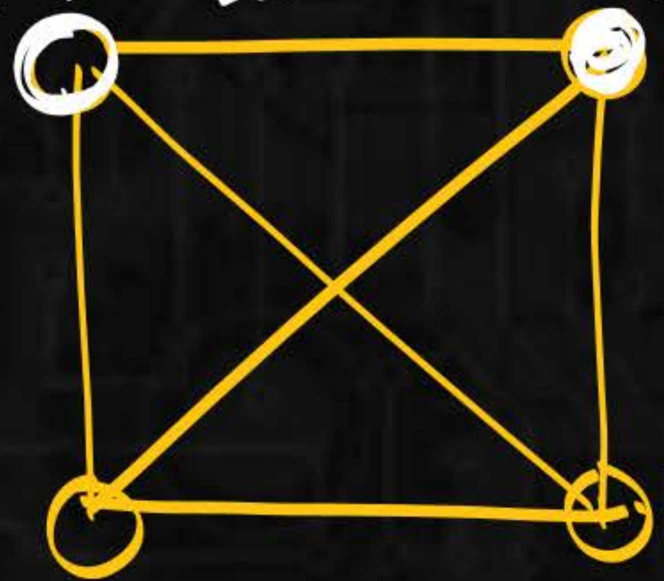
H.G.

no. of distinct Hamiltonian ckt in K_n.

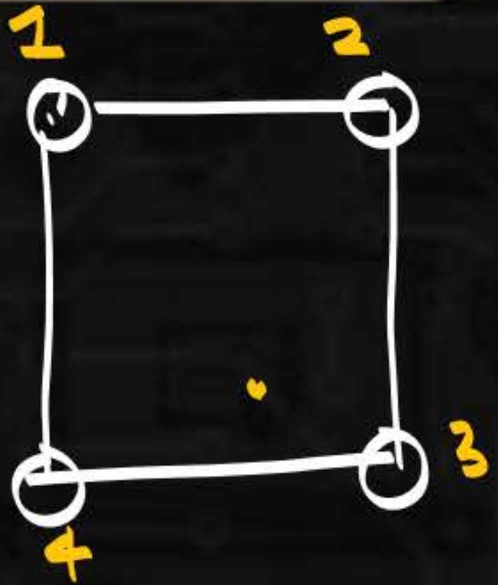
Connectivity in Graphs

1 – 3 ways

2 ways

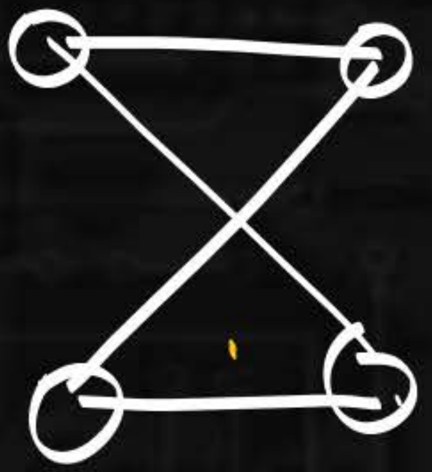


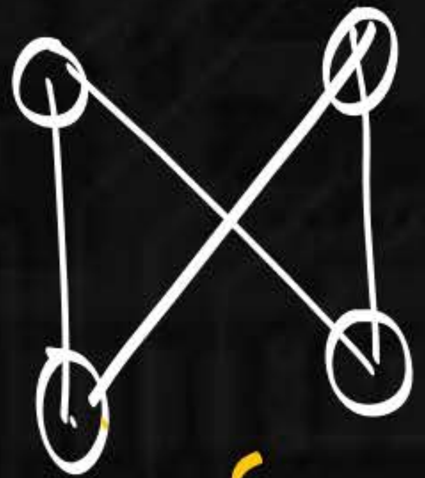
K4



$$\begin{cases} 1 - 2 - 3 - 4 - 1 \\ 1 - 4 - 3 - 2 - 1 \end{cases}$$

$$(n-1) \times (n-2) \ldots 1$$





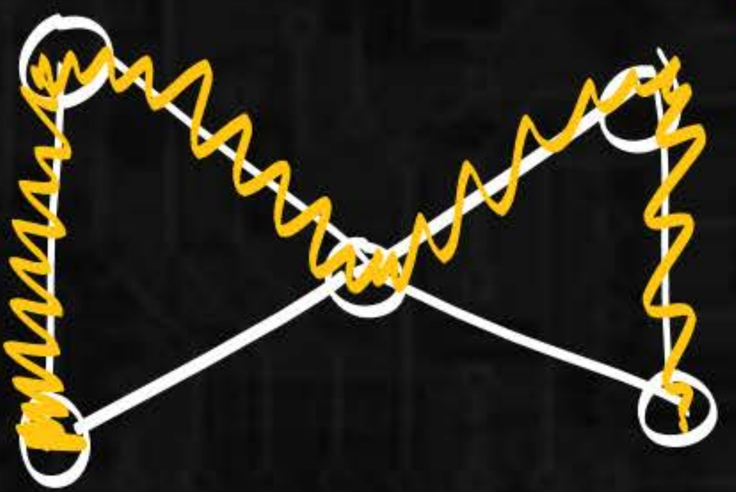
$$\frac{(n-1)!}{2}$$

no. of diff Hamiltonian ckt

Connectivity in Graphs

of diff Hamiltonian ckt in $K_{n,n}$ $(n \geq 2)$

Connectivity in Graphs



H.G ×

H.C ×

H.P ✓

Hamiltonian Path.

open path that covers all vertices.

× closed

✓ all vertices should be cover

Connectivity in Graphs

H.C $\left\{ \begin{array}{l} \text{closed.} \\ \boxed{\text{all vertices}} \end{array} \right.$ H.P.

Every H.C contains H.P.
but viceversa is not true.

H.P ✓
H.C ✗.

Connectivity in Graphs

covering.

- all edges.
  - closed. → Euler ckt.
  - open. → Euler path.
- all vertices.
  - closed. → Hamiltonian ckt
  - open. → Hamiltonian path.


THANK YOU GW
SOLDIERS !