CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

[redacted]

Lecture No. 03

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TOPICS TO BE COVERED

- 01 onto Functions
- 02 1:1 correspondance Functions
- 03 Number of Functions
- 04 Types of Functions
- 05 Various Examples in Functions

$P_1 \; R \; P_2$.

$e_1 \; R \; e_2$ eq: $5 > 3$

$G_1 \; R \; G_2$

Set 1 R set 2 $A \subseteq B$

$\_\_\_ \; R \; \_\_$

$A = \{1, 2, 3\}$ $B = \{1, 2\}$

$A \times B = \{(1,1)\,(1,2)\,(2,1)\,(2,2)\,(3,1)\,(3,2)\}$.

(a, b)

$A \times B$

$(1,1)$

$(1,2)$

$a = b$

$a + b/2$

Result

$\{(1,1)\,(2,2)\}$

Relation:

it is subset of cross product of 2 sets:

$|A| = m \quad |B| = n$

$|A \times B| = m \cdot n$

Total no. of relations = Total no. of diff subsets $= 2^{m \cdot n}$

$|A| = n$

Total no. of relations.

$|A \times A| = n^2$

Total no. of relations.

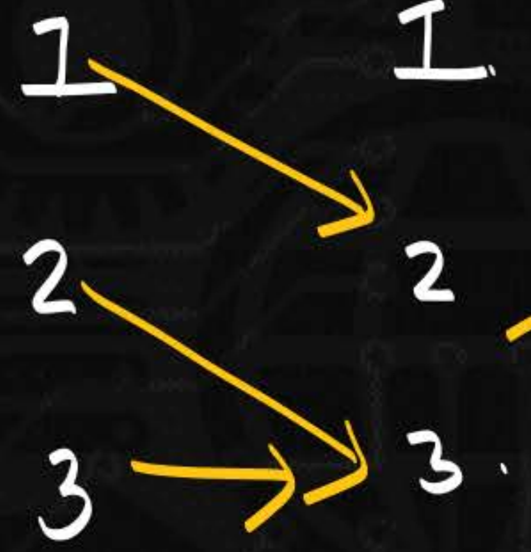
$= 2^{n^2}$

$A = \{1, 2, 3\}$ $B = \{2, 3\}$

$A \times B = \{(1,2)\,(1,3)$
$(2,2)\,(2,3)$
$(3,2)\,(3,3)\}$

$R_1 = \{(1,2)\,(2,3)\,(3,3)\}$

Represent



Fn

$A = \{1, 2, 3\} \rightarrow A \times A \rightarrow \bigcirc \rightarrow$ Result:

$\{ \ldots \ldots \}$

Check.

Symmetric:

$$\forall a \forall b \big((a,b) \in R \rightarrow (b,a) \in R \big)$$

$R_1 = \{ \ \}$

$$\underbrace{\underbrace{(a,b) \in R}_{F} \rightarrow (b,a) \in R}_{\text{True}}$$

$R_2 = \{(1,2)\}$ → not symmetric

$(a,b) \in R \rightarrow (b,a) \in R$

$\underbrace{(1,2) \in R}_{T} \rightarrow \underbrace{(2,1) \in R}_{F} \equiv F$

$a = 1$

$b = 2$

$R_3 = \{(2,3)\ (3,2)\}$

$(a,b) \in R \rightarrow (b,a) \in R.$

$\underbrace{(2,3) \in R}_{T} \longrightarrow \underbrace{(3,2) \in R}_{T.}$

$a = 2.\ b = 3.$

$R_4 = \{(1,1)\}$ ✓

$(a,b) \in R \rightarrow (b,a) \in R.$

$\underbrace{(1,1) \in R}_{T} \rightarrow \underbrace{(1,1) \in R.}_{T.}$

$a = 1$

$b = 1$

<u>Symmetric.</u> :

→ demands flipping.

→ allows same element

$R_1 = \{(1,1)\ (2,2)\}$ ✓

$R_2 = \{\underbrace{(1,3)}_{\times}\ (2,2)\ (3,2)\}$ ✗

$R_3 = \{\underbrace{(1\,1)}_{✓}\ \underbrace{(2\,2)}_{✓}\ \underbrace{(3\,3)}_{✓}\ \underbrace{(1,3)}_{✓}\ \underbrace{(3,1)}_{✓}\}$ ✓

$R_4 = \{\underbrace{(1\,1)}_{✓}\ \underbrace{(2\,2)}_{✓}\ \underbrace{(3\,3)}_{✓}\ \underbrace{(1\,3)}_{\times}\}$ ✗

⋮

$A = \{1, 2, 3\}$

$$A \times A = \begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix}$$

$$\begin{array}{cccccc} & 2 & 2. & & & \\ & \uparrow & \uparrow & & & \\ 11 & 22 & 33 & \boxed{12 \quad 21} & \boxed{23 \quad 32} & \boxed{31 \quad 13} \end{array}$$

11 → 2 choices → p, a.

12 21 → box → a, b.

$A = \{1, 2, 3\}$ $\quad |A| = n$

$$A \times A = \begin{bmatrix} (11) & (12) & (13) \\ (21) & (22) & (23) \\ (31) & (32) & (33) \end{bmatrix} \to 2^n$$

$|A \times A| = n^2$

Diagonal elements $= n$ (2 choices)

non diagonal $= n^2 - n$

boxes $= \dfrac{n^2 - n}{2}$ (2 choices)

boxes $= n^2 - n/2$.

$$\underbrace{11 \quad 22 \quad 33}_{n} \;\Big|\; \underbrace{\boxed{12 \quad 21} \quad \boxed{23 \quad 32} \quad \boxed{13 \quad 31}}_{n^2 - n}$$

n^2.

$$2^n \cdot 2^{\frac{n^2 - n}{2}}$$

2choice. 2choice.

$\{(11)(22)\}$

a. p a. p

$aa \rightarrow \phi$

$ap \rightarrow \{(22)\}$

$pa \rightarrow \{(11)\}$

$pp \rightarrow \{(11)(22)\}$

$2 \times 2.$

$\boxed{1\,1 \quad 2\,2 \quad 3\,3 \quad \boxed{12 \quad 21} \quad \boxed{23 \quad 32} \quad \boxed{13 \quad 31}}$

p p a p a. a.

$\{1\,1 \quad 2\,2 \, (12) \, (21)\}$

$\frac{n^2-n}{2}$

$2^n \cdot 2$

boxes → 2 choices

diagonal → 2 choices

1 1	1 2	1,3
2 1	2 2	2 3
3 1	3 2	3 3

$\rightarrow$ n elements.

$$\text{Total} = n^2$$

$$\text{Diagonal} = n$$

$$\text{non diagonal} = \text{Total} - \text{diagonal}$$

$$= n^2 - n$$

$$\text{boxes} = \frac{n^2 - n}{2}$$

$A = \{1, 2, 3\}$

Reflexive : $\forall a \in A \quad aRa$

$1R1$

$R = \{\ \}$ $\times$

$R_2 = \{\underset{P}{\underline{(1\,1)}}\ \underset{P}{\underline{(2\,2)}}\}$ not reflexive.

$R_3 = \{(1\,1)(2\,2)(3\,3)\}$ ✓

$\{(1\,1)(2\,2)(3\,3)\ \underline{(1\,2)}\}$ ✓

Reflexive

diagonal elements + ___

(1, 1)	(1, 2)	(1, 3)
(2, 1)	(2, 2)	(2, 3)
(3, 1)	(3, 2)	(3, 3)

→ 1 choice present

Diagonal = n.

Total = n^2.

non diagonal = $n^2 - n$.

non diagonal will have 2 choices

$2^{n^2 - n}$

11 22 33 +

$|S| = 2^n \cdot 2^{\frac{n^2-n}{2}}$ $\quad$ $|R| = 2^{n^2-n}$

2^{n^2-n} $\quad$ $2^{n^2-n/2}$

$\underbrace{2^n \cdot 2^{\frac{n^2-n}{2}}}$

$R \cap Sy$

$1^n \cdot 2^{n^2-n/2}$

$$(R \cap \overline{Sy}) = R - (R \cap Sy)$$

$$= 2^{n^2-n} - 2^{\frac{n^2-n}{2}}$$

Antisymmetric.

$$\forall a \forall b\left((a,b) \in R \wedge (b,a) \in R \rightarrow a = b \right)$$

$R_1 = \{ \quad \}$ ✓

$\underline{(a,b) \in R} \wedge (b,a) \in R \rightarrow a = b.$

$F \wedge$

$F \quad \rightarrow$

True.

$R_2 = \{ (1,1) \}$ ✓

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b.$

$\underbrace{(1,1) \in R}_{T} \wedge \underbrace{(1,1) \in R}_{T.} \rightarrow \underbrace{1 = 1}_{T.}$

$a = 1$

$b = 1$

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

not Antisymmetric

$R_1 = \{ (1,2) \}$ $\quad$ $R_2 = \{ (2,1) \}$ $\quad$ $R_3 = \{ (1,2), (2,1) \}$

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\boxed{(1,2) \in R} \wedge \boxed{(2,1) \in R} \rightarrow$

$T \wedge F \longrightarrow$

$F \longrightarrow$

True.

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$(2,1) \in R \wedge (1,2) \in R$

$T \wedge F \longrightarrow$

True

$(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$(1,2) \in R \wedge (2,1) \in R \rightarrow 1 = 2$

$T \wedge T \qquad F$

$T \longrightarrow F$

False.

11	12	13
21	22	23
31	32	33

$\rightarrow 2^n$

(1 2) (2 1)

P P

absent

Antisymmetric

→ allows same element

→ does not allows flipping.

boxes will have 3 choices

$$2^n \cdot 3^{n^2-n/2}$$

$\boxed{(1\,2)\ (2\,1)}$

3 cases.

$\{(1\,2)\}$ ✓

$\{(2\,1)\}$ ✓

$\{(1\,2)(2\,1)\}$ ✗

$\{\ \}$ ✓

$|\text{Sy} \cap \text{Anti}|$

$\begin{Bmatrix} \text{demands flipping} \\ \text{allows same element} \end{Bmatrix} \cap \begin{Bmatrix} \text{no flipping.} \\ \text{allows sam element} \end{Bmatrix} =$ allows same element

↓

2^n

If R_1 & R_2 are antisymmetric relation on a set A.

$\underline{R_1 \cup R_2}$. ✗

$R_1 = \{(1,2)\}$ $R_2 = \{(2,1)\}$

Anti ✓ Anti ✓

$R_1 \cap R_2$ ✓

$R_1 \cup R_2 = \{(1,2)(2,1)\}$ not Anti

	$\cup$	$\cap$
R_1, R_2. Anti	✗	✓
R_1, R_2 reflexive	✓	✓
R_1, R_2 symm	✓	✓

What will be min size of reflexive.

Symmetric.

Antisymme. Relations?


THANK YOU GW
SOLDIERS !