CS & IT ENGINEERING

Discrete mathematics
Set theory

Lecture No.8

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TOPICS TO BE COVERED

01 Lattice

02 bounded lattice

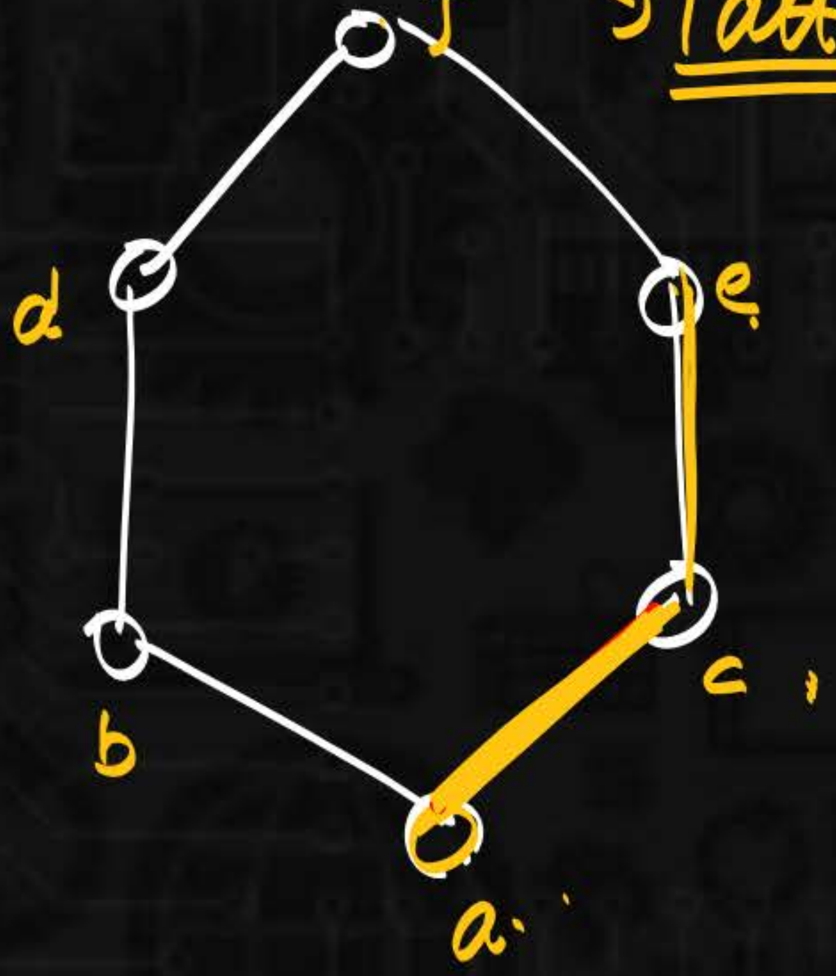
03 Complement lattice

04 Distributive lattice

05 Boolean algebra

(A, R) poset

lattice



lub(a, b) = b

glb(a, b) = a

lub(a, c) = c

glb(a, c) = a

lub(b, e) = f

glb(b, e) = a

lub(d, c) = f

glb(d, c) = a

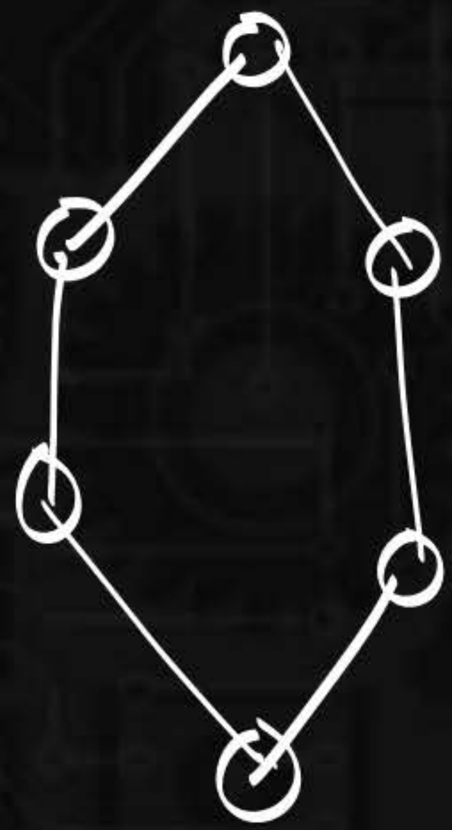
(A, R) poset.

glb & lub exist for all pairs

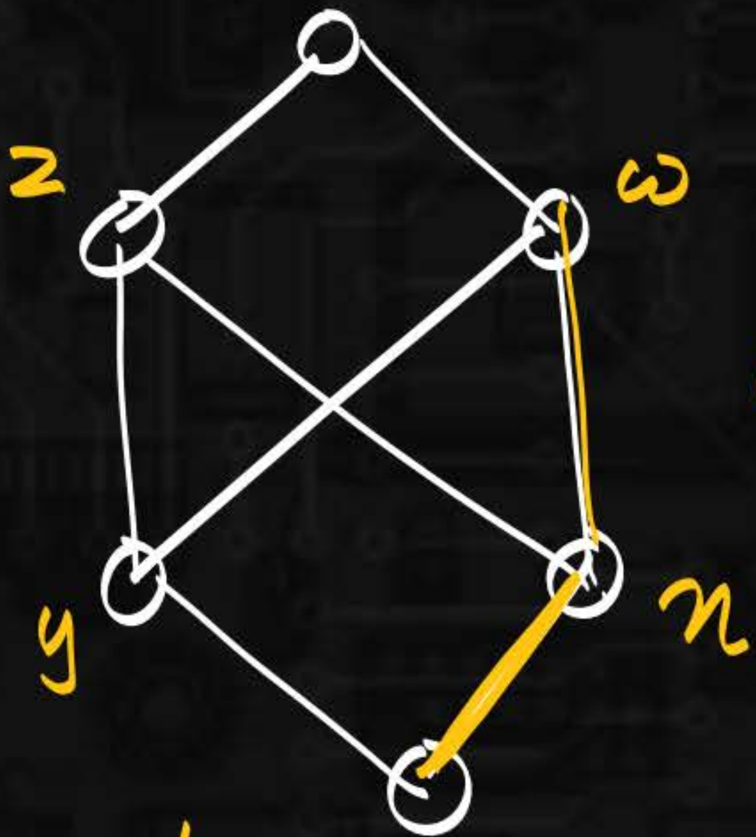
$(A, R) \to$ lattice

$glb(c, d) = NA$.

(MSQ)

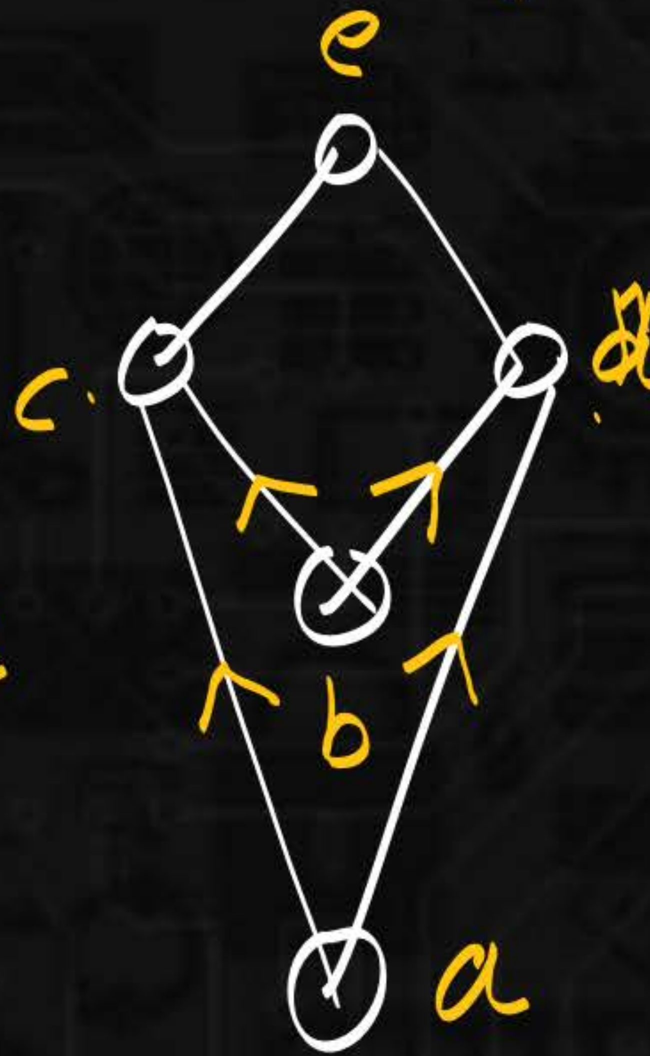


A)



X. B)

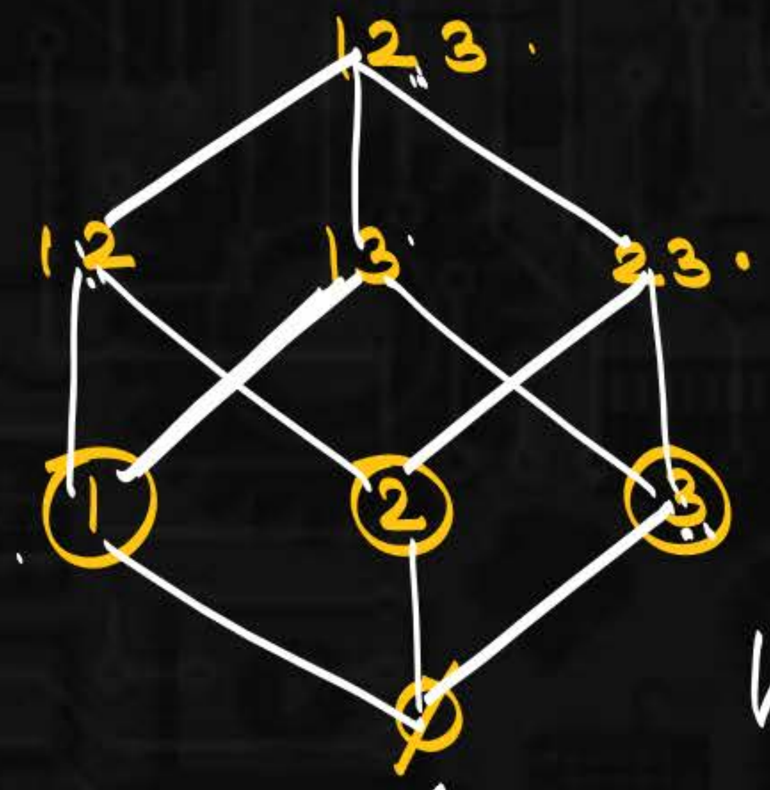
$lub(x, y) = NA$



c)

X.

d)



Lattice

$lub(2,3) = \{23\}$

$glb(2,3) = \phi$

$lub(1,13) = 13$

$glb(1,13) = 1$

$lub(12,3) = 123$

$glb(12,3) = \phi$

$lub\{2,3\} = \{23\}$

$glb(2,3) = \phi$

$lub \rightarrow \cup$

$glb \rightarrow \cap$

lub → ∪

glb → ∩

$$\text{lub}\quad \begin{array}{r} 010 \\ +\ 001 \\ \hline 011 \end{array}$$

lub → +

glb → ·

$$\text{glb}\quad \begin{array}{r} 010 \\ \cdot\ 001 \\ \hline 000 \end{array}$$

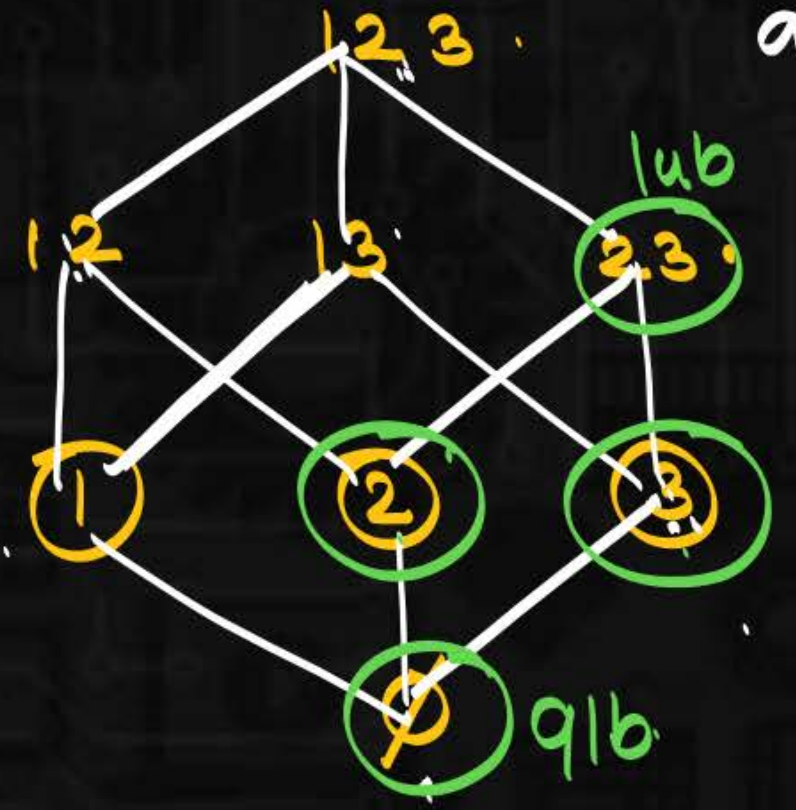
$$\text{lub}\quad \begin{array}{r} FTF \\ \vee\ FFT \\ \hline FTT \end{array}$$

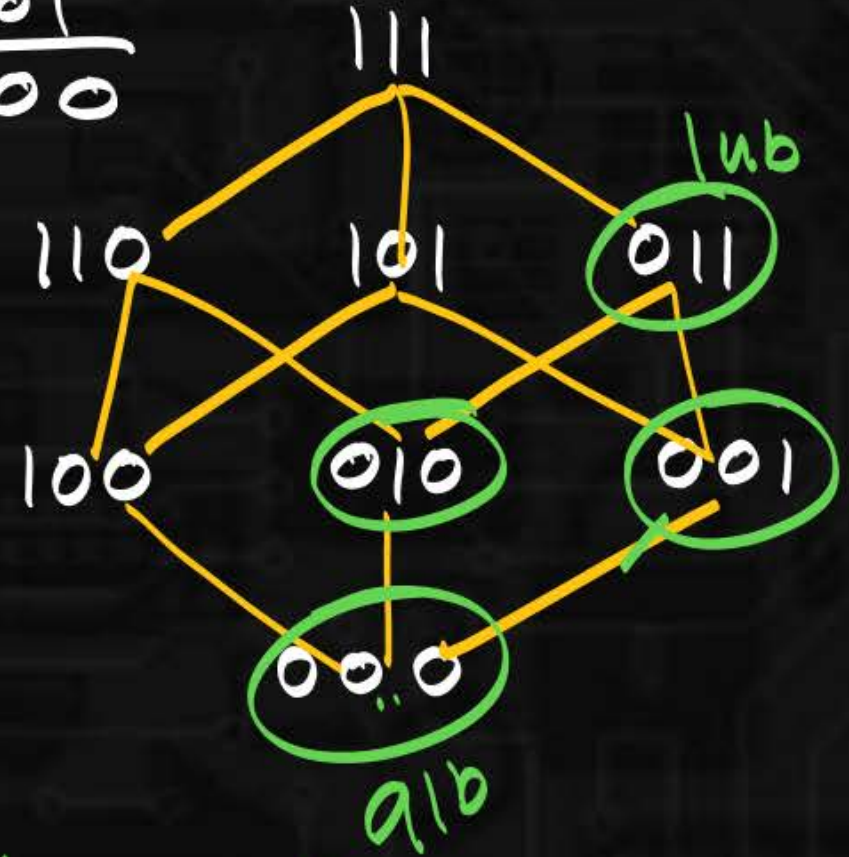
lub → ∨

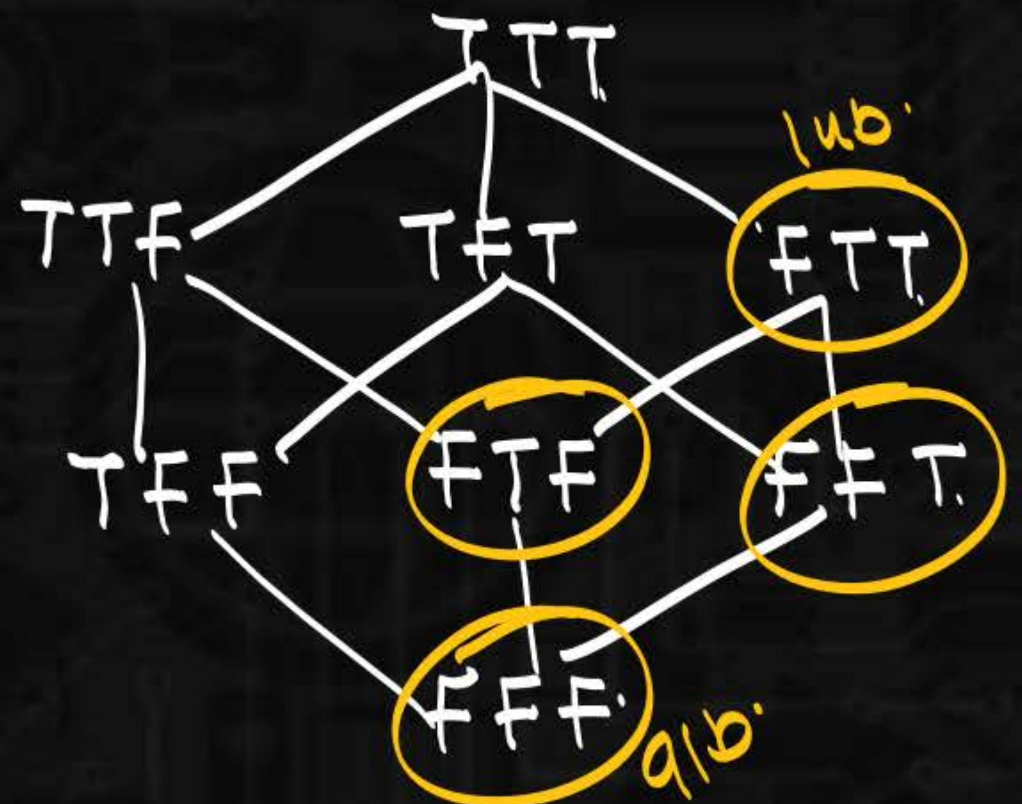
glb → ∧

1 → T

0 → F







lub(2, 3) = {2 3}

glb(2, 3) = ∅

lub(010, 001) = 011

glb(010, 001) = 000

lub(FTF, FFT) = FTT

glb(FTF, FFT) = FFF

$[A, \vee, \wedge] \rightarrow$ lattice.

1) $a \vee a = a \qquad a \wedge a = a.$

$lub(a,a) = a. \qquad glb(a,a) = a.$

2) $a \vee b = b \vee a. \qquad a \wedge b = b \wedge a.$

$lub(a,b) = lub(b,a) \qquad glb(a,b) = glb(b,a)$

3) $a \vee (b \vee c) = (a \vee b) \vee c.$ ✓

4) $a \vee (a \wedge b) = a.$

$a \wedge (a \vee b) = a.$

a, b are any 2 element

OR

lub

$a \vee \underline{(a \wedge b)} = a.$

$\downarrow$

$glb(a,b)$

$\downarrow$

$\underline{a \vee a}$

$a.$

$a \vee \underline{(a \wedge b)} = \underline{a}.$

$\downarrow$

$glb(a,b)$

$\downarrow$

$a \vee b.$

$\underline{lub(a,b)}$

$\downarrow$

a

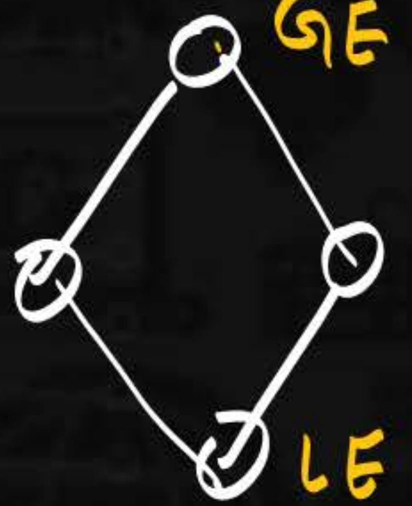
$[L, \vee, \wedge, 1, 0]$

Bounded lattice :

* ~~finite~~ lattice will always be bounded lattice.

lattice → Greatest element (1)

lattice → least element (0) TOR → linear order relation.



$(P(A), \subseteq)$

GE: $\{1\,2\,3\}$

LE: ϕ

$(\{1, 2, 3, 4, 5\}, \leq)$ → Toset → chain.

✓ 5 — 4 — 3 — 2 — 1

$GE \rightarrow 5$

$LE \rightarrow 1$

$(Z^+, \leq)$

LE: 1.

GE : N.A.

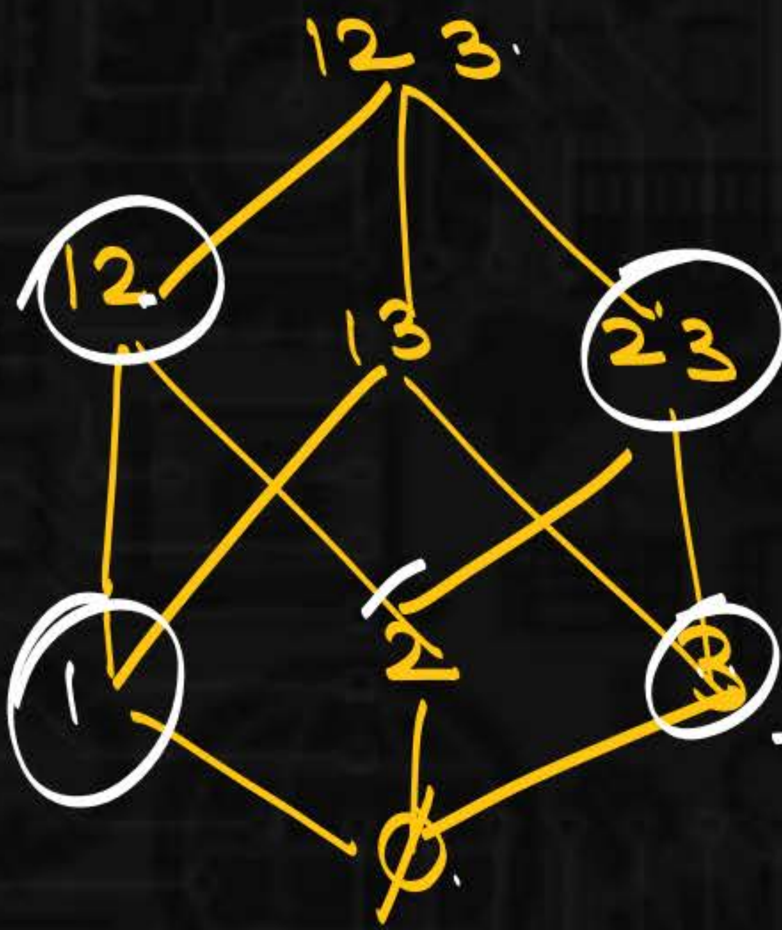
$(Z, \leq)$

LE: NA

GE: NA

+ all elements are having atleast 1 complement
bounded lattice.

Complement lattice



GE → 123

LE → ∅

$1' = ?$

$\cup$

$\text{lub}(1, 23) = GE = \{123\}$

$\text{glb}(1, 23) = LE = \emptyset$

$\cap$

$\text{lub}(23, 1) = \{123\}$

$\text{glb}(23, 1) = \emptyset$

$1' = 23$

$2' = 13$

$3' = 12$

$(123)' = \emptyset$

$\text{lub}(a, b) = GE$

$a + b = 1$

$a \cdot b = 0$

$\text{glb}(a, b) = LE$

a, b are complement to each other.

$b' =$

$lub(b, c) = x$.

$glb(b, c) = a$

$lub(b, d) = x$

$glb(b, d) = a$.

$b' = \{c, d\}$.

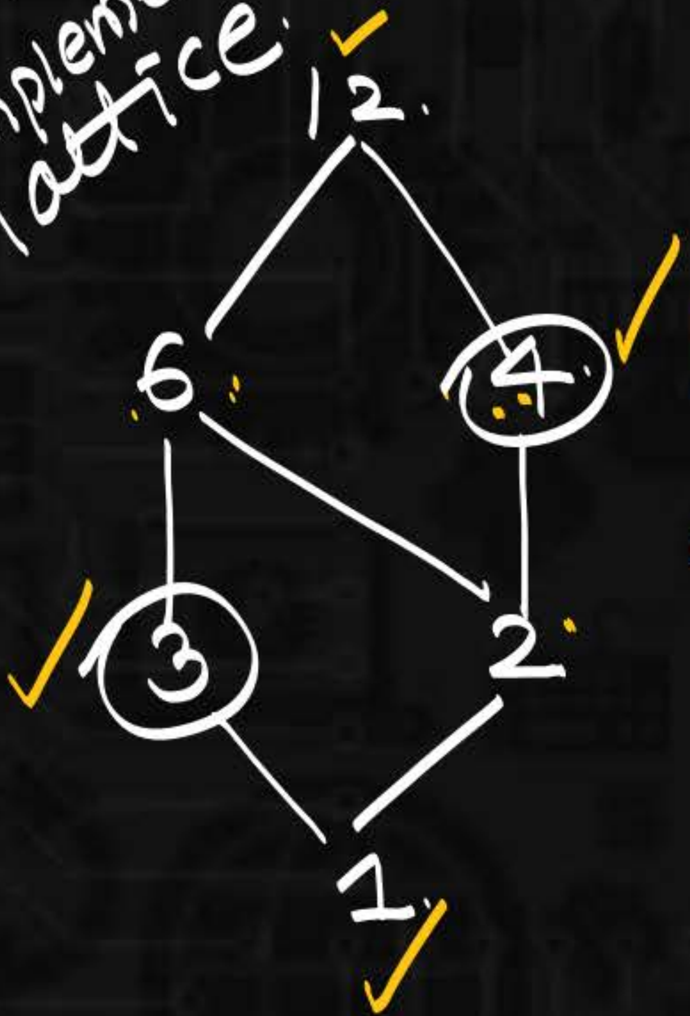
$a + b = 1$.

$lub(\underline{a}, \underline{b}) = GE$.

$glb(a, b) = LE$

$a \cdot b = 0$

not complement lattice. $(D_{12}, |)$



$4' =$

$lub(4, 6) = 12$ ✓

$glb(4, 6) \neq 1$

$= 2$

hence 6 is not complement of 4.

$6' = N.A.$

$4' = 3$

$lub(4, 3) = 12.$

$glb(4, 3) = 1$

$lub(1, 12) = 12.$

$glb(1, 12) = 1$

$1' = 12$

$lub(6, \;) = 12$

$glb(6, \;) = 1.$

$lub(6, 4) = 12$ ✓

$glb(6, 4) = 2$ ✗

$lub(6, 2) = 6$ ✗

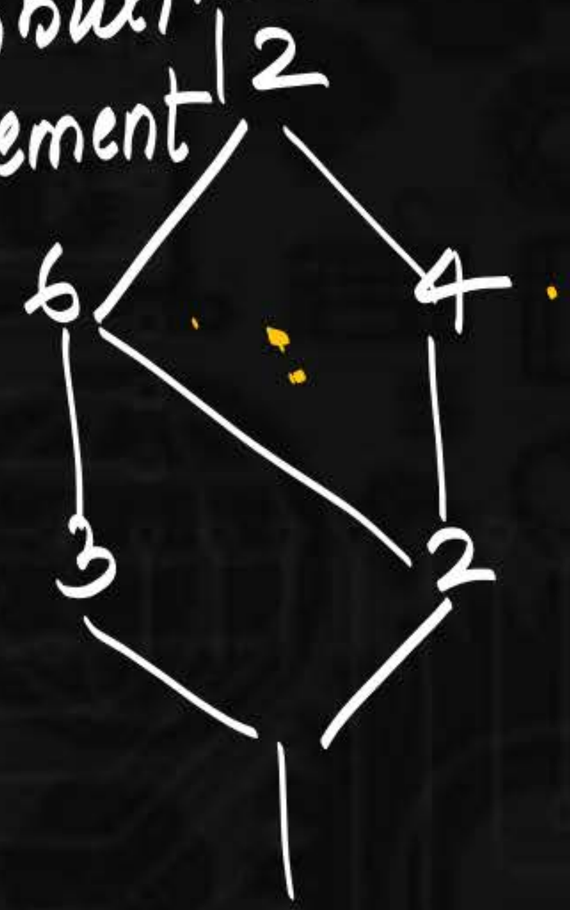
$glb(6, 2)$

Distributive lattice. :

$$\forall a,b,c \begin{cases} a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \\ a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \end{cases}$$

distributive ✓

complement X.

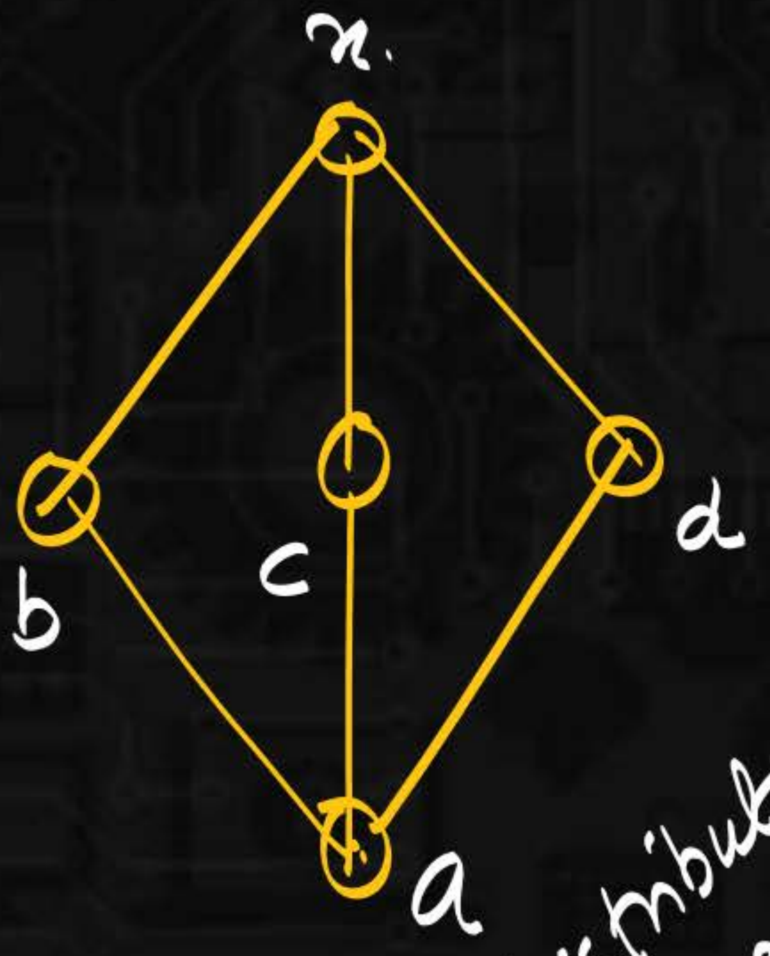


$$6 \vee (\underline{4 \wedge 2}) = (6 \vee 4) \wedge (6 \vee 2)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ glb(4,2) & lub(6,4) & lub(6,2) \\ \downarrow & \downarrow & \downarrow \\ 6 \vee 2 & 12 & \wedge \quad 6 \\ lub(6,2) & & glb(12,6) \\ \downarrow & & \downarrow \\ 6 & & 6 \end{array}$$

$$6 \wedge (4 \vee 2) = (6 \wedge 4) \vee (6 \wedge 2)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \uparrow \\ lub(4,2) & glb(6,4) & glb(6,2) \\ \downarrow & \downarrow & \uparrow \\ 6 \wedge 4 & \underline{2 \vee 2.} & 2 \\ glb(6,4) & \downarrow & \\ \downarrow & 2 & \\ 2 & & \end{array}$$



$$b \vee (c \wedge d) = (b \vee c) \wedge (b \vee d)$$

$$\text{glb}(c,d) \qquad \text{lub}(b,c) \qquad \text{lub}(b,d)$$

$$b \vee a \qquad\qquad x \wedge x$$

$$\text{lub}(a,b) \downarrow b \neq x$$

not a distributive
complement.

Boolean algebra $=$ lattice $+$ complement $+$ distributive.

$a+b=b+a$

$a \cdot b = b \cdot a$

$a+a=a$

$a \cdot a = a$

$a+(b+c)$

$=(a+b)+c$

$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

$a+(a \cdot b) = a$

$a \cdot (a+b) = a$

$a+b=1$

$a \cdot b = 0$

$a+(b \cdot c) = (a+b) \cdot (a+c)$

$a \cdot (b+c) = (a \cdot b)+(a \cdot c)$

$\star$

$(P(A), \subseteq)$

$\downarrow$

$\begin{cases} \mathrm{glb} \to \cap \checkmark \\ \mathrm{lub} \to \cup \checkmark \end{cases}$


THANK YOU GW
SOLDIERS !