CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 05

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 sum rule

02 Product rule

03 Practice

COMBINATORICS

$\langle a_0, a_1, a_2, a_3, a_4 \ldots\ldots\ldots \rangle$

$$G(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 + \ldots\ldots$$

$$\boxed{G(x) = \sum_{i=0}^{\infty} a_i \times x^i}$$

COMBINATORICS

$$\langle \underset{a_0}{1}, \underset{a_1}{1}, \underset{a_2}{1}, \underset{a_3}{1}, \underset{a_4}{1} \ldots\ldots \rangle \qquad \frac{1}{1-x} = 1 + x + x^2 + x^3.$$

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots\ldots$$

$$G(x) = 1.x^0 + 1.x^1 + 1.x^2 + 1.x^3 + 1.x^4.$$

$$G(x) = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots\ldots$$

$$\boxed{G(x) = \frac{1}{1-x}.}$$

COMBINATORICS

$<1, -1, 1, -1, 1, -1 \ldots\ldots>$

$$\boxed{G(x) = \frac{1}{1+x}}$$

$<1, 0, 1, 0, 1, 0 \ldots\ldots>$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$$G(x) = a_0 x^0 + \cancel{a_1 x^1} + a_2 x^2 + \cancel{a_3 x^3} + a_4 x^4 + \cancel{a_5 x^5}$$

$$= 1 \cdot x^0 + 1 \cdot x^2 + 1 \cdot x^4$$

$$= 1 + x^2 + x^4 + x^6 + \ldots\ldots$$

$$= 1 + x^2 + (x^2)^2 + (x^2)^3 \ldots\ldots$$

$$G(x) = \frac{1}{1-x^2}$$

COMBINATORICS

$\langle 0, 0, 1, a, a^2, a^3 \ldots \rangle \quad a \neq 0$

$a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$G(x) = \cancel{a_0 x^0} + \cancel{a_1 x^1} + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$

$= x^2 + a \cdot x^3 + a^2 \cdot x^4 + a^3 \cdot x^5$

$= x^2 (1 + ax + a^2 x^2 + a^3 x^3 + \ldots)$

$= x^2 (1 + ax + (ax)^2 + (ax)^3 + \ldots)$

$= \quad G(x) = \dfrac{x^2}{1 - ax}$

COMBINATORICS

$$\underset{a_0}{0}, \underset{a_1}{0}, \underset{a_2}{0}, \underset{a_3}{6}, \underset{a_4}{-6}, \underset{a_5}{6}, \underset{a_6}{-6} \ldots$$

$$G(x) = \cancel{a_0 x^0} + \cancel{a_1 x^1} + \cancel{a_2 x^2} + a_3 x^3 + a_4 x^4 + a_5 x^5$$

$$G(x) = 6x^3 - 6x^4 + 6x^5 - \ldots$$

$$= 6x^3 (1 - x + x^2 - x^3 + \ldots)$$

$$\boxed{G(x) = \frac{6x^3}{1+x}}$$

COMBINATORICS

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots\cdots$$

$$\boxed{\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3}$$

$$g(x) = \sum_{i=0}^{\infty} (i+1)\, x^i$$

$$G(x) = \sum_{i=0}^{\infty} a_i \cdot x^i$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = -\frac{1}{(1-x)^2}\,\frac{d}{dx}(-x)$$

$$= -\frac{1}{(1-x)^2}(-1)$$

$$= \frac{1}{(1-x)^2}$$

COMBINATORICS

$a_n = 2n+3 \quad n = 0, 1, 2.$

$G(x) = \sum_{n=0}^{\infty} a_n x^n$

$G(x) = \sum_{i=0}^{\infty} a_i x^i$

$= \sum_{n=0}^{\infty} (2n+3) \cdot x^n$

$= \sum_{n=0}^{\infty} 2n \cdot x^n + \sum_{n=0}^{\infty} 3 \cdot x^n$

$= 2 \sum_{n=0}^{\infty} n \cdot x^n + 3 \sum_{n=0}^{\infty} x^n$

$3(x^0 + x^1 + x^2 + x^3 \ldots)$

$3(1 + x + x^2 + x^3 \ldots) = \frac{3}{1-x}$

$2 \sum_{n=0}^{\infty} n \cdot x^n$

$= 2(0 \cdot x^0 + 1 \cdot x + 2 \cdot x^2 + 3 \cdot x^3)$

$= 2(x + 2x^2 + 3x^3 \ldots)$

$= 2x(1 + 2x + 3x^2 + 4x^3 \ldots)$

$= \frac{2x}{(1-x)^2}$

COMBINATORICS

$$\frac{2x}{(1-x)^2} + \frac{3}{(1-x)}\frac{(1-x)}{(1-x)}$$

$$= \frac{2x + 3 - 3x}{(1-x)^2}$$

$$G(x) = \frac{3-x}{(1-x)^2}$$

COMBINATORICS

$$\underset{a_0}{0}, \underset{a_1}{0}, \underset{a_2}{0}, \underset{a_3}{6}, \underset{a_4}{-6}, \underset{a_5}{6}, \underset{a_6}{-6} \ldots$$

$$G(x) = \cancel{a_0 x^0} + \cancel{a_1 x^1} + \cancel{a_2 x^2} + a_3x^3 + a_4x^4 + a_5x^5$$

$$G(x) = 6x^3 - 6x^4 + 6x^5 - \ldots$$

$$= 6x^3(1 - x + x^2 - x^3 + \ldots)$$

$$\boxed{G(x) = \frac{6x^3}{1+x}}$$

COMBINATORICS

a) $G(x) = \dfrac{x^4}{1-x}$

b) $G(x) = \dfrac{x^3}{1-x^2}$

$$G(x) = \frac{x^4}{1-x}$$

$$= x^4\left(\frac{1}{1-x}\right)$$

$$= x^4\left(1 + x + x^2 + x^3 + x^4 + \cdots\right)$$

$$= x^4 + x^5 + x^6 + x^7 \cdots\cdots$$

$$G(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4$$

$$\langle 0, 0, 0, 0, 1, 1, 1 \ldots\ldots \rangle$$

COMBINATORICS

$$G(x) = \frac{1}{1-x} + 3x^7 - 11.$$

$$= (1 + x + x^2 + x^3 + x^4 \cdots x^7 + x^8 + \cdots) + (3x^7 - 11)$$

$$= (1-11) + x + x^2 + \cdots (x^7 + 3x^7) + \cdots$$

$$= -10 + x + x^2 + x^3 \cdots 4x^7 \cdots$$

$a_0 = -10$ $\quad \forall a_i = 1 \, (i \neq 0, 7)$

$a_7 = 4.$

COMBINATORICS

$$G(x) = \frac{1}{3-x}$$

$$\left\langle \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^4} \cdots \right\rangle$$

$$= \frac{1}{3}\left(\frac{1}{1-x/3}\right)$$

$$= \frac{1}{3}\left[1 + x/3 + (x/3)^2 + (x/3)^3 + \cdots\right]$$

$$= \frac{1}{3} + \frac{x}{3^2} + \frac{x^2}{3 \cdot 3^2} + \frac{x^3}{3 \cdot 3^3} + \cdots$$

COMBINATORICS

$$G(x) = \frac{1}{1-x} + \frac{1}{1-ax}.$$

$2,\ 1+a,\ 1+a^2,\ 1+a^3 \ldots\ldots$

b) $7, 8, 9, 10 \ldots\ldots$

Closed form ?

$7, 8, 9, 10 \ldots$

$G(x) = a_0x^0 + a_1x^1 + a_2x^2 + a_3x^3 + \cdots$

$= 7 \cdot x^0 + 8 \cdot x + 9 \cdot x^2 + 10x^3 \ldots$

$= 7 + 8x + 9x^2 + 10x^3 + \cdots$

$= (1 + 6) + (6x + 2x) + (6x^2 + 3x^2) + (6x^3 + 4x^3) + \cdots = \dfrac{7 - 6x}{(1-x)^2}$

$= \underbrace{\left(6 + 6x + 6x^2 + 6x^3 + \cdots\right)}_{\frac{6}{1-x}} + \underbrace{1 + 2x + 3x^2 + 4x^3 + \cdots}_{\frac{1}{(1-x)^2}}$

$G(x) = \dfrac{6}{1-x} + \dfrac{1}{(1-x)^2}$

$= \dfrac{(1-x)\,6}{(1-x)} + \dfrac{1}{(1-x)^2}$

$= \dfrac{6 - 6x + 1}{(1-x)^2}$

COMBINATORICS

$$7 + 8x + 9x^2 + 10x^3.$$

$$\boxed{a_n = 2^n - 1}$$

$$7 + (7x + x) + (7x^2 + 2x^2) + (7x^3 + 3x^3) \ldots$$

$$(7 + 7x + 7x^2 + 7x^3 + \ldots) + (x + 2x^2 + 3x^3 + \ldots)$$

$$7(1 + x + x^2 + x^3 + \ldots) \qquad x(1 + 2x + 3x^2 + 4x^3 \ldots)$$

$$\frac{7}{1-x} \qquad \frac{x}{(1-x)^2} = \frac{(1-x)7}{(1-x)1-x} + \frac{x}{(1-x)^2}$$

$$= \frac{7 - 7x + x}{(1-x)^2} = \frac{7 - 6x}{(1-x)^2}$$

COMBINATORICS

$$a_n = 2^n - 1$$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} (2^n - 1) \cdot x^n$$

$$= \sum_{n=0}^{\infty} 2^n \cdot x^n - \sum_{n=0}^{\infty} 1 \cdot x^n$$

$$= 2^0 x^0 + (2x) + 2^2 x^2 - \frac{1}{(1-x)}$$

$$= \frac{1}{1-2x}$$

$$G(x) = \left(\frac{1}{1-2x}\right) - \left(\frac{1}{1-x}\right)$$


THANK YOU GW
SOLDIERS !