CS & IT ENGINEERING

Discrete maths
Graph Theory

Lecture No. 06

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TOPICS TO BE COVERED

01 Definition In Connectivity

02 Connected vs Disconnected

03 Range of Edges

04 Concepts of tree

05 Connectivity theorem

Connectivity in Graphs

walk: alternating sequences of vertices & edges.

R.V | R.E

1 e_2 2 e_3 5 e_3 2 e_6 3.

Trail: ——— " ———

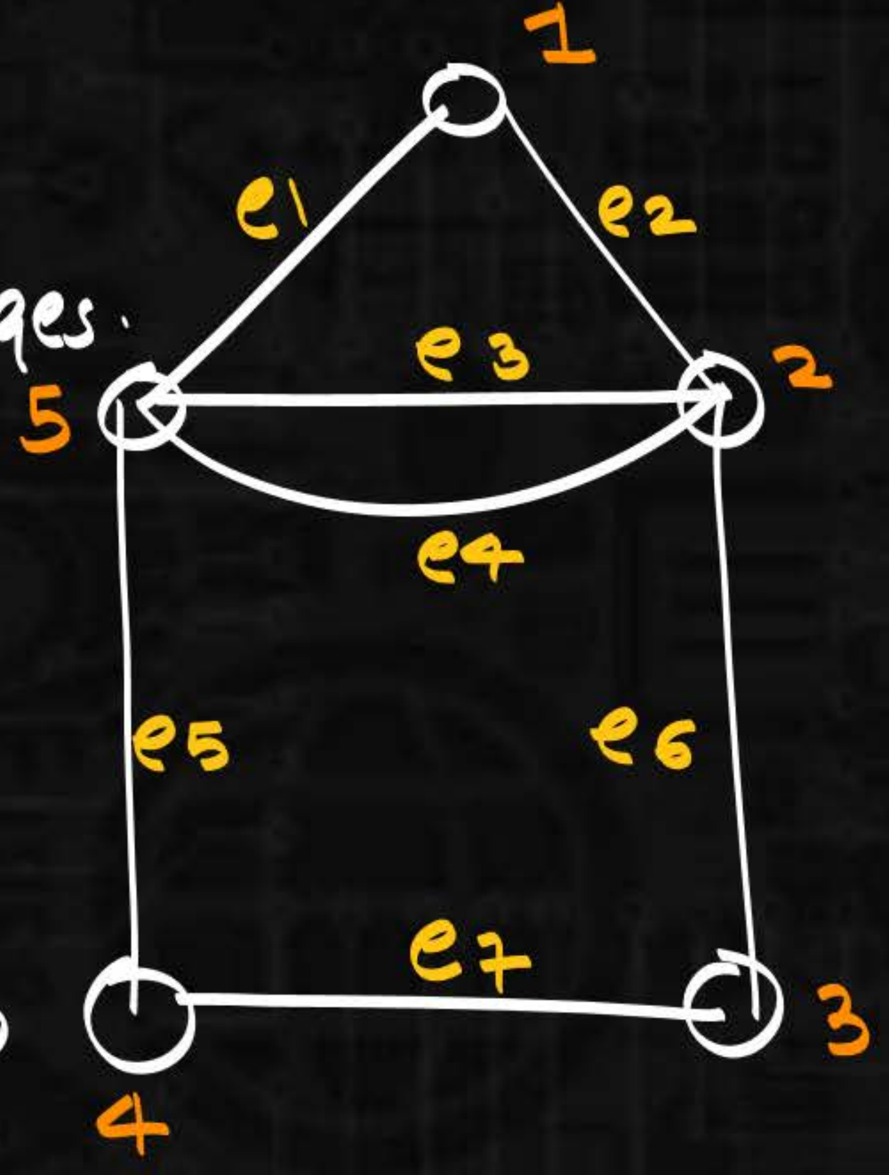
R.V | ~~R.E~~

1 e_2 2 e_3 5 e_4 2 e_6 3

Path: ——— " ———

~~R.V~~ | ~~R.E~~

1 e_2 2 e_6 3.



Connectivity in Graphs

→ 2 isolated.

when path is not available.
atleast 1. pair of vertices.

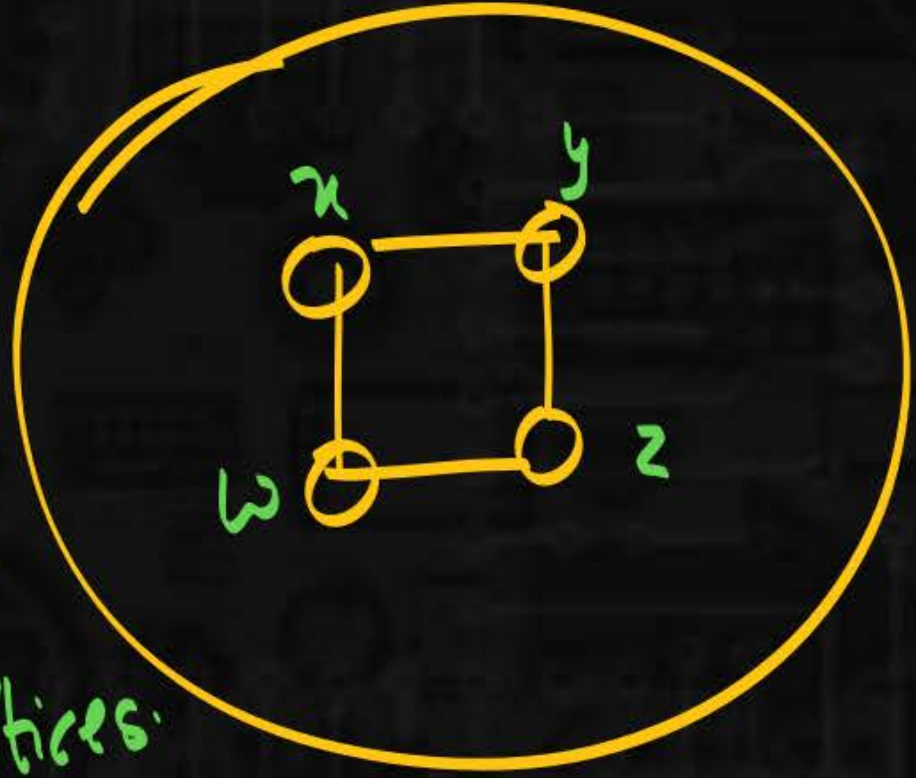
Graph:

Connected.

$G = (V, E)$

$|V| = 4$

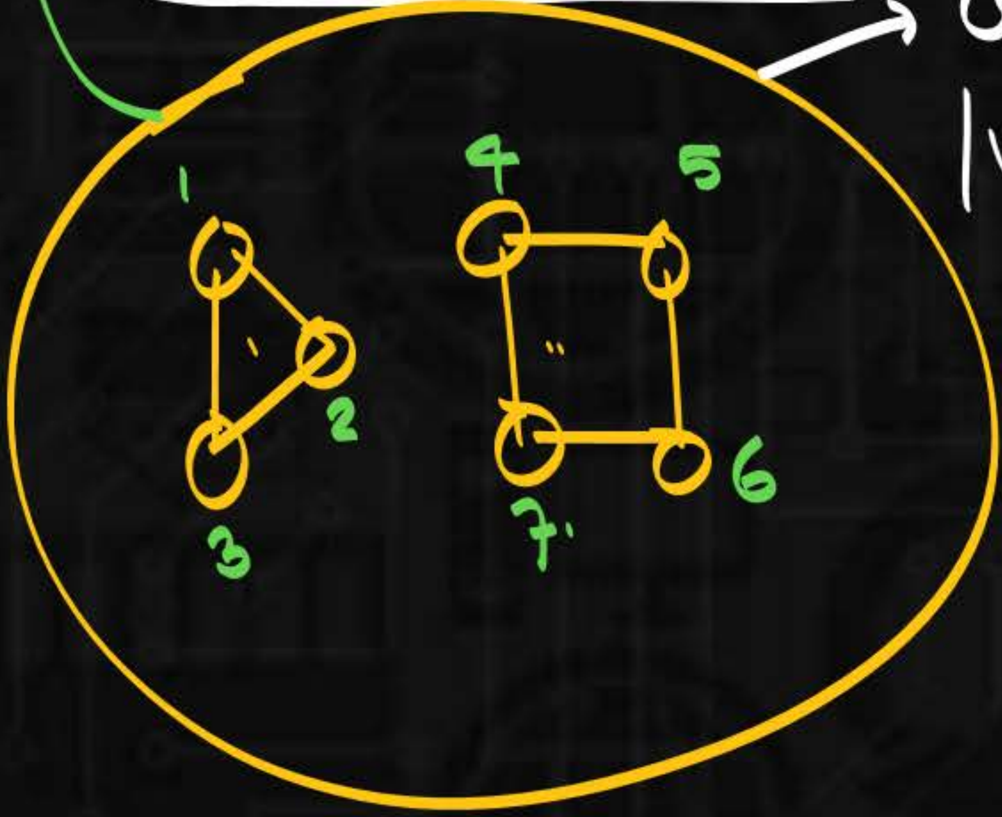
when path is available betn all pair of vertices.



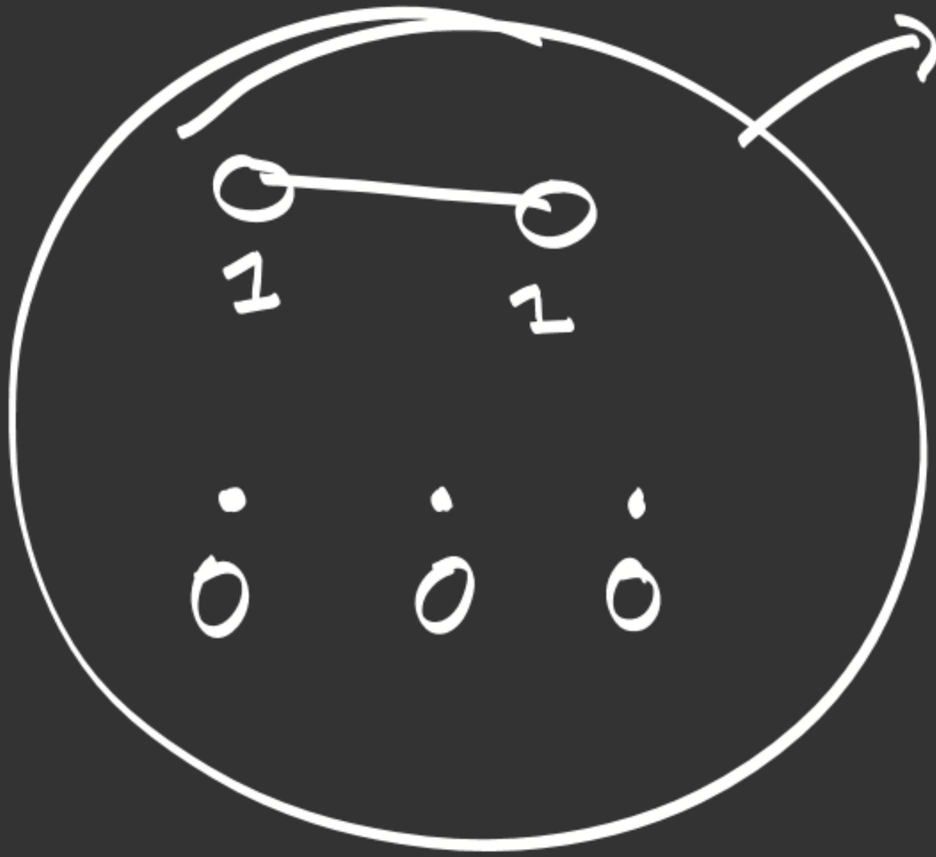
Disconnected.

$G = (V, E)$

$|V| = 7.$



1, 1, 0, 0, 0



Disconnected.

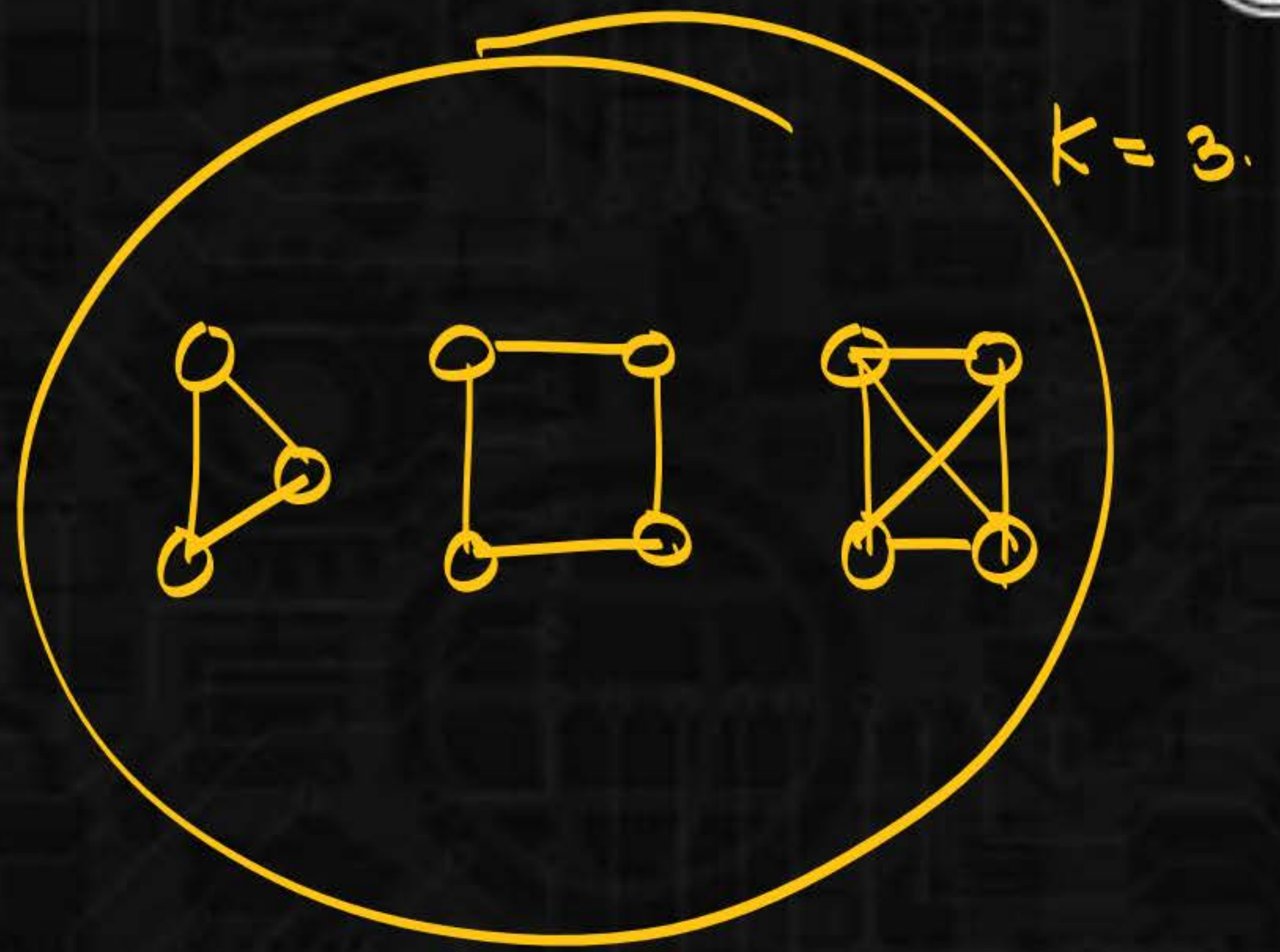
Connectivity in Graphs

Disconnected graph contains Connected subparts.

Component (K)

Connected. (K=1)

Disconnected (K≥2)



Connectivity in Graphs

k = 1. → connected. Graph.

k = 2.
D.C.

K = 4.

Connectivity in Graphs

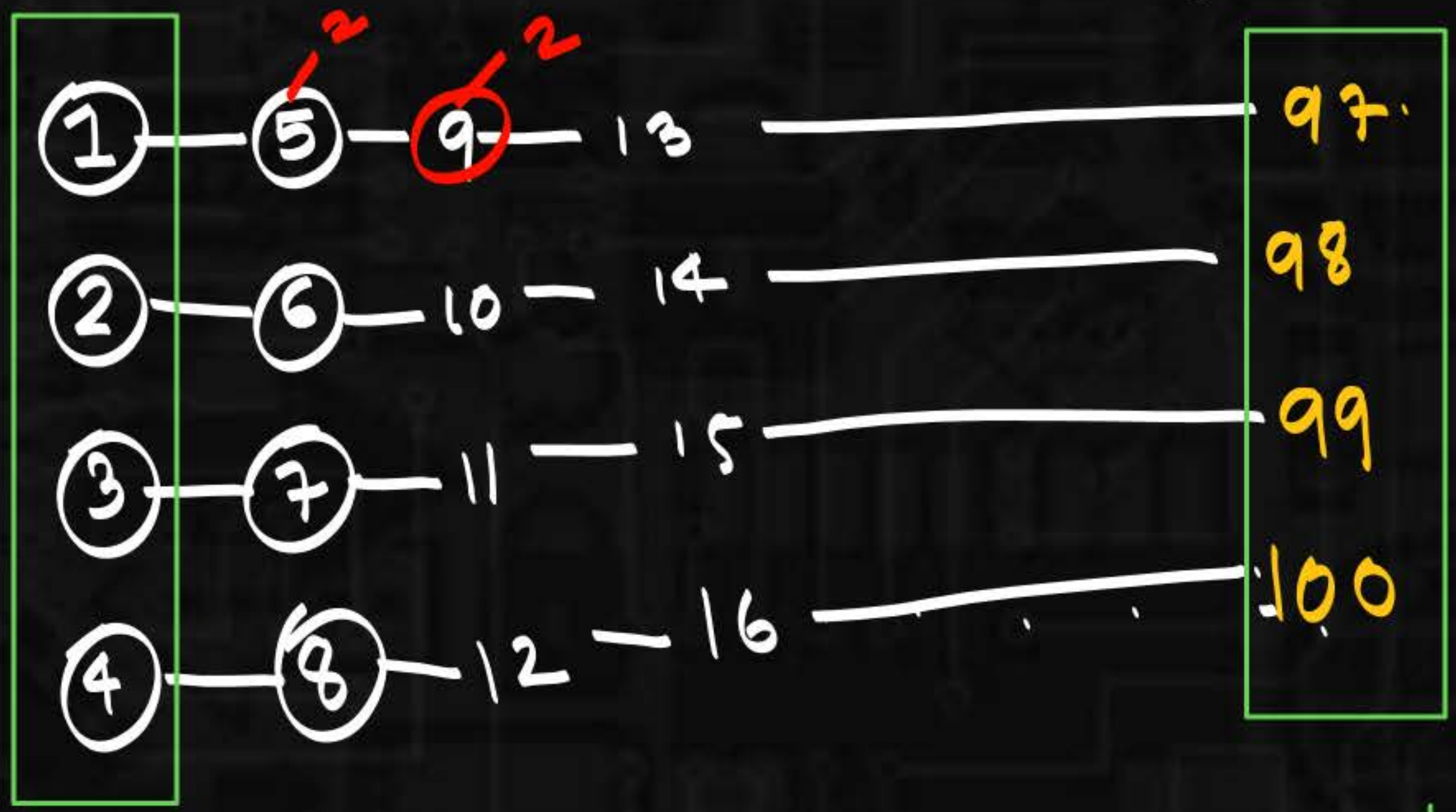
Consider a Graph where vertices are represented as $\{1 \ldots 100\}$ two vertices are connected with each other when $|x-y| = 4$, what will be no. of components?

$K = 4$

Total edges?

$4 \times 1 + 4 \times 1 + 92 \times 2 = 2e$

$e = 96$



Connectivity in Graphs

A is non empty set. $|A| = n$. (GATE)

Graph vertices are represented as elements of (powerset of A)

two vertices are connected if there intersection will give exactly 2 elements, what will be no. of components? ($n \geq 3$)

Connectivity in Graphs

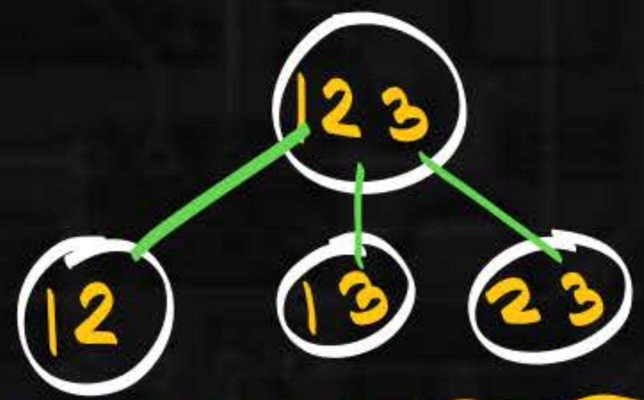
$A = \{1, 2, 3\}$. $|A| = n$.

Total no. of vertices $= 2^n$.

Total elements in $2^A = 2^n$

powerset $(P(A)/2^A)$: $\{\phi, \{1\}, \{2\}, \{3\} \ldots\ldots \{1, 2, 3\}\}$.

$\{1\,2\} \cap \{2\,3\} = \{1\,2\} \neq$



$n \leftarrow$ 1 2 3

$1. \leftarrow \phi$

$n+2$

Connection

$\bigcirc \cap \bigcirc = e_1 \cdot e_2$

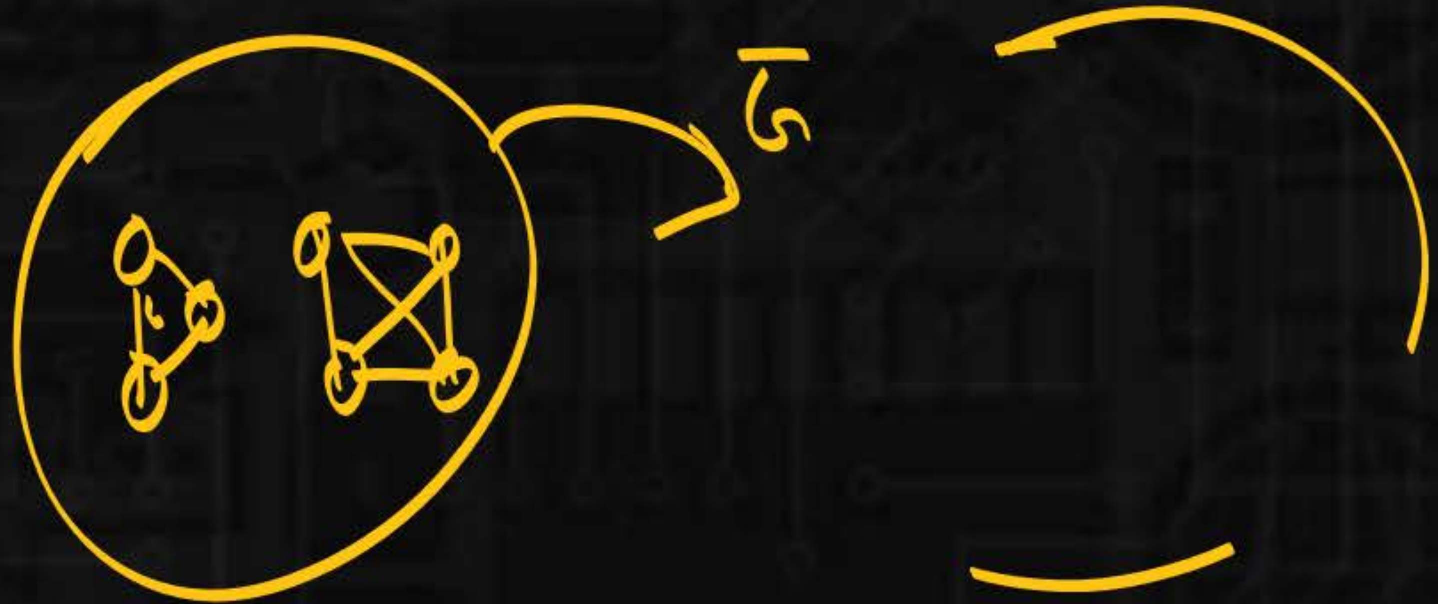
Connectivity in Graphs

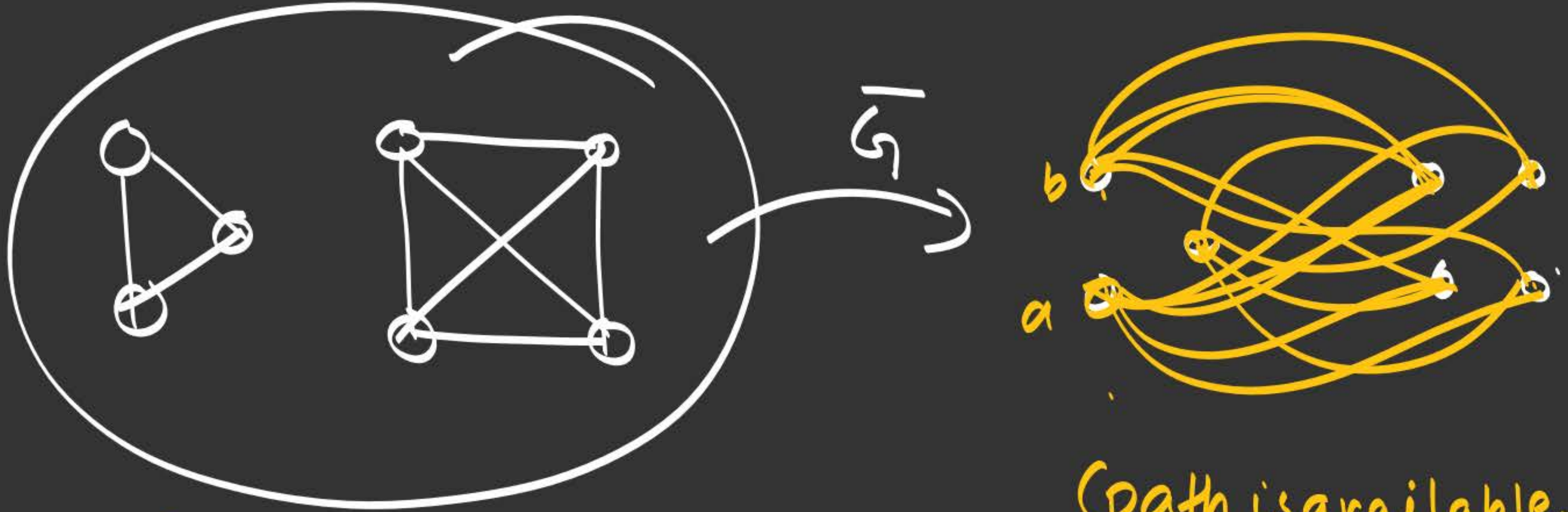
$k=2.$

S_1: if G is connected then $\bar{G}$ will also be connected (false)

S_2. if G is disconnected then $\bar{G}$ will be connected.

Assumption





$G = (V, E)$

$|V| = 7$

path is available.

all pair.

connected.

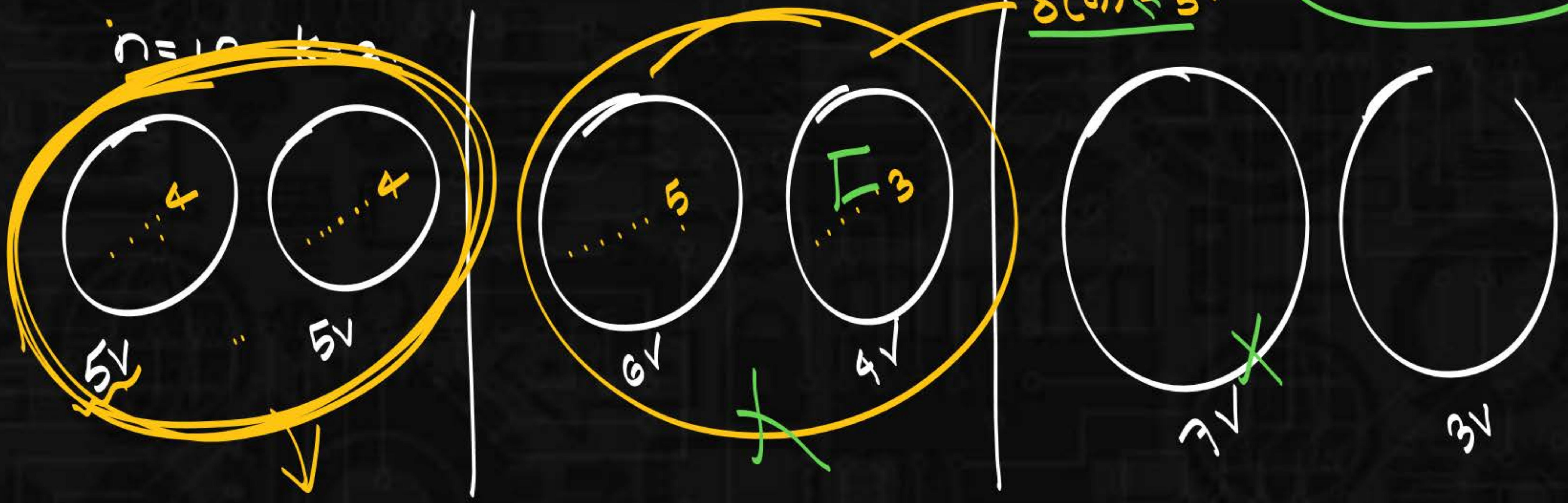
Connectivity in Graphs

If $\delta(G) \geq 5$ with $n = 10$ then G is connected. (True)

Assumption: this is Disconnected with $k = 2$.

$\delta(G) \leq 3$

no division is possible.



6V 4V

minimumum degree ≤ 3

$\delta(G) \geq 5$

Connectivity in Graphs

Thm: if $\delta(G) \geq \frac{n-1}{2}$ then it is connected. (viceversa is not True).

Connectivity in Graphs

C.
D. C.

Consider a Graph is having exactly 2 odd degree vertices

check path is available betⁿ those vertices ?

Graph
↓
Connected
↓

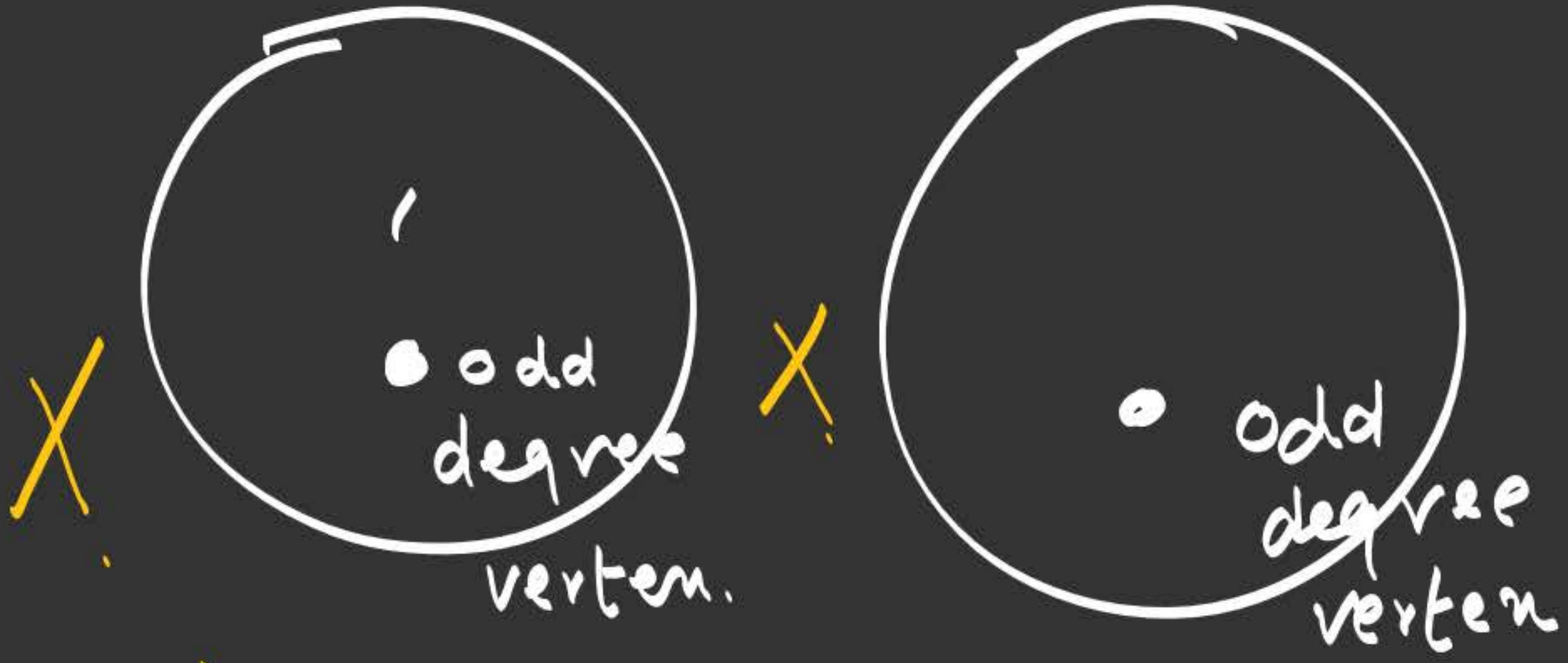
Graph → Disconnected

odd degree vertex

odd deg

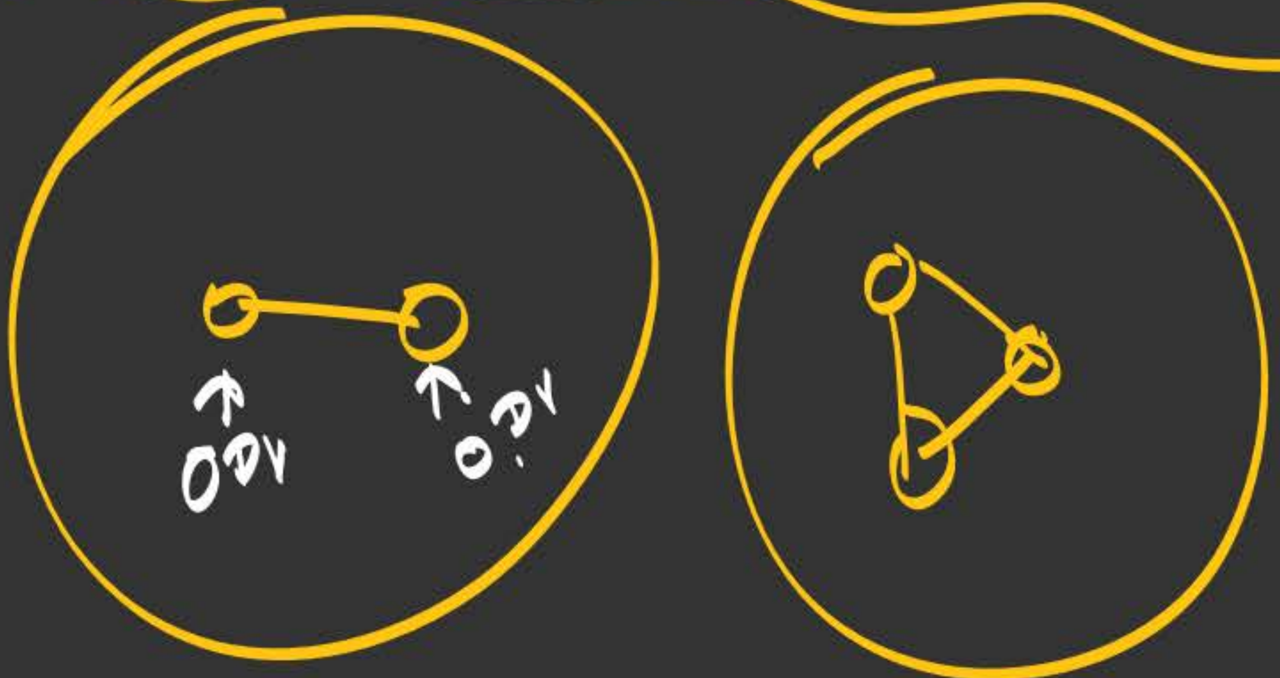
1st comp

2nd componen



Thm2:

no. of odd degree vertices should be even.



In Disconnected Graph.
2 exactly Odd degree vertex.
will be in same component (Thm2)
hence path is available betn them.

Connectivity in Graphs

Range of edges ($k=1$)

$1 \equiv 2 \equiv 3 \equiv 4$.

$G = (V, E)$

$\boxed{1 / 2 / 3 / 4}$

$\downarrow$

Tree

Tree

1. $\begin{cases} n-1 \leq e \leq \frac{n(n-1)}{2} \hookrightarrow (K_n) \\ + \\ \text{connected} \end{cases}$

2. connected acyclic Graph.

3. unique path betn all pair of vertices

4. minimally connected.

$n = 4 \quad e = 0$

x

$n = 4 \quad e = 1$

$n = 4 \quad e = 2$

$n = 4.$

$e = 3$

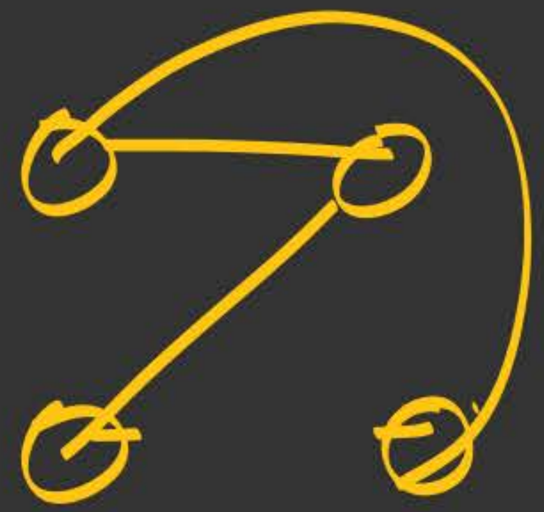
$e = 3 \ (n-1).$

→ no cy

$e = 3$

$n = 4. \quad \overset{n-1}{e = 3} \rightarrow$ connected

acyclic. Graph.



Connectivity in Graphs

Disconnected.

range of edges. $(K \geq 2)$

Forest $n-K \leq e \leq \dfrac{(n-K)(n-K+1)}{2}$

min.

max. no. of edges.

Forest:

Connected.

$e_1 = n_1 - 1$

Tree.

n_1 → min

$e_2 = n_2 - 1$

n_2 → min

$e_3 = n_3 - 1$

n_3 → min

$e_4 = n_4 - 1$

n_4 → min

→ min.

Total vertices = n.

1st component → n_1

2n ——— " —— → n_2.

component = ($K = 4$)

Total edges = $n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1$

$= (n_1 + n_2 + n_3 + n_4) - 4.$

$n - 4 = n - K.$

Forest = collection of Trees.

no. of edges in Forest = $n - K$.

Connectivity in Graphs

$G = (V, E)$ with 10 vertices & 3 components.

min & max no. of edges?

Ans: 7, 28


THANK YOU GW
SOLDIERS !