# CS & IT ENGINEERING

## Discrete Mathematics

### Graph Theory

Lecture No. 07

By- SATISH YADAV SIR

# TOPICS TO BE COVERED

01 Annalysis In Connectivity

02 Various definition in Connectivity

03 Edge Connectivity

04 Vertex Connectivity

05 Largest inequality theorem

# Connectivity in Graphs

$n = 7 \quad k = 2$

min no. of edges:

$e = n - k$

$= 7 - 2$

$= 5$

7v k = 2

3v 4v

5v 2v

6v 1v

⋮

**Case 1:** $n = 7$, $k = 2$.

e = 3 + e = 2 = e = 5

4v 3v

**Case 2:**

4e + 1e. = 5e

5v 2v

**Case 3:**

5e + 0e.

6v 1v

⋮ = 5e.

$\frac{\cancel{4}^{2}\times 3}{\cancel{2}} + \frac{3\times\cancel{2}}{\cancel{2}} = 9e.$

Case 1:

K4    K3

4V    3V

$\frac{6.5}{2} + \frac{1.0}{2} = 15e$

6V    1V

$n = 7 \quad k = 2$

$e = (n-k)(n-k+1)/2$

$= (7-2)(7-2+1)/2$

$= 5.6/2$

$= 15e.$

7V k = 2

4V 3V

5V 2V

$\boxed{6V \; 1V}$

3V 4V

$\frac{5.4}{2} = 10 + \frac{2.1}{2} = 1 = 11$

5V    2V

4V   3V

↓    ↓

6e + 3e = 9e.

5V   2V

↓    ↓

10 + 1 → 11e

# Connectivity in Graphs

$n = 10v \quad k = 2 \longrightarrow$ $K_1$ $K_9$

max edges.

$n = 12v \quad k = 2 \longrightarrow$ $K_1$ $K_{11}$

$n = 10v \quad k = 3.$ $\longrightarrow$ $K_1$ $K_1$ $K_8$.

min will be same for all cases.

max no. of edges

$e = 28$

$n = 12 \quad k = 3$ $K_1$ $K_1$ $K_{10}$

# Connectivity in Graphs

$e = (n-k)(n-k+1)/2 \rightarrow K_1 \; K_1 \; K_1 \; K_{97}$

Consider a Graph with $\boxed{\text{100 vertices}}$ with $\boxed{k=4}$ what will max no. of edges?

1st component → 25V

2nd —— " —— → 30V

3rd —— " —— → 20V

4th —— " —— → 25V

25V

30V

20V

25V

$$\frac{25 \cdot 24}{2} + \frac{30 \cdot 29}{2} + \frac{20 \cdot 19}{2} + \frac{25 \cdot 24}{2} = 1225$$

# Connectivity in Graphs

$K=2$   $K_1$   $K_9$

what will be max no of edges in disconnected graph with 10 vertices?

$(K \geq 2)$

$K=2$

$e = (n-k)(n-k+1)/2$

$= (10-2)(10-2+1)/2$

$= 8 \cdot 9/2$

$= 36$

$K=3$

$e = (10-3)(10-3+1)/2$

$= 7 \cdot 8/2$

$= 28$

$K=4$

$e = (10-4)(10-4+1)/2$

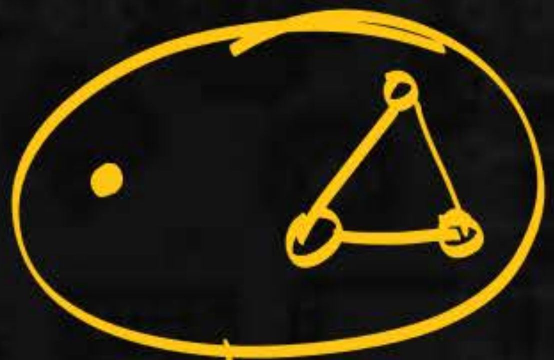
$= 6 \cdot 7/2$

$= 21$

# Connectivity in Graphs

what will be complement of disconnected graph with 10 vertices & max no. of edges.

eg: $K_1$ $K_3$.



↓

$K_{1,3}$

$K_1$ $K_9$.

↓

$\rightarrow K_{1,9}$.

$K_{1,n-1}$ if we take complement of this.

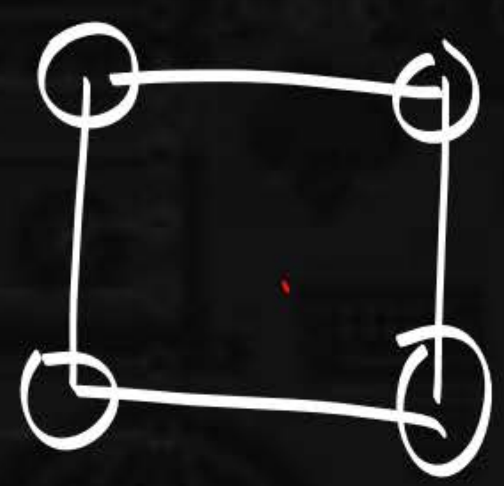
$K_1$ $K_{n-1}$

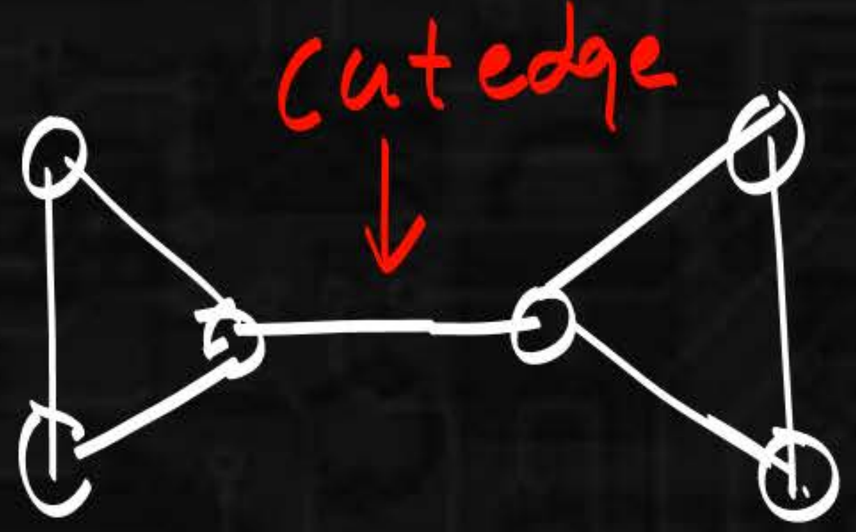
complement

$K_{1,n-1}$

# Connectivity in Graphs

Cut edge/bridge → Removal of single edge from a graph will make graph as a disconnected graph.



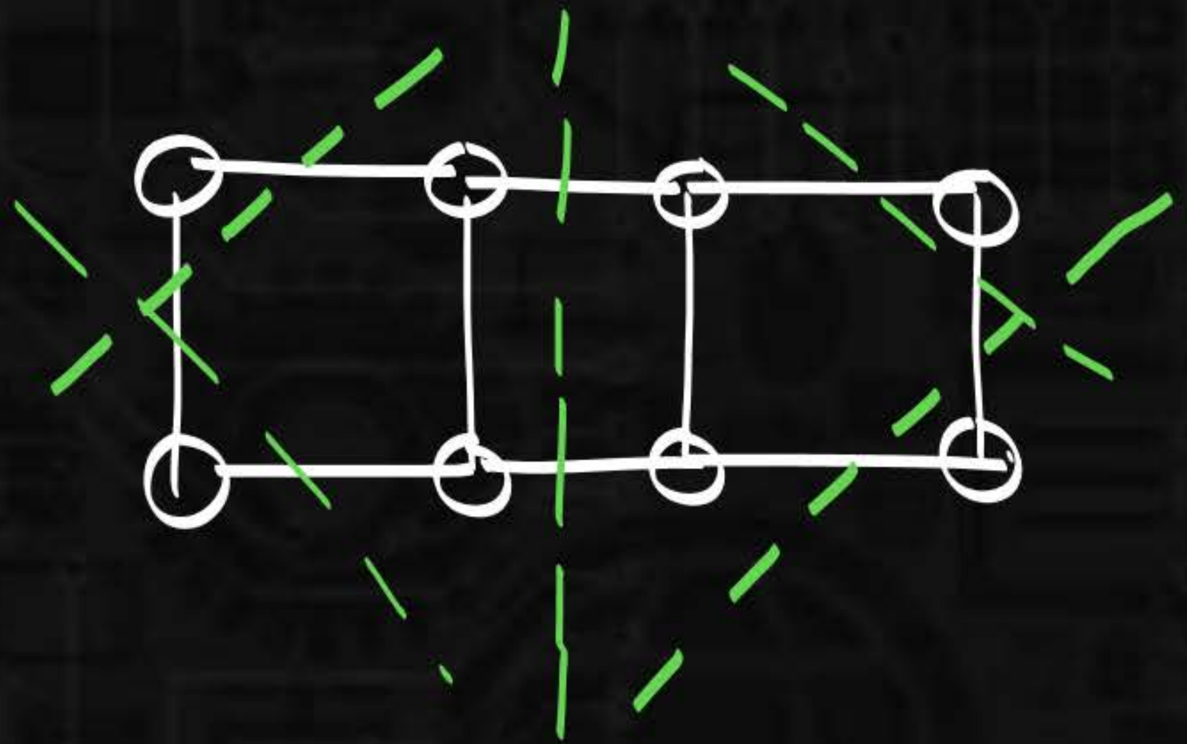



* if cut edge exist then it does not belongs to cycle.

# Connectivity in Graphs

## Cut edge set/cut set:

Removal of set of edges from a graph will make graph as a disconnected.



diff types → diff sizes.

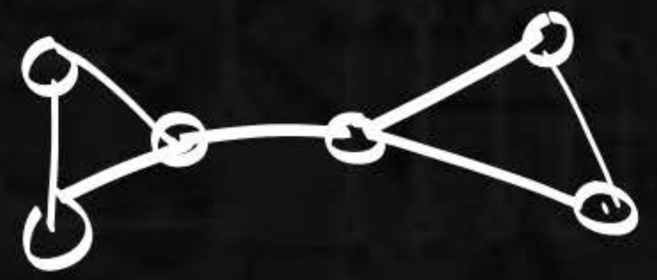
↓

min. sizes.

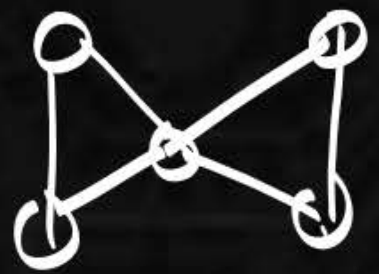
# Connectivity in Graphs

**edge connectivity ($\lambda(G)$):**

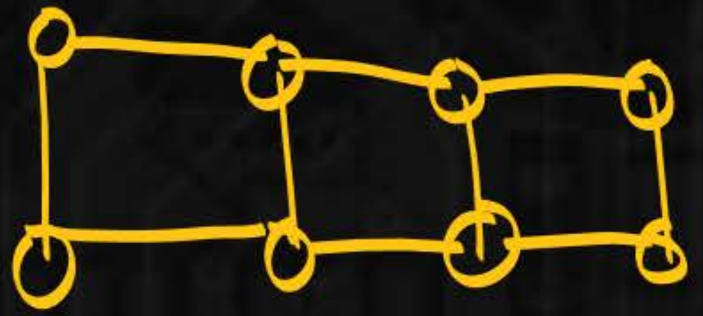
Removal of min no. of edges will make graph as a disconnected graph.



$\lambda(G) = 1$.
$\downarrow$
bridge.



$\lambda(G) = 2$.

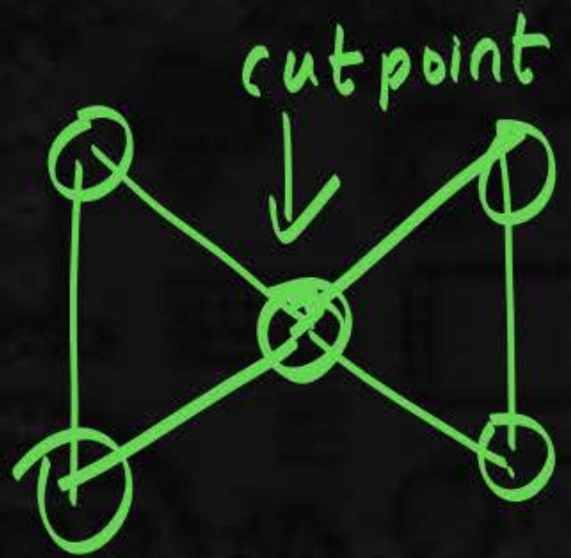


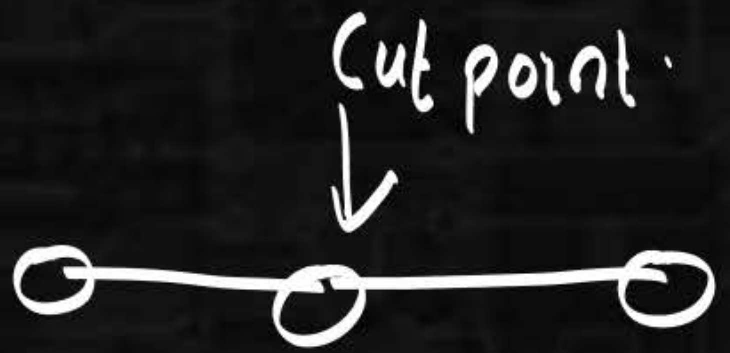
$\lambda(G) = 2$.

# Connectivity in Graphs

Cut vertex / cut point / Articulation point:

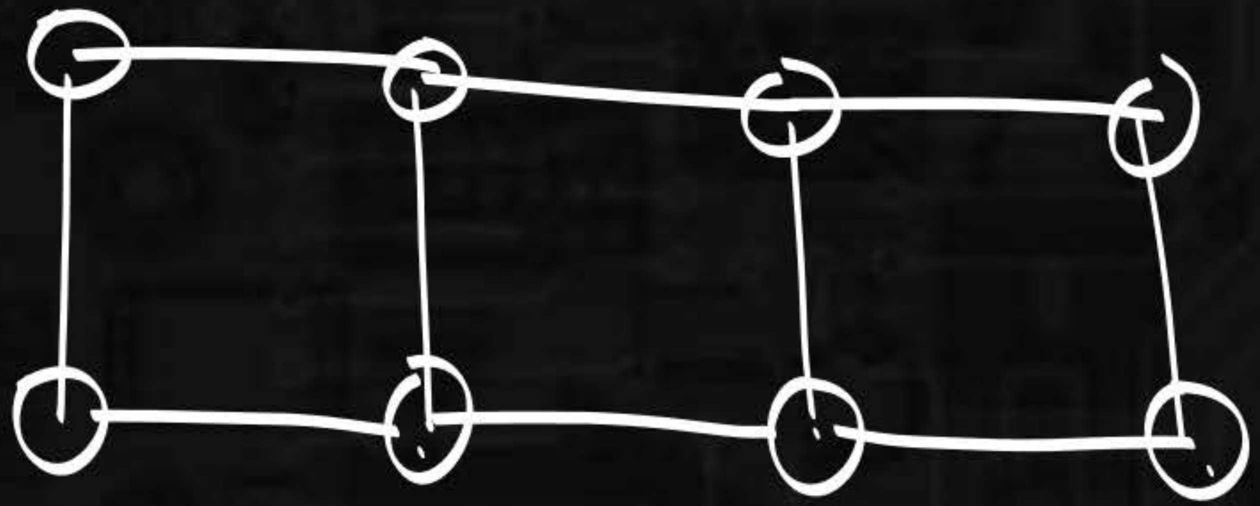
Removal of single vertex from a graph will make graph as a disconnected graph.



# Connectivity in Graphs

<u>Cut vertex set</u>: Removal of set of vertices from a graph will make graph as a disconnected graph.



# Connectivity in Graphs

**Vertex connectivity** $(K(G))$ → Removal of min. no. of vertices from a graph will make graph as a disconnected graph.

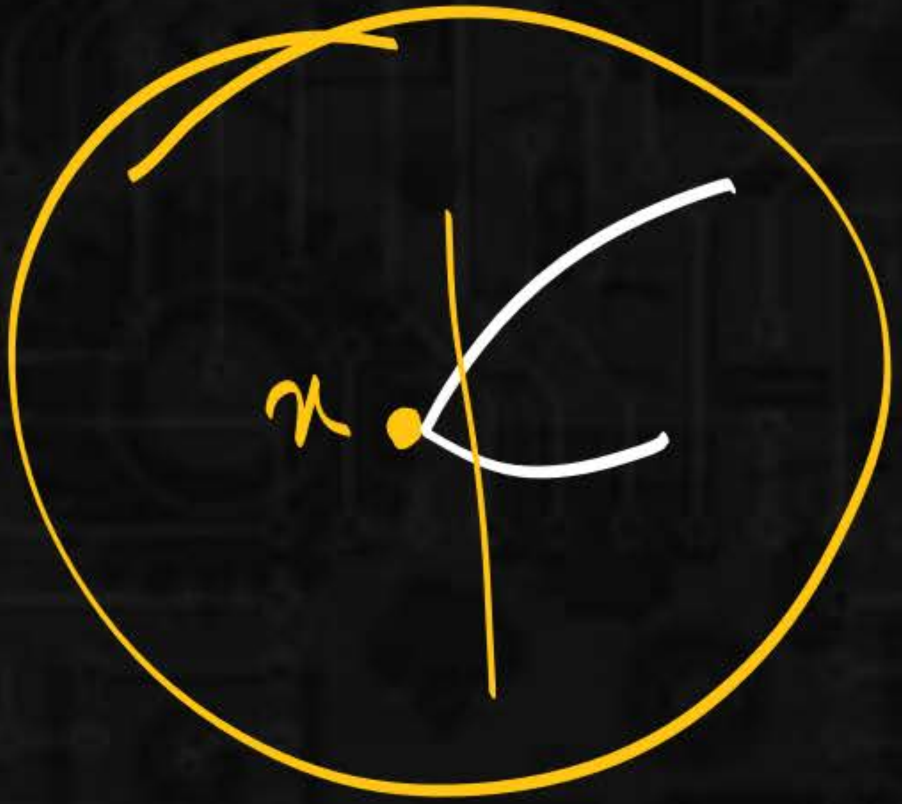
$K_{3,4}$ $\quad \lambda(G) = 3$

$\quad\quad\quad K(G) = 3$

$K_{m,n}$ $\quad \lambda(K_{m,n}) = \min\{m,n\}$

$\quad\quad\quad K(K_{m,n}) = \min\{m,n\}$

# Connectivity in Graphs

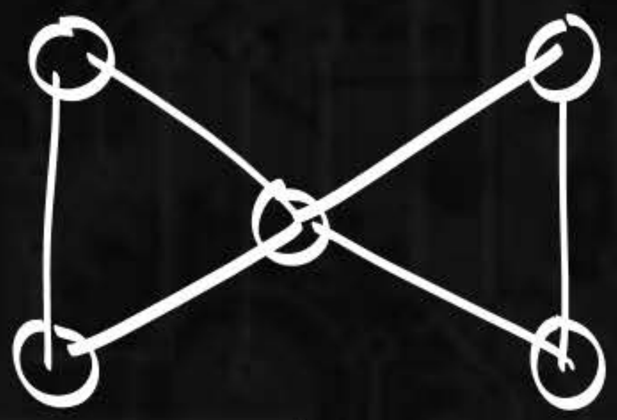



min degree = min ~~no.~~ of edges.

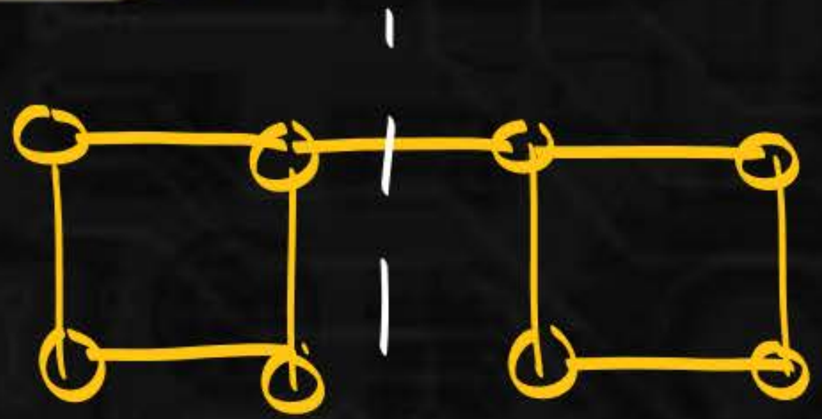
$\lambda(G)$

$\lambda(G) = \delta(G)$.

# Connectivity in Graphs



$\lambda(G) = \delta(G)$



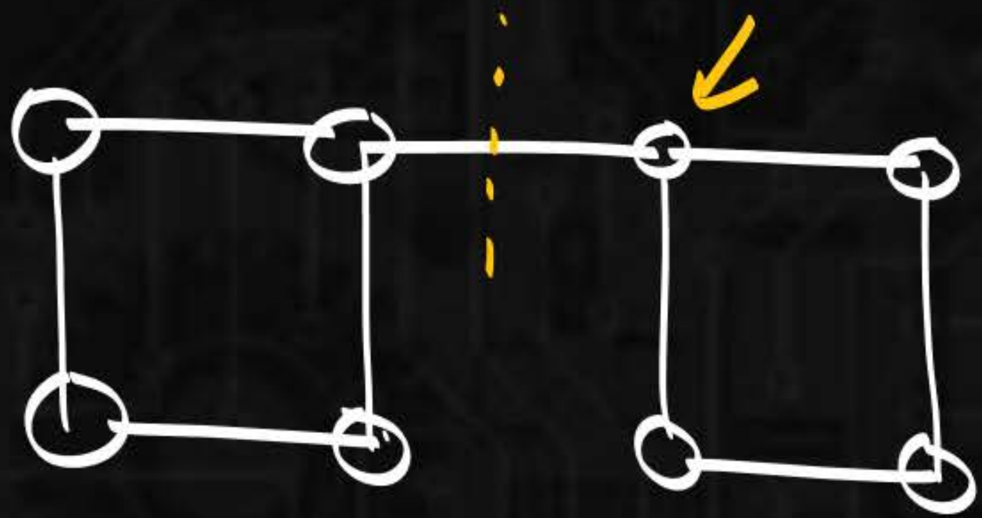
$\delta(G) = 2.$

$\lambda(G) = 1.$

$\lambda(G) < \delta(G)$

$$\boxed{\lambda(G) \leq \delta(G)}$$

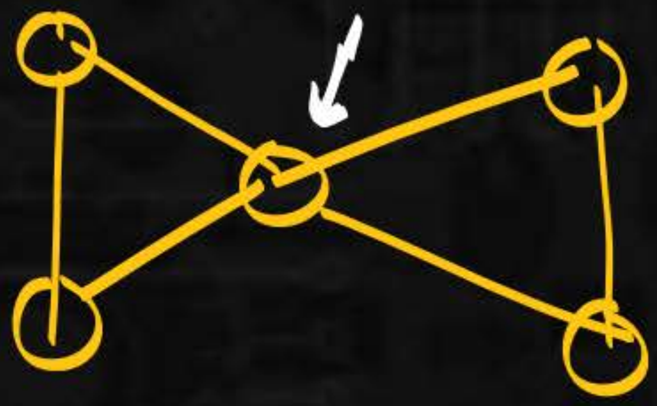
# Connectivity in Graphs



$K(G) = 1.$

$\lambda(G) = 1.$

$\boxed{K(G) = \lambda(G)}$



$K(G) = 1.$

$\lambda(G) = 2.$

$\boxed{K(G) < \lambda(G)}$

$\boxed{K(G) \leq \lambda(G)}$

# Connectivity in Graphs

$\lambda(G) \leq \delta(G)$

$K(G) \leq \lambda(G)$

$K(G) \leq \lambda(G) \leq \delta(G)$

$K(G) \leq \lambda(G) \leq \delta(G) \leq 2e/n \leq \Delta(G) \leq n-1$.

Thms:

$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1$.

# Connectivity in Graphs

G is connected graph with 10 vertices and vertex connectivity 3

min. no. of edges ?

## Connectivity in Graphs

Consider a graph vertices are represented as one of the possibility of linear arrangement of 3 elements, 2 vertices are connected with each other if there adjacent elements are interchanged

in the vertices.

$\lambda(G) = ?$

$K(G) = ?$

# Connectivity in Graphs


$A = \{1, 2, 3\}$

123
132
213
231
312
321

Total vertices $= 3!$

$\lambda(G) = 2$

$K(G) = 2$

# Connectivity in Graphs


THANK YOU GW
SOLDIERS !