CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 4

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TOPICS TO BE COVERED

01 onto Functions

02 1:1 correspondance Functions

03 Number of Functions

04 Types of Functions

05 Various Examples in Functions

$|A| = n.$

Reflexive :

$\{ \boxed{11 \quad 22 \quad 33} \}$

$\hookrightarrow$ min size $= n.$

maxsize $\rightarrow n^2.$

$\{ \boxed{11 \quad 22 \quad 33} \; 12 \quad 21 \quad 13 \quad 31 \quad 32 \quad 23 \}$

Symmetric :

$R_1 = \{ \}$ $|R_1| = 0$

$\overbrace{\overbrace{11 \quad 22}^{n} \quad 33 \quad \underbrace{12 \quad 21 \quad 13 \quad 31 \quad 23 \quad 32}_{n^2-n}}^{n^2}$

$0 \ldots n^2$

Antisymmetric :

$R_1 = \{ \}$ $|R_1| = 0$

$\{ \overbrace{11 \quad 22 \quad 33}^{n} + \overbrace{12 \quad 13 \quad 23}^{\frac{n^2-n}{2}} \}$

$n + \frac{n^2-n}{2}$

Asymmetric :

$$\forall a \forall b \left[(a,b) \in R \rightarrow (b,a) \notin R \right]$$

Asymmetric.
does not allows
same element.
→ no flipping.

$R_1 = \{ \}$

$\underbrace{(a,b) \in R}_{F} \rightarrow (b,a) \notin R.$

$F \rightarrow$

T.

$R_2 = \{ (1\,1) \}$

$(a,b) \in R \rightarrow (b,a) \notin R.$

$\underbrace{(1,1) \in R}_{T} \rightarrow \underbrace{(1,1) \notin R}_{F}.$

a=1
b=1

$R_3 = \{ (1\,2) \}$

$(a,b) \in R \rightarrow (b,a) \notin R.$

$\underbrace{(1,2) \in R}_{T} \rightarrow \underbrace{(2,1) \notin R}_{T}.$

a=1
b=2

$R_3 = \{(1\,2)\ (2\,1)\}$ X.

$(a, b) \in R \rightarrow (b, a) \notin R$

$\underbrace{(1, 2) \in R}_{T} \rightarrow \underbrace{(2, 1) \notin R}_{F}$.

$\boxed{\underbrace{(1\,2)}_{P}\ \underbrace{\overline{(2\,1)}}_{P}}$ absent

11 12 13

21 22 23

31 32 33 → absent

boxes → 3 choices

$n^2 - n/2$.

$\# Asy = 3$

Symmetric	Antisymmetric	Asymmetric
$(a,b)\in R \rightarrow (b,a)\in R$	$(a,b)\in R \wedge (b,a)\in R \rightarrow a=b.$	$(a,b)\in R \rightarrow (b,a)\notin R.$
→ allow same element	→ allows same element	→ no same.
→ Demands flipping.	→ no flipping.	→ no flipping.
2^n. $2^{\frac{n^2-n}{2}}$.	2^n. $3^{\frac{n^2-n}{2}}$.	$3^{\frac{n^2-n}{2}}$

IRREFLEXIVE

$A = \{1, 2, 3\}$

$\forall a \in A \ (a, a) \notin R$

$R_1 = \{\ \}$

IRR ✓

$R_2 = \{(11)(12)\}$

not Irreflexive

not reflexive

$R_3 = \{(12)(21)(22)\}$

not Irreflexive

11	12	13
21	22	23
31	32	33.

absent

1 choice.

Diagonal = n

non diagonal = $n^2 - n$

2 choices

$$\# IRR = 2^{n^2 - n}$$

$\#R = 2^{n^2-n}$

11	22	33
11	22	33

(1 2)

(12) (21)

$\#IRR.$

absent
absent:

(12)

(12) (21)

$= 2^{n^2-n}$

	Ref	Symm	Anti	Asymmetric	IRR
Ref	×				
Sym					
Anti					
Asy					
IRR					

Transitive

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$R_1 = \{\ \}$

Transitive ✓

$R_2 = \{(1,1)\}$ ✓

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$(1,1) \in R \wedge \boxed{(1,\ )}$

F

$a = 1$

$b = 1$

$R_3 = \{(1,2)(2,1)(1,1)\}$ ✓

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$(1,2) \in R \wedge (2,1) \in R \rightarrow (1,1) \in R$

$R = \{(1\,2)\,(2\,1)\,(1\,1)\}$. ✗

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$(1,2) \in R \wedge (2,1) \in R \rightarrow (1,1) \in R$

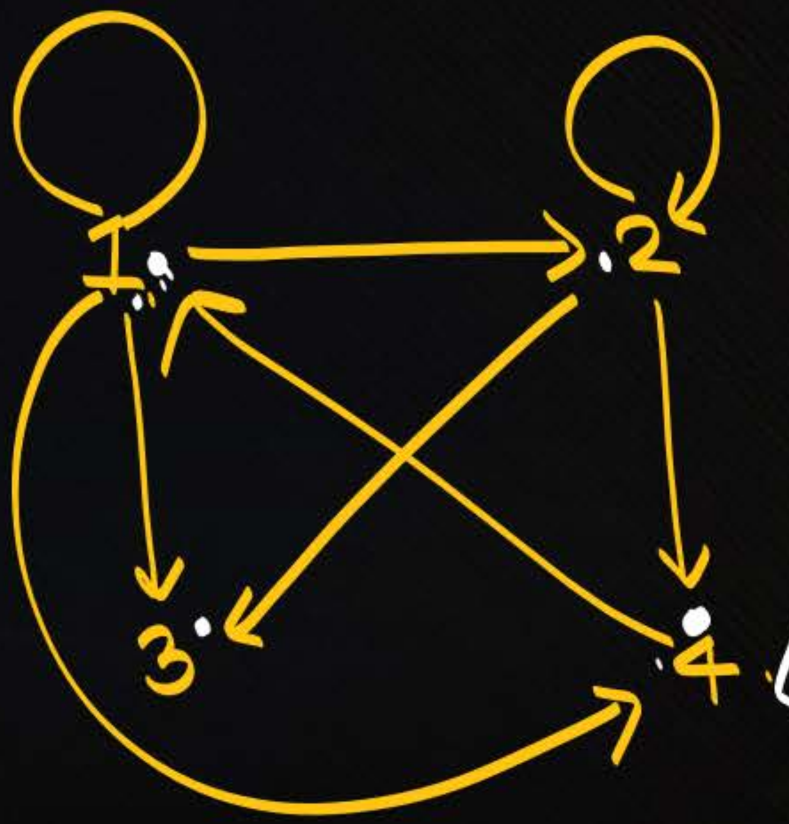
$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\underline{(2,1) \in R} \wedge \underline{(1,2) \in R} \rightarrow \underline{(2,2) \in R}$.

$R_1 = \{(1\,2)\,(2\,1)\,(1\,1)\,(2\,2)\}$ ✓

$A = \{1, 2, 3, 4\}$.

$R = \{ (1\,2)\ (2\,3)\ (1\,3)$
$(2, 4)\ (2, 2)$
$(1, 4)\ (1, 1)$
$(4, 1) \}$.



R'

Stand on 1.

(1, 1) (1, 2) (1, 3) (1, 4)

Stand on 2

(2, 2) (2, 4) (2, 3) (2, 1)

Stand on 3 :

Stand on 4 :

(4, 1) (4, 2) (4, 3) (4, 4)

if $R = R'$ then it is transitive OR not

A =

A × A = .

check.

Result : { ...... }

Z :

Z × Z :

$a = b + 1$

check

Result : { ... }

$R_1 = \{ (a, b) \mid \underline{a = b+1} \}$

$\underline{R}$: aRa $\quad a = a+1$
(False)

$\underline{\text{Sym}}$: $aRb \rightarrow bRa$
$\quad \quad \underline{\quad T \quad} \quad \quad \underline{\quad F \quad}$

X. $a = b+1 \rightarrow b = a+1$

$2 = 1+1 \quad 1 \neq 2+1$

$\underline{(2,1) \in R} \rightarrow (1,2) \notin R$

2×2

$(\underline{a}, b)$

$a=1$ $\quad (1,2)$
$b=2$

$a = b+1$

$1 \neq 2+1$

$a = b+1$

$(1,0)$

$\{ (2,1) \; (3,2)$

$(4,3)$

$(a,b) \in R \rightarrow (b,a) \in R$

$\underline{(2,1) \in R} \rightarrow (1,2) \in R$

T.

$aRb \wedge bRc \rightarrow aRc.$

$$\overset{T}{a = b+1} \wedge \overset{T}{b = c+1}. \rightarrow \overset{F.}{\boxed{a = c+1}}$$

$a = b+1 \quad b = c+1$

$= (c+1)+1$

$a. = c+2$

$R_1 = \{(a,b) \mid a \leq b\}$. $\rightarrow$ not sym. | Transitive

$R_2 = \{(a,b) \mid a+b \leq 3\}$ Sym. | not transitive.

$R_3 = \{(a,b) \mid gcd(a,b)=1\}$. Sym. | not transitive.

$aRb \wedge bRc \rightarrow aRc.$

$a+b \leq 3 \wedge b+c \leq 3 \rightarrow a+c \leq 3.$

$(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$

$(3,0) \in R \wedge (0,3) \in R. \rightarrow (3,3) \in R.$

$3+0 \leq 3 \quad 0+3 \leq 3 \quad \underline{3+3 \leq 3.}$

F

$R = \{ (A, B) \mid A \subseteq B \}$.

R: ARA $\quad A \subseteq A$ ✓

Sy: $ARB \longrightarrow BRA$.

$A \subseteq B \longrightarrow B \subseteq A$

$\{1\} \subseteq \{12\} \rightarrow \{12\} \subseteq \{1\}$

$(\{1\}, \{12\})$

Ant: $ARB \wedge BRA \longrightarrow A = B$.

$A \subseteq B \wedge B \subseteq A \longrightarrow A = B$. (equal set)

Tr: $A \subseteq B \wedge B \subseteq C \longrightarrow A \subseteq C$.

$R = \{ (l_1, l_2) \mid l_1 \parallel l_2 \}$

$z : \{ \ldots\ldots \}$

$z \times z = \{ \ldots\ldots \}$

Result

$z^2 : \{ (\ ), (\ ), (\ ) \ldots \}$

$z^2 \times z^2 = \{ ((\ ), (\ )), ((\ ), (\ )) \ldots$

$$R = \{ (a,b) R (c,d) \mid ad = bc \}$$

$\underline{R}$: $(a,b) R (a,b)$ $\quad a \times b = b \times a$ ✓

$\underline{Sym}$: $(a,b) R (c,d) \rightarrow (c,d) R (a,b)$

$$ad = bc \longrightarrow cb = d \cdot a.$$

$$bc = ad.$$

$(a,b) R (c,d) \mid ad = bc$

$(1,2) R (3,4)$

$1 \times 4 = 2 \times 3$

$(a,b) R (c,d) \wedge (c,d) R (x,y) \rightarrow (a,b) R (x,y)$

$a\boxed{d} = bc. \quad \wedge \quad cy = dx. \quad \longrightarrow \quad ay = bx.$

$\frac{cy}{x} = d.$

$a \times \left[\frac{\cancel{c}y}{x}\right] = b\cancel{c}.$

$ay = bx.$

$$\{(p,q)\,R\,(r,s) \mid p-s = q-r\} \quad (R/Sy)$$


THANK YOU GW
SOLDIERS !