CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No.[redacted]
05

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

D: {1, 2, 3}

P(x): x + 1 = 4

x = 1   1+1=4 (F) P(1)

x = 2   2+1=4 (F)

x = 3   3+1=4 (T)

D: {1, 2, 3}

Q(x): 2x + 1 = 7

x = 1   x = 2   x = 3

F   F   T

$\exists x \to \vee$

$\forall x \to \wedge$

D: {1, 2, 3}

$\exists x\,[P(x) \wedge Q(x)]$

True

x = 1 $\boxed{\overset{F}{P(1)} \wedge \overset{F}{Q(1)}}$ F

$\vee$

x = 2 $\boxed{\overset{F}{P(2)} \wedge \overset{F}{Q(2)}}$ F

$\vee$

x = 3 $\boxed{\overset{T}{P(3)} \wedge \overset{T}{Q(3)}}$ T

True

1 2 3   1 2 3

T $\wedge$ T

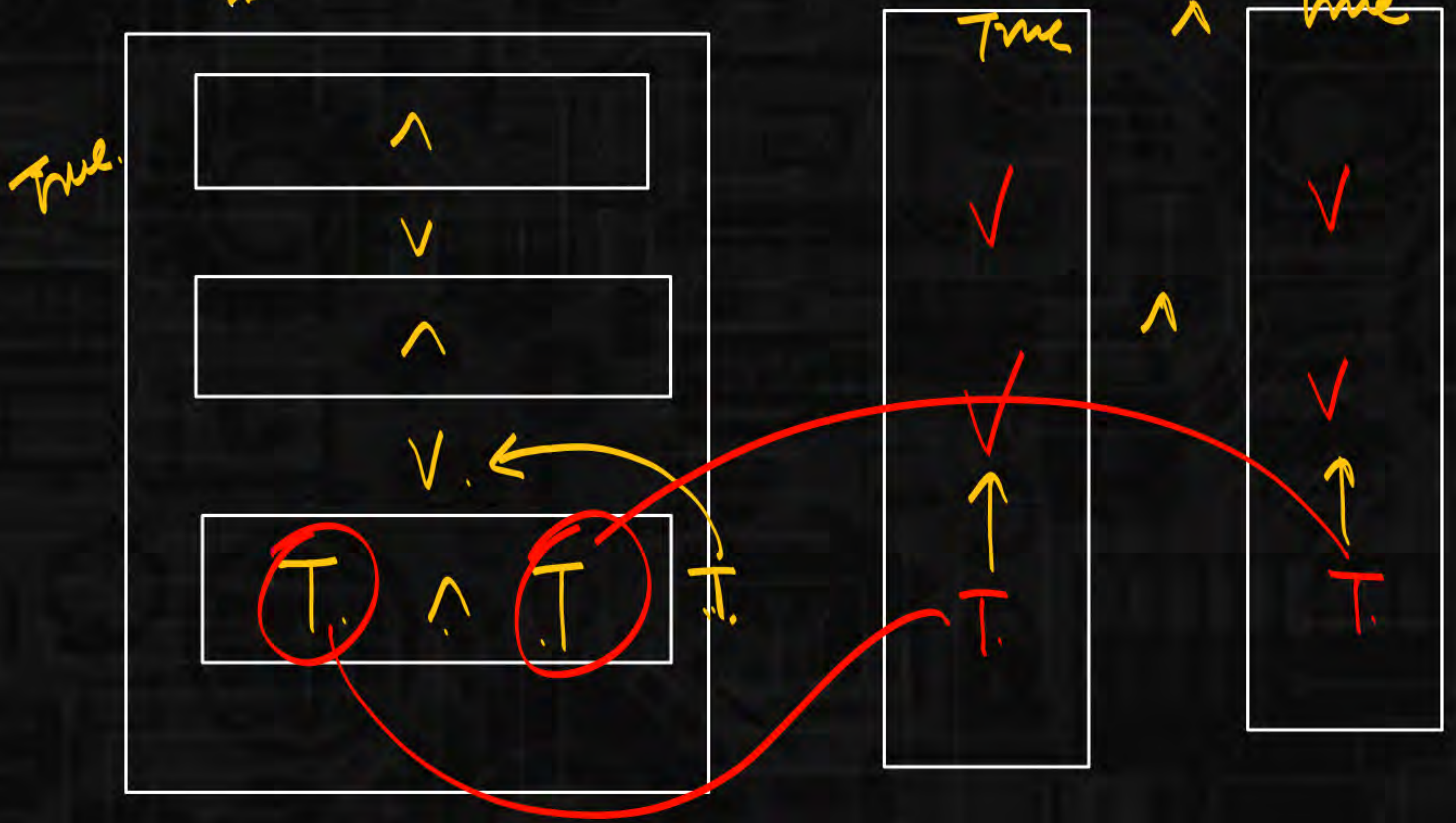
$\exists x P(x) \wedge \exists x Q(x)$

P(1) F
$\vee$
P(2) F
$\vee$
P(3) T

$\wedge$

Q(1) F
$\vee$
Q(2) F
$\vee$
Q(3) T

$$\overset{\text{True}}{\exists x\,[P(x) \wedge Q(x)]} \longrightarrow \exists x P(x) \wedge \overset{\text{True.}}{\exists x Q(x)} \text{ (valid)}$$



D ×

O.S ×

I.R ✓

True

$$\exists x\, P(x) \land \exists x\, Q(x) \not\Rightarrow \exists x\,[P(x) \land Q(x)]$$

$$\begin{array}{ccc} T & & F \\ \lor & \land & \lor \\ F & & T \end{array}$$

True True

$$T \land F \rightarrow F$$

$$\lor \qquad \lor$$

$$F \land T \rightarrow F$$

False

1. $\forall x\,[P(x) \wedge Q(x)] \Leftrightarrow \forall x\, P(x) \wedge \forall x\, Q(x)$

2. $\exists x\,[P(x) \vee Q(x)] \Leftrightarrow \exists x\, P(x) \vee \exists x\, Q(x)$

3. $\exists x\,[P(x) \wedge Q(x)] \longrightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$

4. $\forall x\,[P(x) \rightarrow Q(x)] \rightarrow \forall x\, P(x) \rightarrow \forall x\, Q(x)$

5. $\forall x\,[P(x) \Leftrightarrow Q(x)] \rightarrow \forall x\, P(x) \Leftrightarrow \forall x\, Q(x)$

6. $\forall x\,[P(x) \vee Q(x)] \Leftarrow \forall x\, P(x) \vee \forall x\, Q(x)$

$\forall \rightarrow \wedge$

$$\forall x\,[P(x) \rightarrow Q(x)] \rightarrow \forall x\,P(x) \rightarrow \forall x\,Q(x)$$

$$\boxed{T \rightarrow T} \quad T$$

$$\wedge$$

$$\boxed{F \rightarrow T} \quad T$$

$$\boxed{\begin{matrix} T \\ \wedge \\ F \end{matrix}} \rightarrow \boxed{\begin{matrix} T \\ \wedge \\ T \end{matrix}}$$

$$\frac{F \rightarrow}{\text{True}}$$

5: R → L.

$$\forall x\,[P(x)] \leftrightarrow \forall x\, Q(x) \not\Rightarrow \forall x\,[P(x) \leftrightarrow Q(x)]$$

$$\boxed{\begin{matrix} T \\ \wedge \\ F \end{matrix}} \leftrightarrow \boxed{\begin{matrix} F \\ \wedge \\ T \end{matrix}}$$

$$\frac{F \leftrightarrow F.}{T.}$$

$$\boxed{T \leftrightarrow F} \quad F.$$

$$\wedge$$

$$\boxed{F \leftrightarrow T} \quad F.$$

D: Z.

$p(x)$: $x^2 - 8x + 15 = 0$ $(x = 3, 5)$

$q(x)$: x is odd.

$r(x)$: $x > 0$

a) $\forall x\, [\overset{\text{True}}{p(x) \rightarrow q(x)}]$

$\boxed{P(0) \rightarrow Q(0)}$ T.

$\wedge$

$\boxed{P(1) \rightarrow Q(1)}$ T

$\wedge$

$\boxed{P(2) \rightarrow Q(2)}$ T.

$\wedge$

$\boxed{P(3) \rightarrow Q(3)}$ T.

⋮

those value will satisfy $P(x)$ will become odd $Q(x)$

3, 5 ⟶ 3, 5 odd

$D: \mathbb{Z}$.

$p(x): x^2 - 8x + 15 = 0 \quad (x = 3, 5)$

$q(x)$: x is odd. $q(1)$

$r(x): x > 0$

$\forall x [q(x) \to p(x)]$

$\boxed{q(0) \to p(0)}$

$\boxed{q(1) \to p(1)}$ $\quad T \to F \quad$ F

$\wedge$

$\boxed{q(2) \to p(2)}$

False

Invalid.

b) $\forall x [q(x) \to p(x)]$

c) $\exists x [p(x) \to q(x)]$

d) $\exists x [q(x) \to p(x)]$

e) $\forall x [\neg q(x) \to \neg p(x)]$

f) $\exists x [p(x) \to q(x) \wedge R(x)]$

$D : \mathbb{Z}$

$p(x) : x^2 - 7x + 10$

$q(x) : x^2 - 2x - 3 = 0$

$r(x) : x < 0$

$\forall x \, [p(x) \rightarrow \neg R(x)]$

$\forall x \, [q(x) \rightarrow R(x)]$

$\exists x \, [q(x) \rightarrow R(x)]$

$\exists x \, [p(x) \rightarrow R(x)]$

$$\neg \forall x \, [P(x) \rightarrow \neg R(x)]$$

$$\neg \forall x \, [\neg P(x) \vee \neg R(x)]$$

$$\exists x \, [P(x) \wedge R(x)]$$

negate the stmnts. :

$$\forall x \, [P(x) \rightarrow \neg R(x)]$$

$$\forall x \, [Q(x) \rightarrow R(x)]$$

$$\exists x \, [Q(x) \rightarrow R(x)]$$

$$\exists x \, [P(x) \rightarrow R(x)]$$

negate:

God's Rule:

not logically equivalent:

(GATE-13)

$\neg \exists x [\forall y(\alpha) \wedge \forall z(\beta)]$

$\forall x [\neg \forall y(\alpha) \vee \neg \forall z(\beta)]$

$\forall x [\forall y(\alpha) \rightarrow \neg \forall z(\beta)]$

$\forall x [\forall y(\alpha) \rightarrow \exists z(\neg \beta)]$

$\forall x [\neg \forall z(\beta) \vee \neg \forall y(\alpha)]$

$\forall x [\forall z(\beta) \rightarrow \neg \forall y(\alpha)]$

$\forall x [\forall z(\beta) \rightarrow \exists y(\neg \alpha)]$

A) $\forall x [\exists z(\neg \beta) \rightarrow \forall y(\alpha)]$ ✓

b) $\forall x [\forall z(\beta) \xrightarrow{\text{same}} \exists y(\neg \alpha)]$

c) $\forall x [\forall y(\alpha) \xrightarrow{\text{same}} \exists z(\neg \beta)]$

d) $\forall x [\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta)]$ ✓

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

Establish the validity of the following arguments.

a) $[p \land (p \to q) \land (\neg q \lor r)] \to r$

b)
$$\begin{array}{l} p \to q \\ \neg q \\ \neg r \\ \hline \therefore \neg(p \lor r) \end{array}$$

c)
$$\begin{array}{l} p \to q \\ r \to \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) *P* and *R* only

(c) *P* and *S* only

(d) P, Q, R and S

d) $p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

e) $p \rightarrow (q \rightarrow r)$

$p \vee s$

$t \rightarrow q$

$\neg s$

$\therefore \neg r \rightarrow \neg t$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$ is

(a) $p \wedge \sim q$

(b) $p \vee \sim q$

(c) t

(d) $(p \rightarrow \sim q)$

Let p, q, r and s be four primitive statements. Consider the following arguments:

$P: [(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

$Q: [(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

$R: [(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

$S: [p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

(a) $p \wedge t$

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$S_1: \{(\sim p \to (q \to \sim W)) \land (\sim S \to q) \land \sim t \land (\sim p \lor t)\} \to (w \to s)$

$S_2: \{(q \to t) \land (s \to r) \land (\sim q \to s)\} \to (\sim t \to r)$

The statement formula $\{(a \rightarrow c) \wedge (b \rightarrow d) \wedge (c \rightarrow \sim d)\} \rightarrow (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

The statement formula

$\{((\sim p \vee q) \rightarrow r) \wedge (r \rightarrow (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \rightarrow \sim t)\} \rightarrow p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !