

CS & IT ENGINEERING

Discrete Mathematics

Combinatorics



Lecture No.- 07

By- Satish Yadav sir

Recap of Previous Lecture



Topic

Introduction to Combinatorics



Topics to be Covered



Topic

Pigeonhole Principle





Topic : Combinatorics

Inclusion-Exclusion :

$$= \{ 1 \textcircled{2} 3 \textcircled{4} \textcircled{5} \textcircled{6} 7 \textcircled{8} 9 \textcircled{10} \}$$

Q.1 \Rightarrow what will be no. of elements which are \div by 2 or 5
in a set $\{1 \dots 10\}$. Total - $P(2 \cap 5)$

$$P(2) : \text{no. of elements } \div \text{ by } 2 = \frac{\text{Total elements}}{2} = \frac{10}{2} = 5$$

$$P(5) : \text{no. of elements } \div \text{ by } 5 = \frac{10}{5} = 2$$

$$P(2 \cap 5) = \frac{10}{2 \cdot 5} = 1$$

$$\text{Ans: } P(2) + P(5) = 5 + 2 = 7 - 1 = 6$$



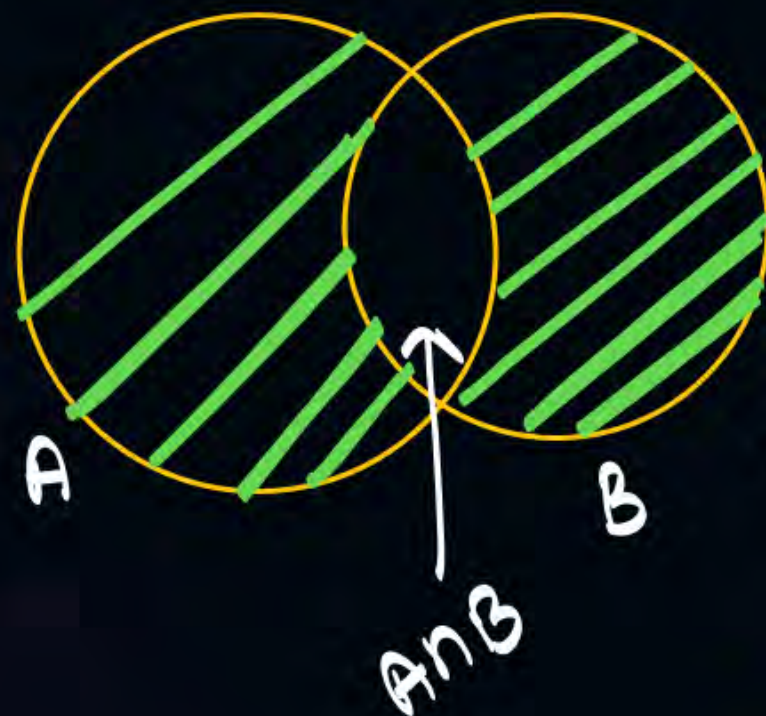
Topic : Combinatorics

$$I = \{e_1, e_2, \dots, e_n\}$$

$$A \cup B = A + B - A \cap B$$

$$A_1 \cup A_2 = A_1 + A_2 - A_1 \cap A_2$$

$$n=2$$



$$A - B = A - (A \cap B)$$

$$B - A = B - (A \cap B)$$

Symmetric Diff (Δ / \oplus) present in A or in B but not in both

$$= (A - B) \cup (B - A)$$

$$= (A \cup B) - (A \cap B)$$



Topic : Combinatorics



$$A \cup B \cup C = A + B + C - A \cap B - B \cap C - A \cap C + A \cap B \cap C$$

$$A_1 \cup A_2 \cup A_3 = \underline{A_1 + A_2 + A_3} - A_1 \cap A_2 - A_2 \cap A_3 - A_1 \cap A_3 + \underline{A_1 \cap A_2 \cap A_3}.$$

$$A_1 \cup A_2 \cup A_3 \cup A_4 = \sum A_i - \sum A_i \cap A_j + \sum A_i \cap A_j \cap A_k - \underline{A_1 \cap A_2 \cap A_3 \cap A_4}.$$



Topic : Combinatorics

$$A_1 \cup A_2 \cup \dots \cup A_n = \sum A_i - \sum A_i \cap A_j + \sum A_i \cap A_j \cap A_k$$

$$\dots \dots (-1)^{n+1} (A_1 \cap A_2 \cap A_3 \dots A_n)$$



Topic : Combinatorics

(GATE)

How many int are not ÷ by 2, 3, or 5 in a set $\{1, \dots, 123\}$?

$$P(2 \text{ OR } 3 \text{ OR } 5) = P(2) + P(3) + P(5) - P(2 \cdot 3) - P(3 \cdot 5) - P(2 \cdot 5) + P(2 \cdot 3 \cdot 5)$$

$$= \left\lfloor \frac{123}{2} \right\rfloor + \left\lfloor \frac{123}{3} \right\rfloor + \left\lfloor \frac{123}{5} \right\rfloor - \left\lfloor \frac{123}{\text{lcm}(2, 3)} \right\rfloor - \left\lfloor \frac{123}{\text{lcm}(3, 5)} \right\rfloor - \left\lfloor \frac{123}{\text{lcm}(2, 5)} \right\rfloor + \left\lfloor \frac{123}{2 \cdot 3 \cdot 5} \right\rfloor$$

$$\text{Ans} = 123 - P(2 \cup 3 \cup 5) = 33$$

no. of elements which are \div by 4, 6 or from a set $\{1 \dots 1000\}$?

$$P(4 \text{ OR } 6 \text{ OR } 8) = \left\lfloor \frac{1000}{4} \right\rfloor + \left\lfloor \frac{1000}{6} \right\rfloor + 1 - \left\lfloor \frac{1000}{\text{lcm}(4,6)} \right\rfloor$$

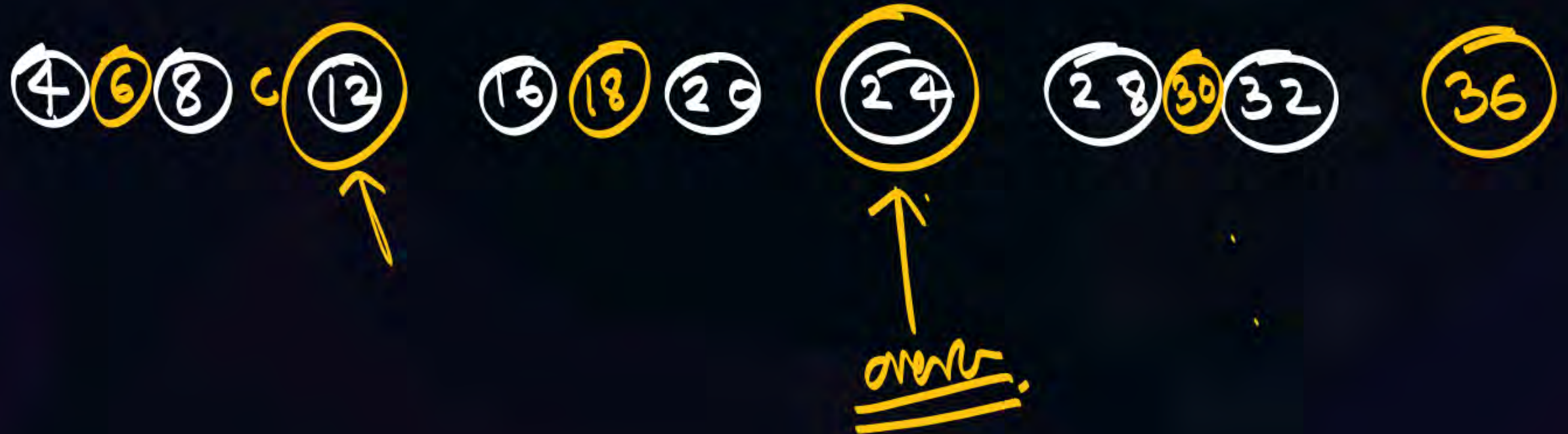
$\{1 \quad 4 \quad 6 \quad 8 \quad 12 \dots 18 \quad 20 \quad 24 \quad 28 \dots 1000\}$



Topic : Combinatorics

no. of elements which are \div by 4 or 6 from a set
 $\{1 \dots 1000\}$

$$\left\lfloor \frac{1000}{4} \right\rfloor + \left\lfloor \frac{1000}{6} \right\rfloor - \left\lfloor \frac{1000}{\text{lcm}(4,6)} \right\rfloor \left(\frac{1000}{24} \right)$$





Topic : Combinatorics

$$x_1 + x_2 + x_3 = 20$$

$$\begin{cases} 2 \leq x_1 \leq 6 \\ 3 \leq x_2 \leq 7 \\ 1 \leq x_3 \leq 5 \end{cases}$$

Required

2	3	1
3	4	2
4	5	3
5	6	4
6	7	5
7	8	6
8	9	7
9	10	8
10	11	9
11	12	10
12	13	11
13	14	12
14	15	13
15	16	14
16	17	15
17	18	16
18	19	17
19	20	18
20	21	19
21	22	20
22	23	21
23	24	22
24	25	23
25	26	24
26	27	25
27	28	26
28	29	27
29	30	28
30	31	29
31	32	30
32	33	31
33	34	32
34	35	33
35	36	34
36	37	35
37	38	36
38	39	37
39	40	38
40	41	39
41	42	40
42	43	41
43	44	42
44	45	43
45	46	44
46	47	45
47	48	46
48	49	47
49	50	48
50	51	49
51	52	50
52	53	51
53	54	52
54	55	53
55	56	54
56	57	55
57	58	56
58	59	57
59	60	58
60	61	59
61	62	60
62	63	61
63	64	62
64	65	63
65	66	64
66	67	65
67	68	66
68	69	67
69	70	68
70	71	69
71	72	70
72	73	71
73	74	72
74	75	73
75	76	74
76	77	75
77	78	76
78	79	77
79	80	78
80	81	79
81	82	80
82	83	81
83	84	82
84	85	83
85	86	84
86	87	85
87	88	86
88	89	87
89	90	88
90	91	89
91	92	90
92	93	91
93	94	92
94	95	93
95	96	94
96	97	95
97	98	96
98	99	97
99	100	98
100	101	99
101	102	100
102	103	101
103	104	102
104	105	103
105	106	104
106	107	105
107	108	106
108	109	107
109	110	108
110	111	109
111	112	110
112	113	111
113	114	112
114	115	113
115	116	114
116	117	115
117	118	116
118	119	117
119	120	118
120	121	119
121	122	120
122	123	121
123	124	122
124	125	123
125	126	124
126	127	125
127	128	126
128	129	127
129	130	128
130	131	129
131	132	130
132	133	131
133	134	132
134	135	133
135	136	134
136	137	135
137	138	136
138	139	137
139	140	138
140	141	139
141	142	140
142	143	141
143	144	142
144	145	143
145	146	144
146	147	145
147	148	146
148	149	147
149	150	148
150	151	149
151	152	150
152	153	151
153	154	152
154	155	153
155	156	154
156	157	155
157	158	156
158	159	157
159	160	158
160	161	159
161	162	160
162	163	161
163	164	162
164	165	163
165	166	164
166	167	165
167	168	166
168	169	167
169	170	168
170	171	169
171	172	170
172	173	171
173	174	172
174	175	173
175	176	174
176	177	175
177	178	176
178	179	177
179	180	178
180	181	179
181	182	180
182	183	181
183	184	182
184	185	183
185	186	184
186	187	185
187	188	186
188	189	187
189	190	188
190	191	189
191	192	190
192	193	191
193	194	192
194	195	193
195	196	194
196	197	195
197	198	196
198	199	197
199	200	198
200	201	199
201	202	200
202	203	201
203	204	202
204	205	203
205	206	204
206	207	205
207	208	206
208	209	207
209	210	208
210	211	209
211	212	210
212	213	211
213	214	212
214	215	213
215	216	214
216	217	215
217	218	216
218	219	217
219	220	218
220	221	219
221	222	220
222	223	221
223	224	222
224	225	223
225	226	224
226	227	225
227	228	226
228	229	227
229	230	228
230	231	229
231	232	230
232	233	231
233	234	232
234	235	233
235	236	234
236	237	235
237	238	236
238	239	237
239	240	238
240	241	239
241	242	240
242	243	241
243	244	242
244	245	243
245	246	244
246	247	245
247	248	246
248	249	247
249	250	248
250	251	249
251	252	250
252	253	251
253	254	252
254	255	253
255	256	254
256	257	255
257	258	256
258	259	257
259	260	258
260	261	259
261	262	260
262	263	261
263	264	262
264	265	263
265	266	264
266	267	265
267	268	266
268	269	267
269	270	268
270	271	269
271	272	270
272	273	271
273	274	272
274	275	273
275	276	274
276	277	275
277	278	276
278	279	277
279	280	278
280	281	279
281	282	280
282	283	281
283	284	282
284	285	283
285	286	284
286	287	285
287	288	286
288	289	287
289	290	288
290	291	289
291	292	290
292	293	291
293	294	292
294	295	293
295	296	294
296	297	295
297	298	296
298	299	297
299	300	298
300	301	299
301	302	300
302	303	301
303	304	302
304	305	303
305	306	304
306	307	305
307	308	306
308	309	307
309	310	308
310	311	309
311	312	310
312	313	311
313	314	312
314	315	313
315	316	314
316	317	315
317	318	316
318	319	317
319	320	318
320	321	319
321	322	320
322	323	321
323	324	322
324	325	323
325	326	324
326	327	325
327	328	326
328	329	327
329	330	328
330	331	329
331	332	330
332	333	331
333	334	332
334	335	333
335	336	334
336	337	335
337	338	336
338	339	337
339	340	338
340	341	339
341	342	340
342	343	341
343	344	342
344	345	343
345	346	344
346	347	345
347	348	346
348	349	347
349	350	348
350	351	349
351	352	350
352	353	351
353	354	352
354	355	353
355	356	354
356	357	355
357	358	356
358	359	357
359	360	358
360	361	359
361	362	360
362	363	361
363	364	362
364	365	363
365	366	364
366	367	365
367	368	366
368	369	367
369	370	368
370	371	369
371	372	370
372	373	371
373	374	372
374	375	373
375	376	374
376	377	375
377	378	376
378	379	377
379	380	378
380	381	379
381	382	380
382	383	381
383	384	382
384	385	383
385	386	384
386	387	385
387	388	386
388	389	387
389	390	388
390	391	389
391	392	390
392	393	391
393	394	392
394	395	393
395	396	394
396	397	395
397	398	396
398	399	397
399	400	398
400	401	399
401	402	400
402	403	401
403	404	402
404	405	403
405	406	404
406	407	405
407	408	406
408	409	407
409	410	408
410	411	409
411	412	410
412	413	411
413	414	412
414	415	413
415	416	414
416	417	415
417	418	416
418	419	417
419	420	418
420	421	419
421	422	420
422	423	421
423	424	422
424	425	423
425	426	424
426	427	425
427	428	426
428	429	427
429	430	428
430	431	429
431	432	430
432	433	431
433	434	432
434	435	433
435	436	434
436	437	435
437	438	436
438	439	437
439	440	438
440	441	439
441	442	440
442	443	441
443	444	442



Topic : Combinatorics



$$1^4 + 2^2 = 16$$

$$\text{Total} = U = \{x_1 + x_2 + x_3 = 20 \mid x_1 \geq 2, x_2 \geq 3, x_3 \geq 1\}$$

$$A_1 \rightarrow 7$$

$$A_2 \rightarrow 8$$

$$A_3 \rightarrow 6$$



Topic : Combinatorics



$$x_1 + x_2 + x_3 = 20$$

$$2 \leq x_1 \leq 5$$

$$4 \leq x_2 \leq 7$$

$$-2 \leq x_3 \leq 9$$

no. of non negative soltⁿ?



Topic : Combinatorics

Derangement : (D_n) nothing is @ right place.

places: 1 2 3

$\left\{ \begin{array}{ccc} 1 & 2 & 3 \\ \textcircled{1} & 3 & 2 \\ 2 & 1 & \textcircled{3} \end{array} \right. \quad \begin{array}{c} \times \\ \times \\ \times \end{array}$

$$D_3 = 2.$$

$D_3 = \text{Total} - \text{atleast 1 element @ right position.}$

$$D_3 = 3! - (\text{atleast 1 ———})$$

$D_3 \left\{ \begin{array}{l} \rightarrow \boxed{2 \quad 3 \quad 1} \checkmark \\ \rightarrow \boxed{3 \quad 1 \quad 2} \checkmark \\ \left\{ \begin{array}{ccc} 3 & \textcircled{2} & 1 \end{array} \right. \times \end{array} \right.$



Topic : Combinatorics

$$D_4 = 4! - (\text{at least 1 element @ right position})$$

$$= 4! - (A_1 \cup A_2 \cup A_3 \cup A_4)$$

$$\underline{A_1} \quad \underline{A_2} \quad \underline{A_3} \quad \underline{A_4}$$

$A_1 \rightarrow$ Total no. of ways.

1st element @ right position.

$A_2 \rightarrow$ — 11 ——— 2nd ——— 11 ———

$A_3 \rightarrow$ ——— 11 ——— 3rd ——— 11 ———

$A_4 \rightarrow$ ——— 11 ——— 4 ———

$A_1 \cap A_2 =$ Total ways
1 & 2 elements
@ right position

$$|A_1| = 3!, |A_2| = 3!, \\ |A_3| = 3!, |A_4| = 3!$$

$A_1 \Rightarrow$

$$|A_1| = 3!$$

1	2	3	4
1	2	4	3

$$3!$$

A_2

1	2	3	4
1	2	4	3

$$3!$$

$$A_1 \cap A_2 = \begin{matrix} 1 & 2 \\ \vdots & \vdots \end{matrix}$$

$$\underline{A_1 \cap A_2} \quad \underline{A_2 \cap A_3} \\ A_1 \cap A_3$$

there are 4 places
we can choose 2
in $4C_2$ ways.



Topic : Combinatorics

$$D_4 = 4! - (A_1 \cup A_2 \cup A_3 \cup A_4)$$

$$= 4! - \left(\sum A_i - \sum A_i \cap A_j + \sum A_i \cap A_j \cap A_k - A_1 \cap A_2 \cap A_3 \cap A_4 \right)$$

$$= 4! - \left(4C_1 \cdot 3! - 4C_2 \cdot 2! + 4C_3 \cdot 1! - 1 \right)$$

$$= 4! - 4 \times 3! + 4C_2 \cdot 2! - 4C_3 - 1$$

$$= \cancel{4!} - \cancel{4!} + \frac{4!}{\cancel{2!} \cdot \cancel{2!}} - \frac{4!}{3!} - \frac{4!}{4!} = \frac{4!}{2!} - \frac{4!}{3!} + \frac{4!}{4!}$$



Topic : Combinatorics

$$D_4 = \frac{4!}{2!} - \frac{4!}{3!} + \frac{4!}{4!} = 4! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right] = 9$$

$$D_n = n! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \dots \dots \frac{(-1)^n}{n!} \right]$$



Topic : Combinatorics

$$D_1 = 0$$

$$D_2 = 1$$

$$D_3 = 2$$

$$\underline{D_4 = 9}$$

$$D_5 = 44$$

$$D_6 = 265$$

$$D_n = n! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \cdots \frac{(-1)^n}{n!} \right] \quad (n \geq 7)$$

$$D_n \approx n! \times 0.367$$

$$D_6 \approx 6! \times 0.367$$

$$Dn \equiv n! \times 0.367$$

$$n = 10$$

$$D_{10} \approx 10! \times 0.367$$

$$\frac{D_{10}}{10!} \approx 0.367$$

$n = 10 \text{ element}$



$$\left\{ \frac{Dn}{n!} \right\} \approx \underline{\underline{0.367}}$$

$$n = 11$$

$$\frac{D_{11}}{11!} \approx 0.367$$





Topic : Combinatorics

$$e = 2.718$$

$$e^{-1} = 0.36$$

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$

put $x = -1$

$$e^{-1} = \cancel{1} + \cancel{\frac{(-1)}{1!}} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!}$$

$$0.3678 \approx \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}$$

Find the number of derangements of the integers from 1 to 10 inclusive, satisfying the condition that the set of elements in the first 5 places is

(a) 1, 2, 3, 4, 5, in some order, ANS : $D_5 \cdot D_5 = 1936$.

(b) 6, 7, 8, 9, 10, in some order. $(5!)^2 = 14,400$

4. An advertising agency has 1,000 clients. Suppose that T is the set of clients that use television advertising, R is the set of clients that use radio advertising, and N is the set of clients who use newspaper advertising. Suppose that $|T| = 415$, $|R| = 350$, $|N| = 280$, 100

clients use all 3 types of advertising, 175 use television and radio, 180 use radio and newspapers, and $|T \cap N| = 165$.

(a) Find $|T \cap R \cap \bar{N}|$.

(b) How many clients use radio and newspaper advertising but not television?

(c) How many use television but do not use newspaper advertising and do not use radio advertising?

(d) Find $|\bar{T} \cap \bar{R} \cap \bar{N}|$.

4. (a) 75.

(b) $|R \cap N \cap \bar{T}| = 80$

(c) $|T \cap \bar{N} \cap \bar{R}| = 175$.

(d) $|\bar{T} \cap \bar{R} \cap \bar{N}| = |\overline{T \cup R \cup N}| = 1,000 - 625 = 375$.

10. Find the number of permutations of the integers 1 to 10 inclusive
- (a) such that exactly 4 of the integers are in their natural positions (that is, exactly 6 of the integers are deranged).
 - (b) such that 6 or more of the integers are deranged.
 - (c) that do not have 1 in the first place, nor 4 in the fourth place, nor 7 in the seventh place.
 - (d) such that no odd integer will be in the natural position.
 - (e) that do not begin with a 1 and do not end with 10.
10. (a) $C(10,6)D_6$.
- (b) $\binom{10}{6}D_6 + \binom{10}{7}D_7 + \binom{10}{8}D_8 + \binom{10}{9}D_9 + \binom{10}{10}D_{10}$
- (c) $10! - (3)9! + (3)8! - 7!$.
- (d) $10! - \binom{5}{1}9! + \binom{5}{2}8! - \binom{5}{3}7! + \binom{5}{4}6! - \binom{5}{5}5!$.
- (e) $10! - (2)9! + 8!$.
17. At a theater 10 men check their hats. In how many ways can their hats be returned so that
- (a) no man receives his own hat?
 - (b) at least 1 of the men receives his own hat?
 - (c) at least 2 of the men receive their own hats?
17. (a) D_{10} .
- (b) $10! - D_{10}$.
- (c) $10! - D_{10} - 10D_9$.
25. The squares of a chessboard are painted 8 different colors. The squares of each row are painted all 8 colors and no 2 consecutive squares in one column can be painted the same color. In how many ways can this be done?
25. The first row can be painted $8!$ ways. Each row after the first can be painted D_8 ways. Hence the number of ways is $8!(D_8)^7$.
5. Determine the number of positive integers n , $1 \leq n \leq 2000$, that are
- a) not divisible by 2, 3, or 5
 - b) not divisible by 2, 3, 5, or 7
 - c) not divisible by 2, 3, or 5, but are divisible by 7

- (a) c_1 : number n is divisible by 2
 c_2 : number n is divisible by 3
 c_3 : number n is divisible by 5
 $N(c_1) = \lfloor 2000/2 \rfloor = 1000$, $N(c_2) = \lfloor 2000/3 \rfloor = 666$,
 $N(c_3) = \lfloor 2000/5 \rfloor = 400$, $N(c_1 c_2) = \lfloor 2000/(2)(3) \rfloor = 333$,
 $N(c_2 c_3) = \lfloor 2000/(3)(5) \rfloor = 133$, $N(c_1 c_3) = \lfloor 2000/(2)(5) \rfloor = 200$,
 $N(c_1 c_2 c_3) = \lfloor 2000/(2)(3)(5) \rfloor = 66$.
 $N(\bar{c}_1 \bar{c}_2 \bar{c}_3) = 2000 - (1000 + 666 + 400) + (333 + 200 + 133) - 66 = 534$
- (b) Let c_1, c_2, c_3 be as in part (a). Let c_4 denote the number n is divisible by 7. Then
 $N(c_4) = 285$, $N(c_1 c_4) = 142$, $N(c_2 c_4) = 95$, $N(c_3 c_4) = 57$, $N(c_1 c_2 c_4) = 47$, $N(c_1 c_3 c_4) = 28$, $N(c_2 c_3 c_4) = 19$, $N(c_1 c_2 c_3 c_4) = 9$. $N(\bar{c}_1 \bar{c}_2 \bar{c}_3 \bar{c}_4) = 2000 - (1000 + 666 + 400 + 285) + (333 + 200 + 133 + 142 + 95 + 57) - (66 + 47 + 28 + 19) + 9 = 458$
- (c) $534 - 458 = 76$.
2. a) List all the derangements of 1, 2, 3, 4, 5 where the first three numbers are 1, 2, and 3, in some order.
b) List all the derangements of 1, 2, 3, 4, 5, 6 where the first three numbers are 1, 2, and 3, in some order.
3. How many derangements are there for 1, 2, 3, 4, 5?
4. How many permutations of 1, 2, 3, 4, 5, 6, 7 are not derangements?
5. a) Let $A = \{1, 2, 3, \dots, 7\}$. A function $f: A \rightarrow A$ is said to have a *fixed point* if for some $x \in A$, $f(x) = x$. How many one-to-one functions $f: A \rightarrow A$ have at least one fixed point?
b) In how many ways can we devise a secret code by assigning to each letter of the alphabet a different letter to represent it?
2. (a) There are only two derangements with this property: 23154 and 31254.
(b) Here there are four such derangements:
(i) 231546 (ii) 231645 (iii) 312546 (iv) 312645
3. The number of derangements for 1,2,3,4,5 is $5![1 - 1 + (1/2!) - (1/3!) + (1/4!) - (1/5!)] = 5![(1/2!) - (1/3!) + (1/4!) - (1/5!)] = (5)(4)(3) - (5)(4) + 5 - 1 = 60 - 20 + 5 - 1 = 44$.
4. There are $7! = 5040$ permutations of 1,2,3,4,5,6,7. Among these there are $7![1 - 1 + (1/2!) - (1/3!) + (1/4!) - (1/5!) + (1/6!) - (1/7!)] = 1854$ derangements. Consequently, we have $5040 - 1854 = 3186$ permutations of 1,2,3,4,5,6,7 that are not derangements.
5. (a) $7! - d_7$ ($d_7 \doteq (7!)e^{-1}$); (b) $d_{26} \doteq (26!)e^{-1}$
6. How many derangements of 1, 2, 3, 4, 5, 6, 7, 8 start with
(a) 1, 2, 3, and 4, in some order? (b) 5, 6, 7, and 8, in some order?
7. For the positive integers 1, 2, 3, \dots , $n - 1$, n , there are 11,660 derangements where 1, 2, 3, 4, and 5 appear in the first five positions. What is the value of n ?

6. (a) There are $(d_4)^2 = 9^2 = 81$ such derangements.
 (b) In this case we get $(4!)^2 = 24^2 = 576$ derangements.
7. Let $n = 5 + m$. Then $11,660 = d_5 \cdot d_m = 44(d_m)$, and so $d_m = 265 = d_6$. Consequently, $n = 11$.

9. In how many ways can Mrs. Ford distribute ten distinct books to her ten children (one book to each child) and then collect and redistribute the books so that each child has the opportunity to peruse two different books?

$$9. \quad (10!)d_{10} \doteq (10!)^2(e^{-1})$$

12. Ms. Pezzulo teaches geometry and then biology to a class of 12 advanced students in a classroom that has only 12 desks. In how many ways can she assign the students to these desks so that (a) no student is seated at the same desk for both classes? (b) there are exactly six students each of whom occupies the same desk for both classes?

$$12. \quad (a) \quad (12!)d_{12} \qquad (b) \quad (12!)\binom{12}{6}d_6$$

1. Determine how many $n \in \mathbb{Z}^+$ satisfy $n \leq 500$ and are not divisible by 2, 3, 5, 6, 8, or 10.

1. We need only consider the divisors 2, 3, and 5. Let c_1 denote divisibility by 2, c_2 divisibility by 3, and c_3 divisibility by 5.

$$N = 500; \quad N(c_1) = \lfloor 500/2 \rfloor = 250; \quad N(c_2) = \lfloor 500/3 \rfloor = 166; \quad N(c_3) = \lfloor 500/5 \rfloor = 100; \\ N(c_1c_2) = \lfloor 500/6 \rfloor = 83; \quad N(c_1c_3) = \lfloor 500/10 \rfloor = 50; \quad N(c_2c_3) = \lfloor 500/15 \rfloor = 33; \\ N(c_1c_2c_3) = \lfloor 500/30 \rfloor = 16.$$

$$N(\bar{c}_1\bar{c}_2\bar{c}_3) = 500 - (250 + 166 + 100) + (83 + 50 + 33) - 16 = 134.$$

19. Caitlyn has 48 different books: 12 each in mathematics, chemistry, physics, and computer science. These books are ar-

ranged on four shelves in her office with all books on any one subject on its own shelf. When her office is cleaned, the 48 books are taken down and then replaced on the shelves — once again with all 12 books on any one subject on its own shelf. In how many ways can this be done so that (a) no subject is on its original shelf? (b) one subject is on its original shelf? (c) no subject is on its original shelf and no book is in its original position? [For example, the book originally in the third (from the left) position on the first shelf must not be replaced on the first shelf and must not be in the third (from the left) position on the shelf where it is placed.]

19. a) $d_4(12!)^4$
 b) $\binom{4}{1}d_3(12!)^4$
 c) $d_4(d_{12})^4$

4. Annually, the 65 members of the maintenance staff sponsor a "Christmas in July" picnic for the 400 summer employees at their company. For these 65 people, 21 bring hot dogs, 35 bring fried chicken, 28 bring salads, 32 bring desserts, 13 bring hot dogs and fried chicken, 10 bring hot dogs and salads, 9 bring hot dogs and desserts, 12 bring fried chicken and salads, 17 bring fried chicken and desserts, 14 bring salads and desserts, 4 bring hot dogs, fried chicken, and salads, 6 bring hot dogs, fried chicken, and desserts, 5 bring hot dogs, salads, and desserts, 7 bring fried chicken, salads, and desserts, and 2 bring all four food items. Those (of the 65) who do not bring any of these four food items are responsible for setting up and cleaning up for the picnic. How many of the 65 maintenance staff will (a) help to set up and clean up for the picnic? (b) bring only hot dogs? (c) bring exactly one food item?

c_1 : Staff member brings hot dogs
 c_2 : Staff member brings fried chicken
 c_3 : Staff member brings salads
 c_4 : Staff member brings desserts
 $N = 65$

$N(c_1) = 21$; $N(c_2) = 35$; $N(c_3) = 28$; $N(c_4) = 32$
 $N(c_1c_2) = 13$; $N(c_1c_3) = 10$; $N(c_1c_4) = 9$; $N(c_2c_3) = 12$; $N(c_2c_4) = 17$; $N(c_3c_4) = 14$
 $N(c_1c_2c_3) = 4$; $N(c_1c_2c_4) = 6$; $N(c_1c_3c_4) = 5$; $N(c_2c_3c_4) = 7$
 $N(c_1c_2c_3c_4) = 2$.

(a) $N(\bar{c}_1\bar{c}_2\bar{c}_3\bar{c}_4) = 65 - [21 + 35 + 28 + 32] + [13 + 10 + 9 + 12 + 17 + 14] - [4 + 6 + 5 + 7] + 2 = 65 - 116 + 75 - 22 + 2 = 4$.

(b) $N(\bar{c}_2\bar{c}_3\bar{c}_4) = N - [N(c_2) + N(c_3) + N(c_4)] + [N(c_2c_3) + N(c_2c_4) + N(c_3c_4)] - N(c_2c_3c_4)$, so
 $N(\bar{c}_2\bar{c}_3\bar{c}_4) = N(c_1) - [N(c_1c_2) + N(c_1c_3) + N(c_1c_4)] + [N(c_1c_2c_3) + N(c_1c_2c_4) + N(c_1c_3c_4)] - N(c_1c_2c_3c_4) = 21 - [13 + 10 + 9] + [4 + 6 + 5] - 2 = 21 - 32 + 15 - 2 = 2$.

(c) $N(\bar{c}_1c_2\bar{c}_3\bar{c}_4) = N(c_2) - [N(c_1c_2) + N(c_2c_3) + N(c_2c_4)] + [N(c_1c_2c_3) + N(c_1c_2c_4) + N(c_2c_3c_4)] - N(c_1c_2c_3c_4) = 35 - [13 + 12 + 17] + [4 + 6 + 7] - 2 = 35 - 42 + 17 - 2 = 8$

$N(\bar{c}_1\bar{c}_2c_3\bar{c}_4) = N(c_3) - [N(c_1c_3) + N(c_2c_3) + N(c_3c_4)] + [N(c_1c_2c_3) + N(c_1c_3c_4) + N(c_2c_3c_4)] - N(c_1c_2c_3c_4) = 28 - [10 + 12 + 14] + [4 + 5 + 7] - 2 = 28 - 36 + 16 - 2 = 6$.
 $N(\bar{c}_1\bar{c}_2\bar{c}_3c_4) = N(c_4) - [N(c_1c_4) + N(c_2c_4) + N(c_3c_4)] + [N(c_1c_2c_4) + N(c_1c_3c_4) + N(c_2c_3c_4)] - N(c_1c_2c_3c_4) = 32 - [9 + 17 + 14] + [6 + 5 + 7] - 2 = 32 - 40 + 18 - 2 = 8$.
 So the answer is $2 + 8 + 6 + 8 = 24$.



THANK - YOU