1500 series

CS & IT ENGINEERING

Discrete Mathematics

Lecture No.- 04

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Recap of Previous Lecture

Topic Pigeonhole – Principle

Topics to be Covered

Topic Generating Function

Generating functⁿ

sequence:

closed form:

coefficient:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{1}{1-ax} = 1 + ax + (ax)^2 + (ax)^3 + \dots \checkmark \quad a = 1, 2, 3 \dots$$

$$\frac{1}{1+ax} = 1 - ax + (ax)^2 - (ax)^3 + \dots \checkmark \quad a = 1, 2, 3 \dots$$

$$\boxed{\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots}$$

$$\frac{1}{1-x} = 1+x+x^2+x^3+x^4+x^5+x^6 \ldots \quad (1, 1, 1, 1 \ldots)$$

$$\boxed{\frac{1}{(1-x)^2} = 1+2x+3x^2+4x^3+5x^4 \ldots} \rightarrow \begin{array}{ccccc} a_0 & a_1 & a_2 & & \\ 1, & 2, & 3, & 4, & 5 \ldots \end{array}$$

(xbyx) $$\frac{x}{(1-x)^2} = \underset{a_0}{0} + \underset{a_1}{x} + \underset{a_2}{2x^2} + 3x^3 + 4x^4 + 5x^5 + \ldots \quad \begin{array}{cccccc} a_0 & a_1 & & & & \\ 0, & 1, & 2, & 3, & 4, & 5 \ldots \end{array}$$

$(1-2x+x^2)$

$$\frac{x+1}{(1-x)^3} = 1+4x+9x^2+16x^3 \quad \left(\begin{array}{l} 1, 4, 9, 16 \ldots \\ 1^2, 2^2, 3^2, 4^2 \ldots \end{array}\right)$$

$$\frac{x+1}{(1-x)^3} = 1+4x+9x^2+16x^3 \dots$$

x by x

$$x\left(\frac{x+1}{(1-x)^3}\right) = x\left(1+4x+9x^2+16x^3+\dots\right)$$

$$= x+4x^2+9x^3+16x^4+\dots$$

$$= x+2^2\cdot x^2+3^2\cdot x^3+4^2\cdot x^4+5^2\cdot x^5\dots$$

$$G(x) = a_0x^0+a_1x^1+a_2x^2$$

$$\left(0, 1^2, 2^2, 3^2, 4^2, 5^2 \dots\right)$$

$a_0 = 0 \quad a_1 = 1 \quad a_2 = 2^2$

Sequence.

GATE

$\langle 0, 1^2, 2^2, 3^2, 4^2 \ldots \rangle$

$a_0 \; a_1 \; a_2 \; a_3 \ldots$

$a_n = n^2$

$G(x) = \sum_{i=0}^{\infty} a_i \cdot x^i$

OR

$G(x) = \sum_{n=0}^{\infty} a_n \, x^n$

$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \ldots$

$= 0x^0 + 1 \cdot x^1 + 2^2 \cdot x^2 + 3^2 \cdot x^3 \ldots$

$G(x) = \sum_{n=0}^{\infty} a_n \cdot x^n$

$G(x) = \dfrac{x(x+1)}{(1-x)^3}$

$= \sum_{n=0}^{\infty} n^2 \cdot x^n = \dfrac{x(x+1)}{(1-x)^3}$

$= 0^2 x^0 + 1^2 x^1 + 2^2 \cdot x^2 + 3^2 x^3$

$$\frac{x(0-2+2x)-(1-x)^2\cdot 1}{(1-x)^4}$$

$$= \frac{-2x(1-x)-(1-x)^2}{(1-x)^4}$$

$$= \frac{(1-x)(-2x-(1-x))}{(1-x)^4} = \frac{(1-x)(-2x-1+x)}{(1-x)^4}$$

$$= \frac{1-x}{(1-x)^3}$$

Q

Given the following sequences, determine the corresponding generating function as a summation and in closed form (as a formula).

a) $0, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, \ldots$

b) $0, 0, 0, 1, 1, 1, 1, 1, 1, \ldots$

c) $0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, \ldots$

d) $2, 4, 8, 16, 32, 64, 128, 256, \ldots$

e) $\binom{7}{0}, \binom{7}{1}, \binom{7}{2}, \ldots, \binom{7}{7}, 0, 0, 0, 0, 0, \ldots$

f) $2, -2, 2, -2, 2, -2, 2, -2, \ldots$

g) $1, 1, 0, 1, 1, 1, 1, 1, 1, 1, \ldots$

h) $0, 0, 0, 1, 2, 3, 4, \ldots$

Q

Given the following sequences, determine the corresponding generating function as a summation and in closed form (as a formula).

a) $-1, -1, -1, -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, \dots$

b) $1, 3, 9, 27, 81, 243, 729, \dots$

c) $0, 0, 3, -3, 3, -3, 3, -3, \dots$

d) $1, 2, 1, 1, 1, 1, 1, 1, 1, \dots$

e) $\binom{7}{0}, 2\binom{7}{1}, 2^2\binom{7}{2}, \dots, 2^7\binom{7}{7}, 0, 0, 0, 0, \dots$

f) $-3, 3, -3, 3, -3, 3, \dots$

g) $0, 1, -2, 4, -8, 16, -32, 64, \dots$

h) $1, 0, 1, 0, 1, 0, 1, 0, \dots$

Given the following sequences, determine the corresponding generating function as a summation and in closed form (as a formula).

0, 2, 6, 12, 20, 30, 42,

$a_0 = 0$ $\qquad a_n = n^2 + n$

$a_1 = 2 = 1^2 + 1$

$a_2 = 6 = 2^2 + 2$

$a_3 = 12 = 3^2 + 3$

$$G(x) = \sum_{n=0}^{\infty} (n^2+n) \cdot x^n$$

$$= \sum_{n=0}^{\infty} n^2 \cdot x^n + \sum_{n=0}^{\infty} n \cdot x^n$$

$$0 + 1 \cdot x + 2 \cdot x^2 + 3 \cdot x^3 + 4x^4$$

$$x(1 + 2x + 3x^2 + 4x^3 \dots$$

$$\frac{x(x+1)}{(1-x)^3} + \frac{x}{(1-x)^2}$$

$$\rightarrow \frac{x(x+1)}{(1-x)^3} + \frac{x}{(1-x)^2}$$

$$\begin{matrix} 0 & 2, & 6, & 12, & 20 \\ a_0 & a_1 & a_2 & a_3 & a_4 \end{matrix}$$

$$G(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots$$

$$= 2x + 6x^2 + 12x^3 + 20x^4$$

$$= x + 4x^2 + 9x^3 + \ldots + \underbrace{x + 2x^2 + 3x^3}_{\frac{x}{1-x}} + 4x^4$$

$$\frac{x}{1-x} \rightarrow n \cdot x^n$$

$$\frac{x}{(1-x)^2} = 0 + x + 2x^2 + 3x^3 + \cdots\cdots \rightarrow n$$

$$\frac{x(x+1)}{(1-x)^3} = 0^2 x^0 + 1^2 \cdot x^1 + 2^2 \cdot x^2 + 3^2 \cdot x^3 \rightarrow n^2$$

$$\frac{x^3 + 4x^2 + x}{(1-x)^4} = 0^3 x^0 + 1^3 x^1 + 2^3 x^2 + 3^3 x^3 + 4^3 \cdot x^4 + \cdots \rightarrow n^3$$

$$G(x) = \sum_{n \geq 0} n^3 \cdot x^n$$

$$\frac{x^4+11x^3+11x^2+x}{(1-x)^5} = 0^4 \quad 1^4 \quad 2^4 \quad 3^4 \quad 4^4 \ldots\ldots$$

$$a_{n+1} = a_n - 100 \qquad a_0 = 50$$

$$\times\, x^{n+1}$$

$$\sum_{n\geq 0} a_{n+1} \cdot x^{n+1} = \sum_{n\geq 0} a_n \cdot x^{n+1} - 100 \sum_{n\geq 0} 1 \cdot x^{n+1}$$

$$\downarrow$$

$$\left(a_1 x^1 + a_2 x^2 + a_3 x^3 + \dots\right)$$

Q

In how many ways can 1000 identical pamphlets be distributed to five different counselling centers, where pamphlets are put in stacks of 50, such that each center receives at least 50 but no more than 500 pamphlets?

(a) 3876
(b) 3246
(c) 3300
(d) 3426

Q

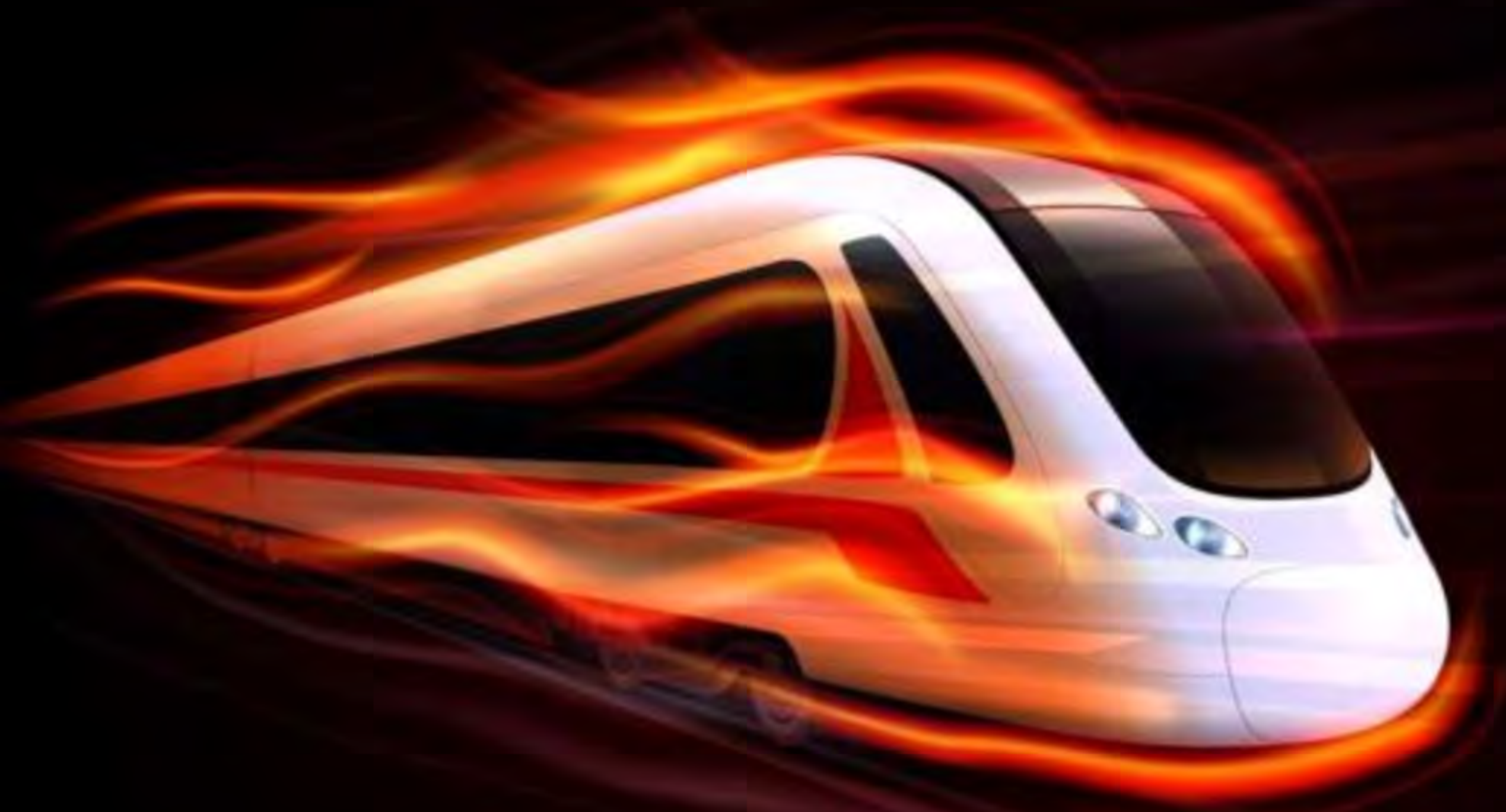
Determine the number of ways that \$12 in loonies can be distributed between a father's three children so that the eldest gets at least four dollars, the middle and youngest child are both guaranteed at least two dollars, but the youngest cannot receive any more than \$5 since he will spend it all on candy and rot his teeth

(a) 38
(b) 12
(c) 13
(d) 15

Q

A restaurant just closed for the night and they had an extra 12 orders of fries and 16 mini-desserts left over. The restaurant manger decides to split this left over food between the four employees closing that night. How can the manager do this so that the head chef receives at least one order of fries and exactly three mini-desserts, while the three other closing-staff are guaranteed at least two orders of fries but less than 5 desserts?

(a) 336
(b) 125
(c) 1333
(d) 1545



THANK YOU