CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No.2

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

Type-2:

logical equivalence:

1. $\begin{cases} p \wedge p \equiv p \\ p \vee p \equiv p \end{cases}$

2) $p \wedge T \equiv p$ $\begin{cases} p = T & T \wedge T \equiv T \\ p = F & F \wedge T \equiv F \end{cases}$

$p \vee F \equiv p$

3) $p \vee T \equiv T$

$p \wedge F \equiv F$

$A \equiv B$

Same behaviour

A	B
T	T
F	F
T	T
F	F

Commutative

$$\begin{cases} p \vee q \equiv q \vee p \\ p \wedge q \equiv q \wedge p \end{cases}$$

Associative.

$$\begin{cases} p \wedge (q \wedge R) \equiv (p \wedge q) \wedge R \\ p \vee (q \vee R) \equiv (p \vee q) \vee R \end{cases}$$

Distributive.

$$\begin{cases} a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c) \\ a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c) \end{cases}$$

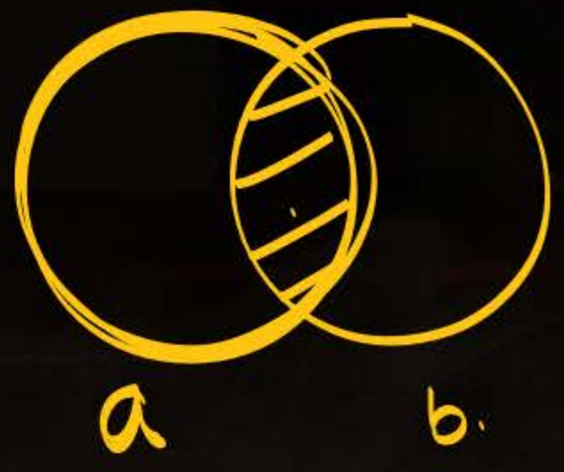
absorption law:

$$\begin{cases} a \vee (a \wedge b) \equiv a \\ a \wedge (a \vee b) \equiv a \end{cases}$$

logic	b.A	set
$\wedge$	$\cdot$	$\cap$
$\vee$	$+$	$\cup$
T	1	U
f	0	ϕ

$a \vee (a \wedge b) \equiv a.$

$a \cup (\underline{a \cap b}) \equiv a.$



$a \wedge (a \vee b) \equiv a.$

$A \cap (\underline{A \cup B}) \equiv A.$

<u>Demorgan's law:</u>

$$\left\{ \begin{array}{l} \neg(a \vee b) \equiv \neg a \wedge \neg b \\ \neg(a \wedge b) \equiv \neg a \vee \neg b. \end{array} \right.$$

$p \to q \equiv \neg p \vee q$

$\neg(p \to q) \equiv \neg(\neg p \vee q)$

$\equiv p \wedge \neg q$

$\neg(p \to q) \equiv p \wedge \neg q$

$p \to q \equiv \neg q \to \neg p$

$\neg p \to q \equiv p \vee q$

$(a \to b) \wedge (a \to c) \equiv a \to (b \wedge c)$

$(\boxed{\neg a \vee} b) \wedge (\boxed{\neg a \vee} c)$

$\neg a \vee (b \wedge c)$

$a \to (b \wedge c)$

$a \to b \equiv \neg a \vee b$

$a \to b \equiv \neg b \to \neg a$

$(a \to c) \wedge (b \to c) \equiv (a \vee b) \to c$

$(\neg a \vee c) \wedge (\neg b \vee c)$

$(\neg a \wedge \neg b) \vee c$

$\neg(a \vee b) \vee c$

$(a \vee b) \to c$

$(a \to b) \wedge (a \to c) \equiv a \to (b \wedge c)$

$(\boxed{\neg a \vee} b) \wedge (\boxed{\neg a \vee} c)$

$\neg a \vee (b \wedge c)$

$a \to (b \wedge c)$

$$\boxed{\begin{array}{l} a \to b \equiv \neg a \vee b \\ a \to b \equiv \neg b \to \neg a \end{array}}$$

$$(a \to b) \wedge (a \to c) \equiv a \to (b \wedge c)$$

$$(a \to c) \wedge (b \to c) \equiv (a \vee b) \to c$$

$$(a \to b) \vee (a \to c) \equiv a \to (b \vee c)$$

$$(a \to c) \vee (b \to c) \equiv (a \wedge b) \to c$$

$(p \vee q) \wedge \neg(\neg p \wedge q)$

$(p \vee q) \wedge \neg(\neg p \wedge q)$

Ans: p.

$\neg[\neg[(p \vee q) \wedge R] \vee \neg q]$

$((p \vee q) \wedge R) \wedge q$

$(p \vee q) \wedge R \wedge q$

$q \wedge (p \vee q) \wedge R$

$q \wedge R$

xOR $\rightarrow \underline{\vee}$

MSQ

$(p \wedge \neg q) \vee (\neg p \wedge q)$

$\neg(p \Leftrightarrow q) \equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$

$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$

$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

a) $p \Leftrightarrow q$

b) $\neg(p \Leftrightarrow q)$ ✓

c) $\neg p \Leftrightarrow q$

d) $p \underline{\vee} q$ ✓

$xor \rightarrow \veebar$

MSQ

$(p \wedge \neg q) \vee (\neg p \wedge q)$

a) $p \Leftrightarrow q$

b) $\neg (p \Leftrightarrow q)$ ✓

(c) $\neg p \Leftrightarrow q$

$(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$

$(p \vee q) \wedge (\neg q \vee \neg p)$

p	q	$\neg p$	$\neg p \Leftrightarrow q$
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	F

c) $\neg p \Leftrightarrow q$ ✓

d) $p \veebar q$ ✓

b, c, d

$$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)]$$

a) $p \vee q$

b) $\neg(p \vee q)$

c) $\neg p \vee \neg q$

d) nota.

$$\neg q \wedge (r \vee \neg q)$$

$$(p \to q) \wedge \frac{\neg q \wedge (\neg q \vee R)}{\neg q}$$

$$(\neg p \vee q) \wedge \neg q \equiv \neg q \wedge (\neg p \vee q)$$

$$\equiv (\neg q \wedge \neg p) \vee \underbrace{(\neg q \wedge q)}_{F} \equiv \neg q \wedge \neg p \equiv \neg(p \vee q)$$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$(\boxed{p \wedge} q) \vee (\boxed{p \wedge} \neg q) \vee (\neg p \wedge \neg q)$

$p \wedge (q \vee \neg q)$

$\frac{p \wedge T}{p} \quad \vee (\neg p \wedge \neg q)$

$p \vee (\neg p \wedge \neg q)$

$(p \vee \neg p) \wedge (p \vee \neg q)$

$T \wedge (p \vee \neg q)$

$(p \vee \neg q)$

P and Q are two propositions. Which of the following logical expressions are equivalent?

I. $P \vee \sim Q$

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III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(GATE-08)

(a) Only I and II

(b) Only I, II and III ✓

(c) Only I, II and IV

(d) All of I, II, III and IV

$(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

$p \wedge (q \vee \neg q)$

$p \wedge T$

$p \quad \vee (\neg p \wedge q)$

$p \vee (\neg p \wedge q) = (p \vee \neg p) \wedge (p \vee q)$

$T \wedge (p \vee q) \equiv p \vee q$

Establish the validity of the following arguments.

a) $[p \wedge (p \rightarrow q) \wedge (\neg q \vee r)] \rightarrow r$

b)
$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \neg r \\ \hline \therefore \neg (p \vee r) \end{array}$$

c)
$$\begin{array}{l} p \rightarrow q \\ r \rightarrow \neg q \\ r \\ \hline \therefore \neg p \end{array}$$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?(GATE - 04)

(a) P and Q only

(b) *P* and *R* only

(c) *P* and *S* only

(d) P, Q, R and S

d) $p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

e) $p \rightarrow (q \rightarrow r)$

$p \vee s$

$t \rightarrow q$

$\neg s$

$\therefore \neg r \rightarrow \neg t$

Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(GATE-15-Set1)

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

The Simplest form of $(p\wedge(\sim r \vee q \vee\sim q))\vee((r \vee t \vee\sim r)\wedge\sim q)$ is

(a) $p\wedge\sim q$

(b) $p\vee\sim q$ ✓

(c) t

(d) $(p\rightarrow\sim q)$

$$\left(p\wedge(\neg r\vee \boxed{q\vee\neg q})\right)\vee\left((r\vee t\vee\neg r)\wedge\neg q\right)$$

$$(\neg r\vee T) \qquad \boxed{r\vee\neg r}\vee t$$

$$p\wedge T \qquad \boxed{T\vee t}$$

$$p \qquad (T\wedge\neg q)$$

$$\neg q$$

Ans: $p\vee\neg q$

Let p, q, r and s be four primitive statements. Consider the following arguments:

P: $[(\sim p \lor q) \land (r \to s) \land (p \lor r)] \to (\sim s \to q)$

Q: $[(\sim p \land q) \land [q \to (p \to r)]] \to \sim r$

R: $[[(q \land r) \to p] \land (\sim q \lor p)] \to r$

S: $[p \land (p \to r) \land (q \lor \sim r)] \to q$

Which of the above arguments are valid?

(GATE - 04)

(a) P and Q only

(b) P and R only

(c) P and S only

(d) P,Q,R and S

The Simplest form of

$(\boxed{p \vee (p \wedge q)} \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$ is

Arrangement.

(a) $p \wedge t$ ✓

(b) $q \wedge t$

(c) $p \wedge r$

(d) $p \wedge q$

$\boxed{p \vee (p \wedge q)}$

$\dfrac{t \vee (t \wedge p \wedge r)}{t}$

$p \vee (p \wedge \boxed{q \wedge \neg R})$

p

$S_1 : \{(\sim p \rightarrow (q \rightarrow \sim W)) \wedge (\sim S \rightarrow q) \wedge \sim t \wedge (\sim p \vee t)\} \rightarrow (w \rightarrow s)$

$S_2 : \{(q \rightarrow t) \wedge (s \rightarrow r) \wedge (\sim q \rightarrow s)\} \rightarrow (\sim t \rightarrow r)$

The statement formula $\{(a \to c) \wedge (b \to d) \wedge (c \to \sim d)\} \to (\sim a \vee \sim b)$ is

(a) satisfiable but not-valid

(b) valid

(c) not satisfiable

(d) none of these

The statement formula

$\{((\sim p \vee q) \rightarrow r) \wedge (r \rightarrow (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \rightarrow \sim t)\} \rightarrow p$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these


THANK YOU GW
SOLDIERS !