CS & IT ENGINEERING

Discrete mathematics
Set theory

Lecture No.7

By- SATISH YADAV SIR

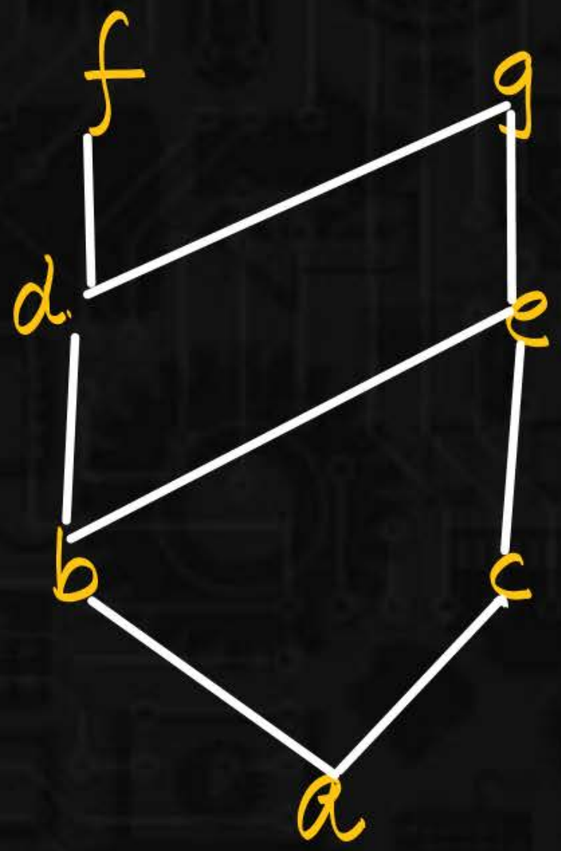
01 Greatest element

02 least element

03 Lower bound

04 Upper bound

05 GLB/LUB



(A, R) poset.

Greatest element (maximum element)

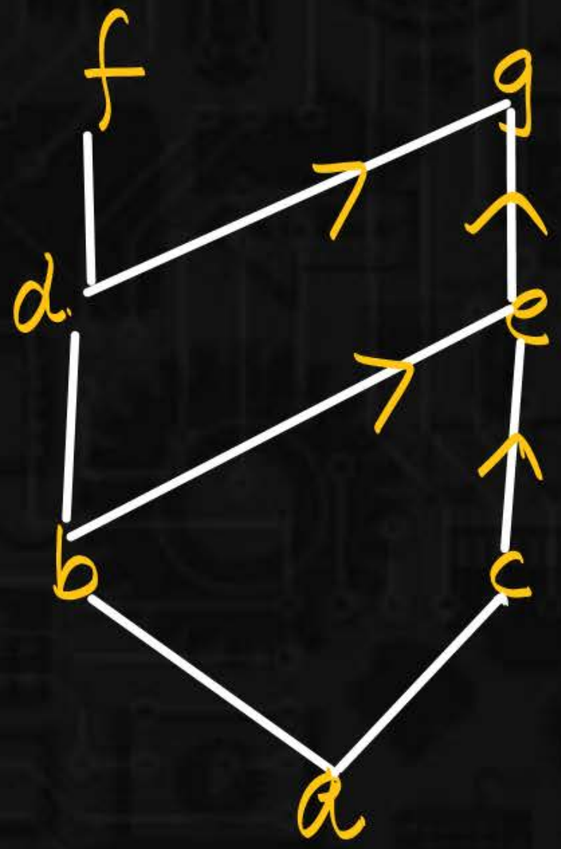
x is called greatest element.

all elements $\in A \leq x \in A$.

aRb

$a \leq b$

a is related to b.



(A, R) poset.

Greatest element (maximum element)

x is called greatest element.

all elements $\in A \leq x \in A$.

check for f. $(x = f)$

$a \leq f$ (T)

$b \leq f$ (T)

$c \leq f$ (F)

$a\,b\,c\,d\,e\,f\,g \leq f$

f is not GE.

G is not GE.

check for g $(x = g)$

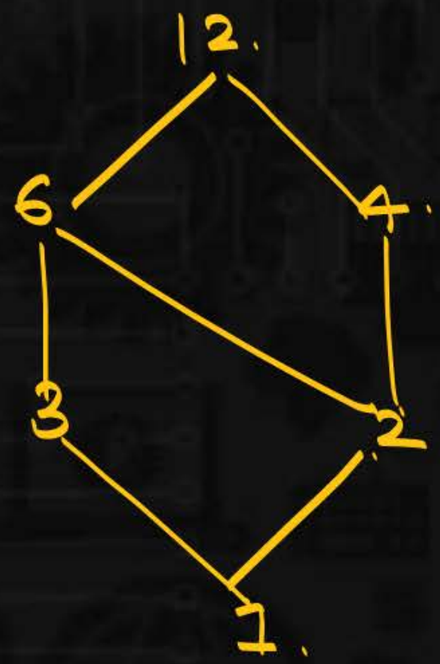
$a \leq g$ (T) $c \leq g$ (T)

$b \leq g$ (T) $d \leq g$ (T)

$e \leq g$ (T) $f \leq g$ (false)

$a\,b\,c\,d\,e\,f\,g \leq g$

$(D_{12}, |)$



$1\ 2\ 3\ 4\ 6\ 12 \leq 12.$

$1 \leq 12$

$2 \leq 12$

$3 \leq 12$

$4 \leq 12$

$6 \leq 12$

$12 \leq 12.$

$a R b \wedge b R a \rightarrow a = b.$

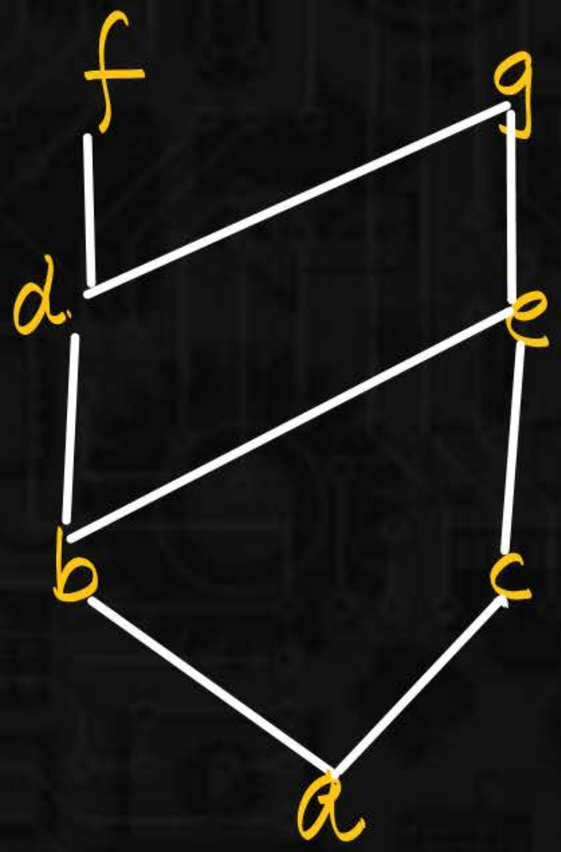
12 is GE

Thm: if GE exist then it will be unique.

Assumption: 2 GE n_1, n_2.

all elements $\leq$ GE	all element $\leq n_2$.
all elements $\leq n_1$	$\ldots n_1 \ldots \leq n_2$

$\ldots n_2 \ldots \leq n_1 \wedge n_1 \ldots \leq n_2 \rightarrow n_1 = n_2$



(A, R) poset.

least element (minimum) element.

x is called least element

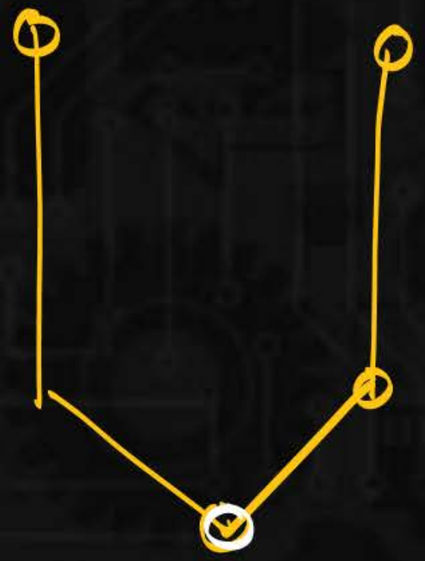
$x \in A \leq$ all elements $\in A$.

check for $(x = a)$ — a is least element.

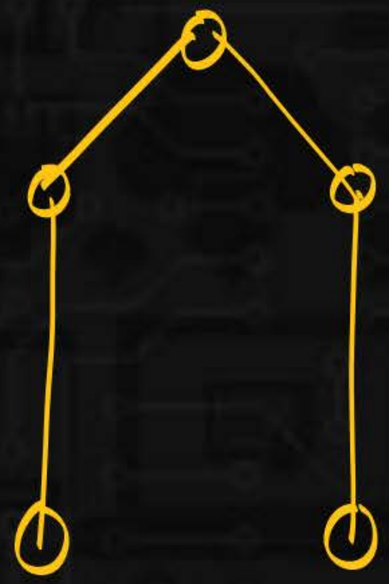
$a \leq abcdefg$

$a \leq a$ $a \leq c$
$a \leq b$ $a \leq d$
$a \leq e$ $a \leq f$
$a \leq g$

* if LE exist then it will be unique



LE: ✓
GE: ✗



GE: ✓
LE: ✗

GE: ✗
LE: ✗

GE: ✓
LE: ✓

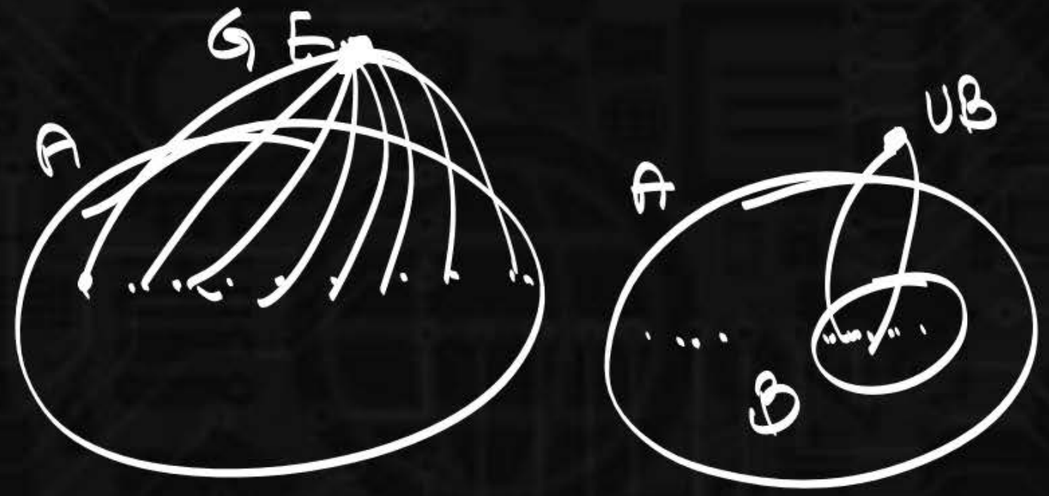
Upper bound :

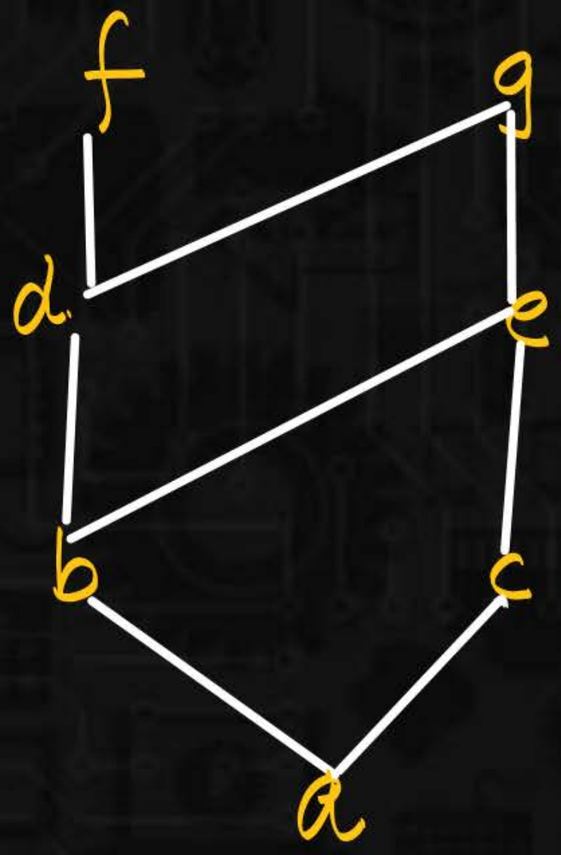
(A, R) poset.

$B \subseteq A$.

x is called upper bound of B.

all elements $\in B \leq x \in A$.





(A, R) poset

$B \subseteq A$.

all elements $\in B \leq x \in A$.

$B = \{b, c\}$

UB's of $\{b, c\}$ is $\{e, g\}$.

all elements $\in B \leq x \in A$

$bc \leq e$	$bc \leq g$
$b \leq e$	$b \leq g$
$c \leq e$	$c \leq g$

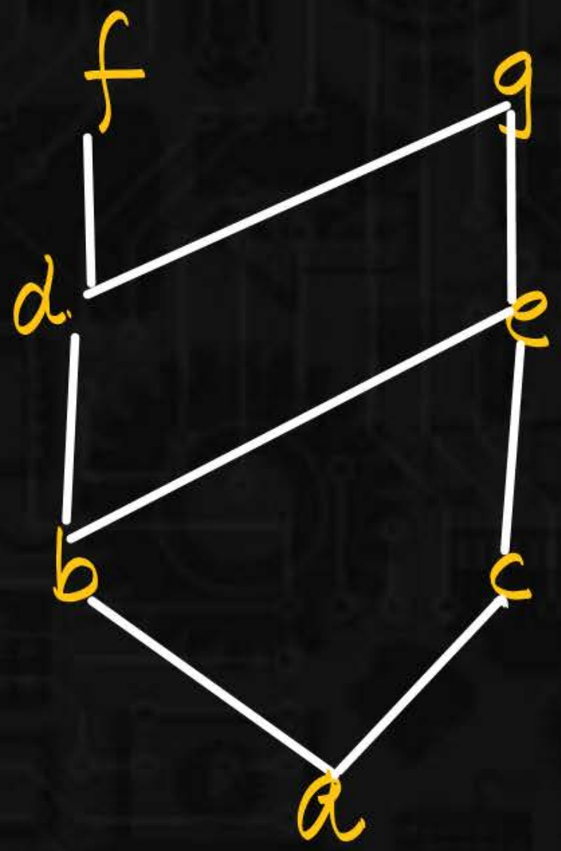
UB of $\{d, c\}$ is $\{g\}$

$\{a, f\} \rightarrow f$

$\{a, b\} \rightarrow \{b, e, g, d, f\}$

$B = \{a, c\}$
$ac \leq e$ ✓
$ac \leq g$ ✓
$ac \leq c$
$a \leq c$ ✓
$c \leq c$

UB's of $\{a, c\}$ is $\{c, e, g\}$.



(A, R) poset

$B \subseteq A$.

$x \in A \leq$ all elements $\in B$. (lower bound)

$B = \{d, e\}$

$b \leq d, e$	$a \leq de$.
$b \leq d$ ✓	$a \leq d$ ✓
$b \leq e$ ✓	$a \leq e$ ✓

LB's of $\{d, e\}$ is $\{a, b\}$.

$\{f, g\} \rightarrow \{d, b, a\}$

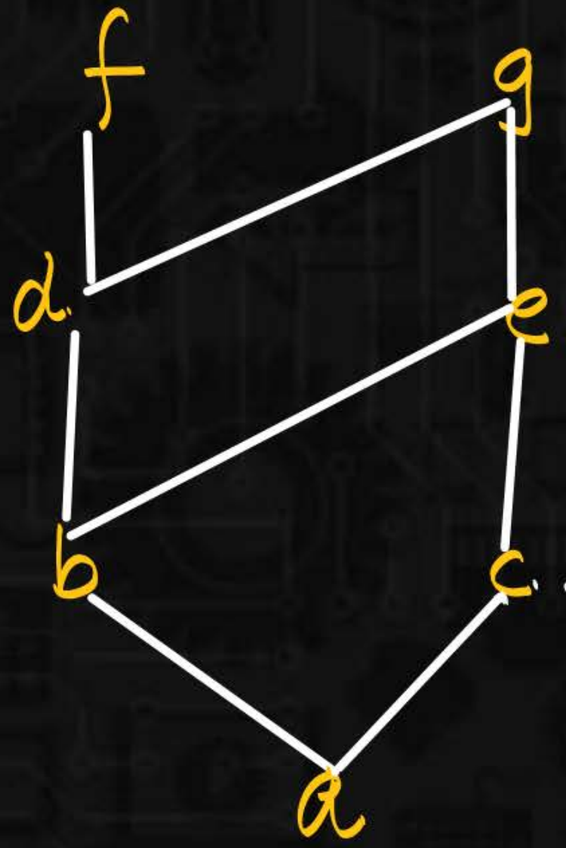
$\{d, g\} \rightarrow \{d, b, a\}$

$\{f, c\} \rightarrow a$.

$a \leq f$ c.

$a \leq f$

$a \leq c$.



c ← lub

least / upper bound (lub) → unique : (join)

all elements $\in B \leq x \in A$.

$x \in A \leq$ all UB's of B.

$B = \{a, c\}$

UB's of $\{a, c\} \rightarrow c, e, g$.

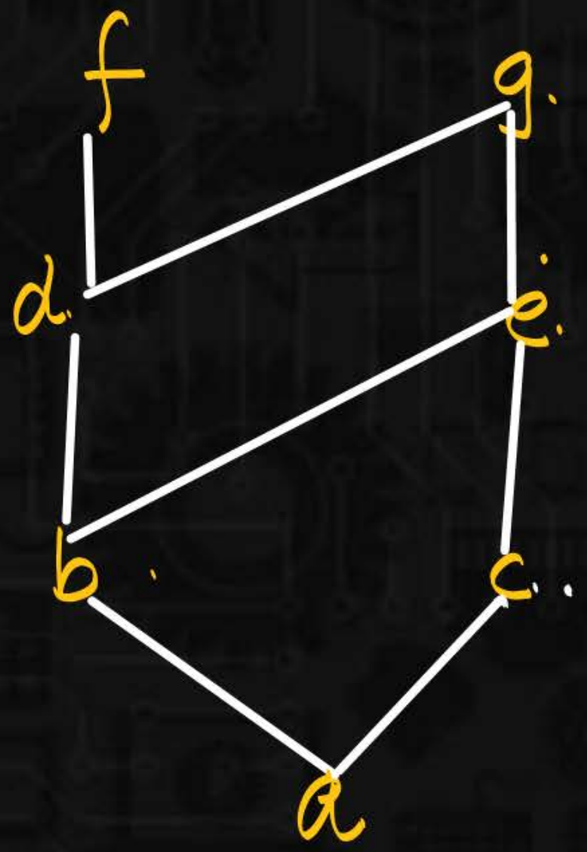
lub ⓒ

$c \leq ceg$	$e \leq ceg$	$g \leq ceg$
$c \leq c$	$e \leq c$ ✗	$g \leq c$ ✗
$c \leq e$		
$c \leq g$		

$B = \{b, c\} \rightarrow$ lub: e

$\{a, f\} \rightarrow f$

$\{b, e\} \rightarrow e$



greatest lower bound (glb) → unique. (meet)

$x \in A \leq$ all elements $\in B$.

all LB's of $B \leq x \in A$.

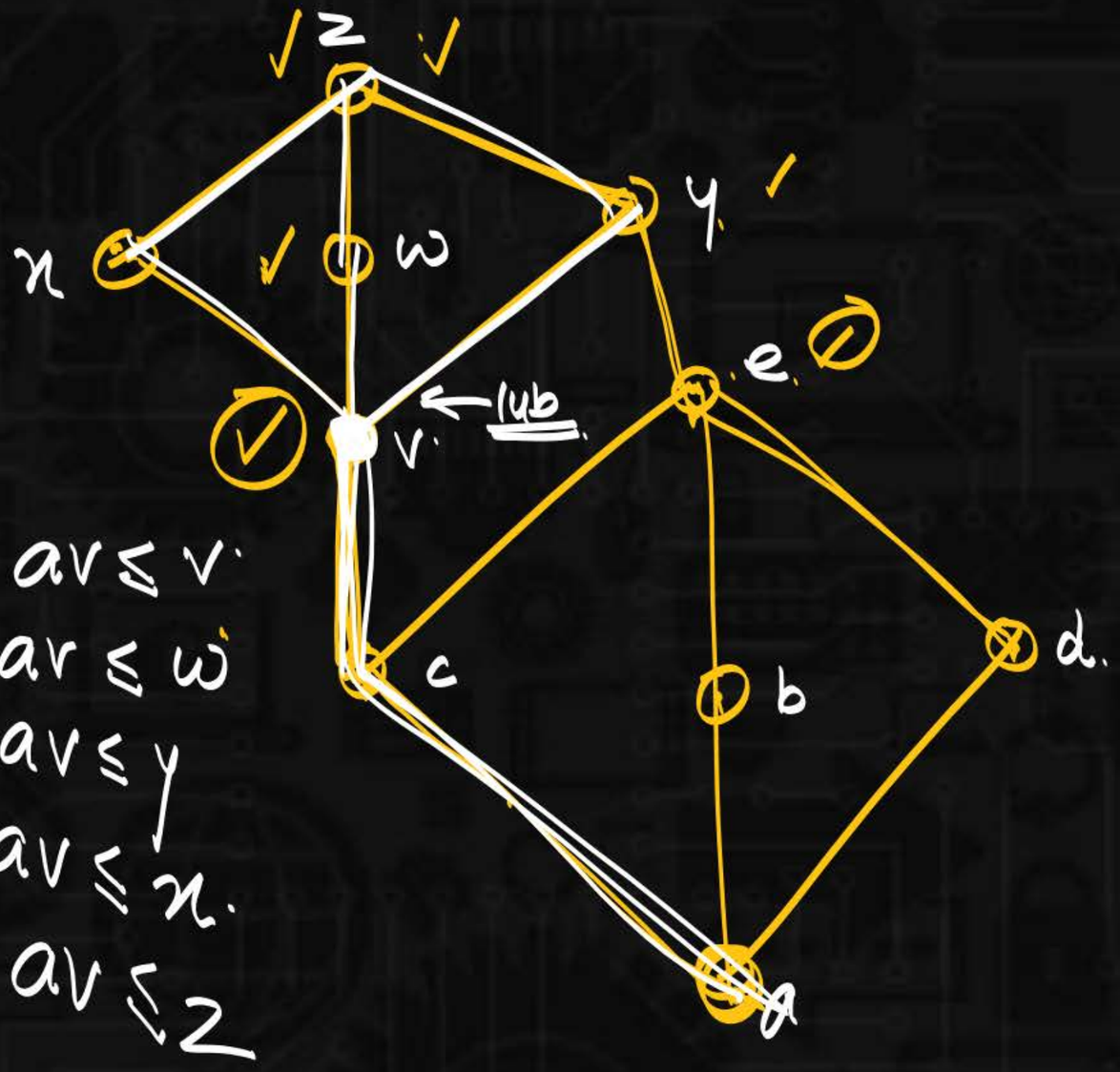
$B = \{g, e\}$.

↳ LB's of $\{g, e\} \rightarrow e, b, c, a$.

$abce \leq e$

$a \leq e$
$b \leq e$
$c \leq e$.
$e \leq e$.

$\{d, c\}$ glb → a



$av \leq v$
$av \leq w$
$av \leq y$
$av \leq x$
$av \leq z$

a) $a \sqcup b \{b, c\} \rightarrow a$.

b) $a \sqcup b \{b, w\} \rightarrow a$.

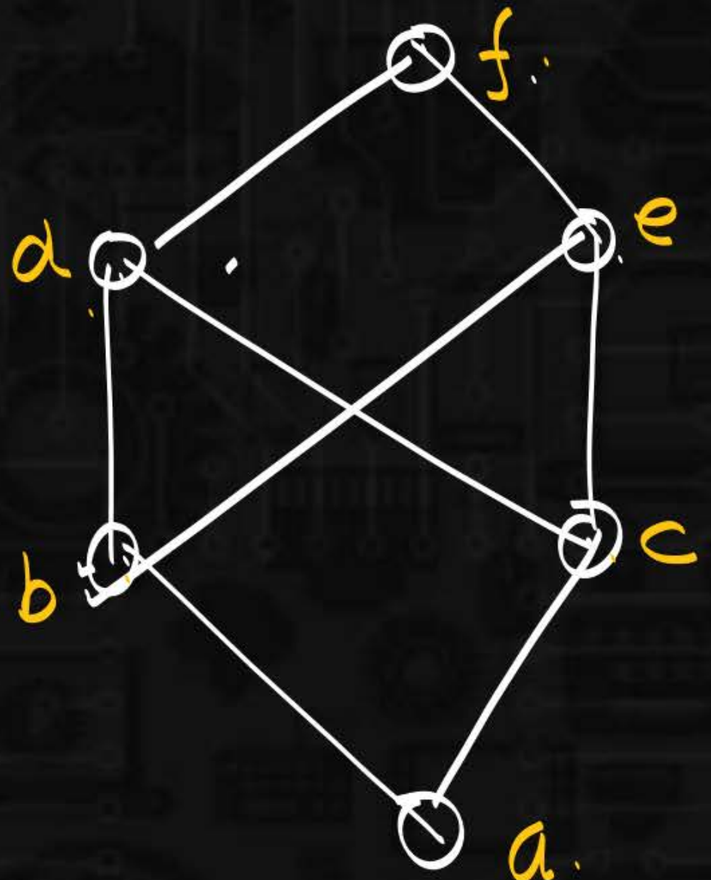
c) $a \sqcup b \{e, x\} \rightarrow c$.

d) $lub \{c, b\} \rightarrow e$

e) $lub \{d, x\} \rightarrow z$.

f) $lub \{c, e\} \rightarrow e$.

g) $lub \{a, v\} \rightarrow v$.



glb(d, e) → NA

lub {b, c} → N.A

d, e, f

Lub

x ∈ A ≤ all UB's of B

check d.	e ≤ def.	f ≤ def
d ≤ def ✓	e ≤ d ✗	✓
d ≤ d ✓		
d ≤ e. ✗		


THANK YOU GW
SOLDIERS !