CS & IT ENGINEERING

Discrete Mathematics

Combinatorics

Lecture No.- 06

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Recap of Previous Lecture

Topic Introduction to Combinatorics

Topics to be Covered

Topic Pigeonhole Principle

Topic : Combinatorics

atleast 2 : H1 H2 H3 H4. atleast 4 atleast 3

some holes → atleast 4. X.

some holes → atleast 3. X.

some holes → atleast 2.

Topic : Combinatorics

Pigeonhole-principle :

if $n+1$ pigeons, we want to distribute to n holes
then some holes contains atleast 2 pigeons :

Topic : Combinatorics

1442 flights are taking off in a single day @ busy airport prove that atleast 2 flights are taking off in single minute?

ID:

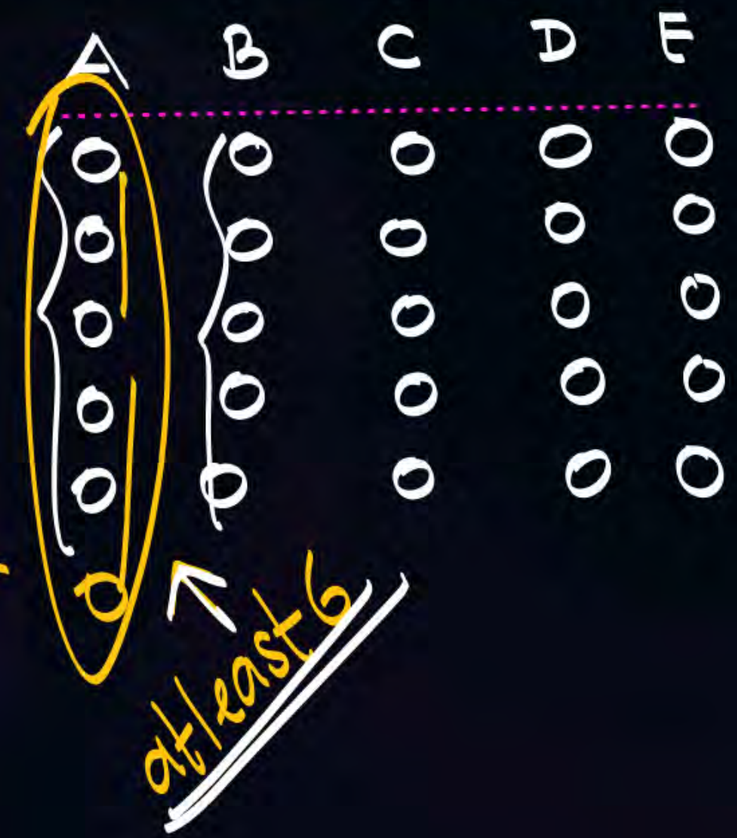
24

24×60

1440

Topic : Combinatorics

In School system, we have 5 diff grades, what will be min no. of students will require such that atleast 6 belongs to same grade ?.



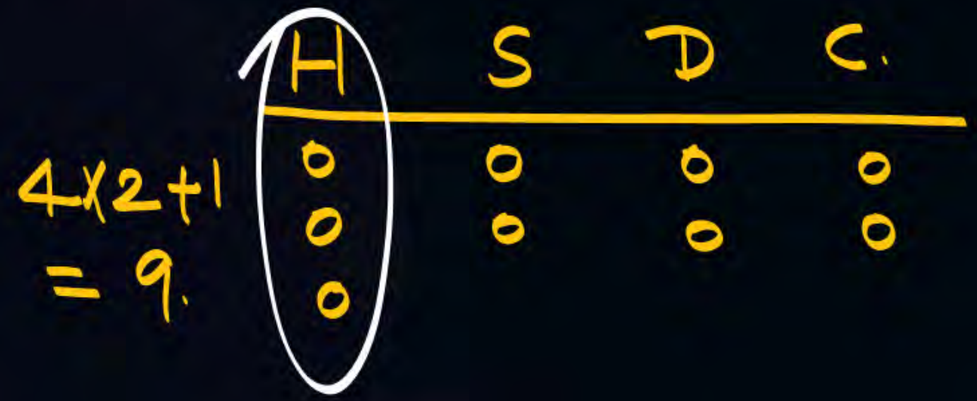
5×5
$= 25.$
$+1.$

$5 \times 5 + 1$
$= \underline{26.}$

Topic : Combinatorics

GATE :

what will be min. no. of cards will be widrawn such that atleast 3 belongs to same suite?



4 diff

4X2+1 = 9.

Topic : Combinatorics

Consider a Graph having 9 vertices. sum of degrees of all vertices are atleast 27. prove that some of vertices will have degree atleast 4.

True/false.

	v_1	v_2	v_3	v_4	v_9
9x3	0	0			0
+1	0	0			0
	0	0			0
	0				

atleast 4

sum of degrees of all.

atleast 27.

$\Sigma d(v_i) = 2e$

$= \text{even}$.

$= (\text{atleast } 27)$

$= (28$

Topic : Combinatorics

A bag contains 6 Red balls, 8 blue, 10 Green, 15 white. 20 Yellow

min no. of balls will be chosen randomly atleast 6 balls of same color?

6R	8 blue	10 G	15 W	20 Y

5×5
$+1$
$= 25+1$
$= 26$

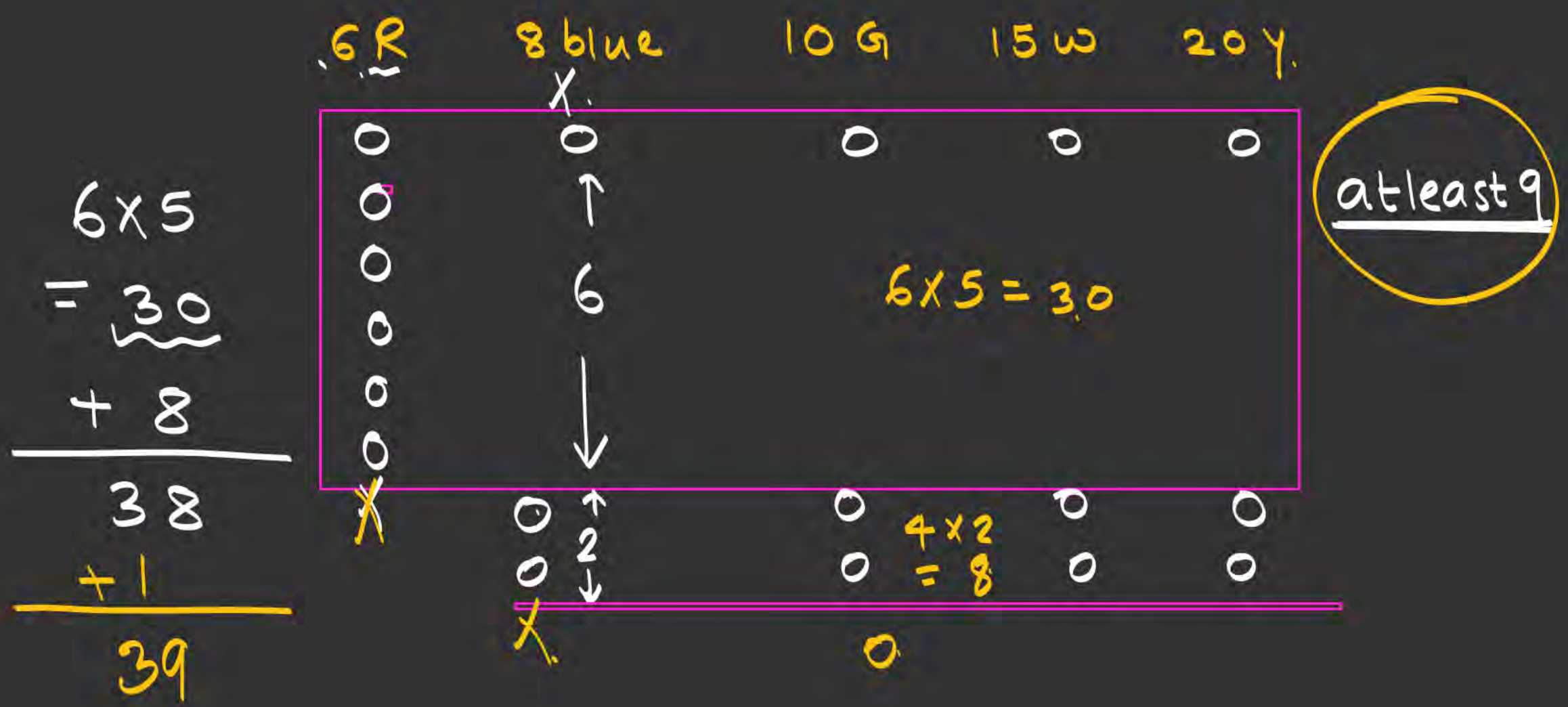
Topic : Combinatorics

A bag contains 6 Red balls, 8 blue, 10 Green, 15 white. 20 Yellow

min no. of balls will be chosen randomly

atleast 9 balls of same color ?

6R	8 blue	10 G	15 W	20 Y


6 R
8 blue
10 G
15 W
20 Y
X
0
0
0
0
0
0
0
0
0
0
0
0
0
0
6
6 X 5 = 30
X
0
0
0
0
0
2
4 X 2
= 8
0
0
0
0
X
0
6 X 5
= 30
+ 8
38
+ 1
39
atleast 9

Topic : Combinatorics

$S = \{0, 1 \ldots\ldots 9\}$

what is smallest positive integer k.

Such that any subset of S of size k.

contains 2 distinct subset of size two

$\{x_1, x_2\}$ $\{y_1, y_2\}$

such that

$x_1 + x_2 = y_1 + y_2 = 9$.

$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. 6 7 8 9.

subset $K = \{0, 1, 2, 3, 4, 5\}$ $|K| = ?$

Ans: 7

0 1 2 3 4 5

$\{x_1, x_2\}$ $\{y_1, y_2\}$

$x_1 + x_2 = y_1 + y_2 = 9$.

$K \geq 4$.

3. An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?

$26^2 + 1 = 677$

4. Let $S = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110?

Subdivide the set S into the 14 subsets: $\{3\}, \{7, 103\}, \{11, 99\}, \{15, 95\}, \ldots, \{43, 67\}$, $\{47, 63\}, \{51, 59\}, \{55\}$. By the Pigeonhole Principle if we select at least 15 elements of S then we must have the elements in one of the two-element subsets and these sum to 110.

20. How many times must we roll a single die in order to get the same score (a) at least twice? (b) at least three times? (c) at least n times, for $n \geq 4$?

20. **(a) 7** **(b) 13** **(c) $6(n-1)+1$**

24. Given 8 Perl books, 17 Visual BASIC† books, 6 Java books, 12 SQL books, and 20 C++ books, how many of these books must we select to insure that we have 10 books dealing with the same computer language?

24. **42**

1. Given a group of n women and their husbands, how many people must be chosen from this group of $2n$ people to guarantee the set contains a married couple?

1. $n + 1$.

There are 20 small towns in a region of west Texas. We want to get three people from one of these towns to help us with a survey of their town. If we go to any particular town and advertise for helpers, we know from past experience that the chances of getting three respondents are poor. Instead, we advertise in a regional newspaper that reaches all 20 towns. How many responses to our ad do we need to assure that the set of respondents will contain three people from the same town?

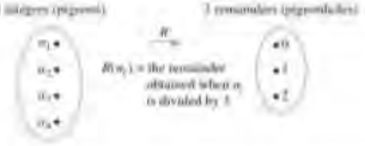
we need more than $2 \times 20 = 40$ responses.

which one is true ?

a.Given any set of four integers, must there be two that have the same remainder when divided by 3? Why?

b. Given any set of three integers, must there be two that have the same remainder when divided by 3?

Ans : a



21. Compute $\phi(n)$ for n equal to (a) 51; (b) 420; (c) 12300.

22. Compute $\phi(n)$ for n equal to (a) 5186; (b) 5187; (c) 5188.

21. (a) 32 (b) 96 (c) 3200

22. (a) $5186 = (2)(2593)$, and $\phi(5186) = (5186)(1/2)(2592/2593) = 2592$.
(b) $5187 = (3)(7)(13)(19)$, so $\phi(5187) = (5187)(2/3)(6/7)(12/13)(18/19) = (2)(6)(12)(18) = 2592$.
(c) $5188 = (2^2)(1297)$, and $\phi(5188) = (5188)(1/2)(1296/1297) = 2592$.
Hence $\phi(5186) = \phi(5187) = \phi(5188)$.

23. Let $n \in \mathbf{Z}^+$. (a) Determine $\phi(2^n)$. (b) Determine $\phi(2^n p)$, where p is an odd prime.

23. (a) 2^{n-1} (b) $2^{n-1}(p-1)$

25. How many positive integers n less than 6000 (a) satisfy $\gcd(n, 6000) = 1$? (b) share a common prime divisor with 6000?

25. (a) $\phi(6000) = \phi(2^4 \cdot 3 \cdot 5^3) = 6000(1-(1/2))(1-(1/3))(1-(1/5)) = 1600$.
(b) $6000 - 1600 - 1$ (for 6000) $= 4399$.

THANK - YOU