CS & IT ENGINEERING

Discrete maths
Mathematical logic

Lecture No 08

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TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

quantifier with I.R:

Universal specifications:

$$\forall x\, P(x) \rightarrow P(a)$$

True. a is for all.

all True

Thm → use

$$\sum d(v_i) = 2e$$

all graphs: → valid

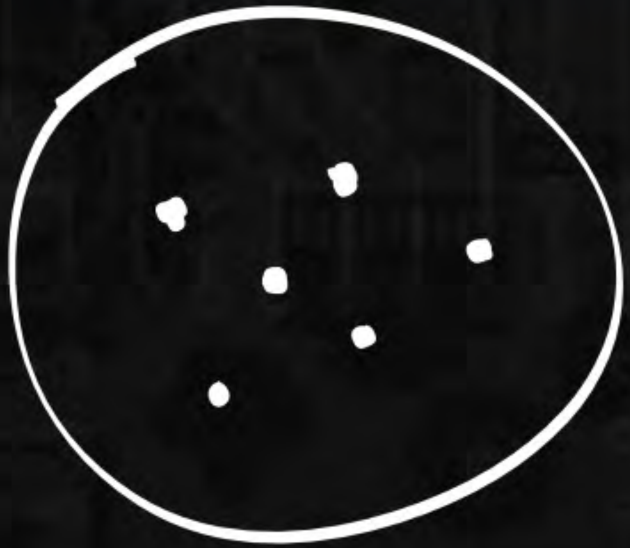
(True)

Domain:

G_1, G_2, G_3, G_4 ....

Universal Generalization :

proof :



$P(a) \longrightarrow \forall x\, P(x)$

a is for all.

True.

True.

Existential specification:

$$\exists x P(x) \rightarrow P(a)$$

True a is fixed.

True

$\exists x (x^2 = 4)$

0 -2 2

Existential Generalization.

$$P(a) \longrightarrow \exists x P(x)$$

a is fixed.

Valid?

$$\left[\forall x\, p(x) \wedge \forall x\,[p(x) \rightarrow Q(x)]\right] \rightarrow \forall x\, Q(x)$$

premise 1: $\forall x\, p(x)$

(U.S) $\downarrow$

2) $p(a)$ [a is for all]

premises: $\forall x\,[p(x) \rightarrow Q(x)]$

(U.S) $\downarrow$

3) $p(a) \rightarrow Q(a)$ [a is for all]

$p(a) \rightarrow 2$

U.G. $Q(a) \rightarrow$ [a is for all]

$\downarrow$

$\forall x\, Q(x)$

Invalid ✓

$$[\exists x\, p(x) \wedge \exists x\, [p(x) \rightarrow Q(x)]] \rightarrow \exists x\, Q(x) \qquad \text{valid?}$$

1. $\exists x\, p(x)$ (Given)

2. $p(a)$ [a is fixed] [E.S]

3. $\exists x\, [p(x) \rightarrow Q(x)]$ (Given)

$p(a) \rightarrow Q(a)$ [E.S / a is fixed]

$p(a)$

$Q(a)$ (False)

$a = 3$

$\exists x\, (x + 1 = 4)$

$\exists x\, (x^2 = 4)$

$a = 2$

$P \to Q$

P

Q

all → all

all

---

all

all → all ($a = 1..10$)

some ($a=2$)

---

some ($a = 2$)

some → some

some

✗

$$\frac{\begin{array}{c}\forall x\,[p(x) \to Q(x)] \\ \forall x\, p(x)\end{array}}{\forall x\, Q(x)}$$

$$\frac{\begin{array}{c}\forall x\,[p(x) \to Q(x)] \\ \exists x\, p(x)\end{array}}{\exists x\, Q(x)}$$

$$\frac{\begin{array}{c}\exists x\,[P(x) \to Q(x)] \\ \exists x\, p(x)\end{array}}{\times}$$

1) $\forall x\,[P(x) \rightarrow Q(x)]$

1A) $P(a) \rightarrow Q(a)$ [a is for all]

2) $\exists x\, P(x)$

2A) $P(a)$ [a is fixed]

1A) $P(a) \rightarrow Q(a)$ [all]

2A) $P(a)$ [fixed]

$Q(a)$ [fixed]

$\downarrow$

$\exists x\, Q(x)$ [E.G]

→ All the students in class will go to IIT (True)

→ some students in class are social Influencer (True)

those who are influencer class will also go to IIT.

$$1.\ \forall x\,[C(x) \rightarrow IIT(x)]\ \checkmark$$

$$2.\ \exists x\,[C(x) \wedge Reel(x)]\ \checkmark$$

$$\exists x\,[Reel(x) \wedge IIT(x)].$$

$\forall x\,[C(x) \rightarrow IIT(x)]$

$\exists x\,[C(x) \wedge Reel(x)]$

$\exists x\,[Reel(x) \wedge IIT(x)]$

1. $\forall x\,[C(x) \rightarrow IIT(x)]$

$C(a) \rightarrow IIT(a)$ [a is for all. $(a = 1 \ldots 10)$

3) $C(a)$ $(a = 2)$

$IIT(a)$ [a is fixed $a = 2$]

2. $\exists x\,[C(x) \wedge Reel(x)]$

2A) $C(a) \wedge Reel(a)$ [a is fixed $(a = 2)$

simplification.

3) $C(a)$ $[a = 2]$

4) $Reel(a)$ $[a = 2]$

$IIT(a) \wedge R(a)$

$\exists x\,[IIT(x) \wedge R(x)]$

all doctors are College graduates. — $\forall x\,[D(x) \rightarrow G(x)]$

Some doctors are not golfers. — $\exists x\,[D(x) \wedge \neg GL(x)]$

Invalid

Some golfers are not college graduates. — $\exists x\,[GL(x) \wedge \neg G(x)]$

$\exists x\,[GL(x) \wedge \neg G(x)]$

1) $\forall x\,[D(x) \rightarrow G(x)]$

$D(a) \rightarrow G(a)$ (a is forall)

$D(a)$ (fixed)

$G(a)$

2) $\exists x\,[D(x) \wedge \neg GL(x)]$

$D(a) \wedge \neg GL(a)$ [a is fixed]

$D(a)$ $\quad$ $\neg GL(a)$

$G(a) \wedge \neg GL(a)$

$\exists x\,[G(x) \wedge \neg GL(x)]$

all doctors are college graduates.

Some doctors are not golfers.

Some golfers are not college graduates.

$\exists x [GL(x) \wedge \neg G(x)]$

1. $\forall x [D(x) \rightarrow G(x)]$
2. $D(a) \rightarrow G(a)$
3. $\exists x [D(x) \wedge \neg GL(x)]$
4. $\underbrace{D(a) \wedge \neg GL(a)}$ (fixed)
5. $D(a)$ | simpl.
6. $\neg GL(a)$ | simpl.

2 & 5) $D(a) \rightarrow G(a)$ (m.p)
$D(a)$

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7) $G(a)$

6 & 7:

$\neg GL(a) \wedge G(a)$

$\exists x [\neg GL(x) \wedge G(x)]$

P_1: $P \wedge Q$

OR.

P

$\wedge$

Q

$(a \wedge b) \wedge (c \wedge d)$

P_1: $a \wedge b$

P_2: $c \wedge d$.

a

$\wedge$ b

$\wedge$ c

$\wedge$ d.

1. $\forall x\,[P(x) \rightarrow (Q(x) \wedge S(x))]$

2. $P(a) \rightarrow Q(a) \wedge S(a)$

3. $\forall x\,[P(x) \wedge R(x)]$

$P(a) \wedge R(a)$

$P(a)$

$R(a)$

$$\frac{\begin{array}{c}\forall x\,[P(x) \rightarrow (Q(x) \wedge S(x))] \\ \forall x\,[P(x) \wedge R(x)]\end{array}}{\forall x\,[R(x) \rightarrow S(x)]}$$

$$\frac{\begin{array}{c}P(a) \rightarrow (Q(a) \wedge S(a)) \\ P(a)\end{array}}{Q(a) \wedge S(a)}$$

$Q(a)$

$S(a)$

$$\frac{\begin{array}{c}\forall x\,[P(x) \vee Q(x)] \\ \forall x\,[\neg P(x) \wedge Q(x) \rightarrow R(x)]\end{array}}{\forall x\,[\neg R(x) \rightarrow P(x)]}$$

$\forall x\,[P(x) \vee Q(x)] \rightarrow P(a) \vee Q(a)$

$\forall x\,[\neg P(x) \wedge Q(x) \rightarrow R(x)] \rightarrow \neg P(a) \wedge Q(a) \rightarrow R(a)$

$\neg(\neg P(a) \wedge Q(a)) \vee R(a)$

$P(a) \vee \neg Q(a) \vee R(a)$

$\neg Q(a) \vee [P(a) \vee R(a)]$

$Q(a) \vee P(a)$

$\forall x\,[P(x) \vee Q(x)]$

$\forall x\,[\neg P(x) \wedge Q(x) \rightarrow R(x)]$

$\forall x\,[\neg R(x) \rightarrow P(x)]$

$P(a) \vee R(a)$

$\forall x\,[p(x) \vee q(x)]$

$\exists x\ \neg p(x)$

$\forall x\,[\neg q(x) \vee r(x)]$

$\forall x\,[S(x) \to \neg R(x)]$

$\exists x\ \neg S(x)$

1. $P(a) \vee q(a)$ [all]

$\neg P(a)$ [fixed]

$q(a)$ [fixed]

$\neg q(a) \vee R(a)$ [all]

$R(a)$ [fixed]

$\neg S(a) \vee \neg R(a)$ [all]

$\neg S(a)$ (fixed) $\to \exists x \neg S(x)$

all $\vee$ all

$\neg$all

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all

$$\begin{array}{l} \forall x\,[p(x) \vee q(x)] \\ \forall x\,[\neg p(x) \wedge q(x) \to R(x)] \\ \hline \forall x\,[\neg R(x) \to p(x)] \end{array}$$

$$\begin{array}{l} P \vee Q \\ \neg p \wedge q \to R. \\ \hline \neg R \Rightarrow P. \end{array}$$

$$\left\{ \begin{array}{l} p(a) \vee Q(a) \\ \neg p(a) \wedge q(a) \to R(a) \\ \hline \neg R(a) \to p(a) \end{array} \right.$$

$$\neg\left(\forall x \forall y \left[(x < y) \rightarrow \exists z (x < z < y) \right]\right)$$

$$\neg\left(\forall x \forall y \left[\neg(x < y) \vee \exists z (x < z < y) \right]\right)$$ God's.

$$\exists x \exists y \left(\neg(\neg(x < y)) \wedge \neg \exists z (x < z < y) \right]$$

$$\exists x \exists y \left((x < y) \wedge \forall z \left(x \geq z \text{ OR } z \geq y \right) \right)$$


THANK YOU GW
SOLDIERS !