CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 03

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TOPICS TO BE COVERED

01 sum rule

02 Product rule

03 Practice

COMBINATORICS

$= a^3b^0 + 3a^2b + 3ab^2 + a^0b^3$

$(a+b)^2 = 1.a^2 + 2ab + 1.b^2$

$ab + ba$

$(a+b)^3 = 1.a^3 + 3a^2b + 3ab^2 + b^3$

$\{a\}\ \{bc\}$
$\{b\}\ \{ac\}$

$\boxed{{}^nC_r = {}^nC_{n-r}}$

$(a+b) \times (a+b)$

a^2

$= {}^3C_0 a^3 b^0 + {}^3C_1 a^2 b^1 + {}^3C_2 a b^2 + {}^3C_3 a^0 b^3$

$\{a, b, c\}$

$(a+b) \times (a+b) \times (a+b)$

$(a+b) \times (a+b) \times (a+b)$

× power will be 3

${}^3C_1 \rightarrow$ out of 3 boxes how many ways we can take 1 b out.

* power a ↓
* power of b ↑

${}^3C_1 = {}^3C_2$

COMBINATORICS

$$(a+b)^3 = {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3.$$

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 \cdots {}^nC_n\, a^0 b^n.$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i\, a^{n-i} \cdot b^i$$

→ no. of ways to take b^i at

COMBINATORICS

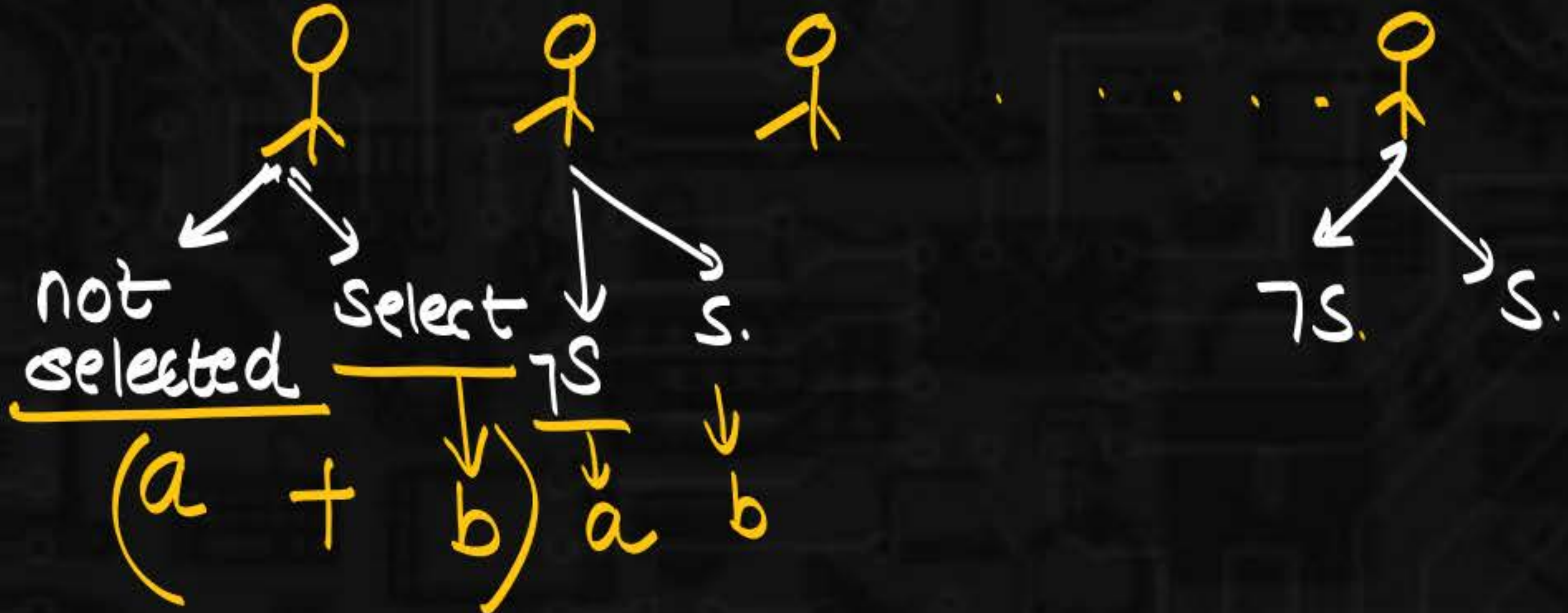
$^{n}C_{i} \cdot a^{n-i} \cdot b^{i}$

$^{10}C_{6}$

$b^{i} = b^{6}$.
$i = 6$. $n = 10$.

How many ways to select 6 students in a class of 10 ?

$\rightarrow$ $^{10}C_{6}$.



$(a+b)\times(a+b)\times(a+b)\cdots\cdots(a+b)$

$b \times b \times b$

$(a+b)^{10}$

Selecting 6 students is same as finding coefficient of b^{6}.

COMBINATORICS

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \, a^{n-i} \cdot b^i$$

$a=1 \quad b=1.$

$$2^n = \sum_{i=0}^{n} {}^nC_i \, (1)^{n-i} (1)^i$$

$$2^n = \sum_{i=0}^{n} {}^nC_i$$

$$2^n = {}^nC_0 + {}^nC_1 \cdots {}^nC_n$$

$a=1 \quad b=2$

$$(1+2)^n = \sum_{i=0}^{n} {}^nC_i \, a^{n-i} \cdot b^i$$

$$3^n = \sum_{i=0}^{n} {}^nC_i \cdot (1)^{n-i} (2)^i$$

$$\boxed{3^n = \sum_{i=0}^{n} {}^nC_i \cdot 2^i}$$

$a=1 \quad b=-1.$

$$(1+(-1))^n = \sum_{i=0}^{n} {}^nC_i \cdot (1)^{n-i} (-1)^i$$

0

$$0 = \sum_{i=0}^{n} {}^nC_i \cdot (-1)^i$$

$$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3$$

$${}^nC_0 + {}^nC_2 + {}^nC_4 \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5$$

COMBINATORICS

Pascal's identity:

$${}^3C_0 + {}^3C_1 = {}^4C_1$$

$$\boxed{{}^nC_x + {}^nC_{x+1} = {}^{n+1}C_{x+1}}$$

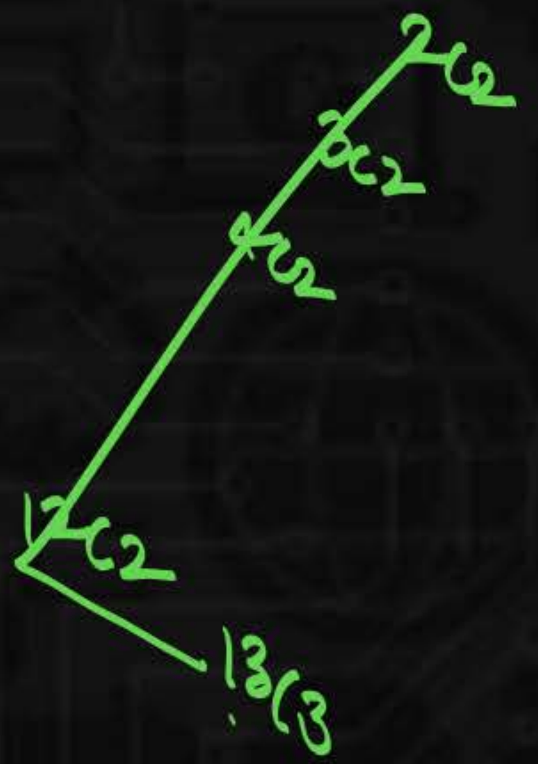
1

$2 \qquad 1 \;\; {}^2C_2$

$1 \;\; {}^3C_0 \qquad 3 \;\; {}^3C_1 \qquad 3 \;\; {}^3C_2 \qquad 1 \;\; {}^3C_3$

$1 \;\; {}^4C_0 \qquad 4 \; ({}^4C_1) \qquad 6 \; ({}^4C_2) \qquad 4 \; ({}^4C_3) \qquad 1 \; ({}^4C_4)$

$1 \qquad 5 \qquad 10 \qquad 10 \;\; {}^5C_3 \qquad 5 \qquad 1$



COMBINATORICS

$x_1 + x_2 + x_3 + a = \underline{10}$

${}^{13}C_3$

$x_1 + x_2 + x_3 \le 10$

$\underline{{}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + \cdots {}^{12}C_2}$

$= {}^{13}C_3$

$x_1 + x_2 + x_3 = 0 \quad {}^{2}C_2$

${}^{3}C_2$

$\vdots$

$10 \quad {}^{12}C_2$

$(x+y)^{50}$ what will be coefficient of x^{30} $= {}^{50}C_{20}$

$${}^{50}C_{20}\, x^{30}\, y^{20}$$

$(2x+3y)^{50}$ what will be coefficient of x^{30} $= {}^{50}C_{20}\, 2^{30} \cdot 3^{20}$

$${}^{50}C_{20}\, (2)^{30} 3^{20}\, x^{30}\, y^{20}$$

COMBINATORICS

$$^{n}C_{K} = \frac{n!}{K! \times (n-K)!}$$

$$= \frac{n \cdot (n-1)(n-2)\cdots(n-K+1)\cancel{(n-K)!}}{K! \times \cancel{(n-K)!}}$$

$$= \frac{n \cdot (n-1)(n-2)\cdots(n-K+1)}{K!}$$

$$^{n}C_{K} = \frac{n \cdot (n-1)\cdots\cdots(n-K+1)}{K!}$$

COMBINATORICS

Extended binomial coefficient:

$$ {}^{-n}C_K = \frac{(-n)(-n-1)(-n-2)\cdots(-n-K+1)}{K!} = $$

$$ = \frac{(-n-K+1)\cdots\cdots(-n-2)(-n-1)\cdot(-n)}{K!} $$

take -1 common.

$$ = (-1)^K \frac{(n+K-1)\cdots(n+2)(n+1)(n)}{K!} \frac{(n-1)!}{(n-1)!} = \frac{(-1)^K (n+K-1)!}{K!\cdot(n-1)!} $$

$$ \boxed{{}^{-n}C_K = (-1)^K \, {}^{n+K-1}C_K} $$

COMBINATORICS

-11_{C_2}

-11_{C_3}

$-14_{C_3} = -$

$-1_{C_0} = 1$

$-1_{C_1} = -1$

$-1_{C_2} = 1.$

$-11_{C_2} = -n_{C_k}$ $\begin{cases} n = 11 \\ k = 2. \end{cases}$

$\begin{cases} k \to \text{even} & \text{Total coefficient } (+ve) \\ k \to \text{odd} & \text{''} \quad \text{''} \quad (-ve) \end{cases}$

$(-1)^k \; {}^{n+k-1}C_k.$

$(-1)^{2} \; {}^{11+2-1}C_{2}.$

$+{}^{12}C_2$

$-11_{C_3} = -n_{C_k}$ $\quad n = 11$, $k = 3$

$(-1)^k \; {}^{n+k-1}C_k$

$(-1)^3 \; {}^{11+3-1}C_3$

$-{}^{13}C_3$

COMBINATORICS

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$$

$$\frac{1}{1-x} = (1-x)^{-1} = (a+b)^n$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \cdot b^i$$

$n = -1 \quad a = 1 \quad b = (-x)$

$$= {}^nC_0 \, a^{n-0} b^0 + {}^nC_1 \, a^{n-1} b^1$$

-ve   -ve   -ve   -ve

$$= {}^{-1}C_0 (1)^{-1-0} (-x)^0 + {}^{-1}C_1 (1)^{-1-1} (-x)^1 + {}^{-1}C_2 (1)^{-1-2} (-x)^2 + {}^{-1}C_3 (1)^{-1-3} (-x)^3$$

$$= 1 \times 1 \times 1 + (-1) \times 1 (-x) + (1)(1)(-x)^2$$

$$1 + x + x^2 + x^3 + x^4 + \dots$$

COMBINATORICS

$$\frac{1}{1-ax} = (1-ax)^{-1}$$

$$= {}^{-1}C_0 (1)^{-1-0} (-ax)^0 + {}^{-1}C_1\, 1^{-1-1} (-ax)^1 + {}^{-1}C_2\, 1^{-1-2} (-ax)^2$$

+ve, -ve, -ve, +ve, +ve

1. 1.

$$= 1 + ax + (ax)^2 + (ax)^3 + (ax)^4 + \ldots\ldots$$

$$\frac{1}{1-ax} = 1 + (ax) + (ax)^2 + (ax)^3 + (ax)^4 + \ldots\ldots$$

COMBINATORICS

$$\frac{1}{1-ax} = 1 + (ax)^1 + (ax)^2 + (ax)^3 + (ax)^4 + \cdots$$

$a = 1$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \cdots$$

$a = 2$

$$\frac{1}{1-2x} = 1 + 2x + (2x)^2 + (2x)^3 + (2x)^4 + \cdots$$

$a = 3$

$\cdots\cdots$

$a = -1$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 \cdots$$

$a = -2$

$$\frac{1}{1+2x} = 1 - 2x + (2x)^2 - (2x)^3 + (2x)^4 \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \cdot$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = 0 + 1 + 2x + 3x^2 + 4x^3 + \ldots$$

$$= \frac{-1}{(1-x)^2}\frac{d}{dx}\left(-x\right)$$

$$= \frac{+1}{(1-x)^2}$$


THANK YOU GW
SOLDIERS !