1500 series

CS & IT ENGINEERING

Discrete Mathematics

Lecture No.- **03**

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Recap of Previous Lecture

Topic Derangement

Topics to be Covered

Topic Pigeonhole – Principle

Pigeonhole → Analysis → Aptitude

→ Number Theory

→ Geometry

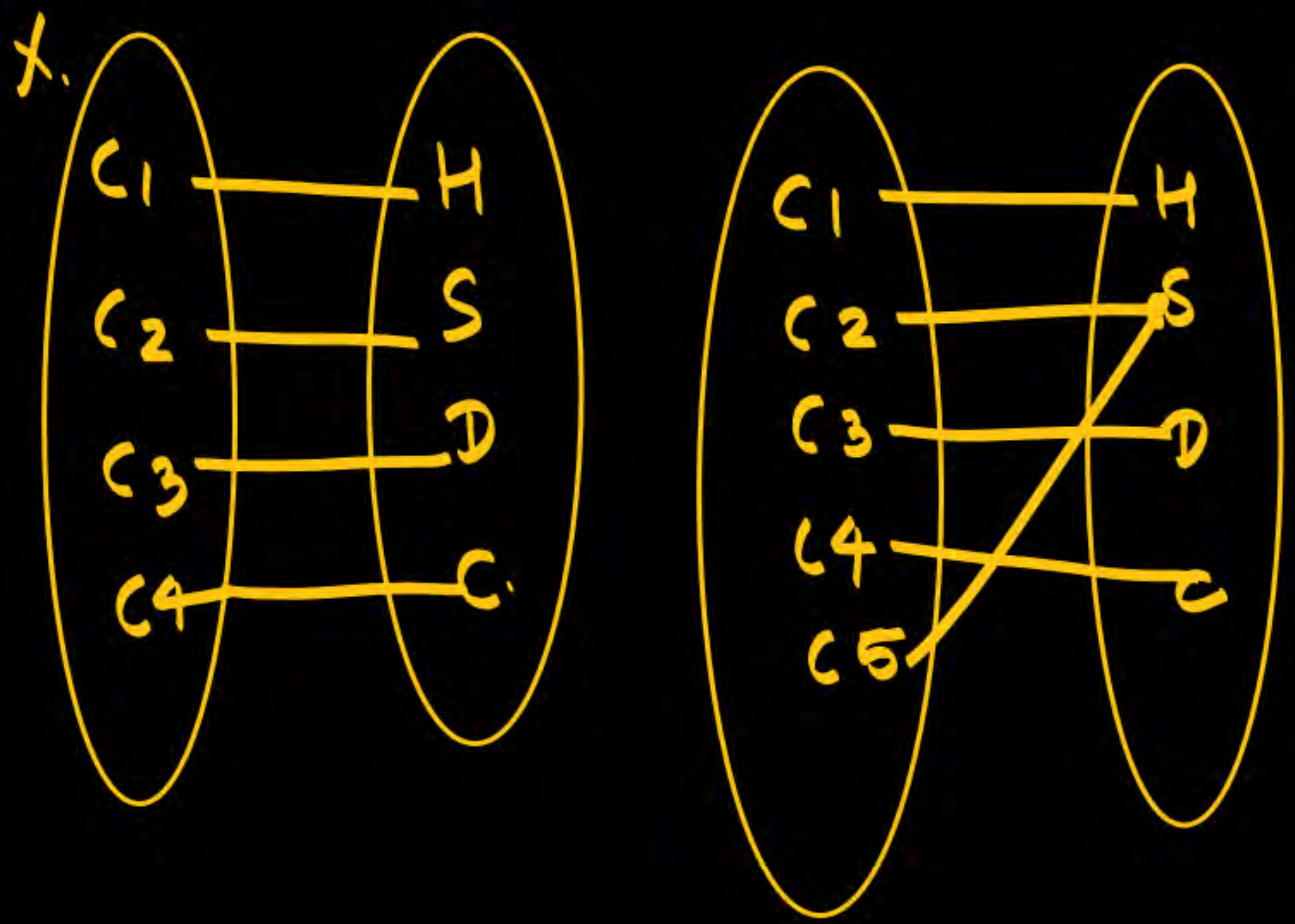
Q

S1 .If 4 cards are selected from a standard 52-card deck, must at least 2 be of the same suit?

S2. If 5 cards are selected from a standard 52-card deck, must at least 2 be of the same suit?

H S D C

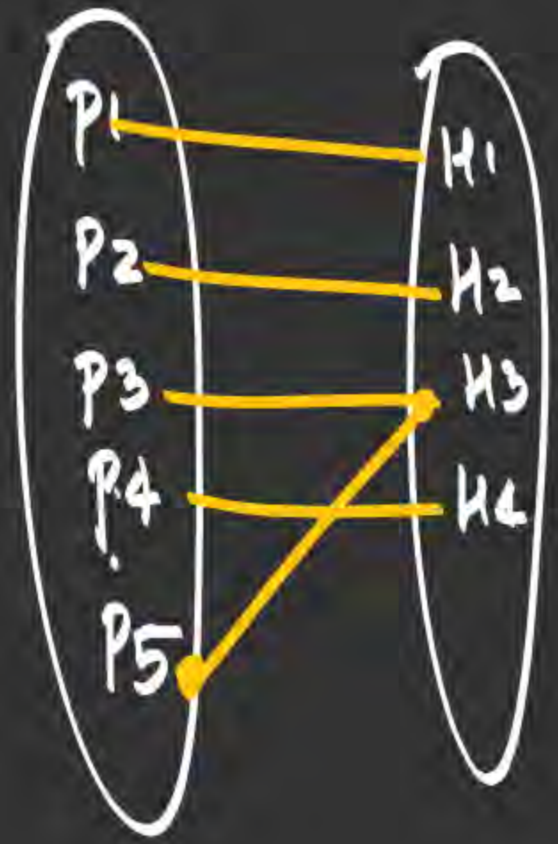
(a) only S1 is valid
(b) only S2 is valid ✓
(c) Both S1 and S2 are valid
(d) Both S1 and S2 are invalid



Pigeonhole → onto:

f : pigeons → holes.

pigeons > holes.



Q

S1 .Given any set of four integers, must there be two that have the same remainder when divided by 3? (T) →

Remainder:
(0, 1, 2)

S2. Given any set of three integers, must there be two that have the same remainder when divided by 3?

0	1	2
e1	e2	e3
e4	e5	e6
e7		

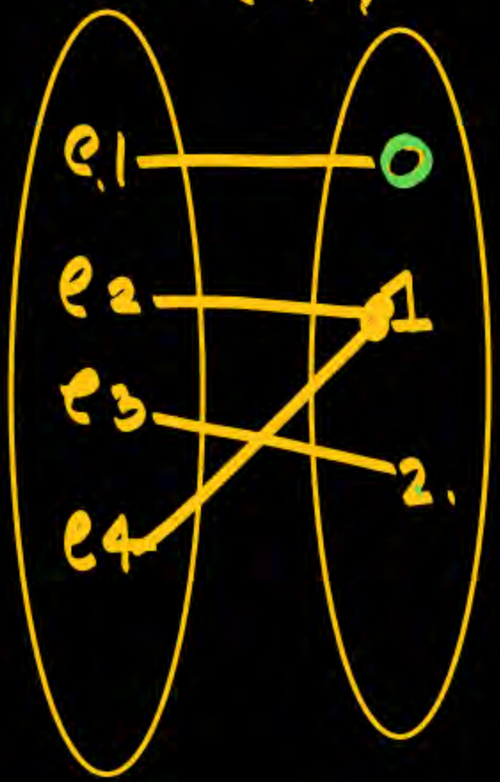
(F)

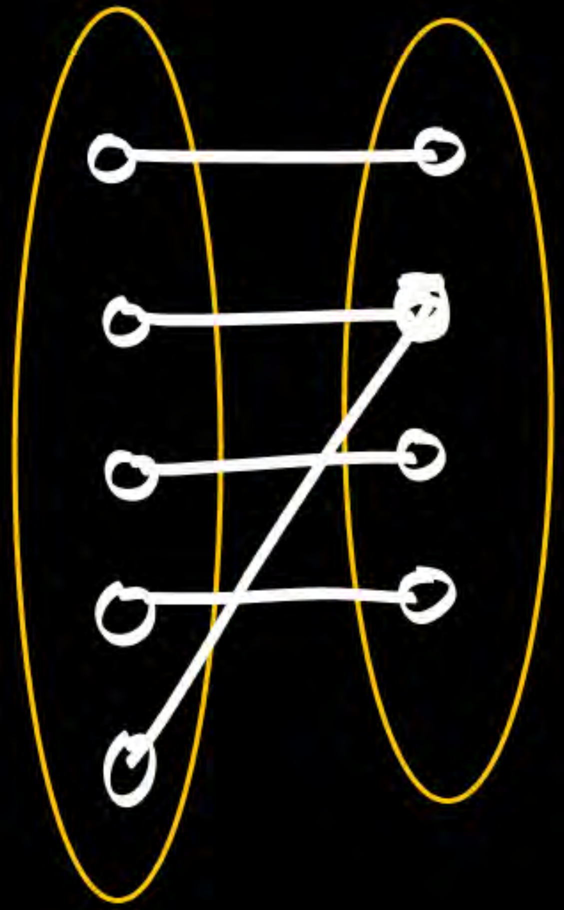
(a) only S1 is valid ✓

(b) only S2 is valid

(c) Both S1 and S2 are valid

(d) Both S1 and S2 are invalid





Suit

(GATE-05)

what is min no. of order pairs of non negative no. should be chosen to ensure two pairs $(a,b) \wedge (c,d)$

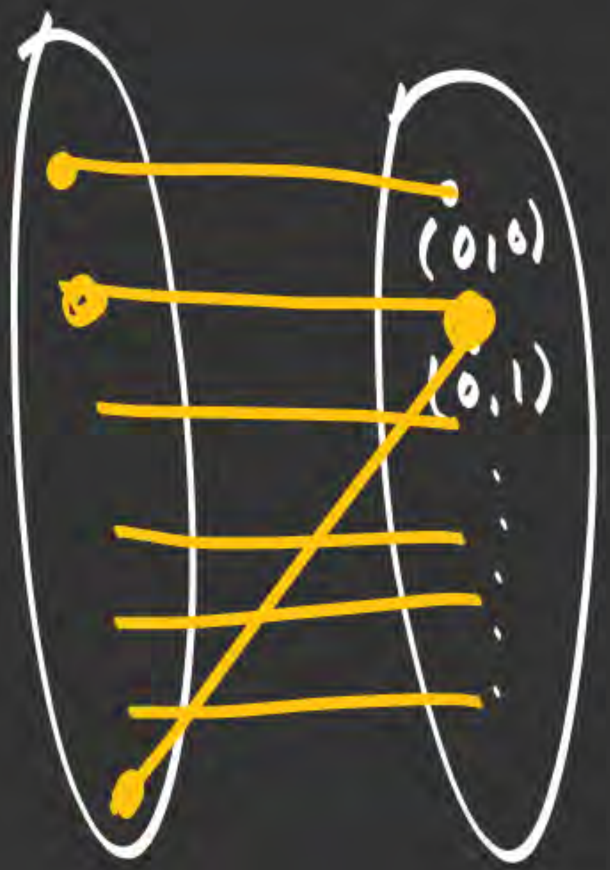
$a \equiv c \pmod 3 \wedge b \equiv d \pmod 5$

a) 4

b) 6

c) 16

d) 24



15

$A = (0, 1, 2)$

$B = \{0, 1 \ldots 4\}$

$A \times B = \{(0,0)\ (0,1) \ldots\ldots (2,4)\}$

Q

Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

S1. If five integers are selected from A, must at least one pair of the integers have a sum of 9? (T)

S2. If four integers are selected from A, must at least one pair of the integers have a sum of 9? (F)

(a) only S1 is valid
(b) only S2 is valid
(c) Both S1 and S2 are valid
(d) Both S1 and S2 are invalid

all pair sum $\neq 9$

$e_1 = 1$
$e_2 = 2$
$e_3 = 3$
$e_4 = 4$

$e_1 \rightarrow \{1, 8\}$
$e_2 \rightarrow \{2, 7\}$
$e_3 \rightarrow \{3, 6\}$
$e_4 \rightarrow \{4, 5\}$
e_5

$e_1 \rightarrow \{1, 8\}$
$e_2 \rightarrow \{2, 7\}$
$e_3 \rightarrow \{3, 6\}$
$e_4 \rightarrow \{4, 5\}$

**

Q

How many integers from 0 through 60 must you pick in order to be sure of getting at least one that is odd? at least one that is even?

$\{0 \ldots\ldots 60\}$

(a) 32,31
(b) 31,31
(c) 32,32
(d) 31,32

Diff even no: $\underbrace{0, 2, 4, 6 \ldots\ldots 60}_{31}$

$e_1 \rightarrow 0$
$e_2 \rightarrow$
$e_3 \rightarrow$
$\vdots$
$e_{31} \rightarrow 60$

Diff even: 31

31 diff no. will get all even no. in worst case if we take 32nd element it will be odd.

$\{0 \ldots 60\}$

Diff odd no: 30

$e_1 \rightarrow 1$

$e_2 \rightarrow 3$

$e_3 \rightarrow$.

⋮

$e_{30} \rightarrow 59$

if we take 31^{st} element it will surely be even.

Q

Given 8 Perl books, 17 Visual BASIC books, 6 Java books, 12 SQL books, and 20 C++ books, how many of these books must we select to insure that we have 10 books dealing with the same computer language? (NAT)

10 ↓ same.

~~8P~~ 17V ~~6J~~ 12S 20C++

6	6	6	6	6

$6 \times 5 = 30$

$+$

$4 \times 2 = 8$

$+3$

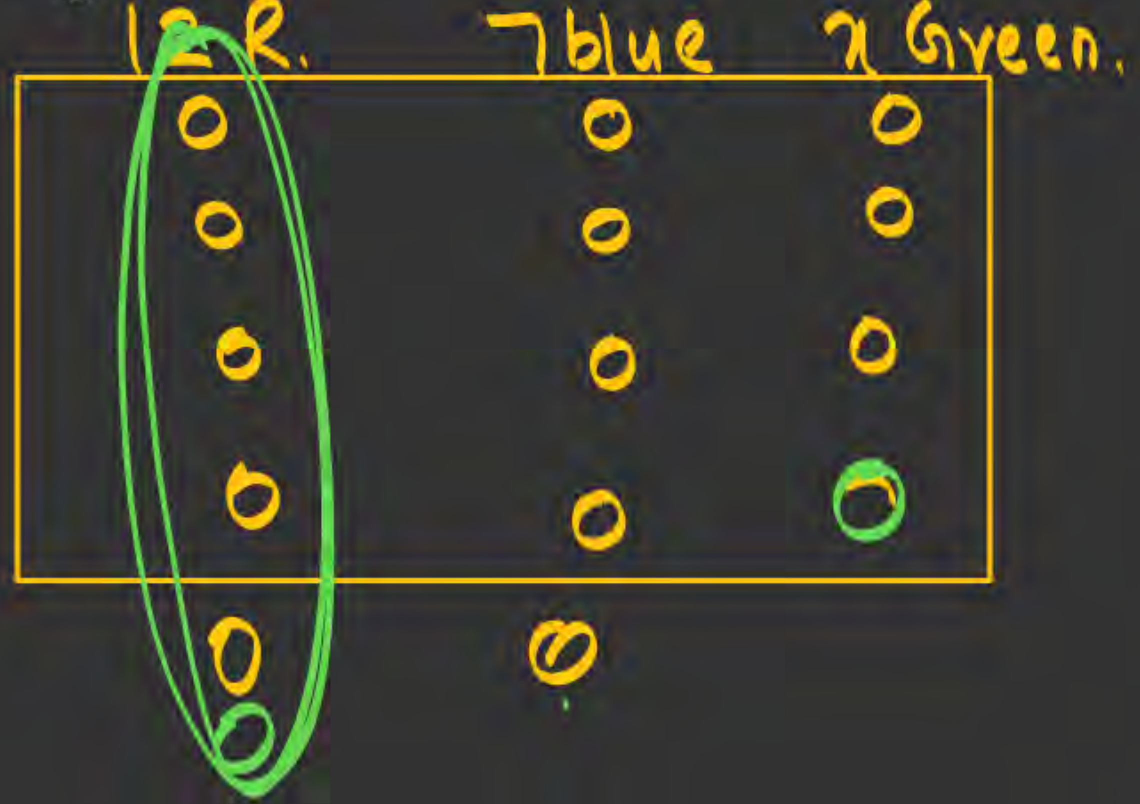
$+1$

Ans: 42

10 → same color.

box contain 12 Red, 7 blue, x green
if min no. of balls we have to choose
from a box that quarantee we have 6 balls
of same color is 15 then x = ____

Ans: 4.



Total operation = 15

6 balls → same color.

$x = 4$.

min no. of non negative int, we have to choose randomly so that there will be atleast 2 int x & y

$\frac{x+y}{10}$ OR $\frac{x-y}{10}$

$\frac{10+100}{10}$ ✓ $\_\_0$

$\frac{11+99}{10}$ $\_\_1 + \_\_9$

$\frac{22+188}{10}$ ✓ $\_\_2 + \_\_8$

Ans: 7

$e_1 \rightarrow \_\_0$ eg: 100

$e_2 \rightarrow \_\_1$ eg: 1211

$e_3 \rightarrow \_\_2$ eg: 22

$e_4 \rightarrow \_\_3$ eg: 133

$e_5 \rightarrow \_\_4$ → 144

$e_6 \rightarrow \_\_5$ → 55

e_7

last digit worst case :

e1	e2	e3	e4	e5	e6
__0	__1	__2	__3	__4	__5

9, 6

$\frac{x+y}{10}$

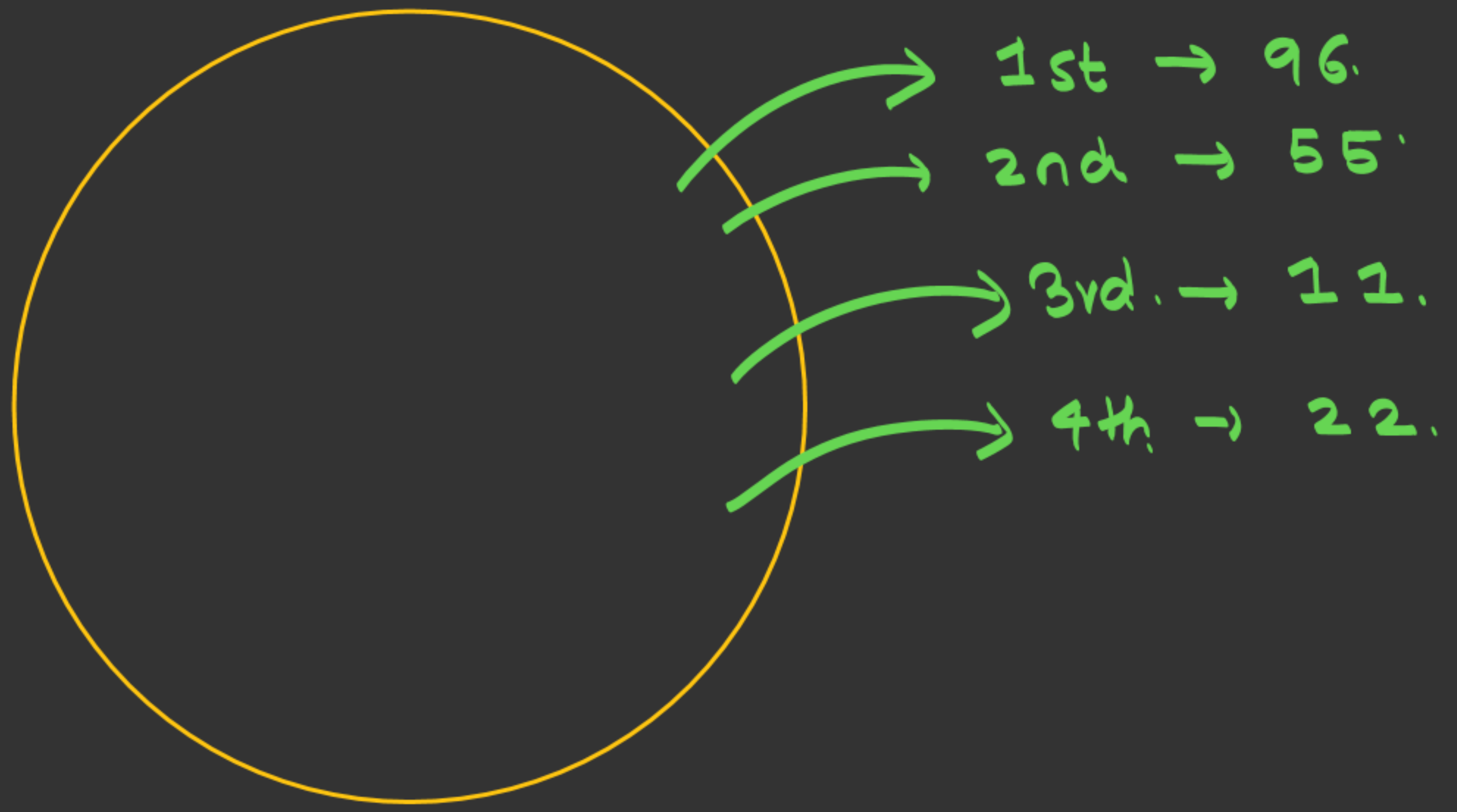
100 → last digit 0 ≡ __0

121 → last digit 1. ≡ __1.

nth element

__0
__1 $x-y/10$ ✓
__2
__3
__9 $x+y/10$ ✓

__1 + __9/10 ✓
__2 + __8/10 ✓
__0 + __0/ ✓
__3 + __7/ ✓
__5 + __5/ ✓


1st → 96.
2nd → 55.
3rd. → 11.
4th → 22.

Q

What is the smallest value of n such that whenever $S \subseteq Z^{+}$ and $|S| = n$, then there exist three elements $x, y, z \in S$ where all three have the same remainder upon division by 1000?

atleast 2 → 1001. $1000 + 1$.

atleast 3 → 2001 $2(1000) + 1$.

atleast n → $n-1(1000) + 1$.

$e_1 \rightarrow$ 0

$e_{1000} \rightarrow$ 999

0

e_1

e_{1001}

e_{2001}

. . . . 999.

e_{1000}

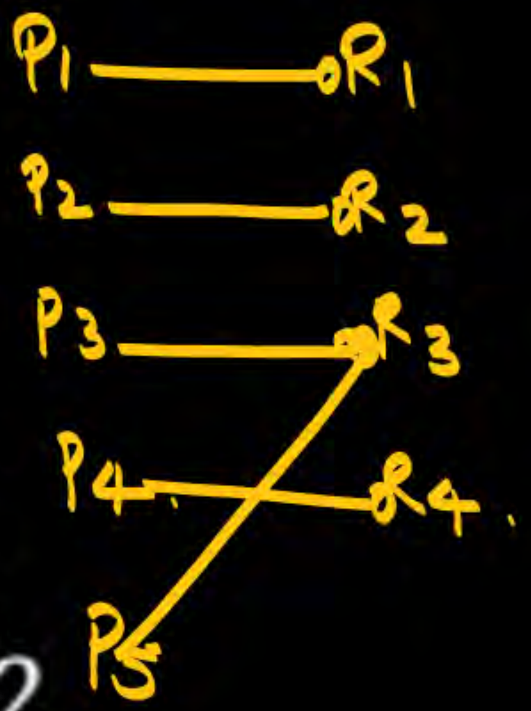
e_{2000}

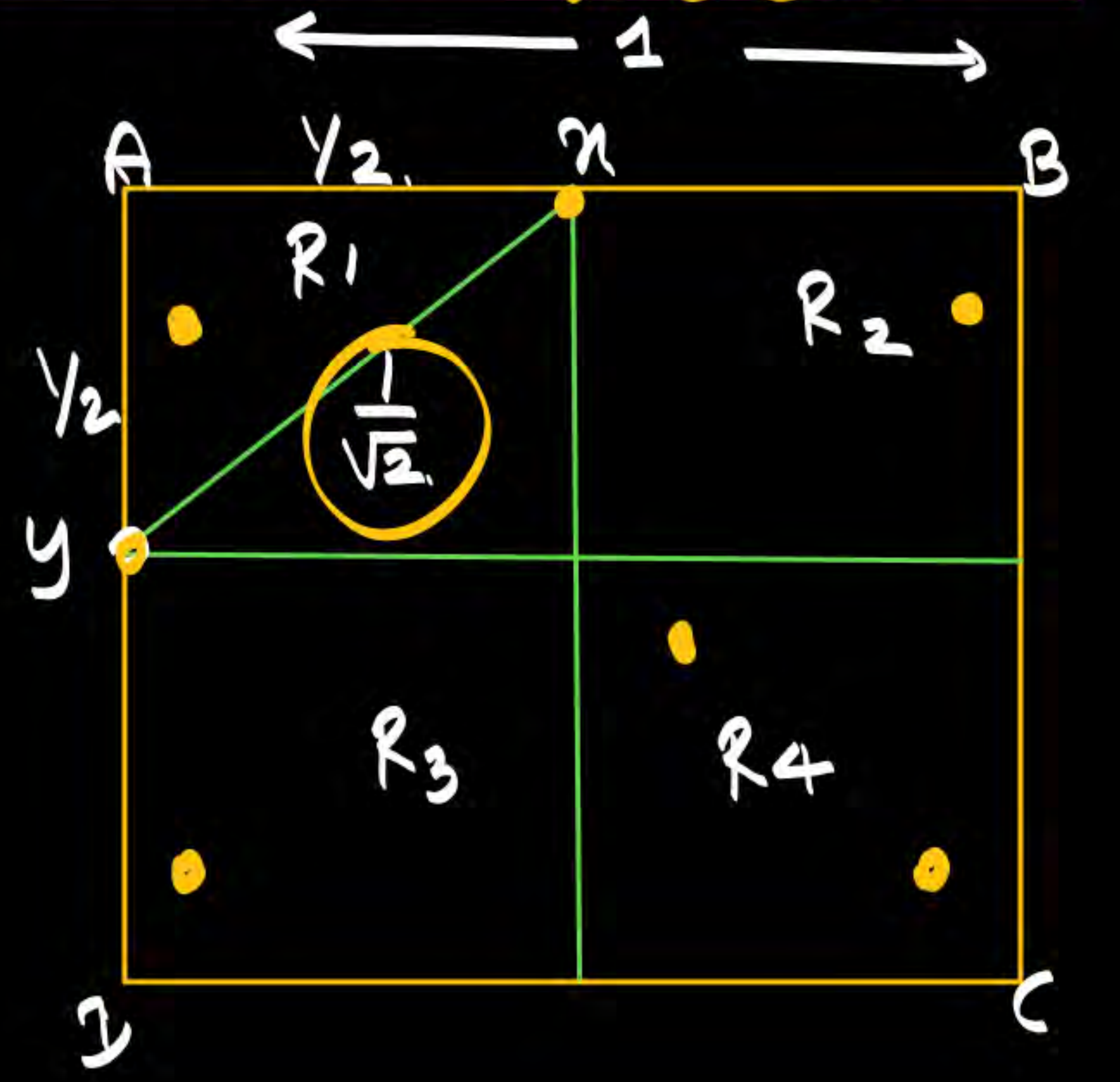
Ans: 2001.

Q

Let ABCD be a square with AB=1. Show that if we select five points in the interior of this square, there are at least two whose distance apart is less than. __________?

(a) 1
(b) 1/2
(c) 1/3
(d) $1/\sqrt{2}$





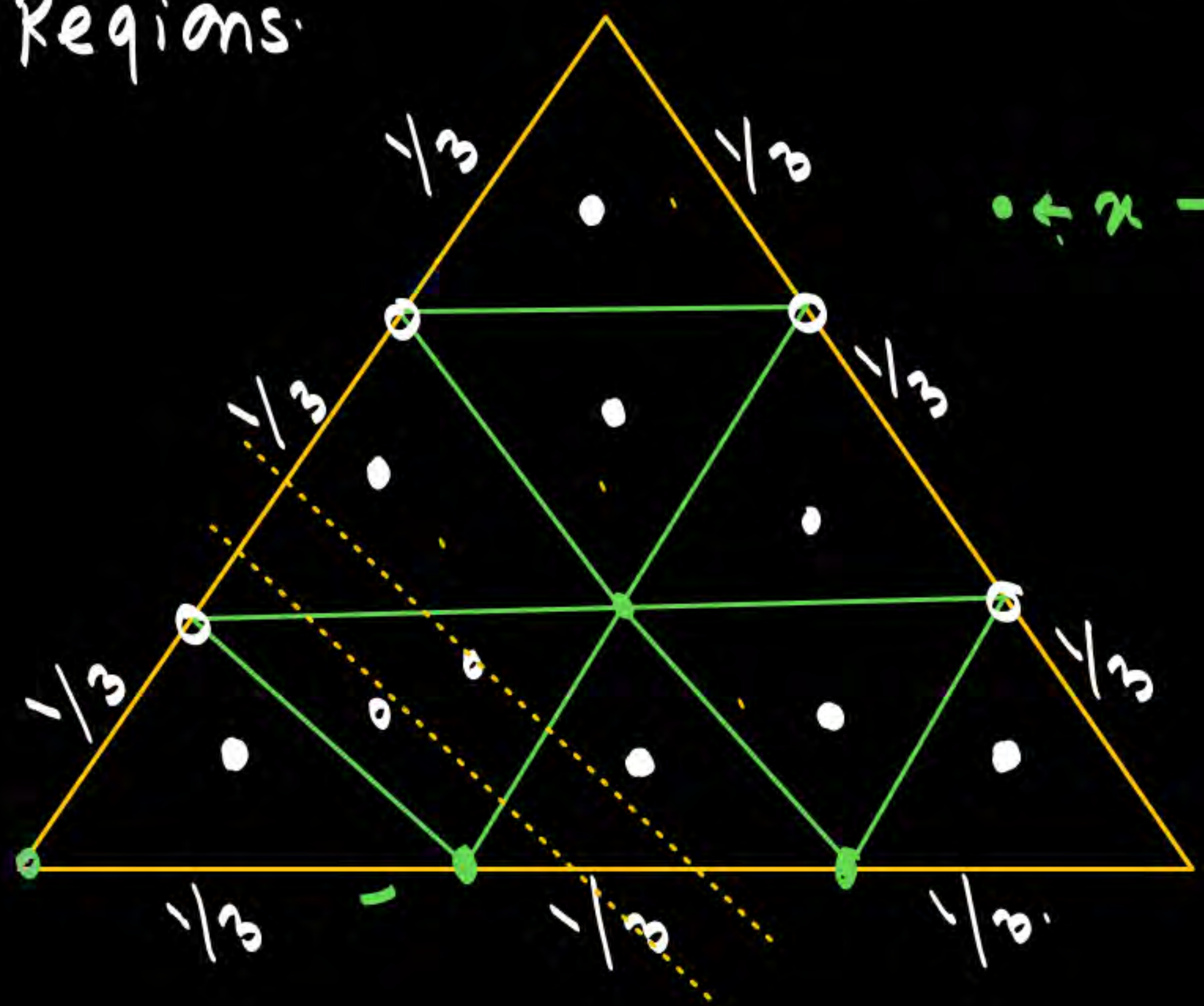
4 points → 4 diff regions.

5 point → will fall in one Regions.

Q

Let triangle ABC be equilateral, with $AB = 1$. Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than ___

10 points > 9 Regions



$\bullet \leftarrow x \rightarrow \bullet \mid < 1/3$

(a) 1
(b) 1/2
(c) 1/3
(d) $1/\sqrt{2}$

3. An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?

$26^2 + 1 = 677$

4. Let $S = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110?

Subdivide the set S into the 14 subsets: $\{3\}, \{7, 103\}, \{11, 99\}, \{15, 95\}, \ldots, \{43, 67\}, \{47, 63\}, \{51, 59\}, \{55\}$. By the Pigeonhole Principle if we select at least 15 elements of S then we must have the elements in one of the two-element subsets and these sum to 110.

20. How many times must we roll a single die in order to get the same score (a) at least twice? (b) at least three times? (c) at least n times, for $n \geq 4$?

20. (a) 7 (b) 13 (c) $6(n-1)+1$

24. Given 8 Perl books, 17 Visual BASIC[†] books, 6 Java books, 12 SQL books, and 20 C++ books, how many of these books must we select to insure that we have 10 books dealing with the same computer language?

24. 42

1. Given a group of n women and their husbands, how many people must be chosen from this group of $2n$ people to guarantee the set contains a married couple?

1. $n + 1$.

There are 20 small towns in a region of west Texas. We want to get three people from one of these towns to help us with a survey of their town. If we go to any particular town and advertise for helpers, we know from past experience that the chances of getting three respondents are poor. Instead, we advertise in a regional newspaper that reaches all 20 towns. How many responses to our ad do we need to assure that the set of respondents will contain three people from the same town?

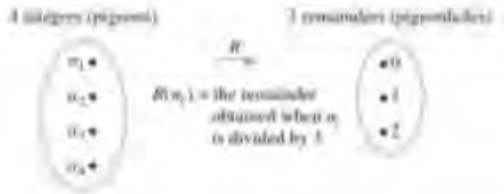
we need more than $2 \times 20 = 40$ responses.

which one is true ?

a.Given any set of four integers, must there be two that have the same remainder when divided by 3? Why?

b. Given any set of three integers, must there be two that have the same remainder when divided by 3?

Ans : a



21. Compute $\phi(n)$ for n equal to (a) 51; (b) 420; (c) 12300.

22. Compute $\phi(n)$ for n equal to (a) 5186; (b) 5187; (c) 5188.

21. (a) 32 (b) 96 (c) 3200

22. (a) $5186 = (2)(2593)$, and $\phi(5186) = (5186)(1/2)(2592/2593) = 2592$.
(b) $5187 = (3)(7)(13)(19)$, so $\phi(5187) = (5187)(2/3)(6/7)(12/13)(18/19) = (2)(6)(12)(18) = 2592$.
(c) $5188 = (2^2)(1297)$, and $\phi(5188) = (5188)(1/2)(1296/1297) = 2592$.
Hence $\phi(5186) = \phi(5187) = \phi(5188)$.

23. Let $n \in \mathbf{Z}^+$. (a) Determine $\phi(2^n)$. (b) Determine $\phi(2^n p)$, where p is an odd prime.

23. (a) 2^{n-1} (b) $2^{n-1}(p-1)$

25. How many positive integers n less than 6000 (a) satisfy $\gcd(n, 6000) = 1$? (b) share a common prime divisor with 6000?

25. (a) $\phi(6000) = \phi(2^4 \cdot 3 \cdot 5^3) = 6000(1-(1/2))(1-(1/3))(1-(1/5)) = 1600$.
(b) $6000 - 1600 - 1$ (for 6000) $= 4399$.

THANK YOU