Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics
Combinatorics

DPP-03

[MCQ]

1. Let A = {1, 2, 3, ..., 7}. A function $f : A \to A$ is said to be have a fixed point if for some $x \in A$, $f(x) = x$. How many one – to – one functions $f : A \to A$ have at least one fixed point?

(a) $8! - d_7$ ($d_7 = (8!)\, e^{-1}$);

(b) $6! - d_7$ ($d_7 = (6!)\, e^{-1}$);

(c) $9! - d_7$ ($d_7 = (9!)\, e^{-1}$);

(d) $7! - d_7$ ($d_7 = (7!)\, e^{-1}$);

[NAT]

2. For the positive integers 1,2, 3, ..., n – 1, n, there are 11,660 derangements where 1, 2, 3, 4, and 5 appear in the first five positions. What is the value of n?

[MCQ]

3. In how many ways can Mrs. Ford distribute ten distinct books to her ten children (one book to each child) and then collect and redistribute the books so that each child has the opportunity to peruse two different books?

(a) $(10!)\, d_{10} = (10!)^2(e^{-1})$

(b) $(11!)\, d_{10} = (10!)^2(e^{-1})$

(c) $(10!)\, d_{10} = (11!)^2(e^{-1})$

(d) $(11!)\, d_{10} = (9!)^2(e^{-1})$

[NAT]

4. Caitlyn has 48 different books: 12 each in mathematics, chemistry, physics, and computer science. These books are arranged on four shelves in her office with all books on any one subject on its own shelf. When her office is cleaned, the 48 books are taken down and then replaced on the shelves — once again with all 12 books on any one subject on its own shelf.

In how many ways can this be done so that

(i) no subject is on its original shelf?

(ii) one subject is on its original shelf?

(iii) no subject is on its original shelf and no book is in its original position?

[For example, the book originally in the third (from the left) position on the first shelf must not be replaced on the first shelf and must not be in the third (from the left) position on the shelf where it is placed.]

(a) (i) $d_3\,(11!)^3$

(ii) $\binom{3}{1} d_2(11!)^3$

(iii) $d_3(d_{11})^3$

(b) (i) $d_4\,(12!)^4$

(ii) $\binom{4}{1} d_3(12!)^4$

(iii) $d_4(d_{12})^4$

(c) (i) $d_5\,(13!)^5$

(ii) $\binom{5}{1} d_3(13!)^4$

(iii) $d_4(d_{13})^4$

(d) (i) $d_4\,(14!)^4$

(ii) $\binom{6}{1} d_3(14!)^4$

(iii) $d_4(d_{14})^4$

[NAT]

5. List all the derangements of 1, 2, 3, 4, 5, 6 where the first three numbers are 1, 2 and 3 in some order.

Answer Key

1. **(d)**
2. **(11)**
3. **(a)**
4. **(b)**
5. **(4)**

Hints and Solutions

1. (d)

$7! - d_7$ $(d_7 = (7!)\, e^{-1})$;

2. (11)

Let n = 5 + m.

Then, 11, 660 = $d_5 \cdot d_m = 44(d_m)$, and

So, $d_m = 265 = d_6$.

Consequently, n = 11.

3. (a)

$(10!)\, d_{10} = (10!)^2(e^{-1})$

4. (b)

(i) $d_4\, (12!)^4$

(ii) $\binom{4}{1} d_3(12!)^4$

(iii) $d_4(d_{12})^4$

5. (4)

Here there are four such derangements:

(i) 231546

(ii) 231645

(iii) 312546

(iv) 312645