CS & IT ENGINEERING

Discrete maths

set theory.

Lecture No.1

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Inference Rule

02 Type 3 Questions in logic

03 Type 3 with Type 1

04 GATE QUESTIONS on 3 &1

05 Practice

$$\forall x\,(p(x) \rightarrow (q(x) \wedge s(x)))$$

$$\forall x\,(p(x) \wedge R(x))$$

$$\overline{\qquad\qquad\qquad\qquad}$$

$$\forall x\,(R(x) \rightarrow s(x))$$

$P \rightarrow Q \wedge S$

$P \wedge R$

$\overline{\qquad\qquad}$

$R \rightarrow S$

$\neg R \vee S$

$P \rightarrow Q \wedge S$

P

$\overline{\qquad\qquad}$

$Q \wedge S$

S

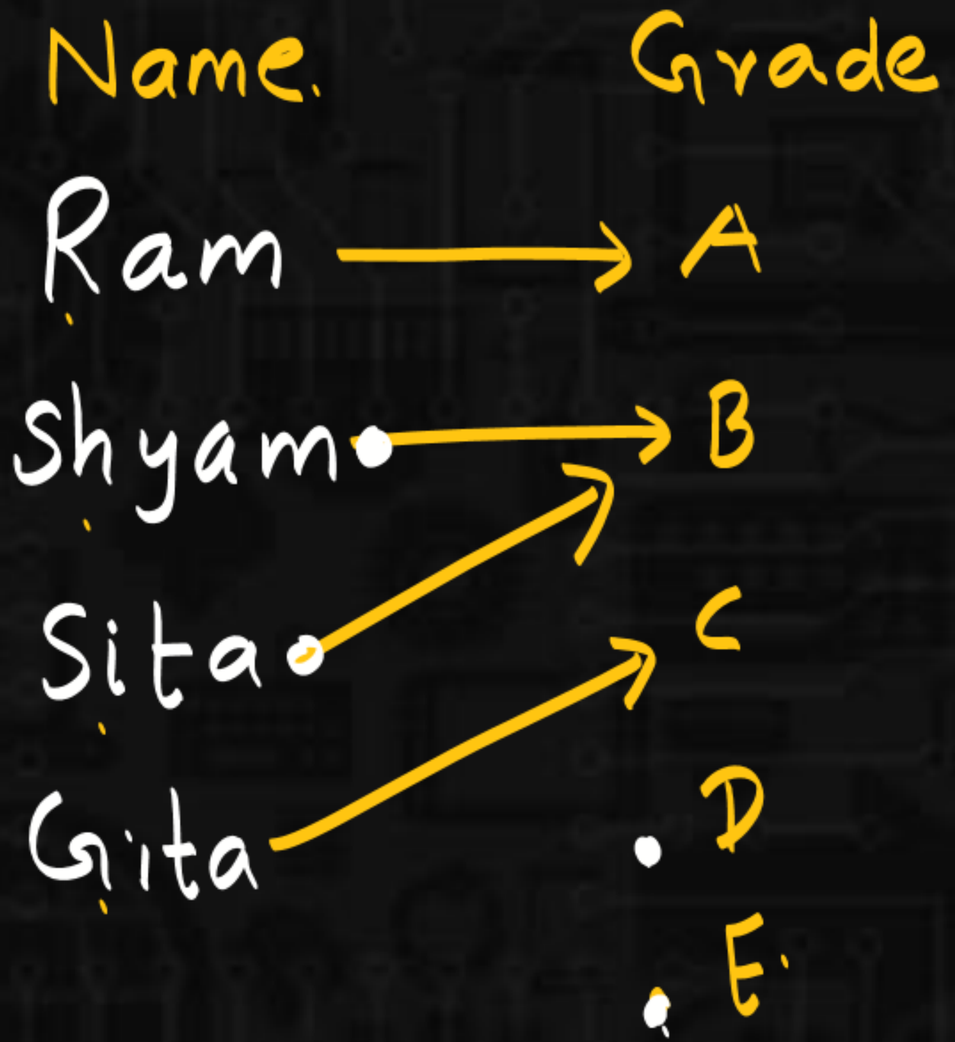
Addition

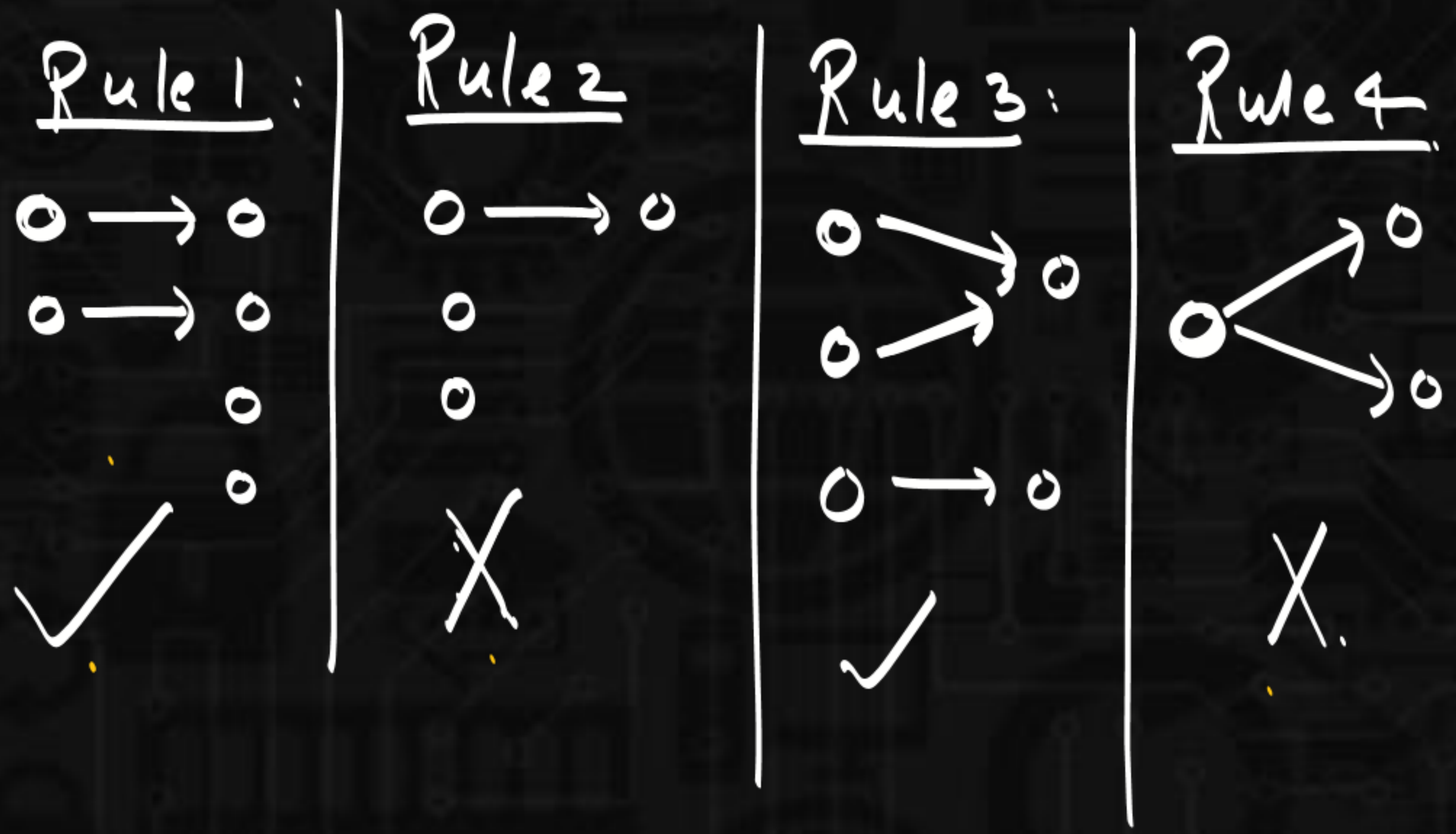
$S \vee \neg R$

$\overline{\qquad\qquad}$

$R \rightarrow S$

Function / assignment / mapping / transformation.



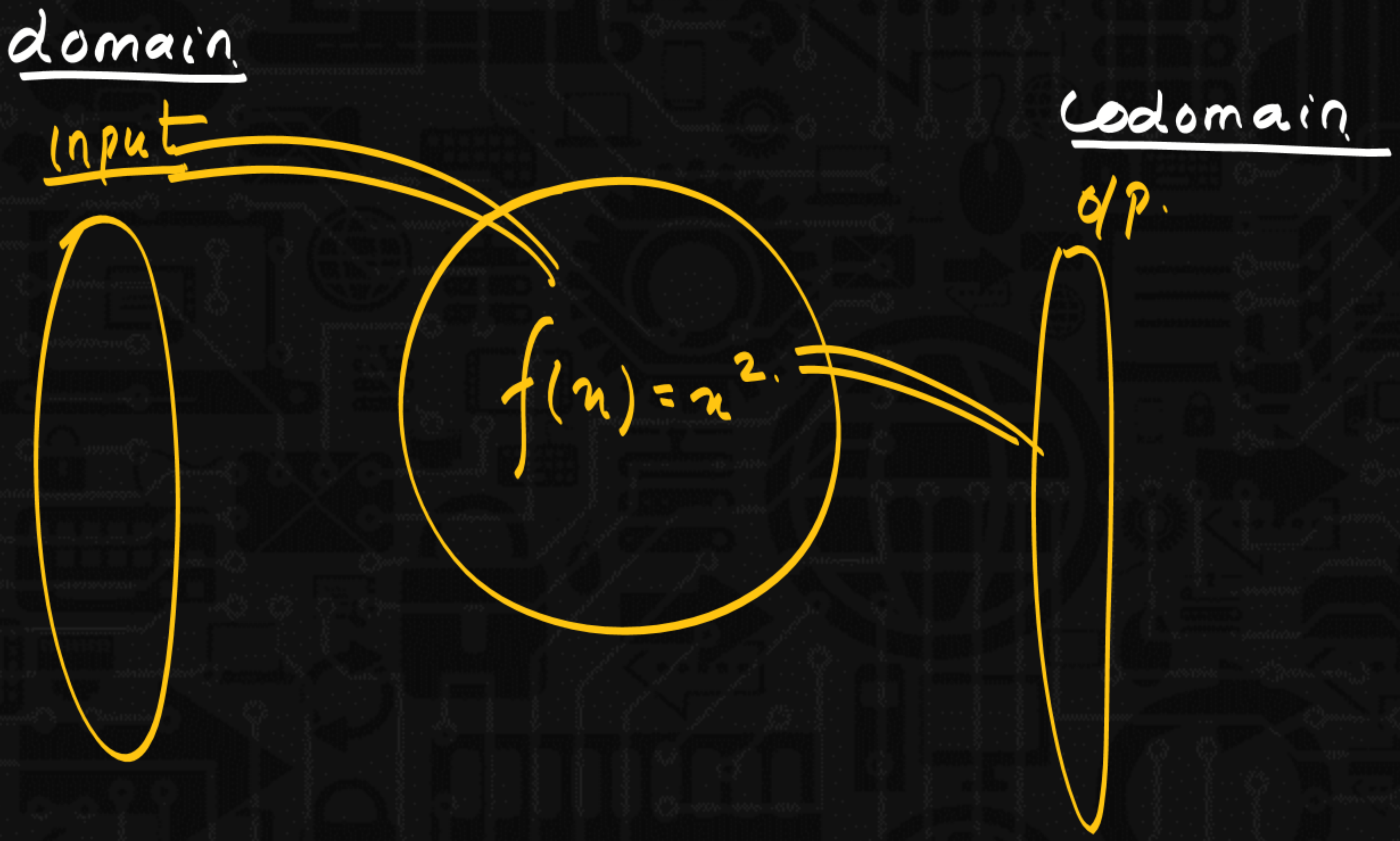


$$\left\{\begin{array}{l} f: Z \rightarrow Z. \\ f(x) = x^2. \end{array}\right.$$

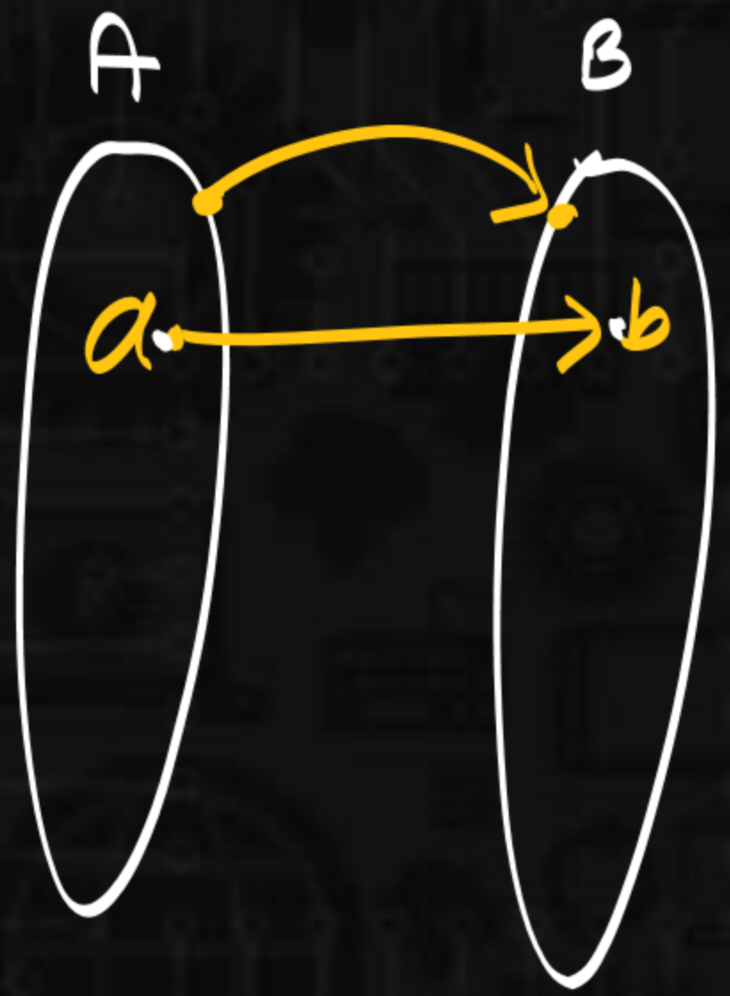
$f(2) = 4$

$f(-2) = 4$

$$\begin{array}{l} 2 \searrow \\ \qquad 4 \\ -2 \nearrow \end{array}$$



$f : A \to B$



$f(a) = b$

Domain

codomain :

$f(a) = b$

b is called image of a.

a is called preimage of b.

Range = collection of images.

$a \longrightarrow 1$

$b \longrightarrow 2$

$\cdot\ 3$

$\cdot\ 4$

Codomain: $\{1, 2, 3, 4\}$

Range = collection of images

$= \{1, 2\}$

$\boxed{\text{Range} \subseteq \text{codomain}}$

Total no. of functions = no. of diff arrows representation.

$f: A \to B$

$|A| = 3$ $\quad$ $|B| = 2$

a	b	c
0	0	0

a	b	c
0	0	1

a	b	c
0	1	0

. . . . .

a	b	c
1	1	1

Total no. of functions

$= 2^3$

$= (R.S)^{L.S.}$

1:1 / one-to-one / injective :

$$\forall a \forall b \left(f(a)=f(b) \longrightarrow a=b \right)$$

$$\neg(a=b) \to \neg\left(f(a)=f(b)\right)$$

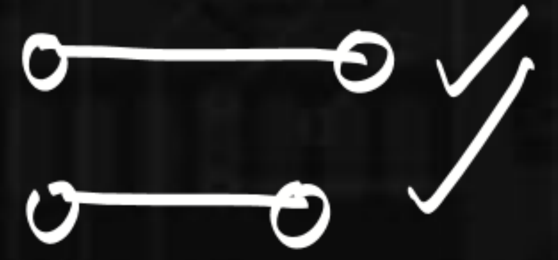
$$\forall a \forall b \left(a \neq b \to f(a) \neq f(b) \right)$$

$$(a=b) \longrightarrow f(a)=f(b)$$

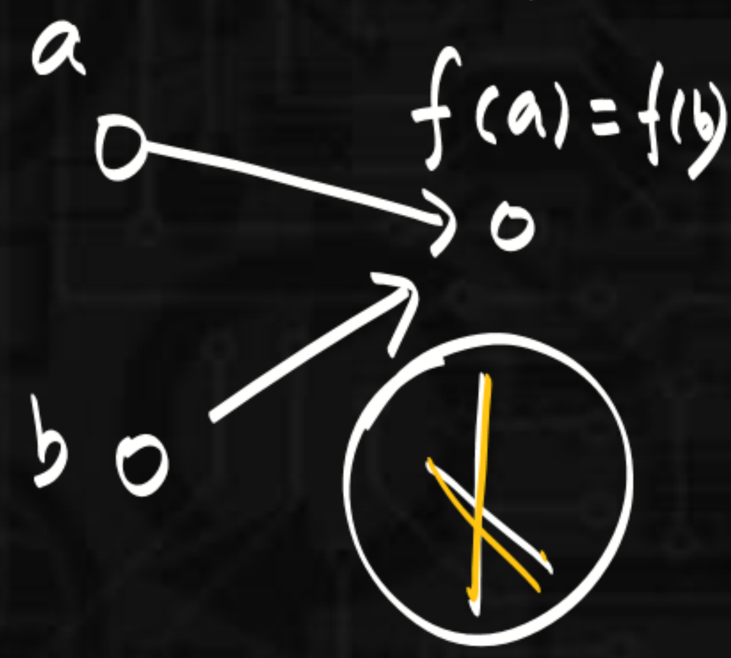
$$a=b \longrightarrow f(a)=f(b)$$

$$a \longrightarrow f(a)$$

$$b \longrightarrow f(b)$$



Invalid



$f: Z \to Z$

$f(x) = x^2$

$$\begin{matrix} 2 \searrow \\ & 4 \\ -2 \nearrow \end{matrix}$$

X.

$a = 2$

$b = -2$

$f(a) = f(b) \longrightarrow a = b$

$f(2) = f(-2)$

$\underbrace{4 = 4}_{T} \longrightarrow \underbrace{2 = -2}_{F}$

it is not 1:1.

$f(x) = x + 1$

$1 \longrightarrow 2$

$2 \longrightarrow 3$

$3 \longrightarrow 4$

$4 \longrightarrow 5$

$f(a) = f(b)$

$a + 1 = b + 1$

$\downarrow$

$a = b$

$f(a) = f(b) \longrightarrow a = b$

$a + 1 = b + 1 \longrightarrow a = b$

$f : R \to R \left(x \neq \frac{1}{4}\right)$

$$f(x) = \frac{2x}{4x-1}$$

Check:

$$f(a) = f(b) \longrightarrow a = b$$

1 : 1

$$\frac{\cancel{2}a}{4a-1} = \frac{\cancel{2}b}{4b-1}$$

$$a(4b-1) = b(4a-1)$$

$$\cancel{4ab} - a = \cancel{4ab} - b$$

$$-a = -b$$

$$a = b$$

Total no. of 1:1 Functions.

= Total diff arrows representation.

$f: A \to B$

$|A| = 5 \quad |B| = 3$

L.S > R.S

1:1 is not possible.

$\boxed{L.S \leq R.S}$

$f: A \to B$

$|A| = 3 \quad |B| = 5$

Total 1:1

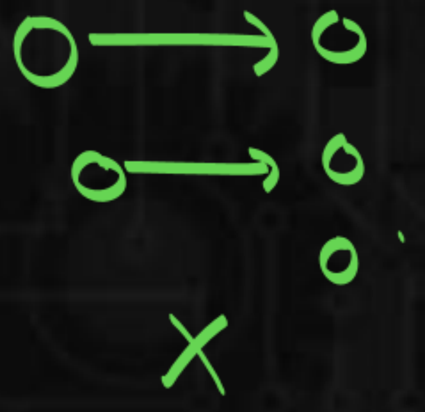
$= 5 \cdot 4 \cdot 3 \left(\frac{2!}{2!}\right)$

$= \frac{5!}{2!}$

$= \frac{5!}{(5-3)!} = {}^{5}P_{3}$

$= {}^{R.S}P_{L.S}$

onto / surjective (Right side must be full) (Range = codomain)



1:1 ✓

onto X

onto ✓

1:1 X

1:1 ✓

onto ✓

$f(x) = x^2$

$0 \longrightarrow 0$ → not onto

$1 \longrightarrow 1$

$\cdot\, 2$

$\cdot\, 3$

$2 \longrightarrow 4$

$f : \mathbb{Z} \to \mathbb{Z}.$

$f(x) = x + 1$

onto

$a \in \mathbb{Z}.$

$a - 1 \in \mathbb{Z}.$

$1 \longrightarrow 2.$

$2 \longrightarrow 3$

$3 \longrightarrow 4$

$a-1 \rightsquigarrow a$

$f : A \to B.$

$\forall B \; \exists A.$

Total no. of onto :

$f : A \to B$

$|A| = 3 \quad |B| = 5$

$L.S < R.S$

not onto

$f : A \to B$

$|A| \geq |B|$

$L.S \geq R.S$

onto is possible.

$f : A \to B$

$|A| = 4 \quad |B| = 3$

Total onto functions :

3^4 –

Total onto = Total function - Total non onto.
(R.S is not full)

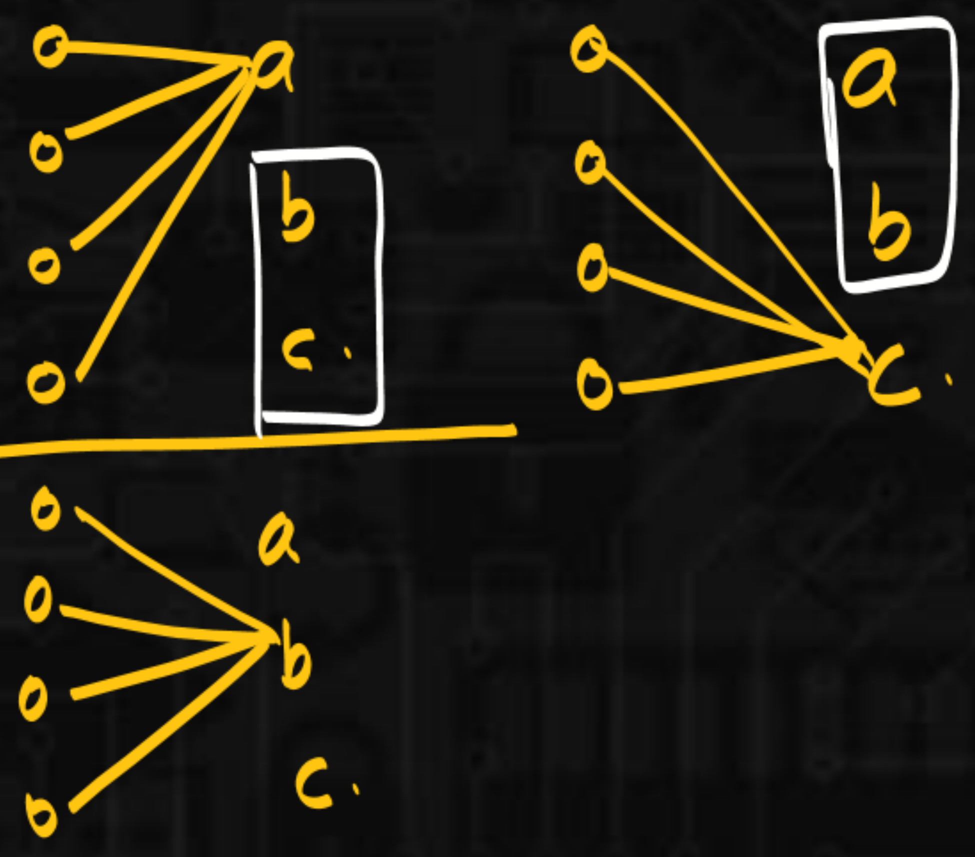
<u>non onto</u> :

no pointing (2 element OR 1 element)

$f: A \to B$

$|A| = 4 \quad |B| = 3.$

all → 1.



$^3C_{\cancel{2}\,1} \cdot 1^{\cancel{4}}$

$= 3.$

- $\boxed{\text{all} \to a}$ ✓
- all $\to$ b

$\downarrow$

Skip $\to$ c.

$\downarrow$

Total Functions.

$= (R.S)^{L.S.}$

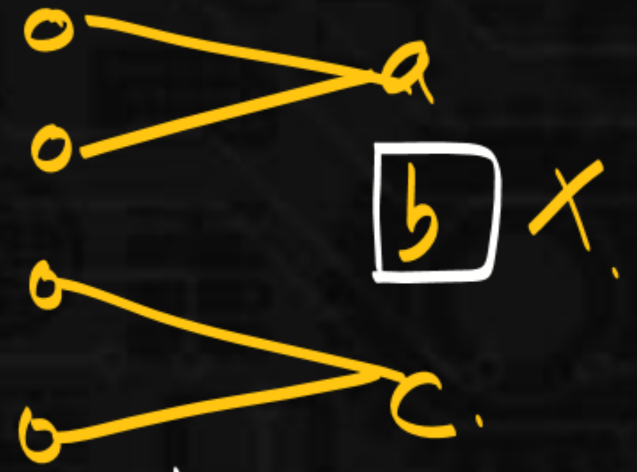
$= 2^4$

- all $\to$ b
- all $\to$ c.

$\downarrow$

2^4

- $\boxed{\text{all} \to a}$ ✓
- all $\to$ c



$\downarrow$

2^4

we are skipping 1 elemen on right side This can be done in ${}^3C_1$ ways.

Hence total Functions after skipping 1 element:

${}^3C_1 \cdot 2^4$

all → 2

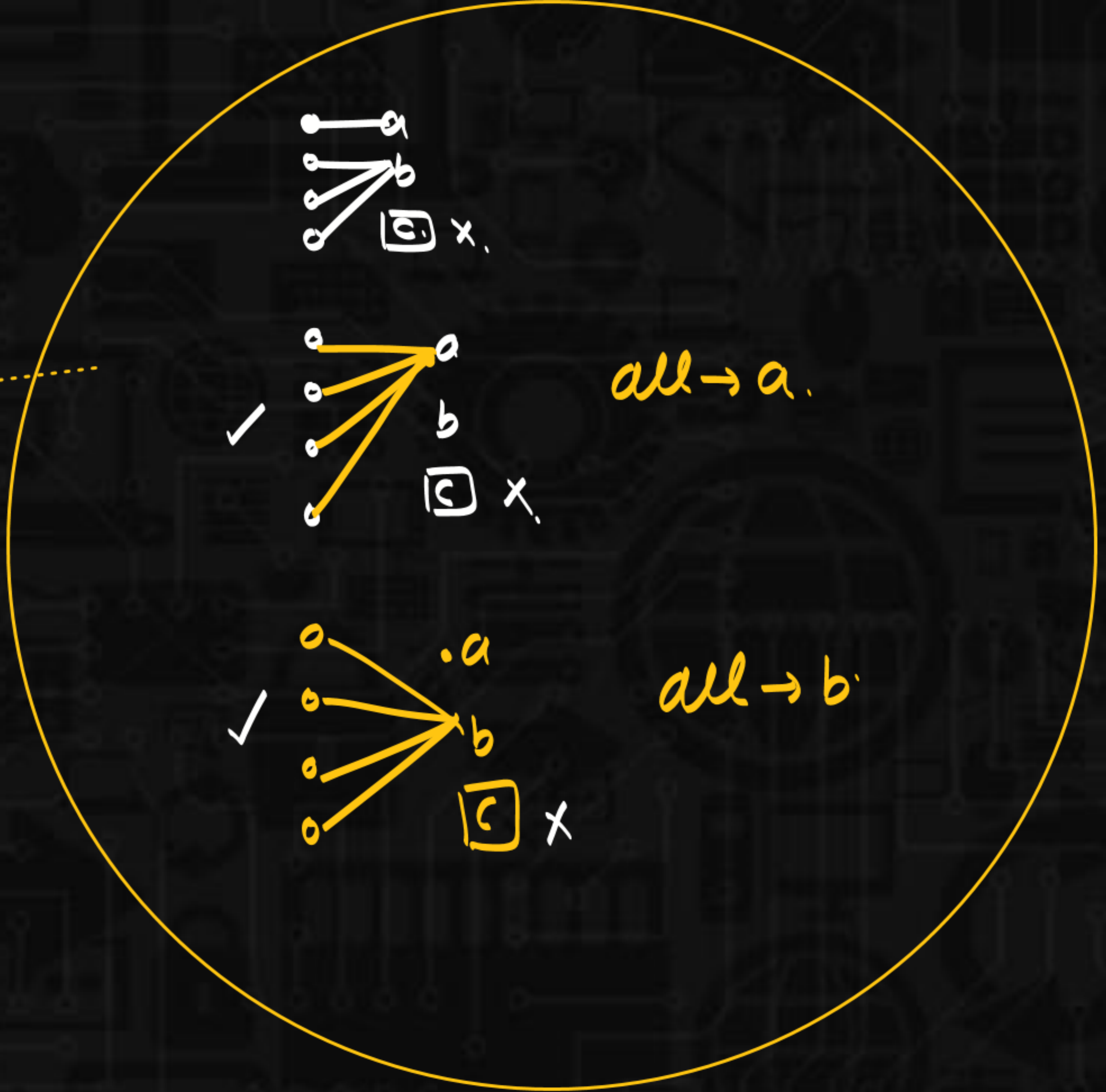
$\rightarrow 2^4$

Skip → c

Total Functions

$= (R.S)^{L.S}$

$= 2^4$



Total onto = Total functions − Total non onto.

$= 3^4 - 3C_1 \cdot 2^4 + 3C_2 \cdot 1^4$

no. of ways to skip 1 element.

no. of ways to skip 2 elements.

2^4

2^4

2^4

$$A \cup B \cup C = A + B + C - A \cap B - B \cap C - A \cap C$$
$$+\ A \cap B \cap C$$

$|A| = 4 \quad |B| = 3$

$m = 4 \quad n = 3$

$$3^4 - {}^3C_1 \cdot 2^4 + {}^3C_2 \cdot 1^4 .$$

$$= {}^3C_0 (3-0)^4 - {}^3C_1 (3-1)^4 + {}^3C_2 (3-2)^4 .$$

$$= {}^nC_0 (n-0)^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - {}^nC_3 (n-3)^m \cdots\cdots$$

$$= \sum_{i=0}^{n} (-1)^i * {}^nC_i * (n-i)^m .$$

$|A| = 7 \quad |B| = 4.$

$m = 7 \quad n = 4.$

$$\sum_{i=0}^{n} (-1)^i \, {}^{n}C_i \, (n-i)^m.$$

$${}^4C_0 (4-0)^7 - {}^4C_1 (4-1)^7 + {}^4C_2 (4-2)^7 - {}^4C_3 (4-3)^7 + {}^4C_4 (4-4)^7.$$

$\underline{\underline{8400}}$

How many ways to adjust 7 diff guests to 4 diff rooms Such that none of the rooms should be empt?

Ans: 8400

$|A| = 3 \quad |B| = 2$

$^2C_0(2-0)^3 - {}^2C_1(2-1)^3 + {}^2C_2(2-2)^3$

$= 2^3 - 2 = \underline{\underline{6}}$

3 diff quest

→ 2 diff room

Ans: 6.

onto:

$\frac{6}{2}$

Rooms → same

[ab] [c]

[ac] [b]

$\therefore$ [bc] [a]

m diff $\longrightarrow$ n diff $\longrightarrow$ onto

3 diff guests $\rightarrow$ 2 diff rooms (none of rooms empty) $\longrightarrow 6$

3 diff guest $\rightarrow$ 2 identical rooms

$\frac{\text{onto}}{2!}$

m diff guest $\rightarrow$ n identical rooms

$\frac{\text{onto}}{n!}$

$$S(m,n) = \frac{1}{n!}\sum_{i=0}^{n} (-1)^i \cdot {}^nC_i \cdot (n-i)^m$$

identical

diff

Sterling's 2^{nd} kind no.


THANK YOU GW
SOLDIERS !