CS & IT ENGINEERING

Discrete maths
Graph theory

Lecture No. 4

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TOPICS TO BE COVERED

01 Complement graph

02 Self complement graph

03 Isomorphic graph

04 Hypercube graph

05 Practice

Types of graph

Iso/morphic :

same — property.

G_1, G_2 are isomorphic to each other when they have same incident property

→ no. of vertices

→ no. of edges.

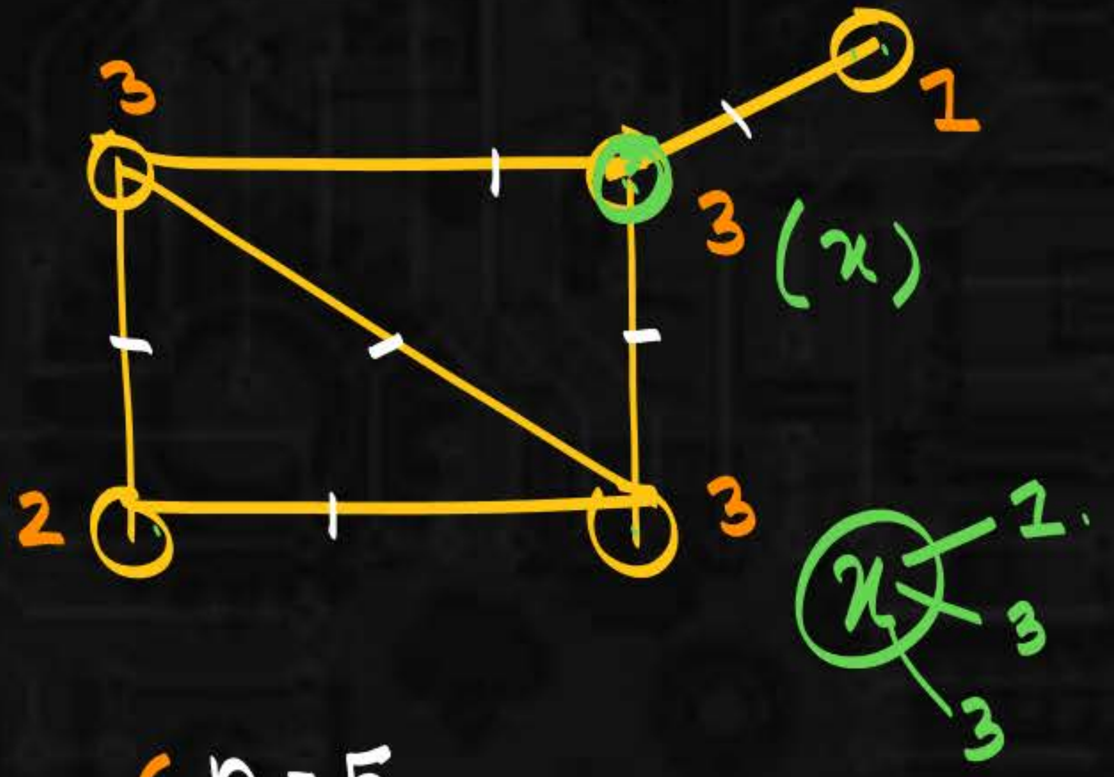
→ degree sequence

isomorphic

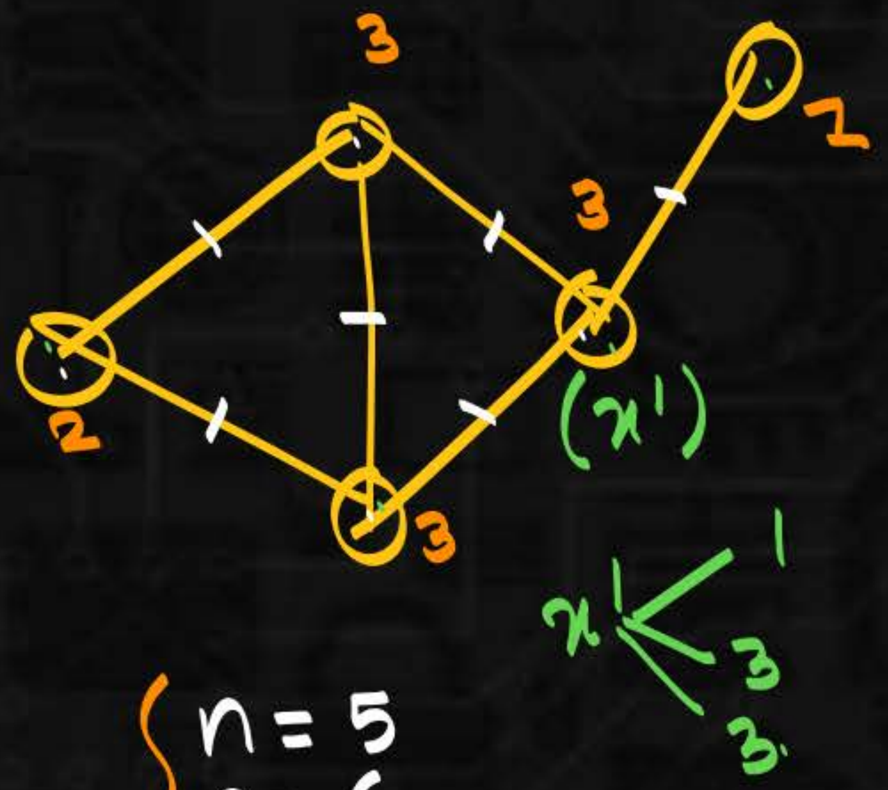
property.

if 2 Graphs are isomorphic then they will have
same no. of vertice, edges.
degree sequence
(viceversa is not
true).

Types of graph



$$\begin{cases} n = 5 \\ e = 6 \\ 3\ 3\ 3\ 2\ 1 \end{cases}$$



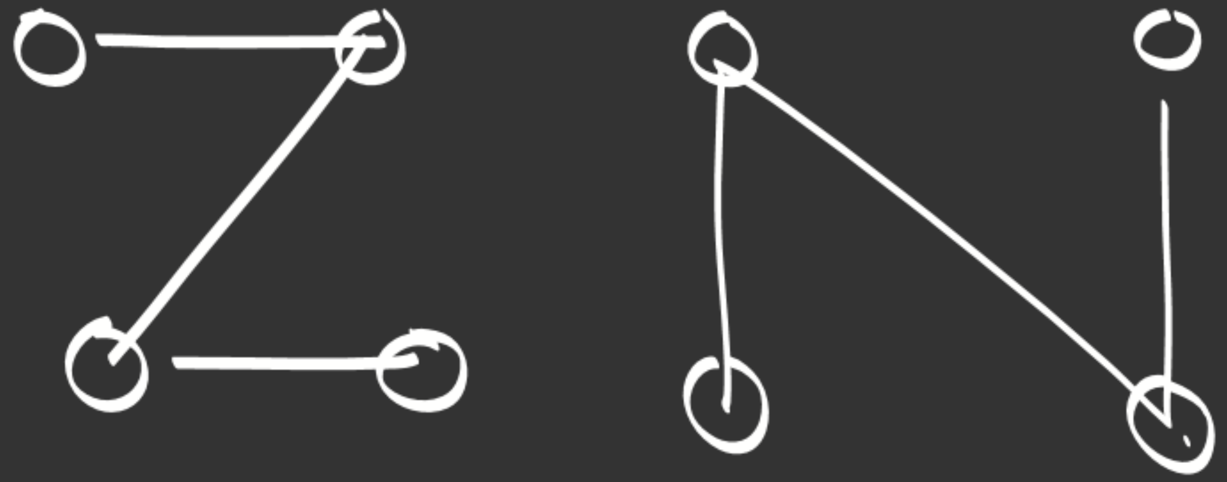
$$\begin{cases} n = 5 \\ e = 6 \\ 3\ 3\ 3\ 2\ 1 \end{cases}$$

Isomorphic:

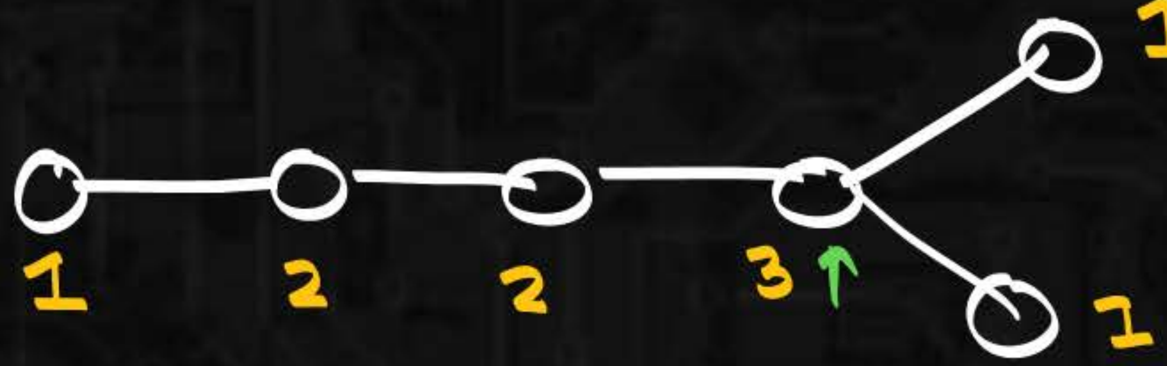
↓

- same graph
- diff
- representation.

Self-complement (: when graph is isomorphic to its own complement)



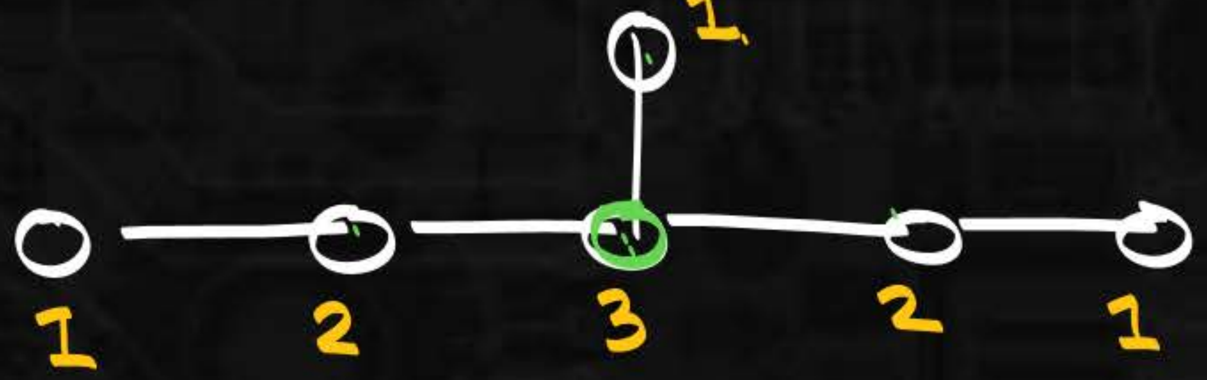
Types of graph



$n = 6$
$e = 5$
$3\,2\,2\,1\,1\,1$

only vertex with degree 3.

3 1 1 2.



only vertex with degree 3.

$n = 6$
$e = 5$
$3\,2\,2\,1\,1\,1$

3 1 2. 2.

Types of graph

$G = (V, E)$

G_1, G_2 are isomorphic to each other.

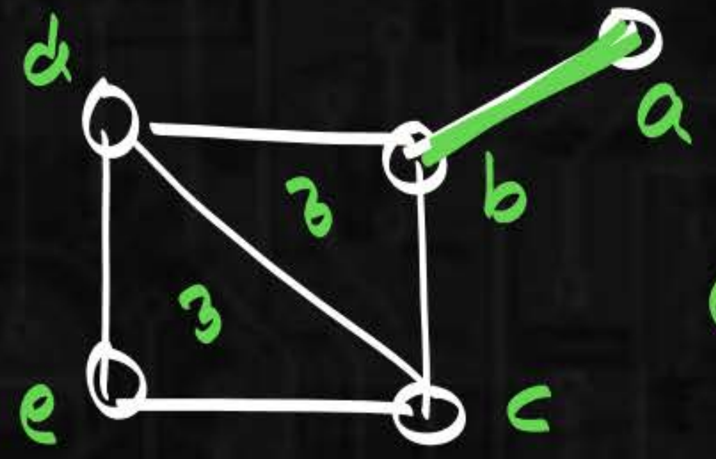
$f : G_1 \to G_2$ (1:1 correspondance)

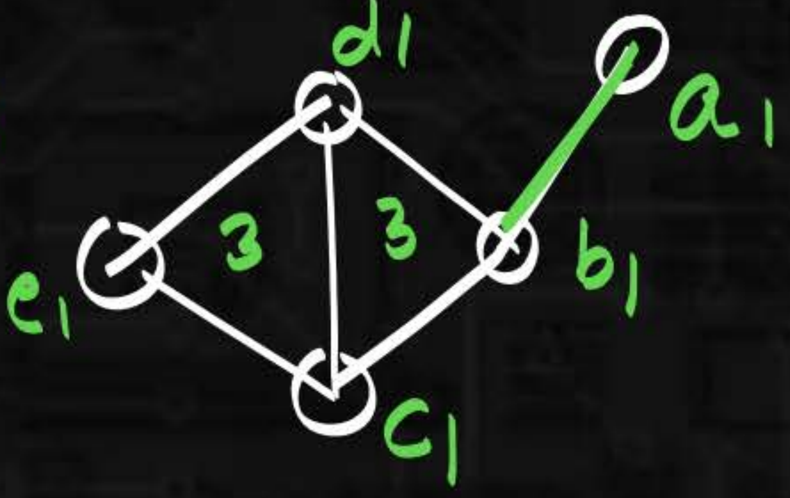
$f : V_1 \to V_2$

$f : E_1 \to E_2$.

Types of graph

$G_1 = (V_1, E_1)$ $\quad$ $G_2 = (V_2, E_2)$





$f : G_1 \to G_2$ (1:1 c)

$f : \underline{V_1} \to \underline{V_2}$.

$a \to a_1$

$b \to b_1$

$c \to c_1$

$d \to d_1$

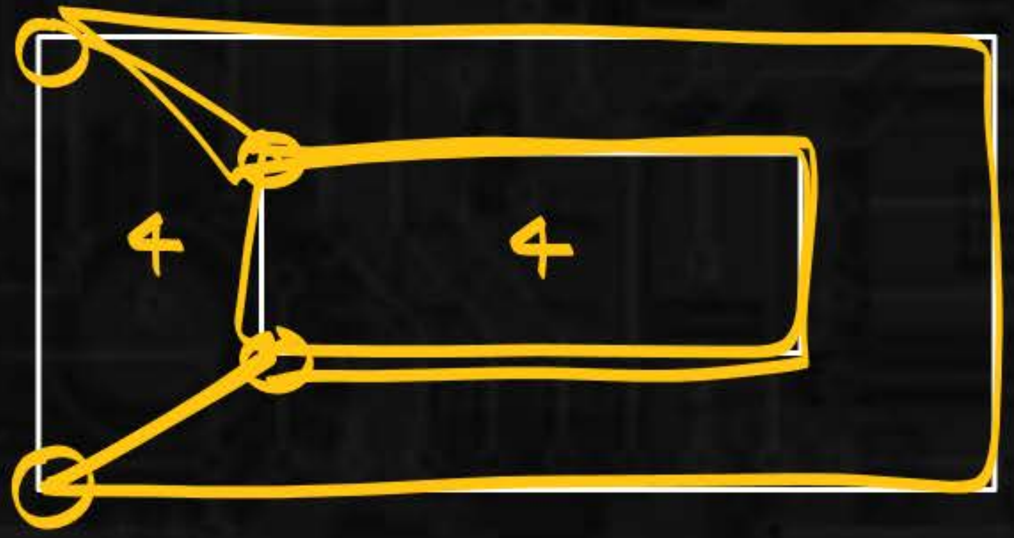
$e \to e_1$.

$f : E_1 \to E_2$

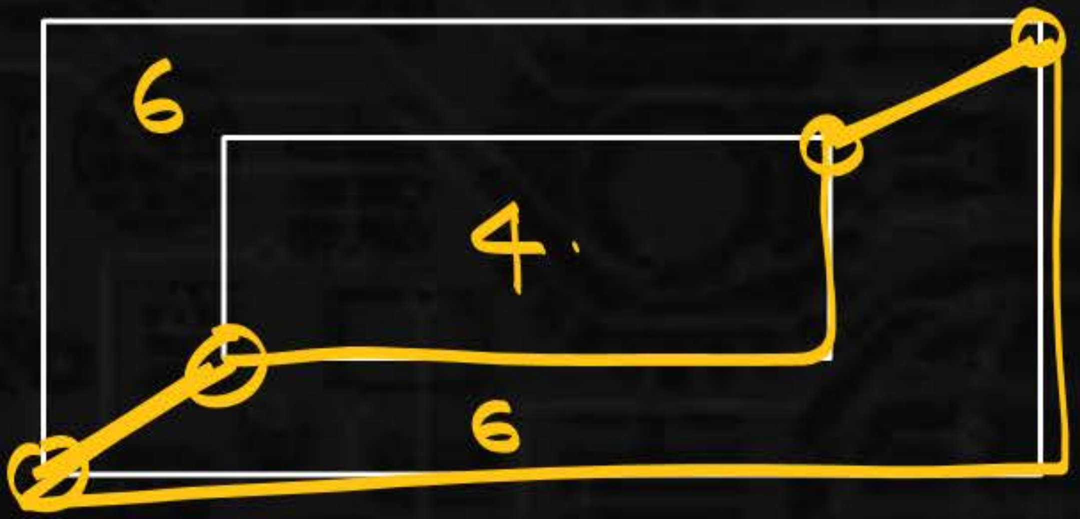
$(a, b) \to a_1 b_1$

$(b c) \to b_1 c_1$.

Types of graph



$\not\equiv$



$$\begin{cases} n = 8 \\ e = 10 \\ \ldots\ldots \end{cases}$$

$$\begin{cases} n = 8 \\ e = 10 \\ \ldots\ldots \end{cases}$$

Types of graph

Bipartite

2 - partition.

$G = (V, E)$

$V \rightarrow V_1, V_2$

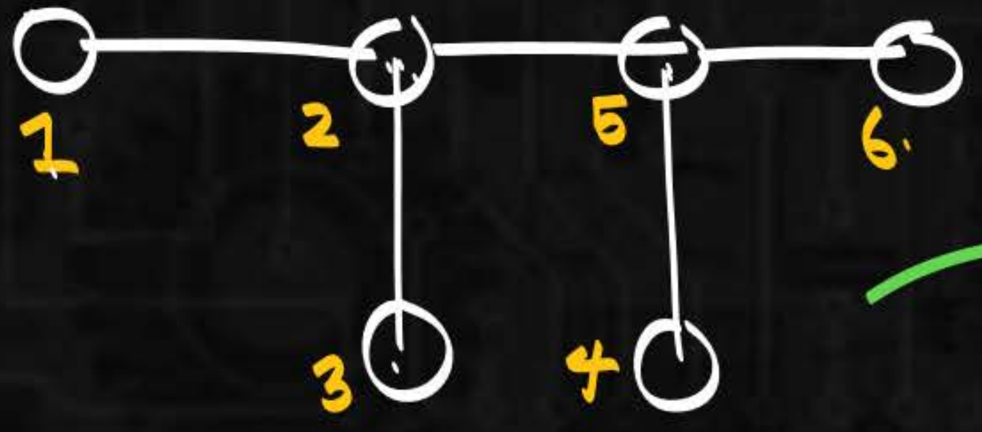
Graph $G = (V, E)$

V can be divided as V_1, V_2.

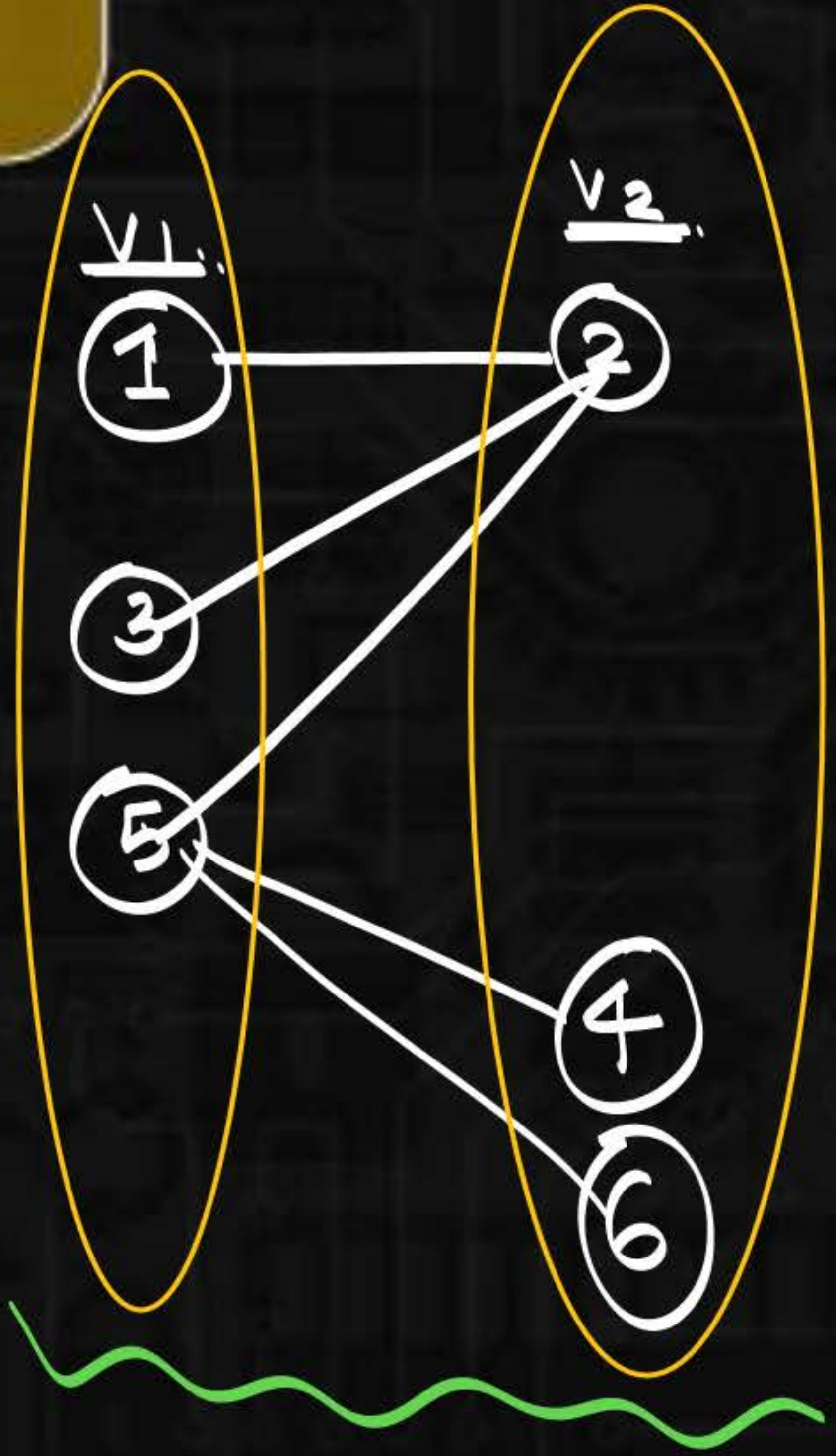
such that all edges will be from one set to another set but not in same set

Types of graph

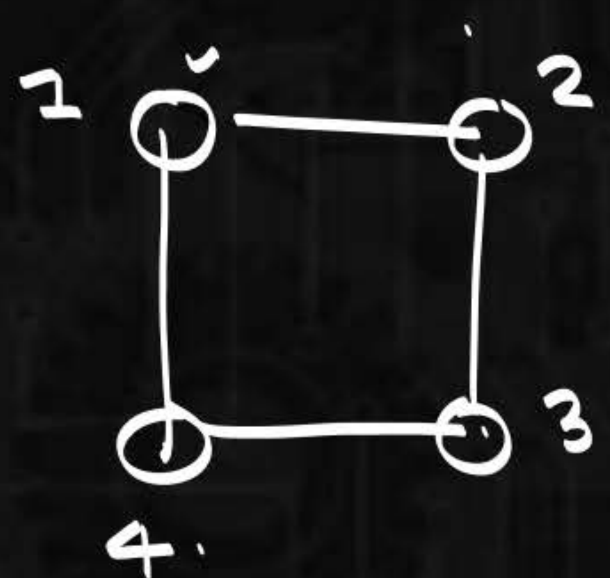
Check bipartite Graph?

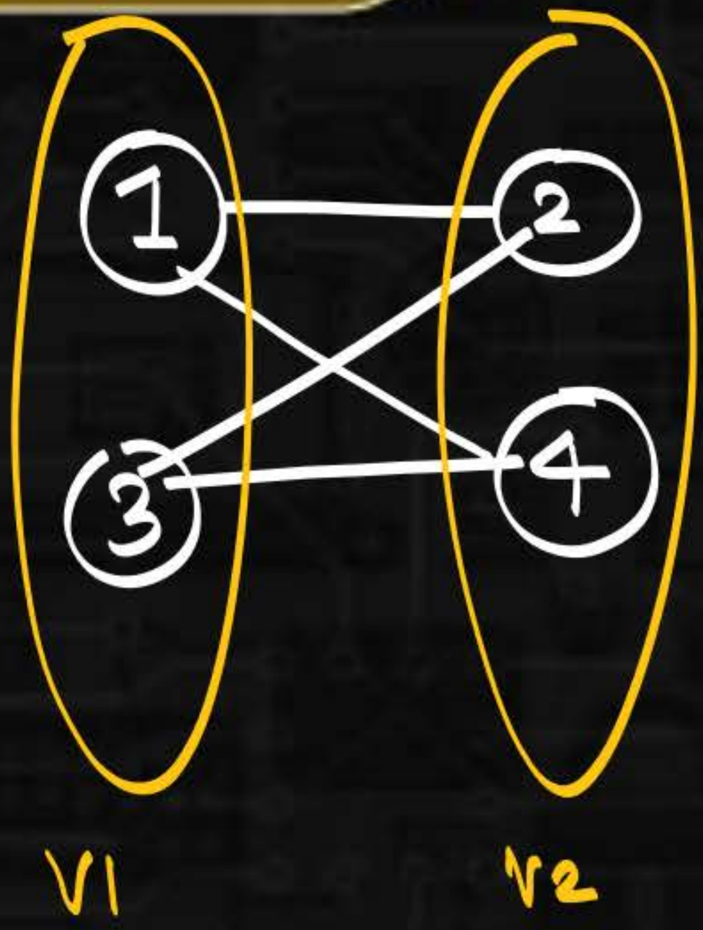


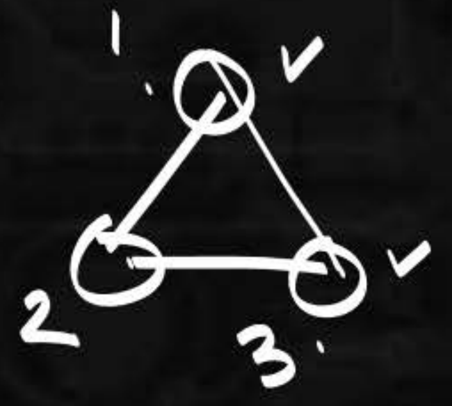
(acyclic Graph)



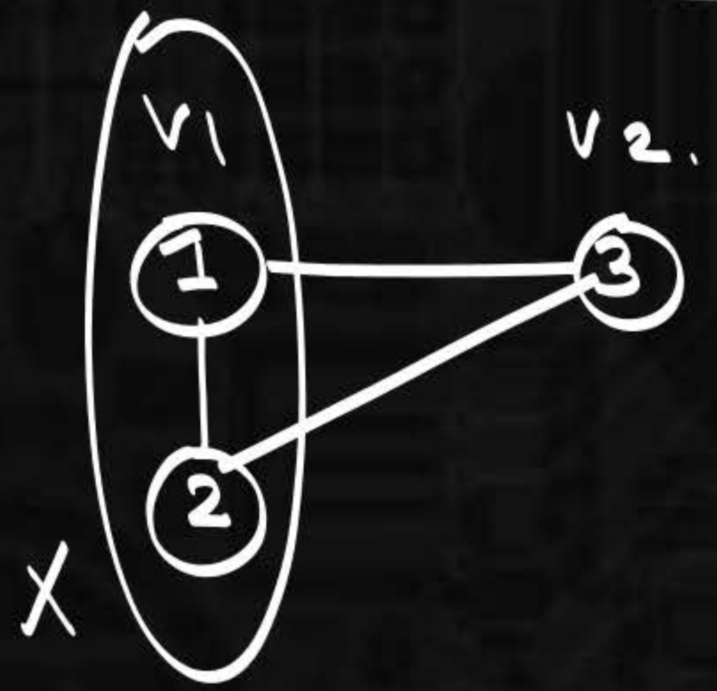
Types of graph

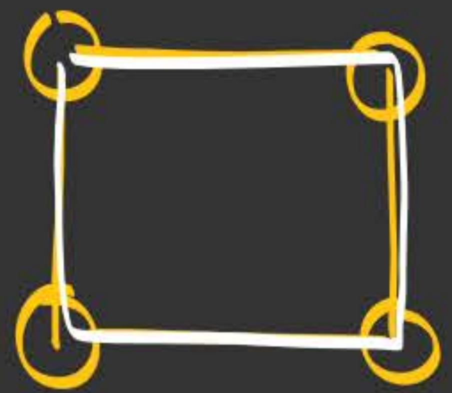






not bipartite Graph.





→ Even length cycle.

→ odd length cycle.

→ not a bipartite Graph.

Thm :

Bipartite Graph does not contains odd length cycle

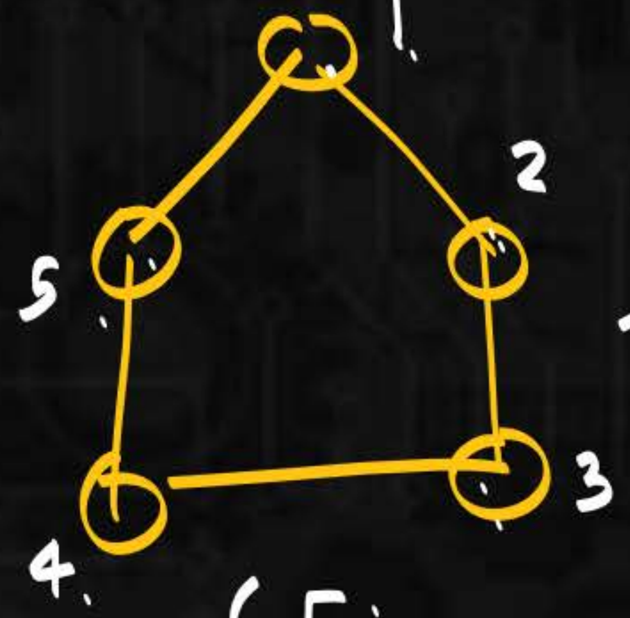
→ Even length cycle ✓

→ no cycle at all ✓

all even length cycles → B.P.G.

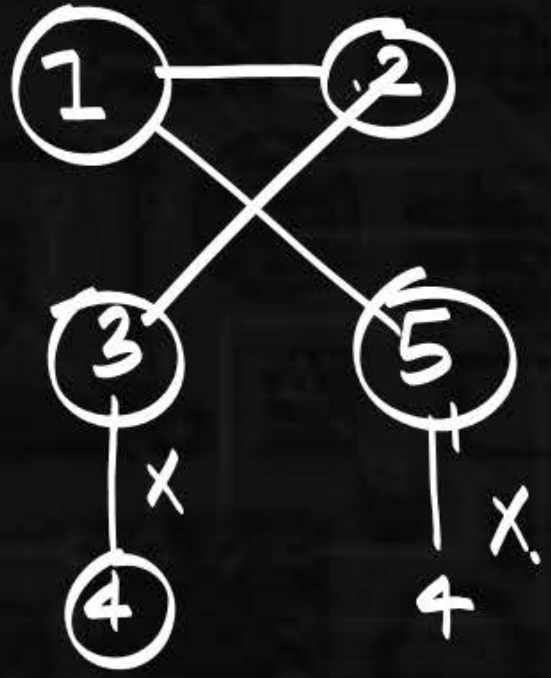
all acyclic graphs → B.P.G.
(Tree)

Types of graph

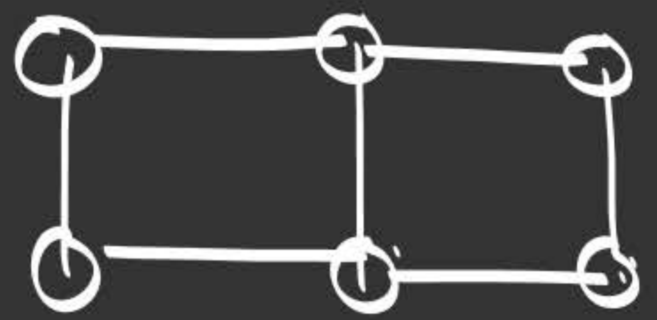


C5.

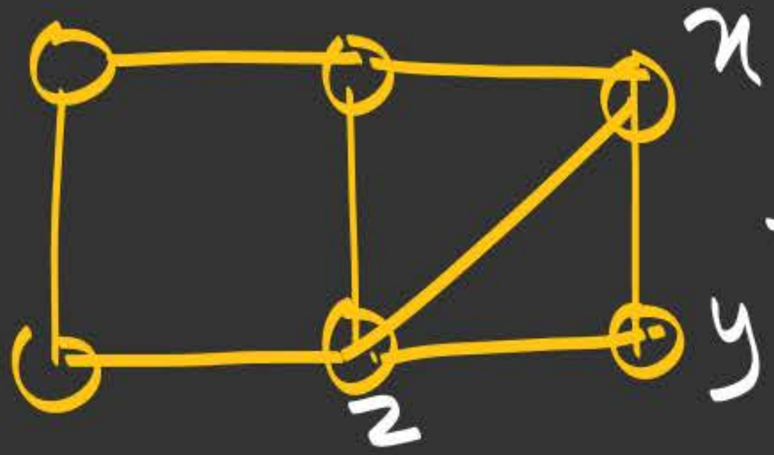
odd length cycle



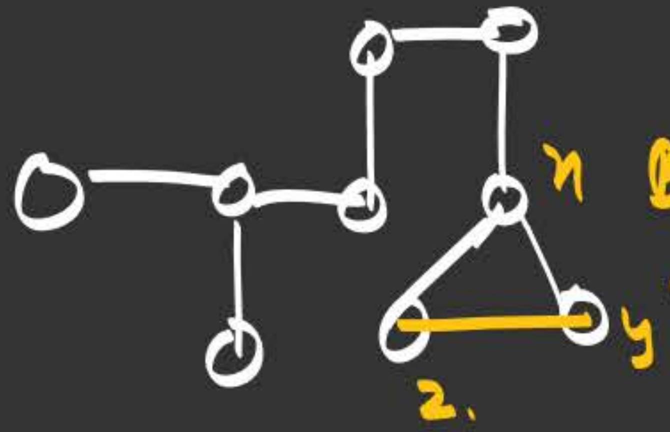
4 can not be adjustable in V1 or in V2.



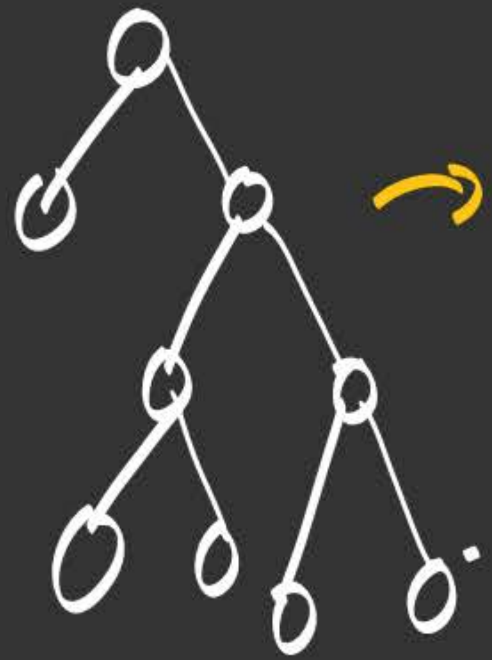
does not contains odd length cycle. → B.P.G.



→ not B.P.G.



~~B.P.G~~ ✓

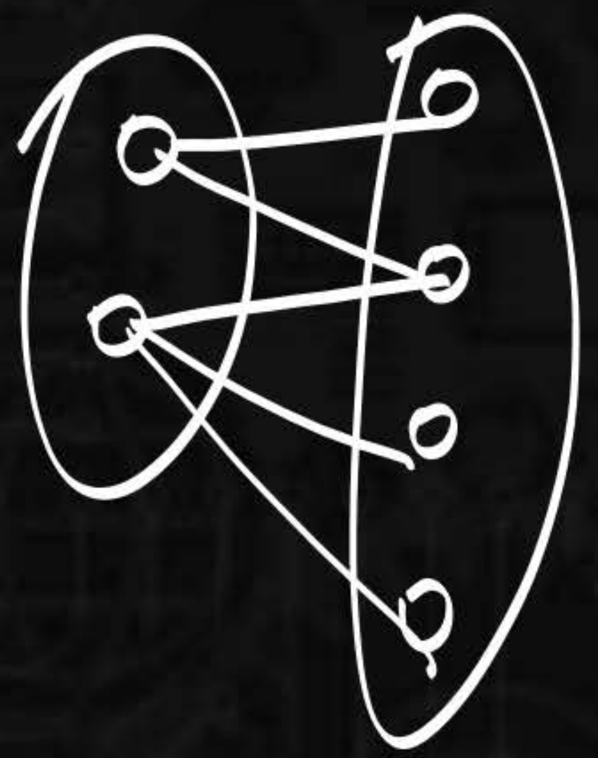


→ B.P.G ✓

Types of graph

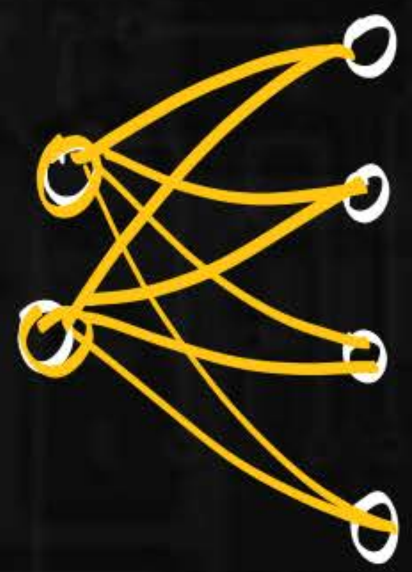
Complete bipartite Graph: $\left(K_{m,n} \right)$ $|V_1| = m$
$|V_2| = n.$

bipartite Graph.



Complete bipartite Graph:

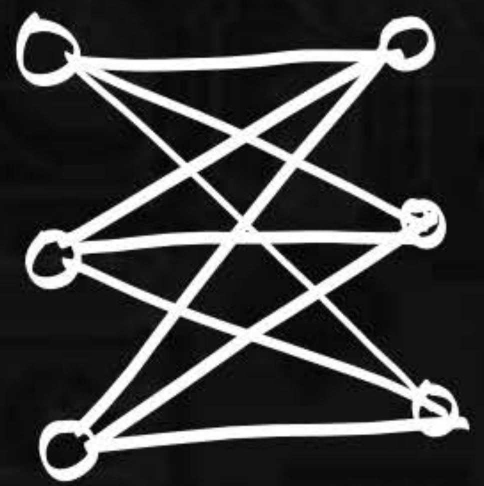
$K_{2,4}$.



all vertices of left side.
must be connected all vertices
of right side.

Types of graph

$K_{3,3}$



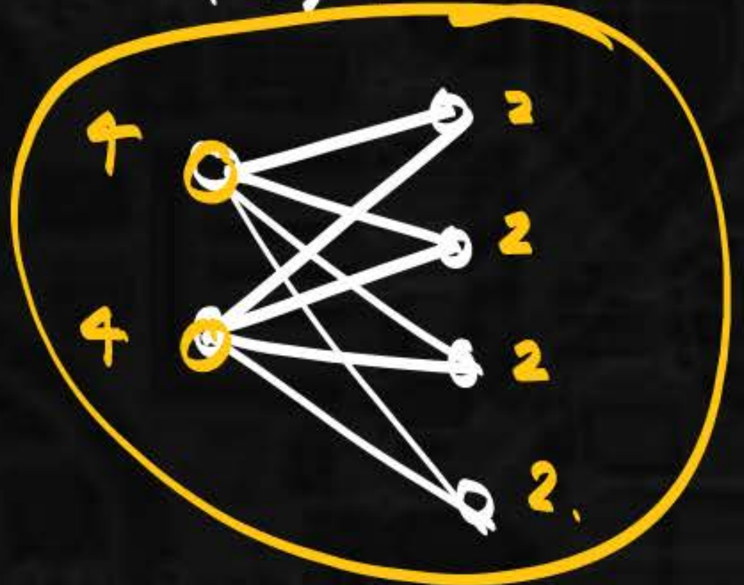
$K_{m,n}$ Total vertices $= m+n$.

Total edges $= m \cdot n$.

$\Delta(K_{m,n}) = \max\{m,n\}$.

$\delta(K_{m,n}) = \min\{m,n\}$.

$K_{2,4}$.



$|V| = 2+4 = 6$.

$E = 2 \cdot 4 = 8$

$\Delta(K_{2,4}) = \max(2,4)$
$= 4$.

$\delta(K_{2,4}) = \min(2,4)$
$= 2$.

Types of graph

Star Graph: $(K_{1,n-1})$

Draw star Graph of 6 vertices.

$K_{1,5}$

What will be no. of edges in the complement of star Graph with 6 vertices. $n = 6$

$\to K_{1,5}$

$e = 1 \times 5$
$= 5$

$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

$5 + e(\bar{G}) = \frac{6 \cdot 5}{2}$

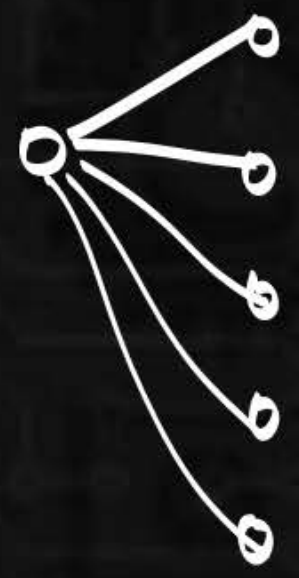
$e(\bar{G}) = 15 - 5 = 10$

$e(\bar{G}) = 10$

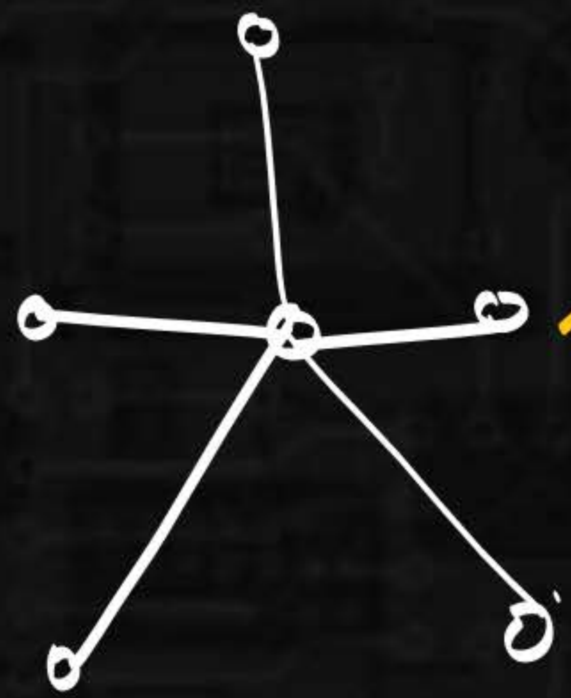
Types of graph

$K_{1,n-1}$ → take complement → we get: K_1 K_{n-1} → complete Graph of remaining vertices

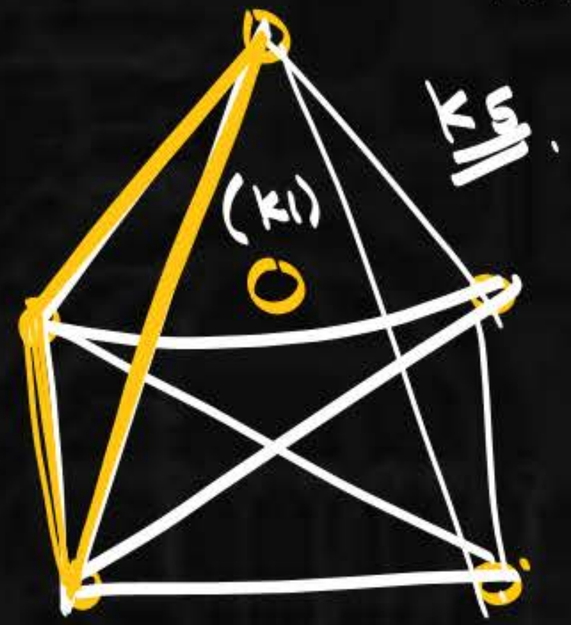
$K_{1,5}$



is same as



take complement →



K_1


THANK YOU GW
SOLDIERS !