CS & IT ENGINEERING

Discrete Mathematics

Combinatorics

Lecture No.- 08

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Recap of Previous Lecture

Topic Introduction to Combinatorics

Topics to be Covered

Topic Pigeonhole Principle

Topic : Combinatorics

Recurrence relation :

a_n : Total bacterias present @ time n.

In a colony, initially 5 bacterias are present at time 0 they are increasing 2 times as the previous. what will be total bacterias at time 100 ?

a_0 : at time 0, 5 bacterias are present

a_1 : at time 1

a_2 : at time 2

$\vdots$

a_{100} : at time 100,

Initial value $a_0, a_1, a_2, a_3, a_{98}, a_{99}, a_{100}$

$a_0 \to 5$

Related.

General Soltⁿ.

$a_0 = 5$

$a_1 = 2 \cdot a_0$

$a_2 = 2 \cdot a_1$

$\vdots$

$a_{100} = 2 \cdot a_{99}$

Recurrence / Relation

$a_1 = 2a_0$

$a_0 \leftarrow a_1$

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$a_0 = 5$

$a_1 = 2 \cdot a_0$

$a_2 = 2 \cdot a_1$

$= 2(2 \cdot a_0)$

$a_2 = 2^2 \cdot a_0$

$a_3 = 2 \cdot a_2$

$a_3 = 2(2^2 \cdot a_0)$

$a_3 = 2^3 \cdot a_0$

$a_4 = 2^4 \cdot a_0$

$\vdots$

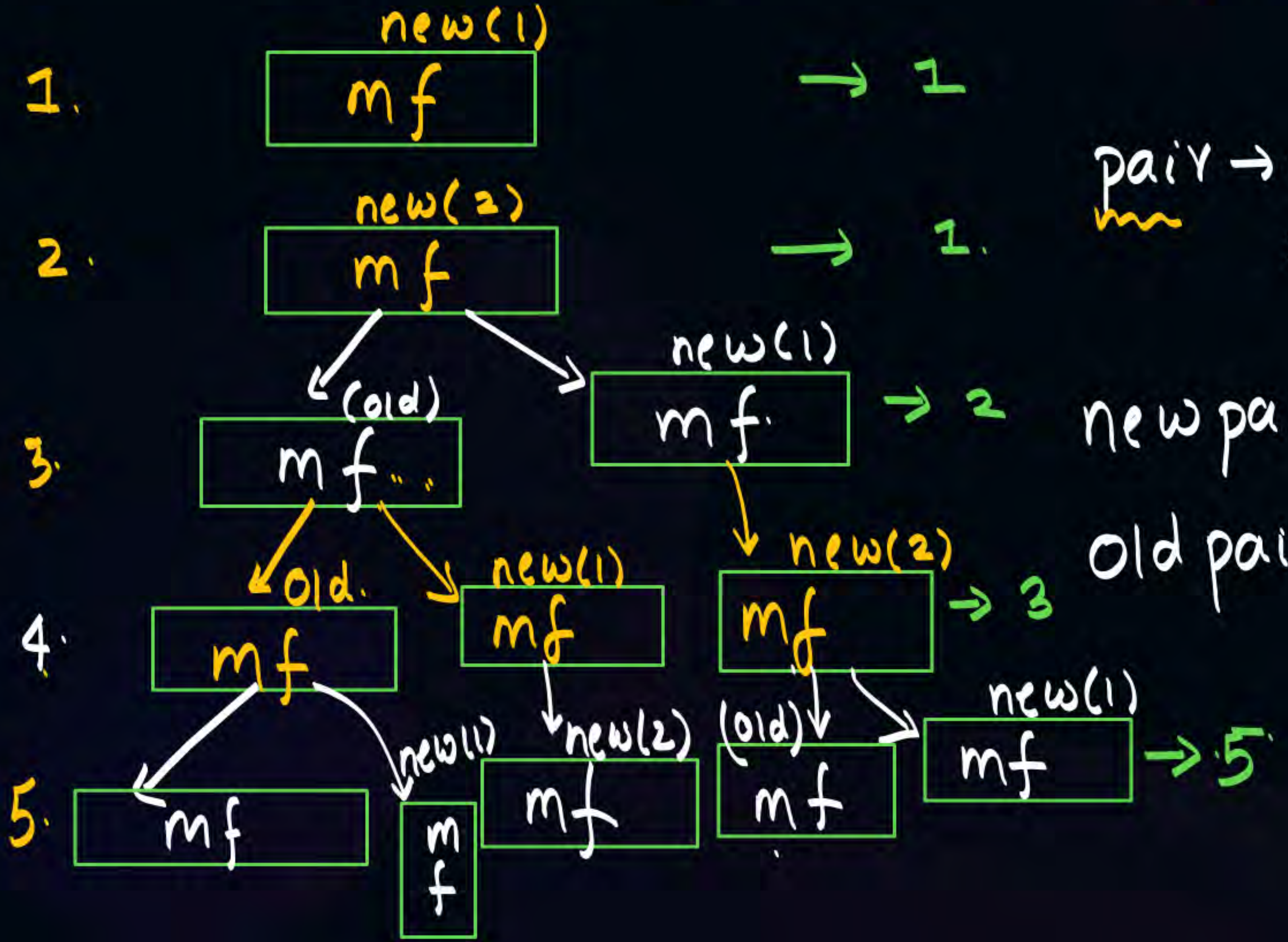
$a_{100} = 2^{100} \cdot a_0$

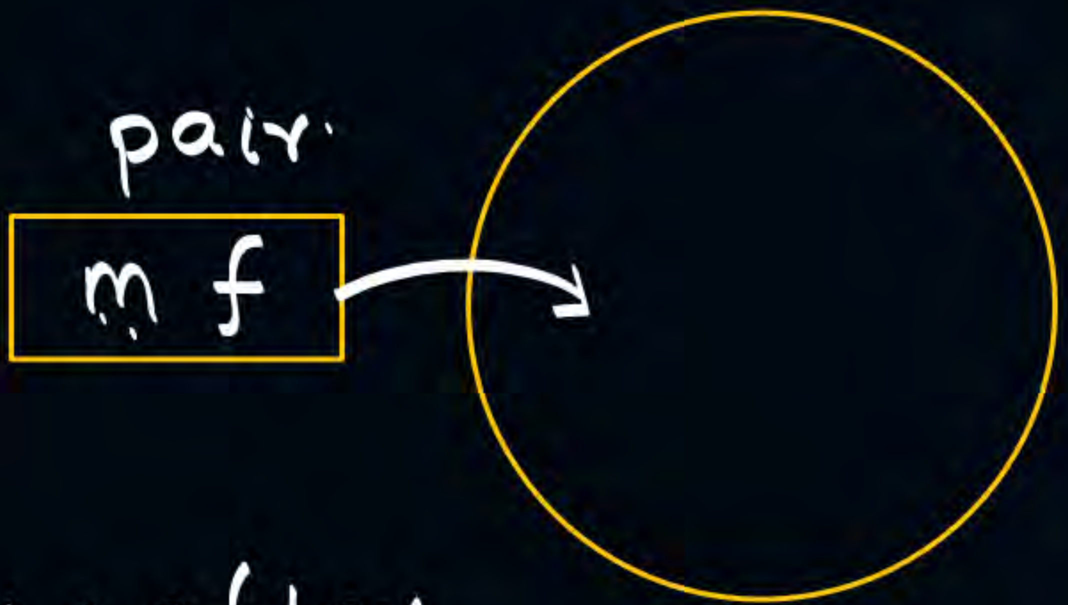
$$\begin{cases} a_n = 2\, a_{n-1} \\ a_0 = 5 \end{cases}$$

$\boxed{a_n = 2^n \cdot a_0}$ (a_0 = initial condition)

Topic : Combinatorics

of pairs = ?





pair → after 2 months → babies in terms of pair.

new pair → 2 months → pair.

old pair → every month → pair.

1, 1, 2, 3, 5

$$a_n = a_{n-1} + a_{n-2}.$$

$$a_n = 8a_{n-1} - 15a_{n-2}.$$

$a_0 = 3 \quad a_1 = 4.$

$c_1 = \frac{11}{2}$

$c_2 = -\frac{5}{2}$

$$a_n - 8a_{n-1} + 15a_{n-2} = 0$$

Step 1:

$$x^2 - 8x + 15x^0 = 0$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

Root: 3, 5

Step 2:

characteristic eqt^n

$$a_n = R_1^n c_1 + R_2^n c_2.$$

$$\boxed{a_n = 3^n c_1 + 5^n c_2}$$

$$a_n = 3^n\left(\frac{11}{2}\right) + 5^n\left(-\frac{5}{2}\right)$$

Step: 3:

$n = 0$ in CE

$$a_0 = 3^0 c_1 + 5^0 c_2.$$

$$\boxed{3 = c_1 + c_2}$$

$n = 1$

$$a_1 = 3^1 c_1 + 5^1 c_2$$

$$\boxed{4 = 3c_1 + 5c_2.}$$

$c_1 = \frac{11}{2}$

$c_2 = -\frac{5}{2}$

$c_1 + c_2 = 3$

$3c_1 + 5c_2 = 4.$

$\underline{-3c_1 + 3c_2 = -9}$

$2c_2 = -5$

$c_2 = -\frac{5}{2}$

$c_1 + c_2 = 3.$

$c_1 - \frac{5}{2} = 3$

$c_1 = 3 + \frac{5}{2} = \frac{11}{2}.$

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$$a_n = 7a_{n-1} - 10a_{n-2}.$$

$$a_0 = 3 \quad a_1 = 4.$$

Step 1:

$$a_n = 7a_{n-1} - 10a_{n-2}$$

$$x^2 = 7x - 10$$

$$x^2 - 7x + 10 = 0$$

Root: $5, 2.$

Step 2:

$$a_n = 5^n c_1 + 2^n c_2$$

$$a_n = 5^n\left(\frac{-2}{3}\right) + 2^n\left(\frac{11}{3}\right)$$

$n = 0$

$$a_0 = c_1 + c_2$$

$$c_1 + c_2 = 3$$

$\times 2$

$$2c_1 + 2c_2 = 6$$

$n = 1.$

$$a_1 = 5c_1 + 2c_2$$

$$\begin{array}{r} 5c_1 + 2c_2 = 4. \\ -\;2c_1 - 2c_2 = -6. \\ \hline 3c_1 = -2. \end{array}$$

$$c_1 = -\frac{2}{3}$$

$$c_1 + c_2 = 3.$$

$$\frac{-2}{3} + c_2 = 3$$

$$c_2 = 3 + \frac{2}{3} = \frac{11}{3}.$$

$$a_n = a_{n-1} + a_{n-2}. \quad a_0 = 0, \; a_1 = 1.$$

(GATE)

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$n = 0 \quad n = 1.$

$$c_1 = \frac{1}{\sqrt{5}} \quad c_2 = -\frac{1}{\sqrt{5}}.$$

$$x^2 - x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$a = 1 \quad b = -1 \quad c = -1.$

$$\boxed{a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$= \frac{-(-1) \pm \sqrt{1^2 - 4(1)(-1)}}{2 \cdot 1}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

Roots: $\dfrac{1 \pm \sqrt{5}}{2}$

CE: $a_n = \left(\dfrac{1+\sqrt{5}}{2}\right)^n c_1 + \left(\dfrac{1-\sqrt{5}}{2}\right)^n c_2.$

Type-1:

$a_n = \alpha a_{n-1}$.

a_0 = initial condt^n.

Solt^n: $a_n = \alpha^n a_0$

Type-2:

$a_n = x a_{n-1} + y a_{n-2}$.

Roots: R_1, R_2.

CE: $a_n = R_1^n c_1 + R_2^n c_2$.

Type-3:

$a_n = x a_{n-1} + y a_{n-2}$.

Roots: R, R

CE:

$a_n = R^n c_1 + n R^n c_2$.

$$\left\{ a_n = 6a_{n-1} - 9a_{n-2} \quad \begin{array}{l} a_0 = 1 \\ a_1 = 2. \end{array} \right.$$

T: 4.

$a_n = x a_{n-1} + y a_{n-2} + z a_{n-3}$.

Roots: R_1, R_2, R_3.

CE: $a_n = R_1^n c_1 + R_2^n c_2 + R_3^n c_2$.

T: 5

Roots: R, R, R_1.

CE: $a_n = R^n c_1 + n \cdot R^n c_2 + R_1^n c_3$.

T: 6. Roots: R, R, R.

$a_n = R^n c_1 + n \cdot R^n c_2 + n^2 \cdot R^n c_3$.

Topic : Combinatorics

$a_n = 6a_{n-1} - 9a_{n-2}$ $a_0 = 1$, $a_1 = 2$

Roots: 3, 3.

CE: $a_n = 3^n c_1 + n \cdot 3^n c_2$

$n = 0$

$a_0 = 3^0 c_1 + 0 \cdot 3^0 c_2$

$1 = c_1$

$n = 1$.

$a_1 = 3^1 c_1 + 1 \cdot 3^1 c_2$.

$2 = 3c_1 + 3c_2$

$2 = 3 + 3c_2$

$c_1 = 1$

$3c_2 = -1$.

$c_2 = -\frac{1}{3}$.

$a_n = 3^n + n\left(-\frac{1}{3}\right) \cdot 3^n$

$= 3^n - \frac{n}{3} \cdot 3^n$

$= 3^n\left(1 - \frac{n}{3}\right)$

THANK - YOU