CS & IT ENGINEERING

DISCRETE MATHS
SET THEORY

Lecture No. 05

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TOPICS TO BE COVERED

01 onto Functions

02 1:1 correspondance Functions

03 Number of Functions

04 Types of Functions

05 Various Examples in Functions

Equivalence Relation:

- Reflexive
- Symmetric
- Transitive

Set: $\mathbb{Z}$.

$R = \{(a,b) \mid a+b = \text{even}\}$

R: aRa $\quad a+a = \text{even}$.

$2a = \text{even}$.

Sym: $aRb \rightarrow bRa$

$a+b = \text{even} \rightarrow b+a = \text{even}$.

$2+4 = \text{even} \longrightarrow 4+2 = \text{even}$

$(2,4) \in R \qquad (4,2) \in R$.

III:

$aRb \wedge bRc \rightarrow aRc$

$a+b = \text{even} \wedge b+c = \text{even} \longrightarrow a+c = \text{even}$

$2+4 = \text{even} \wedge 4+8 = \text{even} \rightarrow 2+8 = \text{even}$

$1+3 = \text{even} \wedge 3+5 = \text{even} \rightarrow 1+5 = \text{even}$

$R_2 = \{(a,b) \mid a \equiv b \pmod n\}$ ✓

<u>R</u>: <u>aRa</u> $a \equiv a \pmod n$

<u>Sy</u>: $a \equiv b \pmod n \rightarrow b \equiv a \pmod n$

T: $a \equiv b \pmod n \wedge b \equiv c \pmod n \rightarrow a \equiv c \pmod n$

$0 \equiv 4 \pmod 4 \wedge 4 \equiv 8 \pmod 4 \rightarrow 0 \equiv 8 \pmod 4$

$a \equiv b \pmod n$

a, b are having same remainder w.r.t (n)

$n = 4$

$1 \equiv 5 \pmod 4$ $(1,5) \in R.$

$2 \equiv 6 \pmod 4$ $(2,6) \in R.$

$0 \equiv 8 \pmod 4$ $(0,8) \in R.$

Set: Z^+

$R = \{(a, b) \mid a \text{ divides } b\}$.

$R = \{(a, b) \mid a | b\}$.

R: aRa $\quad a|a$. ✓

Sym: $a|b \rightarrow b|a$.

a divides b.

$a|b$ or $\frac{b}{a}$

Equivalence Relation creates partition on a set.

Set : A

A x A

EQR

Result: { ...... }

Partition / class

all elements.

Set:

a, b.
c, d.

$R = \{(a, b)\ (c, d)$

$(a,b) \in R$.

aRb.

a, b will go to same partition / class.

$a \not R c \rightarrow$ diff class.

$R = \{(a,b) \mid a+b = \text{even}\}$

$\downarrow$

EQR ✓
(RST)

$\downarrow$

Class

$\downarrow$

set

$(0,0) \in R$

$0+0 = \text{even}$

$(0,2)$

$(0,4)$

$(1,3) \in R$

$1+3 = \text{even}$

set : Z.

$0, 2, 4, 6 \ldots\ldots a_1$ — even

$1, 3, 5, 7 \ldots\ldots b_1$ — odd

2.

$a_1 \cancel{R} b_1$

equivalence Relation.

$R_2 = \{ (a, b) \mid \underline{\underline{a \equiv b \pmod 4}} \}$

$\downarrow$

EQR (RST)

$\downarrow$

class.

$\downarrow$

set = Z.

$[0]_R$	0, 4, 8	$\leftarrow$ 0	0, 4.
$[1]_R$	1, 5, 9	$\leftarrow$ 1	1, 5
$[2]$	2, 6, 10	$\leftarrow$ 2	
$[3]$	3, 7, 11	$\leftarrow$ 3.	

$R = \{ (s_1, s_2) \mid s_1, s_2 \in \text{same state} \}$

end with 1.

$\rightarrow$ EQR $\rightarrow$ Class $\rightarrow$ set of strings.

R: 01 R 01

Sy: 01 R 011 $\rightarrow$ 011 R 01.

T: 01 R 011 $\wedge$ 011 R 01111 $\rightarrow$ 01 R 01111

$R_1 \rightarrow$ EQR

$R_2 \rightarrow$ EQR.

$R_1 \cup R_2 \rightarrow$

$R_1 \cap R_2 \rightarrow$

$R_1 \rightarrow$ Transitive $R_1 = \{(1\,2)\}$ ✓ $(a,b) \in R \wedge \boxed{(b,c) \in R} \rightarrow (a,c) \in R$

$(1,2) \in R \wedge$ F

$R_2 \rightarrow$ Transitive $R_2 = \{(2\,1)\}$ ✓

F $\longrightarrow$

True

$R_1 \cup R_2 = \{(1\,2)(2\,1)\} \rightarrow$ not transitive

$R_1 \cap R_2 = \phi$ ✓

$R_1 \rightarrow EQR$

$\cup$

$R_2 \rightarrow EQR$

R_1	S_1	T_1
$\cup$	$\cup$	$\cup$
R_2	S_2	T_2.
✓	✓	✗

$$R_1 = \{ \underbrace{(11)(22)(33)}_{R} \underbrace{(12)(21)}_{S} \}$$

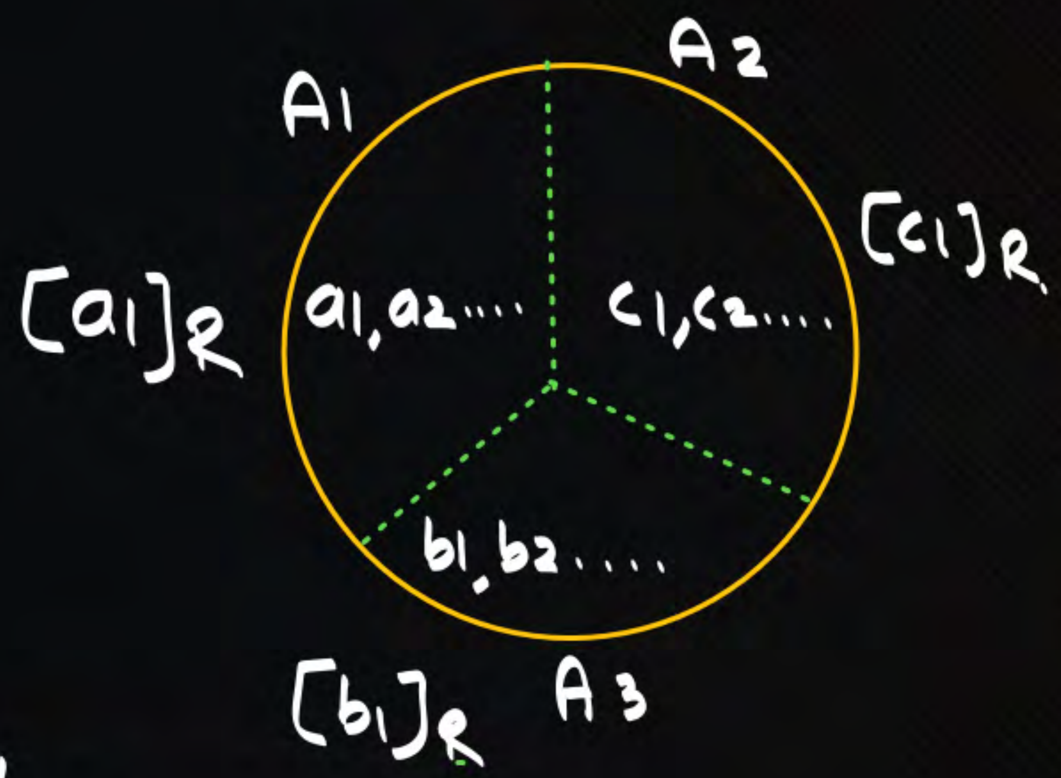
$$R_2 = \{ (11)(22)(33)(13)(31) \}$$

$$R_1 \cup R_2 = \{ (11)(22)(33)\underline{(12)}(21)(13)\underline{(31)} \}$$

EQR ✗

Set : A.

Equivalence relation creates partition on a set



1) $A_1 \cup A_2 \cup A_3 = A$.

2) $A_1 \cap A_2 \cap A_3 = \phi$

$$[a_1] \not R [b_1]$$

$[\ ]_R$ = Representative of a class.

Total no. of equivalence Relation → Total diff no. of partitions.

$S(3,1)+S(3,2)+S(3,3)$

$A = \{1, 2, 3\}$

Total equivalence relation.

$S(3,3)$

$R_1 = \{11\ 22\ 33\}$ → 1 | 2 | 3

$B(3) = \sum_{i=1}^{3} (3, i)$

$R_2 = \{11\ 22\ 33\ 12\ 21\}$ → 1, 2 | 3

$B(n) = \sum_{i=1}^{n} (n, i)$

$S(3,2)$

$R_3 = \{11\ 22\ 33\ 23\ 32\}$ → 2, 3 | 1

$R_4 = \{11\ 22\ 33\ 13\ 31\}$ → 1, 3 | 2

$R_5 = \{11\ 22\ 33\ 12\ 21\ 13\ 31\ 23\ 32\}$ → 1 2 3

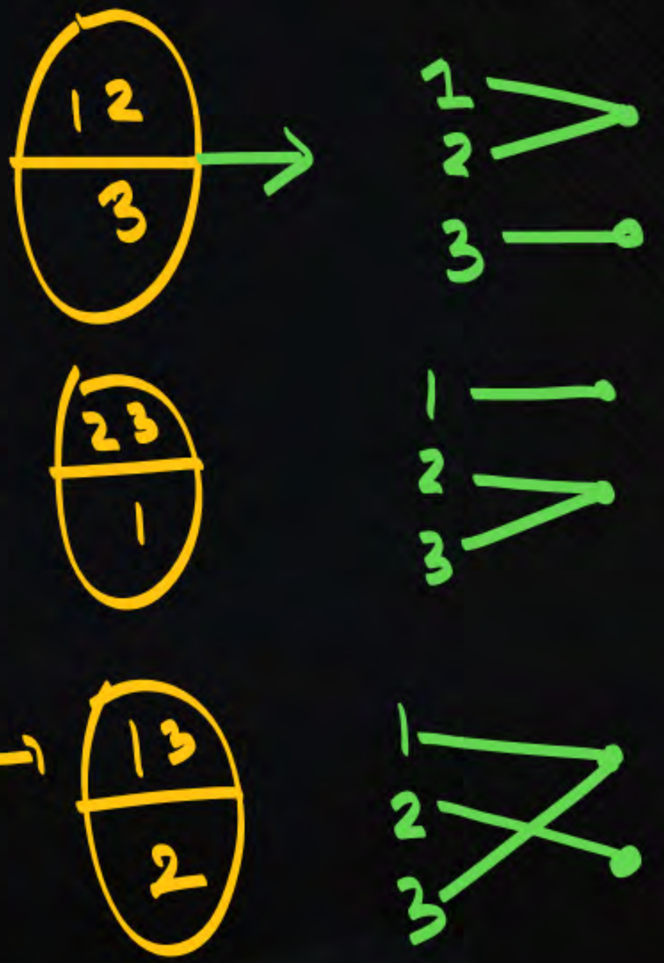
$S(3,1)$

$\boxed{S(3,2)}$

$R_2 = \{11 \;\; 22 \;\; 33 \;\; 12 \;\; 21\} \rightarrow$

$R_3 = \{11 \;\; 22 \;\; 33 \;\; 23 \;\; 32\} \rightarrow$

$R_4 = \{11 \;\; 22 \;\; 33 \;\; 13 \;\; 31\} \rightarrow$

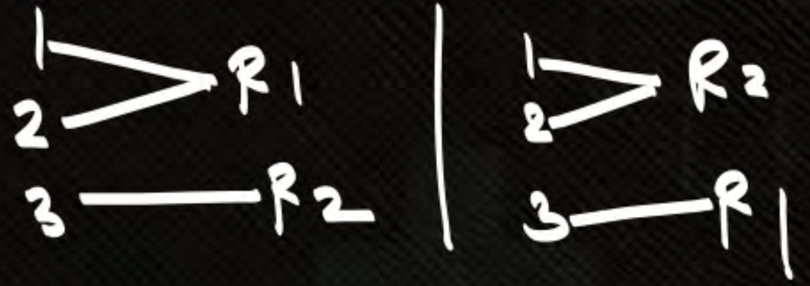


S(3, 2)

$R_1 = R_2$.

onto

$R_1 \neq R_2$



$$
\begin{tikzcd}
B_0 \arrow[r] & 1 \arrow[dr] \\
B_1 \arrow[r] & 1 \arrow[u] \arrow[dr] & 2 \arrow[dr] \\
B_2 \arrow[r] & 2 \arrow[u] \arrow[dr] & 3 \arrow[u] \arrow[dr] & 5 \arrow[dr] \\
B_3 \arrow[r] & 5 \arrow[u] \arrow[dr] & 7 \arrow[u] \arrow[dr] & 10 \arrow[u] \arrow[dr] & 15 \arrow[dr] \\
B_4 \arrow[r] & 15 \arrow[u] & 20 \arrow[u] & 27 \arrow[u] & 37 \arrow[u] & 52
\end{tikzcd}
$$

$B_5 \to 52.$

$$
\begin{tikzcd}
b \arrow[dr] & c \arrow[dr] \\
a \arrow[u] & a+b \arrow[u] & a+b+c
\end{tikzcd}
$$

Completed :

Shortnotes → PYQ → Section Test


THANK YOU GW
SOLDIERS !