

CS & IT ENGINEERING



DISCRETE MATHS
COMBINATORICS

Lecture No. 02



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TOPICS TO BE COVERED

01 sum rule

02 Product rule

03 Practice

COMBINATORICS

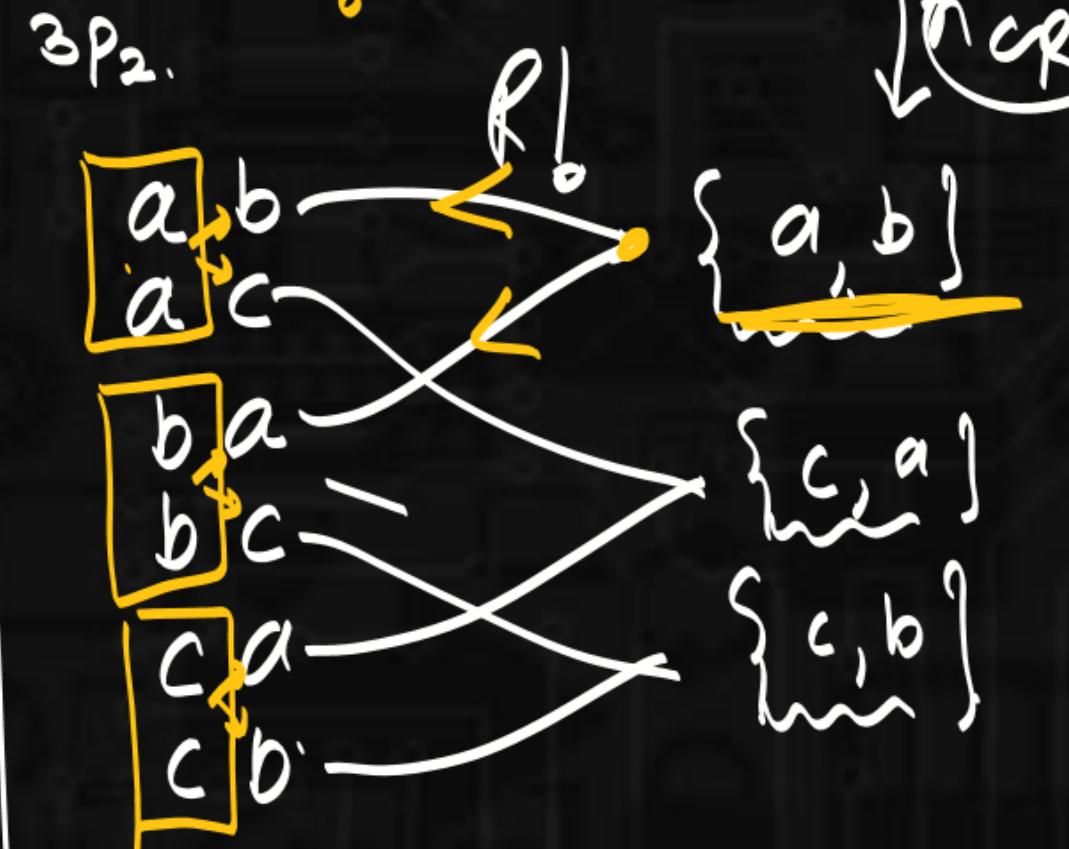
$\{a, b, c\}$.

$$3P_2 = 6$$

How many ways we can arrange 2 elements?

Product Rule:

$$\downarrow \times \downarrow \\ 3 \text{ways} \times 2 \text{ways}$$



$\{a, b, c\}$

nCR

How many ways we can select 2 elements?

$$nCR = \frac{n!}{r!(n-r)!}$$

$$nCR \times R! = \frac{n!}{(n-r)!}$$

$$nCR \times R! = nPR$$

COMBINATORICS

no. of ways to select 2 elements = shifting 2 lines.

$$\{a, b, c\}$$

$$\{a, b\} \rightarrow \left\{ \begin{matrix} 1 & 2 & 3 \\ | & | & - \end{matrix} \right\}$$

$$\{b, c\} \rightarrow \left\{ \begin{matrix} - & | & | \end{matrix} \right\}$$

$$\{a, c\} \rightarrow \left\{ \begin{matrix} | & - & | \end{matrix} \right\}$$

how many ways to select

1 element?

$$\{a\} \rightarrow \left\{ \begin{matrix} | & - & - \end{matrix} \right\}$$

$$\{b\} \rightarrow \left\{ \begin{matrix} - & | & - \end{matrix} \right\}$$

$$\{c\} \rightarrow \left\{ \begin{matrix} - & - & | \end{matrix} \right\}$$

= Places C line.

$$= 3C_2 = 3.$$

→ how many ways to shift 1 line

Places C line.

$$3C_1 = 3.$$

COMBINATORICS

mall, there are collections of Apple, oranges, papaya.
how many ways we can select 4 fruits?

Total ways = 15

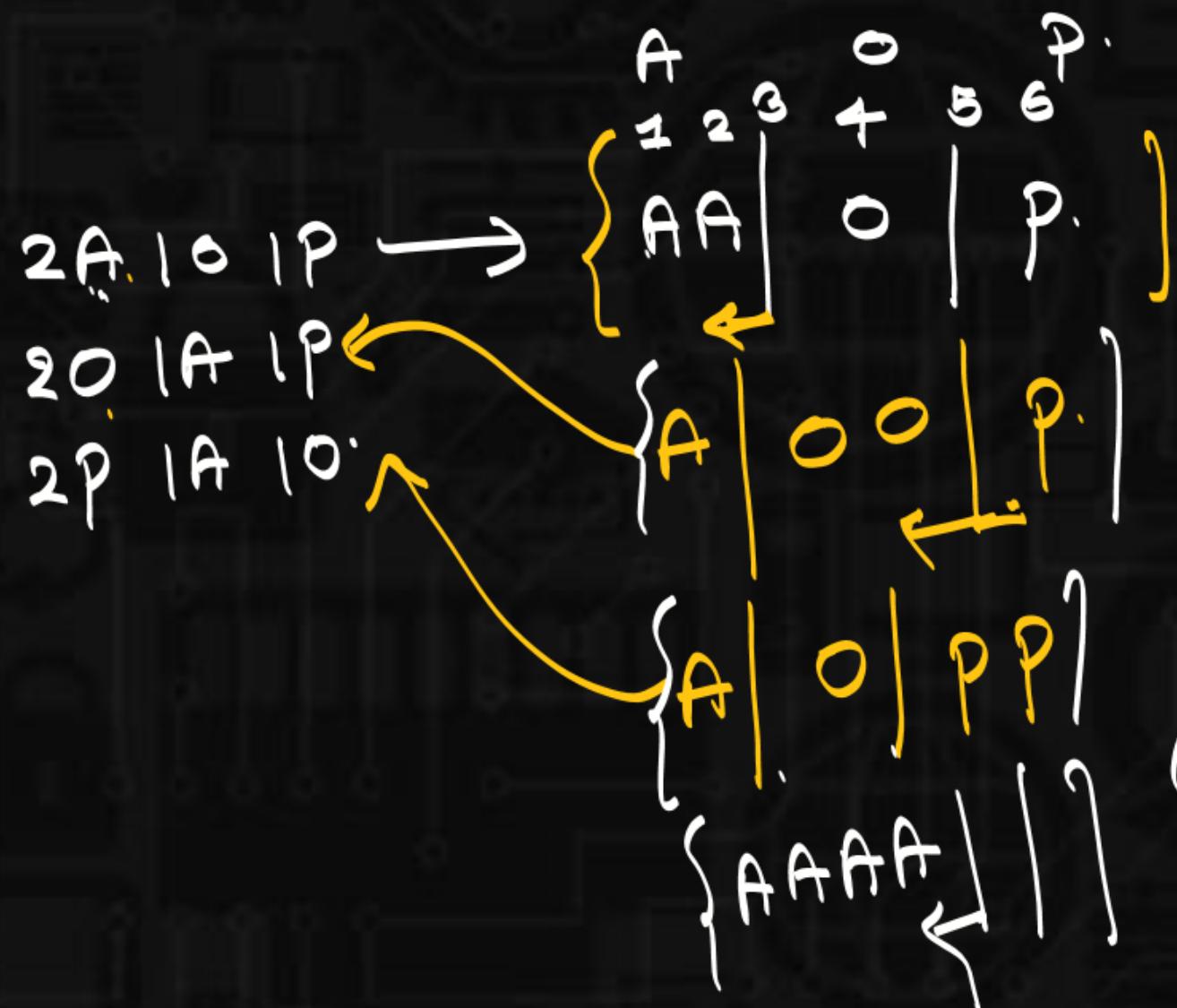
Ans: 15

$$\begin{array}{ll}
 \boxed{4A} & 3A, 1P \\
 \boxed{4P} & \boxed{3A, 1O} \\
 4O & 3P, 1A \\
 3P, 1O & \\
 3O, 1A & \\
 3O, 1P &
 \end{array}$$

A P O

$$\begin{array}{l}
 2A, 2O \\
 2A, 2P \\
 2P, 2O
 \end{array}$$

$$\begin{array}{l}
 2A, 1O, 1P \\
 2O, 1A, 1P \\
 2P, 1A, 1O
 \end{array}
 \longrightarrow$$



now ways
to shift
2 lines
Places C line

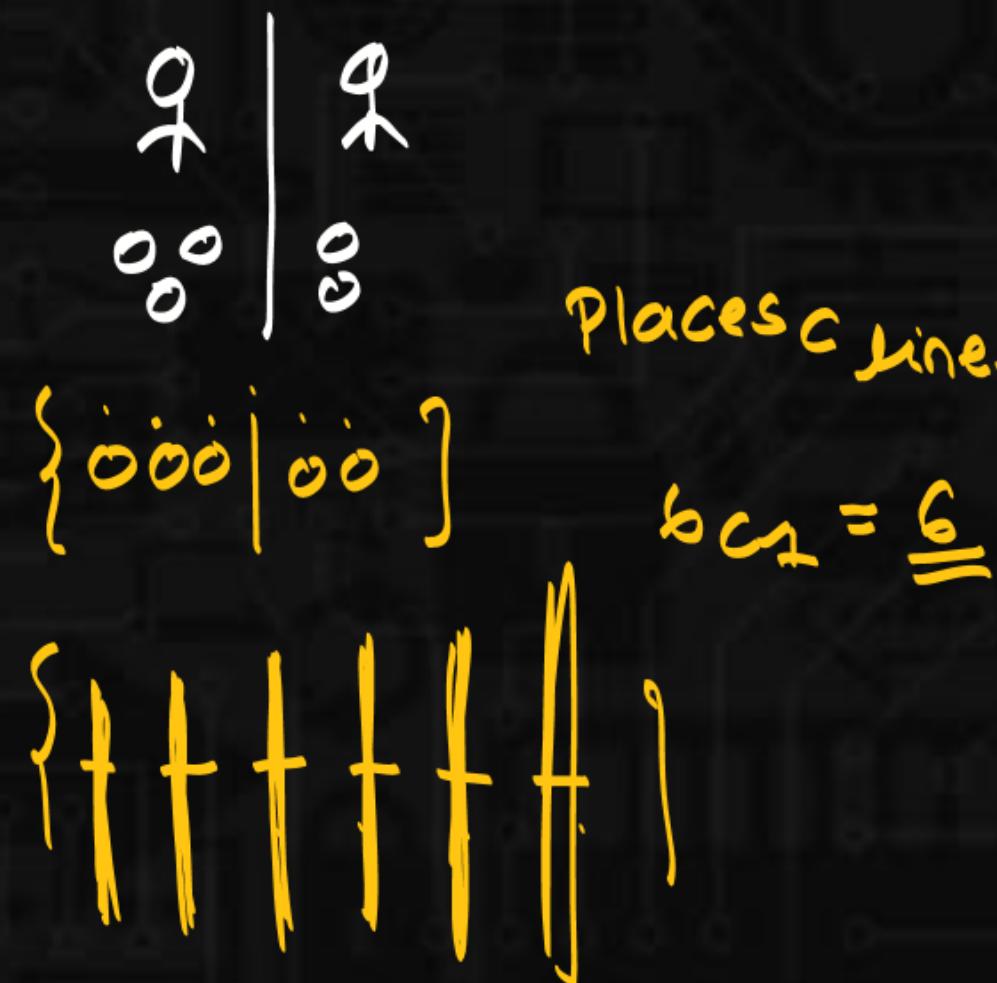
$$\begin{aligned}
 & {}^3C_2 \\
 & = \frac{6}{2!} = 15
 \end{aligned}$$

$$\boxed{\text{AAA|O}}$$

COMBINATORICS

How many ways to distribute 5 flowers among 2 children ?.

5	0
0	5
1	4
4	1
3	2
2	3



COMBINATORICS

How many ways to distribute 10 identical coins among 4 children?



how many nonnegative soltn are ?.

$$x_1 + x_2 + x_3 + x_4 = 10 \quad x_i \geq 0$$

$$x_1 = 10 \quad x_2, x_3, x_4 = 0$$

$$x_1, x_2, x_3 = 0 \quad x_4 = 10$$

10	0	0	0
0	0	0	0
0	0	0	0

$$10 + 3C_3$$

$$= 13C_3$$



$$x_1 + x_2 + x_3 + x_4 = 10$$

$$x_1 + x_2 + x_3 + x_4 = 10.$$

COMBINATORICS

How many ways to distribute 10 identical coins among 4 children, such that each child gets at least 1 coin?

$$= \underline{6}$$

$$\begin{array}{|c|c|c|c|} \hline 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \\ \hline \end{array} \quad 6+3C_3$$

q_{C_3}

how many nonnegative soltn are there?

$$x_1 + x_2 + x_3 + x_4 = 10 \quad x_i \geq 1$$

$$y_i = x_i - 1 \quad y_i \geq 0$$

$$y_i + 1 = x_i \quad \frac{x_i - 1 \geq 0}{y_i}$$

$$y_1 + 1 = x_1$$

$$y_1 + 1 + y_2 + 1 + y_3 + 1 + y_4 + 1 = 10 \quad y_i \geq 0$$

$$y_1 + y_2 + y_3 + y_4 = 10 - 4$$

$$y_1 + y_2 + y_3 + y_4 = 6 \quad y_i \geq 0$$

q_{C_3}

COMBINATORICS

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$\boxed{x_1 \geq 0}$$

$$x_1 > 1 \quad x_2 > 2 \quad x_3 > 3 \quad x_4 > 4$$

$$x_1 \geq 2 \quad x_2 \geq 3 \quad x_3 \geq 4 \quad x_4 \geq 5$$

q_{C_3}

$$x_1 + x_2 + x_3 + x_4 = 20$$

$$x_1 \geq 2 \quad x_2 \geq 3 \quad x_3 \geq 4 \quad x_4 \geq 5$$

$$\frac{x_1 - 2}{y_1} \geq 0 \quad \frac{x_2 - 3}{y_2} \geq 0 \quad \frac{x_3 - 4}{y_3} \geq 0 \quad \frac{x_4 - 5}{y_4} \geq 0$$

$$y_1 + 2 = x_1 \quad y_2 + 3 = x_2 \quad y_3 + 4 = x_3 \quad y_4 + 5 = x_4$$

$$y_1 + 2 + y_2 + 3 + y_3 + 4 + y_4 + 5 = 20$$

$$6 + 3$$

$$= q_{C_3}$$

$$y_1 + y_2 + y_3 + y_4 = 20$$

$$- (2 + 3 + 4 + 5)$$

$$= 6$$

$$y_i \geq 0$$

COMBINATORICS

$$\left\{ \begin{array}{l} x_1 + x_2 + x_3 + x_4 + \dots + x_7 = 30 \\ \text{where} \end{array} \right.$$

$$x_1 + x_2 + x_3 = 10$$

$$\frac{10}{x_1 + x_2 + x_3} + \frac{30 - 10 = 20}{x_4 + x_5 + x_6 + x_7} = 30 \rightarrow$$

$\downarrow \quad \quad \quad \downarrow$

$$12C_2 \quad x \quad 20+3C_3$$

$$12C_2 \times 23C_3.$$

x_1	x_2	x_3	x_4	x_5	x_6	x_7
5	5	0	5	5	5	5
6	4	6	4			
10	0	10	0			

COMBINATORICS



$$x_1 + x_2 + x_3 + x_4 \leq 10$$

$$10+4c_4 = 14c_4$$

$$x_1 + x_2 + x_3 + x_4 + a = \underline{\underline{10}}$$

$$\rightarrow \boxed{x_1 + x_2 + x_3 + x_4 = 0} \quad a = 10$$

$$x_1 + x_2 + x_3 + x_4 = 1 + a = 9$$

$$x_1 + x_2 + x_3 + x_4 = 2 + a = 8$$

⋮

$$x_1 + x_2 + x_3 + x_4 = 10 + a = 0$$

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + x_2 + x_3 + x_4 = 1$$

⋮

$$x_1 + x_2 + x_3 + x_4 = 10$$

COMBINATORICS

How many ways we can distribute 15 sunflowers and 20 daffodils, we can distribute among 2 girls ?.

mistake:

$$\begin{array}{c} 15+20 \\ = 35 \end{array}$$

→ 9 | 9

$$\begin{array}{c} 35+1C_1 \\ = 36C_1 \end{array}$$

$$15 \text{ sun} \rightarrow 15+1C_1 = 16$$

$$20 \text{ D} \rightarrow 20+1C_1 = 21$$

$$16 \times 21$$

= .

COMBINATORICS

$(m \geq nk)$

How many ways we can distribute m identical balls among n boxes, such that each box must contain at least k balls?



(GATE)

n boxes.

$n-1$ lines.

nk balls we have distributed.

Remaining balls

$$= m - nk.$$

$$m - nk + n - 1 < n - 1.$$

COMBINATORICS



Consider set contains an elements between 1 to 1000 (inclusive)
such that the sum of digits must be 10, what will be
total possibility?

{ 1
.
.
.
1000

$$66 - 3 = \underline{63}$$

۱۰۵

(0, · 10)

$$x_1 + x_2 + x_3 = 10$$

↓

66

$$10 + 2 \zeta_2 = 12 \zeta_2 = \frac{12}{\zeta_2}$$

$$= 66$$

	3 6 1	$3+6+1=10$
1 0 0 0	0 5 5	$0+5+5=10$
<u>1 + 0 + 0 + 0 = 1.</u>	5 0 5	$5+0+5=10$

In this 66 ways it includes
3 possibilities \rightarrow (10, 0, 0)
(0, 10, 0)
(0, 0, 10)

COMBINATORICS

$$x_1 + x_2 + x_3 = 10$$

$$12C_2 = 66$$



$$\boxed{5, 0, 5}$$

$$3 \quad 0 \quad 7$$

$$\begin{array}{ccc} 1 & 1 & 8 \\ 2 & 2 & 6 \end{array}$$

⋮

$$\left. \begin{array}{c} 1 \\ 10 \\ 0100 \end{array} \right\} \rightarrow 0+1+0+0 = 1$$

$$505 \rightarrow 5+0+5 = 10$$

$$\boxed{1000} \rightarrow 1+0+0+0 = 1$$

COMBINATORICS

In how many ways a teacher can distribute 8 chocolates candy + jelly candy , among 3 students , if each students wants at least 1 of each kind?

$$\begin{matrix} 8-3 & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & 7-3=4 \\ =5 & | & | & | & | & | & | & \end{matrix}$$

$$5+2C_2$$

$$7C_2 \times 6C_2$$

COMBINATORICS



$$x_1 + x_2 + \dots + x_7 = 37$$

$$x_1 + x_2 + x_3 = 6$$

$$x_1, x_2, x_3 > 0$$

$$2) \quad x_1 + x_2 + x_3 + x_4 + x_5 < 40$$

$$x_i \geq -3$$

How many ways to place
12 marbles among 5 jars.

- a) all marbles are black.
- b) all marbles of diff color.

