CS & IT ENGINEERING

DISCRETE MATHS

Mathematical logic

Lecture No. 1

By- SATISH YADAV SIR

TOPICS

01 Connectives

02 Type 1

03 Type 2

English stmt

Factual

Conclusion.

→ Thm. → Algo → PL

$\downarrow$

S/W.

propositional stmt → Yes

→ no

Simple.

Compound.

breakable

C

S1.

S2.

Simple.

$$\frac{5 > 3}{(T)}$$

Compound.:

$$5 \geq 3$$

$$5 > 3 \text{ OR } 5 = 3$$

Connectives:

Conjunction → (AND/but) → $\wedge$

Disjunction

Conditional

biconditional.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$(\_\_ F \wedge \_\_) \equiv F$

modifier

negation

$(\neg / \sim)$

$F \wedge \_\_ \wedge \_\_$

false

AND HATES FALSE

OR. → Exclusive OR (XOR) ($\oplus$) (one/other but not both)

→ Inclusive OR ($\vee$)

(one/other/both)

p	q	$p \vee q$
T	F	T
F	T	T
T	T	T
F	F	F

$(T \vee \_\_\_) \equiv T$

$(T \vee \_\_ \vee \_\_) \equiv T$

OR loves True

Conditional ($\rightarrow$)

$p \rightarrow q$

if p then q.

if p, q.

q if p

q when p.

q whenever p.

q unless $\neg p$.

p only if q.

p implies q.

if Graph is planar then $e \le 3n-6$.

p	q	$p \to q$ (premise)
planar (T)	$e \le 3n-6$ (T)	T
planar (T)	$e \le 3n-6$ (F)	F
planar (F)	$e \le 3n-6$ (T)	T
planar (F)	$e \le 3n-6$ (F)	T

$T \to T \equiv T$

$T \to F \equiv F$

$F \to T \equiv T$

$F \to F \equiv T$

$\to T \equiv T$

$F \to \quad \equiv T$

promise

if you win the match then i will give pizza

$F \to \equiv T$

$\text{match}(T) \to \cancel{\text{pizza}}(F) \equiv F$.

$\cancel{\text{match}}(F) \to \text{pizza}(T) \equiv T$.

if you clear gate then she will marry you

$F \to \equiv T$

if perfect matching exist then no. of vertices will be even.

p $\qquad$ q

$p \to q \equiv \neg q \to \neg p$

$p \to q \not\equiv \neg p \to \neg q$

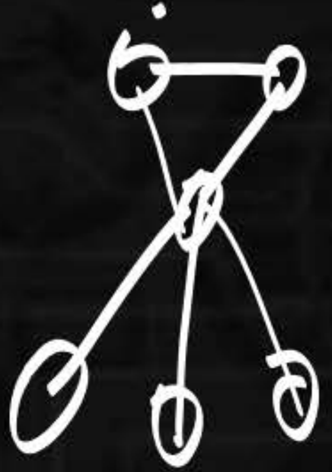
(viceversa)

Converse

$q \to p$

$p \to q \not\equiv q \to p$

even → p.m. ✗

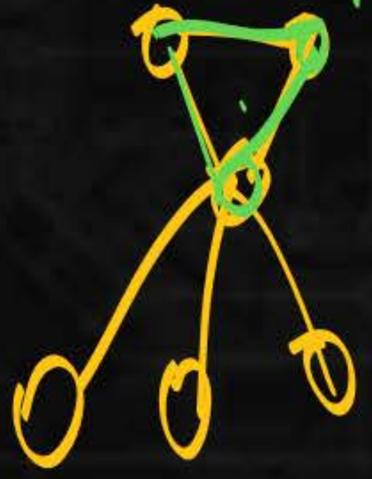


Inverse

$\neg p \to \neg q$

not p.m → not even (false)

$T \to F$



Contrapositive

$\neg q \to \neg p$

not even → not pm.

Thm:

$\boxed{\text{if Perfect matching exist then no of vertice will be even}}$

T. F.

false $\not\to$ True. $\to$ here we can prove aright OR wrong.

$T \to T \equiv T$

$T \to f \equiv F \leftarrow$ Thm is wrong.

$F \to \quad \equiv T.$

if you clear gate the she will marry you.

clear (T)	marry (T)	T
clear (T)	~~marry~~ (F)	F
~~clear~~	marry (T)	T
~~clear~~ F	~~marry~~ (F)	T

F → ≡ T.

GOD

$$\boxed{\begin{aligned} p \to q &\equiv \neg q \to \neg p \\ p \to q &\equiv \neg p \vee q \end{aligned}}$$

$A \equiv B$

1) both are having same behaviour.

2) same column.

p	q		$\neg p$	$p \to q$	$\neg p \vee q$
T	T	$\vee$	F	T	T
T	F	$\vee$	F	F	F
F	T	$\vee$	T	T	T
F	F	$\vee$	T	T	T

if you go late to the office then boss may fire you

$P \longrightarrow q$

Don't go late to the office OR boss may fire you

$\neg p \vee q$

biconditional ($\leftrightarrow$ / $\Longleftrightarrow$)

if and only if

iff

p	q	$p \leftrightarrow q$
T	T	T
F	F	T
T	F	F
F	T	F

G is Euler $\leftrightarrow$ Degrees of all vertices are even.

T $\leftrightarrow$ T

F $\leftrightarrow$ F

Thm: Graph is Euler iff degree

T
T
T
T

tautology / valid.

F
F
F
F

Contradiction

Satisfiable

T

atleast 1 True

All valids are satisfiable (T)

all contingency are satisfiable (T)

Tauto ✗
Contra ✗.

all satisfiable are not Contingency

T
F

Contigency

p	q	$p \wedge q$
T	F	F
F	T	F
F	F	F
T	T	T

→ satisfiable.

Contingency

T
F

atleast 1 pair

Satisfiable.

atleast 1 True

Satis

T
F

Satis

Contingeny

Satis

F
T
T

Satis

Contingen

Satisfiable/valid

T
T
T
T

Satis ✓

Contingencies

$$[\underset{T}{p} \wedge \overset{T}{(p \to q)} \wedge \overset{T}{(\neg q \vee r)}] \to \overset{F}{r}$$

$(T \to T)$

$(F \vee F)$

F

F → tautology

True

$r = F$
$p = T$
$q = T$

Force it to become false

Deny → tautology

Accept → not tautology

$$\left(\overline{\overline{\underset{T}{p}} \wedge \overline{(p \to q)}^{\,T} \wedge \overline{(\underline{\neg q} \vee x)}^{\,T}}^{\,T}\right) \to \overline{x}^{\,F}$$

$$(T \to T)$$

$$(F \vee F)$$

$$F$$

$$x = F$$
$$p = T$$
$$q = T$$
$$\neg q = F$$

$$F \longrightarrow$$

True

left side $= T$

Right side $= F$

$T \to F \equiv F$

$$\left(\overbrace{\overset{T}{p} \wedge (\overset{T}{p \to q}) \wedge (\overset{T}{\neg q \vee w}) \wedge (\overset{T}{w \to m})}^{T}\right) \to \overset{F}{m}$$

$$\underbrace{(T \to F)}_{F} \qquad (T \vee F) \qquad (F \to F)$$

$$\underbrace{\qquad F \qquad \to \qquad}_{\text{True}}$$

$m = F$

$w = F$

$\neg q = T$

$q = F$

$$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$$

a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

b) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$

c) $[(p \vee q) \wedge \neg p] \rightarrow q$

d) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$

a) $\neg(p \vee \neg q) \rightarrow \neg p$

b) $p \rightarrow (q \rightarrow r)$

c) $(p \rightarrow q) \rightarrow r$

d) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

e) $[p \wedge (p \rightarrow q)] \rightarrow q$

f) $(p \wedge q) \rightarrow p$

g) $q \leftrightarrow (\neg p \vee \neg q)$

h) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

a) $[p \wedge (p \rightarrow q) \wedge r] \rightarrow [(p \vee q) \rightarrow r]$

b) $[[(p \wedge q) \rightarrow r] \wedge \neg q \wedge (p \rightarrow \neg r)] \rightarrow (\neg p \vee \neg q)$

c) $[[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \vee r)$

$$\overbrace{[p \to (q \to r)]}^{T} \to \overbrace{[(\underbrace{p \to q}_{T}) \to (\underbrace{p \to r}_{F})]}^{F}$$

$p = T$

$r = F$

$q = T$

$T \to (T \to F)$

$T \to F$

F

$T\ T \quad T\ F$

T


THANK YOU GW
SOLDIERS !