Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics
Mathematical logic

DPP-01

[MCQ]

1. Which of the following is tautology?

(a) $(\sim p \wedge (p \to q)) \to \sim q$

(b) $\sim (p \to q) \to \sim q$

(c) $[(\sim p \wedge q) \wedge [q \to (p \to q)]] \to \sim r$

(d) None of these

[MCQ]

2. The statement $[P \vee (p \leftrightarrow Q) \vee Q]$ is equivalent to

(a) P (b) Q

(c) A tautology (d) $(P \wedge Q)$

[MCQ]

3. Consider the following statement

S_1: $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

S_2: $[((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)]$

Which of the following is/are correct?

(a) S_1 is contingency

(b) S_2 is tautology

(c) S_1 and S_2 both contingency

(d) S_1 and S_2 both Tautology

[MCQ]

4. Which of the following is valid?

S1: $p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

S2: $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

(a) S_1 is valid and s_2 is not valid

(b) S_1 is not valid and s_2 is valid

(c) Both S_1 and S_2 are valid

(d) Neither S_1 nor S_2 is valid

[MCQ]

5. Which of the following is not a tautology?

(a) $((p \to q) \to r) \to (p \to (q \to r))$

(b) $((p \to (r \vee q)) \to ((p \to r) \vee (p \to q))$

(c) $(p \to (r \wedge q)) \to ((p \to r) \vee (p \to q))$

(d) $(p \to (q \to r)) \to ((p \to q) \to r)$

Answer Key

1. **(b)**
2. **(c)**
3. **(a, b)**
4. **(c)**
5. **(d)**

Hints and solutions

1. (b)

$\sim (p \to q) \to \sim q$

$\sim [\sim (p \to q)] \vee \sim q$

$(p \to q) \vee \sim q$

$\sim p \vee q \vee \sim q$

$\sim p \vee 1 \equiv 1$

Hence, option B is tautology.

2. (c)

The statement : $P \vee (P \leftrightarrow Q) \vee Q$

$P \vee P'Q' \vee PQ \vee Q$

$P \vee P'Q' \vee (P \vee 1) Q$

$P \vee P'Q' \vee Q$

Apply absorption LAW

$\underbrace{P \vee \bar{P}\bar{Q}} \vee Q$

$P \vee \bar{Q} \vee Q$

$\therefore P \vee 1 \equiv 1$

Hence, the given statement is tautology.

3. (a, b)

Statement S1 : Contingency

$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (r \Rightarrow p)$

$[(\bar{p} + q) \wedge (\bar{q} + r)] \Rightarrow (\bar{r} + p)$

$\overline{(\bar{p}+q)} + \overline{(\bar{q}+r)} + (\bar{r}+p)$

$\therefore p\bar{q} + q\bar{r} + \bar{r} + p$

$\therefore p + \bar{r}$ [LAW of Absorption]

Hence, S1 is contingency

Statement S2 : Tautology

$[((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)]$

$\therefore p\bar{q} + q\bar{r} + \bar{p} + r$

$\therefore \underbrace{p\bar{q} + \bar{p}} + q\bar{r} + r$

$\therefore \bar{p} + \underbrace{\bar{q} + q} + r$

$\therefore \bar{p} + 1 + r \equiv 1$

Hence, S_2 is tautology

4. (c)

Statement S_1 : valid

$p \Rightarrow (q \vee r) \equiv (p \Rightarrow q) \vee (p \Rightarrow r)$

$\bar{p} + q + r \equiv (\bar{p} + q) + (\bar{p} + r)$

$\bar{p} + q + r = \bar{p} + q + r$

Hence, statement S_1 is valid

Statement S_2 : Valid

$p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$

$\bar{p} + qr \equiv (\bar{p} + q) \cdot (\bar{p} + r)$

$\bar{p} + qr \equiv \bar{p} + qr$

Hence, statement S_2 is also valid.

5. (d)

Statement : $(p \to (q \to r)) \to ((p \to q) \to r)$

$(p \to (\bar{q} + r)) \to ((\bar{p} + q) \to r)$

$(\bar{p} + \bar{q} + r) \to (\overline{(\bar{p} + q)} + r)$

$\overline{(\bar{p} + \bar{q} + r)} + p\bar{q} + r$

$pq\bar{r} + p\bar{q} + r$

$p(q\bar{r} + \bar{q}) + r$

$p(\bar{r} + \bar{q}) + r$

$p\underbrace{\bar{r} + p\bar{q} + r}$

$\therefore \underbrace{p + r + p}\ \bar{q}$

$p(1 + \bar{q}) + r$

$\therefore p + r \neq 1$

Hence, option D is not tautology.