CS & IT ENGINEERING

Minimization

K Map Basics

Lecture No. 2

By- CHANDAN SIR

TOPICS TO BE COVERED

01 THEOREM

02 D-MORGAN'S Law

03 K MAP

04 Questions

05 DISCUSSION

Revision

- INVERTER
- AND, OR
- NAND, NOR
- X-OR
- X-NOR

Q.1 Minimized expression will be $Y = A \oplus (A + B)$

A. $A \oplus B$

B. $A \odot B$

C. $\overline{A} \cdot B$

D. $A + B$

$$\cancel{Y = A \oplus (A+B)}$$
$$\cancel{= A \oplus A + A \oplus B}$$
$$\cancel{= 0 + A \oplus B}$$
$$\cancel{= A \oplus B.}$$

$$Y = A \oplus (A+B)$$
$$Y = \bar{A} \cdot (A+B) + A \cdot (\overline{A+B})$$
$$Y = \cancelto{0}{\bar{A} \cdot A} + \bar{A}B + \cancelto{0}{A \cdot \bar{A} \cdot \bar{B}}$$
$$Y = \bar{A}B$$

Ans

$$A \cdot \bar{A} = 0$$
$$A + \bar{A} = 1$$

Q.2 If the output y = 1
Then correct input is/are-

MSQ.

A B

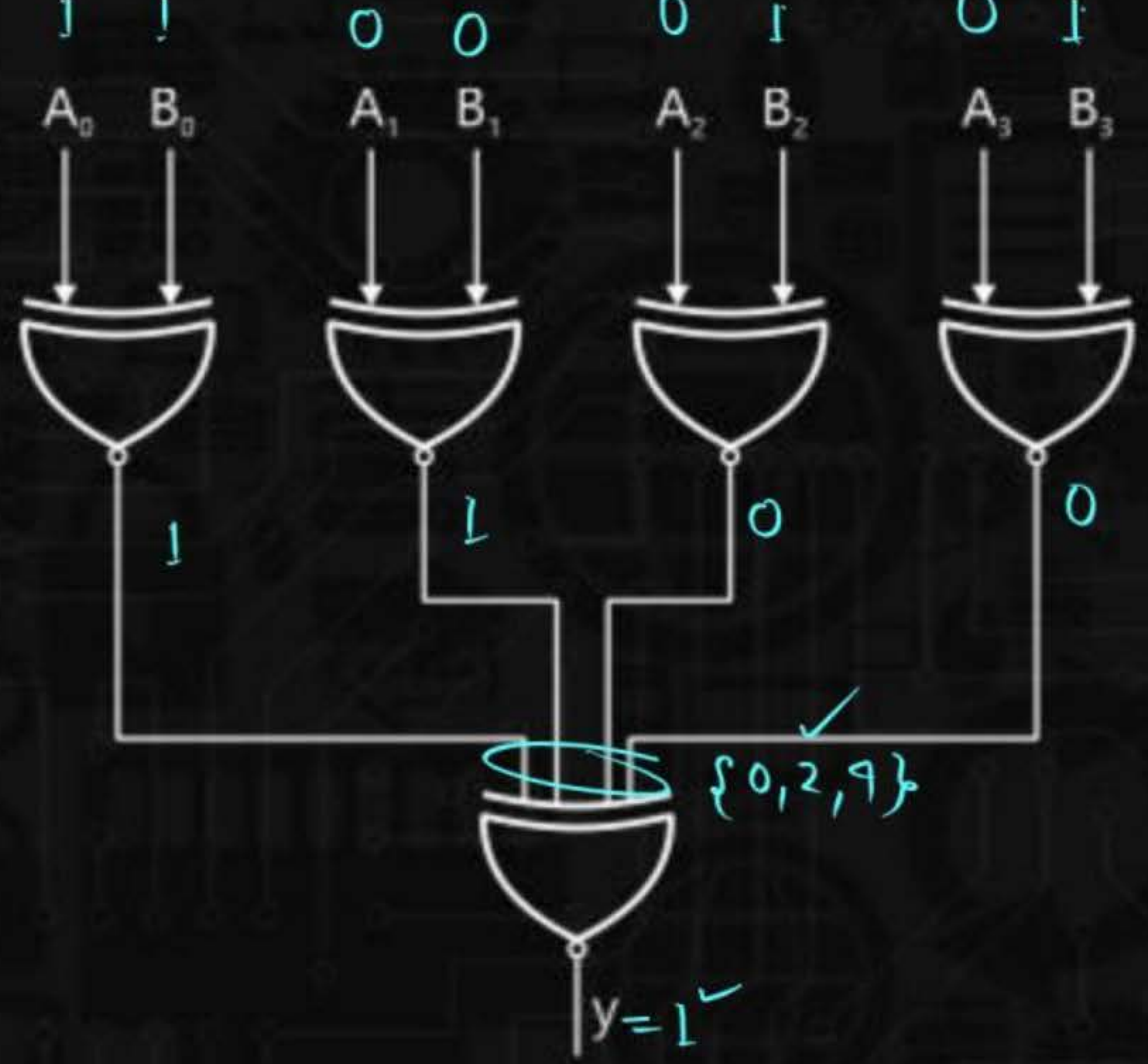
A. $\underbrace{1111}, \underbrace{0000}$ ✓

B. 1010, 0111

C. 0101, 0101

D. 1100, 1110

(E) 1000, 1011 ✓



Even times

one

$\{0, 2, 4\}$

Q.3 Output y will be-

A. 0

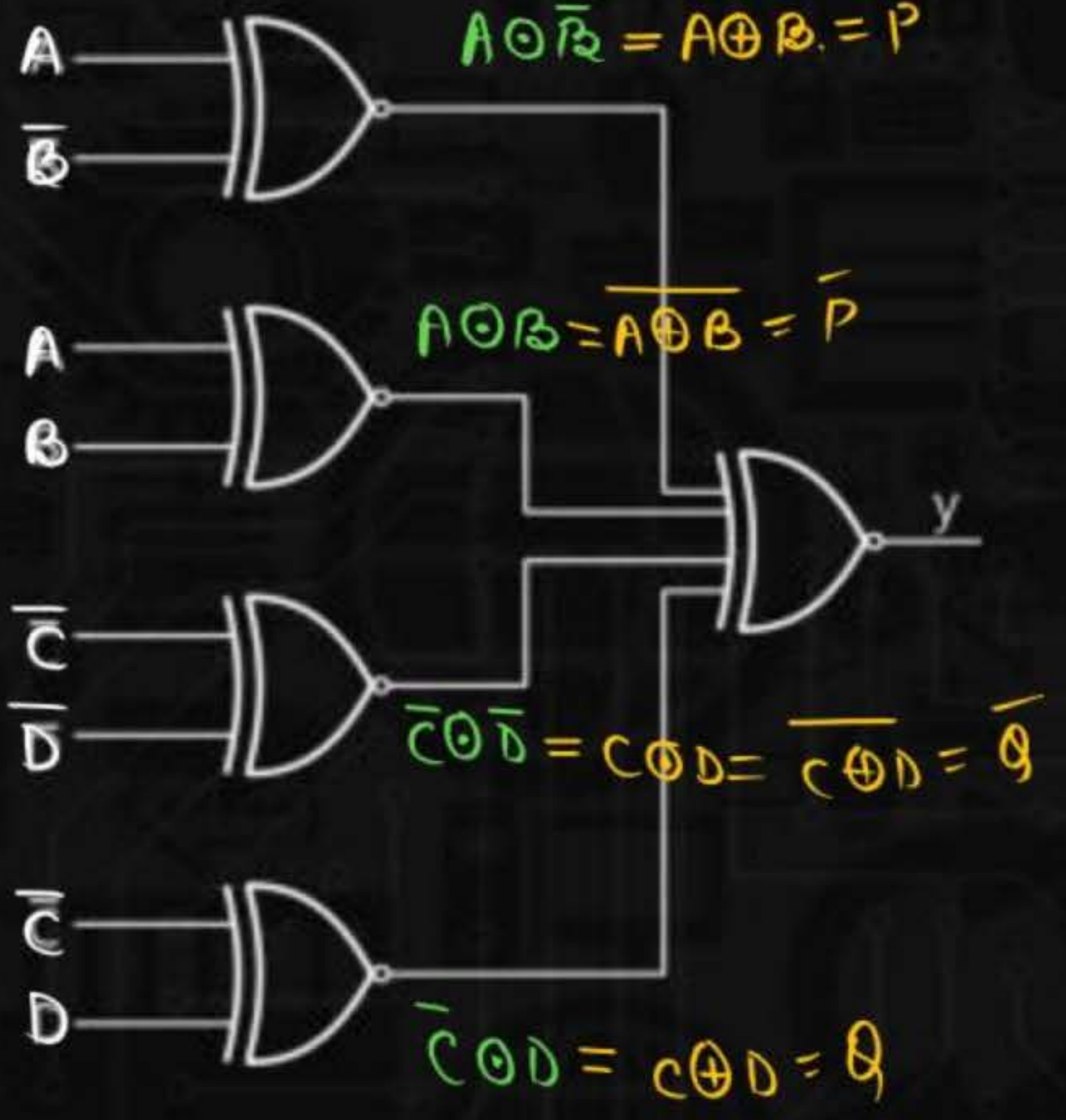
B. 1 ✓

C. $A \oplus B$

D. $A \oplus B \oplus C \oplus D$

$A \oplus B = P$

$C \oplus D = Q$



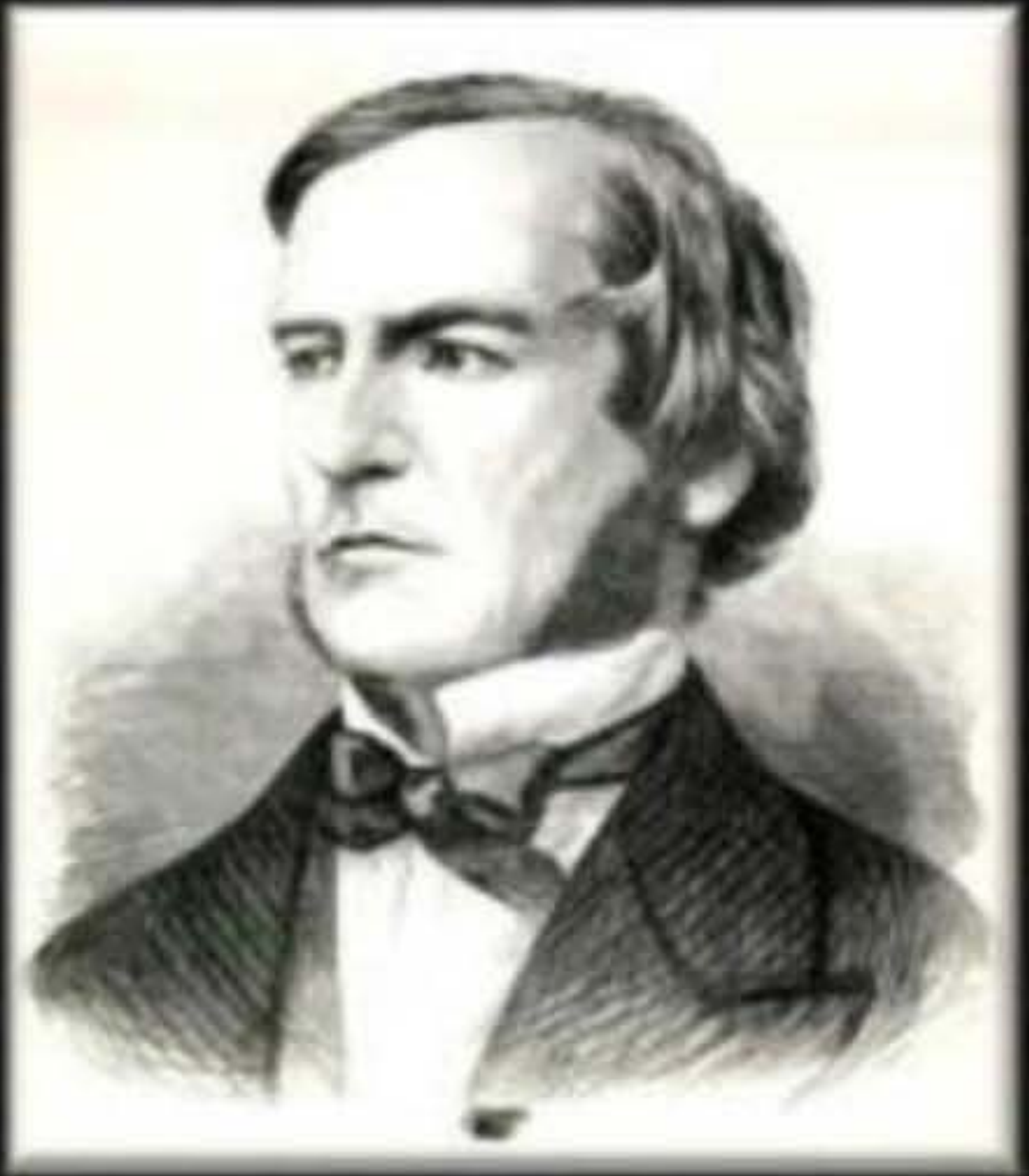
$y = P \odot \bar{P} \odot \bar{Q} \odot Q$

$y = 0 \odot 0 = 1$

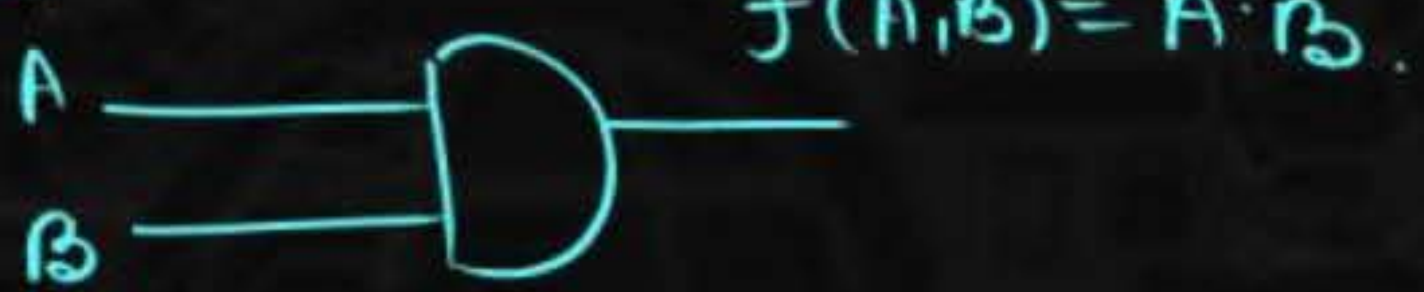
Laws of Boolean Algebra

1854- George Boole

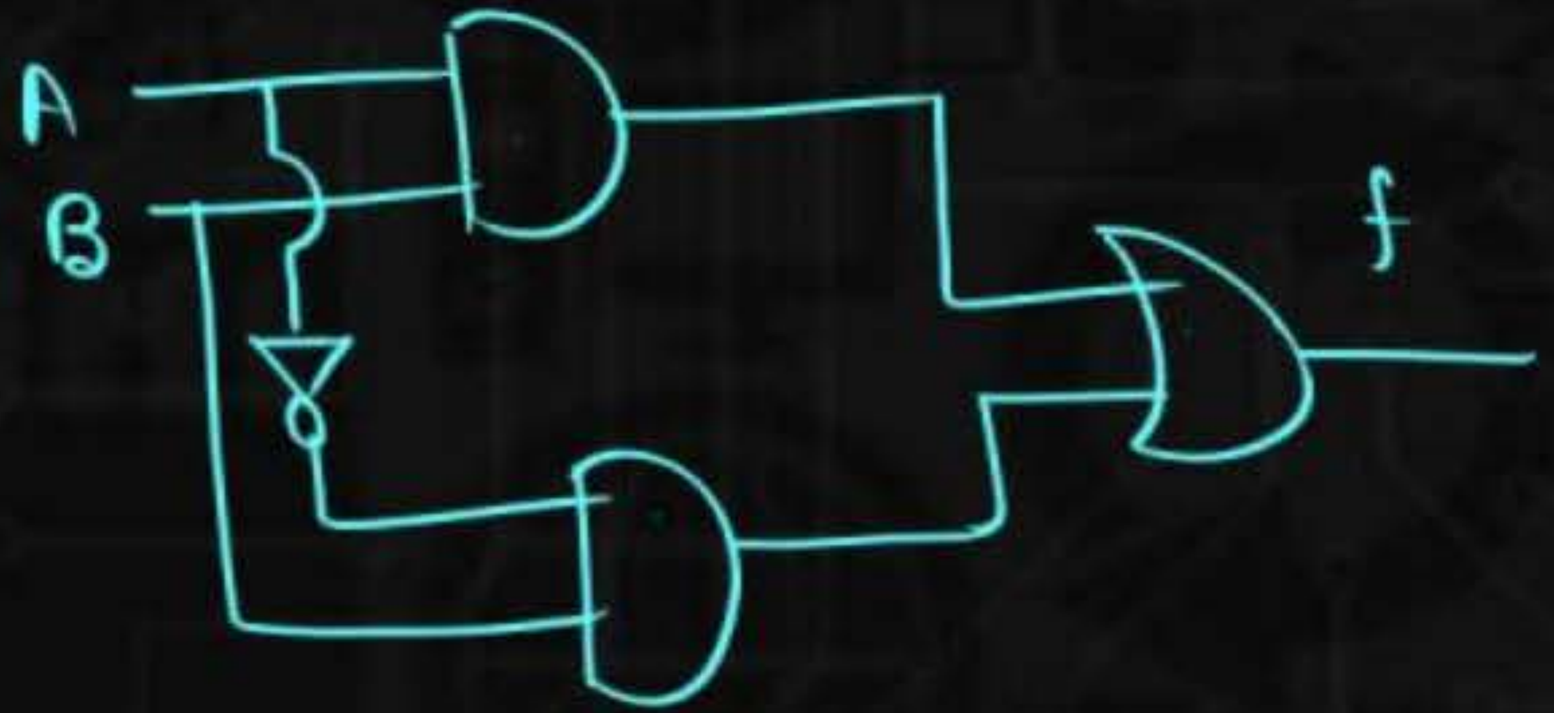
"An Investigation of Law of Thoughts" ✓



Ex. 1 $f(\underbrace{A, B}_{\text{Variables}}) = A \cdot B$ ✓



Ex. 2. $f(A, B) = \underbrace{AB} + \underbrace{\bar{A}B}$



Laws of Boolean Algebra

BOOLEAN ALGEBRA

Boolean function → Combination of inputs on which output will depends.

Laws of Boolean Algebra

BOOLEAN ALGEBRA

Minterm = $\overline{\text{Maxterm}}$

Function

- $\boxed{\text{SOP}}$
  [Sum of product
  $\boxed{\text{Minterm]}}$
- $\boxed{\text{POS}}$
  [product of sum
  $\boxed{\text{Maxterm]}}$

	8	4	2	1
0 →	0	0	0	0
1 →	0	0	0	1
2 →	0	0	1	0
3 →	0	0	1	1
4 →	0	1	0	0
5 →	0	1	0	1
6 →	0	1	1	0
7 →	0	1	1	1
8 →	1	0	0	0
9 →	1	0	0	1
10 →	1	0	1	0
11 →	1	0	1	1
12 →	1	1	0	0
13 →	1	1	0	1
14 →	1	1	1	0
15 →	1	1	1	1

Laws of Boolean Algebra

ऐसे ही Example के लिये लिखा है।
Assume

Decimal	A B C	Min term	Max term	Function
0	0 0 0	$\bar{A}\cdot\bar{B}\cdot\bar{C}$	$A+B+C$	1
1	0 0 1	$\bar{A}\bar{B}C$	$A+B+\bar{C}$	0
2	0 1 0	$\bar{A}B\bar{C}$	$A+\bar{B}+C$	1
3	0 1 1	$\bar{A}BC$	$A+\bar{B}+\bar{C}$	0
4	1 0 0	$A\bar{B}\bar{C}$	$\bar{A}+B+C$	0
5	1 0 1	$A\bar{B}C$	$\bar{A}+B+\bar{C}$	1
6	1 1 0	$AB\bar{C}$	$\bar{A}+\bar{B}+C$	0
7	1 1 1	ABC	$\bar{A}+\bar{B}+\bar{C}$	1

Laws of Boolean Algebra

Boolean Function- It is the combination of inputs on which output is depends.

Standard Canonical SOP form

When each term should contain all the variables.

$$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + ABC$$

$$= m_0 + m_2 + m_5 + m_7$$

$$= \Sigma m(0,2,5,7)$$

$$= \Sigma(0,2,5,7)$$

Standard canonical POS form.

$$F(A,B,C) = \underbrace{\overset{0\;\;0\;\;1}{(A+B+\bar{C})}}_{\text{term-1}} \cdot \underbrace{(A+\bar{B}+\bar{C})}_{\text{term-2}} \cdot \underbrace{(\bar{A}+B+C)}_{\text{term-3}} \cdot \underbrace{(\bar{A}+\bar{B}+C)}_{\text{term4}}$$

$$= M_1 \cdot M_3 \cdot M_4 \cdot M_6$$

$$= \prod M(1,3,4,6)$$

$$= \prod(1,3,4,6)$$

A	B	Minterm	Maxterm	function
0	0	$\bar{A}\bar{B}$	$A+B$	1 ✓
0	1	$\bar{A}B$	$A+\bar{B}$	0 ✓
1	0	$A\bar{B}$	$\bar{A}+B$	0 ✓
1	1	AB	$\bar{A}+\bar{B}$	1

SOP

$$f(A,B) = \bar{A}\bar{B}\cdot 1 + \bar{A}B\cdot 0 + A\bar{B}\cdot 0 + AB\cdot 1$$
$$= \bar{A}\bar{B} + 0 + 0 + AB$$
$$= \bar{A}\bar{B} + AB$$

POS

$$F(A,B) = \cancel{(A+B+1)}^{\,1}\,(A+\bar{B}+0)\cdot(\bar{A}+B+0)\cdot\cancel{(\bar{A}+\bar{B}+1)}^{\,1}$$
$$= 1\cdot(A+\bar{B})\cdot(\bar{A}+B)\cdot 1$$
$$= (A+\bar{B})(\bar{A}+B)$$

1. Distribution theorem :→

$(A+B)(A+C)$

$\Rightarrow A\cdot A + A\cdot C + A\cdot B + B\cdot C$

$\Rightarrow A + AC + AB + BC$

$\Rightarrow A[1+C+B] + BC$

$\Rightarrow A\cdot 1 + BC$

$\Rightarrow A+BC$

$$A + BC = (A+B)\cdot(A+C)$$

$$A + BCD = (A+B)\cdot(A+C)\cdot(A+D)$$

Ex. $\bar{A} + AB = (\bar{A}+A)(\bar{A}+B)$

$= \bar{A}+B$

(2) Concensus Theorem

$$AB + \bar{A}C + BC$$

$$AB + \bar{A}C + 1 \cdot BC$$

$$AB + \bar{A}C + (\bar{A} + A)BC$$

$$AB + \bar{A}C + \bar{A}BC + ABC$$

$$AB[1 + C] + \bar{A}C[1 + B]$$

$$AB + \bar{A}C$$

$$AB + \bar{A}C + \boxed{BC} = AB + \bar{A}C$$

Redundant term

Ex

$$AB + \boxed{A\bar{C} + BC} = A\bar{C} + BC$$

Redundant $= AB$

Ex.

$$\bar{A}\bar{B} + \bar{A}\bar{C} + B\bar{C} = \bar{A}\bar{B} + B\bar{C}$$

Redundant

(3) Transpose Theorem

$\Rightarrow (A+B)(\bar{A}+C)$

$\Rightarrow A\cdot\bar{A} + AC + \bar{A}B + BC$

$\Rightarrow AC + \bar{A}B + BC$

$\Rightarrow AC + \bar{A}B$

$(A+B)(\bar{A}+C) = AC + \bar{A}B$

$(A+B)(\bar{A}+\bar{B}) = A\bar{B} + \bar{A}B$

$(A+\bar{B})(\bar{A}+B) = AB + \bar{A}\bar{B}$

④ D-Morgan's Law.

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

$$\overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

Q.1 Find the minimum number of the NAND gate required to implement the Boolean function given below.

$$f(A, B, C) = A + ABC + AB\overline{C}$$

$$= A\left[1 + BC + B\overline{C}\right]$$

$$= A$$

Zero

I/P O/P

A ——————— A

Q.3 Minimize the expression.

$f(A, B) = A + \bar{A}\bar{B}$

$= (A+\bar{A})(A+\bar{B})$

$= A+\bar{B}$

Ans

Q.4 Minimize the expression.

$f(A, B) = \overline{A} + AB$

$= (\overline{A} + A)(\overline{A} + B)$

$= \overline{A} + B$

Q.5 Minimize the expression.

$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + AB$

$= \overline{A}\,[\overline{B} + B] + AB$

$= \overline{A} + AB$

$= (\overline{A} + A)(\overline{A} + B)$

$= \overline{A} + B$

AB

Q.6 Minimize the expression.

$$f(A, B) = \overline{A}\,\overline{B} + \overline{A}B + A\overline{B} + AB$$

$$= \overline{A}[\overline{B}+B] + A[\overline{B}+B]$$

$$= \overline{A} + A$$

$$= 1$$

$n=1$

$$\left\{\begin{matrix} A \\ \overline{A} \\ 0 \\ 1 \end{matrix}\right\} 4$$

$n=2$

$$\left\{\begin{matrix} \overline{A}\,\overline{B} & A+B & A & 0 \\ \overline{A}B & A+\overline{B} & \overline{A} & 1 \\ A\overline{B} & \overline{A}+B & B & \overline{A}B+A\overline{B} \\ AB & \overline{A}+\overline{B} & \overline{B} & \overline{A}\,\overline{B}+AB \end{matrix}\right\} 16$$

"n" Variables.

→ Total different expression = 2^{2^n} =

$2^4 = 16$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024 = K$

$2^{10} = K$

$2^{20} = M$

$2^{30} = G$

$2^{40} = T$

Ex. If we have '3' Variables Then how many different minimized expression we can form?

Ans ⇒ $2^{2^3} \Rightarrow 2^8 = \underline{\underline{256}}$

Q.7 Minimize the expression.

$f(A, B) = \overline{A}B + A\overline{B}$

already minimized

Q.8 Minimize the expression.

$f(A, B) = AB + \overline{A}C + BC$ $= \cancel{AB + \overline{A}C}$

2 Variable

Wrong format


THANK YOU GW
SOLDIERS !