CS & IT ENGINEERING

Digital Logic

Number system

Lecture No. 1

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01 Latches

02 Practice

03 Discussion

Synchronous counter Design

Q Design a synchronous counter by using T-FF ?

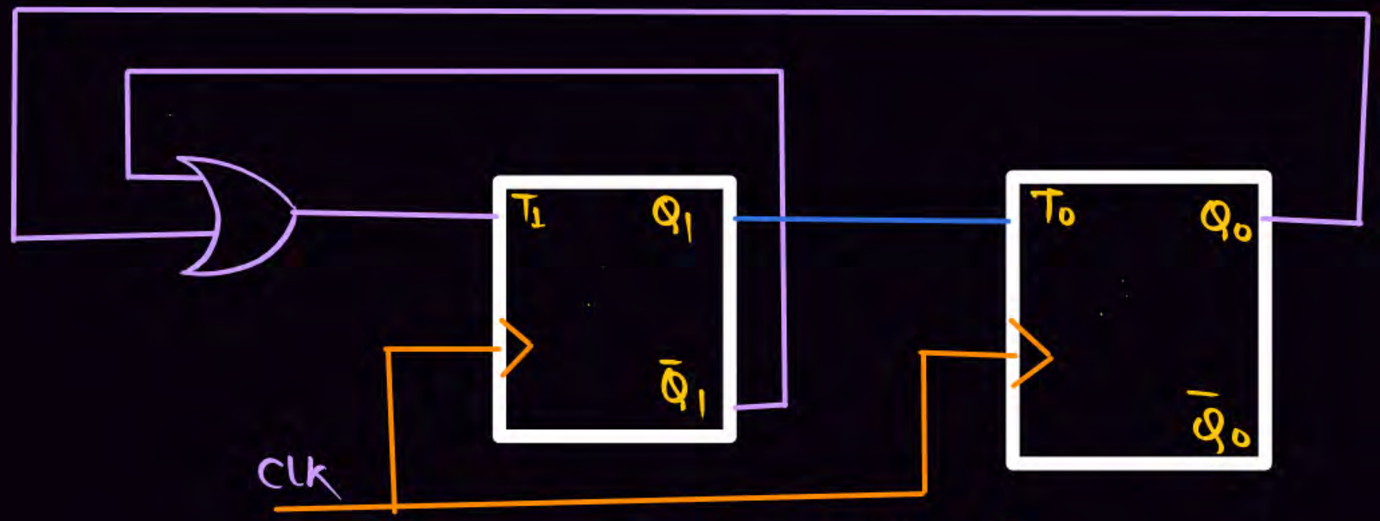
$0 \rightarrow 2 \rightarrow 3 \rightarrow 0$ $\{00 \rightarrow 10 \rightarrow 11 \rightarrow 00\}$

Method (1)

Q_1	Q_0	Q_1^+	Q_0^+	T_1	T_0
0	0	1	0	1	0
0	1	X	X	X	X
1	0	1	1	0	1
1	1	0	0	1	1

$T_1 = \bar{Q}_1 + Q_0$ ✓

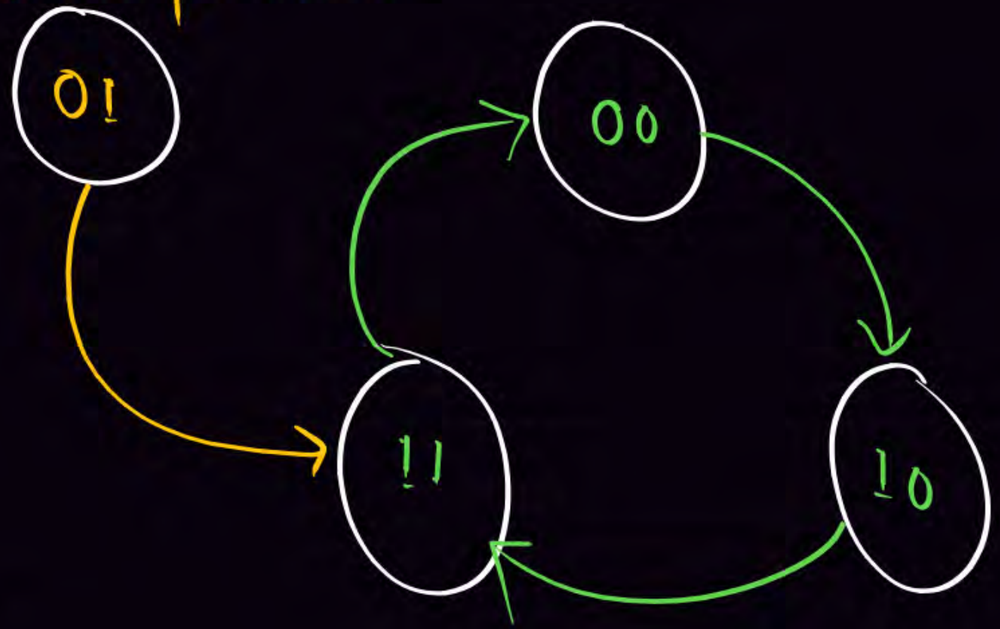
$T_0 = Q_1$ ✓



"No Lockout problem"

Justification

Clock	Q_1	Q_0
0	0	0
1	1	0
2	1	1
3.	0	0



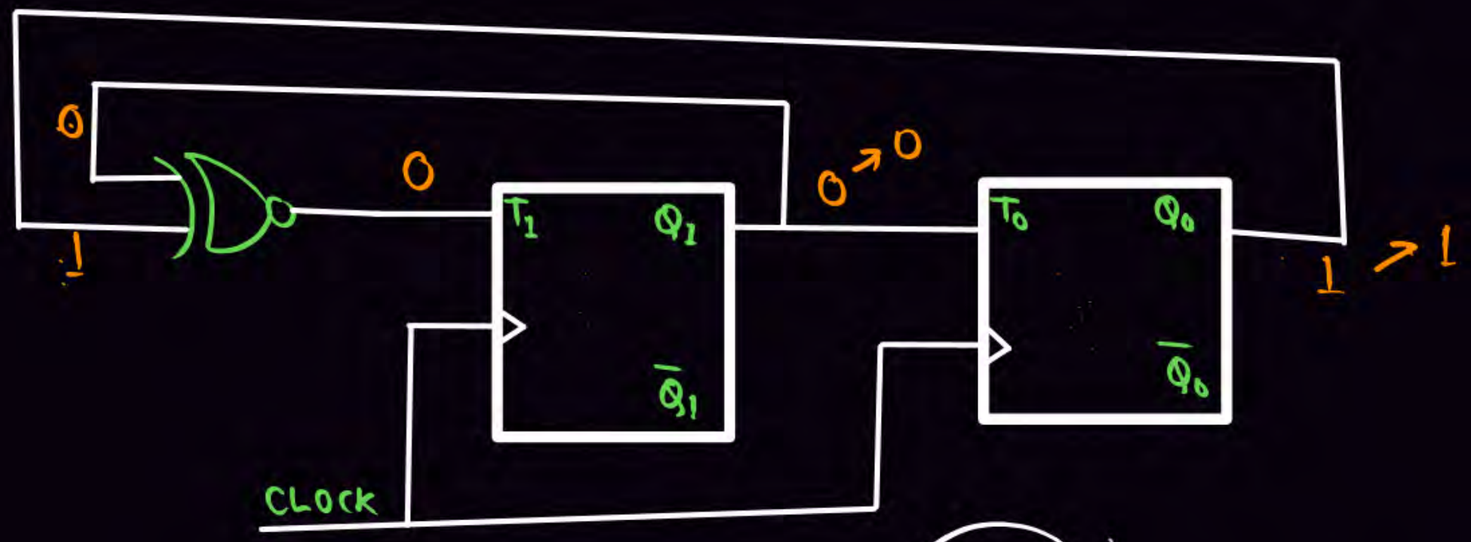
Method ②

$0 \rightarrow 2 \rightarrow 3 \rightarrow 0$ $\{00 \rightarrow 10 \rightarrow 11 \rightarrow 00\}$

Q_1	Q_0	Q_1^+	Q_0^+	T_1	T_0
0	0	1	0	1	0
1	0	1	1	0	1
1	1	0	0	1	1

$T_1 = \bar{Q}_1\bar{Q}_0 + Q_1Q_0 = Q_1 \odot Q_0$

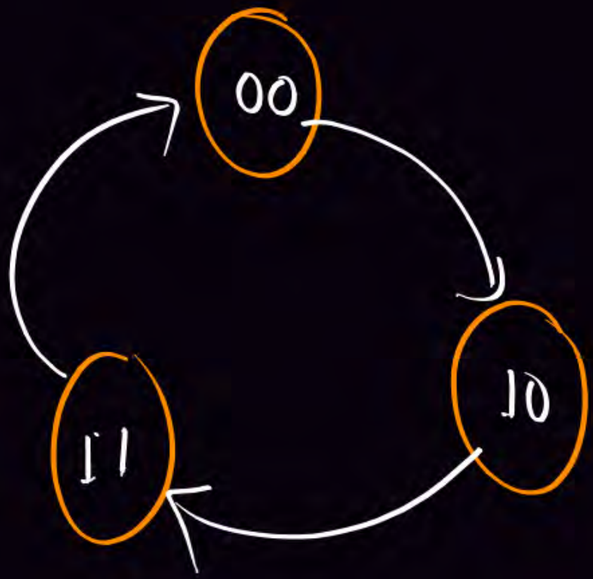
$T_0 = Q_1$



Justification

CLK	Q_1	Q_0
0	0	0
1	1	0
2	1	1
3	0	0
4		

Lock out problem occure



$0 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 1 \rightarrow 6 \rightarrow 0$

Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	1	0	0	X	1	X	0	X
0	0	1	1	1	0	1	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	1	1	X	X	1	X	0
1	0	0	X	X	X	X	X	X	X	X	X
1	0	1	0	0	1	X	1	0	X	X	0
1	1	0	0	0	0	X	1	X	1	0	X
1	1	1	X	X	X	X	X	X	X	X	X

Q	Q^+	JK
0	0	0 X ✓
0	1	1 X ✓
1	0	X 1
1	1	X 0

$0 \to 2 \to 3 \to 5 \to 1 \to 6 \to 0$

Q_2	Q_1	Q_0	Q_2^+	Q_1^+	Q_0^+	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	1	0	0	X	1	X	0	X
0	0	1	1	1	0	1	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	1	1	X	X	1	X	0
1	0	0	X	X	X	X	X	X	X	X	X
1	0	1	0	0	1	X	1	0	X	X	0
1	1	0	0	0	0	X	1	X	1	0	X
1	1	1	X	X	X	X	X	X	X	X	X

J_2

Q_2 \ Q_1Q_0	00	01	11	10
0		1	1	
1	X	X	X	X

$J_2 = Q_0$

K_2

Q_2 \ Q_1Q_0	00	01	11	10
0	X	X	X	X
1	X	1	X	1

$K_2 = 1$

NUMBER SYSTEM

1. Base conversion

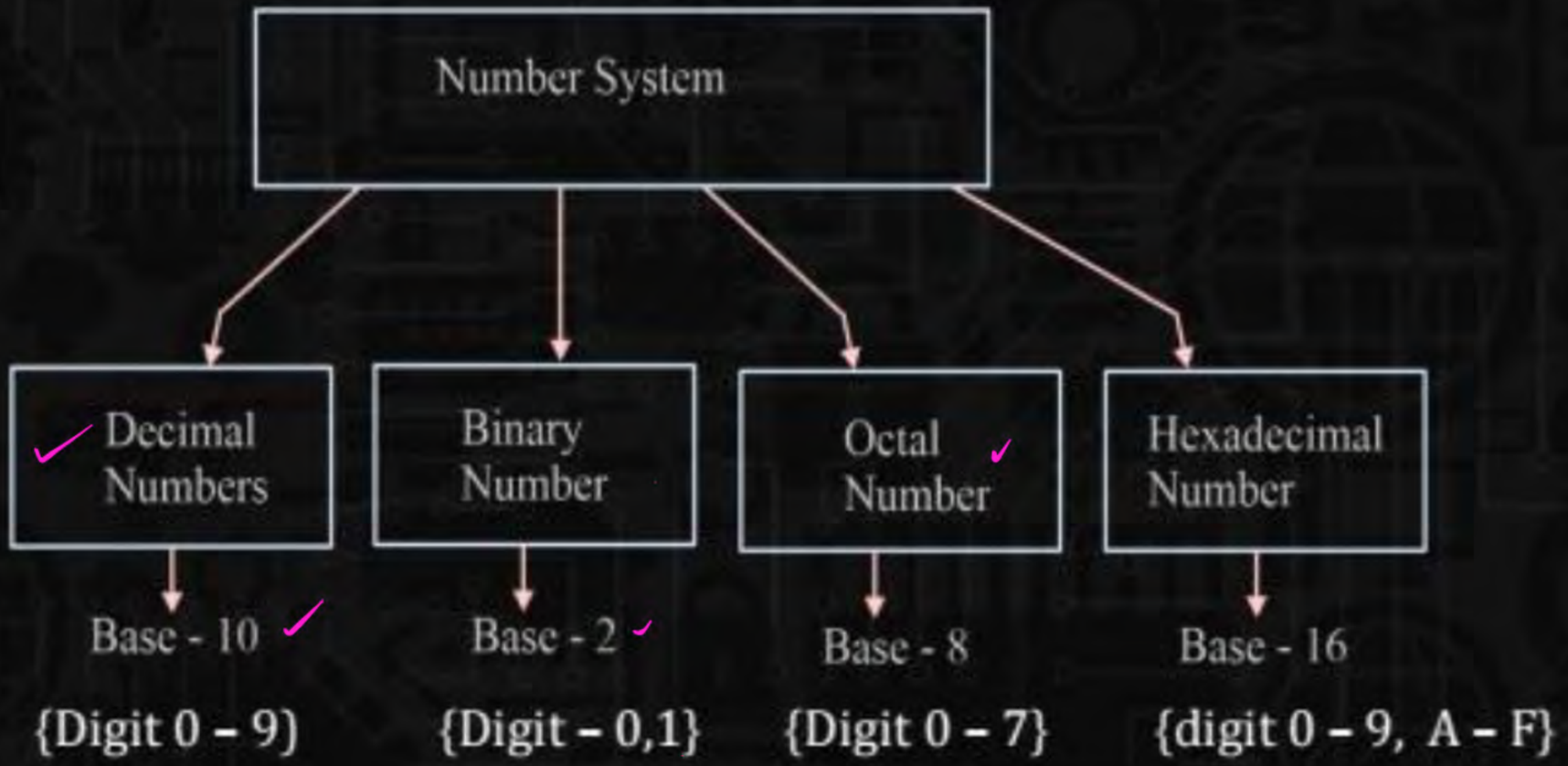
$( )_x \rightarrow ( )_y$

2. Magnitude Representation

Base (Radix)

(2)
Binary → {0, 1}

Total number of digit used in the system



Decimal Number System

...	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3} ...
...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

a^i → Coefficient of decimal number system

10_i → Weight of decimal number system

Example : $(501.23)_{10}$

10^2	10^1	10^0	10^{-1}	10^{-2}
5	0	1	2	3

Base	Digit
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4
6	0, 1, 2, 3,4,5
7	0, 1, 2, 3, 4, 5, 6
8	0, 1, 2, 3, 4, 5, 6, 7
9	0, 1, 2, 3, 4, 5, 6, 7, 8
10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
11	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A
12	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B
13	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C
14	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D
15	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E
16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

$(321)_r$ $r = 4, 5, 6, 7 \cdots$

$(527)_r$ $r = 8, 9, \cdots$

$(101)_r$ $r = 2, 3, 4, \cdots$

$(12C)_r$ $r = 13, 14, 15$

Binary Number System (Base (Radix) = 2)

weight

...	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3} ...

coefficient

...	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3} ...

$2^i \rightarrow$ Weight of Binary number system

$a_i \rightarrow$ Coefficient of Binary number system {0, 1}

$2^2\ 2^1\ 2^0\ 2^{-1}$
1 1 0 . 1

$2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}$

Example:- $(101.11)_2$

2^2	2^1	2^0	2^{-1}	2^{-2}
1	0	1	1	1

Octal Number System (Base (Radix) = 8)

...	8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}	8^{-3} ...
...	a_3	a_2	a_1	a_0		a_{-1}	a_{-2}	a_{-3} ...

$8^i \rightarrow$ Weight of Octal number system

$a_i \rightarrow$ Coefficient of Octal number system {0 -7}

$8^2\ 8^1\ 8^0\ 8^{-1}\ 8^{-2}$

Example:- $(726.64)_8$

8^2	8^1	8^0	8^{-1}	8^{-2}
7	2	6	6	4

Hexadecimal Number System (Base (Radix) = 16)

...	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}	16^{-3}...
...	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}	a_{-3}...

$16^i \rightarrow$ Weight of Hexadecimal number system

$a_i \rightarrow$ Coefficient of Hexadecimal number system {0 – 9, A–F}

$16^2\ 16^1\ 16^0\ 16^{-1}$

Example: $(A2C.F)_{16}$

16^2	16^1	16^0	16^{-1}
A	2	C	F

10 = A
11 = B
12 = C
13 = D
14 = E
15 = F

In base conversion 2 key points are there:

(A) Any base to Decimal conversion ✓

(B) Decimal to any other base conversion ✓

(A) Any base to Decimal conversion :

$r^3\ r^2\ r^1\ r^0\ \ r^{-1}\ r^{-2}$

$$(a_3\ a_2\ a_1\ a_0\ .\ a_{-1}\ a_{-2})_r = (\quad)_{10}$$

$$(a_3 \times r^3 + a_2 \times r^2 + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + a_{-2} \times r^{-2})_{10}$$

Case (1) : Binary to Decimal conversion

$2^3\ 2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}$

Ex. $(1011.11)_2 = (\quad)_{10}$ ✓

$$\Rightarrow \left[(1\times 2^3)+(0\times 2^2)+(1\times 2^1)+(1\times 2^0)+(1\times 2^{-1})+(1\times 2^{-2})\right]$$

$$\Rightarrow \left[8+0+2+1+0.5+0.25\right]_{10}$$

$$\Rightarrow (11.75)_{10}$$ ✓

Case (2) : Octal to Decimal conversion

$8^2\ 8^1\ 8^0\ 8^{-1}$

Ex. $(721.4)_8 = (\quad)_{10}$

$$\Rightarrow \left[(7\times8^2)+(2\times8^1)+(1\times8^0)+(4\times8^{-1})\right]_{10}$$

$$\Rightarrow \left[448+16+1+0.5\right]_{10}$$

$$\Rightarrow (465.5)_{10}$$

✓

Ans

Case (3) : Hexadecimal to Decimal conversion

$16^2\ 16^1\ 16^0\ 16^{-1}$

Ex. $(A2B.C)_{16} = (\quad)_{10}$

$$\Rightarrow \left[(A\times16^2)+(2\times16^1)+(B\times16^0)+(C\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[(10\times256)+(2\times16)+(11\times1)+(12\times16^{-1})\right]_{10}$$

$$\Rightarrow \left[2560+32+11+0.75\right]_{10}$$

$$\Rightarrow (2603.75)_{10}$$

Ans

Case (4) : Base 5 to Decimal conversion

$5^2\ 5^1\ 5^0\ 5^{-1}\ 5^{-2}$

Ex. $(432.22)_5 = (\quad)_{10}$

$$\Rightarrow \left[(4\times5^2)+(3\times5^1)+(2\times5^0)+(2\times5^{-1})+(2\times5^{-2})\right]_{10}$$

$$\Rightarrow [100+15+2+0.4+0.08]_{10}$$

$$\Rightarrow (117.48)_{10}$$

Ans

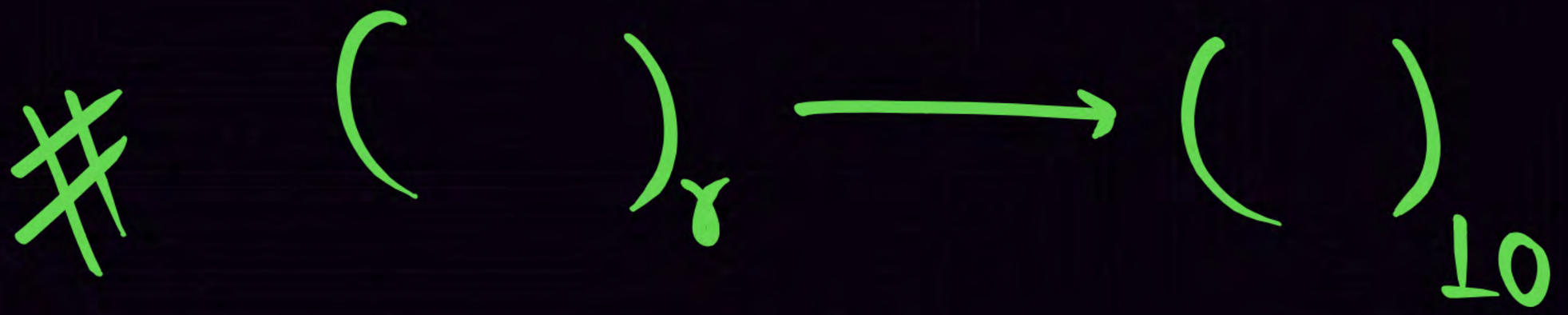
$9^1\ 9^0\ 9^{-1}$

Q $(37.2)_9 = (\ )_{10}$

$[(3\times 9)+(7\times 9^0)+(2\times 9^{-1})]_{10}$

$27+7+\frac{2}{9}$

$(34.22)_{10}$ Ans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( )
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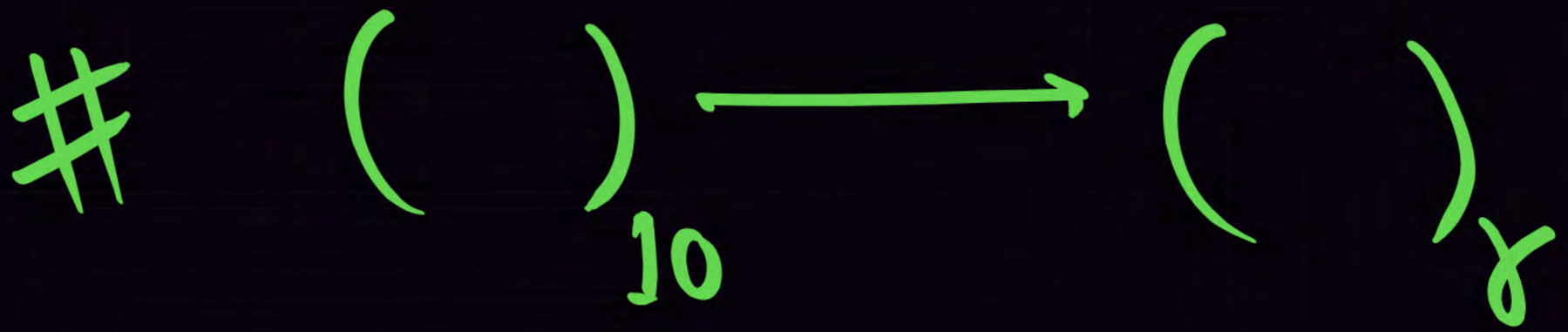
10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(B) Decimal to any other Base conversion

r	a_3 a_2 a_1 a_0	Remainder
		b_0
		b_1
		b_2
		b_3

$(a_3\ a_2\ a_1\ a_0 \cdot a_{-1}\ a_{-2}\ a_{-3})_{10} = (\quad)_r$

Before Decimal — After Decimal

$0 \cdot a_{-1}\ a_{-2}\ a_{-3} \times r = x_0 \cdot x_{-1}\ x_{-2}$

$0 \cdot x_{-1}\ x_{-2} \times r = x_1 \cdot x_{-3}\ x_{-4}$

$0 \cdot x_{-3}\ x_{-4} \times r = x_2 \cdot x_{-5}\ x_{-6}$

x_0
x_1
x_2

$(b_3\ b_2\ b_1\ b_0 \cdot x_0\ x_1\ x_2)_r$

Ans

$(a_3\ a_2\ a_1\ a_0 \cdot a_{-1}\ a_{-2}\ a_{-3})_{10} = (b_3\ b_2\ b_1\ b_0 \cdot x_0\ x_1\ x_2)_r$

Case (1) : Decimal to Binary Base conversion.

Ex. $(19.75)_{10} = (\quad)_2$

Before Decimal R After Decimal

2	19	1
2	9	1
2	4	0
2	2	0
	1	1

$0.75 \times 2 = 1.5$ 1
$0.5 \times 2 = 1.0$ 1

$(19.75)_{10} = (10011.11)_2$

$(19.75)_{10} = (1001 \cdot 11)_2$

Ans

Case (2) : Decimal to Octal Base conversion.

Ex. $(210.23)_{10} = (\quad)_8$

Before Decimal Remainder | After Decimal

8	210	2
8	26	2
	3	3

$0.23 \times 8 = 1.84 \quad 1$

$0.84 \times 8 = 6.72 \quad 6$

$0.72 \times 8 = 5.76 \quad 5$

$(210.23)_{10} = (322.165)_8$

$(210.23)_{10} = (322.165)_8$

Case (3): Decimal to Hexadecimal Base conversion.

Ex. $(1228.55)_{10} = (\quad)_{16}$

Before Decimal Remainder | After Decimal

16	1228	12 (C)
16	76	12 (C)
	4	4

$0.55 \times 16 = 8.8 \quad 8$

$0.8 \times 16 = 12.8 \quad 12(C)$

$(1228.56)_{10} = (4CC.8C)_{16}$

Some Special Case

Case (1): Binary to Octal base conversion

Ex. $(10110111)_2 = (\quad)_8$

Octal → means base 8

$8 = 2^3$

Every three digits of binary represent one digit of octal

010	110	111
2	6	7

Hence $(10110111)_2 = (267)_8$

Some Special Case

Case (2) : Binary to Hexadecimal base conversion

Ex. $(1011011)_2 = (\quad)_{16}$

Hexadecimal → means base 16

$16 = 2^4$

Every four digits of binary represent one digit of Hexadecimal.

0101 1011

5 11(B)

Hence $(1011011)_2 = (5B)_{16}$


THANK YOU GW
SOLDIERS !