DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
*****MA102 Mathematics-II : Test 4*****
Date: June 24, 2021

Total Time: **90** Minutes (10:00 am to 11:30 am) Total Marks: **25** Marks

Instructions:

- The question paper has **FIVE** questions. Answer **ALL** questions. Answers to all subdivisions/ subparts of a question should appear together.
- Read the Questions Carefully. While start writing the answer, ensure that you have noted the mathematical expressions/ equations appearing in the questions correctly.
- Write answers in detail. Do not skip any computational part. Always write reasons for your conclusions.
- No clarification will be given during the test, even if there is an error or missing data in the questions. You are to answer as per your understanding.
- Write answers on plain A4 sheets. **Write your Roll Number on all pages/ sheets**. Also **Put page number on every page** (Example: If you have written a total of 10 pages, then number it as 1/10, 2/10, 3/10, ...).
- At the end of this test, you should scan your answer script including the cover/zeroth page in PDF format and make a single PDF file. Name/Rename the PDF file as your **Rollnumber.pdf**. Then as a response to the assignment, upload the file Rollnumber.pdf immediately. After uploading, you must verify that the file is correctly uploaded and is not of zero-byte. After uploading, do not forget to **Turn In**. Any submission after 11:45 am will NOT be accepted. Extra time will be given to the PWD students as per rules.
- Late submission of answer script will NOT be allowed. Finally, invigilators will verify your uploaded document and confirm its receipt to you. Accordingly, you can then logout from MS Teams

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1. Find a fundamental set of solutions of the linear homogeneous system $\mathbf{x}' = A\,\mathbf{x}$ where

$$A = \begin{bmatrix} -2 & 1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

(**5** marks)

2. Using the **method of variation of parameters**, solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 4\,e^{2t} \\ 4\,e^{4t} \end{bmatrix}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} .$$

(**5** marks)

3. Find the general solution in the neighborhood of the ordinary point $x_0 = 0$ of

$$(1+x^2)\,y'' \;+\; 2x\,y' \;-\; 2y \;=\; 0\,.$$

(**5** marks)

4. Find a series solution $y_1(x)$ corresponding to the root r_1 about the singular point $x_0 = 0$ of the differential equation

$$4x^2\,y'' \;-\; 8x^2\,y' \;+\; (4x^2+1)\,y \;=\; 0$$

and write $y_1(x)$ in a closed form. (**5** marks)

5. (a) Write the system that is equivalent to the equation $\dfrac{d^2x}{dt^2} + 4x = 0$. Then, locate the critical point of the system and determine its type and stability.

(b) Determine the type and stability of the critical point $(0,0)$ of the following nonlinear autonomous system.

$$\begin{aligned} \frac{dx}{dt} &= x + x^2 - 4xy, \\ \frac{dy}{dt} &= -2x + y + 8y^2. \end{aligned}$$

(c) Let $P_n(x)$ denote the Legendre polynomial of degree $n = 0, 1, 2, \ldots$. Compute the value of the integral $\displaystyle\int_{-1}^{1} x\,P_9(x)\,P_{10}(x)\,dx$.

(1.5 + 1.5 + 2 = **5** marks)

Paper Ends