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Tutorial- 4

(a) $T(n) = 3T(n/2) + n^2$

$a=3, b=2;$

$f(n) = n^2$

a & b are constant and f(n) is a function so according to master's theorem

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$ ie $< n^2$

so case 3

$T(n) = \theta(n^2)$

(b) $T(n) = 4T(n/2) + n^2$

$a=4; b=2$ $f(n)=n^2$

$c = \log_b a$

$c = \log_2 4$

$c = 2$

$n^c = n^2$ ie $n^c = f(n)$

case 2

$T(n) = \theta(n^2 \log n)$

(c) $T(n) = T(\frac{n}{2}) + 2^n$

$a=1;$ $b=2$ $f(n) = 2^n$

$c = \log_b a$

$c = \log_2 1 = 0$

$n^c = n^0 = 1$; $f(n) > n^c$

case 3

$T(n) = \theta(2^n)$

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(d) $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a = 2^n$; $b = 2$; $f(n) = n^n$

$\because$ a is not constant, its value depends on n, hence master's method is not applicable.

(e) $T(n) = 16\,T\left(\frac{n}{4}\right) + n$

$c = \log_4^{16} = 2$

$n^c > f(n)$

case 1 is applied

$T(n) = \Theta(n^2)$

(f) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2$; $b = 2$ $f(n) = n\log n$

$c = \log_2^2 = 1$;

$n^c = n^1 = n$

$f(n) > n^c$

case 3 is applied

$T(n) = \Theta(n\log n)$

(g) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a = 2$; $b = 2$; $f(n) = n\log n$.

$c = \log_2^2 = 1$

$n^c = n^1 = n$

$n\log n = f(n) > n$

case 3:

$T(n) = \Theta(n\log n)$

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(h) $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a=2$; $b=2$ $f(n) = n/\log n$

$c = \log_b^a = 1$

$n^c = n^1 = n$

here $f(n) = n/\log n$ is not a polynomial therefore master's method cannot be applied.

(i) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$c = \log_b^a = \log_4^2 = 0.5$

$n^c = n^{0.5}$

$\because f(n) > n^c$

case 3 is applied

$T(n) = \theta(n^{0.51})$

(j) $T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$.

$a<1$, Masters theorem not applicable

(k) $T(n) = 16\,T\left(\frac{n}{4}\right) + n!$

$c = \log_4^{16} = 2$

$n^c = n^2$

$f(n) > n^2$

$T(n) = \theta(n!)$

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(l) $T(n) = \sqrt{n} \cdot T\left(\frac{n}{2}\right) + \log n$

a is not a constant Master's cannot applied

(m) $T(n) = 3T\left(\frac{n}{2}\right) + n$

$c = \log_2 3 = 1.58$
$n^c = n^{1.58} > f(n)$
$T(n) = \theta(n^{1.58})$

(n) $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$c = \log_3 3 = 1$
$n^c = n$
$n^c > f(n)$
case 1 :
$T(n) = \theta(n)$

(o) $T(n) = 4T\left(\frac{n}{2}\right) + c \cdot n$

$c = \log_2 4 = 2, \; n^c = n^2$
$f(n) < n^c$ case 1;
$T(n) = \theta(n^2)$

(p) $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$c = \log_4 3 = 0.79$
$n^c = n^{0.79}$
$f(n) > n^c$ case 3;
$T(n) = \theta(n\log n)$

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(q) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$c = \log_3 3 = 1$

$n^c = n$

$n^c > f(n)$

$T(n) = \theta(n)$

(r) $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$f(n) = n^2 \log n$

$f(n) > n^c$

case 3

$T(n) = \theta(n^2 \log n)$

(s) $T(n) = 4T\left(\frac{n}{2}\right) + n\log n$

$c = \log_2 4 = 2$

$n^c = n^2$, $f(n) = n\log n$

$n^c > f(n)$

case 1

$T(n) = \theta(n^2)$

(t) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

= $f(n)$ is not regular function

= master theorem not applicable