Ques-1. What is the time Complexity of below code and how.

```
void fun(int n)
{ int j=1, i=0;
  while(i<n)
  { i= i+j;
    j++; }
}
```

Sol^n: $i = \overbrace{0, 1, 3, 6, 10, 15, 21, \cdots\cdots n}^{K\text{-terms}}$

let the sum of above K terms is S_K

$$S_K = 1+3+6+10+15+21, \cdots\cdots + T_K \quad \cdots (1)$$

$$S_{K-1} = 1+3+6+10+15+21+\cdots\cdots + T_{K-1} \quad \cdots (2)$$

Subtracting (2) from (1)

$$T_K = S_K - S_{K-1} = 1+2+3+4+5+6+\cdots\cdots + K$$

We have $T_K = n$

$$\therefore 1+2+3+4+5+\cdots\cdots + K = n$$

$$\frac{K(K+1)}{2} = n \Rightarrow K^2+K-2n = 0$$

$$\Rightarrow K = \frac{-1 \pm \sqrt{8n+1}}{2}$$

taking Only positive value we get total no. of times the loop runs for $i = K+1 = \frac{\sqrt{8n+1}}{2}$

$\therefore$ Time complexity, $T(n) = O\left(\frac{\sqrt{8n+1}}{2}\right) = O(\sqrt{n})$.

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Qu.2 Write Recurrance Relation for the recursive function that prints fibonacci series. Solve the recurrance relation to get time complexity of the program. What will be ~~time~~ space complexity of this program and why?

Sol^n:

Recursive function:

```
int fib(int n)
{ if (n<=1)  -> O(1) = c
    return n;
  return fib(n-1)+f(n-2) -> T(n-1)+T(n-2)
}
```

Recurrance Relation, $T(n) = T(n-1) + T(n-2) + C$

$T(n-1) \simeq T(n-2)$

$T(n) = 2T(n-2) + C$

$T(n-2) = 2*(2T(n-2-2)+C) + C$

$= 4T(n-2) + 3C$

$T(n-4) = 2*(4T(n-2) + 3C) + C$

$= 8T(n-3) + 7C$

Generalising $= 2^K T(n-K) + (2^K - 1)C$

Put $n-K=0$

$n = K$

Put $n=K$

$T(n) = 2^n * T(0) + (2^n - 1)C$

$= 2^n * 1 + 2^n C - C$

$= 2^n(1+C) - C$

$= 2^n$

Time Complexity $= O(2^n)$

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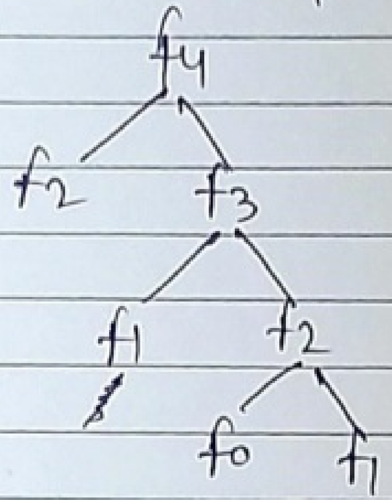
Space Complexity: Space is proportional to the maximum depth of the recursion tree



Hence space complexity of fibonacci recursion is O(N)

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Que-3: Write programs which have complexity.

Sol^n.

1. $n(\log n)$.

```
for (i=1; i<=n; i++)
{ for(j=1; j<=n; j=j*2)
    { sum=sum+j;
    }
}
```

2. n^3

```
for (i=0; i<n; i++)
{ for (j=0; j<n; j++ )
  { for(k=0; k<n; k++ )
      { sum=sum+k;
      }
  }
}
```

3. $\log n(\log n)$.

```
for (i=1; i<=n; i=i*2)
{ for (k=1; k<=n; k=k*2)
    { sum=sum+j;
    }
}
```

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Qus. 4: Solve the Recurrance Relation $T(n) = T(\frac{n}{4}) + T(n/2) + cn^2$

Sol^n

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + cn^2$$

$$\therefore T\left(\frac{n}{4}\right) \simeq T(n/2)$$

$$\Rightarrow T(n) = 2T\left(\frac{n}{2}\right) + cn^2$$

As $a \geq 1$ and $b > 1$

$\therefore$ Using master's Method.

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$c = \log_b a$$

$$c = \log_2 2 = 1$$

$$f(n) > n^c$$

$$\therefore T(n) = O(f(n))$$

$$= O(n^2)$$

Qus 5: What is the time complexity of the following function.

```
int fun(int n)
    { for (int i = 1 ; i <= n ; i++)
        for (int j = 1; j < n; j += i)
            { some O(1) task
    }}}
```

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Solⁿ:

for $i = 1$, j is 1, 2, 3, 4 - - - - - - - - run for n - times
for $i = 2$, j is 1, 3, 5, - - - - - - - - upto $n/2$ times
for $i = 3$, j is 1, 4, 7, - - - - - - - - run for $n/3$ times

$$T(n) = n + n/2 + n/3 + n/4 + \cdots$$
$$n\left(1 + 1/2 + 1/3 + 1/4 + \cdots\right)$$
$$= n \int_1^n dx/x$$
$$= [\log x]_1^n$$

$\Rightarrow$ Time complexity $= n \log n$.

Qu-6: What should be the time complexity of

```
for(int i=2; i<=n; i=pow(i,k))
{ some O(1) expression or statements.
}
```

where k is a constant.

Solⁿ: for first iteration $i = 2$
second iteration $i = 2^k$
third iteration $i = (2^k)^k = 2^{k^2}$
⋮
nth iteration, $i = 2^k$ loop ends at $2^i = n$

apply $\log n = \log_2 k^i$
$$k^i = \log n$$
$$i = \log_c (\log n)$$

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Teacher's Sign ....

Qus. 7 Write a recurrence relation when quick sort repeatedly divides the array in to two parts of 99% and 1%. Derive the time complexity in this case. Show the recursion tree while deriving time complexity and find the difference in heights of both the extreme parts. What do you understand by this analysis.

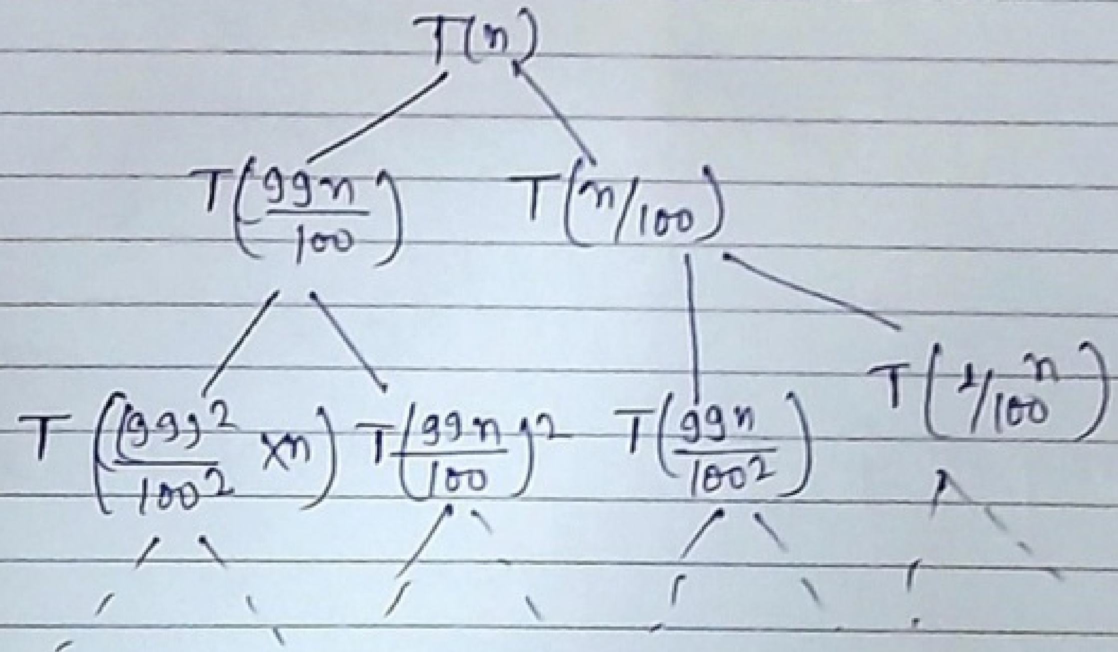
Sol^n

99 to 1 in quick sort
when pivot is where from front or end alway.

So,

$$T(n) = T(99n/100) + T(n^1/100) + O(n)$$

$$T(n) = T(99n/100) + T(n/100) + O(n)$$



$\frac{n}{}$

$$n = (99/100)^K$$

$$\log n = K \log 99/100$$

$$K = \log n \frac{100}{99}$$

$$\therefore T.C = n * \log_{\frac{100}{99}}(n)$$

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Que 8. Arrange the following in increasing order of rate of growth.

Sol^n.

a. $100 < \log\log(n) < \log^2 n < \log n < \log n! < n < n\log n < n^2 < 2^n < 4^n < 2^{\wedge}(2^{\wedge}n) < n!$

b. $1 < \log\log(n) < \sqrt{\log n} < \log(n) < 2\log(n) < \log(2n) < n$
$2n < 4n < \log n! < n\log(n) < 2(2^{\wedge}n)$