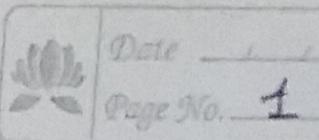


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Tutorial - 4

$$1. \quad T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$\begin{aligned} a &= 3, b = 2, f(n) = n^2 \\ &= n^{\log_b a} = n^{\log_2 3} = n^{1.5} \end{aligned}$$

$$n^{1.5} < n^2 \Rightarrow n^2 < f(n)$$

$$\begin{aligned} T(n) &= \Theta(f(n)) \\ T(n) &= \Theta(n^2) \end{aligned}$$

$$2. \quad T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$\begin{aligned} a &= 4, b = 2, f(n) = n^2 \\ &= n^{\log_b a} = n^{\log_2 4} = n^2 \\ &\quad + f(n) = n^2 \end{aligned}$$

$$T(n) = \Theta(n^2 \log^k n)$$

$$T(n) = \Theta(n^2 \log n)$$

$$3. \quad T(n) = T\left(\frac{n}{2}\right) + 2^n$$

$$\begin{aligned} a &= 1, b = 2, f(n) = 2^n \\ &n^{\log_2 1} = n^0 = 1 \end{aligned}$$

$$f(n) \geq 1$$

$$T(n) = \Theta(f(n))$$

$$= \Theta(2^n)$$

$$4. \quad T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$$

Invalid



3. $T(n) = 16T\left(\frac{n}{4}\right) + b$

$a=8, b=4, f(n)=n$
 $= n^{\log_4 8} = \log_2 2^3 = \frac{\log 8}{\log 4} = \frac{\log 2^3}{\log 2^2}$
 $= \frac{3}{2} \frac{\log 2}{\log 2} = 1.5$

$n^{1.5}$
 $f(n) < n^{1.5} = n < n^{1.5}$

$T(n) = \Theta(n^c) = \Theta(n^{1.5})$

6. $T(n) = 2T\left(\frac{n}{2}\right) + n \log n$

$a=2, b=2, f(n)=n \log n$

$n^{\log_2 2} = n$

$f(n) > n$
 $T(n) = \Theta(f(n)) = \Theta(n \log n)$

7. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

Invalid

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$
 $a=2, b=4, f(n)=n^{0.51}$
 $n^{\log_4 2} = n^{0.5}$
 $f(n) > n^{\log_4 2}$
 $T(n) = \Theta(n^{0.51})$

9. $T(n) = 0.5 T\left(\frac{n}{2}\right) + b$

Invalid



$$10. \quad T(n) = 16T\left(\frac{n}{4}\right) + \ln n$$

$$a=16, b=4, f(n)=\ln n$$

$$n^{\log_4 16} = n^{\log_4 4^2} = n^2$$

$$f(n) > n^2$$

$$T(n) = \Theta(\ln n)$$

$$11. \quad T(n) = \sqrt{2} T\left(\frac{n}{2}\right) + \log n$$

$$a=\sqrt{2}, b=2, f(n)=\log n$$

$$n^{\log_2 \sqrt{2}} = n^{\frac{1}{2}} = \sqrt{n}$$

$$\sqrt{n} > \log n$$
$$T(n) = \Theta(\sqrt{n})$$

$$13. \quad T(n) = 3T\left(\frac{n}{2}\right) + n$$

$$a=3, b=2, f(n)=n$$

$$n^{\log_2 3} = n^{1.5}$$

$$n^{1.5} > f(n)$$

$$T(n) = \Theta(n^{1.5})$$

$$14. \quad T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$$

$$a=3, b=3, f(n)=\sqrt{n}$$

$$n^{\log_3 3} = n$$

$$n > f(n)$$

$$T(n) = \Theta(n)$$



15. $T(n) = 4T\left(\frac{n}{2}\right) + cn$
 $a=4, b=2, f(n)=cn$
 $n^{\log_2 4} = n^2$
 $n^2 \geq f(n)$
 $T(n) = O(n^2)$

16. $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$
 $a=3, b=4, f(n)=n \log n$
 $n^{\log_4 3} = n^{0.2}$
 $f(n) \geq n^{\log_4 3}$
 $T(n) = O(n \log n)$

17. ~~$T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n} \log \sqrt{n}$~~ $T\left(\frac{n}{2}\right) + \log n$

Invalid

17. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$
 $a=3, b=3, f(n)=n$
 $n^{\log_3 3} = n$
 $f(n)=n$
 $T(n) = O(n \log n) = O(n \log n)$

18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$
 $a=6, b=3, f(n)=n^2 \log n$
 $n^{\log_3 6} =$
 $f(n) \geq n^{\log_3 6}$
 $T(n) = O(n^2 \log n)$



19. $T(n) = 4T\left(\frac{n}{2}\right) + n \log n$
 $a=4, b=2, f(n)=\frac{n}{\log n}$
 $n^{\log_2 4} = n^2$
 $n^2 > \frac{n}{\log n}$
 $T(n) = \Theta(n^2)$

20. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$
Invalid

21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$
 $a=7, b=3, f(n)=n^2$
 $n^{\log_3 7}$
 $n^2 > n^{\log_3 7}$
 $T(n) = \Theta(n^2)$

22. $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$
Invalid