

**ASTRO 330 - Galaxies:**  
**Star Formation and Rotation Curves**  
**Class 6 Exercise: Feb. 10**  
**Due Feb. 20 as Homework 3**

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**1. H $\alpha$  as a Star Formation Rate Indicator:**

Pretend there are only two kinds of stars in the Universe: G-stars with  $M = 1 M_{\odot}$  and O-stars with  $M = 100 M_{\odot}$ . According to the IMF, for every O-star that is formed, 900 G-stars are formed. The O-stars have lifetimes of  $t_O = 10$  Myr while the G-stars live for  $t_G = 10$  Gyr (note that these values are similar but not identical to the ones you used in HW2).

Although G-stars do not produce UV photons that can ionize H $\alpha$ , O-stars do; the resulting H $\alpha$  luminosity of a single O-star is  $L_*(H\alpha) = 10^{35}$  ergs s $^{-1}$ . A galaxy has been forming stars with a constant rate of SFR=50  $M_{\odot}$  yr $^{-1}$  for 2 Gyr.

a) [**8 pts**] What is the H $\alpha$  luminosity you observe from the galaxy, assuming none is absorbed by dust? Show all your work.

b) [**2 pts**] Now assume that the galaxy formed stars with a constant rate of SFR=50  $M_{\odot}$  yr $^{-1}$  for 2 Gyr, but in the last 5 Myr this has increased to SFR=500  $M_{\odot}$  yr $^{-1}$ . Would the observed H $\alpha$  luminosity be higher or lower than your answer in part (a) ? Why?

## 2. Galaxy Rotation Curves

For an object of mass  $m$  in uniform circular rotation, the centripetal force equals the gravitational force:

$$m \frac{V_c^2}{r} = \frac{GM(r)m}{r^2}$$

where  $V_c$  is the circular velocity and  $M(r)$  is the mass enclosed by radius  $r$ .

a) [**4 pts**] Suppose that a galaxy has a spherical dark matter density distribution with a constant density  $\rho(r) = C$ . Solve for the dependence of  $V_c$  on  $r$  and sketch the rotation curve.

b) [**6 pts**] Now suppose that the dark matter density distribution scales as  $\rho(r) \propto r^N$ . What value of  $N$  is required for the rotation curve of the galaxy to be flat? Show how you arrive at this answer. To do this problem correctly you'll need to integrate to get  $M(r)$ . (Hint: think of the mass in a thin spherical shell.)