ASTRO 330 - Galaxies:
Star Formation and Rotation Curves
Class 6 Exercise: Feb. 10
Due Feb. 20 as Part 1 of Homework 3

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1. **Hα as a Star Formation Rate Indicator:**

Pretend there are only two kinds of stars in the Universe: G-stars with $M = 1\ M_\odot$ and O-stars with $M = 100\ M_\odot$. According to the IMF, for every O-star that is formed, 900 G-stars are formed. The O-stars have lifetimes of $t_O = 10$ Myr while the G-stars live for $t_G = 10$ Gyr (note that these values are similar but not identical to the ones you used in HW2). Although G-stars do not produce UV photons that can ionize Hα, O-stars do; the resulting Hα luminosity of a single O-star is $L_*(H\alpha) = 10^{35}$ ergs s^{-1}. A galaxy has been forming stars with a constant rate of SFR=50 $M_\odot$ yr^{-1} for 2 Gyr.

1 packet = 1 O star + 900 G stars = 100 M⊙ + 900 M⊙ = 1000 M⊙

a) **[8 pts]** What is the Hα luminosity you observe from the galaxy, assuming none is absorbed by dust? Show all your work.

50 M⊙/yr for 2 Gyr will form $50 \times 2 \times 10^9$ M⊙ $= 1 \times 10^{11}$ M⊙ - not imp

50 M⊙/yr for 10 Myr $= 50 \times 10 \times 10^6$ M⊙ $= 500 \times 10^6 = 5 \times 10^8$ M⊙

$\frac{5 \times 10^{\cancel{8}5} \text{ M⊙}}{\cancel{10^3} \text{ M⊙}} = 5 \times 10^5$ packets formed in 10 Myr.

meaning 5×10^5 O-types stars formed in 10 Myr.

each O-type star has H(α) luminosity of 10^{35} erg/s

∴ Hα luminosity observed from the galaxy

$= 5 \times 10^5 \times 10^{35}$ erg s^{-1}

$= 5 \times 10^{40}$ erg s^{-1}

b) **[2 pts]** Now assume that the galaxy formed stars with a constant rate of SFR=50 $M_\odot$ yr^{-1} for 2 Gyr, but in the last 5 Myr this has increased to SFR=500 $M_\odot$ yr^{-1}. Would the observed Hα luminosity be higher or lower than your answer in part (a) ? Why? → more O-type stars formed

early 5 Myr → 50 M⊙/yr for 5 Myr $= 50 \times 5 \times 10^6$ M⊙ $= 250 \times 10^6 = 2.5 \times 10^8$ M⊙

last 5 Myr → 500 M⊙/yr for 5 Myr $= 500 \times 5 \times 10^6$ M⊙ $= 2500 \times 10^6 = 2.5 \times 10^9$ M⊙

total mass for 10 Myr $= 2.5 \times 10^8 + 2.5 \times 10^9$ M⊙ $= 2.75 \times 10^9$ M⊙

$\frac{2.75 \times 10^{\cancel{9}6} \text{ M⊙}}{\cancel{10^3} \text{ M⊙}} = 2.75 \times 10^6$ O-type stars formed in 10 Myr.

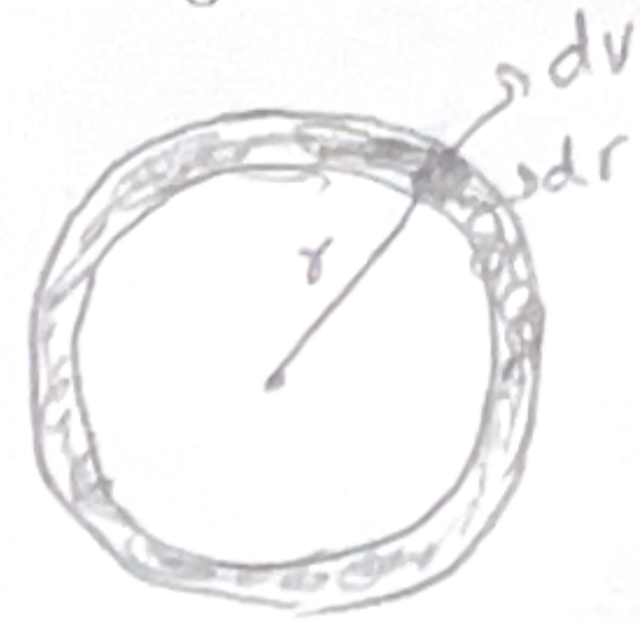
Hα luminosity now observed $= 2.75 \times 10^{41}$ erg s^{-1} → higher than answer for part (a)

2. **Galaxy Rotation Curves**

For an object of mass m in uniform circular rotation, the centripetal force equals the gravitational force:

$$V_c = \sqrt{\frac{GM(r)}{r}}$$

$$\cancel{m}\frac{V_c^2}{\cancel{r}} = \frac{GM(r)\cancel{m}}{r^{\cancel{2}}}$$



where V_c is the circular velocity and $M(r)$ is the mass enclosed by radius r.

a) **[4 pts]** Suppose that a galaxy has a spherical dark matter density distribution with a constant density $\rho(r) = C$. Solve for the dependence of V_c on r and sketch the rotation curve.

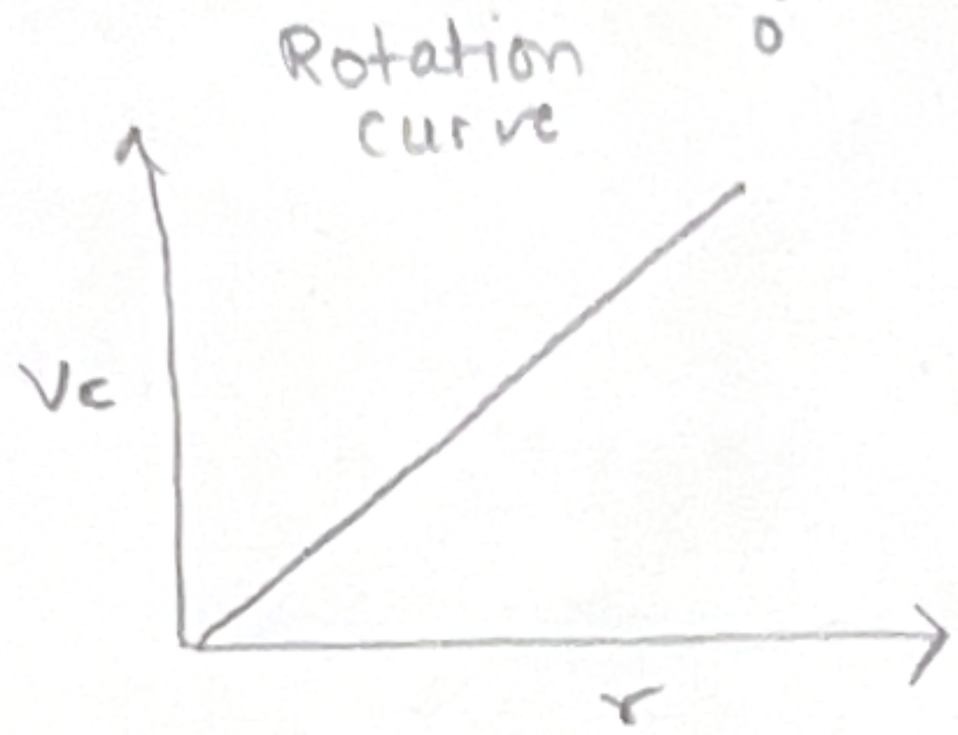
$$dv = 4\pi r^2 dr$$

$$M(r) = \int_0^r \rho(r)\,dV = \int_0^r C \times 4\pi r^2 dr = C 4\pi \int_0^r r^2 dr = C \times 4\pi \left[\frac{r^3}{3}\right]_0^r$$

$$M(r) = \frac{4\pi C r^3}{3}$$

$$V_c = \sqrt{\frac{G \times 4\pi C r^{\cancel{3}2}}{3\cancel{r}}}$$

$$V_c \propto r$$



Physically, this shows a uniform-density sphere, more mass is enclosed as r^3, causing circular velocity to grow proportionally to r.

b) **[6 pts]** Now suppose that the dark matter density distribution scales as $\rho(r) \propto r^N$. What value of N is required for the rotation curve of the galaxy to be flat? Show how you arrive at this answer. To do this problem correctly you'll need to integrate to get $M(r)$. (Hint: think of the mass in a thin spherical shell.)

we can substitute $\rho(r) \propto r^N$ in the enclosed equation

$$M(r) = \int_0^r \rho(r)\,dV = \int_0^r r^N \times 4\pi r^2 dr = 4\pi \int_0^r r^{N+2} dr = 4\pi \left[\frac{r^{N+3}}{N+3}\right]_0^r$$

$$M(r) = \frac{4\pi\, r^{N+3}}{N+3}$$

$$V_c = \sqrt{\frac{G \times 4\pi\, r^{N+\cancel{3}2}}{\cancel{r}(N+3)}}$$

$$V_c = \sqrt{\frac{G \times 4\pi \times r^{N+2}}{(N+3)}}$$

$$r^{N+2} = \text{constant}$$

$$N + 2 = 0$$

$$N = -2$$

with $N = -2$, the dependence of r vanishes and you get a flat rotation curve.

for the rotation curve to be flat, you want V_c to not be proportional to any power of r