1. Calculate the mean molecular weight for a pure hydrogen/helium mixture with $X = 0.4$, assuming the hydrogen and helium are both 50% ionized (i.e., $\mathcal{N}_j = 0.5 \times \mathcal{Z}_j$, where $\mathcal{N}_j$ and $\mathcal{Z}_j$ are defined in *Handout 15*). **4 points**

2. By combining equations [6.4] and [6.8], derive the dependence of temperature T on pressure P for an ideal gas undergoing an adiabatic change. Use this expression to prove equation [13.7]. **4 points**

3. Consider a location within a star where the interior mass and luminosity are given by $m = 0.3\,\mathrm{M}_\odot$ and $\ell = 5\,\mathrm{L}_\odot$, respectively; the pressure and temperature are $P = 10^{17}\,\mathrm{Ba}$ and $T = 10^7\,\mathrm{K}$, respectivey; and the opacity is $\kappa = 1\,\mathrm{cm}^2\,\mathrm{g}^{-1}$.

   - Evaluate the radiative temperature gradient ∇_{rad}.
   - Assuming the stellar material behaves as an ideal gas with $\gamma = 1.4$, evaluate the adiabatic temperature gradient ∇_{ad}.
   - By applying the algorithm given in *Handout 13*, explain why convection will occur at this location.
   - Assuming a convective efficiency $\varphi_{\mathrm{conv}} = 0.5$, evaluate the dimensionless temperature gradient ∇_T at the location.
   - Evaluate the convective (ℓ_{conv}) and radiative (ℓ_{rad}) interior luminosities, in $\mathrm{L}_\odot$.

   7 points

4. The central temperature and density of ZAMS stars can be approximated over the stellar mass interval $0.1\,\mathrm{M}_\odot \lesssim M \lesssim 30\,\mathrm{M}_\odot$ by the fits

$$\log\left(T_\mathrm{c}/\mathrm{K}\right) \approx 7.10 + 0.38\left(M/\mathrm{M}_\odot\right),$$
$$\log\left(\rho_\mathrm{c}/\mathrm{g\,cm^{-3}}\right) \approx 1.77 - 0.77\left(M/\mathrm{M}_\odot\right).$$

   Derive corresponding expressions for the gas pressure $P_{\mathrm{gas}} \equiv P_{\mathrm{ion}} + P_\mathrm{e}$ (assuming an ideal gas with $\mu \approx 0.62$) and the radiation pressure P_{rad} at the center. At what stellar mass does radiation pressure begin to exceed gas pressure? **5 points**

5. For a free electron gas in the completely degenerate limit, use your knowledge of the momentum distribution function $f_e(p)$ to determine what fraction of the electrons have momenta less than half of the Fermi momentum p_F, and what fraction have momenta more than double the Fermi momentum. **5 points**