1. Consider a star with a density that varies linearly from $\rho = \rho_c$ at the center ($r = 0$), to $\rho = 0$ at the surface ($r = R$). Using the mass equation [4.1], derive the interior mass function $m(r)$ for this star. Apply this function to obtain an expression for ρ_c in terms of the star's mass M and radius R. **5 points**

2. For the linear-density star in Q1, derive an expression for the gravitational potential energy Ω of the star in terms of M and R. By comparing this against eqn. [4.8], determine the shape factor f_Ω for the star. How does it compare to the shape factor $f_\Omega = 3/5$ for a uniform density star; is the result expected or unexpected? **5 points**

3. With reference to the equation of hydrostatic equilibrium [5.6], explain why the pressure in a star is a monotonic-decreasing function of radial coordinate. **2 points**

4. An ideal-gas star in hydrostatic equilibrium, with radius R and mass M, is cooled instantaneously down to $T = 0\,\mathrm{K}$. What will happen to the star — and how long will it take? **2 points**

5. Consider the linear-density star from Q1. Assuming the surface pressure vanishes, i.e. $P(R) = 0$, integrate the equation of hydrostatic equilibrium [5.6] to find the central pressure P_c in terms of M and R. **3 points**

6. A ideal-gas sample of fully ionized helium has a density $\rho = 50\,\mathrm{g\,cm^{-3}}$. Evaluate the sample's mean molecular weight and number density of particles n. **2 points**

7. An ideal gas with $\gamma = 5/3$ is compressed adiabatically to one third its volume. By what factor does its density change? Its pressure? Its temperature? **3 points**

8. A star with a mass $4\,M_\odot$ has a hydrogen mass fraction $X = 0.6$ and a metal mass fraction $Z = 0.15$. How many helium nuclei does the star contain? **3 points**