1. Calculate the slope $\mathrm{d}\log L/\mathrm{d}\log T_{\mathrm{eff}}$ of the Hayashi line for fully convective stars in which the opacity is *independent* of pressure and temperature.

4 points

2. In very massive stars ($M \gtrsim 50\,\mathrm{M}_\odot$) the radiation pressure is much greater than the gas pressure, and energy is transported primarily by radiation. For such stars,

   (a) Show that the central temperature follows a scaling $T_\mathrm{c} \sim M^{1/2}/R$ (*hint: assume the scaling relations* $\rho_\mathrm{c} \propto M/R^3$ *and* $P_\mathrm{c} \propto M^2/R^4$ *that apply to polytropes*).

   (b) Show that $L \propto M$ (*hint: follow a similar process to that outlined in* Handout 21).

   (c) With this mass-luminosity relation, argue that the main-sequence lifetime of very massive stars is independent of mass.

6 points

3. For each of the following short-hand reactions comprising the CNO cycle, write out the reaction in full, making sure to conserve baryon number, lepton number and charge, and to have zero net charge on each side of the reaction. Then calculate the total energy released by the reaction (whether as gamma rays, positrons, neutrinos or kinetic energy), in units of MeV. A table of atomic masses is given to the right; remember that $1\,\mathrm{u} = 931.49\,\mathrm{MeV}/c^2$. Confirm that the total energy released by the cycle adds up to the 26.73 MeV for fusion of four hydrogen atoms into one helium atom.

Isotope	Atomic Mass (u)
^{1}H	1.0078250
^{4}He	4.0026033
^{12}C	12.0000000
^{13}C	13.0033548
^{13}N	13.0057386
^{14}N	14.0030740
^{15}N	15.0001089
^{15}O	15.0030654

   (i) $^{12}\mathrm{C}\,(p,\gamma)\,^{13}\mathrm{N}$

   (ii) $^{13}\mathrm{N}\,(\cdot,e^+)\,^{13}\mathrm{C}$

   (iii) $^{13}\mathrm{C}\,(p,\gamma)\,^{14}\mathrm{N}$

   (iv) $^{14}\mathrm{N}\,(p,\gamma)\,^{15}\mathrm{O}$

   (v) $^{15}\mathrm{O}\,(\cdot,e^+)\,^{15}\mathrm{N}$

   (vi) $^{15}\mathrm{N}\,(p,\alpha)\,^{12}\mathrm{C}$

15 points