1. *Look up the parallax of the star Betelgeuse on SIMBAD (an online astronomical database providing basic data, cross-identifications, bibliography and measurements for astronomical objects outside the solar system). Use this parallax to calculate the distance to Betelgeuse, in parsecs.*

   1 point for parallax, **1 point** for distance.

   The SIMBAD database (`http://simbad.u-strasbg.fr/simbad/`) reports the parallax of Betelgeuse as $p = 6.55\,\mathrm{mas}$ (here, 'mas' is a milli-arcsecond), corresponding to $p = 0.006\,55''$. The distance is thus $d = 1/p = 153\,\mathrm{pc}$.

2. *The* New Horizons *spacecraft, which zoomed by Pluto in July 2015, was launched with a speed of* $36\,000\,\mathrm{km\,h^{-1}}$. *Traveling this fast, how long would it take for the spacecraft to reach Gliese 832, a nearby* ($d = 16.16\,\mathrm{pc}$) *star that is orbited by a planet in the star's habitable zone?*

   1 point for distance conversion to cm (or other appropriate unit), **1 point** for time.

   The distance of Gliese 832 is $d = 16.16\,\mathrm{pc} = 4.987 \times 10^{19}\,\mathrm{cm}$. At a velocity $v = 36\,000\,\mathrm{km\,h^{-1}} = 10^6\,\mathrm{cm\,s^{-1}}$, *New Horizons* will take a time $t = d/v = 4.987 \times 10^{13}\,\mathrm{s} = 1.580 \times 10^6\,\mathrm{yr}$ to reach the star.

3. *Prove that* $1\,\mathrm{pc} \approx 206\,265\,\mathrm{au}$. Hint: *starting from Fig. 1.1 of* Handout 1, *apply the small-angle formula* $\tan(p/\mathrm{rad}) \approx (p/\mathrm{rad})$, *where p is the parallax angle and* rad *indicates radians.*

   1 point for finding relationship between p, O and A; **1 point** for choosing appropriate values for these quantities; **1 point** for result.

   Consider a distant star. If the right-angle triangle formed by the Earth-Sun-Star (with the Sun at the right-angle) has an apex angle p, an adjacent-side length A and an opposite-side length O, then $\tan(p/\mathrm{rad}) = O/A$. For $p \ll 1\,\mathrm{rad}$, this becomes $p/\mathrm{rad} \approx O/A$, or $A \approx O/(p/\mathrm{rad})$.

   Apply this to a star at 1 pc distance, the Earth-Sun-Star triangle has $A = 1\,\mathrm{pc}$, $O = 1\,\mathrm{au}$ and $p = 1'' = (2\pi/360)/3600\,\mathrm{rad}$. Thus, $1\,\mathrm{pc} \approx 3600 \times 360/(2\pi)\,\mathrm{au} \approx 206\,265\,\mathrm{au}$.

4. A star has an apparent magnitude $m = 6.4$ and an absolute magnitude $M = 2.5$. How far away is the star, in parsecs?

   1 point for invoking distance modulus equation, **1 point** for result.

   The apparent and absolute magnitude are related by eqn. [1.8]:

   $$m - M = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right).$$

   Solving, $d = 10^{0.78} \times 10\,\mathrm{pc} = 60.3\,\mathrm{pc}$.

5. *Assuming the human body emits radiation like a black body with temperature* $37.5\,^\circ\mathrm{C}$, *at what wavelength is it brightest? Where in the electromagnetic spectrum is this wavelength situated? Estimate the body's total luminosity, assuming a surface area* $1.8\,\mathrm{m^2}$.

   1 point for calculating $\lambda_{\max}$, **1 point** for situating it in electromagnetic spectrum, **1 point** for luminosity.

   For a black-body emitter, Wien's law gives the wavelength of peak flux density as

   $$\lambda_{\max} = \frac{2.898 \times 10^7\,\text{Å}\,\mathrm{K}}{T}.$$

   With $T = 37.5\,^\circ\mathrm{C} = 310.65\,\mathrm{K}$, $\lambda_{\max} = 93\,300\,\text{Å}$. This is in the far infrared part of the electromagnetic spectrum.

The total luminosity can be calculated from the Stefan-Boltzmann law:

$$L = A\sigma T^4.$$

With $A = 1.8\,\mathrm{m}^2 = 1.8\times10^4\,\mathrm{cm}^2$ we find $L = 9.50\times10^9\,\mathrm{erg\,s^{-1}}$ — about the same as ten 100 W light bulbs.

6. *Derive the Stefan-Boltzmann law* [2.3] *of* Handout 2 *from Planck's law* [2.1]. *Be sure to take advantage of the hint given in the handout, and the knowledge that*

$$\int_0^\infty \frac{u^3}{\mathrm{e}^u - 1}\,\mathrm{d}u = \frac{\pi^4}{15}.$$

(*throughout the course, you'll always be given assistance for tricky integrals like this!*)

1 point for writing the flux as an integral of Planck's law, **1 point** for applying the change of variable, **1 points** for the correct result.

The total flux F emitted by a black body can be obtained by integrating Planck's law for the flux density over all wavelengths:

$$F = \int_0^\infty \frac{2\pi hc^2}{\lambda^5}\frac{1}{\mathrm{e}^{hc/\lambda k_\mathrm{B}T} - 1}\,\mathrm{d}\lambda.$$

To evaluate the integral, the hint given in the handout suggests a change of variable to

$$u = \frac{hc}{\lambda k_\mathrm{B} T}.$$

Rearranging,

$$\lambda = \frac{hc}{uk_\mathrm{B}T}$$

and likewise

$$\mathrm{d}\lambda = -\frac{hc}{u^2 k_\mathrm{B} T}\,\mathrm{d}u,$$

so that the flux becomes

$$F = \frac{2\pi k_\mathrm{B}^4 T^4}{h^3 c^2}\int_0^\infty \frac{u^3}{\mathrm{e}^u - 1}\,\mathrm{d}u.$$

The value of the transformed integral is given above, leading to

$$F = \frac{2\pi^5 k_\mathrm{B}^4}{15 h^3 c^2} T^4 = \sigma T^4,$$

which is the Stefan-Boltzmann law.

7. *Toward the end of the Sun's life, it will evolve into a red giant with luminosity* $3.2\times10^3\,\mathrm{L}_\odot$ *and effective temperature* 2600 K. *Will it engulf the Earth?*

2 points for radius calculation, **1 point** for interpretation.

The Stefan-Boltzmann law applied to stars is

$$L = 4\pi R^2 \sigma T_\mathrm{eff}^4.$$

Solving for the radius R in terms of the luminosity L and effective temperature T_eff,

$$R = \sqrt{\frac{L}{4\pi\sigma T_\mathrm{eff}^4}}.$$

Plugging in the given values for L and $T_{\rm eff}$ (in cgs units) gives $R = 1.94 \times 10^{13}$ cm $= 1.30$ au. Since the Earth orbits at a distance 1 au from the Sun's center (more exactly, the center-of-mass of the solar system), it appears that the Sun *will* engulf the Earth during its red giant phase.

8. *Sketch a Hertzsprung-Russell diagram (with appropriate axes), showing the main sequence, giant and white dwarf regions, and illustrating with arrows how stars move between these regions as they evolve.*

2 points for correct axes and regions, **1 point** for arrows.

See Fig. 1.1.

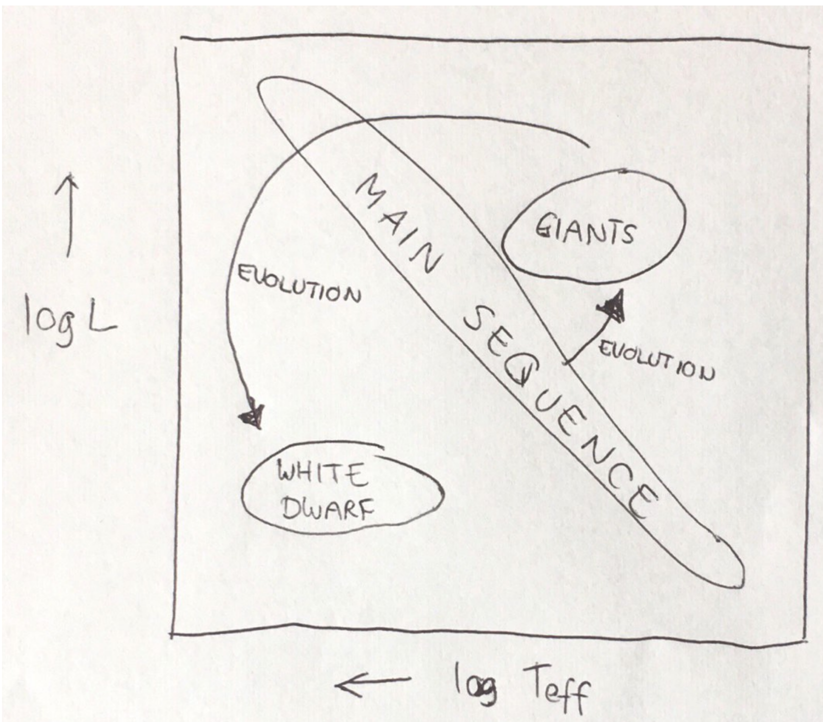

Figure 1.1: HR diagram sketch for Q8.

9. *The Sun is a G2 main-sequence star, while Betelgeuse is an M1 super-giant. Which star is the hotter of the two? Which star is the redder? Which star is the more luminous? Which star is the larger?*

1 point for hotter, **1 point** for redder, **1 point** for more luminous, **1 point** for larger radius.

Recalling that the spectral type sequence is O-A-B-F-G-K-M is in order of decreasing effective temperature, the Sun as a G star is hotter than Betelgeuse as an M star. From Wien's law, this higher effective temperature translates into a shorter wavelength for the flux density peak; so, the Sun is bluer and Betelgeuse is redder. Because Betelgeuse is a red supergiant, it resides in the upper-right portion of the HR diagram; therefore, it is much more luminous and much larger than the Sun.