1. For the solar model introduced MESA-Web demonstration in class last Thursday, create a Python plot of $\log(L/L_\odot)$ as a function of $\log(t/\mathrm{yr})$, where t is the time since the start of the MESA-Web calculation. Be sure to label your axes appropriately. Attach this plot to your submission. (Don't include the code you used to create this plot.). From your plot, estimate by what factor the Sun's luminosity has increased over the period extending from the pre-main sequence luminosity minimum at $\log(t/\mathrm{yr}) \approx 7.10$ through to the present day. **5 points**

2. For a sample of pure hydrogen, write an expression relating the total number density of particles n to the number densities of hydrogen atoms/ions (n_H) and free electrons (n_e). Use this expression, and others appearing in Handout 17, to rewrite equation [17.7] for the ionization fraction x in terms of n and T. Then, evaluate the ionization fraction at a temperature 10 K and pressure 10^5 Ba. (You may neglect radiation pressure and assume the atoms, ions and electrons behave classically). How does the ionization fraction change if the temperature is doubled? Or if the pressure is doubled? **5 points**

3. Consider an ideal-gas star whose pressure and density follow the poltropic relation

$$P = K\rho^{(n+1)/n}$$

   (a) Derive an expression for the temperature in terms of the pressure P and other constant quantities (K, n, μ, etc.).

   (b) Evaluate the dimensionless temperature gradient ∇_T.

   (c) Assuming that $\gamma = 5/3$, show that the star is convectively stable everywhere if $n > 1.5$, and convectively unstable everywhere if $n < 1.5$.

5 points

4. Consider an $n = 1$ polytrope with mass M and radius R.

   (a) Write down an expression for the central density ρ_c, in terms of M and R. (*Hint: apply eqn. [19.11], using the analytic solution for n = 1 to evaluate the term in parentheses.*)

   (b) Evaluate the ratio $\rho_c/\bar{\rho}$, where $\bar{\rho} = 3M/(4\pi R^3)$ is the mean density. Is this ratio larger or smaller than for an $n = 0$ (constant density) polytrope?

   (c) Derive an expression for the interior mass m, in terms of M, R and r.

   (d) Use your $m(r)$ expression to confirm that

$$m(R/2) = \frac{M}{\pi}.$$

5 points

5. Use Poly-Web to create models for $n = 0$ and $n = 1$ polytropes. For each model, plot a graph of w versus z/z_s, and (using a dashed line-style) also show the corresponding analytic solution. Do the Poly-Web and analytic solutions agree? Attach this plot to your submission.

   The following information may prove useful:

   - To use Poly-Web, fill in the Polytropic Index box with the desired value of n, and hit the Submit button. Save the resulting model (a three-column table of data) to disk.
   - To read a model into Python, use the following code:

     ```
     import numpy as np
     data = loadtxt(filename)
     z = data[:,0]
     w = data[:,1]
     dw_dz = data[:,2]
     ```

     (where `filename` is the name of the file you saved the model to.)
   - To plot a dashed line, add `ls='--'` argument to your `plt.plot()` command.

5 points