Homework 1

1) measured parallax of the star Betelgeuse : 0.00763 pc

$$\text{distance (parsec)} = \frac{1}{\text{angle (parsec)}}$$

$$= \frac{1}{0.00763 \text{ pc}}$$

$$= 131.0616 \text{ pc}$$

The distance of Betelgeuse is 131.0616 parsecs.

2) launch speed $= 36{,}000$ km/hr

$$= \cancel{36000}^{10} \times \frac{1000 \text{ m}}{\cancel{3600} \text{ s}} = 10{,}000 \text{ m/s}$$

distance to Gliese 832 (d) $= 16.16$ pc

$$= 16.16 \times 3.086 \times 10^{16} \text{ m}$$

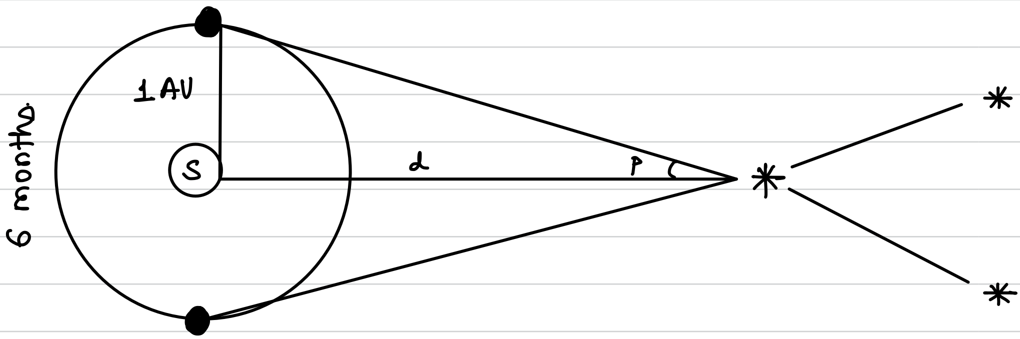
$\cdots$ $(1 \text{ pc} = 3.086 \times 10^{16} \text{ m})$

$$= 4.98698 \times 10^{17} \text{ m}$$

$$\text{time (t)} = \frac{\text{distance}}{\text{speed}} = \frac{4.98698 \times 10^{17} \text{ m}}{10{,}000 \text{ m/s}} = 4.98698 \times 10^{13} \text{ s}$$

$$t \text{ (in years)} = \frac{4.98698 \times 10^{13} \text{ s}}{3.156 \times 10^{7} \text{ s/year}} = 1.580 \times 10^{6} \text{ years.}$$

3)



$$\tan p = \frac{1\,AU}{d}$$

The parallax angle $p \to 0$, so the angle is very small. Using the small-angle approximation formula.

$$\therefore\ p = \frac{1\,AU}{d}$$

1 arcsec (1″) $= \frac{1}{3600}$ degree

1 radian $= \frac{180}{\pi} \times 3600 = 206\,265''$ (arcsec)

$p('')$ parallax angle $= \frac{1\,AU}{d} \times 206265 = 206265\,AU = 1$ parsec (pc)

4) Apparent magnitude (m) = 6.4
of a star

Absolute magnitude of (M) = 2.5
that star

Distance of the star from (d) = ?
us (in pc)

using the distance modulus equation :

$$m - M = 5 \log_{10}\left(\frac{d}{10\,pc}\right)$$

$$6.4 - 2.5 = 5 \log_{10}\left(\frac{d}{10\,pc}\right)$$

$$\frac{3.9}{5} = \log_{10}\left(\frac{d}{10\,pc}\right)$$

$$0.78 = \log_{10}\left(\frac{d}{10\,pc}\right)$$

taking anti log on both sides.

$$6.0256 = \frac{d}{10\,pc}$$

$$d = 10\,pc \times 6.0256$$

$$d = 60.256\ pc$$

5) Temperature (T) = 37.5 °c

brightest wavelength (λ_{bright}) = ?

λ_{bright} in the electromagnetic spectrum = ?

total Luminosity (L) = ? when surface area (dA) = 1.8 m2

$$\lambda_{brightest} = \frac{2.898 \times 10^{7} \text{ Å K}}{T}$$
(λ_{max})

Temp (in kelvin) $= 273.15 + 37.5 = 310\ 65$ K

$$\lambda_{brightest} = \frac{2\ 898 \times 10^{7} \text{ Å } \cancel{K}}{310.65 \cancel{K}}$$

$$\lambda_{brightest} = 93288.26654 \text{ Å}$$
$$= 93288.26654 \times 10^{-10} \text{ m}$$
$$= 9.32882 \times 10^{-6} \text{ m}$$

$\lambda_{brightest}$ falls into the infrared region of the electromagnetic spectrum.

total luminosity (L) $= \sigma \times A \times T^{4}$

$$= 5.67 \times 10^{-5} \frac{\text{erg}}{\cancel{cm^2} \times s \times \cancel{K^4}} \times \frac{(100 \cancel{cm})^2}{(1 \cancel{m})^2} \times 1.8 \cancel{m^2}$$
$$\times (310.65)^4 \cancel{K^4}$$
$$= 5.67 \times 10^{-5} \times 100 \times 100 \times 1.8 \times (310.65)^4$$
$$L = 9.5048 \times 10^{9} \text{ erg/s}$$

6) Planck's Law is given by:

The energy flux in the narrow wavelength interval $[\lambda, \lambda + d\lambda]$ measured is:

$$F_\lambda d\lambda = \frac{2\pi h c^2}{\lambda^5} \times \frac{1}{e^{hc/\lambda K_B T} - 1} d\lambda$$

Integrating Planck's Law over all wavelengths leads to the Stefan-Boltzmann Law:

$$F = \int_0^\infty F_\lambda \, d\lambda$$

$$F = \int_0^\infty \frac{2\pi h c^2}{\lambda^5} \times \frac{1}{e^{hc/\lambda K_B T} - 1} d\lambda$$

Let $x = \dfrac{hc}{\lambda K_B T}$ $\therefore$ $dx = \dfrac{-hc}{\lambda^2 K_B T} d\lambda$

$$F = \frac{2\pi (K_B)^4 T^4}{h^3 c^2} \int_0^\infty \frac{x^3}{(e^x - 1)} dx = \frac{2\pi (K_B)^4 T^4}{h^3 c^2} \times \frac{\pi^4}{15}$$

... (using the hint given in the question).

$$F = \frac{2\pi^5 (K_B)^4 \times T^4}{15 \times h^3 c^2} = \sigma T^4$$, where $\sigma = \dfrac{2\pi^5 (K_B)^4}{15 h^3 c^2} = 5.6704 \times 10^{-5}$ erg cm^{-2}s^{-1}K^{-4}

7) Luminosity of the sun (L) $= 3.2 \times 10^3 \, L_\odot$
in the red gaint phase

$T_{eff} = 2600 \, K$

$1 \, L_\odot = 3.828 \times 10^{23}$ erg/s

$1 AU = 1.496 \times 10^{11}$ m

The Stefan - Boltzmann Law for stars is:

$$L = 4 \pi R^2 \sigma (T_{eff})^4$$

$$3.2 \times 10^3 \times 3.828 \times 10^{23} = 4 \pi \times R^2 \times 5.6704 \times 10^{-5} \times (2600)^4$$

$$R^2 = \frac{3.2 \times 10^3 \times 3.828 \times 10^{23}}{4 \pi \times 5.6704 \times 10^{-5} \times (2600)^4}$$

$$R^2 = \frac{1.22496 \times 10^{27}}{3.25624 \times 10^{10}}$$

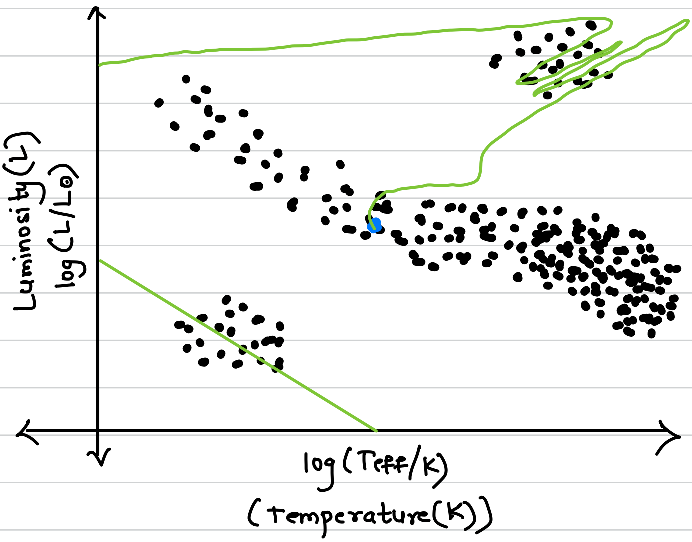
$$R^2 = 3.761885 \times 10^{16}$$

$$R = 1.93956 \times 10^8 \text{ m}$$

$$1.93956 \times 10^8 \text{ m} < 1.496 \times 10^{11} \text{ m}$$

$$\therefore \quad R < 1 AU$$

The radius of of sun in its red gaint phase is lesser than the distance of the sun from the earth. This means that the Earth would not be engulfed by it.

8) 

The Hertzsprung - Russell Diagram

9) Sun : G2 main - sequence star
Betelgeuse : M1 supergiant

(a) The surface temperature of the Sun is approximately 5500°C, while that of Betelgeuse is about 3000°C to 3,500°C. This means Betelgeuse is cooler than the Sun.
The Sun is hotter than Betelgeuse.

(b) The Sun has a whitish - yellow color when viewed from space while Betelgeuse is known for its reddish appearence. So, Betelgeuse is redder of the two stars.

(c) Betelgeuse is a supergiant and is intrinsically much more luminous than the Sun. So, Betelgeuse is more luminous than the sun.

(d) Supergiants are typically much larger in size than main-sequence stars like the Sun.

The radius of Betelgeuse is estimated to be around 1000 times more than the Sun. So, Betelgeuse is larger than the Sun.