1. A star initially has a thermal energy of 2×10^{51} erg. What is its total energy? Suppose that 7×10^{50} erg of heat is added near the center of the star. Assuming the star remains in hydrostatic equilibrium, what is its final total energy? Its final thermal energy? If the central temperature of the star is related to its thermal energy U by

$$T_{\mathrm{c}} = \frac{(\gamma - 1) m_{\mathrm{H}}}{k_{\mathrm{B}} M} U,$$

(where the symbols have their usual meanings), then did the addition of heat make the center of the star hotter or colder? **5 points**

2. Consider a satellite with mass m in a circular orbit around a planet with mass M. Derive expressions for the kinetic and gravitational potential energies of the satellite, in terms of m, M and the satellites orbital radius r. Treating the satellite's kinetic energy as its thermal energy, show that the satellite obeys the virial theorem with $\gamma = 5/3$. **4 points**

3. The Sun will spend approximately 10^{10} yr on the main sequence. Assuming that its luminosity remains fixed at the present-day value $L = L_{\odot}$, how much energy will it radiate into space during this phase? Given that 3.8×10^{24} MeV of energy is released for each gram of hydrogen fused into helium, what total mass of hydrogen must be consumed to provide this energy? From this mass, determine what fraction of the Sun's initial hydrogen content will be converted into helium during the main-sequence phase (assume an initial composition $X = 0.7$, and take $M = M_{\odot}$). **5 points**

4. Sketch a Hertzsprung-Russell diagram (with appropriate axes), showing the main sequence. Draw the track followed by the Sun during its pre-main sequence phase, labeling the Hayashi and Henyey portions of the track. **3 points**

5. As a simple model for the late phases of pre-main sequence evolution, assume that a star with mass M moves along the Henyey track at constant luminosity L. Derive an expression for the time taken for the effective temperature to increase to $T_{\mathrm{eff}}^{\mathrm{b}}$, starting from the value $T_{\mathrm{eff}}^{\mathrm{a}}$. (You can assume the virial theorem applies with $\gamma = 5/3$, and that the shape factor f_{Ω} remains constant). Apply your formula to the Sun (with $L \approx L_{\odot}$) to estimate how long the Sun spent on the Henyey track, assuming $T_{\mathrm{eff}}^{\mathrm{a}} \approx 4300\,\mathrm{K}$, $T_{\mathrm{eff}}^{\mathrm{b}} \approx 5600\,\mathrm{K}$, and $f_{\Omega} \approx 1.7$. **4 points**

6. Stars with masses $M \gtrsim 0.8\,M_{\odot}$ will, at some point in their lives, burn helium in their cores into carbon via the *triple alpha* process

$$3\,{}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C}$$

The rest mass of a helium atom is 4.0026 u, and a carbon atom is 12.0000 u. Calculate the percentage of rest mass converted to energy by this reaction. Compare this percentage against the corresponding value for hydrogen burning (the mass of a hydrogen atom is 1.0079 u) — which is the more efficient fuel? **4 points**