1. Suppose the interior luminosity in a star with mass M and luminosity L is given by
$$\ell(m) = L\left[2\left(\frac{m}{M}\right) - \left(\frac{m}{M}\right)^2\right].$$
Assuming that the star is in thermal equilibrium, and that neutrino losses are negligible, derive an expression for the nuclear energy generation rate $\epsilon_{\rm nuc}$ in terms of m, M and L. (*Hint: start by combining the hydrostatic equilibrium equation* [5.6] *with the thermal equilibrium condition* [10.4].) **5 points**

2. What is the bound-free opacity of a sample of fully ionized hydrogen? What is the electron-scattering opacity of a sample of completely neutral helium? Be sure to explain your answers. **4 points**

3. A sample of stellar material has an average cross section per particle of $10^{-23}\,{\rm cm}^2$, and a mean molecular weight of 2.5. Calculate the opacity of the sample. If the density is $20\,{\rm g\,cm^{-3}}$, calculate the mean free path for photons. **2 points**

4. Suppose the outer layers of the star have a uniform opacity 20 ${\rm cm^2\,g^{-1}}$ and density $10^{-9}\,{\rm g\,cm^{-3}}$. What physical depth below the surface corresponds to an optical depth $\tau = 25$? What total distance must a photon travel in order to random-walk its way from this depth to the surface? **4 points**

5. The *Eddington standard model* is a simplified model for stellar structure, which assumes hydrostatic equilibrium, radiative transport of energy with a constant opacity κ, and a pressure that follows the law
$$P = \frac{aT^4}{3(1-\beta)},$$
where $0 \le \beta < 1$ is an arbitrary constant and a is the radiation constant. Show that $\ell \propto m$ in this model, and derive an expression for the constant of proportionality in terms of β, κ and physical constants. (*Hint: start by substituting the above expression for P into the hydrostatic equilibrium equation* [5.6], *and then use the radiative diffusion equation* [12.4] *to eliminate the resulting* $\mathrm{d}T\,/\,\mathrm{d}r$ *term.*) **5 points**

6. In the cores of red giant stars, nuclear reactions do not take place but there are strong non-nuclear neutrino losses. Assuming thermal equilibrium and radiative transport of energy, create as sketch of $\ell(m)$ in the core. (*Hint: to create the sketch, consider the value of ℓ at $r = 0$, and use the thermal equilibrium condition* [10.4] *to determine the slope of ℓ in the vicinity of $r = 0$*). Use this sketch together with the radiative diffusion equation [12.4] to argue that the dimensionless temperature gradient ∇_T must be negative throughout the core. **5 points**