a let sou so Is it is the desired to (a) we can write this system of equations as an agmented matrix and sow reduce it to see it we find a solution or not.

$$\begin{bmatrix} 3 & 1 & | & 7 \\ 3 & 1 & | & 7 \\ 2 & 5 & | & 5 \\ 1 & -1 & | & 3 \end{bmatrix} \xrightarrow{3K_0 - R_1 = R_3} \begin{bmatrix} 3 & \times 1 & | & 7 \\ 2 & 5 & | & 5 \\ 0 & -4 & | & 2 \end{bmatrix} \xrightarrow{R_3} = R_3$$

$$\begin{bmatrix} 3 & 1 & 1 & 7 \\ 2 & 5 & 1 & 5 \\ 0 & -4/3 & 2/3 \end{bmatrix} \xrightarrow{R_2 - 2R_1 - R_2} \begin{bmatrix} 3 & 1 & 7 & \frac{1}{3} \times R_1 \times R_1 \\ & & & & & & & & \\ & & & & & & & & \\ \end{bmatrix} \xrightarrow{R_2 - 2R_1 - R_2} \begin{bmatrix} 3 & 1 & 7 & \frac{1}{3} \times R_1 \times R_1 \\ & & & & & & \\ & & & & & & \\ \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1/3 & | & 7/3 \\ 2 & 5 & | & 5 \\ 0 & -4/3 & | & 2/3 \end{bmatrix} \xrightarrow{R2-2R4} \xrightarrow{2R2} \begin{bmatrix} 1 & 1/3 & | & 7/3 \\ 0 & | & 13/3 & | & 1/3 \\ 0 & | & -4/3 & | & 2/3 \end{bmatrix} \xrightarrow{R2 \times 3} \xrightarrow{R2} \xrightarrow{$$

$$\begin{bmatrix} 1 & \sqrt{3} & | & 7/3 \\ 0 & 1 & | & \sqrt{13} \\ 0 & -4/3 & | & 2/3 \end{bmatrix} \xrightarrow{R_3 + \frac{4}{3}R_2} \xrightarrow{R_3} \begin{bmatrix} 0 & 1/3 & | & 7/3 \\ 0 & 1 & | & \sqrt{13} \\ 0 & 0 & | & | & \sqrt{13} \\ 0 & 0 & | & | & \sqrt{13} \end{bmatrix}$$

From the third row, we can see that there is a contradiction 10/13 \$ 0 ,. Thus, this system of equations has no solution. This situation is called as inconsistent system.

(b) Let's say "z" is the total absolute deviation: to so anotherpy to marry eith His may soll objective. function: minimize (2) box stetom between s.t. 321 + 22 - 7 52 3x1 +1x2 =7 >7212 -018 1 -1 2x1 + 5x2 -5 52 221 +522 -5 > -2 21 - 22 - 3 22 19 = 19 x - x - 3 > - Z This will be the linear program we can use (c) Let's introduce deviation, variables, el, es, es, for each equation respectively. objective: minimize (e1 + e2 + e3) 813 817 S. E. 3x1 + x2 -7 5e1 3 x1 + 72 - 7 2 - e1 2 ×1 + 5×2 -5 Ke2 EI) 2×1 + 5×2 -5 > -e2 E15 (1)x1 - x2 -3 se3 e de switt duta, = x2 =3 > -e3 or borist sit mora ero: Loups to noteus siei, equates sist ellas anticontras

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(d) Minimize the 2-norm of the violation:

objective: minimize $(e_{1}^{2} + e_{2}^{2} + e_{3}^{2})$ s.t. $3x_{1} + x_{2} - 7 \le e_{1}$ $3x_{1} + x_{2} - 7 \ge e_{1}$ $2x_{1} + 5x_{2} - 5 \le e_{2}$ $2x_{1} + 5x_{2} - 5 \ge e_{2}$ $2x_{1} + 5x_{2} - 5 \ge e_{2}$ $2x_{1} - x_{2} - 3 \le e_{3}$ $x_{1} - x_{2} - 3 \ge -e_{3}$

This type of problem is a quadratic problem because the objective function is quadratic.

(e) To convert this to linear equalities, we introduce auxiliary variables:

objective! minimize $(u_1 + u_2 + u_3)$ s.t. $u_1 \ge e_1^2$, $u_2 \ge e_2^2$. $u_3 \ge e_3^2$ $3x_1 + x_2 - 7 \le e_1$ $3x_1 + x_2 - 7 \ge -e_1$ $2x_1 + 5x_2 - 5 \le e_2$ $2x_1 + 5x_2 - 5 \ge -e_2$ $x_1 - x_2 - 3 \le e_3$ $x_1 - x_2 - 3 \ge -e_3$

This will give us the optimal solution to part (d).