Homework 2

June 24, 2024

1 Homework #2 (4 points)

1.1 Submission requirements

Upload a **single PDF or HTML file** of your IJulia notebook for this entire assigment. Do not submit an .ipynb file. Clearly denote which question each section of your file corresponds to.

1.2 Problem 1 - Blending AvGas

A petroleum corporation that produes fuel for aircraft produces two main types of fuel: Jet A and 100LL. There is a set of raw inputs (different grades of petroleum) P that are used to produce aviation fuel. A subset of these inputs - P_T - must be treated with chemicals in a special production process before they can be blended into fuel. The remaining raw inputs $P \quad P_T$ can be blended directly. Each output (Jet A and 100LL) requires a unique prodution process.

For example, the process to create 20 gallons of 100LL could be 10 gallons of raw input 1 and 10 gallons of raw input 2->blend->20 gallons of 100LL. If a raw input belongs to the set P_T (e.g., raw input 1), the process could be 10 gallons raw input 1->treat raw input 1->blend 10 gallons (treated) raw input 1 and 10 gallons raw input 2->20 gallons 100LL. An equivalent process exists for producing Jet A.

For each raw input $p \in P$, there is a maximum number of gallons available for purchase, u_p. The inputs cost c_p dollars per gallon for each $p \in P$. There is also a cost per gallon associated with each step of the production process: α_{treat}, $\alpha_{jetAprod}$, $\alpha_{100LLprod}$, α_{blend}.

JetA and 100LL have specific parameters they must meet to meet the Federal Aviation Administration standards. There is a set of requirements R such that for each $r \in R$, both Jet A and 100LL must meet a minimum of b_r percentage of the final product. For each raw input $p \in P$, we know that p contains a_{pr} of requirement $r \in R$.

Finally, there is a daily demand for D_1 gallons of Jet A and D_2 gallons of 100LL.

Formulate a linear programming model that minimizes the cost of meeting demand. Be sure to clearly define the decision variables. *Hint: you will need variables representing the amount of each input used to produce each output. Your variables will be indexed over two sets (i.e., have two subscripts).* Implement this general model in Julia and use it to solve the instance with data that is given along with this homework assignment in the file "avgas-data.ipnyb."

1.3 Problem 2 - Brew^2

The brewery from Homework 1 would like to use optimzation for more than just planning how to distribute production across locations. Now they want to determine a production schedule in order to meet projected demand for their most popular beer, LPA (linear programming ale), over the next 6 months.

Month	Demand (kegs)
1	60
2	50
3	80
4	70
5	90
6	50

It costs \$40 to produce one keg of LPA (materials, labor, etc.). Beer has a long shelf life, so kegs of LPA produced in one month can be kept in inventory and used in future months. It costs \$10 per keg per month to store it in inventory. The brewery currently has 15 kegs of LPA in inventory. Since LPA is a seasonal beer, anything left after month 6 will be sold to wholesalers at \$30 per keg.

LPA is produced in stainless steel vats. A vat can produce up to 10 kegs per month. The brewery has the option to either purchase a vat (\$5000 per vat) or lease a vat (\$350 per month). The brewery can even opt to sell used vats if they don't need them anymore. Used vats sell for \$4500 per vat. Owning a vat incurs a \$50 per month maintenance cost (per vat). Assume vats can be purchased, sold, or leased only at the beginning of each month. The brewery currenlty owns 3 vats, and they would also like to own at least 3 vats at the end of month 6.

(a) Formulate a linear program to help the brewery minimize its net cost while meeting demand over the next 6 months. Solve the model in Julia/JuMP and display the production plan (how many kegs to produce each month, how many vats to buy, sell, lease each month). Does your solution make sense?

(b) Now suppose the brewery would like to allow backlogging of demand. Demand not met in a month may be met in a future month (backlogged). The brewery pays a fee of \$19 per keg per month backlogged. Modify your code from part (a) to include the ability to backlog. All demand should be met by the end of month 6. How, if at all, does this change the optimal objective value and solution? Speculate as to the reason for the change (or lack of change).

1.4 Problem 3 - Plane Sorting

This is a real-life problem! Last time Prof. Smith went to the flight school at Wisconsin Aviation (WisAv), Flight School Coordinator Bryan showed her this graph on a whiteboard and said it was stressing him out to figure out how to get all the planes where they needed to go. We can help Bryan with the power of optimization!

There are 7 different maintenance shops WisAv uses for routine 100-hour and Annual inspections: Monroe (EFT), Madison (MSN), Dodge County (UNU), Watertown (RYV), Waukesha (UES), Appleton (ATW), and Waupaca (PCZ).

There is a set of planes currently at MSN and RYV that need to be flown to different maintenance locations. The table below summarizes how many planes are currently at each location and how many planes can be serviced at each maintenance shop.

Location	Planes needing mtx	Mtx capacity
EFT	0	1
MSN	4	2
UNU	0	1
RYV	3	0
EUS	0	1
ATW	0	1
PCZ	0	1

The following table gives the distances (in nautical miles) between each pair of locations. Note that the distances are duplicated for the reverse direction, so those entries are left blank for simplicity. If two airports are farther than 100 nm from each other, that edge cannot exist in the network since the plane would need to refuel (therefore stopping at a different, closer airport on the way). These are denoted by a "x" in the table.

Dist	EFT	MSN	UNU	RYV	UES	ATW	PCZ
EFT	0	33	62	51	65	x	x
MSN	-	0	33	27	49	76	73
UNU	-	-	0	15	31	51	56
RYV	-	-	-	0	23	66	71
UES	-	-	-	-	0	74	85
ATW	-	-	-	-	-	0	22
PCZ	-	-	-	-	-	-	0

Formulate a linear program (as a min-cost network flow problem) to determine where WisAv should send the planes from MSN and RYV to minimize total travel distance. Also draw a network that represents this problem. Solve the model in Julia/JuMP and report the solution (where should planes be sent, and what is the total distance?).

1.5 Problem 4 - Shortest path modeling

Network modeling gives us the power to model many things as optimization problems that might not seem obvious at first, like math puzzles! Start with the number 6. At each step, you can apply one of 3 operations to the current number: * subtract 1 * multiply by 2 * add 3

What is the minimal number of operations (steps) needed to get from 6 to 2024? There are a few different ways to solve this problem. For full credit, build an optimization model structured as a shortest path problem. You may find the following code snippet useful:

[ ]:
```
arcs = []
for i in nodes
    append!(arcs,[(i,i-1)])
end
```