

Homework 6

August 1, 2024

1 Homework #6 (4 points)

Due Friday, August 9, at 11:59pm CT

1.1 Submission requirements

Upload a **single PDF or HTML file** of your IJulia notebook for this entire assignment. Do not submit an .ipynb file. Clearly denote which question each section of your file corresponds to.

1.2 Problem 1 – Machine Scheduling

Consider the following machine scheduling problem (MSP). We have a set $M = \{1, 2, \dots, m\}$ of machines and a set $N = \{1, 2, \dots, n\}$ of jobs that must be performed on the machines. Each machine i has a capacity of b_i units of work, and each job j requires a_{ij} units of work to be completed if it is scheduled on machine i . All jobs must be assigned to exactly one machine.

- (a) Suppose that there is a cost c_{ij} for scheduling job $j \in N$ on machine $i \in M$. There is also fixed cost of operating a machine – that is, in order to schedule any jobs to run on machine $i \in M$, then you must pay a fixed cost of h_i . Write an integer programming model to determine a least cost assignment of jobs to machines not exceeding the machine capacities in this case.
- (b) Modify your answer to (a) to handle the following logical condition:

If you operate at least 3 of the first 10 machines, and you operate at least 3 of the second 10 machines, then you must operate at most 2 of the last 10 machines.

- (c) Now suppose that each job $j \in N$ has a duration (or length) d_j . The *makespan* of a schedule is the length of time it takes to finish all the jobs. You may remove the conditions imposed in (b). Formulate a model that will minimize the makespan of an assignment of jobs to machines not exceeding the machine capacities, and ensuring that you do not operate more than half (15) of the machines. There is no longer a fixed or variable cost.
- (d) For each of the above problems/modifications, we will test different MIP solvers to see the effect on solution time. For a variety of different MIP solvers (at least two; e.g., Gurobi and Cbc or HiGHS) of your choosing, solve (a), (b), and (c).

Report the CPU time and any other observations you make for at least 2 different MIP solvers. What MIP solver did you find most effective? (There isn't a "right answer" here; many factors impact MIP solvers, including how you build your model.)

You can use the code snippet below to generate the instance data for this problem.

```

[25]: using Random
seed = 425 # seed for data generation

N = 1:60 # jobs
M = 1:30 # machines

Random.seed!(seed)

# set time lengths of jobs on each machine
a = zeros(length(M),length(N))
for i in M
    for j in N
        a[i,j] = round(6+Random.rand()*Random.rand(6:25),digits=2)
    end
end

# capacity of each machine
b = Dict{zip(M,[12*sum(a[i,j] for j in N)/length(M) for i in M])}

# cost of running jobs on each machine
c = zeros(length(M),length(N))
for i in M
    for j in N
        c[i,j] = 20+round(Random.rand()*Random.rand(20:60),digits=2)
    end
end

# fixed cost of each machine
h = [30*length(M) for i in M]

# duration of each job (for question (c))
d = [Random.rand(1:10)*Random.rand()+10 for j in N];

```

1.3 Problem 2 – Paint Cleaning

Your friend has decided to start a side hustle as a freelance painter. They are using a set of n different paint batches, which they produce using a single paint mixer. The mixer needs to be cleaned between every pair of batches produced.

The mixer cleaning times depend of the colors and the paint types. For example, a long cleaning period is required if an oil-based paint is produced after a water-based paint, or to produce white paint after a dark color. The times are given in minutes in the following table, where where each entry (i, j) denotes the cleaning time between batch i and batch j . It also takes some amount of time d_i to mix each batch, given in the second table below.

	1	2	3	4	5	6	7	8	9	10
1		11	7	13	11	12	4	9	7	11
2	5		13	15	15	6	8	10	9	8

	1	2	3	4	5	6	7	8	9	10
3	13	15		23	11	11	16	18	5	7
4	9	13	5	3		8	10	12	14	5
5	3	7	7	7		9	10	11	12	13
6	10	6	3	4	14		8	5	11	12
7	4	6	7	3	13	7		10	4	6
8	7	8	9	9	12	11	10		10	9
9	9	14	8	4	9	6	10	8		12
10	11	17	11	6	10	4	7	9	11	

paint	mix time
1	40
2	35
3	45
4	32
5	50
6	42
7	44
8	30
9	33
10	55

Since your friend has other obligations, they want to deal with this weekly production in the shortest possible time (mixing and cleaning). What is the optimal order of paint batches to mix? The order will be applied every week, so the cleaning time between the last batch of one week and the first of the following week needs to be accounted for in the total duration of cleaning.

Solve this problem as a TSP. You may use either MTZ reformulation or subtour elimination (or both, for fun!).

To save you some time, the matrix of times is given in code below.

```
[29]: # matrix representing time between each pair of batches ( i, j)

time_matrix = [0 11 7 13 11 12 4 9 7 11;
               5 0 13 15 15 6 8 10 9 8;
               13 15 0 23 11 11 16 18 5 7;
               9 13 5 0 3 8 10 12 14 5;
               3 7 7 7 0 9 10 11 12 13;
               10 6 3 4 14 0 8 5 11 12;
               4 6 7 3 13 7 0 10 4 6;
               7 8 9 9 12 11 10 0 10 9;
               9 14 8 4 9 6 10 8 0 12 ;
               11 17 11 6 10 4 7 9 11 0]

;
```

[]: