

Homework 1

June 17, 2024

1 Homework #1 (4 points)

1.1 Submission requirements

Upload a **single PDF or HTML file** of your IJulia notebook for this entire assignment. Do not submit an .ipynb file. Clearly denote which question each section of your file corresponds to.

1.2 Problem 1 - Getting Started with Julia/JuMP

Model the following problem in JuMP:

Solve this problem using the HiGHS, ECOS, and SCS solvers.

$$\begin{array}{llll} \max & 4x_1 & +5x_2 & \\ \text{s.t.} & 2x_1 & +3x_2 & \leq 12 \\ & x_1 & +4x_2 & \leq 10 \\ & 3x_1 & +x_2 & \leq 9 \\ & x_1 & & \geq 0 \\ & & x_2 & \geq 0 \end{array}$$

- Which solver is most accurate (gets closest to the “right” answer)?
- Which is fastest (use the @time macro)? NOTE: you should notice it takes longer to solve the problem the first time you run your code, and goes much faster if you re-solve subsequent times. The first time you run the code, the solver takes extra time to compile/“load” the model (it doesn’t repeat the compilation step in future re-solves).
 - Can you speculate as to why?
- If there is no clear difference between the solvers, can you think of some factors that might contribute to solver speed differences?

1.3 Problem 2 - Airport Expansion

The federal government is planning to expand the Dane County Regional Airport to accommodate more flights. It takes 100 hours of labor and 500 pounds of raw material to expand the runway environment by one acre. Each additional acre of land can accommodate 5 new flights per day. It takes 250 hours of labor and 300 pounds of raw material to expand the airport terminal area by

an extra 10 square feet. Each 10 square feet added can accomodate up to 100 new passengers per day. Each flight nets a profit of \\$1000, and each passenger nets a profit of \\$50. The government requires that at least 15 new flights can be added per day. There are 1000 hours of labor and 3000 pounds of raw material available for airport expansion.

- Formulate a linear program (i.e., write a mathematical model using appropriate notation) to help the government plan the airport expansion to maximize profit. State the math model, then code and solve the model using Julia. (Hint: you should have two decision variables). You may type your math model in Markdown or handwrite it, upload it, and embed it as an image in your notebook file.
- Code the same model once again, this time separating the parameters from the solution as we did in class (see Top Brass examples). Confirm that you obtain the same solution as in part (a).
- Solve the problem graphically by plotting the feasible set and at least two isocost lines for the objective function. Confirm that you obtain the same solution as in the previous parts. You may do this by hand or using a graphing calculator (e.g., Desmos), but make sure you label your constraints and isocost line clearly. You may save your solution as an image file and embed it in your notebook file.

1.4 Problem 3 - Craft Beer

A local microbrewery is planning operations for the summer season. They are brewing 25 different rotational beers this season in 5 different locations. During the summer, the brewery has 70,000 total hours of tank fermentation time available at each of the locations. Because of the slight differences in equipment at each brewing location, the cost of producing a keg of beer differs at each location. The time and cost for each brewery are given in the .csv file “brewery_data.csv.” A sample of the data is given in the table below. There is a minimum number of kegs required for each type of beer, also given in the .csv file.

The data is provided in “brewery_data.csv” on Canvas. You can use the code snippet provided below to read the .csv files and load them into Julia, or you can read/load the data in any way you choose.

Beer	Time (hrs)	Min required	Cost/keg	Cost/keg	Cost/keg	...
			(\$) at loc 1	(\$) at loc 2	(\$) at loc 3	
1	20	500	10	12	9	...
⋮	⋮	⋮	⋮	⋮	⋮	⋮

- Formulate a linear program (i.e., write a mathematical model using appropriate notation) to help the microbrewery minimize cost of meeting demand for each type of beer. Give a general form (parameters only – no numbers) of the math model. You may type your math model in Markdown or handwrite it, upload it, and embed it as an image in your notebook file.
- Implement and solve this instance of the model in Julia/JuMP. Display the optimal objective value and the optimal solution (variable values).

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[11]: #You might need to run "Pkg.add(...)" before using these packages
using DataFrames, CSV, NamedArrays

#Load the data file
df = CSV.read("brewery_data.csv",DataFrame,delim=',');

# create a list of beers
beers = convert(Array,df[1:end,1])
# create a list of locations
locs = 1:5
# create a dictionary of the total time it takes to brew each type of beer
time_to_brew = Dict{zip(beers,df[1:end,2])}
# create a dictionary of the minimum kegs required for each beer
min_req = Dict{zip(beers,df[1:end,3])}

brew_cost_matrix = Matrix{df[1:end,4:end]}
# rows are roasting methods, columns are flavors
brew_cost_array = NamedArray(brew_cost_matrix, (beers, locs),("beers","locs"))
;
```

1.5 Problem 4 - Standard Form

Consider the following LP:

$$\begin{aligned}
 &\min 3x_1 + 2x_2 + x_3 + 4x_4 \\
 &\text{s.t. } x_1 + x_2 - x_3 + x_4 \geq 5 \\
 &\quad 2x_1 - 4x_2 + 3x_3 + x_4 = 10 \\
 &\quad -x_1 + 2x_2 + \quad + x_4 \leq 8 \\
 &\quad \quad x_2 + x_4 \geq -3 \\
 &\quad \quad \quad x_1, x_3 \geq 0 \\
 &\quad \quad \quad x_2, x_4 \text{ Unrestricted in Sign (Free)}
 \end{aligned}$$

- Convert the problem to standard form.
- What are A , b , c , and x in this problem? Clearly indicate how the decision variables of your transformed LP relate to those of the original LP.
- Solve the standard-form LP in Julia and report the objective value and the value of each decision variable in an optimal solution to the original LP.

[]: