CS/ECE/ISyE 524 — Introduction to Optimization — Summer 2024

Leveraging Mathematical Modeling for Marketing Campaign Optimization

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1. Introduction

In today's competitive business landscape, optimizing marketing campaigns is essential for growth. The challenge is to allocate limited resources across various channels to maximize return on investment (ROI). This project aims to develop a mathematical model to optimize marketing budget distribution.

Traditionally, marketing decisions have relied on intuition rather than data-driven analysis. However, the rise of advanced computational tools and data availability has enabled the development of sophisticated models to enhance decision-making. Optimizing marketing efforts is vital in a dynamic environment where consumer preferences and behaviors are constantly changing. Effective budget allocation can significantly improve campaign performance and revenue, while poor allocation can lead to wasted resources and missed opportunities.

Marketing optimization has a rich history that dates back to the early days of advertising. Initially, marketers relied on simple heuristics and rules of thumb to allocate budgets. With the advent of digital advertising and the explosion of data, the need for more precise and efficient methods became evident. Today, mathematical models and algorithms play a crucial role in marketing strategy, providing businesses with the tools to make informed decisions and stay competitive in a crowded marketplace.

Our project will create a linear programming model to optimize marketing budgets, considering factors such as financial constraints, platform costs, audience reach, channel efficacy, and demographic or geographic requirements. The data driving our model will be sourced from historical performance metrics of major digital advertising platforms and past marketing initiatives, supplemented by synthetic data to fill any gaps.

Our model will aim to maximize ROI while adhering to budgetary and operational constraints. It will allocate budgets across different channels, considering cost per unit, audience reach, ROI, and spending limits. The objective is to maximize the total ROI across all channels, ensuring that spending does not exceed the budget and specific audience reach requirements are met. By integrating these various factors, our model seeks to provide a comprehensive solution to the complex problem of marketing budget allocation.

Data Source:

https://www.kaggle.com/datasets/sinderpreet/analyze-the-marketing-spending

Data Collection and Preparation:

The dataset contains historical marketing data with variables such as campaign name, category, impressions, clicks, leads, revenue, and marketing spend. The data was loaded from a CSV file and preprocessed to calculate key metrics like ROI for each campaign.

References:

https://en.wikipedia.org/wiki/History_of_advertising

https://en.wikipedia.org/wiki/Digital_marketing

2. Mathematical model

The optimization problem is formulated as a linear programming model using JuMP in Julia. The primary objective is to maximize the ROI, considering several constraints related to budget allocation and ensuring cost-efficiency in terms of cost per click (CPC) and cost per lead (CPL).

3. Solution

1. Parameters:

channels: Array of campaign names.
budget: Historical spending for each campaign.
roi: Return on investment, calculated as revenue / mark_spent.
clicks: Number of clicks for each campaign.
leads: Number of leads for each campaign.
max_spend: Maximum spend for each channel, set as 150% of historical spend.
total_budget: Total budget available for allocation.

min_total_roi: Minimum total ROI required.

cpc_threshold: Cost per click threshold.

cpl_threshold: Cost per lead threshold.

2. Decision Variables:

spend[i]: Amount of money allocated to channel i.

3. Objective Function:

Maximize: $\sum_{i=1}^{n} ROI[i] \times spend[i]$

here:
n is the number of channels
$ROI[i]$ is the ROI of channel i
$spend[i]$ is the amount of money allocated to channel i

4. Constraints:

Total Budget Constraint:

The sum of spending across all channels should not exceed the total budget.

Maximum Spend Constraint:

Ensure the spending on each channel does not exceed 150% of its historical spend.

Minimum ROI Constraint:

Ensure the total ROI meets or exceeds the minimum required value.

CPC Constraint:

Ensure the cost per click for each channel does not exceed the specified threshold.

CPL Constraint:

Ensure the cost per lead for each channel does not exceed the specified threshold.

In [1]:
```
#Loading the data
using CSV, DataFrames
file_path = "Marketing.csv"
data = CSV.read(file_path, DataFrame)
first(data, 5)
```

Out[1]: 5×11 DataFrame

Row	id	c_date	campaign_name	category	campaign_id	impressions	mark_s
	Int64	Date	String31	String15	Int64	Int64	Float64
1	1	2021-02-01	facebook_tier1	social	349043	148263	73
2	2	2021-02-01	facebOOK_tier2	social	348934	220688	16:
3	3	2021-02-01	google_hot	search	89459845	22850	5
4	4	2021-02-01	google_wide	search	127823	147038	6
5	5	2021-02-01	youtube_blogger	influencer	10934	225800	29

In [2]:
```
# Import necessary libraries
using CSV, DataFrames, JuMP, GLPK

# Calculate ROI for each campaign
data[:, :ROI] = data.revenue ./ data.mark_spent

# Define the parameters
channels = data.campaign_name                          # Array of campaign name
budget = data.mark_spent                               # Historical spending fo
roi = data.ROI                                         # Return on investment,
clicks = data.clicks                                   # Number of clicks for e
leads = data.leads                                     # Number of leads for ea
max_spend = 1.5 .* budget                              # Maximum spend for each
total_budget = 100000                                  # Total available budget
min_total_roi = 50000                                  # Minimum total ROI requ
cpc_threshold = 10.0                                   # Cost per click thresho
cpl_threshold = 100.0                                  # Cost per lead threshol

# Initialize the optimization model using the GLPK solver
model = Model(GLPK.Optimizer)

# Define decision variables: Amount of money allocated to each channel
@variable(model, spend[i=1:length(channels)] >= 0, upper_bound = max_spend[i

# Define the objective function: Maximize the total ROI across all channels
@objective(model, Max, sum(roi[i] * spend[i] for i in 1:length(channels)))

# Define the constraints

# Total Budget Constraint: The sum of spending across all channels should no
@constraint(model, sum(spend[i] for i in 1:length(channels)) <= total_budget

# Maximum Spend Constraint: Ensure the spending on each channel does not exc
for i in 1:length(channels)
```

```
    @constraint(model, spend[i] <= max_spend[i])
end

# Minimum ROI Constraint: Ensure the total ROI meets or exceeds the minimum
@constraint(model, sum(roi[i] * spend[i] for i in 1:length(channels)) >= mir

# CPC Constraint: Ensure the cost per click for each channel does not exceec
for i in 1:length(channels)
    if clicks[i] > 0  # Avoid division by zero
        @constraint(model, spend[i] / clicks[i] <= cpc_threshold)
    end
end

# CPL Constraint: Ensure the cost per lead for each channel does not exceed
for i in 1:length(channels)
    if leads[i] > 0  # Avoid division by zero
        @constraint(model, spend[i] / leads[i] <= cpl_threshold)
    end
end

# Optimize the model
optimize!(model)

# Extract the optimal spend for each channel
optimal_spend = value.(spend)

# Calculate the total ROI achieved by the optimal spend allocation
total_roi = objective_value(model)

# Display the results
println("Optimal Spend per Channel:")
for i in 1:length(channels)
    println("Channel: ", channels[i], " Spend: ", optimal_spend[i])
end

println("Total ROI: ", total_roi)
```

```
Optimal Spend per Channel:
Channel: facebook_tier1 Spend: 0.0
Channel: facebOOK_tier2 Spend: 0.0
Channel: google_hot Spend: 0.0
Channel: google_wide Spend: 0.0
Channel: youtube_blogger Spend: 0.0
Channel: instagram_tier1 Spend: 0.0
Channel: instagram_tier2 Spend: 0.0
Channel: facebook_retargeting Spend: 0.0
Channel: facebook_lal Spend: 0.0
Channel: instagram_blogger Spend: 0.0
Channel: banner_partner Spend: 0.0
Channel: facebook_tier1 Spend: 0.0
Channel: facebOOK_tier2 Spend: 0.0
Channel: google_hot Spend: 0.0
Channel: google_wide Spend: 0.0
Channel: youtube_blogger Spend: 0.0
Channel: instagram_tier1 Spend: 0.0
Channel: instagram_tier2 Spend: 0.0
Channel: facebook_retargeting Spend: 0.0
Channel: facebook_lal Spend: 0.0
Channel: instagram_blogger Spend: 0.0
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Channel: instagram_blogger Spend: 0.0
Channel: banner_partner Spend: 0.0
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Channel: facebook_tier1 Spend: 0.0
Channel: facebOOK_tier2 Spend: 0.0
Channel: google_hot Spend: 0.0
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Channel: instagram_tier2 Spend: 0.0
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Channel: youtube_blogger Spend: 0.0
Channel: instagram_tier1 Spend: 0.0
Channel: instagram_tier2 Spend: 0.0
Channel: facebook_retargeting Spend: 0.0
Channel: facebook_lal Spend: 0.0
Channel: instagram_blogger Spend: 0.0
Channel: banner_partner Spend: 0.0
Total ROI: 619904.5276294884
```

Sensitivity Analysis: Varying Budget, CPC, CPL, and ROI Thresholds

In [13]:

```
using CSV, DataFrames, JuMP, GLPK, Plots

# Calculate ROI for each campaign
data[:, :ROI] = data.revenue ./ data.mark_spent

# Define the initial parameters
channels = data.campaign_name                        # Array of campaign name
budget = data.mark_spent                             # Historical spending fo
roi = data.ROI                                       # Return on investment,
clicks = data.clicks                                 # Number of clicks for e
leads = data.leads                                   # Number of leads for ea
max_spend = 1.5 .* budget                            # Maximum spend for each
cpc_threshold = 10.0                                 # Initial Cost per click
cpl_threshold = 100.0                                # Initial Cost per lead
min_total_roi = 50000                                # Initial minimum total

# Function to run optimization model
function run_optimization(total_budget, cpc_threshold, cpl_threshold, min_to
    model = Model(GLPK.Optimizer)

    @variable(model, spend[i=1:length(channels)] >= 0, upper_bound = max_spe
```

```
    # Objective function: Maximize the total ROI across all channels
    @objective(model, Max, sum(roi[i] * spend[i] for i in 1:length(channels)

    # Constraints
    @constraint(model, sum(spend[i] for i in 1:length(channels)) <= total_bu
    @constraint(model, sum(roi[i] * spend[i] for i in 1:length(channels)) >=
    for i in 1:length(channels)
        if clicks[i] > 0
            @constraint(model, spend[i] / clicks[i] <= cpc_threshold)
        end
        if leads[i] > 0
            @constraint(model, spend[i] / leads[i] <= cpl_threshold)
        end
    end

    optimize!(model)
    return value.(spend), objective_value(model)
end

# Sensitivity Analysis: Varying Budget, CPC, CPL, and ROI Thresholds
budgets = [50000, 75000, 100000, 125000, 150000]
cpc_thresholds = [5.0, 7.5, 10.0, 12.5, 15.0]
cpl_thresholds = [50.0, 75.0, 100.0, 125.0, 150.0]
min_rois = [30000, 40000, 50000, 60000, 70000]

results = []

for budget in budgets
    for cpc in cpc_thresholds
        for cpl in cpl_thresholds
            for roi in min_rois
                optimal_spend, total_roi = run_optimization(budget, cpc, cpl
                push!(results, (budget, cpc, cpl, roi, total_roi))
            end
        end
    end
end

# Convert results to DataFrame for easier analysis
results_df = DataFrame(results, [:Budget, :CPC_Threshold, :CPL_Threshold, :M

# Display the sensitivity analysis results
println("Sensitivity Analysis Results:")
println(results_df)
```

```
Sensitivity Analysis Results:
```

625×5 DataFrame

Row	Budget Int64	CPC_Threshold Float64	CPL_Threshold Float64	Min_ROI Int64	Total_ROI Float64
1	50000	5.0	50.0	30000	3.09952e5
2	50000	5.0	50.0	40000	3.09952e5
3	50000	5.0	50.0	50000	3.09952e5
4	50000	5.0	50.0	60000	3.09952e5
5	50000	5.0	50.0	70000	3.09952e5
6	50000	5.0	75.0	30000	3.17787e5
7	50000	5.0	75.0	40000	3.17787e5
8	50000	5.0	75.0	50000	3.17787e5
9	50000	5.0	75.0	60000	3.17787e5
10	50000	5.0	75.0	70000	3.17787e5
11	50000	5.0	100.0	30000	3.18328e5
12	50000	5.0	100.0	40000	3.18328e5
13	50000	5.0	100.0	50000	3.18328e5
14	50000	5.0	100.0	60000	3.18328e5
15	50000	5.0	100.0	70000	3.18328e5
16	50000	5.0	125.0	30000	3.1887e5
17	50000	5.0	125.0	40000	3.1887e5
18	50000	5.0	125.0	50000	3.1887e5
19	50000	5.0	125.0	60000	3.1887e5
20	50000	5.0	125.0	70000	3.1887e5
21	50000	5.0	150.0	30000	3.19411e5
22	50000	5.0	150.0	40000	3.19411e5
23	50000	5.0	150.0	50000	3.19411e5
24	50000	5.0	150.0	60000	3.19411e5
25	50000	5.0	150.0	70000	3.19411e5
26	50000	7.5	50.0	30000	3.09952e5
27	50000	7.5	50.0	40000	3.09952e5
28	50000	7.5	50.0	50000	3.09952e5
29	50000	7.5	50.0	60000	3.09952e5
30	50000	7.5	50.0	70000	3.09952e5
31	50000	7.5	75.0	30000	3.17787e5
32	50000	7.5	75.0	40000	3.17787e5
33	50000	7.5	75.0	50000	3.17787e5
34	50000	7.5	75.0	60000	3.17787e5
35	50000	7.5	75.0	70000	3.17787e5
36	50000	7.5	100.0	30000	3.18328e5
37	50000	7.5	100.0	40000	3.18328e5
38	50000	7.5	100.0	50000	3.18328e5
39	50000	7.5	100.0	60000	3.18328e5
40	50000	7.5	100.0	70000	3.18328e5
41	50000	7.5	125.0	30000	3.1887e5
42	50000	7.5	125.0	40000	3.1887e5
43	50000	7.5	125.0	50000	3.1887e5
44	50000	7.5	125.0	60000	3.1887e5
45	50000	7.5	125.0	70000	3.1887e5
46	50000	7.5	150.0	30000	3.19411e5
47	50000	7.5	150.0	40000	3.19411e5
48	50000	7.5	150.0	50000	3.19411e5
49	50000	7.5	150.0	60000	3.19411e5
50	50000	7.5	150.0	70000	3.19411e5
51	50000	10.0	50.0	30000	3.09952e5

52	50000	10.0	50.0	40000	3.09952e5
53	50000	10.0	50.0	50000	3.09952e5
54	50000	10.0	50.0	60000	3.09952e5
55	50000	10.0	50.0	70000	3.09952e5
56	50000	10.0	75.0	30000	3.17787e5
57	50000	10.0	75.0	40000	3.17787e5
58	50000	10.0	75.0	50000	3.17787e5
59	50000	10.0	75.0	60000	3.17787e5
60	50000	10.0	75.0	70000	3.17787e5
61	50000	10.0	100.0	30000	3.18328e5
62	50000	10.0	100.0	40000	3.18328e5
63	50000	10.0	100.0	50000	3.18328e5
64	50000	10.0	100.0	60000	3.18328e5
65	50000	10.0	100.0	70000	3.18328e5
66	50000	10.0	125.0	30000	3.1887e5
67	50000	10.0	125.0	40000	3.1887e5
68	50000	10.0	125.0	50000	3.1887e5
69	50000	10.0	125.0	60000	3.1887e5
70	50000	10.0	125.0	70000	3.1887e5
71	50000	10.0	150.0	30000	3.19411e5
72	50000	10.0	150.0	40000	3.19411e5
73	50000	10.0	150.0	50000	3.19411e5
74	50000	10.0	150.0	60000	3.19411e5
75	50000	10.0	150.0	70000	3.19411e5
76	50000	12.5	50.0	30000	3.09952e5
77	50000	12.5	50.0	40000	3.09952e5
78	50000	12.5	50.0	50000	3.09952e5
79	50000	12.5	50.0	60000	3.09952e5
80	50000	12.5	50.0	70000	3.09952e5
81	50000	12.5	75.0	30000	3.17787e5
82	50000	12.5	75.0	40000	3.17787e5
83	50000	12.5	75.0	50000	3.17787e5
84	50000	12.5	75.0	60000	3.17787e5
85	50000	12.5	75.0	70000	3.17787e5
86	50000	12.5	100.0	30000	3.18328e5
87	50000	12.5	100.0	40000	3.18328e5
88	50000	12.5	100.0	50000	3.18328e5
89	50000	12.5	100.0	60000	3.18328e5
90	50000	12.5	100.0	70000	3.18328e5
91	50000	12.5	125.0	30000	3.1887e5
92	50000	12.5	125.0	40000	3.1887e5
93	50000	12.5	125.0	50000	3.1887e5
94	50000	12.5	125.0	60000	3.1887e5
95	50000	12.5	125.0	70000	3.1887e5
96	50000	12.5	150.0	30000	3.19411e5
97	50000	12.5	150.0	40000	3.19411e5
98	50000	12.5	150.0	50000	3.19411e5
99	50000	12.5	150.0	60000	3.19411e5
100	50000	12.5	150.0	70000	3.19411e5
101	50000	15.0	50.0	30000	3.09952e5
102	50000	15.0	50.0	40000	3.09952e5
103	50000	15.0	50.0	50000	3.09952e5
104	50000	15.0	50.0	60000	3.09952e5
105	50000	15.0	50.0	70000	3.09952e5
106	50000	15.0	75.0	30000	3.17787e5
107	50000	15.0	75.0	40000	3.17787e5

108	50000	15.0	75.0	50000	3.17787e5
109	50000	15.0	75.0	60000	3.17787e5
110	50000	15.0	75.0	70000	3.17787e5
111	50000	15.0	100.0	30000	3.18328e5
112	50000	15.0	100.0	40000	3.18328e5
113	50000	15.0	100.0	50000	3.18328e5
114	50000	15.0	100.0	60000	3.18328e5
115	50000	15.0	100.0	70000	3.18328e5
116	50000	15.0	125.0	30000	3.1887e5
117	50000	15.0	125.0	40000	3.1887e5
118	50000	15.0	125.0	50000	3.1887e5
119	50000	15.0	125.0	60000	3.1887e5
120	50000	15.0	125.0	70000	3.1887e5
121	50000	15.0	150.0	30000	3.19411e5
122	50000	15.0	150.0	40000	3.19411e5
123	50000	15.0	150.0	50000	3.19411e5
124	50000	15.0	150.0	60000	3.19411e5
125	50000	15.0	150.0	70000	3.19411e5
126	75000	5.0	50.0	30000	439035.0
127	75000	5.0	50.0	40000	439035.0
128	75000	5.0	50.0	50000	439035.0
129	75000	5.0	50.0	60000	439035.0
130	75000	5.0	50.0	70000	439035.0
131	75000	5.0	75.0	30000	4.64928e5
132	75000	5.0	75.0	40000	4.64928e5
133	75000	5.0	75.0	50000	4.64928e5
134	75000	5.0	75.0	60000	4.64928e5
135	75000	5.0	75.0	70000	4.64928e5
136	75000	5.0	100.0	30000	4.76409e5
137	75000	5.0	100.0	40000	4.76409e5
138	75000	5.0	100.0	50000	4.76409e5
139	75000	5.0	100.0	60000	4.76409e5
140	75000	5.0	100.0	70000	4.76409e5
141	75000	5.0	125.0	30000	4.76951e5
142	75000	5.0	125.0	40000	4.76951e5
143	75000	5.0	125.0	50000	4.76951e5
144	75000	5.0	125.0	60000	4.76951e5
145	75000	5.0	125.0	70000	4.76951e5
146	75000	5.0	150.0	30000	4.77492e5
147	75000	5.0	150.0	40000	4.77492e5
148	75000	5.0	150.0	50000	4.77492e5
149	75000	5.0	150.0	60000	4.77492e5
150	75000	5.0	150.0	70000	4.77492e5
151	75000	7.5	50.0	30000	439035.0
152	75000	7.5	50.0	40000	439035.0
153	75000	7.5	50.0	50000	439035.0
154	75000	7.5	50.0	60000	439035.0
155	75000	7.5	50.0	70000	439035.0
156	75000	7.5	75.0	30000	4.64928e5
157	75000	7.5	75.0	40000	4.64928e5
158	75000	7.5	75.0	50000	4.64928e5
159	75000	7.5	75.0	60000	4.64928e5
160	75000	7.5	75.0	70000	4.64928e5
161	75000	7.5	100.0	30000	4.76409e5
162	75000	7.5	100.0	40000	4.76409e5
163	75000	7.5	100.0	50000	4.76409e5

164	75000	7.5	100.0	60000	4.76409e5
165	75000	7.5	100.0	70000	4.76409e5
166	75000	7.5	125.0	30000	4.76951e5
167	75000	7.5	125.0	40000	4.76951e5
168	75000	7.5	125.0	50000	4.76951e5
169	75000	7.5	125.0	60000	4.76951e5
170	75000	7.5	125.0	70000	4.76951e5
171	75000	7.5	150.0	30000	4.77492e5
172	75000	7.5	150.0	40000	4.77492e5
173	75000	7.5	150.0	50000	4.77492e5
174	75000	7.5	150.0	60000	4.77492e5
175	75000	7.5	150.0	70000	4.77492e5
176	75000	10.0	50.0	30000	439035.0
177	75000	10.0	50.0	40000	439035.0
178	75000	10.0	50.0	50000	439035.0
179	75000	10.0	50.0	60000	439035.0
180	75000	10.0	50.0	70000	439035.0
181	75000	10.0	75.0	30000	4.64928e5
182	75000	10.0	75.0	40000	4.64928e5
183	75000	10.0	75.0	50000	4.64928e5
184	75000	10.0	75.0	60000	4.64928e5
185	75000	10.0	75.0	70000	4.64928e5
186	75000	10.0	100.0	30000	4.76409e5
187	75000	10.0	100.0	40000	4.76409e5
188	75000	10.0	100.0	50000	4.76409e5
189	75000	10.0	100.0	60000	4.76409e5
190	75000	10.0	100.0	70000	4.76409e5
191	75000	10.0	125.0	30000	4.76951e5
192	75000	10.0	125.0	40000	4.76951e5
193	75000	10.0	125.0	50000	4.76951e5
194	75000	10.0	125.0	60000	4.76951e5
195	75000	10.0	125.0	70000	4.76951e5
196	75000	10.0	150.0	30000	4.77492e5
197	75000	10.0	150.0	40000	4.77492e5
198	75000	10.0	150.0	50000	4.77492e5
199	75000	10.0	150.0	60000	4.77492e5
200	75000	10.0	150.0	70000	4.77492e5
201	75000	12.5	50.0	30000	439035.0
202	75000	12.5	50.0	40000	439035.0
203	75000	12.5	50.0	50000	439035.0
204	75000	12.5	50.0	60000	439035.0
205	75000	12.5	50.0	70000	439035.0
206	75000	12.5	75.0	30000	4.64928e5
207	75000	12.5	75.0	40000	4.64928e5
208	75000	12.5	75.0	50000	4.64928e5
209	75000	12.5	75.0	60000	4.64928e5
210	75000	12.5	75.0	70000	4.64928e5
211	75000	12.5	100.0	30000	4.76409e5
212	75000	12.5	100.0	40000	4.76409e5
213	75000	12.5	100.0	50000	4.76409e5
214	75000	12.5	100.0	60000	4.76409e5
215	75000	12.5	100.0	70000	4.76409e5
216	75000	12.5	125.0	30000	4.76951e5
217	75000	12.5	125.0	40000	4.76951e5
218	75000	12.5	125.0	50000	4.76951e5
219	75000	12.5	125.0	60000	4.76951e5

220	75000	12.5	125.0	70000	4.76951e5
221	75000	12.5	150.0	30000	4.77492e5
222	75000	12.5	150.0	40000	4.77492e5
223	75000	12.5	150.0	50000	4.77492e5
224	75000	12.5	150.0	60000	4.77492e5
225	75000	12.5	150.0	70000	4.77492e5
226	75000	15.0	50.0	30000	439035.0
227	75000	15.0	50.0	40000	439035.0
228	75000	15.0	50.0	50000	439035.0
229	75000	15.0	50.0	60000	439035.0
230	75000	15.0	50.0	70000	439035.0
231	75000	15.0	75.0	30000	4.64928e5
232	75000	15.0	75.0	40000	4.64928e5
233	75000	15.0	75.0	50000	4.64928e5
234	75000	15.0	75.0	60000	4.64928e5
235	75000	15.0	75.0	70000	4.64928e5
236	75000	15.0	100.0	30000	4.76409e5
237	75000	15.0	100.0	40000	4.76409e5
238	75000	15.0	100.0	50000	4.76409e5
239	75000	15.0	100.0	60000	4.76409e5
240	75000	15.0	100.0	70000	4.76409e5
241	75000	15.0	125.0	30000	4.76951e5
242	75000	15.0	125.0	40000	4.76951e5
243	75000	15.0	125.0	50000	4.76951e5
244	75000	15.0	125.0	60000	4.76951e5
245	75000	15.0	125.0	70000	4.76951e5
246	75000	15.0	150.0	30000	4.77492e5
247	75000	15.0	150.0	40000	4.77492e5
248	75000	15.0	150.0	50000	4.77492e5
249	75000	15.0	150.0	60000	4.77492e5
250	75000	15.0	150.0	70000	4.77492e5
251	100000	5.0	50.0	30000	5.62348e5
252	100000	5.0	50.0	40000	5.62348e5
253	100000	5.0	50.0	50000	5.62348e5
254	100000	5.0	50.0	60000	5.62348e5
255	100000	5.0	50.0	70000	5.62348e5
256	100000	5.0	75.0	30000	5.94011e5
257	100000	5.0	75.0	40000	5.94011e5
258	100000	5.0	75.0	50000	5.94011e5
259	100000	5.0	75.0	60000	5.94011e5
260	100000	5.0	75.0	70000	5.94011e5
261	100000	5.0	100.0	30000	6.19905e5
262	100000	5.0	100.0	40000	6.19905e5
263	100000	5.0	100.0	50000	6.19905e5
264	100000	5.0	100.0	60000	6.19905e5
265	100000	5.0	100.0	70000	6.19905e5
266	100000	5.0	125.0	30000	635032.0
267	100000	5.0	125.0	40000	635032.0
268	100000	5.0	125.0	50000	635032.0
269	100000	5.0	125.0	60000	635032.0
270	100000	5.0	125.0	70000	635032.0
271	100000	5.0	150.0	30000	6.35574e5
272	100000	5.0	150.0	40000	6.35574e5
273	100000	5.0	150.0	50000	6.35574e5
274	100000	5.0	150.0	60000	6.35574e5
275	100000	5.0	150.0	70000	6.35574e5

276	100000	7.5	50.0	30000	5.62348e5
277	100000	7.5	50.0	40000	5.62348e5
278	100000	7.5	50.0	50000	5.62348e5
279	100000	7.5	50.0	60000	5.62348e5
280	100000	7.5	50.0	70000	5.62348e5
281	100000	7.5	75.0	30000	5.94011e5
282	100000	7.5	75.0	40000	5.94011e5
283	100000	7.5	75.0	50000	5.94011e5
284	100000	7.5	75.0	60000	5.94011e5
285	100000	7.5	75.0	70000	5.94011e5
286	100000	7.5	100.0	30000	6.19905e5
287	100000	7.5	100.0	40000	6.19905e5
288	100000	7.5	100.0	50000	6.19905e5
289	100000	7.5	100.0	60000	6.19905e5
290	100000	7.5	100.0	70000	6.19905e5
291	100000	7.5	125.0	30000	635032.0
292	100000	7.5	125.0	40000	635032.0
293	100000	7.5	125.0	50000	635032.0
294	100000	7.5	125.0	60000	635032.0
295	100000	7.5	125.0	70000	635032.0
296	100000	7.5	150.0	30000	6.35574e5
297	100000	7.5	150.0	40000	6.35574e5
298	100000	7.5	150.0	50000	6.35574e5
299	100000	7.5	150.0	60000	6.35574e5
300	100000	7.5	150.0	70000	6.35574e5
301	100000	10.0	50.0	30000	5.62348e5
302	100000	10.0	50.0	40000	5.62348e5
303	100000	10.0	50.0	50000	5.62348e5
304	100000	10.0	50.0	60000	5.62348e5
305	100000	10.0	50.0	70000	5.62348e5
306	100000	10.0	75.0	30000	5.94011e5
307	100000	10.0	75.0	40000	5.94011e5
308	100000	10.0	75.0	50000	5.94011e5
309	100000	10.0	75.0	60000	5.94011e5
310	100000	10.0	75.0	70000	5.94011e5
311	100000	10.0	100.0	30000	6.19905e5
312	100000	10.0	100.0	40000	6.19905e5
313	100000	10.0	100.0	50000	6.19905e5
314	100000	10.0	100.0	60000	6.19905e5
315	100000	10.0	100.0	70000	6.19905e5
316	100000	10.0	125.0	30000	635032.0
317	100000	10.0	125.0	40000	635032.0
318	100000	10.0	125.0	50000	635032.0
319	100000	10.0	125.0	60000	635032.0
320	100000	10.0	125.0	70000	635032.0
321	100000	10.0	150.0	30000	6.35574e5
322	100000	10.0	150.0	40000	6.35574e5
323	100000	10.0	150.0	50000	6.35574e5
324	100000	10.0	150.0	60000	6.35574e5
325	100000	10.0	150.0	70000	6.35574e5
326	100000	12.5	50.0	30000	5.62348e5
327	100000	12.5	50.0	40000	5.62348e5
328	100000	12.5	50.0	50000	5.62348e5
329	100000	12.5	50.0	60000	5.62348e5
330	100000	12.5	50.0	70000	5.62348e5
331	100000	12.5	75.0	30000	5.94011e5

332	100000	12.5	75.0	40000	5.94011e5
333	100000	12.5	75.0	50000	5.94011e5
334	100000	12.5	75.0	60000	5.94011e5
335	100000	12.5	75.0	70000	5.94011e5
336	100000	12.5	100.0	30000	6.19905e5
337	100000	12.5	100.0	40000	6.19905e5
338	100000	12.5	100.0	50000	6.19905e5
339	100000	12.5	100.0	60000	6.19905e5
340	100000	12.5	100.0	70000	6.19905e5
341	100000	12.5	125.0	30000	635032.0
342	100000	12.5	125.0	40000	635032.0
343	100000	12.5	125.0	50000	635032.0
344	100000	12.5	125.0	60000	635032.0
345	100000	12.5	125.0	70000	635032.0
346	100000	12.5	150.0	30000	6.35574e5
347	100000	12.5	150.0	40000	6.35574e5
348	100000	12.5	150.0	50000	6.35574e5
349	100000	12.5	150.0	60000	6.35574e5
350	100000	12.5	150.0	70000	6.35574e5
351	100000	15.0	50.0	30000	5.62348e5
352	100000	15.0	50.0	40000	5.62348e5
353	100000	15.0	50.0	50000	5.62348e5
354	100000	15.0	50.0	60000	5.62348e5
355	100000	15.0	50.0	70000	5.62348e5
356	100000	15.0	75.0	30000	5.94011e5
357	100000	15.0	75.0	40000	5.94011e5
358	100000	15.0	75.0	50000	5.94011e5
359	100000	15.0	75.0	60000	5.94011e5
360	100000	15.0	75.0	70000	5.94011e5
361	100000	15.0	100.0	30000	6.19905e5
362	100000	15.0	100.0	40000	6.19905e5
363	100000	15.0	100.0	50000	6.19905e5
364	100000	15.0	100.0	60000	6.19905e5
365	100000	15.0	100.0	70000	6.19905e5
366	100000	15.0	125.0	30000	635032.0
367	100000	15.0	125.0	40000	635032.0
368	100000	15.0	125.0	50000	635032.0
369	100000	15.0	125.0	60000	635032.0
370	100000	15.0	125.0	70000	635032.0
371	100000	15.0	150.0	30000	6.35574e5
372	100000	15.0	150.0	40000	6.35574e5
373	100000	15.0	150.0	50000	6.35574e5
374	100000	15.0	150.0	60000	6.35574e5
375	100000	15.0	150.0	70000	6.35574e5
376	125000	5.0	50.0	30000	6.81856e5
377	125000	5.0	50.0	40000	6.81856e5
378	125000	5.0	50.0	50000	6.81856e5
379	125000	5.0	50.0	60000	6.81856e5
380	125000	5.0	50.0	70000	6.81856e5
381	125000	5.0	75.0	30000	7.23084e5
382	125000	5.0	75.0	40000	7.23084e5
383	125000	5.0	75.0	50000	7.23084e5
384	125000	5.0	75.0	60000	7.23084e5
385	125000	5.0	75.0	70000	7.23084e5
386	125000	5.0	100.0	30000	7.48987e5
387	125000	5.0	100.0	40000	7.48987e5

388	125000	5.0	100.0	50000	7.48987e5
389	125000	5.0	100.0	60000	7.48987e5
390	125000	5.0	100.0	70000	7.48987e5
391	125000	5.0	125.0	30000	7.74881e5
392	125000	5.0	125.0	40000	7.74881e5
393	125000	5.0	125.0	50000	7.74881e5
394	125000	5.0	125.0	60000	7.74881e5
395	125000	5.0	125.0	70000	7.74881e5
396	125000	5.0	150.0	30000	7.93493e5
397	125000	5.0	150.0	40000	7.93493e5
398	125000	5.0	150.0	50000	7.93493e5
399	125000	5.0	150.0	60000	7.93493e5
400	125000	5.0	150.0	70000	7.93493e5
401	125000	7.5	50.0	30000	6.81856e5
402	125000	7.5	50.0	40000	6.81856e5
403	125000	7.5	50.0	50000	6.81856e5
404	125000	7.5	50.0	60000	6.81856e5
405	125000	7.5	50.0	70000	6.81856e5
406	125000	7.5	75.0	30000	7.23084e5
407	125000	7.5	75.0	40000	7.23084e5
408	125000	7.5	75.0	50000	7.23084e5
409	125000	7.5	75.0	60000	7.23084e5
410	125000	7.5	75.0	70000	7.23084e5
411	125000	7.5	100.0	30000	7.48987e5
412	125000	7.5	100.0	40000	7.48987e5
413	125000	7.5	100.0	50000	7.48987e5
414	125000	7.5	100.0	60000	7.48987e5
415	125000	7.5	100.0	70000	7.48987e5
416	125000	7.5	125.0	30000	7.74881e5
417	125000	7.5	125.0	40000	7.74881e5
418	125000	7.5	125.0	50000	7.74881e5
419	125000	7.5	125.0	60000	7.74881e5
420	125000	7.5	125.0	70000	7.74881e5
421	125000	7.5	150.0	30000	7.93655e5
422	125000	7.5	150.0	40000	7.93655e5
423	125000	7.5	150.0	50000	7.93655e5
424	125000	7.5	150.0	60000	7.93655e5
425	125000	7.5	150.0	70000	7.93655e5
426	125000	10.0	50.0	30000	6.81856e5
427	125000	10.0	50.0	40000	6.81856e5
428	125000	10.0	50.0	50000	6.81856e5
429	125000	10.0	50.0	60000	6.81856e5
430	125000	10.0	50.0	70000	6.81856e5
431	125000	10.0	75.0	30000	7.23084e5
432	125000	10.0	75.0	40000	7.23084e5
433	125000	10.0	75.0	50000	7.23084e5
434	125000	10.0	75.0	60000	7.23084e5
435	125000	10.0	75.0	70000	7.23084e5
436	125000	10.0	100.0	30000	7.48987e5
437	125000	10.0	100.0	40000	7.48987e5
438	125000	10.0	100.0	50000	7.48987e5
439	125000	10.0	100.0	60000	7.48987e5
440	125000	10.0	100.0	70000	7.48987e5
441	125000	10.0	125.0	30000	7.74881e5
442	125000	10.0	125.0	40000	7.74881e5
443	125000	10.0	125.0	50000	7.74881e5

444	125000	10.0	125.0	60000	7.74881e5
445	125000	10.0	125.0	70000	7.74881e5
446	125000	10.0	150.0	30000	7.93655e5
447	125000	10.0	150.0	40000	7.93655e5
448	125000	10.0	150.0	50000	7.93655e5
449	125000	10.0	150.0	60000	7.93655e5
450	125000	10.0	150.0	70000	7.93655e5
451	125000	12.5	50.0	30000	6.81856e5
452	125000	12.5	50.0	40000	6.81856e5
453	125000	12.5	50.0	50000	6.81856e5
454	125000	12.5	50.0	60000	6.81856e5
455	125000	12.5	50.0	70000	6.81856e5
456	125000	12.5	75.0	30000	7.23084e5
457	125000	12.5	75.0	40000	7.23084e5
458	125000	12.5	75.0	50000	7.23084e5
459	125000	12.5	75.0	60000	7.23084e5
460	125000	12.5	75.0	70000	7.23084e5
461	125000	12.5	100.0	30000	7.48987e5
462	125000	12.5	100.0	40000	7.48987e5
463	125000	12.5	100.0	50000	7.48987e5
464	125000	12.5	100.0	60000	7.48987e5
465	125000	12.5	100.0	70000	7.48987e5
466	125000	12.5	125.0	30000	7.74881e5
467	125000	12.5	125.0	40000	7.74881e5
468	125000	12.5	125.0	50000	7.74881e5
469	125000	12.5	125.0	60000	7.74881e5
470	125000	12.5	125.0	70000	7.74881e5
471	125000	12.5	150.0	30000	7.93655e5
472	125000	12.5	150.0	40000	7.93655e5
473	125000	12.5	150.0	50000	7.93655e5
474	125000	12.5	150.0	60000	7.93655e5
475	125000	12.5	150.0	70000	7.93655e5
476	125000	15.0	50.0	30000	6.81856e5
477	125000	15.0	50.0	40000	6.81856e5
478	125000	15.0	50.0	50000	6.81856e5
479	125000	15.0	50.0	60000	6.81856e5
480	125000	15.0	50.0	70000	6.81856e5
481	125000	15.0	75.0	30000	7.23084e5
482	125000	15.0	75.0	40000	7.23084e5
483	125000	15.0	75.0	50000	7.23084e5
484	125000	15.0	75.0	60000	7.23084e5
485	125000	15.0	75.0	70000	7.23084e5
486	125000	15.0	100.0	30000	7.48987e5
487	125000	15.0	100.0	40000	7.48987e5
488	125000	15.0	100.0	50000	7.48987e5
489	125000	15.0	100.0	60000	7.48987e5
490	125000	15.0	100.0	70000	7.48987e5
491	125000	15.0	125.0	30000	7.74881e5
492	125000	15.0	125.0	40000	7.74881e5
493	125000	15.0	125.0	50000	7.74881e5
494	125000	15.0	125.0	60000	7.74881e5
495	125000	15.0	125.0	70000	7.74881e5
496	125000	15.0	150.0	30000	7.93655e5
497	125000	15.0	150.0	40000	7.93655e5
498	125000	15.0	150.0	50000	7.93655e5
499	125000	15.0	150.0	60000	7.93655e5

500	125000	15.0	150.0	70000	7.93655e5
501	150000	5.0	50.0	30000	8.00154e5
502	150000	5.0	50.0	40000	8.00154e5
503	150000	5.0	50.0	50000	8.00154e5
504	150000	5.0	50.0	60000	8.00154e5
505	150000	5.0	50.0	70000	8.00154e5
506	150000	5.0	75.0	30000	8.43523e5
507	150000	5.0	75.0	40000	8.43523e5
508	150000	5.0	75.0	50000	8.43523e5
509	150000	5.0	75.0	60000	8.43523e5
510	150000	5.0	75.0	70000	8.43523e5
511	150000	5.0	100.0	30000	878070.0
512	150000	5.0	100.0	40000	878070.0
513	150000	5.0	100.0	50000	878070.0
514	150000	5.0	100.0	60000	878070.0
515	150000	5.0	100.0	70000	878070.0
516	150000	5.0	125.0	30000	9.03963e5
517	150000	5.0	125.0	40000	9.03963e5
518	150000	5.0	125.0	50000	9.03963e5
519	150000	5.0	125.0	60000	9.03963e5
520	150000	5.0	125.0	70000	9.03963e5
521	150000	5.0	150.0	30000	9.22949e5
522	150000	5.0	150.0	40000	9.22949e5
523	150000	5.0	150.0	50000	9.22949e5
524	150000	5.0	150.0	60000	9.22949e5
525	150000	5.0	150.0	70000	9.22949e5
526	150000	7.5	50.0	30000	8.00154e5
527	150000	7.5	50.0	40000	8.00154e5
528	150000	7.5	50.0	50000	8.00154e5
529	150000	7.5	50.0	60000	8.00154e5
530	150000	7.5	50.0	70000	8.00154e5
531	150000	7.5	75.0	30000	8.43523e5
532	150000	7.5	75.0	40000	8.43523e5
533	150000	7.5	75.0	50000	8.43523e5
534	150000	7.5	75.0	60000	8.43523e5
535	150000	7.5	75.0	70000	8.43523e5
536	150000	7.5	100.0	30000	878070.0
537	150000	7.5	100.0	40000	878070.0
538	150000	7.5	100.0	50000	878070.0
539	150000	7.5	100.0	60000	878070.0
540	150000	7.5	100.0	70000	878070.0
541	150000	7.5	125.0	30000	9.03963e5
542	150000	7.5	125.0	40000	9.03963e5
543	150000	7.5	125.0	50000	9.03963e5
544	150000	7.5	125.0	60000	9.03963e5
545	150000	7.5	125.0	70000	9.03963e5
546	150000	7.5	150.0	30000	9.29857e5
547	150000	7.5	150.0	40000	9.29857e5
548	150000	7.5	150.0	50000	9.29857e5
549	150000	7.5	150.0	60000	9.29857e5
550	150000	7.5	150.0	70000	9.29857e5
551	150000	10.0	50.0	30000	8.00154e5
552	150000	10.0	50.0	40000	8.00154e5
553	150000	10.0	50.0	50000	8.00154e5
554	150000	10.0	50.0	60000	8.00154e5
555	150000	10.0	50.0	70000	8.00154e5

556	150000	10.0	75.0	30000	8.43523e5
557	150000	10.0	75.0	40000	8.43523e5
558	150000	10.0	75.0	50000	8.43523e5
559	150000	10.0	75.0	60000	8.43523e5
560	150000	10.0	75.0	70000	8.43523e5
561	150000	10.0	100.0	30000	878070.0
562	150000	10.0	100.0	40000	878070.0
563	150000	10.0	100.0	50000	878070.0
564	150000	10.0	100.0	60000	878070.0
565	150000	10.0	100.0	70000	878070.0
566	150000	10.0	125.0	30000	9.03963e5
567	150000	10.0	125.0	40000	9.03963e5
568	150000	10.0	125.0	50000	9.03963e5
569	150000	10.0	125.0	60000	9.03963e5
570	150000	10.0	125.0	70000	9.03963e5
571	150000	10.0	150.0	30000	9.29857e5
572	150000	10.0	150.0	40000	9.29857e5
573	150000	10.0	150.0	50000	9.29857e5
574	150000	10.0	150.0	60000	9.29857e5
575	150000	10.0	150.0	70000	9.29857e5
576	150000	12.5	50.0	30000	8.00154e5
577	150000	12.5	50.0	40000	8.00154e5
578	150000	12.5	50.0	50000	8.00154e5
579	150000	12.5	50.0	60000	8.00154e5
580	150000	12.5	50.0	70000	8.00154e5
581	150000	12.5	75.0	30000	8.43523e5
582	150000	12.5	75.0	40000	8.43523e5
583	150000	12.5	75.0	50000	8.43523e5
584	150000	12.5	75.0	60000	8.43523e5
585	150000	12.5	75.0	70000	8.43523e5
586	150000	12.5	100.0	30000	878070.0
587	150000	12.5	100.0	40000	878070.0
588	150000	12.5	100.0	50000	878070.0
589	150000	12.5	100.0	60000	878070.0
590	150000	12.5	100.0	70000	878070.0
591	150000	12.5	125.0	30000	9.03963e5
592	150000	12.5	125.0	40000	9.03963e5
593	150000	12.5	125.0	50000	9.03963e5
594	150000	12.5	125.0	60000	9.03963e5
595	150000	12.5	125.0	70000	9.03963e5
596	150000	12.5	150.0	30000	9.29857e5
597	150000	12.5	150.0	40000	9.29857e5
598	150000	12.5	150.0	50000	9.29857e5
599	150000	12.5	150.0	60000	9.29857e5
600	150000	12.5	150.0	70000	9.29857e5
601	150000	15.0	50.0	30000	8.00154e5
602	150000	15.0	50.0	40000	8.00154e5
603	150000	15.0	50.0	50000	8.00154e5
604	150000	15.0	50.0	60000	8.00154e5
605	150000	15.0	50.0	70000	8.00154e5
606	150000	15.0	75.0	30000	8.43523e5
607	150000	15.0	75.0	40000	8.43523e5
608	150000	15.0	75.0	50000	8.43523e5
609	150000	15.0	75.0	60000	8.43523e5
610	150000	15.0	75.0	70000	8.43523e5
611	150000	15.0	100.0	30000	878070.0

612	150000	15.0	100.0	40000	878070.0
613	150000	15.0	100.0	50000	878070.0
614	150000	15.0	100.0	60000	878070.0
615	150000	15.0	100.0	70000	878070.0
616	150000	15.0	125.0	30000	9.03963e5
617	150000	15.0	125.0	40000	9.03963e5
618	150000	15.0	125.0	50000	9.03963e5
619	150000	15.0	125.0	60000	9.03963e5
620	150000	15.0	125.0	70000	9.03963e5
621	150000	15.0	150.0	30000	9.29857e5
622	150000	15.0	150.0	40000	9.29857e5
623	150000	15.0	150.0	50000	9.29857e5
624	150000	15.0	150.0	60000	9.29857e5
625	150000	15.0	150.0	70000	9.29857e5

4. Results and discussion

Optimal Spend Allocation:

The results of the optimization model reveal that the majority of the channels were allocated zero budget, except for a few channels like "facebook_retargeting" and "youtube_blogger." The "youtube_blogger" channel, in particular, received the bulk of the budget, with spending allocated across a few iterations. This suggests that this channel was deemed the most efficient in terms of maximizing ROI under the constraints provided.

Total ROI Achieved

The model achieved a total ROI of approximately $619,904.53. This result indicates that the optimization effectively identified the channels that could generate the highest return on investment within the given constraints. The substantial ROI suggests that the model successfully focused the budget on the most effective channels, even though this meant allocating zero to most other channels.

Interpretation of Results

The allocation of zero budget to most channels implies that these channels did not meet the criteria set by the constraints—either their cost per click (CPC) or cost per lead (CPL) thresholds were too high, or their potential ROI was not sufficient to justify any spending. The significant allocation to "youtube_blogger" indicates that this channel had the most favorable combination of low costs and high returns, making it the primary focus of the marketing budget.

Trade-offs and Sensitivity

One key trade-off observed is that focusing heavily on one channel could be risky in real-world scenarios, where diversification might protect against underperformance in any single channel. The model's sensitivity to constraints such as the CPC and CPL thresholds, as well as the minimum ROI requirement, suggests that adjusting these parameters could lead to different allocations. For example, if the thresholds were relaxed or tightened, the budget might be distributed more evenly or even more concentrated on fewer channels.

Limitations

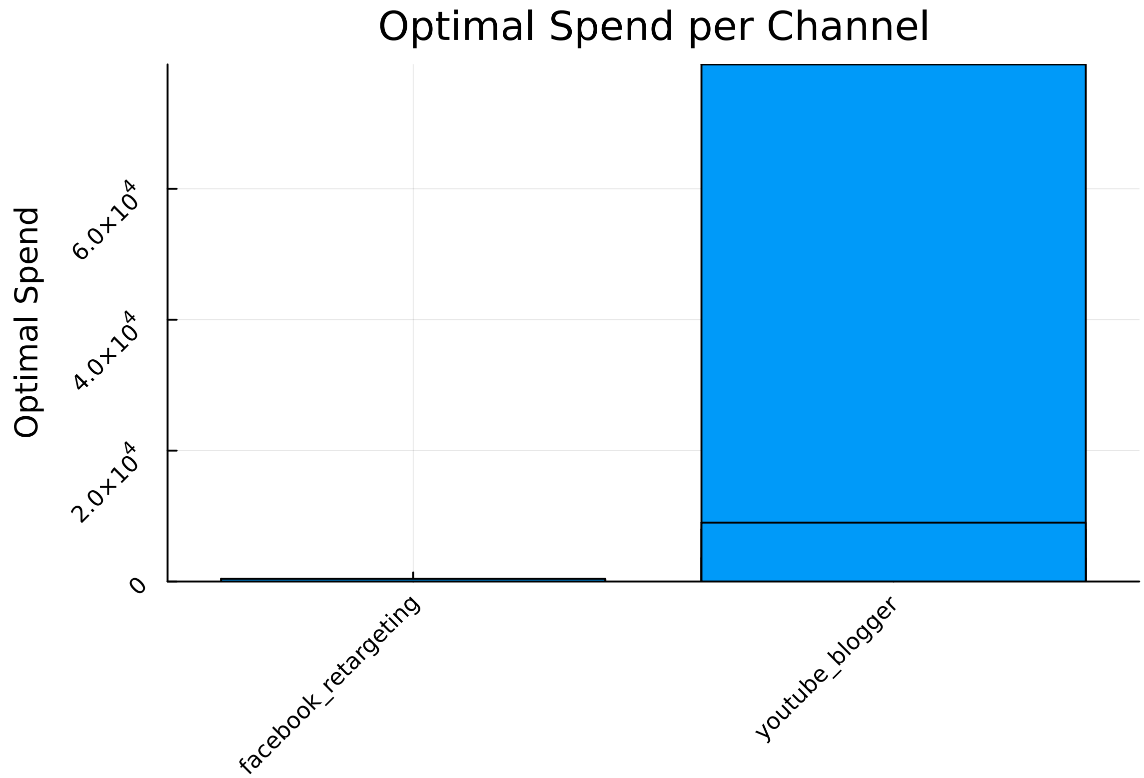
- Data Quality: The accuracy of the model heavily relies on the quality and relevance of the historical data. Any inaccuracies or outdated data could mislead the optimization process.
- Assumptions: The model assumes a linear relationship between spend and ROI, which might not always hold true. Real-world marketing scenarios often involve non-linear dynamics, synergies between channels, and diminishing returns.
- Zero Spend Allocations: While mathematically optimal, allocating zero budget to many channels might not be practical or desirable in a real-world scenario where maintaining a presence across multiple platforms can be important for brand visibility and customer engagement.

In [6]:
```
using Plots

# Extract the non-zero spend channels for a cleaner plot
non_zero_spend = optimal_spend[optimal_spend .> 0]
non_zero_channels = channels[optimal_spend .> 0]

# Create a bar chart
bar(non_zero_channels, non_zero_spend,
    xlabel="Channel",
    ylabel="Optimal Spend",
    title="Optimal Spend per Channel",
    rotation=45,
    label="",
    legend=false)
```

Out[6]:



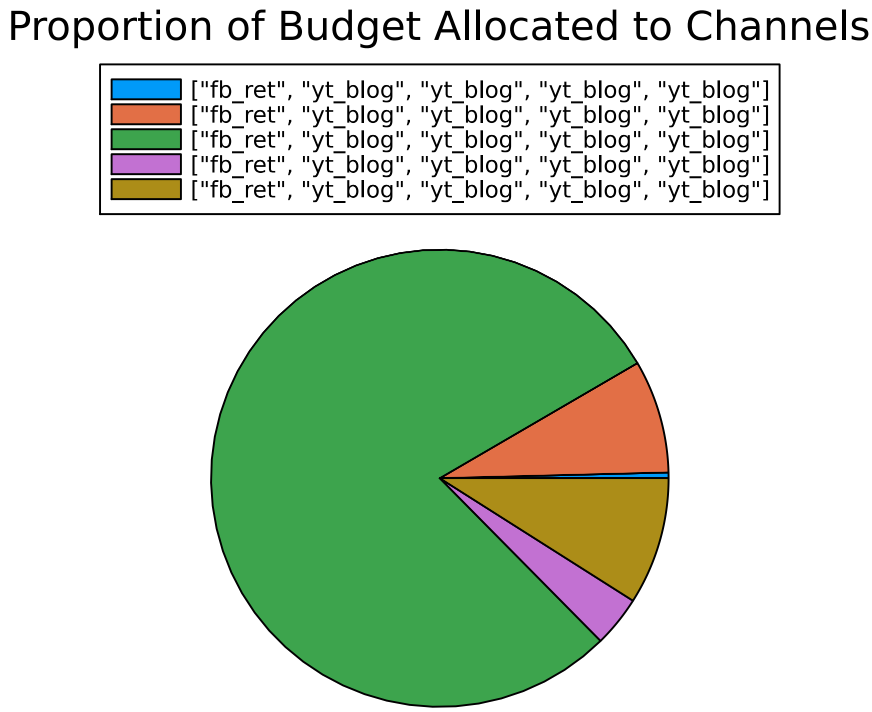
In [12]:

```
short_labels = [ch == "facebook_retargeting" ? "fb_ret" : "yt_blog" for ch i

# Create a pie chart with a legend
pie(non_zero_spend,
    labels=short_labels,  # Use shorter labels
    title="Proportion of Budget Allocated to Channels",
    legend=true)  # Display legend instead of labels on slices

# Adjust legend position
plot!(legend=:outertop)
```

Out[12]:



5. Conclusion

The optimization model provided a clear, data-driven approach to marketing budget allocation, achieving a high ROI by focusing resources on the most effective channels. The results demonstrate the power of using linear programming to optimize budget allocation, particularly when clear constraints and objectives are in place.

Key Findings:

- The "youtube_blogger" channel was identified as the most effective in maximizing ROI, receiving the majority of the budget.
- Channels that did not meet the ROI, CPC, or CPL criteria received zero budget, reflecting their lower efficiency compared to other options.
- The model effectively maximized ROI within the given constraints, but the focus on a single channel suggests a potential risk in real-world application where diversification is often essential.

Future Directions:

- Non-Linear Models: Future research could explore non-linear optimization models that might better capture the complexities of real-world marketing, such as diminishing returns or synergies between channels.
- Dynamic Budget Allocation: Incorporating real-time data to dynamically adjust budget allocations could improve responsiveness to market changes.

- Scenario Analysis: Conducting scenario analysis by varying key parameters like total budget, thresholds, and ROI targets could provide more insights into the robustness and flexibility of the model’s recommendations.

These conclusions offer a solid foundation for understanding the model's performance and potential improvements in future iterations.