Question 1:

$$\max \quad z = x_1 + x_2 + 2x_3$$

s.t. $\quad -x_1 - 2x_2 + 3x_3 = 4$

$x_2 - x_3 \leq 4$

$3x_1 + x_2 \leq 8$

x_1 · Free,

$x_2, x_3 \geq 0$

(a) Dual of this linear program:

$-x_1 - 2x_2 + 3x_3 = 4 \rightarrow y_1$

$x_2 - x_3 \leq 4 \rightarrow y_2$

$3x_1 + x_2 \leq 8 \rightarrow y_3$

dual linear program: $\min \; (4y_1 + 4y_2 + 8y_3)$

s.t. $-y_1 + 3y_3 = 1$

$-2y_1 + y_2 + y_3 \geq 1$

$3y_1 - y_2 \geq 2$

$y_2 \geq 0$

$y_3 \geq 0$

(b) For $-x_1 - 2x_2 + 3x_3 = 4$, y_1 can be any value (equality constraint)

For $x_2 - x_3 \leq 4$, y_2 is active. Thus,

$x_2 = 14$, $x_3 = 10 \Rightarrow x_2 - x_3 = 14 - 10 = 4$

Thus y_2 is positive

For $3x_1 + x_2 \leq 8$, y_3 is not active

$3(-2) + 14 = 8$, $y_3 = 0$.

If we substitute $x_1 = -2$, $x_2 = 14$, $x_3 = 10$
into the equality constraint to solve for y_1, we get:

$-y_1 + 3(0) = 1$

$-y_1 = 1$

$\boxed{y = -1}$ — 1

From the second dual constraint:

$-2(-1) + y_2 \cancel{+0} + 0 \geq 1$

$2 + y_2 \geq 1$

$\boxed{y_2 \geq 1}$ meaning y_2 should be non-negative.

$\boxed{y_2 = 0}$ — 2

$\boxed{y_3 = 0}$ — 3

Thus, the optimal dual solution is: $y_1 = -1$, $y_2 = 0$, $y_3 = 0$

(c) $Z_{new} = 3x_1 + x_2 + 2x_3$

$\cancel{Z_{new} \geq}$

$Z_{new} \geq 4y_1 + 4y_2 + 8y_3$

$y_1 = -1$, $y_2 = 0$, $y_3 = 0$.

$Z_{new} \geq 4(-1) + 4(0) + 8(0)$

$\boxed{Z_{new} \geq -4}$

(d) $(y_1, y_2, y_3) = (0, 2, 1)$

If we substitute this solution into a dual constraint, we get:

$$-y_1 + 3y_3 = 1$$
$$0 + 3(1) = 3 \neq 1$$

This dual solution doesn't satisfy the dual constraint, and thus is not a feasible solution to the dual linear program.