Lecture (8) @ unit I

Bernoulli Distribution (D·RV) — Discrete Random Variable.

⇒ A random variable 'x' is said to be Bernoulli distribution if 'x' takes the values 1 and 0.

i.e. $P(X=1) = P$
$P(X=0) = q = 1-p$ } → $\boxed{P_x(X=x) = p^x(1-p)^{1-x}}$

⇒ In an independent trail of 'n' Bernoulli experiments with 'p' representing the probability of success in each experiment, If 'y' represents the total number of favorable outcomes, then 'y' is said to be Binomial Random Variable.

⇒ Bernoulli trails!
- Finite number of trails (n)
- Trails should be independent
- Probability of success and failure in each trail remains same.
- Every trail will have only two ~~out come~~ outcome.
  - Success
  - Failure.

Binomial Distribution:-

⇒ 'y' is said to be a binomial random variable with parameters 'n' and 'p' if 'y' takes the values 0, 1, 2 --- n with

$$\boxed{P[Y=k] = {}^nC_k\, p^k q^{n-k}} \quad , p+q=1 \quad , k = 0,1,2 \cdots n$$

Mean and variance of Binomial Distribution:

(i) Mean: we know that

$$E[X] = \mu = \sum_{i=0}^{n} x\, P_x(x)$$

$$\Rightarrow E[x] = \mu = \sum_{k=0}^{n} k\, P_x(k)$$

$$\Rightarrow \mu = \sum_{k=0}^{n} k \cdot {}^{n}C_{k} \cdot p^{k} q^{n-k}$$

$$\Rightarrow \mu = \sum_{k=0}^{n} \frac{k \cdot n!}{k!\,(n-k)!} \cdot p^{k} q^{n-k}$$

$$\Rightarrow \mu = \sum_{k=0}^{n} \frac{k \cdot n!}{k\,(k-1)!\,(n-k)!} \cdot p^{k} q^{n-k}$$

$$\cancel{\Rightarrow \mu = \sum_{k=0}^{n} \frac{n(n-1)!}{(k-1)!\,\{(n-1)-(k-1)\}!} \cdot p \cdot p^{k-1} q^{(k-1)}}$$

$$\Rightarrow \mu = \sum_{k=0}^{n} \frac{n(n-1)!}{(k-1)!\,((n-1)-(k-1))!} \cdot p \cdot p^{k-1} q^{(n-1)-(k-1)}$$

$$\Rightarrow \mu = np \sum_{k=0}^{n} \frac{(n-1)!}{(k-1)!\,((n-1)-(k-1))!} \cdot p^{k-1} q^{(n-1)-(k-1)}$$

$$\Rightarrow \boxed{\mu = np}$$

(ii) Variance :-

$$\sigma_n^2 = E[x^2] - \{E[x]\}^2$$

or

$$\sigma_K^2 = E[K^2] - \{E[K]\}^2 \quad \text{— (A)}$$

$\Rightarrow$ since $\boxed{E[K^2] = E[K(K-1)] + E[x]}$ — (1)

R.H.S.

since $E[K(K-1)] + E[K]$

$$= E[K^2] - \cancel{E[K]} + \cancel{E[K]}$$

$$= E[K^2] \quad \text{L.H.S.}$$

$$\Rightarrow E[K(K-1)] = \sum_{K=0}^{n} K(K-1)\, {}^nC_K \, p^K q^{n-K}$$

$$\Rightarrow E[K(K-1)] = \sum_{K=0}^{n} K(K-1) \frac{n!}{K!\,(n-K)!} p^K q^{n-K}$$

$$= \sum_{K=0}^{n} \frac{K(K-1)\, n!}{K(K-1)(K-2)!\,(n-K)!} \cdot p^K q^{n-K}$$

$$= \sum_{K=0}^{n} \frac{n(n-1)(n-2)!}{(K-2)!\,((n-2)-(K-2))!} p^2 p^{K-2} \cdot q^{(n-2)-(n-K)}$$

$$E[K(K-1)] = n(n-1) \cdot p^2$$

From eqn (1)

$$E[K^2] = n(n-1)p^2 + np$$

$$\Rightarrow \quad E[k^2] = n^2p^2 - np^2 + np$$

$$\Rightarrow \quad E[k^2] = n^2p^2 + np(1-p)$$

$$\Rightarrow \quad E[k^2] = n^2p^2 + npq$$

$\Rightarrow$ From eqn. A

$$\sigma_k^2 = n^2p^2 + npq - (np)^2$$

$$\Rightarrow \quad \boxed{\sigma_k^2 = npq}$$