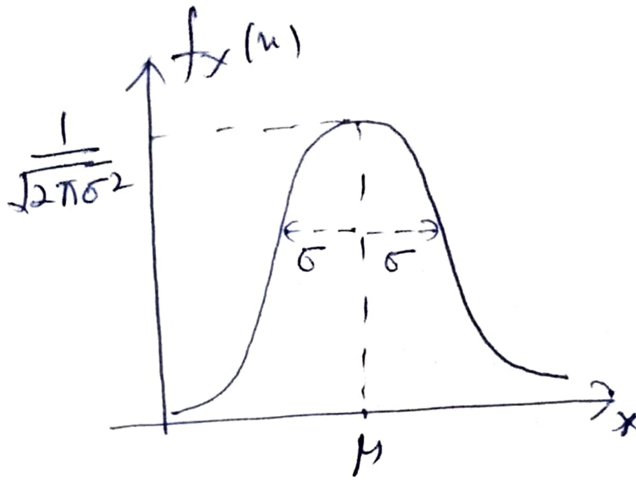
properties of Gaussian | Normal distribution :-

Property 1: The pdf of Normal distribution is symmetrical about its mean (μ)



i.e. $f_X(\mu-\sigma) = f_X(\mu+\sigma)$

Property 2: $P(X \le \mu) = P(X > \mu) = \frac{1}{2}$

because

$$P(X \le \mu) + P(X > \mu) = 1$$

Property 3: peak of Normal distribution is $\frac{1}{\sqrt{2\pi\sigma^2}}$ at $x = \mu$.

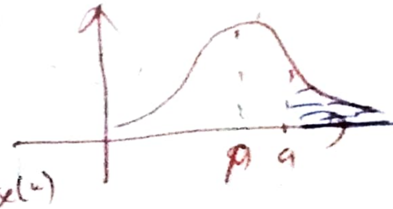
Property 4: $$\left.\frac{d}{dx} f_X(x)\right|_{\text{at } x < \mu} = +ve$$

$$\left.\frac{d f_X(x)}{dx}\right|_{\text{at } x > \mu} = -ve$$

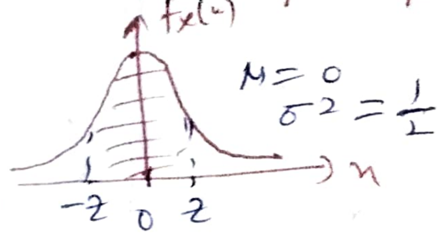
$$\left.\frac{d f_X(x)}{dx}\right|_{\text{at } x = \mu} = 0$$

Q-function, error-function and there relationship

$$P(X>a) = \int_a^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \quad -(1) \longrightarrow$$



$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^{z} e^{-t^2}\, dt \quad -(2) \longrightarrow$$



$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2}\, du \quad -(3)$$

let $t^2 = u^2/2$

$\Rightarrow u = \sqrt{2}\, t$

$du = \sqrt{2}\, dt$

limits! at $u = x$

$x = \sqrt{2}\, t$

$\Rightarrow t = x/\sqrt{2}$

at $u \to \infty$

$t \to \infty$

$$\Rightarrow Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-t^2} \sqrt{2}\, dt$$

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$$\Rightarrow \boxed{Q(x) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]}$$

from equ. (1)

let $z = \frac{x-\mu}{\sigma}$

$\Rightarrow \sigma z = x-\mu \Rightarrow x = \mu + \sigma z$

$\Rightarrow \boxed{dx = \sigma\, dz}$

from eqn. ①

$$P[x>a] = \int_{\frac{a-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-z^2/2}\, \sigma\, dz$$

$$\Rightarrow P[x>a] = \frac{1}{\sqrt{2\pi}} \int_{\frac{a-\mu}{\sigma}}^{\infty} e^{-z^2/2}\, dz$$

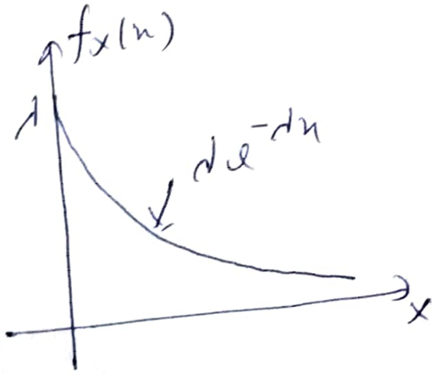
$$\Rightarrow P[x>a] = Q\left[\frac{a-\mu}{\sigma}\right]$$

$$\Rightarrow P[x>a] = \frac{1}{2}\left[1-\operatorname{erf}\left(\frac{a-\mu}{\sqrt{2}\sigma}\right)\right]$$

Exponential Distribution:- A random variable 'X' is called an exponential r.v. with parameter λ ($\lambda>0$), if its pdf is given by:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x>0 \\ 0 & x<0 \end{cases}$$

Corresponding CDF of X is given by



$$F_X(x) = \begin{cases} 1-e^{-\lambda x} & x \geq 0 \\ 0 & x<0 \end{cases}$$

(*) mean and variance of $f_X(x)$:

$$E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$$

$$= \int_0^{\infty} x\lambda e^{-\lambda x}\,dx = \left[-xe^{-\lambda x}\right]_0^{\infty} + \int_0^{\infty} e^{-\lambda x}\,dx$$

$$\Rightarrow \boxed{E[X] = \frac{1}{\lambda}}$$

$$E[X^2] = \int_0^{\infty} x^2 \lambda e^{-\lambda x}\,dx$$

$$\Rightarrow E[X^2] = \left[-x^2 e^{-\lambda x}\right]_0^{\infty} + 2\int_0^{\infty} x e^{-\lambda x}\,dx$$

$$\Rightarrow E[X^2] = \frac{2}{\lambda^2}$$

we know that

$$\sigma_x^2 = E[x^2] - \{E[x]\}^2$$

$$\Rightarrow \sigma_x^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$\Rightarrow \boxed{\sigma_x^2 = \frac{1}{\lambda^2}}$$

$$\Rightarrow \boxed{\sigma_x = \frac{1}{\lambda}}$$