Poisson Distribution:- A Random Variable 'x' is said to be a poisson random variable with parameter λ if 'x' takes the values $0, 1, 2, \ldots \infty$ with pdf.

$$\boxed{\begin{aligned} P_X(X=K) &= \frac{e^{-\lambda}\lambda^K}{K!}, \quad K = 0, 1, 2, \ldots \infty \\ &\text{or} \\ P_X(n) &= \frac{e^{-\lambda}\lambda^n}{n!}, \quad n = 0, 1, 2 \ldots \infty \end{aligned}}$$

or

$$\Rightarrow \quad \boxed{P_K = \frac{e^{-\lambda}\lambda^K}{K!}}$$

$$\Rightarrow \quad P_{K-1} = \frac{e^{-\lambda}\lambda^{K-1}}{(K-1)!}$$

$$\Rightarrow \quad \frac{P_{K-1}}{P_K} = \frac{\frac{e^{-\lambda}\lambda^{K-1}}{(K-1)!}}{\frac{e^{-\lambda}\lambda^K}{K!}} = \frac{K}{\lambda}$$

$$\Rightarrow \quad \boxed{\frac{P_{K-1}}{P_K} = \frac{K}{\lambda}}$$

$\Rightarrow$ Poisson Distribution is closely connected to the ~~Poisson~~ binomial distribution.

⇒ Poisson Distribution gives, the number of occurrences of a rare event in a large no. of trails.

Ex: (i). The number of telephone calls at on exchange over a fixed duration.

(ii) The number of wining tickets among those purched in a large lottery

(iii) Number of printing errors in a book

⇒ For all the above example enterested outcomes is a rare one. The probability distribution of the number of such events is given by poisson distribution.

Mean and varriance of Poisson distribution:

(1) Mean: we know that

$$\mu = E[x] = \sum_{x} x \cdot P_X(x)$$

$$\Rightarrow \mu = E[x] = \sum x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= \sum \frac{x \cdot \lambda^x e^{-\lambda}}{x(x-1)!}$$

$$= \left\{ \sum \frac{\lambda^x}{(x-1)!} \right\} e^{-\lambda}$$

$$\Rightarrow \mu = e^{-d} \sum \frac{d\, d^{n-1}}{(n-1)!}$$

$$= e^{-d} d \sum \frac{d^{n-1}}{(n-1)!}$$

$$= e^{-d} d \cdot e^{d}$$

$$\boxed{\mu = d}$$

(ii) Varriance! since

$$E[n(n-1)] = \sum_{x} n(n-1)\, P_x(n)$$

$$= \sum_{x} \frac{n(n-1)\, d^n e^{-d}}{n!}$$

$$= \sum \frac{\cancel{n(n-1)}\, d^n e^{-d}}{\cancel{n(n-1)}(n-2)!}$$

$$= d^2 e^{-d} \sum \frac{d^{n-2}}{(n-2)!}$$

$$= d^2 e^{-d} \times e^{+d}$$

$$\boxed{E[n(n-1)] = d^2}$$

$$E[x^2] = E[x(x-1)] + E[x]$$

$$= d^2 + d$$

$$= d(\mu + 1)$$

Since

$$\sigma_n^2 = E[x^2] - \{E[x]\}^2$$

$$\Rightarrow \sigma_n^2 = \cancel{d^2} + d - \cancel{d^2}$$

$$\Rightarrow \boxed{\sigma_n^2 = d}$$

$$\Rightarrow \boxed{S.D = \sqrt{\sigma_n^2} = \sqrt{d}}$$

$\Rightarrow$ Characteristic Function: The Characteristic function of a random variable 'x' is given by:

$$\boxed{\begin{gathered} \phi_n(\omega) = E[e^{j\omega n}] \\ \text{or} \\ \phi_n(\omega) = \int_{-\infty}^{\infty} e^{j\omega n} \cdot f_x(n)\, dn \\ \text{where} \quad j = \sqrt{-1} \;\&\; -\infty < \omega < \infty \end{gathered}}$$

$$\Rightarrow \boxed{f_x(n) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_n(\omega)\, e^{-j\omega n}\, d\omega}$$

$\Rightarrow$ Also n^{th} moment of 'x' of $\phi_n(\omega)$ is given by:

$$\boxed{m_n = (-j)^n \left. \frac{d^n \phi_n(\omega)}{d\omega^n} \right|_{\omega=0}}$$

Example! Calculate characteristic function and first moment of exponential density function.

Solution! we know that

$$\phi_x(\omega) = \int_{-\infty}^{\infty}$$