Lecture (10)

Moment Generating Function :- (MGF)

A function which is closely related to the characteristic function is moment generating function. It is represented by $M_X(V)$

i.e. $$\boxed{M_X(V) = E\left[e^{Vx}\right] \qquad -\infty < V < \infty}$$

$$\Rightarrow \boxed{M_X(V) = \int_{-\infty}^{\infty} f_X(x)\, e^{Vx}\, dx}$$

or

⇒. Relation between moment generating function and moments is given by:

$$\boxed{M_n = \left.\frac{d^n M_X(V)}{dV^n}\right|_{V=0}}$$

⇒. Disadvantage of MGF:

$M_X(V)$ exists only if ~~when~~ all moments exists.

<u>Example</u>! Calculate moment generating function and first moment of an exponential distribution function.

⟹ Some important formula of Moments:

(i) Moments about origin.

$$\boxed{m_n = E[x^n] = \int_{-\infty}^{\infty} x^n f_X(x)\,dx}$$

$$\boxed{m_1 = E[x] = \bar{X} = \mu = \int_{-\infty}^{\infty} x f_X(x)\,dx}$$

(ii). Central moments:-

$$\boxed{\mu_n = E[(x-\bar{X})^n] = \int_{-\infty}^{\infty} (x-\bar{X})^n f_X(x)\,dx}$$

for: $n=2$

$$\boxed{\mu_2 = E[(x-\bar{x})^2] = \int_{-\infty}^{\infty} (x-\bar{X})^2 f_X(x)\,dx}$$

The second ~~sent~~ central moments (μ_2) is also known as Variance.

⟹ For D.R.V:

$$m_n = \sum_{i=1}^{N} x_i^n P(x_i)$$

$$\mu_n = \sum_{i=1}^{N} (x_i - \bar{X})^n P(x_i)$$

① Find the mean and variance of a random variable 'x' whose probability density function is given by:

$$f_X(x) = \frac{1}{2b} e^{-|x-m|/b}$$

$b > 0$

$-\infty < m < \infty$

② A submarine attempts to sink an aircraft carrier. It will be successful only if two or more torpedoes hit the carrier. If the submarine fires three torpedoes and probability of a hit is 0.4 for each torpedo, what is the probability that the carrier will be sunk?

③. A Random Variable 'x' is gaussian with $\mu = 0$ and $\sigma_x = 1$

(a). what is the probability that $|x| > 2$

(b). what is the probability that $x > 2$