Design & Analysis of Algorithms.

Tutorial-6.

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Answers

Ans1 A minimum spanning tree is a spanning tree that has all vertices connected together, without any cycles & with the minimum possible total edge count i.e. sum of edge weights is minimum.

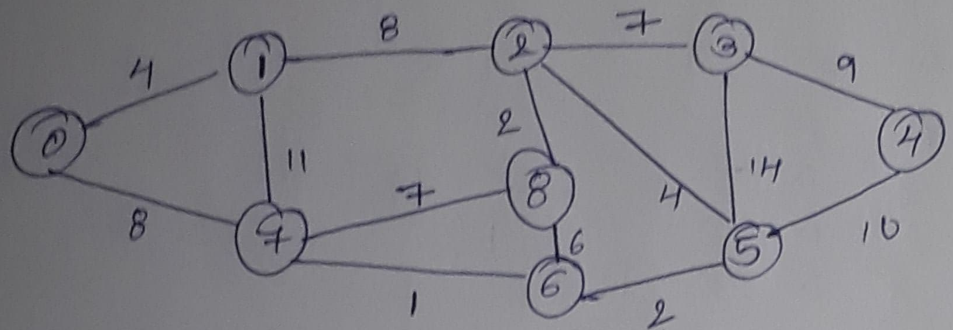
Applications of minimum spanning tree.

- Design of Networks
- Transport System
- Minimum distance problems.

Ans2

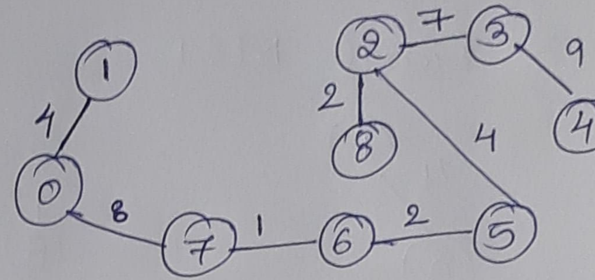
Algorithm	Time Complexity	Space Complexity.
Prim's	$O(V^2)$	$O(1)$
Kruskal's	$O(E\log(V))$	$O(\|E\|+\|V\|)$
Dijkstra's	$O(E\log(V))$	$O(V^2)$
Bellman Ford	$O(\|V\|\|E\|)$	$O(V)$
Prim's (Adjacency)	$O(E\log(V))$	$O(V)$

Ans 3 KRUSKAL.



u	v	w
7	6	1
6	5	2
2	8	2
2	5	4
0	1	4
6	8	6 X
7	8	7 X
2	3	7
0	7	8
1	2	8 X
3	4	9
4	5	10 X
1	7	11 X
3	5	14 X

final MST



Total weight = 37

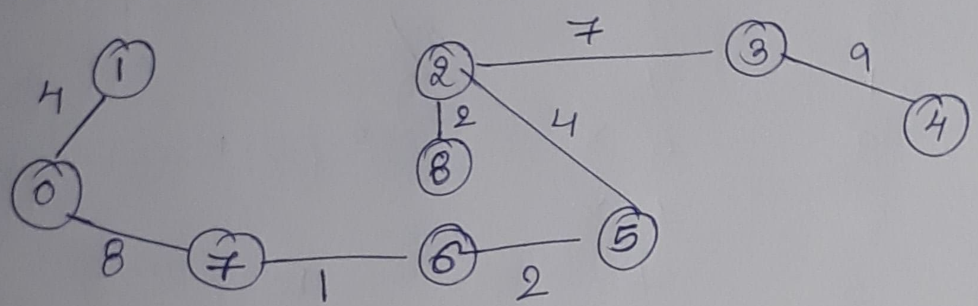
- PRIM'S

	0	1	2	3	4	5	6	7	8
Parent	~~-1~~ 0	~~-1~~ 0	~~-1~~ ~~1~~ 5	-1 ~~5~~ 2	-1 ~~5~~ 3	-1 6	-1 7	-1 0	-1 2

Work Table.

0	1	2	3	4	5	6	7	8
∞	∞	∞	∞	∞	∞	∞	∞	∞
$\boxed{0}$	$\boxed{4}$	∞	∞	∞	∞	∞	8	∞
		8	∞	∞	∞	∞	$\boxed{8}$	∞
		∞	∞	∞	~~∞~~	$\boxed{1}$		~~7~~
		~~8~~	∞	∞	$\boxed{2}$			
		$\boxed{4}$	~~14~~	10				6
			$\boxed{7}$	~~10~~				~~6~~
				$\boxed{9}$				$\boxed{2}$

→ Final MST



Total weight = 37

Ans 4 (i) The shortest path may change. The reason is that there may be different number of edges in different paths from (s) to (t)

(ii) The shortest path doesn't change as it is merely a scaled graph. The number of edges on a path doesn't matter here.

$$D_2 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & \infty & \infty & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ 4 & 1 & \infty & 0 & \infty \\ 7 & 4 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 8 & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & \infty & 1 & 0 & \infty \\ \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array} \quad 4$$

$$D_4 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 5 & 3 & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$D_5 = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$TC = O(|V|^3)$

$SC = O(|V|^2)$