Design and Analysis of Algorithms

Tutorial - 4

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Q1. $T(n) = 3T(n/2) + n^2$.

$a = 3$, $b = 2$, $f(n) = n^2$.

$c = \log_2 3 = 1.585$

$n^{1.585} < n^2 \longleftrightarrow n^c < f(n)$

$\therefore T(n) = \theta(f(n)) = \theta(n^2)$ [Case 3]

Q2. $T(n) = 4T(n/2) + n^2$

$a = 4$, $b = 2$, $f(n) = n^2$.

$c = \log_2 4 = 2$

$n^2 = n^2 \longleftrightarrow n^c = f(n)$ [Case 2]

$\therefore T(n) = \theta(n^2 \log n)$

Q3. $T(n) = T(n/2) + 2^n$

$a = 1$, $b = 2$, $f(n) = 2^n$

$c = \log_2 1 = 0$

$n^c = 1 < 2^n \longleftrightarrow f(n) > n^c$ [Case 3]

$\therefore T(n) = \theta(2^n)$

Q4. $T(n) = 2^n T(n/2) + n^n$

Master's Theorem can't be applied as $a = 2^n$ is not constant but depends on (n).

Q5. $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4,\ f(n) = n$

$c = \log_4 16 = 2$

$n^2 > n \longleftrightarrow n^2 > f(n)$ [case 1]

$\therefore T(n) = \theta(n^2)$

Q6. $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2,\ f(n) = n\log n$

$c = \log_2 2 = 1$

$n^1 = n \longleftrightarrow f(n) = n^c$ [case 2]

$T(n) = \theta(n\log n)$

Q7. $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^1 > n/\log n \longleftrightarrow n^c > f(n)$ [case 1]

$T(n) = \theta(n)$

Q8. $T(n) = 2T(n/4) + n^{0.51}$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$c = \log_4 2 = 0.5$

$n^{0.5} < n^{0.51} \longleftrightarrow f(n) > n^c$ [case 3]

$T(n) = \theta(n^{0.51})$

Q9. $T(n) = 0.5T(n/2) + 1/n$

$a = 0.5,\ b = 2,\ f(n) = 1/n$

$c = \log_2 0.5 = -1$

$n^{-1} = 1/n \longleftrightarrow f(n) = n^c$ [case 2]

$T(n) = \theta(\frac{1}{n}\log n)$

Q10. $T(n) = 16T(n/4) + n!$

$a = 16,\ b = 4,\ c = n!$

$c = \log_4 16 = 2$

$n! > n^2 \longleftrightarrow f(n) > n^c$ [Case 3]

$\therefore T(n) = \Theta(n!)$

Q11. $T(n) = 4T(n/2) + \log n$

$a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_2 4 = 2$

$n^2 > \log n \longleftrightarrow n^c > f(n)$ [case 1]

$\therefore T(n) = \Theta(n^2)$

Q12. $T(n) = \sqrt{n}\, T(n/2) + \log n$

Master's Theorem is not applicable as $a = \sqrt{n}$ is not a constant but depends on n.

Q13. $T(n) = 3T(n/2) + n$

$a = 3,\ b = 2,\ f(n) = n$

$c = \log_2 3 = 1.585 \longrightarrow n^c = n^{1.585}$

$n^{1.585} > n \longleftrightarrow n^c > f(n)$ [Case 1]

$\therefore T(n) = \Theta(n^{1.585})$

Q14. $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3,\ b = 3,\ f(n) = \text{sqrt}(n)$

$c = \log_3 3 = 1 \longrightarrow n^1$

$\sqrt{n} < n \longleftrightarrow f(n) < n^c$ [case 3]

$T(n) = \Theta(n)$

Q15. $T(n) = 4T(n/2) + cn$

$a = 4,\ b = 2,\ f(n) = cn$

$c = \log_2 4 = 2 \rightarrow n^2.$

$cn < n^2 \longrightarrow Tn = \theta n^2.$

On the assumption that (c) is a constant.

Q16. $T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n.$

$c = \log_4 3 = 0.792 \longrightarrow n^c = n^{0.792}$

$n\log n > n^{0.792}. \longleftrightarrow n\log n > n^{0.792}$ [case 3]

$\therefore T(n) = \theta(n\log n)$

Q17. $T(n) = 3T(n/3) + n/2.$

$a = 3,\ b = 3,\ f(n) = n/2$

$c = \log_3 3 = 1 \longrightarrow n^1$

$n > n/2 \longleftrightarrow n > f(n)$ [case 1].

$T(n) = \theta(n)$

Q18. $T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$c = \log_3 6 = 1.631 \longrightarrow n^c = n^{1.631}$

$n^2\log n > n^{1.631} \longleftrightarrow f(n) > n^c$ [case 3]

$\therefore T(n) = \theta(n^2\log n)$

19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, \ b = 2, \ f(n) = n/\log n$

$c = \log_2 4 = 2 \longrightarrow n^c = n^2.$

$\frac{n}{\log n} < n^2 \longleftrightarrow f(n) < n^c$ [case 1]

$\therefore T(n) = \Theta(n^2)$

Q20. $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, \ b = 8, \ f(n) = n^2 \log n$

$c = \log_8 64 = 2 \longrightarrow n^c = n^2$

$n^2 \log n > n^2 \longleftrightarrow f(n) > n^2.$ [case 3]

$\therefore T(n) = \Theta(n^2 \log n)$

Q21. $T(n) = 7T(n/3) + n^2.$

$a = 7, \ b = 3, \ f(n) = n^2.$

$c = \log_3 7 = 1.771 \longrightarrow n^c = n^{1.771}$

$n^2 > n^{1.771} \longleftrightarrow f(n) > n^c$ [case 3]

$\therefore T(n) = \Theta(n^2)$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \ b = 2, \ f(n) = n(2 - \cos n)$

$c = \log_2 1 = 0, \ n^c = 1$

$n(2 - \cos n) > 1 \longleftrightarrow f(n) > n^c$ [case 3]

$\therefore T(n) = \Theta(n(2 - \cos n))$

$n \leq f(n) \leq 3n \longrightarrow T(n) = \Theta(n)$