DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer   Py.Ass.2   6 September 2017

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1. Write a function that takes as its argument two matrices and returns their commutator and anti-commutator. Ensure that your function is such that on asking for "help" it should return a meaningful description.

2. Create a three dimensional complex array, containing complex numbers, and whose real and imaginary parts of uniformly distributed in $[0, 1)$. Take its conjugate, transpose, inverse, determinant, hermetian adjoint, and 100th power. Verify that the inverse is correct, by multiplying it with the original.

3. Consider the question: given a $n \times n$ real matrix containing independent standard normal random numbers, what is the probability that *all* the eigenvalues are real?

   Write a function that will take as its input the dimensionality of the matrix and the number of realizations used and returns the probability that all the eigenvalues are real.

   Here you will have to test if a complex number is real or not. Assume that if the imaginary part is less than ϵ it is real. What value of ϵ will you naturally use?

   The analytic answer is $2^{-n(n-1)/4}$. Vary the number of realizations for $n = 2$ and $n = 3$ dimensional matrices from 100 to 10^6 and see how close to this answer you get.

4. Consider the "Hamiltonian matrix":

$$H = \sigma_z + h_x\sigma_x + h_y\sigma_y,$$

   where $\sigma_{x,y,z}$ are Pauli matrices (also defined in class). Taking $h_x = h\cos(\theta)$ and $h_y = h\sin(\theta)$ find (of course, using a Python code!) and plot the energy levels (eigenvalues of H) as a function of θ $(-\pi \leq \theta < \pi)$ for $h = 1, 2, 5$.