DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer Assignment 5 8 September 2017

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1. **Forced damped oscillations and resonance**

The equation of motion for a forced oscillator is

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t,$$

where β is the damping coefficient, ω_0 is the natural frequency of the oscillator and ω is the frequency of the driving force. Correspondingly, $\tau_0 = 2\pi/\omega_0$ is the natural time-period of the oscillator and $\tau = 2\pi/\omega$ is the time-period of the driving force.

(a) Solve (symbolically) this equation of motion for $x(t)$ subject to the initial conditions $x(0) = \dot{x}(0) = 0$, using $\beta = 0.1$ and $\tau = 1$. Plot $x(t)$ for the driving force parameters $A = 1$ and $\tau = 1$.

(b) Use the "Manipulate" command to study the motion for different values of the damping parameter β. What is the range of values of β for which you observe no oscillations?

(c) Fixing the value of $\beta = 0.1$, use the "Manipulate" command to study $x(t)$ of the forced oscillator for different value of the driving period τ. What is the value of τ at which you observe resonance?

(d) Use "Manipulate" and "Module" or "Block" to solve numerically and plot the phase-space $\{x(t), \dot{x}(t)\}$, with a "locator" and slider for β, ω_0, ω and A. Observe what happens across resonance and also what happens when ω_0/ω is a rational number (try 1/2, 2, 1/3, 3 for example).

2. **"Humped" potential well**

An object of mass m moves in one dimension according to Newton's second law $F = m\ddot{x}$, with a force $F(x) = ax^2 - bx$, where a and b are positive constants.

(a) Numerically solve for $x(t)$ with initial conditions $x(0) = x_0$, $\dot{x} = 0$, and plot the results for $0 \leq t \leq t_{\max}$. Choose $t_{\max}$ large enough so that you can observe the $t \to \infty$ behavior. Choose three values for x_0 which satisfy each of these constraints:

i. $|x_0| \ll b/2a$

ii. x_0 close to $-b/2a$, but a little larger.

iii. x_0 close to $b/2a$, but a little smaller.

You may pick any appropriate numerical values for a, b and m.

(b) What is special about the value $x_0 = b/2a$? To understand this, solve for the potential energy $U(x) = -\int_0^x F(u)du$.

3. **"Double-well" potential**

Consider the a particle of unit mass in the one-dimensional potential $V(x) = -ax^2/2 + bx^4/4$, with $b > 0$.

(a) Use Manipulate and plot the potential, visualizing it as a and b vary. In particular pay attention to what happens when a crosses 0.

(b) In the case when $a > 0$ there are two potential minima and the origin is unstable. In this case write a code to visualize various phase-space orbit, paying attention to two types of orbits, those that are stuck to either one of the wells or those that traverse both. What is the condition on energy that separates these?

(c) Now add dissipation with a term that is proportional to the velocity. Orbits that were traversing both wells now dissipate to either one of the wells. From your code vary the initial conditions to see which of them go to which well?

(d) Add forcing as in Problem (1) above and visualize the dissipating orbits.