DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer Assignment 5 15 Sep 2017

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1. Consider the matrix representing the y component of spin 1 angular momentum operator

$$J_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

1. Show that J_y is Hermitian.
2. Find the eigenvalues and *normalized* eigenvectors of J_y.
3. Show that $U^\dagger U = I$, where U is the matrix formed by taking above vectors as it's columns.
4. Show that the matrix $U^\dagger J_y U$ is diagonal. Identify the diagonal elements.

2. Check out the command `PauliMatrix[k]`, and using it, establish the following results for the Pauli matrices $\sigma^{(k)}$:

1. $\text{trace}(\sigma^{(k)}\sigma^{(m)}) = 2\delta_{km}$
2. $\sum_{k=1}^{3} \sigma_{ab}^{(k)}\sigma_{ij}^{(k)} = 2\delta_{aj}\delta_{bi} - \delta_{ab}\delta_{ij}$

3. Establish the following identity:

$$U = \exp\left(\frac{i\theta\boldsymbol{\sigma}\cdot\boldsymbol{n}}{2}\right) = I_{2\times 2}\cos(\theta/2) + i(\boldsymbol{\sigma}\cdot\boldsymbol{n})\sin(\theta/2)$$

where $\boldsymbol{\sigma}$ represents a "vector" whose components are the three Pauli matrices; i.e., $\boldsymbol{\sigma} \equiv [\sigma_x, \sigma_y, \sigma_z]$. You might want to remember to following points:

1. $\exp(M)$ for a matrix M is defined via the series: $\exp(M) = I + \sum_{k=1}^{\infty} M^k/k!$, where M^k is M multiplied k times with itself.
2. The vector $\boldsymbol{n}$ above is by definition a unit vector in 3 dimensions; i.e., $\boldsymbol{n}\cdot\boldsymbol{n} = 1$. Make sure you take this into account properly.

Comment: The above identity provides an alternate way of describing rotations (by the angle θ about the axis $\boldsymbol{n}$) in 3 dimensions, the usual description being in terms of the 3×3 rotation matrices $R_{\boldsymbol{n}}(\theta)$.