DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH2140 Mathematics on the Computer Py.Ass.1 28 September 2017

1. Narayana's cows (for example in the Online Encyclopedia of Integer Sequences: https://oeis.org/A000930) is defined by the recursion

$$x_{n+1} = x_n + x_{n-2}$$

Write a function (module) using "def" and arguments containing the first 3 elements $x(0), x(1)$ and $x(2)$ and number of terms n such that the final entry is $x(n-1)$. The function should return the List $[x(0), \cdots, x(n-1)]$. Produce the first 100 elements of the sequence if the initial 3 were $(1, 1, 1)$. Your program should utilze tuples and lists and be modeled after the Fibonacci example.

2. Write a function with arguments $x(0)$ and number of iterates N, that will iterate the following map:

$$x_{n+1} = 2x_n \,(\text{modulo } 1).$$

Run for several choices of $x(0)$ and N, especially when $x(0)$'s binary representation is terminating or not and especially if it is irrational. Show that for any choice of $x(0)$ there is a time M beyond which $x(n > M) = 0$. Does this reflect the actual situation? What is the maximum value of M, and why?

3. Write at least two functions that will return $\pi(n)$, the number of primes $< n$ and compute it for $n = 10, 10^2, 10^3, 10^4, 10^5$. Also time the programs.