

Homework#2 - Dice experiment

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2025-09-11

SUMMARY

In this experiment, we will try to prove the fairness of a die that was heated in an oven for 20 minutes. We want to claim that this die is not fair.

- We first have null hypothesis to make claim that our dice is fair and we try to prove this claim wrong.
 - **Null Hypothesis:** The die is fair. The probability of getting 6 in one roll is $1/6$ ($P(\text{getting } 6) = 1/6$)
- We then have alternative hypothesis which if proven right will contradict our null hypothesis. This will disprove that the dice is fair.
 - **Alternative Hypothesis:** $P(\text{getting } 6) \neq 1/6$
- We have random variable for the experiment.
 - **Random variable:** The number of times we get 6 in our total (n) rolls.
- Here we know that probability of getting 6 if the dice is fair. It is $1/6$. We also want to see true or false for getting an outcome. Given that we can use Bernoulli trial here.

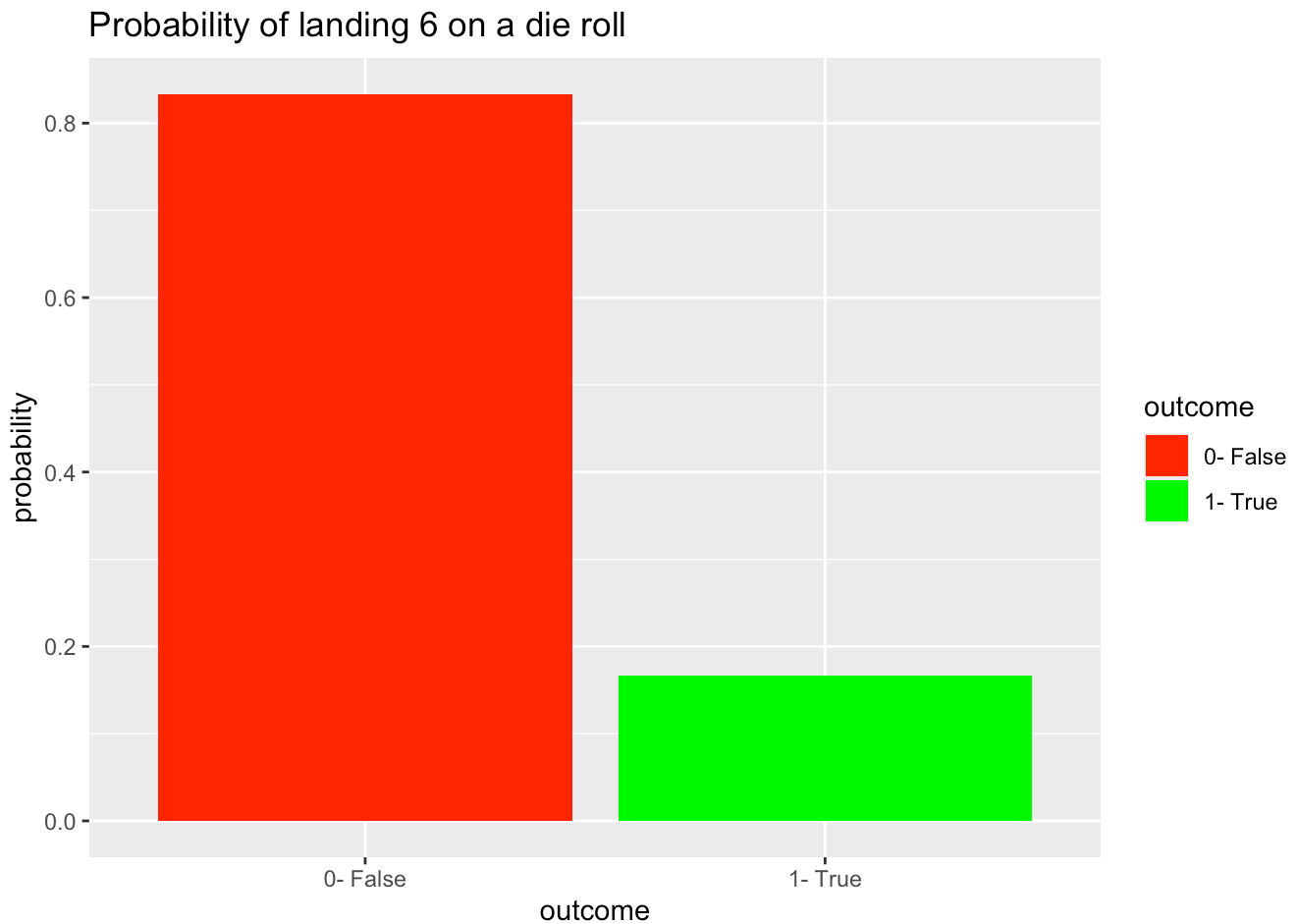
TEST

Below is how our Bernoulli distribution would look like:

```
library(ggplot2)

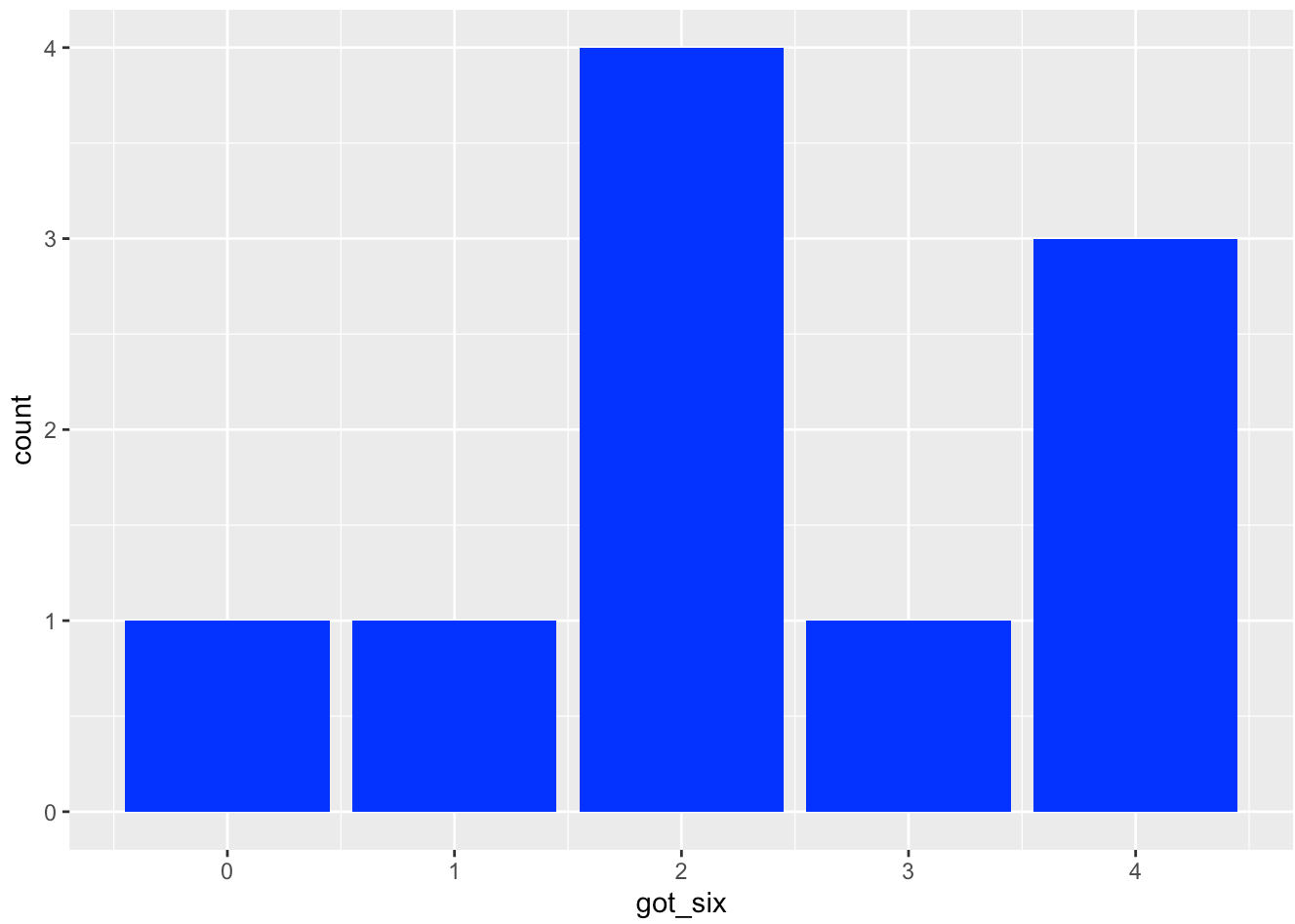
probability_of_rolling_6 <- 1/6
data <- data.frame(
  outcome=c("0- False", "1- True"),
  probability=c(1-probability_of_rolling_6, probability_of_rolling_6)
)

ggplot(data, aes(x=outcome, y=probability, fill=outcome)) +
  geom_bar(stat = "identity") +
  scale_fill_manual(values = c("red", "green")) +
  labs(title="Probability of landing 6 on a die roll")
```

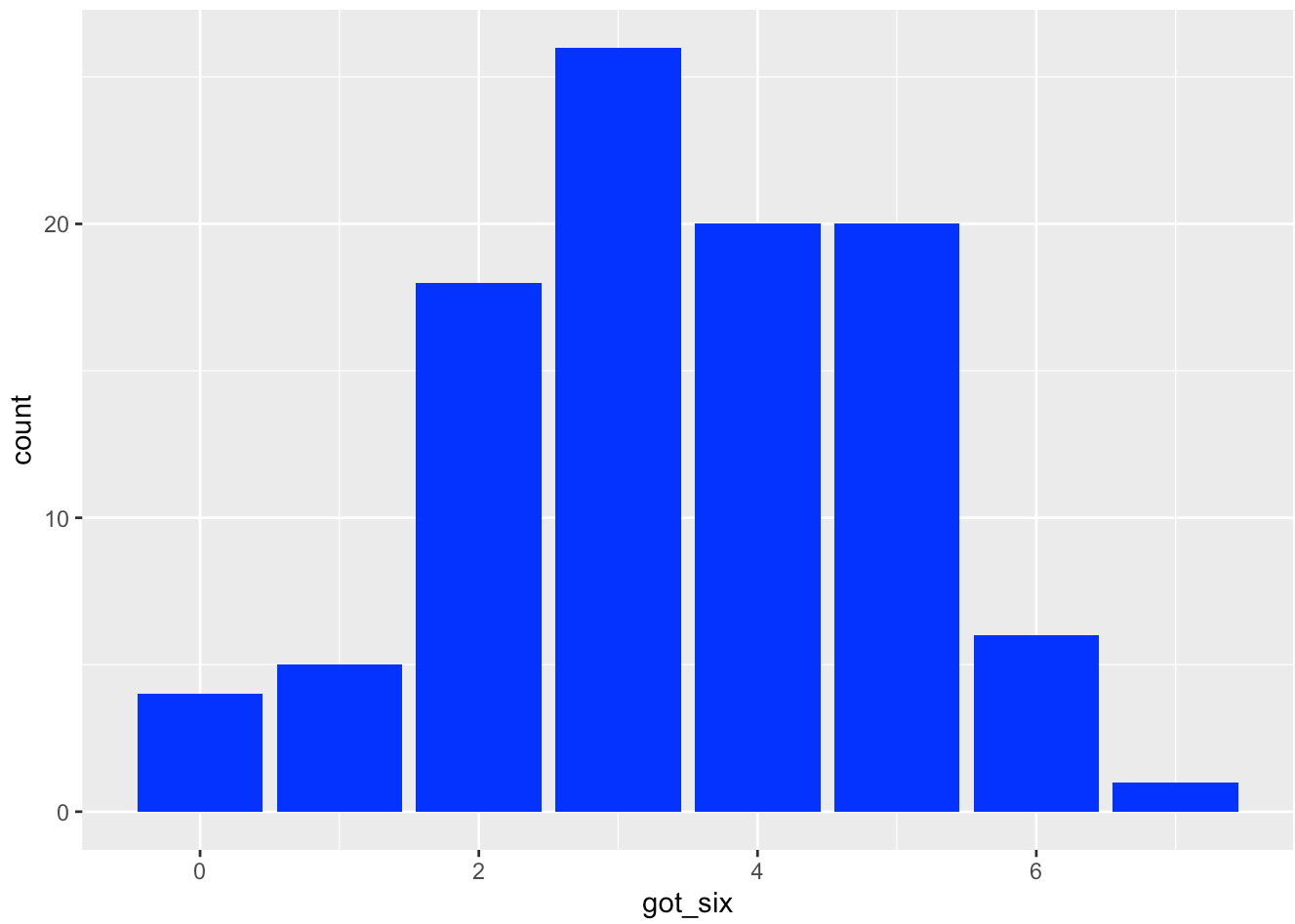


Running multiple experiments with multiple trials in each experiment, we can see how the distribution would look like. Here we will conduct the experiment using a small number of trials and then a large number of trials. This will show that having a larger trial size better represents all data including the tails. **THINK OF INCREASING THE NUMBER OF TRIALS ALLOWS YOU TO SEE COMPLETE RANGE AND INCREASING THE EXPERIMENT GIVES CLEARER PICTURE DUE TO CENTRAL LIMIT THEOREM.**

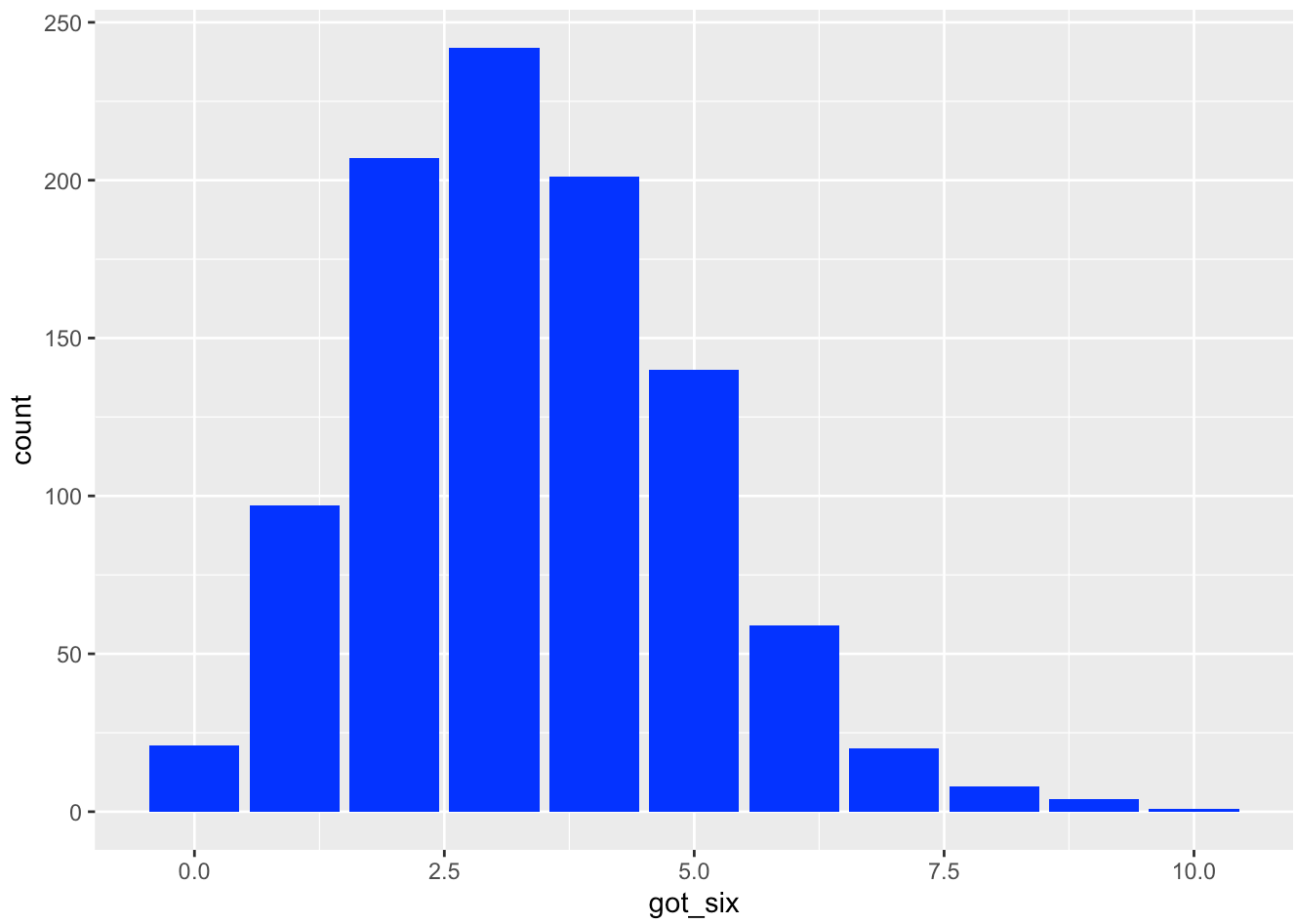
```
create_binomial_dist<- function(num_of_experiment, num_of_trials, probability) {  
  results <- rbinom(num_of_experiment, num_of_trials, probability)  
  data <- data.frame(got_six = results)  
  
  ggplot(data, aes(x=got_six)) +  
    geom_bar(fill = "blue")  
}  
  
create_binomial_dist(10, 20, probability_of_rolling_6)
```



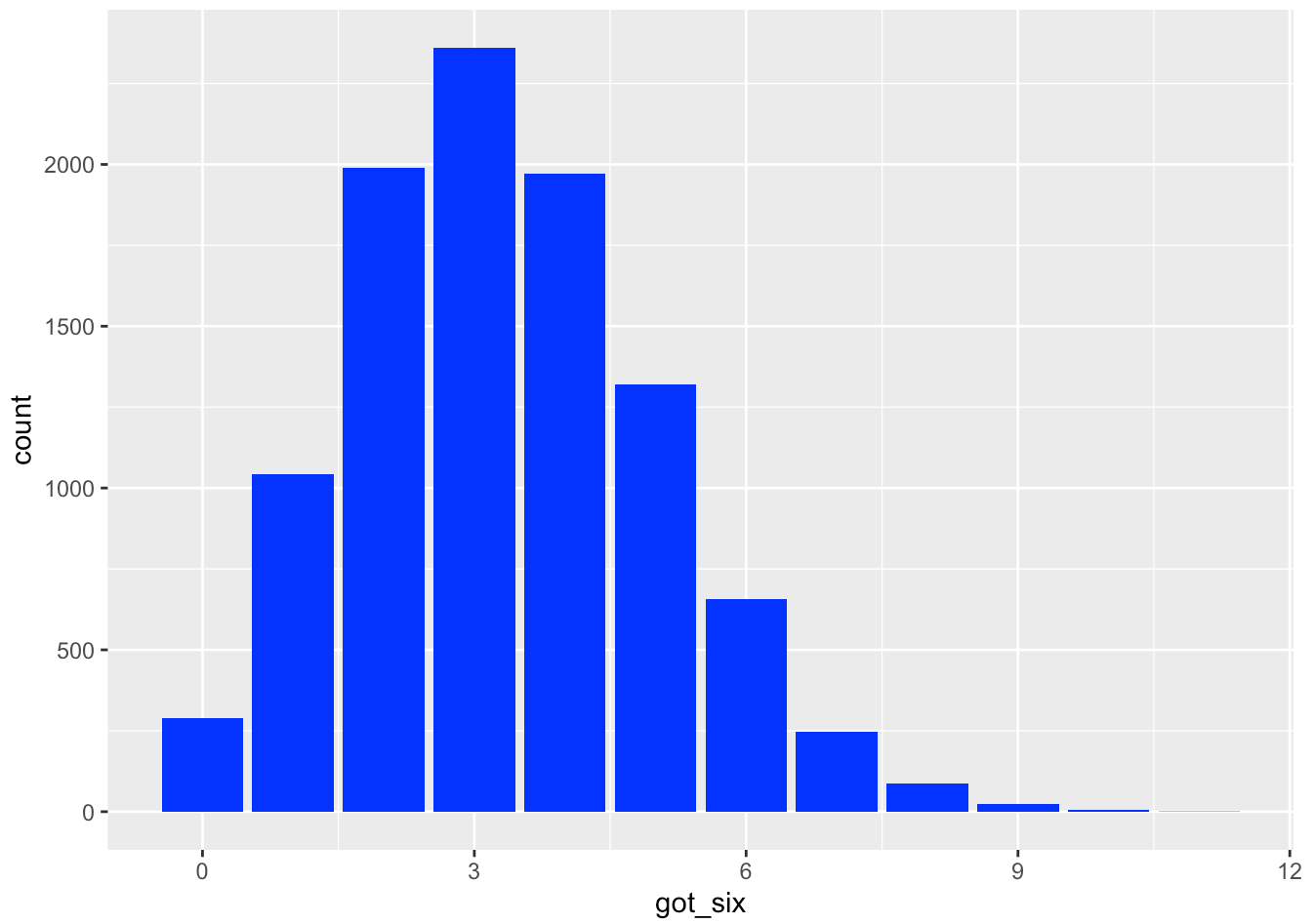
```
create_binomial_dist(100, 20, probability_of_rolling_6)
```



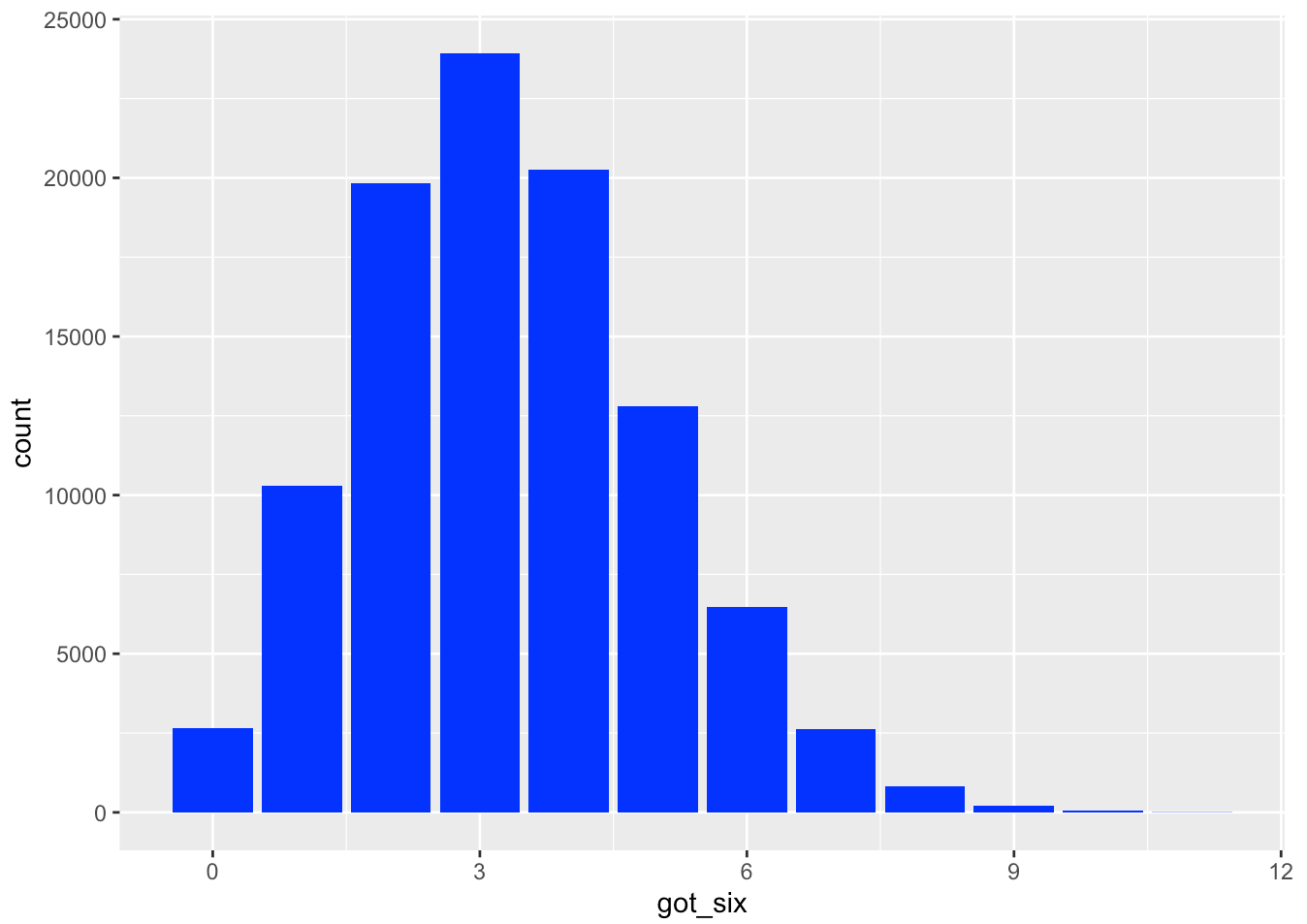
```
create_binomial_dist(1000, 20, probability_of_rolling_6)
```



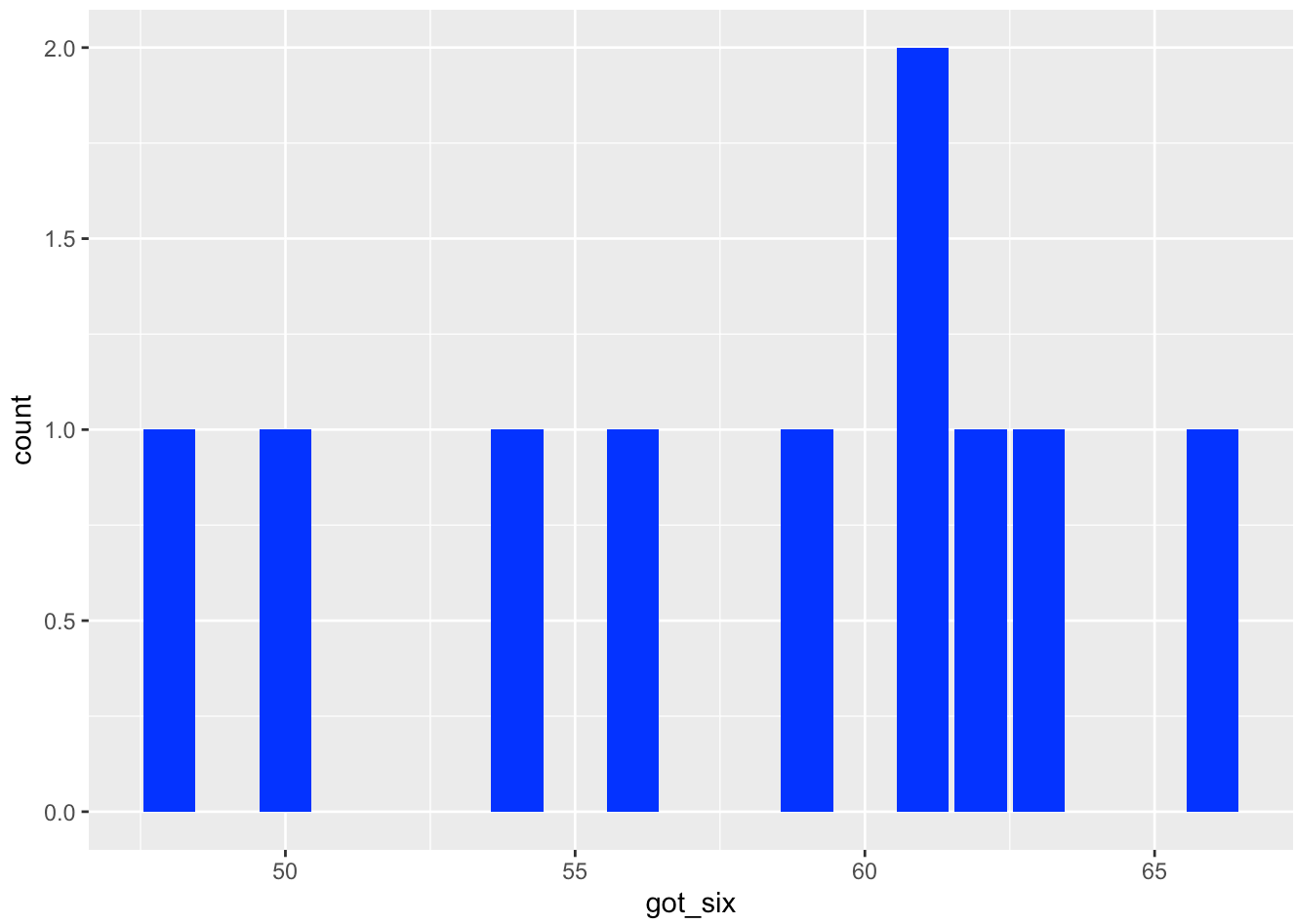
```
create_binomial_dist(10000, 20, probability_of_rolling_6)
```



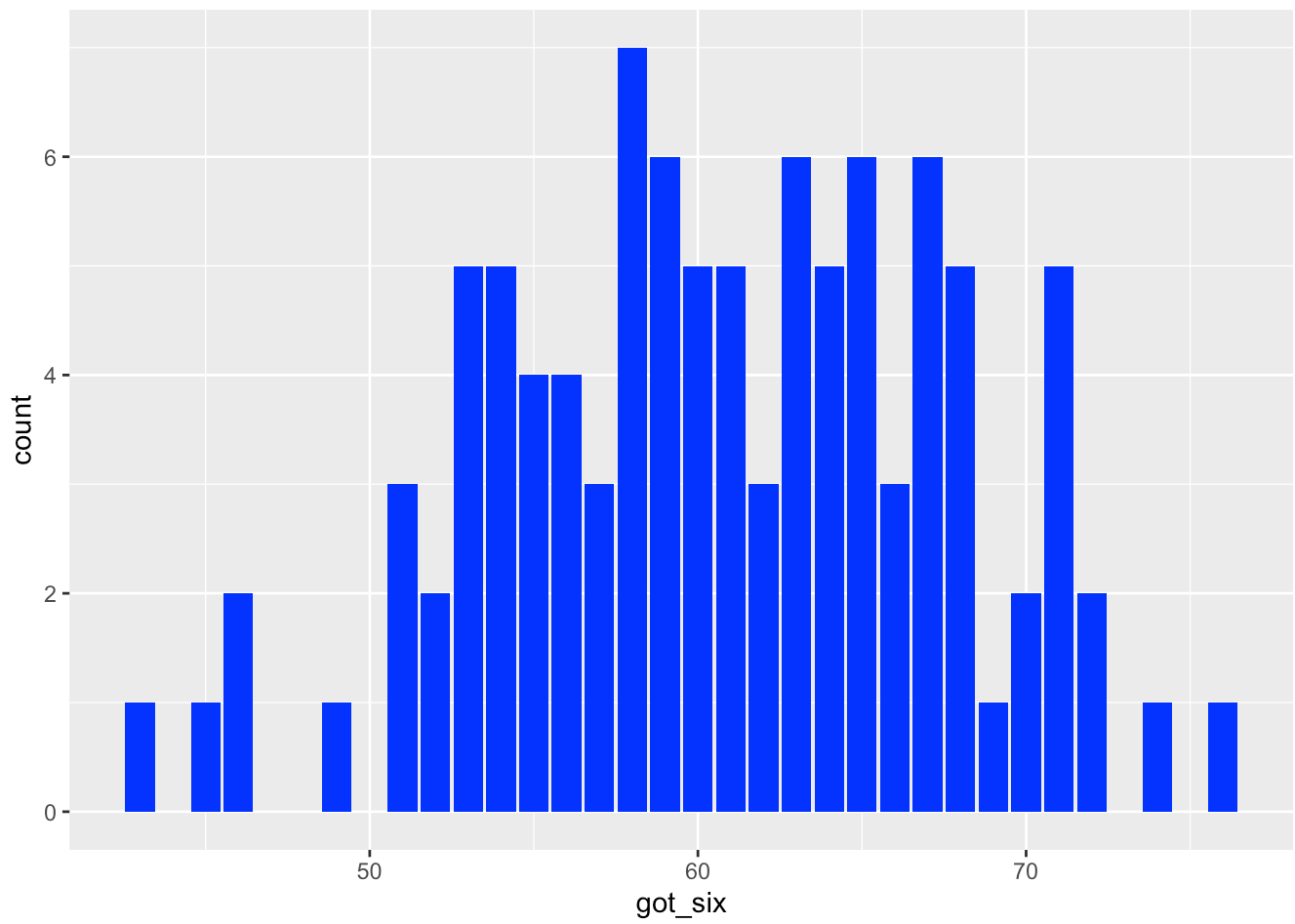
```
create_binomial_dist(100000, 20, probability_of_rolling_6)
```



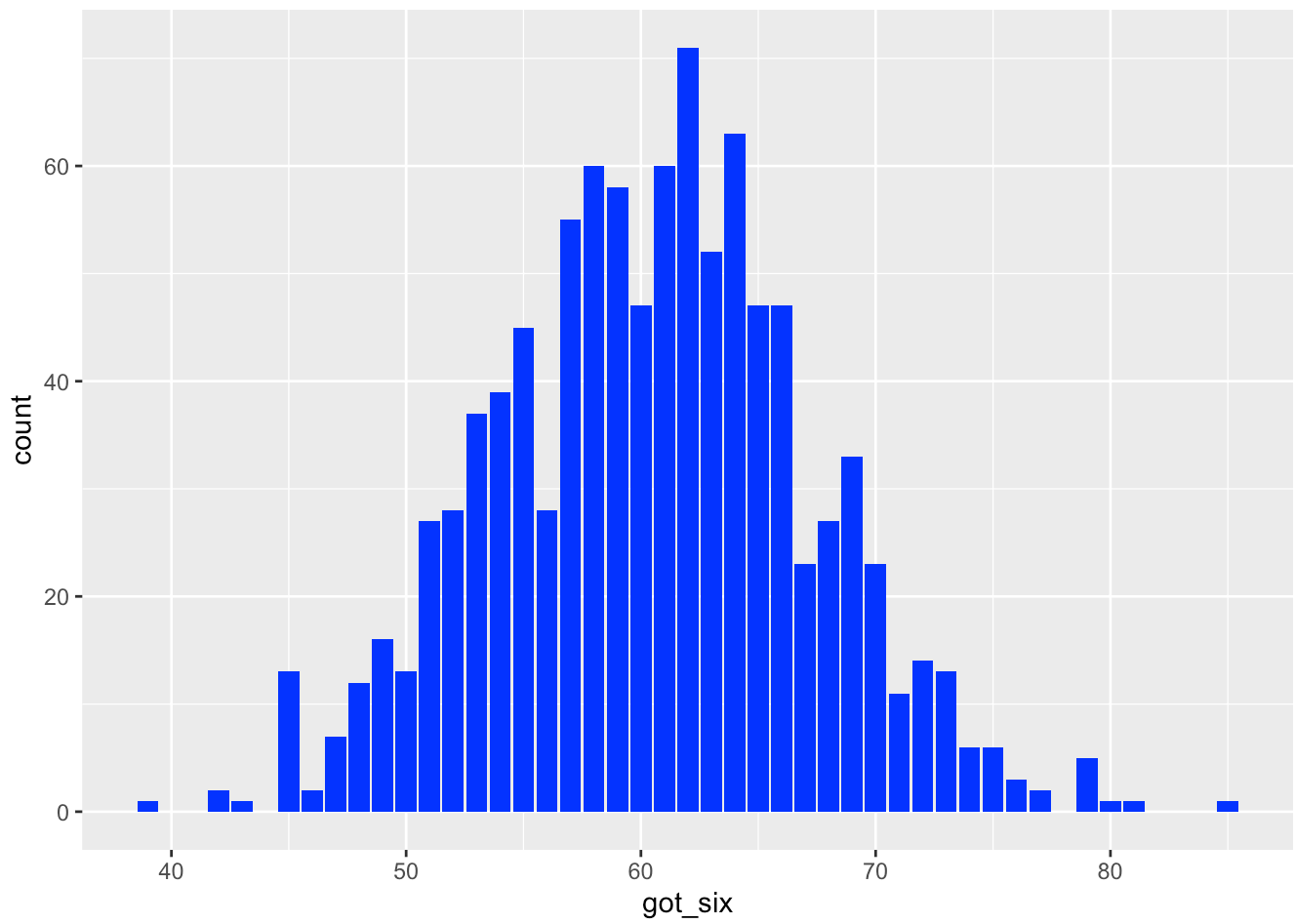
```
create_binomial_dist(10, 360, probability_of_rolling_6)
```



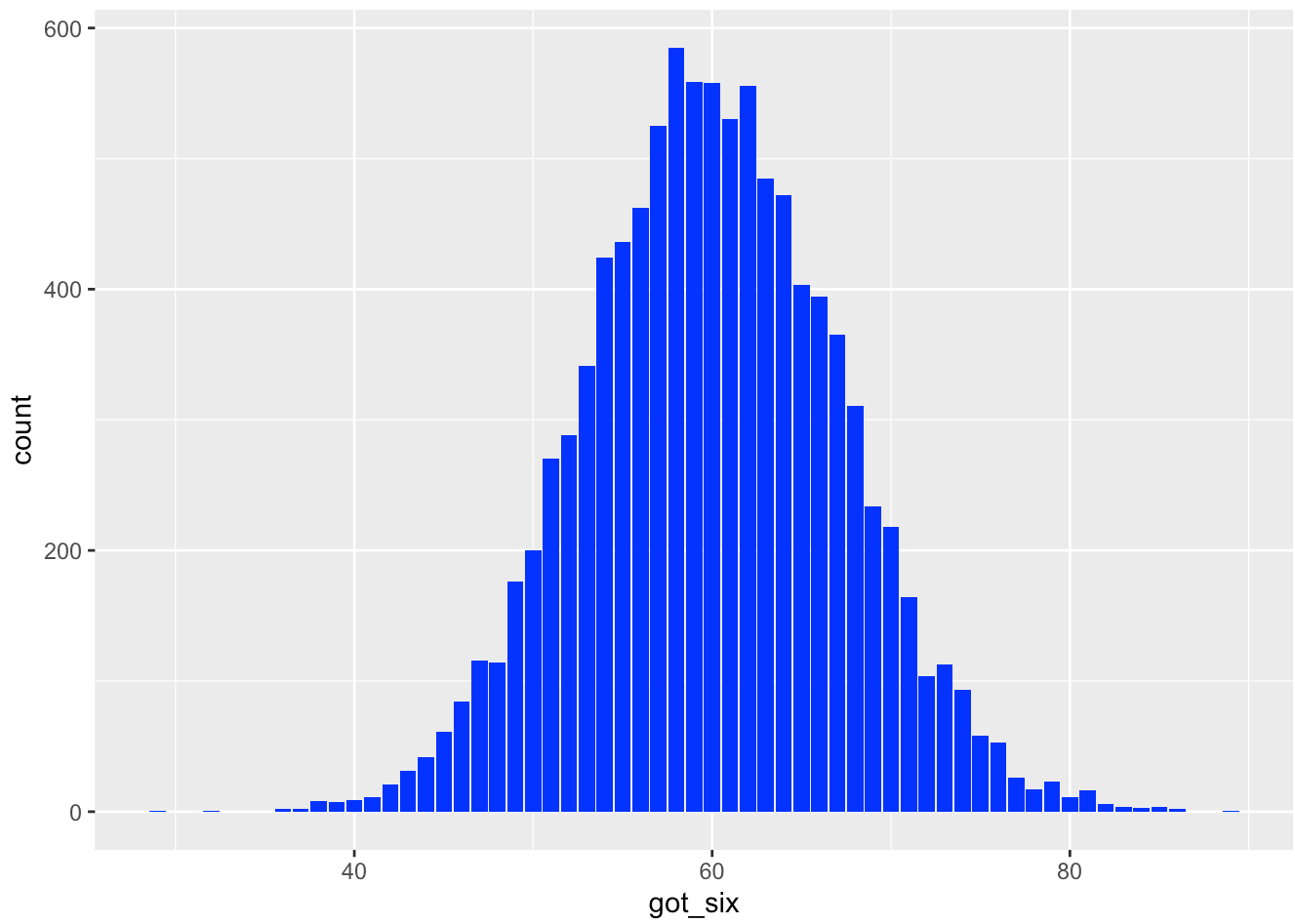
```
create_binomial_dist(100, 360, probability_of_rolling_6)
```

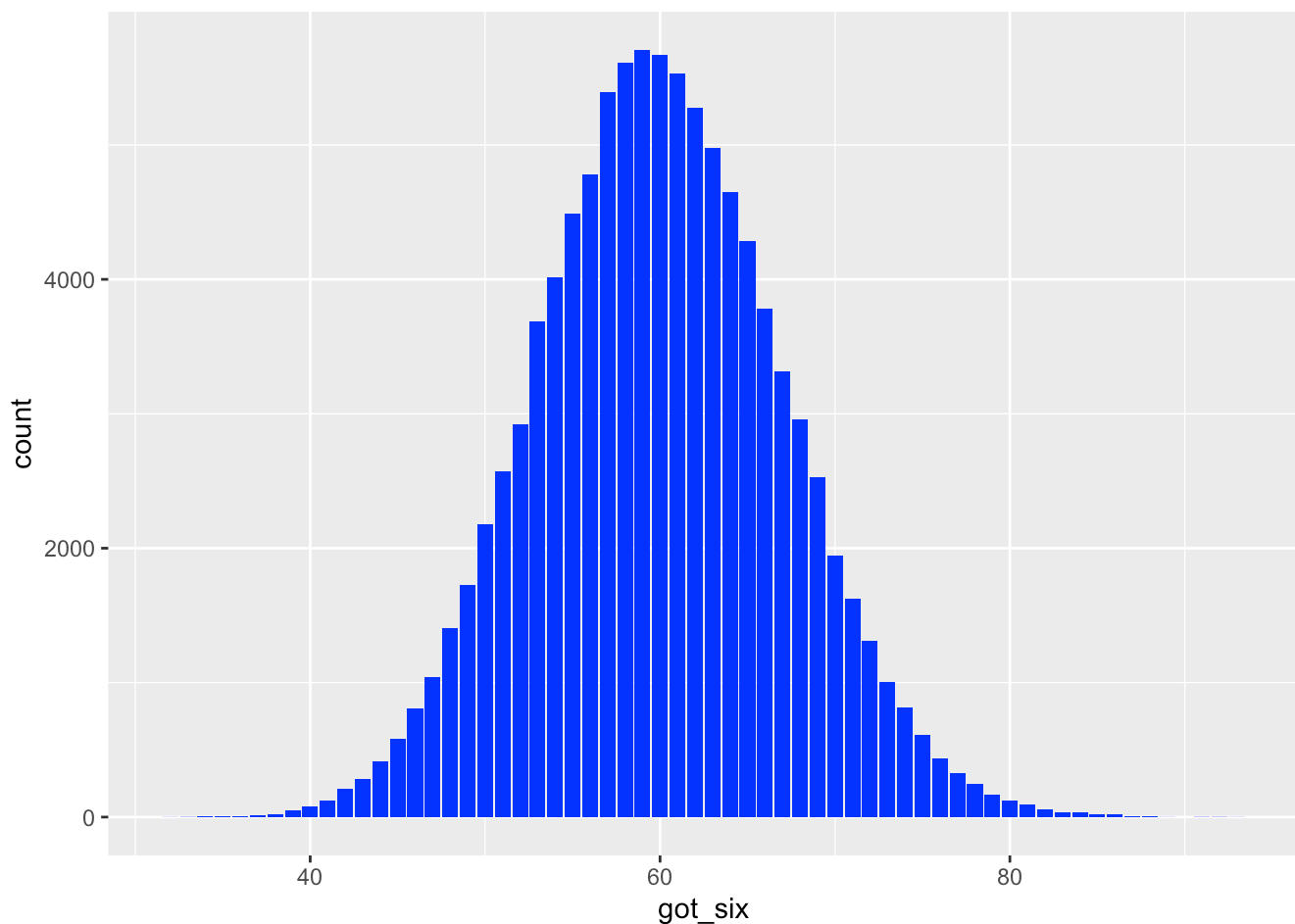
```
create_binomial_dist(1000, 360, probability_of_rolling_6)
```



```
create_binomial_dist(10000, 360, probability_of_rolling_6)
```



```
create_binomial_dist(100000, 360, probability_of_rolling_6)
```



ANALYSIS

In our experiment, we rolled the die 360 times and got 66 6s, 60 5s, 64 4s, 58 3s, 63 2s, 49 1s. Using binomial formula we get: $360C66 * (1/6)^{66} * (5/6)^{294}$

```
probability_result <- dbinom(66, size = 360, prob = 1/6)
print(probability_result)
```

```
## [1] 0.03820861
```

However this does not give us anything to show that 6 was rolled unfair amount of times. We can either use our Gaussian like distribution to fit our result and find p-value based on z score or just use binomial test. **I will use binomial test**

HYPOTHESIS TESTING

Here we can do binomial test to find the p-value with significance level of 5%. We will do two tailed test to find the fairness of the die.

```
binom.test(x = 66, n = 360, p = 1/6, alternative = "two.sided")
```

```
##  
## Exact binomial test  
##  
## data: 66 and 360  
## number of successes = 66, number of trials = 360, p-value = 0.3961  
## alternative hypothesis: true probability of success is not equal to 0.1666667  
## 95 percent confidence interval:  
## 0.1447213 0.2272455  
## sample estimates:  
## probability of success  
## 0.1833333
```

We see that the p-value is no less than or equal to .05.

Furthermore we do see very little amount of 1s with a value of 49 rolls. However this also gives us p-value more than .05 so nothing odd here either.

```
binom.test(x = 49, n = 360, p = 1/6, alternative = "two.sided")
```

```
##  
## Exact binomial test  
##  
## data: 49 and 360  
## number of successes = 49, number of trials = 360, p-value = 0.1371  
## alternative hypothesis: true probability of success is not equal to 0.1666667  
## 95 percent confidence interval:  
## 0.1024205 0.1759250  
## sample estimates:  
## probability of success  
## 0.1361111
```

CONCLUSION: CANNOT DISPROVE NULL HYPOTHESIS