③ Mixture Model on Bernoulli

X: 20000 × 784

Bernoulli

$X \rightarrow$ 784 pixels

$\theta_i \rightarrow$ 784 dimensional

$$p(x \mid \theta_i) = \boxed{\prod_{j=1}^{784} \theta_{ij}^{x_j} (1-\theta_{ij})^{1-x_{ij}}}$$

Total Data Likelihood

$$P(D \mid \theta_i) = \prod_{j=1}^{N} \prod_{j=1}^{784} \theta_{ij}^{x_j} (1-\theta_{ij})^{1-x_{ij}}$$

20000 data → 784 pixels

Alternative matrix operation that gives whole thing at once

$a = [1, 2, 3, 4]$

$b = [2, 3, 4, 5]$

$$a^b = [1^2 \quad 2^3 \quad 3^4 \quad 4^5]$$

np. power (a, b)

$\Rightarrow a^b$

suppose cluster 1

$$\text{matrix 1} = \begin{bmatrix} \theta_{11}^{x_{11}} & \theta_{12}^{x_{12}} & \theta_{13}^{x_{13}} & \cdots & \theta_{1,784}^{x_{1,784}} \\ \theta_{11}^{x_{21}} & \theta_{12}^{x_{22}} & \theta_{13}^{x_{23}} & \cdots & \theta_{1,784}^{x_{2,784}} \\ \vdots & & & & \end{bmatrix}$$

$$\text{matrix 2} = \begin{bmatrix} (1-\theta_{11})^{1-x_{11}} & (1-\theta_{12})^{1-x_{12}} & \cdots \\ (1-\theta_{11})^{1-x_{21}} & (1-\theta_{12})^{1-x_{22}} & \cdots \end{bmatrix}$$

matrix1 * matrix2 = matrix

$$= \begin{bmatrix} \theta_{11}^{x_{11}}(1-\theta_{11})^{1-x_{11}} & \theta_{12}^{x_{12}}(1-\theta_{12})^{1-x_{12}} & \cdots \\ \theta_{11}^{x_{21}}(1-\theta_{11})^{1-x_{21}} & \theta_{12}^{x_{22}}(1-\theta_{12})^{1-x_{22}} & \cdots \end{bmatrix}$$

$$\begin{bmatrix} \theta_1^{x_1}(1-\theta_1)^{1-x_1} \\ \theta_1^{x_2}(1-\theta_1)^{1-x_2} \\ \vdots \end{bmatrix} = \text{np.prod(matrix, axis=1)}$$

h[:, i] = np.prod(matrix, axis=1) * pi[i]

h = np.divide(h, np.sum(h, axis=1))

$$\gamma_{ik} = \frac{\pi_k^0 \, P(x_i \mid z=k;\ \theta_k^0)}{\sum^{10} \ldots}$$

$$\sum_{m=1} \pi_m \, P(x_i \mid z = m \,;\, \theta_m^{0})$$

Data point $\Rightarrow x_i$

cluster K

$$P(x_i \mid z = K \,;\, \theta_K^{0})$$

$$= \theta_K^{x_i} (1-\theta_K)^{1-x_i}$$

$\gamma_{iK} \Rightarrow$ element wise

$\gamma[:, K] =$ (np.prod(matrix, axis=1)) * pi(K)

```
(- 2 * X @ Kmus.T + np.sum(np.multiply(Kmus, Kmus),
                                      axis = 1).T).T

                 + np.sum(np.multiply(X, X), axis=1)
                                                    .T
```