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Keywords: market risk, backtesting, conditional predictive ability, GARCH, volatility, capital requirements, Basel II.

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Comparing univariate and multivariate models to forecast portfolio value-at-risk

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Abstract

This article addresses the problem of forecasting portfolio value-at-risk (VaR) with multivariate GARCH models vis-à-vis univariate models. Existing literature has tried to answer this question by analyzing only small portfolios and using a testing framework not appropriate for ranking VaR models. In this work we provide a more comprehensive look at the problem of portfolio VaR forecasting by using more appropriate statistical tests of comparative predictive ability. Moreover, we compare univariate vs. multivariate VaR models in the context of diversified portfolios containing a large number of assets and also provide evidence based on Monte Carlo experiments. We conclude that, if the sample size is moderately large, multivariate models outperform univariate counterparts on an out-of-sample basis.
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1 Introduction

Market risk management has been receiving increased attention in the last few years due to the importance devoted by the Basel II Accord to the regulation of the financial system. Basel II explicitly recognizes the role of standard financial risk measures, such as VaR, that financial institutions must implement and report in order to monitor their risk exposure and to determine the amount of capital subjected to regulatory control (Berkowitz and O'Brien 2002). Consequently, VaR is now established as one of the most popular risk measures designed to controlling and managing market risk. The Accord also establishes penalties for inadequate models, and consequently, there are incentives to pursue accurate approaches to estimate the VaR. A myriad of models are currently available for modeling the VaR, but no consensus has been reached on which model or method is the best.

The first decision one has to take when trying to predict the VaR of a portfolio is whether to use a multivariate model for the system of asset returns contained in it or, alternatively, assuming a known vector of weights and modeling the univariate time series of portfolio returns. The question that immediately emerges is: Which method is able to provide more accurate VaR forecasts? Each

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of these two alternatives have several pros and cons. First, the univariate model has to be estimated each time that the vector of weights changes because this yields a different univariate time series of portfolio returns; see Bauwens et al. (2006). This requirement is not necessary when a multivariate model is fitted. Moreover, one can possibly argue that modeling the joint dynamics of the assets contained in the portfolio via a multivariate model can also lead to forecast improvements due to the use of more information. However, as the dimension of the problem increases, the estimation of multivariate models becomes more complicated due to the usually large number of parameters involved; see McAleer (2009). As a consequence, the predictive ability of these models can be compromised. Therefore, the trade-off between estimation difficulties and forecasting performance is not clear at a first glance.

Bauwens et al. (2006) conjecture that, under the current state-of-the-art, it is probably better to adopt univariate models. More recently, Christoffersen (2009) also argues that univariate models are more appropriate if the purpose is risk measurement (e.g. VaR computation) whereas multivariate models are more suitable for risk management (e.g. portfolio selection). Furthermore, most of the existing empirical papers have focused on the analysis of only one class of models, without any comparative analysis among competing approaches. For instance, Engle and Manganelli (2001), Giot and Laurent (2004) and Kuester et al. (2006) analyze VaR forecasting performance among univariate models while Engle (2002), McAleer and da Veiga (2008a) and Chib et al. (2006) provide a similar analysis among multivariate models. In any case, they did not provide a direct comparison of VaR predictive performance among univariate and multivariate volatility models when implemented to the same data set. One exception is the work of Berkowitz and O'Brien (2002), who conclude that a simple univariate model is able to improve the accuracy of portfolio VaR estimates delivered by large US commercial banks. On the other hand, Brooks and Persand (2003) also conclude that there are no gains from using multivariate models, while, more recently, McAleer and da Veiga (2008b) found mixed evidence. However, the empirical analysis of these authors are based on portfolios composed of very few assets (3 and 4, respectively), while in real-world situations, financial institutions are usually faced with much larger portfolios. Furthermore, they compared univariate and multivariate VaRs by using traditional backtesting tests based on coverage/independence criteria (Kupiec 1995; Christoffersen 1998). These tests, though very useful to evaluate the accuracy of a single model, can provide an ambiguous decision about which candidate model is better. Therefore, it is better to use formal statistical tests designed to evaluate the comparative predictive performance among candidate models or, in other words, to compare in a straightforward way the performance of one model versus the other. Finally, Brooks and Persand (2003) and McAleer and da Veiga (2008b) only consider in their empirical analysis multivariate models with constant conditional correlations. There is, however, large evidence that, in practice, conditional correlations move over time; see, for example, Engle (2002), Tse and Tsui (2002) and Cappiello et al. (2006), among many others.

The goal of this paper is to compare univariate and multivariate GARCH models when implemented to forecast the VaR of large portfolios. The comparison among the alternative models considered in this paper is done by using the formal statistical tests of superior predictive ability proposed by Giacomini and White (2006). We conduct several Monte Carlo experiments using a very general specification for data generating process (DGP) that include stylized facts such as asymmetric effects. Finally, we also provide empirical evidence by estimating the portfolio VaR of three data sets of real market portfolios containing a large number of assets. We show that even in very large systems, if the sample size is moderately large, it could be worth to model the second order dynamics by fitting multivariate models to predict the VaR of a portfolio.

The paper is organized as follows. Section 2 provides a brief description of the multivariate and univariate VaR models considered in this paper. In Section 3 we compare both approaches using simulated data, while Section 4 reports results based on real market data. Section 5 concludes.

2 Univariate and multivariate VaR models

In this Section, we describe several alternative procedures to obtain portfolio VaR forecasts using univariate and multivariate GARCH models. Throughout the paper we focus on the portfolio VaR for a long position in which traders have bought the assets and wish to measure the risk associated to decreases in asset prices.

2.1 VaR estimation

Denote by $\mathbf{Y}_t = (y_{t1}, \ldots, y_{tN})'$ the vector of returns of the N assets contained in the portfolio at time t and by $y_{p,t} = \mathbf{W}'_{t-1}\mathbf{Y}_t$ the portfolio return, where $\mathbf{W}_{t-1}$ is the vector of portfolio weights, which is assumed to be known at time $t-1$. Then, the portfolio VaR for a long position is defined as the α-quantile of the conditional distribution of the portfolio $y_{p,t}$. This means that, with probability α, the portfolio return will be smaller than the VaR. Therefore, the VaR can be defined as:

$$VaR_t^{\alpha} = \sup\left[r|P\left(y_{p,t} \leq r\right) \leq \alpha\right] \tag{1}$$

where the probability P is taken with respect to the distribution function of the portfolio returns conditional on the information up to $t-1$. Throughout the paper we focus on the portfolio VaR at $\alpha = 1\%$, which is the relevant level that banks must compute and report their risk exposure. Therefore, from now on, we eliminate the superscript α from the definition of VaR. We can consider two alternative conditioning sets. First, we can consider the distribution of portfolio returns conditional on past portfolio returns, i.e. the distribution of $y_{p,t}$ conditional on a linear combination of past asset returns, $\mathbf{W}'_{t-h-1}\mathbf{Y}_{t-h}$. Alternatively, we can consider the distribution of $y_{p,t}$ conditional on the whole vector of past asset returns, $\mathbf{Y}_{t-h}$. The former case leads to a univariate model for the portfolio returns while the latter leads to a multivariate analysis. In any of both cases there are two possibilities for the specification of the conditional distribution of portfolio returns. First, the VaR can be estimated without assuming any particular parametric form of this distribution, thus estimating directly its 1% quantile. Alternatively, we can assume a parametric specification by assuming a particular model for the conditional mean and variance and a particular distribution for the standardized returns. Therefore, the portfolio returns can be represented by the following process,

$$y_{p,t} = \mu_{p,t} + \sigma_{p,t}\varepsilon_{p,t} \tag{2}$$

where $\mu_{p,t}$ and $\sigma_{p,t}$ are the portfolio conditional mean and standard deviation at time t, respectively, and $\epsilon_{p,t}$ is a disturbance. Then, the portfolio VaR is given by:

$$VaR_t = \mu_{p,t} + \sigma_{p,t} q \tag{3}$$

where q is the 1% quantile of the conditional distribution function f of the centered and standardized returns. To simplify the problem, we assume that q is time invariant and that $\mu_{p,t} = 0$. This last assumption is reasonable when one is dealing with daily data. The specification of $\sigma_{p,t}$ depends on whether we consider a univariate or a multivariate model. Therefore, when computing the VaR using a univariate model, $\sigma_{p,t}$ is given by

$$\sigma_{p,t}^2 = E\left[y_{p,t}^2 | y_{p,1}, ..., y_{p,t-1}\right] \tag{4}$$

and q is the 1% quantile of the conditional distribution of $y_{p,t}$ given $\{y_{p,1}, ..., y_{p,t-1}\}$. On the other hand, when dealing with multivariate models, then

$$\sigma_{p,t}^2 = W_{t-1}' H_t W_{t-1} \tag{5}$$

where H_t is the positive definite conditional covariance matrix. In this case, the variable q in (3) is the 1% quantile of the conditional distribution of the linear combination $\mathbf{W}_{t-1}' \mathbf{Y}_t$ given $\{\mathbf{Y}_1, ..., \mathbf{Y}_{t-1}\}$. This conditional distribution is, in general, unknown. It takes a tractable form when the distribution of returns is closed under linear transformations, i.e. when, for example, all linear combinations of $\mathbf{Y}$ have the same distribution as the marginal distribution of returns. This is the case of the standardized multivariate Normal and Student t distribution; see Pesaran et al. (2008) and Christoffersen (2009). Therefore, in this paper, we consider two alternative specifications for the conditional distribution: the Gaussian distribution and, in order to take into account the presence of fatter tails, the Student's t distribution[1].

2.2 Univariate VaR models

As we mentioned above, parametric univariate models for calculating the VaR are based on assuming a particular variance and distribution of portfolio returns given past portfolio returns. We consider the specification of returns in (2) with $\mu_{p,t} = 0$ where the distribution of $\varepsilon_{p,t}$ can be either a Gaussian or a Student's t distribution with v degrees of freedom. Moreover, we consider two different specifications for the portfolio conditional standard deviation $\sigma_{p,t}$: the GARCH model (Bollerslev 1986) and the asymmetric GJR-GARCH model (Glosten et al. 1993). The GARCH model is given by:

$$\sigma_{p,t}^2 = \omega + \alpha y_{p,t-1}^2 + \beta \sigma_{p,t-1}^2 \tag{6}$$

[1]Note that, when considering a Student's t distribution (in both multivariate and univariate models), the 1% quantile of the conditional distribution function, q, in (3) is given by $q = \sqrt{\frac{v-2}{v}}\tilde{q}$, where $\tilde{q}$ is the 1% quantile of a Student's t distribution with v degrees of freedom; see Pesaran and Pesaran (2007, section 5).

where $\omega > 0$, $\beta, \alpha \geq 0$ and $\alpha + \beta < 1$ to guarantee the positivity of conditional variances and stationarity of returns. The asymmetric GJR-GARCH model is described as:

$$\sigma^2_{p,t} = \omega + \alpha y^2_{p,t-1} + \beta \sigma^2_{p,t-1} + \delta I(\varepsilon_{t-1} < 0) y^2_{p,t-1} \tag{7}$$

where $I(\cdot)$ is an indicator function that takes value 1 when the argument is true. The restriction to ensure that $\sigma^2_{p,t}$ is positive is $\omega > 0$, $\alpha, \beta, \delta \geq 0$. The model is stationary if $\delta < 2(1 - \alpha - \beta)$; see Hentschel (1995).

We also consider the semiparametric conditional autoregressive VaR model (known as CAViaR) proposed by Engle and Manganelli (2004) which is designed to estimate directly the 1% quantile of the conditional distribution of the returns, which is given by the following expression:

$$VaR_{p,t} = (\omega + \alpha y^2_{p,t-1} + \beta VaR^2_{p,t-1})^{1/2}. \tag{8}$$

The parameters of the univariate GARCH models considered in this work are estimated via quasi maximum likelihood (QML). A review of estimation issues of univariate GARCH models, such as choice of initial values, numerical algorithms and accuracy, is provided by Zivot (2009). It is important to note that even when the normality assumption is inappropriate, maximizing the Gaussian log likelihood results in QML estimates that are consistent and asymptotic normally distributed provided that the conditional mean and variance functions of the GARCH model are correctly specified; see Bollerslev and Wooldridge (1992). Finally, the estimation of the parameters of the CAViaR model is performed by means of regression quantiles; see Engle and Manganelli (2004).

2.3 Multivariate VaR models

We now consider the multivariate modeling of the VaR. In this case, we only focus on parametric VaR models as the semi-parametric specifications are rather complicated in a multivariate system with a large number of asset returns. We consider the following specification of the system of returns Y_t,

$$Y_t = \epsilon_t H_t^{1/2} \tag{9}$$

where H_t is the $N \times N$ conditional covariance matrix and ϵ_t is a $1 \times N$ vector of disturbances. We consider five different specifications for H_t: the diagonal VEC model of Bollerslev et al. (1988) and its asymmetric version, the constant conditional correlation (CCC) model of Bollerslev (1990), the dynamic conditional correlation (DCC) model of Engle (2002) and the asymmetric DCC (AsyDCC) model of Cappiello et al. (2006). Moreover, two alternative multivariate distributions for the system of standardized residuals ϵ_t are considered: the Gaussian and the Student's t distribution with v degrees of freedom.

The diagonal VEC(1,1) model (hereafter DVEC(1,1)) of Bollerslev et al. (1988) is given by:

$$H_t = C + A \odot Y_{t-1} Y'_{t-1} + B \odot H_{t-1} \tag{10}$$

where $\odot$ denotes the Hadamard (elementwise) product, and C, A and B are positive definite squared symmetric matrices. In this model each covariance depends on its own past values and shocks. Besides, the model is covariance-stationary if the eigenvalues of $A+B$ are all less than 1 in modulus. In order to represent stylized facts such as conditional asymmetries, Engle and Sheppard (2008) proposed an asymmetric version of the DVEC(1,1) model, hereafter AsyDVEC(1,1), which is given by:

$$H_t = C + A \odot Y_{t-1}Y'_{t-1} + B \odot H_{t-1} + G \odot \eta_{t-1}\eta'_{t-1} \tag{11}$$

where A, B and G are positive definite matrices and $\eta_t = I\,(Y_t < 0) \odot Y_t$. By taking expectations, the matrix C can be rewritten as $\bar{H} \odot (\iota\iota' - A - B) - \bar{N} \odot G$, where ι is a vector of ones, $\bar{N} = E[\eta_t \eta'_t]$ and $\bar{H}$ is the unconditional covariance matrix. A sufficient condition to ensure the positive definiteness of H_t in the AsyVEC(1,1) model is that $\bar{H} \odot (\iota\iota' - A - B) - \bar{N} \odot G$ and the matrices H_0 are positive definite; see Cappiello et al. (2006) and Engle and Sheppard (2008). Both DVEC(1,1) and AsyDVEC(1,1) models are highly parameterized. For example, for a 10-asset portfolio the DVEC(1,1) has 75 parameters. Therefore, in this work we only consider these two models as data generating processes (DGPs) in Section 3.

Models of conditional correlations are now one of the most promising alternatives to model and forecast conditional covariances. These models are based on the decomposition of the conditional covariance matrix into conditional standard deviations and correlations; see Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009). One of their greatest advantages is that they have a smaller number of parameters than traditional VEC models. In models of conditional correlations the conditional covariance matrix H_t is defined as:

$$H_t = D_t R_t D_t \tag{12}$$

where

$$D_t = diag\left(h_{t1}^{1/2} ... h_{tN}^{1/2}\right) \tag{13}$$

with $diag(\cdot)$ being the operator that transforms a $N \times 1$ vector into a $N \times N$ diagonal matrix and h_{tj} follows univariate GARCH models such as those considered in the previous Section. R_t is a symmetric positive definite conditional correlation matrix with elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1$.

The CCC model of Bollerslev (1990) assumes that the conditional correlation matrix R_t is constant over time, i.e. $R_t = R$ where R is the unconditional correlation matrix of the standardized returns. The CCC model was further extended by Engle (2002)[2] in order to allow time-varying dynamic conditional correlations. In the DCC model the conditional correlation matrix R_t is given by:

$$R_t = diag(Q_t^{-1/2}) Q_t diag(Q_t^{-1/2}) \tag{14}$$

where $diag(Q_t)$ is a diagonal matrix containing diagonal elements of a $N \times N$ positive definite

[2]An alternative conditional correlation model with time-varying correlation matrices was also proposed by Tse and Tsui (2002).

matrix Q_t with elements given by the following GARCH dynamics:

$$Q_t = (1 - \alpha - \beta)\,\bar{Q} + \alpha\epsilon_{t-1}\epsilon'_{t-1} + \beta Q_{t-1} \tag{15}$$

where $\bar{Q}$ is the $N \times N$ unconditional covariance matrix of ϵ_t and α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.

More recently, Cappiello et al. (2006) extended the DCC model to incorporate asymmetric effects in the conditional correlations, yielding the asymmetric DCC (AsyDCC) model. In the AsyDCC model the dynamics of Q_t are now described by:

$$Q_t = \left(\bar{Q} - \alpha\bar{Q} - \beta\bar{Q} - \delta\bar{\Gamma}\right) + \alpha\epsilon_{t-1}\epsilon'_{t-1} + \beta Q_{t-1} + \delta n_{t-1}n'_{t-1} \tag{16}$$

where $n_t = I\left(\epsilon_t < 0\right) \odot \epsilon_t$ and $\bar{\Gamma} = E\left[n_t n'_t\right]$. Cappiello et al. (2006) note that a necessary condition for Q_t to be positive definite is that $\alpha + \beta + \lambda\delta < 1$, where λ is the maximum eigenvalue of $\bar{Q}^{-1/2}\bar{N}\bar{Q}^{-1/2}$.

When assuming a Gaussian distribution for the errors, we use the two-step procedure proposed by Engle and Sheppard (2001) for the QML estimation of the DCC models considered in this work. Theoretical and empirical properties of this estimation procedure are detailed in Engle and Sheppard (2001) and Sheppard (2003). Some functions available in the Matlab-based toolbox USCD_GARCH were used in the QML estimation of multivariate models[3].

2.4 Forecast evaluation of VaR models

Forecast evaluation of VaR models is usually done by means of the backtesting analysis of coverage and independence tests proposed by Kupiec (1995) and Christoffersen (1998). However, if the objective is the comparison among competing models, these tests may not be the best option to provide an unambiguous ranking regarding which candidate model offers superior VaR predictive performance. Instead, it is probably better to use an statistical test to compare in a straightforward way the performance of one model versus the other. In order to achieve this goal, a number of VaR-based comparative predictive ability tests have been proposed; see, for instance, Christoffersen et al. (2001), Giacomini and Komunjer (2005) and, in a more general context of predictive ability, Giacomini and White (2006). In this paper, we use this last test, known as conditional predictive ability (CPA) test, because it can be applied to interval forecasts and it allows the comparison between nested and nonnested models and among several alternative estimation procedures.

In this paper, the CPA test is carried out by assuming an asymmetric linear (tick) loss function $\mathcal{L}$ of order α defined as:

$$\mathcal{L}_\alpha\left(e_{t+1}\right) = \left(\alpha - \mathbf{1}\left(e_{t+1} < 0\right)\right) e_{t+1} \tag{17}$$

where $e_{t+1} = y_{p,t+1} - \mathrm{VaR}_{t+1}$. As Giacomini and Komunjer (2005) argue, the tick loss function is the implicit loss function whenever the object of interest is a forecast of a particular α-quantile, where $\alpha \in (0, 1)$. Therefore, this function can be considered the relevant loss function for the

[3] The toolbox is available in the link `http://www.kevinsheppard.com/wiki/UCSD_GARCH`.

VaR problem[4]. Moreover, there are at least two important features regarding the use of the tick loss function vis-à-vis traditional backtesting techniques. First, as Lemma 1 in Giacomini and Komunjer (2005) shows, Christoffersen's (1998) correct conditional coverage criterion can be alternatively expressed as $E_t[(\alpha - \mathbf{1}(e_{t+1} < 0)) e_{t+1}] = 0$. Thus, "*correct conditional coverage condition is equivalent to requiring optimality of an interval forecast with respect to the tick loss function*" (Giacomini and Komunjer 2005, p.419). Second, the tick loss function takes into account the magnitude or the *implicit cost* associated to VaR forecasting errors, in this case e_{t+1}. Since VaR estimates are frequently used to help strategic financial decision-making process and to manage market risk, VaR forecasting errors can imply financial distresses such as misestimation of capital subjected to regulatory control. Therefore, finding the model that minimizes the relevant cost function is an intuitive, appealing criterion to compare predictive ability.

Under the null hypothesis of equal predictive ability, the loss difference between two models follows a martingale difference sequence. A Wald-type test of the following form is conducted:

$$CPA = T\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\mathcal{LD}_{t+1}\right)'\hat{\Omega}^{-1}\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\mathcal{LD}_{t+1}\right) \tag{18}$$

where T is the sample size, $\mathcal{LD}$ is the loss difference between the two models, $\mathcal{I}$ is the set of instruments that help predicting differences in forecast performance between the two models, and $\hat{\Omega}$ is a matrix that consistently estimate the variance of $\mathcal{I}_t\mathcal{LD}_{t+1}$. Following Giacomini and White (2006) we assume $\mathcal{I}_t = (1, \mathcal{LD}_t)$. The null hypothesis of equal predictive ability is rejected when $CPA > \chi^2_{T,1-\alpha}$. Giacomini and White (2006) note that the statistic CPA can be alternatively computed as TR^2, where R^2 is the uncentered squared multiple correlation coefficient for the artificial regression of the constant unity on $(1, \mathcal{LD}_t)$.

3 Monte Carlo evidence

In this Section we perform Monte Carlo experiments in order to compare the in-sample and out-of-sample performance of multivariate versus univariate models. Our Monte Carlo experiment consists in the following steps

1. Simulate a multivariate system with 10 assets and sample size of T=5,000 observations. The DGP used is the AsyDVEC(1,1) model in (11) with Gaussian errors. We chose this model as DGP because it is a very general specification for the dynamics of asset returns that takes into account time-varying second moments and also asymmetric effects. The parametrization of the simulated model is shown in Table 1. As we comment next, the results are not affected by the choice of the DGP and the parametrization. Furthermore, it is worth noting that since the AsyDVEC is the true DGP, we do not use this model to estimate the parameters using

[4] To see how the tick loss function works in practice, consider a simple example involving two different VaR models. Suppose that the portfolio return in day t is -4% and that the VaR in day t (forecasted in $t-1$) obtained from the two models is -2% and -6%, respectively. Obviously, for the first model there is a VaR violation whereas for the second there is not. For the first model, the value of the tick loss function in (17) is $(0.01-1)(-2) \cong 2$ whereas for the second model the value is $(0.01-0)2 = 0.02$. (Recall that, since we are considering only a long position in the portfolio, the VaR will be always a negative number). Therefore, according to the tick loss function, a model is more penalized when a VaR violation is observed. Moreover, the greater is the magnitude of the violation the greater is the penalization.

the simulated data since this would give an unfair advantage to this approach in comparison to the other competing models. Finally, we focus on the case of a long position in an equally-weighted portfolio. This portfolio composition has been been extensively used in the empirical literature; see, for instance, DeMiguel et al. (2009);

2. Use the first 2,500 observations of the simulated data to estimate each of the multivariate and univariate models described in Section 2, except the DVEC and AsyDVEC models;

3. For each estimated model, obtain in-sample one-step-ahead forecasts for the portfolio VaR using the first 2,500 observations;

4. For each estimated model, use the remaining 2,500 observations to provide one-step-ahead out-of-sample forecasts. These forecasts are nonadaptative, i.e. the parameters estimated using the first half of the sample were kept fixed in the second half of the sample. We also considered the case in which the parameters of all models are re-estimated in a rolling window basis. This procedure, however, is very time consuming. Furthermore, the results are very similar to those of fixed window estimates.

Steps 1 to 4 are repeated 100 times. In each Monte Carlo simulation we compute the average mean squared error (MSE) of the estimated portfolio VaR with respect to the true portfolio VaR obtained from the simulated data, and the CPA test for the pairwise comparisons between multivariate and univariate models. Therefore, after the last 100th Monte Carlo simulation, we have a 100×1 vector of average MSEs and a 100×1 vector of CPA statistics for each pairwise comparison among multivariate and univariate models. This analysis allows us to evaluate the moments of the distribution of the MSEs and also the number of times a model outperformed the other according to the CPA test.

Figure 1 plots the Monte Carlo in-sample and out-of-sample distribution of the MSE of the differences between the estimated and true portfolio VaR. Obviously, a higher MSE can be interpreted as a high deviation from the true portfolio VaR, indicating a poor performance. Multivariate models systematically achieved lower MSEs for both in-sample and out-of-sample periods. This can be understood as an indication that multivariate models can perform better than univariate models for the problem of portfolio VaR forecasting. The worse in-sample and out-of-sample performance among univariate models was achieved by the GJR model.

The Monte Carlo results of the Giacomini and White (2006) CPA test are summarized in Table 2, which reports the number of times in which each multivariate model outperformed each univariate model; the threshold confidence level is 10%. For instance, the out-of-sample comparison of the DCC versus the GARCH model indicated that the multivariate model was preferred 40 times, the univariate model was preferred in 3 times and in 57 times the performance of the two models was statistically equal. The results in Table 2 show that when comparing univariate and multivariate models within-sample, in approx. 80% of the times, both approaches are similar. However, among the cases in which one of the models is better than the other, the selected model is multivariate. One exception to this conclusion is when the multivariate models are compared to the univariate GJR model. In this case, the multivariate and univariate GJR model are only indifferent in around 51% of the simulated systems, with the multivariate model being preferred in 45% of the systems.

Therefore, by looking at the within-sample performance of the models, it may seem that with the exception of the GJR model, the advantages of the multivariate versus univariate models is very mild. However, the advantage of the multivariate models appear much clearly when looking at the out-of-sample results. In this case, the multivariate models outperform the multivariate GJR model in nearly all of the simulated systems. For the rest of multivariate models, approximately half of the times, the multivariate and univariate models are similar. When one of the models is selected, with few exceptions, the selected model is the multivariate. For instance, the DCC model was chosen in 40 of the simulated systems in comparison to the GARCH model, which was chosen only in 3 of them. In comparison to the CAVIAR model, the DCC model was chosen in 49 of the simulated systems whereas the CAVIAR model was chosen in 2 of them.

Table 2 also reports the pairwise comparisons among only multivariate and among only univariate models. The comparison among multivariate models indicates that dynamic conditional correlation models are preferred to the constant conditional correlation model nearly the same number of times as the CCC model is preferred to the DCC. In any case, it is clear that both models are preferred to the AsyDCC model. The comparison among univariate models delivered mixed results. The CAVIAR and GARCH models performed similarly in approx. 50% of the times, but the latter was selected more often than the former in the within-sample period, while the opposite result was observed in the out-of-sample period. In both periods, however, these two univariate models outperform the GJR model.

The Monte Carlo results indicate that multivariate models perform better than univariate models when applied to the problem of portfolio VaR forecasting. However, one can possibly argue that these results might be driven by a specific choice of the parameters and the choice of DGP specification. In order to rule out this possibility, we have performed the same analysis with different parameter sets, and also with different specifications for the DGP. The results are very similar to those reported here, and are not reported to save space.

4 Empirical evaluation with real market data

In this Section we compare multivariate and univariate models by using real market data to forecast one-day-ahead VaR for a long position in equally-weighted portfolios. We are now interested in evaluating the performance of each model under more realistic situations, i.e. when the portfolio has a very large number of assets and is diversified, including not only stocks but also bonds, commodities and foreign currencies. This is usually the case in most financial institutions.

We analyze three systems of real market returns. The first data set is composed of daily returns of 48 US industry portfolios[5]. The second data set is a 30-asset global portfolio composed of daily

[5]The industry sectors included in the portfolio are: agriculture, food products, candy & soda, beer & liquor, tobacco products, recreation, entertainment, printing and publishing, consumer goods, apparel, healthcare, medical equipment, pharmaceutical products, chemicals, rubber and plastic products, textiles, construction materials, construction, steel, fabricated products, machinery, electrical equipment, automobiles and trucks, aircraft, shipbuilding and railroad equipment, defense, precious metals, non-metallic and industrial metal mining, coal, petroleum and natural gas, utilities, communication, personal services, business services, computers, electronic equipment, measuring and control equipment, business supplies, shipping containers, transportation, wholesale, retail, restaurants/hotels/motels, banking, insurance, real estate, trading, and other (sanitary services, steam, air conditioning supplies, irrigation systems, and cogeneration). The data set was downloaded from the web page of Kenneth French (`http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/`)

returns of 30 futures contracts grouped into 14 equity futures indices, four commodities, eight currencies, and four 10 year government bonds[6]. Finally, the third data set is composed of daily returns of all stocks belonging to the S&P100 index with common available observations during the time period considered in this paper. This yields a total of 81 stocks. The three systems of returns have been observed daily from 01/03/2000 to 31/07/2008 and the returns are computed as the differences in log prices. The data was downloaded from the Reuters Ecowin database. Table 3 shows the number of observations, the mean, standard deviation, skewness, and kurtosis for each data set. The statistics are based on an equally-weighted portfolio and the sample is divided into in- and out-of-sample observations. The fist $T - 500$ observations correspond to the in-sample period whereas the remaining 500 observations correspond to the out-of-sample period, where T is the length of each data set.

4.1 VaR estimation

For each of the three data sets analyzed, the within-sample observations are used to estimate the parameters and the remaining 500 observations are used to obtain out-of-sample forecasts. We obtain out-of-sample forecasts using a fixed estimation window similar as in the Monte Carlo simulations. In unreported results, we also analyzed the case in which forecasts were obtained via rolling windows, with similar conclusions. It is worth noting, however, that the computational effort of obtaining rolling windows forecasts is extremely high, since all multivariate models need to be re-estimated 500 times.

The following multivariate and univariate models are considered: DCC, AsyDCC, CCC, GARCH, GJR, and the semiparametric CAViaR model, yielding a total of three univariate and three multivariate models. For each of the parametric models, we fit both the model with Gaussian and with Student's t errors. Tables 4, 5 and 6 report the estimated parameters of all six models for each of the three data sets considered in this paper, respectively. The parameter estimates are similar to those found in previous works by other authors. For instance, the values of the DCC and AsyDCC parameters are similar to those reported in Cappiello et al. (2006), whereas the value of the CAViaR parameters are similar to those reported in Engle and Manganelli (2004). Similar as in Engle and Sheppard (2001), we found that the estimated news parameter, $\hat{\alpha}$, in the DCC models are small ($\hat{\alpha} < .01$), although significant. It is also worth highlighting some interested findings revealed in the estimation results. The estimated number of degrees of freedom v differ across Student's t distributed models. In general, univariate models tend to estimate a larger value for the degrees of freedom in comparison to their multivariate counterparts. The number of degrees of freedom in the multivariate Student's t models ranged from 10 to 19, similar to those reported in Pesaran and Pesaran (2007). Furthermore, the parameter δ associated to the asymmetric term in both univariate and multivariate models is significant in the majority of the cases. In particular, the asymmetric term in the AsyDCC model has a higher significance when a Student's t distribution

[6]The global portfolio is composed of 14 equity futures indices: S&P500, NASDAQ, DJIA, Canada 60, FTSE, CAC, DAX, IBEX, MIB, Nikkei, Hang Seng, SGX, Bovespa, IPC; four 10 year government bond futures contracts: US, UK, Germany, and Japan; four commodities futures contracts: gold, silver, wheat, and crude; eight currencies futures contracts: EUR, GBP, JPY, CAD, CHF, AUD, MXN, BRL. The overall portfolio is measured in dollars with returns on the futures contracts appropriately adjusted for rollovers.

is used to fit the model, suggesting that the asymmetric multivariate model is better fitted when fat-tailedness is taken into account. Finally, the estimated $\hat{\alpha}$ parameters in the GJR model is not significant in any of the three portfolios considered.

Figures 2 to 4 plot the out-of-sample VaRs predicted by each of the models for the three data sets according to each model. The evolution of the VaR estimates obtained by multivariate models tended to be smoother in comparison to multivariate models. In general, the VaR estimations obtained by all models have a similar evolution. However, albeit useful, this visual inspection does not allow us to draw an appropriate statistical evaluation of the accuracy of multivariate and univariate models. Therefore, we now proceed to the analysis of the Giacomini-White CPA test as described in Subsection 2.4.

Table 7 reports the results of the Giacomini-White CPA test for the 48 industry portfolio. In this Table, after each CPA coefficient, a left (up) arrow means that the model in the row outperforms (underperforms) the model in the column. p-values appear bellow each CPA coefficient. The upper and middle panels show the result for the Normally and Student's t distributed models, respectively, whereas the lower panel shows a comparison among them. The results for the Normally distributed models indicate that multivariate models outperformed univariate models. The best performance was achieved by the DCC model. The results for the Student's t distributed models also indicate that multivariate models performed better, and that the best performance was achieved by the CCC-t model. Finally, the lower panel in Table 7 indicates that Normally distributed multivariate models performed better than Student's t distributed univariate models. Overall, the best performance was achieved by the DCC model with Gaussian errors.

Table 8 reports the results of the Giacomini-White CPA test for the 30-asset global portfolio. The results for Normally distributed models show that multivariate models performed better than univariate models, and the best performing models are the DCC and CCC models. For the case of Student's t distributed models, multivariate models also outperformed univariate counterparts, and the best performance was achieved by the CCC-t model. Furthermore, similar as in the previous case, the lower panel results indicate that the overall best performance was also achieved by the DCC with Gaussian errors and CCC-t models.

Table 9 reports the results of the Giacomini-White CPA test for the S&P100 stocks. The results are very similar to those reported in Table 8: The Normally distributed multivariate models outperformed their univariate counterparts and among Student's t distributed models, the best performance was achieved by the CCC-t model. Finally, the comparison among the Normally and Student's t distributed models indicated that, similar as in the 30-asset global portfolio, the best overall performance was achieved by the DCC model with Gaussian errors.

Finally, it is worth noting that although the asymmetric term in the AsyDCC model is significant in the majority of the cases, this model is usually outperformed by the (symmetric) DCC model. This result is in line with our previous findings obtained via Monte Carlo simulations in Section 3. Moreover, we found that among Normally distributed models dynamic conditional correlations models are preferred, whereas among Student's t distributed model the preferred model is the one with constant conditional correlations. In any of the two cases, the best performing model is always multivariate.

4.2 Capital requirement analysis

Our aim in this Subsection is to compare, or relate, the statistical results previously obtained with the requirements established by the current regulatory framework set by the Basel II Accord. Under the framework of Basel II, the VaR estimates of the banks must be reported to the domestic regulatory authority. These estimates are used to compute the amount of regulatory capital requirements in order to control and monitor financial institutions' market risk exposure and to act as a cushion for adverse market conditions.

The empirical evidence presented by Berkowitz and O'Brien (2002) and Pérignon et al. (2008) show that banks systematically overestimate their VaR, which leads to an excessive amount of regulatory capital. Pérignon et al. (2008) conjecture that the causes of this overinflated VaR can be due to difficulties in aggregating the VaR across different business lines, or because banks don't want neither to put their reputation at risk nor to attract attention (internally and externally). In any of the cases, there is a cost of misestimating the VaR. As Pérignon et al. (2008) argue, one consequence of the exaggeration of banks' own level of risk is that they appear more risky than they actually are. Therefore, pursing models that deliver accurate estimates of this capital can lead to an increase in efficiency and in the accuracy of risk assessments made by investors.

Basel II allows banks to use internal models to obtain their VaR estimates. However, as McAleer and da Veiga (2008b) point out, if this is the case, the banks have to demonstrate that their models are accurate. This is done by means of a backtesting analysis based on the number of VaR violations, i.e. the number of times in which losses exceeded the estimated VaR. The Accord also establishes penalties for bad models in terms of a multiplicative factor k, which is never lower than 3 and it is based on the VaR estimates over the last 250 business days. The penalty zones are described in Table 10. The amount of capital charge is thus obtained by the following formula:

$$\text{Capital Requirement}_t = \max\left\{-VaR_{t-1}, -(3+k)\cdot\overline{VaR}_{60}\right\} \tag{19}$$

where $\overline{VaR}_{60}$ is the average VaR over the last 60 business days.

It is worth noting that, from a financial institution's point of view, it is desired to pursue a VaR model that yields minimum capital requirements, since this amount of regulatory of capital has an opportunity cost and could be employed in profitable activities. However, given the characteristics of the capital requirement in (19), lower levels of capital requirements could be achieved by adopting a VaR model that delivers a high number of violations, which is definitely not a desired outcome. Therefore, there is an important trade-off between capital requirements and number of VaR violations that should be taken into account when evaluating a set of models according to the Basel II criterion.

Table 11 reports the mean daily capital requirements (MDCC) and the number of VaR violations for the out-of-sample period. To facilitate the analysis, the number of VaR violations is reported separately for the first and for the second half of the out-of-sample period, i.e. the number of violations is based on 250 observations, which is equivalent to one trading year. Note that, according to the Basel II accord, this is the time period required to evaluate the number of VaR violations.

A result that immediately emerges from the Table 11 is that multivariate models delivered lower

MDCC in comparison to univariate models in the three data sets considered in this paper. For the industry portfolio the model that delivered lower MDCC is the DCC-t, whereas for the global portfolio and for the S&P100 stocks the model is the CCC and CCC-t, respectively. The superior performance of multivariate models in terms of MDCC coincides with our previous backtesting analysis based on the Giacomini and White (2006) CPA test. Another important result is that the number of VaR violations is higher in the second half of the out-of-sample period, in comparison to the first half. This is a reflection of a higher volatility clustering in this period (as can be seen in Figures 2, 3, and 4), thus increasing the occurrences of VaR exceptions.

5 Conclusions

Obtaining accurate risk measures can be seen as the most important objective of a VaR model. This paper addressed the question of whether multivariate or univariate models are most appropriate for the problem of portfolio VaR forecasting. We compare both types of models in the context of large and diversified portfolios as those are usually encountered in practice. We also consider complex dynamics of variances and covariances with asymmetries and dynamic correlations. Finally, the models are compared by using more appropriate statistics than those used in previous works. The results of comparative predictive performance for one-step-ahead portfolio VaR obtained with both Monte Carlo simulations and with real market data indicate that multivariate GARCH models outperformed competing univariate models on an out-of-sample basis. Furthermore, the results based on the backtesting analysis established by the Basel II Accord indicate that multivariate models delivered lower levels of daily capital requirements in comparison to univariate models. Considering that previous empirical evidence show that banks systematically overestimate their VaR and the amount of regulatory capital, we conclude that the use of multivariate models can improve the estimation of capital requirements, thus attenuating the costs associated to the overestimation of regulatory capital. As a recommendation for further research, it could be interesting to consider models with conditional means different from zero.

References

Bauwens, L., S. Laurent, and J. Rombouts (2006). Multivariate GARCH models: a survey. *Journal of Applied Econometrics 21*(1), 79–109.

Berkowitz, J. and J. O'Brien (2002). How accurate are value-at-risk models at commercial banks? *The Journal of Finance 57*(3), 1093–1111.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics 31*(3), 307–327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics 72*(3), 498–505.

Bollerslev, T., R. Engle, and J. Wooldridge (1988). A capital asset pricing model with time-varying covariances. *The Journal of Political Economy 96*(1), 116.

Bollerslev, T. and J. Wooldridge (1992). Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric reviews 11*(2), 143–172.

Brooks, C. and G. Persand (2003). Volatility forecasting for risk management. *Journal of Forecasting 22*(1), 1–22.

Cappiello, L., R. Engle, and K. Sheppard (2006). Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics 4*(4), 537–572.

Chib, S., F. Nardari, and N. Shephard (2006). Analysis of high dimensional multivariate stochastic volatility models. *Journal of Econometrics 134*(2), 341–371.

Christoffersen, P. (1998). Evaluating interval forecasts. *International Economic Review 39*(4), 841–862.

Christoffersen, P. (2009). Value-at-risk models. In T. Andersen, R. Davis, J.-P. Kreiss, and T. Mikosch (Eds.), *Handbook of Financial Time Series*. Springer Verlag.

Christoffersen, P., J. Hahn, and A. Inoue (2001). Testing and comparing value-at-risk measures. *Journal of Empirical Finance 8*(3), 325–342.

DeMiguel, V., L. Garlappi, and R. Uppal (2009). Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy? *Review of Financial Studies 22*(5), 1915–1953.

Engle, R. (2002). Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics 20*(3), 339–350.

Engle, R. and S. Manganelli (2001). Value-at-risk models in finance. *Working Paper n. 75, European Central Bank*.

Engle, R. and S. Manganelli (2004). CAViaR: conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics 22*(4), 367–382.

Engle, R. and K. Sheppard (2001). Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. *NBER Working Paper*.

Engle, R. and K. Sheppard (2008). Evaluating the specification of covariance models for large portfolios. *Working Paper, Department of Economics, University of Oxford*.

Giacomini, R. and I. Komunjer (2005). Evaluation and combination of conditional quantile forecasts. *Journal of Business & Economic Statistics 23*(4).

Giacomini, R. and H. White (2006). Tests of conditional predictive ability. *Econometrica 74*(6), 1545–1578.

Giot, P. and S. Laurent (2004). Modelling daily value-at-risk using realized volatility and ARCH type models. *Journal of Empirical Finance 11*(3), 379–398.

Glosten, L., R. Jagannathan, and D. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance 48*, 1779–1801.

Hentschel, L. (1995). All in the family: nesting symmetric and asymmetric GARCH models. *Journal of Financial Economics 39*(1), 71–104.

Kuester, K., S. Mittnik, and M. Paolella (2006). Value-at-risk prediction: a comparison of alternative strategies. *Journal of Financial Econometrics 4*(1), 53–89.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. *The Journal of Derivatives 3*(2), 73–84.

McAleer, M. (2009). The ten commandments for optimizing value-at-risk and daily capital charges. *Forthcoming, Journal of Economic Surveys*.

McAleer, M. and B. da Veiga (2008a). Forecasting value-at-risk with a parsimonious portfolio spillover GARCH (PS-GARCH) model. *Journal of Forecasting 27*, 1–19.

McAleer, M. and B. da Veiga (2008b). Single-index and portfolio models for forecasting value-at-risk thresholds. *Journal of Forecasting 27*(3), 217.

Pesaran, B. and M. Pesaran (2007). Modelling volatilities and conditional correlations in futures markets with a multivariate t distribution. *CESifo Working Paper No. 2056*.

Pesaran, M., C. Schleicher, and P. Zaffaroni (2008). Model averaging and value-at-risk based evaluation of large multi-asset volatility models for risk management. *Forthcoming, Journal of Empirical Finance*.

Pérignon, C., Z. Deng, and Z. Wang (2008). Do banks overstate their value-at-risk? *Journal of Banking & Finance 32*(5), 783–794.

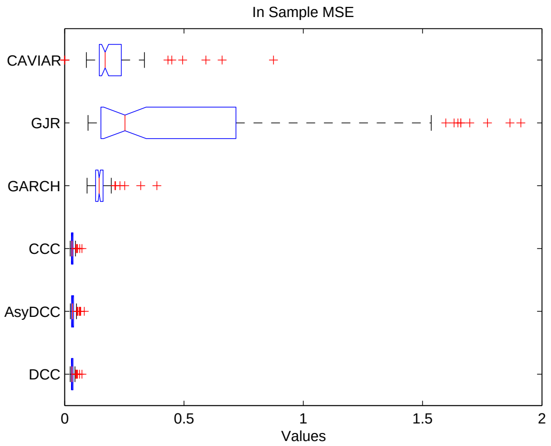
Sheppard, K. (2003). Multi-step estimation of multivariate GARCH models. In *Proceedings of the International ICSC. Symposium: Advanced Computing in Financial Markets.*

Silvennoinen, A. and T. Teräsvirta (2009). Multivariate GARCH models. In T. Andersen, R. Davis, J.-P. Kreiss, and T. Mikosch (Eds.), *Handbook of Financial Time Series.* Springer Verlag.

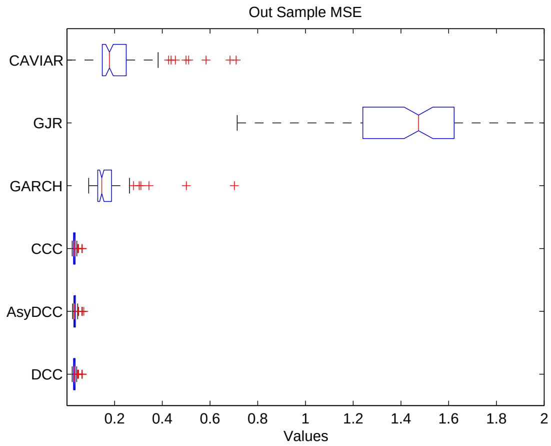
Tse, Y. and A. Tsui (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics 20*(3), 351–362.

Zivot, E. (2009). Practical issues in the analysis of univariate GARCH models. In T. Andersen, R. Davis, J.-P. Kreiss, and T. Mikosch (Eds.), *Handbook of Financial Time Series.* Springer Verlag.

Figure 1: Monte Carlo distribution of the mean squared error (MSE) of estimated VaR

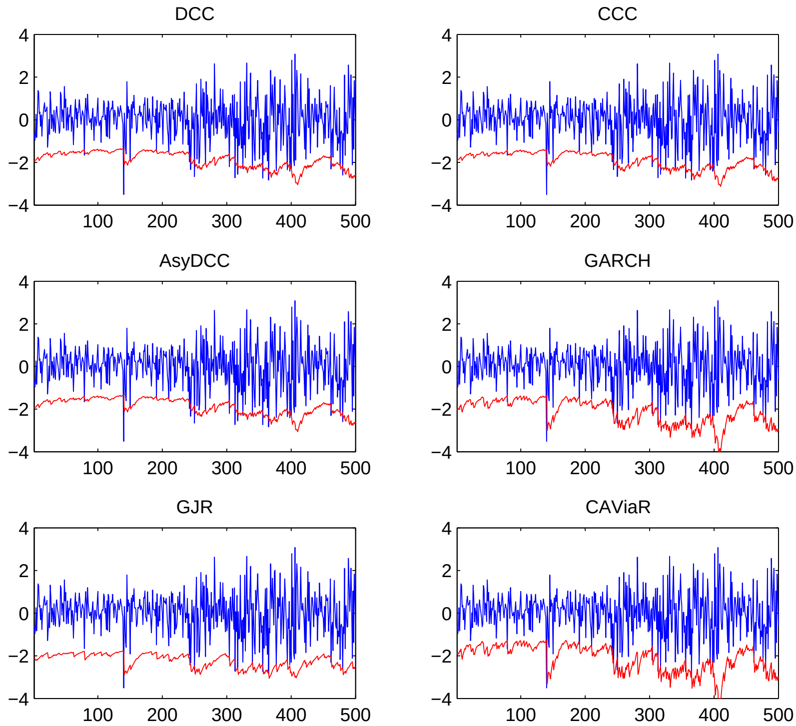


(a) In-sample

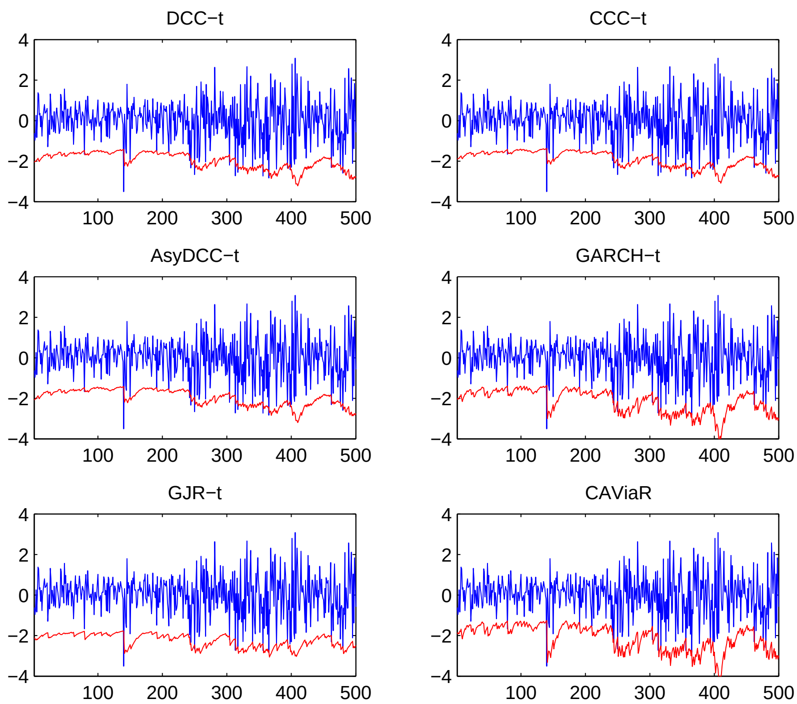


(b) Out-of-sample

Figure 2: Out-of-sample estimated VaRs for the industry portfolios

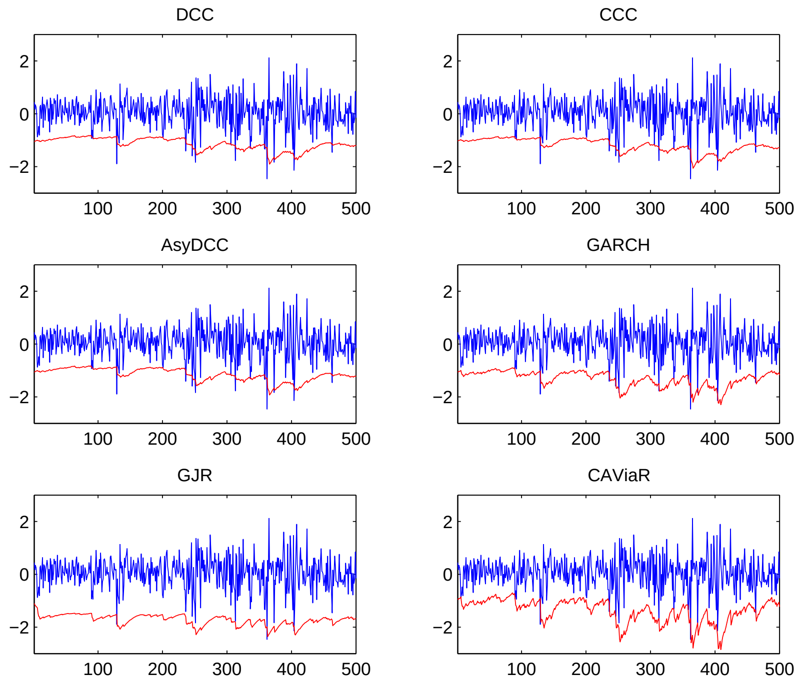


(a) Normally distributed models

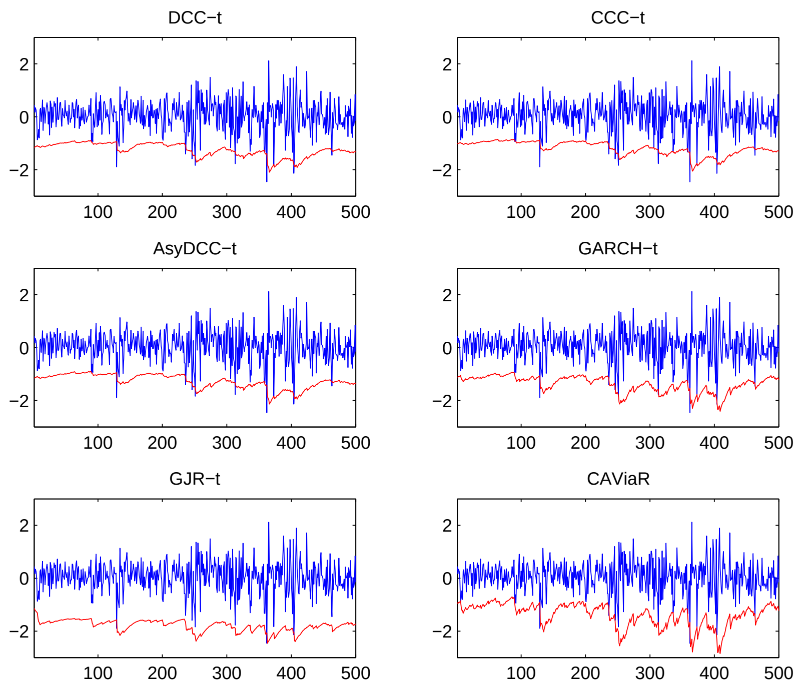


(b) Student's t distributed models

Figure 3: Out-of-sample estimated VaRs for the global portfolio

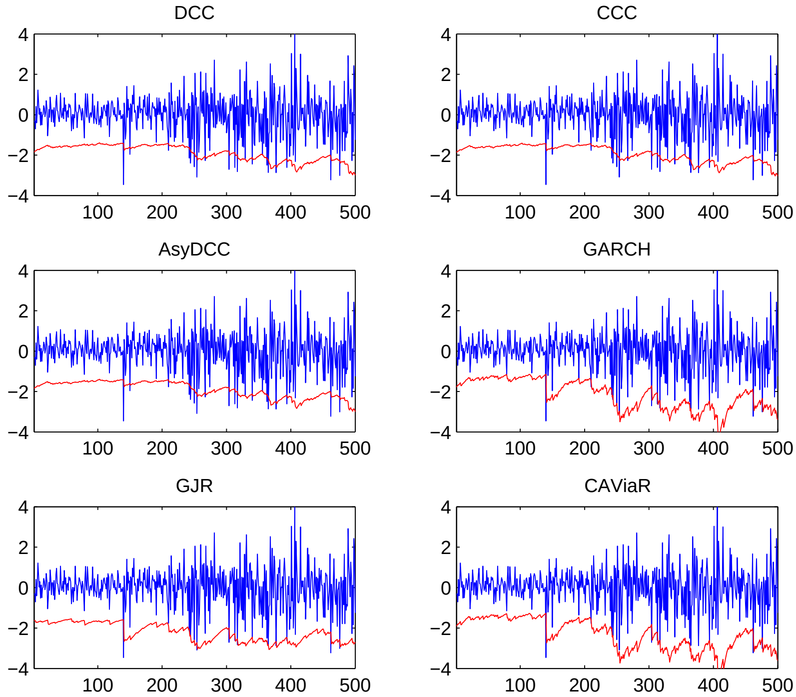


(a) Normally distributed models

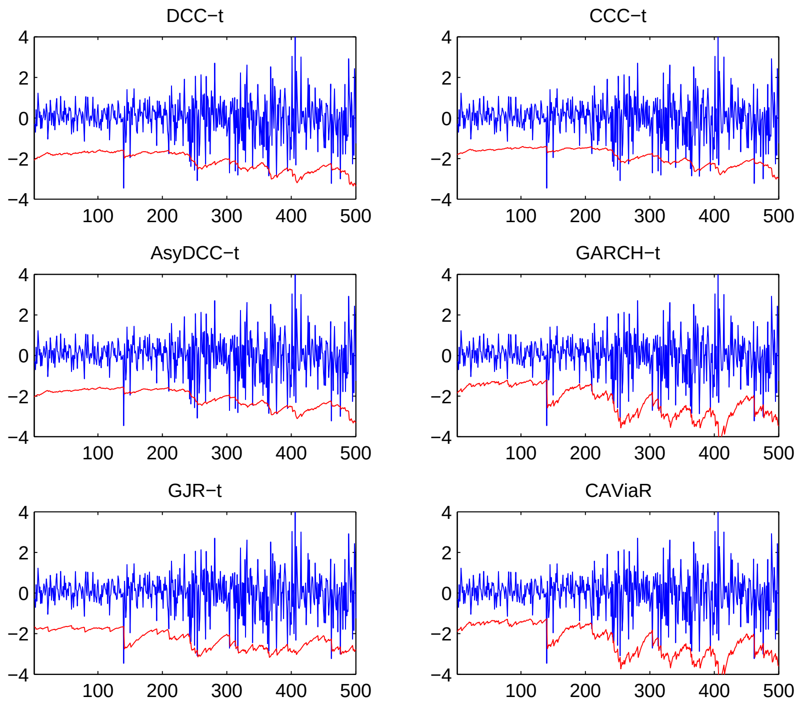


(b) Student's t distributed models

Figure 4: Out-of-sample estimated VaRs for the S&P 100 stocks



(a) Normally distributed models



(b) Student's t distributed models

Table 1: Parametrization of the simulated AsyDVEC(1,1) model with 10 assets

					C					
	1	2	3	4	5	6	7	8	9	10
1	4.121									
2	0.983	2.726								
3	0.769	-0.836	4.036							
4	1.426	0.011	-1.954	3.713						
5	0.189	0.640	-0.336	-0.721	4.529					
6	0.497	0.616	0.838	-0.279	1.139	2.158				
7	-0.226	0.227	0.886	-2.511	1.312	0.739	3.160			
8	-1.156	-0.439	-0.780	-0.518	-0.158	-1.213	-0.330	1.604		
9	1.307	0.537	0.978	-0.567	-1.060	1.154	0.936	-0.539	3.682	
10	0.607	-0.266	0.924	0.876	-0.179	0.607	-0.656	-0.376	0.216	2.790
					A					
	1	2	3	4	5	6	7	8	9	10
1	0.114									
2	0.054	0.104								
3	-0.026	0.001	0.132							
4	-0.033	-0.070	0.014	0.148						
5	0.007	0.039	-0.010	0.011	0.126					
6	-0.043	-0.003	0.102	-0.019	-0.016	0.133				
7	-0.054	0.003	-0.016	-0.019	0.031	0.030	0.100			
8	-0.009	-0.027	-0.017	0.074	0.005	-0.010	-0.003	0.085		
9	0.043	-0.014	-0.023	0.025	-0.003	-0.039	-0.045	0.010	0.093	
10	-0.043	-0.064	0.045	-0.026	-0.022	0.038	-0.009	-0.069	0.004	0.186
					B					
	1	2	3	4	5	6	7	8	9	10
1	0.757									
2	0.748	0.742								
3	0.744	0.736	0.734							
4	0.762	0.755	0.750	0.772						
5	0.775	0.768	0.762	0.782	0.796					
6	0.766	0.760	0.755	0.774	0.787	0.781				
7	0.757	0.749	0.744	0.766	0.776	0.769	0.761			
8	0.740	0.733	0.728	0.747	0.759	0.752	0.741	0.728		
9	0.771	0.764	0.760	0.779	0.791	0.783	0.773	0.755	0.789	
10	0.756	0.750	0.744	0.764	0.775	0.767	0.758	0.742	0.772	0.759
					G					
	1	2	3	4	5	6	7	8	9	10
1	5.6×10^{-5}									
2	4.3×10^{-5}	3.9×10^{-5}								
3	4.2×10^{-5}	3.5×10^{-5}	3.7×10^{-5}							
4	4.8×10^{-5}	4.0×10^{-5}	3.8×10^{-5}	4.5×10^{-5}						
5	4.8×10^{-5}	3.8×10^{-5}	3.8×10^{-5}	4.4×10^{-5}	4.5×10^{-5}					
6	4.3×10^{-5}	3.6×10^{-5}	3.8×10^{-5}	4.0×10^{-5}	4.0×10^{-5}	4.2×10^{-5}				
7	4.7×10^{-5}	3.8×10^{-5}	3.6×10^{-5}	4.1×10^{-5}	4.0×10^{-5}	3.7×10^{-5}	4.2×10^{-5}			
8	4.5×10^{-5}	4.0×10^{-5}	4.0×10^{-5}	4.3×10^{-5}	4.2×10^{-5}	4.3×10^{-5}	4.0×10^{-5}	4.7×10^{-5}		
9	4.7×10^{-5}	3.9×10^{-5}	3.7×10^{-5}	4.4×10^{-5}	4.3×10^{-5}	3.9×10^{-5}	4.0×10^{-5}	4.2×10^{-5}	4.3×10^{-5}	
10	4.8×10^{-5}	3.9×10^{-5}	3.7×10^{-5}	4.2×10^{-5}	4.0×10^{-5}	3.8×10^{-5}	4.2×10^{-5}	4.2×10^{-5}	4.1×10^{-5}	4.3×10^{-5}

Table 2: Monte Carlo results on the Giacomini and White (2006) CPA test for the comparison among multivariate and univariate models

The Table reports the number of times (over 100 Monte Carlo simulations) in which multivariate and univariate models outperform each other according to the CPA test.

Multivariate versus Univariate	Multivariate preferred	Univariate Preferred	Indifferent
In Sample			
DCC versus GARCH	14	2	84
DCC versus GJR	45	4	51
DCC versus CAVIAR	14	5	81
AsyDCC versus GARCH	14	3	83
AsyDCC versus GJR	44	5	51
AsyDCC versus CAVIAR	13	4	83
CCC versus GARCH	14	2	84
CCC versus GJR	45	4	51
CCC versus CAVIAR	14	3	83
Out Sample			
DCC versus GARCH	40	3	57
DCC versus GJR	99	0	1
DCC versus CAVIAR	49	2	44
AsyDCC versus GARCH	42	3	55
AsyDCC versus GJR	99	0	1
AsyDCC versus CAVIAR	48	3	44
CCC versus GARCH	40	3	57
CCC versus GJR	100	0	0
CCC versus CAVIAR	49	2	44

Only multivariate	Model 1 preferred	Model 2 preferred	Indifferent
In Sample			
DCC versus AsyDCC	58	29	13
DCC versus CCC	38	34	28
AsyDCC versus CCC	34	49	17
Out Sample			
DCC versus AsyDCC	48	37	15
DCC versus CCC	38	35	27
AsyDCC versus CCC	28	54	18

Only univariate	Model 1 preferred	Model 2 preferred	Indifferent
In Sample			
GARCH versus GJR	43	15	42
GARCH versus CAVIAR	0	45	55
GJR versus CAVIAR	2	70	28
Out Sample			
GARCH versus GJR	100	0	0
GARCH versus CAVIAR	34	21	45
GJR versus CAVIAR	4	90	6

Table 3: Descriptive statistics for the three data sets considered in this paper

	Number of assets	Number of obs.	Mean ($\times$ 100)	Std. dev.	Kurtosis	Skewness
Industry portfolio	48					
In sample		1,617	0.087	0.866	4.582	-0.235
Out sample		500	0.014	0.993	3.772	-0.327
S&P100 stocks	81					
In sample		1,656	0.013	1.045	5.983	0.013
Out sample		500	0.007	0.997	4.688	-0.331
Global portfolio	30					
In sample		1,694	0.020	0.549	4.088	-0.084
Out sample		500	0.047	0.595	4.496	-0.407

Note: descriptive statistics are based on an equally-weighted portfolio.

Table 4: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: industry portfolios

	ω	α	β	δ	v
DCC		0.0046	0.9482		
		(0.0007)	(0.0112)		
DCC-t		0.0053	0.9333		18.8528
		(0.0006)	(0.0109)		(1.1192)
AsyDCC		0.0032	0.9510	0.0045	
		(0.0005)	(0.0093)	(0.0017)	
AsyDCC-t		0.0037	0.9373	0.0054	19.0262
		(0.0005)	(0.0086)	(0.0009)	(1.1678)
GARCH	0.0370	0.1098	0.8408		
	(0.0107)	(0.0179)	(0.0262)		
GARCH-t	0.0366	0.1069	0.8438		69.0487
	(0.0108)	(0.0181)	(0.0265)		(64.0360)
GJR	0.0395	0.0170	0.8633	0.1464	
	(0.0085)	(0.0118)	(0.0218)	(0.0245)	
GJR-t	0.0395	0.0169	0.8635	0.1462	531.3362
	(0.0085)	(0.0116)	(0.0216)	(0.0245)	(217.0499)
CAVIAR	0.2086	0.8018	0.7756		
	(0.1672)	(0.0602)	(0.3360)		

Table 5: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: global portfolio

	ω	α	β	δ	v
DCC		0.0080	0.9581		
		(0.0016)	(0.0145)		
DCC-t		0.0080	0.9615		10.085
		(0.0013)	(0.0109)		(0.7691)
AsyDCC		0.0074	0.9584	0.0018	
		(0.0016)	(0.0141)	(0.0016)	
AsyDCC-t		0.0072	0.9594	0.0036	10.211
		(0.0011)	(0.0111)	(0.0012)	(0.7914)
GARCH	0.0114	0.0652	0.8970		
	(0.0041)	(0.0124)	(0.0194)		
GARCH-t	0.0100	0.0626	0.9047		12.4795
	(0.0036)	(0.0120)	(0.0183)		(3.3764)
GJR	0.0131	0.0039	0.9086	0.0894	
	(0.0036)	(0.0121)	(0.0182)	(0.0199)	
GJR-t	0.0126	0.0046	0.9095	0.0896	15.586
	(0.0035)	(0.0109)	(0.0191)	(0.0209)	(5.2779)
CAVIAR	0.032	0.8661	0.7026		
	(0.0527)	(0.0408)	(0.8170)		

Table 6: Estimated parameters with corresponding asymptotic standard errors in parenthesis. Data set: S&P100

	ω	α	β	δ	v
DCC		0.0020	0.8869		
		(0.0007)	(0.0480)		
DCC-t		0.0044	0.7403		13.341
		(0.001)	(0.069)		(0.5680)
AsyDCC		0.0020	0.8869	0.000	
		(0.0006)	(0.0507)	(0.0011)	
AsyDCC-t		0.0039	0.7538	0.001	13.340
		(0.001)	(0.079)	(0.001)	(0.5706)
GARCH	0.0120	0.0802	0.908		
	(0.0043)	(0.0129)	(0.0142)		
GARCH-t	0.0120	0.0754	0.9123		16.275
	(0.0046)	(0.0137)	(0.0153)		(5.5639)
GJR	0.0108	0.0012	0.9271	0.1224	
	(0.0031)	(0.0087)	(0.0120)	(0.0192)	
GJR-t	0.0117	0.0000	0.9257	0.1251	19.160
	(0.0034)	(0.0052)	(0.0111)	(0.0200)	(7.0425)
CAVIAR	0.0922	0.9041	0.4983		
	(0.0496)	(0.0124)	(0.2253)		

Table 7: Giacomini and White (2006) CPA test results. Data set: industry portfolios. The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAViaR
DCC	**16.222**$^{\leftarrow}$	**17.324**$^{\leftarrow}$	**15.810**$^{\leftarrow}$	**18.245**$^{\leftarrow}$	**13.433**$^{\leftarrow}$
	0.000	0.000	0.000	0.000	0.001
AsyDCC		**16.289**$^{\leftarrow}$	**15.860**$^{\leftarrow}$	**17.926**$^{\leftarrow}$	**13.396**$^{\leftarrow}$
		0.000	0.000	0.000	0.001
CCC			**17.649**$^{\leftarrow}$	**15.918**$^{\leftarrow}$	**14.165**$^{\leftarrow}$
			0.000	0.000	0.001
GARCH				**27.875**$^{\uparrow}$	**9.487**$^{\leftarrow}$
				0.000	0.009
GJR					**29.676**$^{\leftarrow}$
					0.000

Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	3.264$^{\uparrow}$	**18.511**$^{\uparrow}$	**16.647**$^{\leftarrow}$	**13.592**$^{\leftarrow}$	**11.572**$^{\leftarrow}$
	0.196	0.000	0.000	0.001	0.003
AsyDCC-t		**15.255**$^{\uparrow}$	**16.857**$^{\leftarrow}$	**13.757**$^{\leftarrow}$	**11.628**$^{\leftarrow}$
		0.000	0.000	0.001	0.003
CCC-t			**17.812**$^{\leftarrow}$	**16.529**$^{\leftarrow}$	**14.137**$^{\leftarrow}$
			0.000	0.000	0.001
GARCH-t				**31.700**$^{\uparrow}$	**11.486**$^{\uparrow}$
				0.000	0.003
GJR-t					**29.367**$^{\leftarrow}$
					0.000

Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**19.846**$^{\leftarrow}$	**17.636**$^{\leftarrow}$	**10.976**$^{\leftarrow}$	**16.150**$^{\leftarrow}$	**18.345**$^{\leftarrow}$
	0.000	0.000	0.004	0.000	0.000
AsyDCC		**17.233**$^{\leftarrow}$	**6.625**$^{\leftarrow}$	**16.204**$^{\leftarrow}$	**18.031**$^{\leftarrow}$
		0.000	0.036	0.000	0.000
CCC			**8.875**$^{\uparrow}$	**17.995**$^{\leftarrow}$	**16.056**$^{\leftarrow}$
			0.012	0.000	0.000
GARCH				**12.900**$^{\leftarrow}$	**27.291**$^{\uparrow}$
				0.002	0.000
GJR					**29.784**$^{\leftarrow}$
					0.000

Table 8: Giacomini and White (2006) CPA test results. Data set: global portfolio.
The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAVIAR
DCC	**4.883**$^{\leftarrow}$	2.728$^{\leftarrow}$	**13.360**$^{\leftarrow}$	**49.510**$^{\leftarrow}$	**11.162**$^{\leftarrow}$
	0.087	0.256	0.001	0.000	0.004
AsyDCC		2.295$^{\leftarrow}$	**13.671**$^{\leftarrow}$	**50.847**$^{\leftarrow}$	**11.038**$^{\leftarrow}$
		0.318	0.001	0.000	0.004
CCC			**13.469**$^{\leftarrow}$	**56.558**$^{\leftarrow}$	**10.374**$^{\leftarrow}$
			0.001	0.000	0.006
GARCH				**69.851**$^{\leftarrow}$	**3.259**$^{\leftarrow}$
				0.000	0.196
GJR					**61.645**$^{\uparrow}$
					0.000

Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	**20.533**$^{\leftarrow}$	**20.639**$^{\uparrow}$	**14.317**$^{\leftarrow}$	**69.136**$^{\leftarrow}$	**5.571**$^{\leftarrow}$
	0.000	0.000	0.001	0.000	0.062
AsyDCC-t		**18.935**$^{\uparrow}$	**12.644**$^{\leftarrow}$	**70.958**$^{\leftarrow}$	**4.349**$^{\leftarrow}$
		0.000	0.002	0.000	0.114
CCC-t			**18.352**$^{\leftarrow}$	**64.459**$^{\leftarrow}$	**10.384**$^{\leftarrow}$
			0.000	0.000	0.006
GARCH-t				**89.394**$^{\leftarrow}$	2.885$^{\uparrow}$
				0.000	0.236
GJR-t					**78.280**$^{\uparrow}$
					0.000

Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**10.050**$^{\leftarrow}$	**10.223**$^{\leftarrow}$	2.831$^{\leftarrow}$	**15.649**$^{\leftarrow}$	**56.825**$^{\leftarrow}$
	0.007	0.006	0.243	0.000	0.000
AsyDCC		**10.817**$^{\leftarrow}$	2.426$^{\leftarrow}$	**16.229**$^{\leftarrow}$	**58.237**$^{\leftarrow}$
		0.005	0.297	0.000	0.000
CCC			**4.473**$^{\leftarrow}$	**18.360**$^{\leftarrow}$	**64.353**$^{\leftarrow}$
			0.107	0.000	0.000
GARCH				**25.513**$^{\leftarrow}$	**80.937**$^{\leftarrow}$
				0.000	0.000
GJR					**109.325**$^{\leftarrow}$
					0.000

Table 9: Giacomini and White (2006) CPA test results. Data set: S&P100 stocks.
The Table shows test statistics of Giacomini and White (2006) conditional predictive ability test (with p-values). A left (up) arrow means that the model in the row outperforms (underperforms) the model in the column.

Gaussian models	AsyDCC	CCC	GARCH	GJR	CAViaR
DCC	**14.205**$^{\leftarrow}$	**16.432**$^{\leftarrow}$	**4.727**$^{\leftarrow}$	**13.995**$^{+}$	**10.388**$^{\leftarrow}$
	0.001	0.000	0.094	0.001	0.006
AsyDCC		**16.432**$^{\leftarrow}$	**4.727**$^{\leftarrow}$	**13.995**$^{\leftarrow}$	**10.388**$^{\leftarrow}$
		0.000	0.094	0.001	0.006
CCC			3.894$^{\leftarrow}$	**12.949**$^{\leftarrow}$	**9.800**$^{\leftarrow}$
			0.143	0.002	0.007
GARCH				**16.279**$^{\uparrow}$	**29.379**$^{\leftarrow}$
				0.000	0.000
GJR					**22.241**$^{\leftarrow}$
					0.000
Student's t models	AsyDCC-t	CCC-t	GARCH-t	GJR-t	CAVIAR
DCC-t	**8.166**$^{\uparrow}$	**18.973**$^{\uparrow}$	0.660$^{\leftarrow}$	3.892$^{\leftarrow}$	3.122$^{\leftarrow}$
	0.017	0.000	0.719	0.143	0.210
AsyDCC-t		**18.593**$^{\uparrow}$	0.664$^{\leftarrow}$	**5.714**$^{\leftarrow}$	3.420$^{\leftarrow}$
		0.000	0.718	0.057	0.181
CCC-t			**7.185**$^{\leftarrow}$	**15.883**$^{\leftarrow}$	**9.993**$^{\leftarrow}$
			0.028	0.000	0.007
GARCH-t				**24.386**$^{\uparrow}$	**31.471**$^{\leftarrow}$
				0.000	0.000
GJR-t					**30.572**$^{\leftarrow}$
					0.000
Gaussian vs. Student's t models	DCC-t	AsyDCC-t	CCC-t	GARCH-t	GJR-t
DCC	**18.752**$^{\leftarrow}$	**16.908**$^{\leftarrow}$	**2.730**$^{\leftarrow}$	**7.429**$^{\leftarrow}$	**16.429**$^{\leftarrow}$
	0.000	0.000	0.255	0.024	0.000
AsyDCC		**16.908**$^{\leftarrow}$	2.730$^{\leftarrow}$	**7.429**$^{\leftarrow}$	**16.429**$^{\leftarrow}$
		0.000	0.255	0.024	0.000
CCC			**8.386**$^{\uparrow}$	**6.670**$^{\leftarrow}$	**15.652**$^{\leftarrow}$
			0.015	0.036	0.000
GARCH				**24.785**$^{\leftarrow}$	**12.802**$^{\leftarrow}$
				0.000	0.002
GJR					**28.130**$^{\leftarrow}$
					0.000

Table 10: Basel II penalty zones

Zone	Number of violations	Increase in k
Green	0-4	0.00
Yellow	5	0.40
	6	0.50
	7	0.65
	8	0.75
	9	0.85
Red	>10	1.00

Note: based on the number of violations for 250 business days.

Table 11: Mean daily capital requirements and number of VaR violations

	Mean daily capital requirement (%)			VaR violations first 250 out-sample obs.			VaR violations last 250 out-sample obs.		
	Industry portfolios	Global portfolio	S&P100 stocks	Industry portfolio	Global portfolio	S&P100 stocks	Industry portfolios	Global portfolio	S&P100 stocks
Gaussian models									
DCC	5.02	3.06	4.61	7	5	5	15	10	14
AsyDCC	5.04	3.08	4.61	7	5	5	15	10	14
CCC	5.10	3.04	4.66	7	4	5	13	9	13
GARCH	5.30	3.31	5.10	5	3	5	11	8	8
GJR	5.29	3.67	5.17	1	1	3	9	2	9
CAVIAR	5.24	3.34	5.13	5	4	4	11	5	5
Student's t models									
DCC-t	4.95	3.07	5.17	4	3	5	12	7	12
AsyDCC-t	4.95	3.10	5.11	4	3	5	12	6	12
CCC-t	5.05	3.05	4.58	7	5	5	13	8	14
GARCH-t	5.34	3.44	5.15	5	3	4	10	7	7
GJR-t	5.30	3.82	5.20	1	1	2	9	2	8
CAVIAR	5.24	3.34	5.13	5	4	4	11	5	5