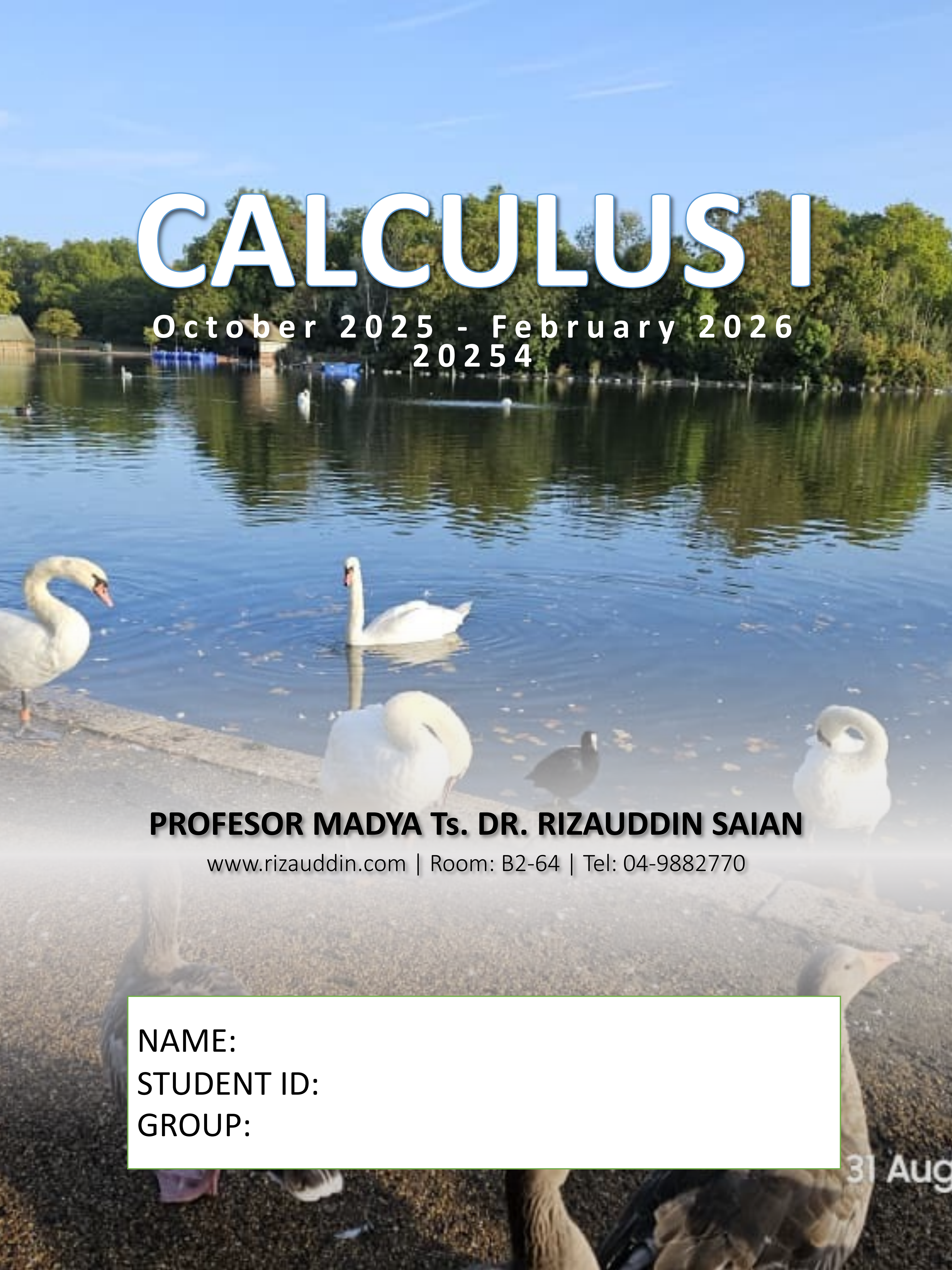

CALCULUS I

October 2025 - February 2026
20254

PROFESOR MADYA Ts. DR. RIZAUDDIN SAIAN
www.rizauddin.com | Room: B2-64 | Tel: 04-9882770

NAME:
STUDENT ID:
GROUP:

Calculus I Workbook

Profesor Madya Ts. Dr. Rizauddin Saian
https://www.rizauddin.com | Room: B2-64 | Tel: 04-9882770

Semester October 2025 - February 2026 (20254)

Scheme of Works

MAT183-20254

Week	Date	Topic
		Updated on 23/9/2025
1	6/10/25 - 10/10/25	**1. FUNCTIONS, LIMITS AND CONTINUITY** 1.1 Functions - Properties of functions - Operations on functions - Graphs (Family) of functions 1.2 Limits - Introduction (An intuitive approach) - One sided limits and two sided limits
2	13/10/25 - 17/10/25	- Infinite limits and limits at infinity - Computing limits - Computing limits: End behaviour
3	20/10/25 - 24/10/25	1.3 Continuity - Continuity at a point - Limits and Continuity of Trigonometric Functions
	20/10/25 (Monday)	**Deepavali**
4	27/10/25 - 31/10/25	**2. DIFFERENTIATION** 2.1 An Introduction to the Derivative: Tangents 2.2 The Definition of Derivative - Differentiation from the first principle - Differentiability and continuity 2.3 Techniques of Differentiation
5	3/11/25 - 7/11/25	2.4 Derivatives of Trigonometric, Exponential and Logarithmic Functions 2.5 The Chain Rule
		Assessment 1 - Test (20%) - 1.2, 1.3, 2.1, 2.2
6	10/11/25 - 14/10/25	2.6 Implicit Differentiation 2.7 Linear Approximations and Differentials **3. APPLICATIONS OF DIFFERENTIATION** 3.1 Related Rates

Week	Date	Topic
		3.1 Related Rates
		Assessment 2 - Video Presentation (15%) Ch 2.3-2.7
7	12/5 - 16/6/2025	3.2 Analysis of Functions I: - Intervals of increasing and decreasing functions - Concavity and inflection points 3.3 Analysis of Functions II: - Relative maxima and minima - Critical numbers - First and Second Derivative Tests - Graphs of Polynomial Functions
	12/5/2025 (Mon)	**Wesak Day**
8	19/5 - 23/5/2025	- Graphs of Rational Functions 3.4 Maximum and Minimum Values - Absolute maxima and minima 3.5 Applied Maximum and Minimum Problems
9	26/5 - 29/5/2025	3.12 Rolle's Theorem; Mean-Value Theorem **4. INTEGRATION** 4.1 Antiderivatives 4.2 The indefinite integral of algebraic functions, trigonometric functions, exponential and logarithmic Functions
	30/5 - 8/6/2025	**Mid-Semester Break / Special Break**
		Raya Haji: 7/6 - 8/6/2025, **Pesta Menuai**: 30/5 – 31/5/2025, **Gawai**: 1/6 – 2/6/2025
10	9/6 - 13/6/2025	4.3 Integration by Substitution 4.4 Sigma notation: Area as a Limit and the definite integrals
11	16/6 - 20/6/2025	- Properties of Definite Integrals 4.5 Fundamental Theorems of Calculus - First Fundamental Theorem of Calculus

Week	Date	Topic
12	23/6 - 27/6/2025	- Evaluating Definite Integrals by Substitution **5. APPLICATIONS OF INTEGRATION** 5.1 Area Between Two Curves
		Assessment 3 - Lab Assignment (10%) - Ch 3.1-3.6
	27/6/2025 (Fri)	**Awal Muharram**
13	30/6 - 4/7/2025	5.2 Volumes: Solids of Revolution - Volume by Disks Method - Volumes by Washer Method
14	7/7 - 11/7/2025	5.4 Volumes by Cylindrical Shells Method
	14/7 - 18/7/2025	**Revision Week**
	21/7 - 10/8/2025 (3 weeks)	**Final Examination**
	11/8 - 28/9/2025 (7 weeks)	**Semester Break**

Ref: https://hea.uitm.edu.my/v4/index.php/calendars/academic-calendar
https://www.perlis.gov.my/index.php/suk-perlis/info-umum/hari-kelepasan-am-negeri-perlis

Assessment:

Final Assessment : **40% (2 hours)**

Continuous assessment : **60%**

1. Test : 20% - 1 hour - Ch 1.2, 1.3, 2.1, 2.2
2. Video Presentation (Group) : 15% - 2 weeks - Ch 2.3-2.7
3. Lab Assignment (Individual) : 25% - 2 weeks - Ch 3.1-3.7

Recommended Text:

1. Stewart, J., Clegg, D. (2020). Calculus: Early Transcendentals. Singapore: Cengage Learning. [ISBN: 9780357113516]

References

1. Anton, H., Bivens, I. C., Davis, S. (2005). Calculus: Early Transcendentals Single Variable. United States: Wiley. [ISBN: 9781119244912]
2. Shamsatun Nahar Ahmad, Farah Suraya Md Nasrudin, Muhammad Yassar Yusri 2020, Fundamentals Of Calculus, 1 Ed., UiTM Cawangan Johor [ISBN: 9789673636044]
3. Hass, J. R., Heil, C. E., & Weir, M. D. (2019). Thomas' Calculus: Early Transcendentals in SI Units (14th edition). Pearson. [ISBN: 9781292253114]
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5. Adams, R., & Essex, C. (2009). Calculus: A Complete Course, Seventh Edition (7th edition). Pearson Education Canada. [ISBN: 9780321549280].
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Preface

Welcome to the world of Calculus I! This workbook is designed to be your trusted companion on your journey through the fundamental concepts of calculus. Whether you are a student gearing up for your first encounter with calculus, an educator looking for comprehensive teaching materials, or someone seeking to refresh their calculus skills, this workbook is tailored to meet your needs.
This workbook is carefully crafted to guide you through the essential topics of Calculus I, starting from the basic principles of limits and continuity and progressing to the heart of calculus: differentiation and integration. Each chapter is structured to provide a clear explanation of key concepts, accompanied by exercises that reinforce your understanding. The exercises are designed to challenge you and encourage active learning, allowing you to practice and master the skills necessary to solve calculus problems with confidence.

Key Features of This Workbook:

1. **Comprehensive Coverage:** This workbook covers all the fundamental concepts of Calculus I, ensuring that you have a strong foundation for advanced calculus and related subjects.
2. **Clarity and Accessibility:** Complex topics are explained in a clear and concise manner, making the material accessible to learners of all levels.
3. **Practice Exercises:** Ample practice exercises are provided throughout the workbook, ranging from basic to advanced levels of difficulty.

Remember, learning calculus is a gradual process that requires patience, practice, and perseverance. By working through this workbook diligently, you will not only grasp the principles of calculus but also develop the confidence to apply your knowledge to diverse challenges.
Best of luck in your studies, and may your exploration of calculus be both rewarding and enlightening!
Happy Learning Calculus!

Acknowledgements

As I embark on the task of acknowledging those whose support and inspiration have been instrumental in the creation of this workbook, I find myself deeply grateful to the remarkable individuals who have shaped my journey in mathematics education.

First and foremost, I express my heartfelt gratitude to my wife, whose unwavering support, encouragement, and patience have been my constant pillars. Her belief in my work and her boundless love have provided me with the strength to undertake this endeavor. To my daughters, who have brought immense joy and laughter into our lives, thank you for your understanding during the long hours spent crafting these pages. Your presence has been my source of inspiration, reminding me of the importance of education for future generations.

I extend my sincere appreciation to my teachers, whose passion for teaching ignited the spark of curiosity within me. Their dedication to nurturing young minds has been a guiding light, and I am forever indebted for their wisdom and guidance. It is through their teachings that I found my love for mathematics, a passion I aim to instill in others through this workbook.

To my previous students, your enthusiasm, questions, and thirst for knowledge have been the driving force behind this project. Your engagement in the classroom has challenged me to find innovative ways to explain complex concepts, and your success stories continue to inspire me. Each interaction with you has reinforced my belief in the transformative power of education.

I am also grateful for the support and encouragement I have received from colleagues, friends, and mentors. Your insights and discussions have enriched my understanding of calculus and teaching methodologies, shaping the content of this workbook.

Lastly, I extend my thanks to the countless authors, researchers, and educators whose contributions to the field of mathematics have paved the way for innovative teaching approaches. Your work has been a wellspring of knowledge and ideas, shaping the content of this workbook and enriching the learning experience for readers.

To all of you, I offer my deepest gratitude. This workbook stands as a testament to the collective effort and shared passion for education. May it serve as a valuable resource for learners, igniting the same love for calculus that has been kindled in me by the remarkable individuals in my life.

With heartfelt appreciation,
Rizauddin Saian
8 Oct 2023

Contents

1 Functions, Limits and Continuity

1.1 Find the Domain of a Function

Find the domain of the following functions.

1 $f(x) = x + 4$

Ans: $(-\infty, +\infty)$

2 $f(x) = x^2 + 4x + 5$

Ans: $(-\infty, +\infty)$

3 $f(x) = 4$

Ans: $(-\infty, +\infty)$

4 $f(x) = \frac{1}{4}$

Ans: $(-\infty, +\infty)$

5 $f(x) = -\sqrt{3}$

Ans: $(-\infty, +\infty)$

6 $f(x) = 0$

Ans: $(-\infty, +\infty)$

7 $f(x) = x^3 - 5$

Ans: $(-\infty, +\infty)$

8 $f(x) = \frac{2}{x+3}$

Ans: $(-\infty, -3) \cup (-3, +\infty)$

9 $f(x) = \frac{2}{x^2+3}$

Ans: $(-\infty, +\infty)$

10 $f(x) = \sqrt{x+3}$

Ans: $[-3, +\infty)$

11 $f(x) = \sqrt{x^2+3}$

Ans: $(-\infty, +\infty)$

12 $f(x) = \sqrt{x^2+2x-8}$

Ans: $(-\infty, -4] \cup [2, +\infty)$

13 $f(x) = \log(x+3)$

Ans: $(-3, +\infty)$

14 $f(x) = \log\left(x^2+3\right)$

Ans: $(-\infty, +\infty)$

15 $f(x) = \log(x^2+2x-8)$

Ans: $(-\infty, -4) \cup (2, +\infty)$

16 $f(x) = 2^{x+3}$

Ans: $(-\infty, +\infty)$

17 $f(x) = \dfrac{\sqrt{x+3}}{x-2}$

Ans: $[-3, 2) \cup (2, +\infty)$

18 $f(x) = \dfrac{x+3}{\log(x-2)}$

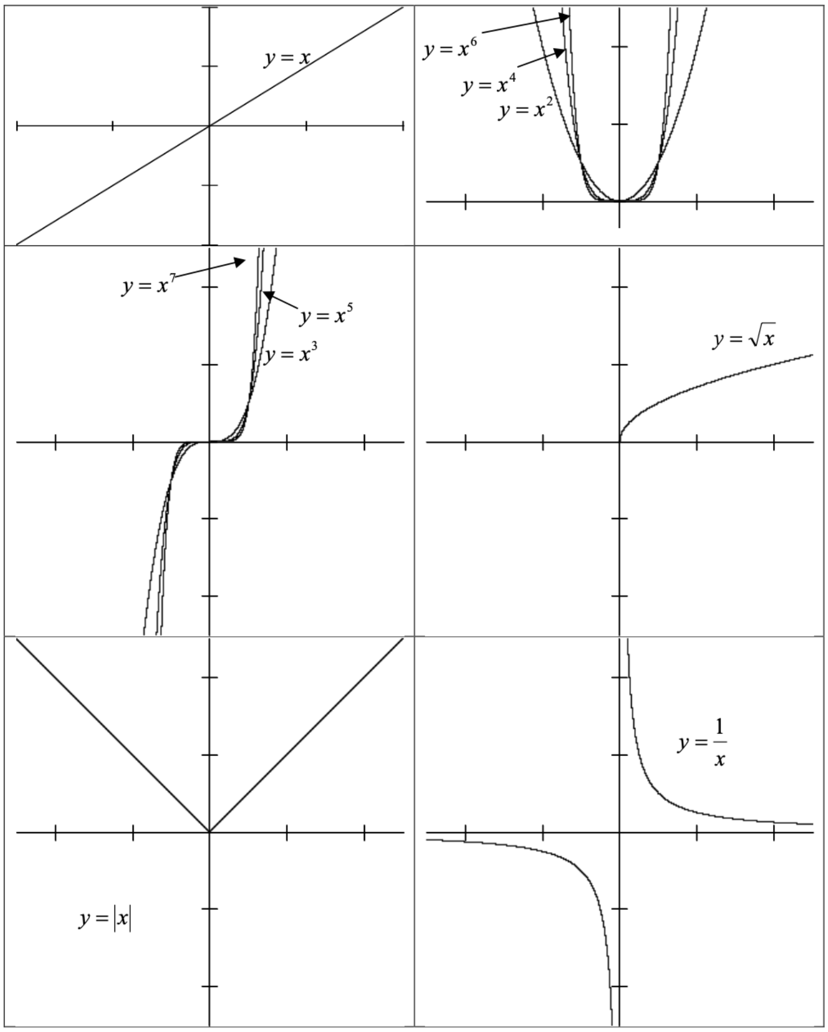
Ans: $(2, 3) \cup (3, +\infty)$

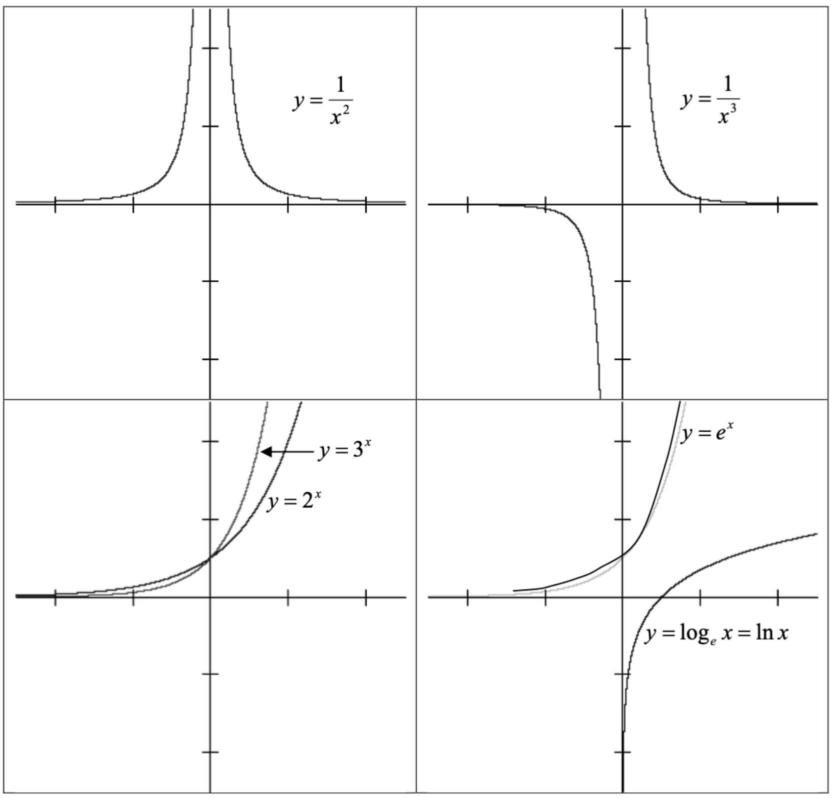
19 $f(x) = \dfrac{\sqrt{x+3}}{x^2-16}$

Ans: $[-3, 4) \cup (4, +\infty)$

Scan for guides

1.2 Graphs of Functions





1.3 Limits

1. One sided limits:

   (a) the limit of $f(x)$ as x approaches a from the right is L.

   $$\lim_{x\to a^+} f(x) = L$$

   (b) the limit of $f(x)$ as x approaches a from the left is L.

   $$\lim_{x\to a^-} f(x) = L$$

2. Two sided limits:

   (a) What is the limit of the following function as x approaches a?

   $$\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

   (b) If there is no sign in the limit notation, then you're being asked for a two-sided limit. You only have a two-sided limit if your left and right limits agree. The existence of a limit from the left-hand side does not imply that you have a right-sided limit. When we say that something has a limit, then we mean that it has an actual numeric value.

3. Infinite limits

   (a) Increases without bound

   $$\lim_{x\to a} f(x) = +\infty$$

   (b) Decreases without bound

   $$\lim_{x\to a} f(x) = -\infty$$

4. Limits at infinity

   (a) $\lim_{x\to+\infty} f(x) = L$: $f(x) \to L$ as $x \to +\infty$

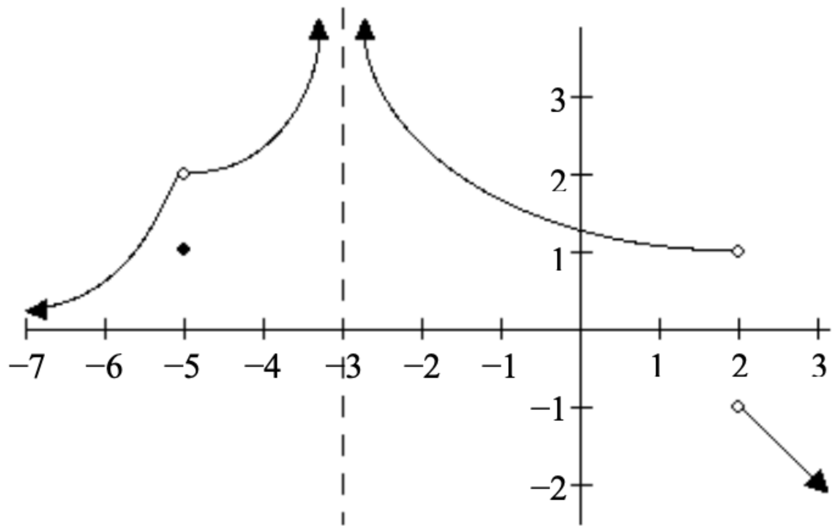
   (b) $\lim_{x\to-\infty} f(x) = L$: $f(x) \to L$ as $x \to -\infty$

5. Vertical asymptote

   If $\left(\lim_{x\to a^-} f(x) = +\infty \text{ or } \lim_{x\to a^-} f(x) = -\infty\right)$ and $\left(\lim_{x\to a^+} f(x) = +\infty \text{ or } \lim_{x\to a^+} f(x) = -\infty\right)$, then **the line** $x = a$ is called the **vertical asymptote** of the graph of a function f.

6. Horizontal asymptote If $\lim_{x\to+\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$, then **the line** $y = L$ is called the **horizontal asymptote** of the graph of a function f

Consider the graph of a function



Find:

1 $f(2)$

Ans: undefined

2 $\lim_{x \to 2^-} f(x)$

Ans: 1

3 $\lim_{x \to 2^+} f(x)$

Ans: −1

4 $\lim_{x \to 2} f(x)$

Ans: undefined

5 $f(-5)$

Ans: 1

6 $\lim_{x \to -5^-} f(x)$

Ans: 2

7 $\lim_{x \to -5^+} f(x)$

Ans: 2

8 $\lim_{x \to -5} f(x)$

Ans: 2

9 $f(-3)$

Ans: undefined

10 $\lim_{x \to -3^-} f(x)$

Ans: $+\infty$

11 $\lim_{x \to -3^+} f(x)$

Ans: $+\infty$

12 $\lim_{x \to -3} f(x)$

Ans: $+\infty$

13 $\lim_{x \to -\infty} f(x)$

Ans: 0

14 $\lim_{x \to +\infty} f(x)$

Ans: $-\infty$

15 The vertical asymptote

Ans: $x = -3$

1.4 Computing Limits

Evaluate the following limits.

1 $\lim_{x \to 3} (x+2)$

Ans: 5

2 $\lim_{x \to 3} \left(x^2 + x + 2\right)$

Ans: 14

3 $\lim_{x \to 3} \frac{x+2}{4-x}$

Ans: 5

4 $\lim_{x \to 4} (4-x)$

Ans: 0

5 $\lim_{x \to 4} \frac{4-x}{x-2}$

Ans: 0

6 $\lim_{x \to 4} \frac{x-2}{4-x}$

Ans: does not exists

Scan for guides

1.5 Computing Limits - Case $\lim_{x\to a} f(x) = \frac{0}{0}$

Evaluate the following limits.

1 $\lim_{x\to -3} \frac{x^2+5x+6}{x+3}$

Ans: -1

2 $\lim_{x\to 0} \frac{x}{\sqrt{x+4}-2}$

Ans: 4

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1.6 Computing Limits at ∞

1.6.1 Odd vs Even Positive Integer Powers

Evaluate the following limits.

1 $\lim_{x\to 2} x$

Ans: 2

2 $\lim_{x\to +\infty} x$

Ans: $+\infty$

3 $\lim_{x\to -2} x$

Ans: -2

4 $\lim_{x\to -\infty} x$

Ans: $-\infty$

5 $\lim_{x\to 2} 3$

Ans: 3

6 $\lim_{x\to +\infty} 3$

Ans: 3

7 $\lim_{x\to -2} x^3$

Ans: -8

8 $\lim_{x\to -\infty} x^3$

Ans: $-\infty$

9 $\lim_{x\to -2} x^2$

Ans: $+4$

10 $\lim_{x\to -\infty} x^2$

Ans: $+\infty$

11 $\lim_{x\to -2} (-x^2)$

Ans: -4

12 $\lim_{x\to -\infty} (-x^2)$

Ans: $-\infty$

13 $\lim_{x\to -2} (-x^3)$

Ans: $+8$

14 $\lim_{x\to -\infty} (-x^3)$

Ans: $+\infty$

15 $\lim_{x\to +\infty} (-3x)$

Ans: $-\infty$

Scan for guides

1.6.2 Polynomials

Limits at ∞ for polynomials matches limits at ∞ of its highest degree term.

Evaluate the following limits.

1 $\lim\limits_{x\to+\infty}(3x^4+2x+3)$

Ans: $+\infty$

2 $\lim\limits_{x\to+\infty}(-3x^4+2x+3)$

Ans: $-\infty$

3 $\lim\limits_{x\to-\infty}(3x^4+2x+3)$

Ans: $+\infty$

4 $\lim\limits_{x\to-\infty}(-3x^4+2x+3)$

Ans: $-\infty$

5 $\lim\limits_{x\to-\infty}(2x^3+2x+3)$

Ans: $-\infty$

6 $\lim\limits_{x\to-\infty}(-2x^3+2x+3)$

Ans: $+\infty$

Scan for guides

1.6.3 Rational Functions

1. Divide the numerator and denominator by the highest power x that occurs in the denominator.
2. $\lim_{x\to+\infty} \frac{1}{x} = \lim_{x\to-\infty} \frac{1}{x} = 0$

Evaluate the following limits.

$\boxed{\mathbf{1}}$ $\lim_{x\to+\infty} \frac{2x+4}{5x-3}$

Ans: $\frac{2}{5}$

$\boxed{\mathbf{2}}$ $\lim_{x\to-\infty} \frac{2x^2+4}{5x-3}$

Ans: $-\infty$

Scan for guides

1.6.4 Radical Functions

1. Find the limit first, and subsequently, calculate the square root.
2. $\lim\limits_{x\to+\infty}\dfrac{1}{x}=\lim\limits_{x\to-\infty}\dfrac{1}{x}=0$

Evaluate the following limits.

$\boxed{\mathbf{1}}$ $\lim\limits_{x\to+\infty}\sqrt{\dfrac{2x+4}{5x-3}}$

Ans: $\sqrt{\frac{2}{5}}$

$\boxed{\mathbf{2}}$ $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+1}}{2x-3}$

Ans: $\frac{1}{2}$

$\boxed{\mathbf{3}}$ $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{2x-3}$

Ans: $-\frac{1}{2}$

Scan for guides

1.6.5 Indeterminate Form of Type $\infty - \infty$

Evaluate the following limits.

1 $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-3x}-x\right)$

Ans: $-\frac{3}{2}$

2 $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+3}-x\right)$

Ans: 0

Scan for guides

1.7 Limits of Trigonometric Functions

Evaluate the following limits.

1 $\lim_{x\to 0} (\sin(x))$

Ans: 0

2 $\lim_{x\to 0} (\cos(x+1))$

Ans: cos (1)

3 $\lim_{x\to +\infty} \left(\cos\left(\frac{1}{x}\right)\right)$

Ans: 1

1. $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{x}{\sin x}=1$
2. $\displaystyle\lim_{x\to 0}\frac{\cos x-1}{x}=\lim_{x\to 0}\frac{x}{1-\cos x}=0$

Evaluate the following limits.

4 $\displaystyle\lim_{x\to 0}\left(\frac{\sin(3x)}{x}\right)$

Ans: 3

5 $\displaystyle\lim_{x\to 0}\left(\frac{\sin(3x)-x\cos(3x)}{x}\right)$

Ans: 2

Evaluate the following limits.

6 $\lim_{x \to \frac{\pi}{2}} \frac{\pi + x\cos(x)}{2x - \sin(x)}$

Ans: $\frac{\pi-1}{\pi}$

7 $\lim_{x \to 0} \frac{\sin(5x)}{\sin(4x)}$

Ans: $\frac{5}{4}$

8 $\lim_{x \to 0} \frac{x}{\tan(3x)}$

Ans: $\frac{1}{3}$

9 $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = \cos(x)$

Ans: $-\sin(x)$

Scan for guides

1.8 Continuity

f is continuous at $x = a$ if:

1. $f(a)$ is defined.
2. $\lim\limits_{x \to a} f(x)$ exists $\Leftrightarrow \lim\limits_{x \to a^+} f(x) = \lim\limits_{x \to a^-} f(x)$.
3. $\lim\limits_{x \to a} f(x) = f(a) = L$.

1 Determine whether the function $f(x) = \dfrac{4x+12}{x^2-9}$ is continuous at $x = 0$.

Ans: continuous

2 Determine whether the function $f(x) = \dfrac{4x+1}{x^2-9}$ is continuous at $x = -3$.

Ans: discontinuous

3 Determine whether the following function is continuous at $x = 3$.

$$f(x) = \begin{cases} 2x^2 - 5 & x < 3 \\ 4x + 1 & x \geq 3 \end{cases}$$

Ans: continuous

4 Find the value(s) of k if f is continuous at $x = 1$.

$$f(x) = \begin{cases} x^2 - 2 & x < 1 \\ kx - 4 & x \geq 1 \end{cases}$$

Ans: 3

Scan for guides

2 | Differentiation

2.1 Definition of Derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Find the derivative of f using the definition of derivative.

1 $f(x) = x + 4$

Ans: 1

2 $f(x) = x^2 + 4$

Ans: $2x$

$\boxed{\textbf{3}}$ $f(x) = \sqrt{x+4}$

Ans: $\frac{1}{2\sqrt{x+4}}$

$\boxed{\textbf{4}}$ $f(x) = \dfrac{1}{x+4}$

Ans: $-\frac{1}{(x+4)^2}$

5 $f(x) = \sin(x)$

Ans: $\cos(x)$

6 $f(x) = \cos(x)$

Ans: $-\sin(x)$

Scan for guides

2.2 Techniques of Differentiation

1. Power Rule: $(x^n)' = nx^{n-1}; n \in \Re$
2. Product Rule: $[u(x)\,v(x)]' = v(x)u'(x) + u(x)v'(x)$
3. Quotient Rule: $\left[\dfrac{u(x)}{v(x)}\right]' = \dfrac{v(x)u'(x) - u(x)v'(x)}{v(x)^2}$

Find the derivative $f'(x)$.

$\boxed{\mathbf{1}}$ $f(x) = 3x^8 - 2x^5 + 6x + 1$

Ans: $24x^7 - 10x^4 + 6$

$\boxed{\mathbf{2}}$ $f(x) = \left(4x^2 - 1\right)\left(7x^3 + x\right)$

Ans: $140x^4 - 9x^2 - 1$

$\boxed{\mathbf{3}}$ $f(x) = \dfrac{x^2 - 1}{x^4 + 1}$

Ans: $\dfrac{-2x^5 + 4x^3 + 2x}{(x^4 + 1)^2}$

$\boxed{\mathbf{4}}$ $f(x) = x^2 \tan x$

Ans: $(\tan x)(2x) + x^2 \sec^2 x$

1. Derivative of logarithmic function: $\left[\log_b(u(x))\right]' = \dfrac{u'(x)}{u(x)\ln b}$
2. Derivative of natural logarithmic function: $[\ln(u(x))]' = \dfrac{u'(x)}{u(x)}$
3. Derivative of exponential function: $\left[b^{u(x)}\right]' = b^{u(x)}(\ln b)\,u'(x)$
4. Derivative of exponential function: $\left[e^{u(x)}\right]' = b^{u(x)}\,u'(x)$

$\boxed{\mathbf{5}}$ $f(x) = \log_2(x^2+1)$

Ans: $\dfrac{2x}{(x^2+1)\ln 2}$

$\boxed{\mathbf{6}}$ $f(x) = \ln(x^2+1)$

Ans: $\dfrac{2x}{x^2+1}$

$\boxed{\mathbf{7}}$ $f(x) = 2^{\sin(x)}$

Ans: $2^{\sin x}(\cos(x))\ln 2$

$\boxed{\mathbf{8}}$ $f(x) = e^{\cos x}$

Ans: $-e^{\cos x}\sin x$

2.3 The Chain Rule

Chain Rule: $(f \circ g)'(x) = f'(g(x)) \times g'(x)$

1 $f(x) = (x^3 + 2x - 3)^4$

Ans: $4(x^3 + 2x - 3)^3(3x^2 + 2)$

2 $f(x) = 4\sin(x^3)$

Ans: $12x^2 \cos(x^3)$

3 $f(x) = \dfrac{1}{x^3 + 2x - 3}$

Ans: $-\dfrac{3x^2 + 2}{(x^3 + 2x - 3)^2}$

4 $f(x) = \sqrt{x^3 + 2x - 3}$

Ans: $\dfrac{3x^2 + 2}{2\sqrt{x^3 + 2x - 3}}$

Scan for guides

2.4 Implicit Differentiation

1 $\dfrac{d}{dx}\left(x^2+3x+4\right)$

Ans: $2x+3$

2 $\dfrac{d}{dx}\left[\left(x^2+3x+4\right)^5\right]$

Ans: $5\left(x^2+3x+4\right)^4(2x+3)$

3 Find $\dfrac{dy}{dx}$ for $y = f(x)$.

Ans: $\dfrac{df}{dx}$

4 Find $\dfrac{dy}{dx}$ for $y = [f(x)]^5$.

Ans: $5[f(x)]^4\dfrac{df}{dx}$

5 Find $\dfrac{dy}{dx}$ for $y^3 = [f(x)]^5$.

Ans: $\dfrac{5[f(x)]^4 \frac{df}{dx}}{3y^2}$

6 Find $\dfrac{dy}{dx}$ for $y^3 = \left(x^2+3x+4\right)^5$.

Ans: $\dfrac{5\left(x^2+3x+4\right)^4(2x+3)}{3y^2}$

7 Find $\dfrac{dy}{dx}$ for $5y^2+\sin y = x^2$.

Ans: $\dfrac{2x}{10y+\cos y}$

8 Find $\dfrac{dy}{dx}$ for $x^3+y^3 = 3xy$ (Folium of Descartes).

Ans: $\dfrac{y-x^2}{y^2-x}$

Scan for guides

2.5 Equation of Tangent Line

$$y - y_1 = m(x - x_1)$$

1 Find the equation of tangent line to $f(x) = x^2 + 1$ at $x = 2$.

Ans: $y = 4x - 3$

2 Find the equation of tangent line to $f(x) = 2x^2$ at the point $(3, 18)$.

Ans: $y = 12x - 18$

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2.6 Linear Approximations and Differentials

1. Local Linear Approximation of f at x_0

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

2. Differentials: For the function y = f (x), we define the following:

   (a) dx, called the differential of x, given by the relation $dx = \delta x$

   (b) dy, called the differential of y, given by the relation $dy = f'(x)dx$

$\boxed{\mathbf{1}}$ Find the local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 4$.

Ans: $\frac{x+4}{4}$

$\boxed{\mathbf{2}}$ Use differentials to approximate $\sqrt{3.98}$.

Ans: $1\frac{199}{200}$

$\boxed{\mathbf{3}}$ Use differentials to approximate $\sqrt{4.02} - \dfrac{1}{\sqrt{4.02}}$.

Ans: $1\frac{81}{160}$

Scan for guides

3 | Applications of Differentiation

3.1 Related Rates

Find the rate at which some quantity is changing by relating the quantity to other quantities whose rates of change is known.

1. Given $y = f(x)$ and $x = g(t)$
   and a constant rate of change $\dfrac{dx}{dt}$.
2. Find the changes of y in time (or rate of change or how fast is the changing) when $x = \text{‘}something'$.
3. $\dfrac{dy}{dt} = \left.\dfrac{df}{dx}\right|_{x=\text{‘}something'} \times \dfrac{dx}{dt}$.

$\boxed{1}$ The value of x is increasing at a constant rate of 4. How fast is $y = 3x^2 + 2$ changing at the instant $x = 2$.

Ans: $\frac{dy}{dt} = 48$

$\boxed{2}$ The value of y is decreasing at a constant rate of 1. $x^2 + y^2 = 625$. How fast is x changing at the instant $x = 7$.

Ans: $\frac{dx}{dt} = \frac{24}{7}$

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3.2 Critical Points

$x = c$ is a critical point of $f(x)$ if:

1. $f'(c) = 0$, or
2. $f'(c)$ does not exists

Determine all the critical point(s) for the following functions:

1 $x^2 - 4x + 3$

Ans: $x = 2$

2 $x^3 - 3x + 2$

Ans: $x = \{-1, 1\}$

3 $\dfrac{x-1}{x+2}$

Ans: $x = -2$

Scan for guides

3.3 Intervals of Increasing and Decreasing Functions

Let f be a function that is continuous on a closed interval $[a,b]$ and is differentiable on the open interval (a,b).

	x on interval (a,b)		
$f'(x)$	$+$	$-$	0
$f(x)$	increasing, $f\uparrow$	decreasing $\downarrow$	constant

Determine the intervals where the following functions are decreasing or decreasing.

1 x^2-4x+3

Ans: decreasing: $\{(-\infty,2)\}$; increasing: $\{(2,+\infty)\}$

2 x^3-3x+2

Ans: decreasing: $\{(-1,1)\}$; increasing: $\{(-\infty,-1),(1,+\infty)\}$

3 $\dfrac{x-1}{x+2}$

Ans: decreasing: none; increasing: $\{(-\infty,-2),(-2,+\infty)\}$

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3.4 Concavity and Inflection Points

Let f be twice differentiable on the open interval (a, b).

	x on interval (a, b)	
$f''(x)$	$+$	$-$
$f(x)$	concave up, $f \cup$	concave down $\cap$

$x = c$ is an **inflection point** of $f(x)$ if the concavity changes at $x = c$.

1 $x^2 - 4x + 3$

Ans: concave up: $\{(-\infty, +\infty)\}$; concave down: none; no inflection point

2 $x^3 - 3x + 2$

Ans: concave up: $\{(0, +\infty)\}$; concave down: $\{(-\infty, 0)\}$; inflection point: $x = 0$

3 $\dfrac{x-1}{x+2}$

Ans: concave up: $\{(-\infty, -3)\}$; concave down: $\{(-2, +\infty)\}$; no inflection point

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3.5 Asymptotes

1. **vertical asymptotes:** vertical lines which correspond to the zeroes of the denominator of rational function.

2. **horizontal asymptotes:** $\lim_{x\to\pm\infty} f(x)$

$\boxed{\mathbf{1}}$ $f(x) = \dfrac{x-1}{x+2}$

Ans: vertical asymptotes: $x = -2$; horizontal asymptotes: $y = 1$

Scan for guides

3.6 Curve Sketching: Even Polynomial Function

Steps:

1. $y-$intercepts
2. $x-$intercepts
3. Intervals of decrease and increase
4. Concavity and inflection points
5. Relative extrema
6. Sketch graph

Sketch the graph of the following function.

1 $f(x) = x^2 - 4x + 3$

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3.7 Curve Sketching: Odd Polynomial Function

Steps:

1. $y-$intercepts
2. $x-$intercepts
3. Intervals of decrease and increase
4. Concavity and inflection points
5. Relative extrema
6. Sketch graph

Sketch the graph of the following function.

1 $f(x) = x^3 - 3x + 2$

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3.8 Curve Sketching: Rational Function

Steps:

1. y – intercepts
2. x – intercepts
3. Intervals of decrease and increase
4. Concavity and inflection points
5. Relative extrema
6. **Asymptotes**
7. Sketch graph

Sketch the graph of the following function.

1 $f(x) = \dfrac{x-1}{x+2}$

Scan for guides

3.9 Rolle's Theorem; Mean-Value Theorem

Mean Value Theorem: f is differentiable on (a, b) and continuous on $[a, b]$. Then, there is at least one number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$\boxed{1}$ Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for $f(x) = x^3 + 1$ on $(0, 2)$.

Ans: $x = +1.1547$

Rolle's Theorems: f is differentiable on (a, b).

$$f(a) = f(b) = 0$$

Then, there is at least one number c in (a, b) such that

$$f'(c) = 0$$

$\boxed{2}$ Determine all the numbers c which satisfy the conclusions of the Rolle's Theorem for $f(x) = x^3 - x$ on $[0, 1]$.

Ans: $x = +0.5774$

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3.10 Maximum and Minimum Values of a Function

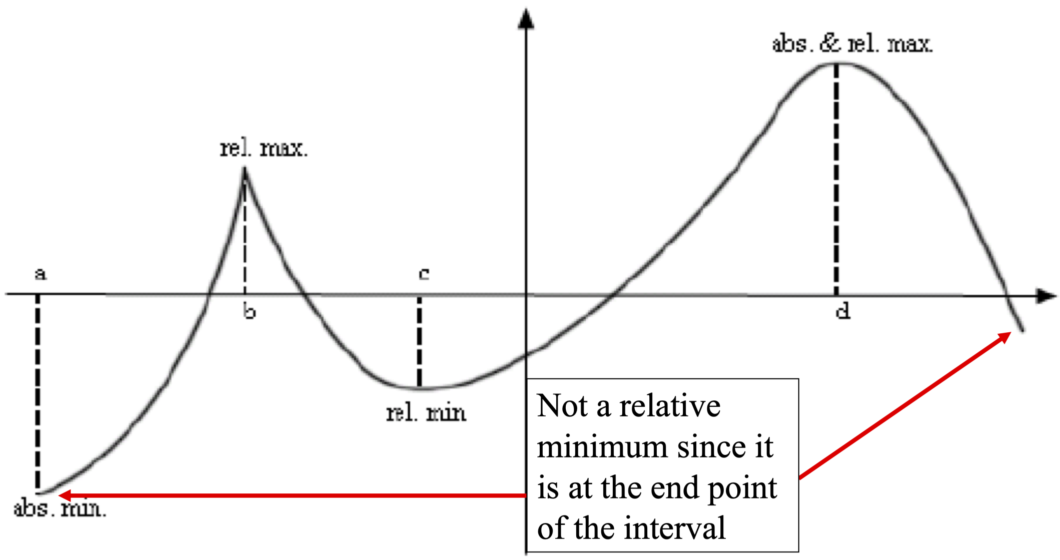

Figure 3.1: Maximum and minimum values.

1. **First Derivative test:** Suppose that f is continuous at a critical number x_0.
   (a) **Relative maximum** at x_0: $f'(x < x_0) > 0$ and $f'(x > x_0) < 0$
   (b) **Relative minimum** at x_0: $f'(x < x_0) < 0$ and $f'(x > x_0) > 0$
   (c) **No relative extremum** at x_0: No changes in sign of $f'(x_0)$
2. **Second Derivative test:** Suppose that f is twice differentiable x_0 and $f'(x_0) = 0$.
   (a) **Relative minimum** at x_0: $f'' > 0$
   (b) **Relative maximum** at x_0: $f'' < 0$
   (c) **Inconclusive:** $f'' = 0$

$\boxed{\mathbf{1}}$ Find the relative and absolute extremum values for $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$.

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4 | Integration

4.1 The Indefinite Integral

$$\int b\,dx = bx + C$$

1 $\int 4\,dx$

Ans: $4x + C$

2 $\int 0.4\,dy$

Ans: $0.4y + C$

3 $\int \frac{1}{3}\,dx$

Ans: $\frac{x}{3} + C$

4 $\int e\,d\theta$

Ans: $e\theta + C$

5 $\int \sqrt{2}\,dx$

Ans: $\sqrt{2}x + C$

6 $\int (3+\sqrt{2})\,dz$

Ans: $(3+\sqrt{2})z + C$

7 $\int \pi\,dx$

Ans: $\pi x + C$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c;\, r \neq -1$$

$\boxed{\mathbf{8}}$ $\int x\,dx$

Ans: $\frac{x^2}{2} + C$

$\boxed{\mathbf{9}}$ $\int 4x\,dx$

Ans: $2x^2 + c$

$\boxed{\mathbf{10}}$ $\int x^2\,dx$

Ans: $\frac{x^3}{3} + C$

$\boxed{\mathbf{11}}$ $\int 3x^2\,dx$

Ans: $x^3 + c$

$\boxed{\mathbf{12}}$ $\int x^3\,dx$

Ans: $\frac{x^4}{4} + C$

$\boxed{\mathbf{13}}$ $\int 0.5x^3\,dx$

Ans: $\frac{x^4}{8} + C$

$\boxed{\mathbf{14}}$ $\int x^{0.5}\,dx$

Ans: $\frac{x^{1.5}}{1.5} + C$

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + c; r \neq -1$$

15 $\int (x + x^2) \, dx$

Ans: $\frac{x^2}{2} + \frac{x^3}{3} + C$

16 $\int (3x^6 - 2x^2 + 7x + 1) \, dx$

Ans: $3\frac{x^7}{7} - 2\frac{x^3}{3} + 7\frac{x^2}{2} + x + C$

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + c; r \neq -1$$

17 $\int \sqrt{x}\, dx$

Ans: $\frac{2}{3}x^{\frac{3}{2}} + C$

18 $\int \sqrt[3]{x}\, dx$

Ans: $\frac{3}{4}x^{\frac{4}{3}} + C$

19 $\int \sqrt{x^3}\, dx$

Ans: $\frac{2}{5}x^{\frac{5}{2}} + C$

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + c; r \neq -1$$

20 $\displaystyle\int (x+2)(x-3)\, dx$

Ans: $\frac{x^3}{3} - \frac{x^2}{2} - 6x + C$

21 $\displaystyle\int \frac{x^3+2x^2}{x}\, dx$

Ans: $\frac{x^3}{3} + x^2 + C$

22 $\displaystyle\int \frac{1}{x^3}\, dx$

Ans: $-\frac{1}{2x^2} + C$

23 $\displaystyle\int \frac{3}{x^2}\, dx$

Ans: $-\frac{3}{x} + C$

1. $\displaystyle\int x^r\,dx = \frac{x^{r+1}}{r+1} + c; r \neq -1$

2. $\displaystyle\int \frac{1}{u}\,du = \ln u + C$

3. $\displaystyle\int e^u\,du = e^u + C$

4. $\displaystyle\int b^u\,du = \frac{b^u}{\ln b} + C$

24 $\displaystyle\int \frac{2}{x}\,dx$

Ans: $2\ln x + C$

25 $\displaystyle\int 5e^x\,dx$

Ans: $5e^x + C$

26 $\displaystyle\int 2^x\,dx$

Ans: $\dfrac{2^x}{\ln 2} + C$

27 $\displaystyle\int \pi^x\,dx$

Ans: $\dfrac{\pi^x}{\ln \pi} + C$

1. $\displaystyle\int \sin(x)\,dx = -\cos(x) + C$
2. $\displaystyle\int \cos(x)\,dx = \sin(x) + C$
3. $\displaystyle\int \sec^2(x)\,dx = \tan(x) + C$
4. $\displaystyle\int \csc^2(x)\,dx = -\cot(x) + C$
5. $\displaystyle\int \sec(x)\tan(x)\,dx = \sec(x) + C$
6. $\displaystyle\int \csc(x)\cot(x)\,dx = -\csc(x) + C$

28 $\displaystyle\int 2\sin(x)\,dx$

Ans: $-2\cos(x) + C$

29 $\displaystyle\int 10\cos(x)\,dx$

Ans: $10\sin(x) + C$

30 $\displaystyle\int 5\sec^2(x)\,dx$

Ans: $5\tan(x) + C$

31 $\displaystyle\int 2\csc^2(x)\,dx$

Ans: $-2\cot(x) + C$

32 $\displaystyle\int 10\sec(x)\tan(x)\,dx$

Ans: $10\sec(x) + C$

33 $\displaystyle\int 2\csc(x)\cot(x)\,dx$

Ans: $-2\csc(x) + C$

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4.2 Integration by Substitution

Evaluate the following integral:

1 $\displaystyle\int (x^2+1)^{50} 2x\,dx$

Ans: $\dfrac{(x^2+1)^{51}}{51}+C$

2 $\displaystyle\int (x^2+1)^{50} x\,dx$

Ans: $\dfrac{(x^2+1)^{51}}{102}+C$

3 $\displaystyle\int (x-8)^5\,dx$

Ans: $\dfrac{(x-8)^6}{6}+C$

4 $\displaystyle\int \frac{1}{\left(\frac{x}{3}-8\right)^5}\,dx$

Ans: $-\dfrac{3}{4(\frac{x}{3}-8)^4}+C$

5 $\displaystyle\int \sin(x+9)\,dx$

Ans: $-\cos(x+9)+C$

6 $\displaystyle\int \cos(5x)\,dx$

Ans: $\dfrac{\sin(5x)}{5}+C$

$\boxed{\mathbf{7}}$ $\displaystyle\int \left(\frac{1}{x^2} + \sec^2(\pi x)\right) dx$

Ans: $-\frac{1}{x} + \frac{\tan(\pi x)}{\pi} + C$

$\boxed{\mathbf{8}}$ $\displaystyle\int \sin^2(x)\cos(x)\,dx$

Ans: $\frac{\sin^3(x)}{3} + C$

$\boxed{\mathbf{9}}$ $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

Ans: $2\sin(\sqrt{x}) + C$

$\boxed{\mathbf{10}}$ $\displaystyle\int t^4 \sqrt[3]{3-5t^5}\,dt$

Ans: $-\frac{3}{100}\left(3-5t^5\right)^{\frac{4}{3}} + C$

$\boxed{\textbf{11}}$ $\displaystyle\int x^2\sqrt{x-1}\,dx$

Ans: $\frac{2}{7}(x-1)^{\frac{7}{2}}+\frac{4}{5}(x+1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+C$

$\boxed{\textbf{12}}$ $\displaystyle\int \frac{3x^2}{\left(x^3-1\right)^5}\,dx$

Ans: $-\frac{\left(x^3-1\right)^{-4}}{4}+C$

$\boxed{\textbf{13}}$ $\displaystyle\int \cos^3(x)\,dx$

Ans: $\sin(x)-\frac{\sin^3(x)}{3}+C$

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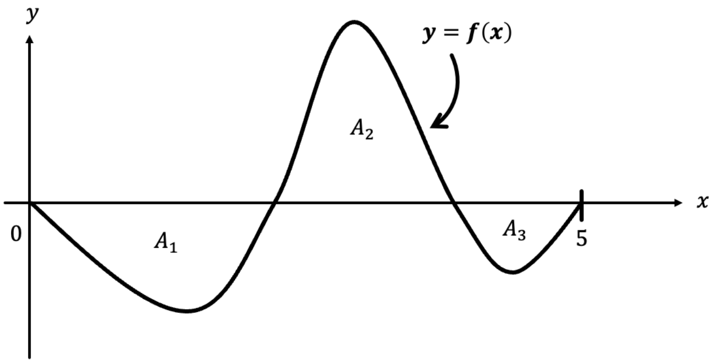
4.3 The Definite Integral

1. **First Fundamental Theorem of Calculus:** If f is continuous on $[a,b]$ and F is antiderivative of f on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

2. **Area** =
The sum of the areas **above the x-axis** and **under** the graph −
The sum of the areas **under the x-axis** and **above** the graph
$= A_2 - A_1 - A_3$
$= \int_0^5 f(x)\,dx$



$\boxed{1}$ $\int_{-1}^{4} \frac{1}{x^2}\,dx$

Ans: undefined

$\boxed{2}$ $\int_1^2 x\,dx$

Ans: $\frac{3}{2}$

3 $\displaystyle\int_0^3 \left(9 - x^2\right) dx$

Ans: 18

4 $\displaystyle\int_0^{\frac{\pi}{3}} \sec^2(x)\, dx$

Ans: $\sqrt{3}$

5 $\displaystyle\int_1^1 x^2\, dx$

Ans: 0

6 $\displaystyle\int_4^0 x\, dx$

Ans: -8

$\boxed{\mathbf{7}}$ Evaluate the integral $\int_0^6 f(x)\,dx$ if $f(x) = \begin{cases} x^2 & x < 2 \\ 3x - 2 & x \geq 2 \end{cases}$.

Ans: $\frac{128}{3}$

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4.4 Evaluating Definite Integrals by Substitution

1 $\displaystyle\int_0^2 x\left(x^2+1\right)^3\,dx$

Ans: 78

2 $\displaystyle\int_0^{\frac{\pi}{8}} \sin^5(2x)\cos(2x)\,dx$

Ans: $\frac{1}{96}$

3 $\displaystyle\int_2^5 (2x-5)(x-3)^9\,dx$

Ans: $\frac{52233}{110}$

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4.5 The Second Fundamental Theorem of Calculus

If f is continuous on interval I, A is any number in I, F is an antiderivative of F on I.

1. $F(x) = \int_a^x f(t)\,dt \implies F'(x) = \dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$
2. $F(x) = \int_a^{g(x)} f(t)\,dt \implies F'(x) = \dfrac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))g'(x)$

1 $\dfrac{d}{dx}\int_1^x t^3\,dt$

Ans: x^3

2 $\dfrac{d}{dx}\int_1^x \dfrac{\sin(t)}{t}\,dt$

Ans: $\dfrac{\sin(x)}{x}$

3 $\dfrac{d}{dx}\int_2^{3x^2} 4u\,du$

Ans: $72x^3$

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4.6 Mean-Value Theorem for Integrals

If f is continuous on $[a, b]$, then there is at least one number c in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c)(b-a)$$

1 Find the value of c in $[1, 4]$, if $f(x) = x^2$ that satisfy the Mean-Value theorem for integrals.

Ans: $c = +\sqrt{7}$

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5 | Applications of Integration

5.1 Area Between Two Curves

Area between two curves $y = f(x)$ (upper function) and $y = g(x)$ (lower function) on the interval $[a, b]$ is given by

$$A = \int_a^b \big(f(x) - g(x)\big)\, dx$$

$\boxed{\textbf{1}}$ Find the area of the region
bounded above by $y = x + 6$,
bounded below by $y = x^2$, and
bounded of the sides by the lines $x = 0$ and $x = 2$.
Ans: $\frac{34}{3}$

$\boxed{\textbf{2}}$ Find the area of the region
that is enclosed between the curves
$y = x^2$ and $y = x + 6$.
Ans: $\frac{125}{6}$

$\boxed{\mathbf{3}}$ Find the area of the region that is enclosed between the curves $x = y^2$ and $y = x - 2$.

Ans: $\frac{9}{2}$

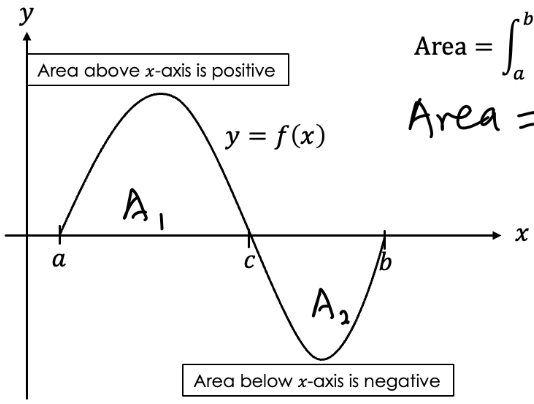
Area between two curves $x = f(y)$ (right function) and $x = g(y)$ (left function) on the interval $[c, d]$ is given by

$$A = \int_c^d \big(f(y) - g(y)\big)\, dy$$

$\boxed{\mathbf{1}}$ Find the area of the region that is enclosed between the curves $x = y^2$ and $y = x - 2$.

Ans: $\frac{9}{2}$

Area between a curve and the x-axis.



$$\text{Area} = \int_a^b f(x)\, dx$$

$$\text{Area} = A_1 - A_2$$

Find the area under the curve $y = \cos(x)$ over the following intervals:

1 $\left[0, \frac{\pi}{2}\right]$

Ans: 1

2 $\left[\frac{\pi}{2}, \pi\right]$

Ans: 1

3 $[0, \pi]$

Ans: 2

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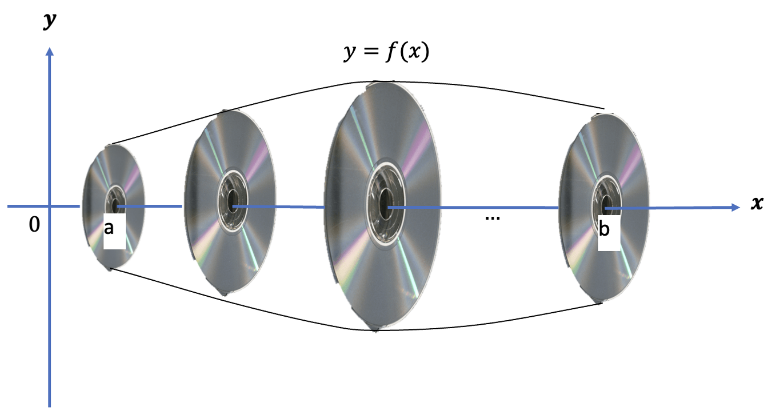
5.2 Method of Disks/Washers (Perpendicular to the x-axis)



1. $\text{Area} = \pi\left(f(x) - 0\right)^2 = \pi\left(f(x)\right)^2$

2. $\text{Volume} = \pi \int_{x=a}^{b} \left[f(x)\right]^2 \, dx$

$\boxed{1}$ Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval $[1,4]$ is revolved about the x-axis.

Ans: $\frac{15}{2}\pi$

2 Find the volume of the solid that is obtained when the region between the graph $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x-axis.

Ans: $\frac{69}{10}\pi$

$\boxed{\mathbf{3}}$ Find the volume of the solid that is obtained when the region bounded by the graph $x = y^2$ and $y = x^2$ is revolved about the x-axis.

Ans: $\frac{3}{10}\pi$

4 Find the volume of the solid that is obtained when the region bounded by the graph $y = x^2$ and $y = x^3$ is revolved about the line $y = -1$.

Ans: $\frac{47}{210}\pi$

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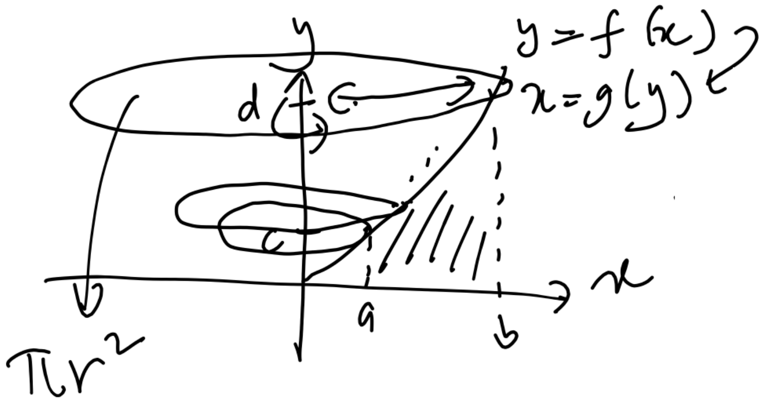
5.3 Method of Disks/Washers (Perpendicular to the y-axis)



1. Area $= \pi\left(g(y) - 0\right)^2 = \pi\left[g(y)\right]^2$

2. Volume $= \pi \int_{y=c}^{d} \left[g(y)\right]^2\, dy$

$\boxed{1}$ Find the volume of the solid that is generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y-axis.

Ans: $\frac{32}{5}\pi$

2 Find the volume of the solid that is generated when the region
enclosed by $x = y^2$ and $y = x^2$
is revolved about the y-axis.

Ans: $\frac{3}{10}\pi$

$\boxed{\mathbf{3}}$ Find the volume of the solid that is generated when the region bounded by $y = x^2$ and $y = x^3$
is revolved about the line $x = -1$.

Ans: $\frac{4}{15}\pi$

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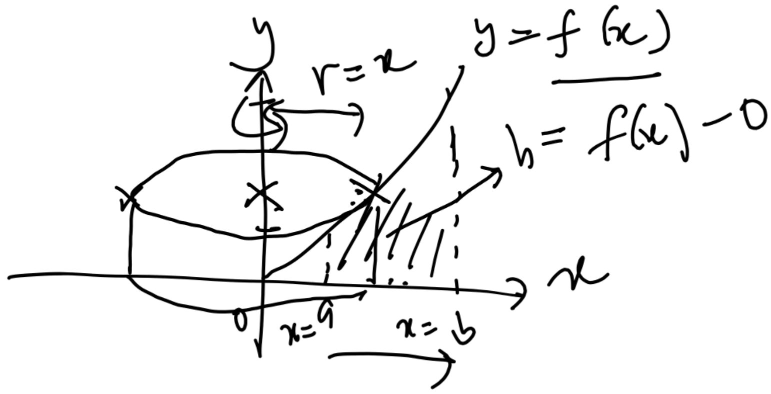
5.4 Cylindrical Shells (Revolved about the y-axis)



1. Volume of a cylinder, $V = 2\pi r h$
2. $r = x$; $h = f(x)$
3. $V = 2\pi \int_{x=a}^{b} x f(x)\, dx$

$\boxed{1}$ Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis is revolved about the y-axis.

Ans: $\frac{124}{5}\pi$

$\boxed{\mathbf{2}}$ Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed by $y = x$ and $y = x^2$ is revolved about the y-axis.

Ans: $\frac{\pi}{6}$

3 Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed by $y = x$ and $y = x^2$ is revolved about the line $x = -1$.

Ans: $\frac{\pi}{2}$

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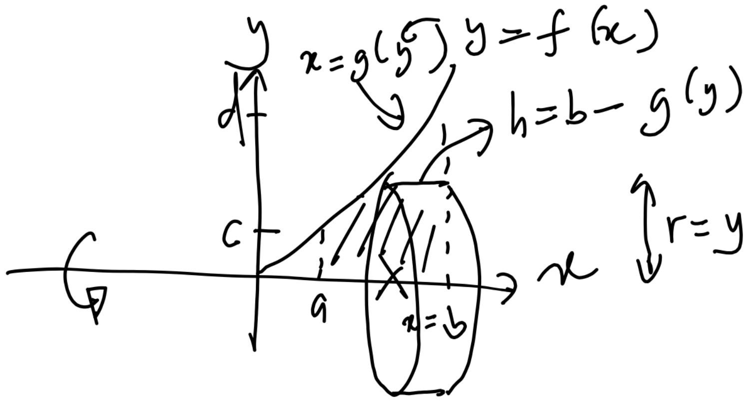
5.5 Cylindrical Shells (Revolved about the x-axis)



1. Volume of a cylinder, $V = 2\pi r h$
2. $r = y$; $h = b - g(y)$
3. $V = 2\pi \int_{y=c}^{d} y\big(b - g(y)\big)\,dy$

1 Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval $[0,2]$ is revolved about the x-axis.

Ans: $\frac{32}{5}\pi$

2 Use cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$ and $y = x$ about the line $y = -1$.

Ans: $\frac{63}{2}\pi$

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5.6 Washer vs. Cylindrical Shell

a) Find the volume of the solid generated
when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x-axis
is revolved about the y-axis.

$\boxed{1}$ Washer

Ans: $\frac{124}{5}\pi$

$\boxed{2}$ Cylindrical Shell

Ans: $\frac{124}{5}\pi$

b) Find the volume of the solid generated
when the region R in the first quadrant enclosed by $y = x$ and $y = x^2$
is revolved about the x-axis.

$\boxed{1}$ Washer

Ans: $\frac{2\pi}{15}$

$\boxed{2}$ Cylindrical Shell

Ans: $\frac{2\pi}{15}$

c) Find the volume of the solid generated
when the region R in the first quadrant enclosed by $y = x$ and $y = x^2$
is revolved about the y-axis.

$\boxed{3}$ Washer

Ans: $\frac{\pi}{6}$

$\boxed{4}$ Cylindrical Shell

Ans: $\frac{\pi}{6}$

d) Use cylindrical shells to find the volume of the solid generated when the region R in the first quadrant enclosed by $y = x$ and $y = x^2$ is revolved about the line $x = -1$.

$\boxed{1}$ Washer

Ans: $\frac{\pi}{2}$

$\boxed{2}$ Cylindrical Shell

Ans: $\frac{\pi}{2}$

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A Past Year Questions

A.1 July, 2025

CONFIDENTIAL CD/JUL 2025/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	JULY 2025
TIME	:	2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of three (3) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of:

   i) the Question Paper
   ii) a two – page Appendix 1
   iii) an Answer Booklet – provided by the Faculty

5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 4 printed pages

 CONFIDENTIAL

QUESTION 1

a) Find $f'(x)$ using the definition of derivative for the function $f(x) = 2x^2 + 4x$.

(4 marks)

b) Find $\dfrac{dy}{dx}$ of the following functions:

i) $y = e^{x+1}\cos(3x+4)$

(3 marks)

ii) $y = \dfrac{3x^2 - 1}{\sin x}$

(3 marks)

c) Given $\cos 2x + x\ln y = 3y - 2$. Find $\dfrac{dy}{dx}$ using implicit differentiation.

(5 marks)

d) Use linear approximation to estimate the value of $\sqrt[6]{64.01} - \dfrac{1}{\sqrt[3]{64.01}}$ correct to four decimal places.

(5 marks)

QUESTION 2

a) Given $\int_0^1 f(x)\,dx = 5$ and $\int_1^0 g(x)\,dx = -3$. Use the properties of integral to find the value of k such that $\int_0^1 g(x)\,dx + \int_0^1 \left[kx^2 + 5f(x)\right]dx = 30$.

(5 marks)

b) Evaluate the following integrals:

i) $\int \left[e^{-2x} + \sec^2(1+x)\right]dx$

(2 marks)

ii) $\int_1^2 \dfrac{(5x^2+4)(7x-3)}{x}\,dx$

(5 marks)

c) Evaluate $\int \tan^3(x)\sec^2(x)\,dx$ using an appropriate substitution.

(5 marks)

d) Given $F(x)=\int_0^{2x^2} \frac{t^2-1}{t+2}dt$. Use the Second Fundamental Theorem of Calculus to find $F'(1)$.

(5 marks)

QUESTION 3

a) Figure 1 shows the shaded region **R** is bounded by the curve $y=2x^3+2$, the line $x=1$ and x-axis. Given $A(-1,0)$ and $B(1,4)$ are the intersection points.

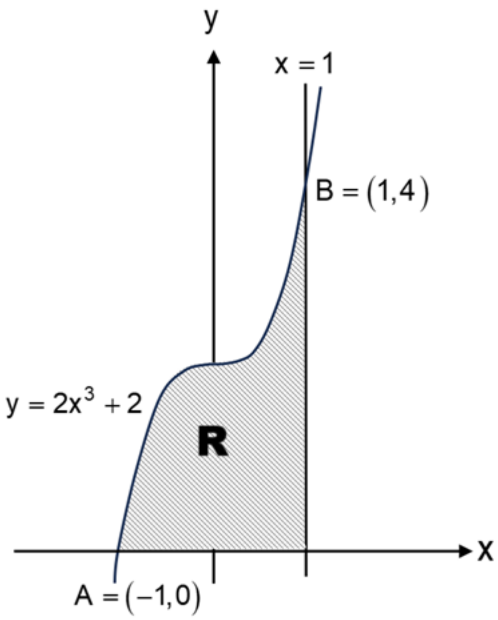

Figure 1

i) Find the area of the shaded region **R**.

(4 marks)

ii) Find the volume of the solid obtained by revolving the region **R** about the x-axis by using **Disk method**.

(4 marks)

b) Figure 2 shows the shaded region **S** is bounded by the line $y = -x$, and the curve $y = 6 - x^2$. Given $A(-2,2)$ and $B(3,-3)$ are the intersection points.

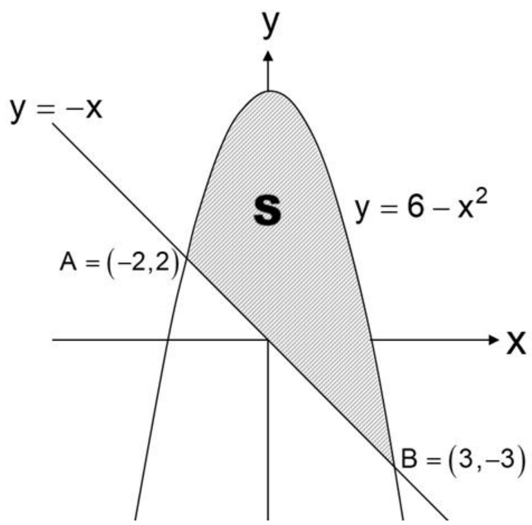

Figure 2

i) Find the volume of the solid generated when the shaded region **S** is revolved about the line $y = -3$ by using **Washer method**.

(5 marks)

ii) Find the volume of the solid generated when the shaded region **S** is revolved about the line $x = 3$ by using **Shell method**.

(5 marks)

END OF QUESTION PAPER

A.2 February, 2025

CONFIDENTIAL CD/FEB 2025/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	FEBRUARY 2025
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

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This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limit:

i) $\lim\limits_{x \to 2} \dfrac{x^2-6x+8}{x^2-5x+6}$

(3 marks)

ii) $\lim\limits_{x \to \infty} \sqrt[3]{\dfrac{3x^3-5x^2+x}{x^3+2x-4}}$

(4 marks)

iii) $\lim\limits_{x \to 0} \dfrac{\sin 6x}{3x(1+\cos 3x)}$

(4 marks)

b) The function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} \dfrac{4-x}{3} & , \quad x<1 \\ \dfrac{1}{2}x^2-kx & , \quad 1 \le x < 4 \\ \sqrt{x-3} & , \quad x \ge 4 \end{cases}$$

i) Find $\lim\limits_{x \to 1^-} f(x)$.

(2 marks)

ii) Find the value of k if $\lim\limits_{x \to 1} f(x)$ exists.

(3 marks)

iii) Determine whether the function is continuous at $x=4$.

(4 marks)

QUESTION 2

a) Given the function $f(x)=3x-x^2$.

i) Use the definition of derivative to find $f'(x)$.

(5 marks)

ii) Find the equation of tangent line to the function at the point $(3,0)$.

(3 marks)

b) Given $2e^{3x}+2y^2=x\ln y$. Find $\dfrac{dy}{dx}$ using implicit differentiation.

(5 marks)

c) Use linear approximation to estimate the value of $\sqrt{63.8}-\frac{3}{\sqrt[3]{63.8}}$ correct to four decimal places.

(5 marks)

QUESTION 3

a) A tank filled with water is in the shape of an inverted cone has 30 cm high and a circular base (on top) whose radius is 10 cm. Water is leaking out from the bottom of the tank at constant rate of 1 cm^3s^{-1}. How fast is the water level falling when the water is 20 cm deep?

[Hint: Volume of cone: $V=\frac{1}{3}\pi r^2 h$]

(5 marks)

b) Given a function $f(x)=\frac{3-x}{x+3}$.

i) Find the x – intercept and y – intercept of $f(x)$.

(2 marks)

ii) Find the vertical asymptote and horizontal asymptote of $f(x)$.

(3 marks)

iii) Find the interval(s) where $f(x)$ is increasing or decreasing.

(4 marks)

iv) Find the interval(s) where $f(x)$ is concave up or concave down. Hence, determine the inflection point of $f(x)$ (if any).

(4 marks)

v) Sketch the graph of $f(x)$ using the above information.

(2 marks)

c) A closed rectangular container is to be made to hold 800 cm^3 of liquid. The base is a square with x cm.

i) Show that the surface area of the container is $A=2x^2+\frac{3200}{x}$.

(3 marks)

ii) Find the minimum surface area of the container.

(4 marks)

QUESTION 4

a) Given $\int_{1}^{3} 2f(x)\,dx = 12$ and $\int_{-1}^{3} f(x) = 20$. Use the properties of integrals to evaluate $\int_{-1}^{1} f(x)\,dx$.

(4 marks)

b) Evaluate each of the following integral using an appropriate substitution.

i) $\int 6x\left(3x^2+5\right)^7 dx$

(5 marks)

ii) $\int \frac{\sec^2 x}{(4+\tan x)^3}\,dx$

(5 marks)

c) Given $F(x) = \int_{0}^{3x} \left(\sqrt{2t^2+t+2}\right) dt$. Use the Second Fundamental Theorem of Calculus to find $F'(1)$.

(5 marks)

QUESTION 5

a) Figure 1 shows the shaded region **R** is bounded by the line $x = 2+y$ and the curve $x = -4+y^2$. Given $A(5,3)$ and $B(0,-2)$ are the intersection points.

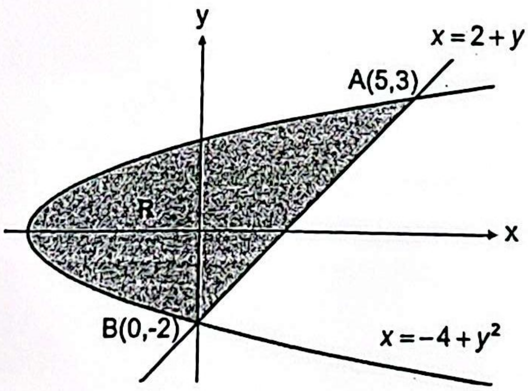

Figure 1

Find the area of the shaded region **R**.

(4 marks)

b) Figure 2 shows the shaded region **S** is bounded by the line $y = x$ and the curve $y = \frac{x^2}{5}$.

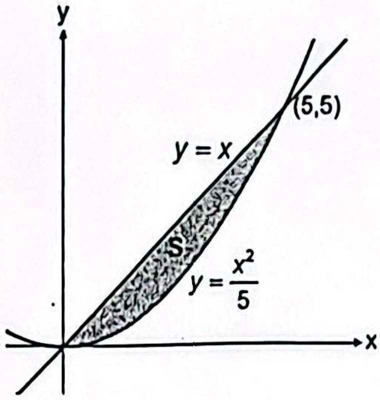

Figure 2

i) Find the volume of the solid generated when the shaded region **S** is revolved about the line $y = 0$ by using **Washer** method.

(6 marks)

ii) Find the volume of the solid generated when the shaded region **S** is revolved about the line $x = 0$ by using **Shell** method.

(6 marks)

END OF QUESTION PAPER

RULES OF DIFFERENTIATION

1. Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

2. Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

3. Power Rule

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$$

4. Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

DEFINITION OF DIFFERENTIATION

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

DIFFERENTIATION RULES FOR TRIGONOMETRIC FUNCTIONS

1. $\frac{d}{dx}(\sin x) = \cos x$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(\tan x) = \sec^2 x$

4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$

5. $\frac{d}{dx}(\sec x) = \sec x \tan x$

6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

THE SQUEEZING THEOREM

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

LINEAR APPROXIMATION

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f[g(x)]g'(x)$$

GEOMETRY FORMULA

1. Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
2. Area of a circle $= \pi r^2$
3. Surface area of a sphere $= 4\pi r^2$
4. Surface area of a cylinder $= 2\pi r^2 + 2\pi rh$
5. Surface area of a cone $= \pi r^2 + \pi rl$
6. Volume of a cylinder $= \pi r^2 h$
7. Volume of a cone $= \frac{1}{3}\pi r^2 h$
8. Volume of a sphere $= \frac{4}{3}\pi r^3$

LIST OF INTEGRALS

1. $$\int (ax+b)^n\,dx = \begin{cases} \dfrac{(ax+b)^{n+1}}{a(n+1)} + C; & n \neq -1 \\ \dfrac{1}{a}\ln|ax+b| + C & n = -1 \end{cases}$$

2. $$\int \frac{1}{x}\,dx = \ln|x| + C$$

3. $$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

4. $$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$

5. $$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$

6. $$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$$

7. $$\int \csc^2 ax\,dx = -\frac{1}{a}\cot ax + C$$

AREA UNDER A CURVE

1. $$A = \int_a^b [f(x) - g(x)]\,dx$$

2. $$A = \int_c^d [w(y) - v(y)]\,dy$$

A.3 July, 2024

CONFIDENTIAL **CD/JUL 2024/MAT183**

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	**:**	**CALCULUS I**
COURSE CODE	**:**	**MAT183**
EXAMINATION	**:**	**JULY 2024**
TIME	**:**	**3 HOURS**

INSTRUCTIONS TO CANDIDATES

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This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limit:

i) $\lim_{x \to 2} \dfrac{3x-6}{x^2+2x-8}$

(3 marks)

ii) $\lim_{x \to \infty} \dfrac{3x^3-2x^2-1}{x^3-x+1}$

(4 marks)

iii) $\lim_{x \to 0} \dfrac{x^2+5\sin x}{x}$

(4 marks)

b) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 3-kx & , \quad x < -1 \\ \dfrac{2x-8}{x^2-x-12} & , \quad -1 \le x < 3 \\ x^2+9 & , \quad x \ge 3 \end{cases}$$

i) Find the $\lim_{x \to 2} f(x)$.

(2 marks)

ii) Find the value of k if $\lim_{x \to -1} f(x)$ exists.

(3 marks)

iii) Determine whether the function is continuous at $x = 3$.

(4 marks)

QUESTION 2

a) Given the function $f(x) = \dfrac{2x}{3-x}$.

i) Use the definition of derivative to find $f'(x)$.

(5 marks)

ii) Find the equation of the tangent line to the function at the point $(2, 4)$.

(3 marks)

b) Given $\ln 2y + y\sin 2x = 2x^2 + 3$. Find $\dfrac{dy}{dx}$ using implicit differentiation.

(6 marks)

c) Use differentials to approximate the value of $\sqrt[3]{7.99.}+(7.99)^2$ correct to three decimal places.

(4 marks)

QUESTION 3

a) Water is flowing out of a conical tank at a rate of $6cm^3/s$. The radius of the tank is $3cm$ and the height is $9cm$. How fast is the radius changing when the radius of the water is $2cm$?

[Hint: Volume of cone, $V=\frac{1}{3}\pi r^2 h$]

(5 marks)

b) Given a function $f(x)=x^3+3x^2-2$.

i) Find the critical point of $f(x)$.

(3 marks)

ii) Find the interval(s) where $f(x)$ is increasing or decreasing. Hence, determine the relative extremum of $f(x)$ (if any).

(4 marks)

iii) Find the interval(s) where $f(x)$ is concave up or concave down. Hence, determine the inflection point of $f(x)$ (if any).

(4 marks)

v) Sketch the graph of $f(x)$ using the above information.

(2 marks)

c) An open rectangular box (no top) has a volume of $288cm^3$. The length of the box is two times of its width as shown in Figure 1.

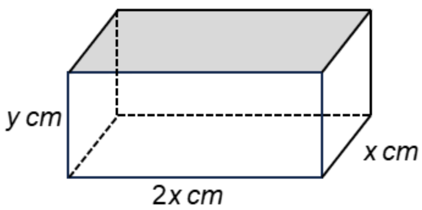

Figure 1

i) Show that the surface area of the rectangular box is given by $A=2x^2+\frac{864}{x}$.

(3 marks)

ii) Find the length of x and y that will minimize the surface area of the box.

(5 marks)

QUESTION 4

a) Given $\int_{0}^{-2} f(x)dx = -4$ and $\int_{0}^{3} f(x)dx = 6$. Use the properties of integrals to evaluate $\int_{-2}^{3} \left[f(x) + x^2\right]dx$.

(5 marks)

b) Evaluate each of the following integral using an appropriate substitution.

i) $\int (x+1)\left(x^2+2x\right)^3 dx$

(5 marks)

ii) $\int \frac{(\ln x)^3}{3x} dx$

(5 marks)

c) Given $F(x) = \int_{-1}^{2x} \sqrt{t^3+1}\,dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$

(4 marks)

ii) $F'(1)$

(1 mark)

QUESTION 5

Figure 2 shows the shaded region R is bounded by the line $y = 4 - x$ and the curve $y = \frac{1}{2}x^2$. Given $A(-4, 8)$ and $B(2, 2)$ are the intersection points.

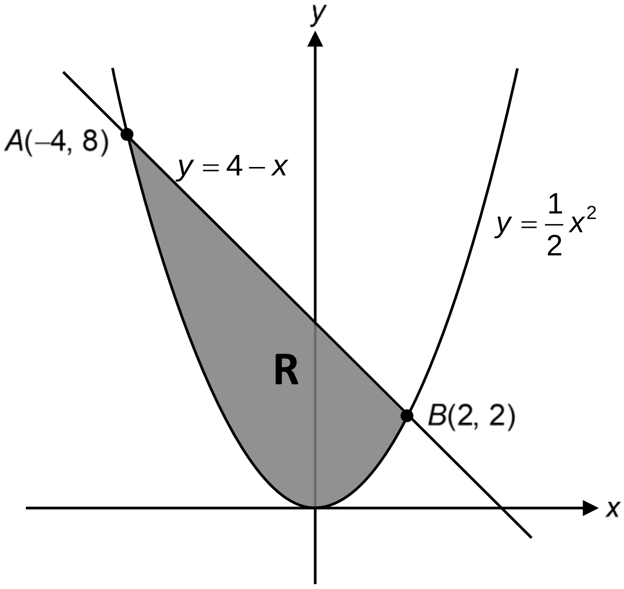

Figure 2

a) Find the area of the shaded region R.

(4 marks)

b) Find the volume of the solid generated when the shaded region R is revolved about the line $y = -1$ by using **Washer** method.

(6 marks)

c) Find the volume of the solid generated when the shaded region R is revolved about the line $x = 3$ by using **Shell** method.

(6 marks)

END OF QUESTION PAPER

A.4 January, 2024

CONFIDENTIAL CD/JAN 2024/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	**CALCULUS I**
COURSE CODE	:	**MAT183**
EXAMINATION	:	**JANUARY 2024**
TIME	:	**3 HOURS**

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This examination paper consists of 6 printed pages

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 3} \dfrac{4x-12}{x^2+x-12}$

(3 marks)

ii) $\lim\limits_{x \to -4} \dfrac{\sqrt{x+8}-2}{(x+4)}$

(4 marks)

iii) $\lim\limits_{x \to 0} \dfrac{2x+\sin 3x}{x}$

(4 marks)

b) The function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} ax+7 & , \quad x<-3 \\ \dfrac{x^2-16}{x-4} & , \quad -3 \le x < 4 \\ 8x-5 & , \quad x \ge 4 \end{cases}$$

i) Find the $\lim\limits_{x \to 5} f(x)$.

(2 marks)

ii) Find the value of a if $\lim\limits_{x \to -3} f(x)$ exists.

(3 marks)

iii) Determine whether the function is continuous at $x=4$.

(4 marks)

QUESTION 2

a) Given the function $f(x)=x^2+3x-2$.

i) Use the definition of derivative to find $f'(x)$.

(5 marks)

ii) Find the equation of the tangent line to the function at the point $(0,-2)$.

(3 marks)

b) Given $y^3 + 2x\ln(2y) = e^{x+2}$. Find $\dfrac{dy}{dx}$ using implicit differentiation.

(5 marks)

c) Use differentials to estimate the value of $(4.02)^2 - \ln(4.02)$ correct to three decimal places.

(5 marks)

QUESTION 3

a) A water tank in the shape of a cone is filled by water at a constant rate of $3\text{m}^3\text{h}^{-1}$. The radius of the tank is 3 m and the height of the tank is 9 m. At what rate is the depth of the water in the tank changing when the depth of the water is 6m.

[Hint: $v_{cone} = \frac{1}{3}\pi r^2 h$]

(6 marks)

b) Given a function $f(x) = \dfrac{x}{2-x}$.

i) Find the x-intercept and y-intercept of *f*.

(2 marks)

ii) Find the vertical asymptote and horizontal asymptote of *f*.

(3 marks)

iii) Find the interval(s) where f is increasing or decreasing.

(4 marks)

iv) Find the interval(s) where f is concave up or concave down. Hence, determine the inflection point of f (if any).

(4 marks)

v) Sketch the graph of *f* using the above information.

(2 marks)

c) An open cuboid made of paper has a volume of 120cm^3. The height of the cuboid is h and the length of the cuboid is triple the width, w.

cuboid

i) Show that the surface area of the ~~water tank~~ is $A = 3w^2 + \dfrac{320}{w}$.

(2 marks)

ii) Hence, find the dimensions of the cuboid so that the surface area is minimized.

(3 marks)

QUESTION 4

a) Given $\int_{-2}^{0} f(x)\,dx = 3$ and $\int_{0}^{5} f(x)\,dx = 5$. Find $\int_{-2}^{5}\left[\frac{1}{8}f(x)+2x\right]dx$ by using the properties of the integral.

(5 marks)

b) Evaluate each of the following integrals using appropriate substitution.

i) $\int \frac{4x^2}{x-1}\,dx$

(5 marks)

ii) $\int \frac{\cos x}{(2-\sin x)^5}\,dx$

(5 marks)

c) The Mean Value Theorem of differentiation states that if $f(x)$ is differentiable on (a,b) and continuous on $[a,b]$, then there is at least one point c in (a,b) where

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

If $f(x)=-2x^3+6x-2$, find the value of c satisfying the conclusion of the above theorem for $(-2,2)$.

(5 marks)

QUESTION 5

a) Figure 1 shows the shaded region R is bounded by the curve $y = 6x - 2x^2$ and the line $y = \dfrac{x}{2}$.

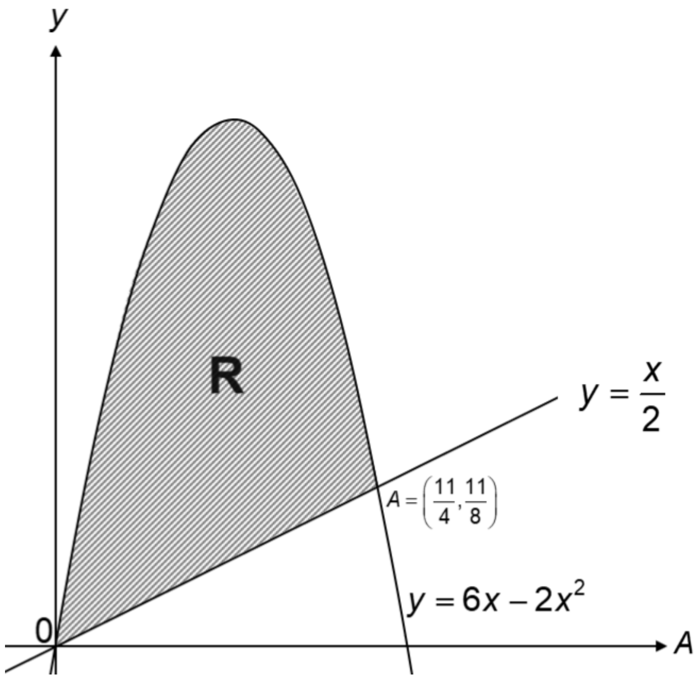

Figure 1

Find the area of the shaded region R.

(4 marks)

b) Figure 2 shows the region S is bounded by the curves $4y = x^2$ and $y^2 = 4x$.

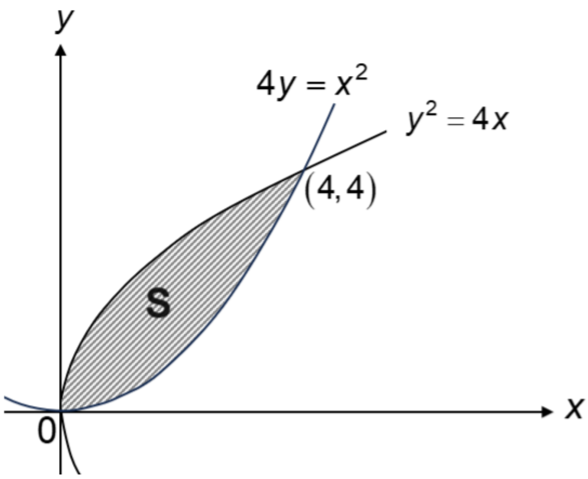

Figure 2

i) Find the volume of the solid generated when the shaded region S is revolved about the line x-axis using **Washer** method.

(6 marks)

ii) Find the volume of the solid generated when the shaded region S is revolved about the line $y = 4$ using **Shell** method.

(6 marks)

END OF QUESTION PAPER

A.5 February, 2023

CONFIDENTIAL CD/FEB 2023/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

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COURSE CODE	:	MAT183
EXAMINATION	:	FEBRUARY 2023
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 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$

(3 marks)

ii) $\lim_{x \to 2} \frac{\sqrt{x+2} - 2}{3x - 6}$

(4 marks)

iii) $\lim_{x \to 0} \frac{2\sin 5x}{3x}$

(4 marks)

b) The function $g(x)$ is defined as follows:

$$g(x) = \begin{cases} 4m + x & , \quad x < -1 \\ \dfrac{x^2 - 9}{x - 3} & , \quad -1 \le x < 3 \\ 3x + 2 & , \quad x \ge 3 \end{cases}$$

i) Find $\lim_{x \to 1} g(x)$.

(2 marks)

ii) Find the value of m if $g(x)$ is continuous at $x = -1$.

(3 marks)

iii) Determine whether the function $g(x)$ is continuous at $x = 3$.

(4 marks)

QUESTION 2

a) Given $f(x) = \frac{2x}{3 - x}$.

i) Use definition of derivative to find $f'(x)$.

(5 marks)

ii) Find the equation of a tangent line to the function at point $\left(1, \frac{3}{2}\right)$.

(3 marks)

b) Given the function $e^{y^2-4} - \frac{x}{y^3} = 1 - \cos(5x)$. Find $\frac{dy}{dx}$ for the function using implicit differentiation.

(5 marks)

c) Use differentials to estimate the value of $\sqrt[4]{15.98} - \left(\frac{15.98}{4}\right)^2$ correct to four decimal places.

(5 marks)

QUESTION 3

a) Water is poured into a conical tank at the rate of 9 m^3s^{-1}. The radius of the tank is 4 m and the height is 12 m. At what rate is the water level rising when the water reached 4 m deep.

[Hint: $v_{cone} = \frac{1}{3}\pi r^2 h$]

(6 marks)

b) Given a function $f(x) = \frac{1}{3}x^2(x-6)$.

i) Find the *x*-intercept(s) and *y*-intercept.

(2 marks)

ii) Find the critical points of *f*.

(3 marks)

iii) Find the interval(s) where *f* is increasing or decreasing. Hence, determine the relative extremum of *f*.

(4 marks)

iv) Find the interval(s) where *f* is concave up or concave down. Hence, determine the inflection point of *f*.

(4 marks)

v) Sketch the graph of *f* using the above information.

(2 marks)

c) A factory plans to manufacture opened cylindrical steel cases that can hold 450cm^3. Let *r* and *h* be the radius and height of the cylinder respectively.

i) Express *h* in terms of *r*.

(2 marks)

ii) Show that the surface area of the cylindrical steel cases is $A = \pi r^2 + \frac{900}{r}$.

(3 marks)

QUESTION 4

a) If $\int_{1}^{4} f(x)dx = 2$, evaluate $\int_{1}^{4}\left[1+3f\left(x\right)\right]dx$ using the properties of integral.

(3 marks)

b) Evaluate each of the following integrals using appropriate substitution.

i) $\int \frac{3x^2}{x-2}\,dx$

(4 marks)

ii) $\int \frac{\sin x}{(2+\cos x)^4}\,dx$

(5 marks)

c) The Mean Value Theorem for differentiation states that if f(x) is differentiable on (a,b) and continuous on [a,b], then there is at least one point c in (a,b) where

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

If $f(x) = 3x^2 - x^3$, find the value of c in the interval (1,3) that satisfies the above theorem.

(4 marks)

d) Given $F(x) = \int_{0}^{\sin(2x)} (3\cos(t) - 5t^2)dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$.

(3 marks)

ii) $F'(0)$.

(1 mark)

QUESTION 5

a) Figure 1 below shows a shaded region **R** bounded by curves $y = x^2 - 4x$ and $y = -x^2 + 2x + 8$.

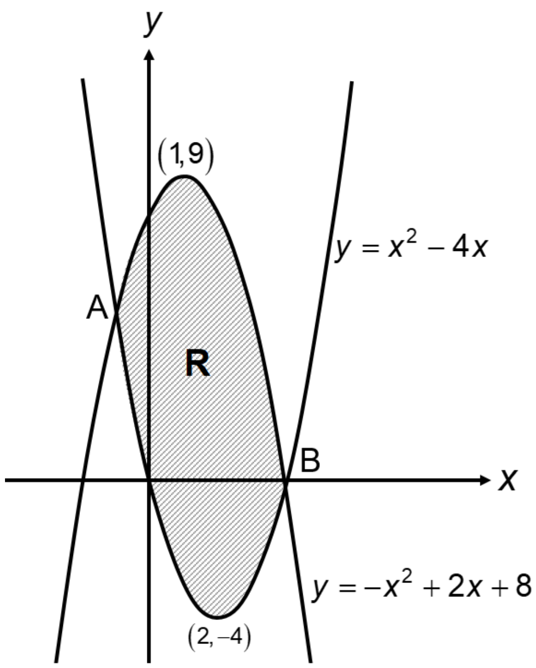

Figure 1

i) Find the coordinates of A and B.

(2 marks)

ii) Find the area of the shaded region **R**.

(4 marks)

b) Figure 2 below shows the shaded region **S** bounded by the curves $x = y^2 - 4y$ and $y = \sqrt{-x}$.

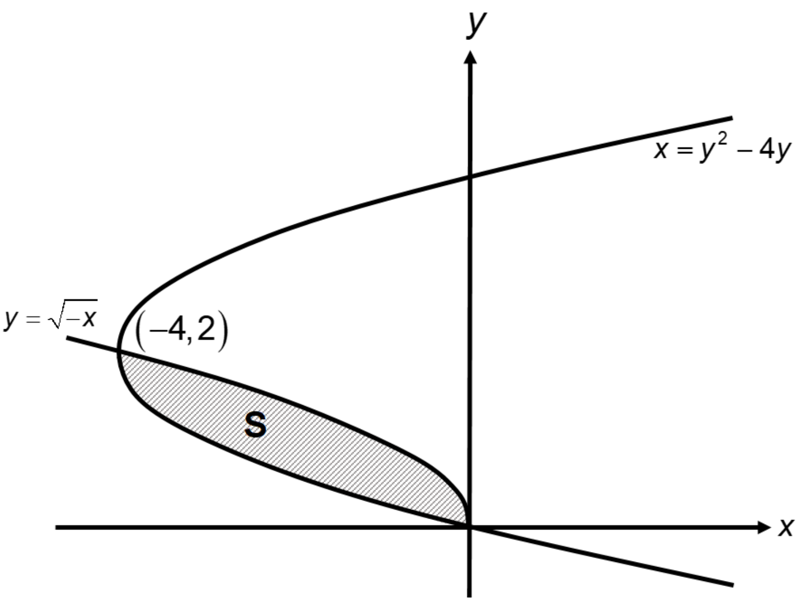

Figure 2

i) Find the volume of the solid generated when the shaded region **S** is revolved about the line $x = 0$ using the **Washer Method**.

(5 marks)

ii) Find the volume of the solid generated when the shaded region **S** is revolved about the line $y = 2$ using **Shell Method.**

(5 marks)

END OF QUESTION PAPER

A.6 December, 2019

CONFIDENTIAL CS/DEC 2019/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	DECEMBER 2019
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :

   i) the Question Paper
   ii) an Answer Booklet – provided by the Faculty

5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 6 printed pages

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim_{x \to 4} \frac{8-2x}{x^2-x-12}$

(3 marks)

ii) $\lim_{x \to 0} \frac{x^3+4\sin 5x}{x}$

(4 marks)

iii) $\lim_{x \to 0} \frac{\sqrt{3+x}-\sqrt{3}}{x}$

(5 marks)

b) Given that

$$f(x) = \begin{cases} 3k-4x & , \quad x < -1 \\ kx^2+2x & , \quad -1 \le x \le 1 \\ -e^{(x-1)} & , \quad x > 1 \end{cases}$$

i) Find the value of k such that $\lim_{x \to -1} f(x)$ exists.

(3 marks)

ii) Determine whether the function is continuous at $x = 1$.

(5 marks)

QUESTION 2

a) Use definition of derivative to differentiate $y = 2x - \frac{1}{x}$.

(5 marks)

b) Given the function $e^{2xy} - \sin y = \ln(4x+1)$.

i) Find $\frac{dy}{dx}$ using implicit differentiation.

(4 marks)

ii) Find the equation of a tangent line to the function at the point (-2,0).

(3 marks)

c) Figure 1 shows a right cylinder closed at both ends has a radius, x and height, y.

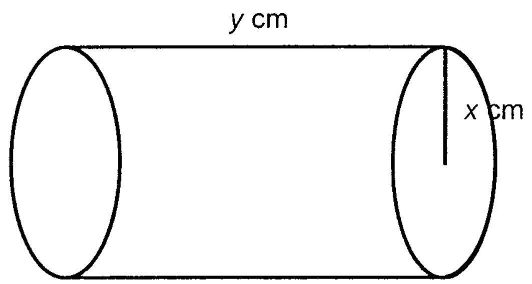

Figure 1

i) If the surface area of the cylinder is 150π cm^2, write y in terms of x.

(3 marks)

ii) Prove that the volume of the cylinder is $V = 75\pi x - \pi x^3$.

(2 marks)

iii) Find the value of x such that the volume of the cylinder is maximum.

(5 marks)

QUESTION 3

a) Given a function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 4$.

i) Find the y-intercept of f.

(1 mark)

ii) Find the critical points of f.

(4 marks)

iii) Find the interval(s) where f is increasing and decreasing. Hence, determine the relative extremum of f (if any).

(4 marks)

iv) Find the interval(s) where f is concave up and concave down. Hence, determine the inflection point of f (if any).

(4 marks)

v) Sketch the graph of f using the above information.

(2 marks)

b) Use differentials to estimate the value of $\sqrt{15.8}-\dfrac{2}{\sqrt[4]{15.8}}$ correct to four decimal places.

(4 marks)

c) Wheat is poured into a conical pile at the rate of 20 cm^3/min whose bottom radius is always half the height. How fast will the area of the base be increasing when the pile is 16 cm height?

(Hint: Volume of conical pile $=\dfrac{1}{3}\pi r^2 h$)

(6 marks)

QUESTION 4

a) Evaluate each of the following integrals using the appropriate substitution.

i) $\displaystyle\int x(x+6)^7\,dx$

(5 marks)

ii) $\displaystyle\int \frac{5\sec^2 x}{2(1-\tan x)}\,dx$

(4 marks)

b) Given $f(x)=\begin{cases} x-2 & x\le 1 \\ kx^2 & x>1 \end{cases}$. Find the value of k if $\displaystyle\int_0^3 \frac{f(x)}{3}\,dx=6$.

(4 marks)

c) Let $F(x)=\displaystyle\int_0^{\sin x}\frac{2}{1+t^2}\,dt$. Use the Second Fundamental Theorem of Calculus to find

i) $F'(x)$

(3 marks)

ii) $F'(0)$

(1 mark)

QUESTION 5

a) The shaded region in Figure 2 is bounded by the curve $y = -2x^2 + 7x$ and $y = -x + 6$.

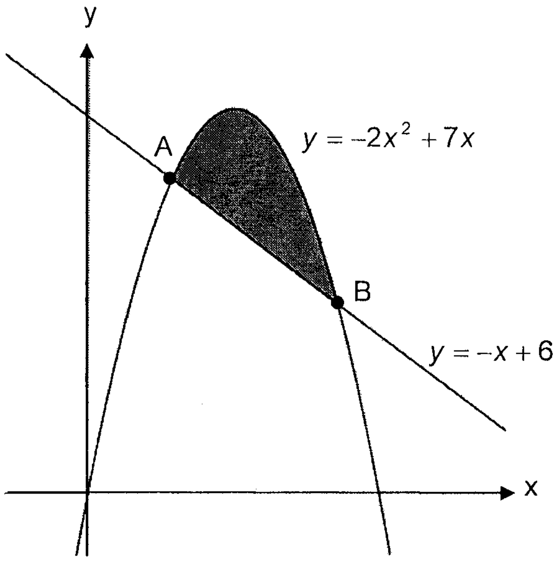

Figure 2

i) Find the coordinates of **A** and **B**.

(2 marks)

ii) Find the area of the shaded region.

(4 marks)

b) Figure 3 below shows the region **R** bounded by the curve $y = x^3$ and the line $y = 1$.

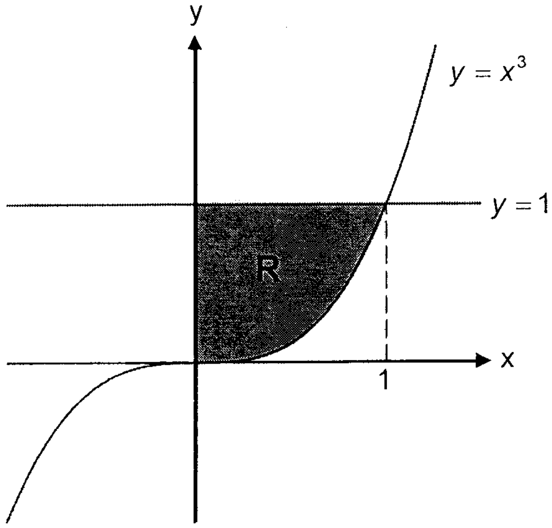

Figure 3

i) Find the volume of the solid by using **Disk method** when the shaded region is revolved about the line $y = 1$.

(5 marks)

ii) Using the **Shell method**, find the volume of the solid generated when the shaded region is revolved about x-axis.

(5 marks)

END OF QUESTION PAPER

A.7 June, 2019

CONFIDENTIAL CS/JUN 2019/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	JUNE 2019
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :
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   ii) an Answer Booklet – provided by the Faculty
5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 6 printed pages

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x\to 3} \dfrac{3-x}{x^2+2x-15}$

(3 marks)

ii) $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+3}-2}{x-1}$

(4 marks)

iii) $\lim\limits_{x\to 0} \dfrac{x\cos 2x-\sin 3x}{6x}$

(4 marks)

b) The function $f(x)$ is defined as follows:

$$f(x)=\begin{cases} 6-kx\ , & x\le 2 \\ x^2-10\ , & 2<x\le 4 \\ \dfrac{12}{x-2}\ , & x>4 \end{cases}$$

i) Find $\lim\limits_{x\to\infty} f(x)$.

(2 marks)

ii) Find the value of k if $\lim\limits_{x\to 2} f(x)$ exists.

(3 marks)

iii) Determine whether the function is continuous at $x=4$.

(4 marks)

QUESTION 2

a) Given $f(x)=x^2+3x+2$. Find $f\,'(x)$ using the definition of derivative.

(5 marks)

b) Given the function $\ln(2-e^{1-x})+xy^2+1=\dfrac{2x}{x+1}+3y$.

i) Find $\dfrac{dy}{dx}$ using implicit differentiation.

(4 marks)

ii) Find the equation of the tangent line to the function at the point $(1,3)$.

(3 marks)

c) Figure 1 below shows a piece of wire of length 200 cm is bent into the shape of a triangle ABF and a rectangle CDEF as shown. Given that $BC = DE$.

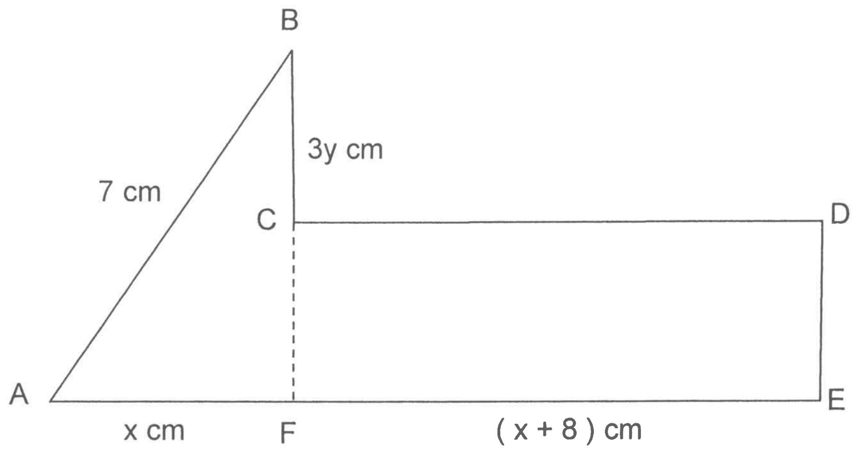

Figure 1

i) Express x in terms of y.

(2 marks)

ii) Show that the area $A\text{cm}^2$, enclosed by the wire is $A = 378y - 12y^2$.

(3 marks)

iii) Find the maximum area enclosed.

(5 marks)

QUESTION 3

a) Given a function $f(x) = \dfrac{3x-6}{x+2}$.

i) Find the x-intercept(s) and y-intercept(s).

(2 marks)

ii) Find the vertical asymptote and horizontal asymptote.

(3 marks)

iii) Find the interval(s) where f is increasing or decreasing.

(4 marks)

iv) Find the interval(s) where f is concave up or concave down. Hence, determine the inflection point.

(4 marks)

v) Sketch the graph of f using the above information.

(2 marks)

b) Use differentials to estimate the value of $\ln(0.98)+(0.98)^5$.

(4 marks)

c) A spherical balloon is deflated at a rate of $12\ cm^3\ s^{-1}$. Find the rate of change of the surface area when the volume is $36\pi\ cm^3$.

[Hint: $V_{sphere} = \frac{4}{3}\pi r^3$ and $A_{sphere} = 4\pi r^2$]

(6 marks)

QUESTION 4

a) Evaluate each of the following integrals using the appropriate substitution.

i) $\int (2+\sin^2\theta)^2 \cos\theta\, d\theta$

(4 marks)

ii) $\int \frac{x}{\sqrt{2x+1}}\, dx$

(5 marks)

b) Given $\int_{-3}^{4} g(x)\, dx = \frac{5}{2}$ and $\int_{1}^{4} g(x)\, dx = -1$. Find $\int_{1}^{4} [x+3+g(x)]\, dx + \int_{-3}^{1} \frac{g(x)}{2}\, dx$.

(4 marks)

c) Given $F(x) = \int_{0}^{x^2-4} e^{\sin t}(2-t)\ dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$

(3 marks)

ii) $F'(2)$

(1 mark)

QUESTION 5

a) Figure 2 below shows the region **R** bounded by the curve $x = y^2 + 1$ and the line $x = 7 - y$.

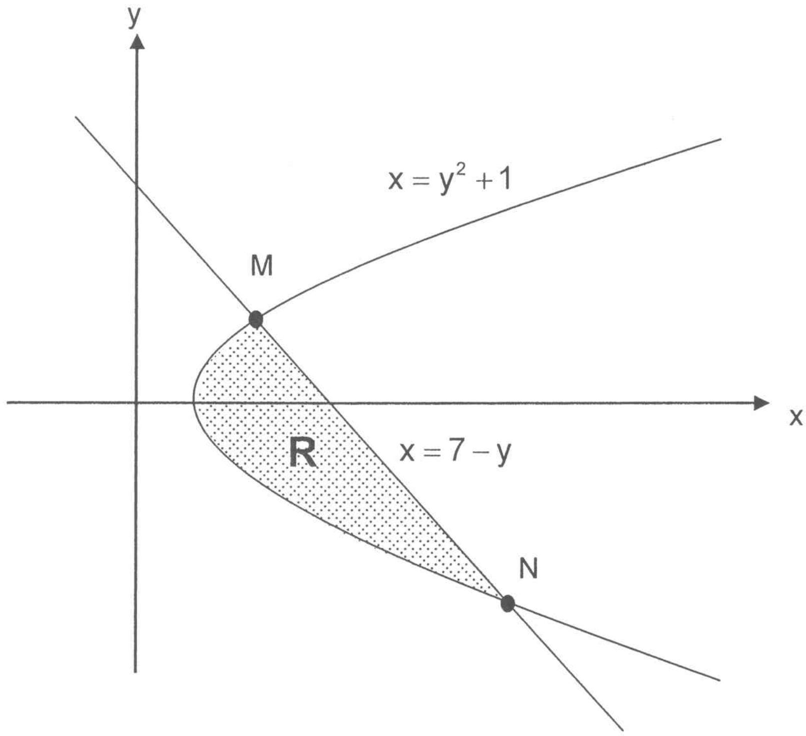

Figure 2

i) Find the coordinates of **M** and **N**.

(2 marks)

ii) Find the area of the shaded region **R**.

(4 marks)

b) Figure 3 below shows the shaded region **R** bounded by the curve $y = x^2$ and the line $y = x + 2$.

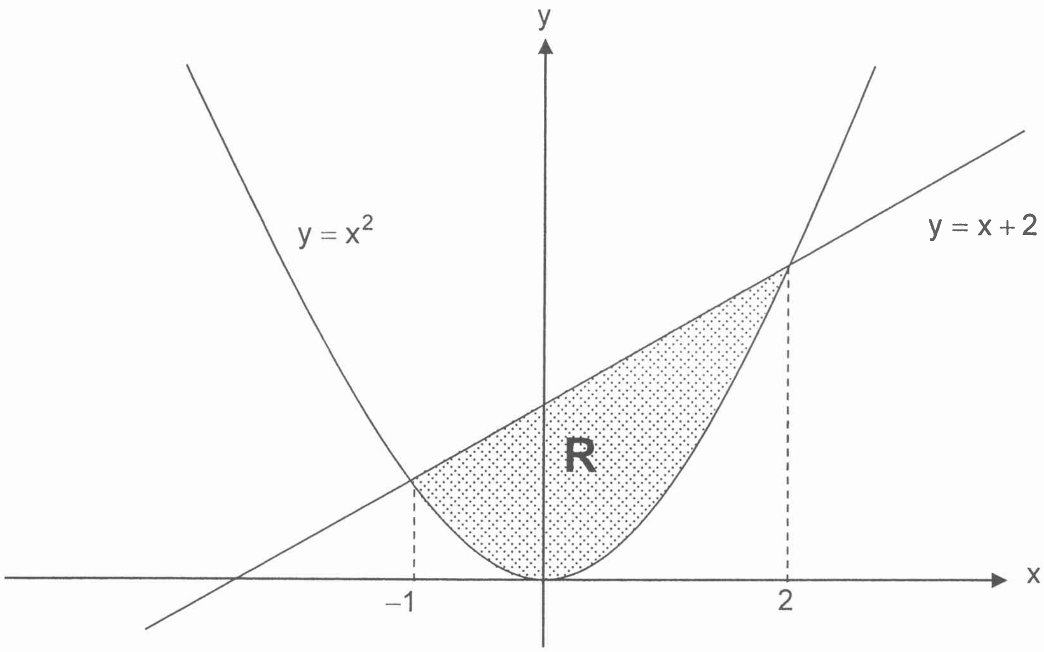

Figure 3

i) Find the volume of the solid generated when the shaded region **R** is revolved about the line $y = 5$ using the **Washer Method**.

(5 marks)

ii) Find the volume of the solid generated when the shaded region **R** is revolved about the line $x = -2$ using the **Shell Method**.

(5 marks)

END OF QUESTION PAPER

A.8 December, 2018

CONFIDENTIAL CS/DEC 2018/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	DECEMBER 2018
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
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   ii) an Answer Booklet – provided by the Faculty
5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 7} \dfrac{x^2 - 49}{x^2 - 4x - 21}$

(3 marks)

ii) $\lim\limits_{x \to \infty} \dfrac{\sqrt{3x^2 - 5}}{7 + 2x}$

(5 marks)

iii) $\lim\limits_{x \to 0} \dfrac{\sin 4x}{2x(1 + \cos 3x)}$

(5 marks)

b) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} kx^2 & , \quad x \le 2 \\ (1-k)x & , \quad 2 < x < 3 \\ \dfrac{1}{2x+6} & , \quad x \ge 3 \end{cases}$$

i) Find the value of k such that $\lim\limits_{x \to 2} f(x)$ exists.

(3 marks)

ii) Determine whether the function is continuous at $x = 3$.

(5 marks)

QUESTION 2

a) Given $f(x) = \sqrt{5x - 1} + 3$. Find $f'(x)$ using the definition of derivative.

(5 marks)

b) Given the function $4ye^{2x} + \sin(xy) = \ln(2x)$.

i) Find $\dfrac{dy}{dx}$ using implicit differentiation.

(5 marks)

ii) Find the gradient of the tangent line for the function at the point $(1,0)$.

(2 marks)

c) Given the function $f(x) = \dfrac{2x^2 - 2}{1 - 9x}$. Find the equation of the tangent line to the curve at the point (1,1).

(5 marks)

QUESTION 3

a) Given a function $f(x) = 3x^2 - x^3$.

i) Find the x-intercept(s) of f.

(2 marks)

ii) Find the interval(s) where f is increasing or decreasing. Hence, determine the relative extremum of f (if any).

(5 marks)

iii) Find the interval(s) where f is concave up or concave down. Hence, determine the inflection point of f (if any).

(5 marks)

iv) Sketch the graph of f using the above information.

(3 marks)

b) Use differentials to estimate $\sqrt{3.99} + (3.99)^3$ correct to **two** decimal places.

(4 marks)

c) Figure 1 shows a closed small cylinder can with radius r cm and height h cm is placed on a large cylinder. The radius of large cylinder is $2r$ cm and height $2h$ cm.

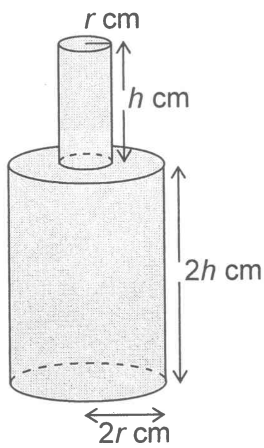

Figure 1

i) If the surface area of the both cylinders are 350π cm^2, express h in terms of r.

(2 marks)

ii) Shows that the volume of both cylinders are given by $V = 315\pi r - 180r^3$.

(3 marks)

iii) Determine r (in cm) so that the volume is a maximum. Hence, find the maximum volume of both cylinders.

(5 marks)

QUESTION 4

a) Evaluate each of the following integrals using the appropriate substitution.

i) $\int (3 - \cos^3 x)\sin x \, dx$

(4 marks)

ii) $\int \frac{3}{4(4-3x)^7} dx.$

(5 marks)

b) Find $\int_{-1}^{2} \frac{5x^{-3} + 2x^2}{x} dx.$

(4 marks)

c) Given $F(x) = \int_{0}^{2x^2} (1 - 3t^2) dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$

(3 marks)

ii) $F'(1)$

(1 mark)

QUESTION 5

a) The region **R** in Figure 2 is bounded by the curve $y^2 = 1 - x$ and the line $y = x + 1$.

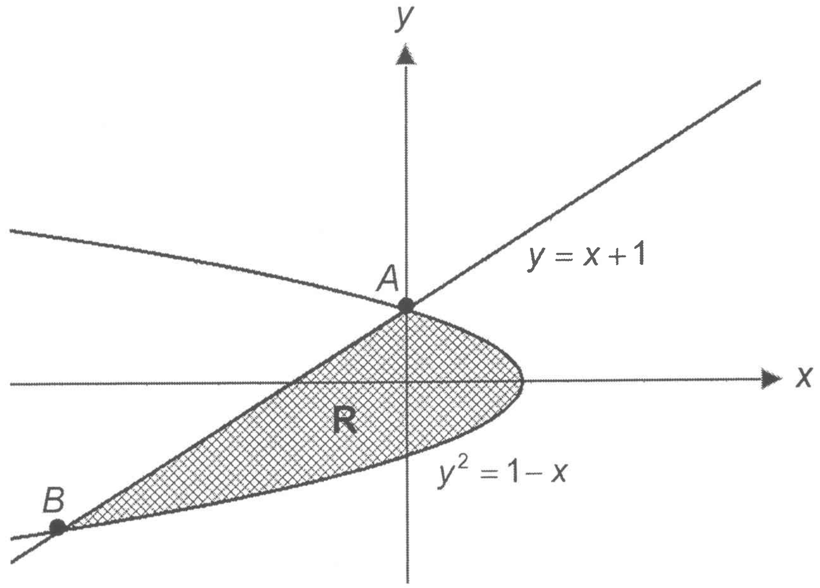

Figure 2

i) Find the coordinates of **A** and **B**.

(2 marks)

ii) Find the area of the shaded region **R.**

(4 marks)

b) Find the volume of the solid generated when the shaded region **R** is revolved about the line:

i) $x = 1$ using the **Washer Method**.

(5 marks)

ii) $y = -2$ using the **Shell Method**.

(5 marks)

END OF QUESTION PAPER

A.9 June, 2016

 CS/JUN 2018/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	JUNE 2018
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless the invigilator gives you permission to do so.
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5. Answer ALL questions in English.

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This examination paper consists of 6 printed pages

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 4} \dfrac{x^2 - 3x - 4}{x^2 - 16}$

(3 marks)

ii) $\lim\limits_{x \to \infty} \sqrt[3]{\dfrac{8x^6 - 3}{x^6 + 2x^3}}$

(4 marks)

iii) $\lim\limits_{x \to 0} \dfrac{x^2 + 3\sin(5x)}{x}$

(5 marks)

b) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x^3 + x + 4 & , \quad x \le -1 \\ \dfrac{2x}{3} + 1 + w & , \quad -1 < x \le 0 \\ 3x^2 + \dfrac{5}{3} & , \quad x > 0 \end{cases}$$

i) Find the value of w such that $\lim\limits_{x \to -1} f(x)$ exists.

(3 marks)

ii) Determine whether the function is continuous at $x = 0$.

(5 marks)

QUESTION 2

a) Given $f(x) = \dfrac{x^2}{x+3}$. Find $f'(x)$ using the definition of derivative.

(5 marks)

b) Given the function $\ln(\cos 2y) + ye^{3x} - 5x^2 = 4$.

i) Find $\dfrac{dy}{dx}$ using implicit differentiation.

(4 marks)

ii) Find the equation of the tangent line to the function at the point $\left(0, \dfrac{\pi}{2}\right)$.

(3 marks)

c) An open-top box is to have a square base and a volume of 10 m^3. Let x and y be the box's width and height respectively.

i) Express y as a function of x.

(2 marks)

ii) Show that the surface area of the box is given by $A = x^2 + \frac{40}{x}$.

(3 marks)

iii) Find the minimum surface area of the box.

(5 marks)

QUESTION 3

a) Given a function $f(x) = 2x^3 - 12x^2 + 18x$.

i) Find the x-intercept(s) of f.

(1 mark)

ii) Find the critical points of f.

(4 marks)

iii) Find the interval(s) where f is increasing or decreasing. Hence, determine the relative extremum of f (if any).

(4 marks)

iv) Find the interval(s) where f is concave up or concave down. Hence, determine the inflection point of f (if any).

(4 marks)

v) Sketch the graph of f using the above information.

(2 marks)

b) Use differentials to estimate $\sqrt{63.99} - \frac{2}{\sqrt[3]{63.99}}$ correct to four decimal places.

(4 marks)

c) A block of ice in the shape of a cylinder has a radius of $r\,cm$ and height $(2r+3)\,cm$. The ice is melting at a constant rate of $4\,cm^3/\text{sec}$. [Hint: $V_{cylinder} = \pi r^2 h$].

i) Show that the volume of ice is given by $V = 2\pi r^3 + 3\pi r^2$.

(2 marks)

ii) How fast is the radius changing at the instant when the radius is $2\,cm$?

(4 marks)

QUESTION 4

a) Evaluate each of the following integrals using the appropriate substitution.

i) $\int \frac{(3+\ln x)^3+12}{4x}\,dx$

(4 marks)

ii) $\int_0^{\frac{\pi}{3}} e^{2\cos(3x)}\sin(3x)\,dx$

(5 marks)

b) Given $\int_{-2}^{0} f(x)dx = 4$ and $\int_{3}^{4} f(x)dx = 2$. Find k if $\int_{-2}^{0} kf(x)dx + \int_{1}^{3} kx^2 dx + \int_{3}^{4} \frac{f(x)}{2}dx = 12$.

(4 marks)

c) Given $F(x) = \int_{0}^{x-1} (t+5t^3)dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$

(3 marks)

ii) $F'(3)$

(1 mark)

QUESTION 5

a) The region **R** in Figure 1 is bounded by the curve $y = x^2 + 2$ and the line $x + y = 4$.

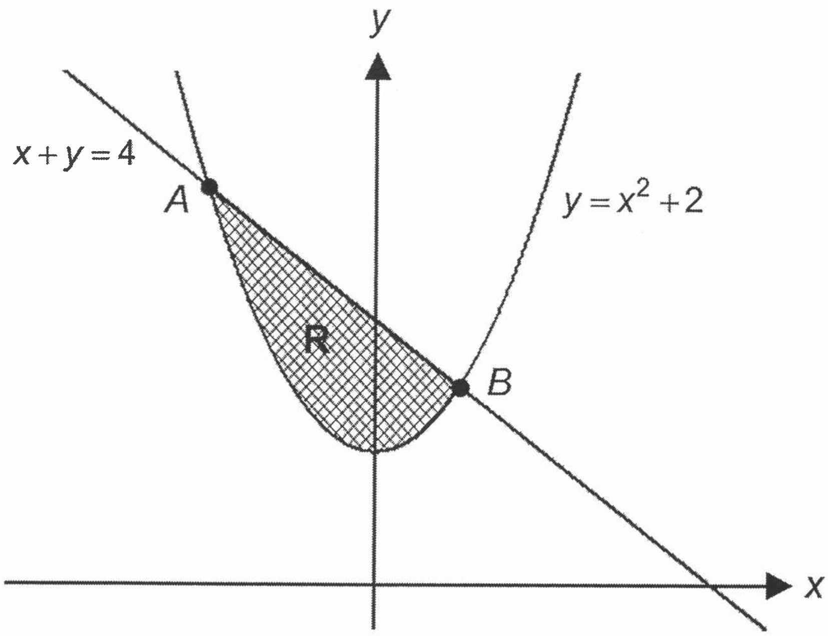

Figure 1

i) Find the coordinates of **A** and **B**.

(2 marks)

ii) Find the area of the shaded region **R.**

(4 marks)

b) The region **B** in Figure 2 is bounded by the curve $x = y^2 + 3$, the line $2y = x - 11$ and the x-axis.

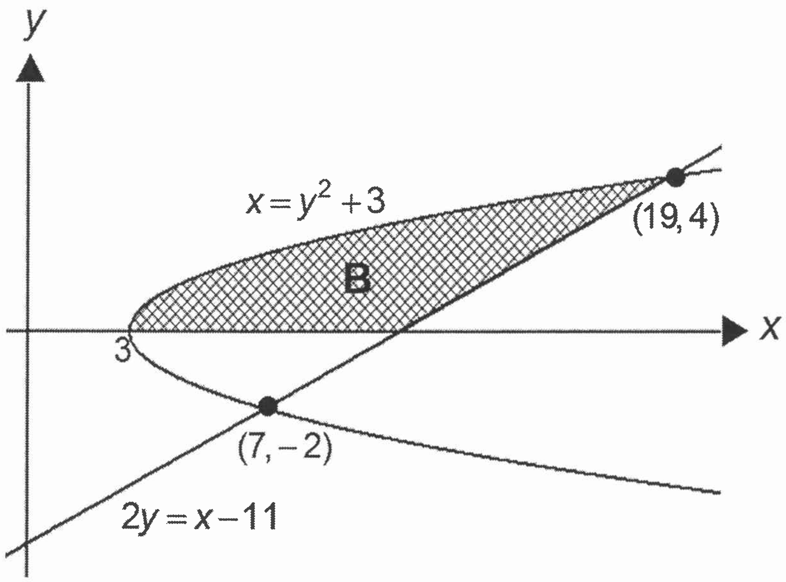

Figure 2

i) Find the volume of the solid generated when the shaded region **B** is revolved about the line $x = 3$ using the **Washer Method.**

(5 marks)

ii) Find the volume of the solid generated when the shaded region **B** is revolved about the line $y = -1$ using the **Shell Method.**

(5 marks)

END OF QUESTION PAPER

A.10 January, 2018

CONFIDENTIAL CS/JAN 2018/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	JANUARY 2018
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

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 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 1}$

(3 marks)

ii) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{1 + 9x^2}}{1 - 3x}$

(4 marks)

iii) $\lim\limits_{x \to 0} \dfrac{6x - \sin(4x)}{2x}$

(5 marks)

b) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \dfrac{6}{x+1} & , x < 2 \\ |x - 4| & , x = 2 \\ \dfrac{x+2}{\sqrt{x+2}} & , x > 2 \end{cases}$$

i) Find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to 7} f(x)$.

(3 marks)

ii) Determine whether the function is continuous at $x = 2$.

(5 marks)

QUESTION 2

a) Given $f(x) = \dfrac{1}{3x - 2}$. Find $f'(x)$ by using the definition of derivative.

(5 marks)

b) Given the function $ye^{x-1} + \cos(3y) = 2 - x^2$.

i) Find $\dfrac{dy}{dx}$ using implicit differentiation.

(4 marks)

ii) Find the equation of the tangent line to the function at the point (1,0).

(3 marks)

c) Figure 1 shows a piece of wire of length 130 cm is bent into the shape ABCDEFG. Given that BC = DE.

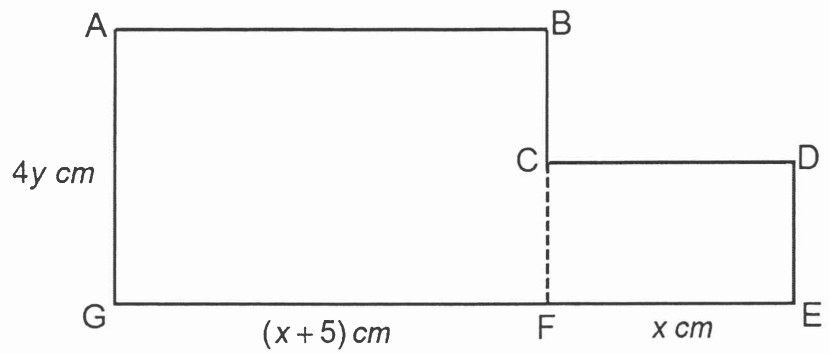

Figure 1

i) Express x in term of y.

(2 marks)

ii) Show that the area enclosed by the wire is $A = 200y - 12y^2$.

(3 marks)

iii) Hence, find the maximum area of the shape ABCDEFG.

(5 marks)

QUESTION 3

a) Given a function $f(x) = -x^3 + 6x^2 - 9x$.

i) Find the y – intercept for f.

(1 mark)

ii) Find the critical points of f.

(4 marks)

iii) Find the interval(s) where f is increasing or decreasing. Hence, determine the relative extremum of f (if any).

(4 marks)

iv) Find the interval(s) where f is concave up and concave down. Hence, determine the inflection point of f (if any).

(4 marks)

v) Sketch the graph of f.

(2 marks)

b) Given $f(x) = \frac{8}{x^3} + \ln(2x-3)$. Estimate the value of $f(2.01)$ using differentials.

(4 marks)

c) A conical water tank has a radius of $3cm$ at the top and $15cm$ high as shown in Figure 2. Water flows into the tank at a rate of $6cm^3s^{-1}$. [Hint: $V_{cone} = \frac{1}{3}\pi r^2 h$].

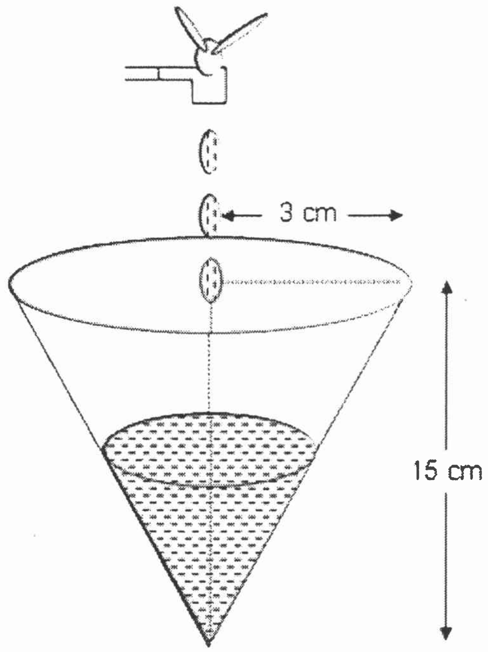

Figure 2

i) Show that the volume of water is given by $V = \frac{\pi h^3}{75}$.

(2 marks)

ii) How fast is the height changing at an instant when the height of the water is 5cm?

(4 marks)

QUESTION 4

a) Evaluate each of the following integral using appropriate substitution.

i) $\int x(x-1)^7 \, dx$

(4 marks)

ii) $\int \frac{\sin(4x)}{(2+\cos(4x))^2} \, dx$

(5 marks)

b) If $\int_{-1}^{2} f(x)dx = 5$ and $\int_{0}^{2} f(x)dx = -2$, evaluate $\int_{0}^{2}\left[x + \frac{f(x)}{3}\right]dx + \int_{-1}^{0} f(x)dx$.

(4 marks)

c) Given $F(x) = \int_{0}^{2x+1} \frac{t^2 - 1}{5 + \sqrt{3t}}\,dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$

(3 marks)

ii) $F'(1)$

(1 mark)

QUESTION 5

a) The region **R** in Figure 3 is bounded by the curve $x = y^2 - 4y + 2$ and the line $x = y - 2$.

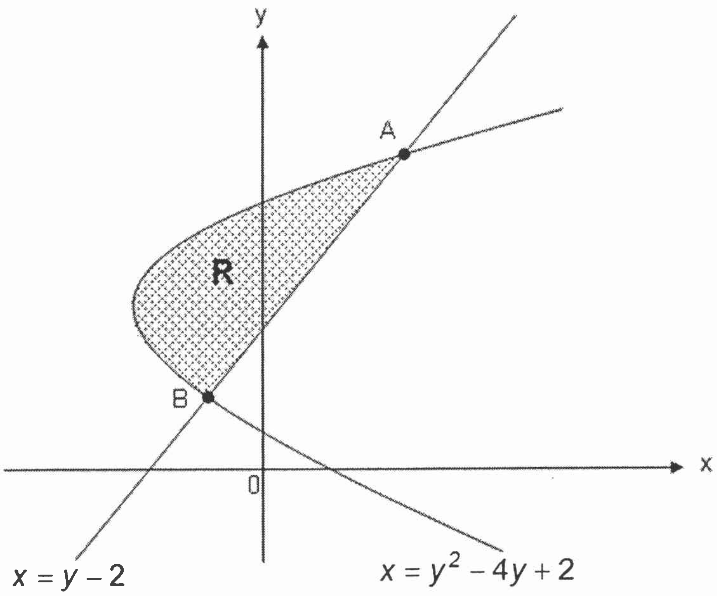

Figure 3

i) Find the coordinates of **A** and **B**.

(2 marks)

ii) Find the area of the shaded region **R**.

(4 marks)

b) The region **W** in Figure 4 is bounded by the curve $y = x^2$, the line $x + y = 6$ and the y-axis.

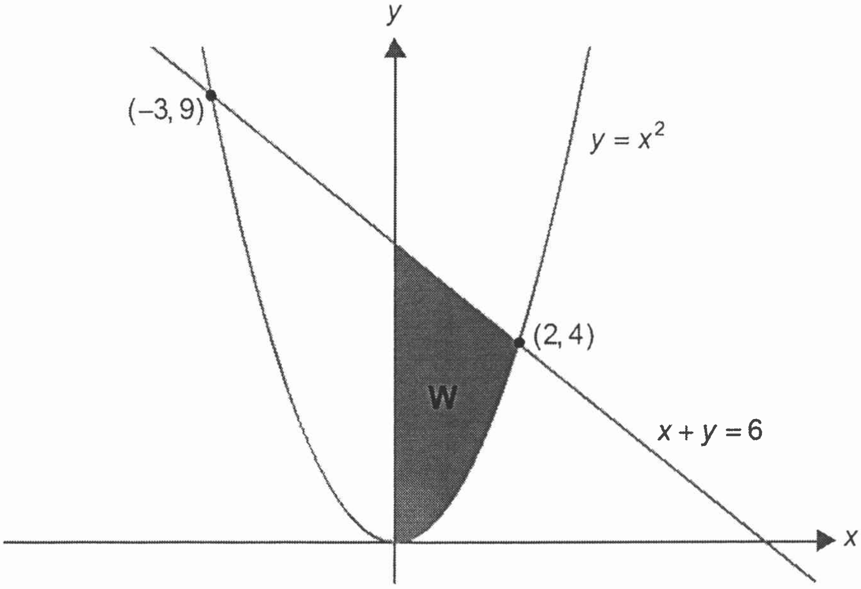

Figure 4

i) Find the volume of the solid generated when the shaded region **W** is revolved about the x–axis using **Washer Method**.

(5 marks)

ii) Find the volume of the solid generated when the shaded region **W** is revolved about the $x = -1$ by using the **Shell Method**.

(5 marks)

END OF QUESTION PAPER

A.11 March, 2017

CONFIDENTIAL CS/MAR 2017/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	MARCH 2017
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless the invigilator gives you permission to do so.
4. Please check to make sure that this examination pack consists of :
   i) the Question Paper
   ii) an Answer Booklet – provided by the Faculty
5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x\to 3} \dfrac{\sqrt{3}-\sqrt{x}}{x-3}$

(3 marks)

ii) $\lim\limits_{x\to -\infty} \sqrt{\dfrac{x^3-3x^2+x}{4x^3+2x-7}}$

(3 marks)

iii) $\lim\limits_{x\to 0} \dfrac{x+\tan 4x}{2x}$

(5 marks)

b) Let $f(x)=\begin{cases} 4+\sqrt{k-x} & , \quad x<-1 \\ x^2-5x & , \quad -1\le x<3 \\ -4x+6 & , \quad x\ge 3 \end{cases}$

i) Find the value of k such that $\lim\limits_{x\to -1} f(x)$ exists.

(4 marks)

ii) Determine whether the function is continuous at $x=3$.

(5 marks)

QUESTION 2

a) Use differential to estimate the value of $\sqrt[3]{63.99}+\dfrac{1}{\sqrt{63.99}}$ to three decimal places.

(5 marks)

b) Find $\dfrac{dy}{dx}$ for $\ln(\sin y^2)-\dfrac{x}{y}=e^{3y}$ by using implicit differentiation.

(7 marks)

c) Given the function $f(x)=\sqrt{x^2+25}$.

i) Find the gradient of the tangent to the curve at $x=2$.

(3 marks)

ii) Find the equation of the tangent line to the curve at the point (2,0).

(2 marks)

iii) Use the definition of derivative to find $f'(x)$ for the function.

(5 marks)

QUESTION 3

a) Given the function $f(x)=\dfrac{5-2x}{x+3}$.

i) Find the x–intercept and y–intercept.

(2 marks)

ii) Find the vertical asymptote and horizontal asymptote.

(3 marks)

iii) Find the interval(s) where $f(x)$ is increasing or decreasing.

(4 marks)

iv) Find the interval(s) where $f(x)$ is concave up or down.

(3 marks)

v) Sketch the graph of $f(x)$ using the above information.

(3 marks)

b) The volume of a cylindrical can open at the top with height, h cm and radius, r cm is 1200 cm^3. Find the minimum surface area of the cylindrical can.

[Hint: Volume of cylinder: $V=\pi r^2 h$; Surface area of cylinder open at the top: $A=\pi r^2+2\pi rh$]

(5 marks)

c) If the radius of a sphere is increasing at the rate of $4\,cms^{-1}$, how fast is the volume changing when the surface area is $16\,cm^2$?

(Hint : Volume of sphere $=\dfrac{4}{3}\pi r^3$; surface area of a sphere $=4\pi r^2$)

(5 marks)

QUESTION 4

a) Evaluate each of the following integral using appropriate substitution.

i) $\int \csc^2(2x)\cot(2x)\,dx$

(5 marks)

ii) $\int \dfrac{\ln(y^2)}{3y}\,dy$

(4 marks)

b) If $\int_{-2}^{4} f(x)\,dx=18$ and if $\int_{-2}^{0} f(x)\,dx=6$, evaluate $\int_{0}^{4}(f(x)+x^2)\,dx$.

(4 marks)

c) Given $F(x) = \int_0^{\sqrt{x}} t \ln(t^2)\, dt$. Use the Second Fundamental Theorem of Calculus to find

i) $F'(x)$

(3 marks)

ii) $F'(1)$

(1 mark)

QUESTION 5

The shaded region in Figure 1 is bounded by the curve $y = x^2 + x + 1$ and $y = -x$.

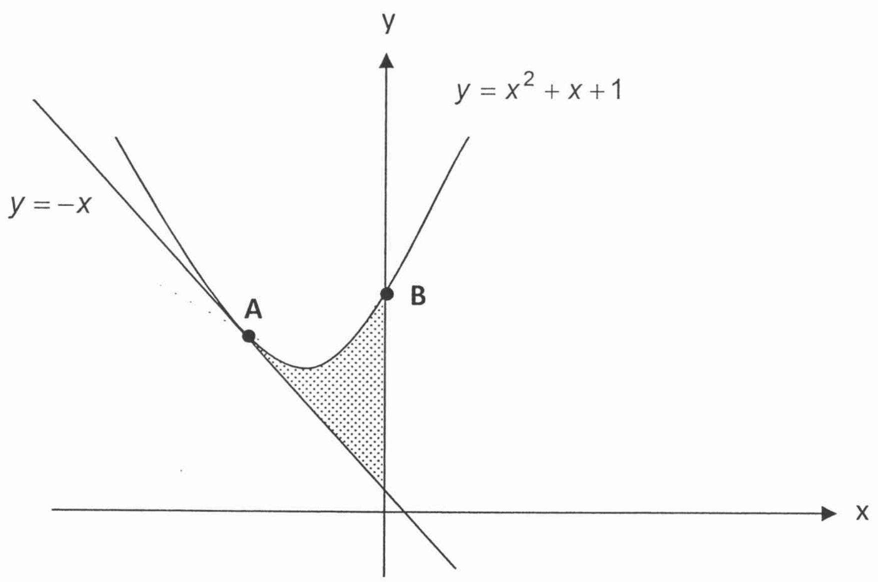

Figure 1

a) Find the coordinates of **A** and **B**.

(2 marks)

b) Find the area of the shaded region.

(4 marks)

c) Consider the shaded region **R** enclosed by the curves $x = -y$ and $x = 2 - y^2$ as shown in Figure 2

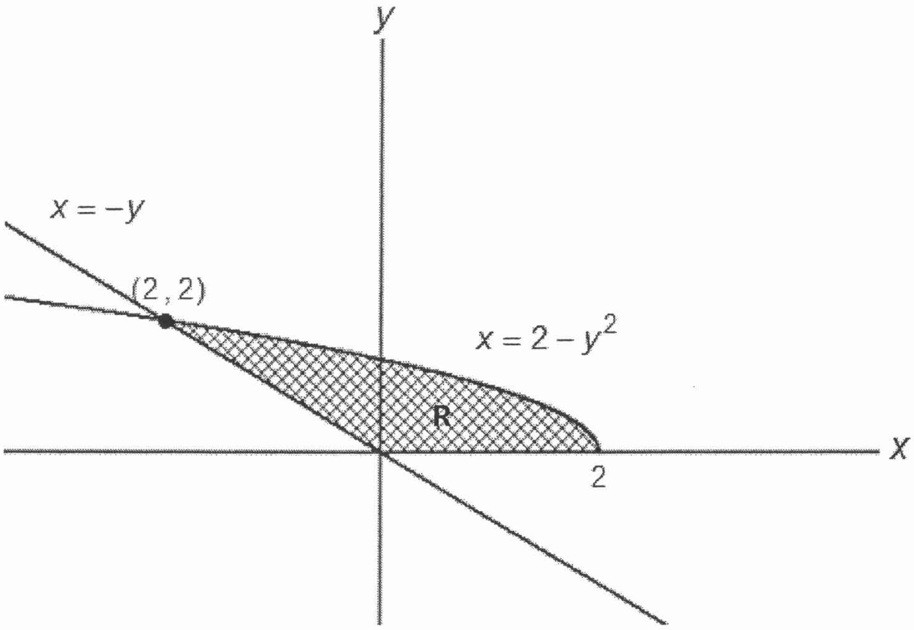

Figure 2

i) Using Shell method, find the volume of the solid generated when the shaded region is revolved about the x-axis.

(5 marks)

ii) Find the volume of the solid by using Washer Method when the shaded region is revolved about the line $x = 2$.

(5 marks)

END OF QUESTIONS PAPER

A.12 October, 2016

CONFIDENTIAL CS/OCT 2016/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	OCTOBER 2016
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :
   i) the Question Paper
   ii) an Answer Booklet – provided by the Faculty
5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 4 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim_{x \to -2} \dfrac{x^2 - 4x - 12}{x^2 - 2x - 8}$

(3 marks)

ii) $\lim_{x \to \infty} \dfrac{3 + 2x}{8 + 10x^2}$

(3 marks)

iii) $\lim_{x \to 0} \dfrac{\sin 6x}{\tan 5x}$

(5 marks)

b) Let $f(x) = \begin{cases} 3x^3 + 2x + 8 & , \quad x \le -2 \\ \dfrac{3}{2}(2x+2) + k & , \quad -2 < x \le 0 \\ 2x^2 + 6 & , \quad x > 0 \end{cases}$

i) Find the value of k such that $\lim_{x \to -2} f(x)$ exists.

(4 marks)

ii) Determine whether the function is continuous at $x = 0$.

(5 marks)

QUESTION 2

a) Use differentials to estimate the value of $(15.99)^{\frac{3}{4}} + (15.99)^{\frac{1}{2}}$ to three decimal places.

(5 marks)

b) Find $\dfrac{dy}{dx}$ for $\dfrac{3y}{2x} + 7e^x \ln(\sin y) = 4x^2$ by using implicit differentiation.

(7 marks)

c) Given the function $f(x) = \sqrt{3x - 12} + 9$.

i) Find the gradient of the tangent to the curve at $x = 5$.

(3 marks)

ii) Find the equation of the tangent line to the curve at the point (5,0).

(2 marks)

iii) Use the definition of derivative to find $f'(x)$ for the function.

(5 marks)

QUESTION 3

a) Given the function $f(x) = x^3 - 6x^2 + 9x + 2$.

i) Find y-intercept(s) of $f(x)$.

(1 marks)

ii) Find the interval(s) where the function is increasing or decreasing. Hence, determine the relative extremum of $f(x)$, if any.

(6 marks)

iii) Find the interval(s) where $f(x)$ is concave up or concave down. Hence, determine the inflection point(s), if any.

(5 marks)

iv) Sketch the graph of $f(x)$ using the above information

(3 marks)

b) The area of a triangle is decreasing at rate of 6 cm^2/min. Find the rate of change of the side, x, when the area of the triangle is 338 cm^2.

$$\left[\text{Hint : Area of triangle} = \frac{1}{2}x^2\right]$$

(5 marks)

c) The Mean Value Theorem for differentiation states that if $f(x)$ is continuous on a closed interval $[a,b]$ and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

If $f(x) = \dfrac{x+1}{x}$, find the value of c in the interval $\left[\dfrac{1}{2}, 2\right]$.

(5 marks)

QUESTION 4

a) Evaluate each of the following integral using appropriate substitution.

i) $\int \tan^3 4x \sec^2 4x dx$

(5 marks)

ii) $\int \dfrac{6}{5(4-7x)^3} dx$

(4 marks)

b) Find $\int_2^5 \left(x + \dfrac{2}{x}\right)^2 dx$

(4 marks)

c) Given $F(x)=\int_0^{\sqrt{x}} \frac{t}{t^2-1}\,dt$. Use the Second Fundamental Theorem of Calculus to find

i) $F'(x)$

(3 marks)

ii) $F'(3)$

(1 mark)

QUESTION 5

The shaded region in Figure 1 is enclosed by the curve $y=4-x^2$ and $y=-2x+4$.

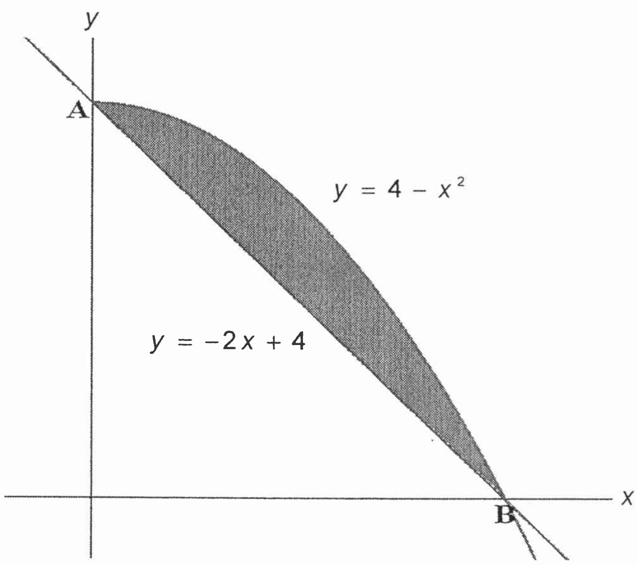

Figure 1

a) Find the coordinates of **A** and **B**.

(2 marks)

b) Find the area of the shaded region.

(4 marks)

c) i) Using Washer method, set up the integral to find the volume of the solid obtained when the shaded region is revolved about the line $y=0$.

(5 marks)

ii) Find the volume of the solid by using Shell Method when the shaded region is revolved about the line $x=2$.

(5 marks)

END OF QUESTIONS PAPER

A.13 March, 2016

CONFIDENTIAL CS/MAR 2016/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	MARCH 2016
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :

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   ii) an Answer Booklet – provided by the Faculty

5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate the following:

i) $\lim_{t\to 0} \dfrac{2t^3+3t^2}{3t^4-2t^2}$.

(3 marks)

ii) $\lim_{x\to \infty} \sqrt[3]{\dfrac{27x^6+5}{x^6-11x^3}}$.

(3 marks)

iii) $\lim_{x\to 0} \dfrac{x^3+4\sin 7x}{x}$.

(4 marks)

b) The function $f(x)$ is defined as follows.

$$f(x)=\begin{cases} kx+1 & , \quad x\le 2 \\ kx^2-3 & , \quad 2<x<4 \\ \dfrac{3x^2}{2}+5 & , \quad x\ge 4 \end{cases}$$

i) Compute the value of k so that $\lim_{x\to 2} f(x)$ exists.

(4 marks)

ii) Hence, determine whether f is continuous at $x=4$.

(5 marks)

QUESTION 2

a) Use the definition of derivative to find $f'(x)$ for $f(x)=2\sqrt{x}$.

(5 marks)

b) Find $\dfrac{dy}{dx}$ for $y\ln(x^4)+\cos(2y)=x^3y^2$ by using implicit differentiation.

(7 marks)

c) Let C be a curve represented by $y=\dfrac{x-1}{x+1}$. Find an equation of the tangent line to C at the point P(0,-1).

(5 marks)

d) Use differentials to estimate the value of $\sqrt{63.8}-\sqrt[3]{63.8}$ to three decimal places.

(4 marks)

QUESTION 3

a) Given a function $f(x) = 2x^3 + 3x^2 - 12x - 2$.

i) Find the x- and y-intercepts for f.

(2 marks)

ii) Find the intervals where f is increasing and decreasing. Hence, determine the relative extremum of f (if any).

(6 marks)

iii) Find the intervals where f is concave up and concave down. Hence, determine the inflection point of f (if any).

(5 marks)

iv) Sketch the graph of f.

(3 marks)

b) A spherical balloon is being inflated at the rate of 8 cm^3/sec. Determine the rate at which the radius of the balloon is changing when the radius is 10 cm.

$$\left[\text{Hint : Volume of sphere} = \frac{4}{3}\pi r^3\right]$$

(5 marks)

c) An open box with a square base and a volume of $256\,cm^3$ is to be constructed from the same material. Show that the surface area of the box is $S = x^2 + \frac{1024}{x}$, where x is the length of the base of the box. Hence, find the dimensions of the box so that the minimum amount of material was used in its construction.

(6 marks)

QUESTION 4

a) Find the following:

i) $\displaystyle\int \frac{\sin x}{(1+\cos x)^3}\,dx$.

(5 marks)

ii) $\displaystyle\int t^{\frac{1}{3}}(t-1)^2\,dt$.

(4 marks)

b) Given $f(x)=\begin{cases}2x+3 & , \; x \le 0 \\ x^3 & , \; x>0\end{cases}$

Find the value of k if $\int_{-1}^{4} k\,f(x)\,dx = 33$.

(4 marks)

c) Let $F(x)=\int_{0}^{x^4}\sqrt{t}\;dt$. Use the Second Fundamental Theorem of Calculus to find:

i) $F'(x)$.

(3 marks)

ii) $F'(1)$.

(1 mark)

$\left[\text{Hint: } \frac{d}{dx}\int_{a}^{x} f(t)\,dt = f(x)\right]$

QUESTION 5

a) **Figure 1** shows the curves $y=x^2-4x+3$ and $y=-x^2+2x+3$ intersects at points **A** and **B**.

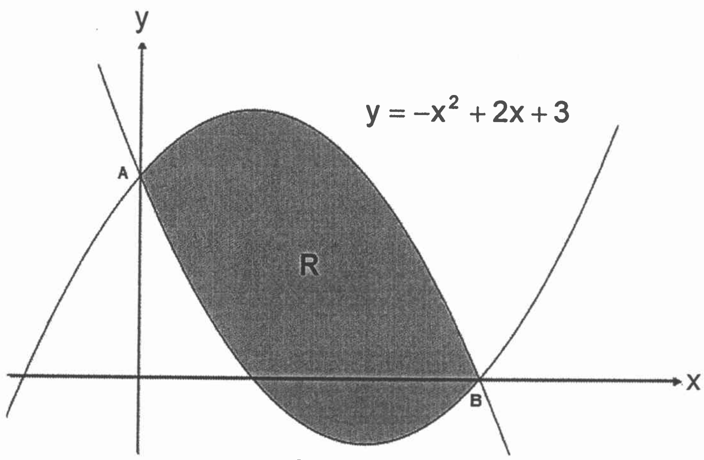

Figure 1

i) Find the coordinates of **A** and **B**.

(2 marks)

ii) Find the area of the shaded region **R**.

(4 marks)

b) Consider the shaded region R enclosed by the curves $y = x^2$ and $y^2 = 8x$ as shown in **Figure 2**.

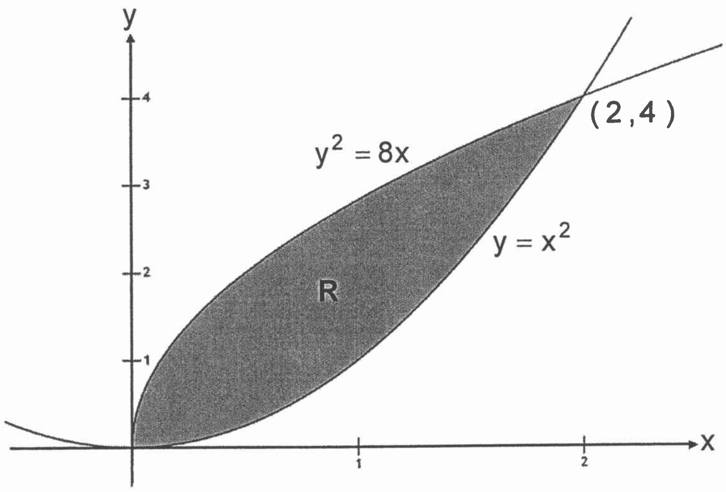

Figure 2

Find the volume of the solid generated by revolving R:

i) about the $x = 0$ using **Washer Method**.

(5 marks)

ii) about the $y = 0$ using **Shell Method**.

(5 marks)

END OF QUESTION PAPER

A.14 September, 2015

CONFIDENTIAL CS/SEP 2015/MAT183

UNIVERSITI TEKNOLOGI MARA
FINAL EXAMINATION

COURSE	:	CALCULUS I
COURSE CODE	:	MAT183
EXAMINATION	:	SEPTEMBER 2015
TIME	:	3 HOURS

INSTRUCTIONS TO CANDIDATES

1. This question paper consists of five (5) questions only.
2. Answer ALL questions in the Answer Booklet. Start each answer on a new page.
3. Do not bring any material into the examination room unless permission is given by the invigilator.
4. Please check to make sure that this examination pack consists of :
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5. Answer ALL questions in English.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

This examination paper consists of 5 printed pages

 CONFIDENTIAL

QUESTION 1

a) Evaluate each of the following limits:

i) $\lim\limits_{x \to 4} \dfrac{16-4x}{x^2-x-12}$

(3 marks)

ii) $\lim\limits_{x \to +\infty} \dfrac{4x^5-x+1}{3x^4-2x^2}$

(3 marks)

iii) $\lim\limits_{x \to 0} \left(\dfrac{\sin 5x}{3x} - \dfrac{x \cos x}{\tan x} \right)$

(5 marks)

b) $$f(x) = \begin{cases} 3x & , \quad x < \dfrac{1}{2} \\ \dfrac{x^2-4x-5}{x-5} & , \quad \dfrac{1}{2} \le x < 5 \\ x^2-2k & , \quad x \ge 5 \end{cases}$$

i) Find the value of k such that $\lim\limits_{x \to 5} f(x)$ exist.

(5 marks)

ii) Determine whether the function is continuous at $x = \dfrac{1}{2}$.

(5 marks)

QUESTION 2

a) Use the definition of derivative to find $f'(x)$ for the function $f(x) = \dfrac{x}{3x-1}$.

(5 marks)

b) Find $\dfrac{dy}{dx}$ for $2y\ln\left(x^2\right) + 3x^2 = 2\tan(x-y) - e^{x^2}$ by using implicit differentiation.

(7 marks)

c) A rectangular door with a semicircular top as shown in **Figure 1** is to have a perimeter of 6 meters.

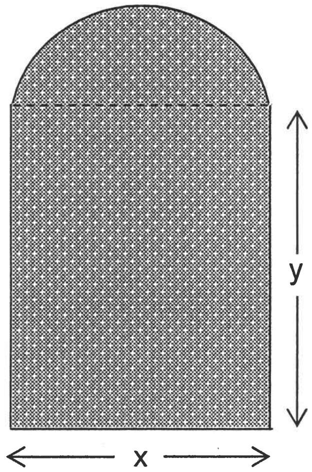


Figure 1

i) Express y in terms of x.

(3 marks)

ii) Show that area of the door is given by $A = \dfrac{24x - 4x^2 - \pi x^2}{8}$.

(2 marks)

iii) Determine the maximum area of the door.

(5 marks)

QUESTION 3

a) Consider the function $f(x) = x^3 - 5x^2 + 3x + 2$

i) State the y - intercept(s) of f.

(1 mark)

ii) Find the intervals where f is increasing and decreasing. Hence, determine the relative extremum of f, if any.

(8 marks)

iii) Find the intervals where f is concave up or down. Hence, determine the inflection point(s) of f, if any.

(5 marks)

iv) Sketch the graph of f using the above information.

(3 marks)

b) The Mean-Value Theorem for derivatives states that if $f(x)$ is continuous on (a, b) and differentiable on [a, b], then there exists at least one point c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Find the value(s) of c that satisfies the theorem for $f(x) = \sqrt{100 - x^2}$ on the interval $[-10, 6]$.

(7 marks)

QUESTION 4

a) Evaluate each of the following integrals using appropriate substitution.

i) $$\int_0^1 \frac{x}{(x+1)^3}\,dx$$

(5 marks)

ii) $$\int \cos 2x\, e^{\sin 2x}\,dx$$

(4 marks)

b) Find $\displaystyle\int \frac{\left(4+\sqrt{x}\right)^2}{2x}\,dx$.

(4 marks)

c) Given $F(x) = \displaystyle\int_0^{4x^2} \left(3t^2 + 5\right)dt$.

i) Find $F'(x)$ by using the Second Fundamental Theorem of Calculus.

(3 marks)

ii) Hence, evaluate $F'(1)$

$$\left[\text{Hint}: F'(x) = \frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f[g(x)]\,g'(x)\right]$$

(1 mark)

QUESTION 5

Figure 2 shows the shaded region **R** bounded by the parabola $y^2 = 4x$ and the line $x = 1$.

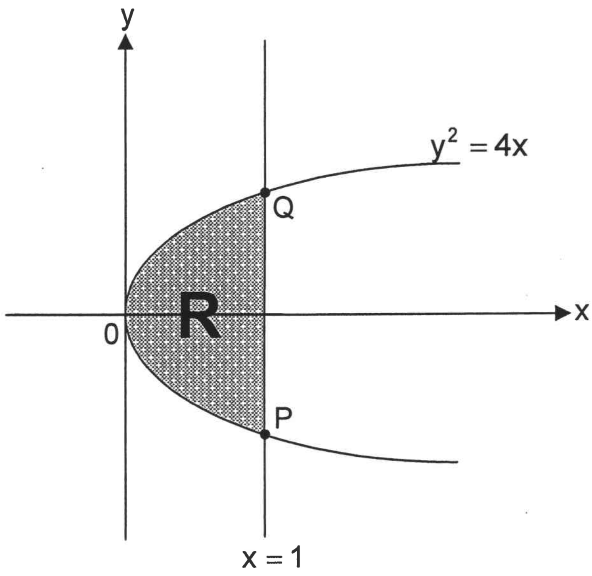


Figure 2

a) Find the coordinates of P and Q.

(2 marks)

b) Determine the area of **R**.

(3 marks)

c) Find the volume of the solid of revolution obtained by revolving **R** about:

i) y-axis using the method of washers.

(5 marks)

ii) the line $y = 2$ using the method of cylindrical shells.

(6 marks)

END OF QUESTION PAPER

B Important Formulas

B.1 Derivatives and Integrals

Derivatives:

$$\frac{d}{dx}c = 0$$
$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}a^x = \ln(a)\cdot a^x$$
$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$
$$\frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$
$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$
$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$
$$\frac{d}{dx}e^{u(x)} = u'(x)e^{u(x)}$$
$$\frac{d}{dx}\ln|u(x)| = \frac{u'(x)}{u(x)}$$

Integrals:

$$\int 0\,dx = C$$
$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$
$$\int e^x\,dx = e^x + C$$
$$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int \frac{1}{x\ln(a)}\,dx = \log_a|x| + C$$
$$\int \cos(x)\,dx = \sin(x) + C$$
$$\int \sin(x)\,dx = -\cos(x) + C$$
$$\int \sec^2(x)\,dx = \tan(x) + C$$
$$\int \csc^2(x)\,dx = -\cot(x) + C$$
$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$
$$\int \csc(x)\cot(x)\,dx = -\csc(x) + C$$
$$\int e^{u(x)}\,dx = \int e^{u(x)}u'(x)\,dx \quad \text{(use substitution)}$$
$$\int \frac{1}{u(x)}u'(x)\,dx = \ln|u(x)| + C$$

B.2 Trigonometric Identities, Index (Exponent) Rules and Logarithmic Properties

Index (Exponent) Rules:

$$a^m \cdot a^n = a^{m+n}$$
$$a^m / a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$a^0 = 1 \quad (a \neq 0)$$
$$a^{-n} = 1/a^n$$

Logarithmic Properties:

$$\log_a(xy) = \log_a(x) + \log_a(y)$$
$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$
$$\log_a(x^n) = n\log_a(x)$$
$$\log_a(a) = 1$$
$$\log_a(1) = 0$$

Trigonometric Identities:

$$\sin^2(x) + \cos^2(x) = 1$$
$$1 + \tan^2(x) = \sec^2(x)$$
$$1 + \cot^2(x) = \csc^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$
$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$
$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$
$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

C | Cheat Sheets

C.1 Computing Limits

LIMIT

One-sided limit

$$\lim_{x\to 3^-}\frac{x}{x-3}=\frac{2.9}{2.9-3}=\frac{+}{-}=-\infty$$

$$\lim_{x\to 3^+}\frac{x}{x-3}=\frac{3.1}{3.1-3}=\frac{+}{+}=+\infty$$

$$\lim_{x\to 3}\frac{x}{x-3}=\frac{3}{0}=\text{undefined}$$

Continuity

$$f(x)=\begin{cases} x-3 & ; \quad x<1 \\ 2x & ; \quad 1\le x<2 \\ 4 & ; \quad x\ge 2\end{cases}$$

$$\lim_{x\to 3}f(x)=\lim_{x\to 3}2x=2(3)=6$$

$f(1)=2(1)=2$

$$\lim_{x\to 1^-}f(x)=\lim_{x\to 1}(x-3)=1-3=-2$$

$$\lim_{x\to 1^+}f(x)=\lim_{x\to 1}2x=2(1)=2$$

$$\lim_{x\to 1}f(x)=\text{undefined}$$

$\therefore$ f is discontinuous at $x=1$

$f(2)=4$

$$\lim_{x\to 2^-}f(x)=\lim_{x\to 2}2x=2(2)=4$$

$$\lim_{x\to 2^+}f(x)=\lim_{x\to 2}4=4$$

$$\lim_{x\to 2}f(x)=4$$

Since $f(2)=\lim_{x\to 2}f(x)=4$,

$\therefore$ f is continuous at $x=2$

Two-sided limit

$$\lim_{x\to 2}\left(x^2+3x-3\right)=2^2+3(2)-3=7$$

$$\lim_{x\to 2}\frac{x+3}{x-4}=\frac{2+3}{2-4}=\frac{5}{-2}$$

$$\lim_{x\to 2}\sqrt{x+2}=\sqrt{2+2}=2$$

$$\lim_{x\to 2}\frac{x+3}{x-2}=\frac{5}{0}=\text{undefined}$$

$$\lim_{x\to 2}\sqrt{x-5}=\sqrt{2-5}=\sqrt{-3}=\text{undefined}$$

$$\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}\frac{(x+3)(x-2)}{x-2}=\lim_{x\to 2}(x+3)=2+3=5$$

$$\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}=\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}\times\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+4}+2\right)}{x+4-4}=\lim_{x\to 0}\frac{x\left(\sqrt{x+4}+2\right)}{x}$$

$$=\lim_{x\to 0}\left(\sqrt{x+4}+2\right)$$

$$=\sqrt{0+4}+2=2+2=4$$

Limit at infinity

$$\lim_{x\to+\infty}2x^3+4x-3=\lim_{x\to+\infty}2x^3=+\infty$$

$$\lim_{x\to-\infty}2x^3+4x-3=\lim_{x\to-\infty}2x^3=-\infty$$

$$\lim_{x\to+\infty}-2x^3=\lim_{x\to+\infty}-2x^3=-\infty$$

$$\lim_{x\to-\infty}-2x^3=\lim_{x\to-\infty}-2x^3=+\infty$$

$$\lim_{x\to+\infty}\frac{3x+4}{7x+5}=\lim_{x\to+\infty}\frac{(3x+4)/x}{(7x+5)/x}=\lim_{x\to+\infty}\frac{3+4/x}{7+5/x}=\frac{3}{7}$$

$$\lim_{x\to+\infty}\sqrt[3]{\frac{3x+4}{7x+5}}=\lim_{x\to+\infty}\sqrt[3]{\frac{(3x+4)/x}{(7x+5)/x}}=\lim_{x\to+\infty}\sqrt[3]{\frac{3+4/x}{7+5/x}}=\sqrt[3]{\frac{3}{7}}$$

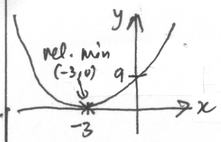
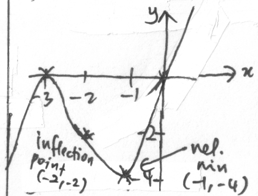
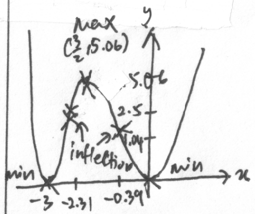
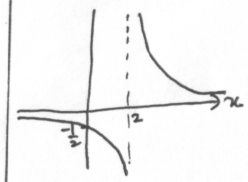
$$\lim_{x\to+\infty}\frac{\sqrt{x^2+3}}{2x-4}=\lim_{x\to+\infty}\frac{\sqrt{x^2+3}/\sqrt{x^2}}{(2x-4)/x}=\lim_{x\to+\infty}\frac{\sqrt{x^2/x^2+3/x^2}}{2x/x-4/x}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{1+3/x^2}}{2-4/x}=\frac{\sqrt{1}}{2}=\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+3}}{2x-4}=\lim_{x\to-\infty}\frac{\sqrt{x^2+3}/\sqrt{x^2}}{(2x-4)/(-x)}=\lim_{x\to-\infty}\frac{\sqrt{x^2/x^2+3/x^2}}{2x/(-x)-4/(-x)}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{1+3/x^2}}{-2+4/x}=\frac{\sqrt{1}}{-2}=\frac{1}{-2}$$

C.2 Curve Sketching

CURVE SKETCHING	$f(x) = (x+3)^2$	$f(x) = x(x+3)^2$	$f(x) = x^2(x+3)^2$	$f(x) = \frac{1}{x-2}$
① Intercept(s) a) x-intercept b) y-intercept	 $y=0, \quad x=-3$ $x=0, \quad y=9$	 $y=0, x=0, x=-3$ $x=0, y=0$	 $y=0, x=0, x=-3$ $x=0, y=0$	 $y=0$, none $x=0, y=-\frac{1}{2}$
② Behaviour as a) $x \to \infty$ b) $x \to -\infty$	$\lim_{x\to+\infty} f(x) = +\infty$ $\lim_{x\to-\infty} f(x) = +\infty$	$\lim_{x\to+\infty} f(x) = +\infty$ $\lim_{x\to-\infty} f(x) = -\infty$	$\lim_{x\to\infty} f(x) = +\infty$ $\lim_{x\to-\infty} f(x) = +\infty$	$\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to-\infty} f(x) = 0$
③ Critical points a) where f is not differentiable b) stationary points	$f'(x) = 2(x+3)$ $f'(x) = 0 \Rightarrow x = -3$ -3 f': − | + f: ↓ | ↑	$f'(x) = (x+3)^2 + 2x(x+3)$ $= (x+3)(x+3+2x)$ $= (x+3)(3x+3)$ $f'(x) = 0 \Rightarrow x=-3, x=-1$ -3 -1 f': + | − | + f: ↑ | ↓ | ↑	$f'(x) = 2x(x+3)^2$ $+ 2x^2(x+3)$ $= 2x(x+3)(2x+3)$ $f'(x) = 0 \Rightarrow x=0, x=-3, x=-\frac{3}{2}$ -3 -3/2 0 f': − | + | − | + f: ↓ | ↑ | ↓ | ↑	$x-2=0$ $x=2$ critical point no stationary point 2 f': − | + f: ↓ | ↑
④ Interval(s) of a) decrease b) increase	interval of decrease : $(-\infty,-3)$ interval of increase : $(-3,\infty)$	interval of decrease : $(-3,-1)$ interval of increase : $(-\infty,-3)\cup(-1,\infty)$	intervals of decrease : $(-\infty,-3)\cup(-\frac{3}{2},0)$ intervals of increase : $(-3,-\frac{3}{2})\cup(0,\infty)$	interval of decrease : $(-\infty,2)$ interval of increase : $(2,\infty)$
⑤ Relative extremum	rel. min : $(-3,0)$ rel. max : none	rel. min : $(-1,-4)$ rel. max : $(-3,0)$	rel. min : $(-3,0), (0,0)$ rel. max : $(-\frac{3}{2}, 5.06)$	rel. min : none rel. max : none ∴ no stationary point
⑥ Concavity	$f''(x) = 2$ -3 f'': + | + f: ∪ | ∪	$f''(x) = 6x+12$ $f''(x) = 0 \Rightarrow x = -2$ -2 f'': − | + f: ∩ | ∪	$f''(x) = 10x^2+27x+9$ $f''(x) = 0 \Rightarrow x = -2.31$ & -0.39 -2.31 -0.39 f'': + | − | + f: ∪ | ∩ | ∪	$f'(x) = -\frac{1}{(x-2)^2}$ $f''(x) = \frac{2}{(x-2)^3}$ 2 f'': − | + f: ∩ | ∪
⑦ Inflection point(s)	no inflection point	$(-2,-2)$	$(-2.31, 2.5), (-0.39, 1.04)$	no inflection point
⑧ Vertical asymptote(s)	none	none	none	$x-2=0, x=2$ $\lim_{x\to2^+} f(x) = +\infty, \lim_{x\to2^-} f(x) = -\infty$
⑨ Horizontal asymptote(s)	none	none	none	From ②, $y=0$