LIMIT

One-sided limit

$$\lim_{x\to 3^-}\frac{x}{x-3}=\frac{2.9}{2.9-3}=\frac{+}{-}=-\infty$$

$$\lim_{x\to 3^+}\frac{x}{x-3}=\frac{3.1}{3.1-3}=\frac{+}{+}=+\infty$$

$$\lim_{x\to 3}\frac{x}{x-3}=\frac{3}{0}=\text{undefined}$$

Continuity

$$f(x)=\begin{cases} x-3 & ; \quad x<1 \\ 2x & ; \quad 1\le x<2 \\ 4 & ; \quad x\ge 2 \end{cases}$$

$$\lim_{x\to 3} f(x)=\lim_{x\to 3}2x=2(3)=6$$

$$f(1)=2(1)=2$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}(x-3)=1-3=-2$$

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1}2x=2(1)=2$$

$$\lim_{x\to 1} f(x)=\text{undefined}$$

$\therefore$ f is discontinuous at $x=1$

$$f(2)=4$$

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2}2x=2(2)=4$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2}4=4$$

$$\lim_{x\to 2} f(x)=4$$

Since $f(2)=\lim_{x\to 2} f(x)=4$,

$\therefore$ f is continuous at $x=2$

Two-sided limit

$$\lim_{x\to 2}\left(x^2+3x-3\right)=2^2+3(2)-3=7$$

$$\lim_{x\to 2}\frac{x+3}{x-4}=\frac{2+3}{2-4}=\frac{5}{-2}$$

$$\lim_{x\to 2}\sqrt{x+2}=\sqrt{2+2}=2$$

$$\lim_{x\to 2}\frac{x+3}{x-2}=\frac{5}{0}=\text{undefined}$$

$$\lim_{x\to 2}\sqrt{x-5}=\sqrt{2-5}=\sqrt{-3}=\text{undefined}$$

$$\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}\frac{(x+3)(x-2)}{x-2}=\lim_{x\to 2}(x+3)=2+3=5$$

$$\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}=\lim_{x\to 0}\frac{x}{\sqrt{x+4}-2}\times\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+4}+2\right)}{x+4-4}=\lim_{x\to 0}\frac{x\left(\sqrt{x+4}+2\right)}{x}$$

$$=\lim_{x\to 0}\left(\sqrt{x+4}+2\right)$$

$$=\sqrt{0+4}+2=2+2=4$$

Limit at infinity

$$\lim_{x\to +\infty}2x^3+4x-3=\lim_{x\to +\infty}2x^3=+\infty$$

$$\lim_{x\to -\infty}2x^3+4x-3=\lim_{x\to -\infty}2x^3=-\infty$$

$$\lim_{x\to +\infty}-2x^3=\lim_{x\to +\infty}-2x^3=-\infty$$

$$\lim_{x\to -\infty}-2x^3=\lim_{x\to -\infty}-2x^3=+\infty$$

$$\lim_{x\to +\infty}\frac{3x+4}{7x+5}=\lim_{x\to +\infty}\frac{(3x+4)/x}{(7x+5)/x}=\lim_{x\to +\infty}\frac{3+4/x}{7+5/x}=\frac{3}{7}$$

$$\lim_{x\to +\infty}\sqrt[3]{\frac{3x+4}{7x+5}}=\lim_{x\to +\infty}\sqrt[3]{\frac{(3x+4)/x}{(7x+5)/x}}=\lim_{x\to +\infty}\sqrt[3]{\frac{3+4/x}{7+5/x}}=\sqrt[3]{\frac{3}{7}}$$

$$\lim_{x\to +\infty}\frac{\sqrt{x^2+3}}{2x-4}=\lim_{x\to +\infty}\frac{\sqrt{x^2+3}/\sqrt{x^2}}{(2x-4)/x}=\lim_{x\to +\infty}\frac{\sqrt{x^2/x^2+3/x^2}}{2x/x-4/x}$$

$$=\lim_{x\to +\infty}\frac{\sqrt{1+3/x^2}}{2-4/x}=\frac{\sqrt{1}}{2}=\frac{1}{2}$$

$$\lim_{x\to -\infty}\frac{\sqrt{x^2+3}}{2x-4}=\lim_{x\to -\infty}\frac{\sqrt{x^2+3}/\sqrt{x^2}}{(2x-4)/(-x)}=\lim_{x\to -\infty}\frac{\sqrt{x^2/x^2+3/x^2}}{2x/(-x)-4/(-x)}$$

$$=\lim_{x\to -\infty}\frac{\sqrt{1+3/x^2}}{-2+4/x}=\frac{\sqrt{1}}{-2}=\frac{1}{-2}$$