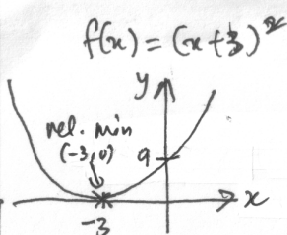


# CURVE SKETCHING

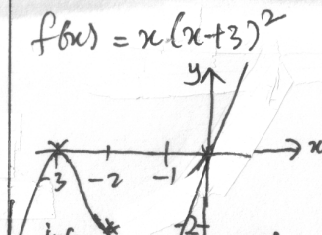
## ① Intercept(s)

- a) x-intercept
- b) y-intercept



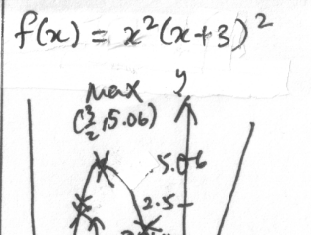
$$y=0, x=-3$$

$$x=0, y=9$$



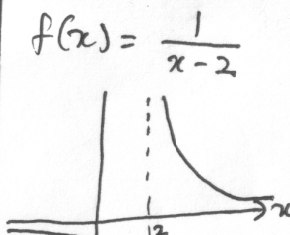
$$y=0, x=0, x=-3$$

$$x=0, y=0$$



$$y=0, x=0, x=-3$$

$$x=0, y=0$$



$$y=0, \text{ none}$$

$$x=0, y=-\frac{1}{2}$$

## ② Behaviour as

- a)  $x \rightarrow \infty$
- b)  $x \rightarrow -\infty$

$$\lim_{x \rightarrow \infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow \infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$

$$\lim_{x \rightarrow \infty} f(x) = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = +\infty$$

$$\lim_{x \rightarrow \infty} f(x) = 0$$

$$\lim_{x \rightarrow -\infty} f(x) = 0$$

## ③ Critical points

- a) where  $f$  is not differentiable
- b) stationary points

$$f'(x) = 2(x+3)$$

$$f'(x) = 0 \Rightarrow x = -3$$

	-3	
$f'$	-	+
$f$	↓	↑

$$f'(x) = (x+3)^2 + 2x(x+3)$$

$$= (x+3)(x+3+2x)$$

$$= (x+3)(3x+3)$$

$$f'(x) = 0 \Rightarrow x = -3, x = -1$$

	-3	-1	
$f'$	+	-	+
$f$	↑	↓	↑

$$f'(x) = 2x(x+3)^2 + 2x^2(x+3)$$

$$= 2x(x+3)(2x+3)$$

$$f'(x) = 0 \Rightarrow x = 0, x = -3, x = -\frac{3}{2}$$

	-3	-3/2	0	
$f'$	+	-	+	
$f$	↑	↓	↑	

$$x-2=0$$

$$x=2$$

Critical point

no stationary point

	2	
$f'$	-	+
$f$	↓	↑

## ④ Interval(s) of

- a) decrease
- b) increase

interval of decrease:  $(-\infty, -3)$

interval of increase:  $(-3, \infty)$

interval of decrease:  $(-3, -1)$

interval of increase:  $(-\infty, -3) \cup (-1, \infty)$

interval of decrease:  $(-\infty, -3) \cup (-\frac{3}{2}, 0)$

interval of increase:  $(-3, -\frac{3}{2}) \cup (0, \infty)$

interval of decrease:  $(-\infty, 2)$

interval of increase:  $(2, \infty)$

## ⑤ Relative extremum

rel. min:  $(-3, 0)$

rel. max: none

rel. min:  $(-1, -4)$

rel. max:  $(-3, 0)$

rel. min:  $(-3, 0), (0, 0)$

rel. max:  $(-\frac{3}{2}, 5.06)$

rel. min: none

rel. max: none

no stationary point

## ⑥ Concavity

$$f''(x) = 2$$

	-3	
$f''$	+	+
$f$	∪	∪

$$f''(x) = 6x + 12$$

$$f''(x) = 0 \Rightarrow x = -2$$

	-2	
$f''$	-	+
$f$	∩	∪

$$f''(x) = 10x^2 + 27x + 9$$

$$f''(x) = 0 \Rightarrow x = -2.31, -0.39$$

	-2.31	-0.39	
$f''$	+	-	+
$f$	∪	∩	∪

$$f''(x) = -\frac{1}{(x-2)^3}$$

	2	
$f''$	-	+
$f$	∩	∪

## ⑦ Inflection point(s)

no inflection point

$(-2, -2)$

$(-2.31, 2.5), (-0.39, 1.04)$

no inflection point

## ⑧ Vertical asymptote(s)

none

none

none

$x-2=0, x=2$

$\lim_{x \rightarrow 2^+} f(x) = +\infty, \lim_{x \rightarrow 2^-} f(x) = -\infty$

## ⑨ Horizontal asymptote(s)

none

none

none

From ②,

$y = 0$