BITS Pilani
Pilani Campus

Artificial & Computational Intelligence

AIMLCZG557

Indumathi V
Guest Faculty
BITS -WILP

Module 5:

Probabilistic Representation and Reasoning

A. Inference using full joint distribution

B. Bayesian Networks

I. Knowledge Representation

II. Conditional Independence

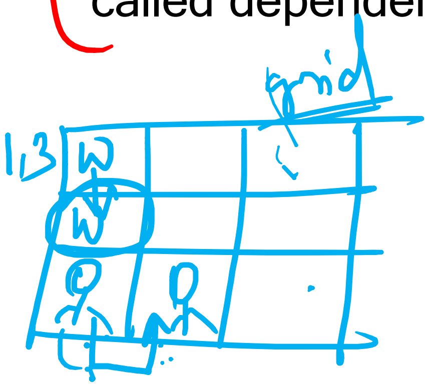
III. Exact Inference

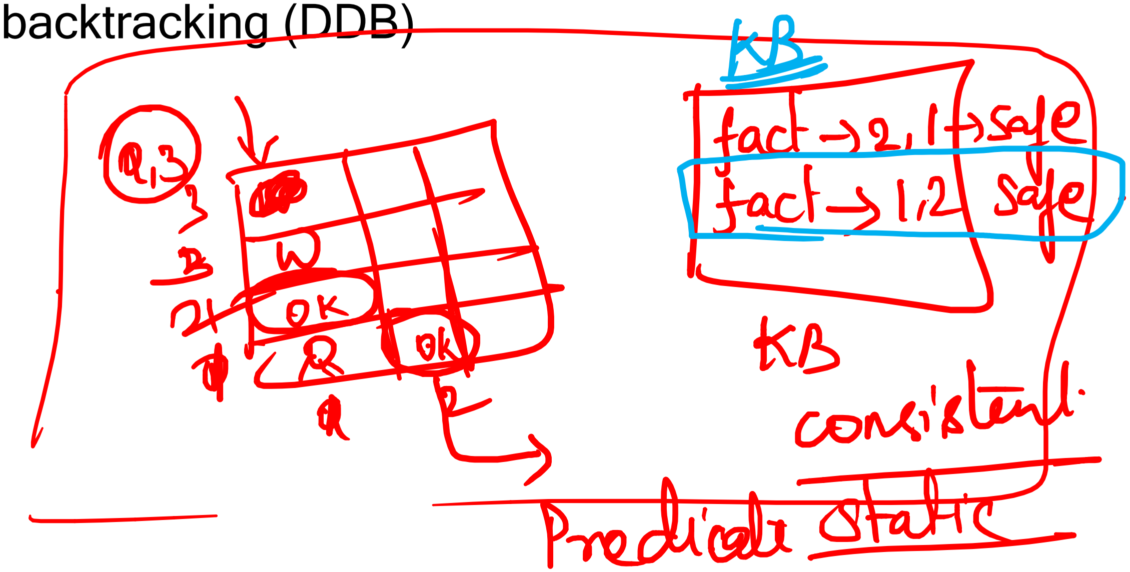
IV. Introduction to Approximate Inference

Reasoning

- Monotonic Reasoning
- Non- Monotonic Reasoning

  Dependency Directed Backtracking: when a statement is deleted as “ no more valid”, other related statements have to be backtracked and they should be either deleted or new proofs have to be found for them. This is called dependency directed backtracking (DDB)





Building a Bayesian Network

AI → dental surgery

① Random Var 2^4

Example Bayesian Net #1

W T V C

- A simple world with four random variables
  - Weather, Toothache, Cavity, Catch
  - Weather is independent of other variables
  - Toothache and Catch are conditionally independent given Cavity
  - P(Toothache, Catch | Cavity) = P(Toothache | Cavity) . P(Catch | Cavity)
  - Cavity is a direct cause of Toothache and Catch
  - No direct relation between Toothache and Catch exists

	P(W)	P(¬W)
	-	-

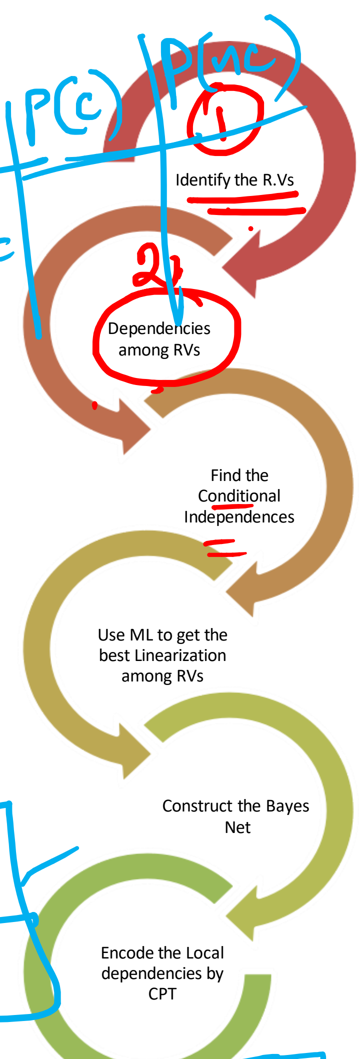
	P(V)	P(¬V)

$P(T/\neg V)$ $P(T/V)$

$P(\neg T/\neg V)$ $P(\neg T/V)$

T CPT

V	P(T)	P(¬T)
T		
F		



P → simply apply ~~ø~~ chain rule

Example Bayesian Net #1

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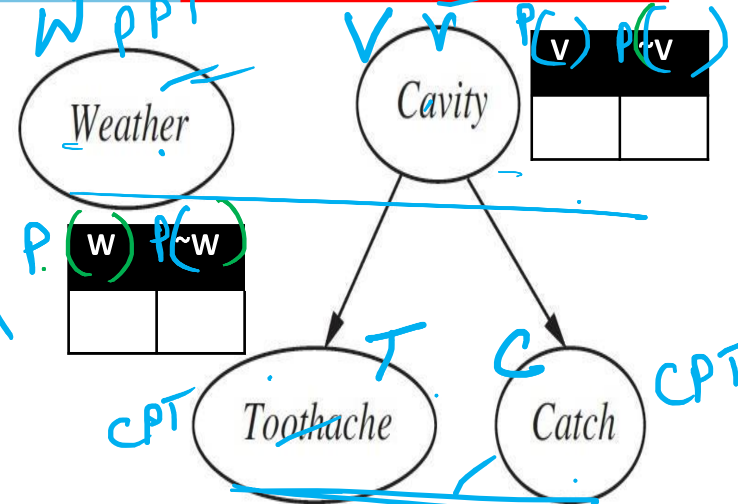
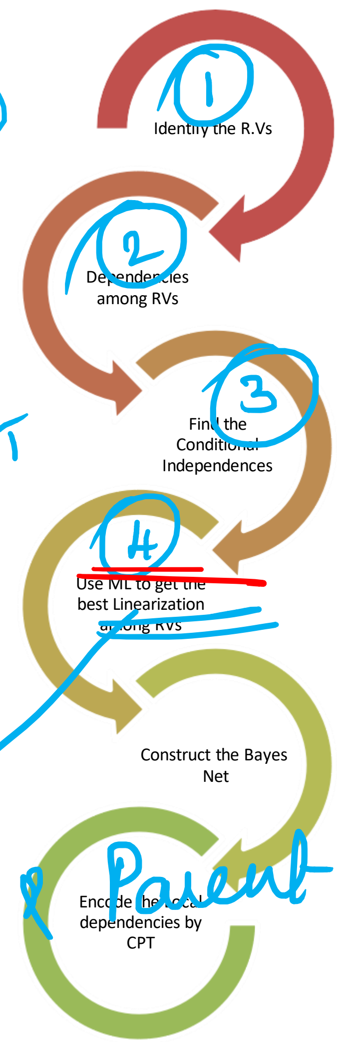


P(W)	P(~W)

P(V)	P(~V)

V	T	~T
T	–	–
F	–	–

V	C	~C
T	–	–
F	–	–



WVTC

child should proceed first & Parent

WVTC

4

TCVW → CTVW → WTCV → WCTV

Q: $P(W, nV, nT, C) \Longrightarrow P(W, nT, C, nV)$

chain rule

Example Bayesian Net #1

A simple world with four random variables

- Weather, Toothache, Cavity, Catch
- Weather is independent of other variables   1-0.2
- Toothache and Catch are conditionally independent given Cavity
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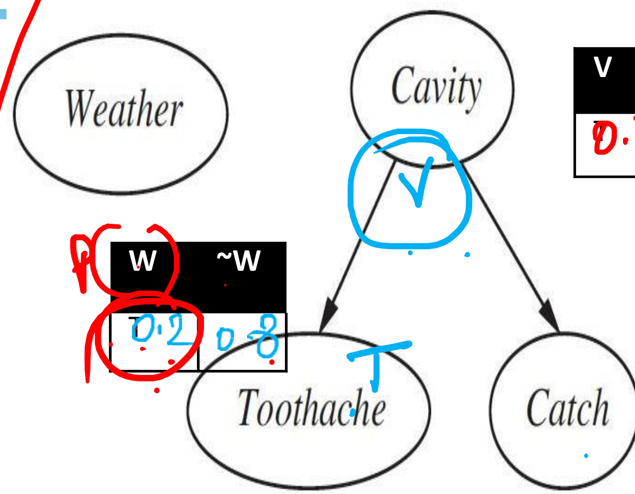
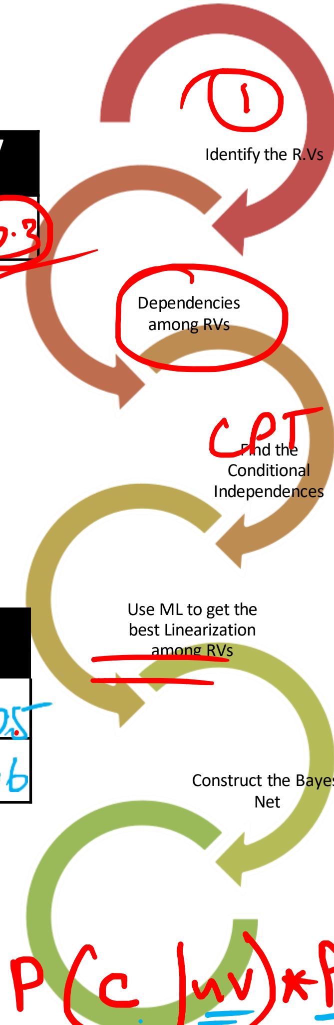


W	~W
0.2	0.8

V	~V
0.7	0.3

V	P(T)	~T
T	0.6	0.4
F	8 → 0.2	8 → 0.8

V	P(C)	~C
T	0.5	0.5
F	0.4	0.6



$$P(W \mid \cancel{nT}, \cancel{C}, \cancel{nV}) * P(nT \mid \cancel{C}, nV) * P(C \mid nV) * P(nV)$$

$$\Downarrow$$

$$P(W) * P(nT \mid nV) * P(C \mid nV) * P(nV)$$

$$0.2 * 0.8 * 0.4 * 0.3 \Rightarrow 0.0192$$

Example Bayesian Net #1

A simple world with four random variables

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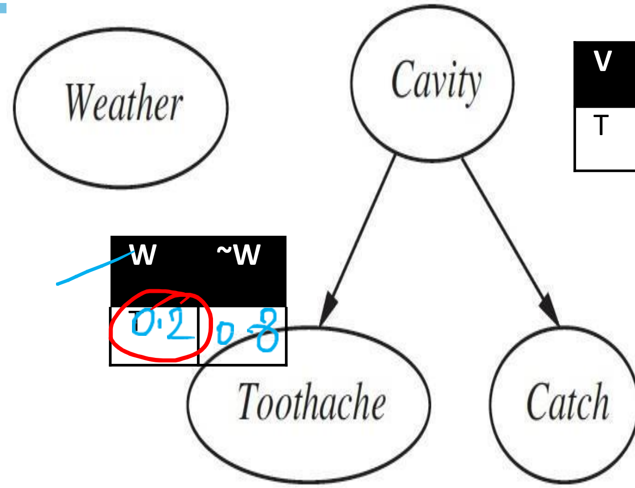
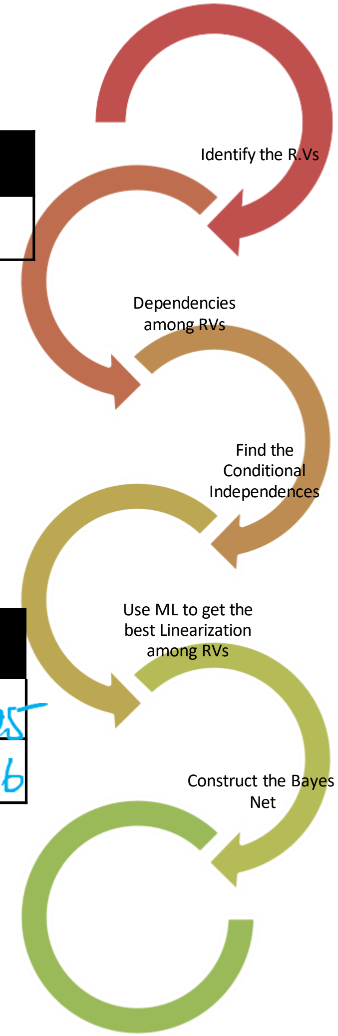


V	~V
T	

W	~W
T 0.2	0.8

V		~T
T		
F		

V		~C
T	0.5	0.5
F	0.4	0.6



Example Bayesian Net #2

5

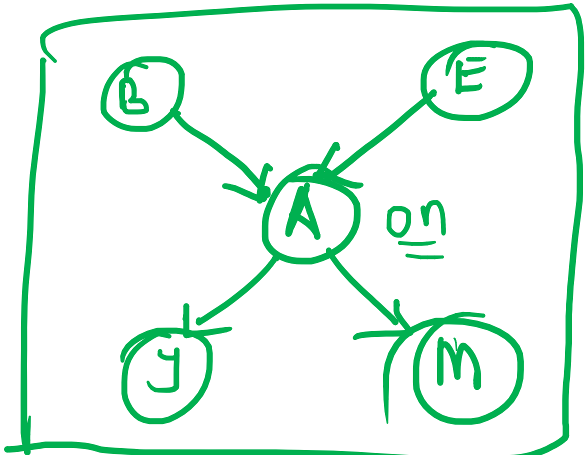
B, E, J, M, A

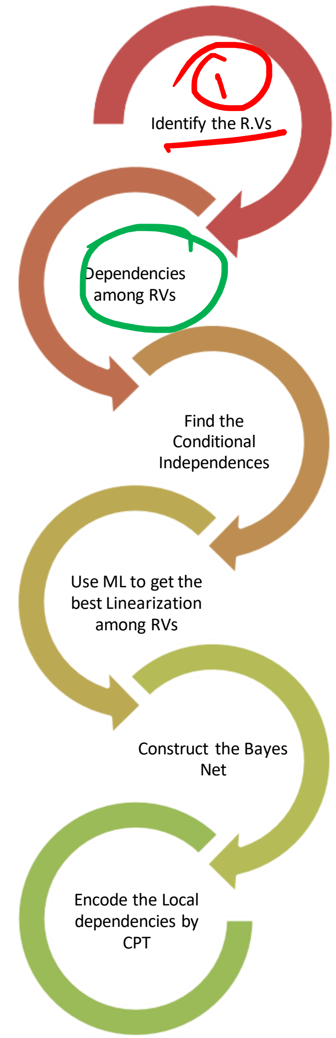
A Burglary Alarm System

- Fairly reliable on detecting a burglary B
- Also responds to earthquakes E
- Two neighbors John and Mary are asked to call you at work when Burglary happens and they hear the Alarm A
  J M
- John nearly always calls when he hears the alarm, however sometimes confuses the telephone ring with alarm and calls then too
- Mary like loud music and often misses the alarm altogether
- **Problem**: Given the information that who has / has not called we need to estimate the probability of a burglary

(A J)

(A, M)





Example Bayesian Net #2

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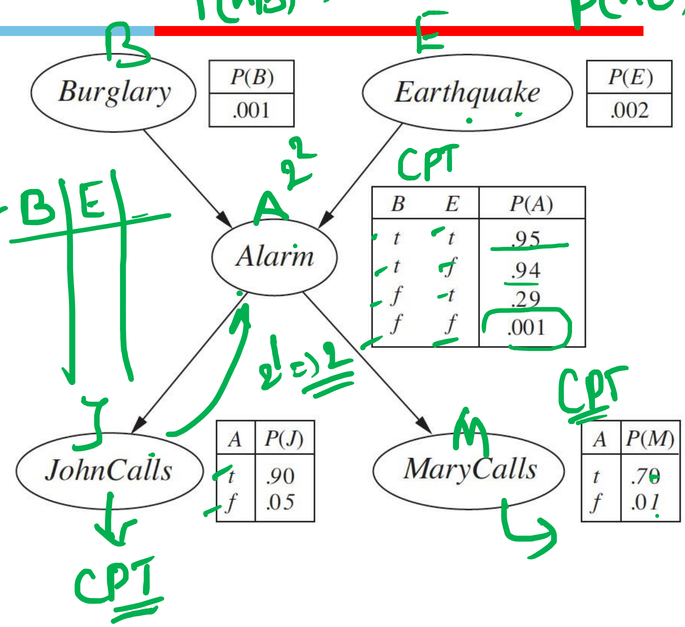


P(B)
.001

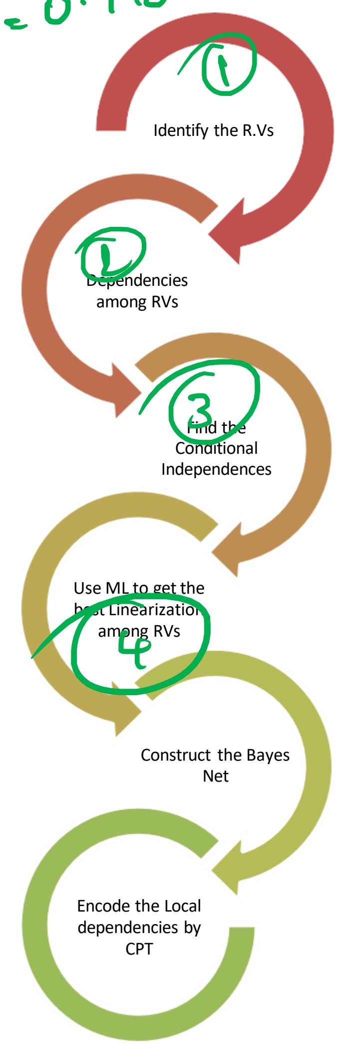
P(E)
.002

B	E	P(A)
t	t	.95
t	f	.94
f	t	.29
f	f	.001

A	P(J)
t	.90
f	.05

A	P(M)
t	.70
f	.01

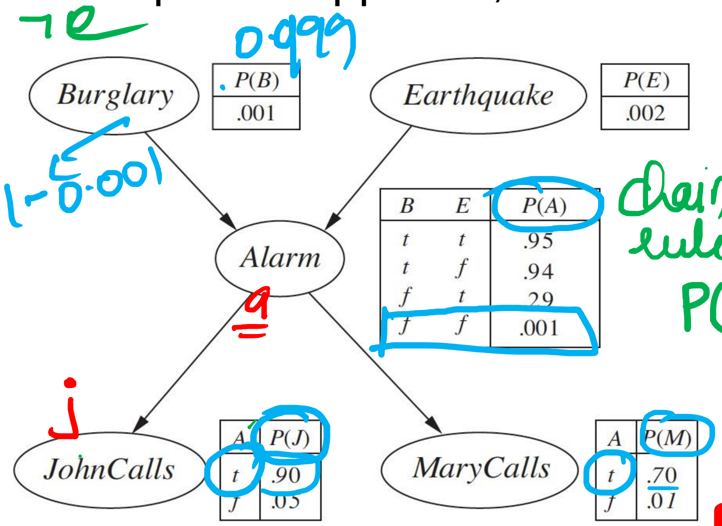
JMABE



Examples

$P(a \neg b \neg e\, j m) \rightarrow P(j m a \neg b \neg e)$

1. Calculate the probability that alarm has sounded, but neither burglary nor earthquake happened, and both John and Mary called



$$P(j, m, a, \neg b, \neg e) = P(j \mid a)P(m \mid a)P(a \mid \neg b \wedge \neg e)P(\neg b)P(\neg e)$$

$$= 0.90 \times 0.70 \times 0.001 \times 0.999 \times 0.998 = 0.000628$$

chain rule

$P(j \mid \cancel{m}\, a \neg b \cancel{\neg e}) * P(m \mid a\, \cancel{\neg b \neg e}) * P(a \mid \neg b \neg e) * P(\neg b \mid \cancel{\neg e}) * P(\neg e)$

$P(j \mid a) * P(m \mid a) * P(a \mid \neg b \neg e) * P(\neg b) * P(\neg e)$

F F

$0.90 * 0.70 * 0.001 * 0.999 * 0.998$

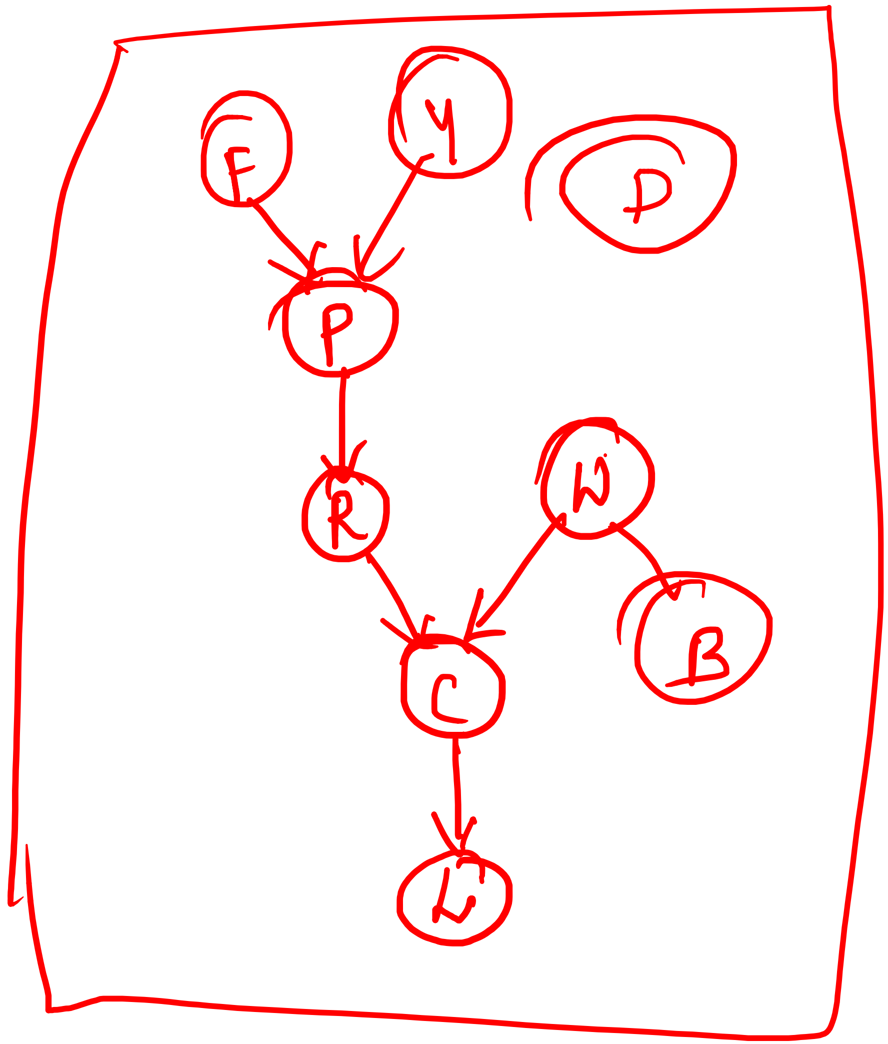
$\Rightarrow 0.000628$

1

Example Bayesian Net #3

Traffic Prediction -Travel Estimation

X =
- AI system reminds traveler regarding start time
- Travel plan is to reach Delhi and the weather of Delhi D may influence the accommodation plans
- Traveler always take car to reach airport C
- Car may be rerouted either due to road R block or weekday W traffic during working hours which delays the arrival to airport L
- Bars are always observed to be full on weekends B
- Authorities block roads to safe the processions P
- Processions observed during festive F season or due to the political rally Y
- **Problem**: Given the information that there is a political rally expected estimate the probability of late arrival



Example Bayesian Net #3

Traffic Prediction -Travel Estimation

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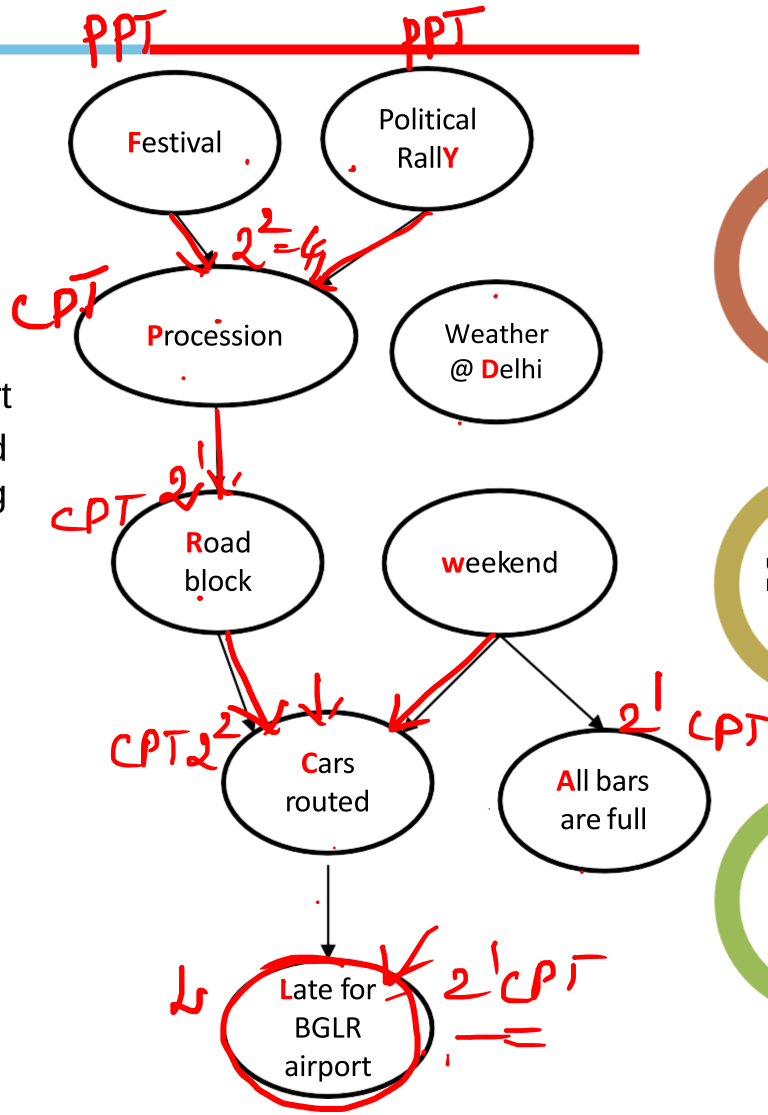
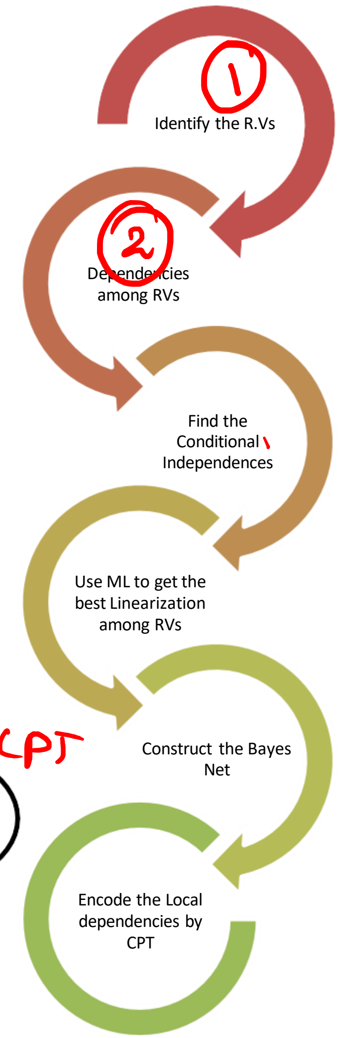


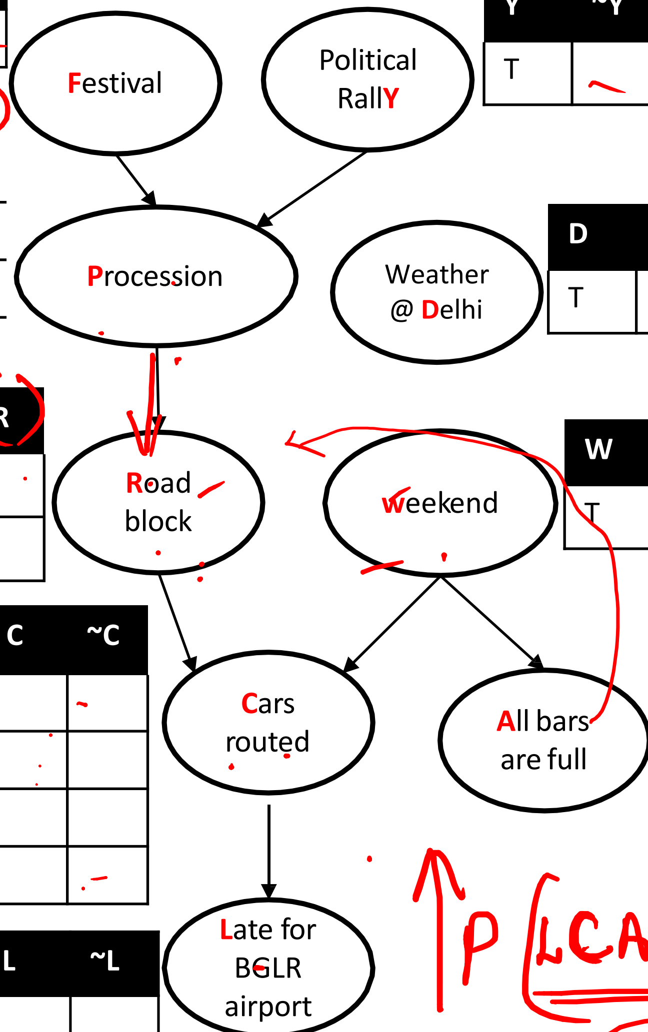


P(L|CARWPFYD) * P(C|ARWPFYD) *

Example Bayesian Net #3

L C A R W P F Y
T T T T T T T T
F F F F F F F F

2^9



F	~F
T	

F	y	P(P)	P(~P)
T	T		
T	F		
F	T		
F	F		

CPT

P	P(R)	P(~R)
T		
F		

Y	~Y
T	

D	~D
T	

W	~W
T	

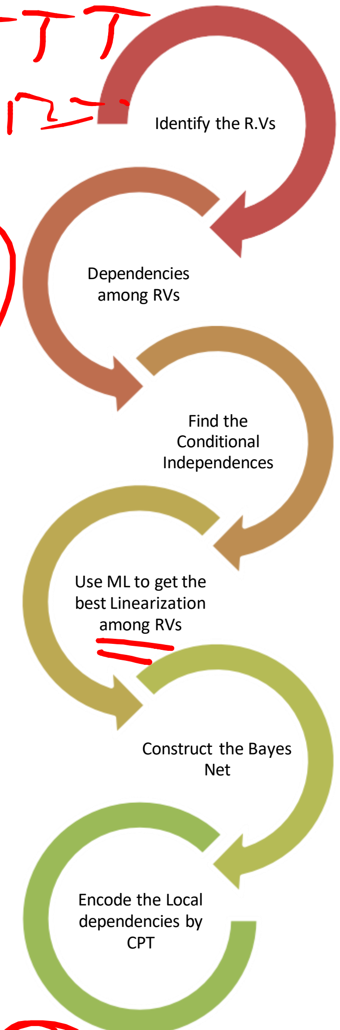
R	W	C	~C
T	T		
T	F		
F	T		
F	F		

W	A	~A
T		
F		

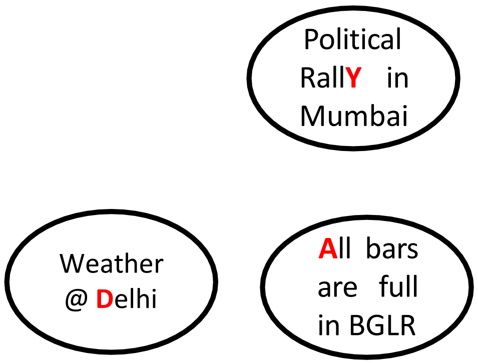
C	L	~L
T		
F		

P(LCARWPFYD) 9

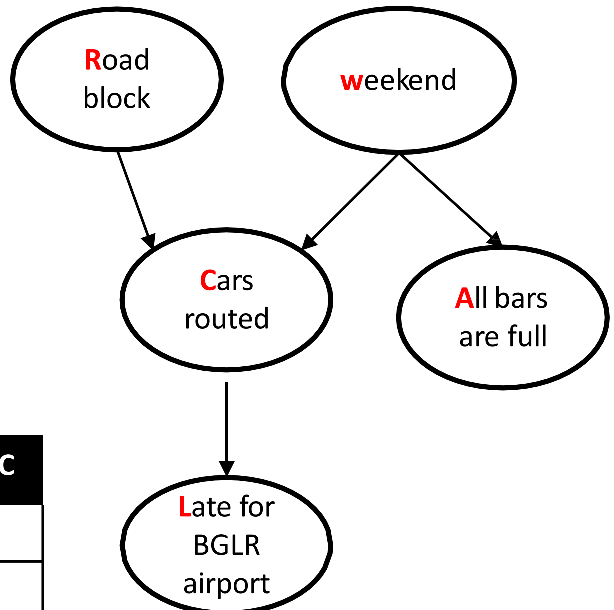
2^9



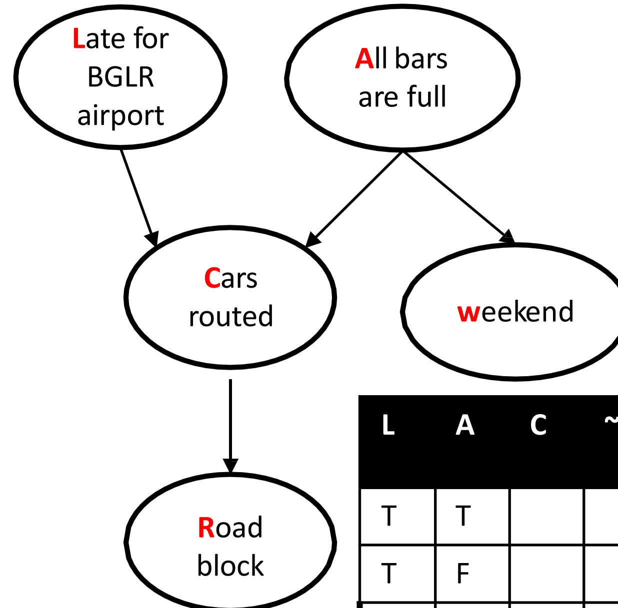
Example Bayesian Nets



R	W	C	~C
T	T		
T	F		
F	T		
F	F		



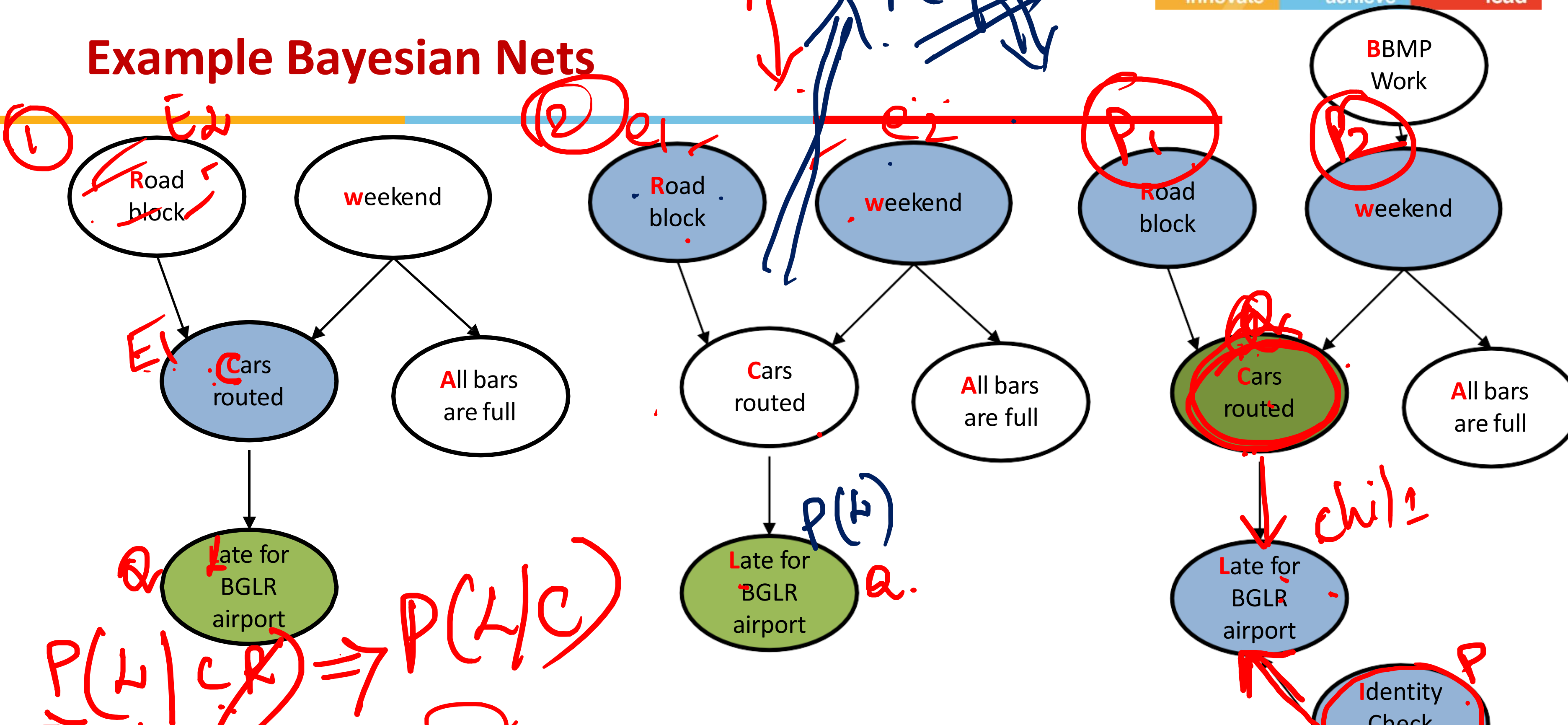
C	L	~L
T		
F		



C	R	~R
T		
F		

L	A	C	~C
T	T		
T	F		
F	T		
F	F		

Example Bayesian Nets



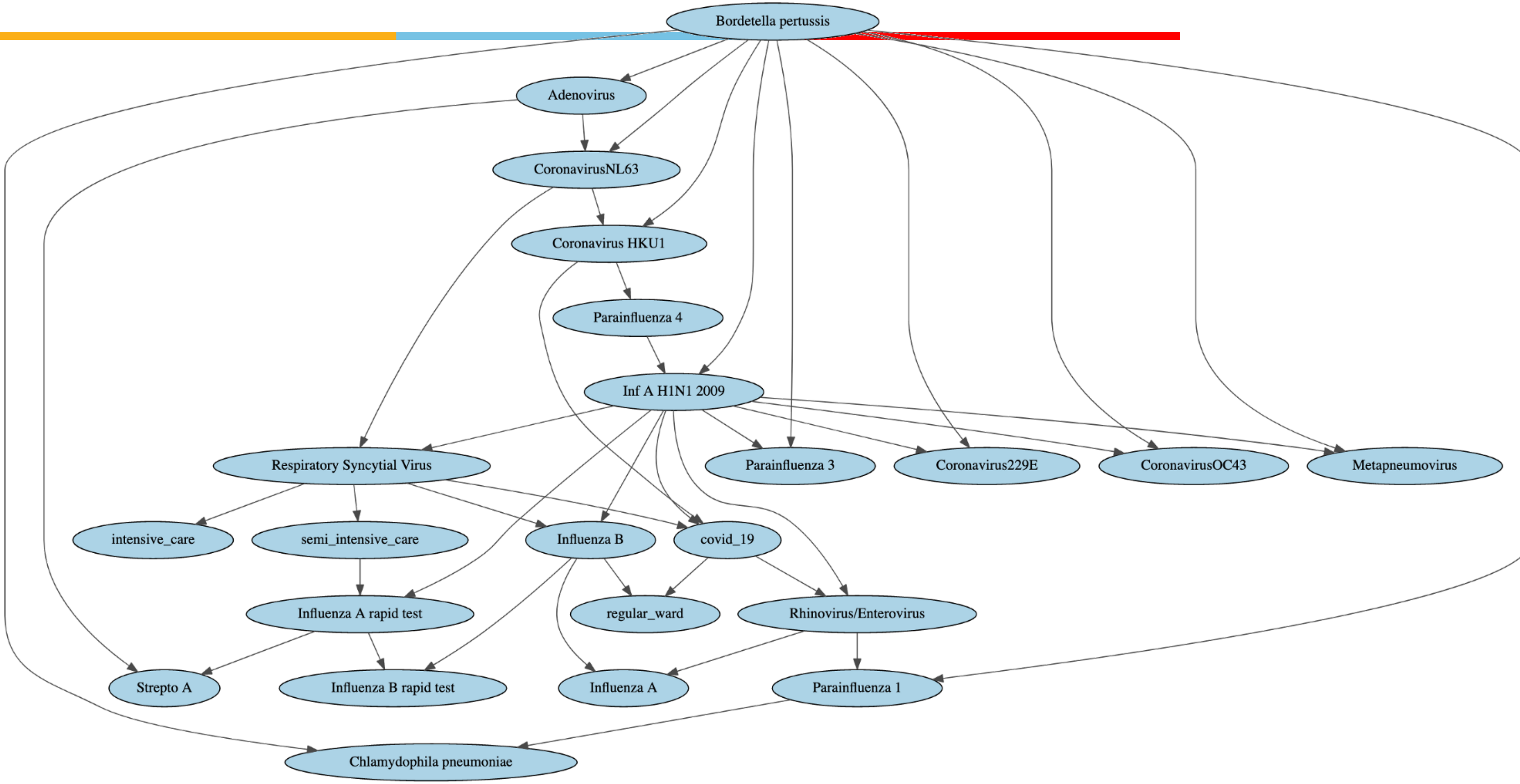
- A node is conditionally independent of its non-descendants given its parents
- A node is conditionally independent of all other nodes in the net , given its parents, children and children's parents.

Markov blanket P(C|

Bayesian Nets

Interesting Case Study

111-column wide dataset : 6347497291776 entries to store the JPD
817 entries. Memory gain :99.99999998712879%!



Source Credit : https://www.kaggle.com/einsteindata4u/covid19

Bayesian Network

Cyber Security

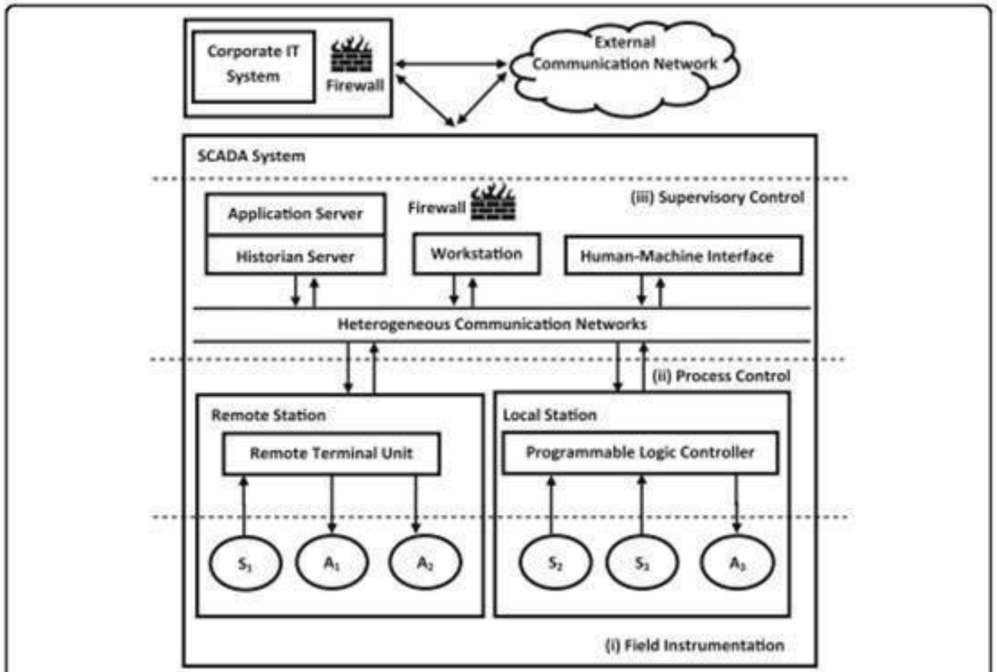


Fig. 1 Typical ICS Architecture and Layers

[Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.]

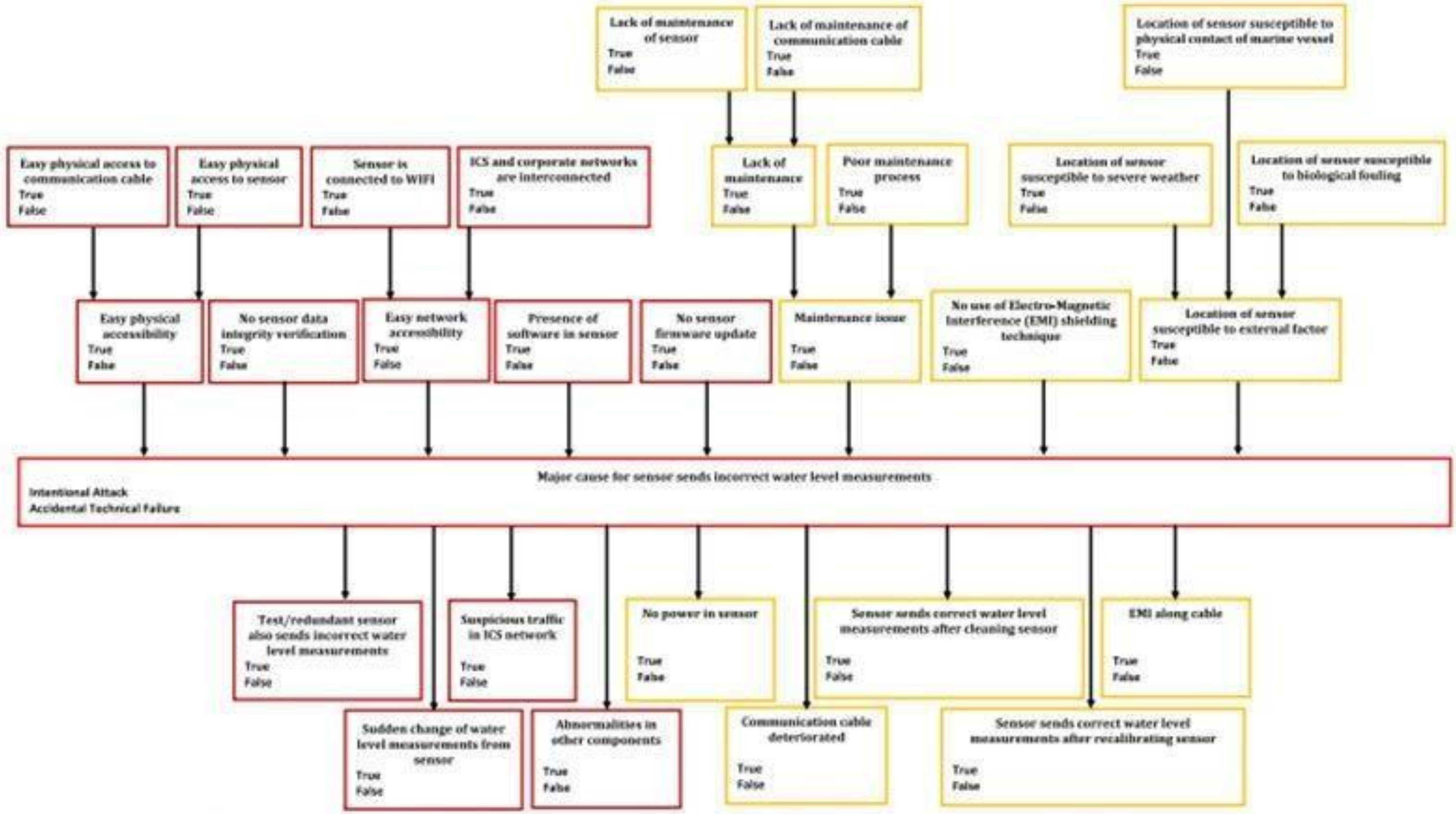


Fig. 4 Constructed Qualitative BN Model. (In this Figure, the presence of contributory factors and observations (or test results) colored in dark red would increase the likelihood of the problem (colored in red) due to an attack on the sensor. Furthermore, the presence of contributory factors and observations (or test results) colored in orange would increase the likelihood of the problem due to sensor failure)

Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.

Bayesian Network

Cyber Security

Table 2 CPT Excerpt – Problem Variable

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	Y: Attack	Y: Failure
True	True	True	True	True	True	True	True	0.02	0.98
True	True	True	True	True	True	True	False	0.09	0.91
True	True	True	True	True	True	False	True	0.06	0.94
True	True	True	True	True	True	False	False	0.24	0.76
True	True	True	True	True	False	True	True	0.09	0.91
True	True	True	True	True	False	True	False	0.38	0.62
True	True	True	True	True	False	False	True	0.24	0.76
True	True	True	True	True	False	False	False	0.97	0.03
True	True	True	True	False	True	True	True	0.02	0.98
True	True	True	True	False	True	True	False	0.09	0.91

In this table, C_1: Easy physical accessibility, C_2: No sensor data integrity verification, C_3: Easy network accessibility, C_4: Presence of software in sensor, C_5: No sensor firmware update, C_6: Maintenance issue, C_7: No use of EMI shielding technique, C_8: Location of sensor susceptible to external factor and Y: Major cause for sensor sends incorrect water level measurements

Source Credit : 2021 : Chockalingam, S., Pieters, W., Teixeira, A. *et al.* Bayesian network model to distinguish between intentional attacks and accidental technical failures: a case study of floodgates.

Inferences in Bayesian Nets

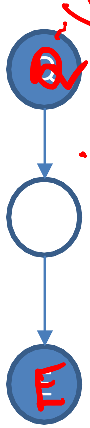
Enumeration

Exact → 0.00925

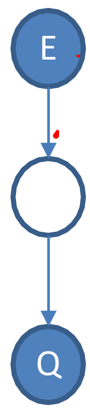
Approx

Belief Nets

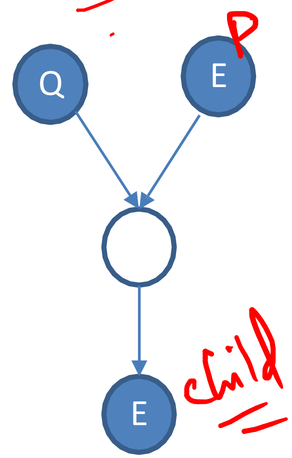
Diagnostic



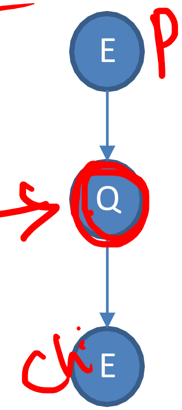
Causal

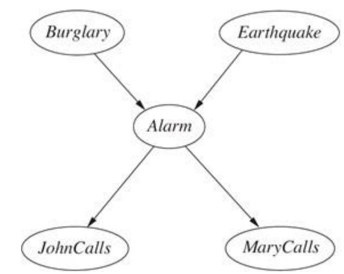


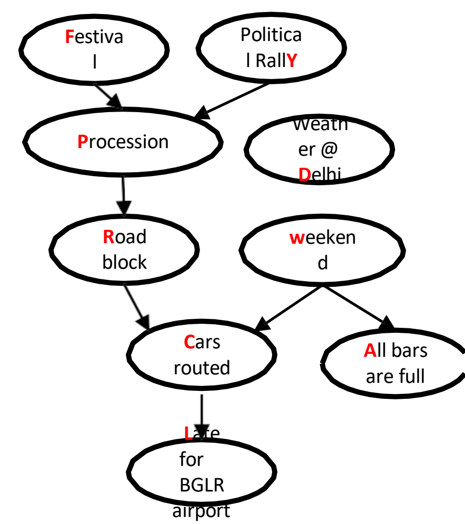
Inter-Casual



Mixed Inferences

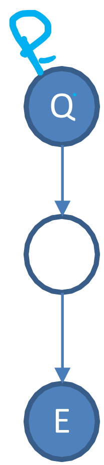


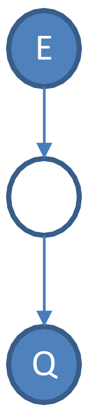


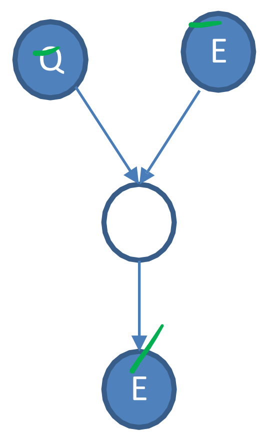


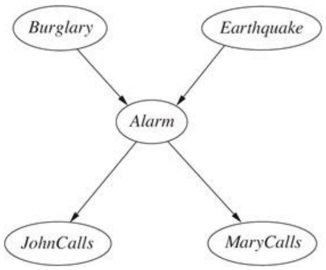
Belief Nets

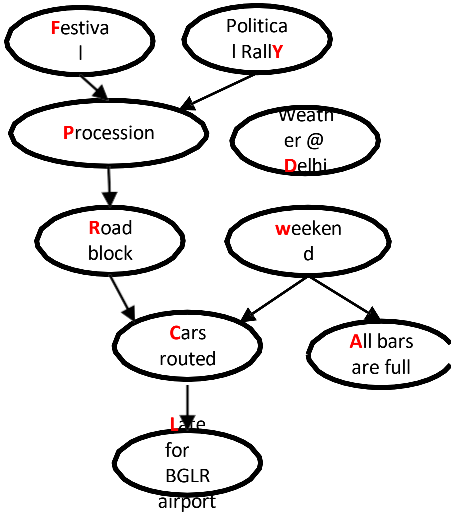
Diagnostic **Causal** **Inter-Casual** **Mixed Inferences**





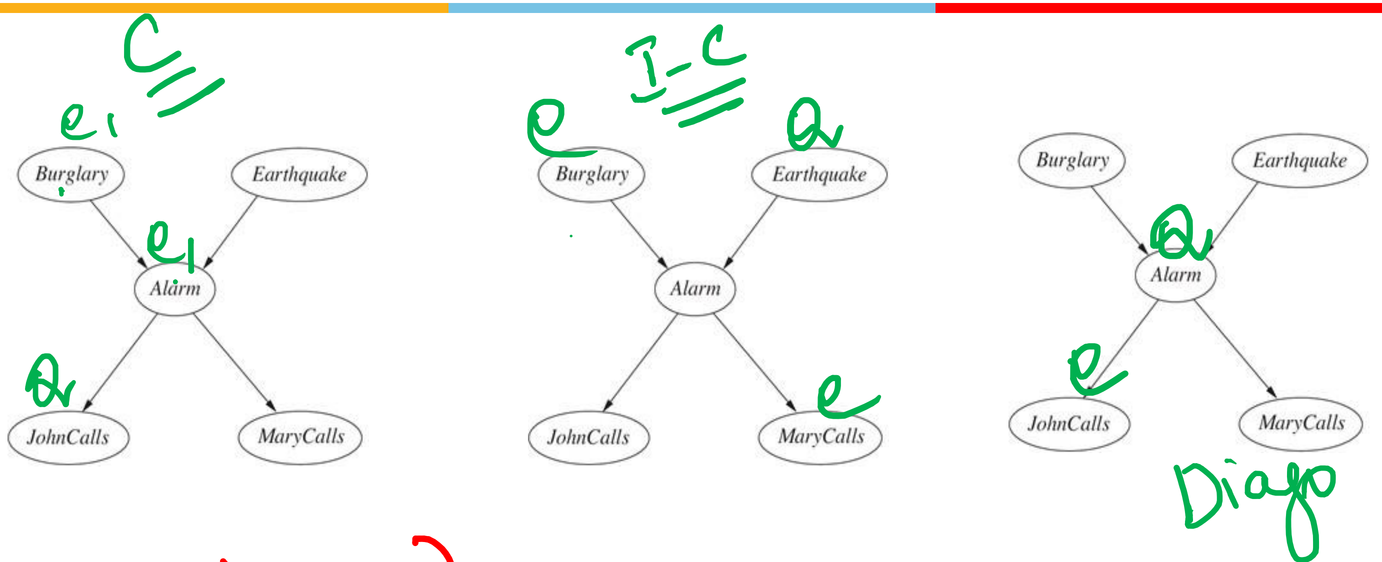






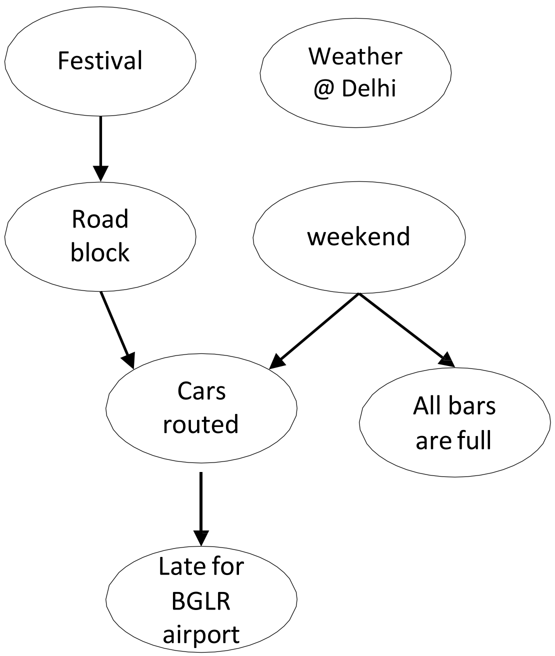
Q. $P(A|JM)$

Belief Nets



1) $P(J \mid B \wedge A)$
2) $P(E \mid BM)$
3) $P(A \mid J)$

Examples



$P(L \mid W \cap R \cap F \cap C)$

1. Calculate the probability that arrival at airport was delayed during a weekend but there was no road block or festival and car was not routed anywhere.

2. What is the probability that it is a festival season given cars where routed?

$P(F \mid C)$

3. What is the probability that car arrived late at airport given it's a festival day?

$P(L \mid F)$

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1, #9

Next Session Plan:

- (Prerequisite Reading : Refresh the basics of probability , Bayes Theorem , Conditional Probability, Product Rule, Conditional Independence, Chain Rule)

- Bayesian Network
- Representation
- Inferences (Exact and approximate-only Direct sampling)

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials