Artificial & Computational Intelligence

AIML CLZG557

M6: Reasoning over time

Indumathi V

Guest Faculty,

BITS - WILP

BITS Pilani

Pilani Campus

Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

Module 6:
Reasoning over time

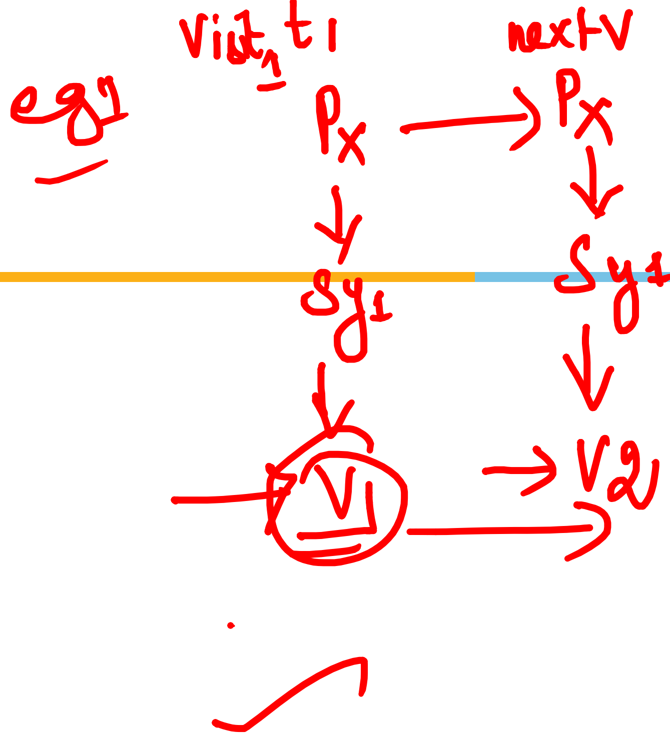
Reasoning Over Time

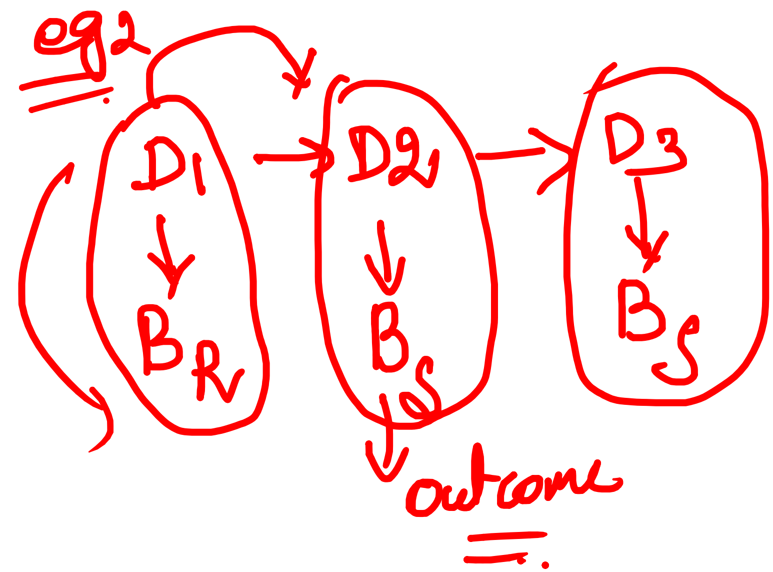
A. Time and Uncertainty

B. Inference in temporal models

C. Introduction to Hidden Markov Model

D. Applications of HMM





Time & Uncertainty

innovate achieve lead

Learning Objective

1) Understand the relationship between Time & Uncertainty

2. Recognize the transition model of Markov Model

3. Relate to the application of the Hidden Markov Model

Sequential Decision Problems & Markov Decision Process

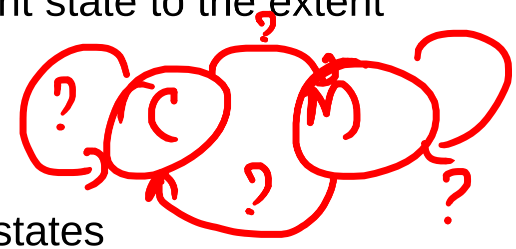
Markov Decision Process

Sequential Problem | Partial Observability | Belief System

Modelling sequences of random events and transitions between states over time is known as Markov chain

Agents in partially observable environment should keep a track of current state to the extent allowed by sensors

E.g., Robot moving in a new maze



Agent maintains a **belief state** representing the current possible world states

Transition Model / Probability Matrix : D1 D2

Using belief state and transition model,
the agent can how the world might evolve in
next time step. To capture the degree of belief
we will use Probability Theory. We model the change in world
using a variable for each aspect of state and
at each point in time.
Current state depends only finite number of previous states.

C	M	
0.40	0.20	C
0.60	0.80	M

Markov Decision Process

Time - Uncertainty | States - Observations

Static World: Each random variable would have a single fixed value

E.g., Diagnosing a broken car

Dynamic World: The state information keeps changing with time

E.g., treating a diabetic patient, tracking the location of robot, tracking economic activity of a nation

Time slices: World is observed in time slices. Each slice has a set of random variables, some observable and some not.

Assumption: We will assume same subset of random variables are observable in each time slice

$\mathbf{E_t}$ – set of observable random variables at time t

$\mathbf{X_t}$ – set of unobserved random variables at time t

$$\begin{array}{cc}
\begin{array}{cc} \mathbf{C} & \mathbf{M} \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} \mathbf{C} \\ \mathbf{M} \end{array}
\end{array}$$

Markov Process

1st order

one previous state

States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Markov chain

Transition Model / Probability Matrix :

Current state depends only finite number of previous states. :

one → 1st order MM

2 → 2nd order MM

current

t-1	t	t+1
C	C	
M	M	
M	C	
C	M	

t → 2

CC	MM	MC	CM	
				C
				M

t → t+1

C	M	
0.40	0.20	C
0.60	0.80	M

t+1

current t+1

Markov Model- Example 1

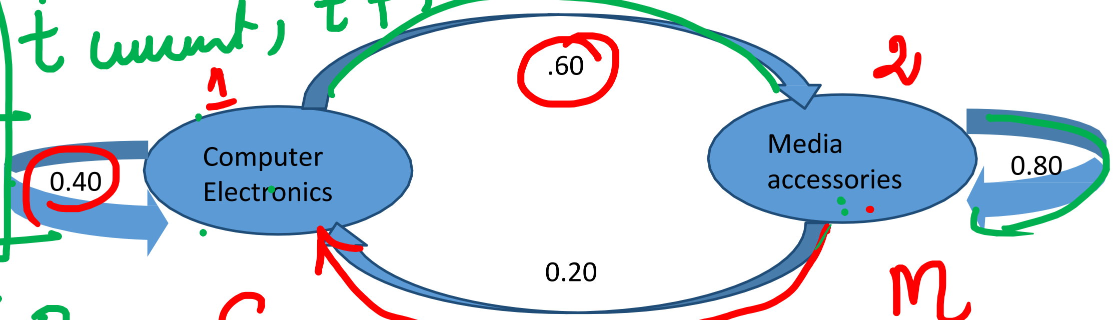
Previous $t-1$, t

C	M	
0.4	0.2	C
0.6	0.8	M

t current, t+1

0.2

$P(C_{t+1} \mid M_t)$



C

$0.60 \leftarrow P(M_{t+1} \mid C_t)$

1st → C, 2nd → M } 0.60

	C	M	M
	C	C	M
	0.40	0.2	0.8

Transition Model

	C	M	
	0.40	0.20	C
	0.60	0.80	M

$P(M_{t+1} \mid M_t) \Rightarrow 0.8$

$P(C_{t+1} \mid C_t) \Rightarrow 0.4$

$P(M_t \rightarrow M_{t+1})$ (1st, 2nd)

V1 V2 V3 V4
C C M C

C1 → C2 → M3 → C4

1st

2

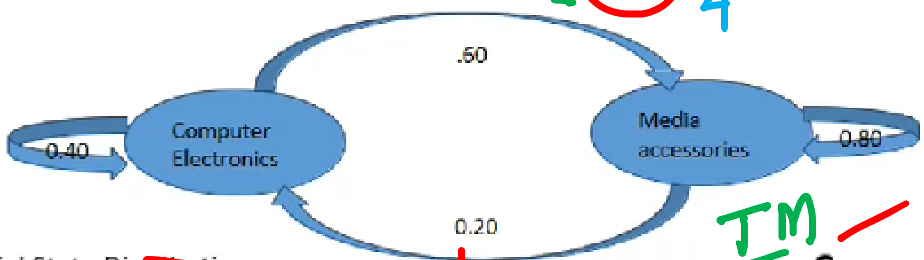
(2)

Markov Model

(1) ✓

(2) 1 previous
1st markov model

P(C) = 1
P(M) = 0



Current State: Initial State Distribution

I   A X I   1st   C

P(C) →	1	C
P(M) →	0	M

TM

A	C	M		
	0.40	0.20	C	1
	0.60	0.80	M	0

State transition prob

Next State : Likely to buy Media accessories on next visit

2nd   M

0.40	C
0.60	M

→ I X A

Next State : Likely to buy Media accessories on next visit

0.28	C
0.72	M

M

Inference in temporal Models

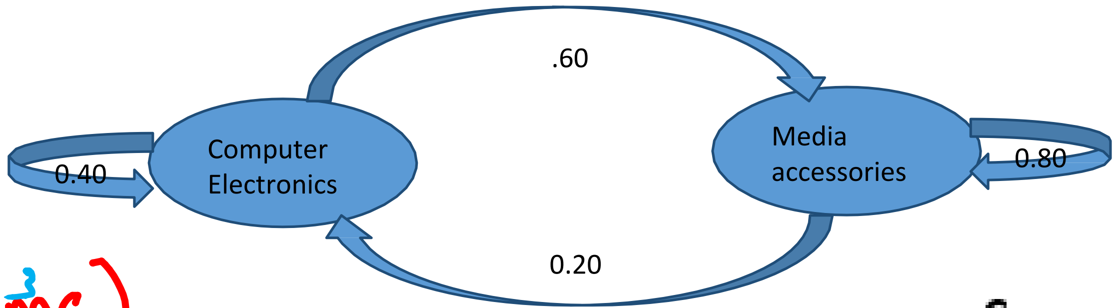
Markov Model

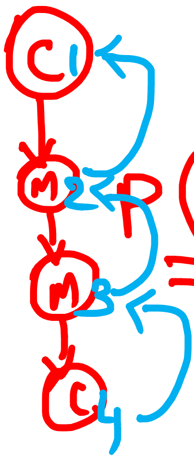
$I = \begin{array}{c} \begin{bmatrix}1\\0\end{bmatrix} \end{array}\begin{array}{l} C\\ M\end{array}$

Inference Type 1

$P(C) = 1$

$P(M) = 0$





Prev

C	M	
0.40	0.20	C
0.60	0.80	M

Curr

$P(C\,M\,M\,C_4)$

Curr Prev C P

$P(C_1) * P(M_2|C_1) * P(M_3|M_2) * P(C_4|M_3)$

$1 * 0.60 * 0.80 * 0.20$

1. What is the probability that the purchasing behaviour of the customer is in below sequential order only? Initial Probability Matrix is $\boxed{P(C) = 1,\ P(M) = 0}$

✓ (Computer, Media, Media, Computer)

v_1 v_2 v_3 v_4

<u>Apply Bayes chain rule:</u>

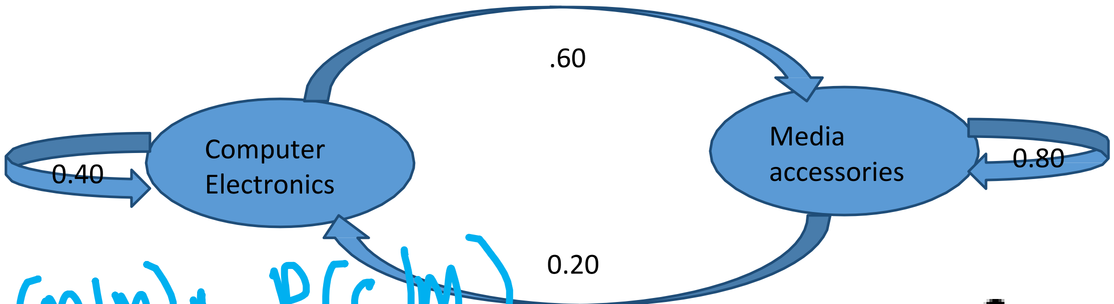
P(Computer, Media, Media, Computer) = P(C) * P(M|C) * P(M|M) * P(C|M) = $\boxed{0.096}$

$P(CMMC) = 0.096$

Markov Model

Inference Type 2

P(M) = 1
P(C) = 0



P(M) * P(M|M) * P(M|M) * P(C|M)

1 * 0.80 * 0.80 * 0.20

P(M M M C) =>

C	M	
0.40	0.20	C
0.60	0.80	M

2. What is the probability that the customer who purchased ~~Media accessories~~ will keep coming back to purchase media accessories in the next 2 consecutive visits only?

<u>Derive Initial prob values & Apply Bayes chain rule on the pattern exhibited:</u>

Initial Probability Matrix is P(M) = 1 , P(C) = 0

P(Media, Media, Media, Computer) = P(M) * P(M|M) * P(M|M) * P(C|M) = 0.128

Markov Model

Inference Type 3

$P(C) = 1 \quad P(M) = 0$

$1 * 0.40 \Rightarrow 0.40$

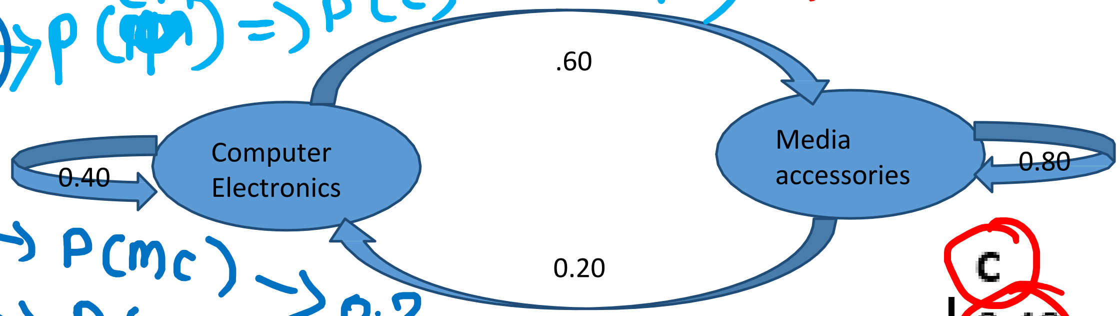
$P(CC) \Rightarrow P(C) \cdot P(C|C)$

$1 * 0.60 \cdot P(M|C) \Rightarrow 0.60$

$P(CM) \Rightarrow P(C) \cdot P(M|C)$

$P(MC) \to 0.2$

$P(MM) \to 0.8$



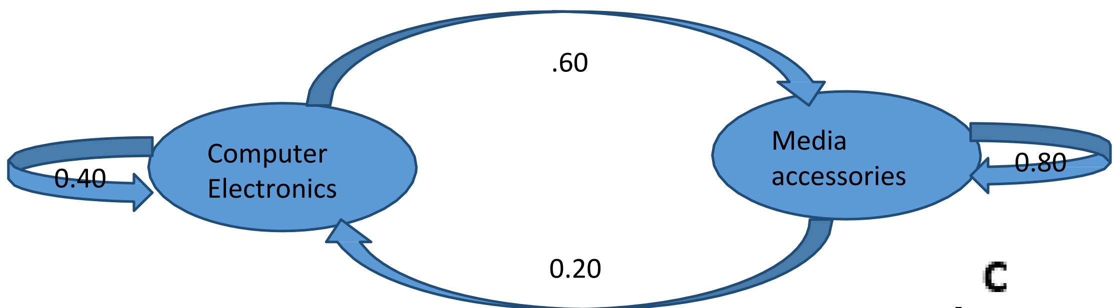
	C	M
C	0.40	0.20
M	0.60	0.80

Given the evidence that the customer walked into the store and bought a computer electronics, find the expected purchase pattern in the next 3 visits

$P(CMMM)$

Markov Model

Inference Type 3



C	M	
0.40	0.20	C
0.60	0.80	M

Given the evidence that the customer walked into the store and bought a computer electronics, find the expected purchase pattern in the next 3 visits

<u>Derive Initial prob values & Apply Bayes chain rule and reverse predict the combination on the most likely pattern (Similar to Viterbi Algorithm):</u>

Initial Probability Matrix is P(C) = 1 , P(M) = 0

P(Computer , X, Y, Z) = P(Computer) * P(X|Computer) * P(Y|X) * P(Z|X) =

1*0.6*0.8*0.8 → Produces max values P(CMMM)

Ans : Pattern = (Computer, Media, Media, Media)

Markov Process

States | Observations | Assumptions

Standard Mathematical Example:
Urn & Ball Model

facilitator
Player

t_1 t_2 t_3

V_2 V_0 V_2

X_t or V_t

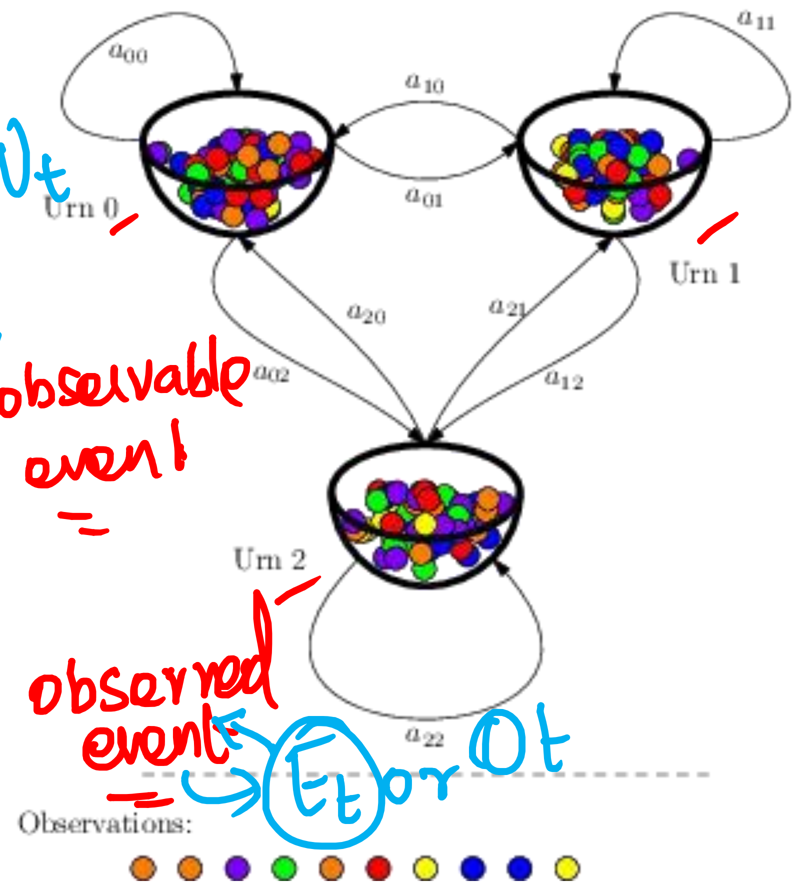
unobservable = event

O_1 O_2 O_3

Player Player Player

observed event

E_t or O_t



Hidden Markov Process

States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Morkov chain

X_t

Hidden Markov Process models events as the state sequences that are not directly observable but only be approximated from the sequence of observations produced by the system

E_t

Transition Model / Probability Matrix :

Current state depends only finite number of previous states. :

1, 2

Evidence / Sensor Model/ Emission Probability Matrix :

Current Evidence or Observation depends Current State of the world. Given the Current State Knowledge of the world, observation doesn't depend on history:

Markov Process

HMM

States | Observations | Assumptions

t

Standard Mathematical Example:

Urn & Ball Model

t+1

	U_0	U_1	U_2
U_0	0.2	0.4	0.2
U_1	0.5	0.4	0.3
U_2	0.3	0.2	0.5
	1	1	1

$P(U_1 \mid U_0) \Rightarrow 0.5$

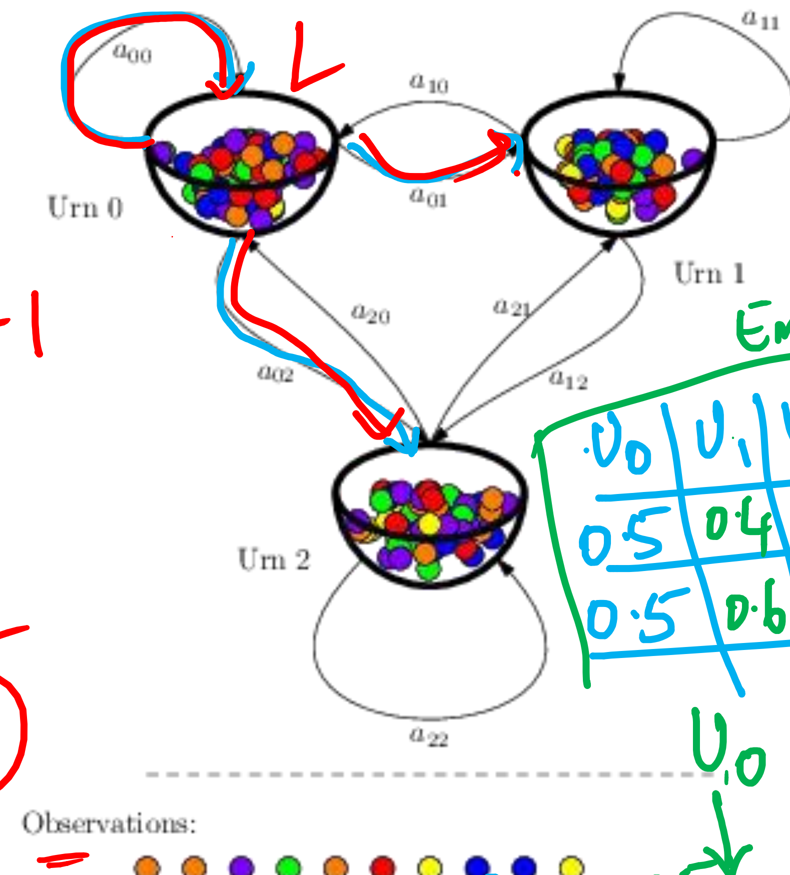
t , t+1

U_0
U_1
U_2

3

Previ next

$U_0 \to U_0, U_1, U_2$



Emission

	U_0	U_1	U_2
Or	0.5	0.4	0.3
Bl	0.5	0.6	0.7

2 < Orange, blue

U_0: 0.5 O, 0.5 b

Hidden Markov Model

States | Observations | Assumptions

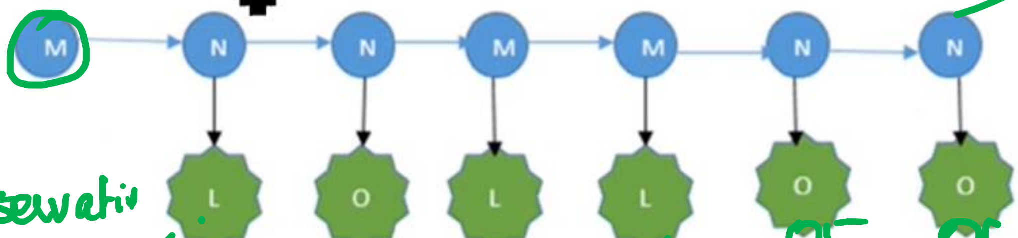
2

t_0 t_1

Time Slice (t)	0	1	2	3	4	5	6	$P(O_t \mid O_{t-1})$
Observed Evidence (O_t/ E_t)	-	Late	OnTime	Late	Late	Ontime	Ontime	....
Unobserved State (U_t /X_t/ Q_t)	Meeting	No Meeting	No Meeting	Meeting	Meeting	No Meeting	No Meeting	......

E_t L O m N

unobserved X_t



Observation

O_1 O_2 O_3 O_4 O_5 O_6

Observed event E_t

Transition Model / Probability Matrix

t-1

$P(U_{t-1}$ = No Meeting)	$P(U_{t-1}$ = Meeting)	←Previous
0.5	0.67	$P(U_t$ = No Meeting)
0.5	0.33	$P(U_t$ = Meeting)

t

P(N|M)

P(N|N)

P(M|N)

P(M|M)

Evidence / Sensor Model/ Emission Probability Matrix

$P(U_t$ = No Meeting)	$P(U_t$ = Meeting)	←Unobserved Evidence v
0.9	0.3	$P(O_t$ = OnTime)
0.1	0.7	$P(O_t$ = Late)

innovate achieve lead

Hidden Markov Model

States | Observations | Assumptions

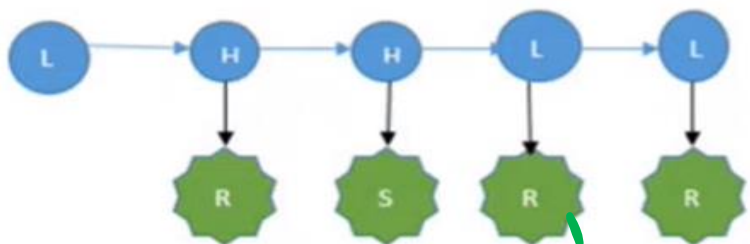
R S R R

Time Slice (t)	0	1	2	3	4	....	$P(O_t \mid O_{t-1}, O_{t-2})$
Observed Evidence (O_t)	-	Rainy	Sunny	Rainy	Rainy		
Unobserved State(U_t)	Low Pressure	High Pressure	High Pressure	Low Pressure	Low Pressure		

L H H L L

L H

t-2 t-1
LP LP
LP HP
HP HP
HP LP



2nd Markov model = t-2, t-1

Transition Model / Probability Matrix

$P(U_{t-2} = LP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = LP)$	$P(U_{t-2} = LP, U_{t-1} = LP)$	←Previous
0.2	0.40	0.85	0.5	$P(U_t = LP)$
0.8	0.60	0.15	0.5	$P(U_t = HP)$

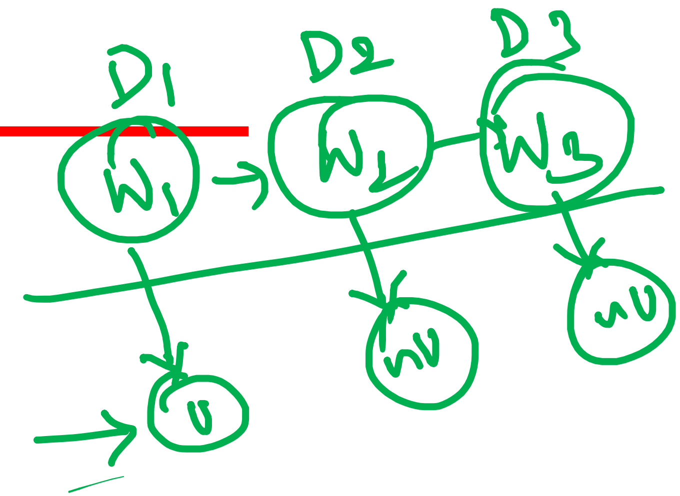
current t

Evidence / Sensor Model/ Emission Probability Matrix

1st

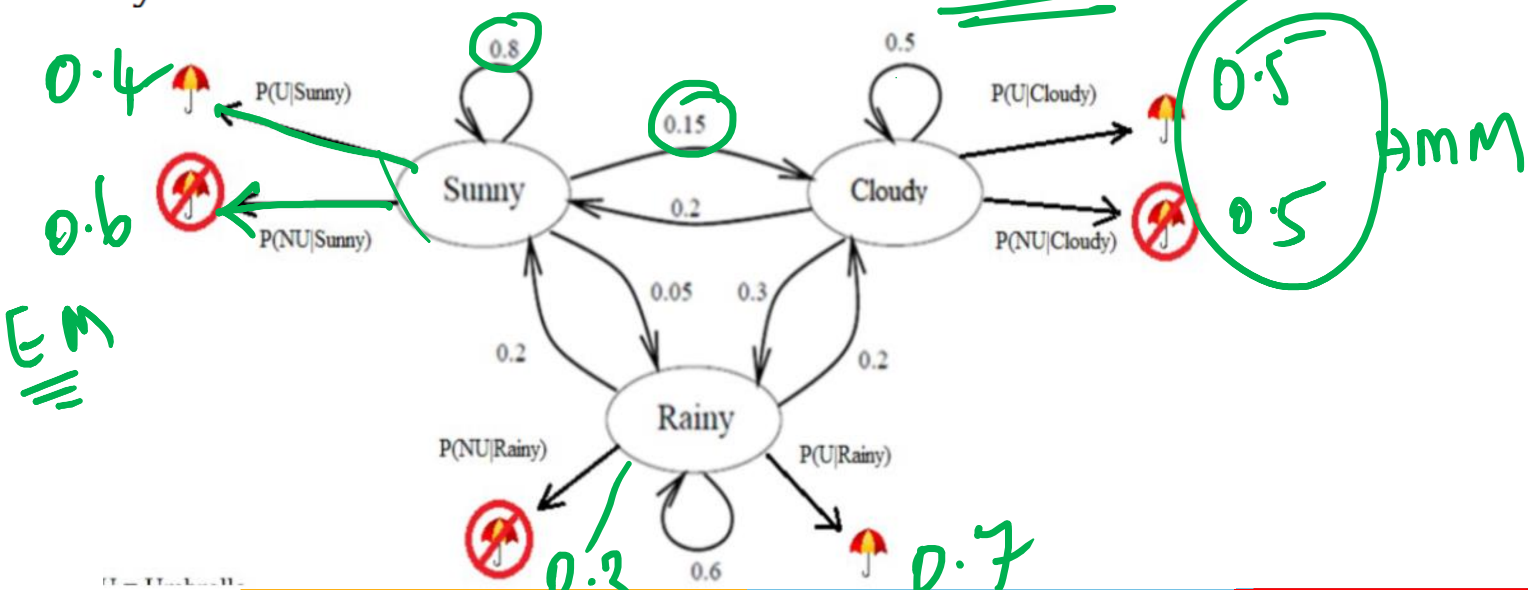
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

3rd



- Imagine: You were locked in a room for several days and you were asked about the weather outside. The only piece of evidence you have is whether the person who comes into the room bringing your daily meal is carrying an umbrella or not.
- What is hidden? {Sunny, Rainy, Cloudy}
- What can you observe? Umbrella or Not

TM



Hidden Markov Model →

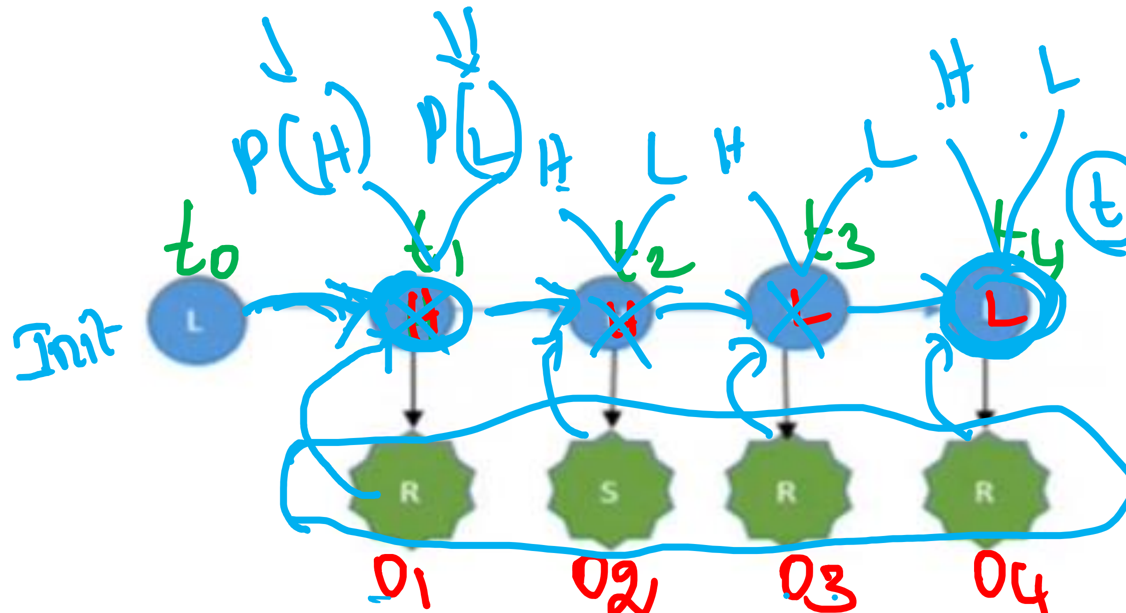
Veterbi Alg

1 Filtering	2 Prediction	3 Smoothing	4 Most Likely Explanation
$P(L_4 \mid R_1\text{-}S_2\text{-}R_3\text{-}R_4)$	$P(L_3 \mid R\text{-}S)$	$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$	$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$
$P(X_t \mid E_{1\ldots t})$	$P(X_{t+k} \mid E_{1\ldots t})$	$P(X_{k,\ o>k>t} \mid E_{1\ldots t})$	$\text{argmax}\ X_{1\ldots t} : P(X_{1\ldots t} \mid E_{1\ldots t})$

$P(X_t \mid E_{1\ldots t})$

$P(L_4 \mid \underline{R_1 S_2 R_3 R_4})$

$4 \mid 1 \_\_ 4$



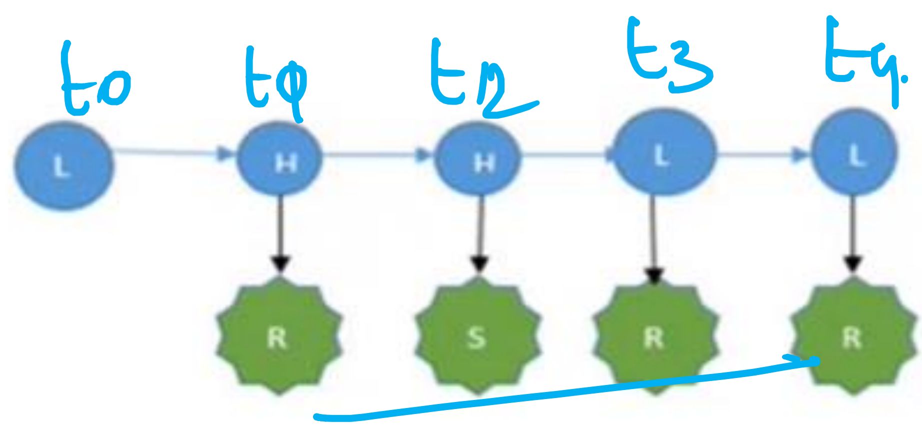
Hidden Markov Model

Current

Filtering	Prediction	Smoothing	Most Likely Explanation
$P(L_3 \mid R\text{-}S\text{-}R\text{-}R)$	$P(L_3 \mid R\text{-}S)$	$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$	$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$
$P(X_t \mid E_{1\ldots t})$	$P(X_{t+k} \mid E_{1\ldots t})$	$P(X_{k,\ o>k>t} \mid E_{1\ldots t})$	$\text{argmax } X_{1\ldots t} : P(X_{1\ldots t} \mid E_{1\ldots t})$

$$P(H_7 \mid \underline{R_1 S_2 R_3 R_4})$$

future st

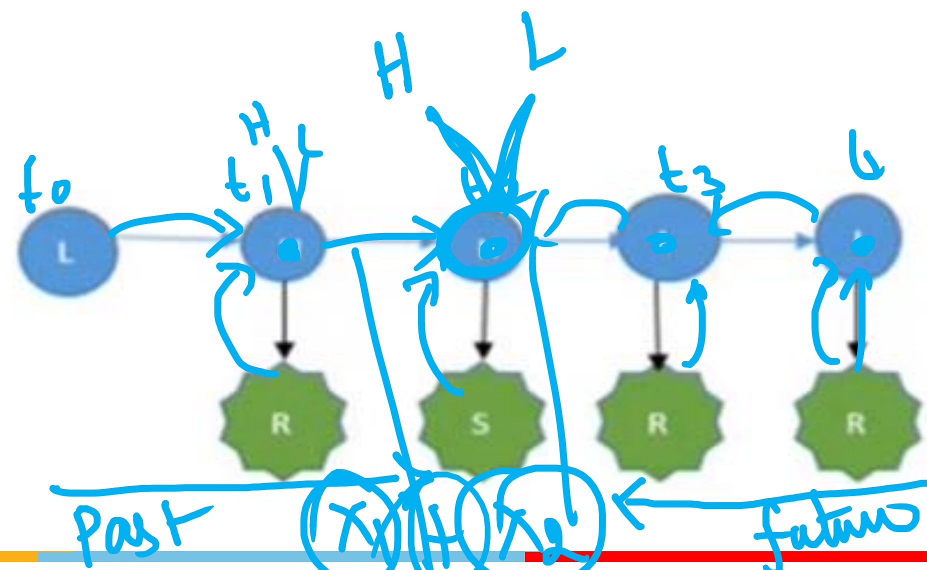


Hidden Markov Model

Filtering	Prediction	Smoothing	Most Likely Explanation
$P(L_3 \mid R\text{-}S\text{-}R\text{-}R)$	$P(L_3 \mid R\text{-}S)$	$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$	$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$
$P(X_t \mid E_{1\ldots t})$	$P(X_{t+k} \mid E_{1\ldots t})$	$P(X_{k,\ 0>k>t} \mid E_{1\ldots t})$	$\text{argmax}\ X_{1\ldots t} : P(X_{1\ldots t} \mid E_{1\ldots t})$

P

2nd state < H, L



Hidden Markov Model

Filtering	Prediction	Smoothing	Most Likely Explanation
$P(L_3 \mid R\text{-}S\text{-}R\text{-}R)$	$P(L_3 \mid R\text{-}S)$	$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$	$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$
$P(X_t \mid E_{1\ldots t})$	$P(X_{t+k} \mid E_{1\ldots t})$	$P(X_{k,\ o>k>t} \mid E_{1\ldots t})$	$\text{argmax } X_{1\ldots t} : P(X_{1\ldots t} \mid E_{1\ldots t})$

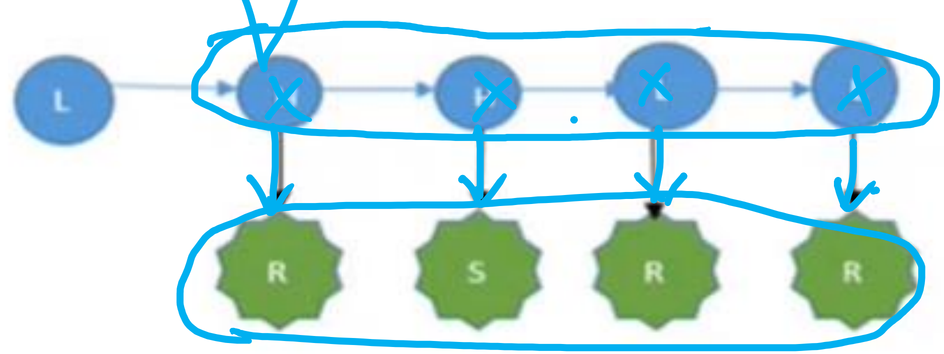
2^4

H H H H → P(c1)

H H H L → P(c2)

L L L L → P(cn)

H L

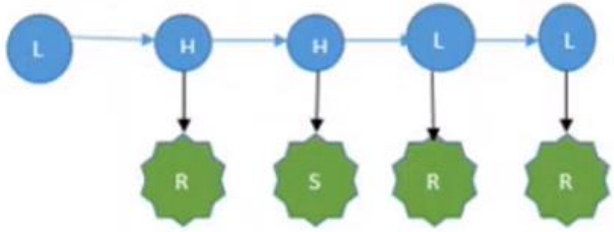


Hidden Morkov Model

Inference: Type -1

Sequence Evaluation : Likely hood Computation : Forward Algorithm

Find the probability of occurrence of
this weather sequence observation: **S-S-R**



Intuition: $P(E_{1...t}) = \sum_{i=1}^{N} P(E_{1...t} \mid X_{1...t}) * P(X_{1...t}) =$

$= \sum_{i=1}^{N} \prod_{j=1}^{t} P(Ej \mid Xj) * P(Xj \mid Xj_{-1})$

Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	← Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	← Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

P(SSR)

$$= \sum_{X} P(SSR, X) = \sum_{X} P(SSR, X_1X_2X_3)$$

$$= \sum_{X} P(R, X_3, S, X_2, S, X_1) = \sum_{X} P(R \mid X_3) * P(X_3 \mid X_2) * P(S \mid X_2) * P(X_2 \mid X_1) * P(S \mid X_1) * P(X_1 \mid X_0)$$

$$= \sum_{X} P(R \mid X_3) * P(S \mid X_2) * P(S \mid X_1) * P(X_3 \mid X_2) * P(X_2 \mid X_1) * P(X_1 \mid X_0)$$

$$= \sum_{X} \prod_{j=1}^{t} P(Ej \mid X_j) * P(Xj \mid X_{j-1})$$

Hidden Markov Model

P(HP) = 0.5
P(LP) = 0.5

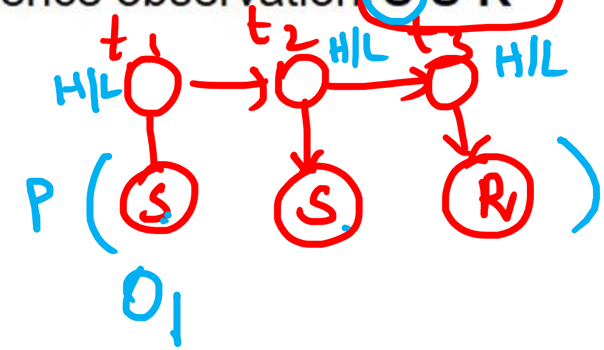
Forward Propagation Algorithm

Find the probability of occurrence of this Pressure sequence observation S-S-R

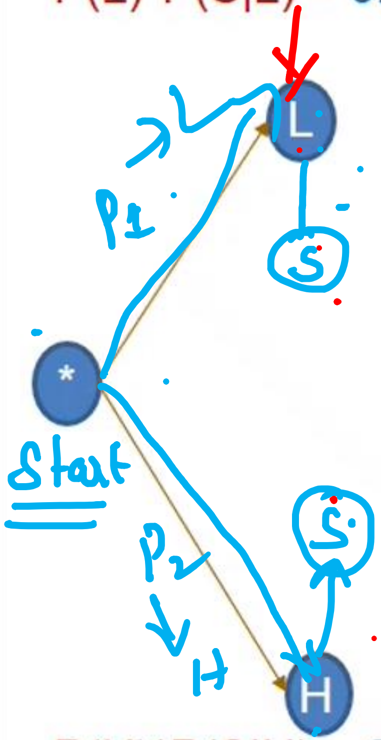
P(SSR)

Initialization Phase:

Normalize

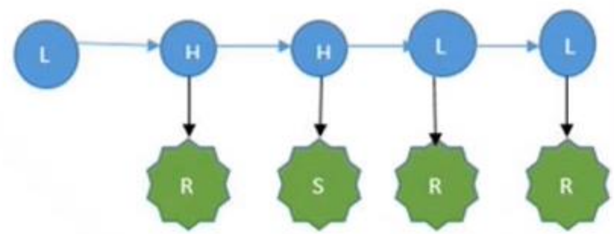


① P(L)*P(S|L) = 0.5*0.2 = 0.1 □□0.25



P(L) * P(S|L)
0.5 * 0.2 => 0.1

$\frac{0.1}{0.1+0.3}$ => 0.25



Transition Model / Probability Matrix

P(U_{t-1} = HP)	P(U_{t-1} = LP)	←Previous
0.2	0.5	P(U_t = LP)
0.8	0.5	P(U_t = HP)

P(H) * P(S|H)
0.5 * 0.6 => 0.3

$\frac{0.3}{0.1+0.3}$ => 0.75

Evidence / Sensor Model/ Emission Probability Matrix

P(X_t = LP)	P(X_t = HP)	←Unobserved Evidence v
0.8	0.4	P(E_t = Rainy)
0.2	0.6	P(E_t = Sunny)

P(H)*P(S|H) = 0.5*0.6 = 0.3 □□0.75

O1 => S

P_1 L L
P_2 H L
L H P_{11}
H H P_{12}

H|L H|L

Hidden Markov Model

O_1, O_2, O_3

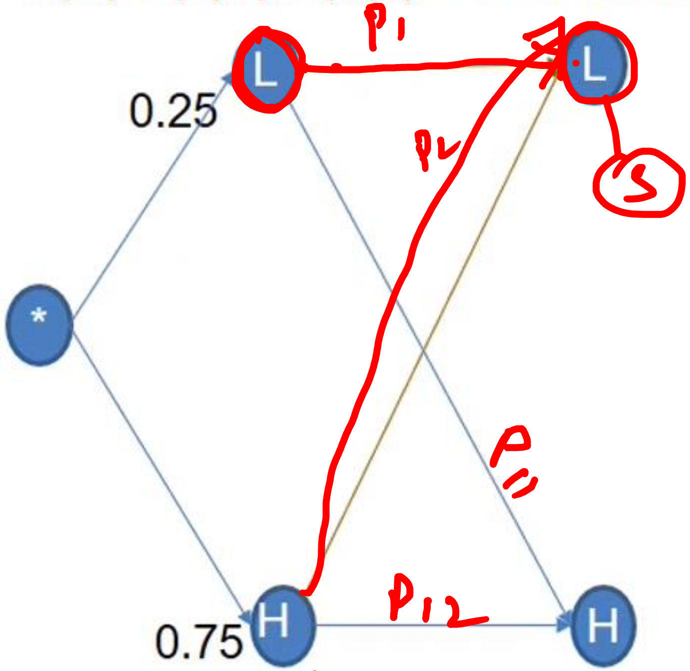
Forward Propagation Algorithm : S-S-R

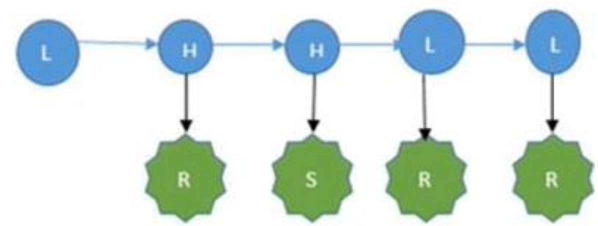
P(L)*P(L|L)*P(S|L) = 0.25*0.5*0.2 = 0.025 + Margin

P(H)*P(L|H)*P(S|L) = 0.75*0.2*0.2 = 0.03

$P_1 \Rightarrow P(L) * P(L|L) * P(S|L)$

$P_2 \Rightarrow P(H) * P(L|H) * P(S|L)$





$O_1 \Rightarrow S$ $O_2 \Rightarrow S$

P(L)*P(H|L)*P(S|H) = 0.25*0.5*0.6 = 0.075

P(H)*P(H|H)*P(S|H) = 0.75*0.8*0.6 = 0.36

Transition Model / Probability Matrix

P(U_{t-1} = HP)	P(U_{t-1} = LP)	←Previous
0.2	0.5	P(U_t = LP)
0.8	0.5	P(U_t = HP)

Evidence / Sensor Model/ Emission Probability Matrix

P(X_t = LP)	P(X_t = HP)	←Unobserved Evidence v
0.8	0.4	P(E_t = Rainy)
0.2	0.6	P(E_t = Sunny)

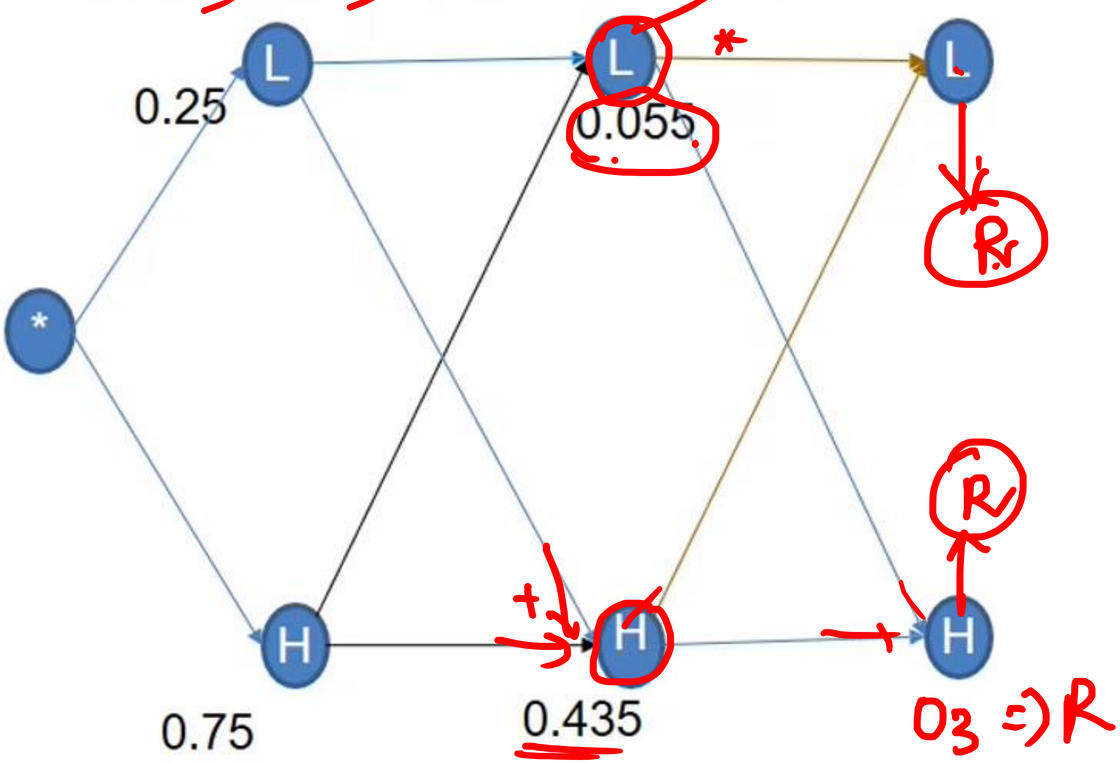
Hidden Markov Model

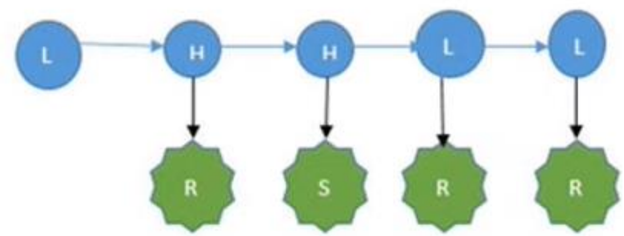
Forward Propagation Algorithm : S-S-R

$O_1\ O_2\ O_3$

$P(L)*P(L|L)*P(R|L) = 0.055*0.5*0.8 = 0.022$ +

$P(H)*P(L|H)*P(R|L) = 0.435*0.2*0.8 = 0.0696$





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

$P(L)*P(H|L)*P(R|H) = 0.055*0.5*0.4 = 0.011$ +

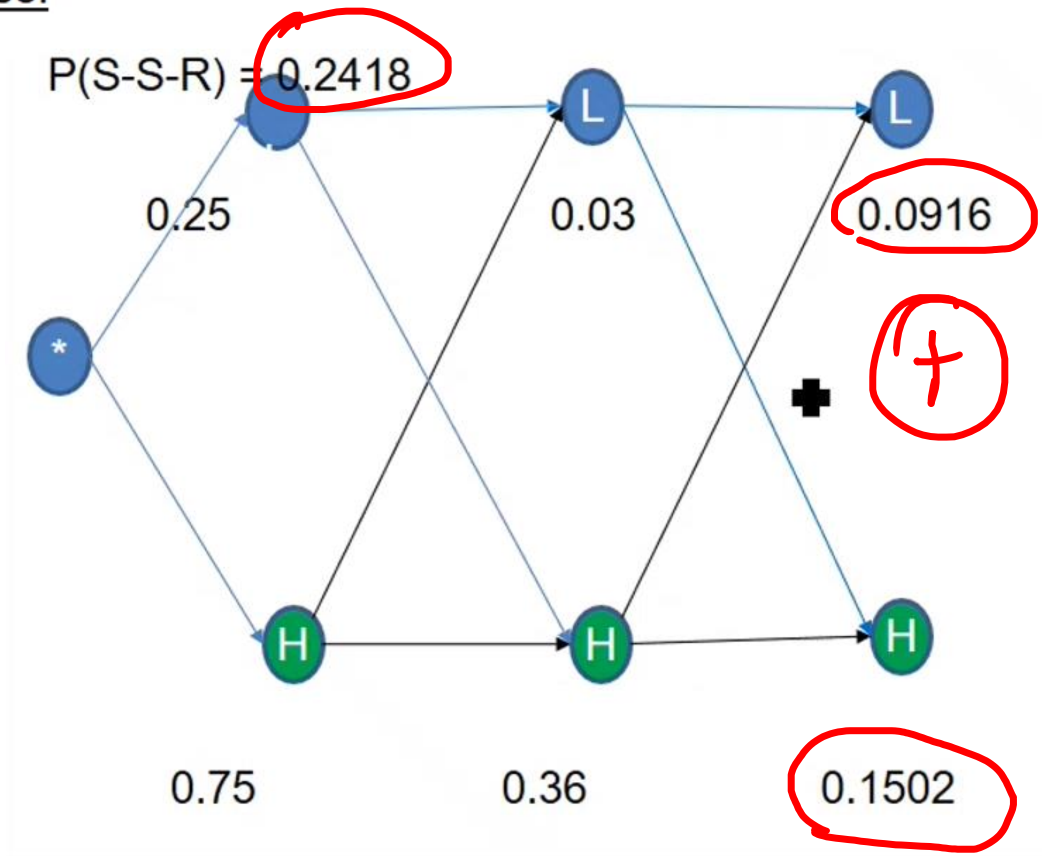
$P(H)*P(H|H)*P(R|H) = 0.435*0.8*0.4 = 0.1392$

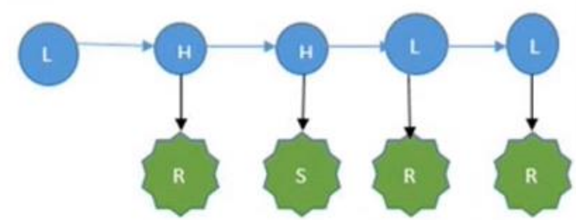
Hidden Markov Model

Forward Propagation Algorithm : S-S-R

P (SSR)

Termination Phase:





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

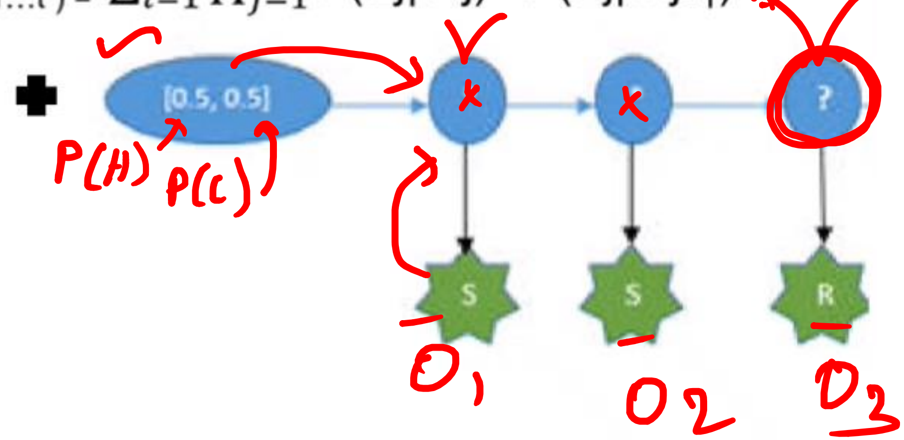
Hidden Markov Model –

Inference: Type -3

Filtering : Forward Propagation Algorithm

Find the Current Pressure if sequence of weather observations recorded are: **S-S-R**

Intuition: $P(E_{1...t}) = \sum_{i=1}^{N} P(E_{1...t} \mid X_{1...t}) * P(X_{1...t}) = \sum_{i=1}^{N} \prod_{j=1}^{t} P(Ej \mid Xj) * P(Xj \mid Xj_{-1})$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

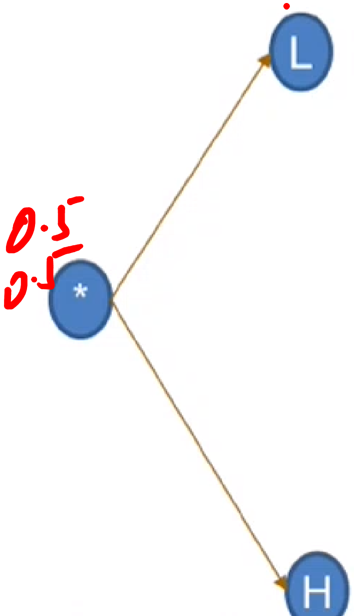
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

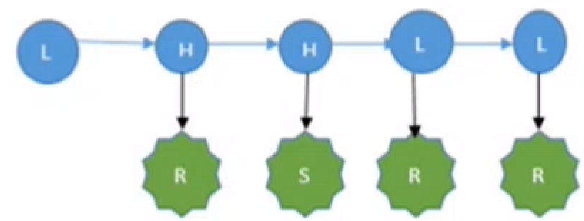
Pressure sequence observation: **S-S-R**

Initialization Phase:

$P(L)*P(S|L) = 0.5*0.2 = 0.1 \rightarrow 0.25$



$P(H)*P(S|H) = 0.5*0.6 = 0.3 \rightarrow 0.75$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

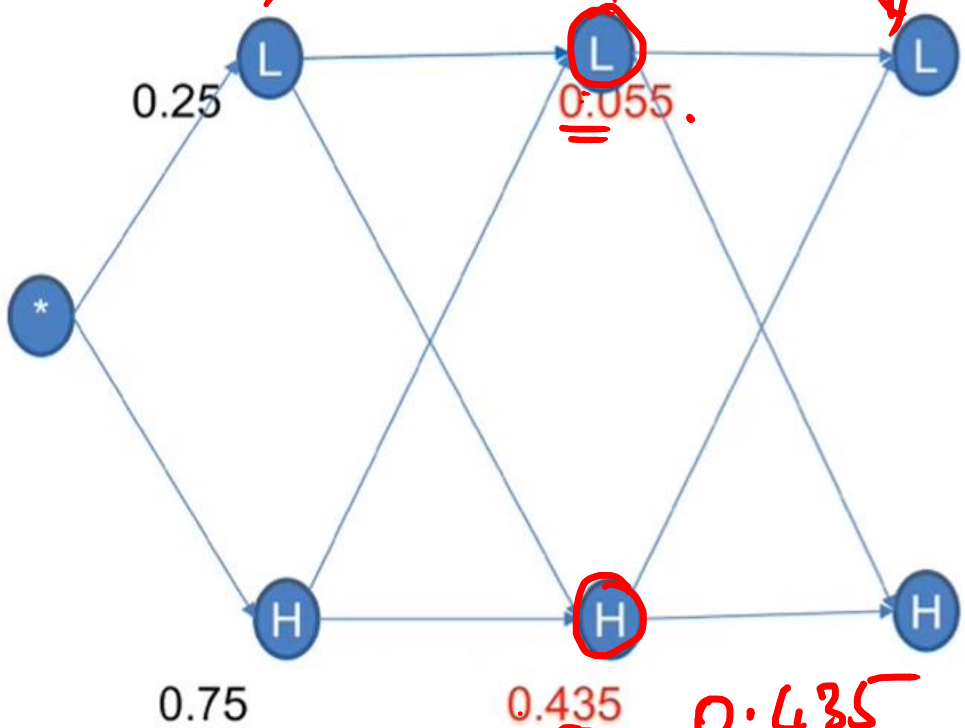
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

forward → Normalization.

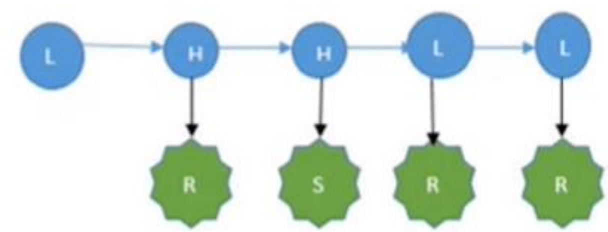
Hidden Markov Model

Filter.

$$\frac{0.055}{0.055+0.435} \Rightarrow$$



$$= \oplus \cdot \frac{0.435}{0.055+0.435}$$



Transition Model / Probability Matrix

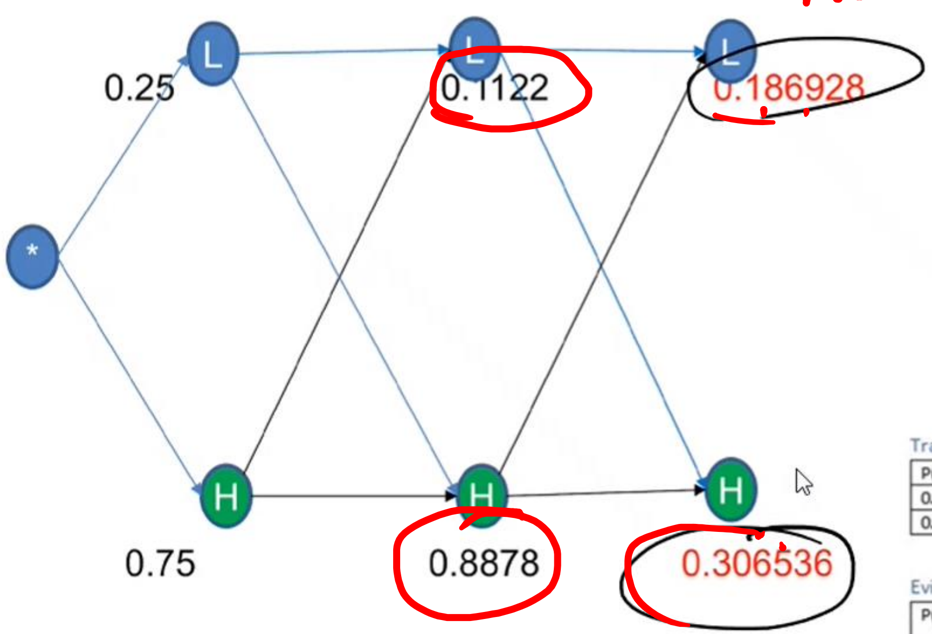
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

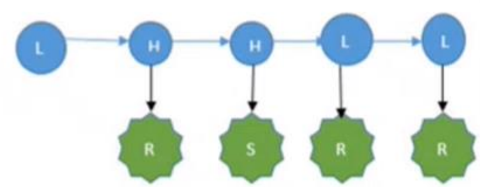
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

Termination Phase:



$$\frac{0.186}{0.186+0.306} \Rightarrow$$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

$$\frac{0.306}{0.306+0.186} \Rightarrow$$

$=$

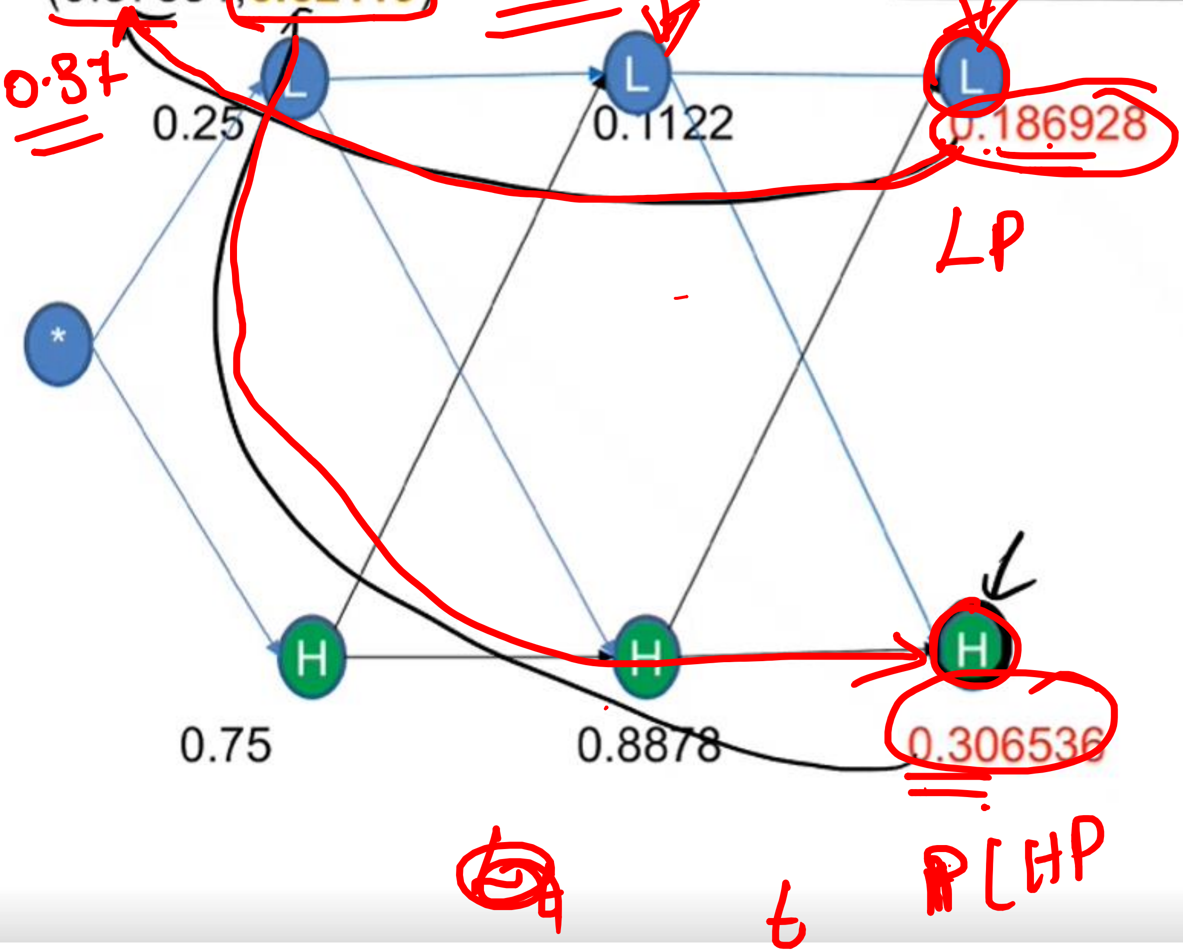
innovate achieve lead

Hidden Markov Model

Termination Phase:

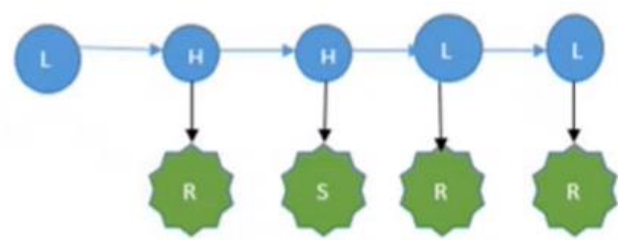
(0.37881, 0.62119)

0.62



$P(H_3 | S_1 S_2 R_3) > P(L_3 | S_1 S_2 R_3)$

$P(H | SSR) = \boxed{0.62119}$



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

Inference: Type -2

$O_1 \; O_2 \; O_3$

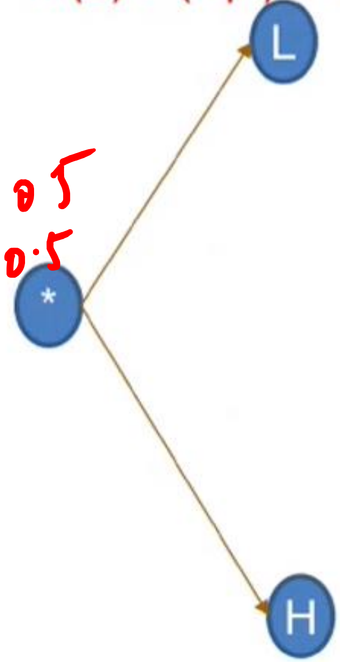
Most Likely Explanation : Veterbi Algorithm

Find the pattern in pressure that might have caused this observation: **S-S-R**.

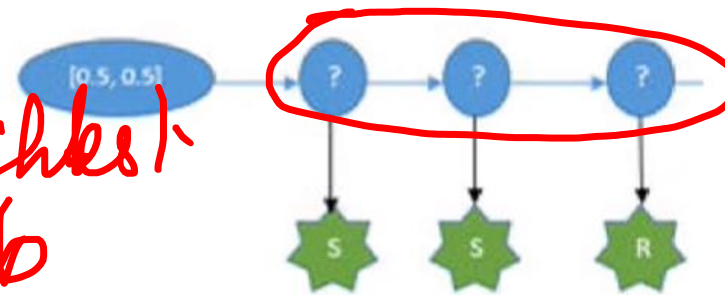
argmax $X_{1...t}$: $P(X_{1...t} \mid E_{1...t})$

MM Inf

$P(L)*P(S|L) = 0.5*0.2 = 0.1 \rightarrow 0.25$



P(HHH/SSR) => highest prob



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

$P(H)*P(S|H) = 0.5*0.6 = 0.3 \rightarrow 0.75$

Hidden Markov Model

Veterbi Algorithm : S-S-R

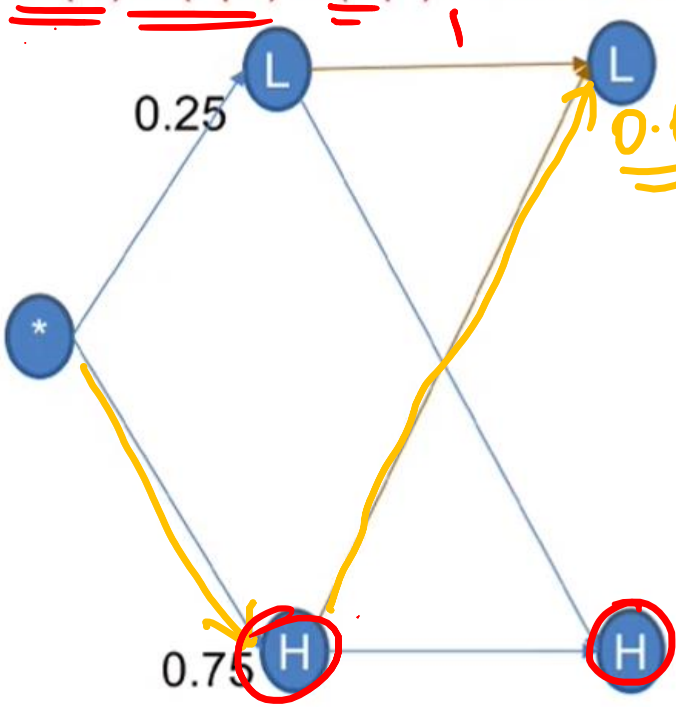
H L

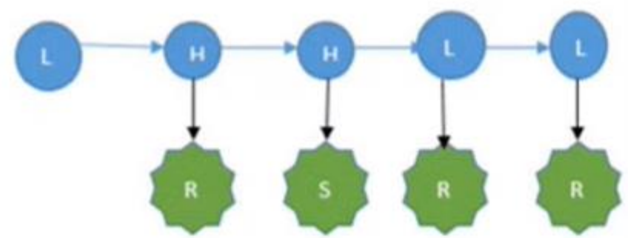
P1

$P(L)*P(L|L)*P(S|L) = 0.25*0.5*0.2 = 0.025$

P2

$P(H)*P(L|H)*P(S|L) = 0.75*0.2*0.2 = 0.03$





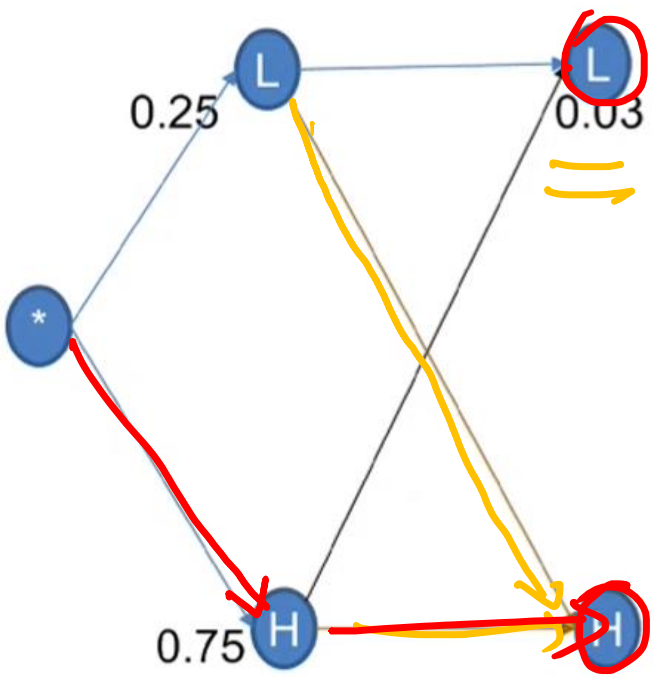
Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

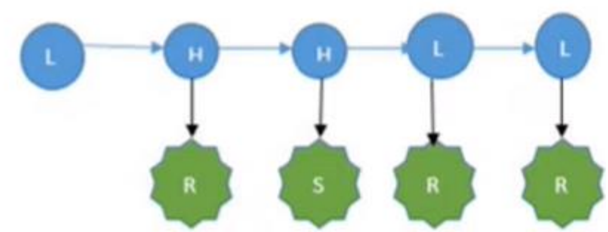


P(L)*P(H|L)*P(S|H) = 0.25*0.5*0.6 = 0.075

P(H)*P(H|H)*P(S|H) = 0.75*0.8*0.6 = 0.36

P(H|H)

max



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

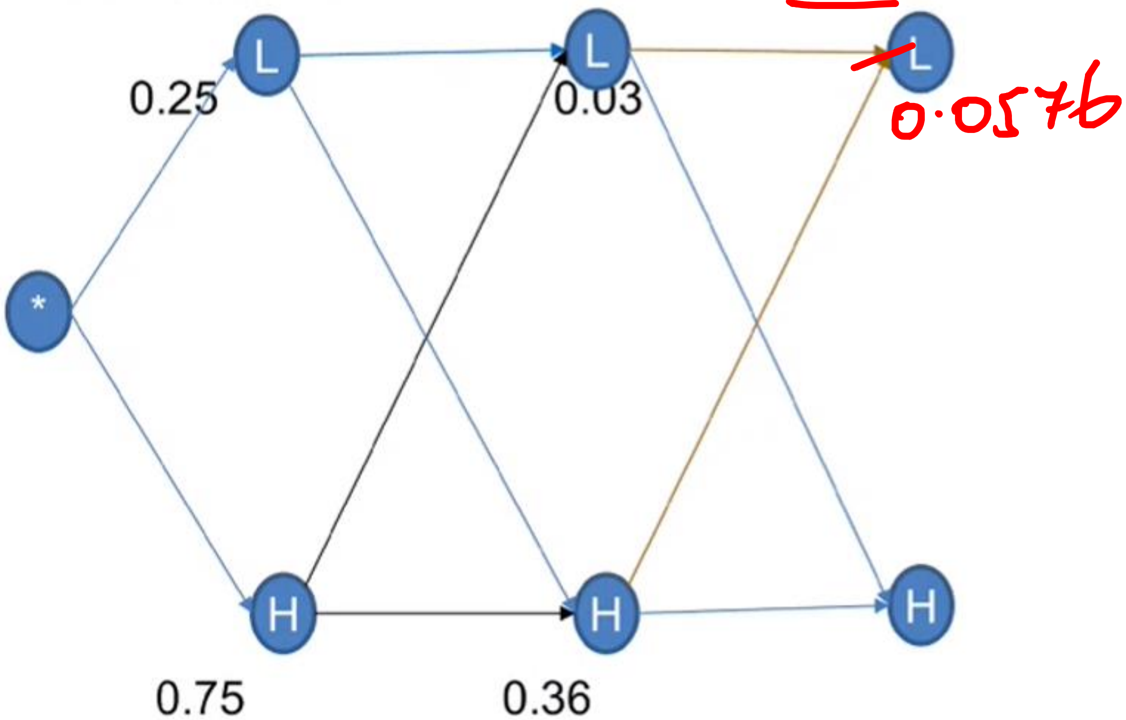
Evidence / Sensor Model/ Emission Probability Matrix

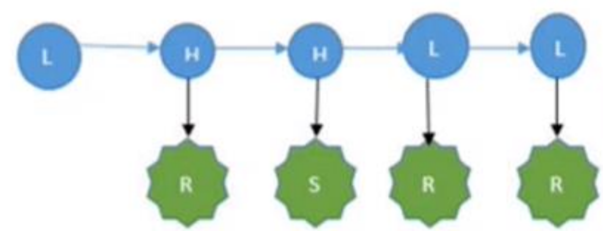
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

$P(L)*P(L|L)*P(R|L) = 0.03*0.5*0.8 = 0.012$

$P(H)*P(L|H)*P(R|L) = 0.36*0.2*0.8 = \mathbf{0.0576}$





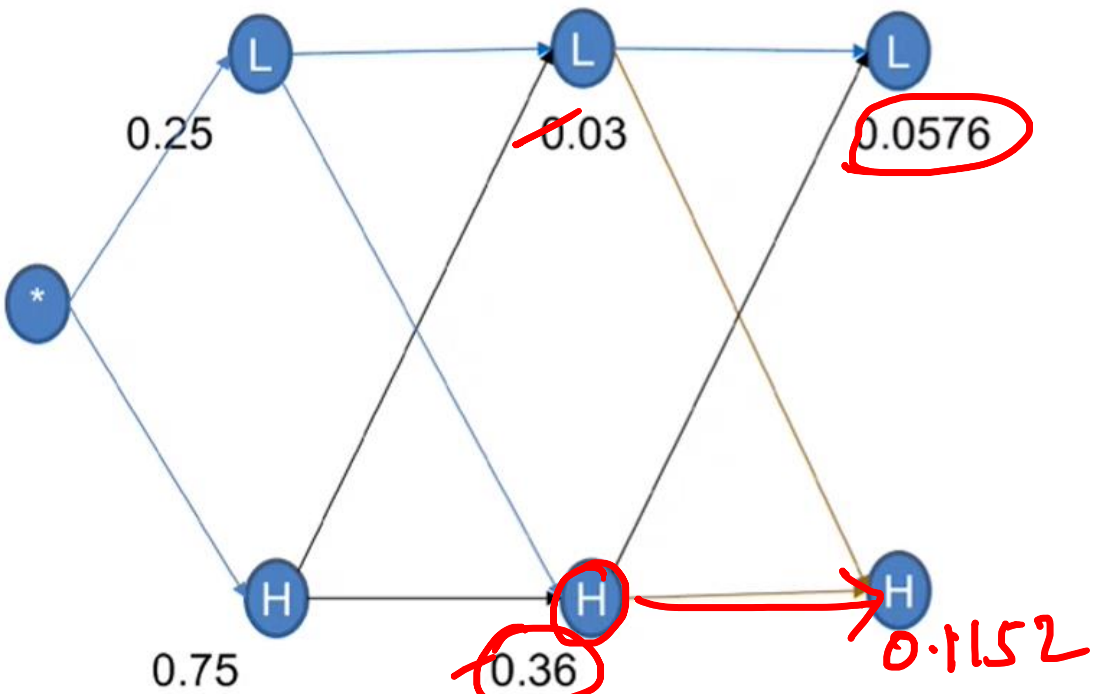
Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = \text{Rainy})$
0.2	0.6	$P(E_t = \text{Sunny})$

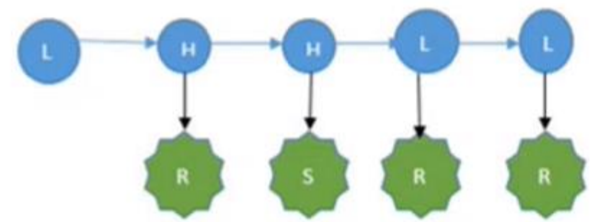
Hidden Markov Model



P(L)*P(H|L)*P(R|H) = 0.03*0.5*0.4 = 0.006

P(H)*P(H|H)*P(R|H) = 0.36*0.8*0.4 = 0.1152

P(H|H)



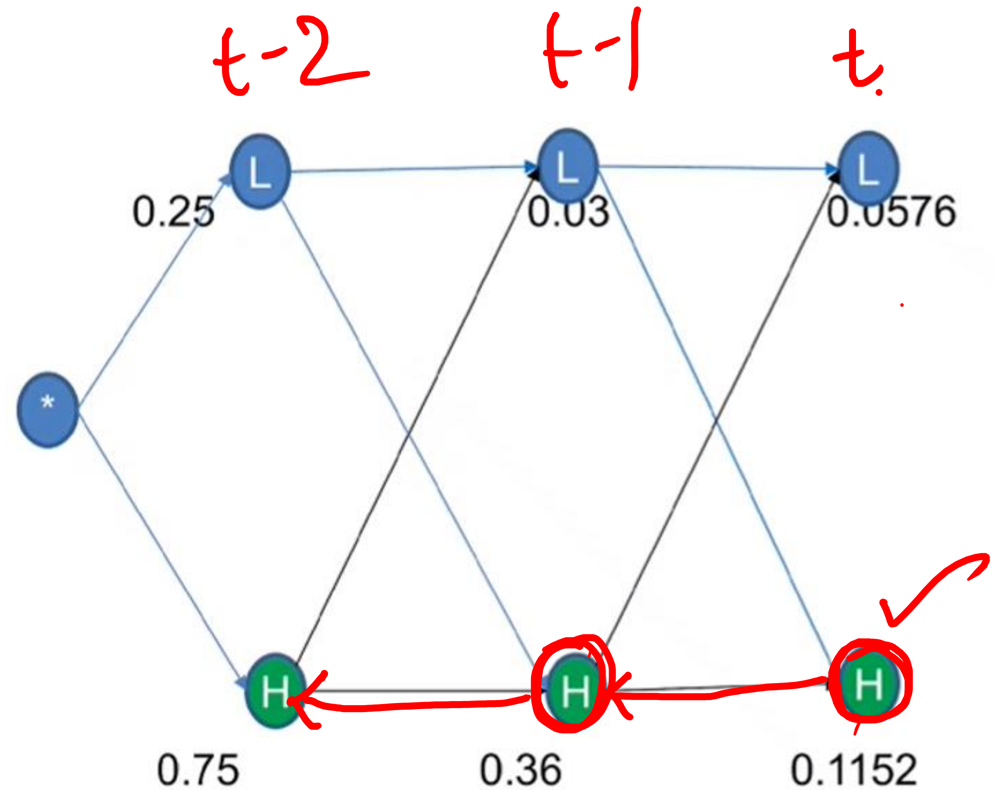
Transition Model / Probability Matrix

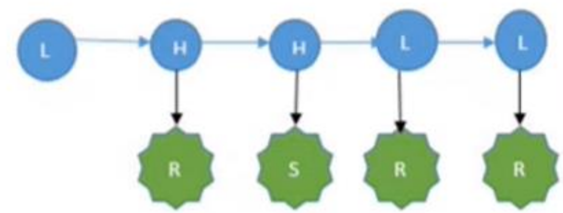
$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

HMM in Prevention of Network Security Threat

(Interesting Case Studies)

Hidden Morkov Model

Cyber Security

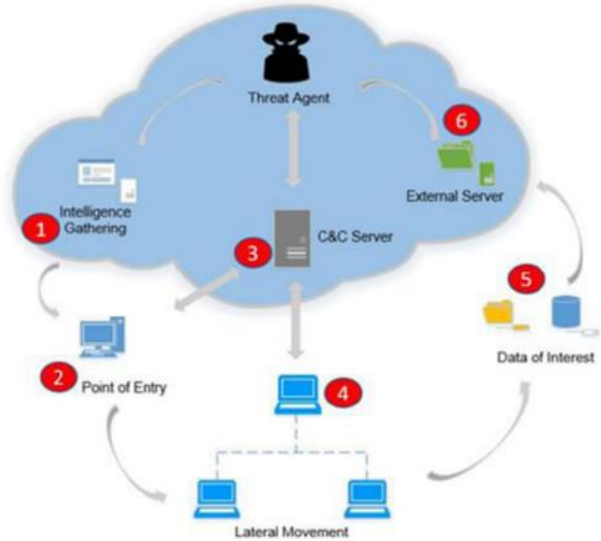
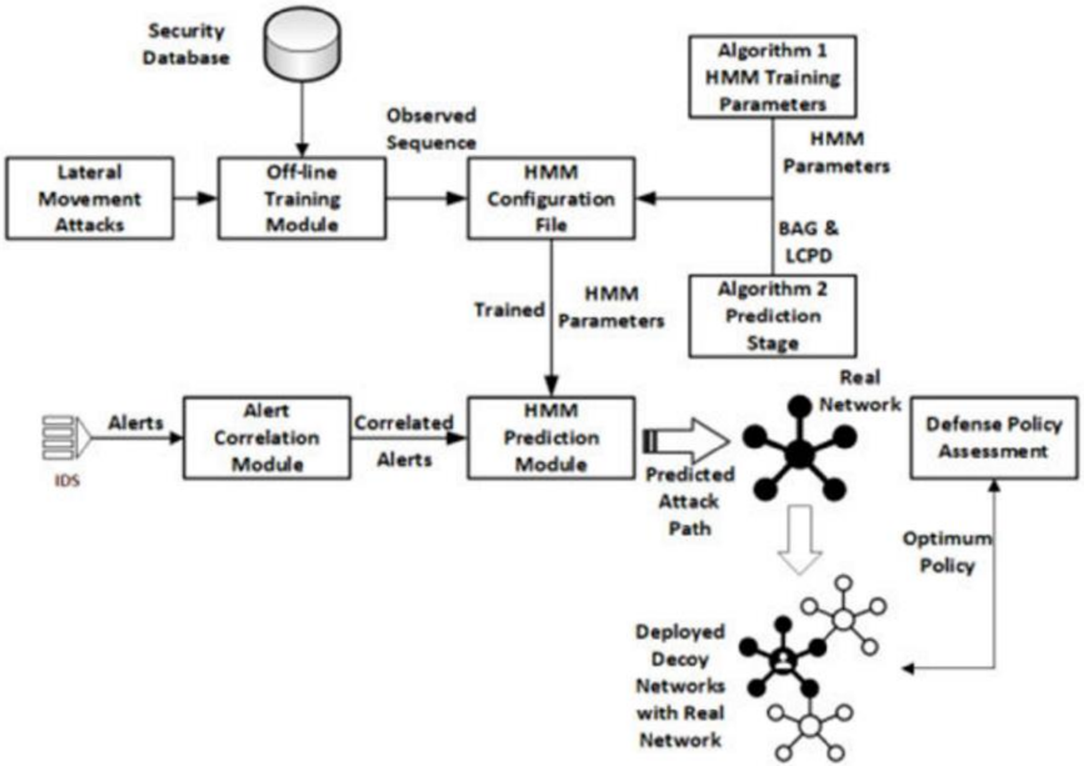


FIGURE 1. Typical stages of APT attack.

Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]

Hidden Morkov Model

Cyber Security



Source Credit : [2021 : Hidden Markov Model and Cyber Deception for the Prevention of Adversarial Lateral Movement]

Required Reading: AIMA - Chapter #15.1, #15.2, #15.3

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials