Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Mathematical Foundations for AIML

II Semester 2022-23

Homework - 2

Instructions

- Do not copy, either from someone or some internet resourse / book.
- This is for your understanding and you need not submit for correction.
- This is not to be taken as sample questions for the examinations.

Q1 Let $\mathbf{B} = (\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_{r-1}}, \mathbf{b_r}, \mathbf{b_{r+1}}, \ldots, \mathbf{b_n})$ be a non-singular matrix. If column $\mathbf{b_r}$ is replace by $\mathbf{a}$ and that the resulting matrix is called $\mathbf{B_a}$ along with $\mathbf{a} = \sum_{i=1}^{n} y_i \mathbf{b_i}$, then state the necessary and sufficient condition for $\mathbf{B_a}$ to be non-singular.

Q2 Let V be a finite dimensional vector space over $\mathbb{R}$. If S is a set of elements in V such that $\text{Span}(S) = V$, what is the relationship between S and the basis of V?

Q3a) Let P be a real square matrix satisfying $P = P^T$ and $P^2 = P$.

i) Can the matrix P have complex eigenvalues? If so, construct an example, else, justify your answer.

ii) What are the eigenvalues of P?

b) Given the following matrix $A = \begin{pmatrix} 1 & 2 & r \\ c & 1 & 7 \\ c & 1 & 7 \end{pmatrix}$ where c and r are arbitrary real numbers and $5.5 < r \leq 6.5$, and the fact that $\lambda_1 = 3$ is one of the eigenvalues, is it possible to determine the other two eigenvalues? If so, compute them and give reasons for your answer.

Q4 The Fibonacci sequence is defined by $V_n = V_{n-1} + V_{n-2}$ for $n \geq 2$ with starting values $V_0 = 1$ and $V_1 = 1$. Observe that the calculation of V_k requires the calculation of $V_2, V_3, \ldots, V_{k-1}$. To avoid this, could this problem be written as an eigenvalue problem and solved for V_n directly? If so, find the explicit formula for V_n.

Q5 Prove that if A is a square matrix of size $n \times n$, then $A^k \to 0$ as $k \to \infty$ if and only if $|\lambda_i| < 1 \quad \forall i$.

Q6 Construct examples of matrices for which the **defect** is positive, negative and zero wherever possible.