Mathematical Foundations for Machine Learning Key Points

Lecture 1

- 1. Understand the key properties of addition and multiplication and note that in general $AB \neq BA$.
- 2. Determine rank of a matrix using elementary row operations and converting to REF / RREF.
- 3. Use rank to deduce the consistency of linear systems and derive general solution(s) of linear systems.
- 4. Properties of transpose and inverse operations.
- 5. Gauss elimination method of solving a system of linear equations of the form $A_{n\times n}x=b$.

Lecture 2

- 1. Define groups and fields.
- 2. Understand the concept of a vector space over a field F.
- 3. Determine whether a given subset of a vector space is a subspace or not.
- 4. Linear span, linear independence and dependence of vectors.
- 5. Basis and dimension of a vector space.

Lecture 3

- 1. Define norms of vectors and matrices.
- 2. Define innerproduct and differentiate from dot product.
- 3. Understand innerproduct space as a vector space with an innerproduct defined.
- 4. Derive positive definiteness of A from the positive definiteness of Ω .
- Angles and orthogonality of vectors and the Cauchy-Schwarz inequality.
- 6. Gram Schmidt orthogonalization process.
- 7. LU decomposition through elementary matrices.

Lecture 4

- 1. Determinants and its properties.
- 2. Determine eigenvalues and eigenvectors of a square matrix.
- 3. Diagonalize a matrix using linearly independent eigenvectors (eigenvalue decomposition (EVD)).
- 4. Use the fact that a symmetric matrix has an orthogonal (and hence) an orthonormal set of eigenvectors.
- 5. Observe that eigenvectors corresponding to distinct eigenvalues are linearly independent.
- 6. Understand the nature of eigenvalues for Hermitian, Skew-Hermitian and Unitary matrices.
- 7. Apply the properties of eigenvalues and eigenvectors in solving problems.

Lecture 5

- 1. Derive SVD of any given matrix and understand it geometrically.
- 2. Note the key differences between EVD and SVD.
- 3. Write a matrix A of rank r and a sum of r rank 1 matrices.
- 4. Understand the spectral norm of a matrix.