Birla Institute of Technology and Science, Pilani

Work Integrated Learning Programmes Division

Mathematical Foundations for Machine Learning

II Semester 2022-23

Homework - 1

Instructions

- Do not copy, either from someone or some internet resourse / book.
- This is for your understanding and you need not submit for correction.
- This is not to be taken as sample questions for the examinations.

Q1 Let $A_{m\times n}$ be a given matrix with $m > n$. If the time taken to compute the determinant of a square matrix of size j is j^3, find upper bound on the

a) total time taken to find the rank of A using determinants. Note that if you are given a $k \times l$ matrix, you may find the determinants of all p *timesp* sub-matrices, where $p = \min(k, l)$ and if at least one of them has a non-zero value, then the rank of A is p. If all of them are zero, you should set p to $p - 1$ and continue.

b) number of divisions, additions (subtraction is to be counted as addition) and multiplications required to determine the rank using the elementary row operations. Note that exchanging of rows does not include any these operations.

Q2 Let $A_{n\times n}$ be a given square matrix. Compute the number of multiplications and additions required to evaluate A^{28} using

a) the naive method, $A^{28} = \underbrace{A \cdot A \cdots A}_{28 \text{ times}}$

b) A^2, $A^4 = A^2 \cdot A^2$, etc.

Q3 Let $A_{n\times k}$ and $B_{k\times k}$ be two given matrices with $\text{rank}(B) = k$. Estimate the rank of AB in terms of the ranks of A and B. Note that the rank of a matrix is the dimension of the vector space spanned by its rows / columns.

Q4 Compute the total number of divisions, multiplications and additions required to perform the forward elimination and back substitution in solving a system of linear equations $A_{n\times n}x = b$ using the Gauss elimination method.