

# BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

## Work Integrated Learning Programmes Division

Cluster Programme - M. Tech in AI & ML

II Semester , 2022 – 23(July,2023)

Mid semester Examination (**Regular**)

Course No : AIMLC ZC418

Course Title : Introduction to Statistical Methods

Nature of Exam. : Open Book (Online)

Weightage : 30 Marks

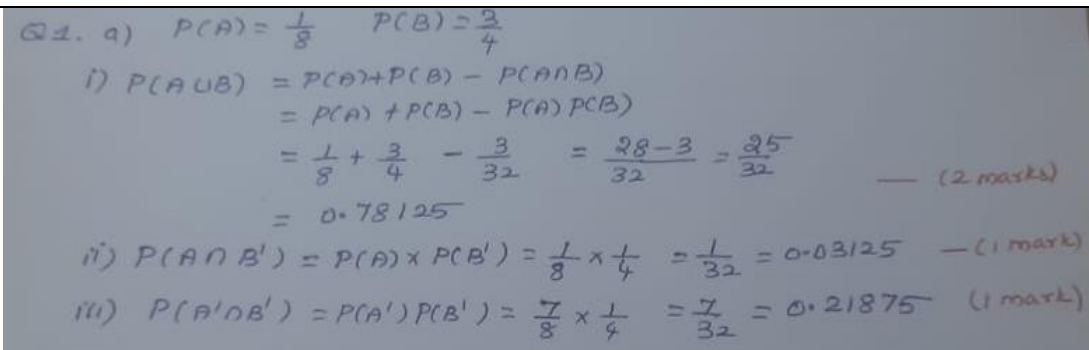
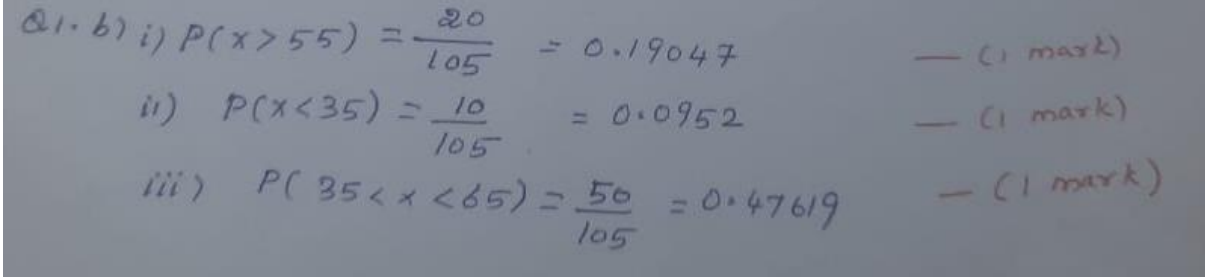
Duration : 120 minutes

Date : 16<sup>th</sup> July,2023\_FN

Number of questions:4

Number of Pages: 3

## SET- B Answer Key

Q. No	Question	Marks																					
Q.1.a)	Let A be an event of a student passing the examination and B be the event of getting preplacement offer(PPO) with probabilities 1/8 and 3/4 respectively in a university. Then find the probability that a student i).passing the examination or getting PPO ii).passing the examination but not getting PPO iii).neither passing the examination nor getting PPO	4 M																					
Solution:	 <p>Q.1. a) <math>P(A) = \frac{1}{8}</math> <math>P(B) = \frac{3}{4}</math>  i) <math>P(A \cup B) = P(A) + P(B) - P(A \cap B)</math>  <math>= P(A) + P(B) - P(A)P(B)</math>  <math>= \frac{1}{8} + \frac{3}{4} - \frac{3}{32} = \frac{28-3}{32} = \frac{25}{32}</math> — (2 marks)  <math>= 0.78125</math>  ii) <math>P(A \cap B') = P(A) \times P(B') = \frac{1}{8} \times \frac{1}{4} = \frac{1}{32} = 0.03125</math> — (1 mark)  iii) <math>P(A' \cap B') = P(A')P(B') = \frac{7}{8} \times \frac{1}{4} = \frac{7}{32} = 0.21875</math> (1 mark)</p>																						
Q.1.b	Consider the following data and answer the questions if possible. Otherwise state reasons. <table border="1" data-bbox="360 1453 1272 1568"><tr><td>Marks (X)</td><td>25</td><td>35</td><td>45</td><td>55</td><td>65</td><td>75</td></tr><tr><td>Number of Students</td><td>10</td><td>25</td><td>20</td><td>30</td><td>15</td><td>5</td></tr><tr><td>Grade</td><td>E</td><td>D</td><td>C</td><td>B</td><td>A</td><td>A+</td></tr></table> i).Find $P(X > 55)$ ii).Find $P(X < 35)$ iii).Find $P(35 < X < 65)$	Marks (X)	25	35	45	55	65	75	Number of Students	10	25	20	30	15	5	Grade	E	D	C	B	A	A+	3 M
Marks (X)	25	35	45	55	65	75																	
Number of Students	10	25	20	30	15	5																	
Grade	E	D	C	B	A	A+																	
Solution:	 <p>Q.1. b) i) <math>P(X &gt; 55) = \frac{20}{105} = 0.19047</math> — (1 mark)  ii) <math>P(X &lt; 35) = \frac{10}{105} = 0.0952</math> — (1 mark)  iii) <math>P(35 &lt; X &lt; 65) = \frac{50}{105} = 0.47619</math> — (1 mark)</p>																						

Q.2.a)	If two events, A and B, are such that $P(A) = 0.3$ , $P(B) = 0.5$ , and $P(A \cap B) = 0.10$ , find the Following: i) $P(A A \cup B)$ ii) $P(A A \cap B)$ iii) $P(A \cap B A \cup B)$	4 M
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Solution	<p style="text-align: center;"><u>SET B</u></p> <p><u>Question 2. a)</u></p> $P(A) = 0.3 \quad P(B) = 0.5 \quad P(A \cap B) = 0.10$ <p>i) <math>P(A A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B) - P(A \cap B)}</math></p> $= \frac{0.3}{0.3 + 0.5 - 0.10} = \frac{0.3}{0.7} = 0.4285$ <p><math>P(A A \cup B) = 0.4285</math> — 2 marks</p> <p>ii) <math>P(A A \cap B) = \frac{P(A \cap (A \cap B))}{P(A \cap B)} = \frac{P(A \cap B)}{P(A \cap B)} = 1</math></p> <p><math>P(A A \cap B) = 1</math> — 1 mark</p> <p>iii) <math>P(A \cap B A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)}</math></p> $= \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.10}{0.7} = 0.143$ <p><math>P(A \cap B A \cup B) = 0.143</math> — 1 mark</p>	
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Q.2.b)	An e-commerce company has three delivery boys A, B and C who delivers 30%, 40% and 25% of items daily from the warehouse. It is observed that they take more time than the expected with probabilities 5%, 10% and 3% respectively. i). Find that the probability that the delivery is always delayed by the company ii). The probability that the delay in delivery is by B	3 M
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Solution	<p><u>Question 2. b)</u></p> <p>i) Find the probability that the delivery is always delayed</p> $P(D) = P(A) \cdot P(D A) + P(B) \cdot P(D B) + P(C) \cdot P(D C)$ $= (0.30)(0.05) + (0.40)(0.10) + (0.25)(0.03)$ $= 0.0625 = 0.063$ <p><math>P(D) = 0.063</math> — 1 mark</p> <p>ii) The Probability that the delay in delivery is B</p> $P(B D) = \frac{P(B \cap D)}{P(D)} = \frac{P(B) \cdot P(D B)}{P(D)} = \frac{(0.40)(0.10)}{0.063} = 0.6349$ <p><math>P(B D) = 0.635</math> — 2 marks</p>	
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Q.3.a)	Probability distribution of two random variables X and Y are given below.					4 M
	Y	X				
		0	1	2	3	
	0	0.15	0.30	0.05	0	
	1	0.05	0.15	2k	0.05	
	2	0	0.05	0.10	k	
	i).Find the probability P(Y > X).					

	ii).Validate the statement" X and Y are independent"											
Solution	<p>i) <math>P(Y &gt; X)= P(0,1)+P(0,2)+P(1,2)=0.05+0+0.05=0.1</math> (2marks)</p> <p>ii) As <math>P(0,0)=0.15</math> and <math>P(x=0).P(y=0)=0.2*0.5=0.1</math> Thus <math>P(0,0) \neq P(x=0).P(y=0)</math> (1mark) X and y are not independent (1mark)</p>											
Q.3.b)	<p>Consider the following probability distribution.</p> <table><tr><td>X</td><td>-2</td><td>-1</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>0.20</td><td>0.40</td><td>0.25</td><td>0.15</td></tr></table> <p>i)." Probability distribution is not valid because x is negative". Validate. ii.If the distribution is valid then find <math>E(X)</math> ,<math>E(X^2)</math> and hence variance of X.</p>	X	-2	-1	1	2	P(X)	0.20	0.40	0.25	0.15	4 M
X	-2	-1	1	2								
P(X)	0.20	0.40	0.25	0.15								
Solution	<p>i) As <math>0 &lt; P(x_i) &lt; 1</math> and summation of <math>P(x_i)=1</math> (1mark)</p> <p>ii) <math>E(x) = - 0.25</math> (1mark) <math>E(x^2)=2.05</math> (1mark) <math>V(x)=1.9875</math> (1mark)</p>											
Q.4.a)	<p>Let X be a random variable which follows binomial distribution with <math>n = 500</math> and <math>p = 0.20</math>. Then find the following i). <math>P(X &gt; 290)</math> ii) <math>P(X = 250)</math> iii). <math>P(120 &lt; X &lt; 180)</math>.</p>	4 M										
Solution	<p>As <math>n.p= 500*0.2=100 &gt; 15</math> and <math>n.q= 500*0.8=400 &gt; 15</math> , hence we can use Binomial approximation to Normal distribution <math>\mu = np = 100</math> and <math>\sigma = \sqrt{npq} = 8.94</math> Let <math>z = \frac{x-\mu}{\sigma} = \frac{x-100}{8.94}</math> (1mark)</p> <p>i). <math>P(X &gt; 290) = P(X &gt; 290.5) = P(Z &gt; 21.3)=0</math> (1mark) ii) <math>P(X = 250) = P(249.5 &lt; X &lt; 250.5) = P(16.72 &lt; Z &lt; 16.83)= 0</math>(1mark) iii). <math>P(120 &lt; X &lt; 180) = P(120.5 &lt; X &lt; 179.5) = P(2.29 &lt; Z &lt; 8.89)=0.5-0.4890=0.011</math> (1mark)</p>											
Q.4.b)	<p>The rain fall (in cms) in a Country during every July month is normally distributed with mean and standard deviation of rainfall are respectively as 12cms and 1.25cms. For the July month of 2022, calculate the probabilities of having rainfall i) less than 15 cms ii).in between 13cms and 18cms. iii) of 13 cms</p>	4 M										
Solution:	<p><math>\mu = 12</math> and <math>\sigma = 1.25</math> Let <math>z = \frac{x-\mu}{\sigma} = \frac{x-12}{1.25}</math> (1mark)</p> <p>i) <math>P(x &lt; 15)=P(z &lt; 2.4)=0.9918</math> (1mark) ii) <math>P(13 &lt; x &lt; 18)=P(0.8 &lt; z &lt; 4.8)= 0.5 - 0.2881=0.2119</math> (1mark) iii) <math>P(x=13) = 0</math> (1mark)</p>											

