BITS Pilani

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Introduction to Statistical Methods

ISM Team

IMP Note to Self

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BITS Pilani

Session No 2

Axioms of Probability, Probability basics, mutually exclusive and independent events,

(Session 2: 27th/28th MAY 2023)

Contact Session 2

Contact Session 2: Module 1(Module 1:Basic Probability & Statisitcs)

Contact Session	List of Topic Title	Reference
CS - 2	Axioms of Probability,Mutually exclusive and independent events,Problem solving to understand basic probability concepts	T1 & T2
HW	Problems on probability	T1 & T2
Lab		

Agenda

- Experiments, assignment of probabilities
- Events and their probability
- Some basic relationships of probability
- Basic problem solving

RECALL: Random Experiment

Term "**random experiment**" is used to describe any action whose outcome is not known in advance. Here are some examples of experiments dealing with statistical data:

- Tossing a coin
- Counting how many times a certain word or a combination of words appears in the text of the “King Lear” or in a text of Confucius
- counting occurrences of a certain combination of amino acids in a protein database.
- pulling a card from the deck

Sample spaces, sample sets and events

The ***sample space*** of a random experiment is a set S that includes all possible outcomes of the experiment.

For example, if the experiment is to throw a die and record the outcome, the sample space is S = { 1,2,3,4,5,6}

- Discrete sample spaces.
- Continuous sample spaces

Discrete Random Variables

- A discrete random variable is one which may take on only a countable number of distinct values such as 0,
  1, 2, 3, 4,....
- • Discrete random variables are usually (but not necessarily) counts.

Examples:

- number of children in a family
- the Friday night attendance at a cinema
- the number of patients a doctor sees in one day
- the number of defective light bulbs in a box of ten
- the number of “heads” flipped in 3 trials

Continuous Random Variable

- A continuous random variable is one which takes an infinite number of possible values.

- Examples:
  - ✓ height
  - ✓ weight
  - ✓ the amount of sugar in an orange
  - ✓ the time required to run a mile.

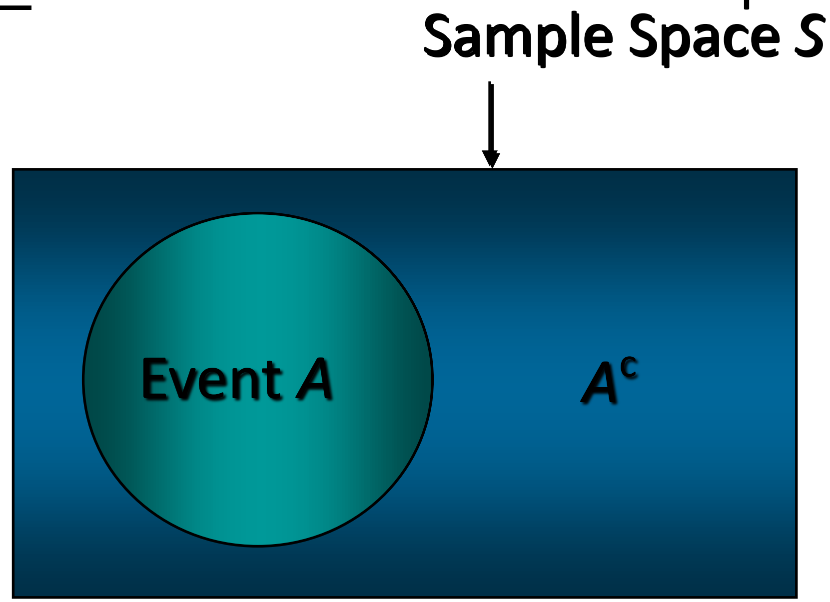
Event

An event is a subset of the sample space of a random experiment.

An event is a set of outcomes of the experiment. This includes the *null* (empty) set of outcomes and the set of *all* outcomes. Each time the experiment is run, a given event *A* either *occurs*, if the outcome of the experiment is an element of *A*, or *does not occur*, if the outcome of the experiment is not an element of *A*.

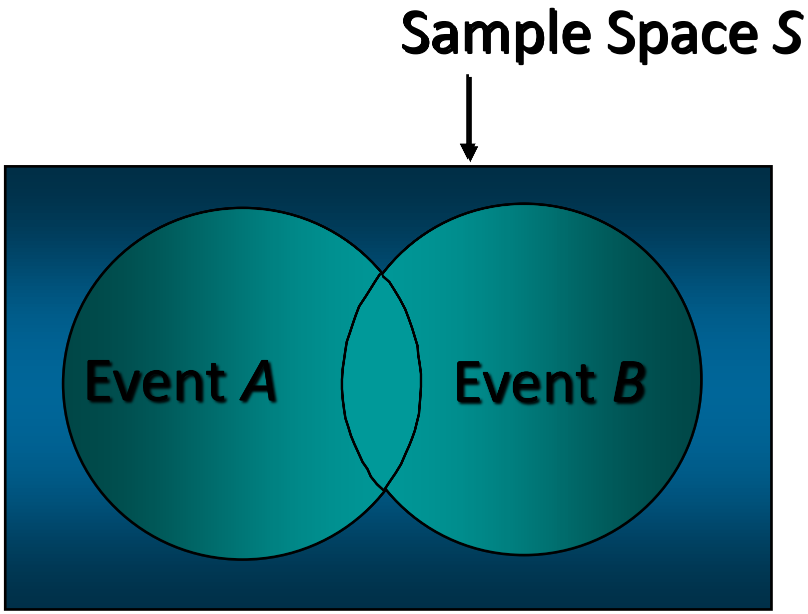
Complement of an Event

- The <u>complement</u> of event *A* is defined to be the event consisting of all sample points that are not in *A.*
- The complement of *A* is denoted by A^c.
- The <u>Venn diagram</u> below illustrates the concept of a complement.



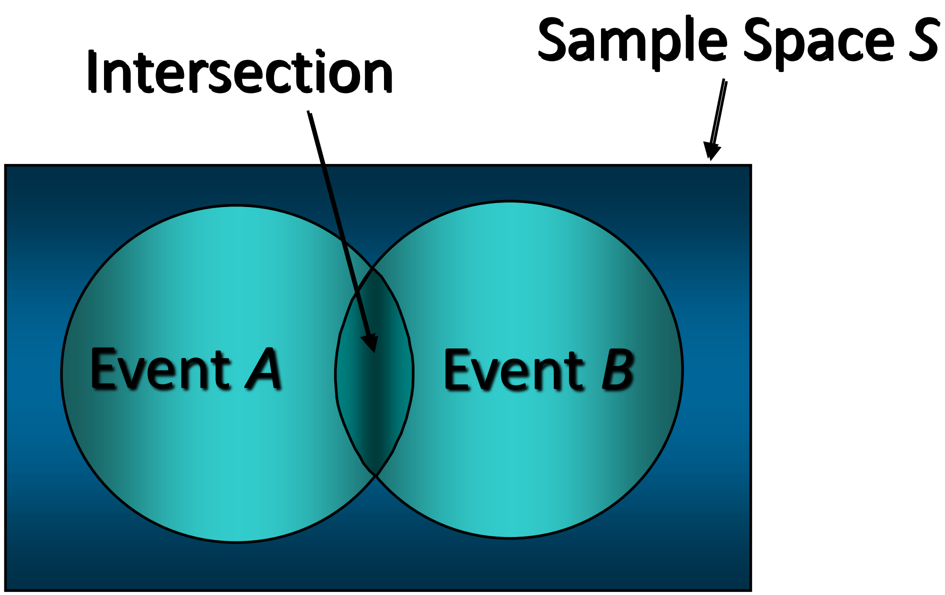
Union of Two Events

- The <u>union</u> of events *A* and *B* is the event containing all sample points that are in *A* or *B* or both.
- The union is denoted by *A* U *B*
- The union of *A* and *B* is illustrated below.



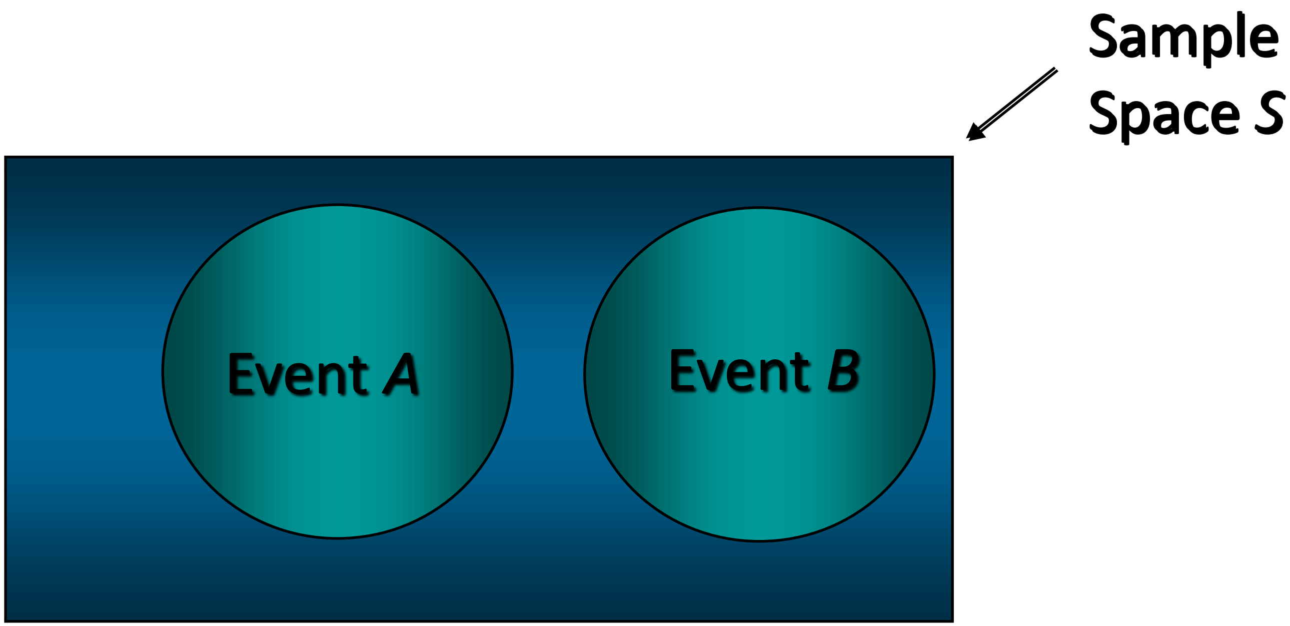
Intersection of Two Events

- The intersection of events *A* and *B* is the set of all sample points that are in both *A* and *B*.
- The intersection of *A* and *B* is the area of overlap in the illustration below.



Mutually Exclusive Events

- Two events are said to be <u>mutually exclusive</u> if the events have no sample points in common. That is, two events are mutually exclusive if, when one event occurs, the other cannot occur.



Axioms of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

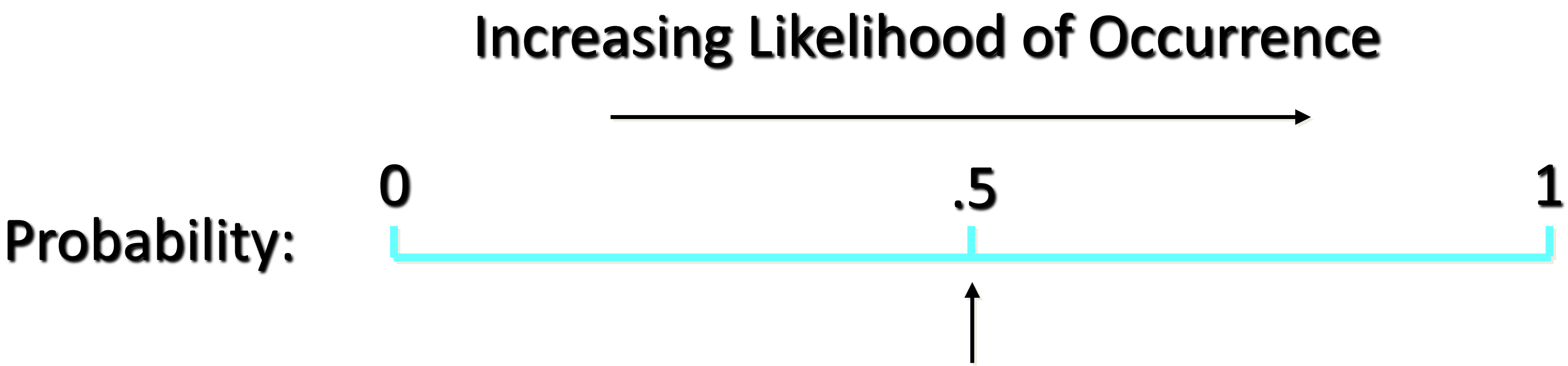
(1) $P(S) = 1$

(2) $0 \leq P(E) \leq 1$

(3) For two events E_1 and E_2 with $E_1 \cap E_2 = \varnothing$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Probability as a Numerical Measure of the Likelihood of Occurrence



The occurrence of the event is just as likely as it is unlikely.

THE ADDITION RULE

- The probability that event *A* or *B* will occur is given by

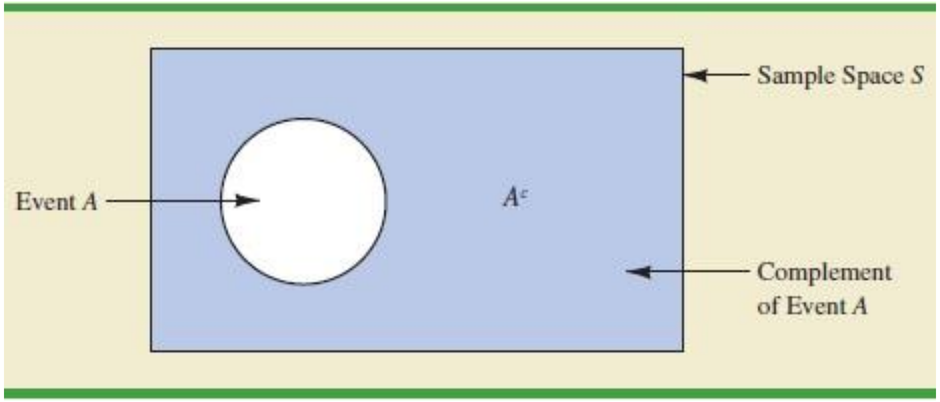
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

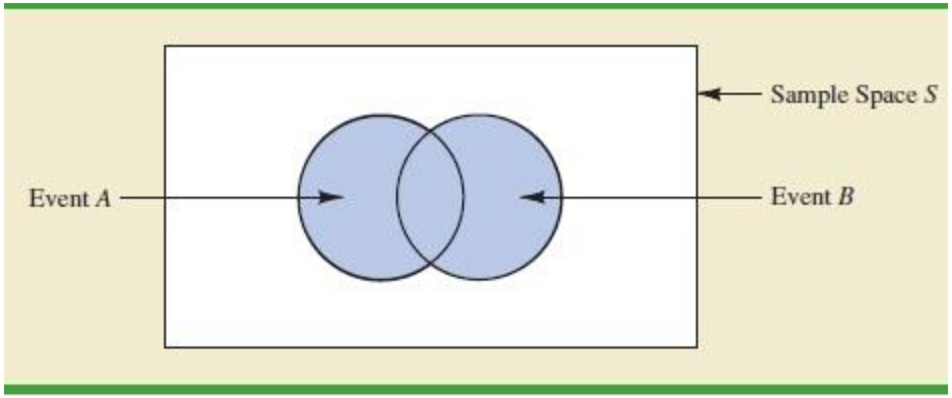
- If events *A* and *B* are **mutually exclusive**, then the rule can be simplified to

  $P(A \cup B) = P(A) + P(B)$.

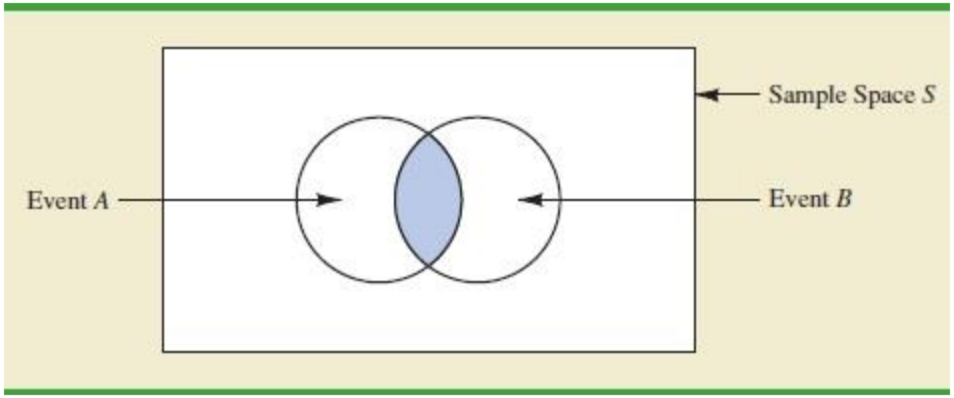
Probability and Venn Diagram

$$P(A) = 1 - P(A^c)$$





$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Independent & Dependent

Events are either

- ***Independent*** (the occurrence of one event has no effect on the probability of occurrence of the other) or
- ***Dependent*** *(*the occurrence of one event gives information about the occurrence of the other)

Example

An experiment has the four possible mutually exclusive outcomes A, B, C, D. Check whether the following assignments of probability are permissible:

(a) $P(A)= 0.38$, $P(B) = 0.16$, $P(C) =0.11$, $P(D) = 0.35$ Permissible
(b) $P(A)= 0.31$, $P(B) = 0.27$, $P(C) =0.28$, $P(D) = 0.16$ NOT
(c) $P(A)= 0.32$, $P(B) = 0.27$, $P(C) = -0.06$, $P(D) = 0.47$ NOT
(d) $P(A)= 1/2$, $P(B) = 1/4$, $P(C) = 1/8$, $P(D) = 1/16$ NOT
(e) $P(A)= 5/8$, $P(B) = 1/6$, $P(C) = 1/3$, $P(D) = 2/9$ NOT

EXAMPLE: 1

If two dice are thrown , what is the probability that the sum is

a) Greater than 8

b) Less than 6

c) Neither 7 nor 11

Solution:

$$P(X = 2) = P\{(1, 1)\} = \frac{1}{36}$$

$$P(X = 3) = P\{(1, 2), (2, 1)\} = \frac{2}{36}$$

$$P(X = 4) = P\{(1, 3), (2, 2), (3, 1)\} = \frac{3}{36}$$

$$P(X = 5) = P\{(1, 4), (2, 3), (3, 2), (4, 1)\} = \frac{4}{36}$$

$$P(X = 6) = P\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} = \frac{5}{36}$$

$$P(X = 7) = P\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\} = \frac{6}{36}$$

$$P(X = 8) = P\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\} = \frac{5}{36}$$

$$P(X = 9) = P\{(3, 6), (4, 5), (5, 4), (6, 3)\} = \frac{4}{36}$$

$$P(X = 10) = P\{(4, 6)\ (5, 5), (6, 4)\} = \frac{3}{36}$$

$$P(X = 11) = P\{(5, 6), (6, 5)\} = \frac{2}{36}$$

$$P(X = 12) = P\{(6, 6)\} = \frac{1}{36}$$

If two dice are thrown , what is the probability that the sum is

a) Greater than 8

b) Less than 6

c) Neither 7 nor 11

a) $P(\text{sum} > 8)$

$$= P(9) + P(10) + P(11) + P(12)$$

$$= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{10}{36}$$

b) $P(\text{sum} < 6)$

$$= P(5) + P(4) + P(3) + P(2)$$

$$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{10}{36}$$

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If two dice are thrown , what is the probability that the sum is

a) Greater than 8

b) Less than 6

c) Neither 7 nor 11

$P(\text{neither } 7 \text{ nor } 11)$

$= 1 - [P(7) + P(11)]$

$= 1 - \left[\frac{6}{36} + \frac{2}{36}\right]$

$= 28/36 = 7/9$

Example:2

The probability that a student passes in statistics examination is 2/3 and the probability that he /she will not pass in mathematics examination is 5/9. The probability that he/she will pass in at least one of the examination is 4/5. Find the probability that he /she will pass in both the examinations

Solution:

The probability of passing in statistics P(S)=2/3

The probability of passing in Mathematics P(M)=1-5/9 =4/9

Probability of Passing at least one of these examination

= P(S∪M) = 4/5

***P* (*S* U M) = *P* (*S*) + *P* (M) – *P* (*S* ∩ *M*).**

$\frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P\,(S \cap M\,).$

$P\,(S \cap M\,) = \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{45}\,.$

The probability of passing both examinations $P\,(S \cap M\,) = \frac{14}{45}$

Example: 3

Suppose that 75% of all investors invest in traditional annuities and 45% of them invest in the stock market. If 85% invest in the stock market and/or traditional annuities, what percentage invest in both?

Solution:

Let A be the event that a randomly selected investor invests in traditional annuities.

Let B be the event that he or she invests in the stock market.

Then $P(A) = 0.75$, $P(B) = 0.45$, and $P(A \cup B) = 0.85$

Since,

$$P(AB) = P(A) + P(B) - P(A \cup B)$$
$$= 0.75 + 0.45 - 0.85$$
$$= 0.35.$$

Example: 4

Suppose the manufacturer's specifications for the length of a certain type of computer cable are 2000 ± 10 millimeters. In this industry, it is known that small cable is just as likely to be defective (not meeting specifications) as large cable. That is, the probability of randomly producing a cable with length exceeding 2010 millimeters is equal to the probability of producing a cable with length smaller than 1990 millimeters. The probability that the production procedure meets specifications is known to be 0.99.

(a) What is the probability that a cable selected randomly is too large?

(b) What is the probability that a randomly selected cable is larger than 1990 millimeters?

Solution:

***Solution*:** Let M be the event that a cable meets specifications. Let S and L be the events that the cable is too small and too large, respectively. Then

(a) $P(M) = 0.99$ and $P(S) = P(L) = (1 - 0.99)/2 = 0.005$.

(b) Denoting by X the length of a randomly selected cable, we have

$$P(1990 \leq X \leq 2010) = P(M) = 0.99.$$

Since $P(X \geq 2010) = P(L) = 0.005$,

$$P(X \geq 1990) = P(M) + P(L) = 0.995.$$

This also can be solved by using Theorem 2.9:

$$P(X \geq 1990) + P(X < 1990) = 1.$$

Thus, $P(X \geq 1990) = 1 - P(S) = 1 - 0.005 = 0.995$.

Exercise problems:

Q 1: A Survey conducted by a bank revealed that 40% of the accounts are savings accounts and 35% of the accounts are current accounts and the balance are loan accounts.

- What is the probability that an account taken at random is a loan account ?
- What is the probability that an account taken at random is **NOT** savings account ?
- What is the probability that an account taken at random is **NOT** a current account
- What is the probability that an account taken at random is a current account or a loan account?

Exercise problems

Q 2. A speaks truth in 80% cases and B speaks in 60% cases. What percentage of cases are they likely to contradict each other in stating the same fact.

Q 3. In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected,

i) What is the probability that it gets at least one of these two services from the company, and

ii) What is the probability that it gets exactly one of these services from the company?

Exercise problems

Q 4: The next generation of miniaturised wireless capsules with active locomotion will require two miniature electric motors to manoeuvre each capsule. Suppose 10 motors have been fabricated but that, in spite of test performed on the individual motors 2 will not operate satisfactorily when placed into capsule, to fabricate a new capsule,2 motors will be randomly selected(that is, each pair of motors has the same chance of being selected) find the probability that

a) Both motors will operate satisfactorily in the capsule.

a) One motor will operate satisfactorily and other will not.

Exercise problems

Q 5. Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products.

i) What is the probability that a randomly selected adult regularly consumes both coffee and soda?

ii) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products?

Exercise problems

Q 6. Suppose a student is selected at random from 80 students where 30 are taking mathematics, 20 are taking chemistry and 10 are taking both. Find the probability ‘p’ that the student is taking Mathematics or chemistry?.

Q7: If A and B are events with $P(A \cup B) = 7/8$, $P(A \cap B) = 1/4$ and $P(A') = 5/8$, find $P(A)$, $P(B)$ and $P(A \cap B')$.

Thanks