

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Work Integrated Learning Programmes Division

Cluster Programme - M. Tech in AI & ML

II Semester , 2022 – 23(July,2023)

Mid semester Examination (**Regular**)

Course No : AIMLC ZC418
 Course Title : Introduction to Statistical Methods
 Nature of Exam. : Open Book (Online)
 Weightage : 30 Marks
 Duration : 120 minutes
 Date : 16th July,2023_FN

Number of questions:4

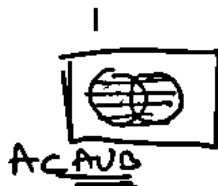
Number of Pages: 3

SET- A: Answer Key

| Q. No | Question | Marks | | | | | | | | | | | | | | | | | | | | | |
|------------------------|--|------------|----|----|----|----|----|----|--------------------|----|----|----|----|---|---|-------|---|---|---|---|---|----|------------|
| SET-A Q.1.a) | Let A be an event of a student passing the examination and B be the event of getting preplacement offer(PPO) with probabilities $\frac{3}{4}$ and $\frac{1}{8}$ respectively in a university. Then find the probability that a student i).passing the examination and also getting PPO, ii).failing in the examination but getting PPO iii).neither passing the examination nor getting PPO | 4 M | | | | | | | | | | | | | | | | | | | | | |
| Solution | <p>Q.1.a) $P(A) = \frac{3}{4}$ $P(B) = \frac{1}{8}$</p> <p>i) $P(A \cap B) = P(A)P(B)$ (A and B are independent) $= \frac{3}{4} \times \frac{1}{8}$ $= \frac{3}{32} = 0.09375$ — (2 mark)</p> <p>ii) $P(A' \cap B) = P(A')P(B)$ $= (1 - P(A)) P(B) = \frac{1}{4} \times \frac{1}{8} = \frac{1}{32}$ — (1.5 mark) $= 0.03125$</p> <p>iii) $P(A' \cap B') = P(A')P(B') = \frac{1}{4} \times \frac{7}{8} = \frac{7}{32} = 0.21875$ — (1.5 mark)</p> | | | | | | | | | | | | | | | | | | | | | | |
| Q.1.b) | Consider the following data and answer the questions if possible. Otherwise state reasons. <table><tr><td>Marks (X)</td><td>25</td><td>35</td><td>45</td><td>55</td><td>65</td><td>75</td></tr><tr><td>Number of Students</td><td>10</td><td>20</td><td>25</td><td>35</td><td>5</td><td>5</td></tr><tr><td>Grade</td><td>E</td><td>D</td><td>C</td><td>B</td><td>A</td><td>A+</td></tr></table> i).Find $P(X > 55)$, ii).Find $P(X < 35)$, iii).Find $P(35 < X < 65)$ | Marks (X) | 25 | 35 | 45 | 55 | 65 | 75 | Number of Students | 10 | 20 | 25 | 35 | 5 | 5 | Grade | E | D | C | B | A | A+ | 3 M |
| Marks (X) | 25 | 35 | 45 | 55 | 65 | 75 | | | | | | | | | | | | | | | | | |
| Number of Students | 10 | 20 | 25 | 35 | 5 | 5 | | | | | | | | | | | | | | | | | |
| Grade | E | D | C | B | A | A+ | | | | | | | | | | | | | | | | | |
| Solution | <p>Q.1.b) $P(X > 55) = \frac{10}{100} = 0.1$ — (1 mark)</p> <p>$P(X < 35) = \frac{10}{100} = 0.1$ — (1 mark)</p> <p>$P(35 < X < 65) = \frac{60}{100} = 0.6$ — (1 mark)</p> | | | | | | | | | | | | | | | | | | | | | | |
| SET-A Q.2.a) | If two events, A and B, are such that $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.15$, find the Following:i) $P(A A \cup B)$, ii) $P(A A \cap B)$, iii) $P(A \cap B A \cup B)$ | 4 M | | | | | | | | | | | | | | | | | | | | | |

Solution

$$Q.2a) i) P(A|A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)}$$



$$= \frac{P(A)}{P(A) + P(B) - P(A \cap B)}$$

$$= \frac{0.3}{0.3 + 0.5 - 0.15}$$

$$= \frac{0.3}{0.65} = \underline{\underline{0.4615}}$$

$$ii) P(A|A \cap B) = \frac{P[A \cap (A \cap B)]}{P(A \cap B)}$$

$$= \frac{P[A \cap B]}{P[A \cap B]} = \underline{\underline{1}}$$

$$iii) P(A \cap B|A \cup B) = \frac{P[(A \cap B) \cap (A \cup B)]}{P(A \cup B)}$$

$$= \frac{P[A \cap B]}{P(A \cup B)} = \frac{0.15}{0.65}$$

$$= \underline{\underline{0.231}}$$



Q.2.b)

An e-commerce company has three delivery boys A, B and C who delivers 30%, 40% and 30% of items daily from the warehouse. It is observed that they take more time than the expected with probabilities 5%, 10% and 3% respectively.

3 M

- a). Find that the probability that the delivery is always delayed by the company.
b). The probability that the delay in delivery is by A

Solution

Q.2 b) i) By Total probability

$$P(D) = P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)$$

$$= (0.30)(0.05) + (0.40)(0.10) + (0.30)(0.03)$$

$$= \underline{\underline{0.064}}$$

$$ii) P[A|D] = \frac{P(A \cap D)}{P(D)} = \frac{P(A) \cdot P(D|A)}{P(D)}$$

$$= \frac{0.30 \times 0.05}{0.064} = \underline{\underline{0.234}}$$

SET-A

Q.3.a)

Probability distribution of two random variables X and Y are given below.

| Y | X | | | |
|---|------|------|------|------|
| | 0 | 1 | 2 | 3 |
| 0 | 0.15 | 0.30 | 0.05 | 0 |
| 1 | 0.05 | 0.15 | 2k | 0.05 |
| 2 | 0 | 0.05 | 0.10 | k |

4 M

- i). If possible find the value of k, ii). Find marginal distribution of X
iii). Find marginal distribution of Y

Solution

i) To find k, $0.15 + 0.30 + 0.05 + 0 + 0.05 + 0.15 + 2k + 0.05 + 0 + 0.05 + 0.10 + k = 1$
 $3k = 0.1 \Rightarrow k = \frac{1}{30}$ — (1 mark)

Marginal distribution of X

$P(X=0) = 0.15 + 0.05 + 0 = 0.20$

$P(X=1) = 0.30 + 0.15 + 0.05 = 0.50$

$P(X=2) = 0.05 + 2(0.033) + 0.10 = 0.22$

$P(X=3) = 0 + 0.05 + 0.03 = 0.08$ — (1.5 marks)

Marginal distribution of Y

$P(0) = 0.50$

$P(1) = 0.32$

$P(2) = 0.18$ — (1.5 marks)

Q.3.b)

Consider the following probability distribution.

| | | | | |
|------|------|------|------|------|
| X | -2 | -1 | 1 | 2 |
| P(X) | 0.20 | 0.40 | 0.25 | 0.15 |

- i). "Probability distribution is not valid because x is negative". Validate.
 ii). If the distribution is valid then find mean and variance.
 iii). $P(X > -1)$

4 M

| | | |
|--------------------------------|--|------------|
| <p>Solution</p> | <p>Sum of the probabilities = 1</p> $0.20 + 0.40 + 0.25 + 0.15 = 1$ <p>Hence Valid. — 1 Mark</p> <p>To find Mean: $\sum_{i=1}^4 x P(x)$</p> $= (-2)(0.20) + (-1)(0.40) + (0)(0.25) + (2)(0.15)$ $= -0.40 - 0.40 + 0.25 + 0.30$ <p>Mean = $-0.80 + 0.55 = -0.25$ — 1 Mark</p> <p>To find Variance: $\sum x^2 P(x) - \mu^2$</p> $= [(-2)^2(0.20) + (-1)^2(0.40) + (0)^2(0.25) + (2)^2(0.15)] - (-0.25)^2$ $= 2.05 - 0.0625$ <p>Variance = 1.9875 — 1 Mark</p> <p>$P(X > -1) \Rightarrow P(1) + P(2) = 0.25 + 0.15 = 0.40$ — 1 Mark</p> | |
| <p>SET-A Q.4.a)</p> | <p>Let X be a random variable which follows binomial distribution with $n = 500$ and $p = 0.25$. Then find the following</p> <p>i). $P(X > 290)$ ii) $P(X = 250)$ iii). $P(120 < X < 180)$.</p> | <p>4 M</p> |
| <p>Solution</p> | <p>As $n.p = 500 \times 0.25 = 125 > 15$ and $n.q = 500 \times 0.75 = 375 > 15$, hence we can use Binomial approximation to Normal distribution</p> <p>$\mu = np = 125$ and $\sigma = \sqrt{npq} = 9.6825$</p> <p>Let $z = \frac{x - \mu}{\sigma} = \frac{x - 125}{9.6825}$ (1mark)</p> <p>i). $P(X > 290) = P(X > 289.5) = P(Z > 16.989) = 0$ (1mark)</p> <p>ii) $P(X = 250) = P(249.5 < X < 250.5) = P(12.86 < Z < 12.96) = 0$ (1mark)</p> <p>iii). $P(120 < X < 180) = P(119.5 < X < 180.5) = P(-0.57 < Z < 5.73) = 0.5 + 0.2157 = 0.7157$ (1mark)</p> | |
| <p>Q.4.b)</p> | <p>The rain fall (in cms) in a Country during every July month is normally distributed with mean and standard deviation of rainfall are respectively as 12cms and 1.25cms. For the July month of 2022, calculate the probabilities of having rainfall</p> <p>i) more than 15 cms, ii). in between 13cms and 18cms, iii) of 12 cms</p> | <p>4 M</p> |
| <p>Solution</p> | <p>$\mu = 12$ and $\sigma = 1.25$</p> <p>Let $z = \frac{x - \mu}{\sigma} = \frac{x - 12}{1.25}$ (1mark)</p> <p>(i) $P(x > 15) = P(z > 2.4) = 0.0082$ (1mark)</p> <p>(ii) $P(13 < x < 18) = P(0.8 < z < 4.8) = 0.5 - 0.2881 = 0.2119$ (1mark)</p> <p>$P(x = 12) = 0$ (or) can't find by normal distribution. (1mark)</p> | |