40. Count Inversions in an Array | Brute and Optimal

Problem Statement Suggest Edit

There is an integer array 'A' of size 'N'.
Number of inversions in an array can be defined as the number of pairs of 'i', 'j' such that 'i' < 'j' and 'A[i]' > 'A[j]'.
You must return the number of inversions in the array.
For example,

```
Input:
A = [5, 3, 2, 1, 4], N = 5
Output:
7
Explanation:
The pairs satisfying the condition for inversion are (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), and (3, 4).
The number of inversions in the array is 7.
```

Count Inversion

arr[]: [5 3 2 4 1]

$i < j$ && $a[i] > a[j]$

no of pairs should be left element > right element

(5, 3) ⇒ 5 > 3

(5, 2) ⇒ 5 > 2

(5, 1) ⇒ 5 > 1

(4, 2) X

because $i < j$ X

left element > right element

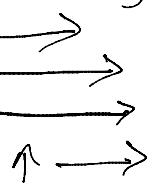
(3,2) (2,1)

(3,1) (4,1)

count = 8 ⟹ return this.

1) Brute Force:

arr[] = { 5 3 2 4 1 }



```
cnt = 0
for ( i = 0 ⟶ n-1)
{   for ( j = i+1 ⟶ j++)
    {
        if ( a[i] > a[j])
            cnt++;
    }
}
return cnt;
```

TC ⇒ ≃ O(n^2) SC ⇒ O(1)

2) Better Approach $n^2 \longrightarrow n \log n$ or n

Suppose there are two sorted array:

[2, 3, 5, 6] [2, 2, 4, 4, 8]

Now to find how many no of pair you can form where left element is from left array & right element is from right array and it's greater.

e.g.

Take two pointers.

[2, 3, 5, 6] [2, 2, 4, 4, 8]

here 1st 2 == 2, so move

3 > 2 ⇒ pair

left > right

we need to find this

so that means every element right of '3' will make pair with '2' as array is sorted.

[5 > 2], [6, 2] cnt += 3

So, now move the right array to 2

again all of the three element will make pair so again cnt += 3 ⇒ 3 + 3 = 6

So, now move the right array to 4

(3 < 4) X so move left array

(5 > 4), so all the element from 'T' 's' will make pair with the '4'

(5, 4) (6, 4) cnt = 6 + 2 = 8

Now move the right array again it's '4'.

(5 > 4) so cnt += 2 ⇒ 8 + 2 = 10

move the left array,

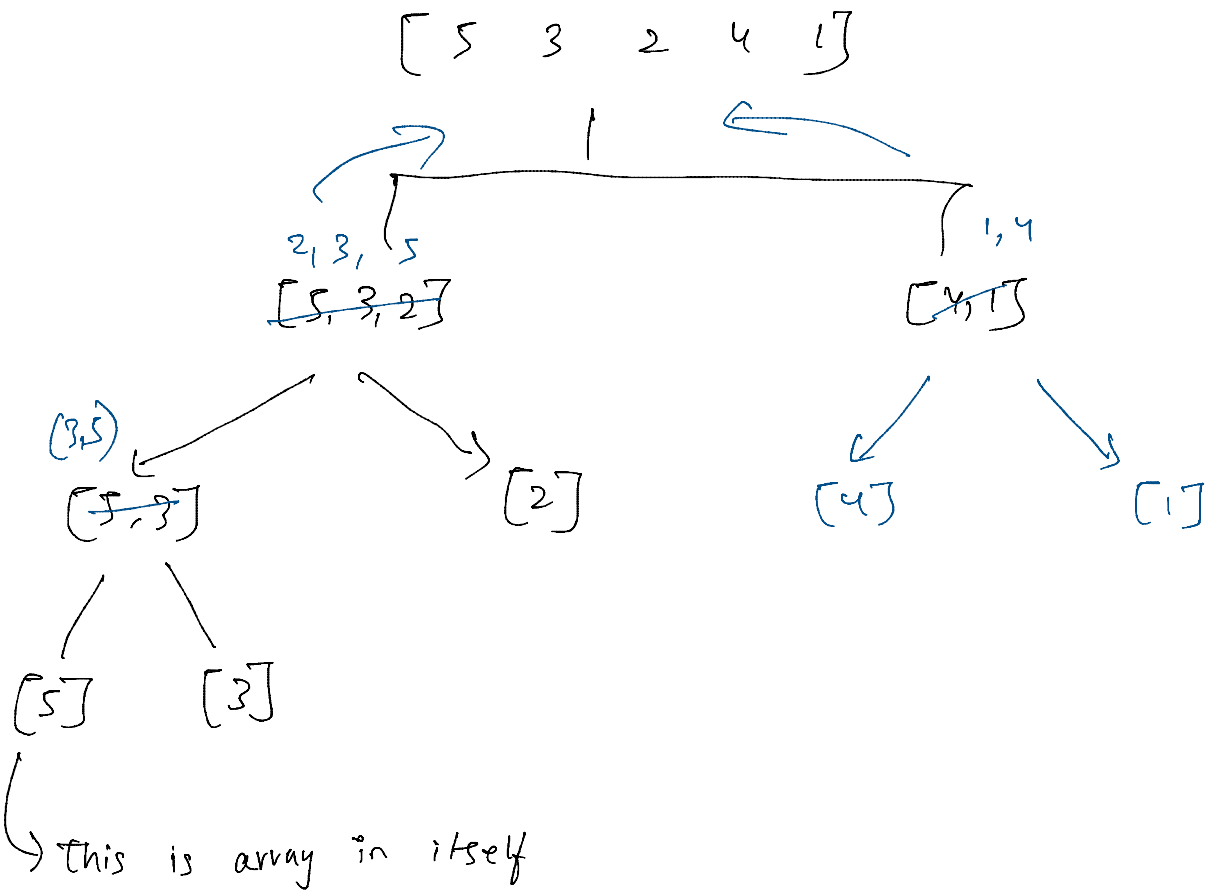
(6 > 8) X so array got exhausted.

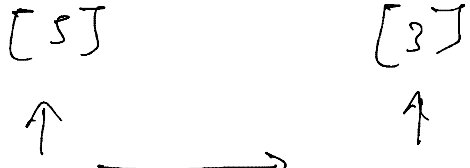
2 2 2 3 4 4 5 6 8

So now if we map the array into some sorted

So now if we map the array into some sorted form and get the left section & right section sorted, so here we think of merge sort algorithm

Approach + Dry Run





everyone right of 5 can form a pair.

here only 3 is there, so cnt +1.

[3, 5] [2]

3 can pair with 2.

So that mean all the element will form pair now +2,

+1

[2, 3, 5] [1, 4]

```cpp
int cnt = 0;

void merge(vector<int> &arr, int low, int mid, int high) {
    vector<int> temp; // temporary array
    int left = low;      // starting index of left half of arr
    int right = mid + 1;   // starting index of right half of arr

    //storing elements in the temporary array in a sorted manner//

    while (left <= mid && right <= high) {
        if (arr[left] <= arr[right]) {
            temp.push_back(arr[left]);
            left++;
        }
        else {
            temp.push_back(arr[right]);
            cnt += (mid - left + 1);
            right++;
        }
    }

    // if elements on the left half are still left //

    while (left <= mid) {
        temp.push_back(arr[left]);
        left++;
    }

    //  if elements on the right half are still left //
    while (right <= high) {
        temp.push_back(arr[right]);
        right++;
    }

    // transfering all elements from temporary to arr //
    for (int i = low; i <= high; i++) {
        arr[i] = temp[i - low];
    }
}
```

```cpp
void mergeSort(vector<int> &arr, int low, int high) {
    if (low >= high) return;
    int mid = (low + high) / 2 ;
    mergeSort(arr, low, mid);  // left half
    mergeSort(arr, mid + 1, high); // right half
    merge(arr, low, mid, high);  // merging sorted halves
}

int numberOfInversions(vector<int>&a, int n) {
 mergeSort(a, 0, n-1);
 return cnt;
}
```

T.C ⇒ $O(n \log n)$

S.C ⇒ $O(n)$