

Fuzzy Systems

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Logic [RUS95]

Jenis <i>logic</i>	Apa yang ada di dunia nyata	Apa yang dipercaya <i>Agent</i> tentang fakta
<i>Propositional logic</i>	fakta	benar/salah /tidak diketahui
<i>First-order logic</i>	fakta, objek, relasi	benar/salah /tidak diketahui
<i>Temporal logic</i>	fakta, objek, relasi, waktu	benar/salah /tidak diketahui
<i>Probability theory</i>	fakta	derajat kepercayaan [0,1]
<i>Fuzzy logic</i>	derajat kebenaran	derajat kepercayaan [0,1]

Peringatan:

1 dari 50 botol ini berisi cairan kimia mematikan yang warna dan rasanya sama dengan air mineral. Anda akan mati seketika jika meminumnya.

Probability

1

Peringatan:

Satu plastik cairan kimia mematikan dicampurkan ke dalam 50 botol ini secara tidak merata. Anda tidak akan mati jika cuma minum satu botol, tetapi anda akan menderita pusing ringan/berat.

Fuzziness

2

Fuzziness & Probability

- Banyak peneliti berbeda pendapat tentang **teori fuzzy** dan **teori probabilitas**
- Sebenarnya, kedua teori tersebut memang sama-sama untuk menangani masalah ketidakpastian
- Tetapi, perbedaannya adalah pada **jenis ketidakpastian** yang ditangani

Fuzzy Systems

- Ide dasar *fuzzy systems* adalah *fuzzy logic* dan *fuzzy sets*
- *Fuzzy logic* sudah lama dipikirkan oleh para filsuf Yunani kuno
- Plato: filsuf pertama yang meletakkan fondasi *fuzzy logic*
- Plato: “Terdapat area ketiga selain Benar dan Salah”
- Rumi: “Jauh di balik benar dan salah, ada sebuah padang terbuka. Kutemui kau di sana.”
- *Fuzzy logic* menghilang selama 2 milenium
- Muncul kembali pada era 1960-an
- Konsep *fuzzy logic* yang sangat sistematis pertama kali diusulkan oleh Lotfi A. Zadeh, the University of California, Berkeley, Amerika Serikat

Classical Sets

- Teori himpunan klasik: suatu himpunan secara intuitif adalah setiap kumpulan elemen-elemen
- Himpunan klasik dikenal juga sebagai ***crisp set***
- *Crisp = clear and distinct* [Oxford 1995]
- *Crisp set* : himpunan yang membedakan anggota dan non anggotanya dengan batasan yang jelas

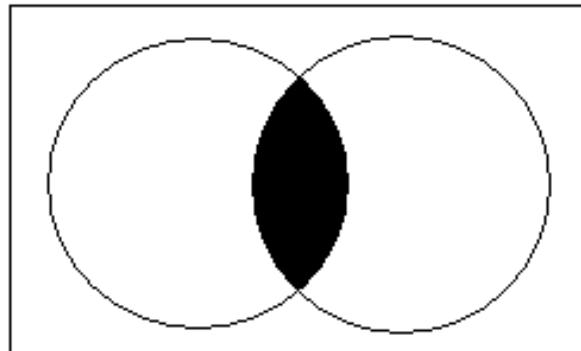
Contoh

$$A = \{x \mid x \text{ bilangan bulat}, x > 6\}$$

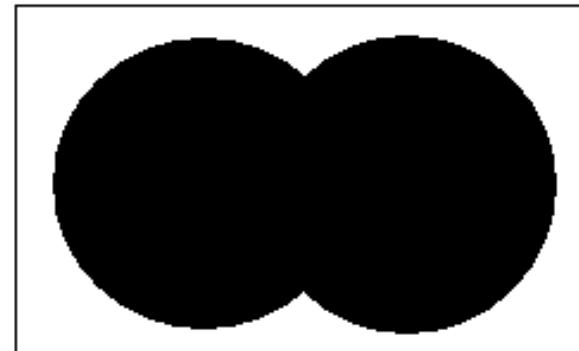
Anggota himpunan A adalah 7, 8, 9, ...

Bukan anggota A adalah 6, 5, 4, ...

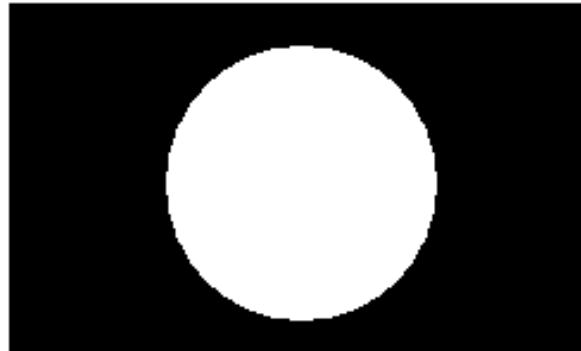
Intersection, union, complement, difference



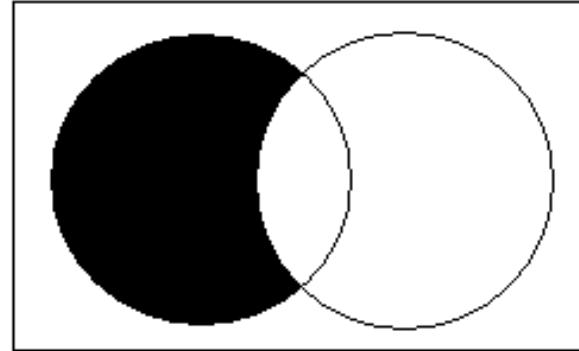
(a)



(b)



(c)



(d)

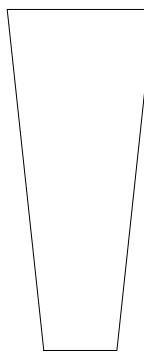
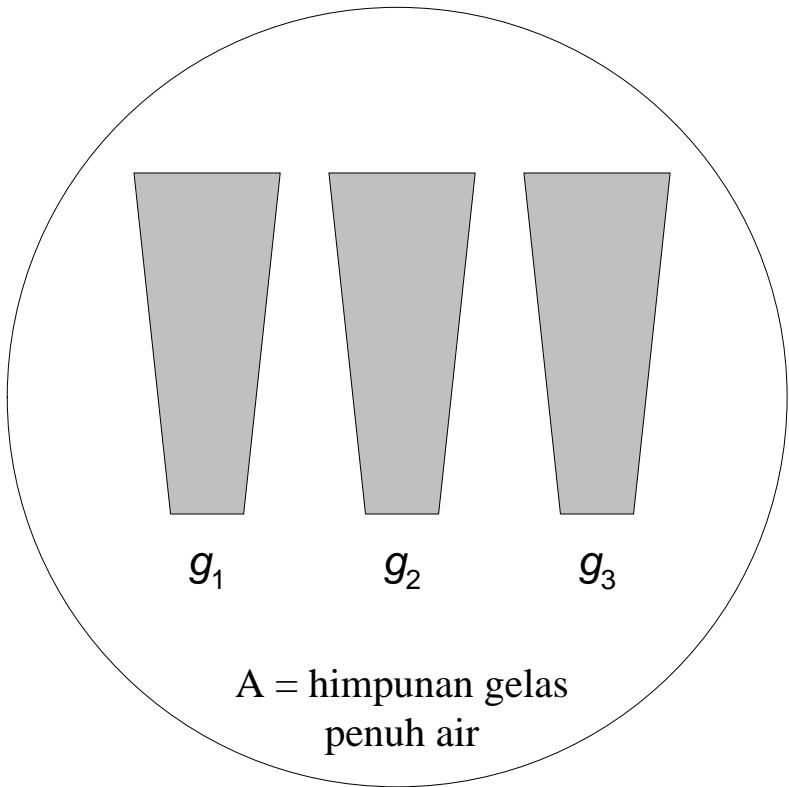
Excluded Middle Laws

$$A \cup \overline{A} = U$$

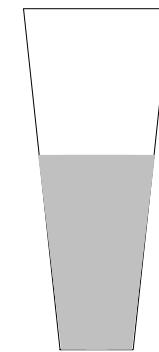
Law of Contradiction

$$A \cap \overline{A} = \emptyset$$

Semesta U



Gelas kosong
termasuk \bar{A}



= ?

Gelas yang berisi air
setengah bagian tidak
termasuk A maupun \bar{A}

Aku adalah pembohong.
Jangan percaya padaku.

Pernyataan P dan negasinya
sama-sama benar. $T(P) = T(\bar{P})$



Fungsi Karakteristik

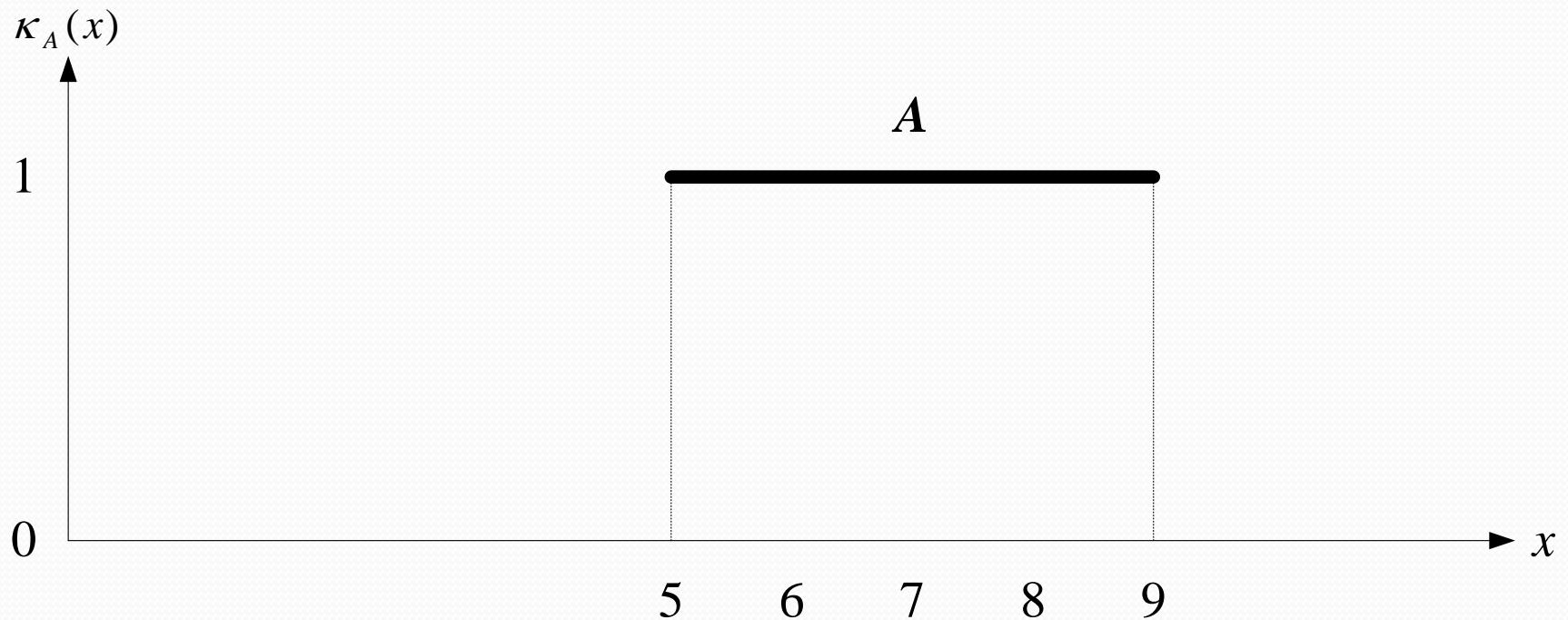
Fungsi karakteristik dari himpunan A adalah suatu pemetaan

$$\kappa_A : U \rightarrow \{0, 1\}$$

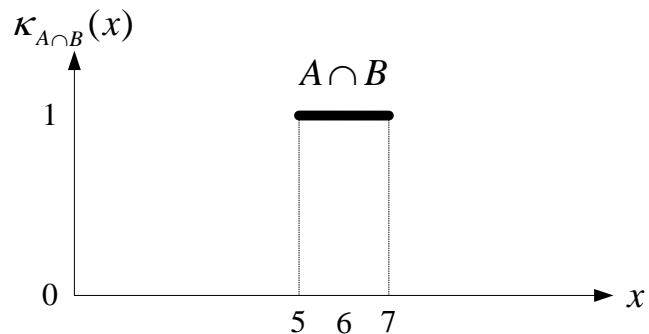
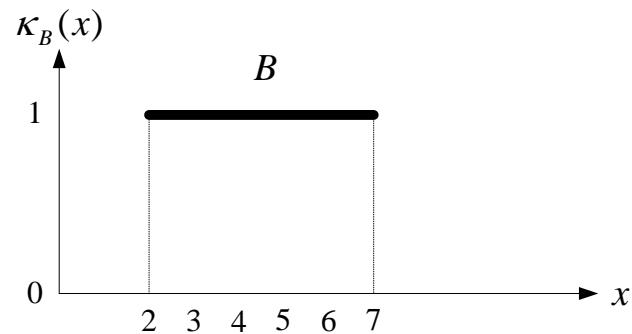
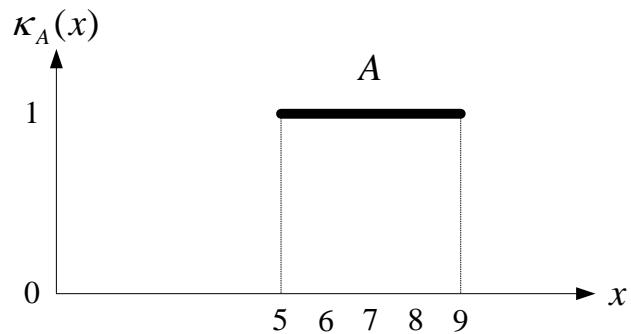
sedemikian hingga, untuk semua x ,

$$\kappa_A(x) = \begin{cases} 1, & \text{jika } x \in A; \\ 0 & \text{untuk kasus lainnya.} \end{cases}$$

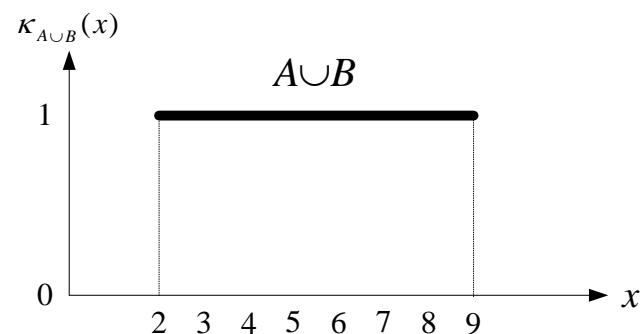
Fungsi Karakteristik *Classical Set*



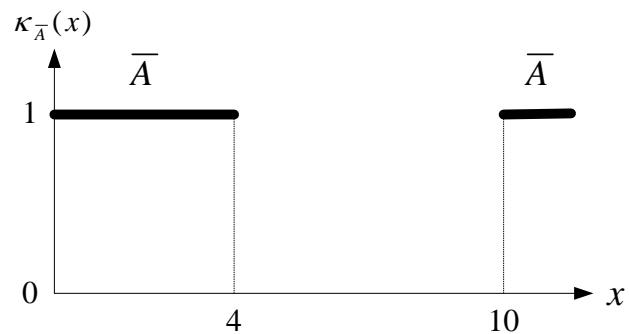
$A = \text{himpunan klasik semua bilangan bulat positif lebih dari } 4 \text{ dan kurang dari } 10 \text{ atau } \{5, 6, \dots, 9\}.$



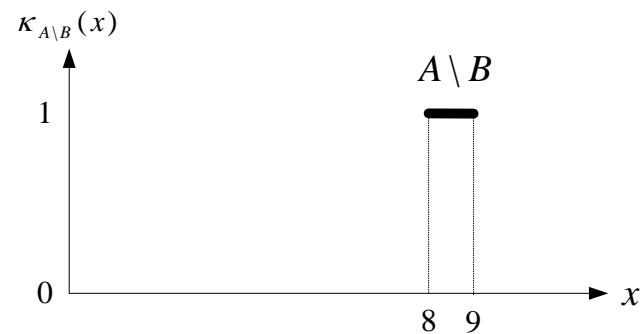
Intersection



Union



Complement



Difference

Kasus 1: Pemberian Beasiswa

Mahasiswa	IPK	Gaji Ortu (Rp/bulan)
A	3,00	10 juta
B	2,99	1 juta

if $IPK \geq 3,00$ and $Gaji \leq 10$ juta
then Dapat Beasiswa

Fuzzy Sets

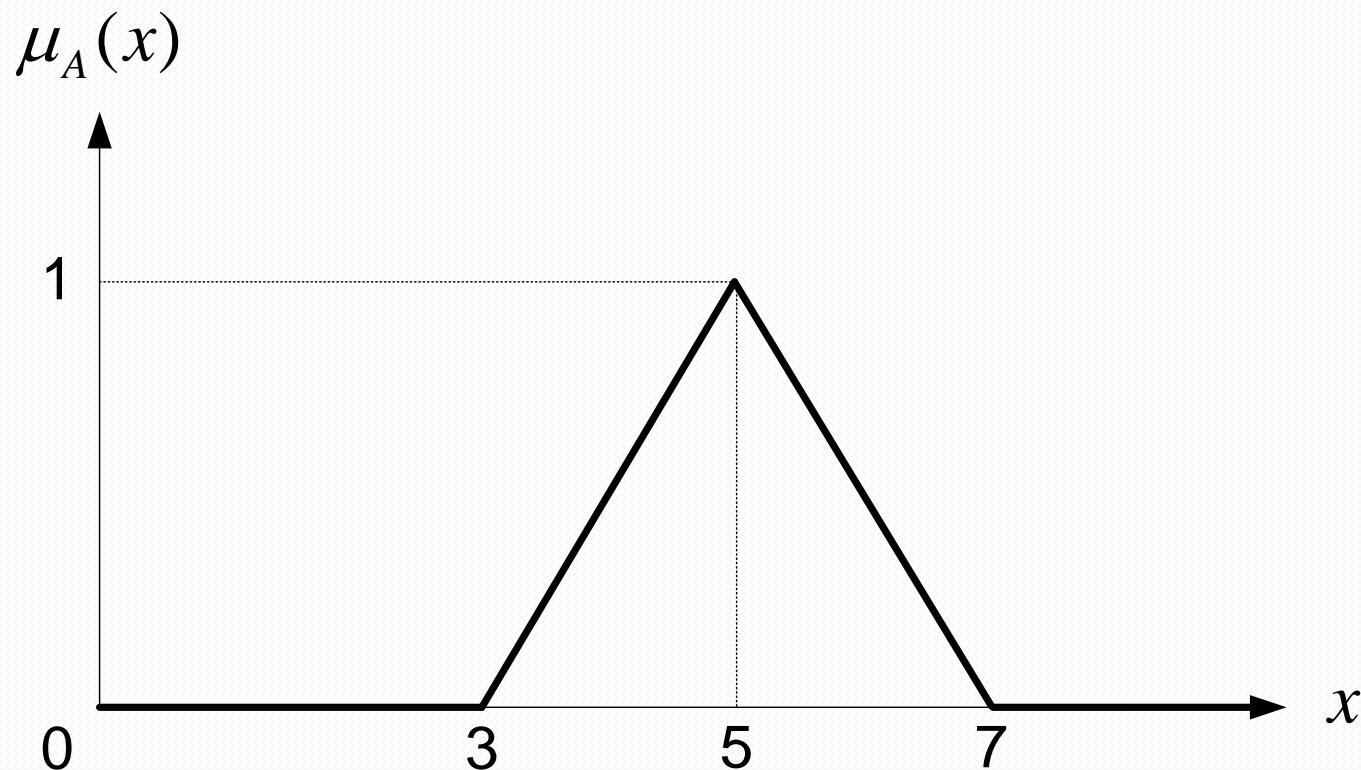
- Digunakan untuk penalaran yang lebih manusiawi.
- Misalkan U adalah *universe of discourse* (semesta pembicaraan) dan x adalah anggota U .
- Suatu *fuzzy set* A di dalam U didefinisikan sebagai suatu *membership function* atau fungsi keanggotaan, yang memetakan setiap objek di U menjadi suatu nilai real dalam interval $[0, 1]$.
- Nilai-nilai menyatakan derajat keanggotaan x di dalam A .

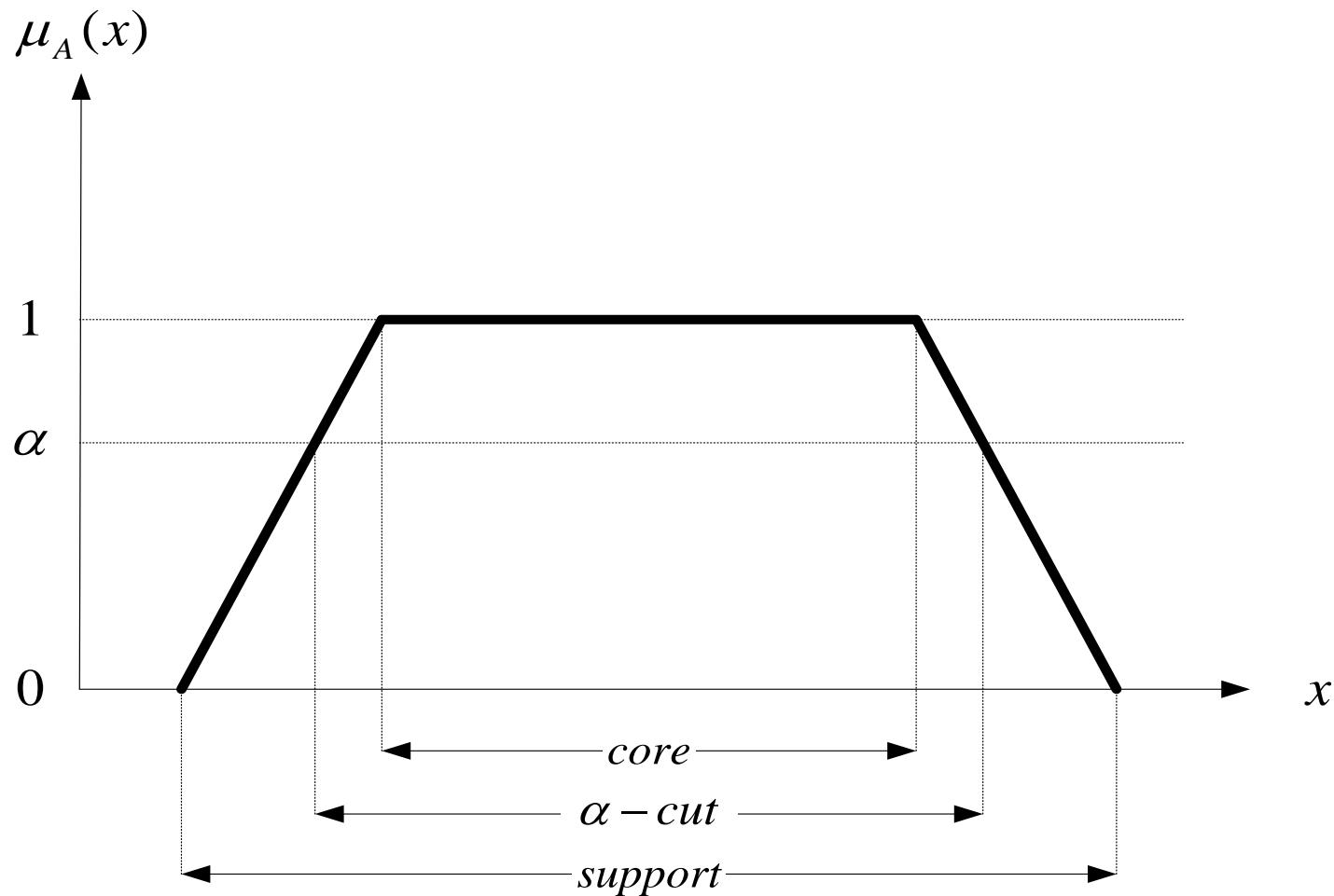
Suhu ($^{\circ}$ C)	Dingin	Hangat	Panas
5	1	0,1	0
15	0,9	0,8	0
25	0,5	1	0,6
35	0,1	0,6	0,9
45	0	0,2	1

Fuzzy Set

- Dingin = {5, 15, 25, 35} dan derajat keanggotaannya dinyatakan oleh $\mu_{Dingin} = \{1; 0,9; 0,5; 0,1\}$
- Hangat = {5, 15, 25, 35, 45} dan derajat keanggotaannya dinyatakan oleh $\mu_{Hangat} = \{0,1; 0,8; 1; 0,6; 0,2\}$
- Panas = {25, 35, 45} dan derajat keanggotaannya dinyatakan oleh $\mu_{Panas} = \{0,6; 0,9; 1\}$

Graphical Representation

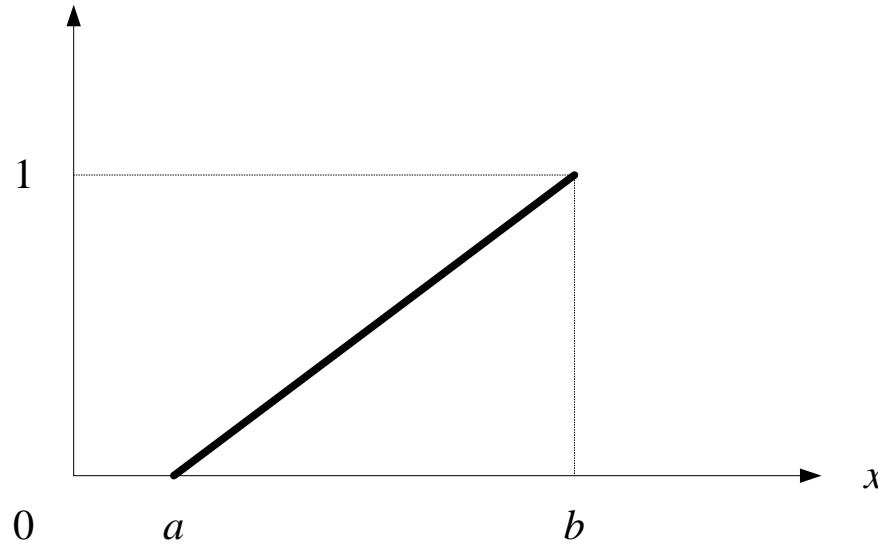




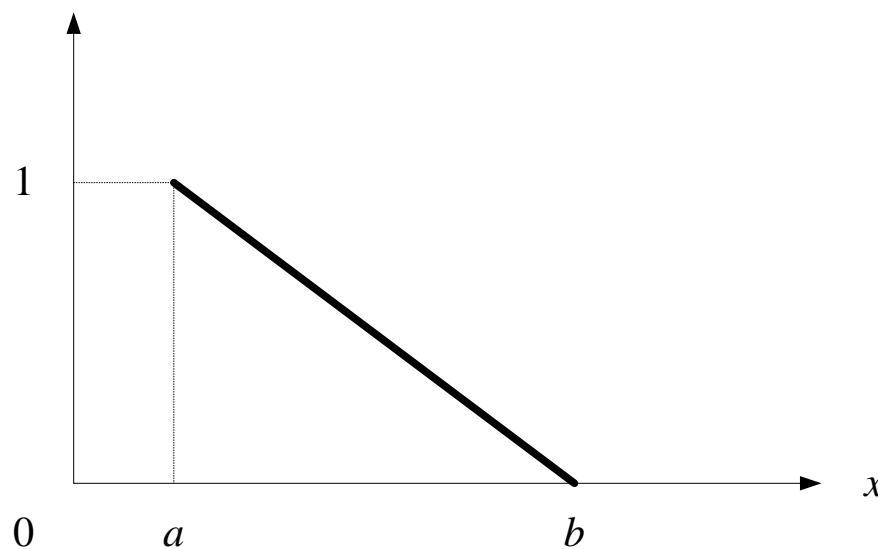
Fungsi keanggotaan μ_A dengan *core*, α - *cut* dan *support*

Bentuk Fungsi Keanggotaan

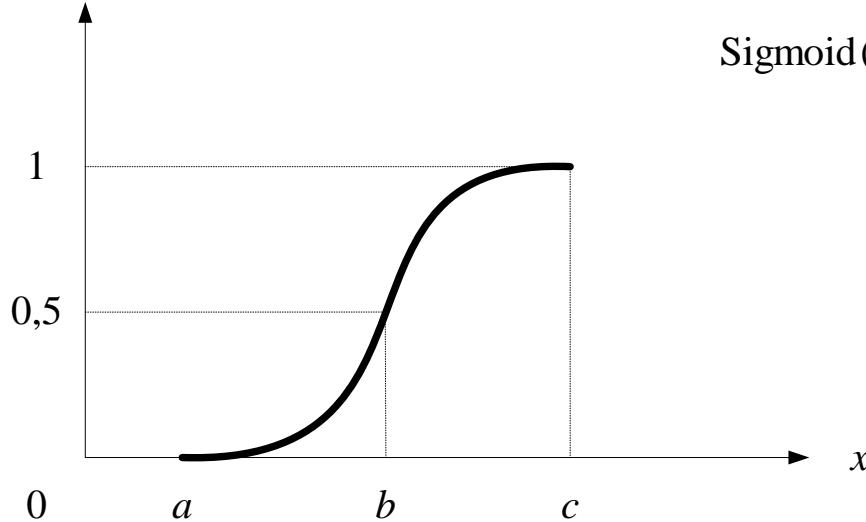
- Fungsi Linier
- Fungsi *Sigmoid*
- Fungsi Segitiga
- Fungsi Trapezium
- Fungsi Berbentuk *Bell*:
 - *Phi*
 - *Beta*
 - *Gauss*

$\mu(x)$ 

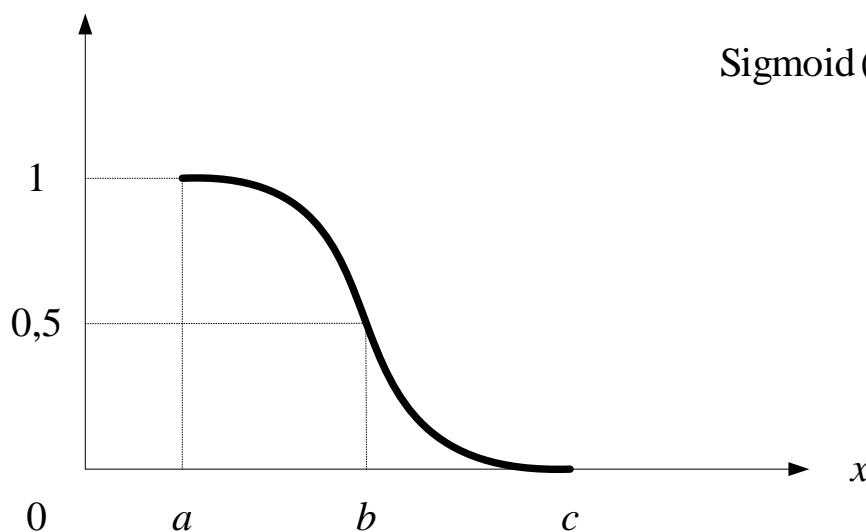
$$\text{LinierNaik}(x, a, b) = \begin{cases} 0, & x \leq a \\ (x - a)/(b - a), & a < x \leq b \end{cases}$$

 $\mu(x)$ 

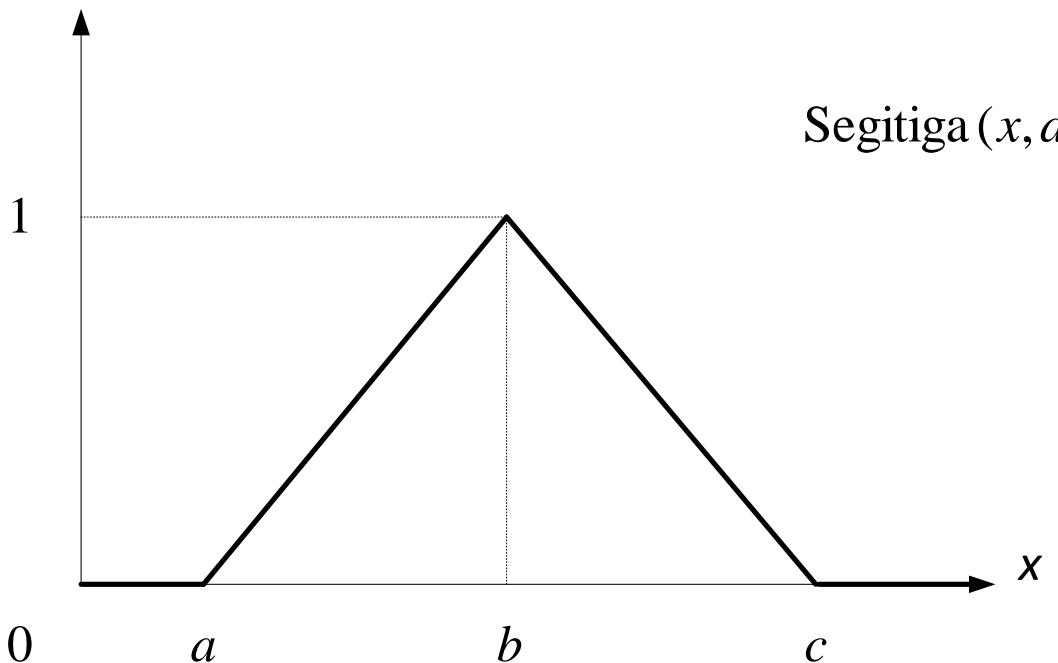
$$\text{LinierTurun}(x, a, b) = \begin{cases} (b - x)/(b - a), & a \leq x < b \\ 0, & x \geq b \end{cases}$$

$\mu(x)$ 

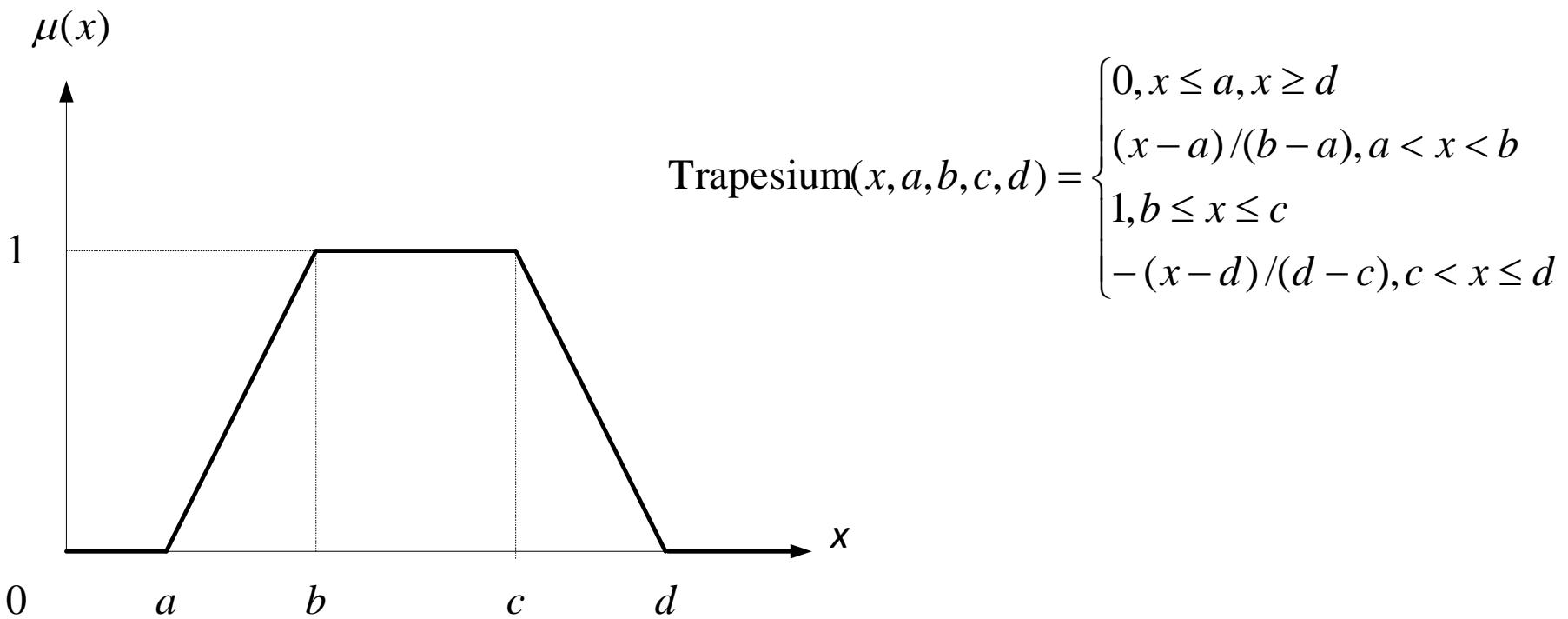
$$\text{Sigmoid}(x, a, b, c) = \begin{cases} 0, & x \leq a \\ 2((x-a)/(c-a))^2, & a < x \leq b \\ 1 - 2((c-x)/(c-a))^2, & b < x < c \\ 1, & c \leq x \end{cases}$$

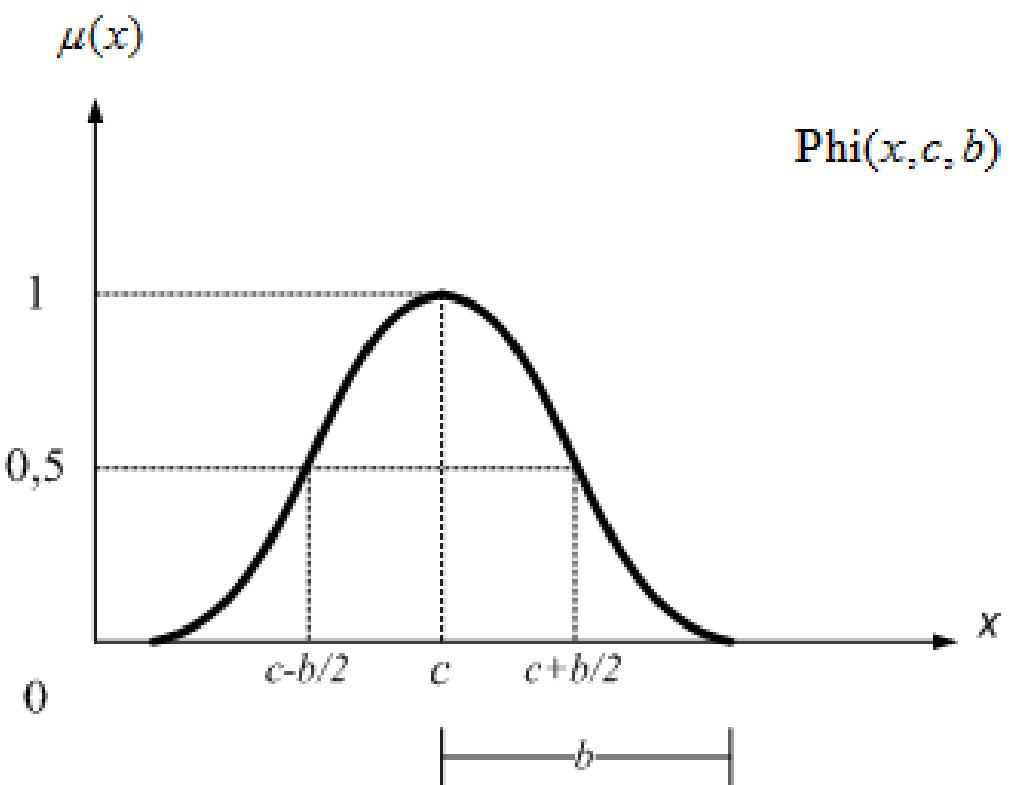
 $\mu(x)$ 

$$\text{Sigmoid}(x, a, b, c) = \begin{cases} 1, & x \leq a \\ 1 - 2((x-a)/(c-a))^2, & a < x \leq b \\ 2((c-x)/(c-a))^2, & b < x < c \\ 0, & x \geq c \end{cases}$$

$\mu(x)$ 

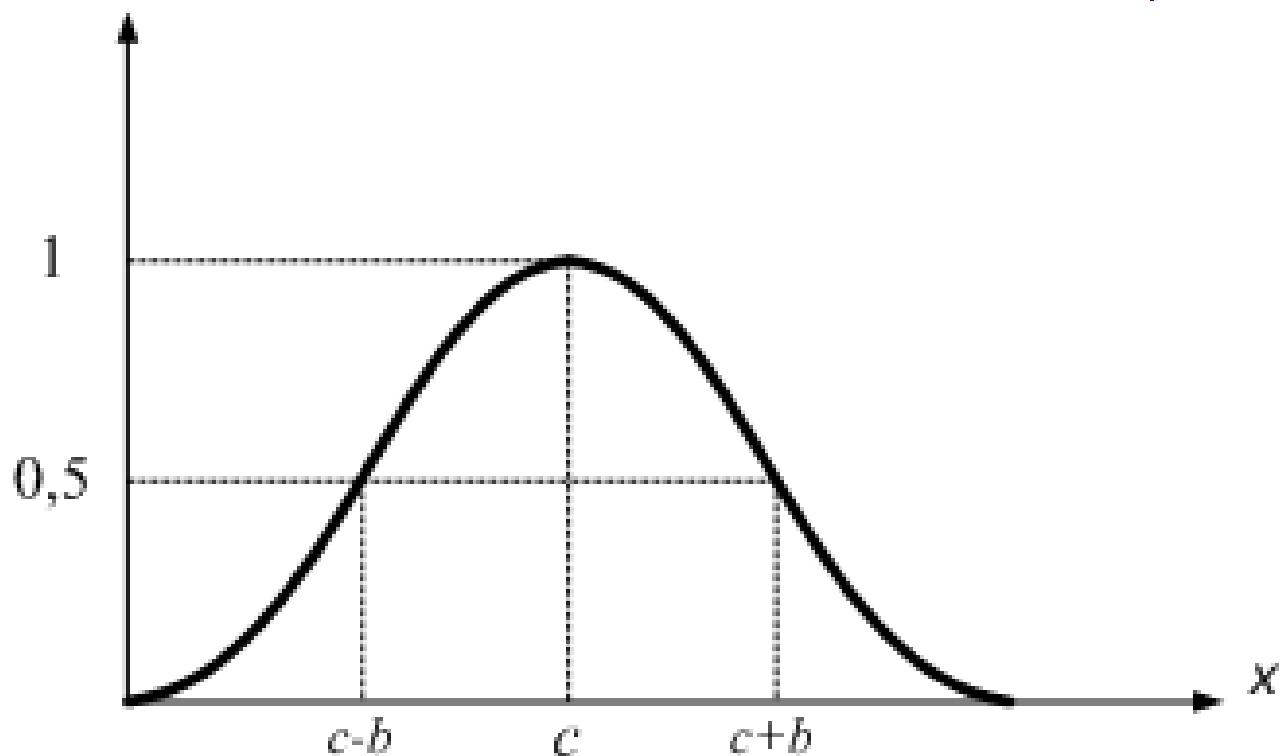
$$\text{Segitiga}(x, a, b, c) = \begin{cases} 0, & x \leq a, x \geq c \\ (x - a)/(b - a), & a < x \leq b \\ -(x - c)/(c - b), & b < x \leq c \end{cases}$$





$$\text{Phi}(x, c, b) = \begin{cases} \text{Sigmoid}\left(x, c-b, c-\frac{b}{2}, c\right), & x \leq c \\ 1 - \text{Sigmoid}\left(x, c, c+\frac{b}{2}, c+b\right), & x > c \end{cases}$$

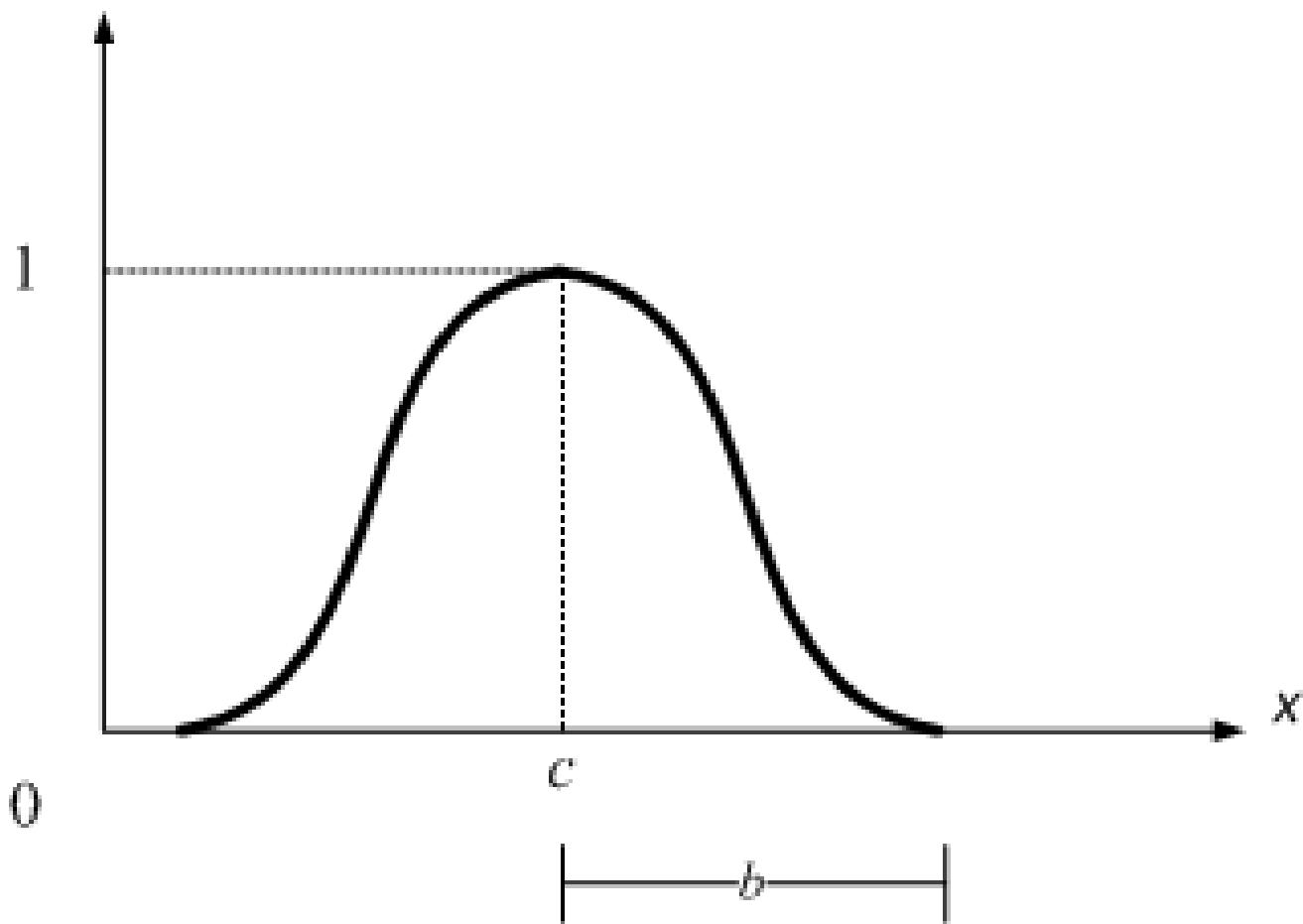
$\mu(x)$

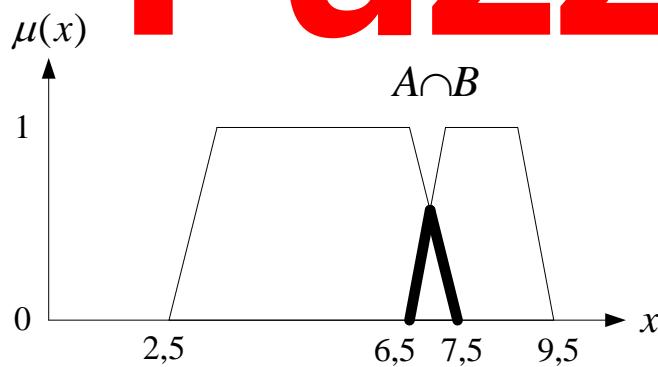
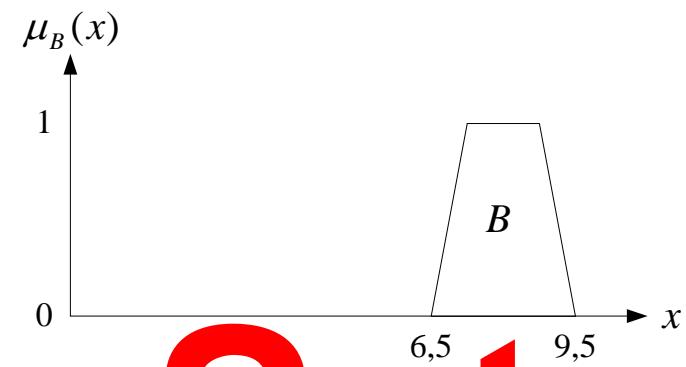
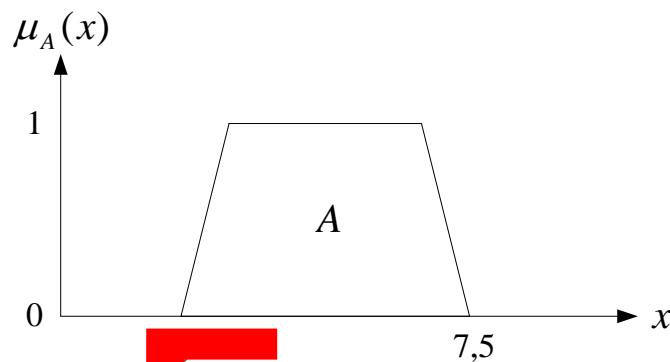


$$\text{Beta}(x, c, b) = \frac{1}{\left(1 + \left|\frac{x-c}{b}\right|^2\right)}$$

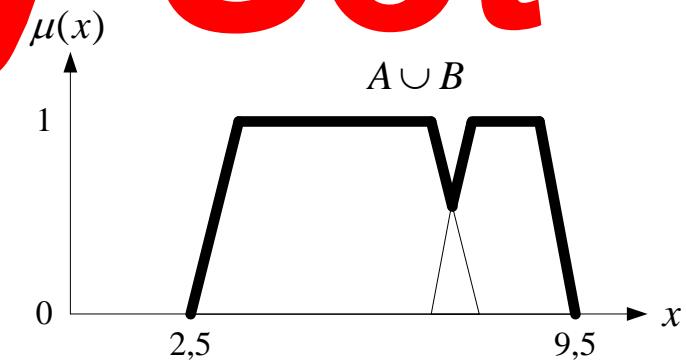
$\mu(x)$

$$\text{Gauss}(x, c, b) = e^{-b(c-x)^2}$$

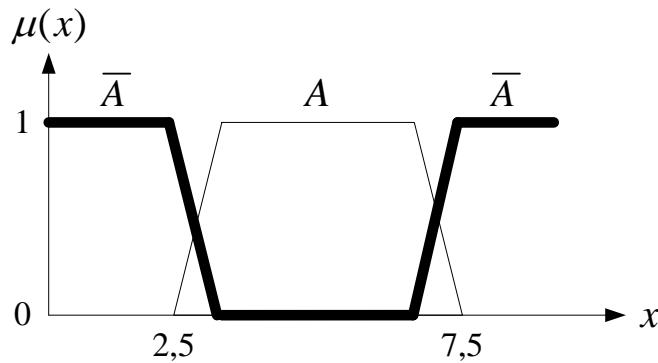




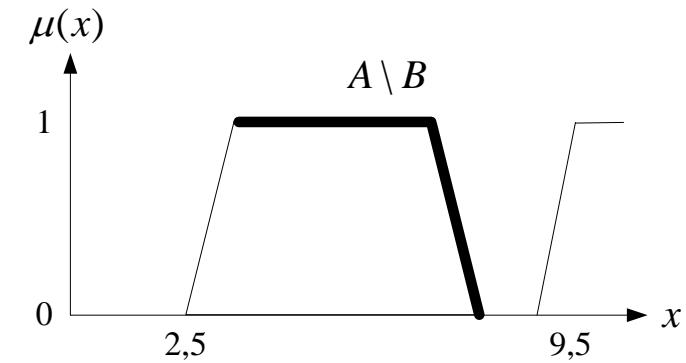
Intersection



Union



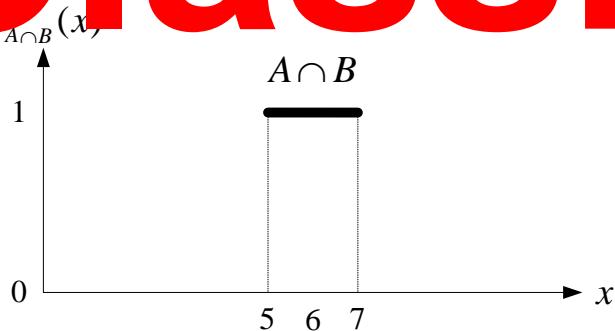
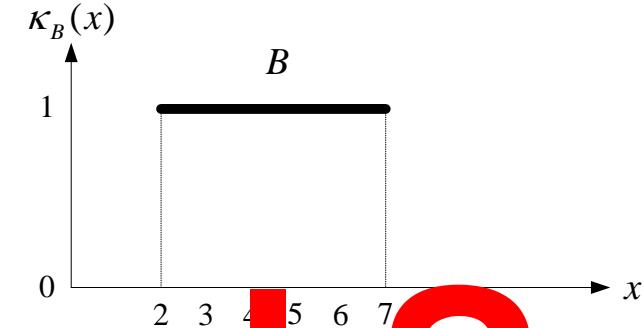
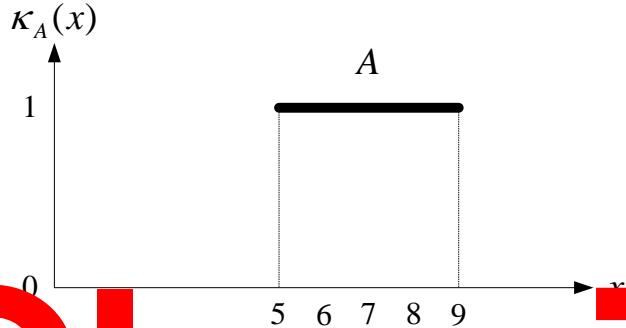
Complement



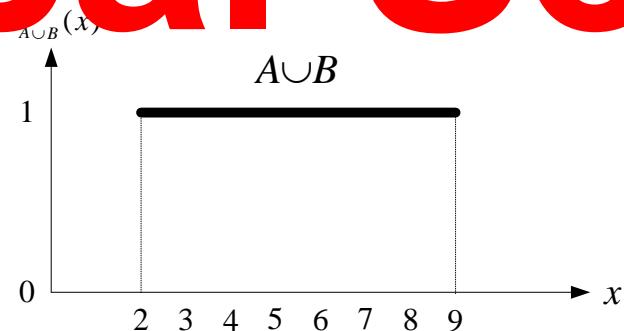
Difference

Fuzzy Set

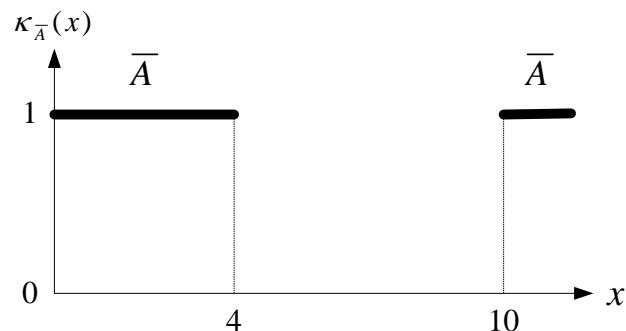
Classical Set



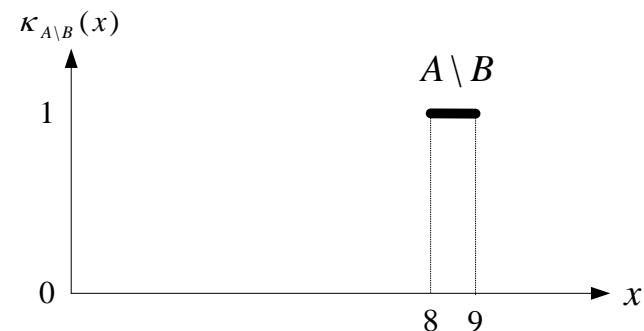
Intersection



Union



Complement



Difference

Logical connectives & Implication

- Dalam bahasa manusia, banyak percakapan yang menggunakan kalimat yang tidak pasti kebenarannya
 - '**Hampir semua** orang suka permen'
 - '**Sepertinya** dia anak yang pintar'
- Misalkan P adalah suatu *fuzzy logic proposition*
- Nilai kebenaran P adalah $[0, 1]$.
 - Nilai 0 menyatakan bahwa P adalah salah
 - Nilai 1 menyatakan bahwa P adalah benar

Logical connectives & Implication

$$T : P \rightarrow [0, 1]$$

T adalah fungsi kebenaran yang memetakan P ke suatu nilai dalam interval $[0, 1]$.

Logical Connectives

- *Negation* $T(\neg P) = 1 - T(P)$
- *Disjunction* $T(P \vee Q) = \max\{T(P), T(Q)\}$
- *Conjunction* $T(P \wedge Q) = \min\{T(P), T(Q)\}$

Approximate Reasoning

A : 'Apakah dia anak yang **pintar**?'

B : '**Sepertinya begitu.**'

A : 'Apakah Indeks Prestasi dan hasil tes psikologinya **bagus**?'

B : '**Ya, keduanya sangat bagus.**'

A : 'Apakah dia layak mendapatkan beasiswa?'

B : '**Ya, sepertinya itu adalah keputusan yang baik.**'

Approximate Reasoning

P_1 : Sebagian besar mahasiswa suka membaca

P_2 : Dani adalah mahasiswa

P_3 : Sepertinya Dani suka membaca

Reasoning yang Pasti

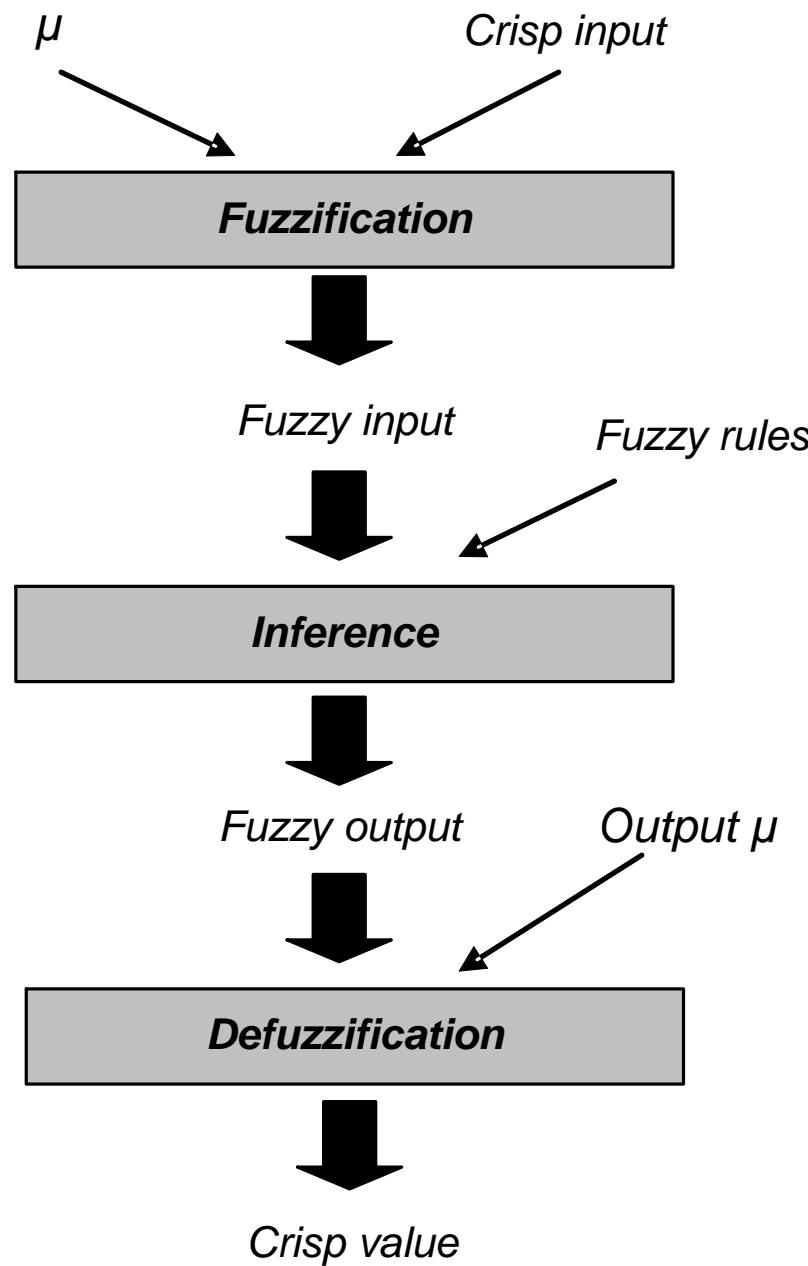
P_1 : Semua manusia pasti akan mati

P_2 : Dani adalah manusia

P_3 : Dani pasti akan mati

Fuzzy Inference Systems

- Variabel linguistik adalah suatu interval numerik dan mempunyai nilai-nilai linguistik, yang semantiknya didefinisikan oleh fungsi keanggotaannya.
- Misalnya, *Suhu* adalah suatu variabel linguistik yang bisa didefinisikan pada interval $[-10^{\circ}\text{C}, 40^{\circ}\text{C}]$.
- Variabel tersebut bisa memiliki nilai-nilai linguistik seperti 'Dingin', 'Hangat', 'Panas' yang semantiknya didefinisikan oleh fungsi-fungsi keanggotaan tertentu.



Model Inferensi

- Mamdani → *Intuitive*
- Sugeno → *Control*

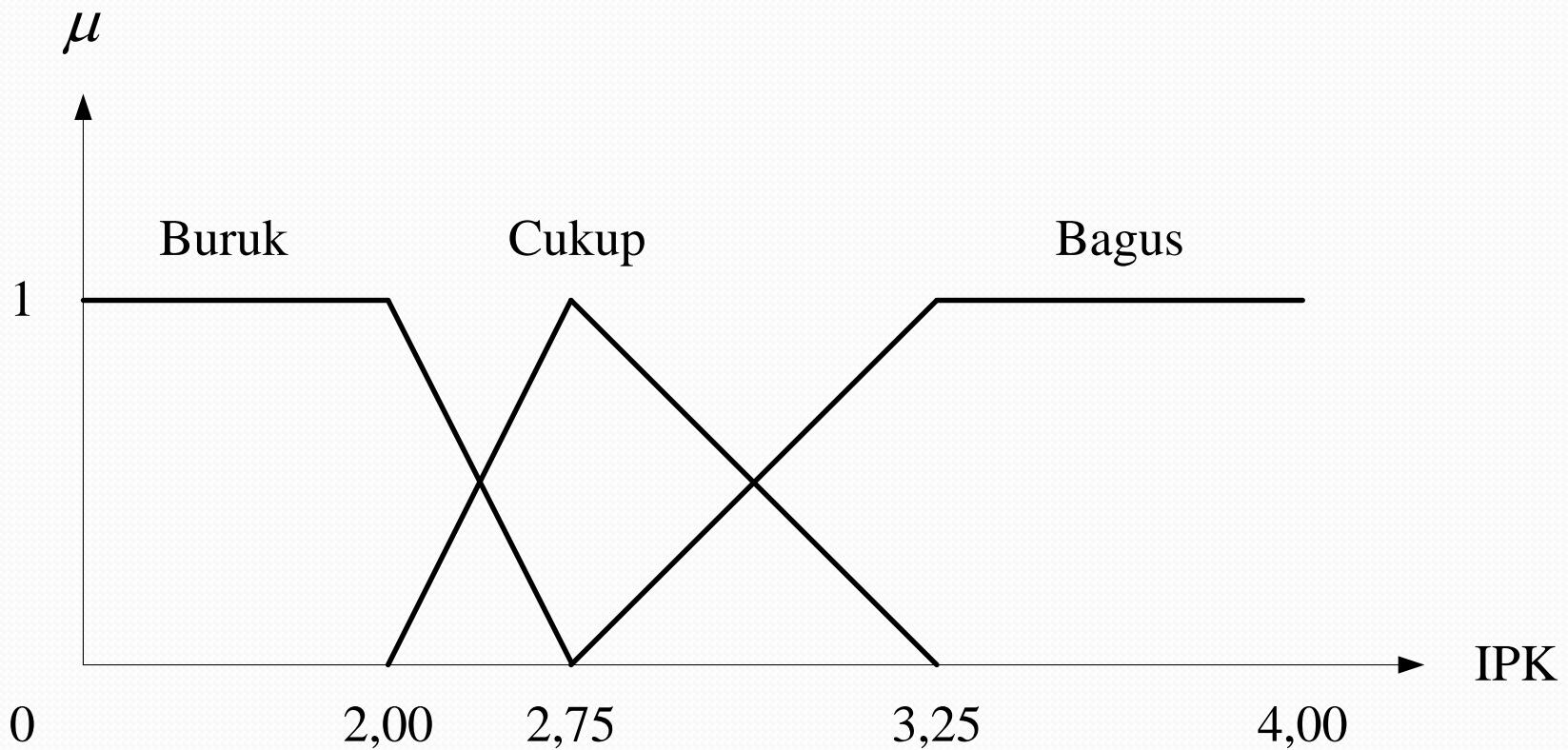
Kasus 1: Pemberian Beasiswa

Mahasiswa	IPK	Gaji Ortu (Rp/bulan)
A	3,00	10 juta
B	2,99	1 juta

Model Mamdani

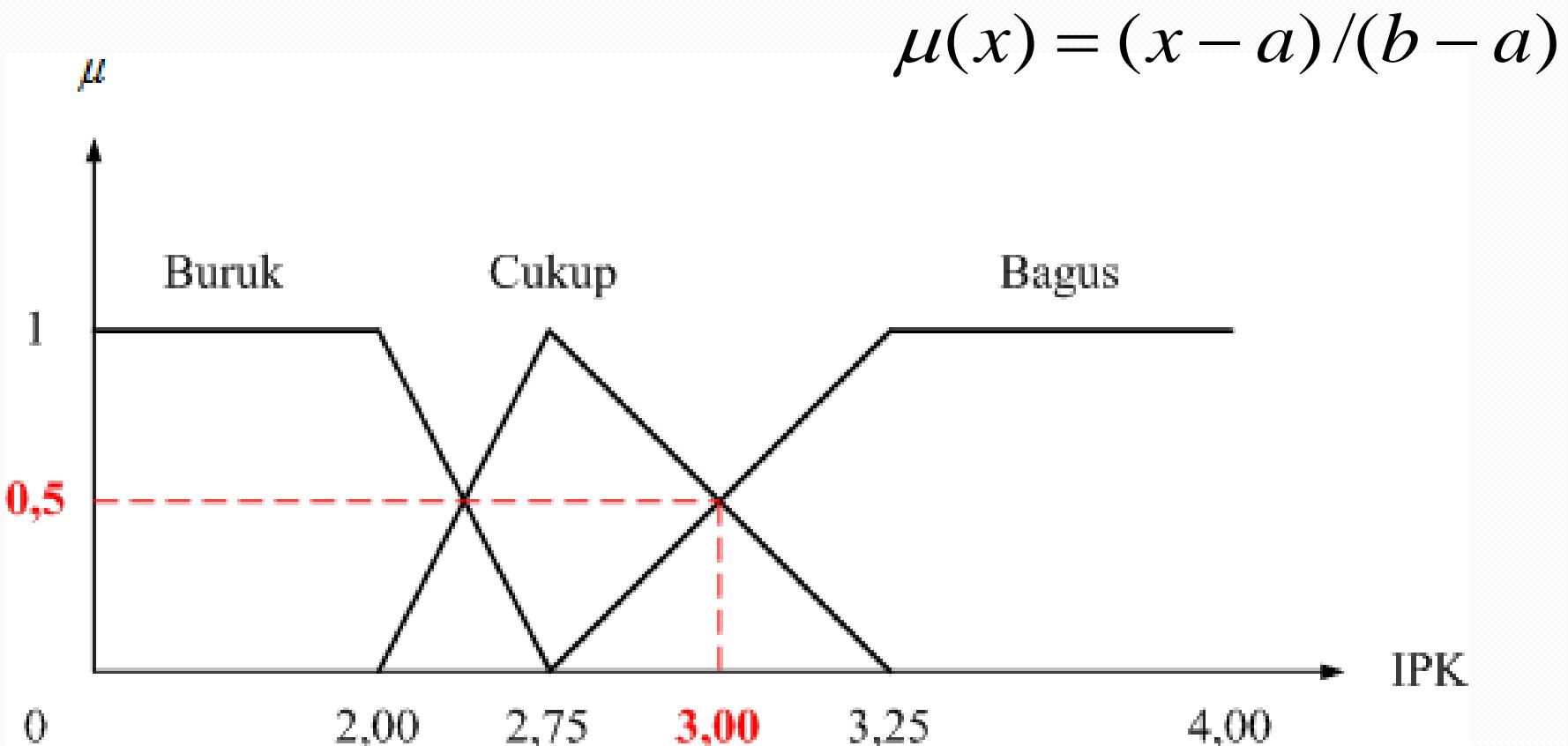
- Untuk membangun sistem yang penalarannya menyerupai perasaan manusia
- Perhitungannya kompleks sehingga membutuhkan waktu relatif lama
- Ketelitiannya tinggi

FK untuk IPK

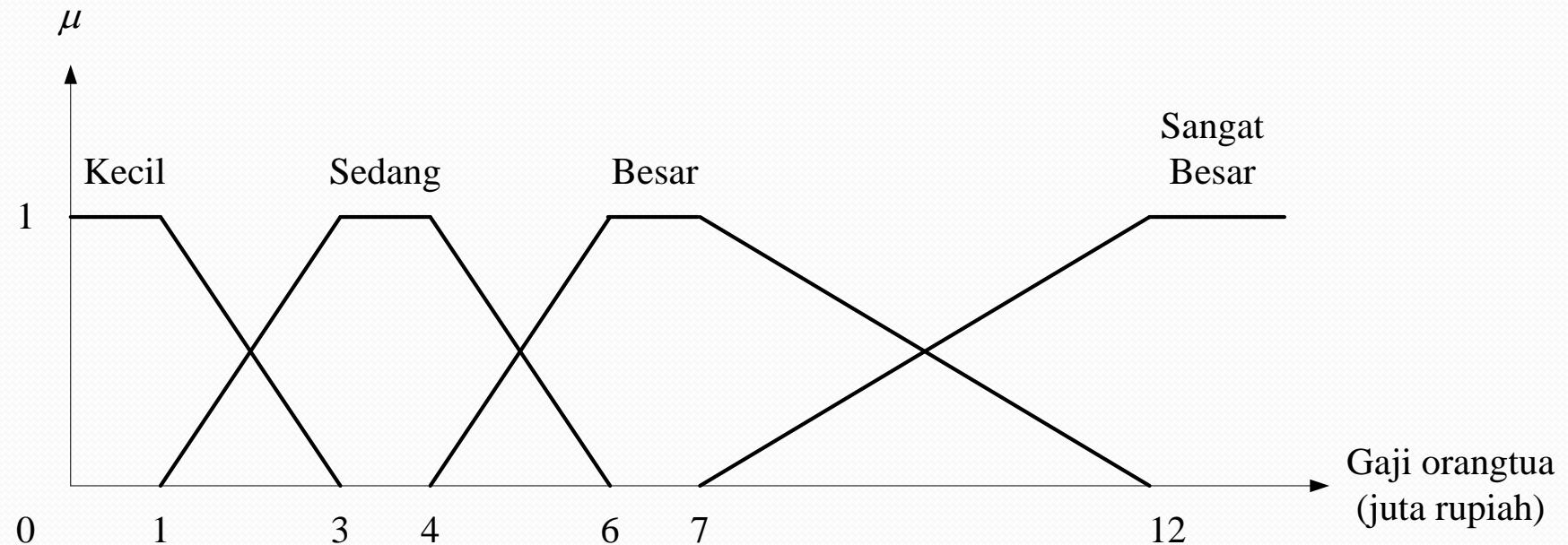


IPK mahasiswa A

$$\mu(x) = -(x - c)/(c - b)$$



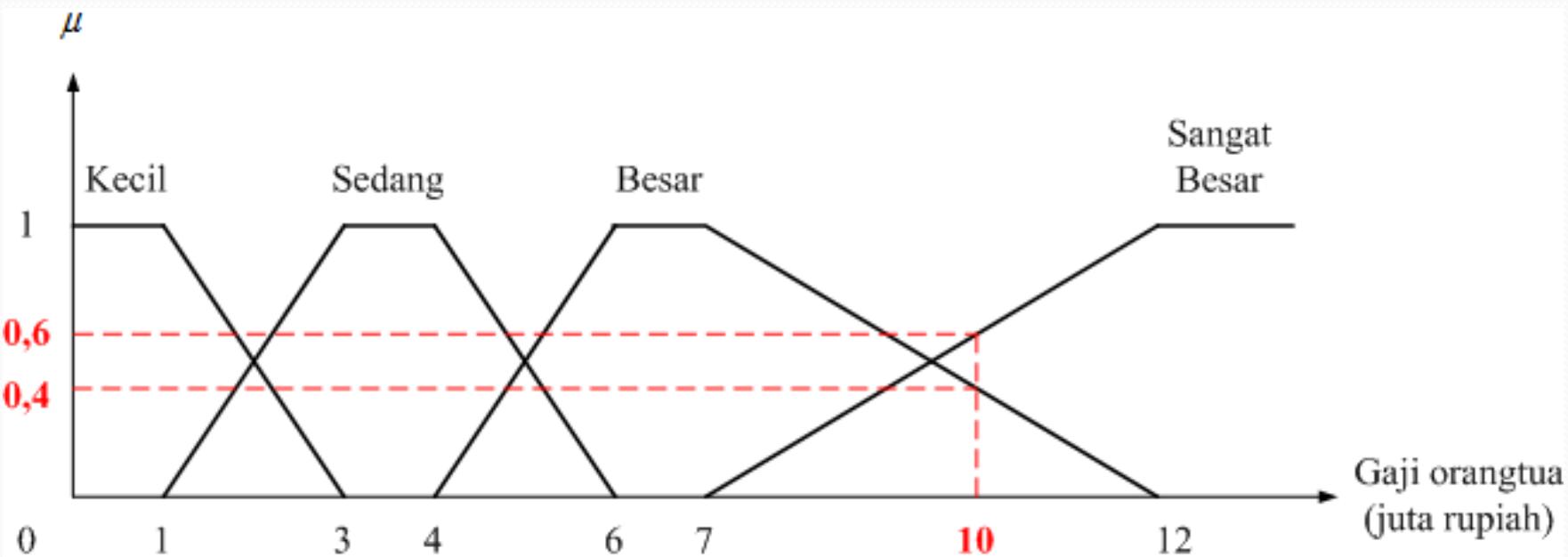
FK Gaji Orangtua



Gaji Ortu mhs A

$$\mu(x) = -(x - d)/(d - c), c < x \leq d$$

$$\mu(x) = (x - a)/(b - a), a < x < b$$



Fuzzification untuk mhs A

IPK = 3,00

Gaji Orangtua = 10 juta/bulan



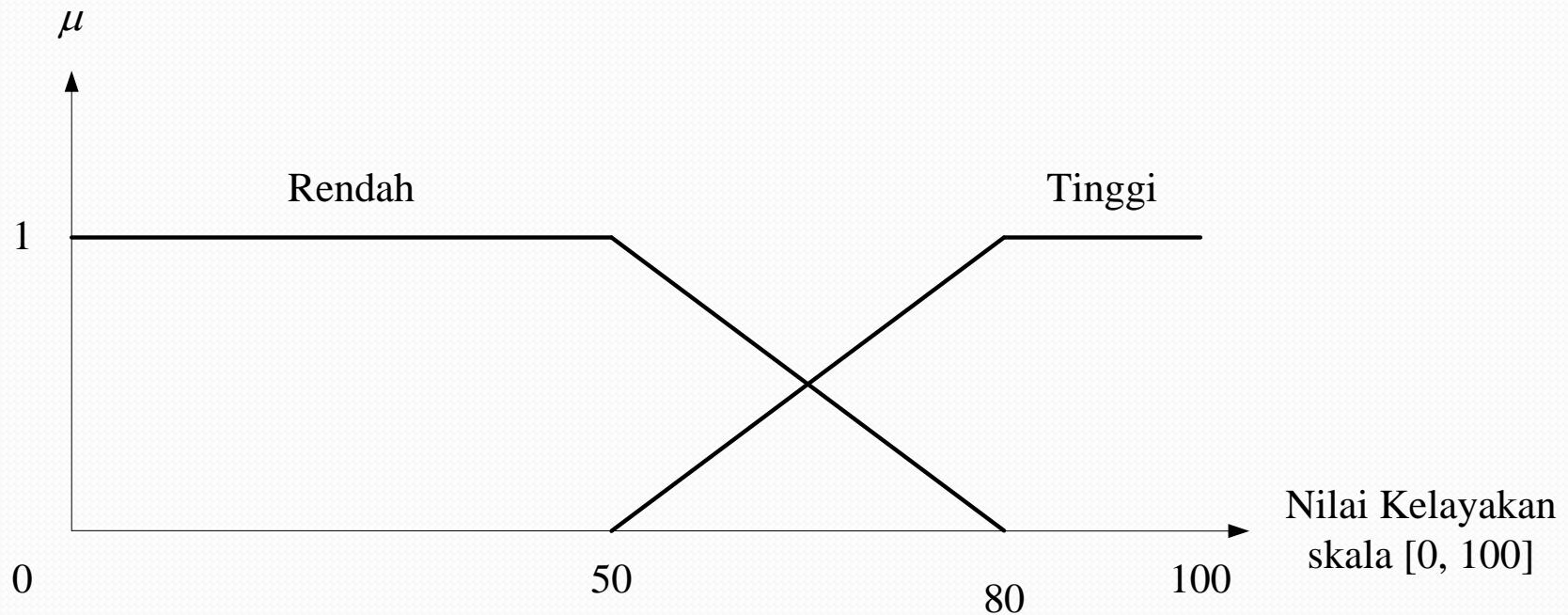
IPK = **Cukup** (0,5)

IPK = **Bagus** (0,5)

Gaji Orangtua = **Besar** (0,4)

Gaji Orangtua = **Sangat Besar** (0,6)

Fungsi Keanggotaan Nilai Kelayakan

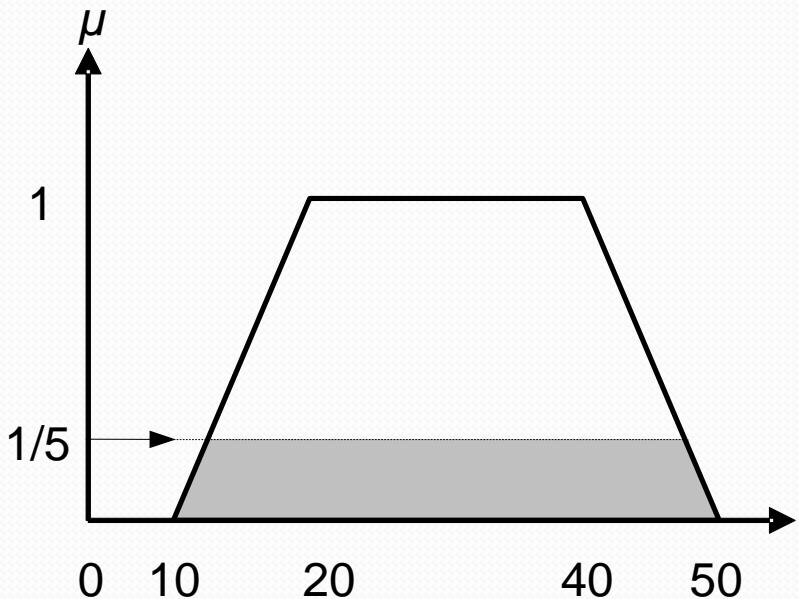


Aturan Fuzzy untuk Nilai Kelayakan

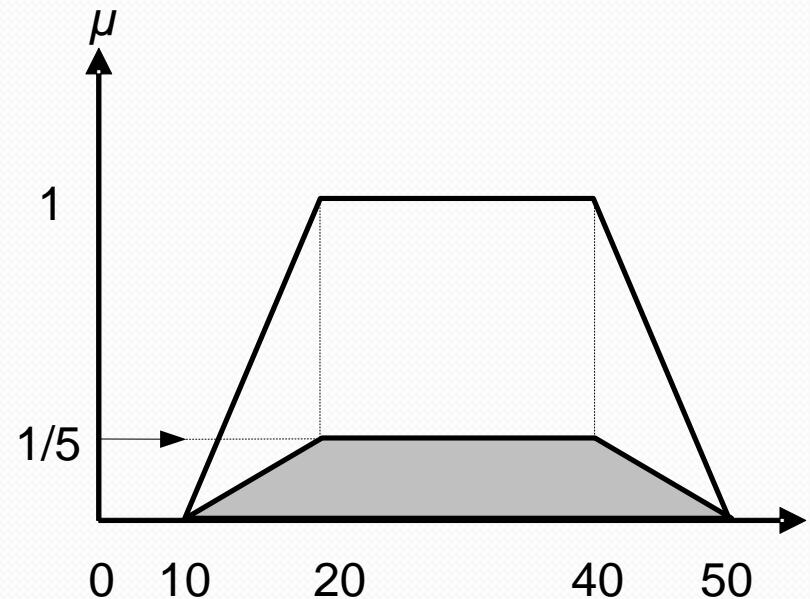
IPK \ Gaji	Kecil	Sedang	Besar	Sangat Besar
Buruk	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah

1. IF IPK = Buruk AND $Gaji$ = Kecil THEN NK = Rendah
2. IF IPK = Buruk AND $Gaji$ = Sedang THEN NK = Rendah
3. IF IPK = Buruk AND $Gaji$ = Besar THEN NK = Rendah
4. IF IPK = Buruk AND $Gaji$ = Sangat Besar THEN NK = Rendah
5. IF IPK = Cukup AND $Gaji$ = Kecil THEN NK = Tinggi
6. IF IPK = Cukup AND $Gaji$ = Sedang THEN NK = Rendah
7. IF IPK = Cukup AND $Gaji$ = Besar THEN NK = Rendah
8. IF IPK = Cukup AND $Gaji$ = Sangat Besar THEN NK = Rendah
9. IF IPK = Bagus AND $Gaji$ = Kecil THEN NK = Tinggi
10. IF IPK = Bagus AND $Gaji$ = Sedang THEN NK = Tinggi
11. IF IPK = Bagus AND $Gaji$ = Besar THEN NK = Tinggi
12. IF IPK = Bagus AND $Gaji$ = Sangat Besar THEN NK = Rendah

Inferensi pada model Mamdani: *Clipping* dan *Scaling*



(a) *Clipping*



(b) *Scaling*

Aturan *fuzzy* yang diaplikasikan

7. IF $IPK = \text{Cukup}$ AND $Gaji = \text{Besar}$ THEN $NK = \text{Rendah}$
8. IF $IPK = \text{Cukup}$ AND $Gaji = \text{Sangat Besar}$ THEN $NK = \text{Rendah}$
11. IF $IPK = \text{Bagus}$ AND $Gaji = \text{Besar}$ THEN $NK = \text{Tinggi}$
12. IF $IPK = \text{Bagus}$ AND $Gaji = \text{Sangat Besar}$ THEN $NK = \text{Rendah}$

Nilai *fuzzy* untuk mhs A

IPK = 3,00

Gaji Orangtua = 10 juta/bulan



IPK = **Cukup** (0,5)

IPK = **Bagus** (0,5)

Gaji Orangtua = **Besar** (0,4)

Gaji Orangtua = **Sangat Besar** (0,6)

Conjunction (\wedge) & Disjunction (\vee)

IF $IPK = \text{Cukup}(0,5)$ AND $Gaji = \text{Besar}(0,4)$ THEN $NK = \text{Rendah}(0,4)$

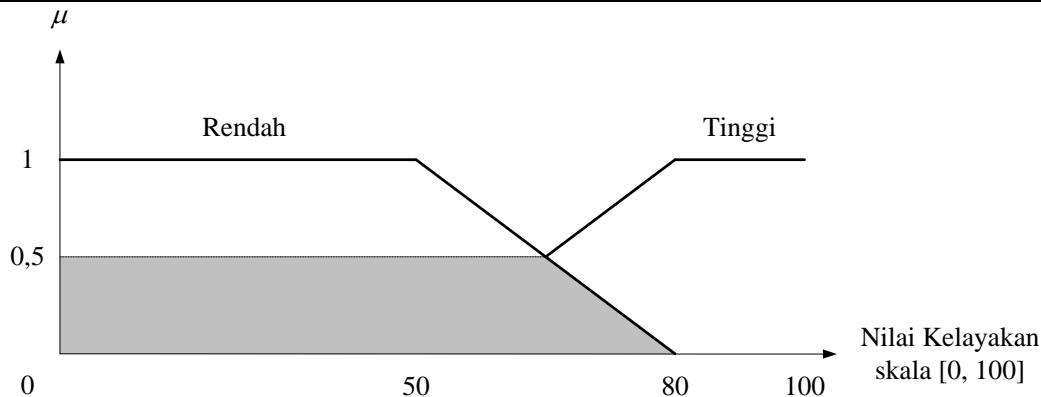
IF $IPK = \text{Cukup}(0,5)$ AND $Gaji = \text{Sangat Besar}(0,6)$ THEN $NK = \text{Rendah}(0,5)$

IF $IPK = \text{Bagus}(0,5)$ AND $Gaji = \text{Besar}(0,4)$ THEN $NK = \text{Tinggi}(0,4)$

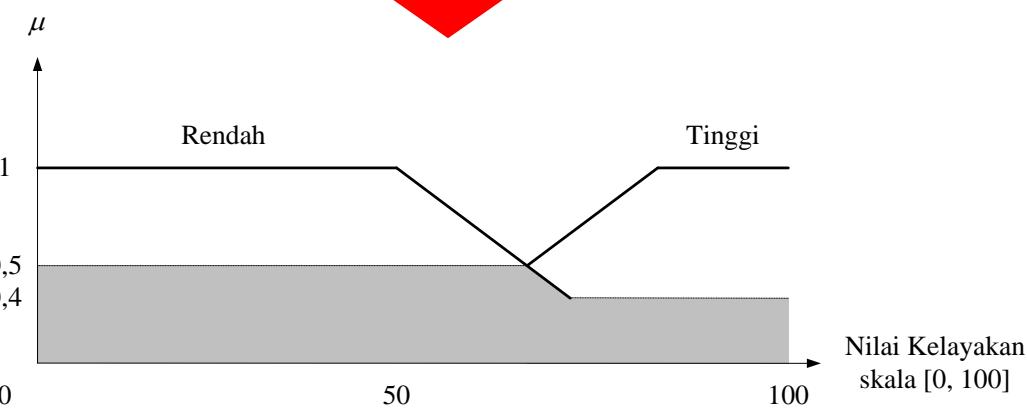
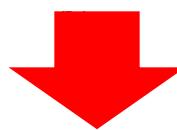
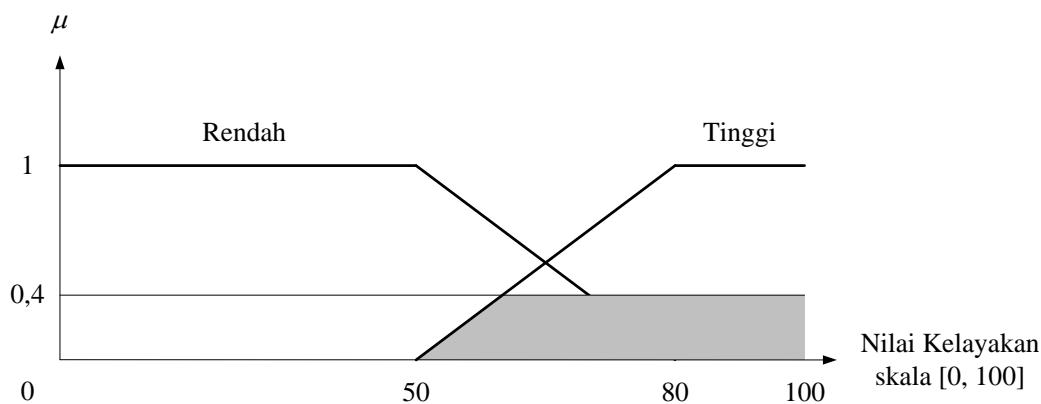
IF $IPK = \text{Bagus}(0,5)$ AND $Gaji = \text{Sangat Besar}(0,6)$ THEN $NK = \text{Rendah}(0,5)$

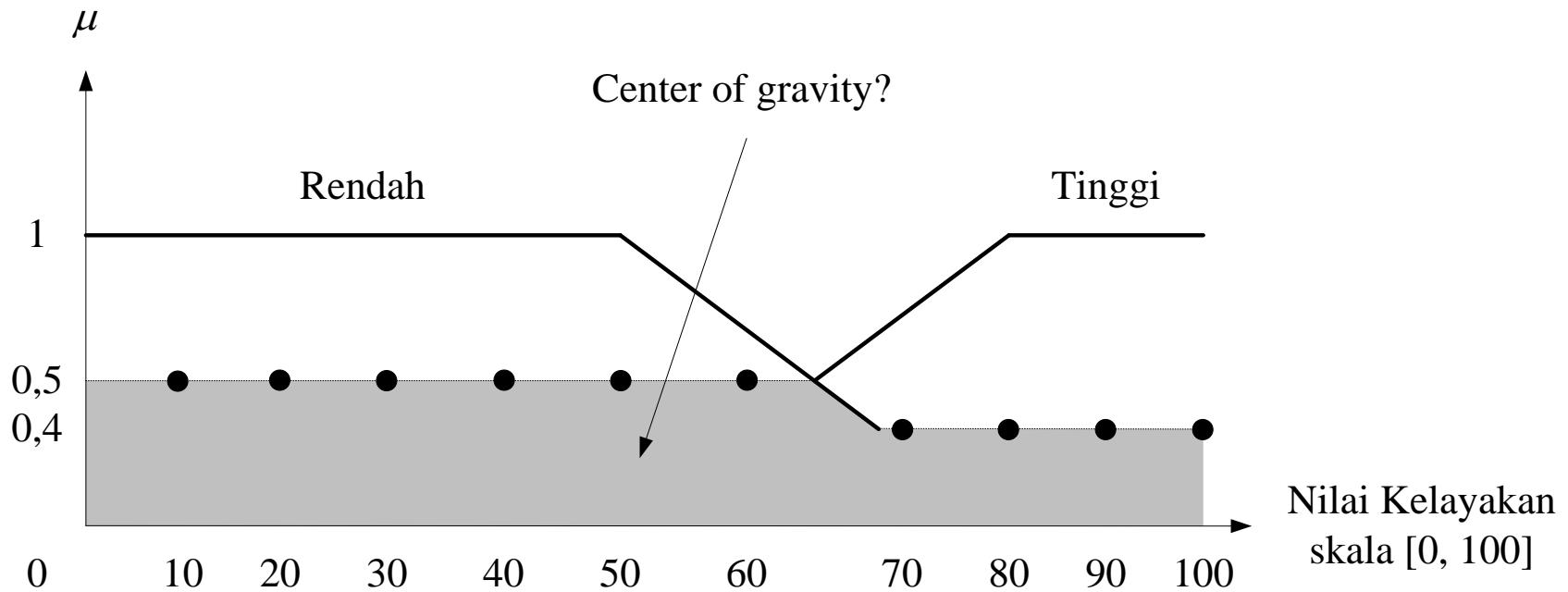


$NK = \text{Rendah} (0,5)$
 $NK = \text{Tinggi} (0,4)$



(a)

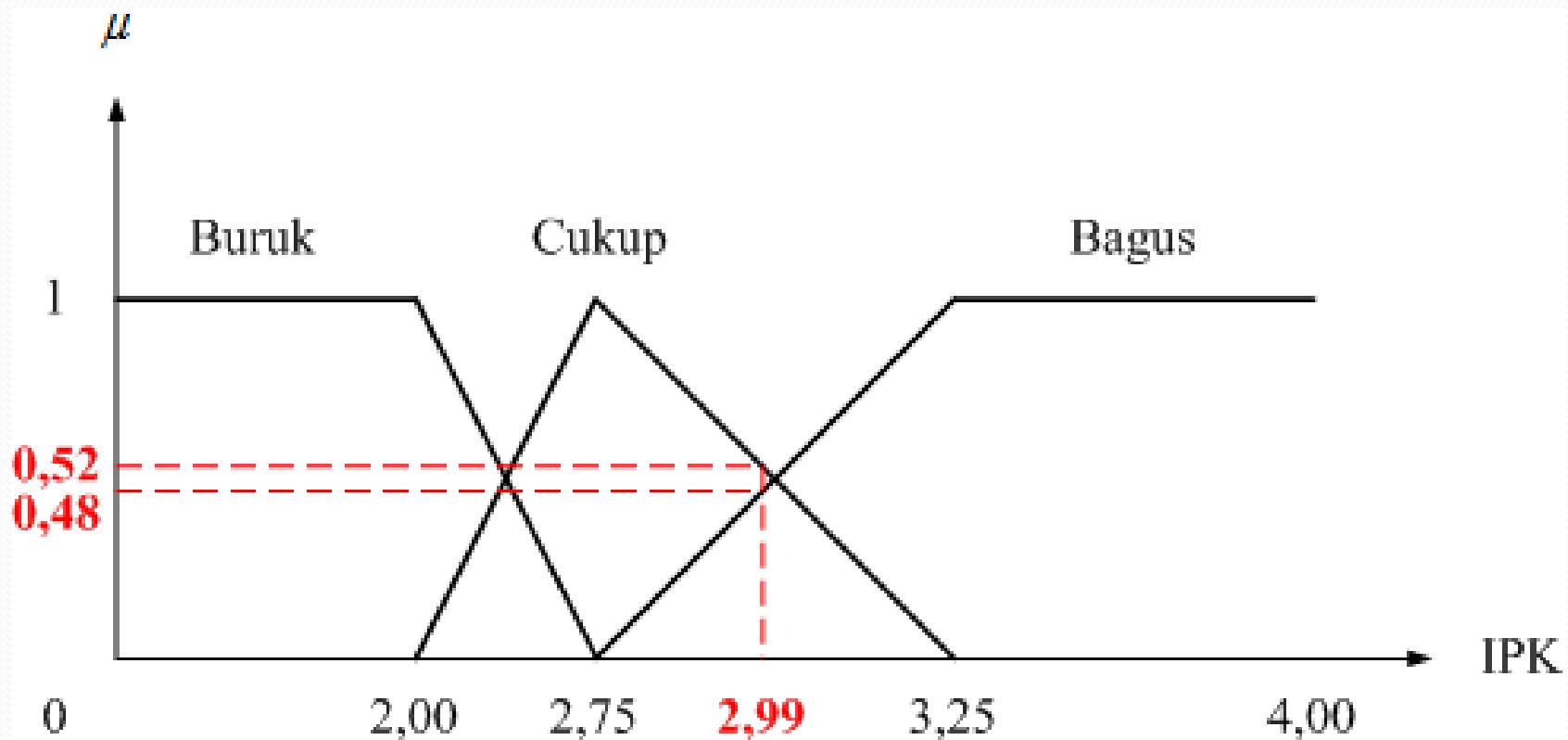




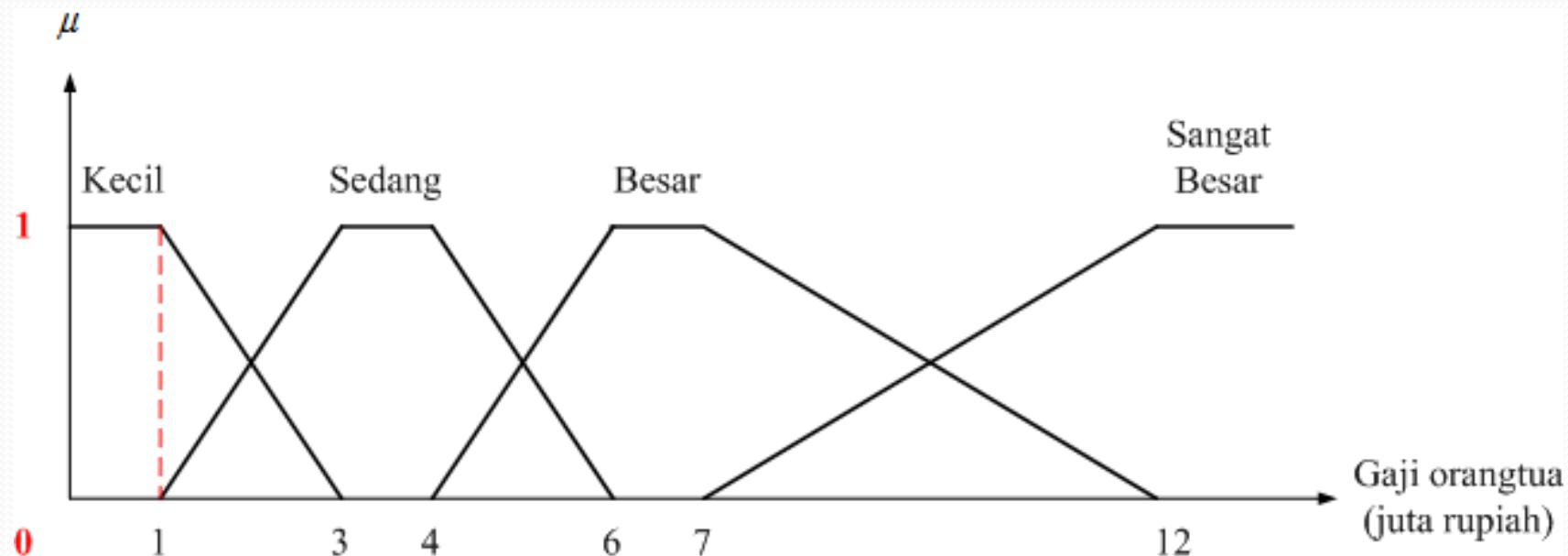
$$y^* = \frac{(10 + 20 + 30 + 40 + 50 + 60)0,5 + (70 + 80 + 90 + 100)0,4}{6(0,5) + 4(0,4)}$$

$$y^* = \frac{105 + 136}{4,6} = 52,39$$

IPK mahasiswa B



Gaji Orangtua mhs B



Conjunction (\wedge) & Disjunction (\vee)

IF $IPK = \text{Cukup}(0,52)$ AND $Gaji = \text{Kecil}(1)$ THEN $NK = \text{Tinggi}(0,52)$

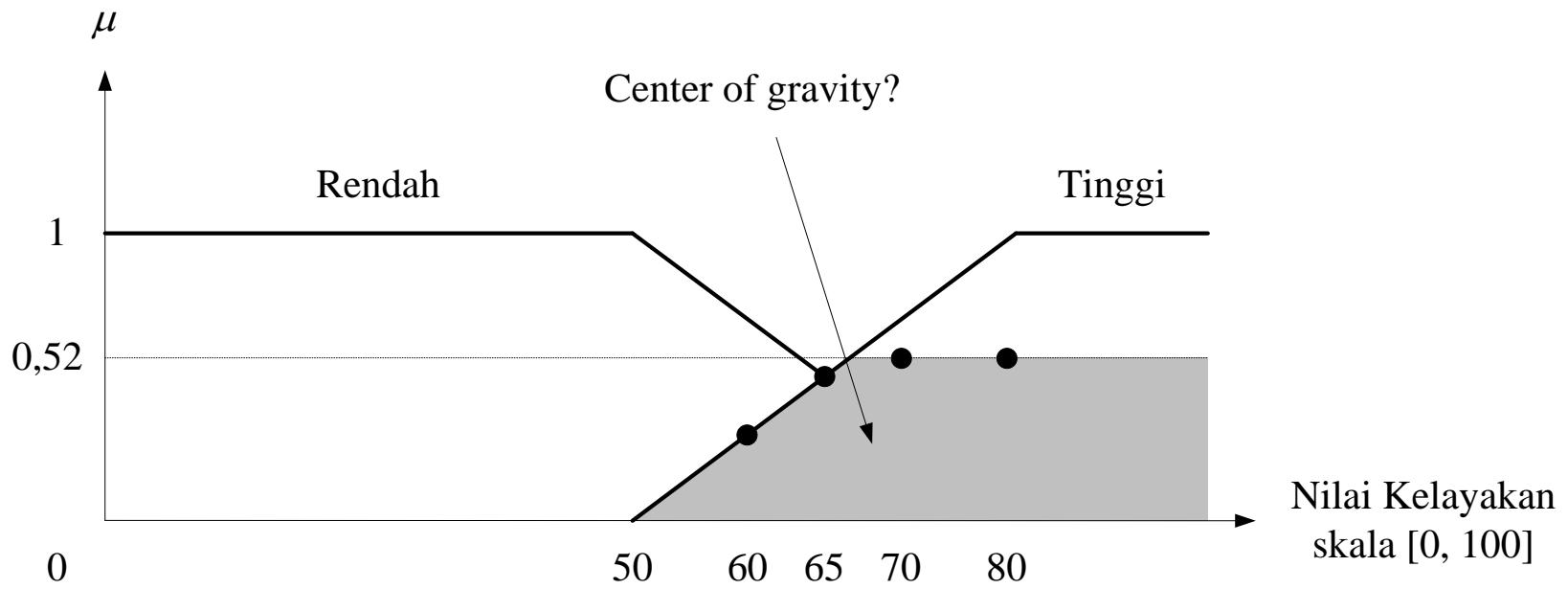
IF $IPK = \text{Cukup}(0,52)$ AND $Gaji = \text{Sedang}(0)$ THEN $NK = \text{Rendah}(0)$

IF $IPK = \text{Besar}(0,48)$ AND $Gaji = \text{Kecil}(1)$ THEN $NK = \text{Tinggi}(0,48)$

IF $IPK = \text{Besar}(0,48)$ AND $Gaji = \text{Sedang}(0)$ THEN $NK = \text{Tinggi}(0)$



$NK = \text{Rendah} (0)$
 $NK = \text{Tinggi} (0,52)$



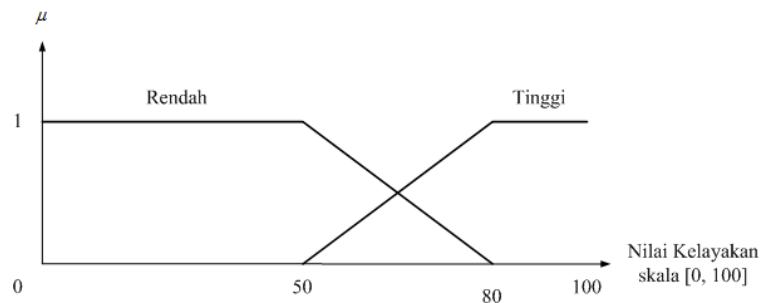
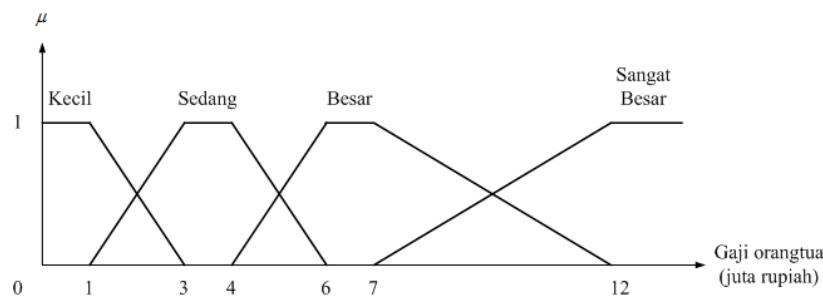
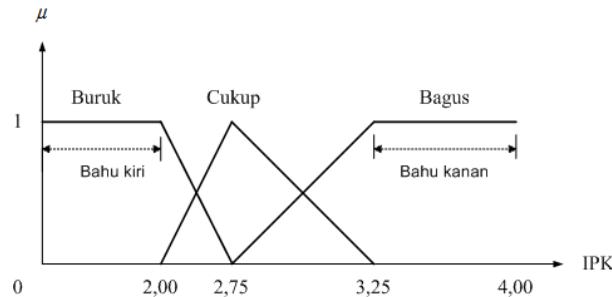
$$y^* = \frac{60(1/3) + 65(1/2) + (70 + 80)(0,52)}{(1/3) + (1/2) + (0,52)2}$$

$$y^* = \frac{20 + 32,5 + 78}{2,87334} = 69,66$$

Keputusan Model Mamdani

- Mahasiswa B dengan IPK = 2,99 dan Gaji orangtuanya sebesar 1 juta rupiah per bulan memperoleh Nilai Kelayakan sebesar **69,66**.
- Lebih besar dibandingkan dengan Nilai Kelayakan mahasiswa A yang sebesar **52,39**.
- Jadi, mahasiswa B layak mendapatkan beasiswa.

Model Mamdani



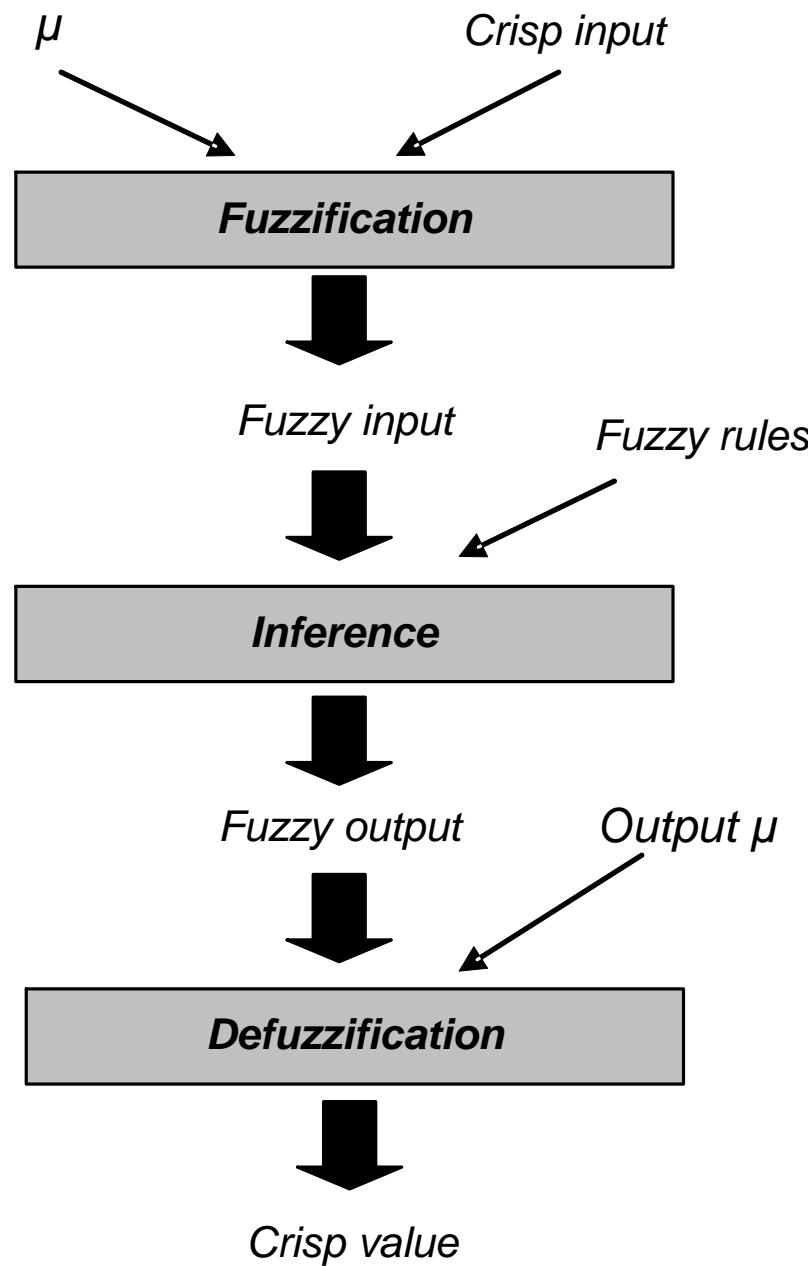
IPK \ Gaji	Kecil	Sedang	Besar	Sangat Besar
Buruk	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah

Model Sugeno

- Model ini sering digunakan untuk membangun sistem kontrol yang membutuhkan respon cepat
- Proses perhitungannya sangat sederhana sehingga membutuhkan waktu relatif cepat sehingga sangat sesuai untuk sistem kontrol
- Bagaimana jika digunakan untuk masalah pemberian beasiswa?

Masalah: Pemberian Beasiswa

Mahasiswa	IPK	Gaji Ortu (Rp/bulan)
A	3,00	10 juta
B	2,99	1 juta



Fuzzification & Rule Evaluation

- Misalkan proses *fuzzification*-nya sama persis dengan model Mamdani.
- Misalkan *Rule* yang digunakan juga sama persis dengan model Mamdani.

Mahasiswa A

IF $IPK = \text{Cukup}(0,5)$ AND $Gaji = \text{Besar}(0,4)$ THEN $NK = \text{Rendah}(0,4)$

IF $IPK = \text{Cukup}(0,5)$ AND $Gaji = \text{Sangat Besar}(0,6)$ THEN $NK = \text{Rendah}(0,5)$

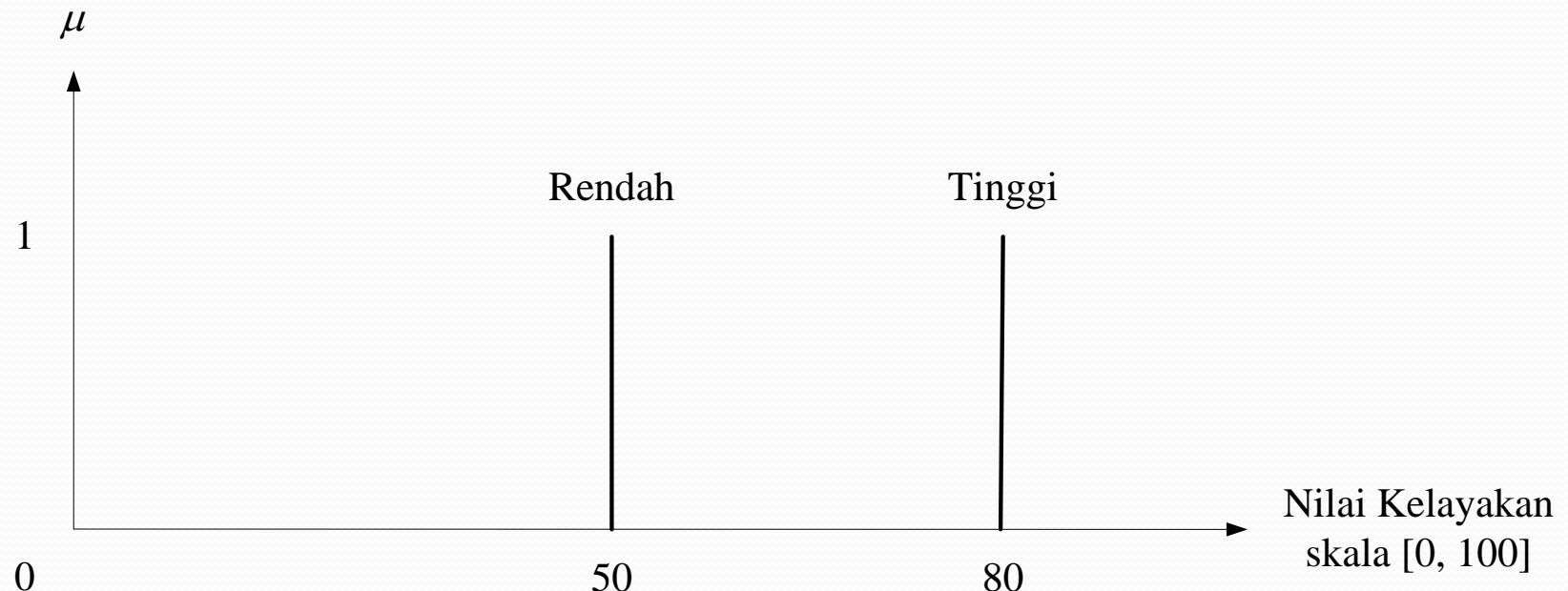
IF $IPK = \text{Bagus}(0,5)$ AND $Gaji = \text{Besar}(0,4)$ THEN $NK = \text{Tinggi}(0,4)$

IF $IPK = \text{Bagus}(0,5)$ AND $Gaji = \text{Sangat Besar}(0,6)$ THEN $NK = \text{Rendah}(0,5)$

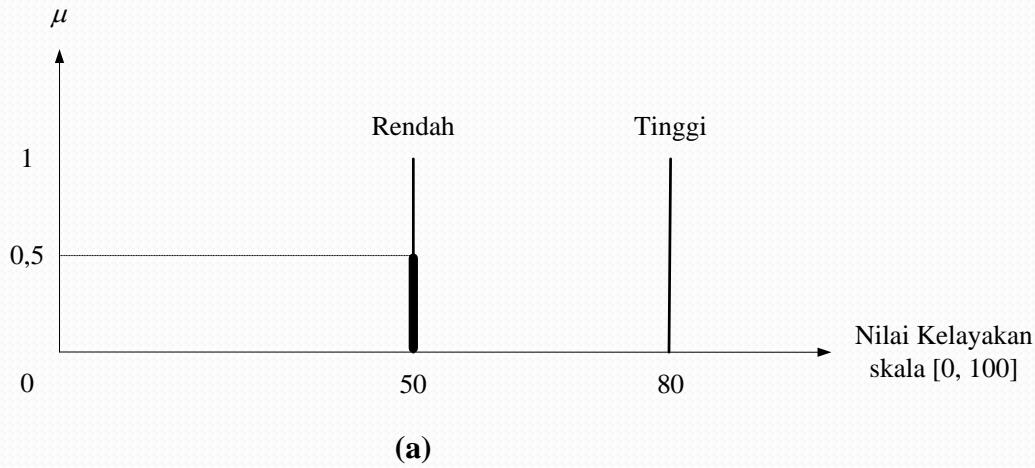


$NK = \text{Rendah} (0,5)$
 $NK = \text{Tinggi} (0,4)$

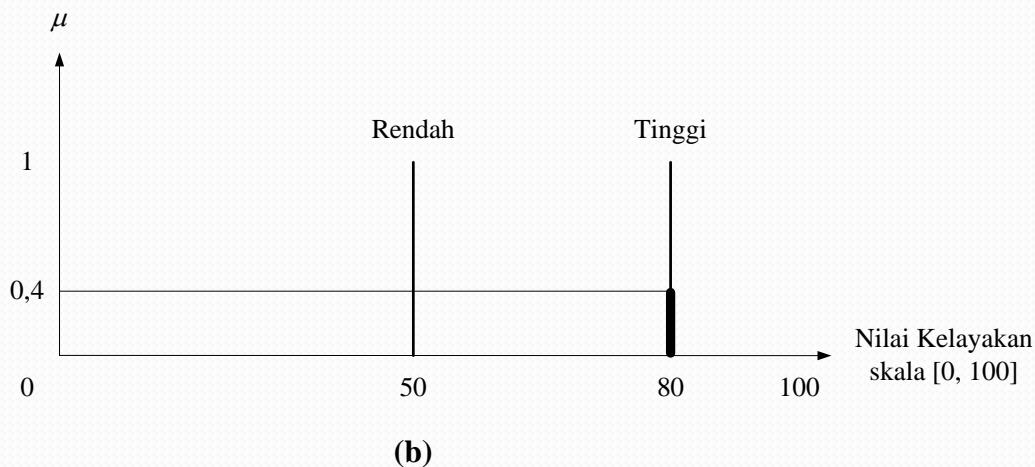
FK *singleton* untuk Nilai Kelayakan



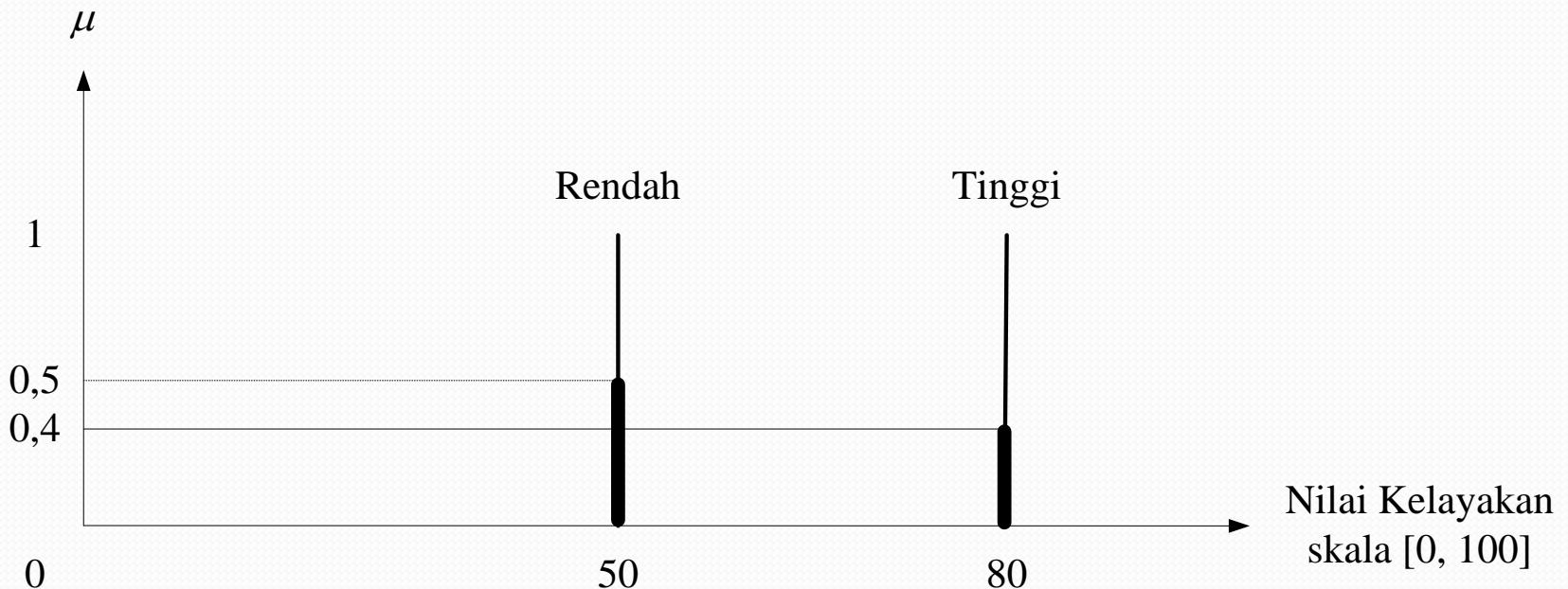
Untuk mahasiswa A



- NK = Rendah (0,5)
- NK = Tinggi (0,4)



Proses *Composition*



Defuzzification: Weighted Average

$$y^* = \frac{(0,5)50 + (0,4)80}{(0,5) + (0,4)} = 63,33$$

Mahasiswa B

IF $IPK = \text{Cukup}(0,52)$ AND $Gaji = \text{Kecil}(1)$ THEN $NK = \text{Tinggi}(0,52)$

IF $IPK = \text{Cukup}(0,52)$ AND $Gaji = \text{Sedang}(0)$ THEN $NK = \text{Rendah}(0)$

IF $IPK = \text{Besar}(0,48)$ AND $Gaji = \text{Kecil}(1)$ THEN $NK = \text{Tinggi}(0,48)$

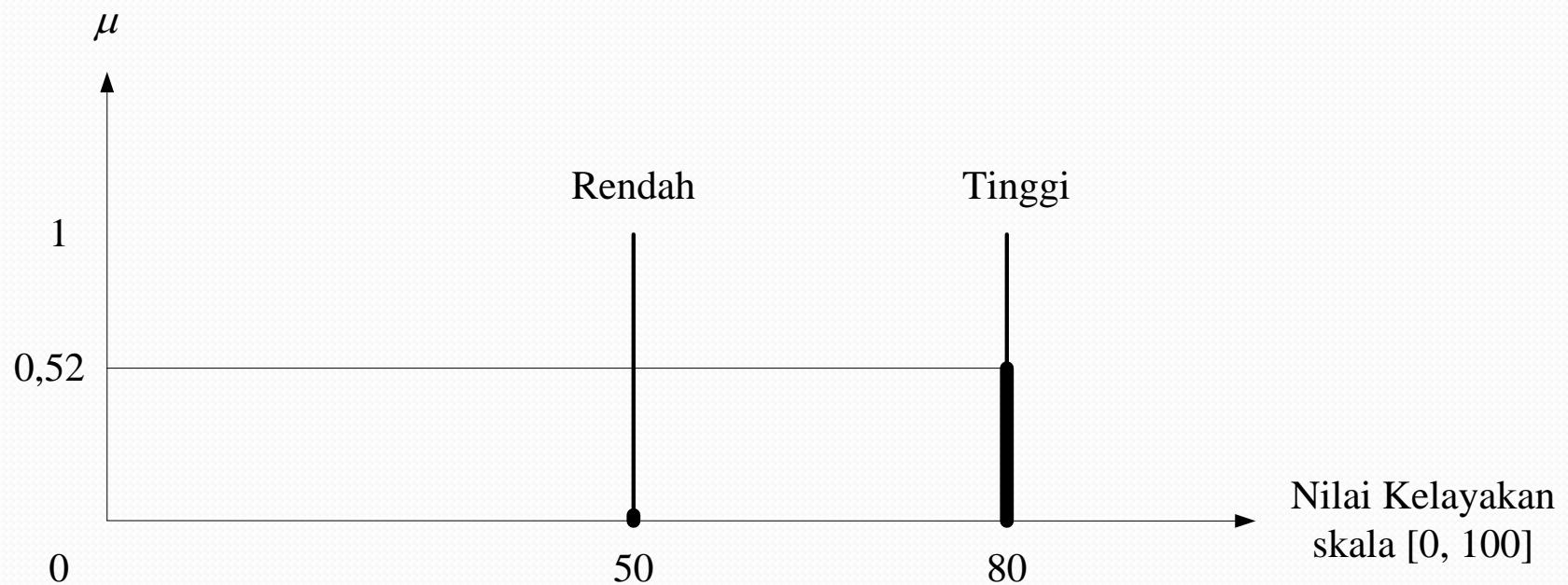
IF $IPK = \text{Besar}(0,48)$ AND $Gaji = \text{Sedang}(0)$ THEN $NK = \text{Tinggi}(0)$



$NK = \text{Rendah} (0)$
 $NK = \text{Tinggi} (0,52)$

Untuk Mahasiswa B

- NK = Rendah (0)
- NK = Tinggi (0,52)



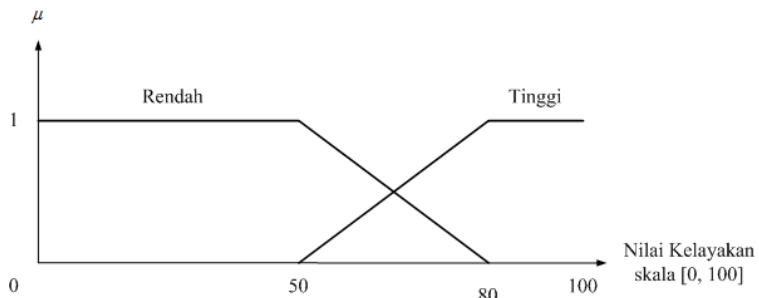
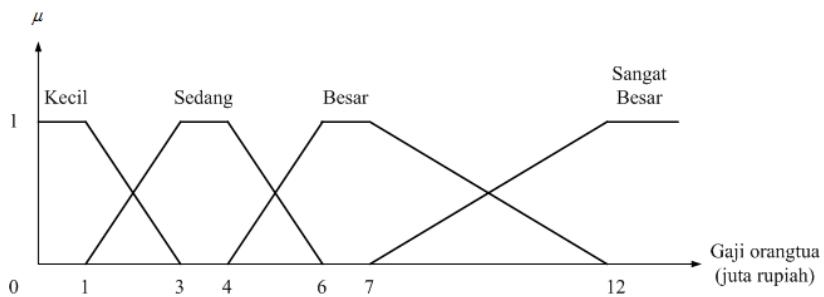
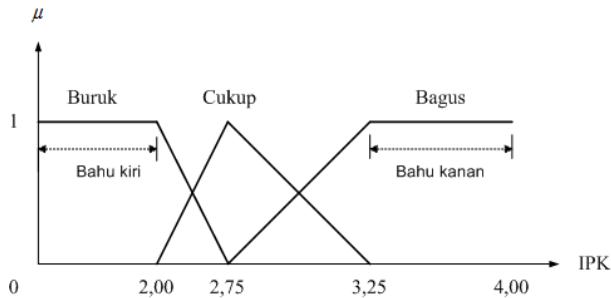
Defuzzification: Weighted Average

$$y^* = \frac{(0)50 + (0,52)80}{0 + 0,52} = 80$$

Keputusan Model Sugeno

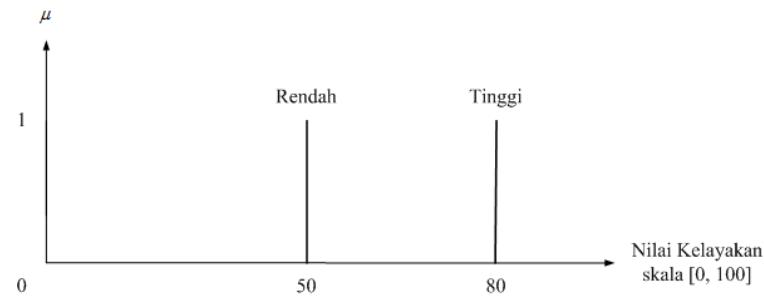
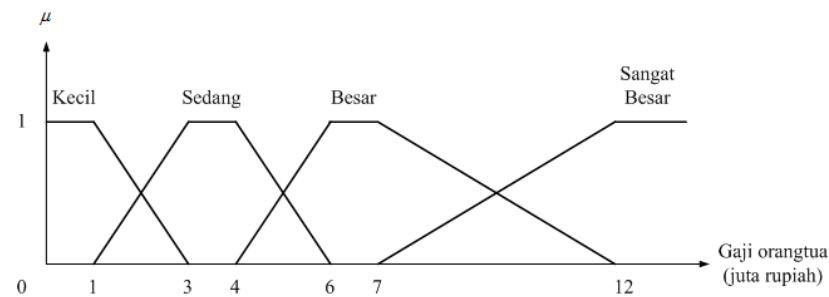
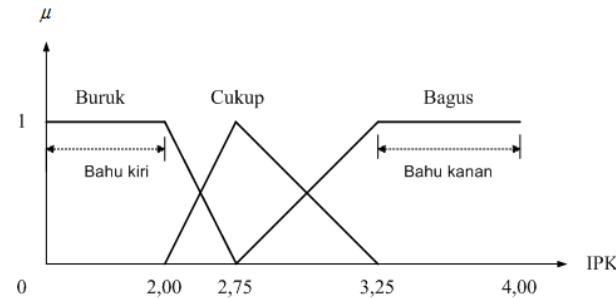
- Mahasiswa B dengan IPK = 2,99 dan Gaji orangtuanya sebesar Rp 1 juta per bulan memperoleh Nilai Kelayakan sebesar **80**.
- Lebih besar dibandingkan dengan Nilai Kelayakan mahasiswa A yang sebesar **63,33**.
- Jadi, mahasiswa B layak mendapatkan beasiswa.

Model Mamdani



	Gaji	Kecil	Sedang	Besar	Sangat Besar
IPK					
Buruk	Rendah	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah	Rendah

Model Sugeno



	Gaji	Kecil	Sedang	Besar	Sangat Besar
IPK					
Buruk	Rendah	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Tinggi	Rendah

Nilai Kelayakan mahasiswa A & B

Mahasiswa	Nilai Kelayakan mendapat beasiswa	
	Model Mamdani	Model Sugeno
A	52,39	63,33
B	69,66	80
Selisih A dan B	17,72	16,67

Sistem Berbasis *Crisp Sets* dan FOL

IF $IPK = 2,57$ AND $Gaji = 1,20$ juta THEN $NK = 62,5$

IF $IPK = 2,57$ AND $Gaji = 1,25$ juta THEN $NK = 62,3$

...

...

...

Aturan FOL untuk proses inference

IPK \ Gaji	G1	G2	G3	G4	G5
P1	70	60	40	30	20
P2	80	70	50	40	30
P3	90	80	60	50	40
P4	100	90	70	60	50

Aturan FOL untuk proses inference

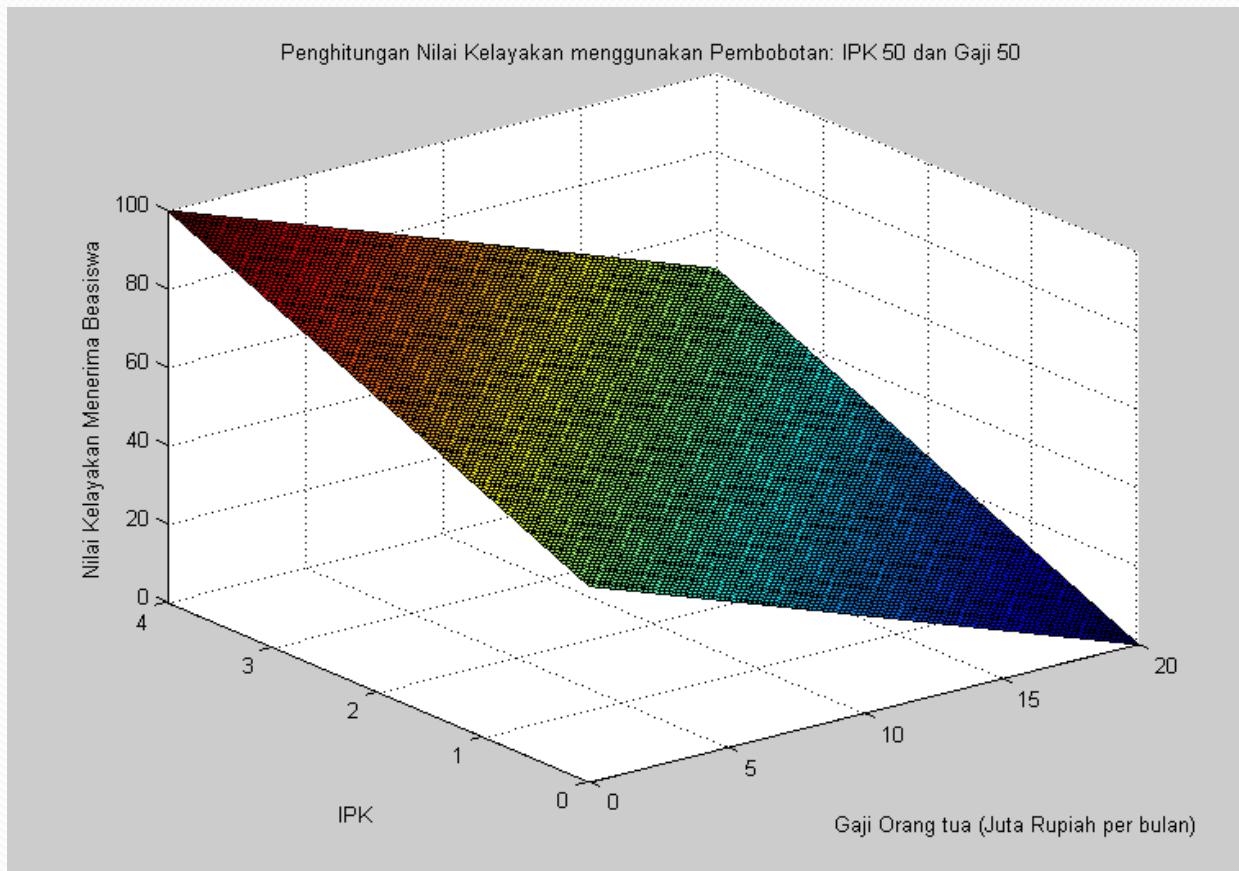
1. IF $\text{Interval}(IPK, P1) \wedge \text{Interval}(Gaji, G1) \Rightarrow NK = 70$
2. IF $\text{Interval}(IPK, P1) \wedge \text{Interval}(Gaji, G2) \Rightarrow NK = 60$
- ...
- ...
- ...
20. IF $\text{Interval}(IPK, P4) \wedge \text{Interval}(Gaji, G5) \Rightarrow NK = 50$

Sistem yang Linier

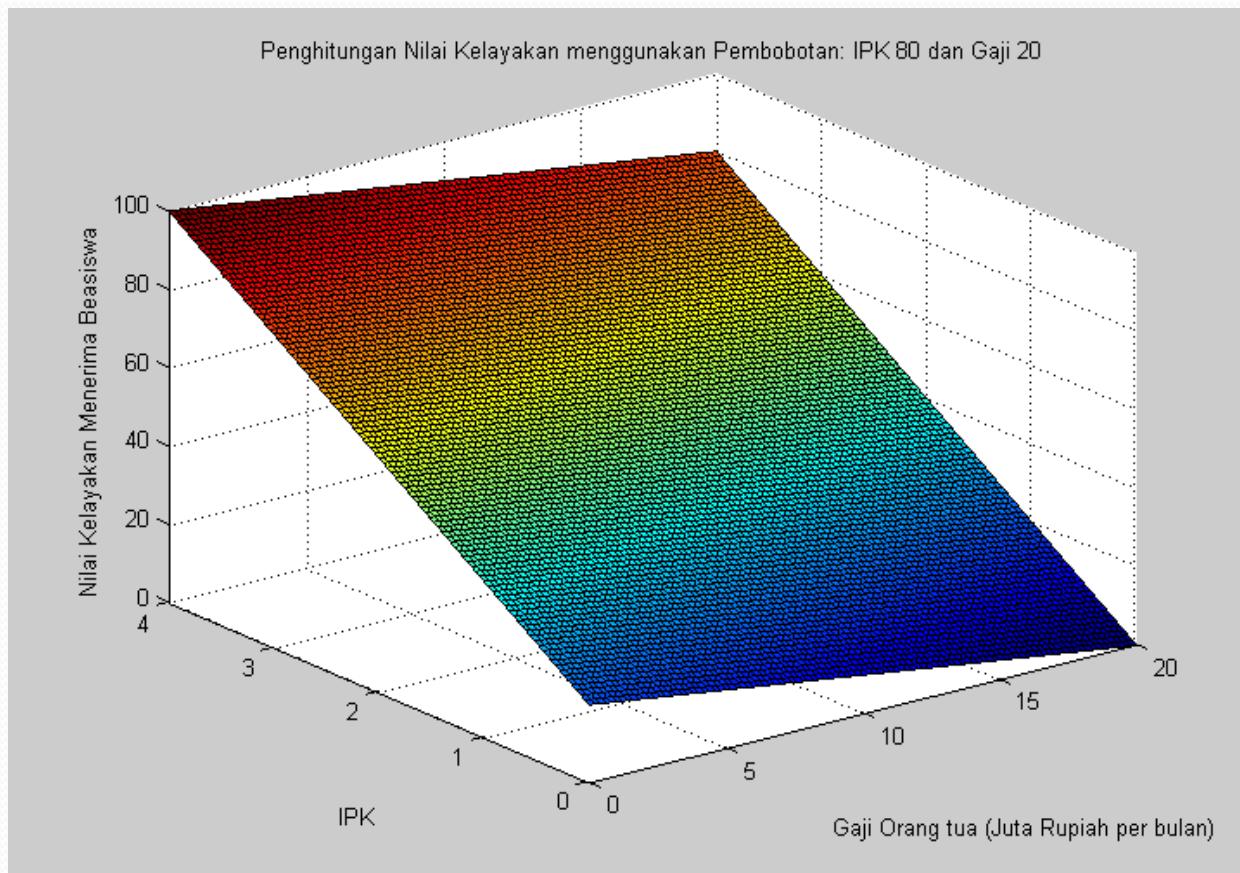
$$NK = W_1(IPK / 4) + W_2((20 - Gaji) / 20)$$

- W_1 adalah bobot untuk IPK, W_2 adalah bobot untuk Gaji.
- Asumsi: IPK maksimum adalah 4,00
- Asumsi: Gaji Orang tua maksimum adalah Rp 20 jt/bln.
- Karena skala untuk NK adalah [0, 100], maka $W_1 + W_2$ harus sama dengan 100.

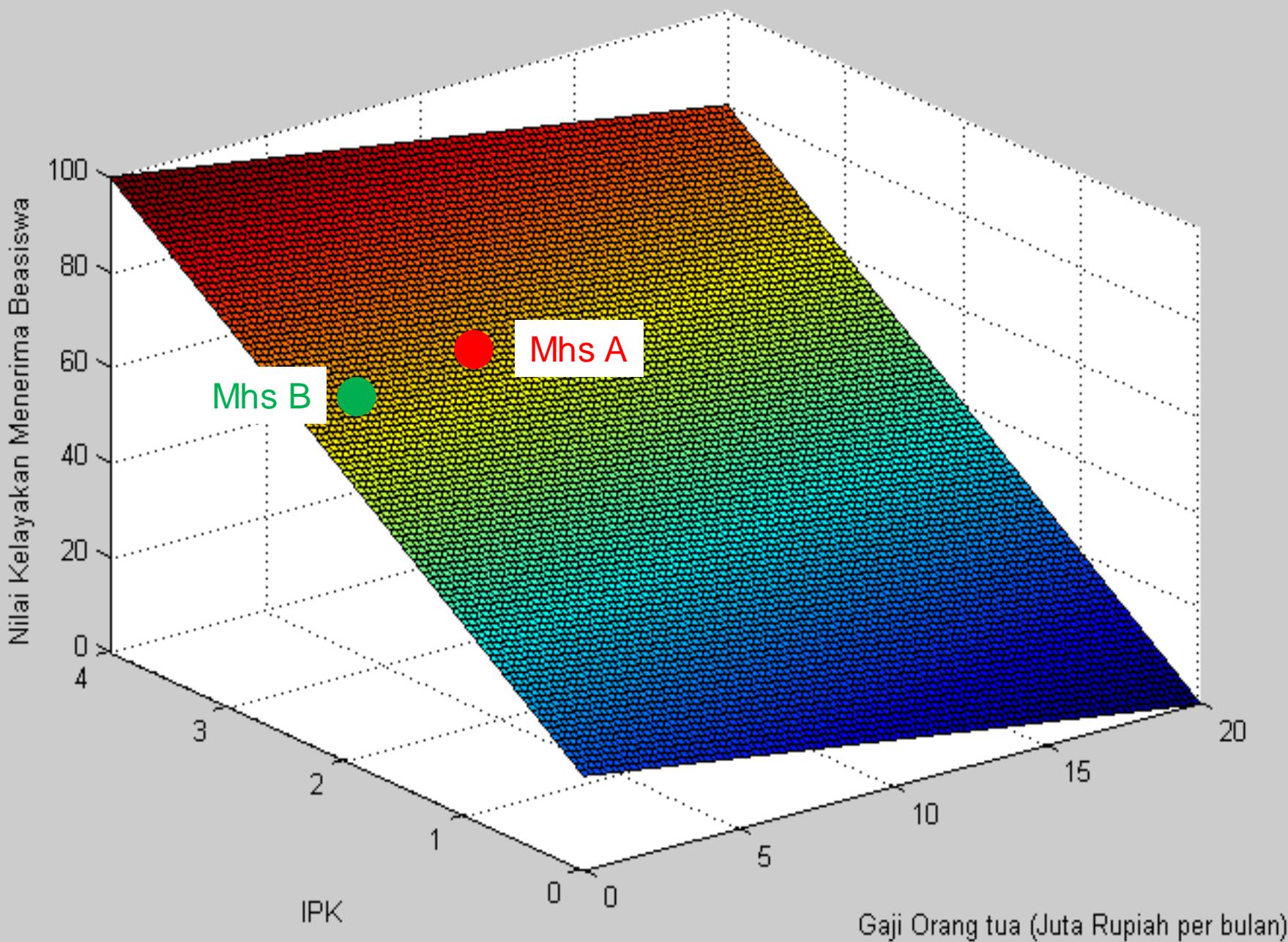
$W_1 = 50$ dan $W_2 = 50$



$$W_1 = 80 \text{ dan } W_2 = 20$$

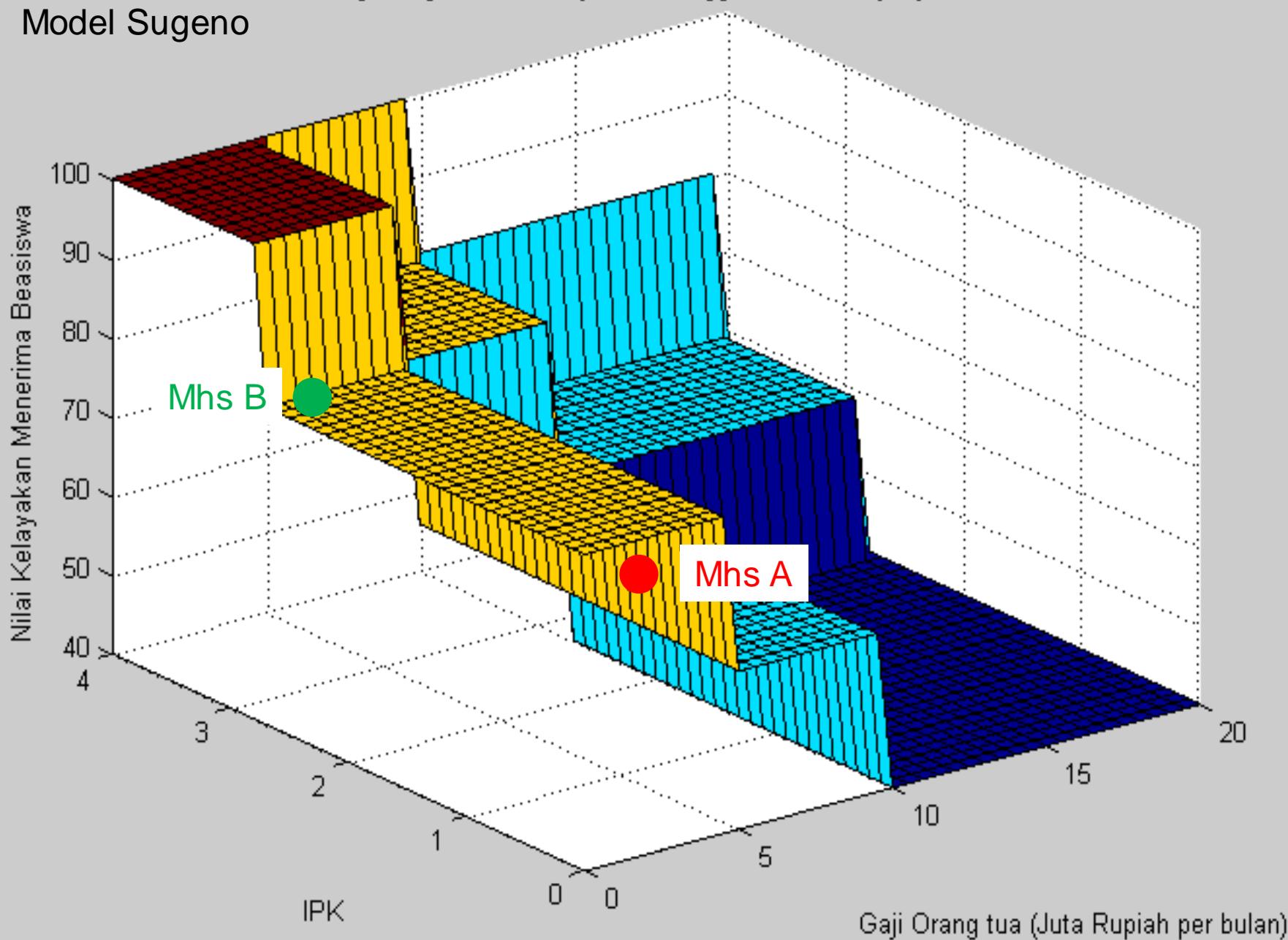


Penghitungan Nilai Kelayakan menggunakan Pembobotan: IPK 80 dan Gaji 20

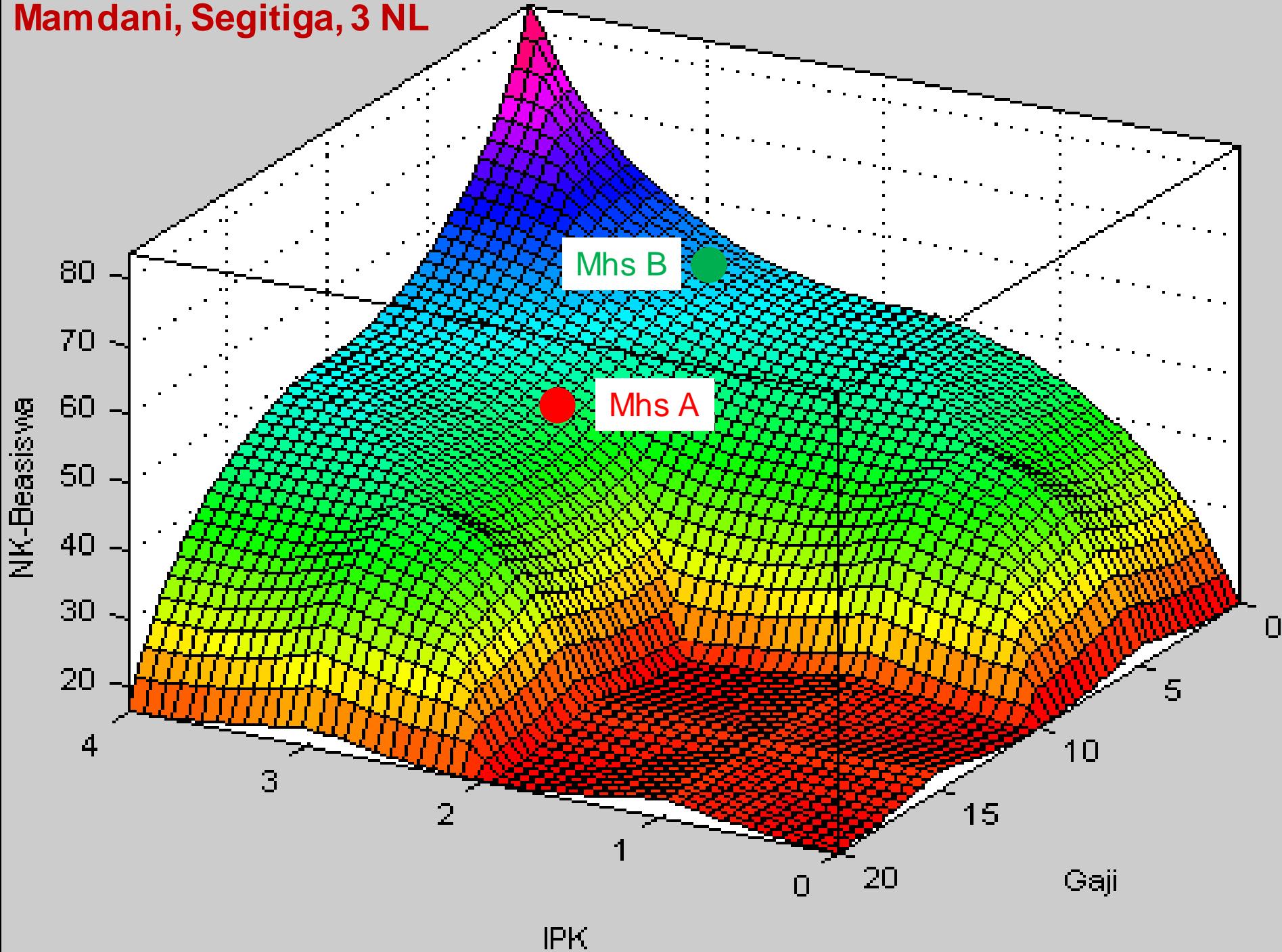


Penghitungan Nilai Kelayakan menggunakan Fuzzy Systems

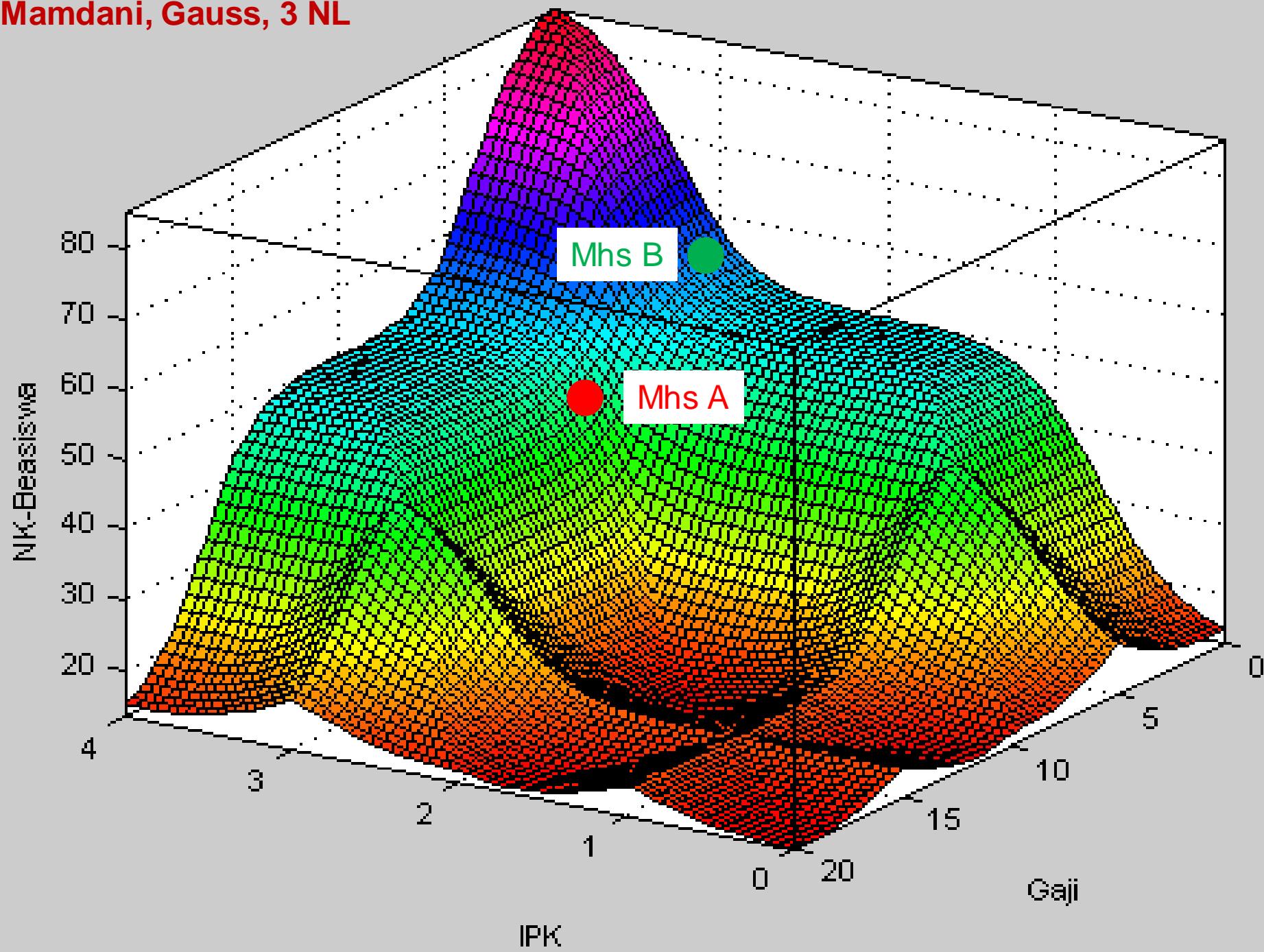
Model Sugeno



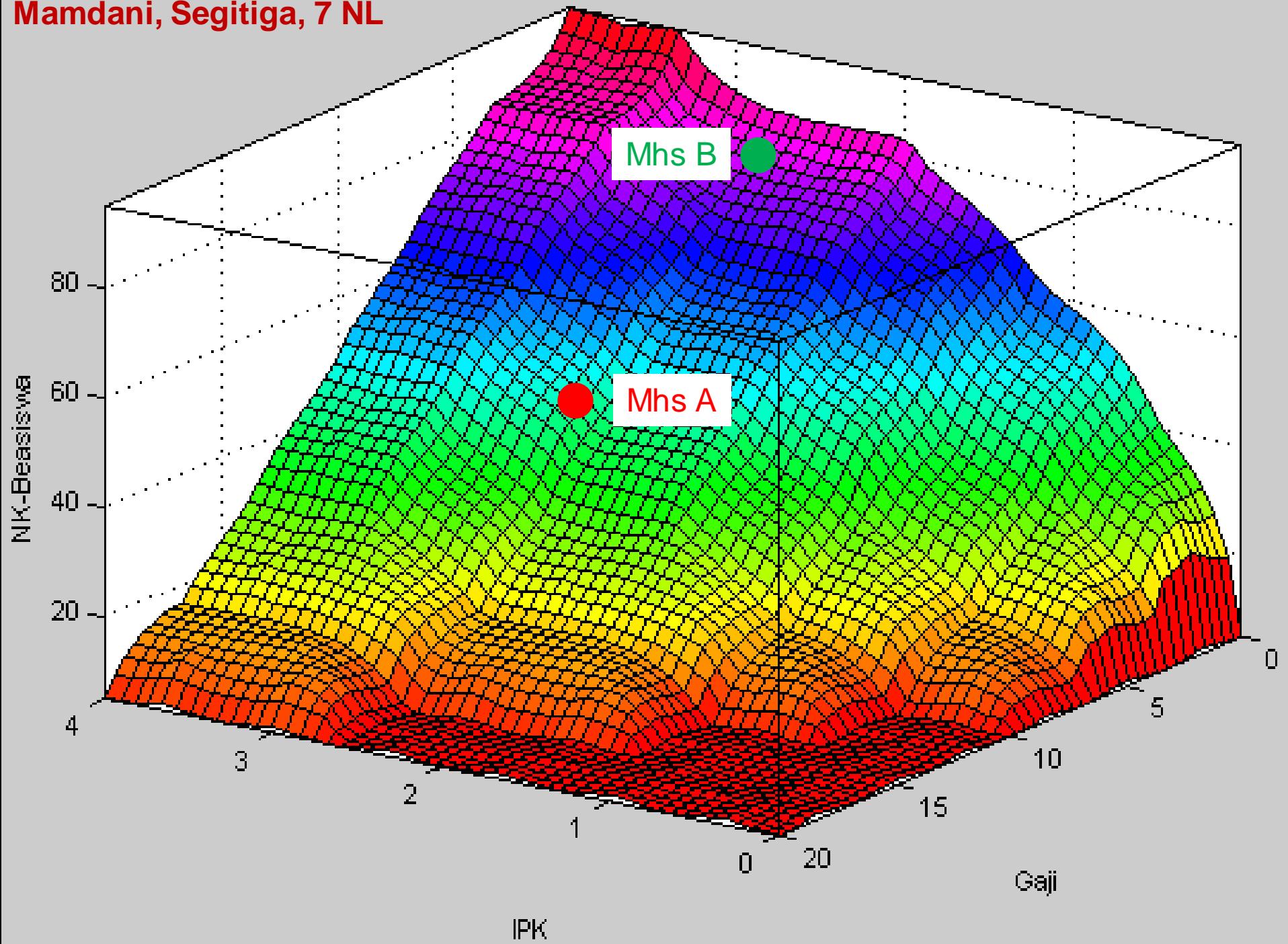
Mamdani, Segitiga, 3 NL



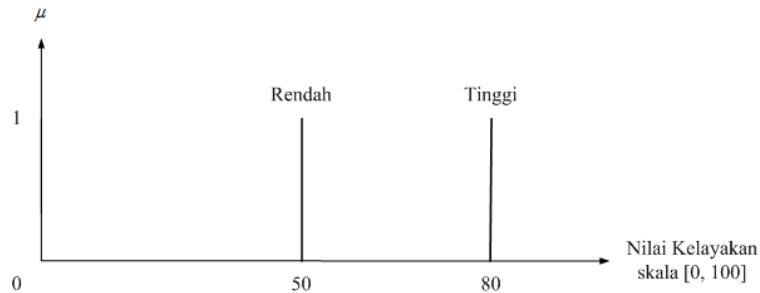
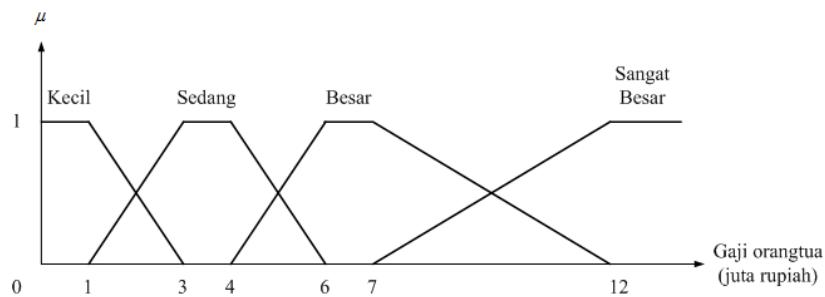
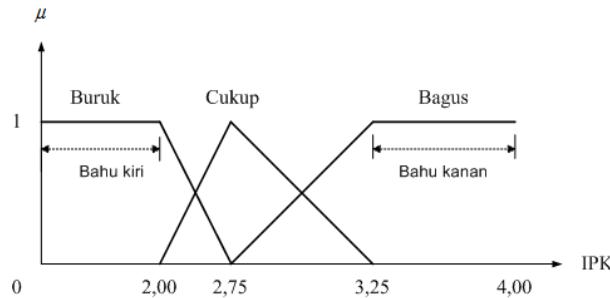
Mamdani, Gauss, 3 NL



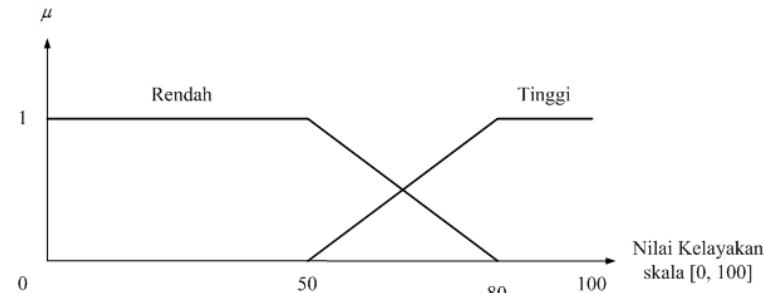
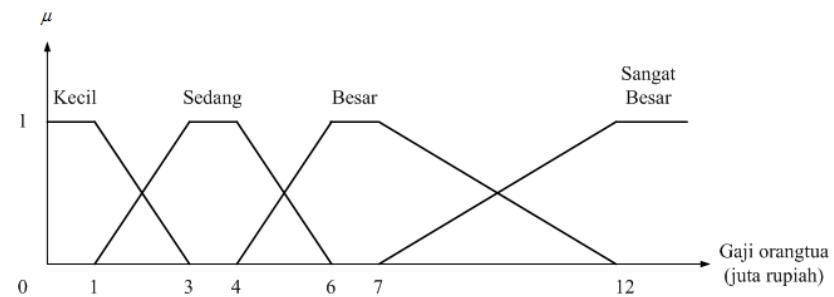
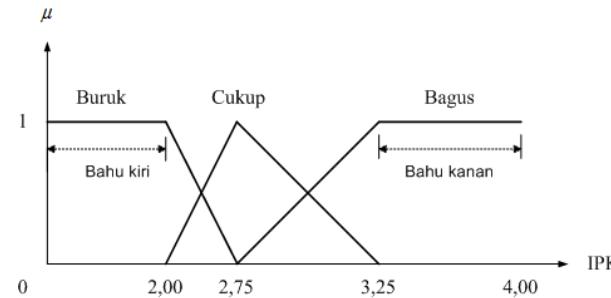
Mamdani, Segitiga, 7 NL



Model Sugeno



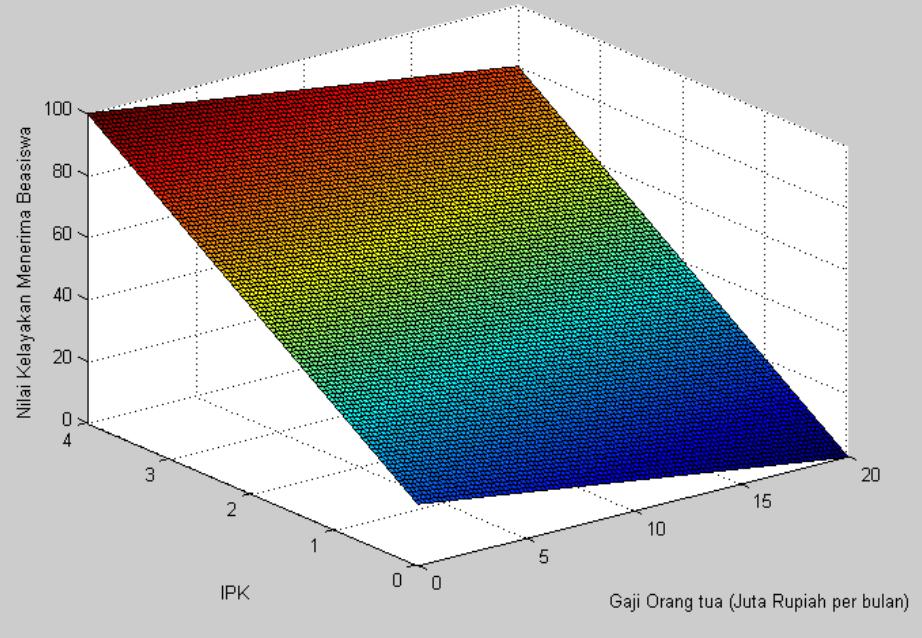
Model Mamdani



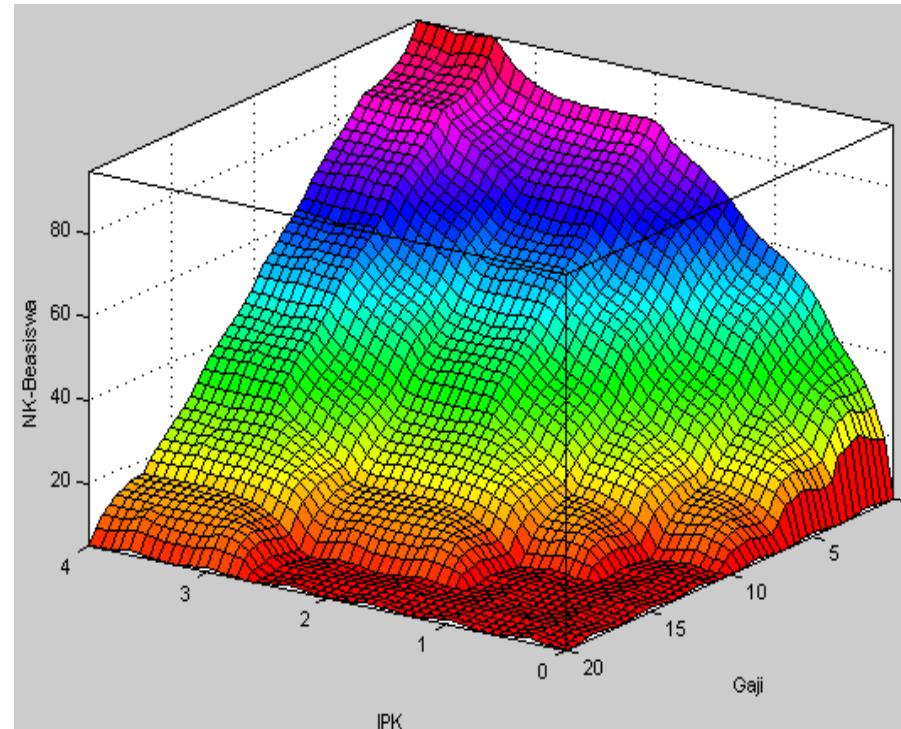
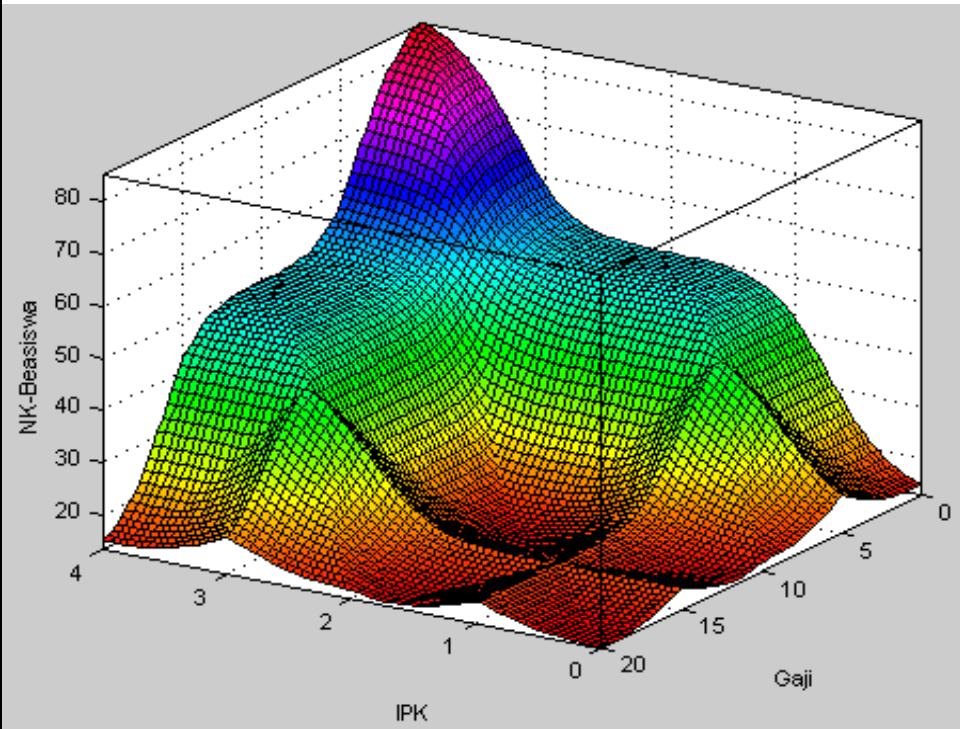
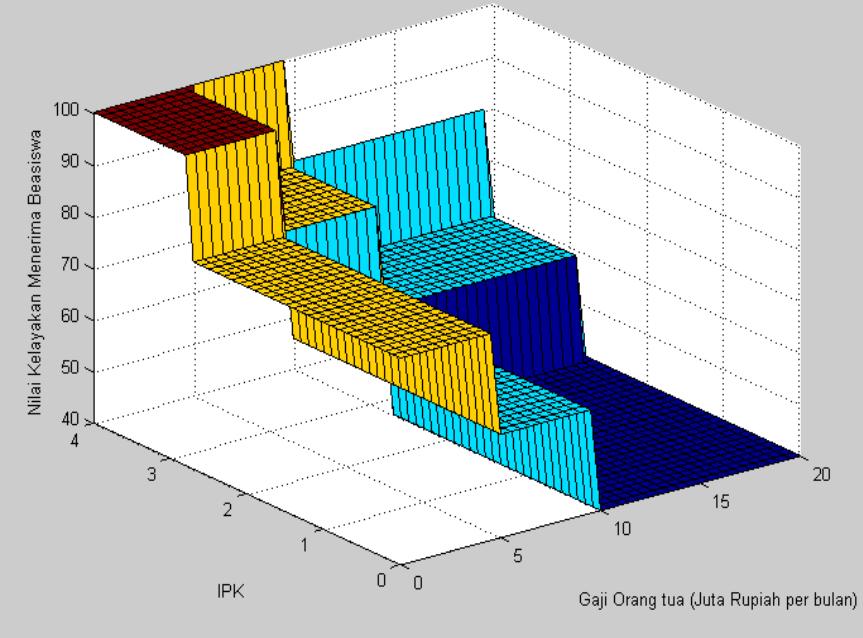
IPK \ Gaji	Kecil	Sedang	Besar	Sangat Besar
Buruk	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah

IPK \ Gaji	Kecil	Sedang	Besar	Sangat Besar
Buruk	Rendah	Rendah	Rendah	Rendah
Cukup	Tinggi	Rendah	Rendah	Rendah
Bagus	Tinggi	Tinggi	Tinggi	Rendah

Penghitungan Nilai Kelayakan menggunakan Pembobotan: IPK 80 dan Gaji 20



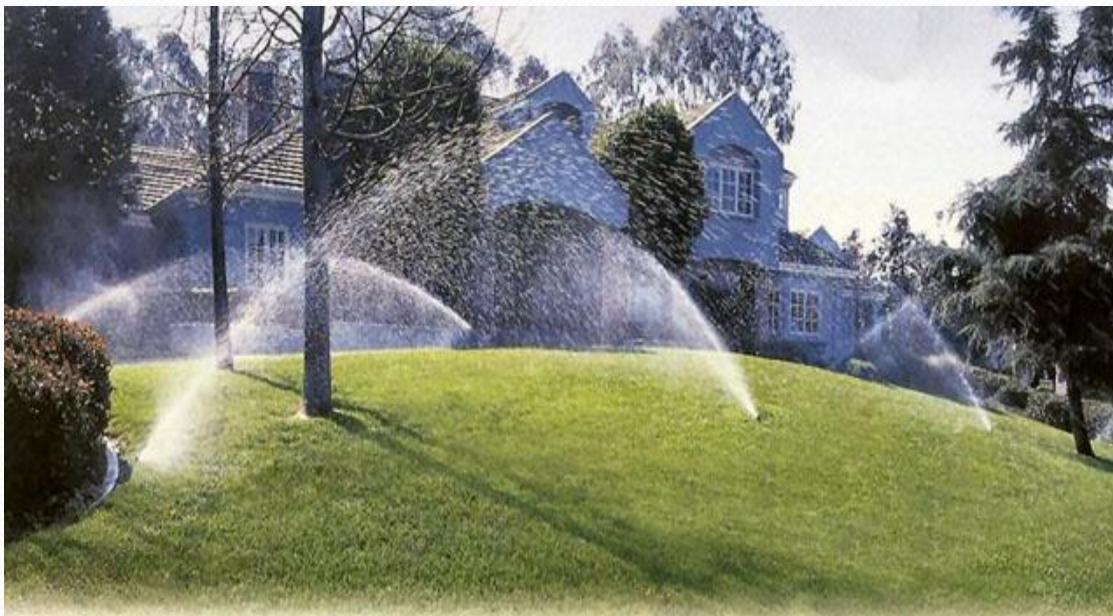
Penghitungan Nilai Kelayakan menggunakan Fuzzy Systems





Kasus 2: *Sprinkler CS*

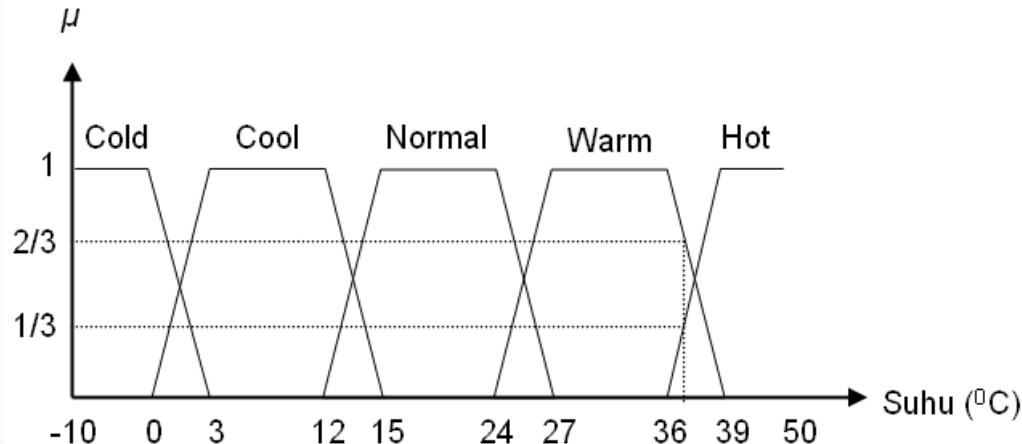
- Sistem kontrol untuk penyiraman air.
 - *Input:* Suhu Udara ($^{\circ}\text{C}$) & Kelembaban Tanah (%)
 - *Output:* Durasi Penyiraman (menit)
-
- Nilai *crisp* yang diterima oleh sensor suhu adalah 37°C dan sensor kelembaban adalah 12%.
 - Berapa lama durasi penyiraman yang dilakukan?



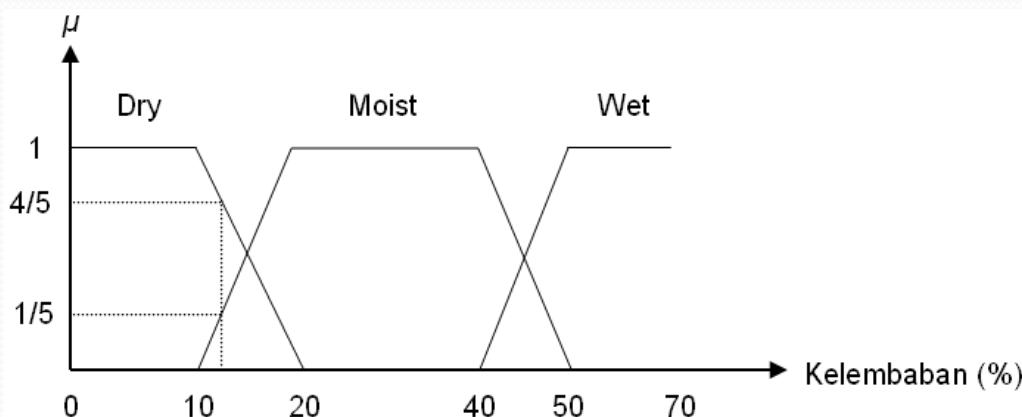
Kasus 2: *Sprinkler CS*

- Model Mamdani atau Sugeno?
- Jumlah Nilai Linguistik untuk setiap variabel?
- Fungsi Keanggotaan: Segitiga, trapesium, phi, ...?
- Batas-batas Nilai Linguistik?
- *Fuzzy rule* yang tepat?

Proses fuzzification



- Warm = $-(37-39)/(39-36) = 2/3$.
- Hot = $(37-36)/(39-36) = 1/3$.



- Dry = $-(12-20)/(20-10) = 4/5$
- Moist = $(12-10)/(20-10) = 1/5$

Proses inferensi

Antecedent 1 (Temperature)

Antecedent 2
(Moisture)

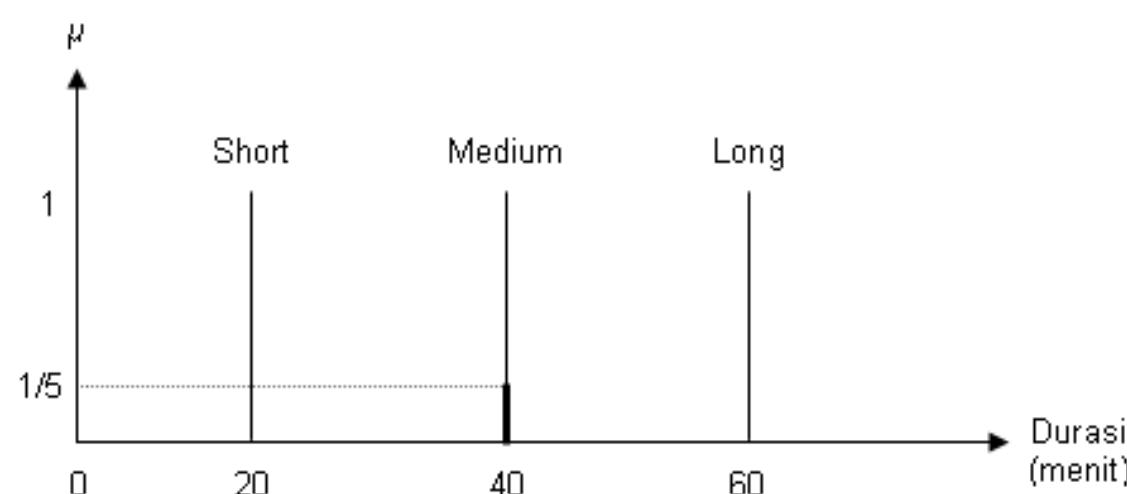
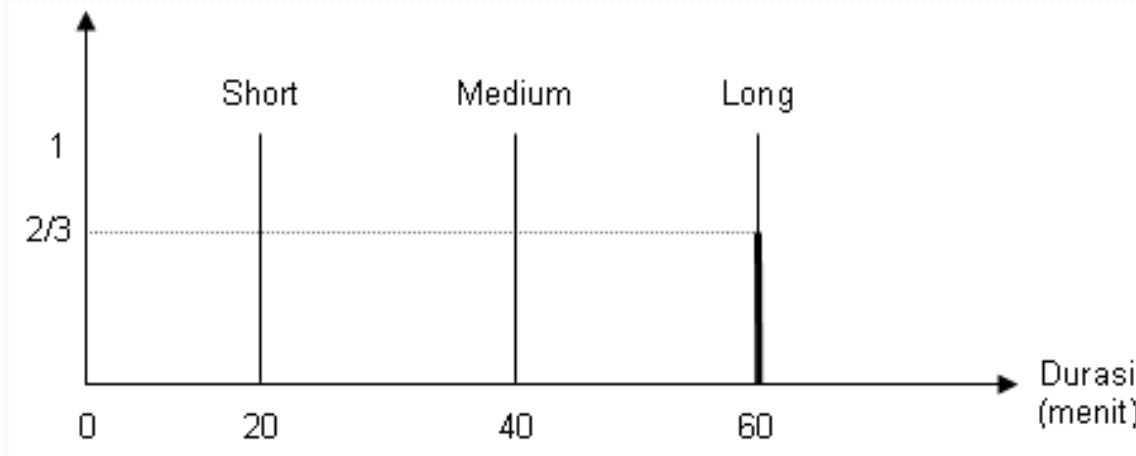
	Cold	Cool	Normal	Warm	Hot
Dry	L	L	L	L	L
Moist	L	M	M	M	M
Wet	S	S	S	S	S

Note: L = Long, M = Medium, S = Short

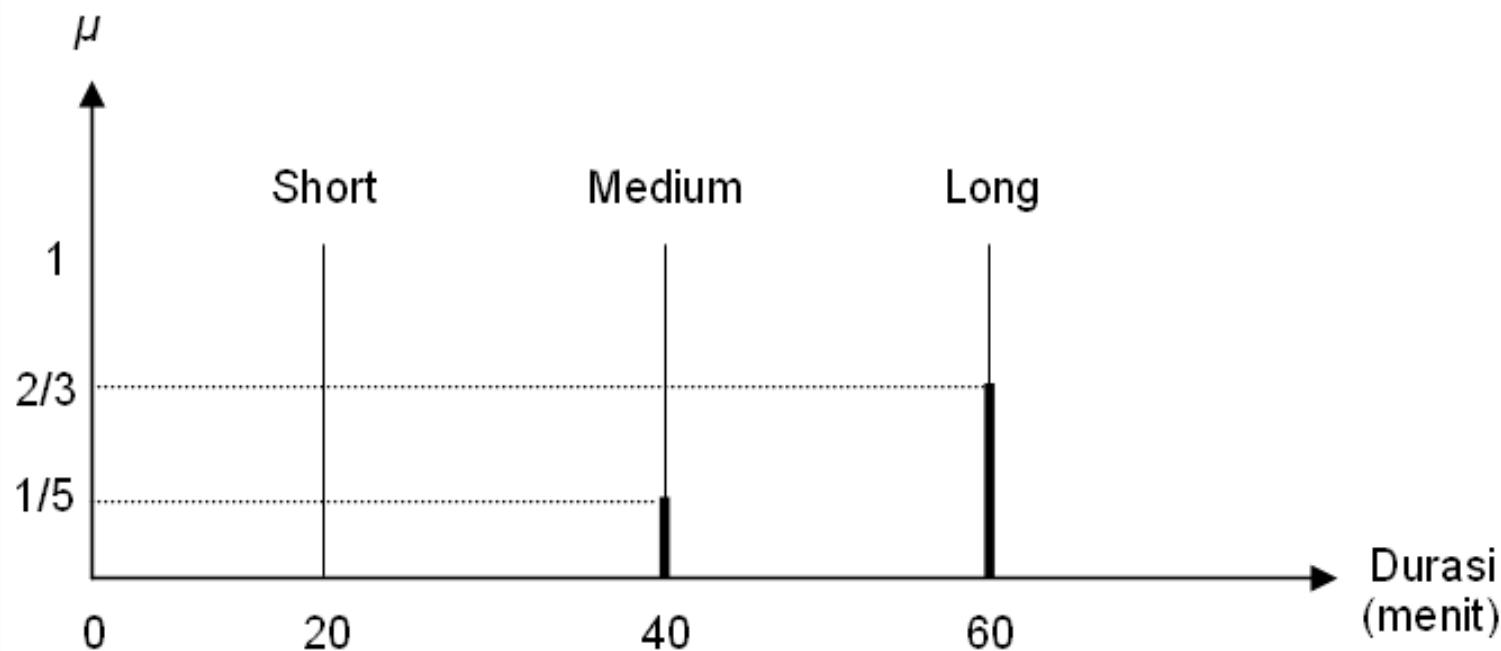
Proses *inference*

- Dari empat data fuzzy *input* di atas, Warm (2/3), Hot (1/3), Dry (4/5) dan Moist (1/5), maka kita mendapatkan 4 aturan (dari 15 aturan) yang dapat diaplikasikan:
 - IF Suhu is Warm (2/3) AND Kelembaban is Dry (4/5)
THEN Durasi is **Long** (2/3)
 - IF Suhu is Warm (2/3) AND Kelembaban is Moist (1/5)
THEN Durasi is **Medium** (1/5)
 - IF Suhu is Hot (1/3) AND Kelembaban is Dry (4/5)
THEN Durasi is **Long** (1/3)
 - IF Suhu is Hot (1/3) AND Kelembaban is Moist (1/5)
THEN Durasi is **Medium** (1/5)

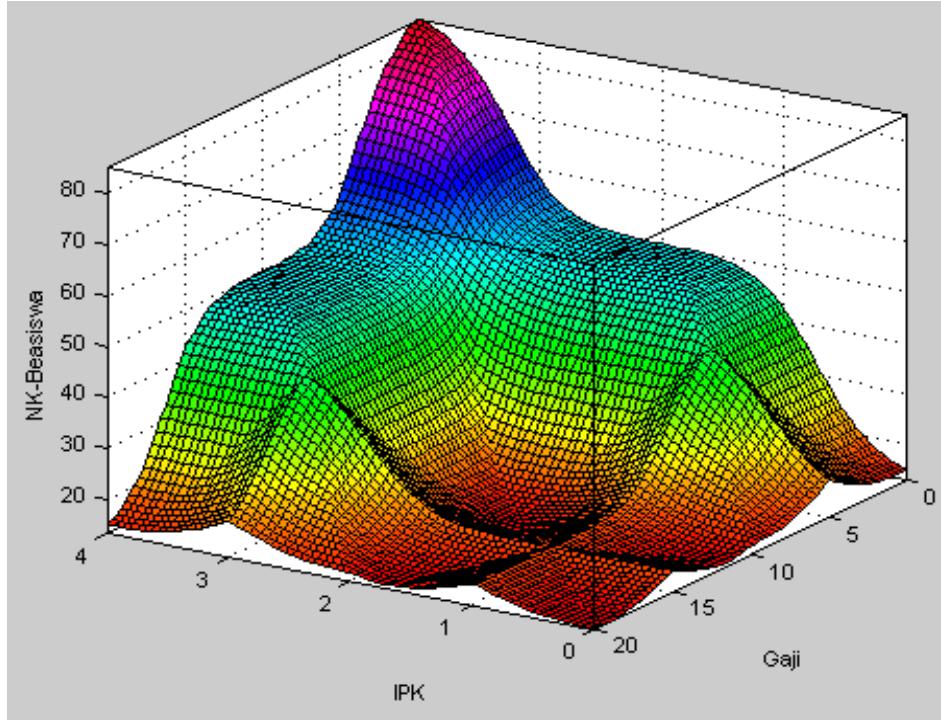
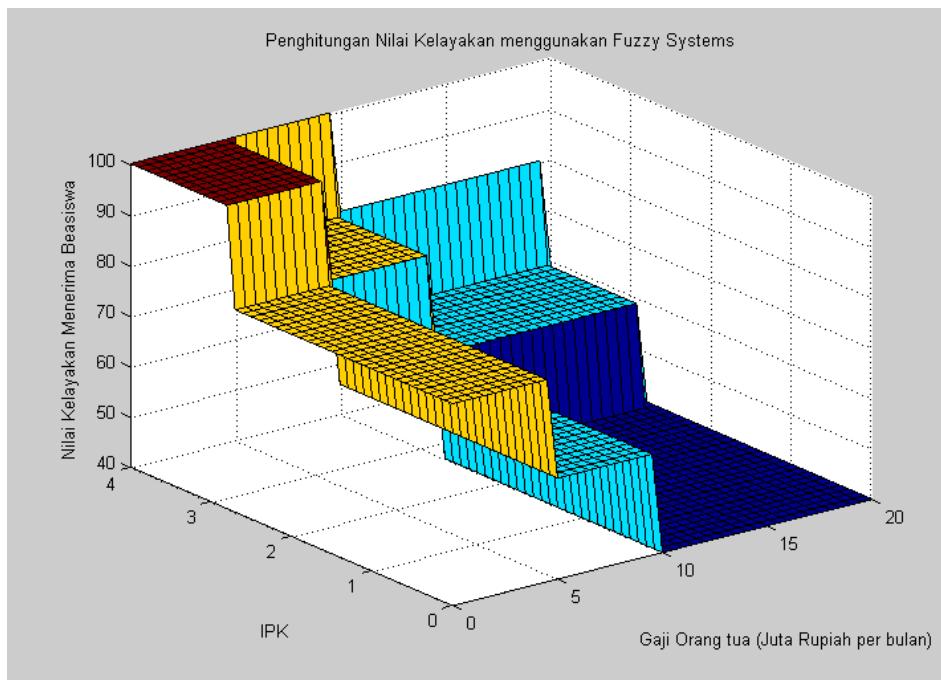
Inference: Model Sugeno



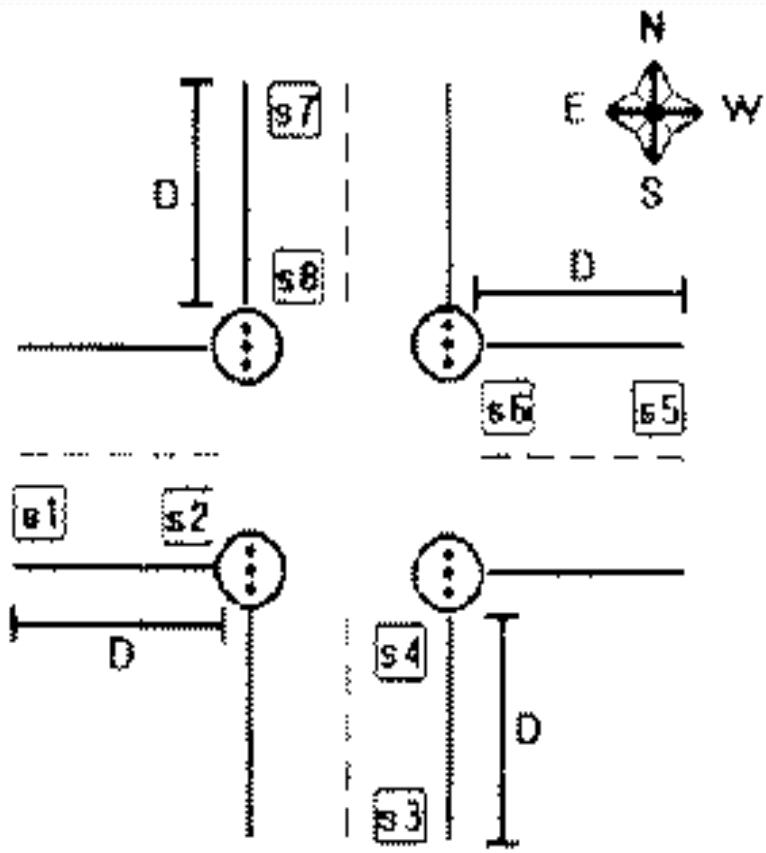
Defuzzyfication: Weighted Average



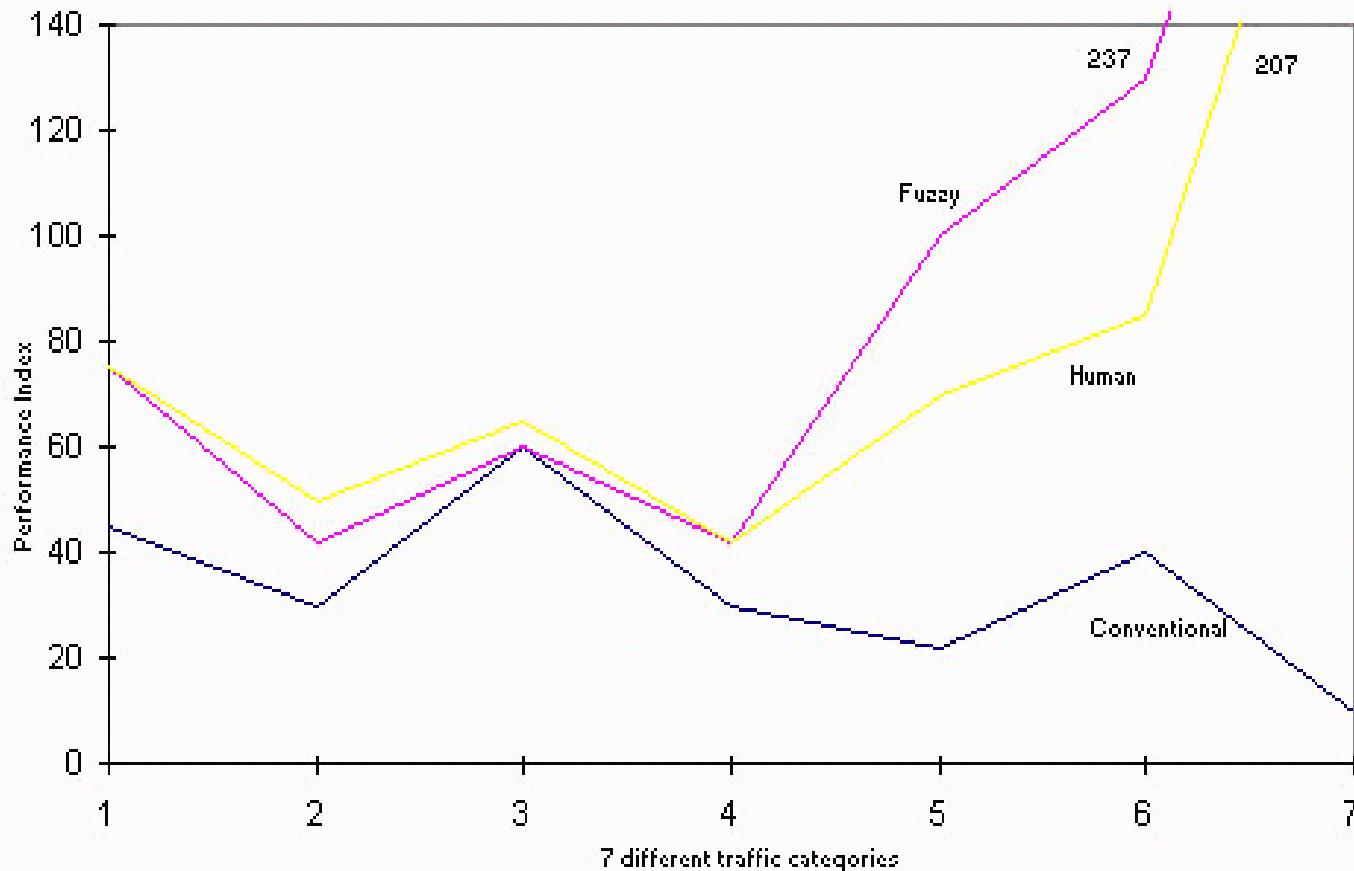
$$y^* = \frac{1/5(40) + 2/3(60)}{1/5 + 2/3} = 55,38$$



Kasus 3: Traffic Light Controller



Kasus 3: *Traffic Light Controller*



Kasus 4: Akreditasi Prodi

- Pendidikan dan Pengajaran
 - Pendidikan Dosen: S₁, S₂, S₃?
 - Jabatan Akademik: AA, L, LK, GB?
 - Rasio Dosen : Mhs?
- Penelitian
 - Dana proyek?
 - Paper Ilmiah?
- Pengabdian Masyarakat
 - Jumlah kegiatan per tahun?
 - Manfaat bagi masyarakat?

Kasus 5: *University Rankings*

- Bagaimana mengelompokkan universitas ke dalam ***cluster*** yang secara statistik mirip untuk semua kriteria yg ada?
- ***Clustering harus*** dilakukan tanpa asumsi apapun tentang tingkat kepentingan relatif dari semua kriteria.
- Kriteria:
 - Jumlah profesor?
 - Jumlah mhs asing?
 - Rasio Dosen:Mhs?
 - Besarnya Dana proyek?
 - Jumlah Paper Ilmiah?

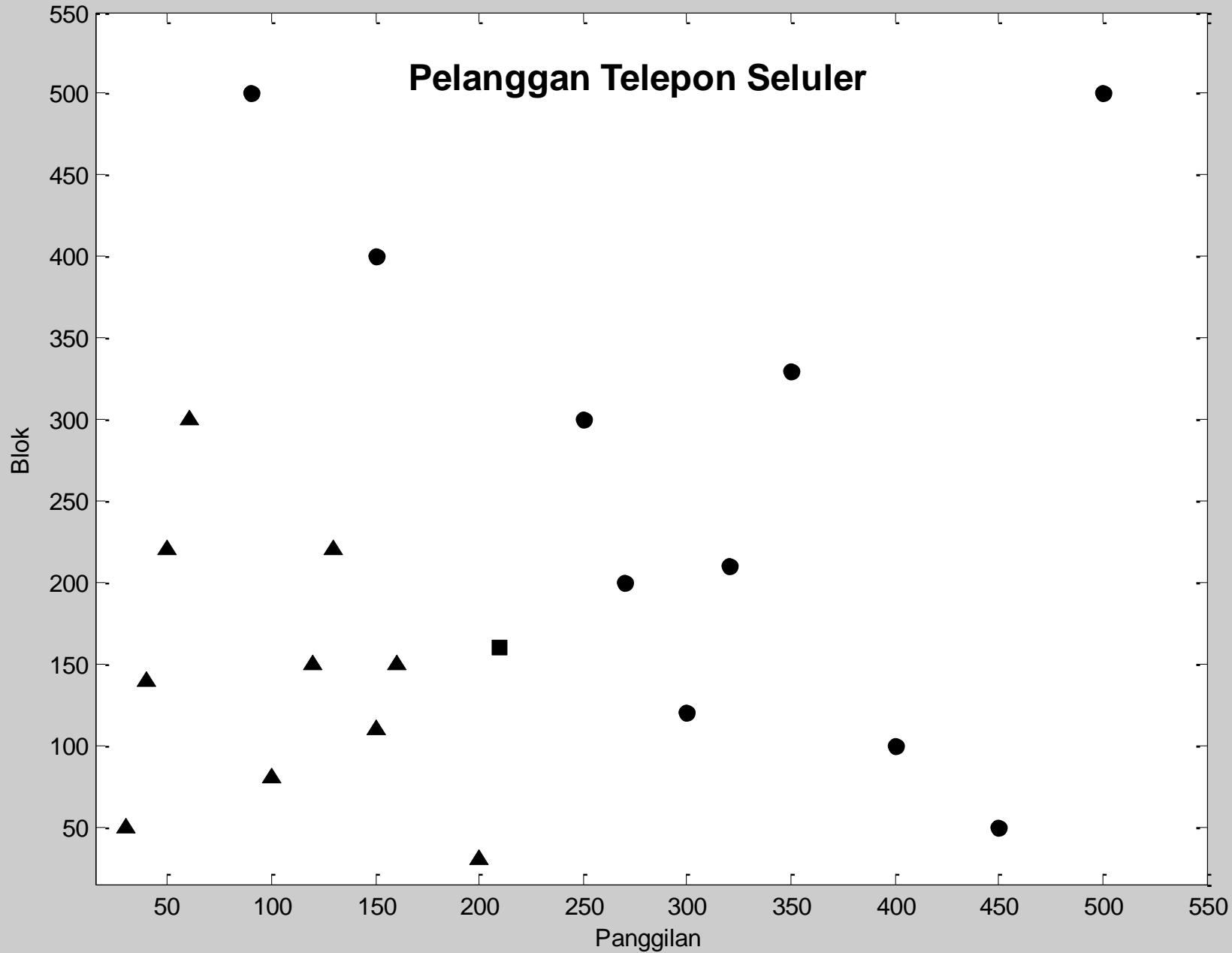
Kasus 5: *University Rankings*

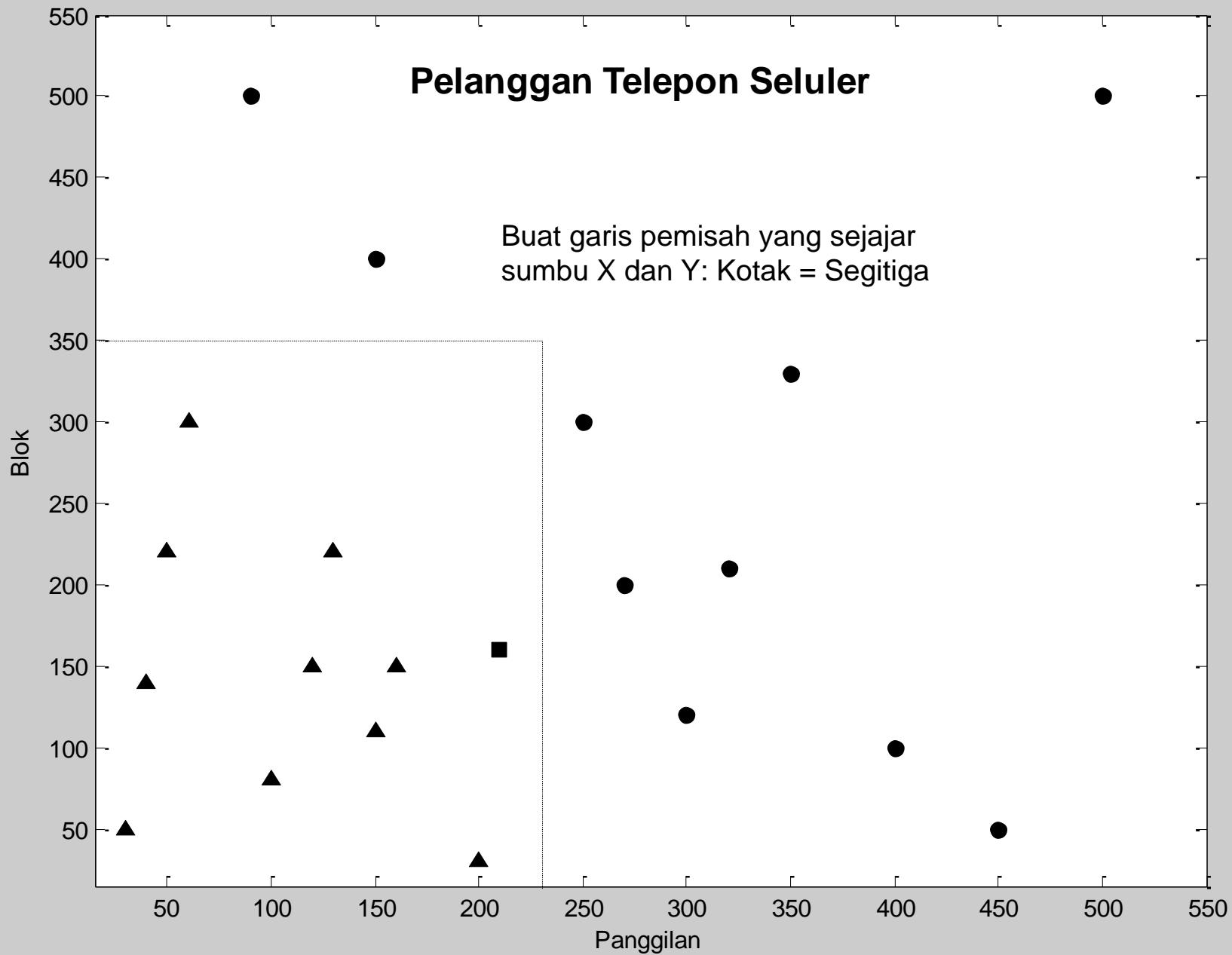
- Shanghai Jao Tong University (SJTU)
- The Time Higher League Table (THES)
- Metode: ***Fuzzy Clustering***
- Top 500 Univerisities in the world
 1. Harvard
 2. Stanford
 3. Cambridge

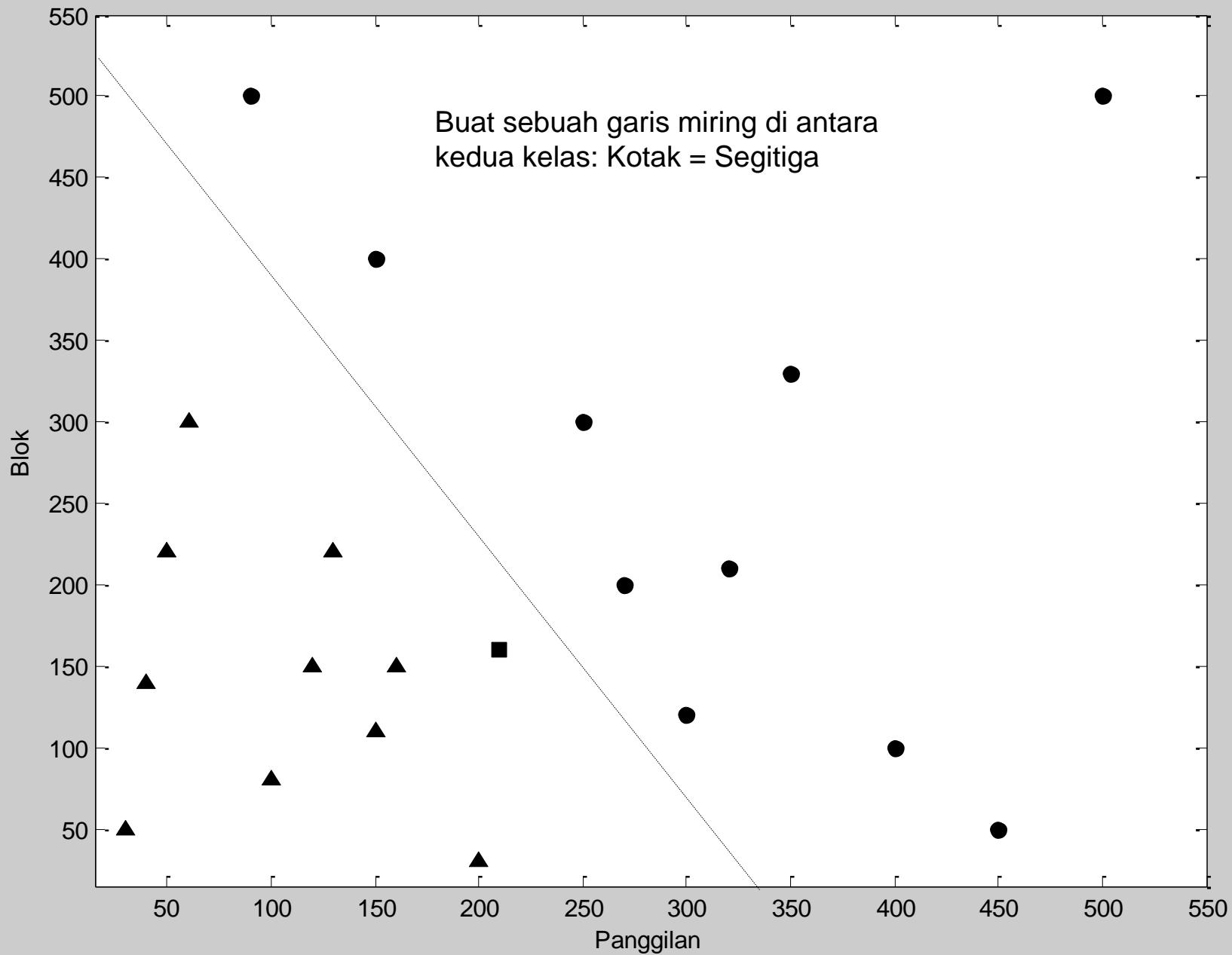
Data Pelanggan Telepon Seluler

Panggilan	Blok	Bonus
30	50	Tidak
40	140	Tidak
50	220	Tidak
60	300	Tidak
100	80	Tidak
120	150	Tidak
130	220	Tidak
150	110	Tidak
160	150	Tidak
200	30	Tidak

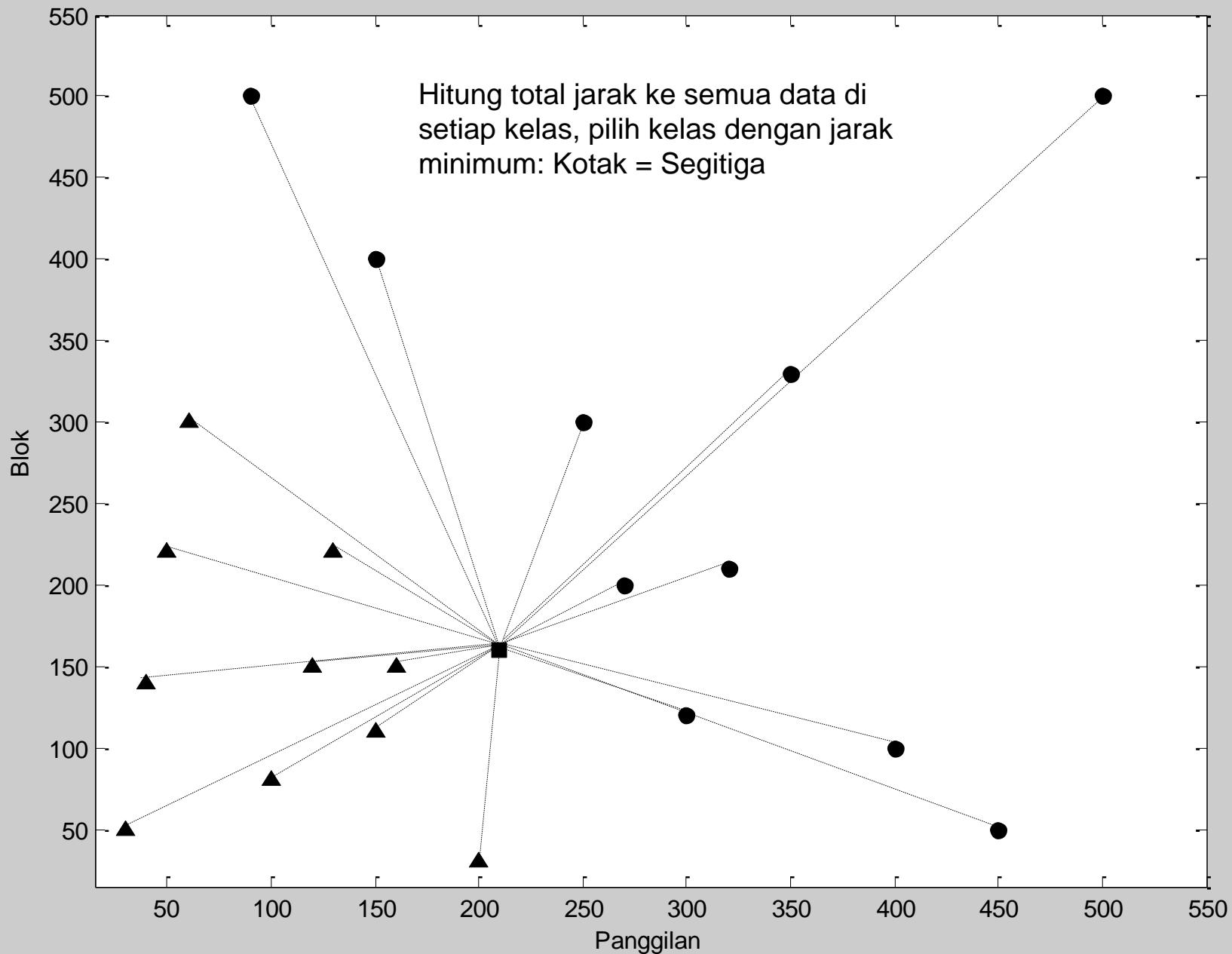
Panggilan	Blok	Bonus
90	500	Ya
150	400	Ya
250	300	Ya
270	200	Ya
300	120	Ya
320	210	Ya
350	330	Ya
400	100	Ya
450	50	Ya
500	500	Ya

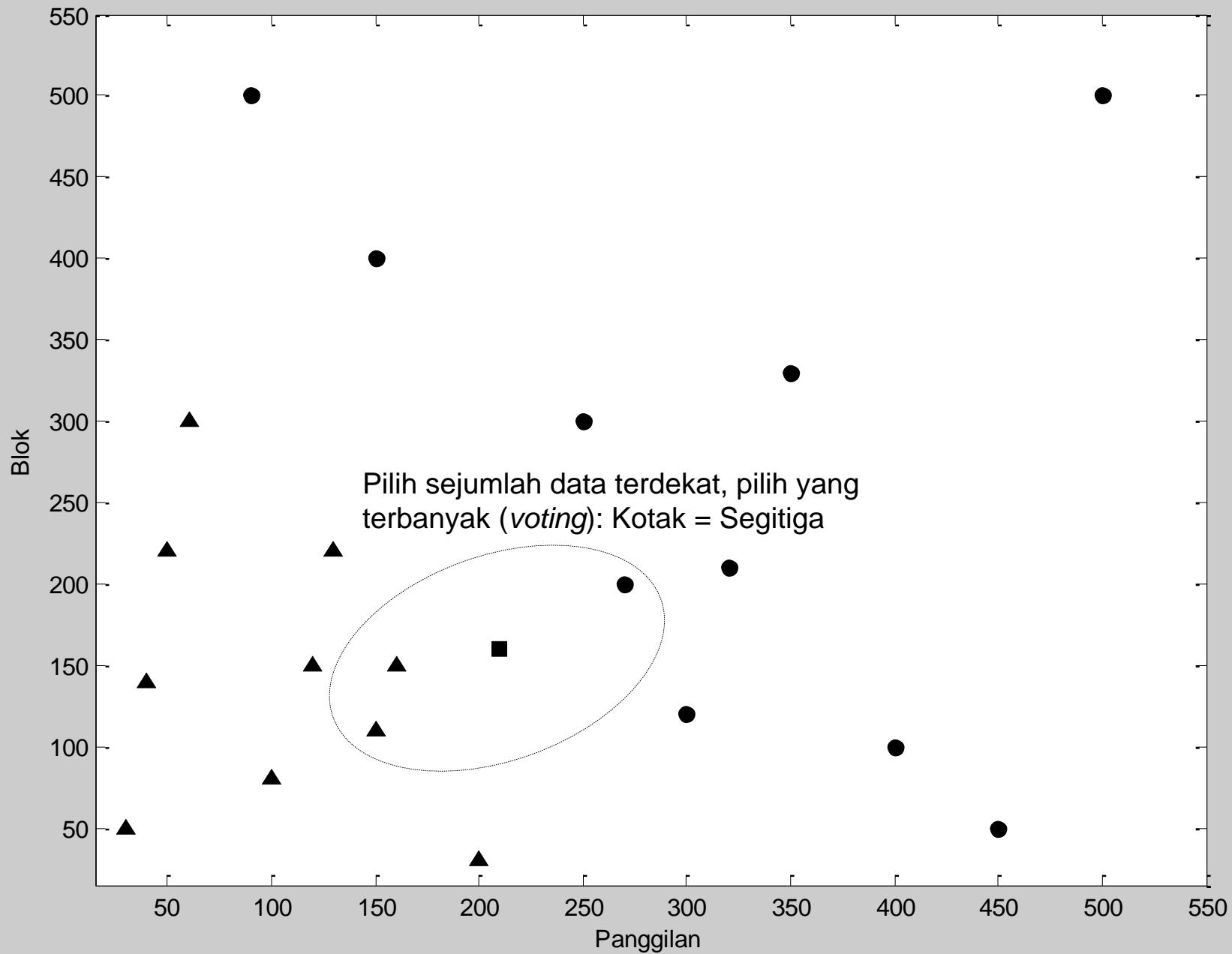


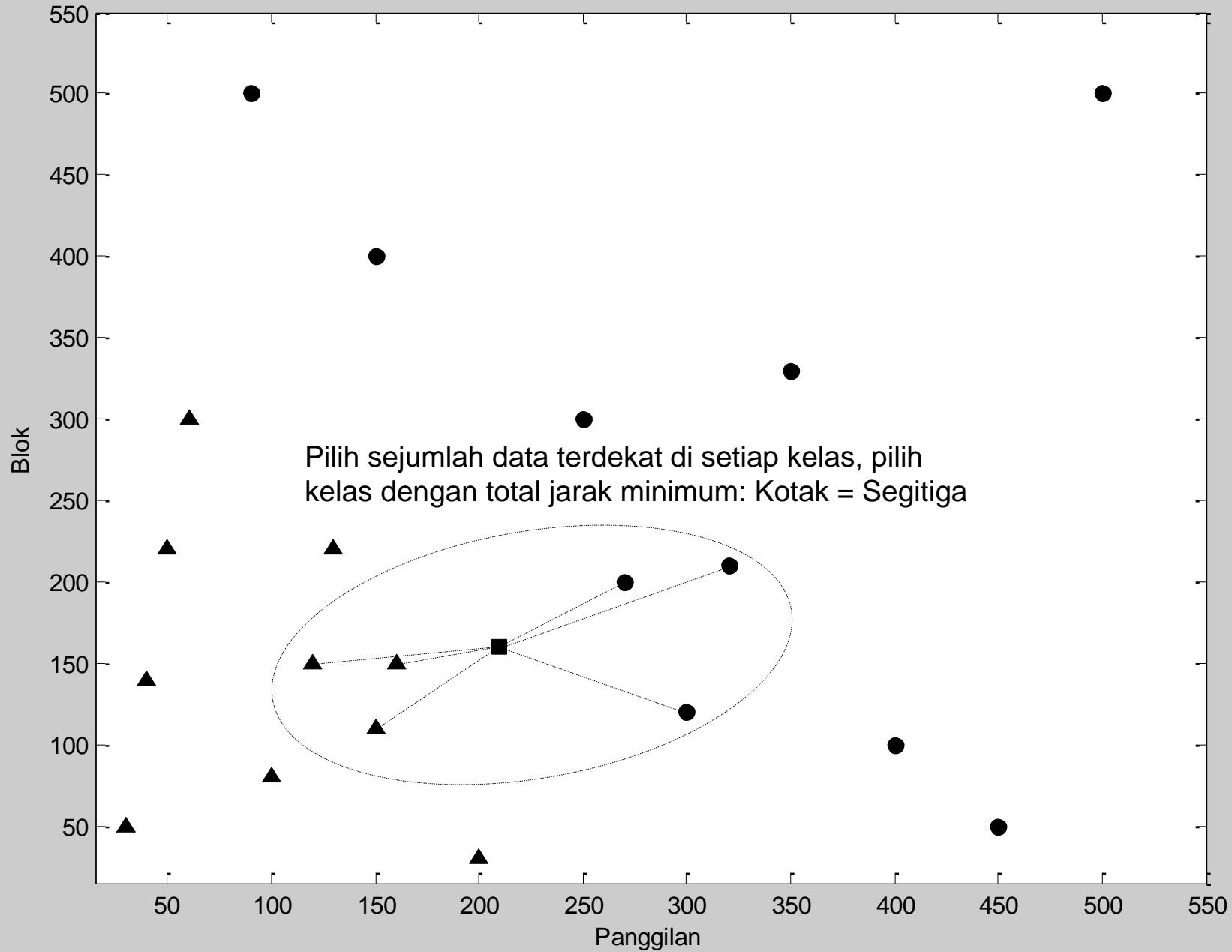


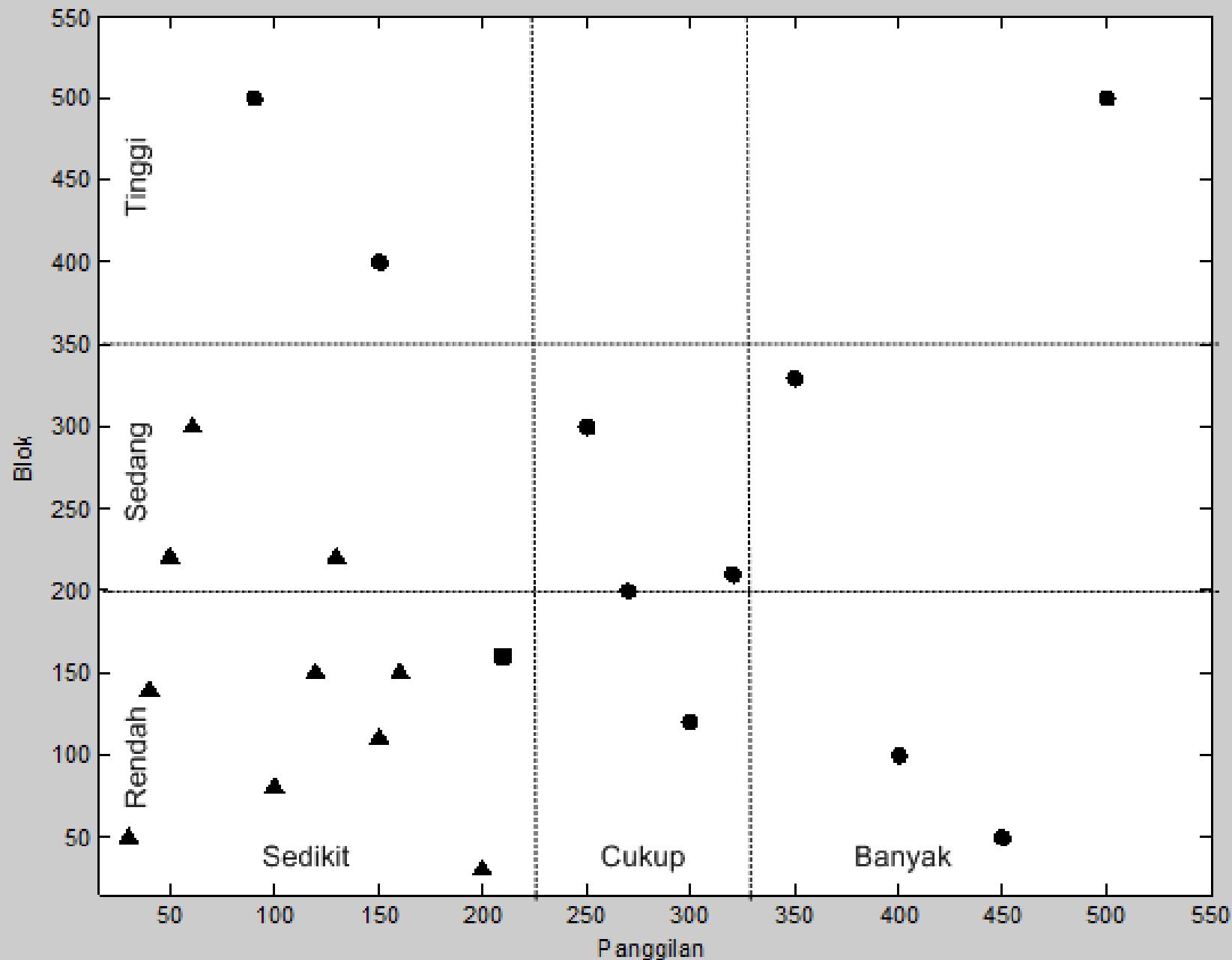


Buat sebuah garis miring di antara
kedua kelas: Kotak = Segitiga

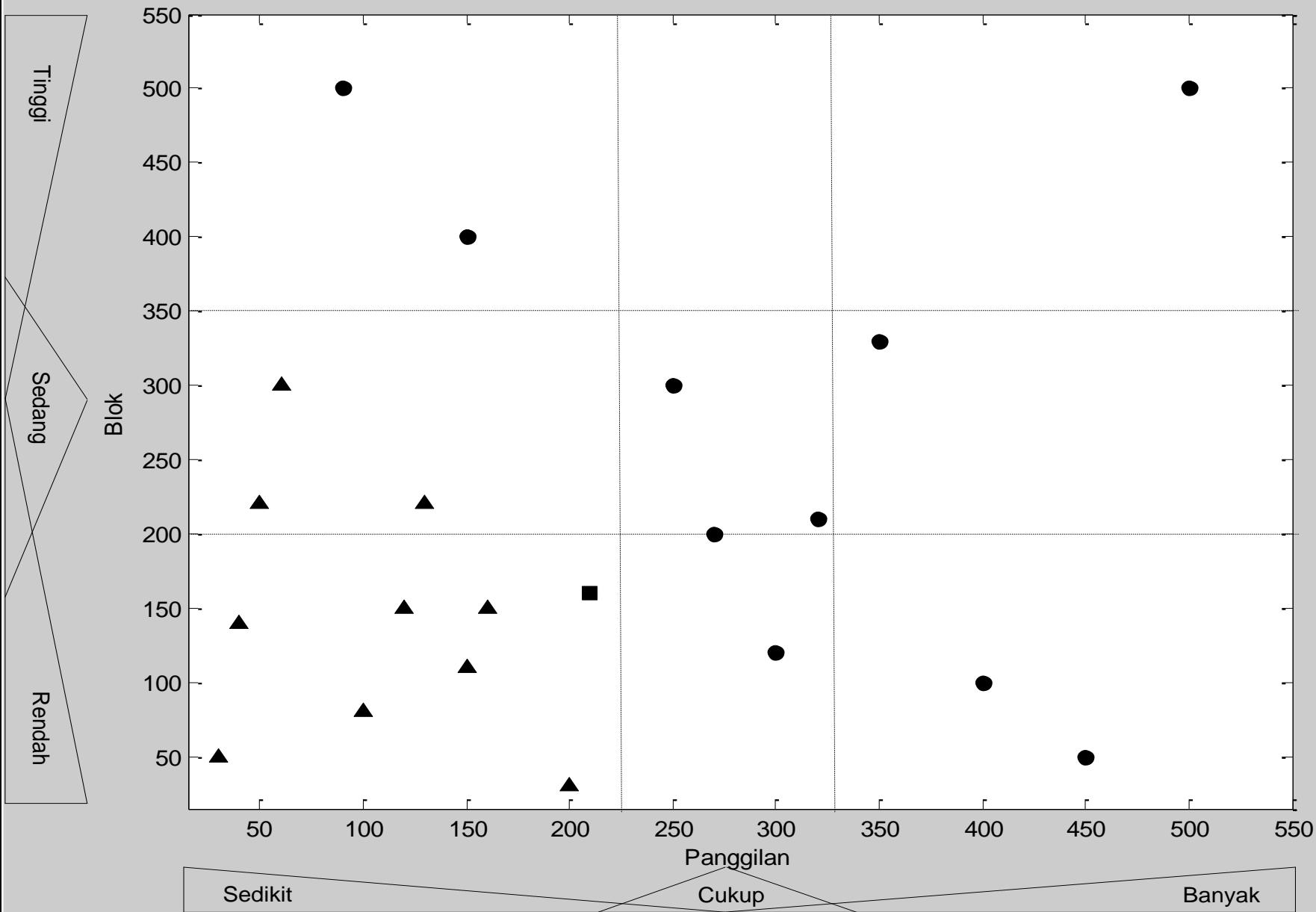




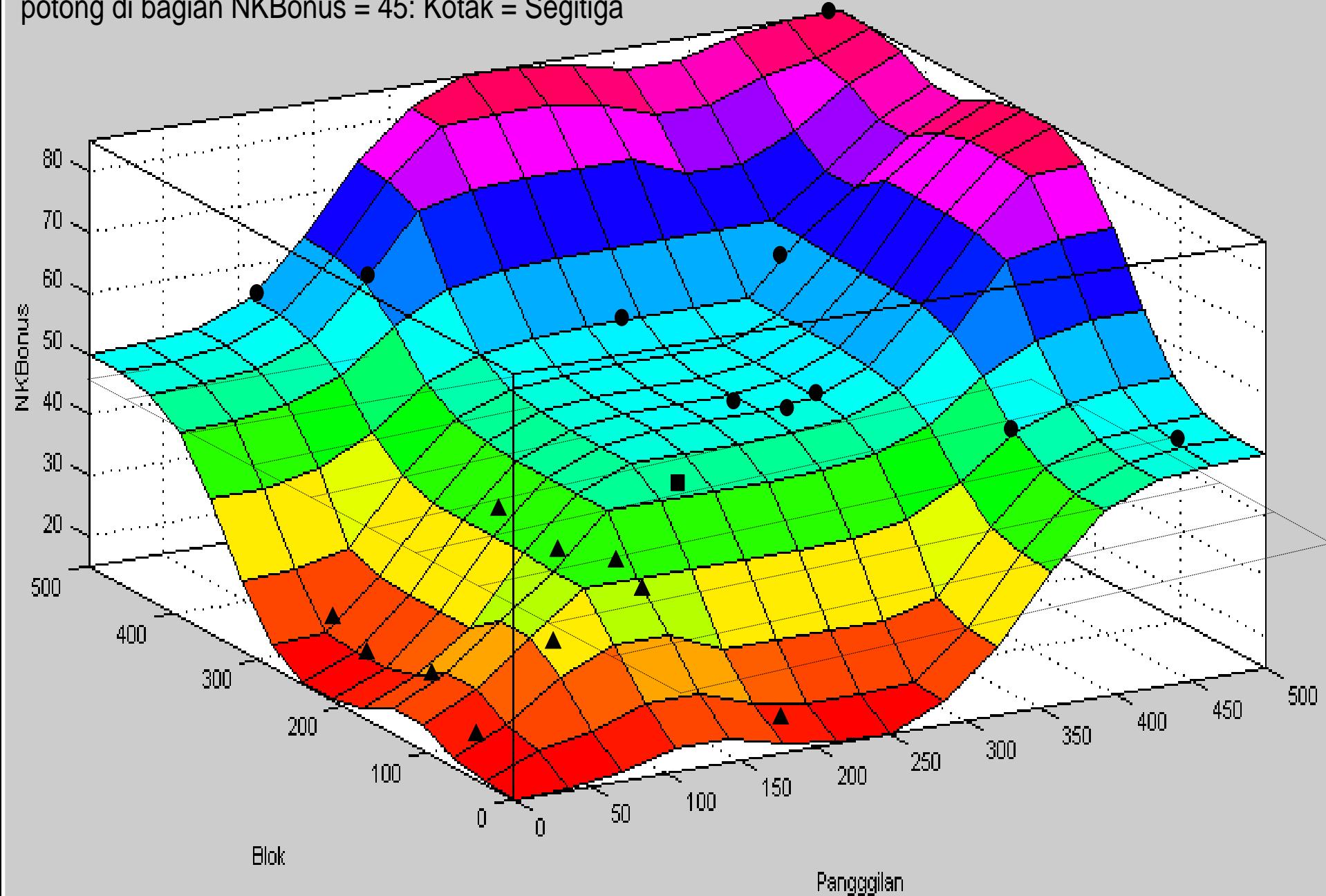


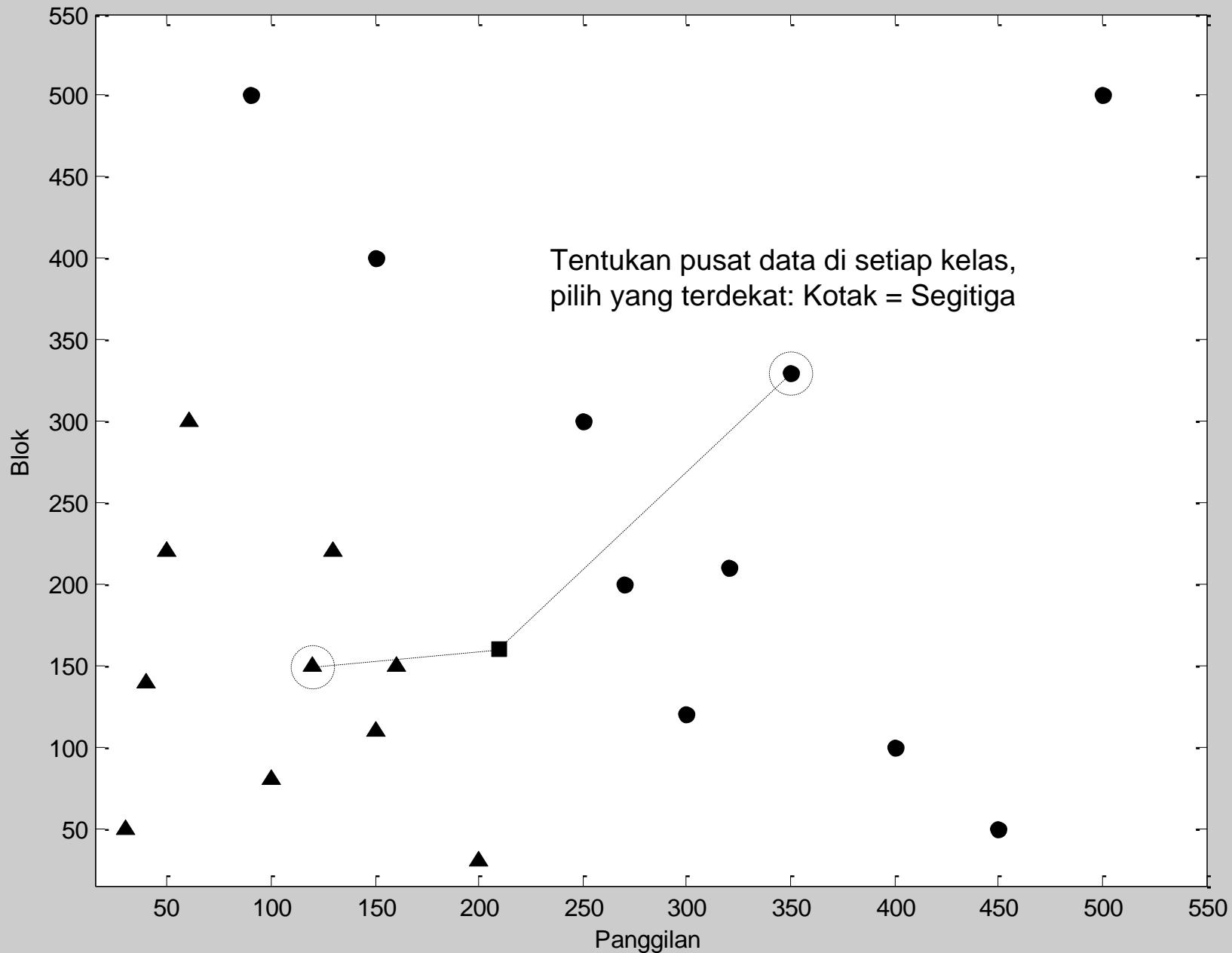


Bagi Panggilan dan Blok ke dalam tiga nilai linguistik, lalu lakukan *fuzzification* dan inferensi *berbasis fuzzy rules* untuk menentukan kelas. Kotak = Segitiga



Buat fungsi yang menarik semua data ke atas, lalu potong di bagian $NKBonus = 45$: Kotak = Segitiga



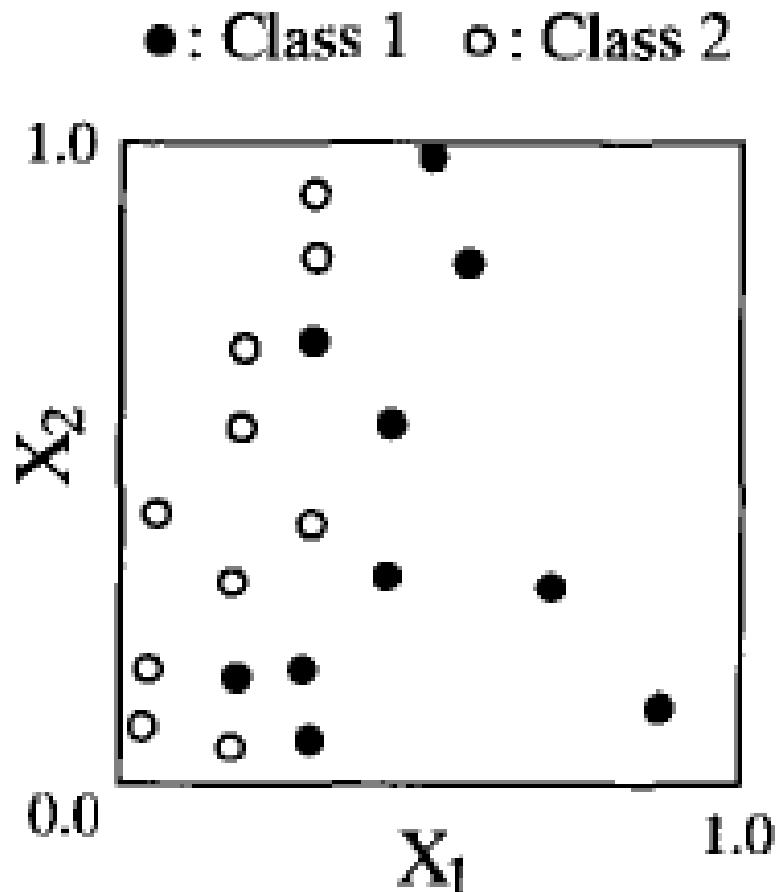


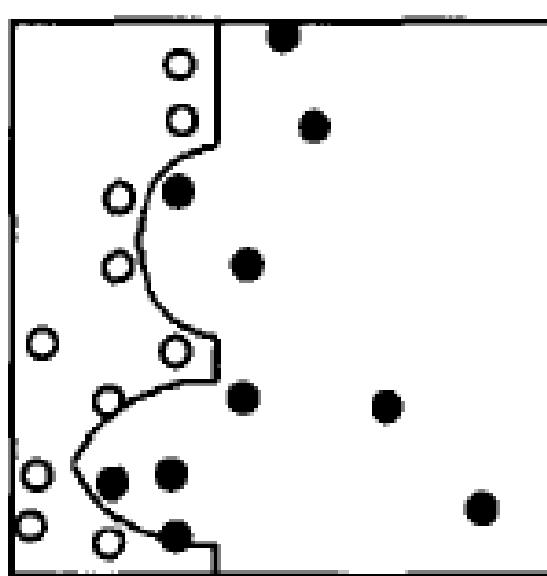
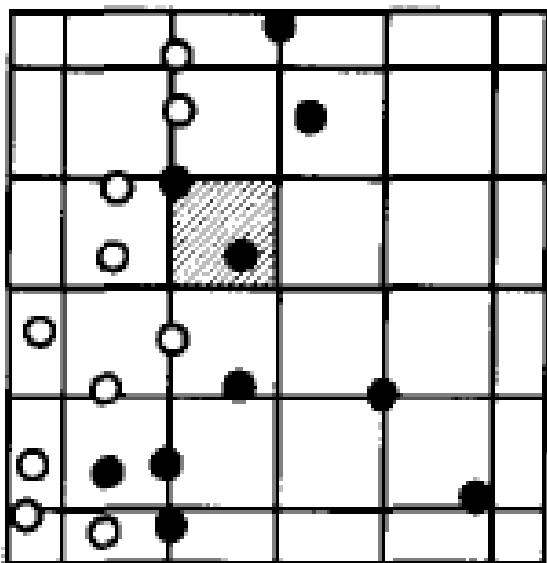
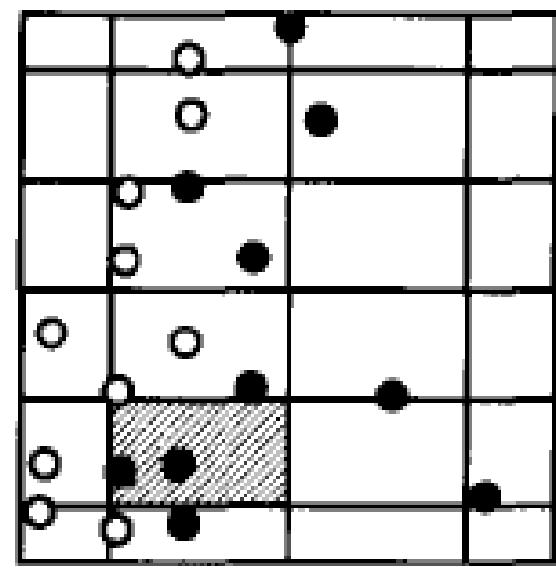
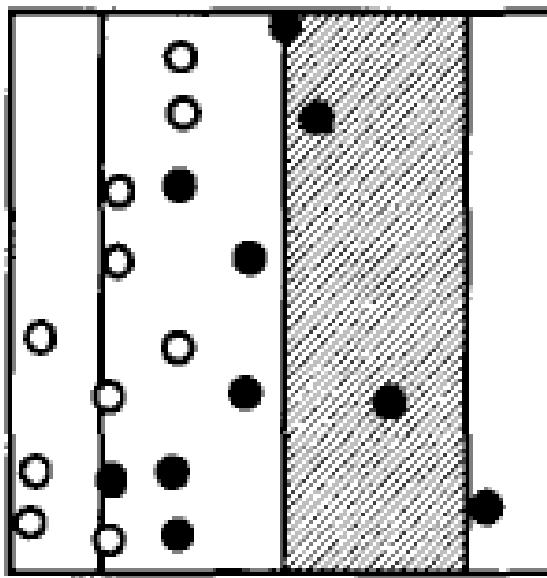
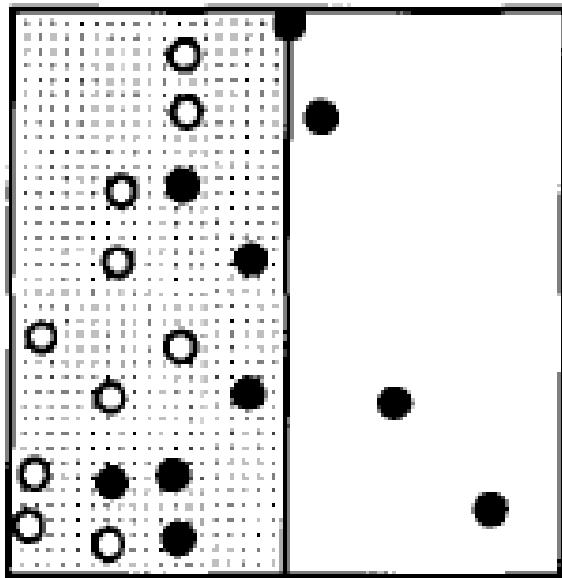
Construction of fuzzy classification systems with rectangular fuzzy rules using genetic algorithms

Hisao Ishibuchi*, Ken Nozaki, Naohisa Yamamoto, Hideo Tanaka

Department of Industrial Engineering, University of Osaka Prefecture, Gakuen-cho 1-1, Sakai, Osaka 593, Japan

Received January 1994; revised February 1994

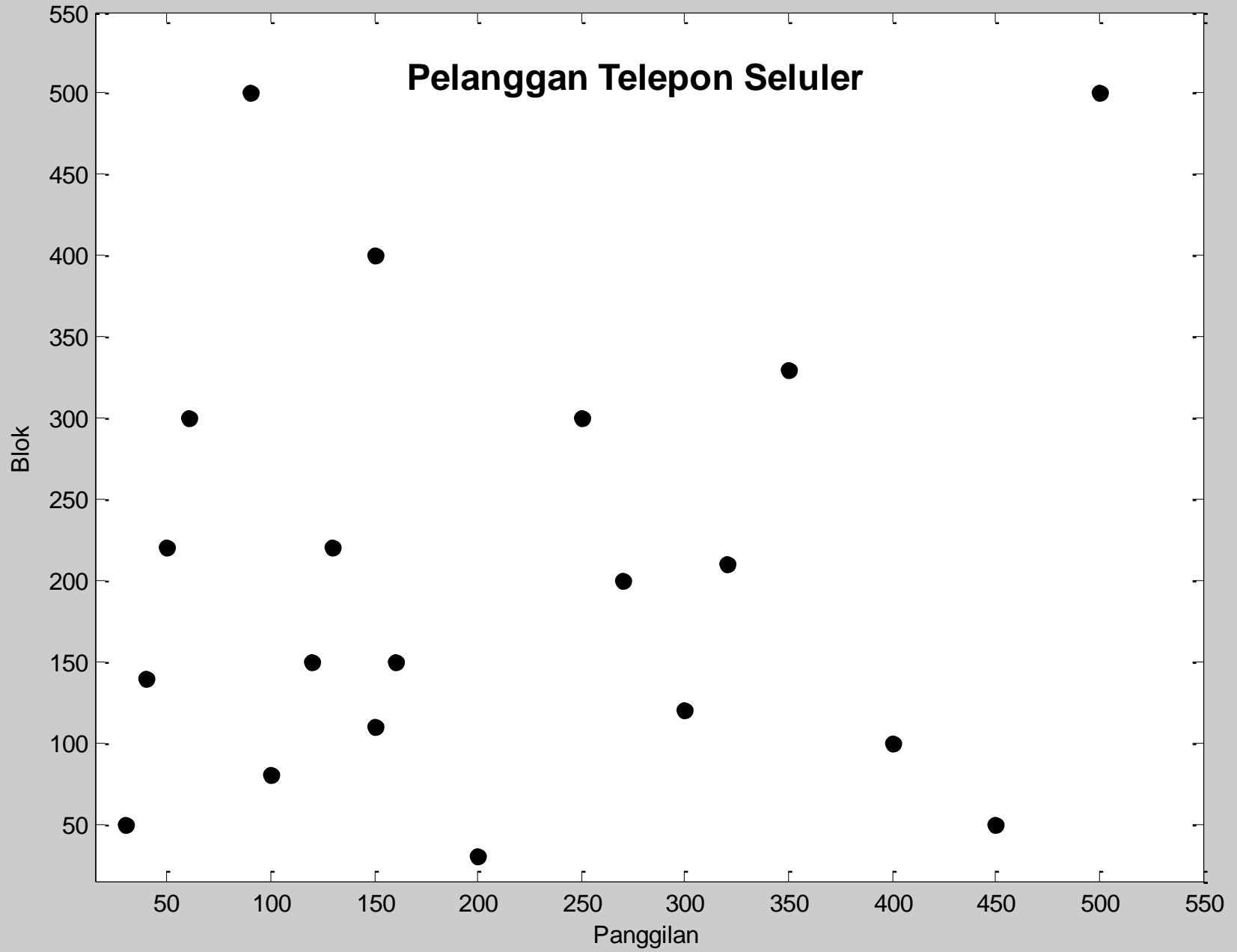


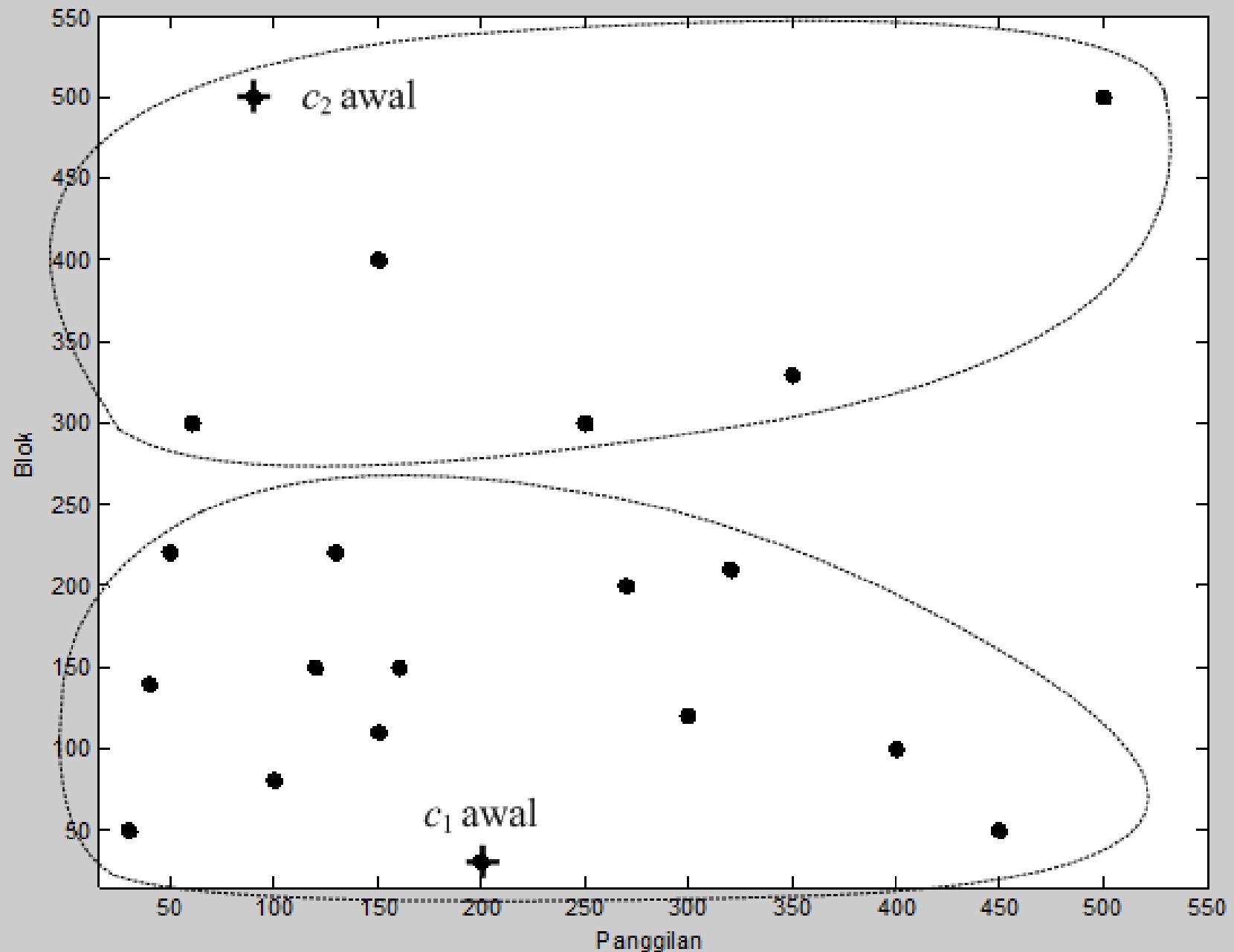


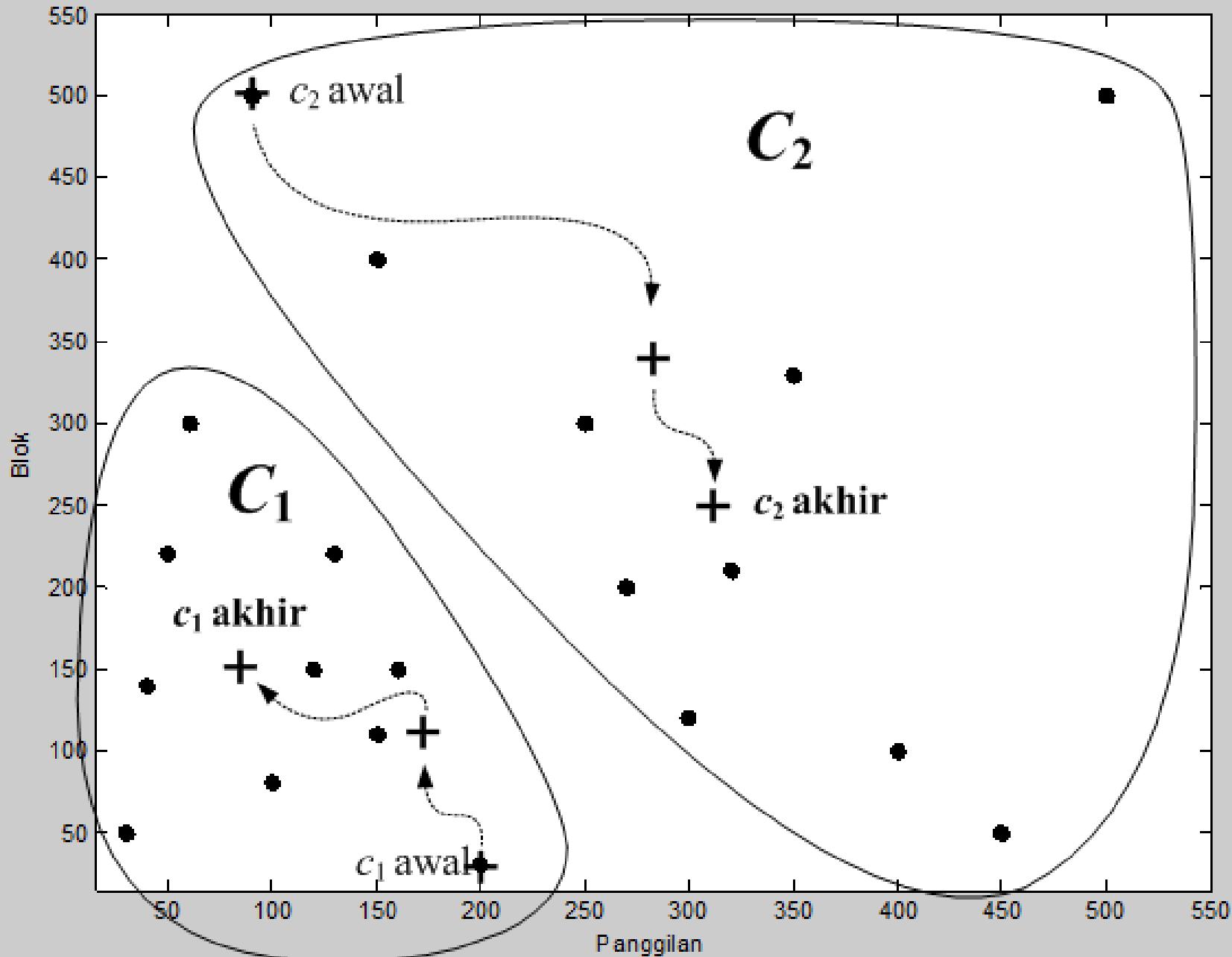
Data Pelanggan Telepon Seluler

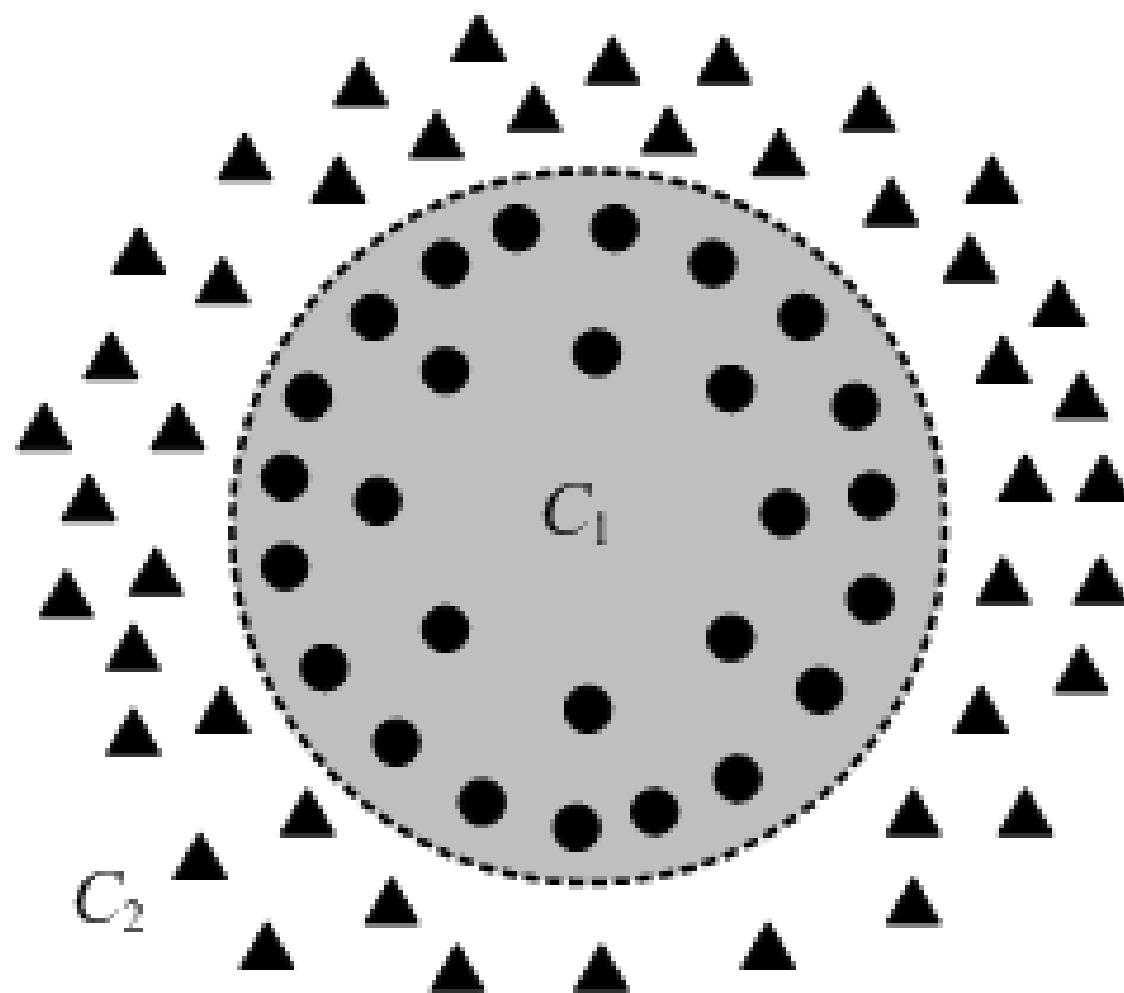
Panggilan	Blok
30	50
40	140
50	220
60	300
100	80
120	150
130	220
150	110
160	150
200	30

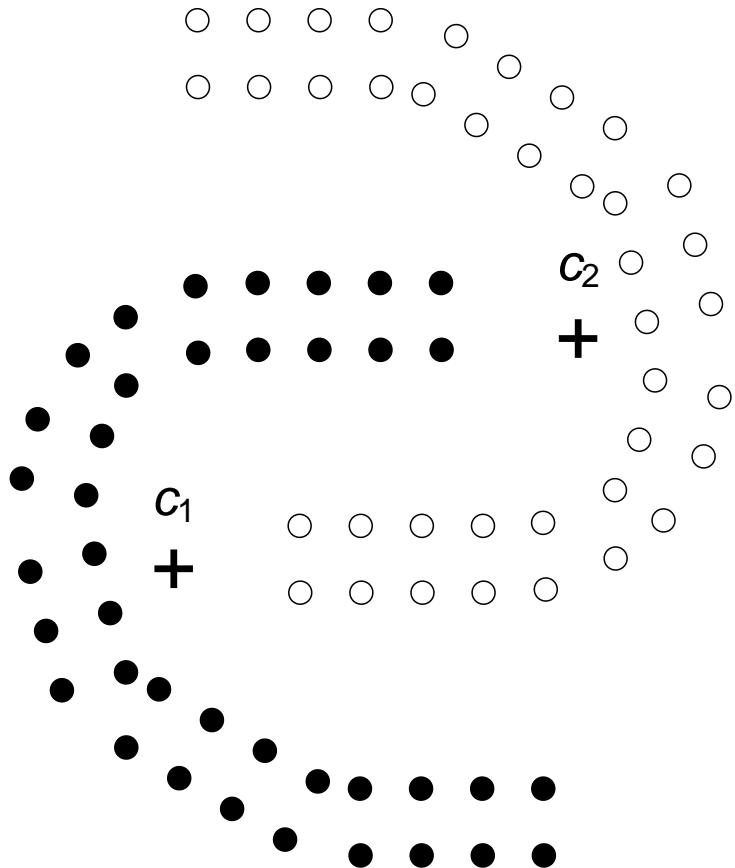
Panggilan	Blok
90	500
150	400
250	300
270	200
300	120
320	210
350	330
400	100
450	50
500	500

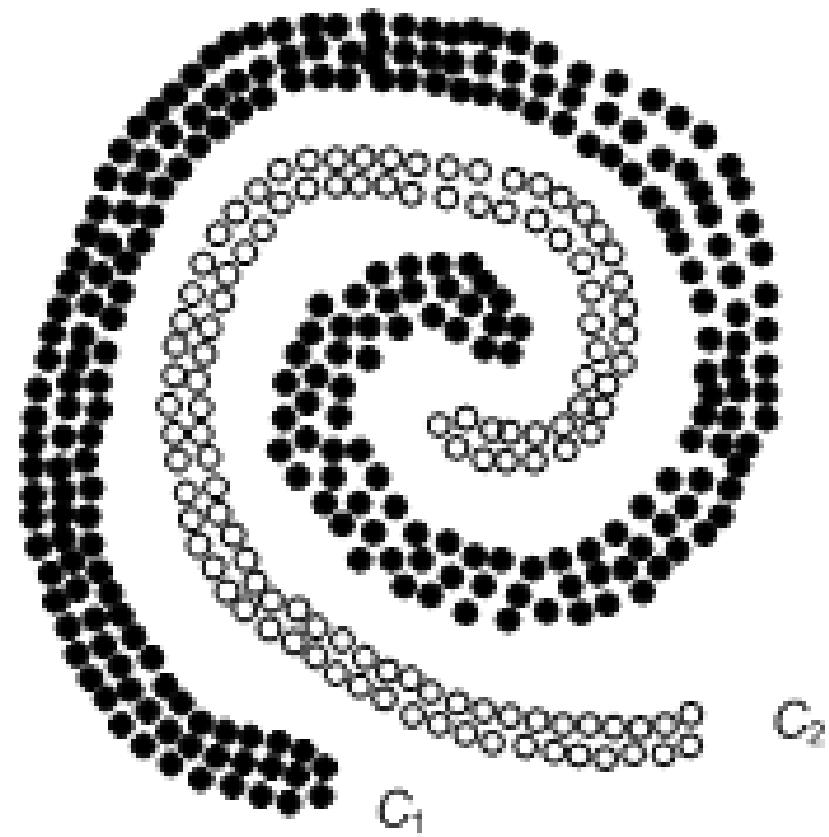












Fuzzy Clustering

- To extract rules from data
- Fuzzy c-means
- Aplikasi:
 - Cracked Tile Detection
 - Finding Cancer Cells
 - Image Segmentation
 - Document clustering

Cluster

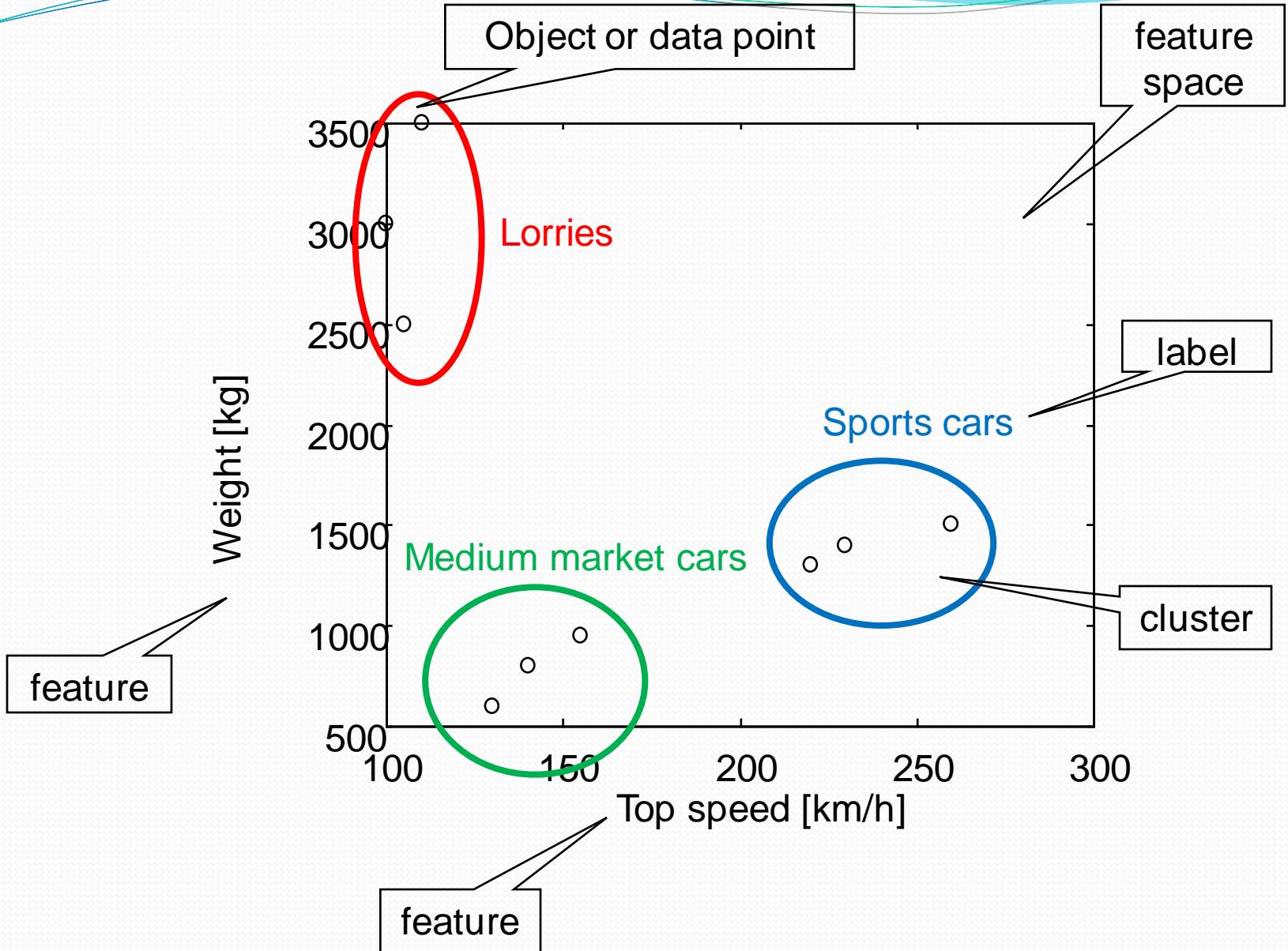
- Sejumlah individu yang memiliki sifat hampir sama atau terjadi bersama-sama
- Kelompok universitas kelas dunia
- Kelompok mobil kategori tertentu
- Kumpulan spam email
- Kumpulan tipe penyakit kanker

Cluster Analysis

- A statistical classification technique
- for discovering whether the individuals of a population fall into different groups
- by making **quantitative comparisons** of multiple characteristics.

Vehicle

Vehicle	Top speed km/h	Colour	Air resistance	Weight Kg
V1	220	red	0.30	1300
V2	230	black	0.32	1400
V3	260	red	0.29	1500
V4	140	gray	0.35	800
V5	155	blue	0.33	950
V6	130	white	0.40	600
V7	100	black	0.50	3000
V8	105	red	0.60	2500
V9	110	gray	0.55	3500



Kasus 6: *Tile Clustering*

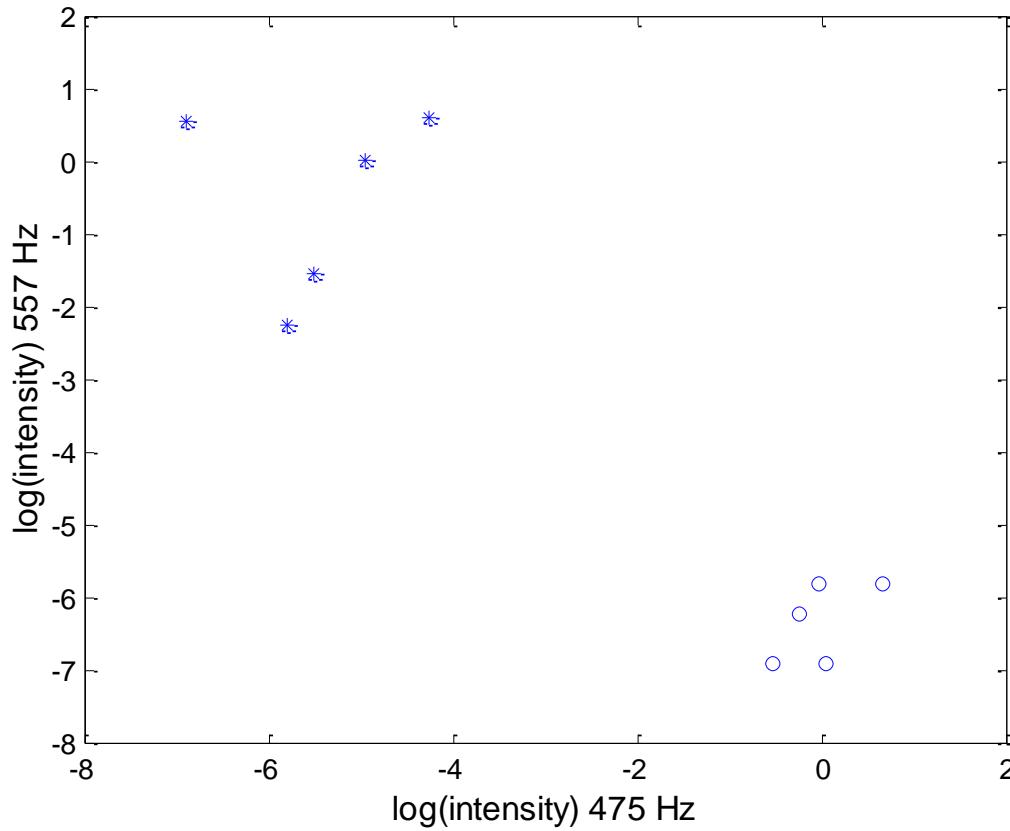
- Tiles are made from clay moulded into the right shape, brushed, glazed, and baked.
- Unfortunately, the baking may produce invisible cracks.
- Operators can detect the cracks by hitting the tiles with a hammer, and in an automated system the response is recorded with a microphone, filtered, Fourier transformed, and normalised.
- A small set of data is given in the TABLE below.

Kasus 6: *Tile Clustering*

475Hz	557Hz	Ok?
0.958	0.003	Yes
1.043	0.001	Yes
1.907	0.003	Yes
0.780	0.002	Yes
0.579	0.001	Yes
0.003	0.105	No
0.001	1.748	No
0.014	1.839	No
0.007	1.021	No
0.004	0.214	No

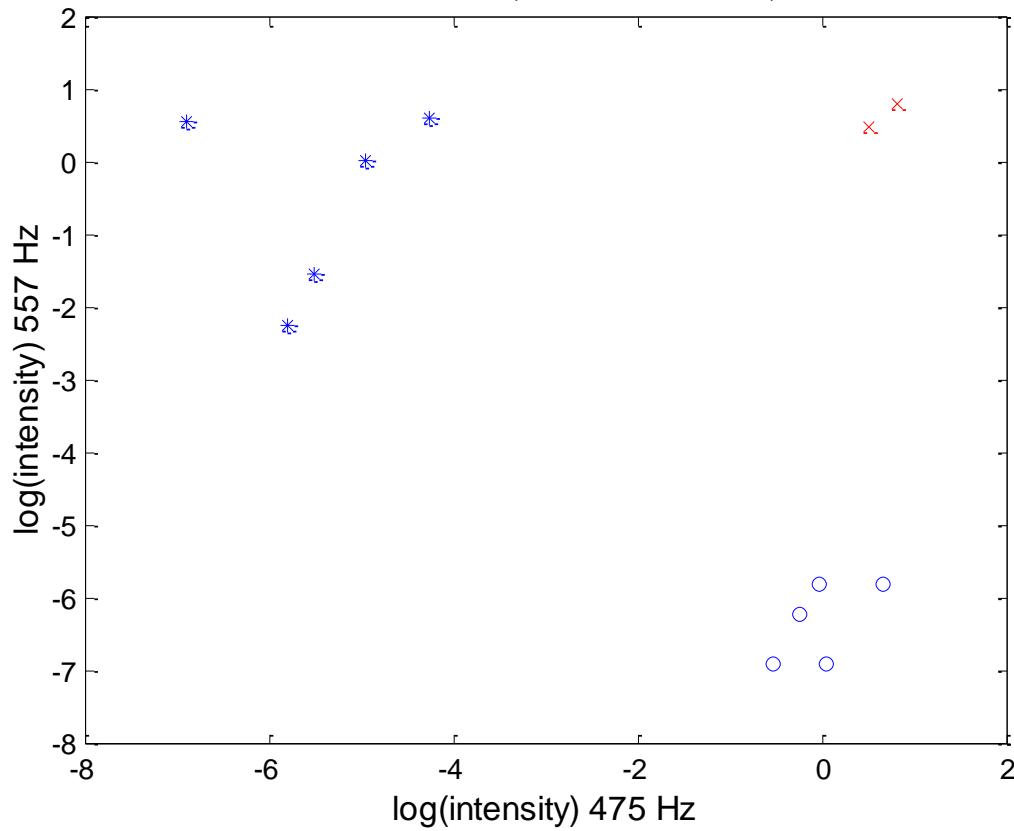
Hard c-means (HCM) (biasa disebut *k-means*)

Tiles data: o = whole tiles, * = cracked tiles, x = centres

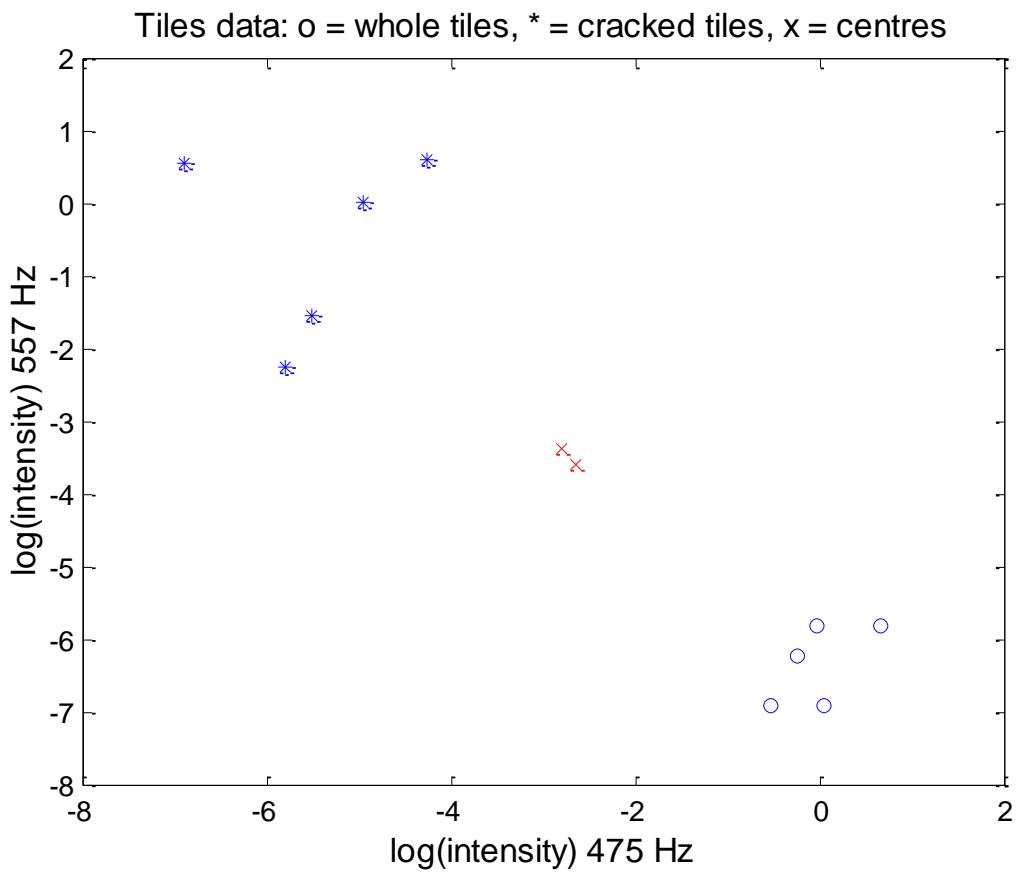


Plot of tiles by frequencies (logarithms). The whole tiles (o) seem well separated from the cracked tiles (*). The **objective** is to find the two clusters.

Tiles data: o = whole tiles, * = cracked tiles, x = centres

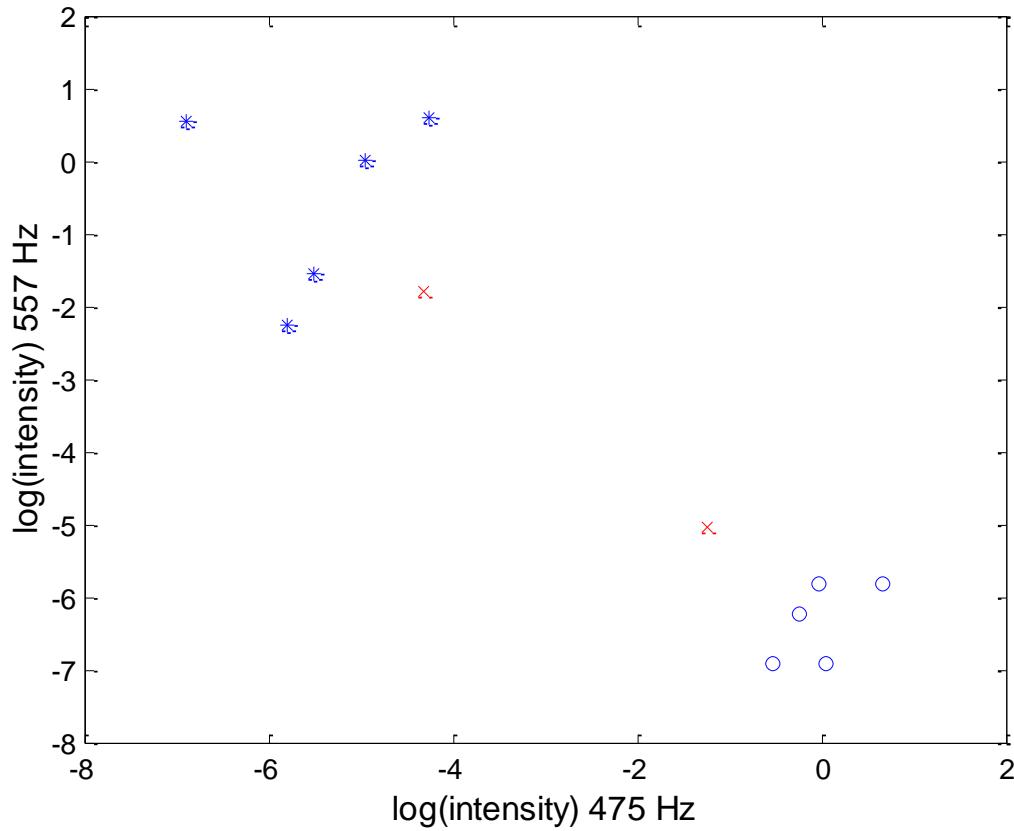


1. Place two cluster centres (x) at random.
2. Assign each data point (*) and (o) to the nearest cluster centre (x)



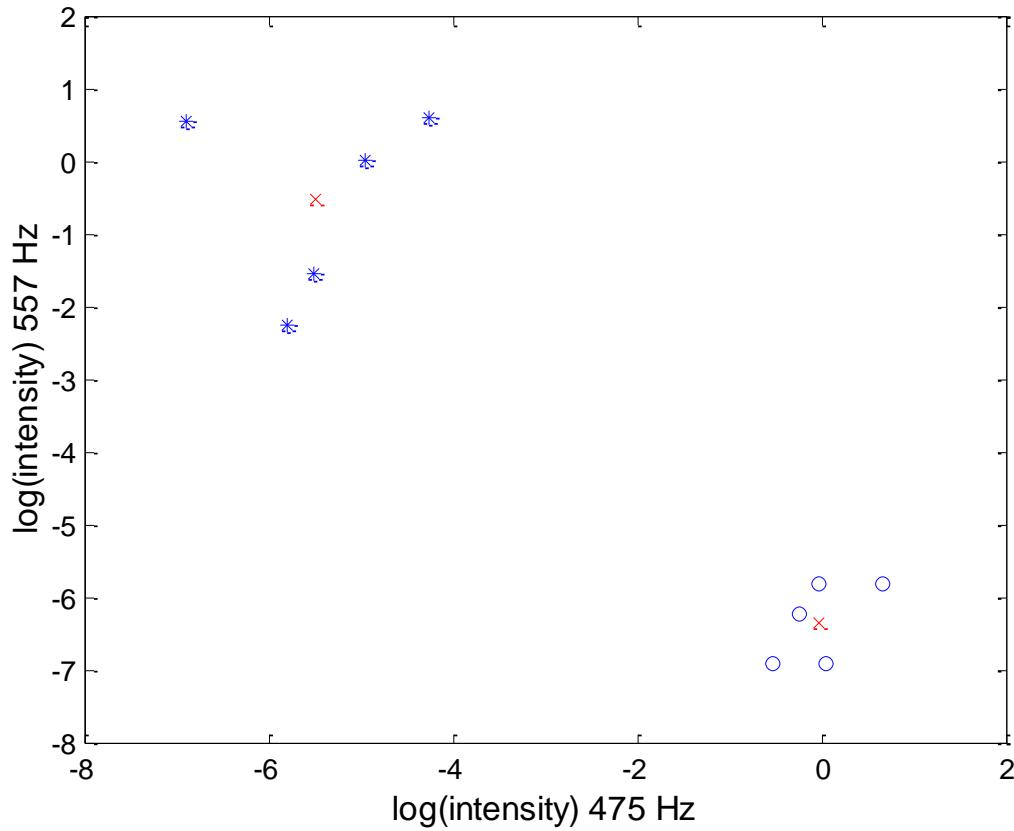
1. Compute the new centre of each class
2. Move the crosses (x)

Tiles data: o = whole tiles, * = cracked tiles, x = centres

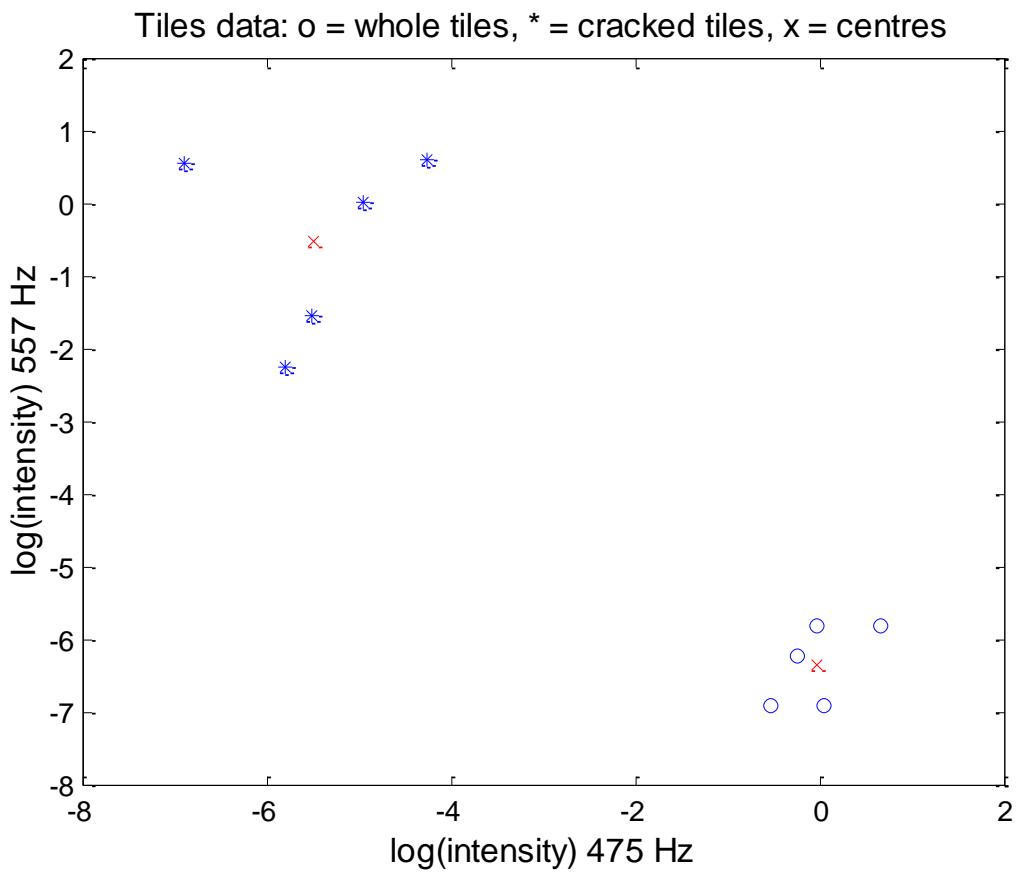


Iteration 2

Tiles data: o = whole tiles, * = cracked tiles, x = centres



Iteration 3



Iteration 4 (then stop, because no visible change)
Each data point belongs to the cluster defined by the nearest centre

M =

0.0000	1.0000
0.0000	1.0000
0.0000	1.0000
0.0000	1.0000
0.0000	1.0000
1.0000	0.0000
1.0000	0.0000
1.0000	0.0000
1.0000	0.0000
1.0000	0.0000

The membership matrix M:

1. The last five data points (rows) belong to the first cluster (column)
2. The first five data points (rows) belong to the second cluster (column)

Membership matrix \mathbf{M}

$$m_{ik} = \begin{cases} 1 & \text{if } \|\mathbf{u}_k - \mathbf{c}_i\|^2 \leq \|\mathbf{u}_k - \mathbf{c}_j\|^2 \\ 0 & \text{otherwise} \end{cases}$$

Diagram illustrating the components of the membership matrix formula:

- data point k** : Points to the variable \mathbf{u}_k .
- cluster centre i** : Points to the term \mathbf{c}_i .
- cluster centre j** : Points to the term \mathbf{c}_j .
- distance**: Points to the squared distance term $\|\mathbf{u}_k - \mathbf{c}_i\|^2$.

c-partition

All clusters C together fills the whole universe U

$$\bigcup_{i=1}^c C_i = U$$

Clusters do not overlap

$$C_i \cap C_j = \emptyset \quad \text{for all } i \neq j$$

$$\emptyset \subset C_i \subset U \quad \text{for all } i$$

A cluster C is never empty and it is smaller than the whole universe U

$$2 \leq c \leq K$$

There must be at least 2 clusters in a c-partition and at most as many as the number of data points K

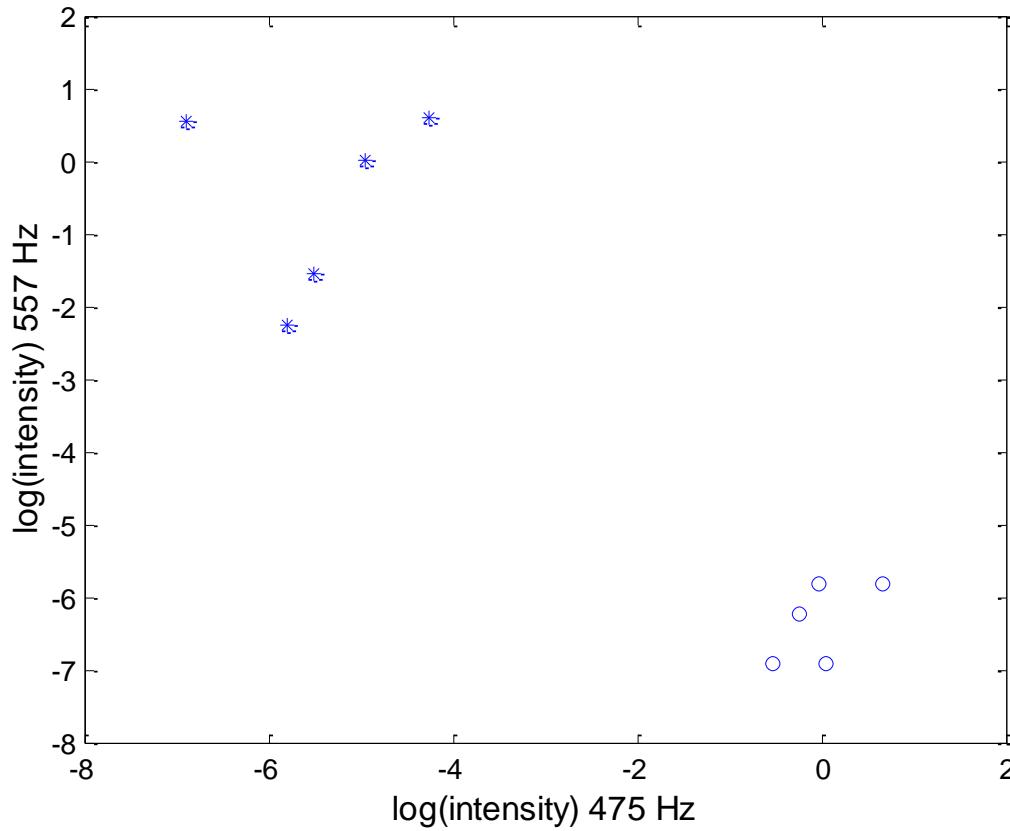
Objective function

Minimise the total sum
of all distances

$$J = \sum_{i=1}^c J_i = \sum_{i=1}^c \left(\sum_{k, \mathbf{u}_k \in C_i} \|\mathbf{u}_k - \mathbf{c}_i\|^2 \right)$$

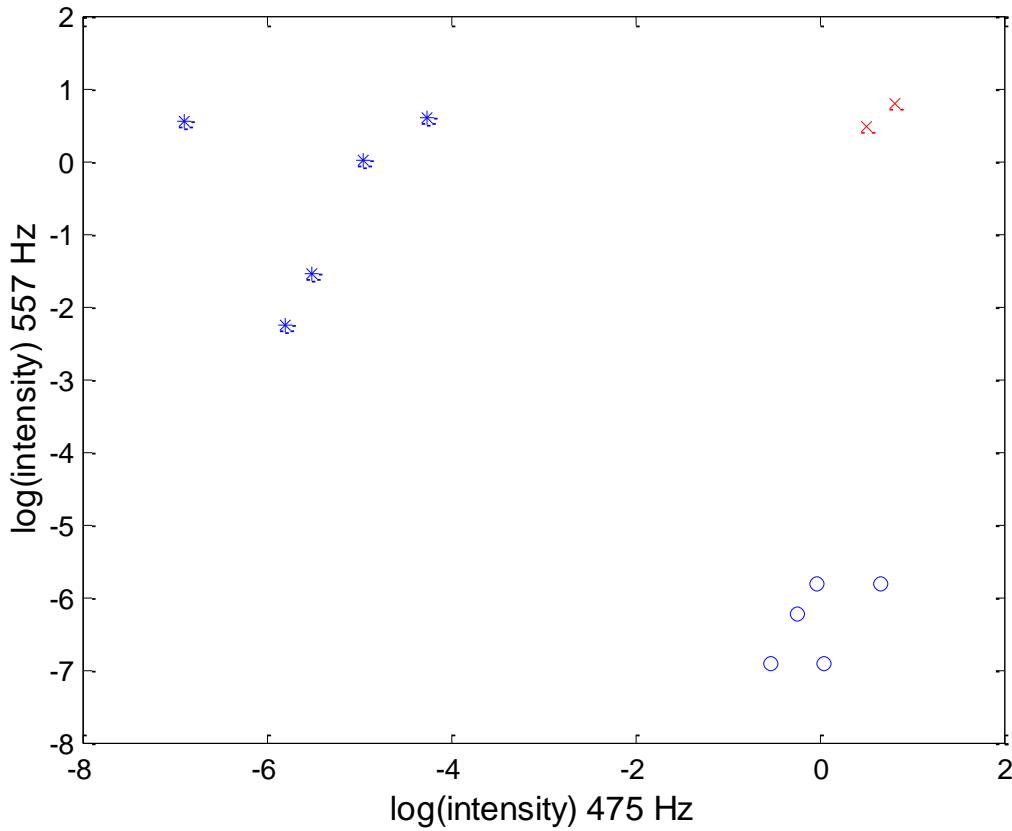
Fuzzy c-means (FCM)

Tiles data: o = whole tiles, * = cracked tiles, x = centres



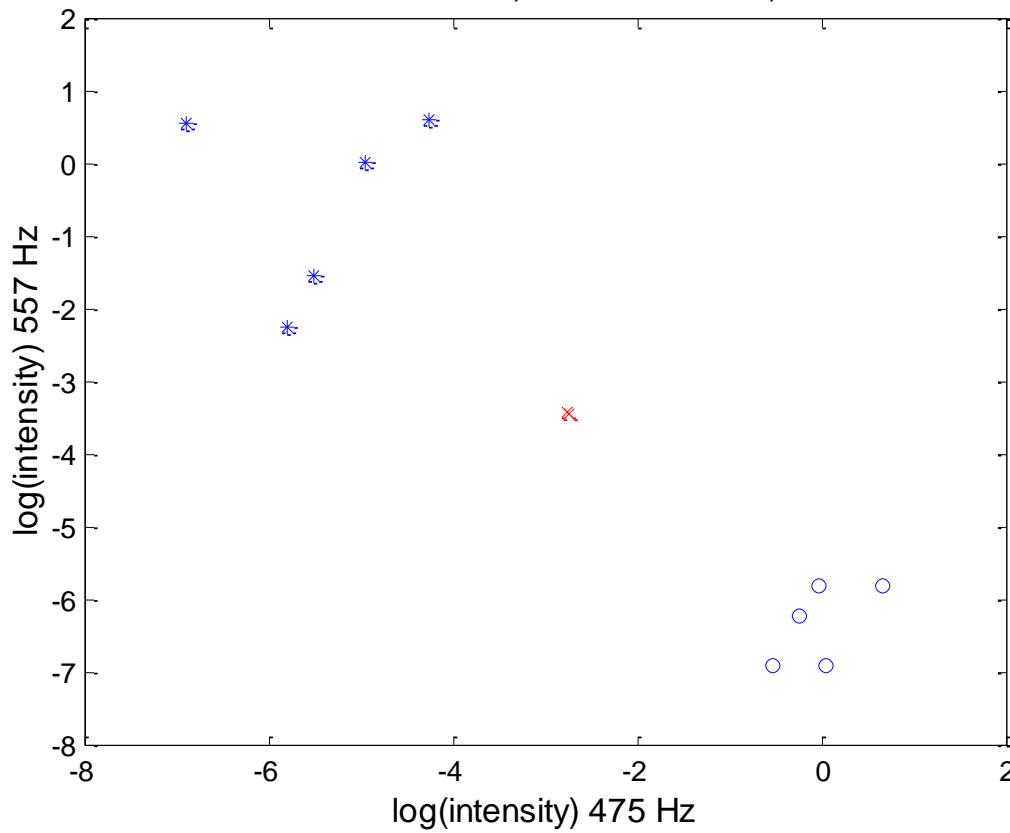
Each data point belongs to two clusters to different degrees

Tiles data: o = whole tiles, * = cracked tiles, x = centres



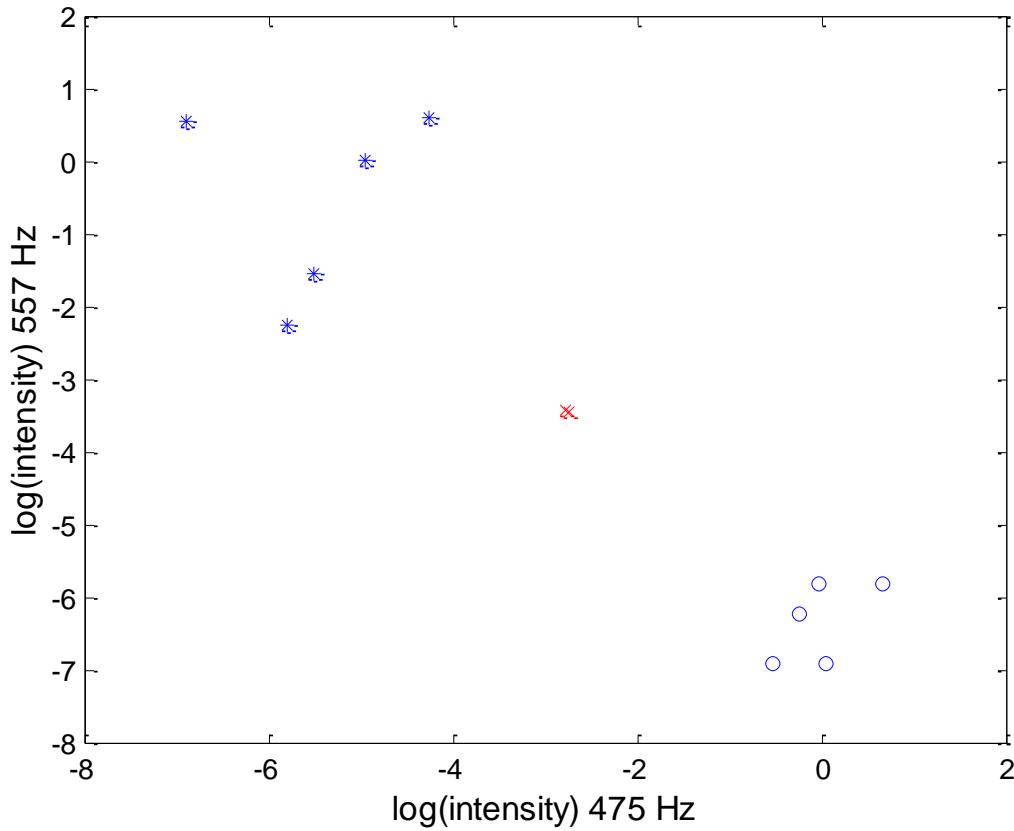
1. Place two cluster centres
2. Assign a fuzzy membership to each data point depending on distance

Tiles data: o = whole tiles, * = cracked tiles, x = centres



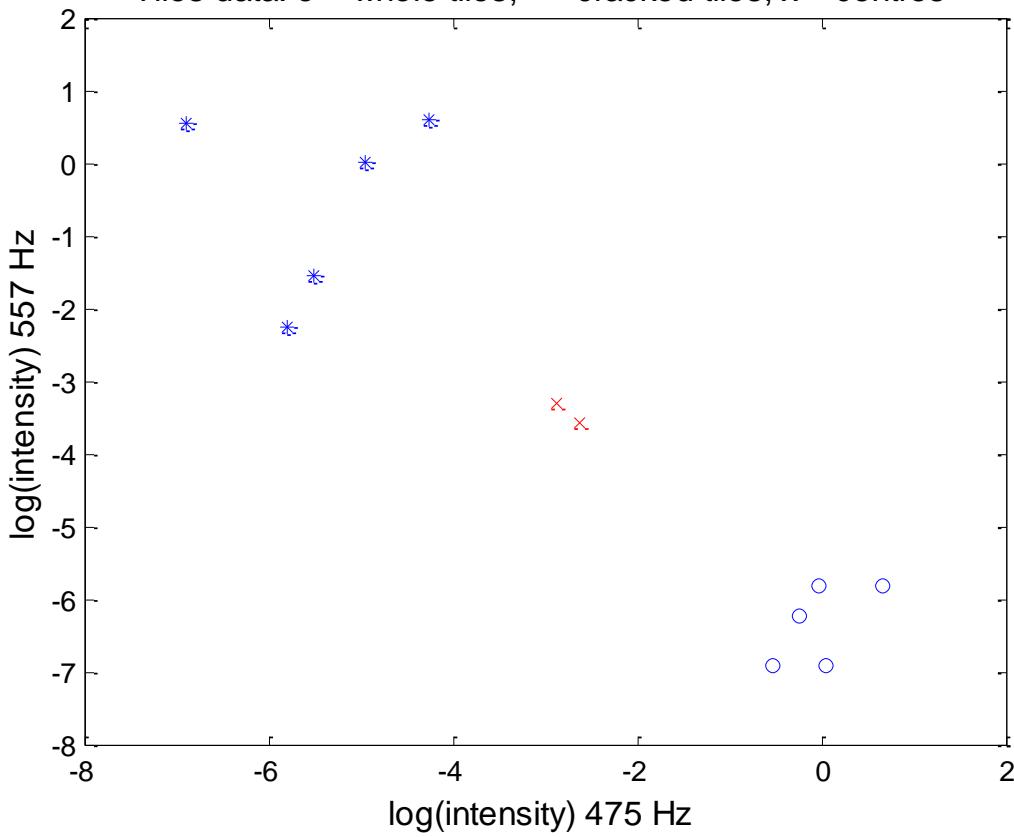
1. Compute the new centre of each class
2. Move the crosses (x)

Tiles data: o = whole tiles, * = cracked tiles, x = centres



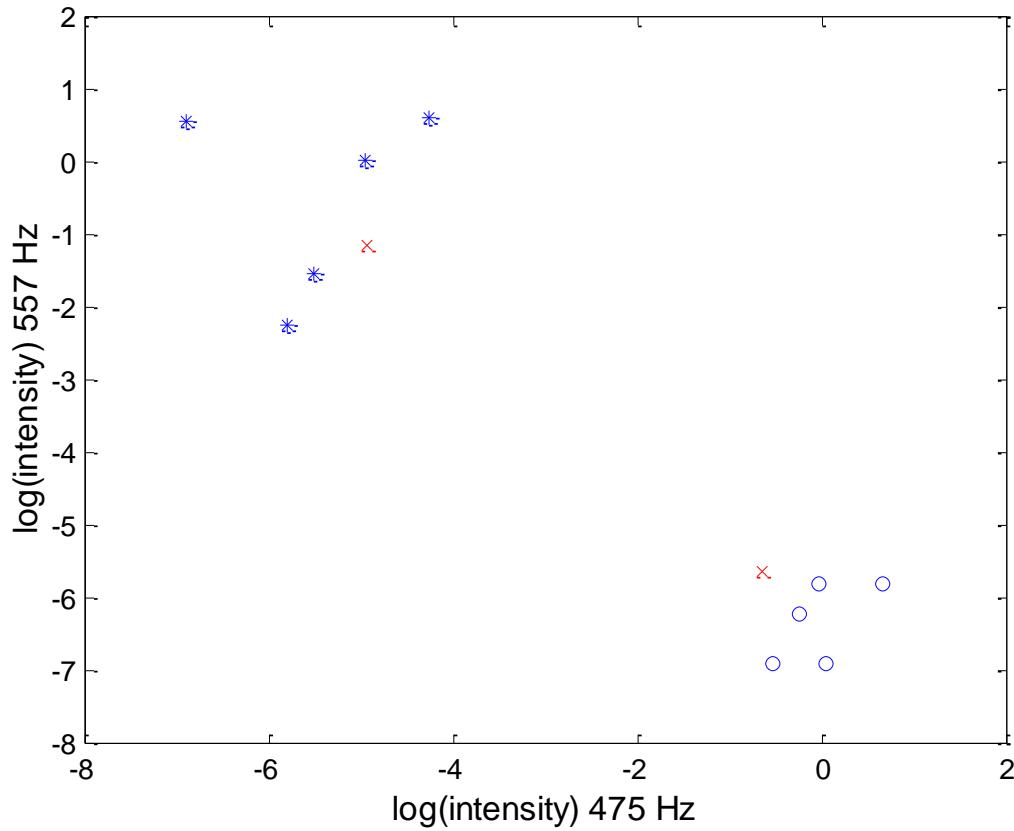
Iteration 2

Tiles data: o = whole tiles, * = cracked tiles, x = centres



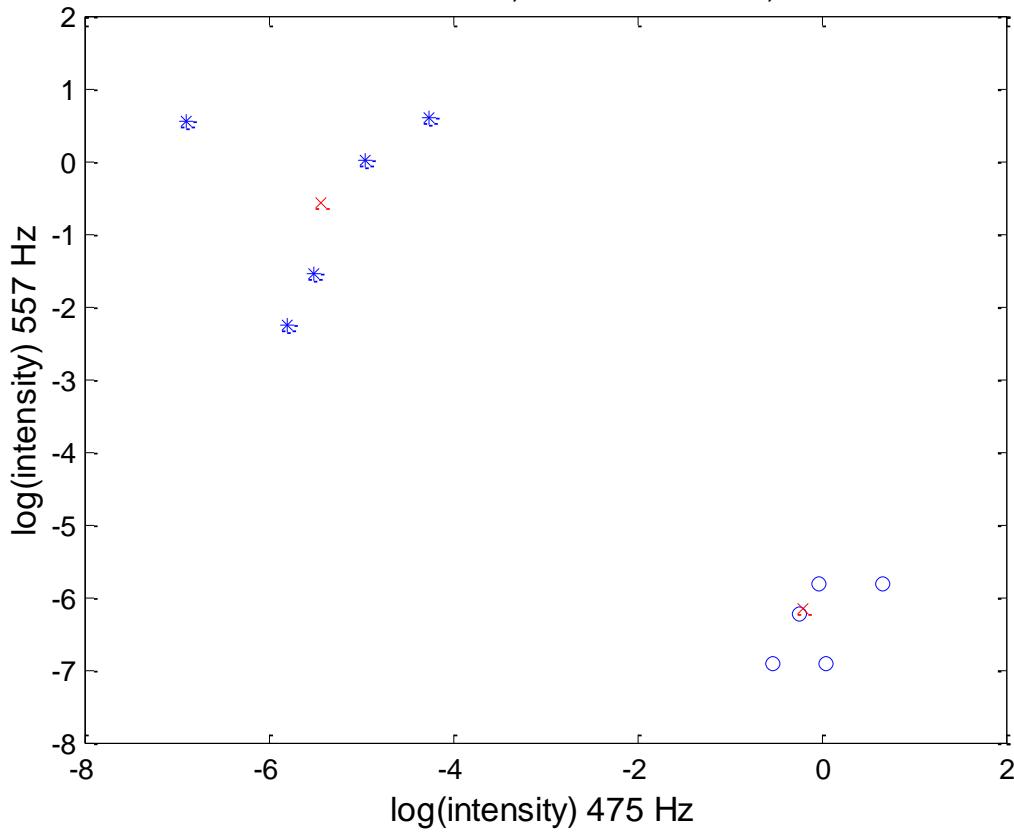
Iteration 5

Tiles data: o = whole tiles, * = cracked tiles, x = centres



Iteration 10

Tiles data: o = whole tiles, * = cracked tiles, x = centres



Iteration 13 (then stop, because no visible change)
Each data point belongs to the two clusters to a degree

M =

0.0025	0.9975
0.0091	0.9909
0.0129	0.9871
0.0001	0.9999
0.0107	0.9893
0.9393	0.0607
0.9638	0.0362
0.9574	0.0426
0.9906	0.0094
0.9807	0.0193

The membership matrix M:

1. The last five data points (rows) belong mostly to the first cluster (column)
2. The first five data points (rows) belong mostly to the second cluster (column)

Fuzzy membership matrix \mathbf{M}

Point k 's
membership
of cluster i

$$m_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}} \right)^{2/(q-1)}}$$

Fuzziness
exponent

Distance from point
 k to current cluster
centre i

$$d_{ik} = \|\mathbf{u}_k - \mathbf{c}_i\|$$

Distance from point
 k to other cluster
centres j

Fuzzy membership matrix \mathbf{M}

$$\begin{aligned}m_{ik} &= \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}} \right)^{2/(q-1)}} \\&= \frac{1}{\left(\frac{d_{ik}}{d_{1k}} \right)^{2/(q-1)} + \left(\frac{d_{ik}}{d_{2k}} \right)^{2/(q-1)} + \cdots + \left(\frac{d_{ik}}{d_{ck}} \right)^{2/(q-1)}} \\&= \frac{\frac{1}{d_{ik}^{2/(q-1)}}}{\frac{1}{d_{1k}^{2/(q-1)}} + \frac{1}{d_{2k}^{2/(q-1)}} + \cdots + \frac{1}{d_{ck}^{2/(q-1)}}}\end{aligned}$$

Gravitation to
cluster i relative to
total gravitation

Fuzzy c-partition

All clusters C together fill the whole universe U .
Remark: The sum of memberships for a data point is 1, and the total for all points is K

$$\bigcup_{i=1}^c C_i = U$$

$$C_i \cap C_j = \emptyset \quad \text{for all } i \neq j$$

$$\emptyset \subset C_i \subset U \quad \text{for all } i$$

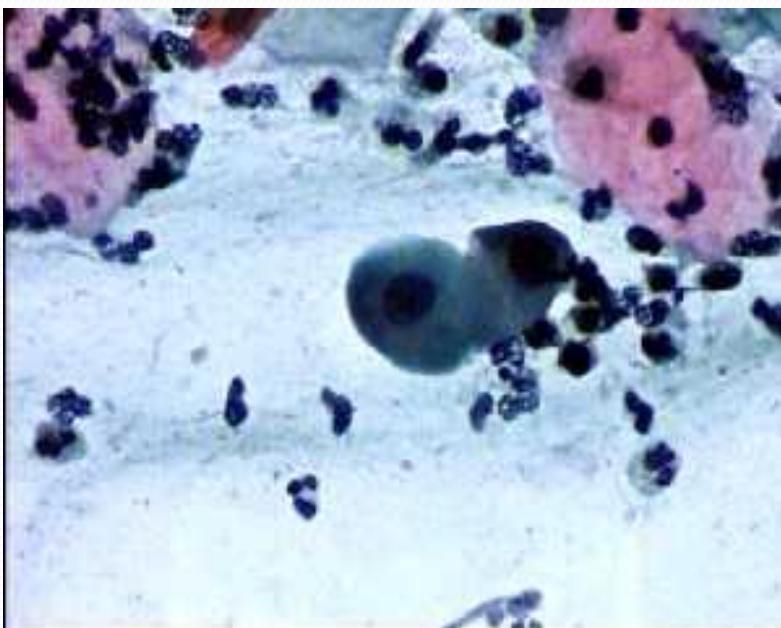
A cluster C is never empty and it is smaller than the whole universe U

$$2 \leq c \leq K$$

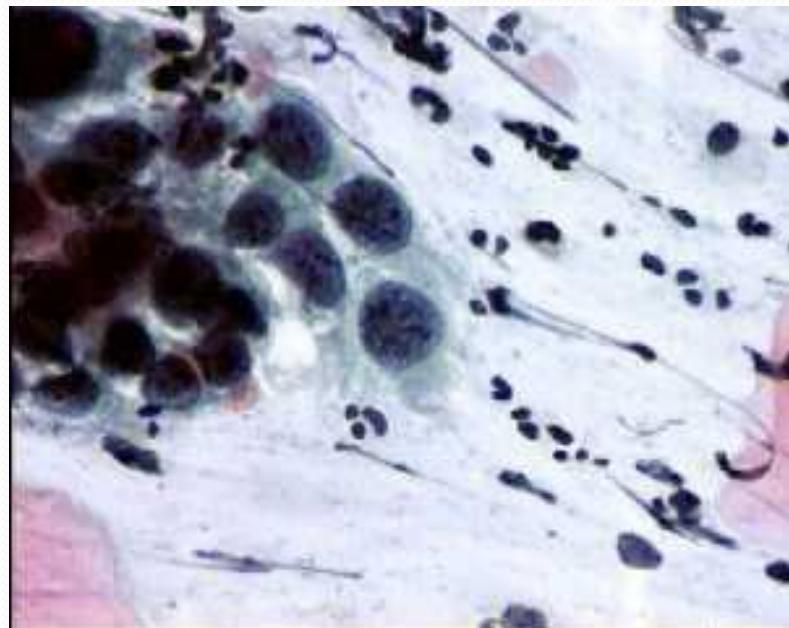
*Not valid:
Clusters do overlap*

There must be at least 2 clusters in a c-partition and at most as many as the number of data points K

Kasus 7: Klasifikasi Sel Kanker



Normal smear

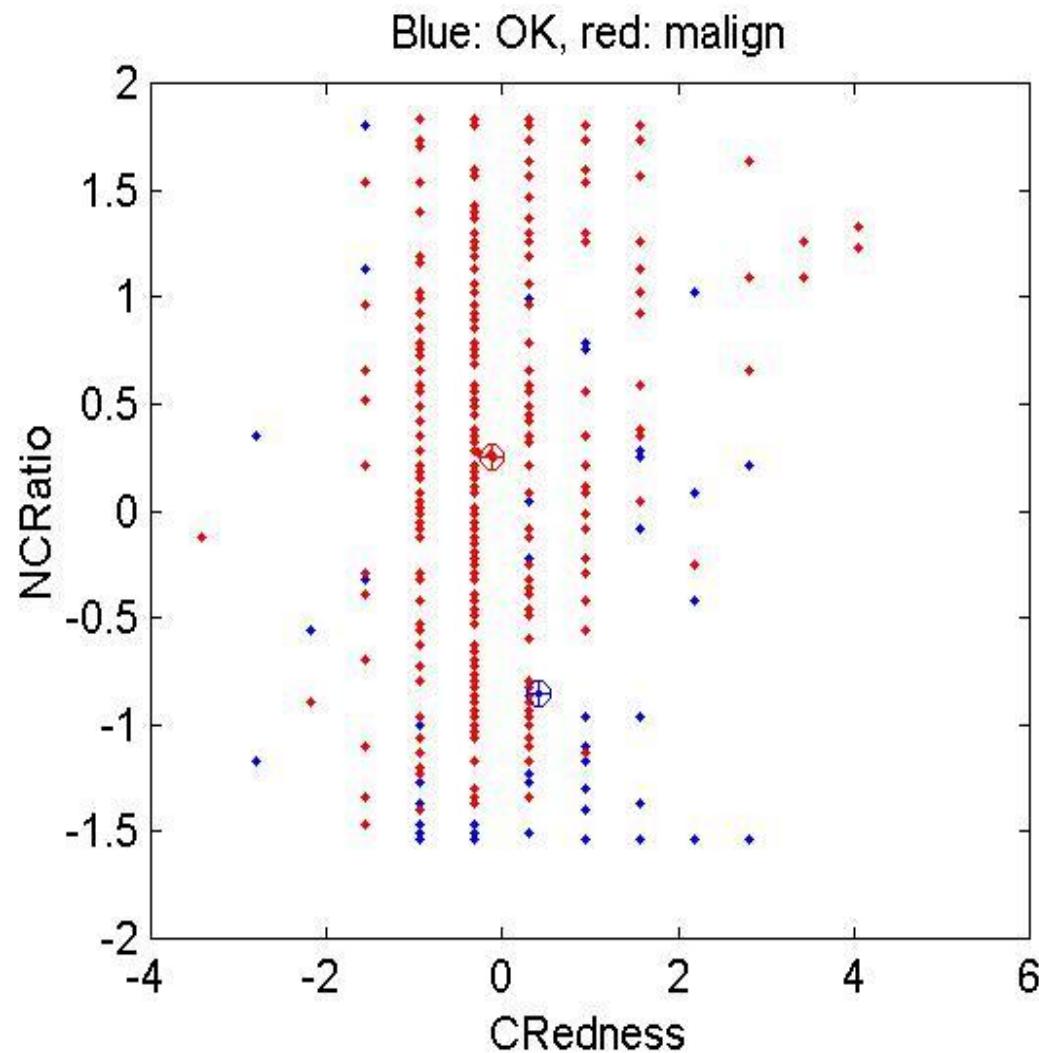


Severely dysplastic smear

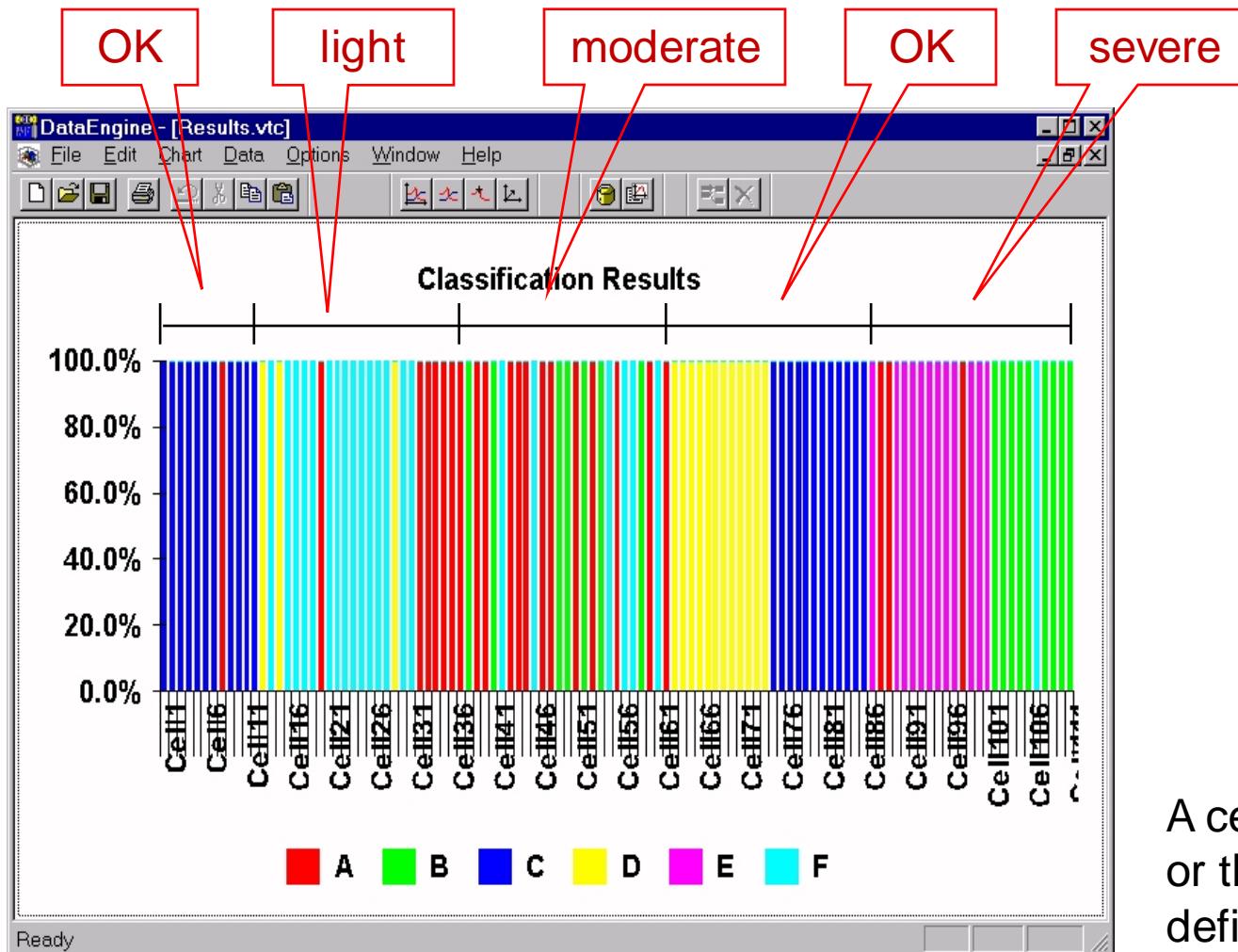
Possible Features

- Nucleus and cytoplasm area
- Nucleus and cyto brightness
- Nucleus shortest and longest diameter
- Cyto shortest and longest diameter
- Nucleus and cyto perimeter
- Nucleus and cyto no of maxima
- ...

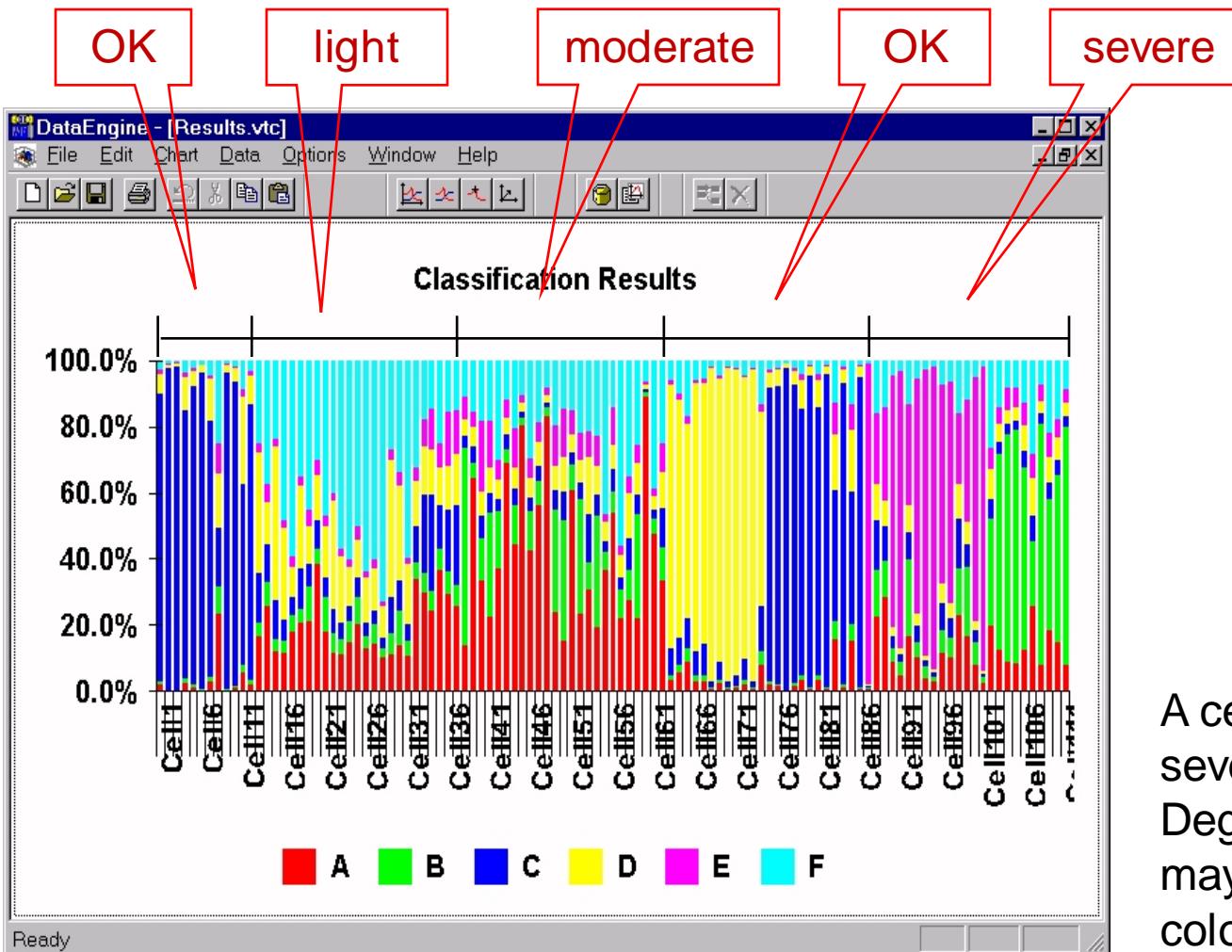
Classes are nonseparable



Hard Classifier (HCM)



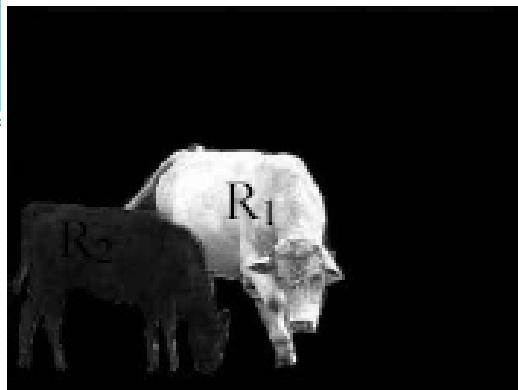
Fuzzy Classifier (FCM)



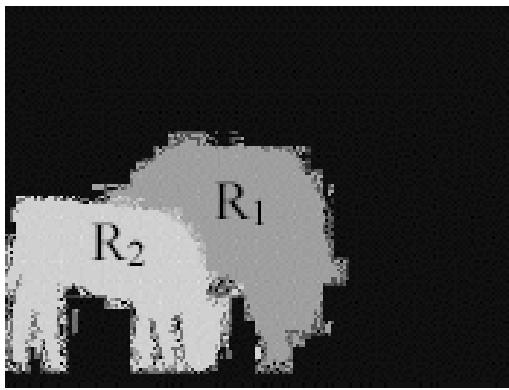
A cell can belong to several classes to a Degree, i.e., one column may have several colours.

Kasus 8: *Image Segmentation*

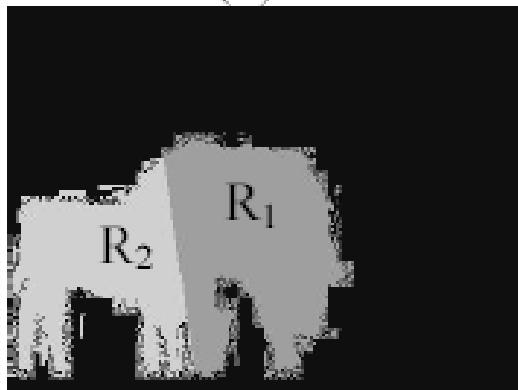
- Sangat penting untuk *image analysis, understanding, dan coding*
- Bagaimana memisahkan satu objek yang diinginkan dari objek2 lainnya?
- Pada image terdapat banyak ambiguitas



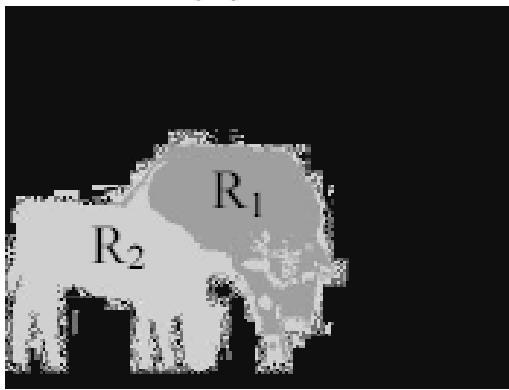
(a)



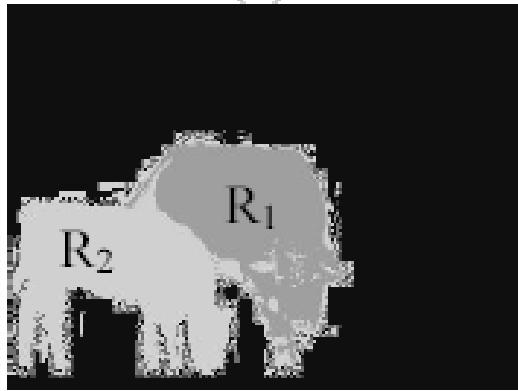
(b)



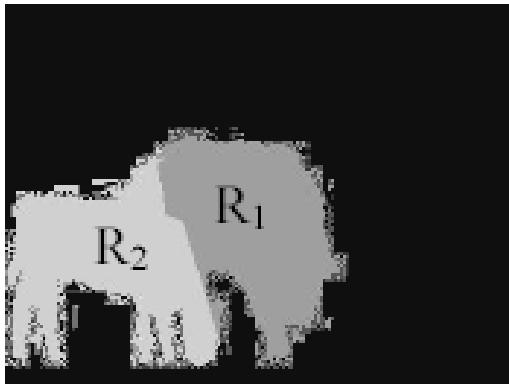
(c)



(d)



(e)



(f)

- (a) Original cow image,
- (b) Manually segmented reference image for (a).
- (c) FCM with pixel locations
- (d) FCM with pixel intensity
- (e) FCM with both features
- (f) Fuzzy Clustering Incorporating Spatial Information (FCSI).

[M. Ameer Ali, Gour C Karmakar and Laurence S Dooley, "Image Segmentation Using Fuzzy Clustering Incorporating Spatial Information, Monash University, Australia]

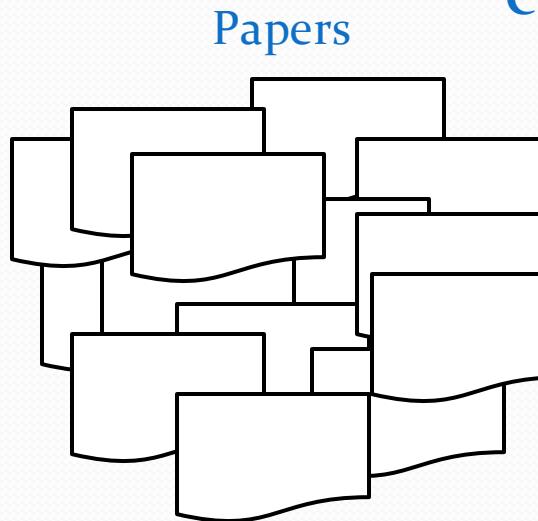
Kasus 9: *Documents Clustering*

- Text represented as an unordered collection of words
- Using tf-idf (term frequency-inverse document frequency)
- Document = one vector in high dimensional space
- Similarity = cosine similarity between vectors

PROBLEM

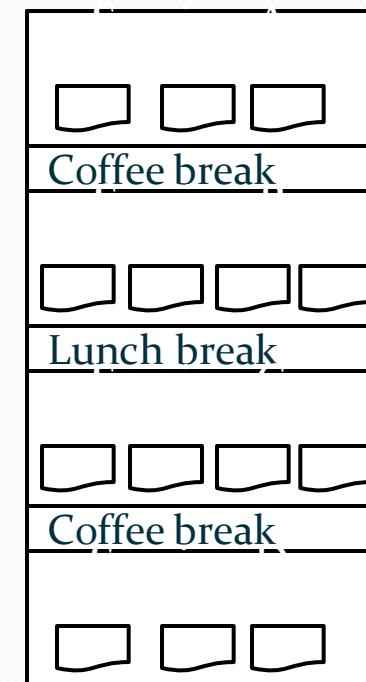
Grouping conference papers with regard to their contents into predefined sessions schedule.

EXAMPLE



Constraint-based clustering

Sessions schedule



Combining CBC & Fuzzy Clustering

- **PHASE 1 SOLUTION**

- constrained-based clustering (CBC)

- **DIFFICULTIES**

- CBC can get stuck in local minimum
 - often low quality result (created schedule)
 - user interaction needed to repair schedule

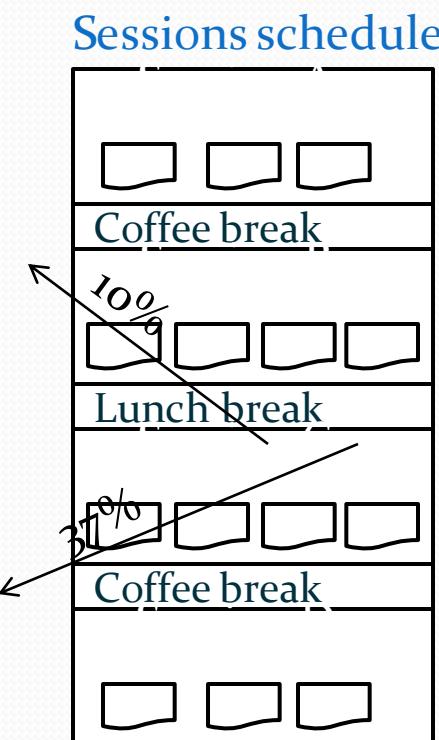
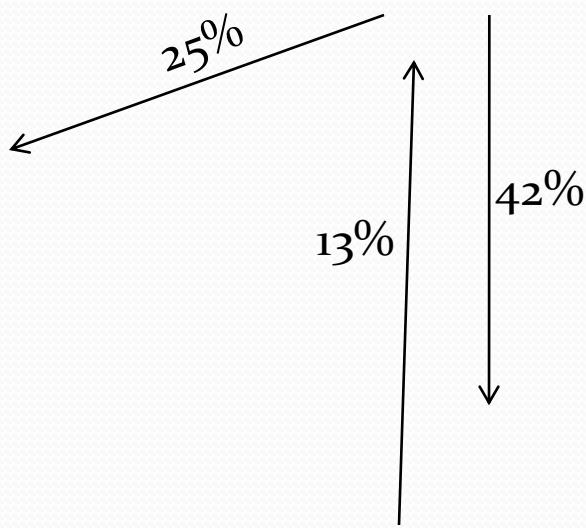
- **PHASE 2 NEEDED**

- run fuzzy clustering (FC) with initial clusters from CBC
 - if output clusters of FC differ from CBC repeat everything
 - if the clusters of FC equal to CBC show new info to user

RUN FUZZY CLUSTERING ON PHASE 1 RESULTS

- insight into result quality
- identify problematic papers

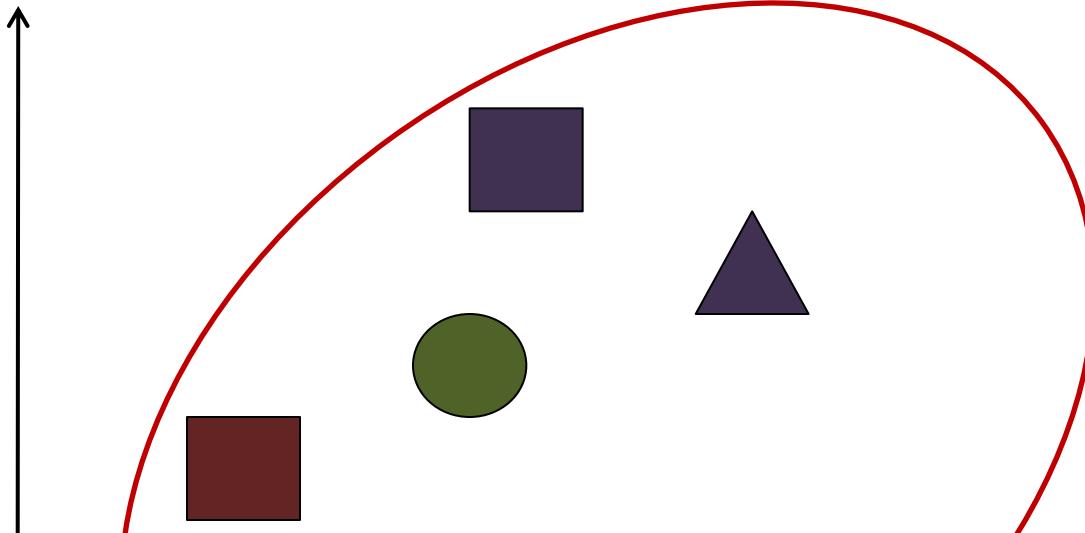
EXAMPLE



Masalah Tugas Akhir IF

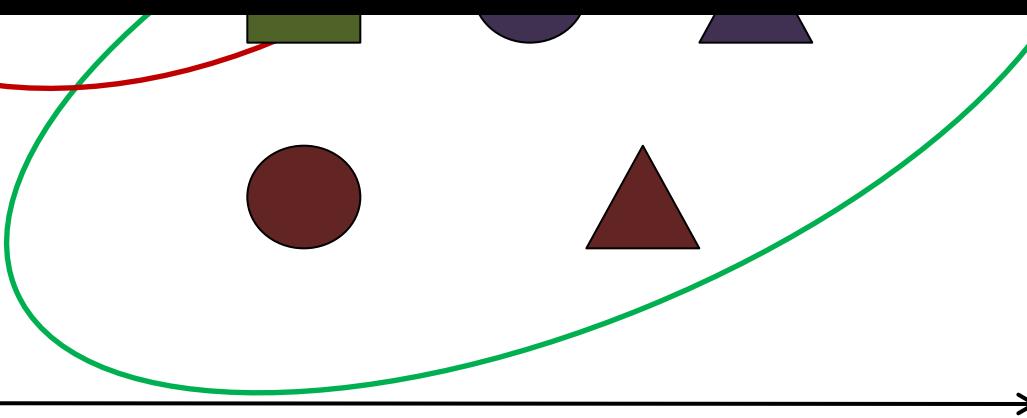
- Clustering dokumen TA berdasarkan KBK
- Clustering Dosen berdasarkan Keahliannya
- Penentuan Dosen Penguji Sidang TA

Hard Classifier



Ketidakpastian didekati dengan Kepastian

→ Ketidakpastian



Fuzzy Classifier



Kesulitan *Fuzzy Systems*

- Model Mamdani atau Sugeno atau model lain?
- Jumlah Nilai Linguistik untuk setiap variabel?
- Fungsi Keanggotaan: Segitiga, trapesium, phi, ...?
- Batas-batas Nilai Linguistik?
- *Fuzzy rule* yang tepat?

Solusi

- Semua komponen *fuzzy* bisa didefinisikan secara otomatis menggunakan EAs atau ANN.
- *Evolving Fuzzy Systems*
- *Neuro-Fuzzy*
- *Fuzzy Systems* yang bisa **LEARNING**
- Perlu data latih yang **MEMADAI**

Pemberian Beasiswa

NIM	IPK	Gaji Ortu (juta rupiah)	Nilai Kelayakan
070001	2,0	16	45
070002	2,5	12	50
070003	3,6	30	55
070004	1,5	140	10
070005	2,7	10	40
070006	3,9	0,5	95
070007	1,9	1	52
070008	2,8	8	68
070009	3,5	6	72
070010	2,0	7	53

Sprinkler Control System

Tanggal	Waktu	Suhu (°C)	Kelembaban Tanah (%)	Durasi Penyiraman (menit)
01-02-2006	08:00	20	16	55,7
01-02-2006	13:00	25	12	59,3
01-02-2006	18:00	16	30	5,6
02-02-2006	07:00	15	14	30,1
02-02-2006	12:00	27	10	43,4
02-02-2006	19:00	12	19	18,6
03-02-2006	06:30	19	12	22,1
03-02-2006	10:00	28	8	47,3
03-02-2006	13:30	35	6	76,4
03-02-2006	16:00	20	17	20,9

Evolving Fuzzy Systems

- Representasi Individu?
- Fungsi fitness-nya?
- Algoritma EAs?
- Operator-operator evolusi?

Masa Depan *Fuzzy*

- Computers that understand and respond to normal human language.
- Machines that write interesting novels and screenplays in a selected style, such as Hemingway's.
- Huge expert decision makers, theoretically able to extract the wisdom of every document ever written.
- Molecule-sized soldiers of health that will roam the blood-stream, killing cancer cells and slowing the aging process.

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