Register Allocation

Live Ranges and Live Intervals

- Recall: A variable is **live** at a particular program point if its value may be read later before it is written.
  - Can find this using global liveness analysis.
- The **live range** for a variable is the set of program points at which that variable is live.
- The **live interval** for a variable is the smallest subrange of the IR code containing all a variable's live ranges.
  - A property of the IR code, **not** the CFG.
  - Less precise than live ranges, but simpler to work with.

Computing Live Variables

- To know if a variable will be used at some point, we iterate across the statements in a basic block in reverse order.
- Initially, some small set of values are known to be live (which ones depends on the particular program).
- When we see the statement **a = b + c**:
  - Just before the statement, **a** is not alive, since its value is about to be overwritten.
  - Just before the statement, both **b** and **c** are alive, since we're about to read their values.
  - *(what if we have* **a = a + b***?)*

Liveness Analysis

```
a = b;
c = a;
d = a + b;
e = d;
d = a;
f = e;
```

Liveness Analysis

```
  a = b;

  c = a;

d = a + b;

  e = d;

  d = a;

  f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
```
`{ b, d, e }`
```
f = e;
```
`{ b, d }`

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Live Ranges and Live Intervals

Live Ranges and Live Intervals

```
    e = d + a
    f = b + c
    f = f + b
    IfZ e Goto _L0
    d = e + f
    Goto _L1;
_L0:
    d = e - f
_L1:
    g = d
```

Live Ranges and Live Intervals

```
    e = d + a
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    f = f + b
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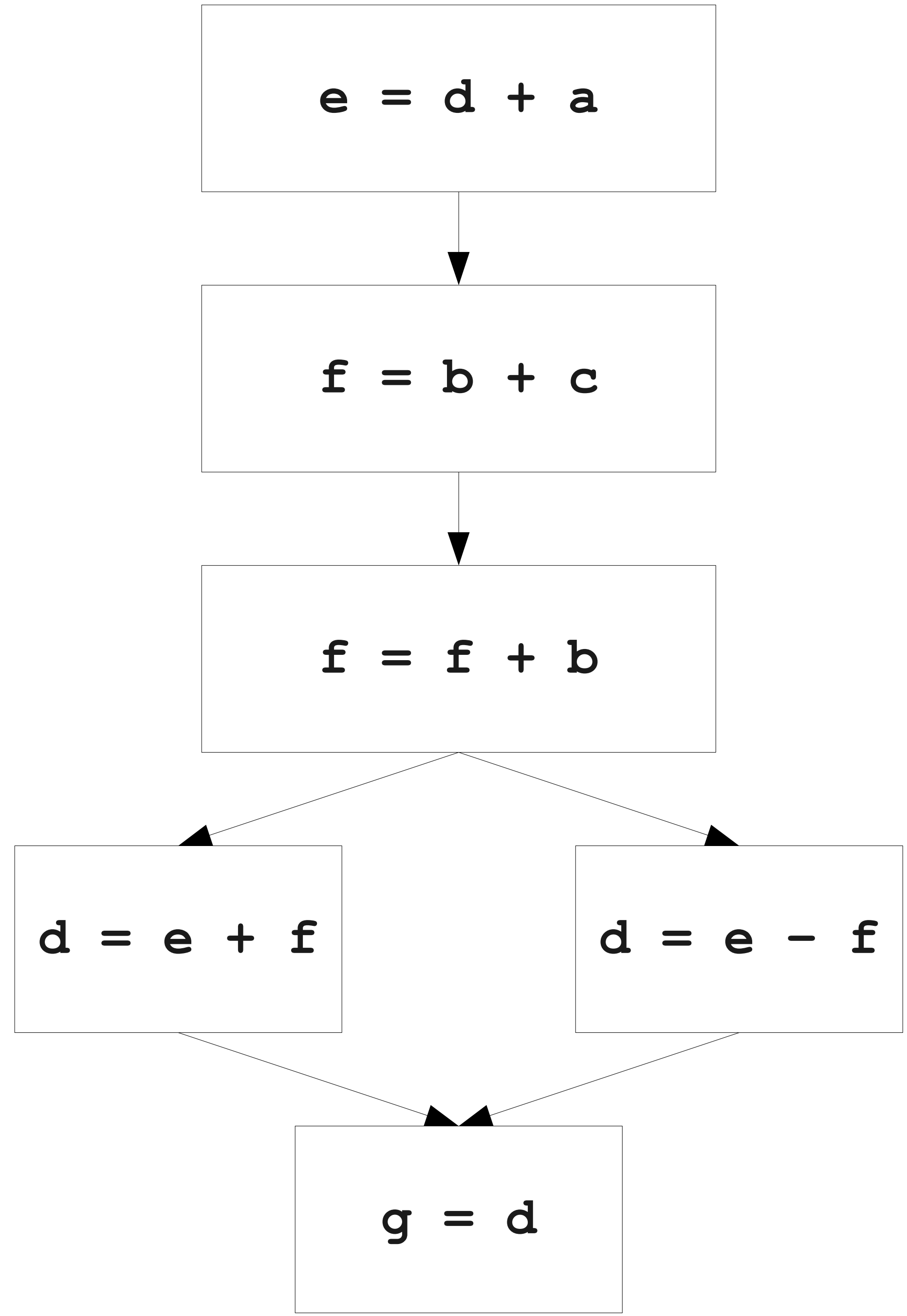

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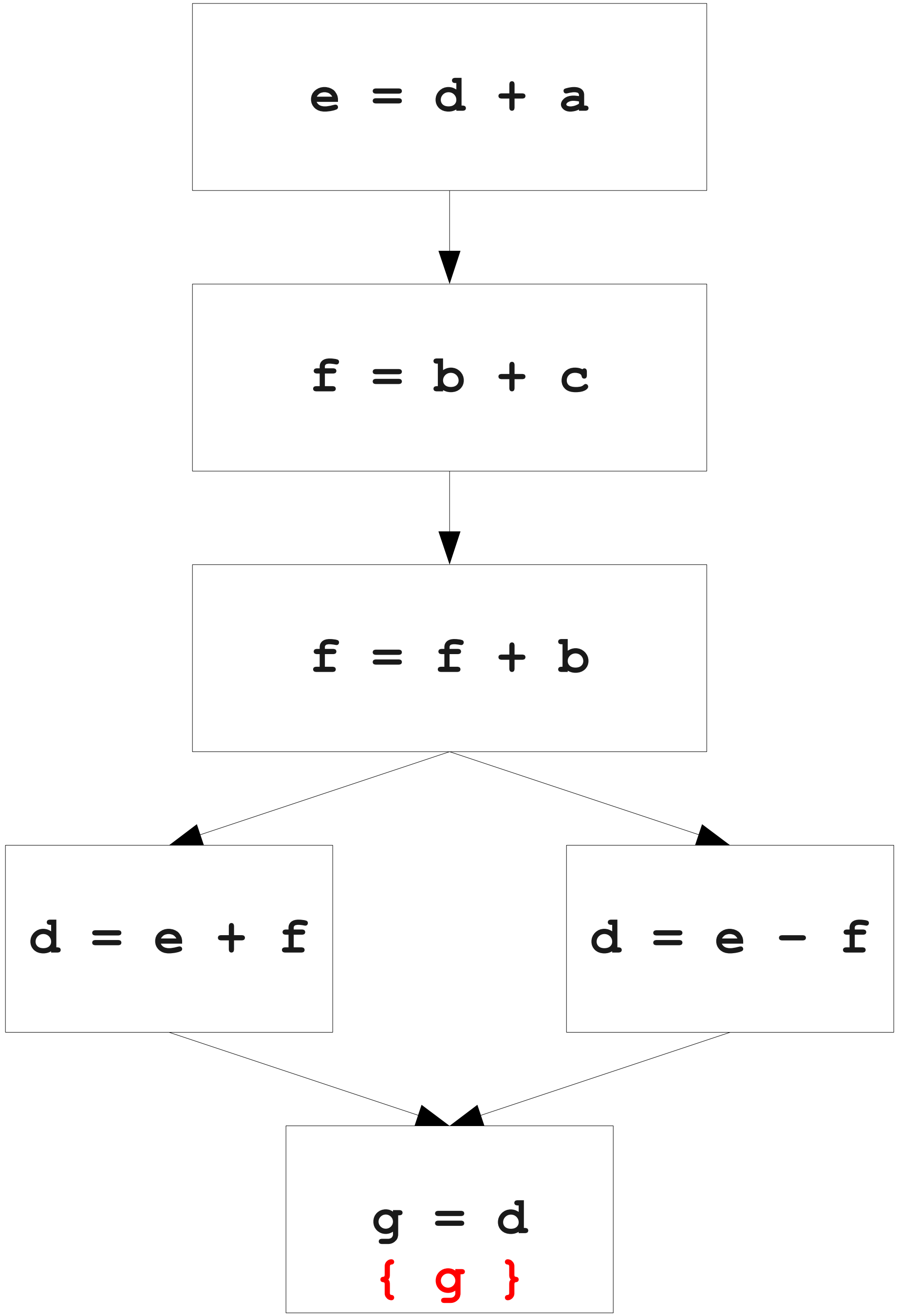

Live Ranges and Live Intervals

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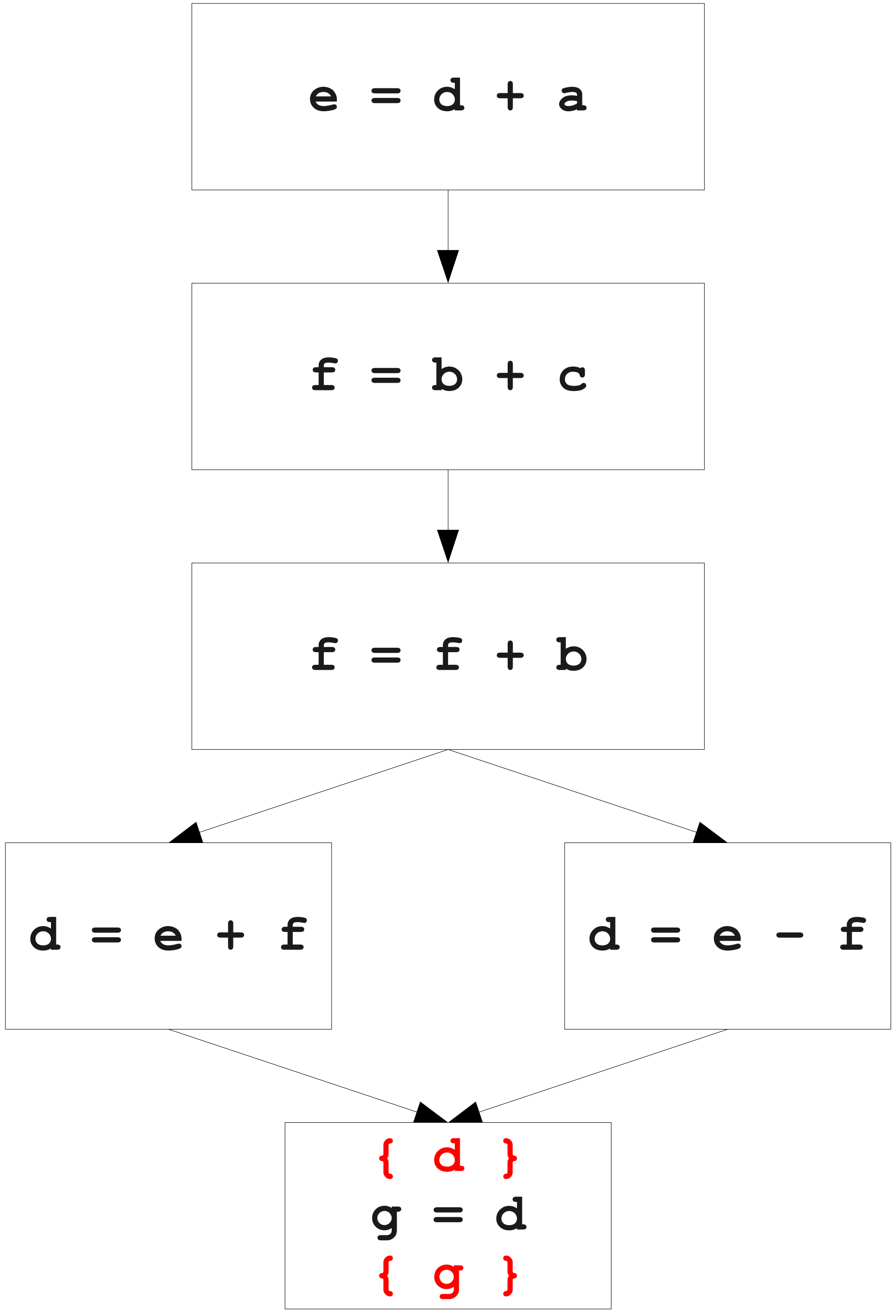

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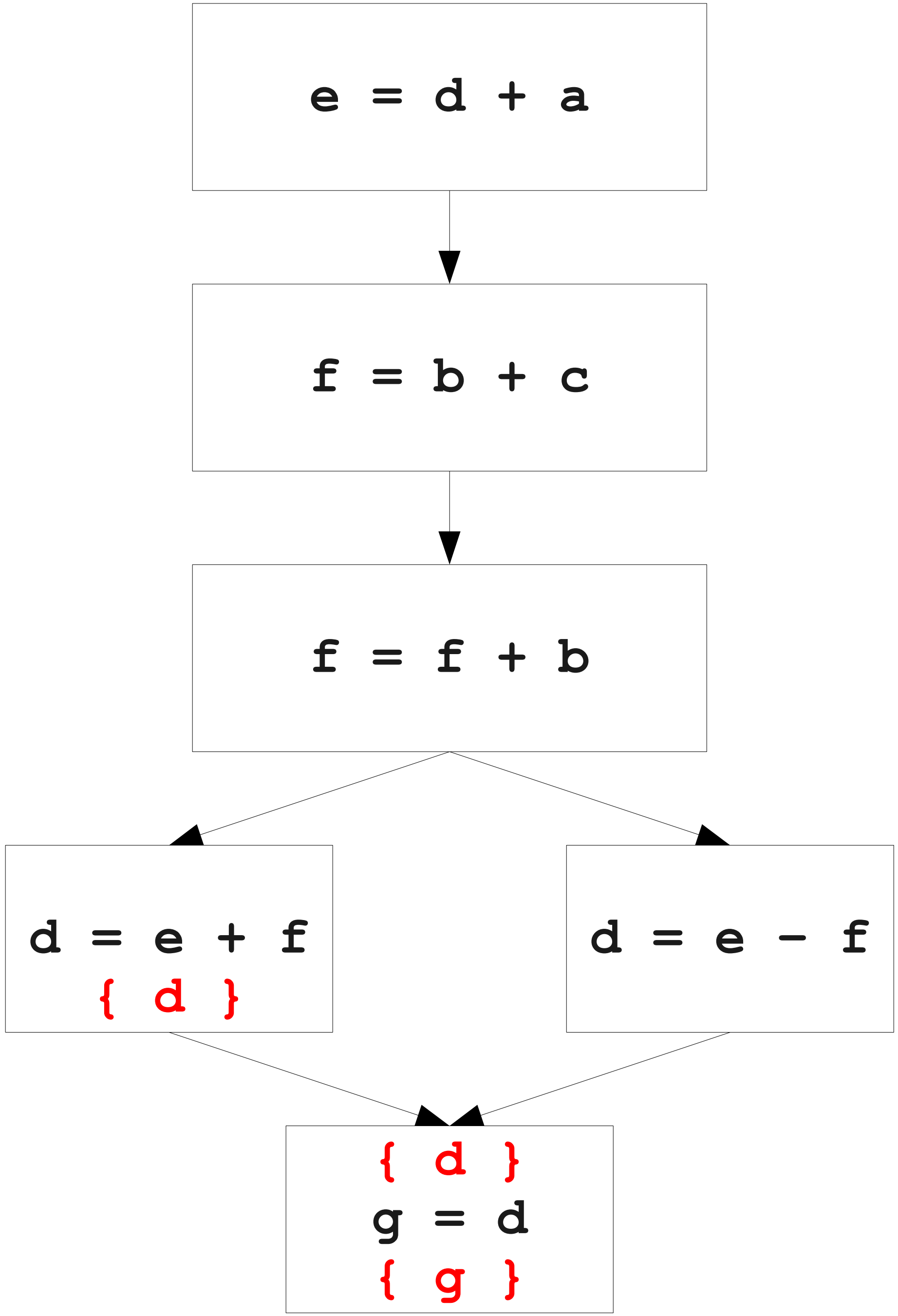

Live Ranges and Live Intervals

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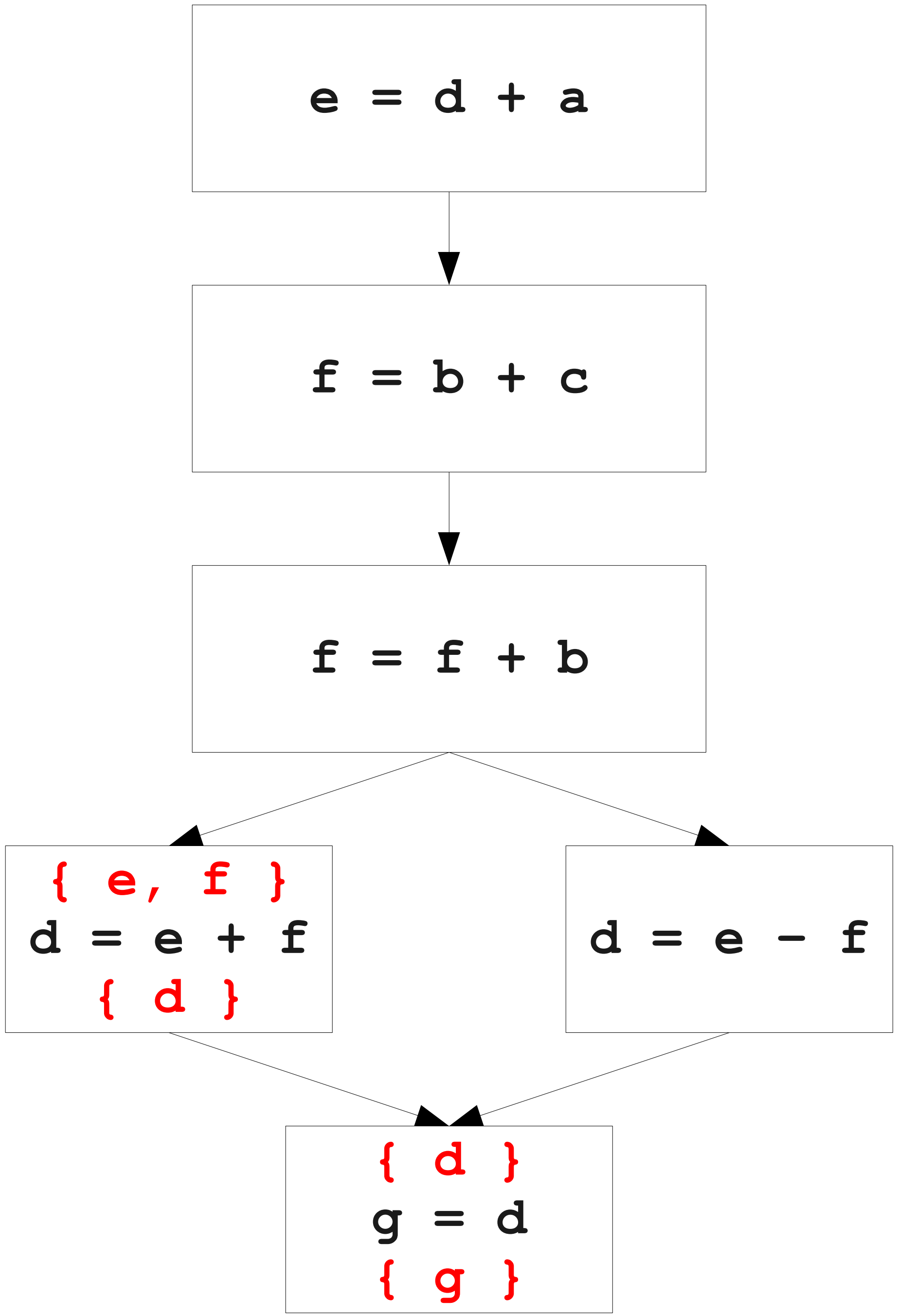

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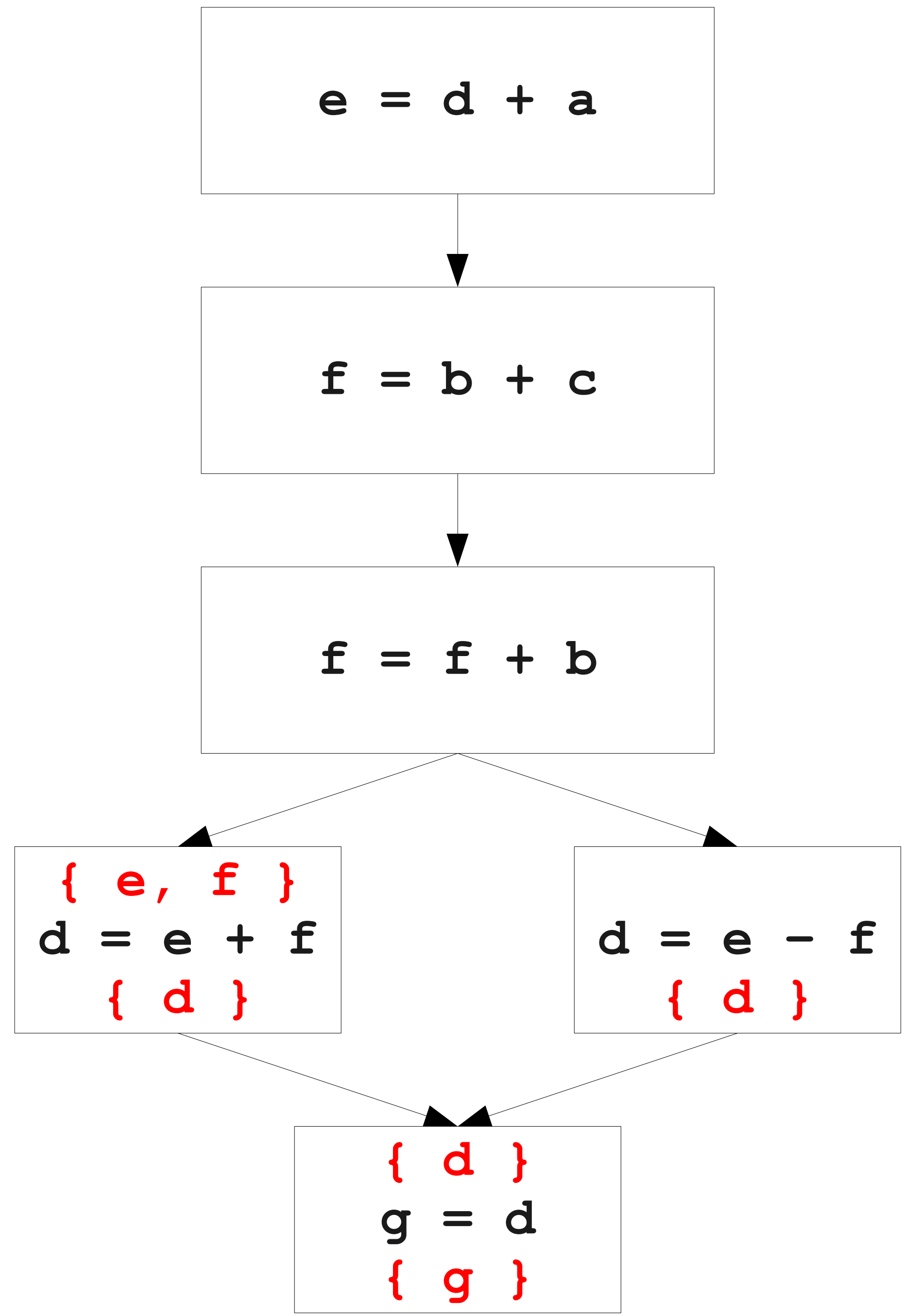

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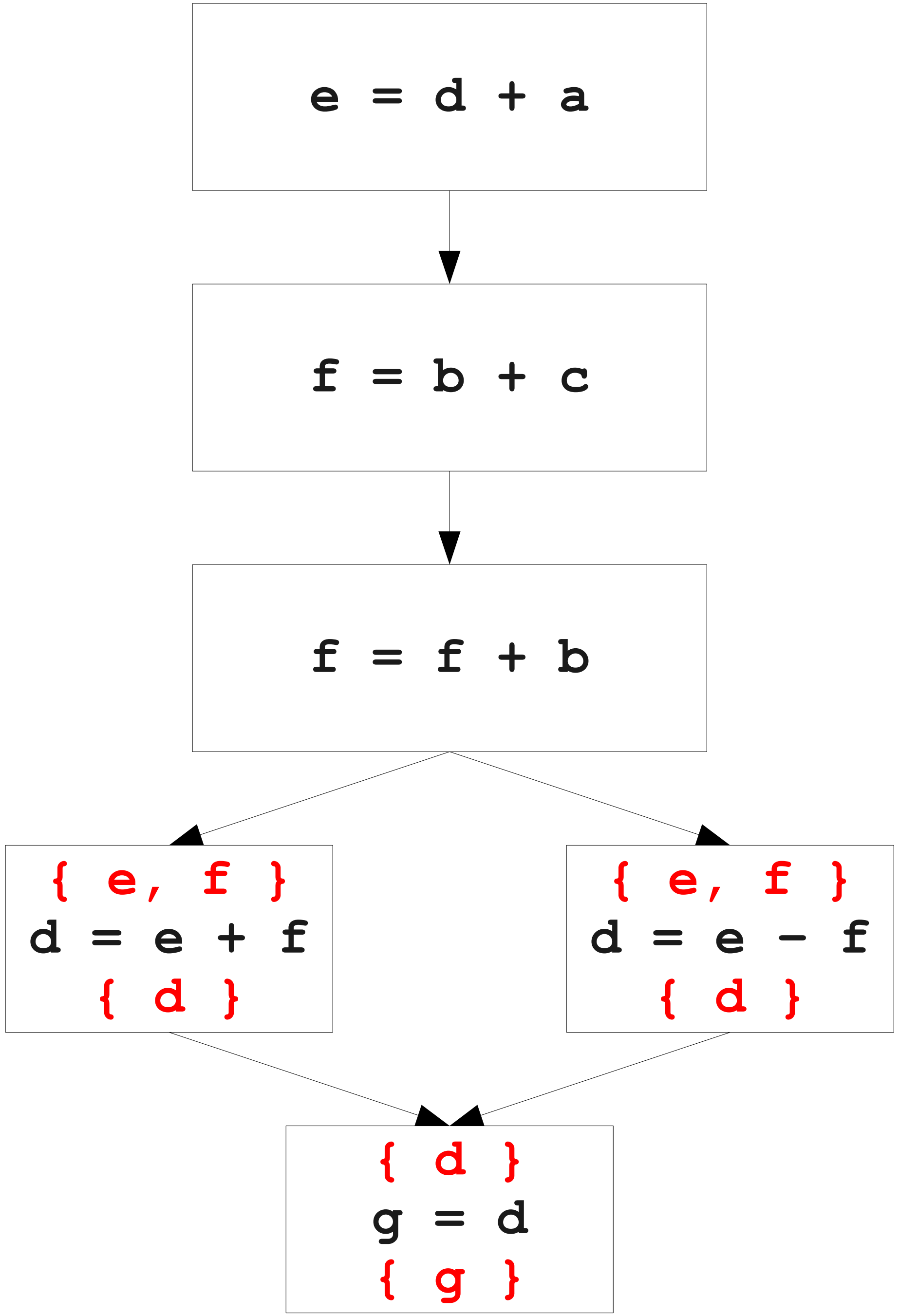

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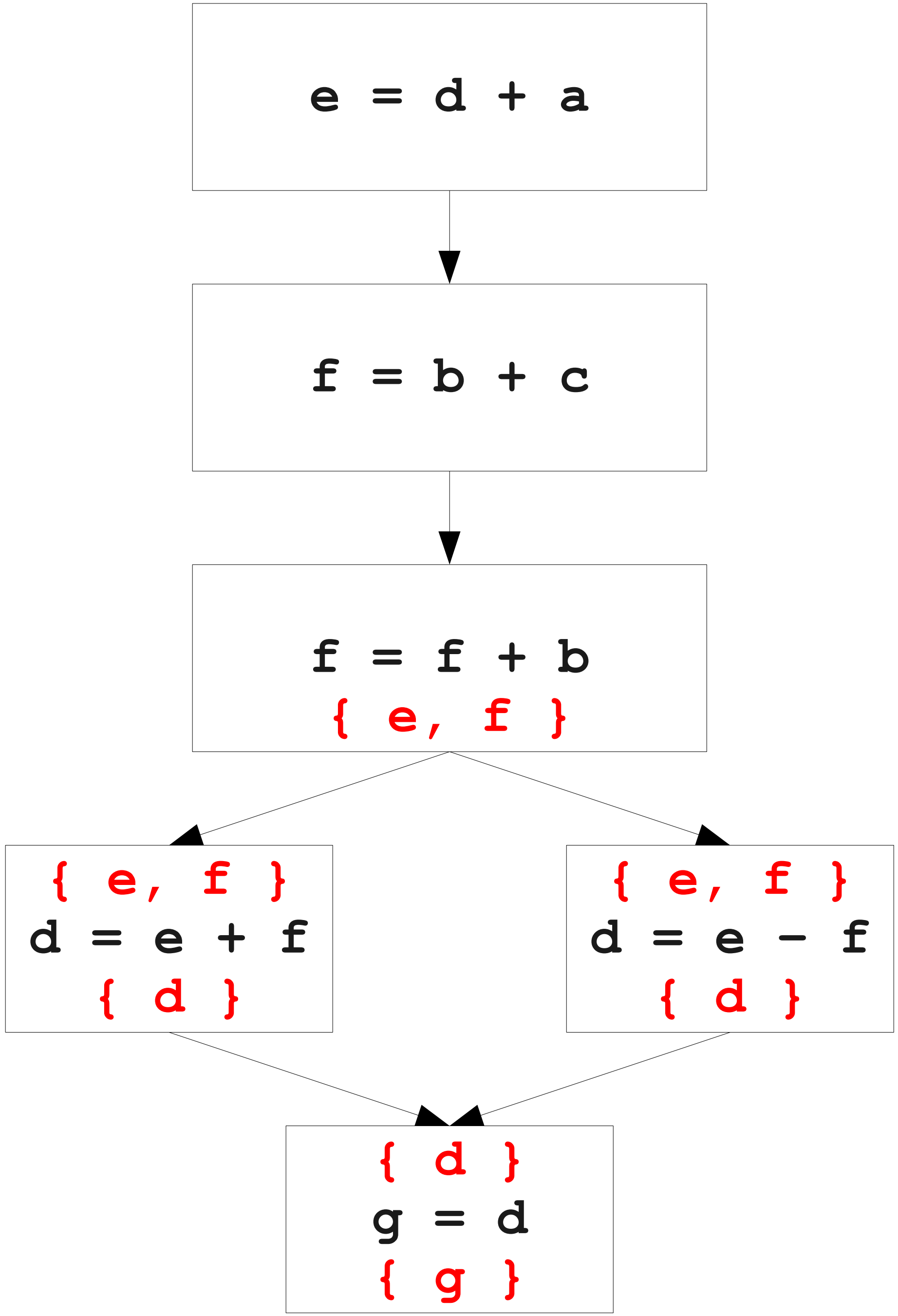

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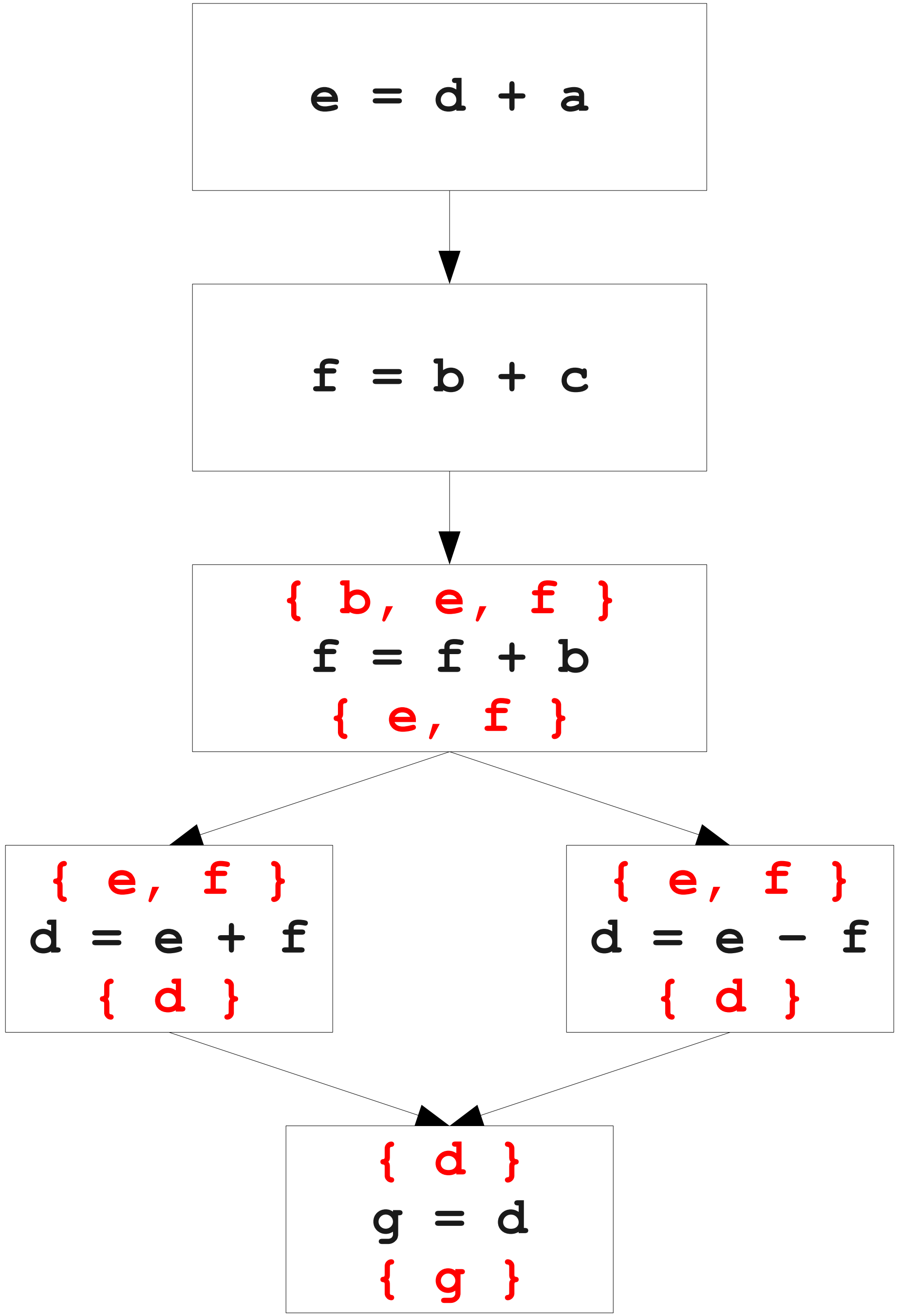

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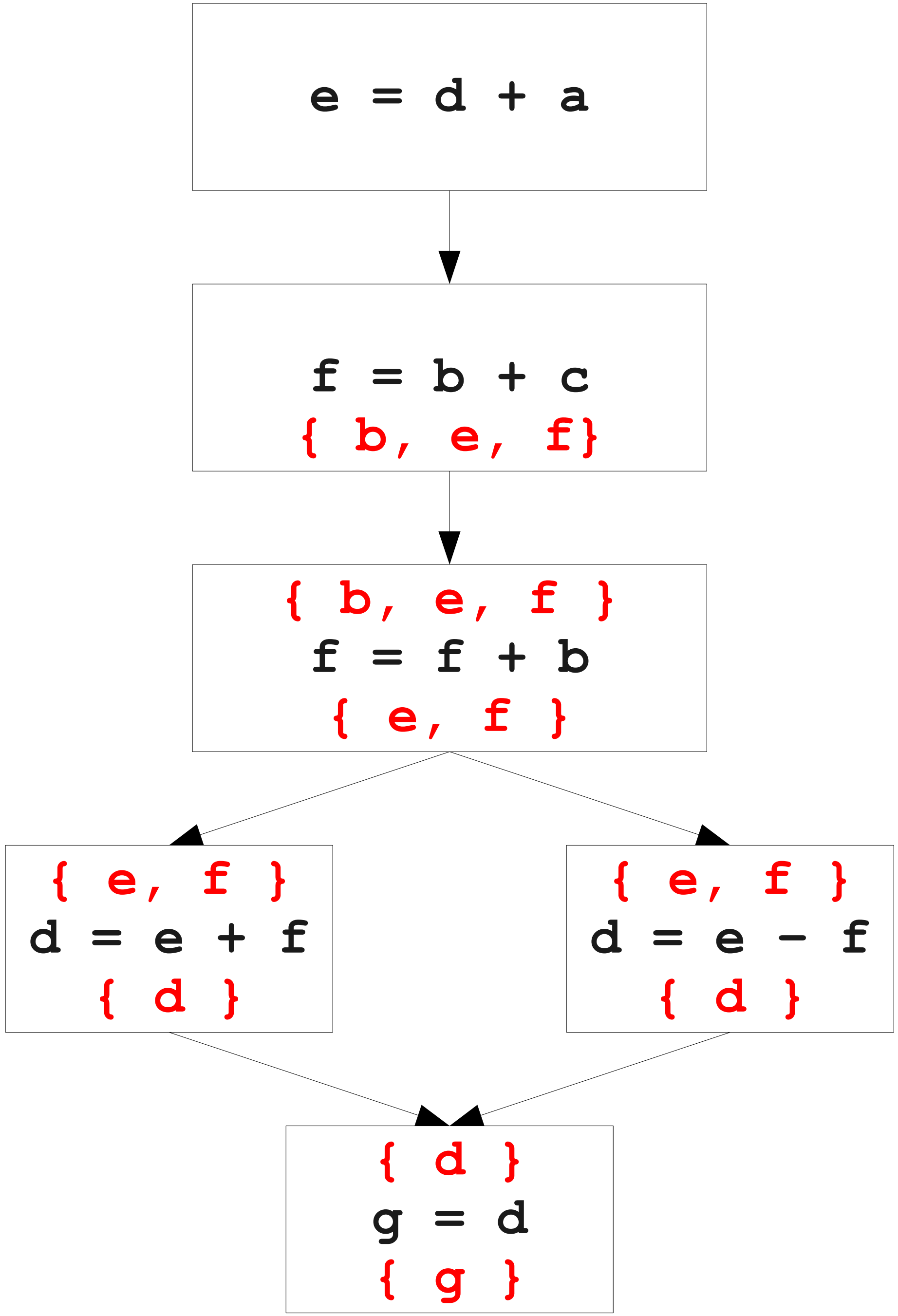

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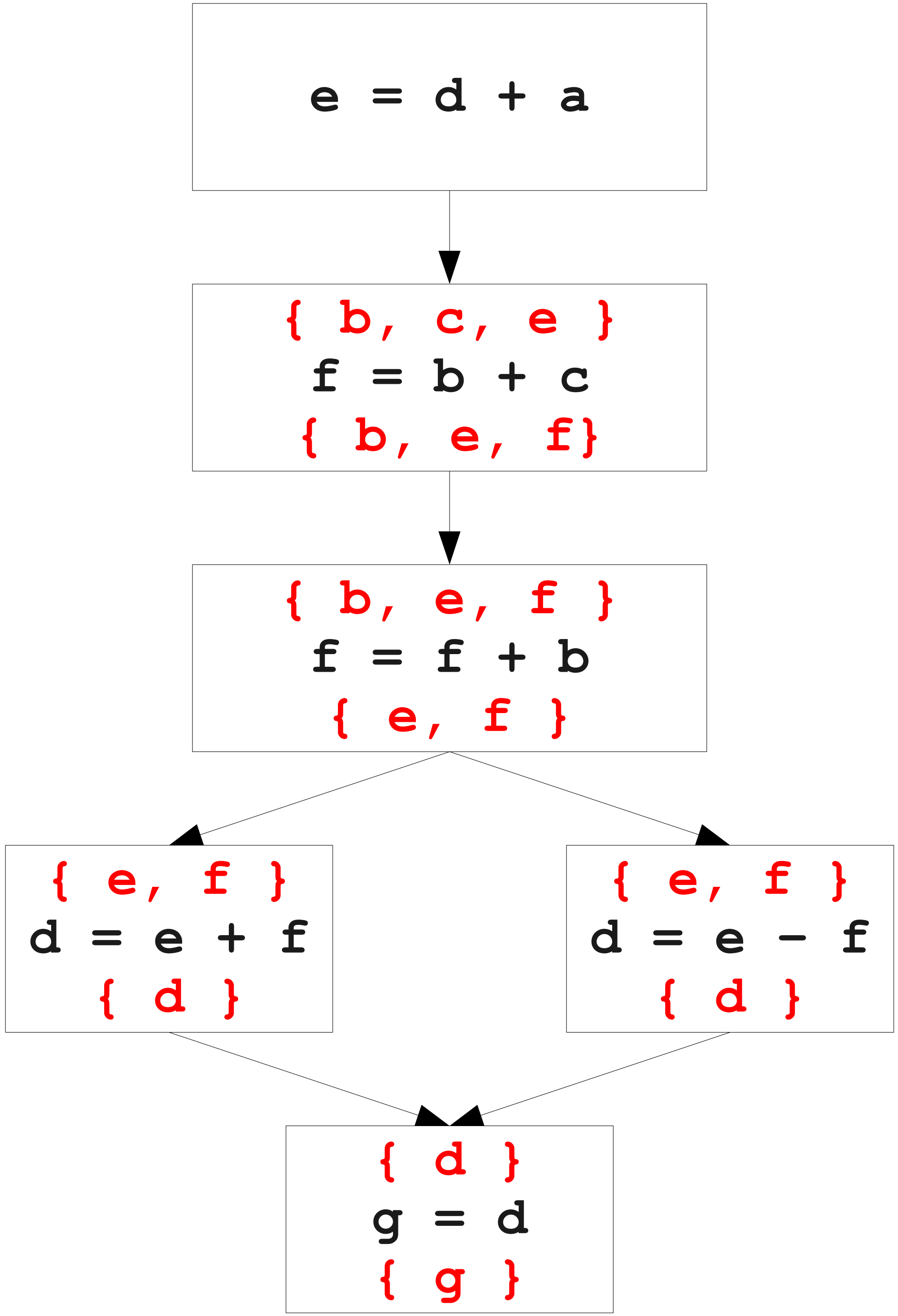

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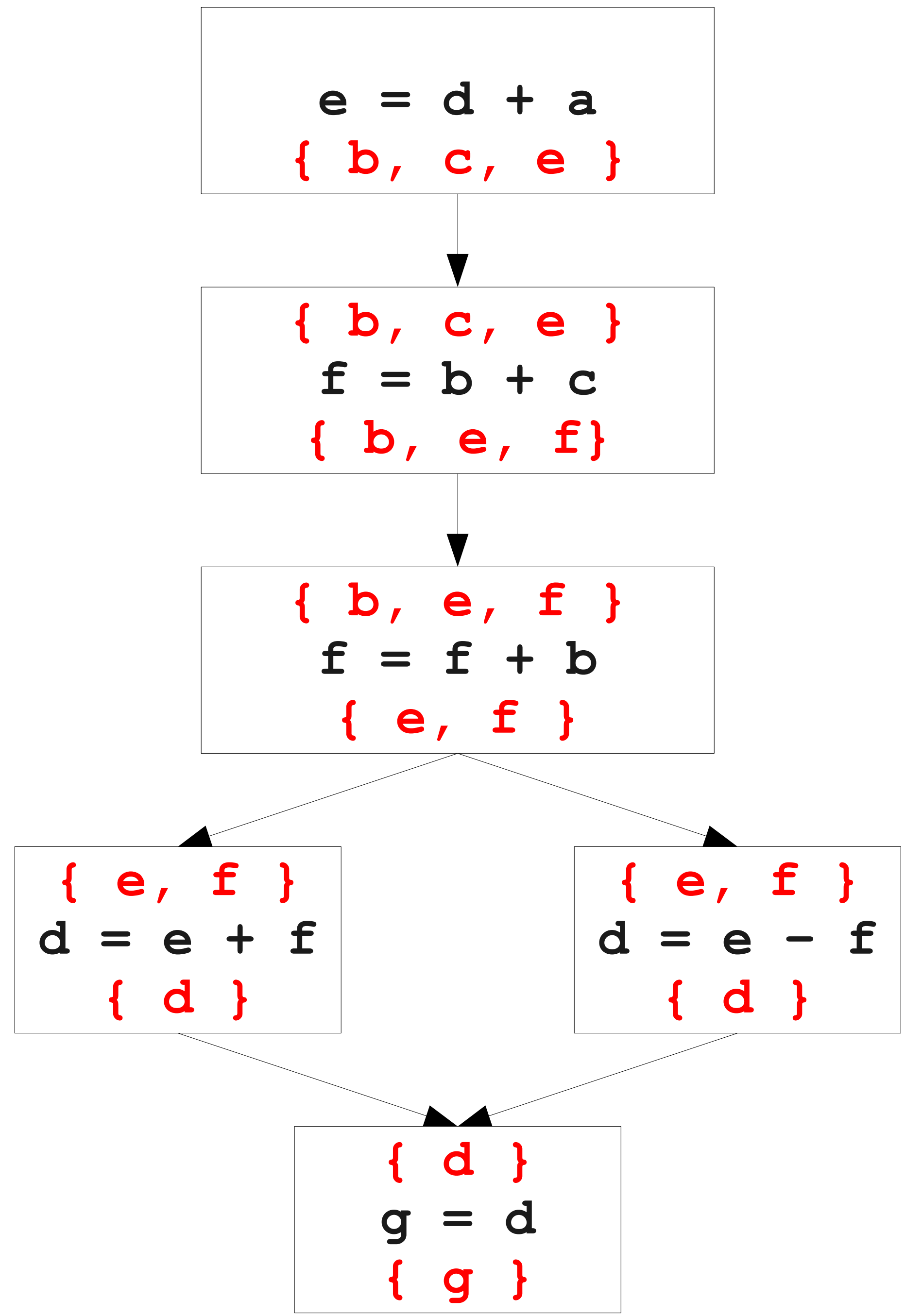

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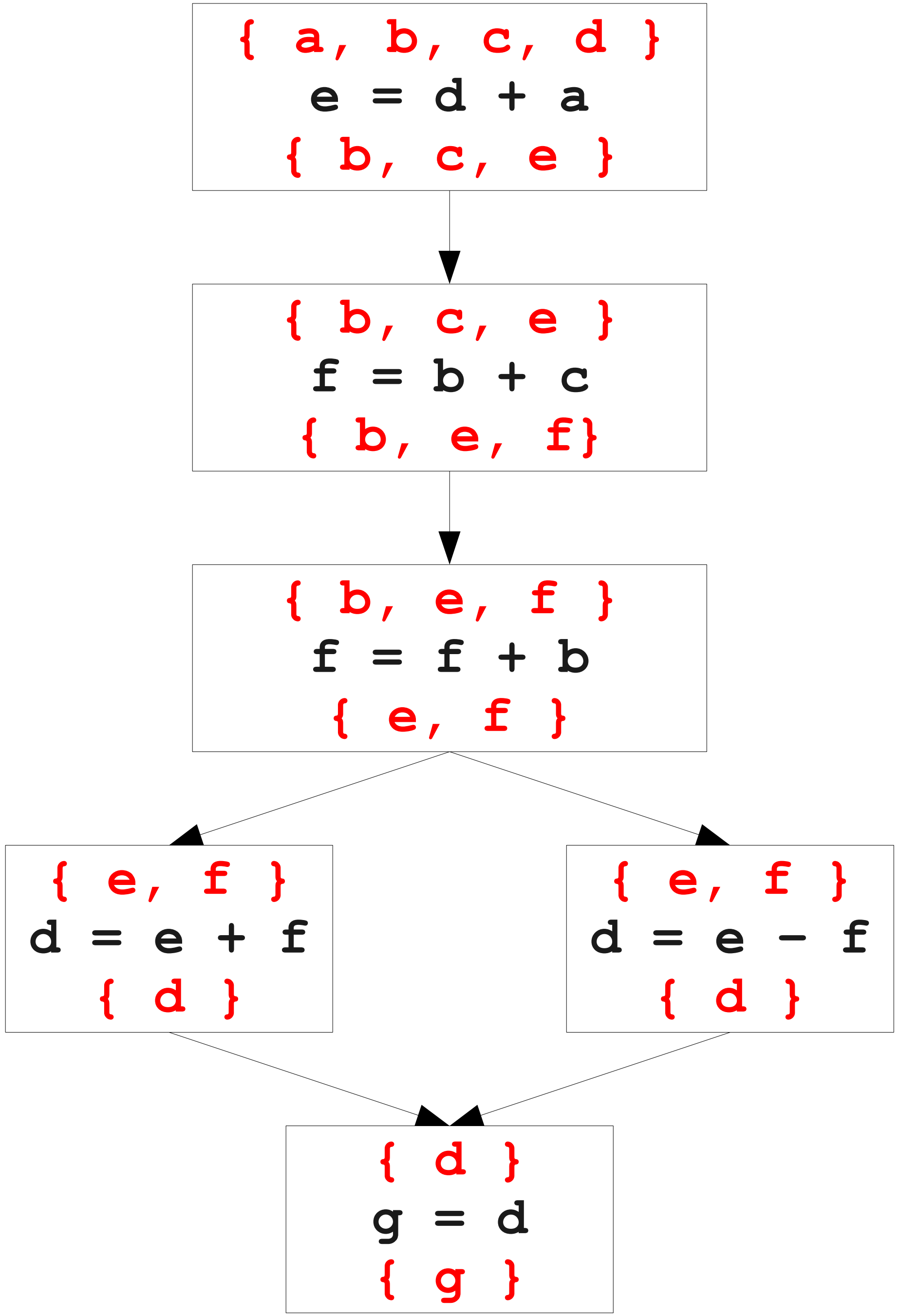

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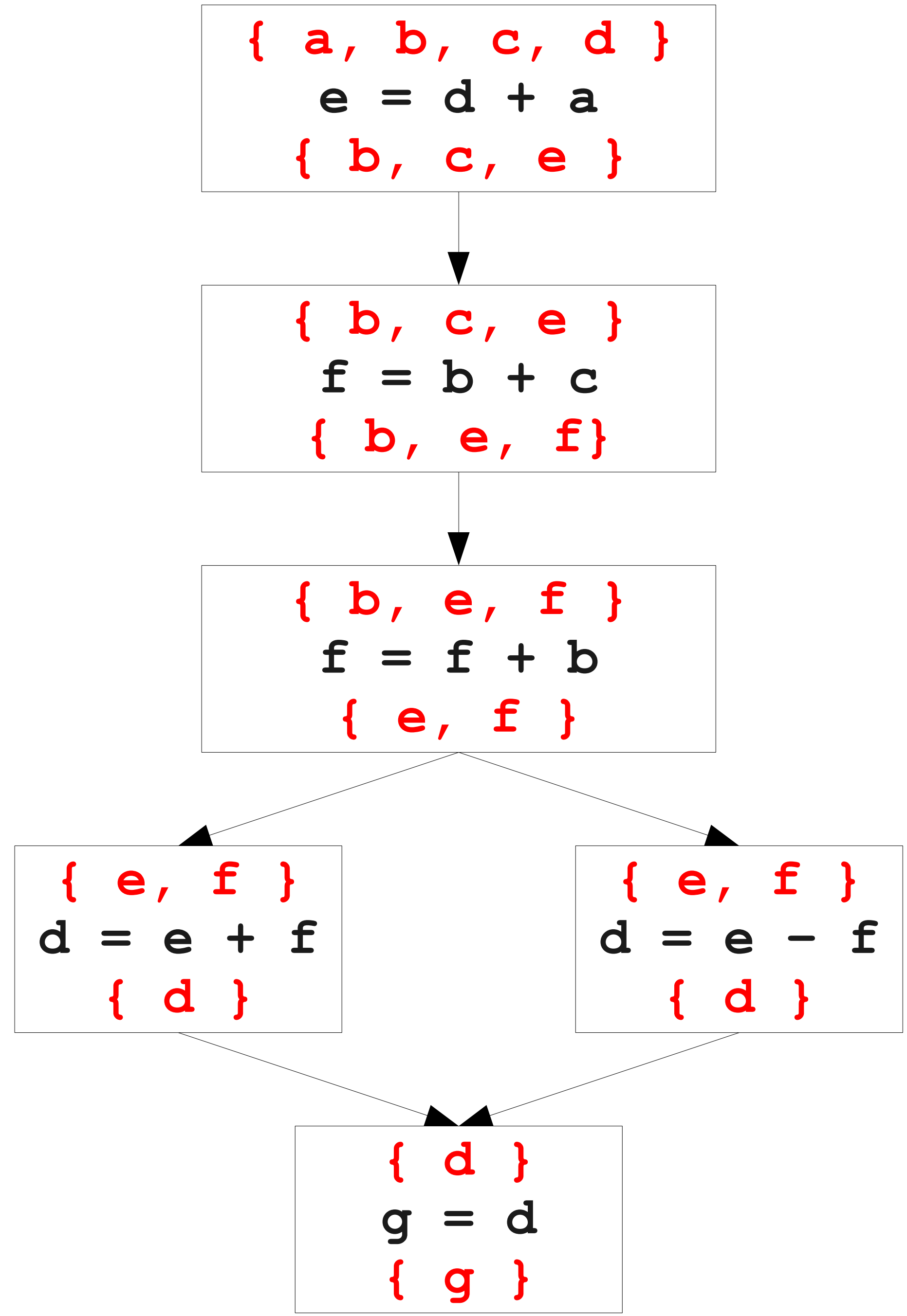

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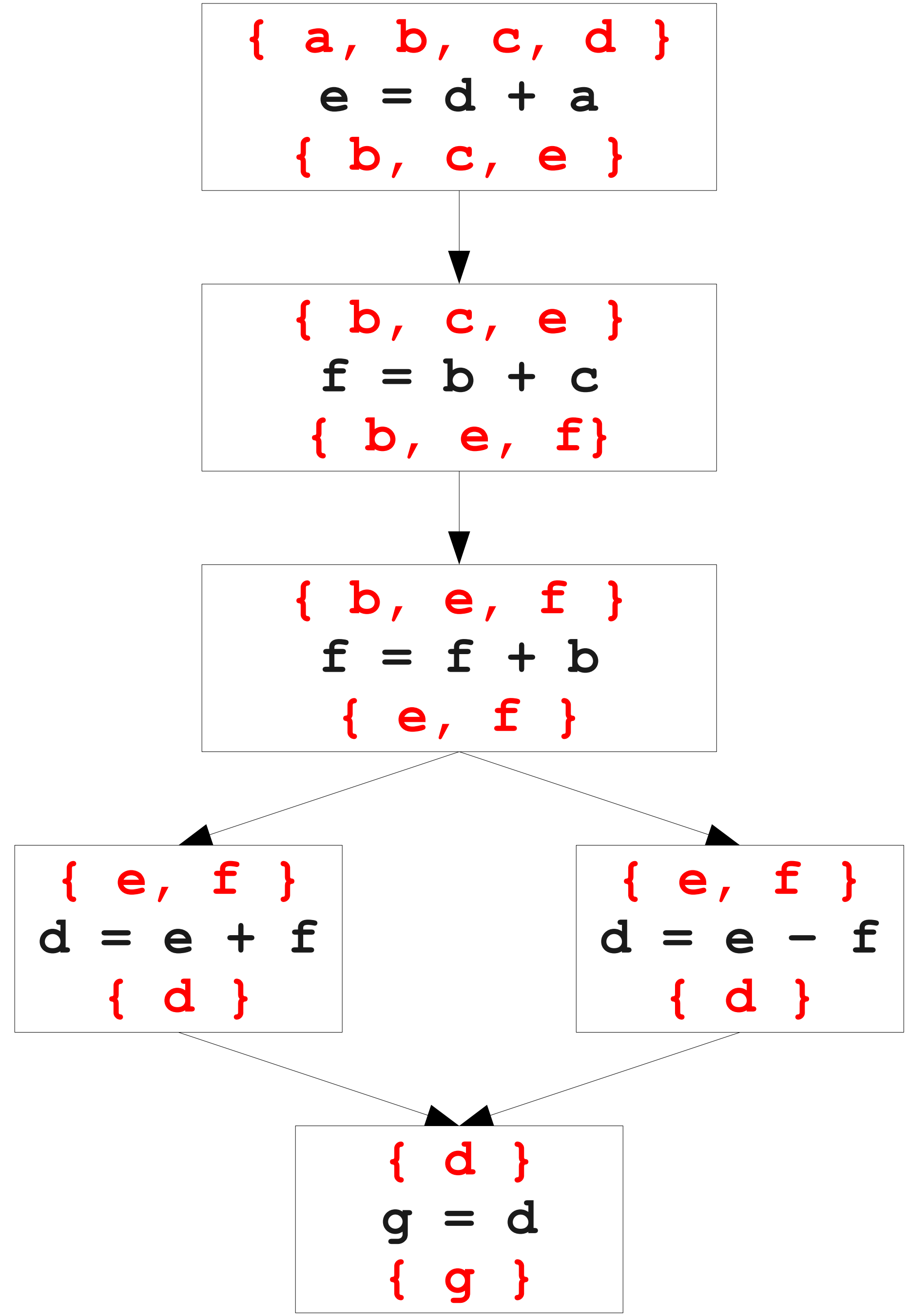

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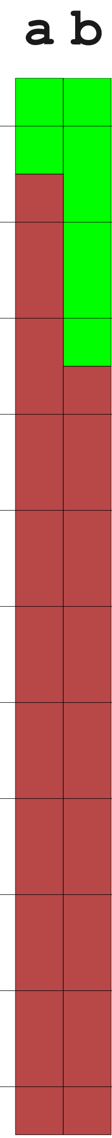

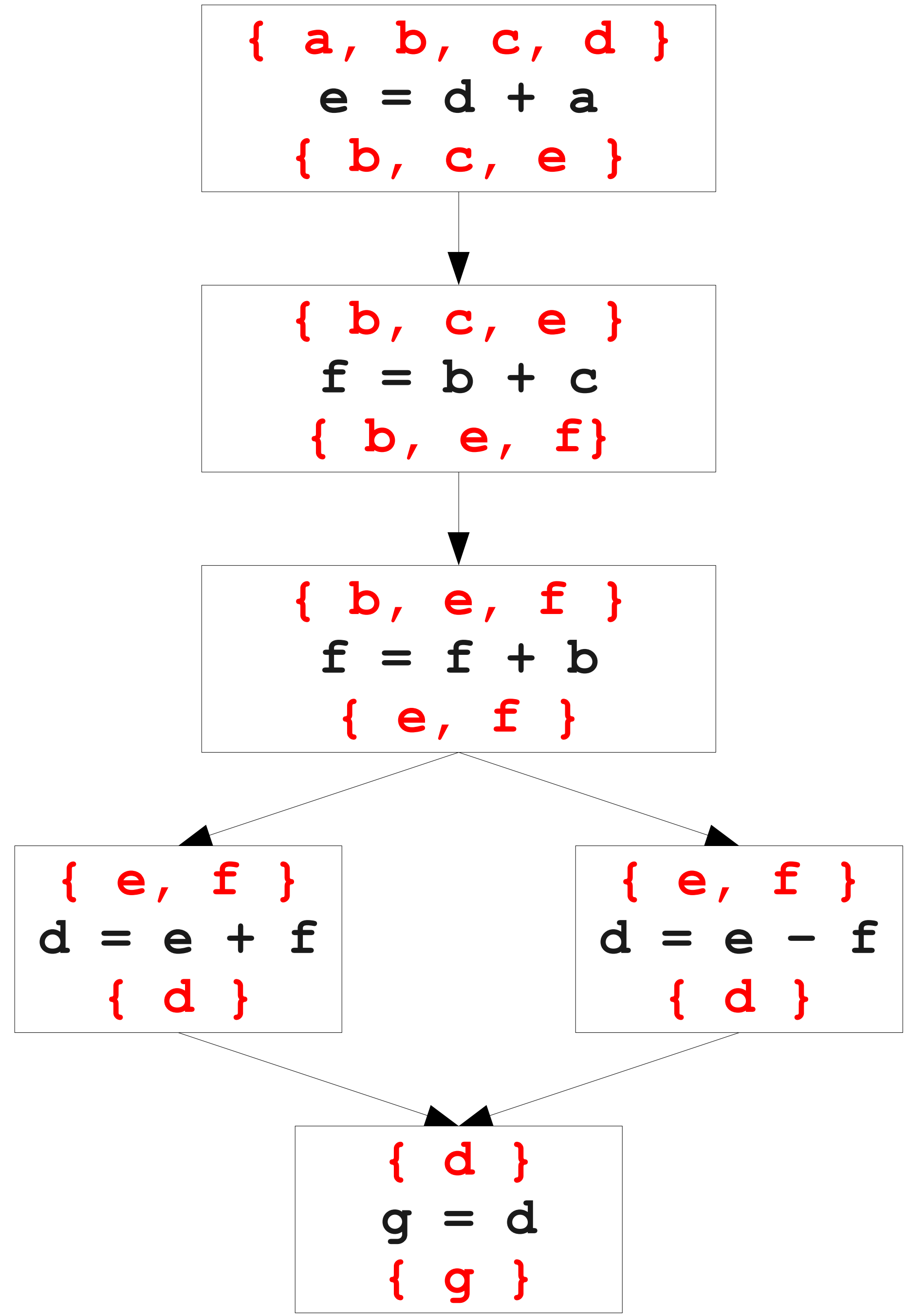

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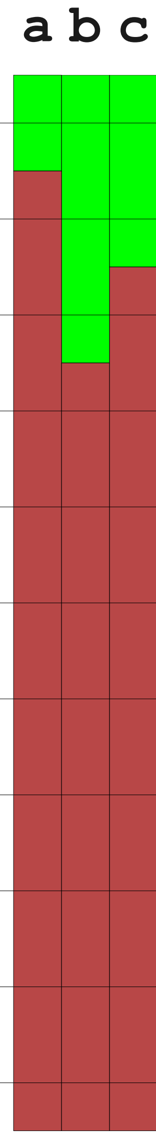

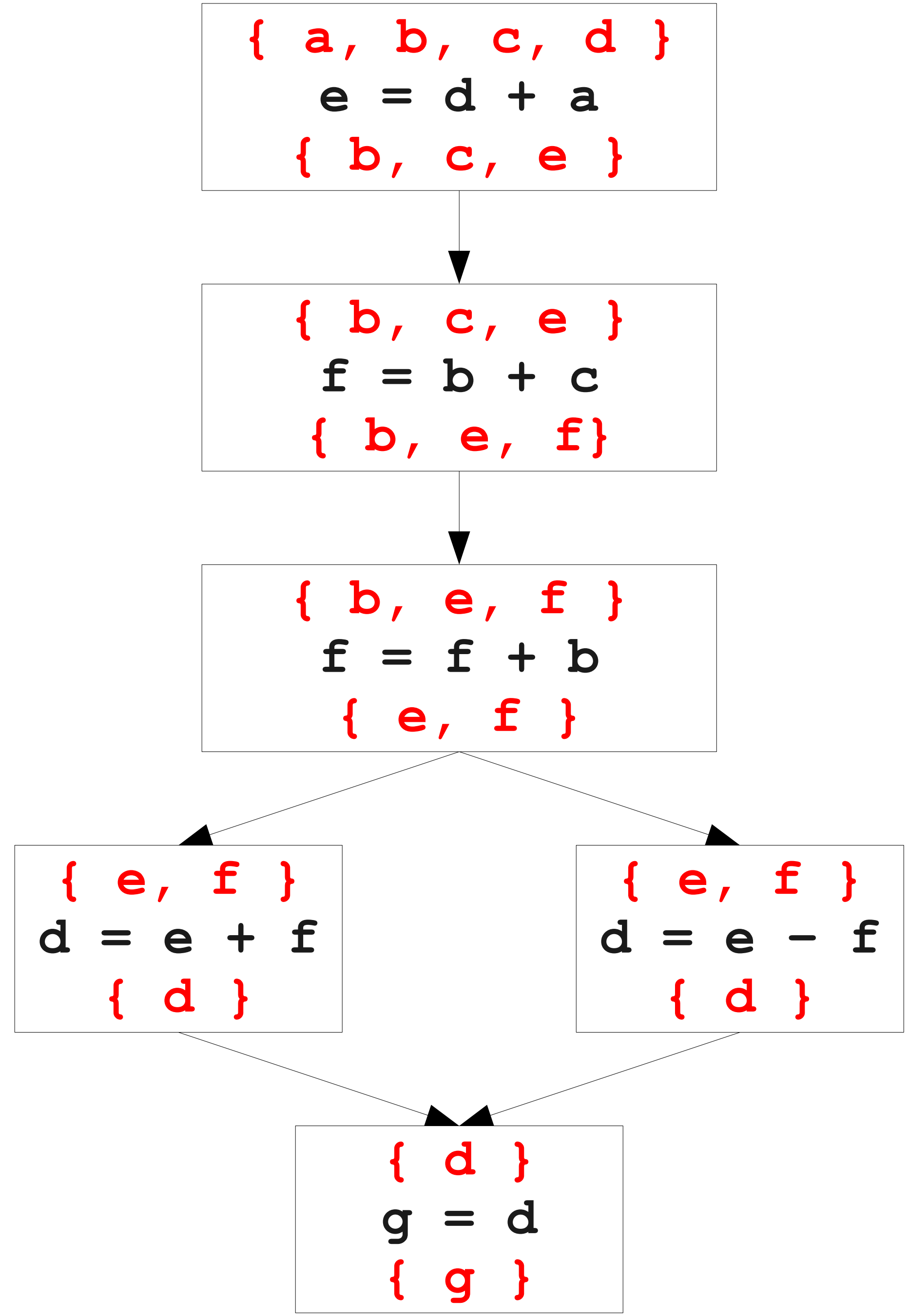

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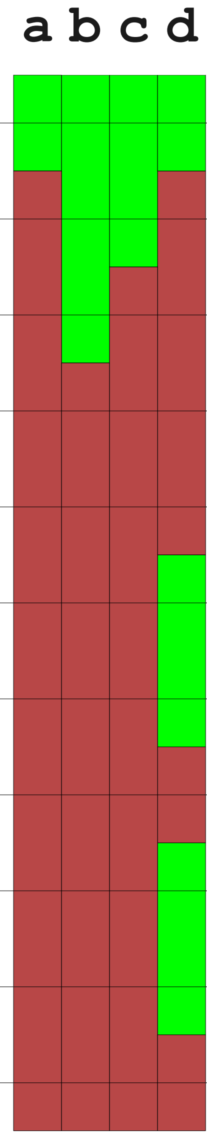

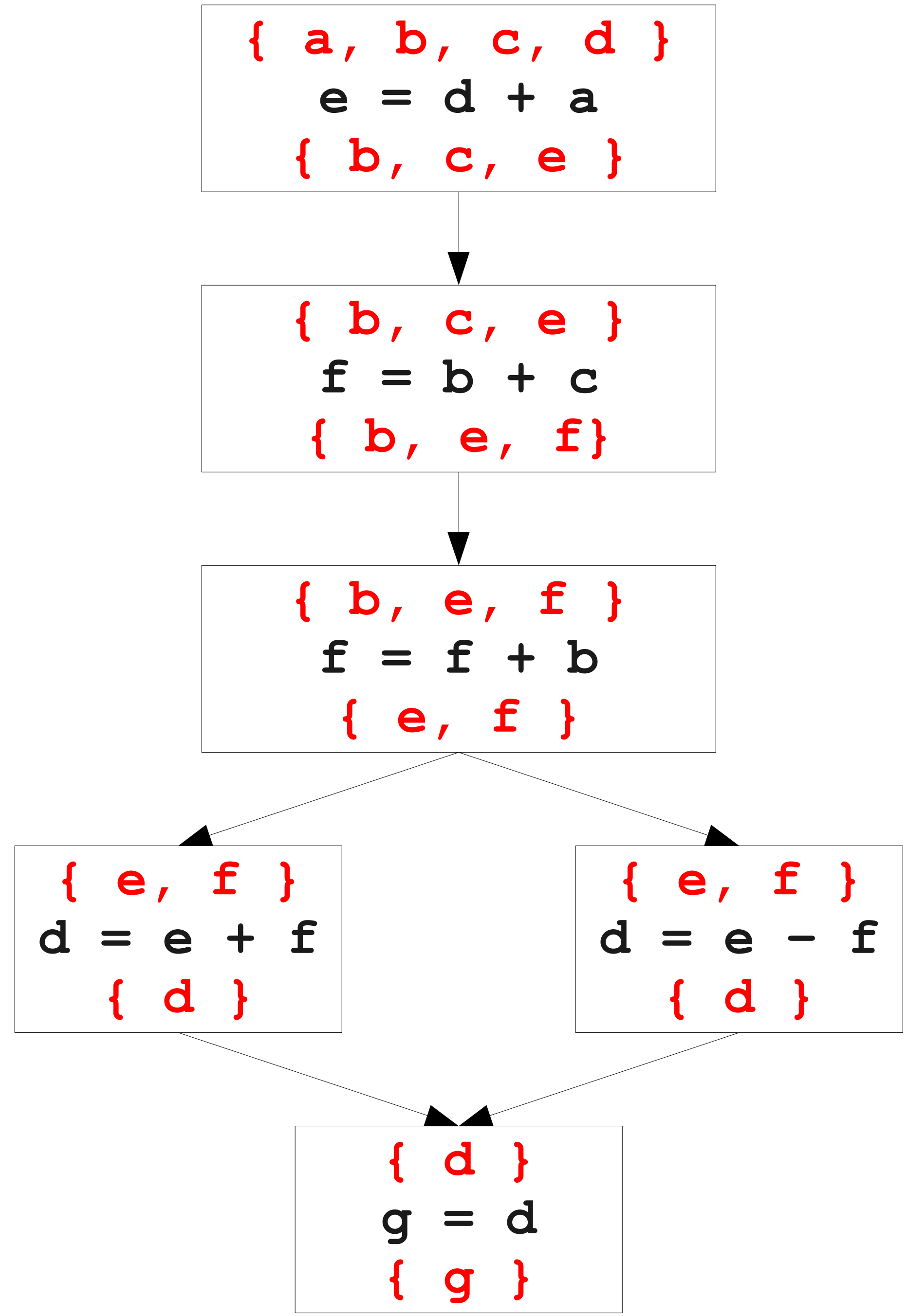

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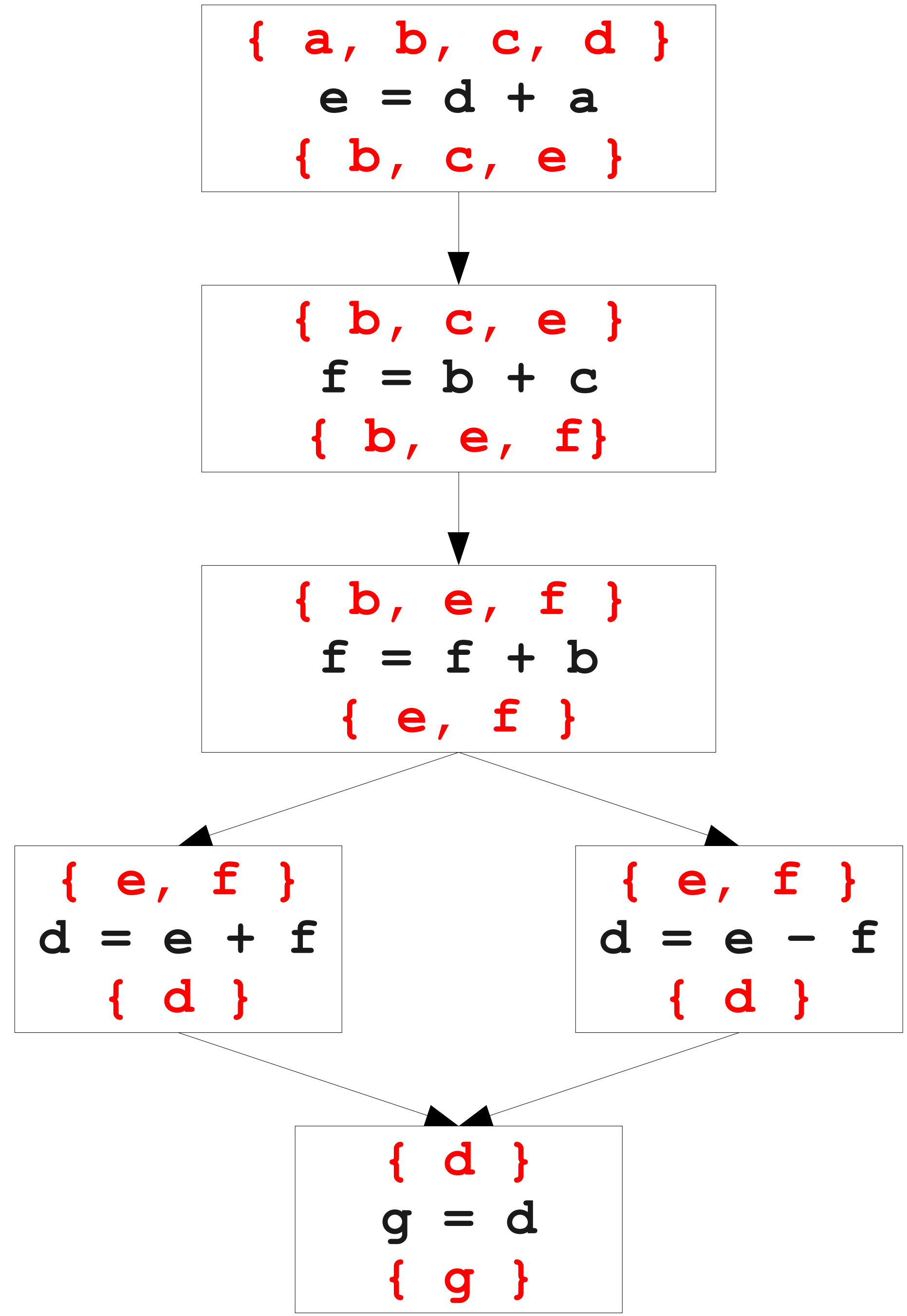
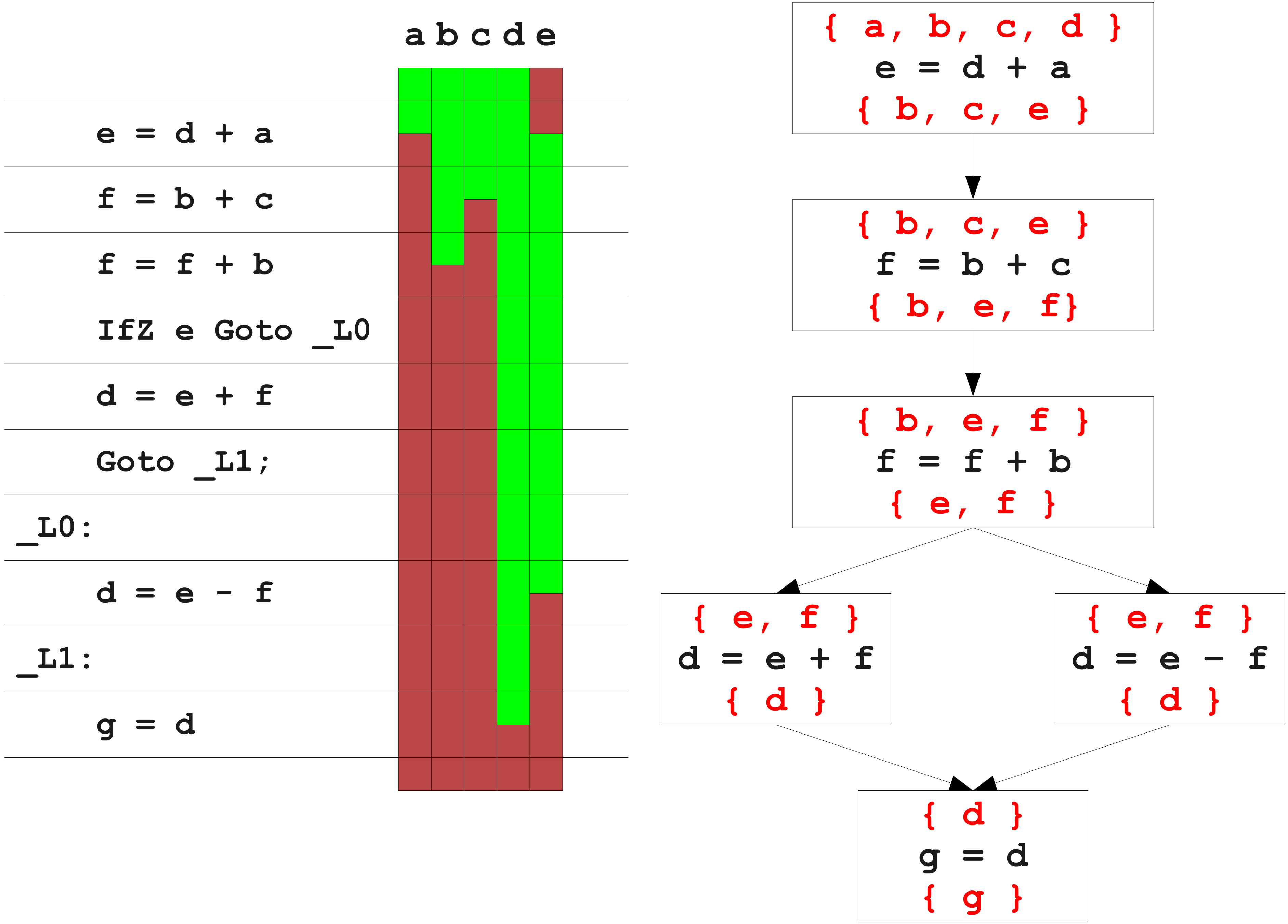


Live Ranges and Live Intervals



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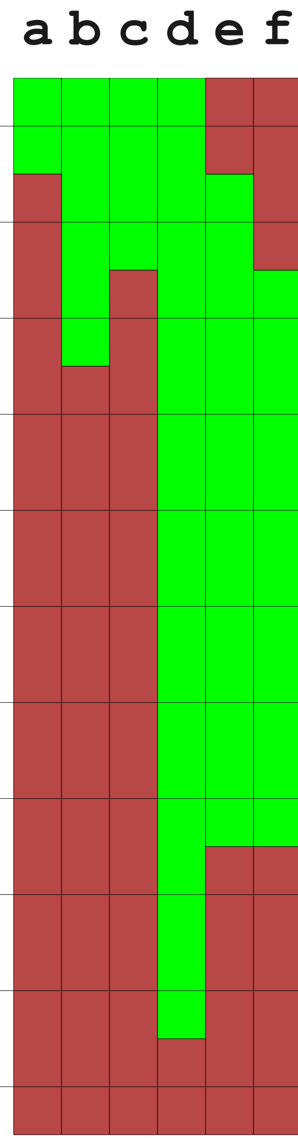

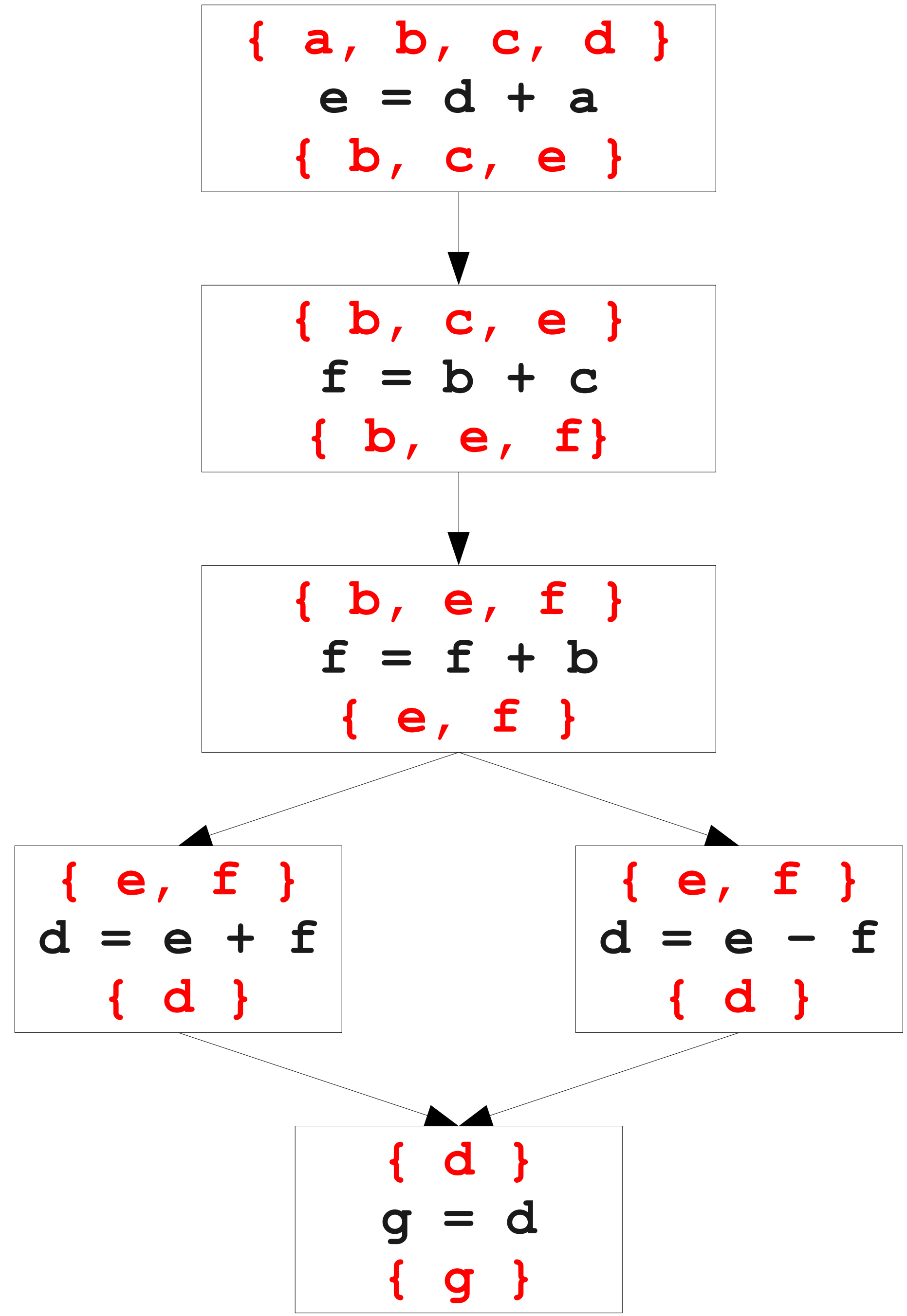

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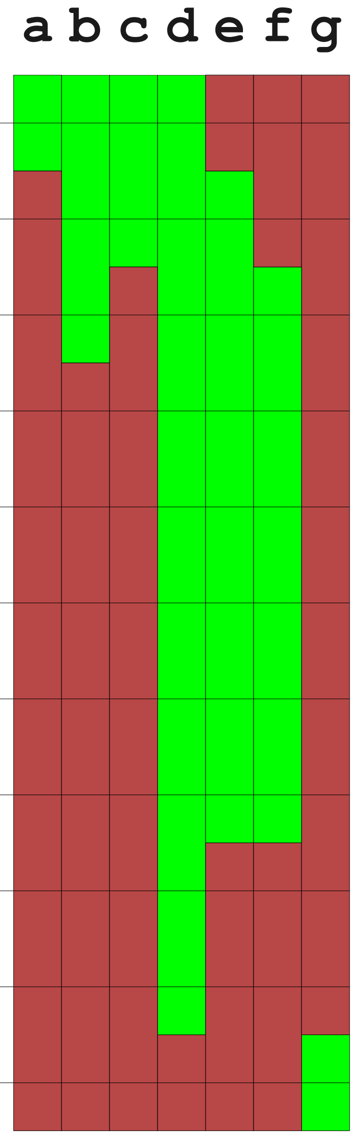
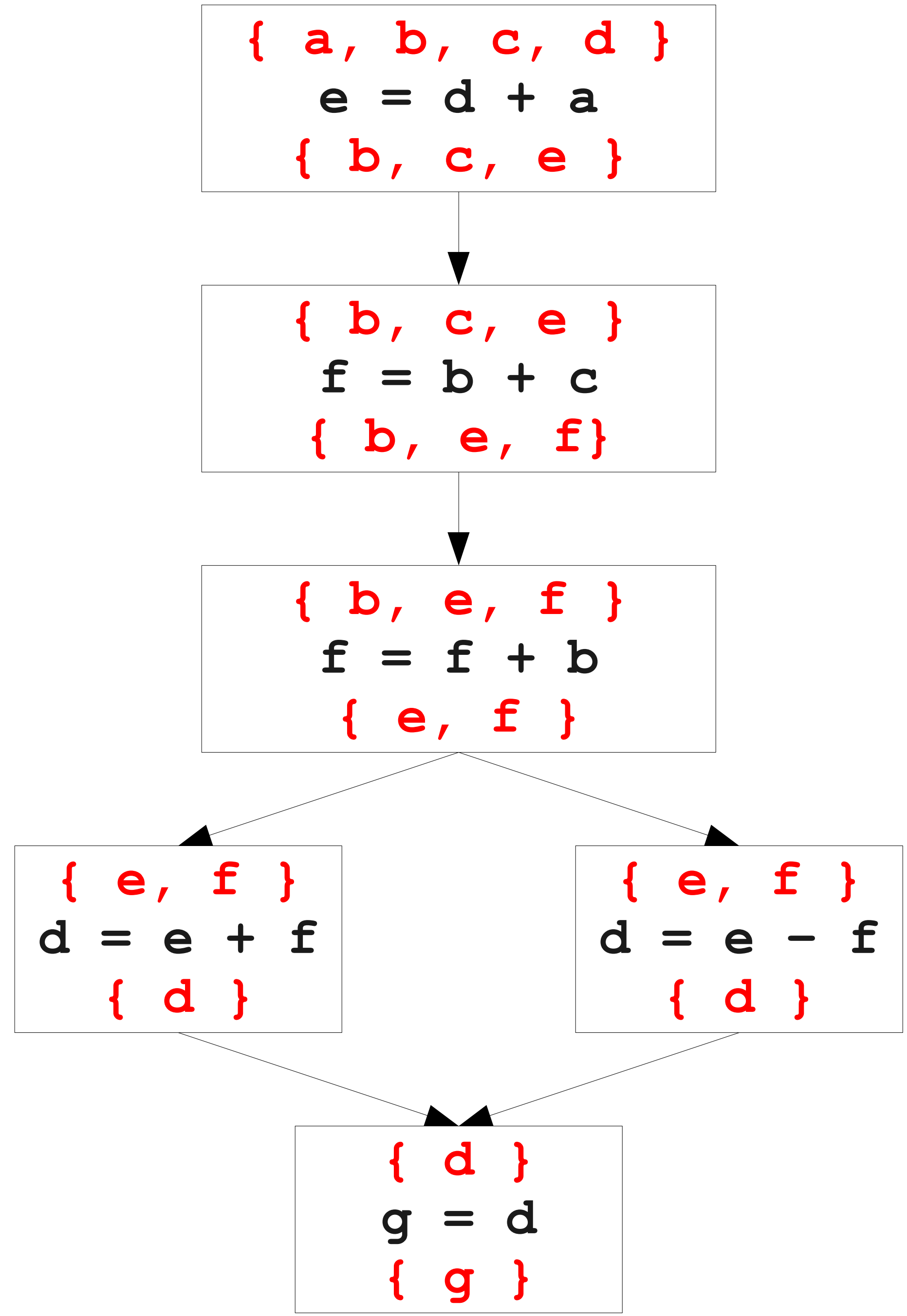
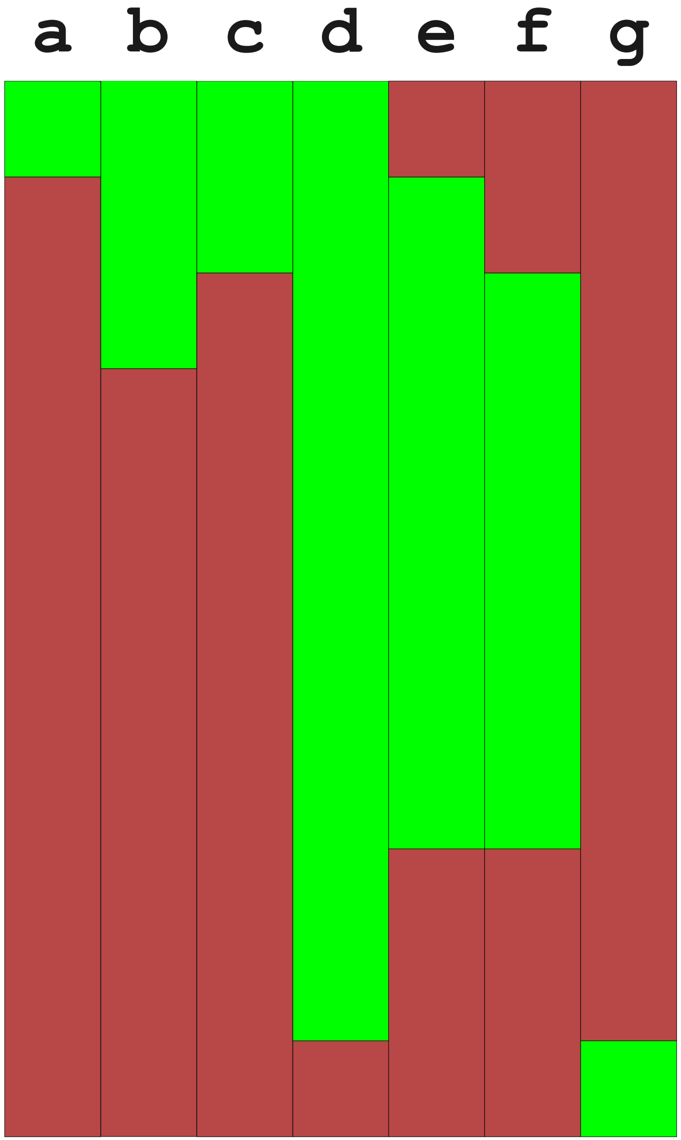




Register Allocation with Live Intervals

- Given the live intervals for all the variables in the program, we can allocate registers using a simple greedy algorithm.
- Idea: Track which registers are free at each point.
- When a live interval begins, give that variable a free register.
- When a live interval ends, the register is once again free.
- We can't always fit everything into a register; we'll see what do to in a minute.



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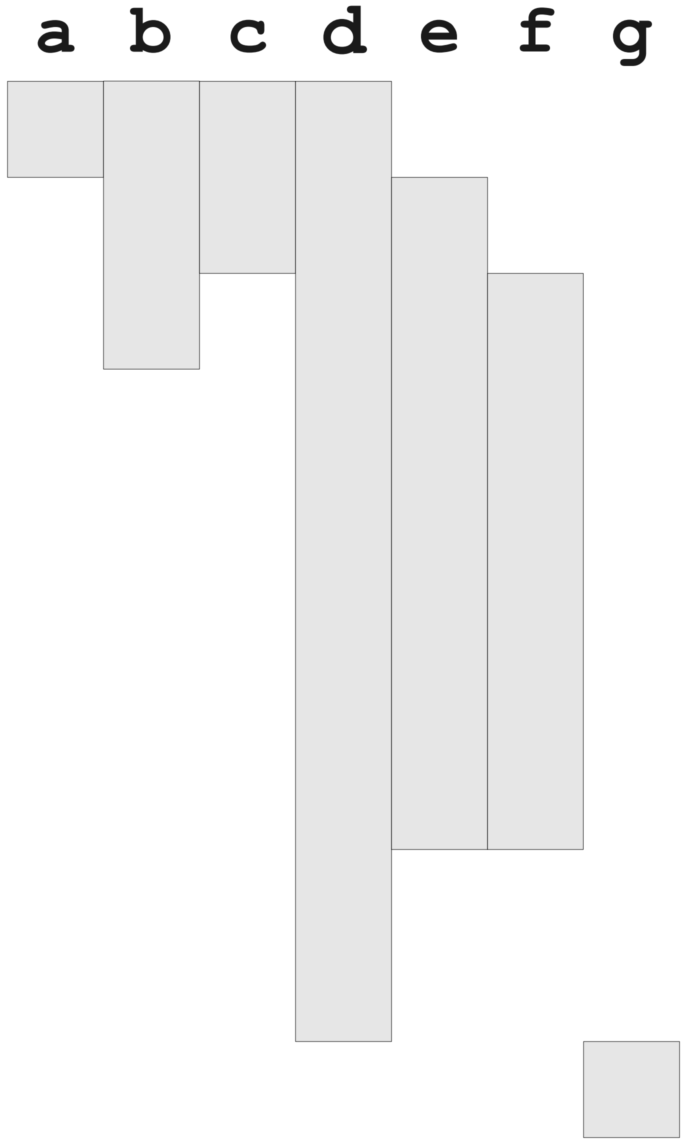

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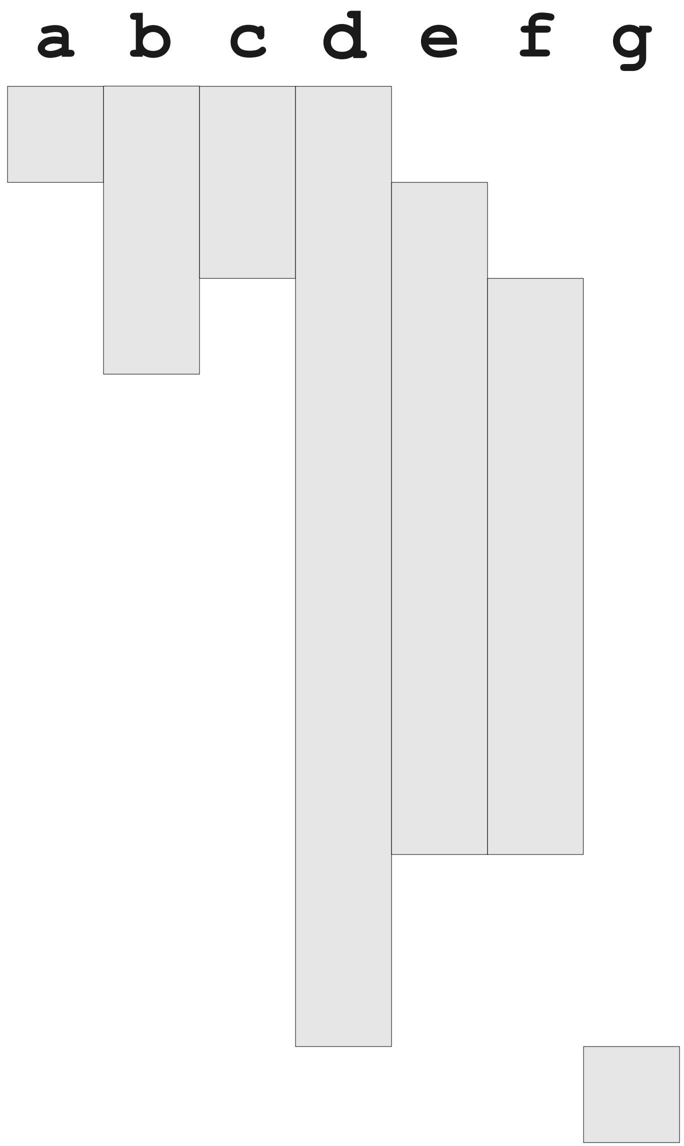

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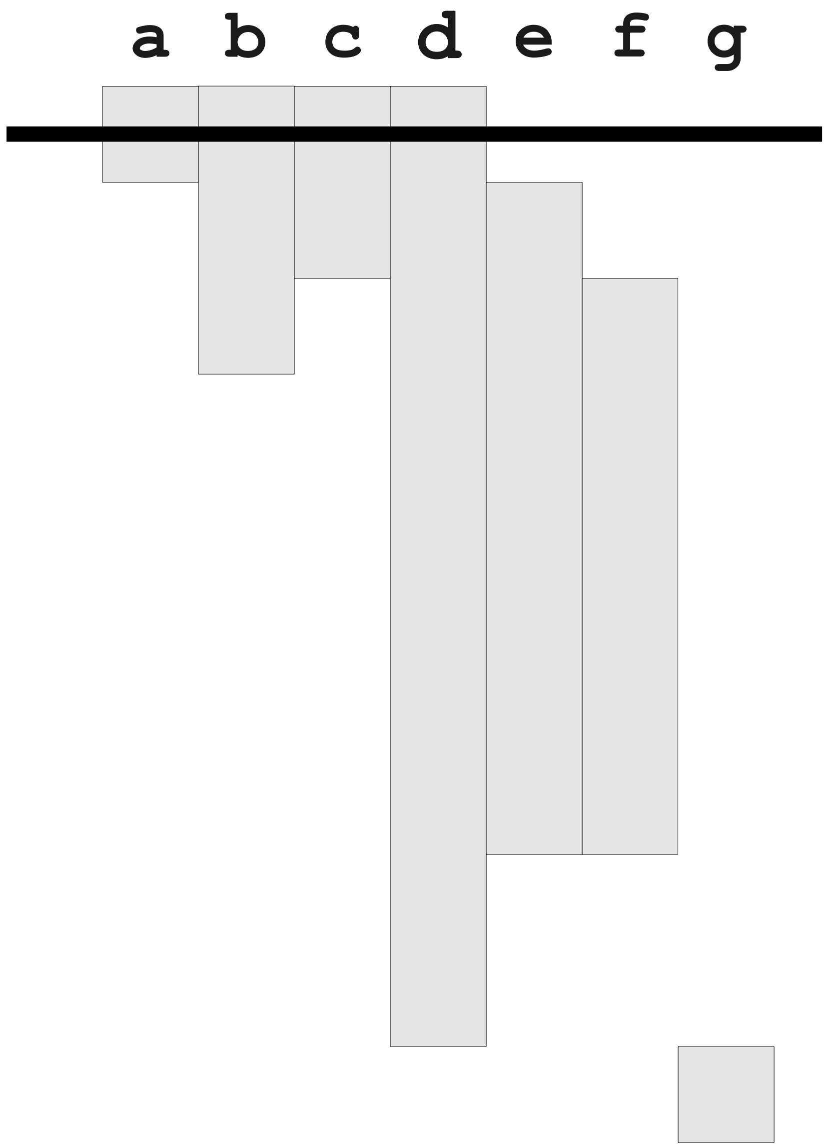
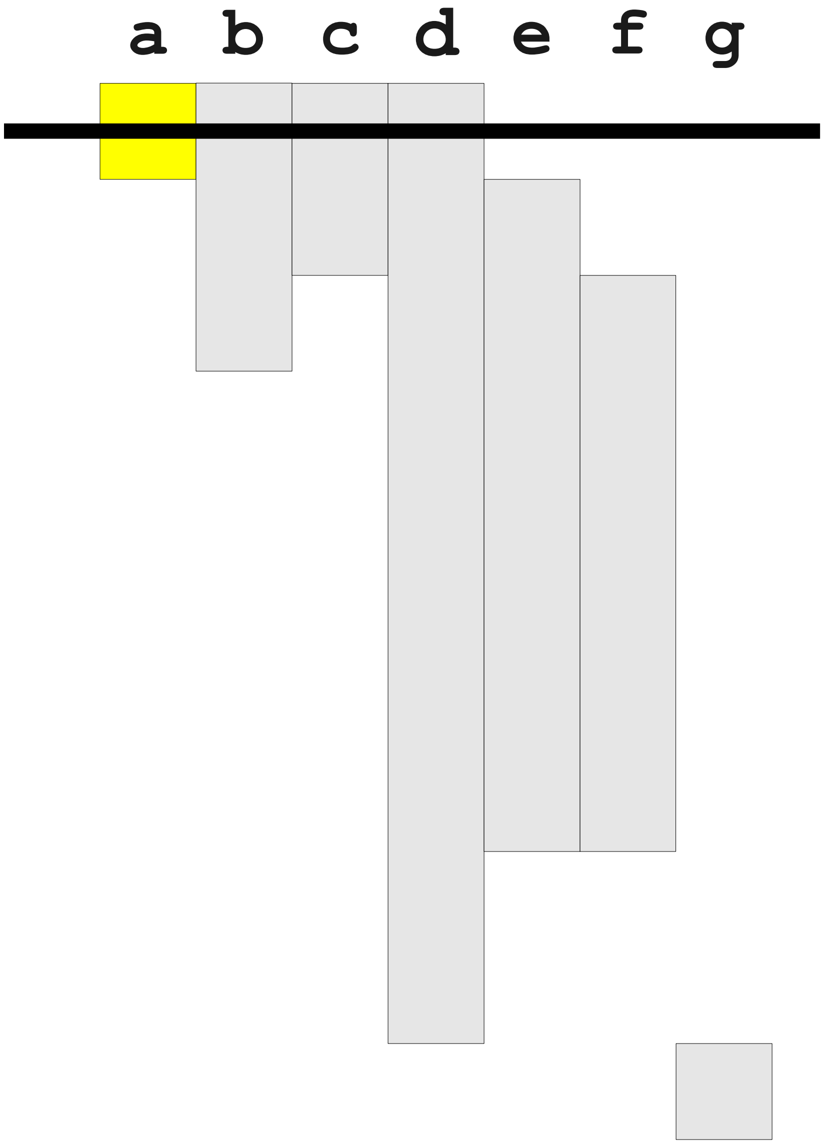


Free Registers

R_0	R_1	R_2	R_3

Register Allocation with Live Intervals



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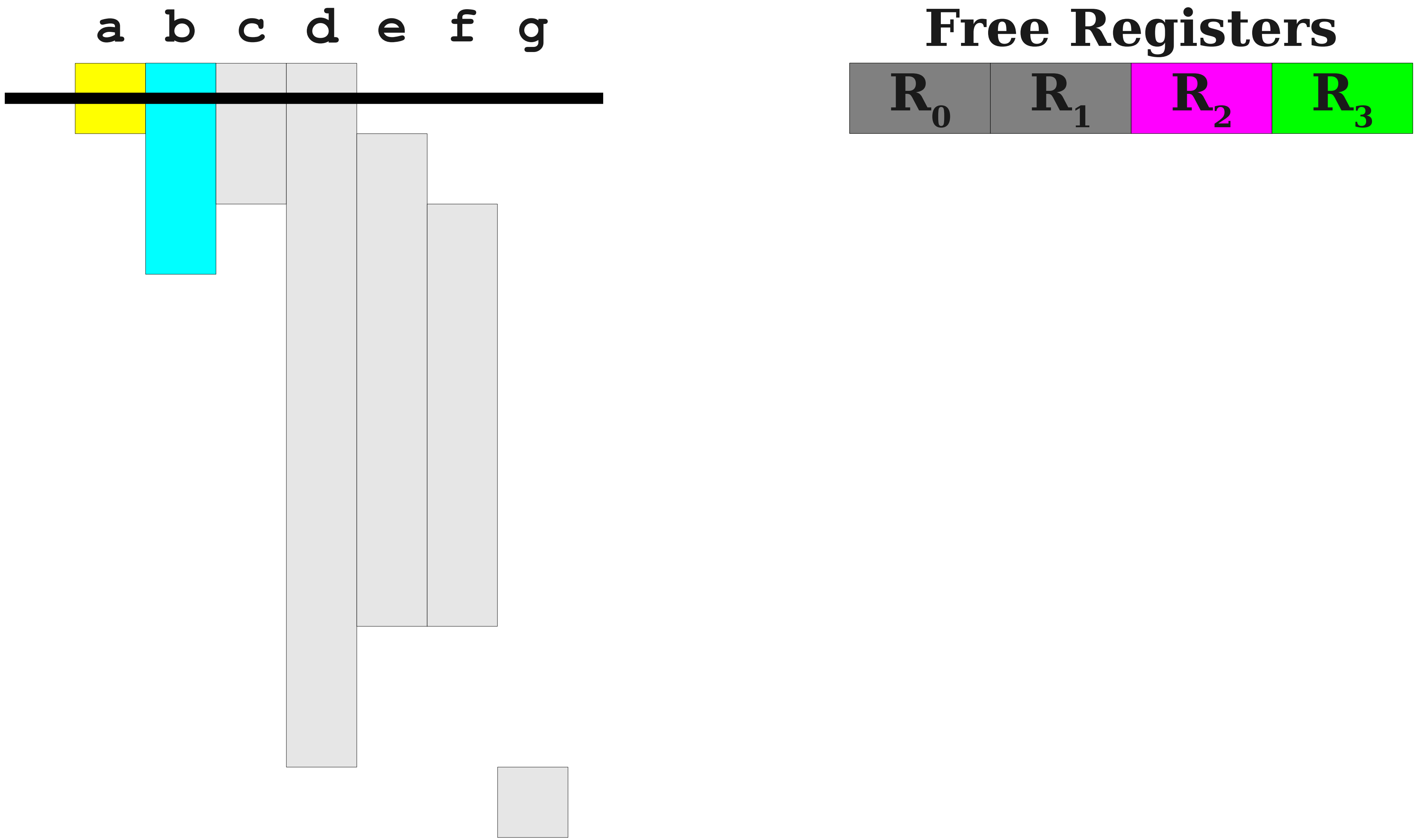

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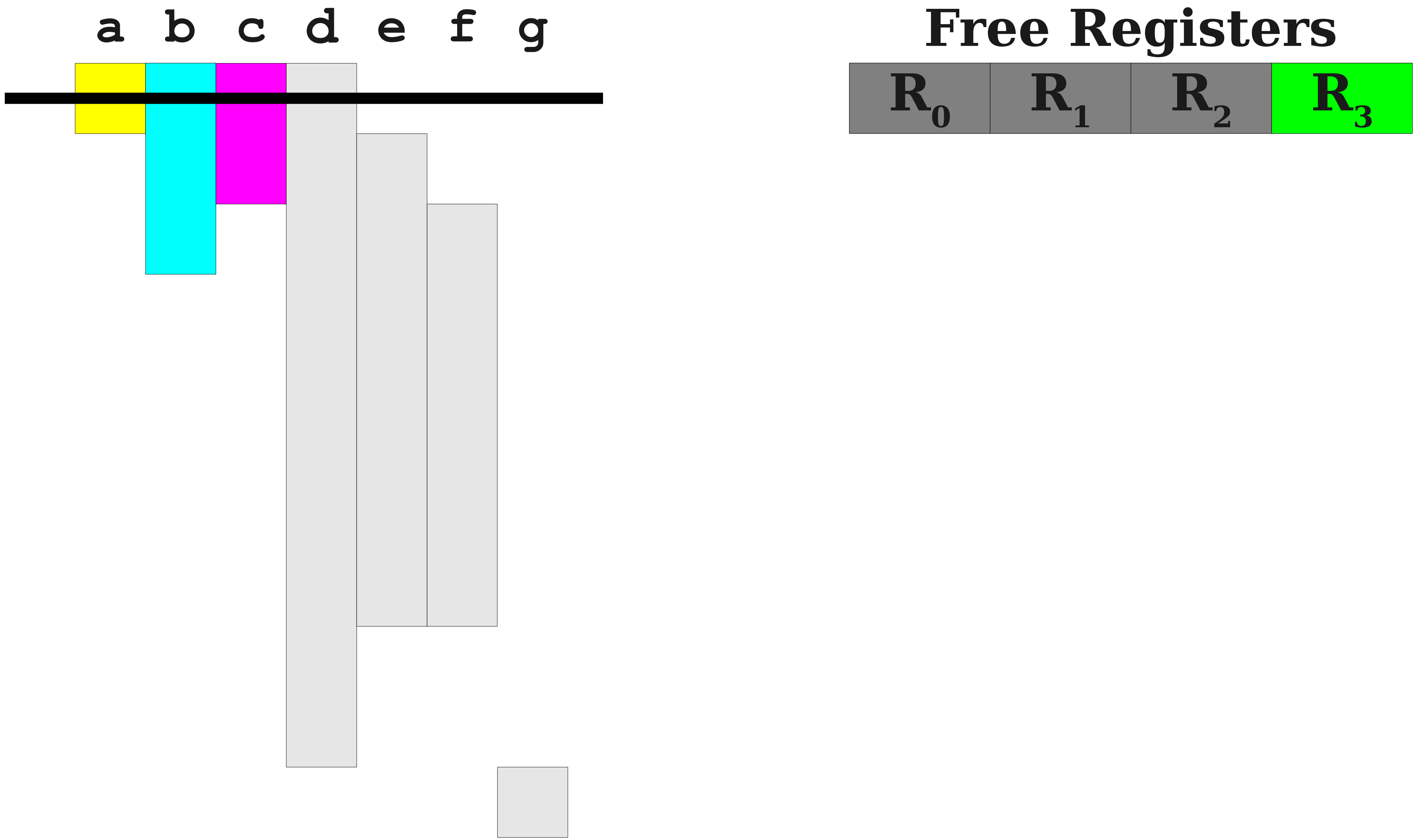

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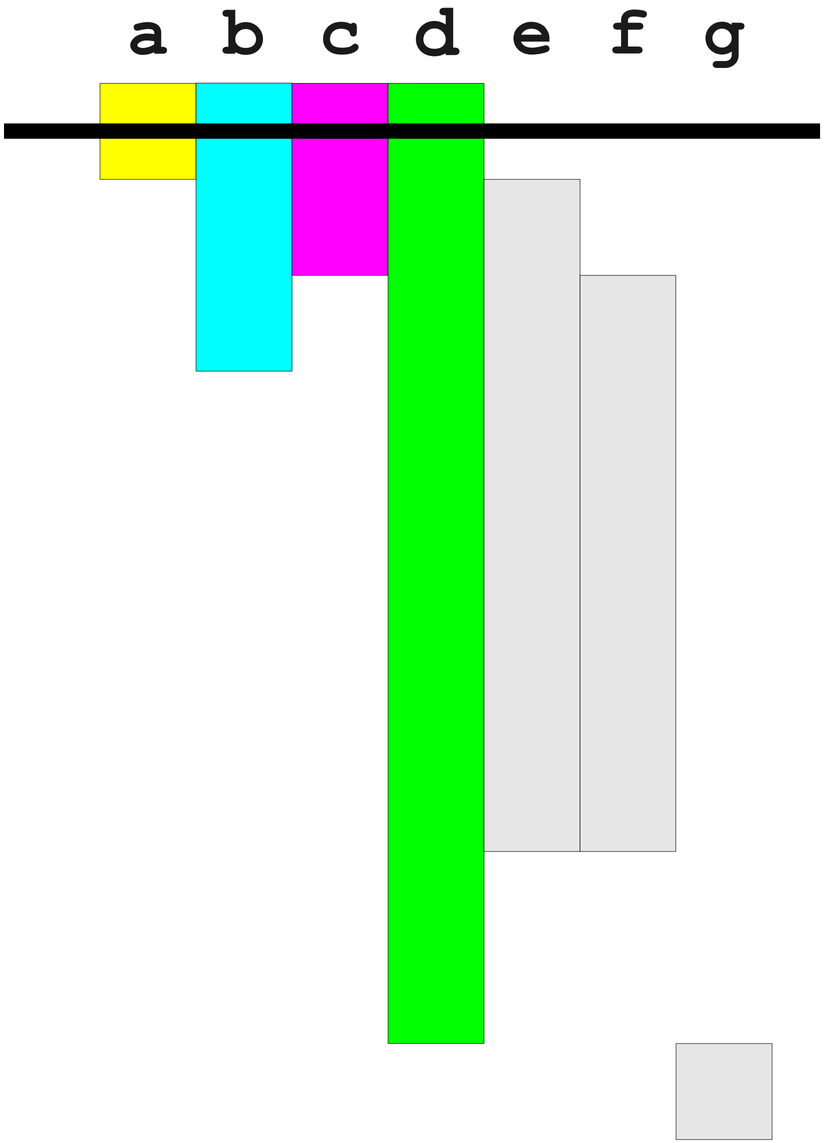
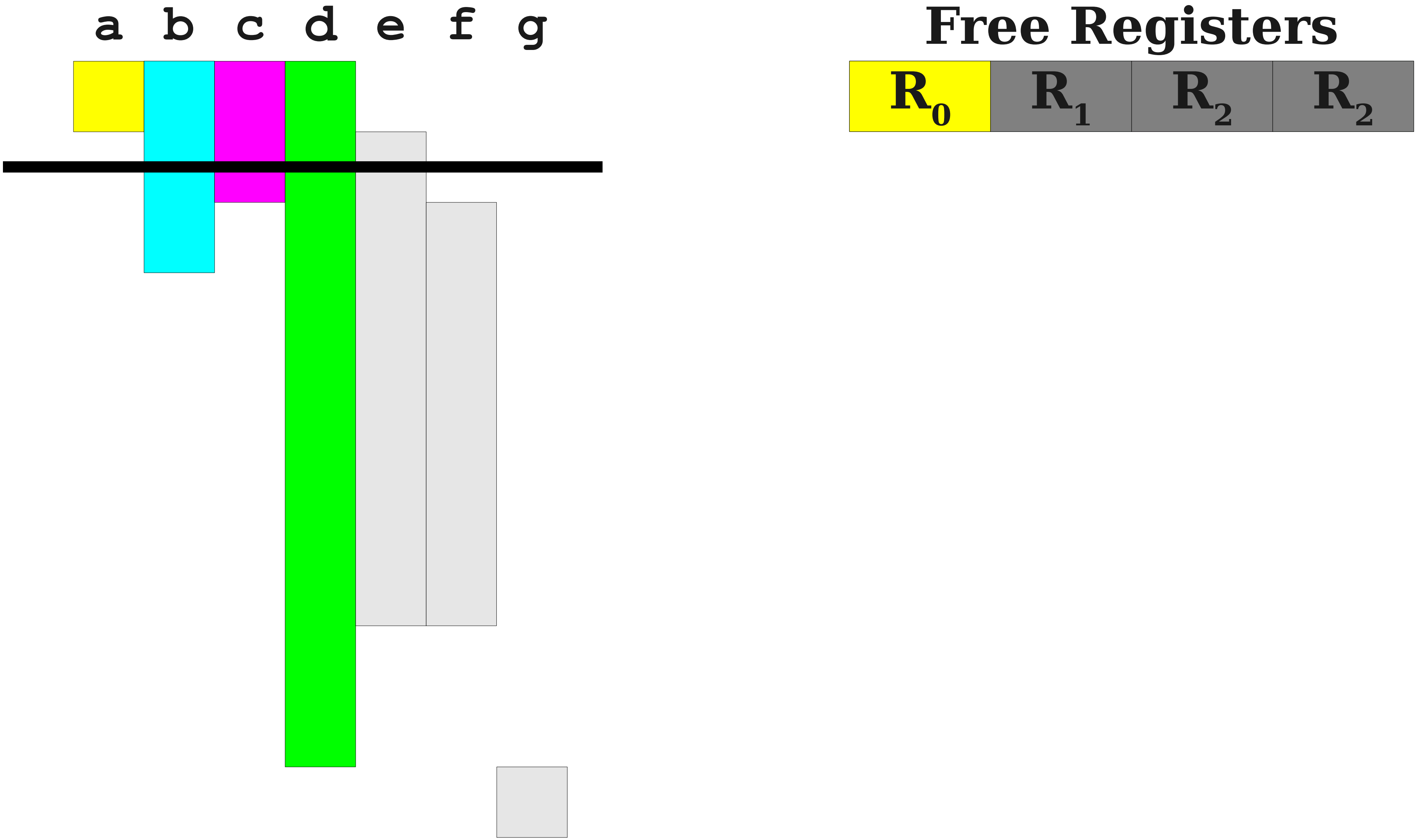


Free Registers

R_0	R_1	R_2	R_2

Register Allocation with Live Intervals



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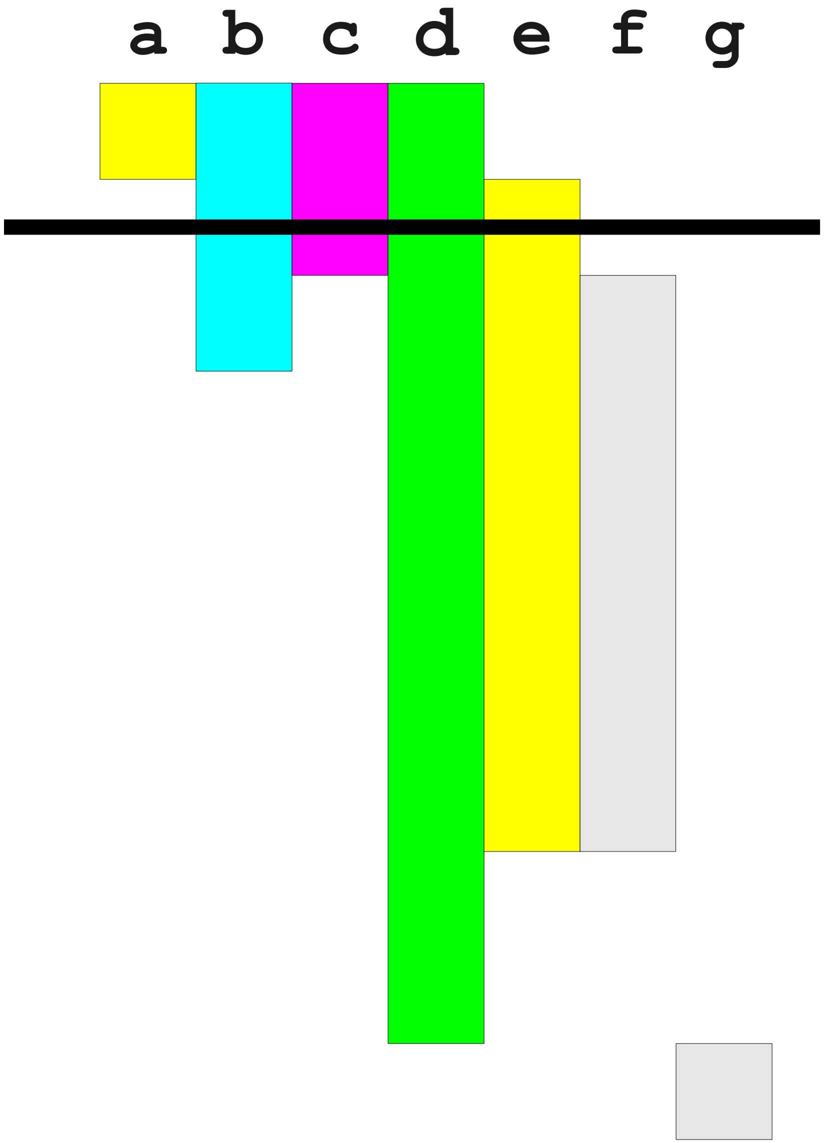

Free Registers

R_0	R_1	R_2	R_2

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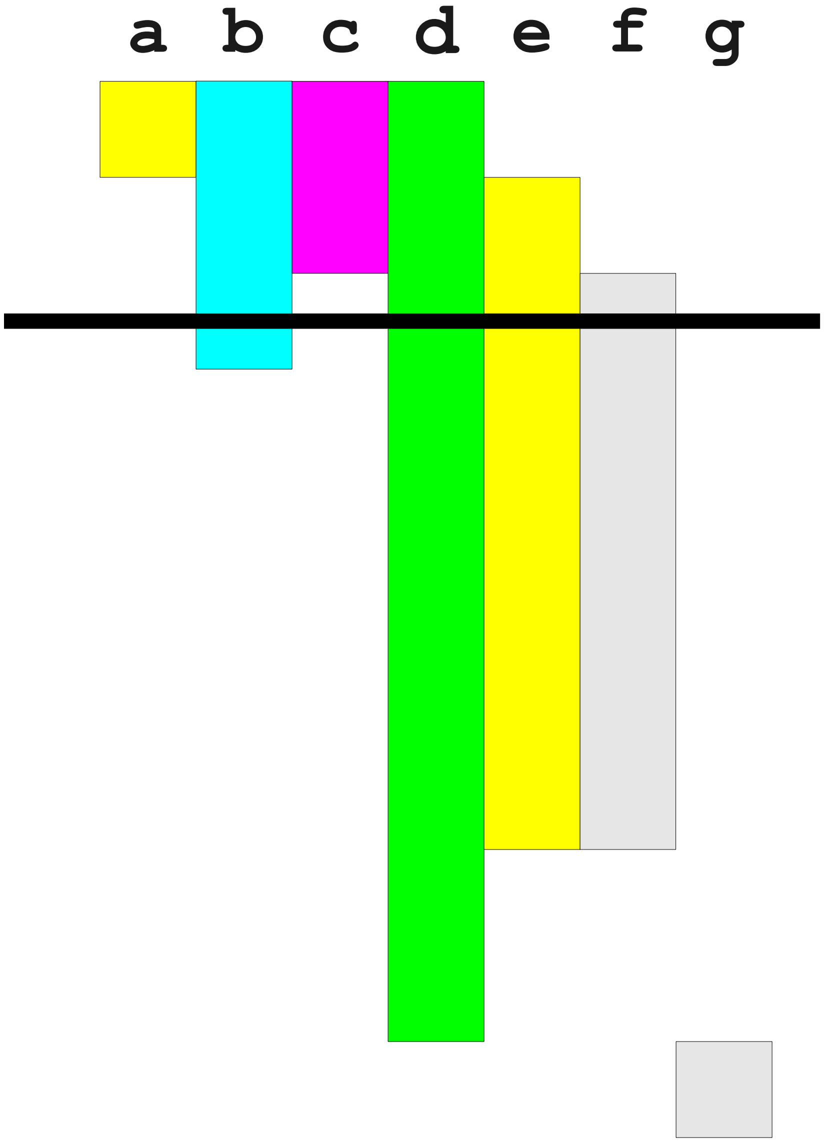

Free Registers

R_0	R_1	R_2	R_2

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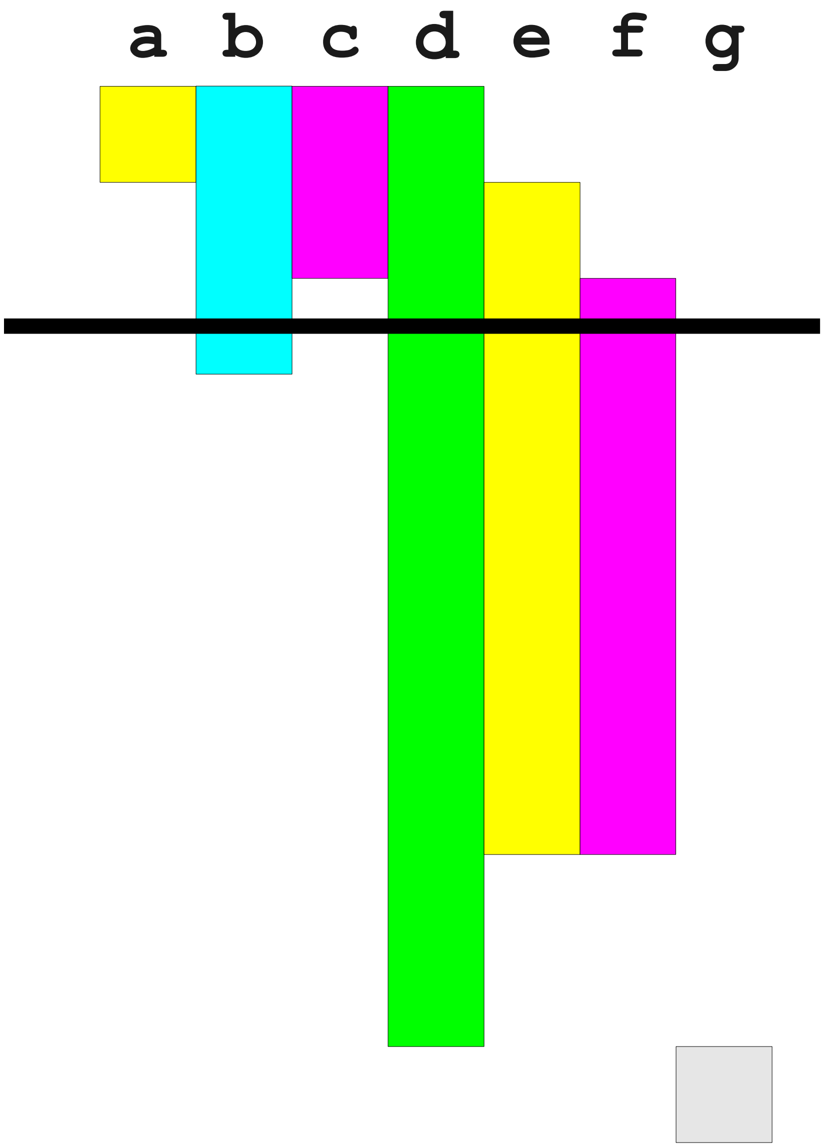

Free Registers

R_0	R_1	R_2	R_2

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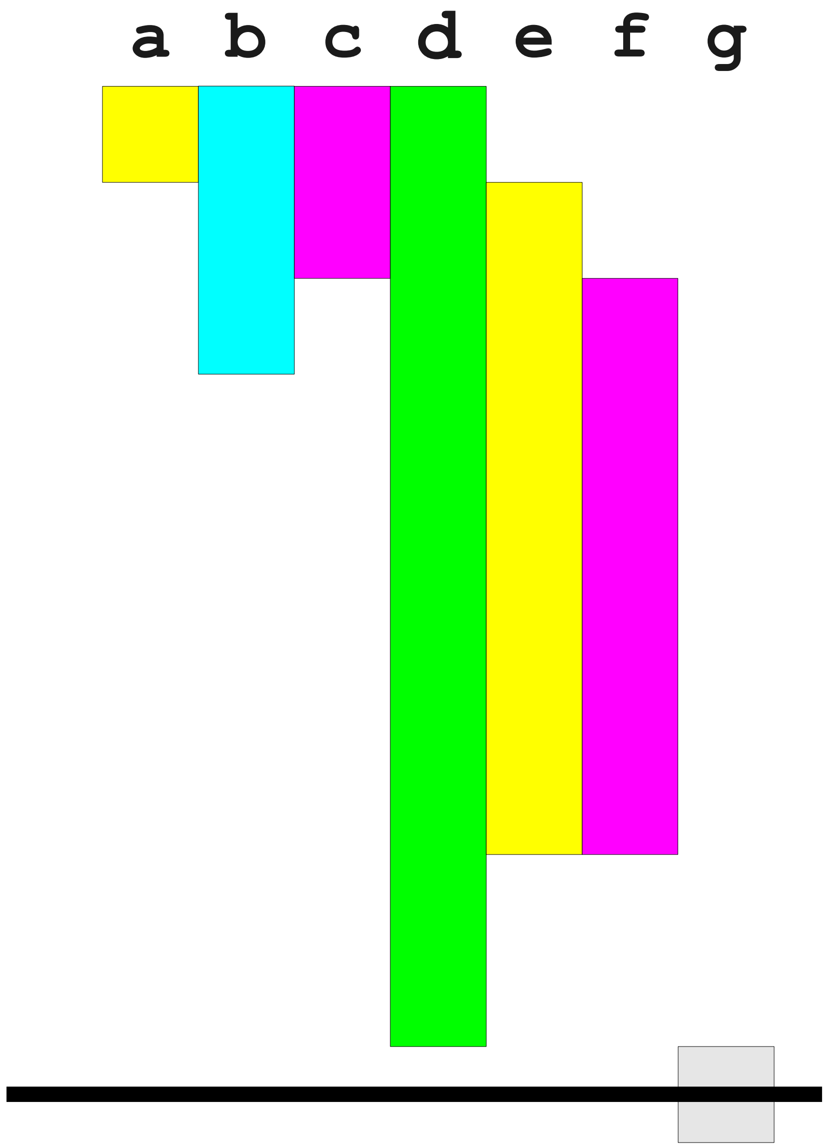
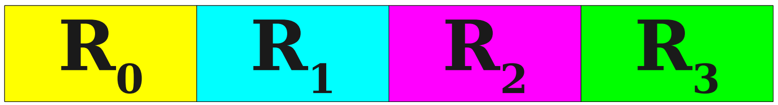


Free Registers



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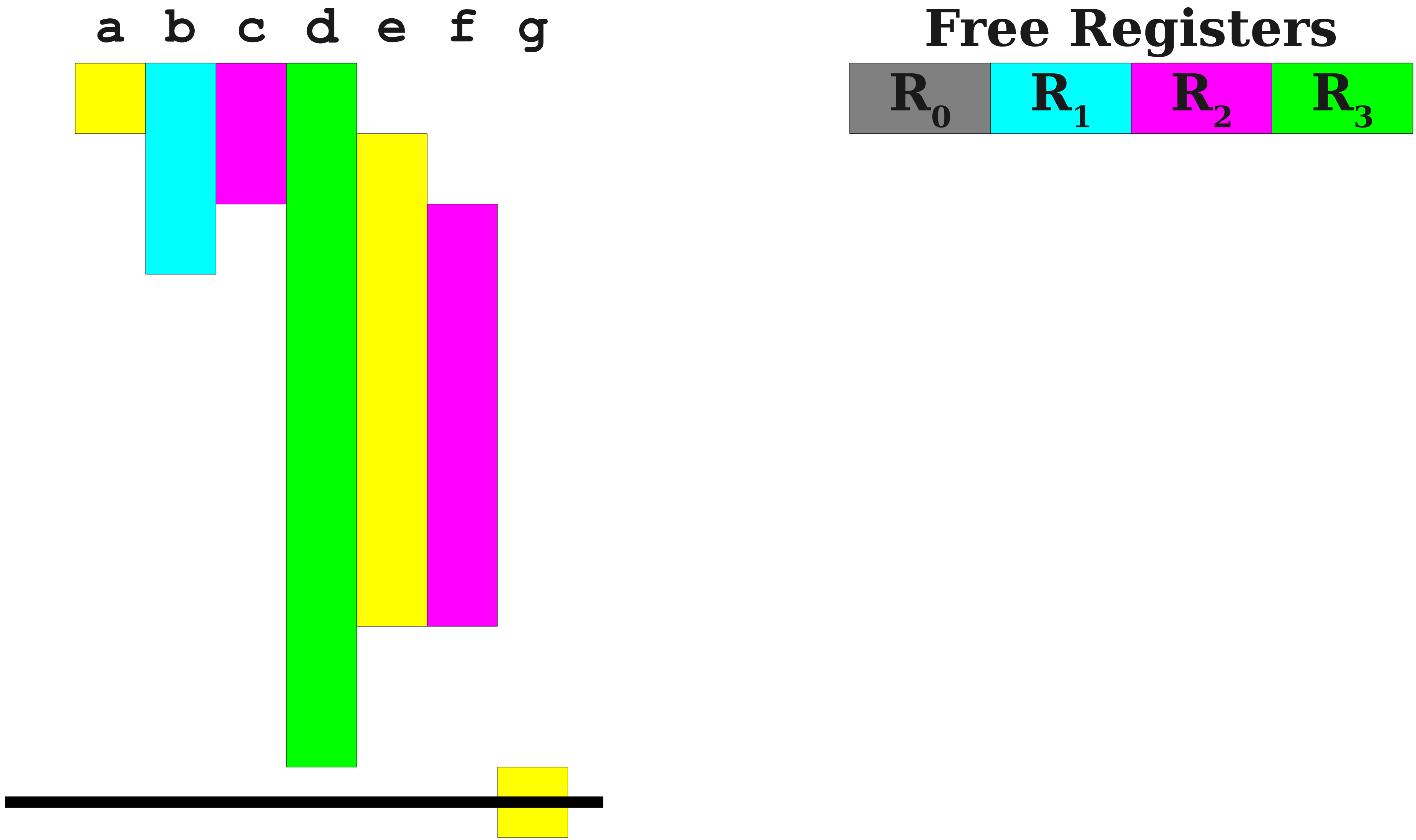

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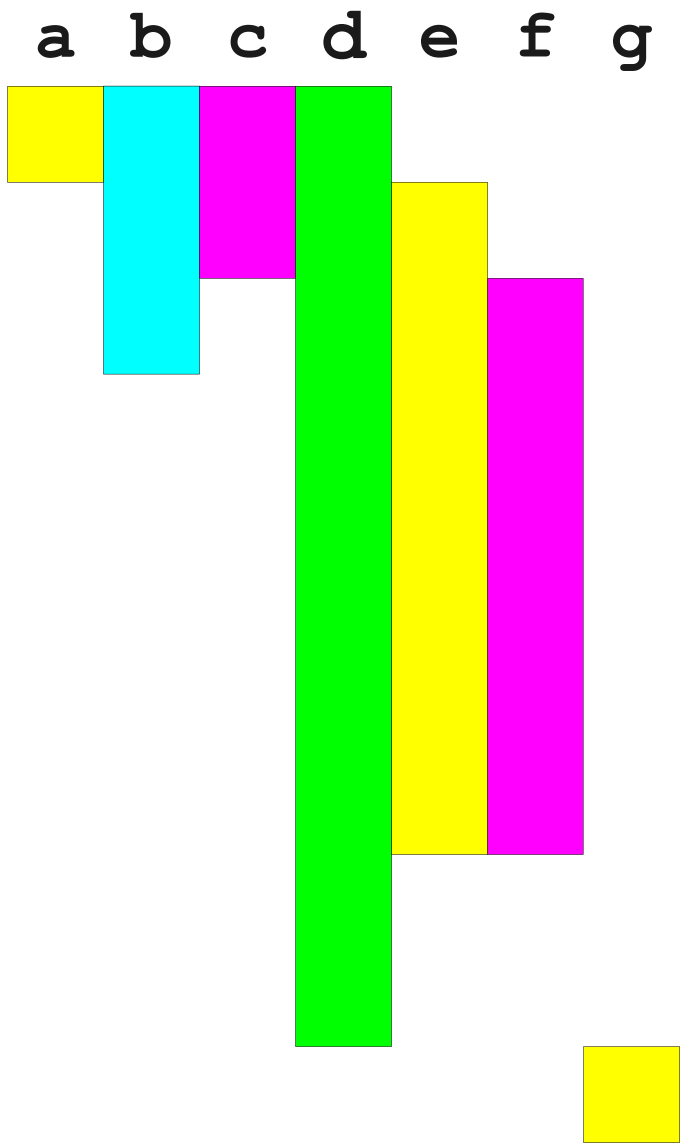
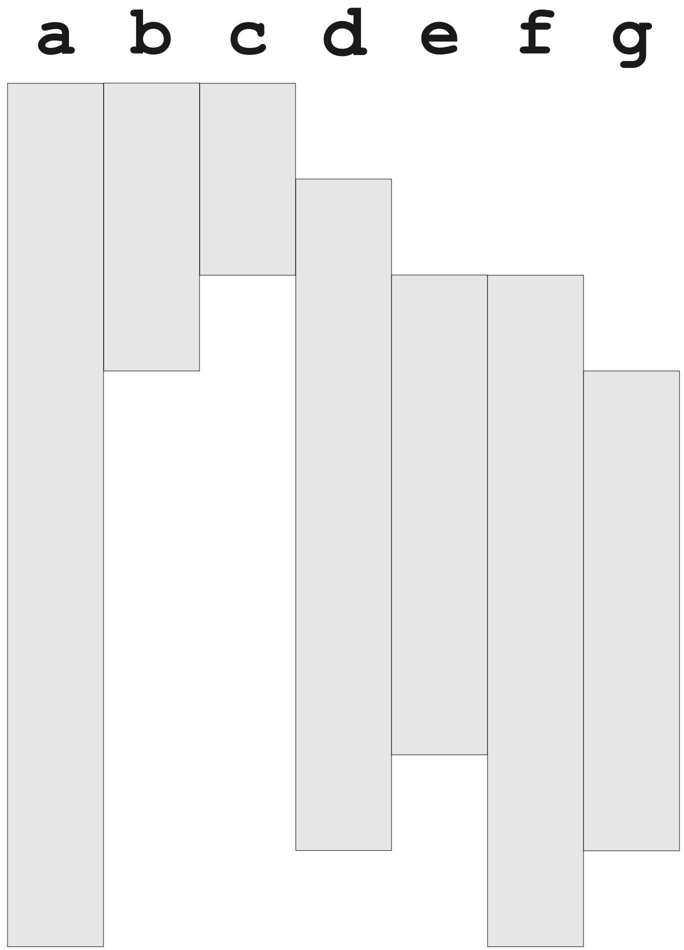


Free Registers

R_0	R_1	R_2	R_3

Another Example

Another Example



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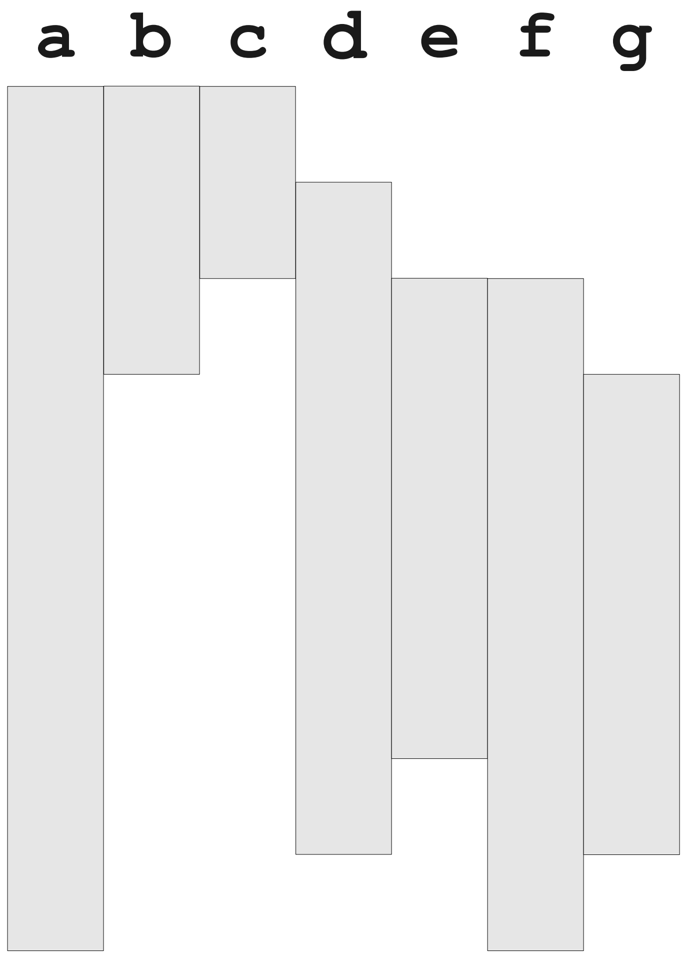
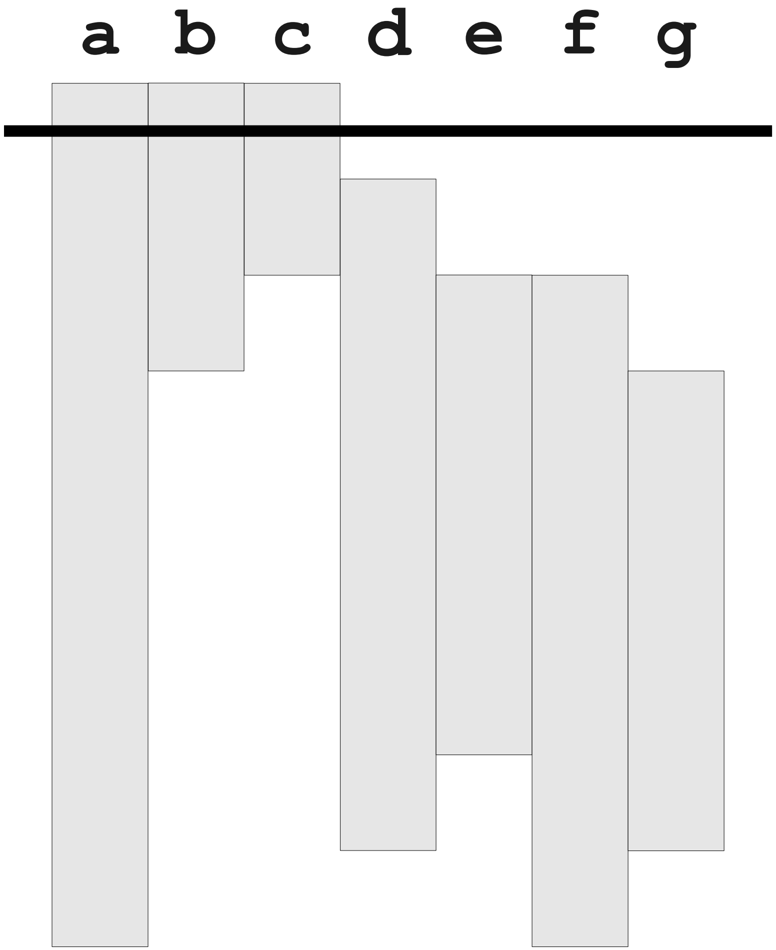


Free Registers

R_0	R_1	R_2

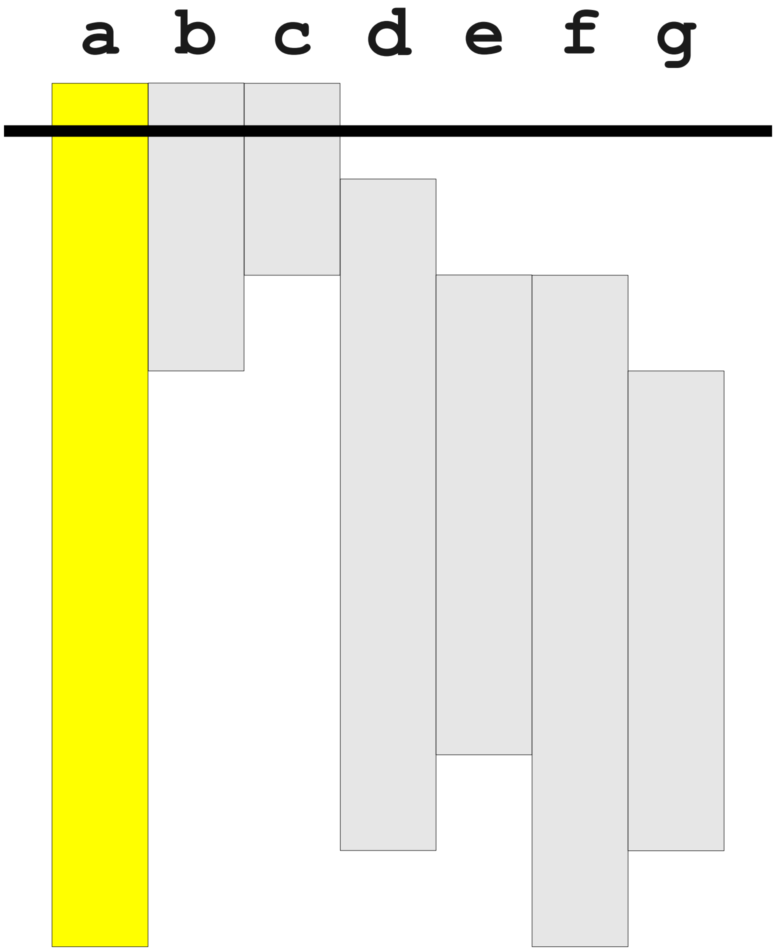
Another Example



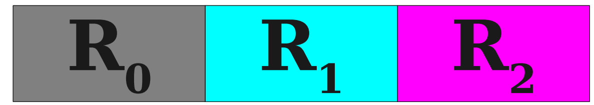
Free Registers

R_0	R_1	R_2

Another Example



Free Registers



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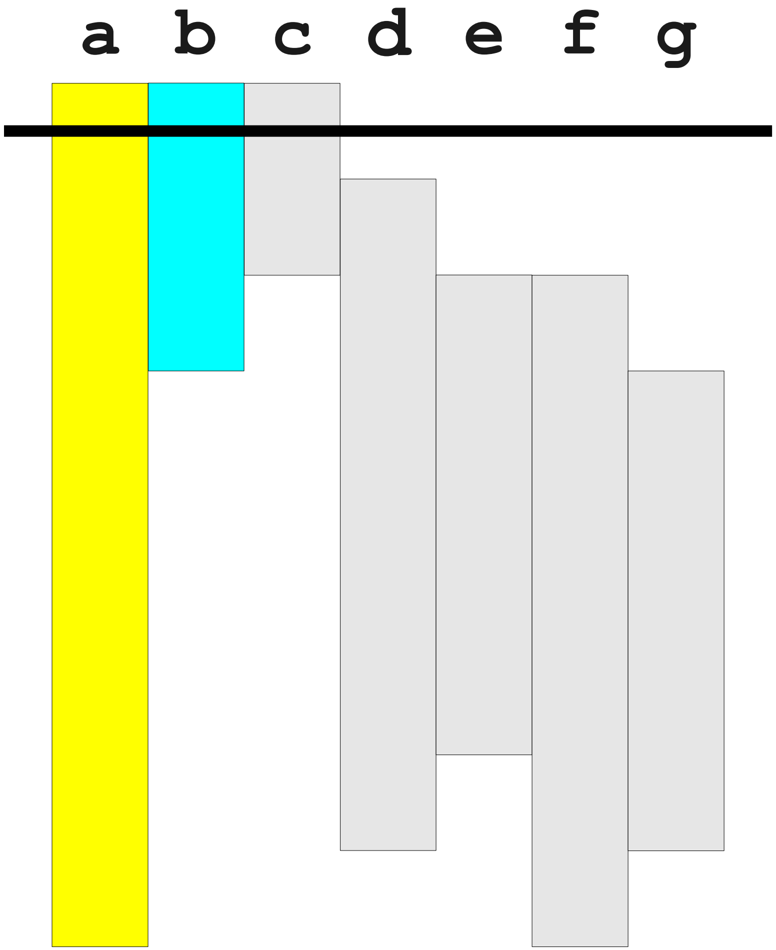

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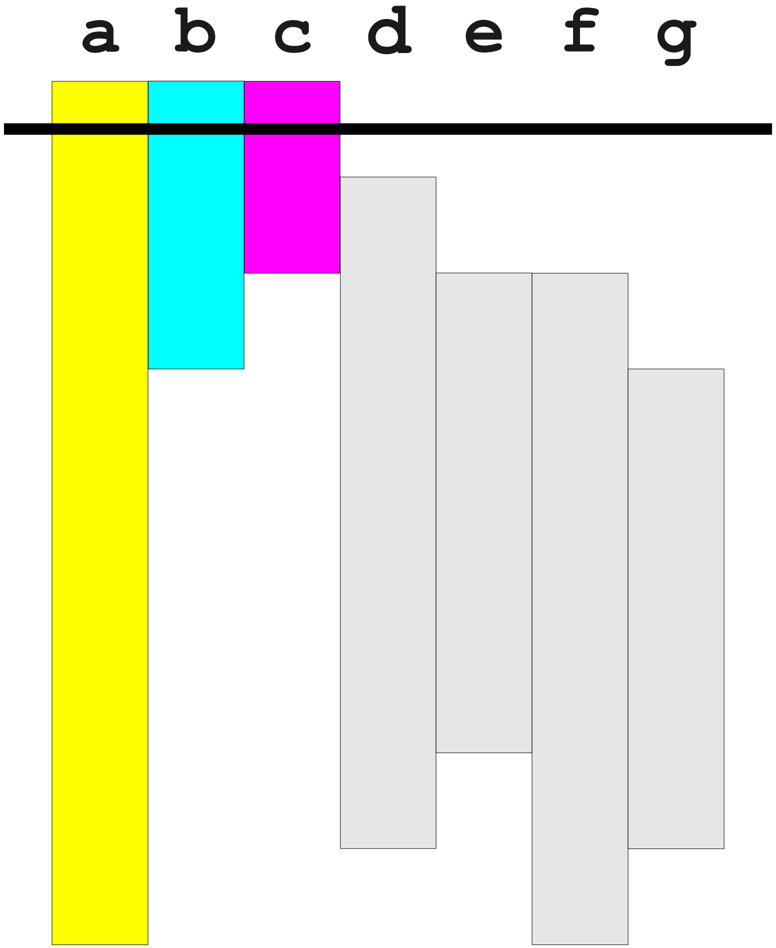

Free Registers

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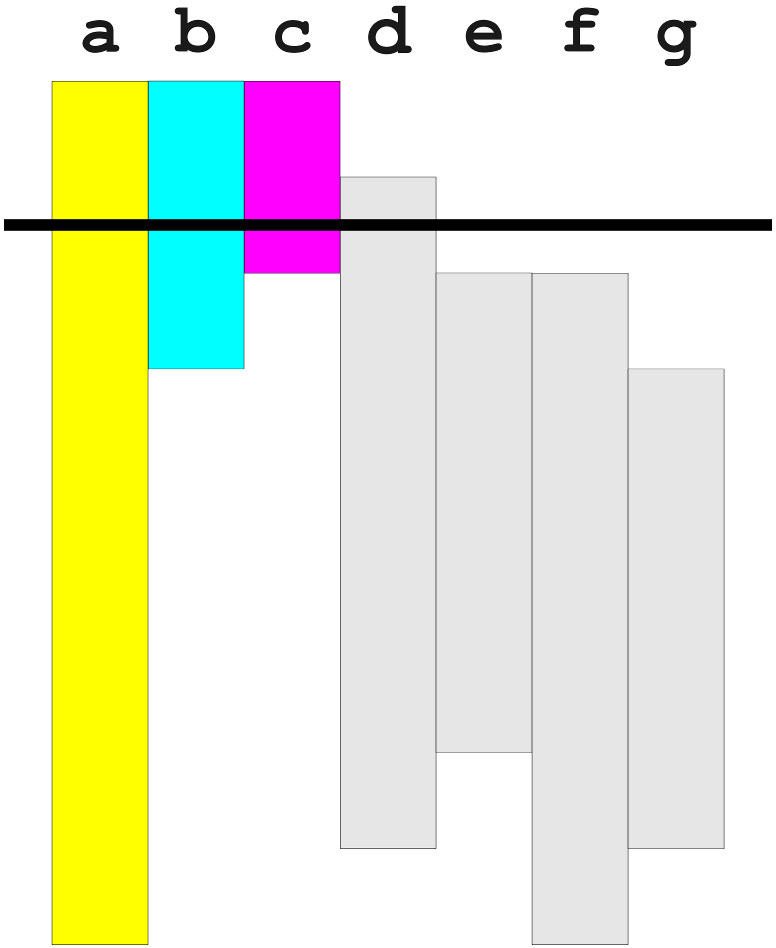

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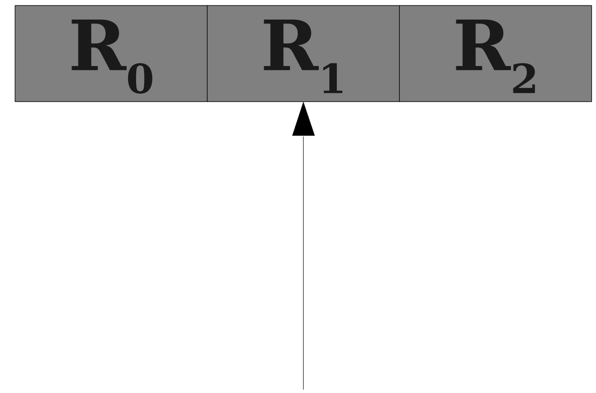
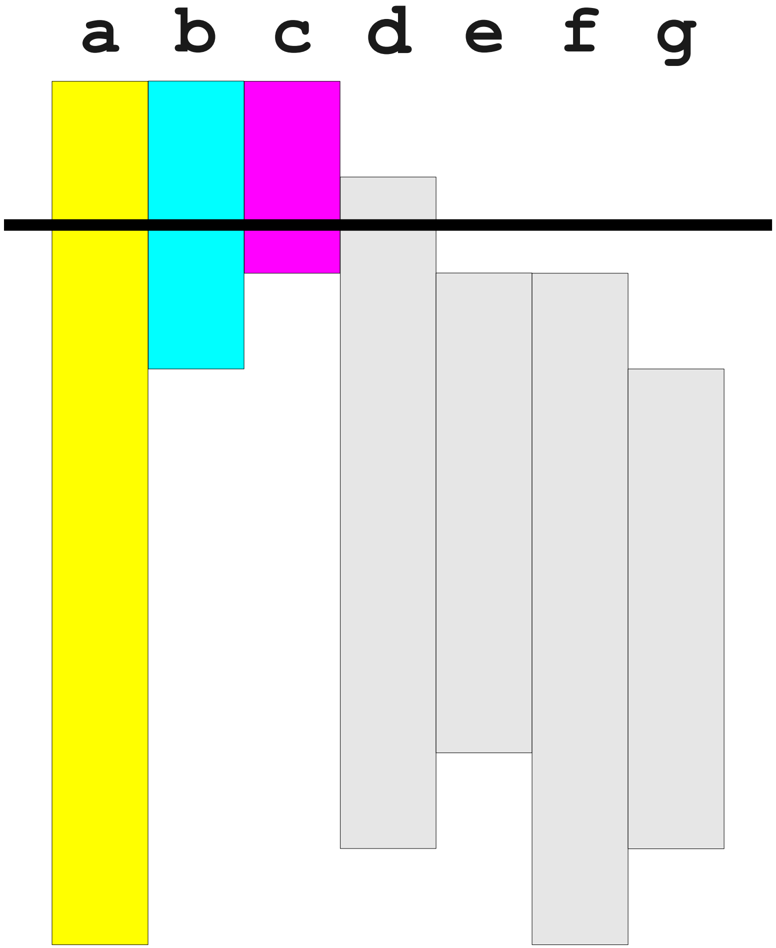


What do we do now?

Register Spilling

- If a register cannot be found for a variable *v*, we may need to **spill** a variable.
- When a variable is spilled, it is stored in memory rather than a register.
- When we need a register for the spilled variable:
  - Evict some existing register to memory.
  - Load the variable into the register.
  - When done, write the register back to memory and reload the register with its original value.
- Spilling is slow, but sometimes necessary.

Another Example



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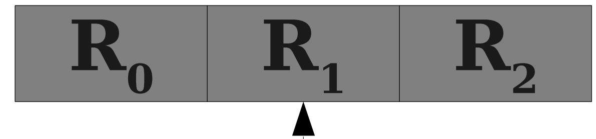

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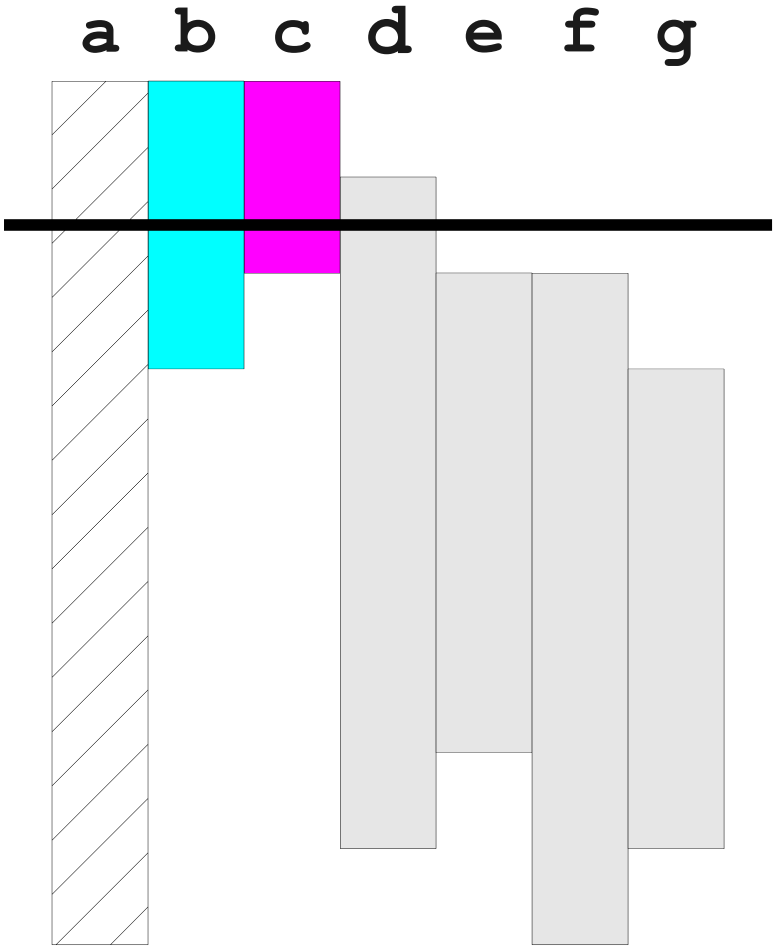

Free Registers

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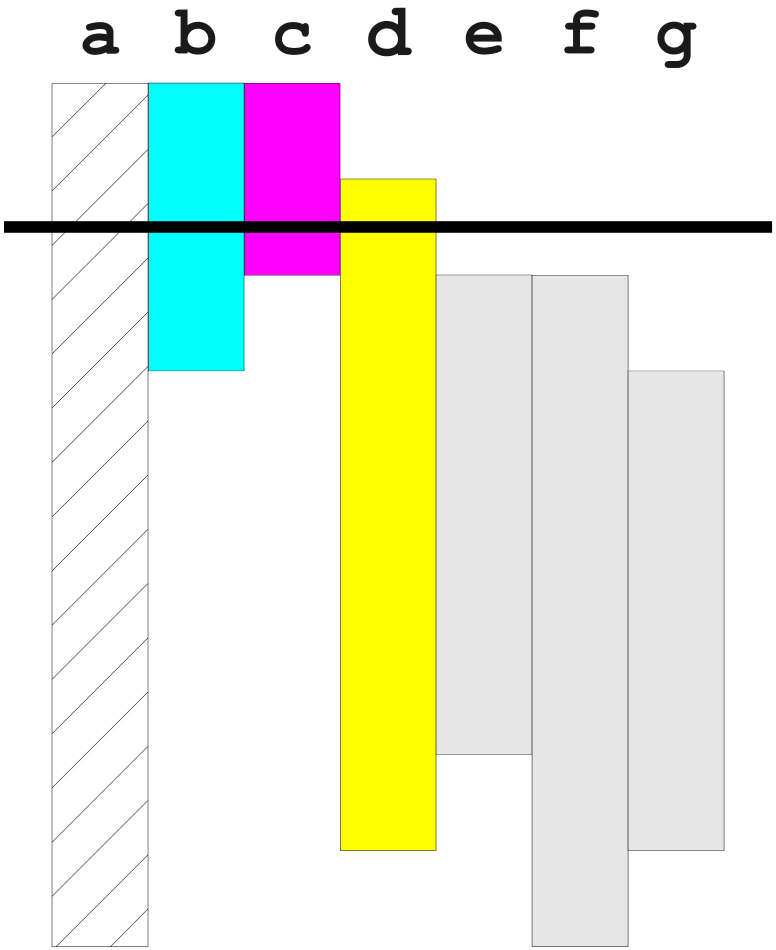

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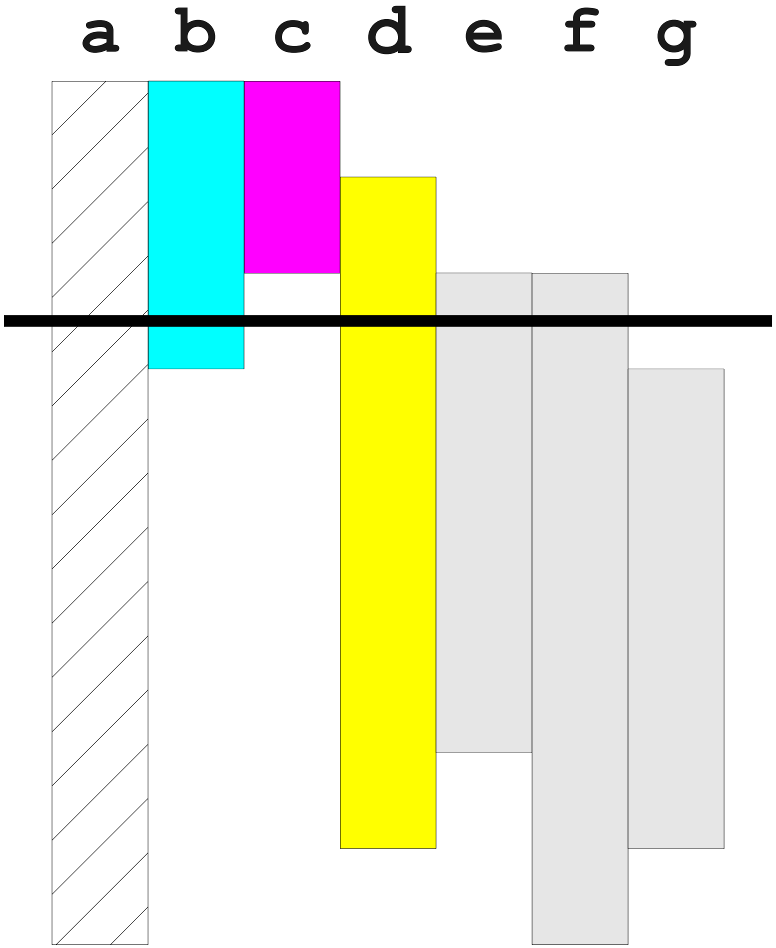

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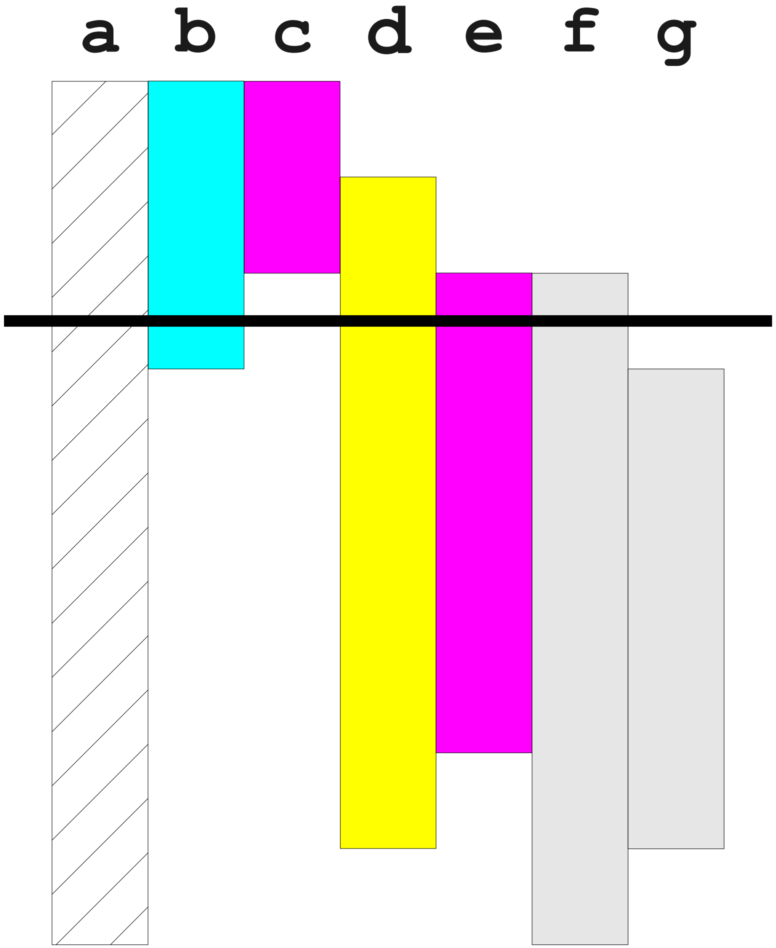

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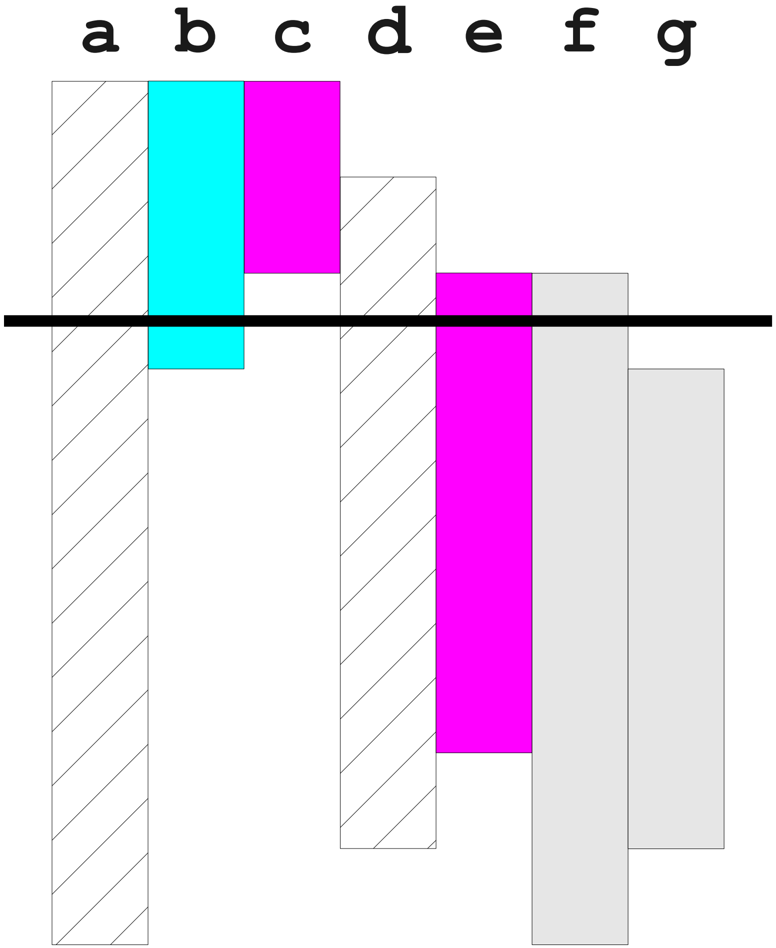

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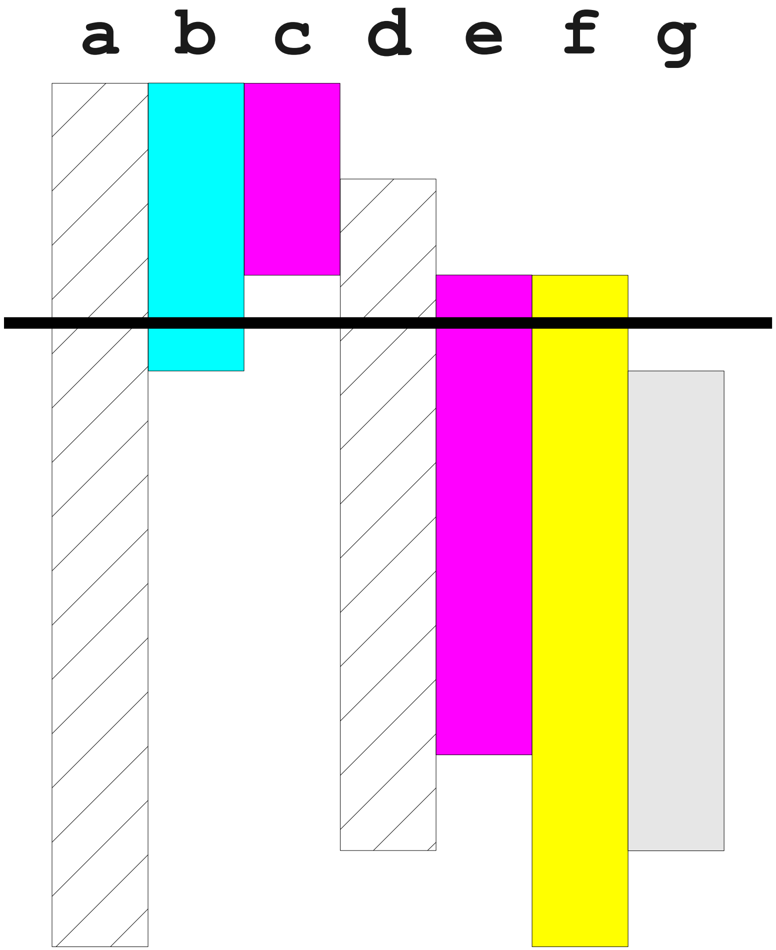

Free Registers

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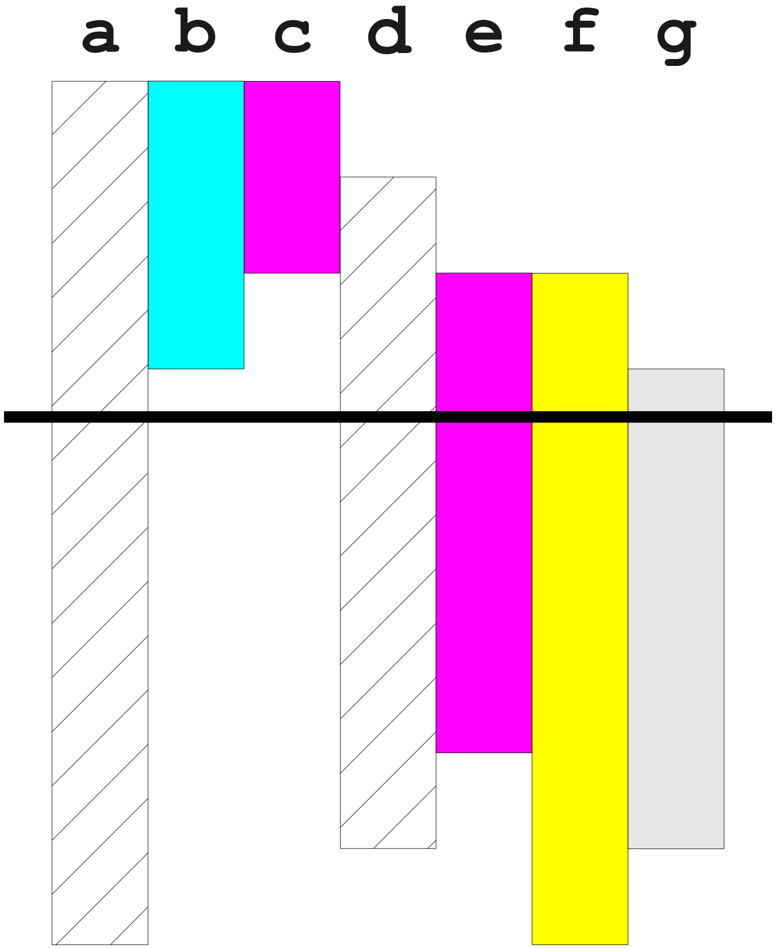

Free Registers

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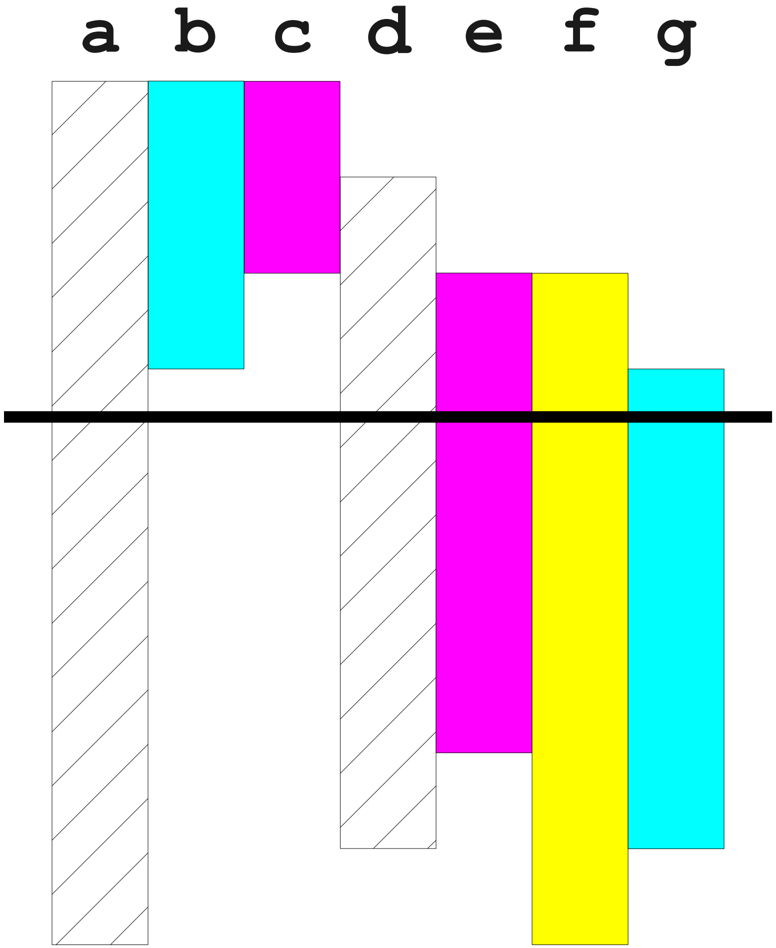

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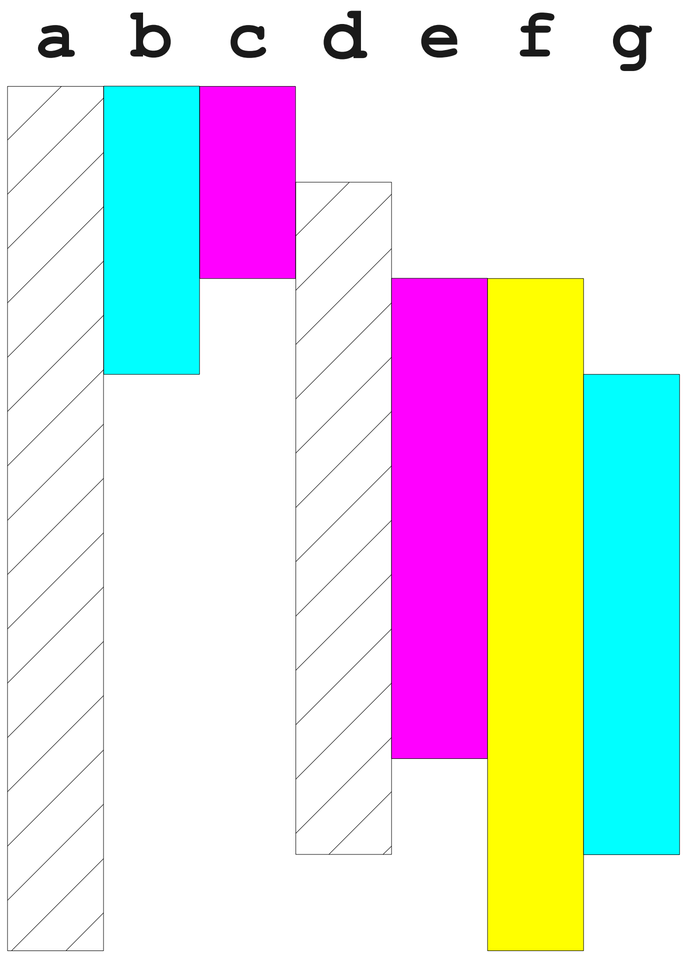


Free Registers

R_0	R_1	R_2

Linear Scan Register Allocation

- This algorithm is called **linear scan register allocation** and is a comparatively new algorithm.
- Advantages:
  - Very efficient (after computing live intervals, runs in linear time)
  - Produces good code in many instances.
  - Allocation step works in one pass; can generate code during iteration.
  - Often used in JIT compilers like Java HotSpot.
- Disadvantages:
  - Imprecise due to use of live **intervals** rather than live **ranges**.
  - Other techniques known to be superior in many cases.

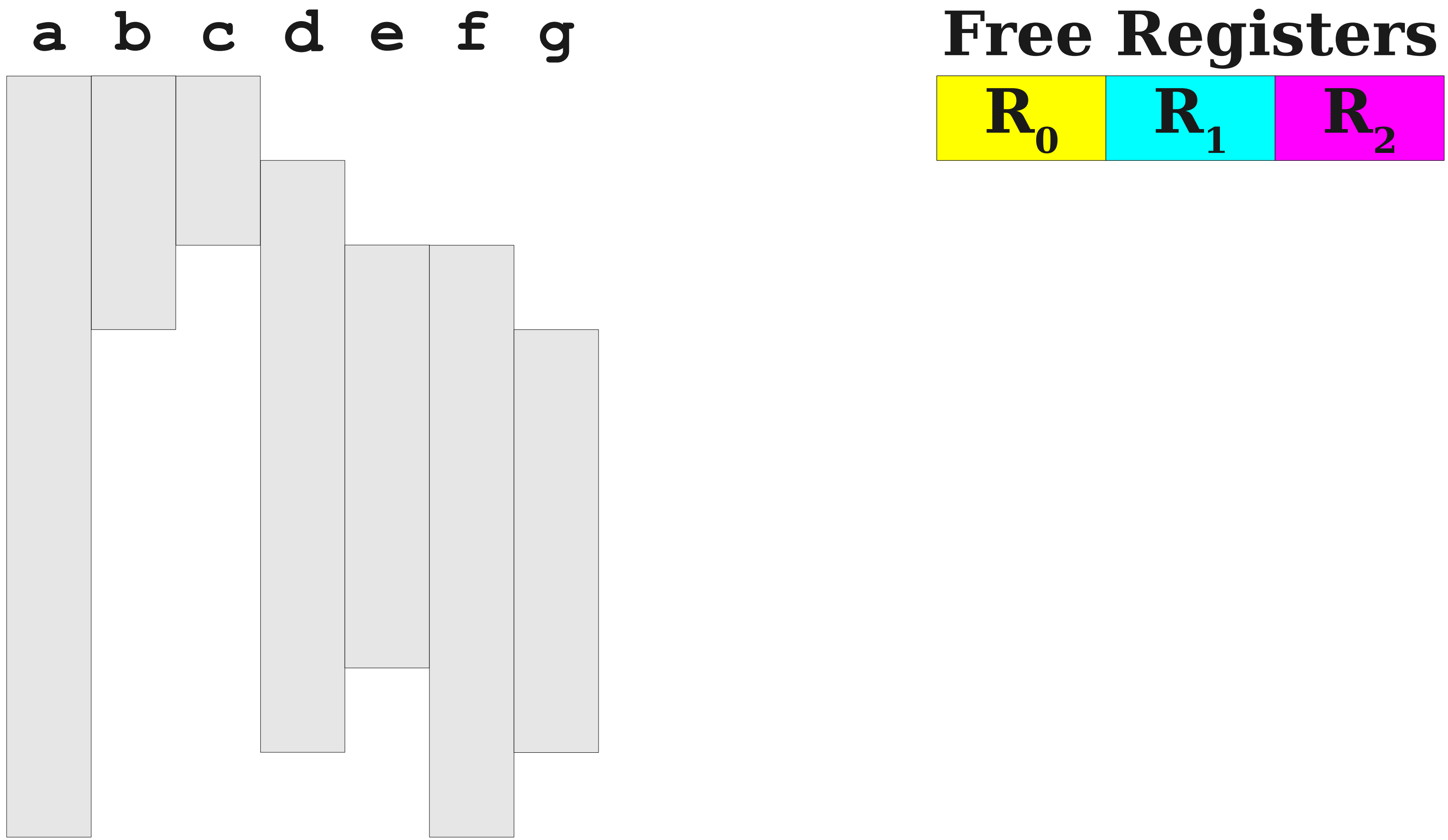
Correctness Proof Sketch

- No register holds two live variables at once:
  - Live intervals are conservative approximations of live ranges.
  - No two variables with overlapping live ranges placed in the same register.
- At each program point, every variable is in the same location:
  - All variables assigned a unique location.

Second-Chance Bin Packing

- A more aggressive version of linear-scan.
- Uses live **ranges** instead of live **intervals**.
- If a variable must be spilled, don't spill all uses of it.
  - A later live range might still fit into a register.
- Requires a final data-flow analysis to confirm variables are assigned consistent locations.
- See “Quality and Speed in Linear-scan Register Allocation” by Traub, Holloway, and Smith.

Second-Chance Bin Packing



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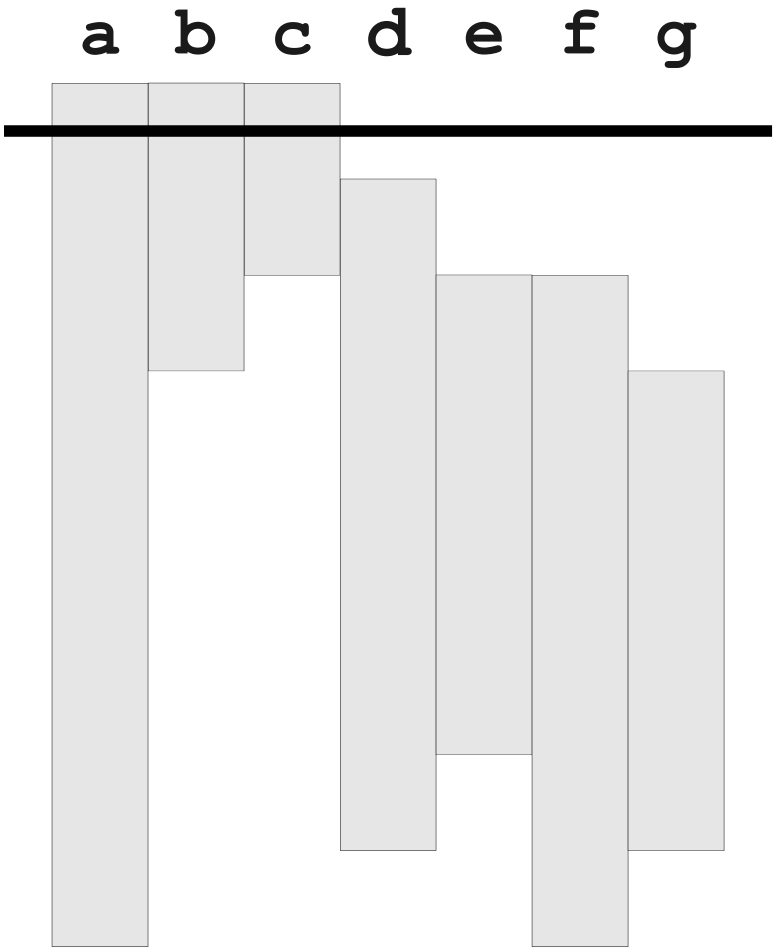

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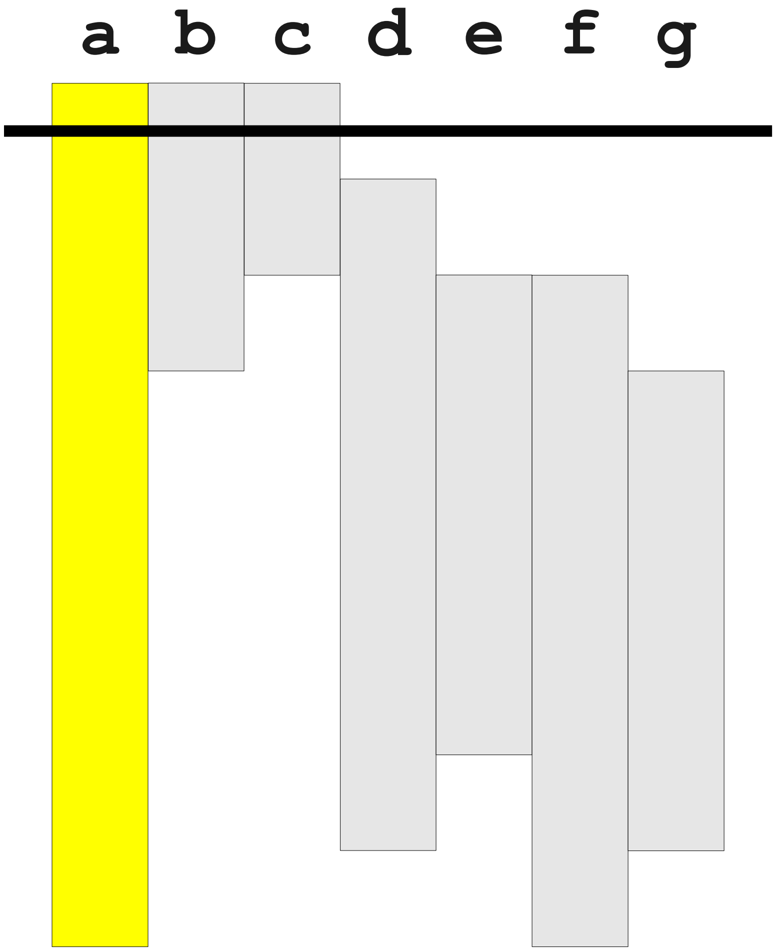

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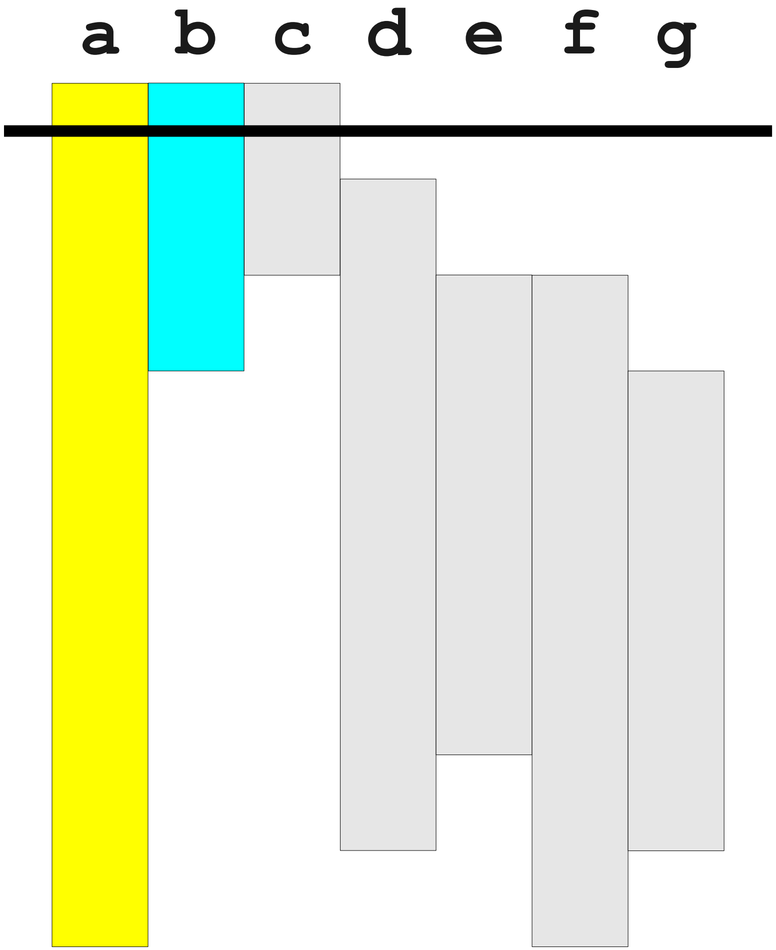

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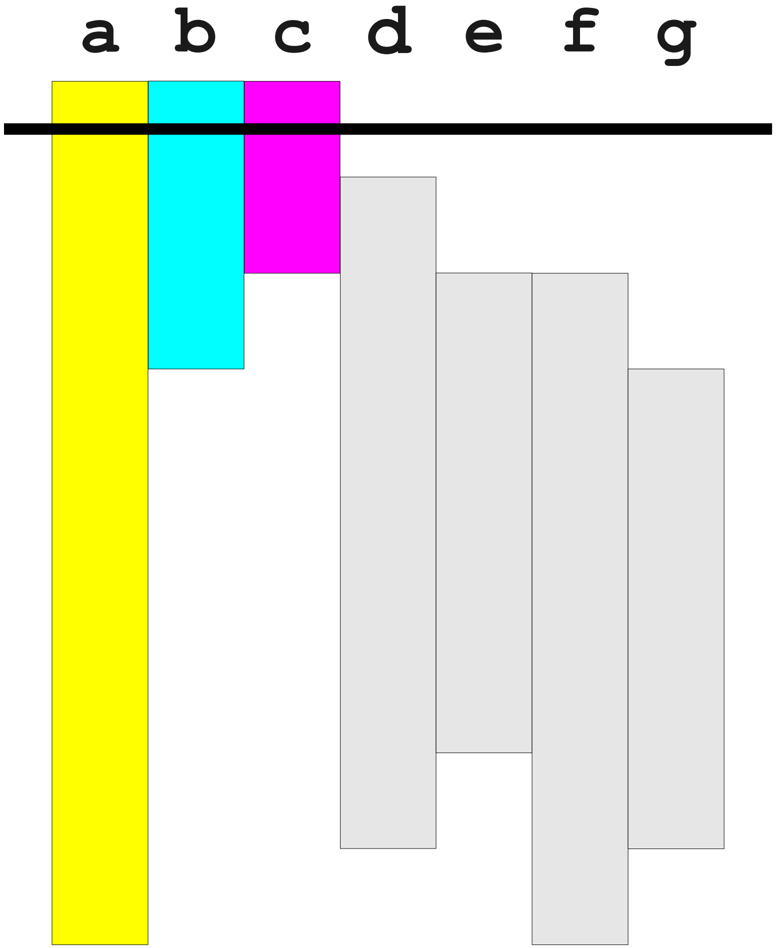

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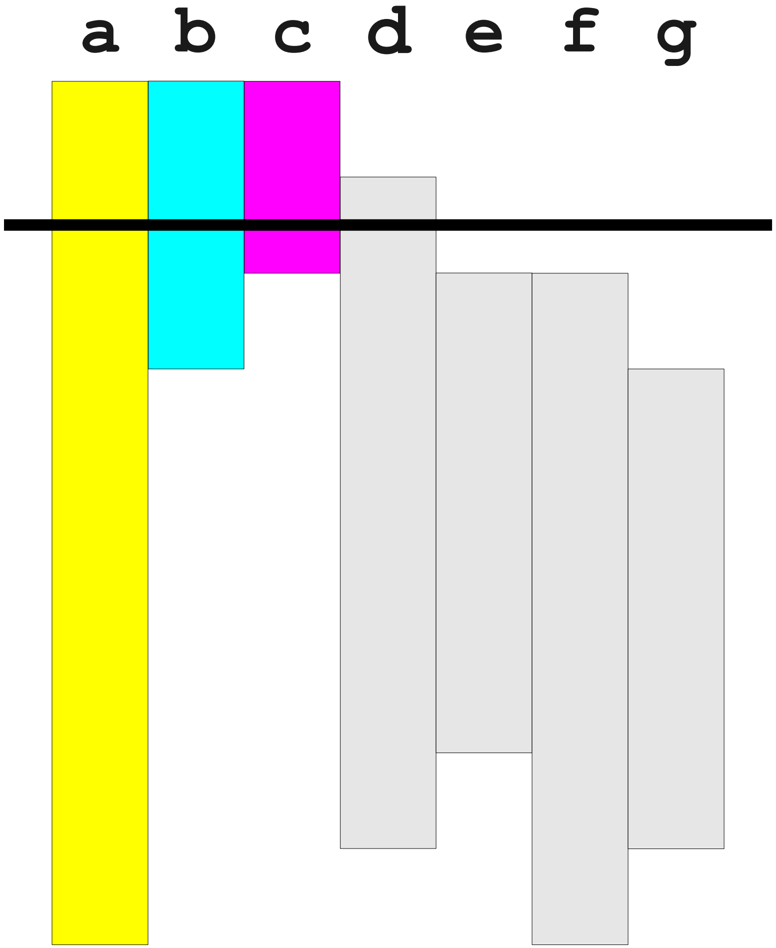

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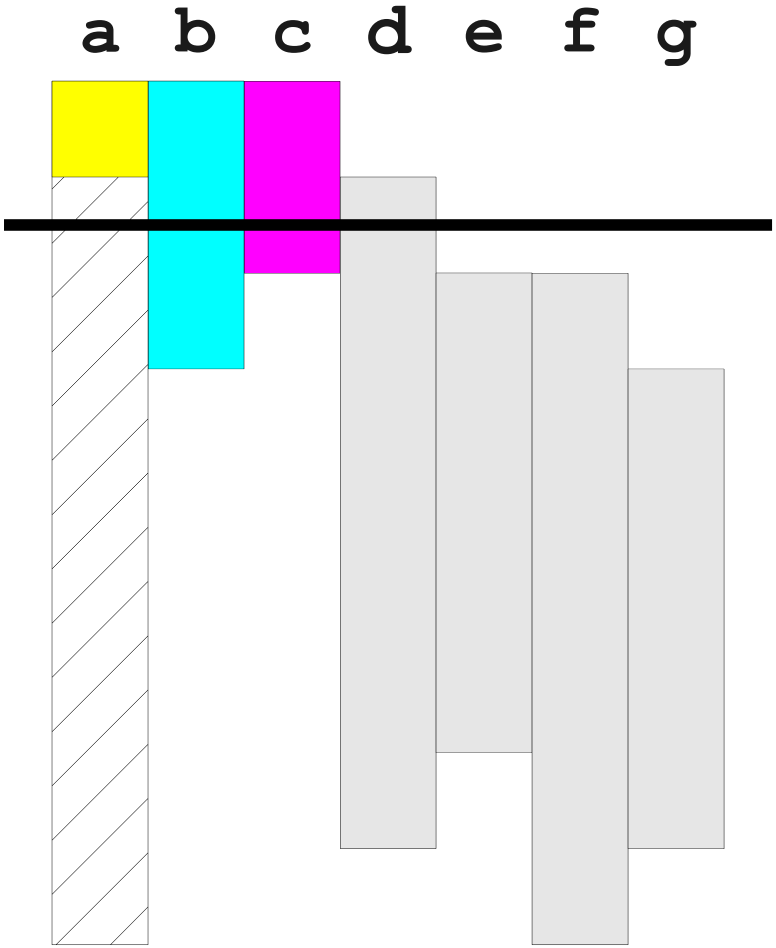

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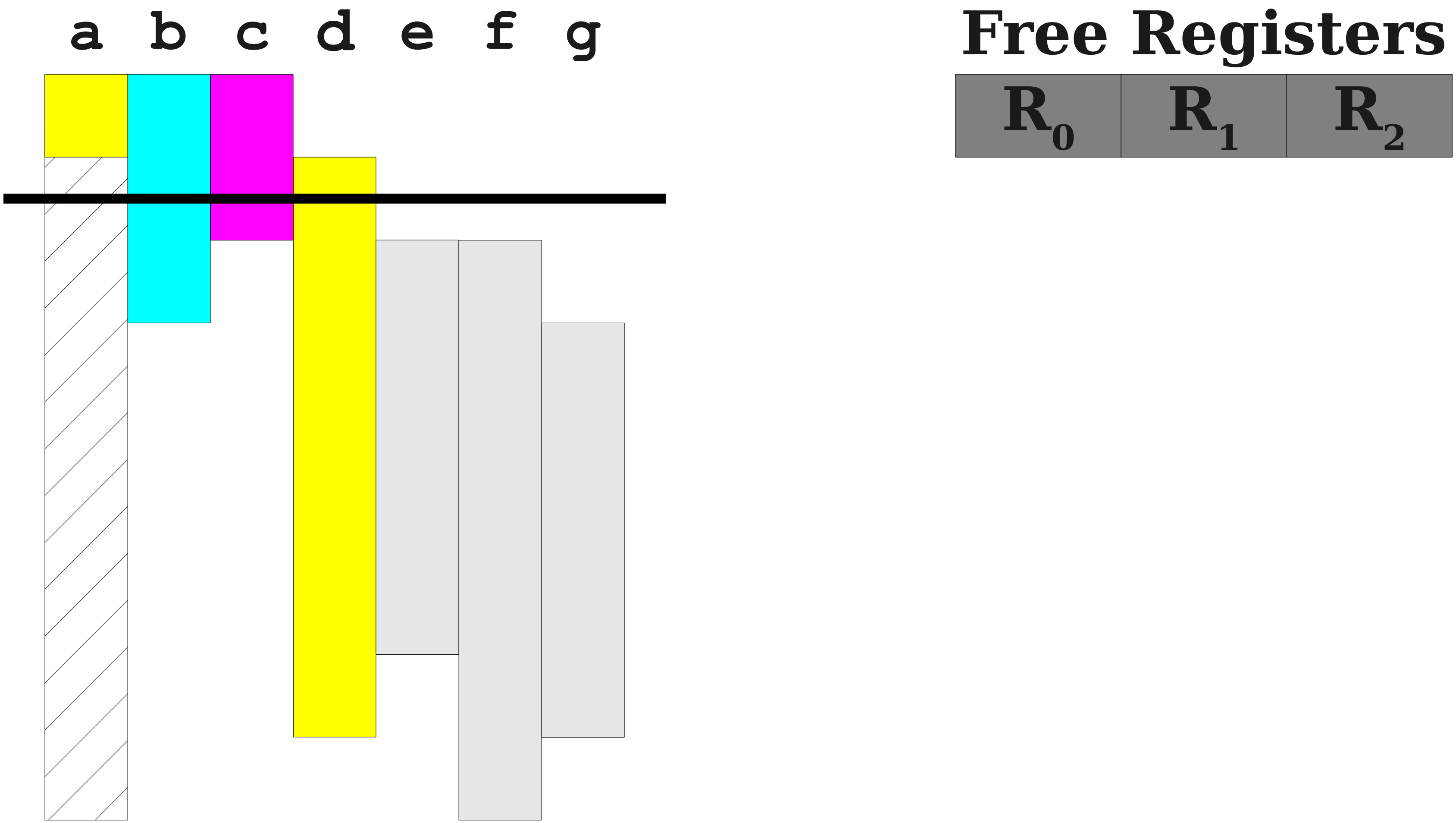

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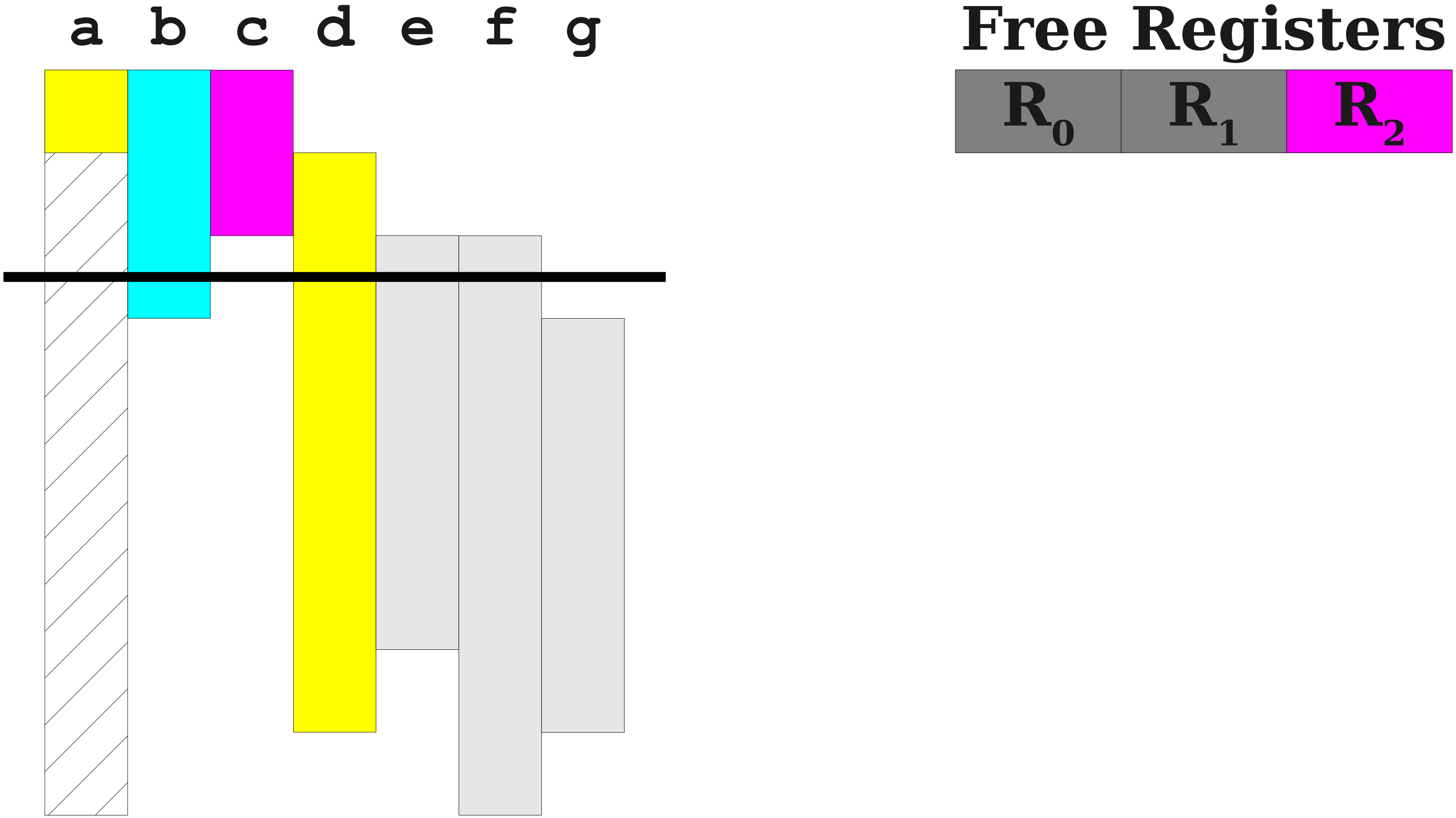

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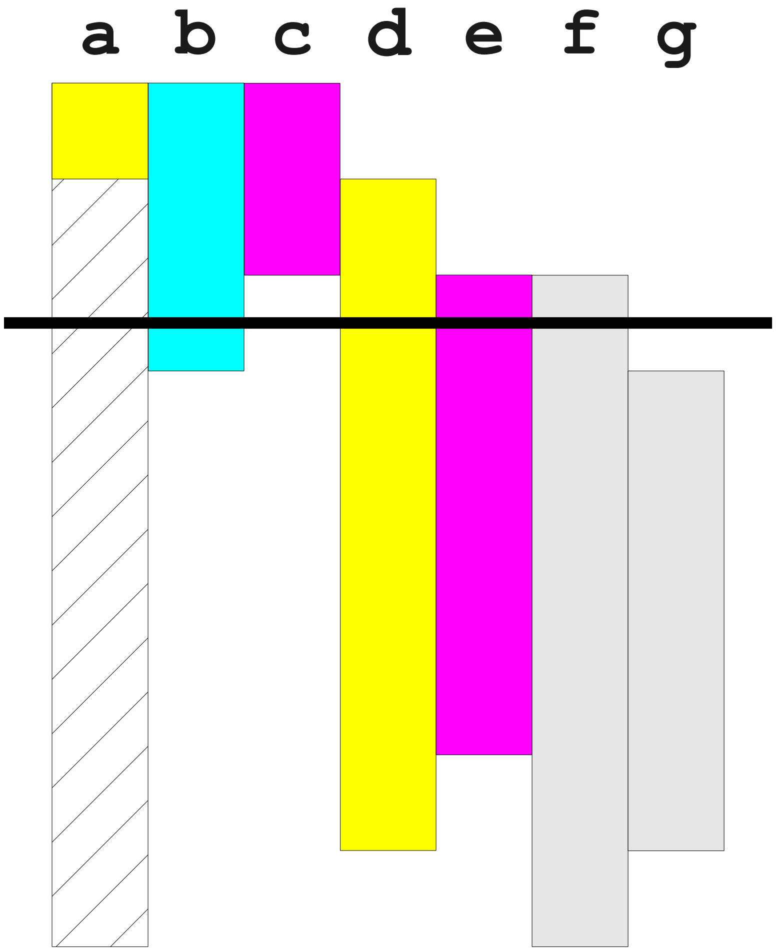

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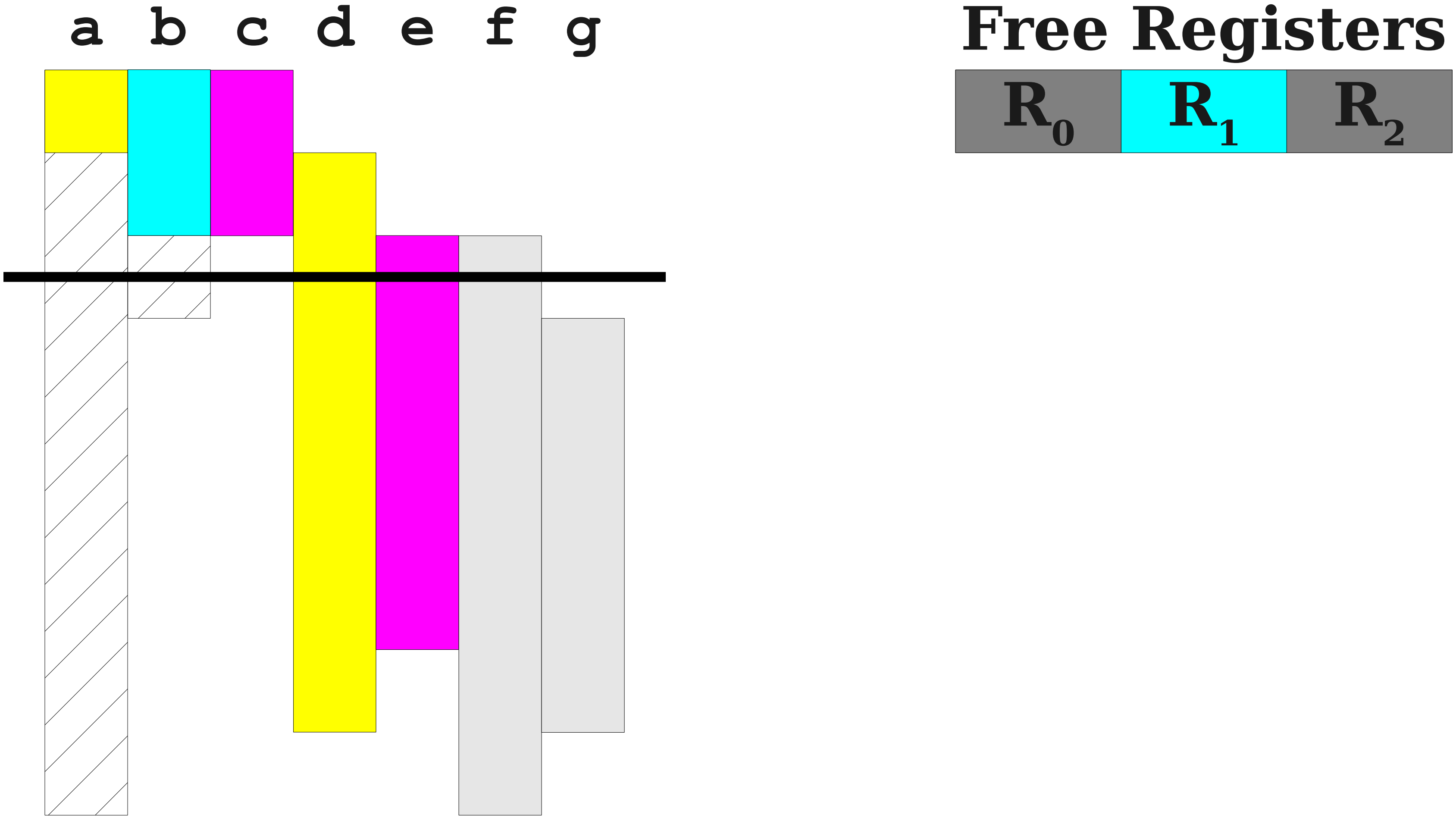

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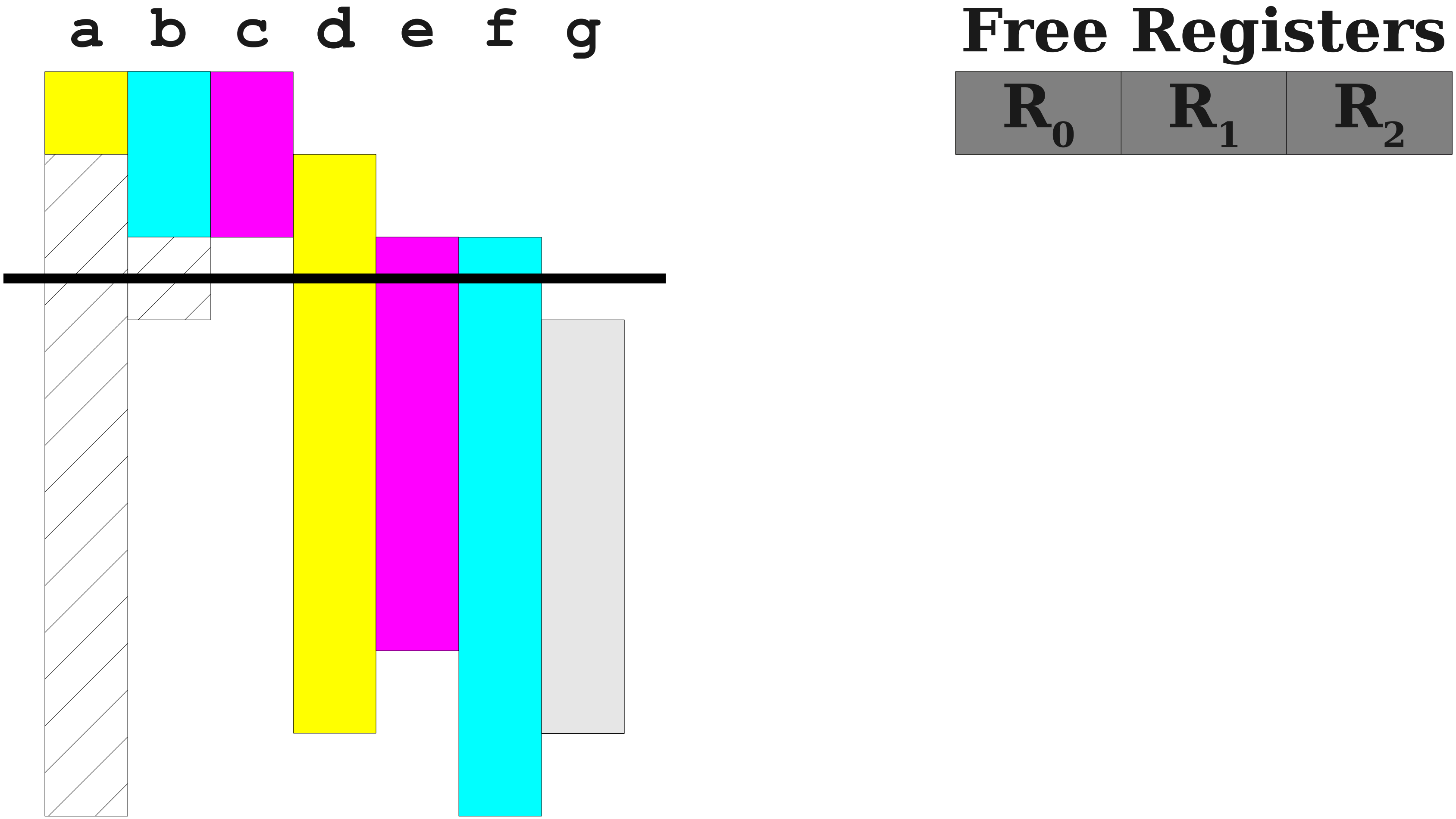

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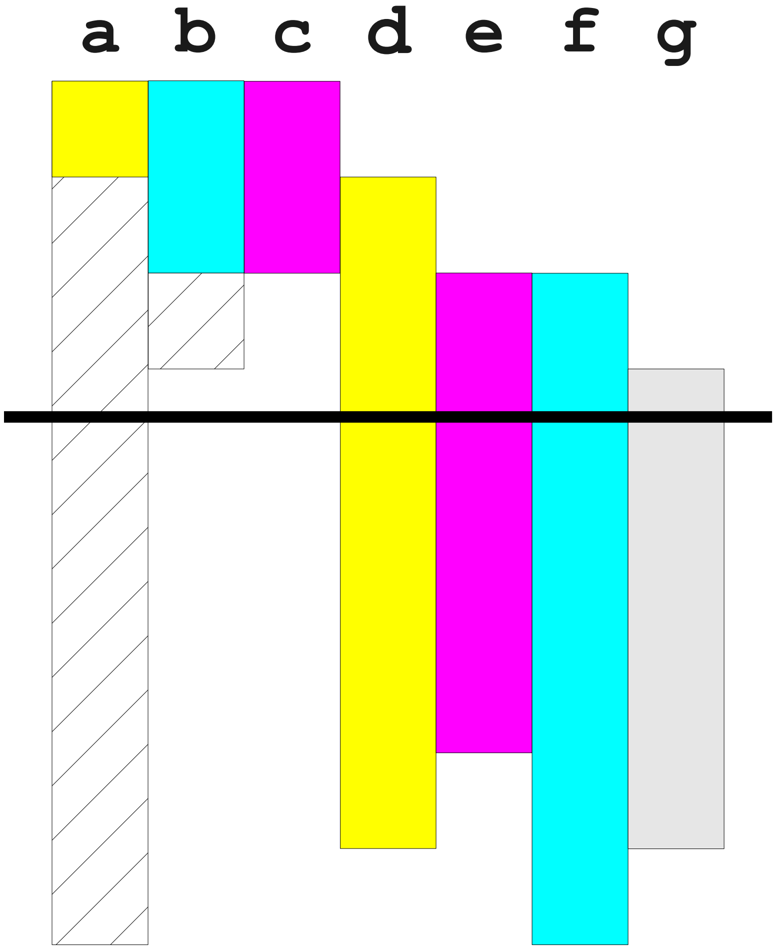

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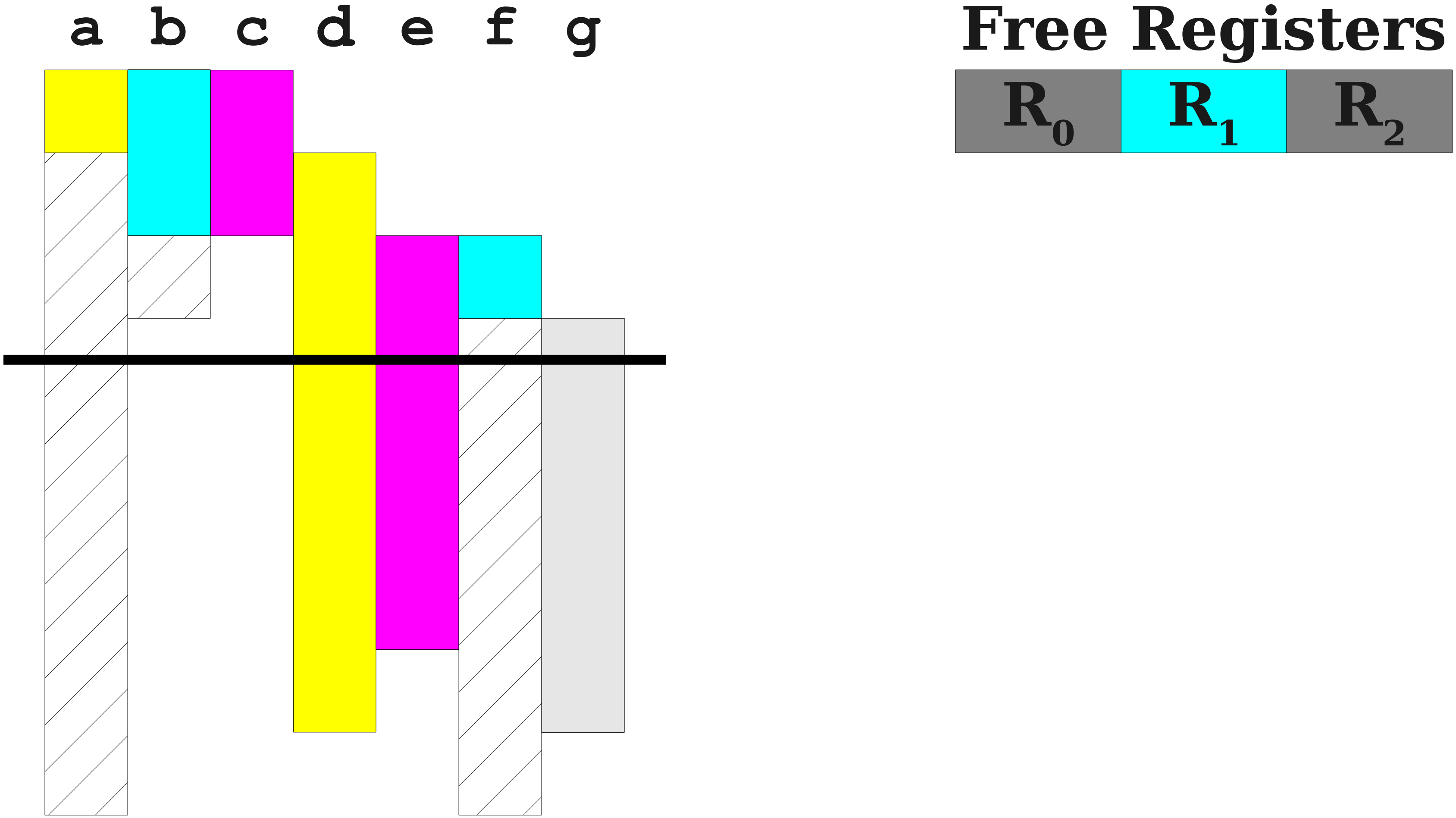

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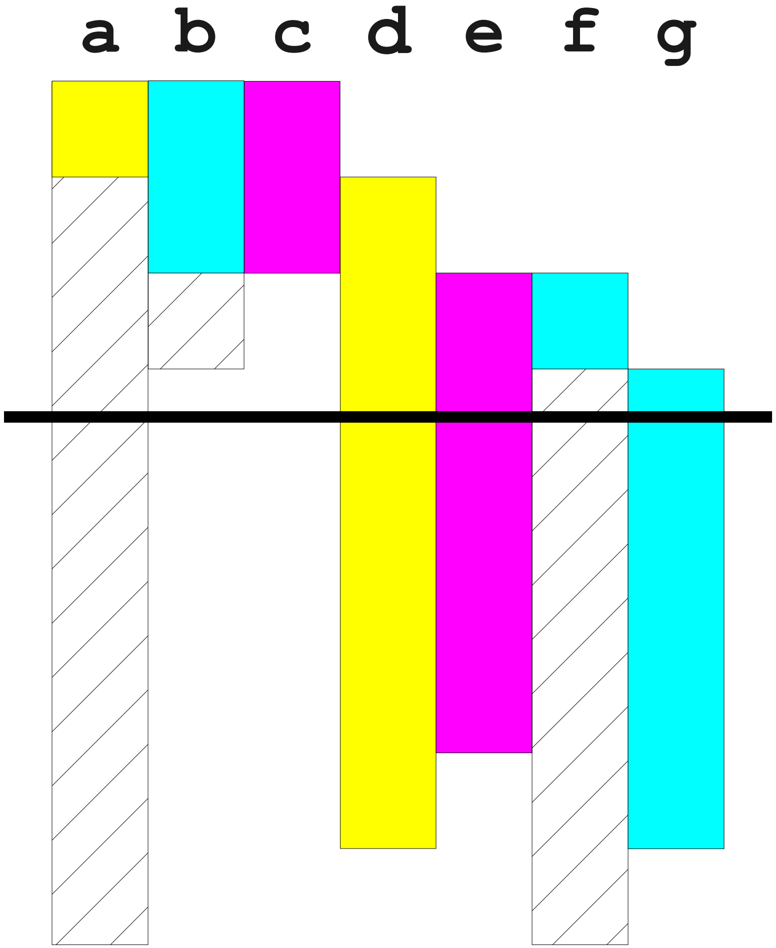

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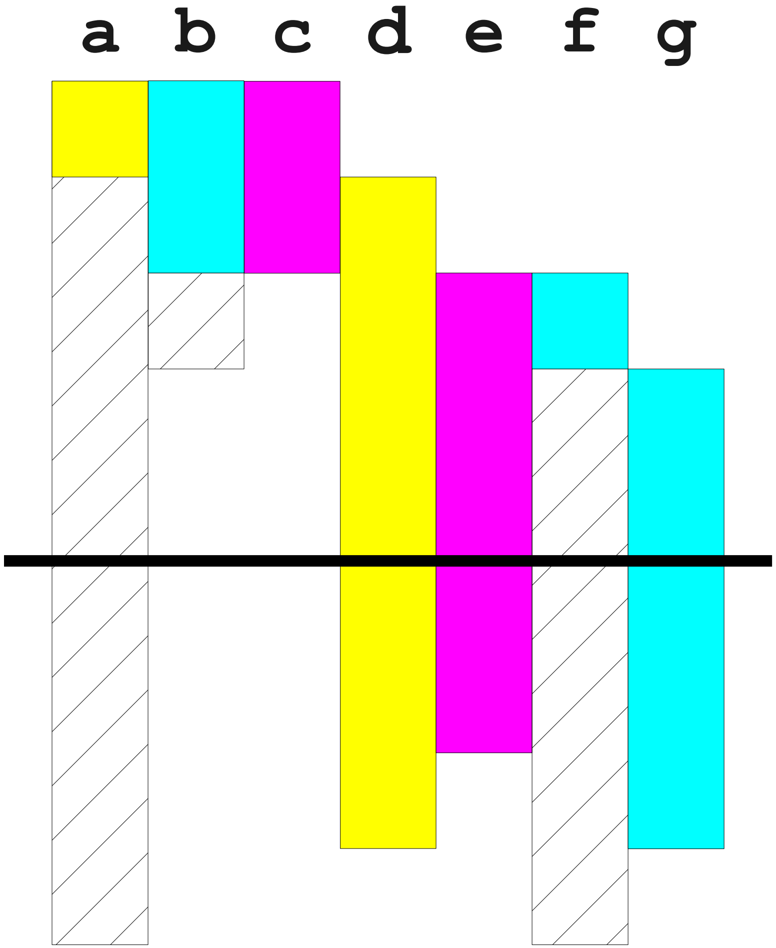

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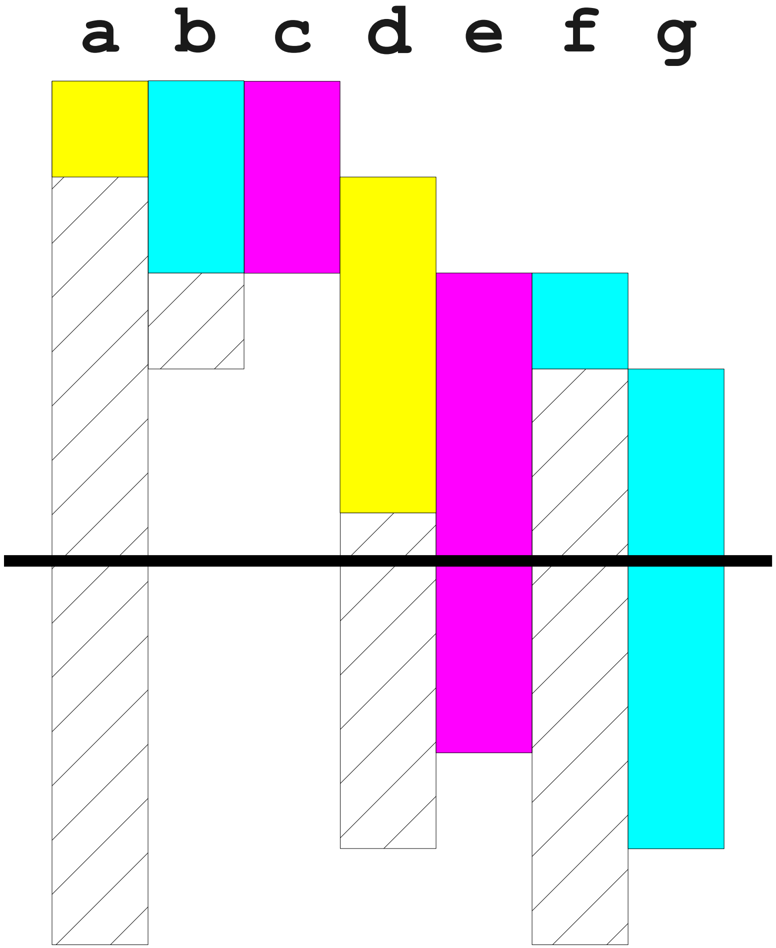

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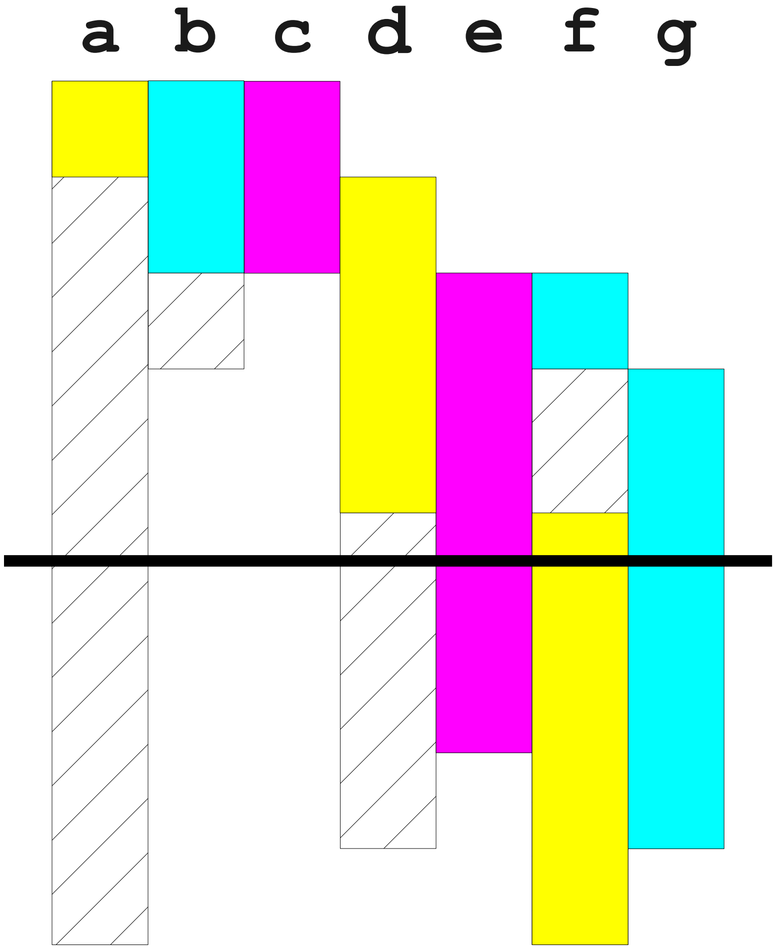

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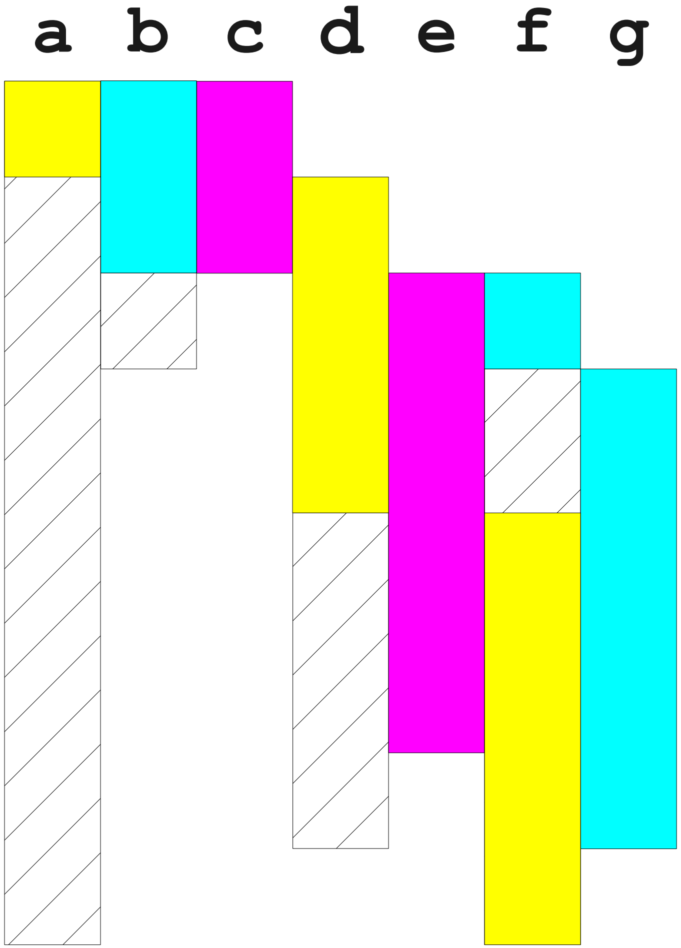

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An Entirely Different Approach

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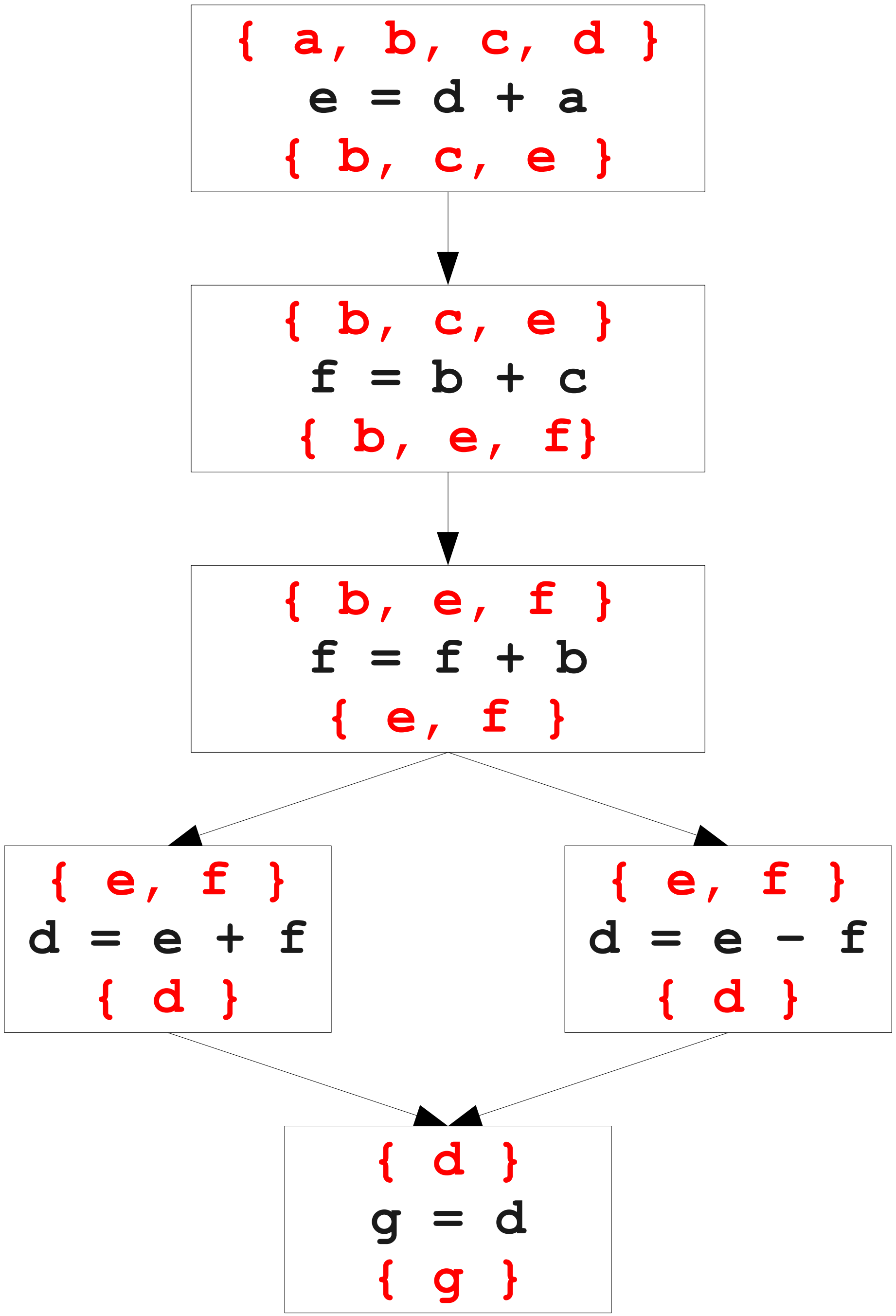

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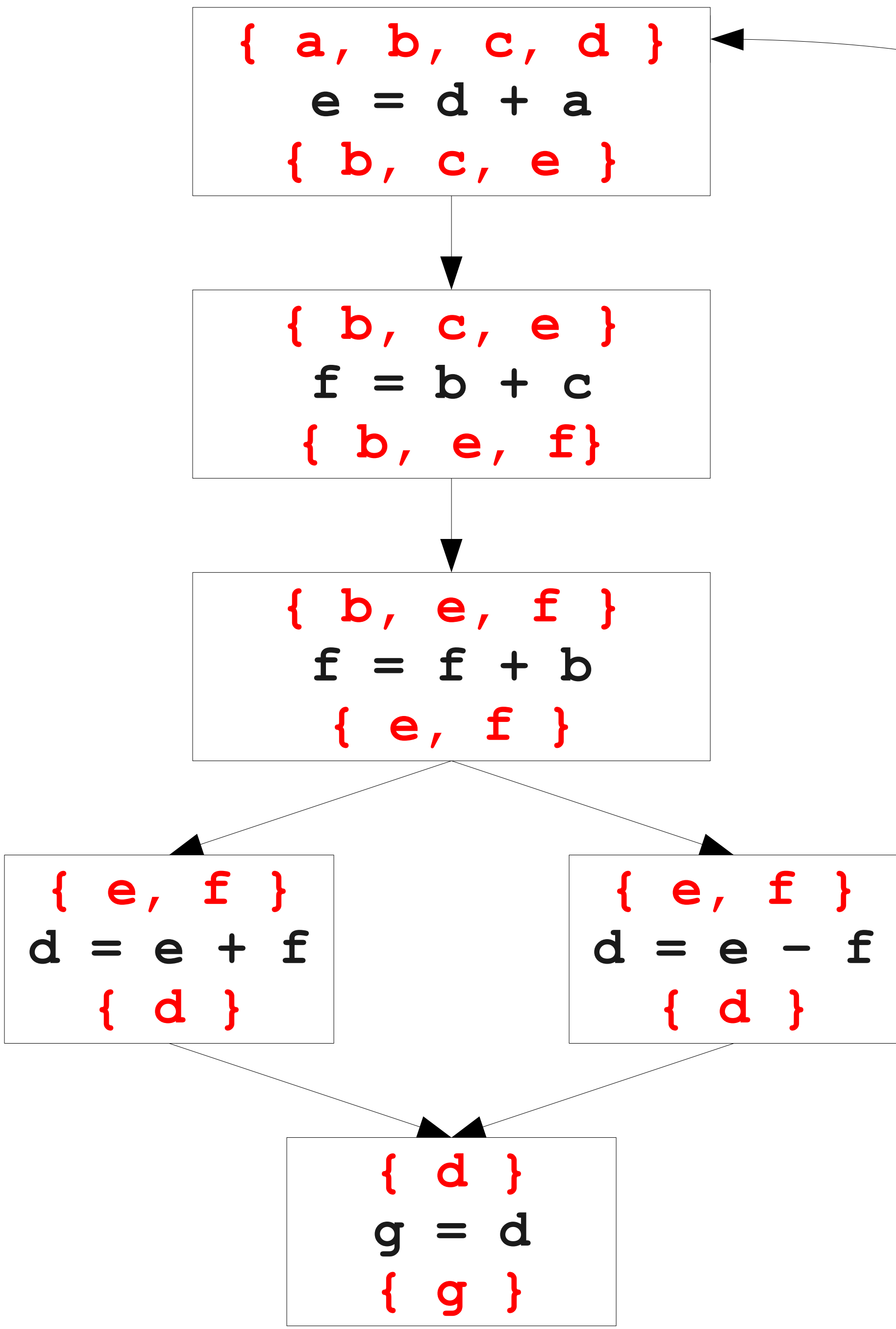
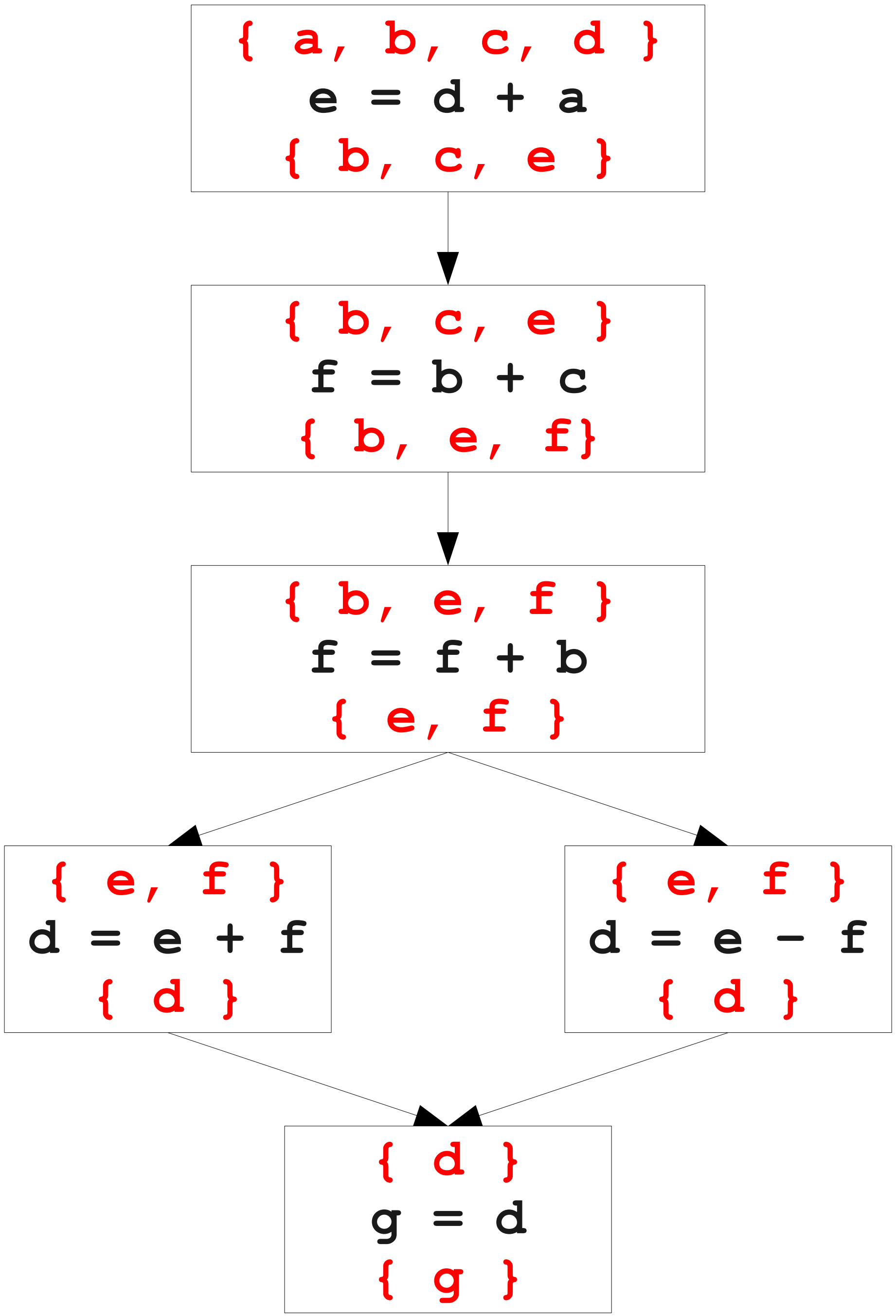


What can we infer from all these variables being live at this point?

An Entirely Different Approach



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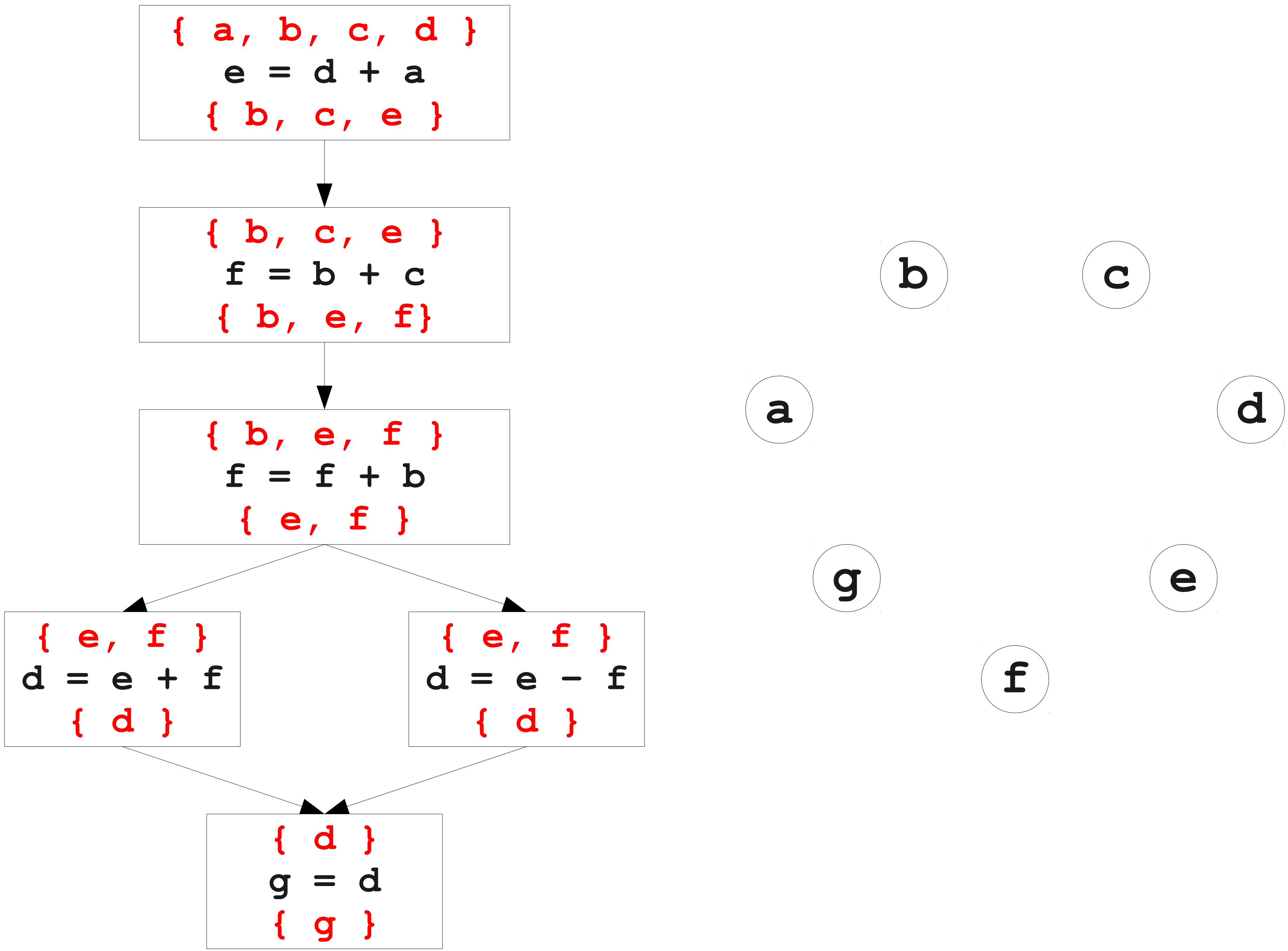

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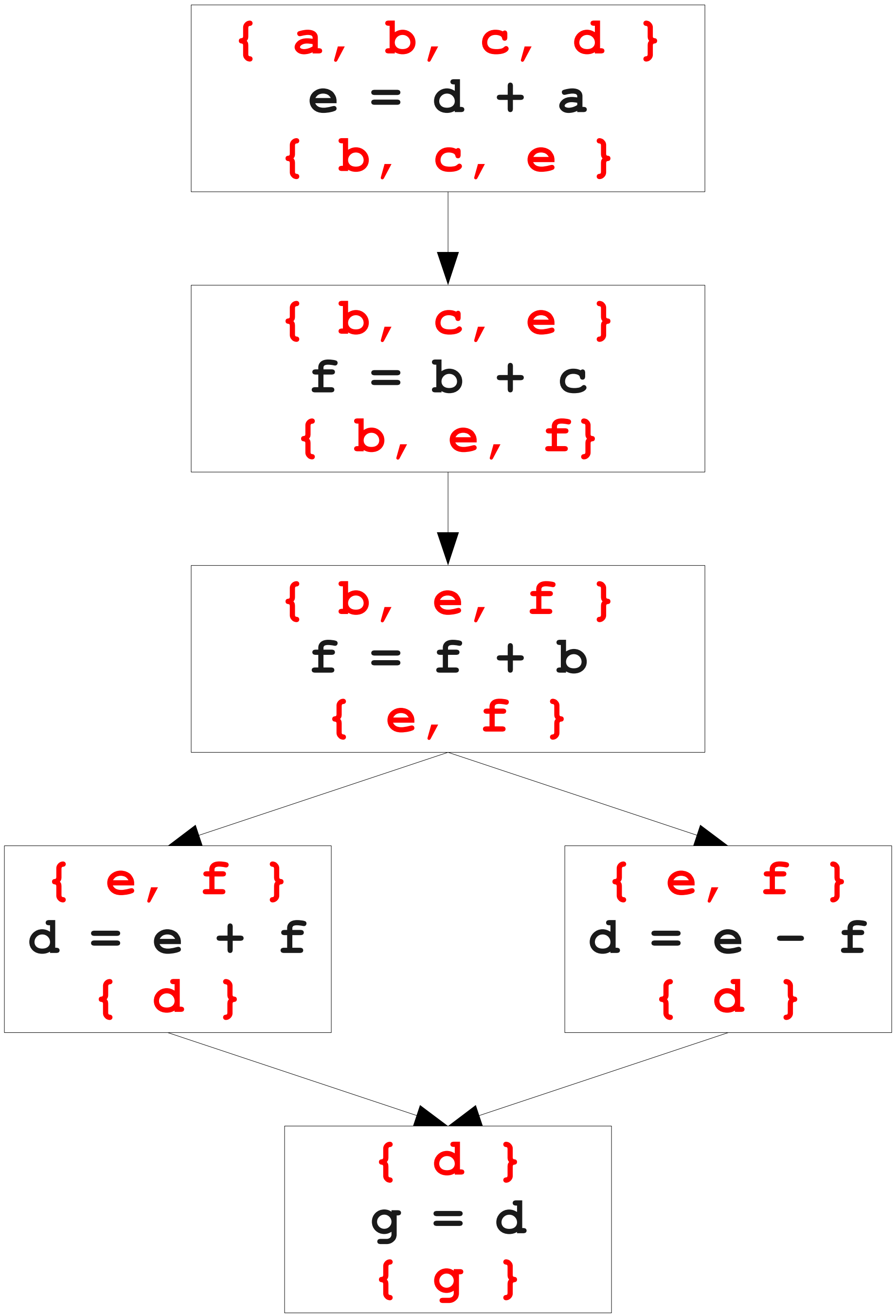

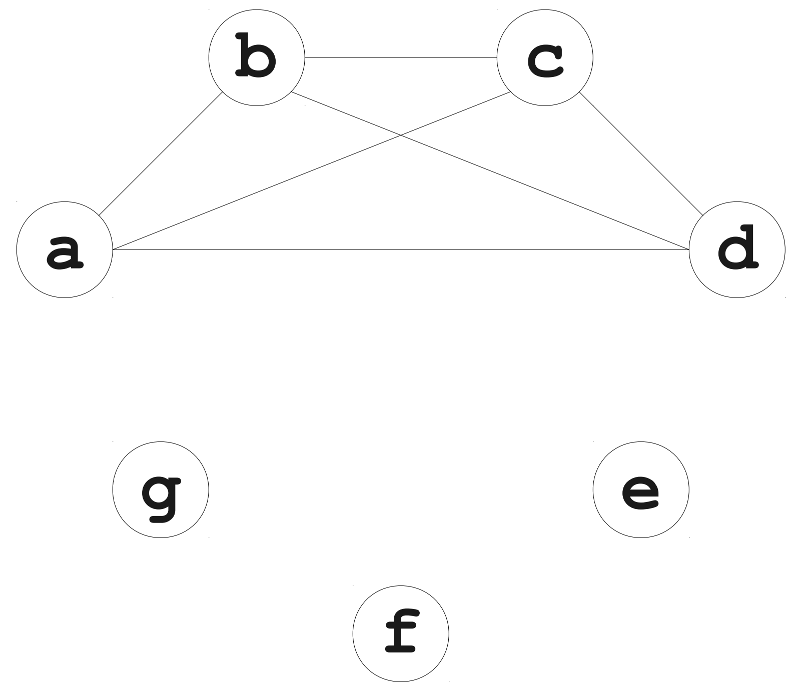

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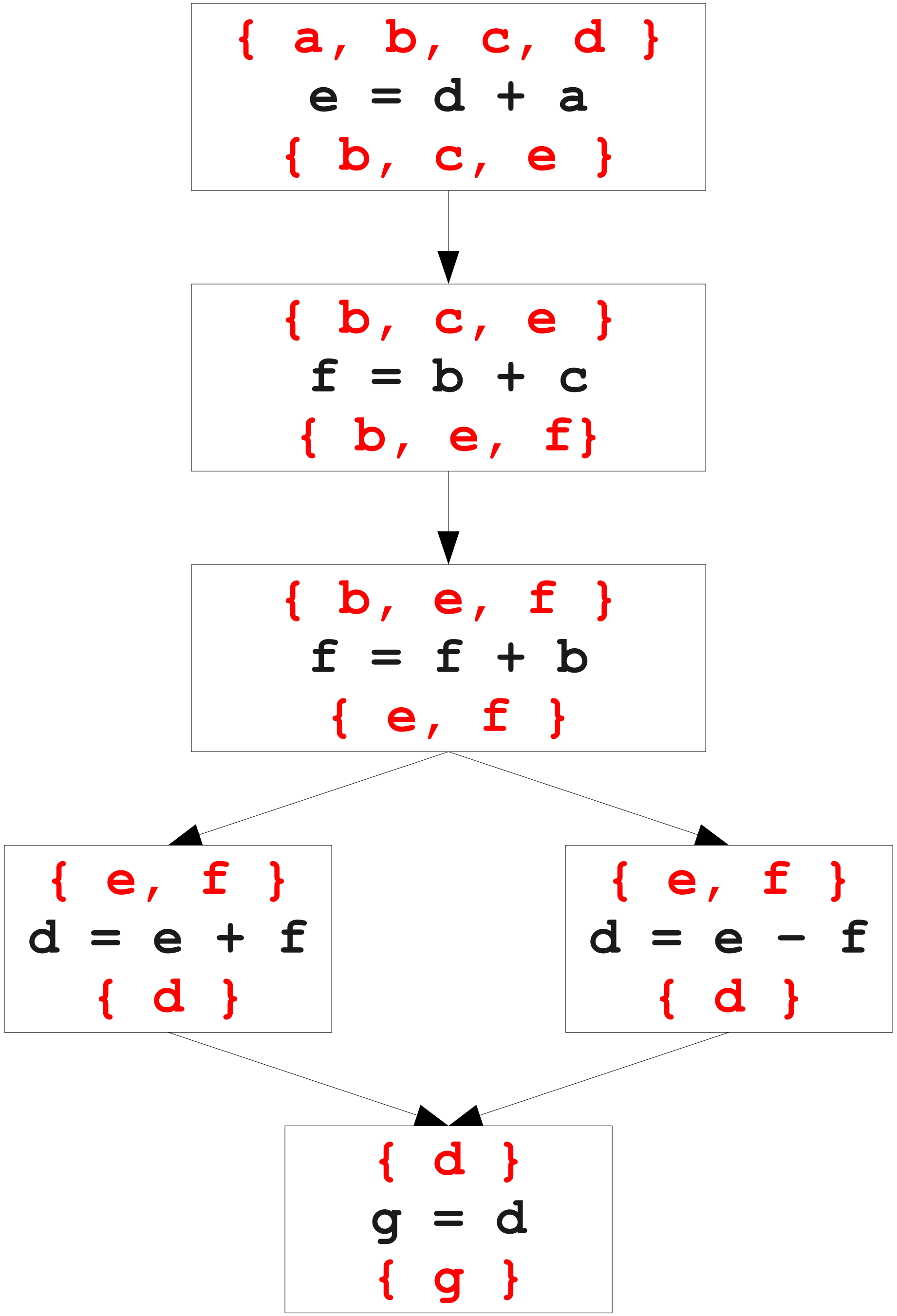

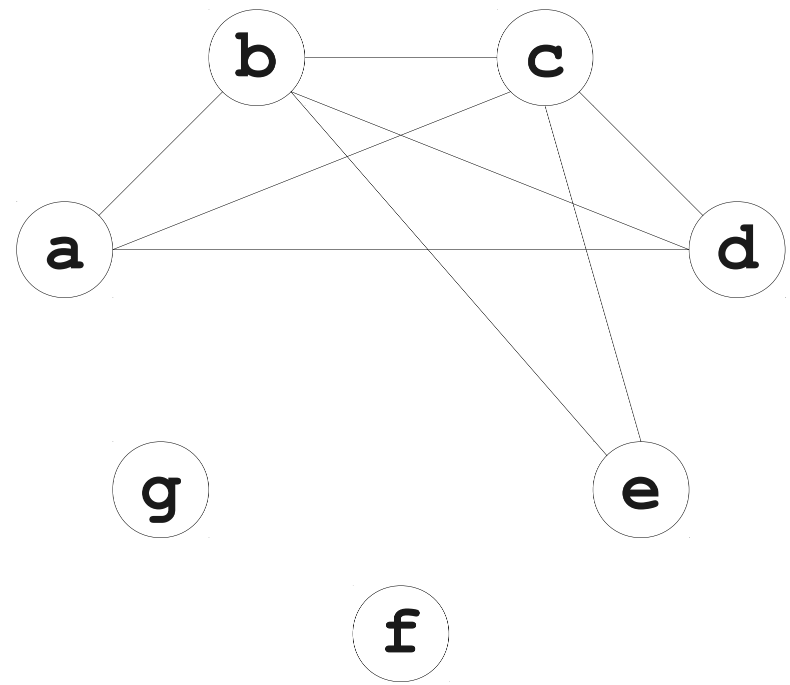

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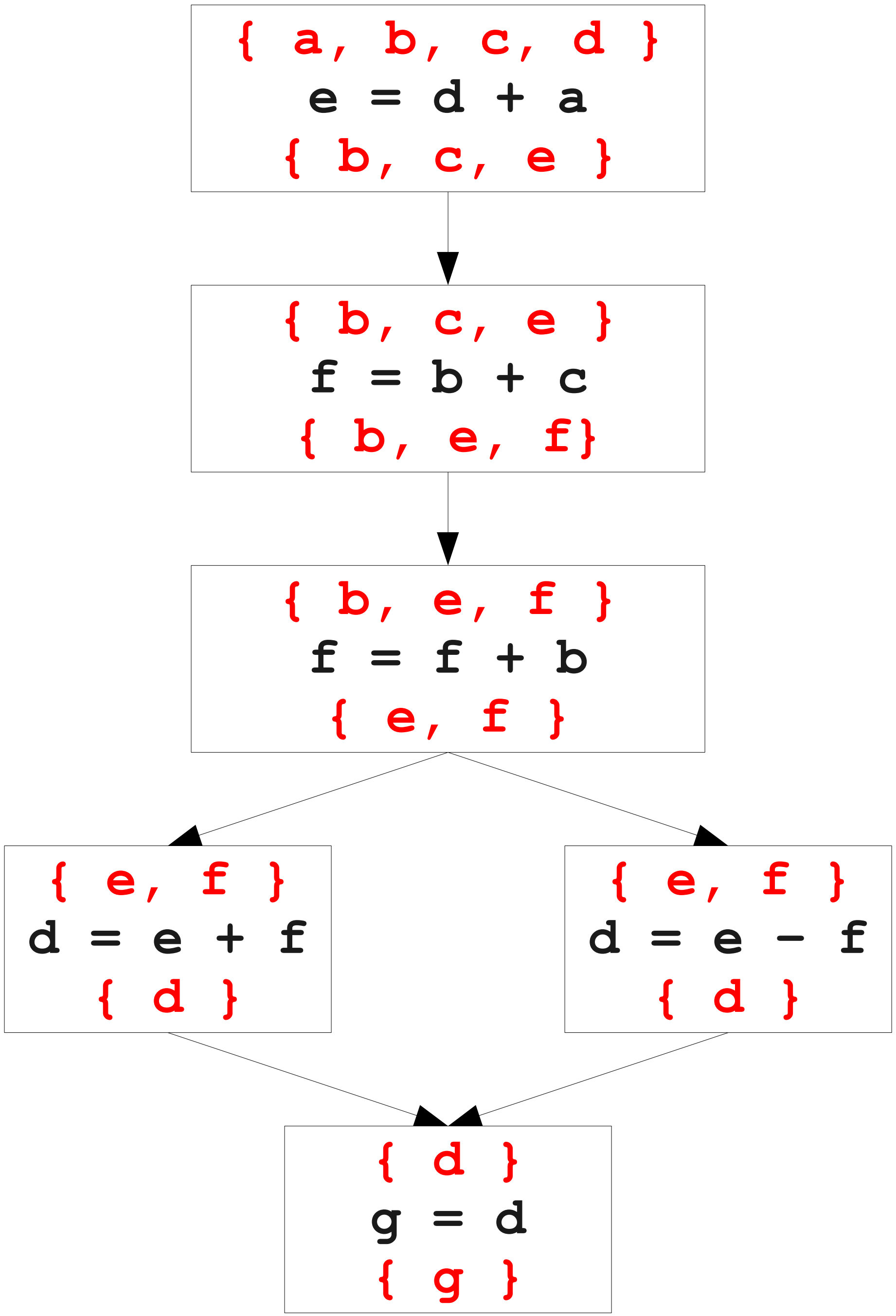

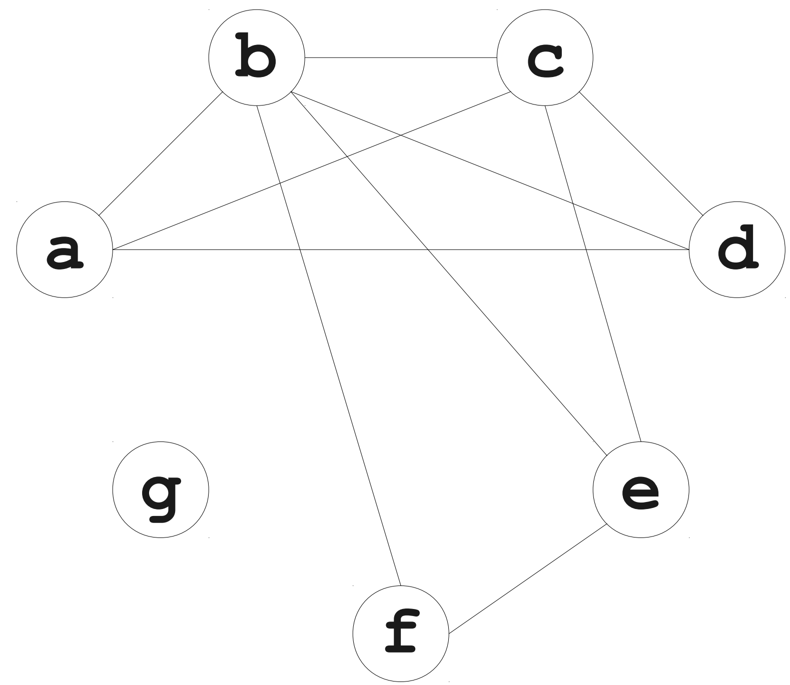

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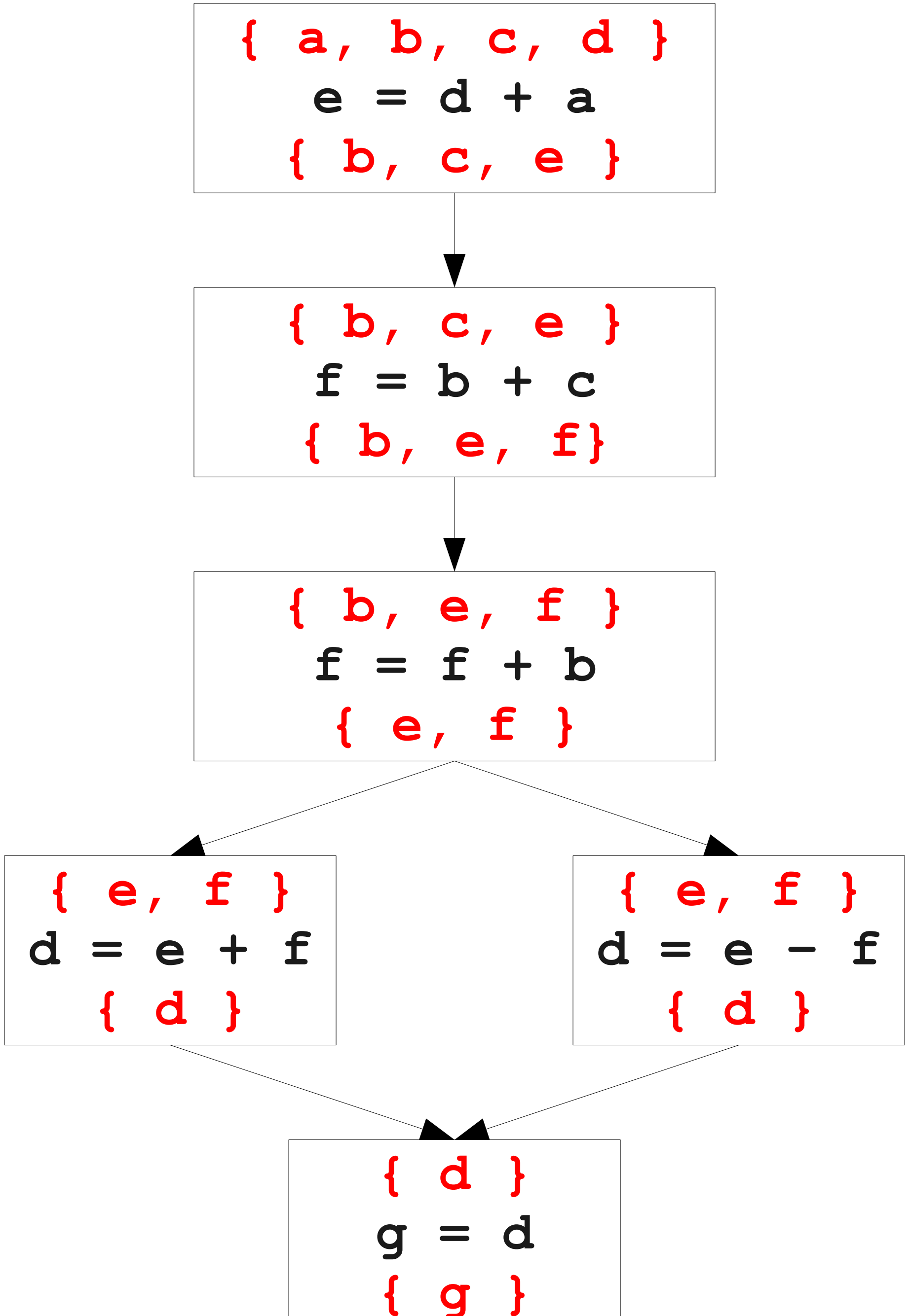

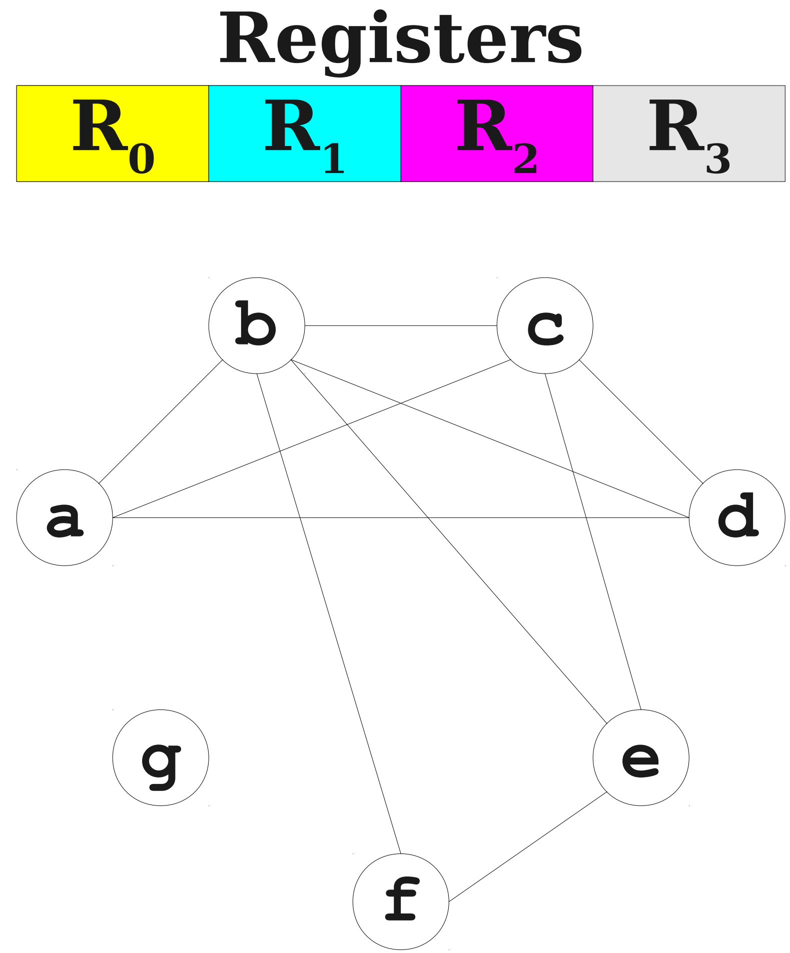

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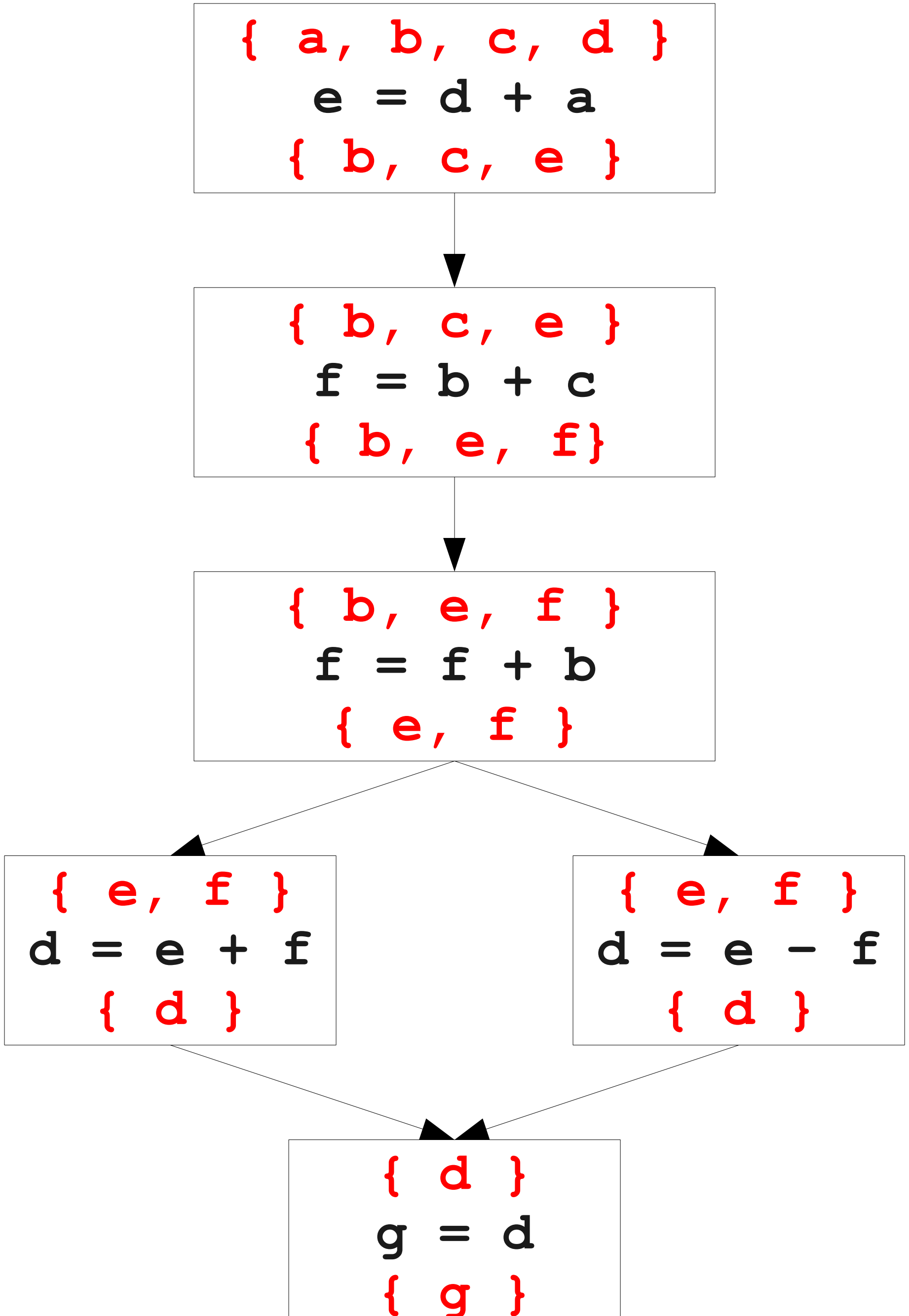

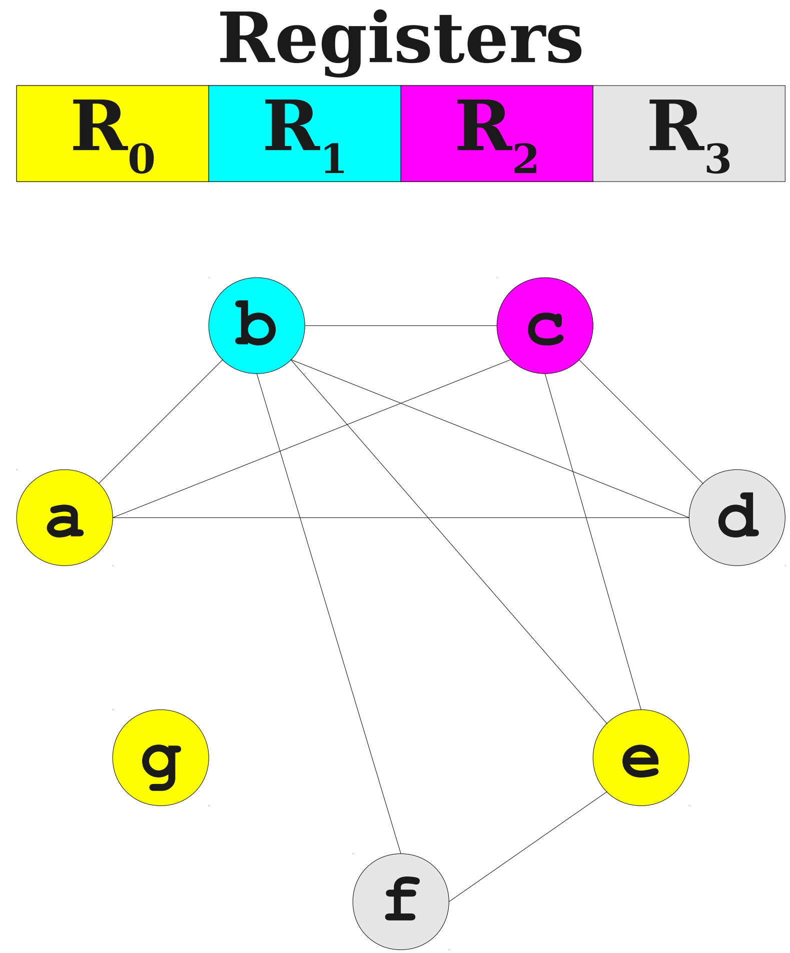

The Register Interference Graph

- The **register interference graph** (RIG) of a control-flow graph is an undirected graph where
  - Each node is a variable.
  - There is an edge between two variables that are live at the same program point.
- Perform register allocation by assigning each variable a different register from all of its neighbors.
- There's just one catch...

The One Catch

- This problem is equivalent to **graph-coloring**, which is **NP-hard** if there are at least three registers.
- No good polynomial-time algorithms (or even good approximations!) are known for this problem.
- We have to be content with a heuristic that is good enough for RIGs that arise in practice.

The One Catch to The One Catch

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If you can figure out a way to assign registers to arbitrary RIGs, you've just proven **P = NP** and will get a **$1,000,000 check** from the Clay Mathematics Institute.

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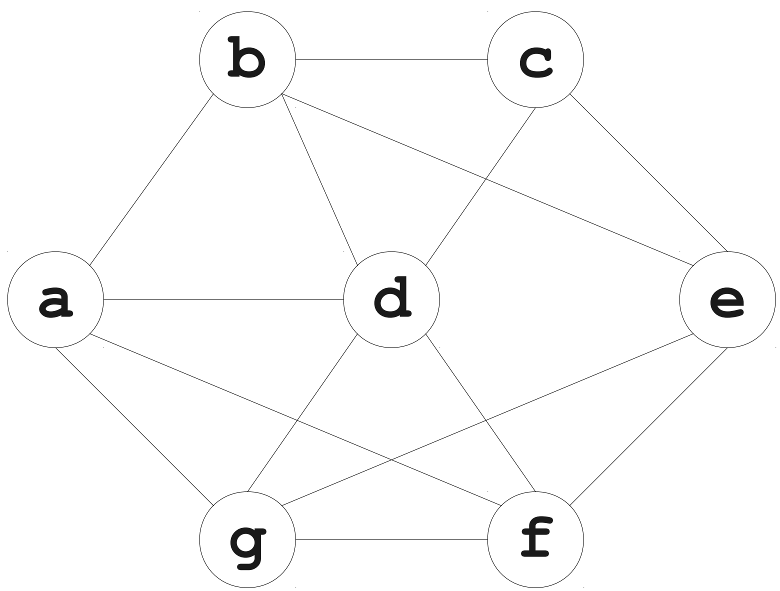
Battling **NP**-Hardness

Chaitin's Algorithm

- Intuition:
  - Suppose we are trying to k-color a graph and find a node with fewer than k edges.
  - If we delete this node from the graph and color what remains, we can find a color for this node if we add it back in.
  - Reason: With fewer than k neighbors, some color must be left over.
- Algorithm:
  - Find a node with fewer than k outgoing edges.
  - Remove it from the graph.
  - Recursively color the rest of the graph.
  - Add the node back in.
  - Assign it a valid color.

Chaitin's Algorithm

Chaitin's Algorithm



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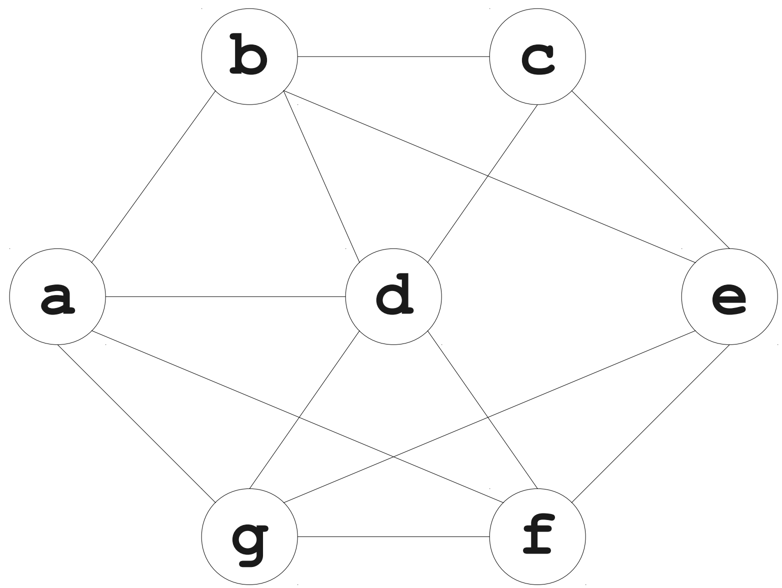
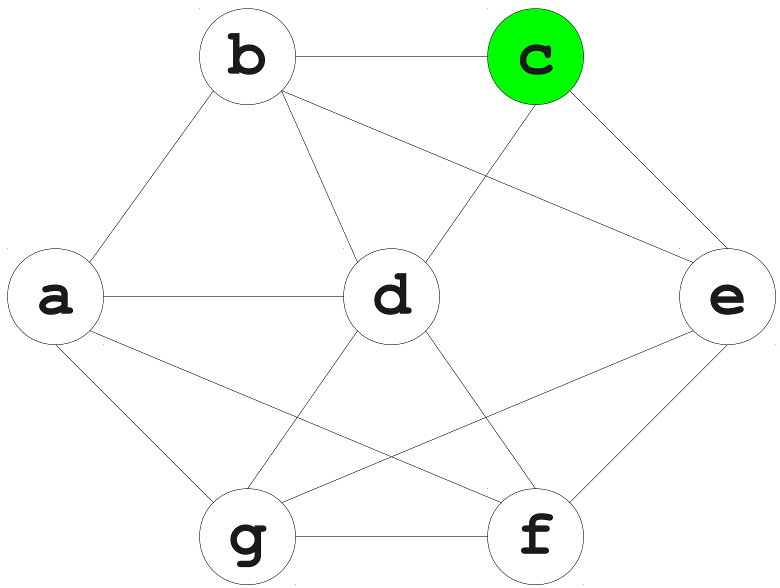


Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm



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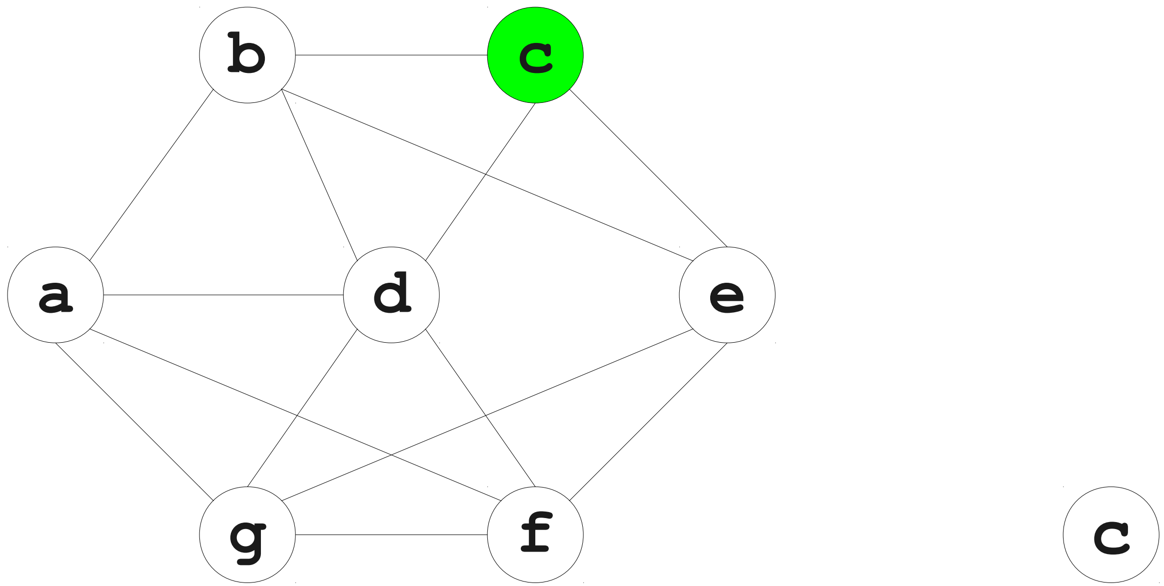

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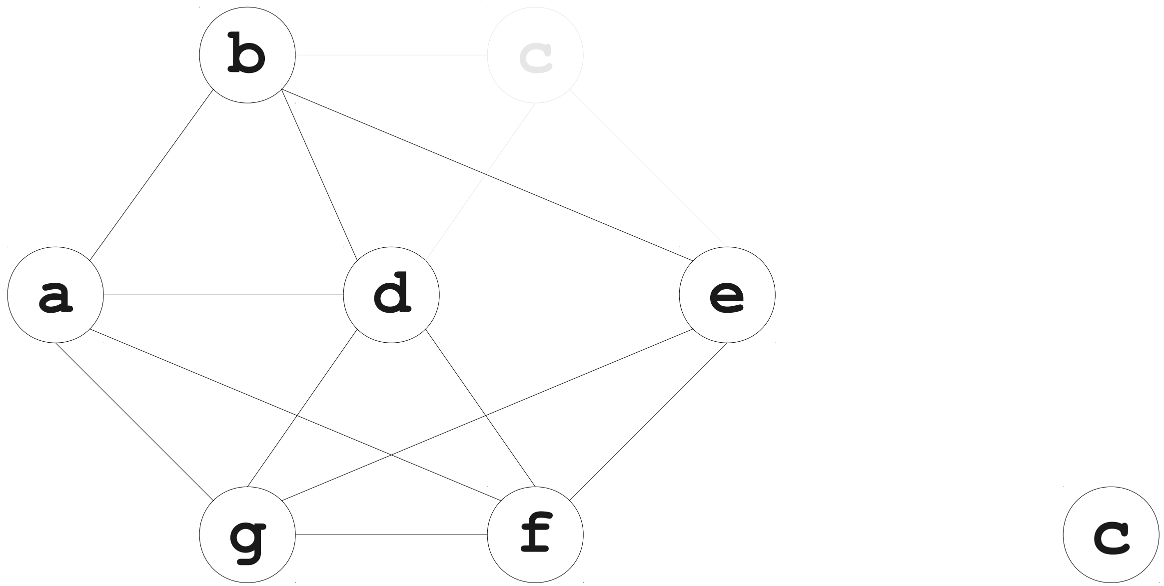

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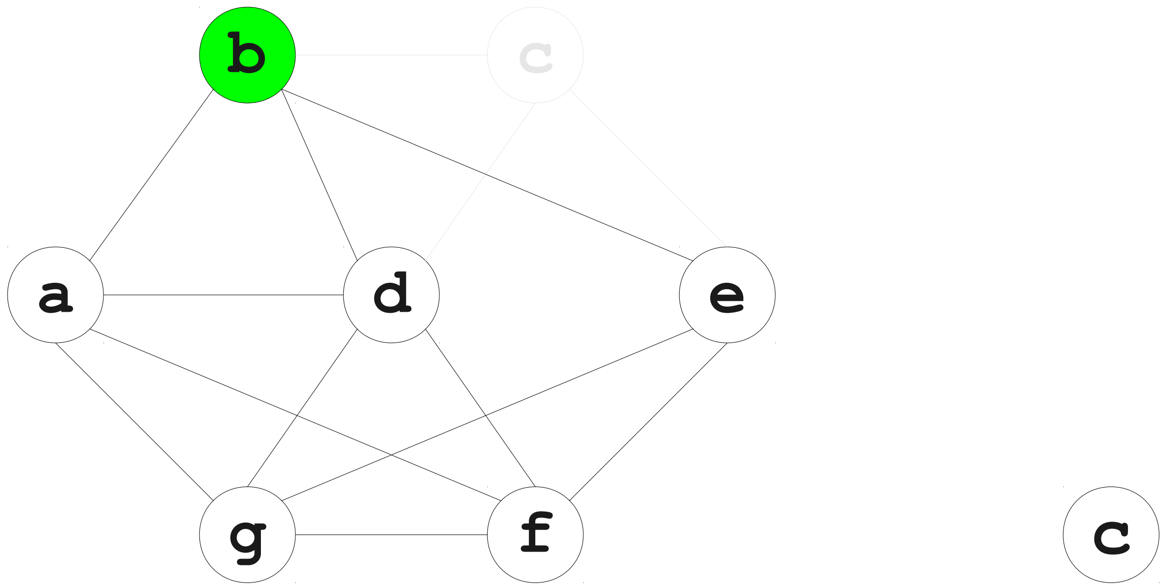

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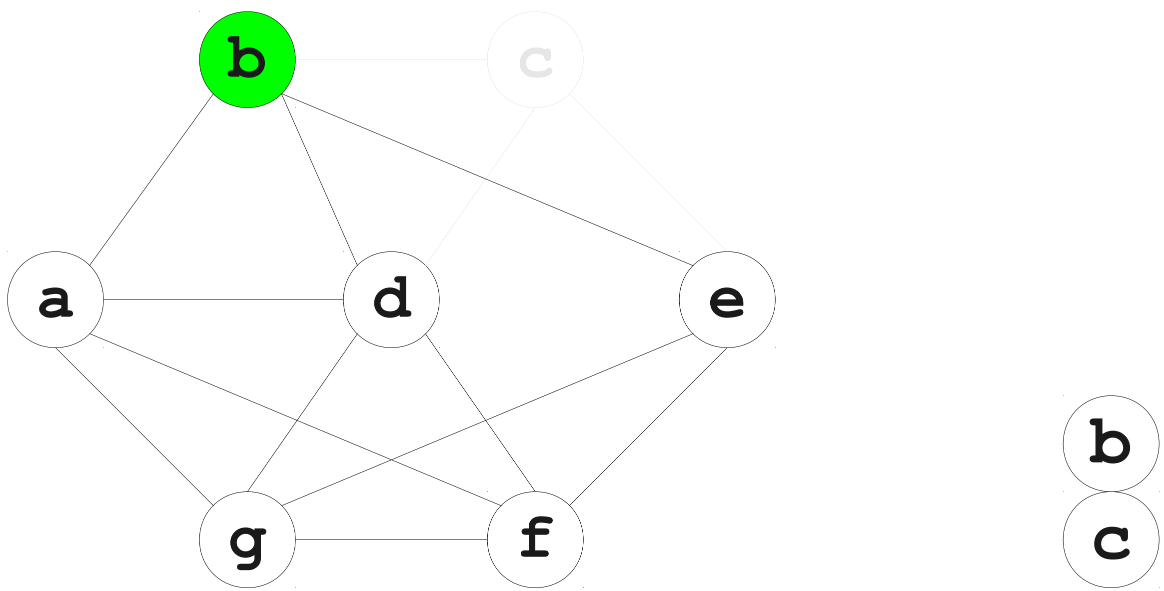

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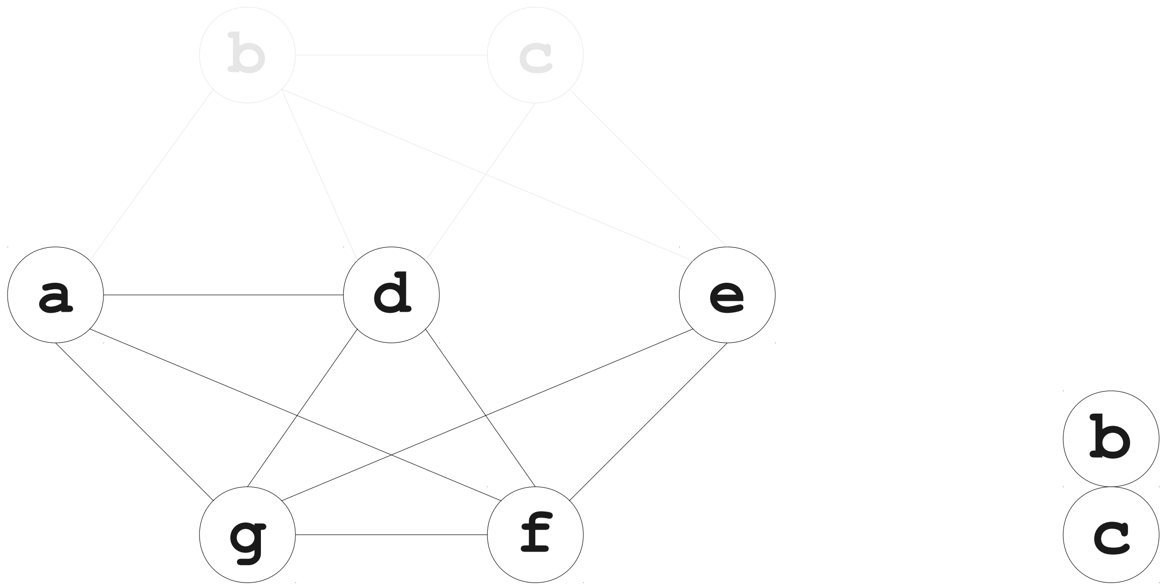

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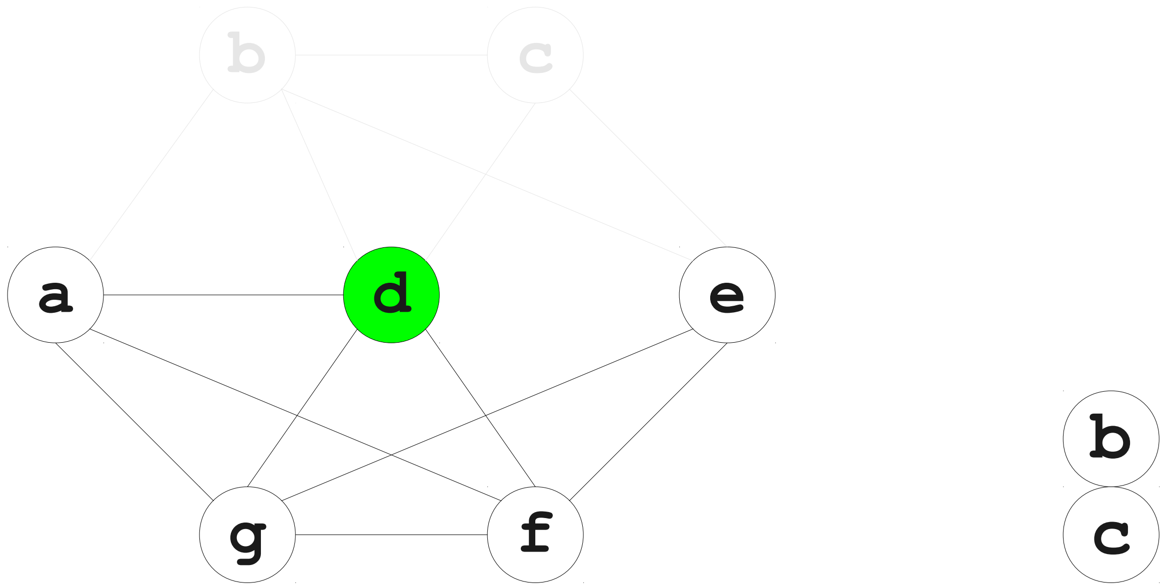
R_0	R_1	R_2	R_3

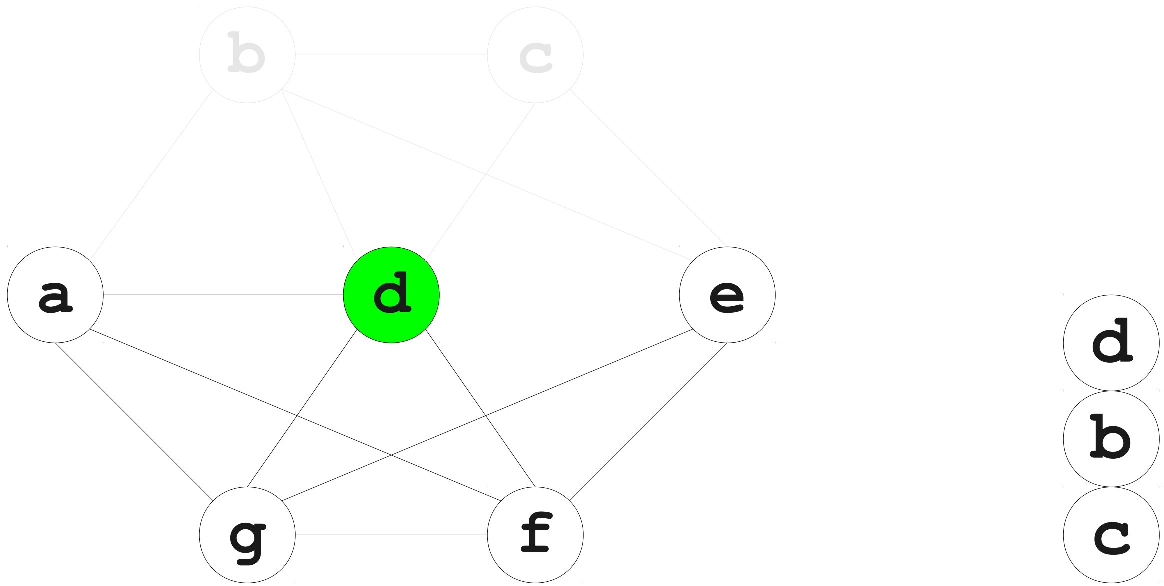
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

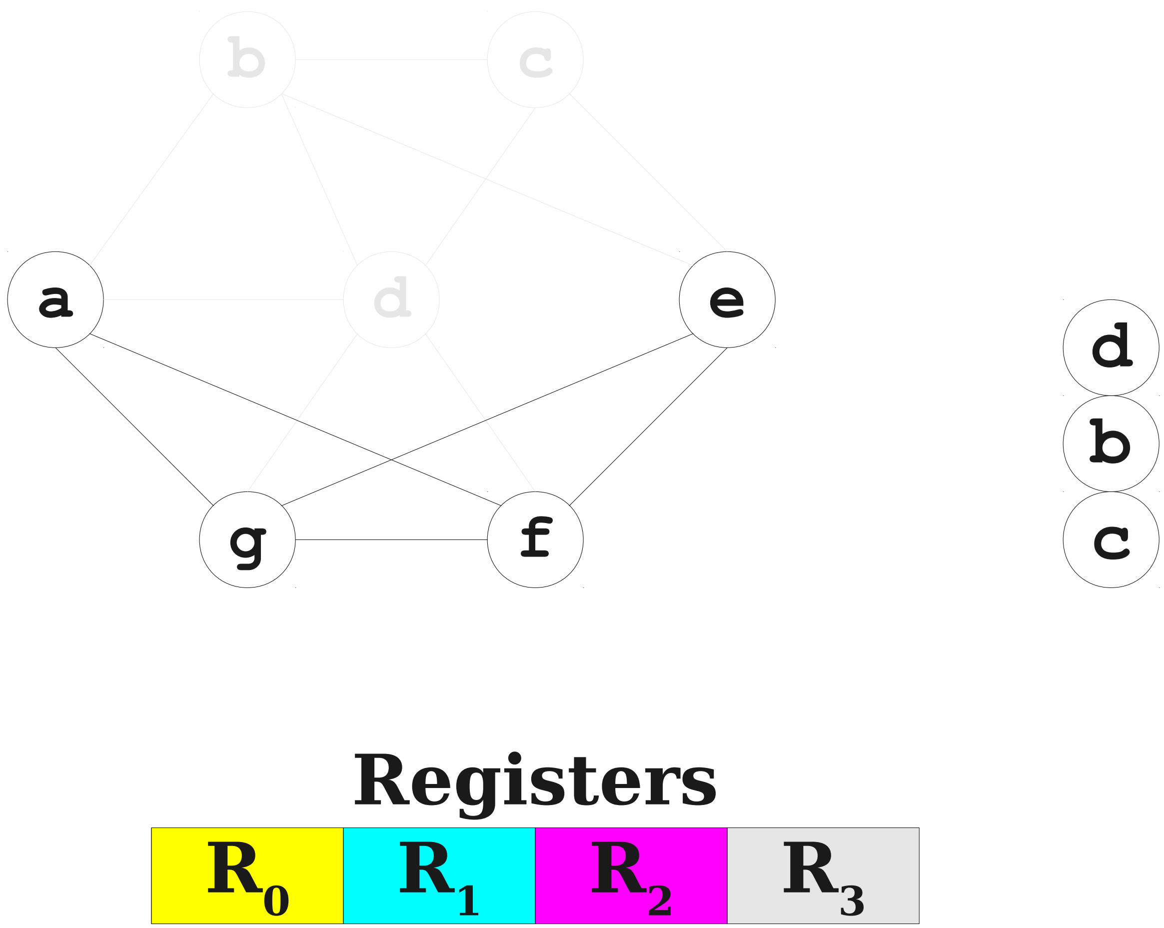
Chaitin's Algorithm



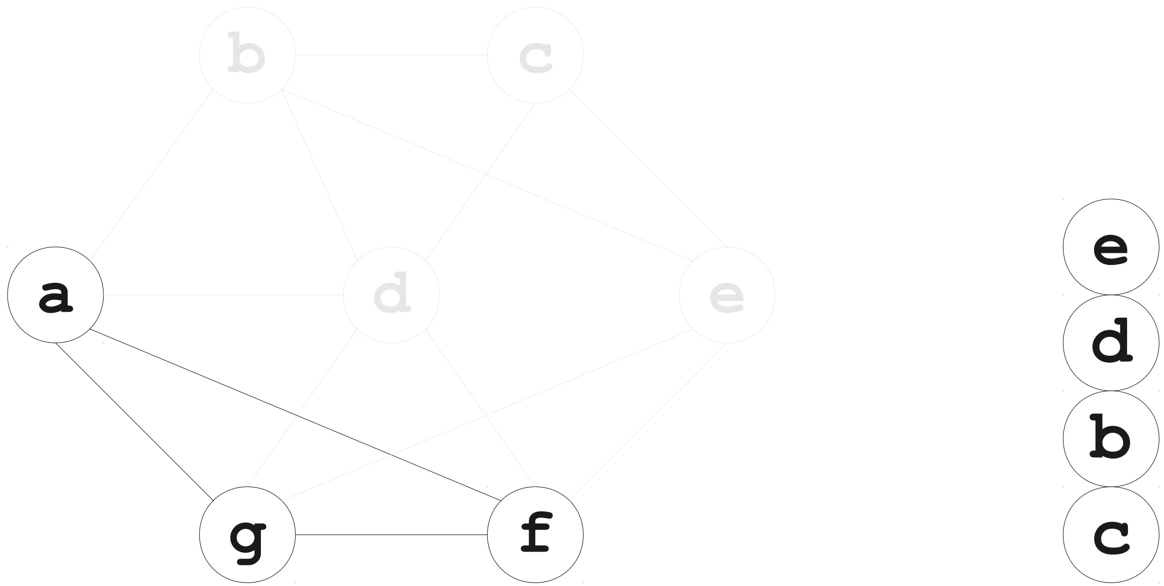
Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm



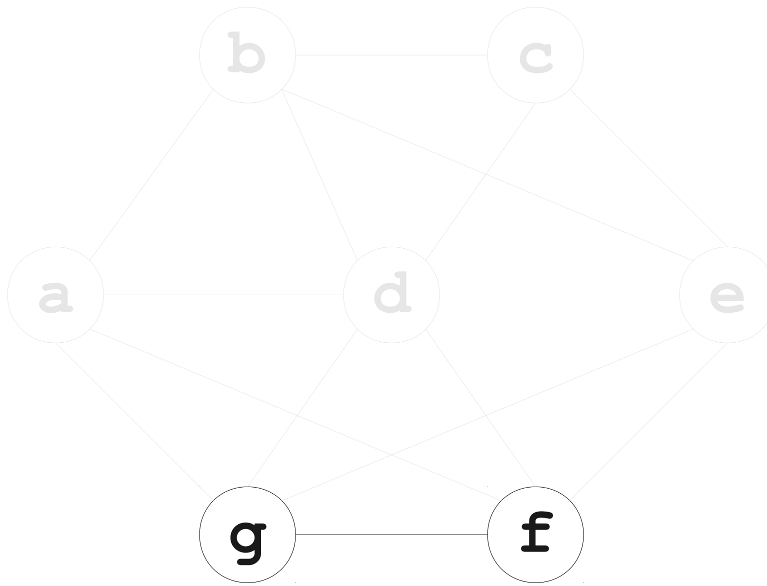
Chaitin's Algorithm

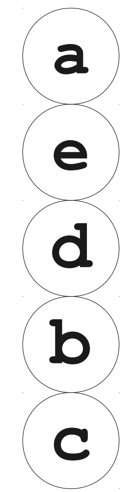


Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm

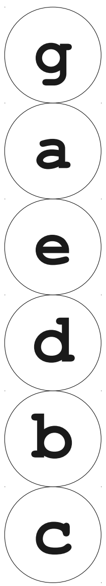




Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm



Registers

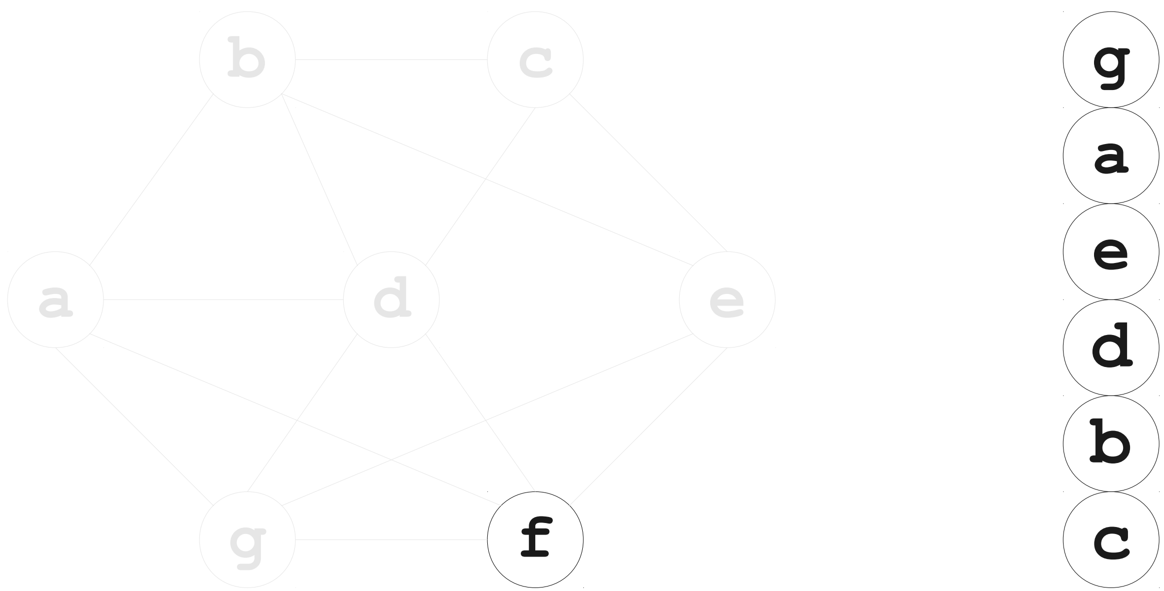
R_0	R_1	R_2	R_3

Chaitin's Algorithm

Registers

R_0	R_1	R_2	R_3

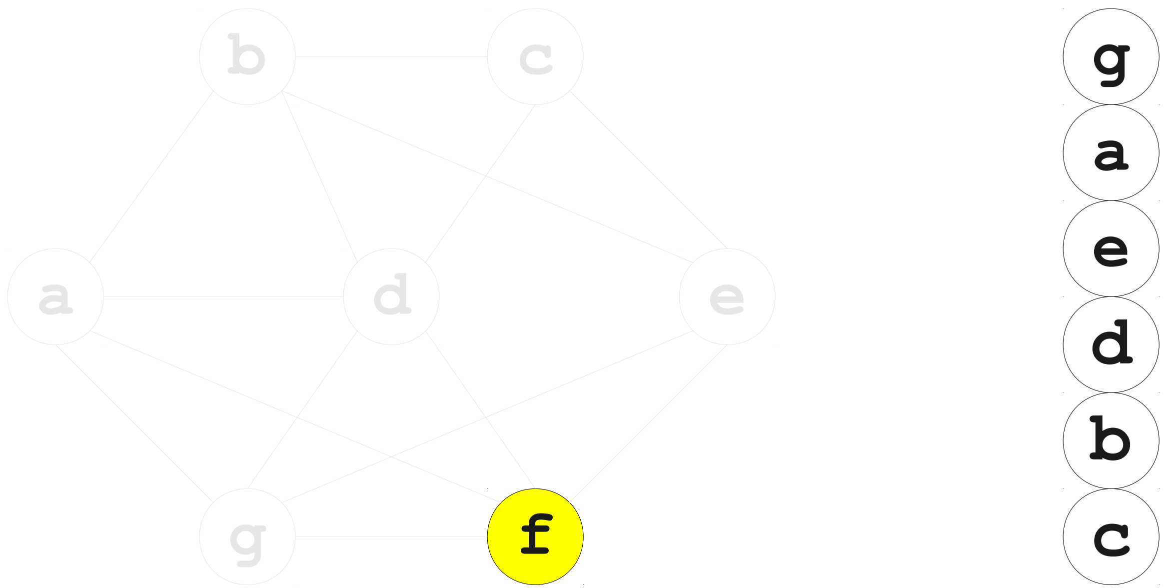
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

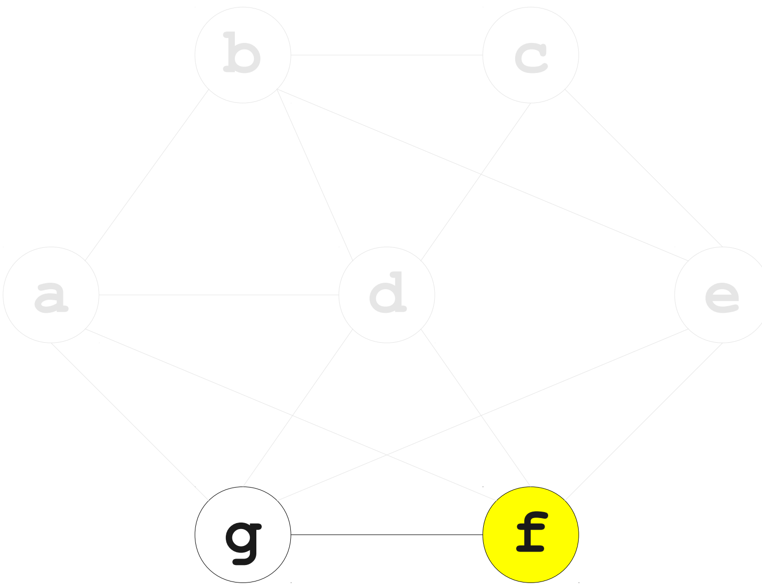
Chaitin's Algorithm

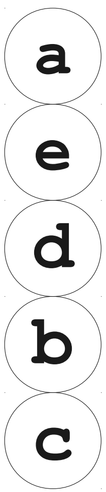


Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm

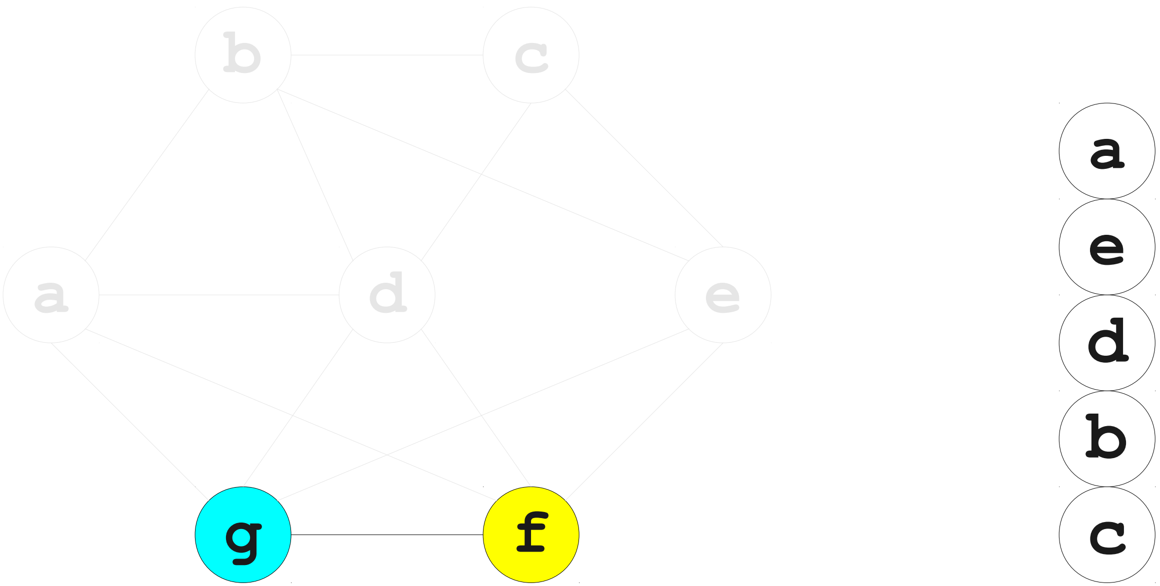




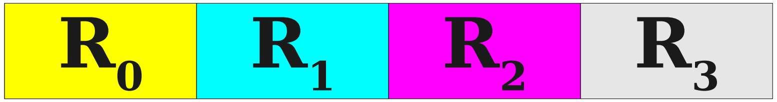
Registers

R_0	R_1	R_2	R_3

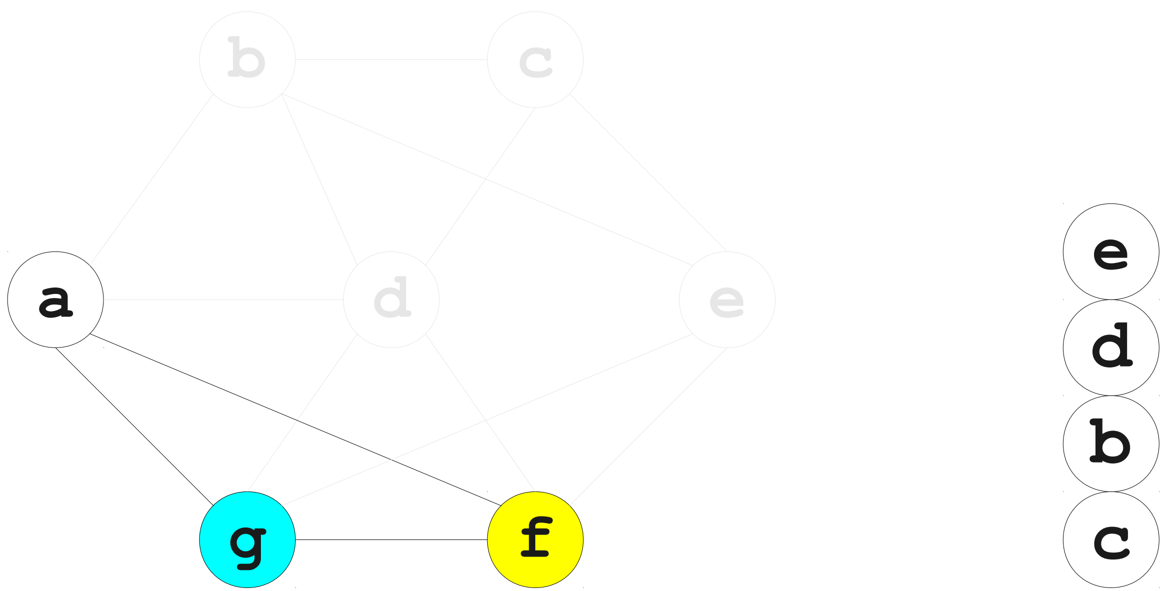
Chaitin's Algorithm



Registers



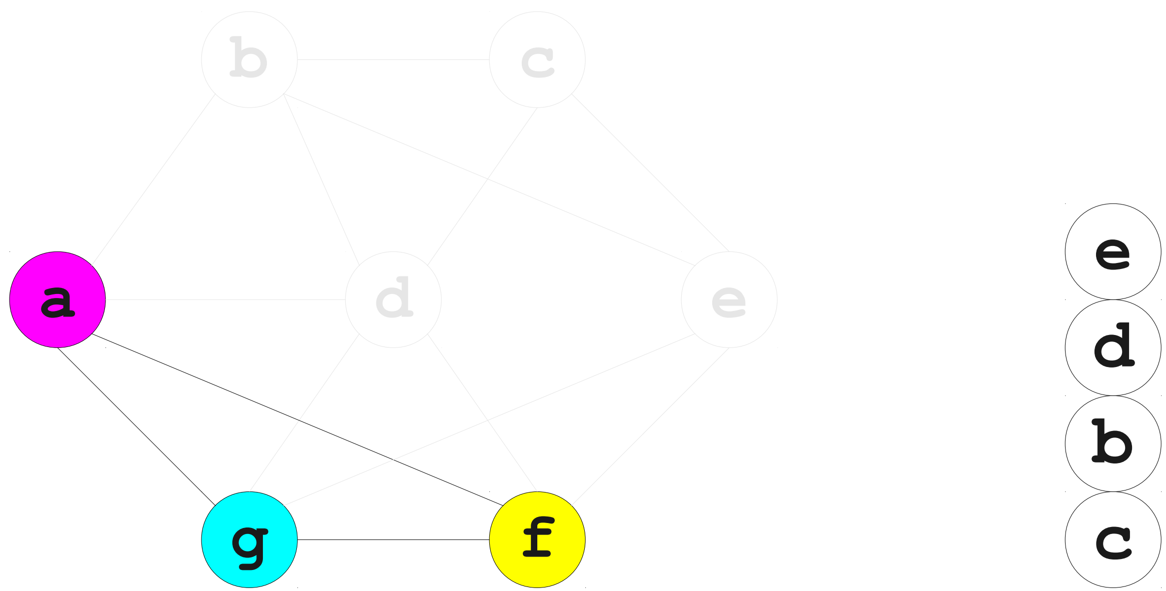
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

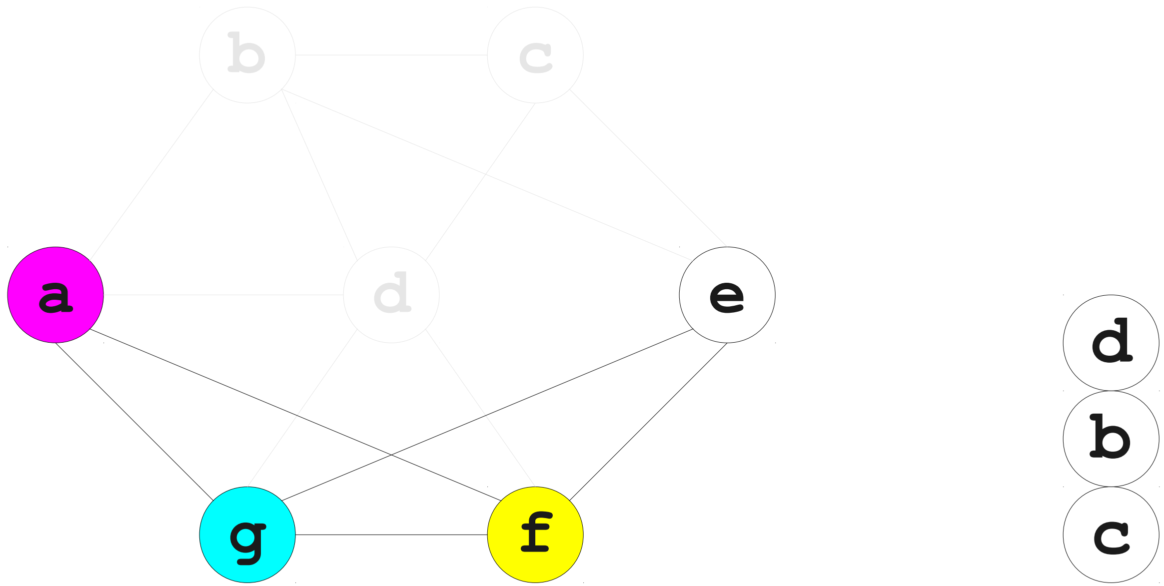
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

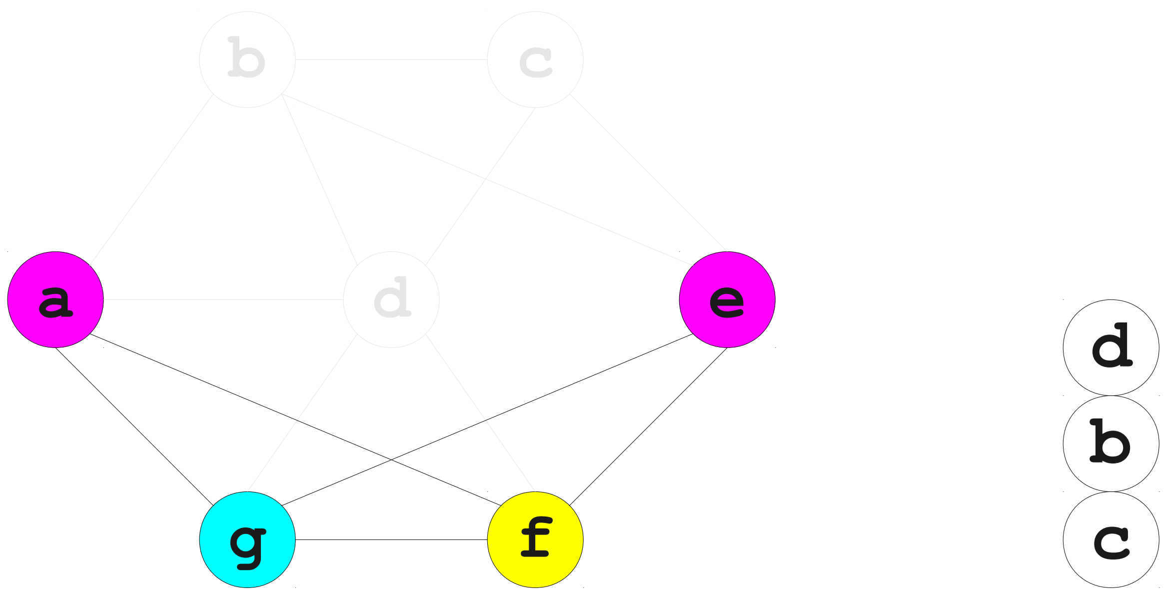
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

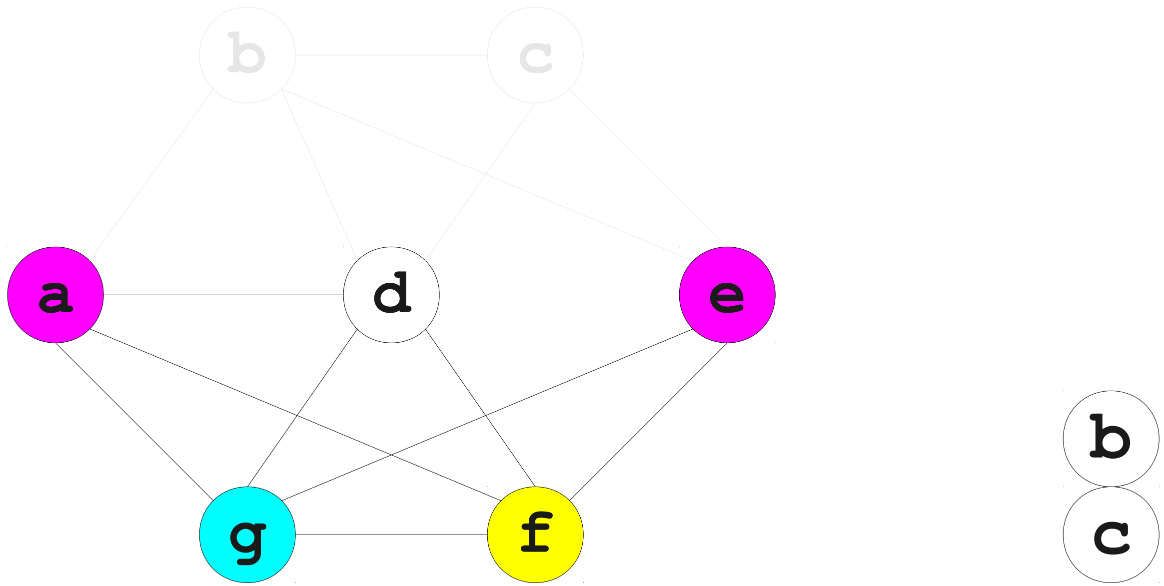
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

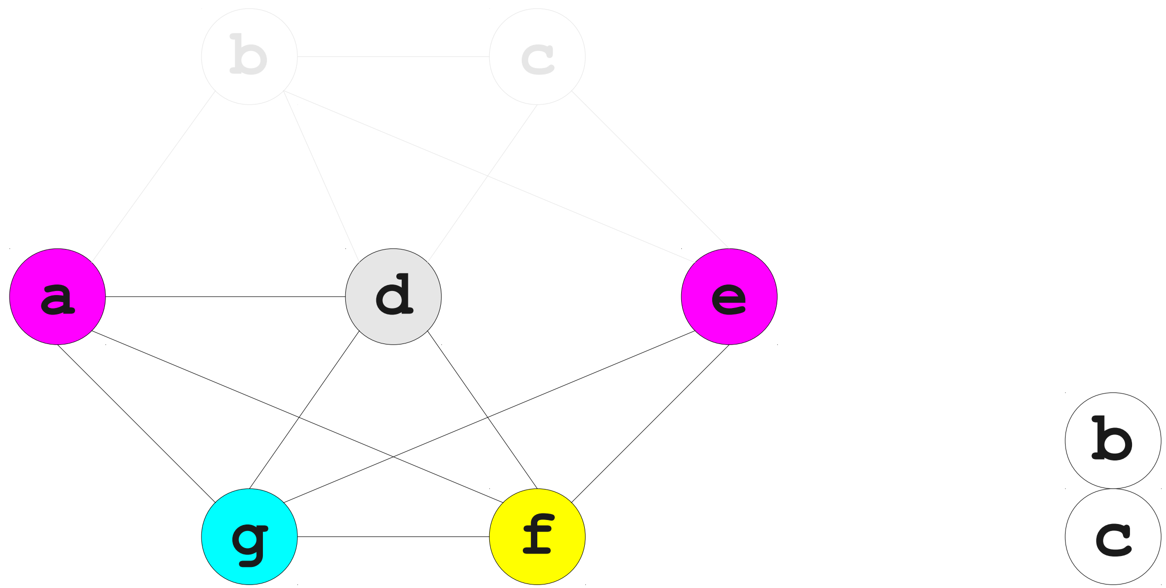
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm



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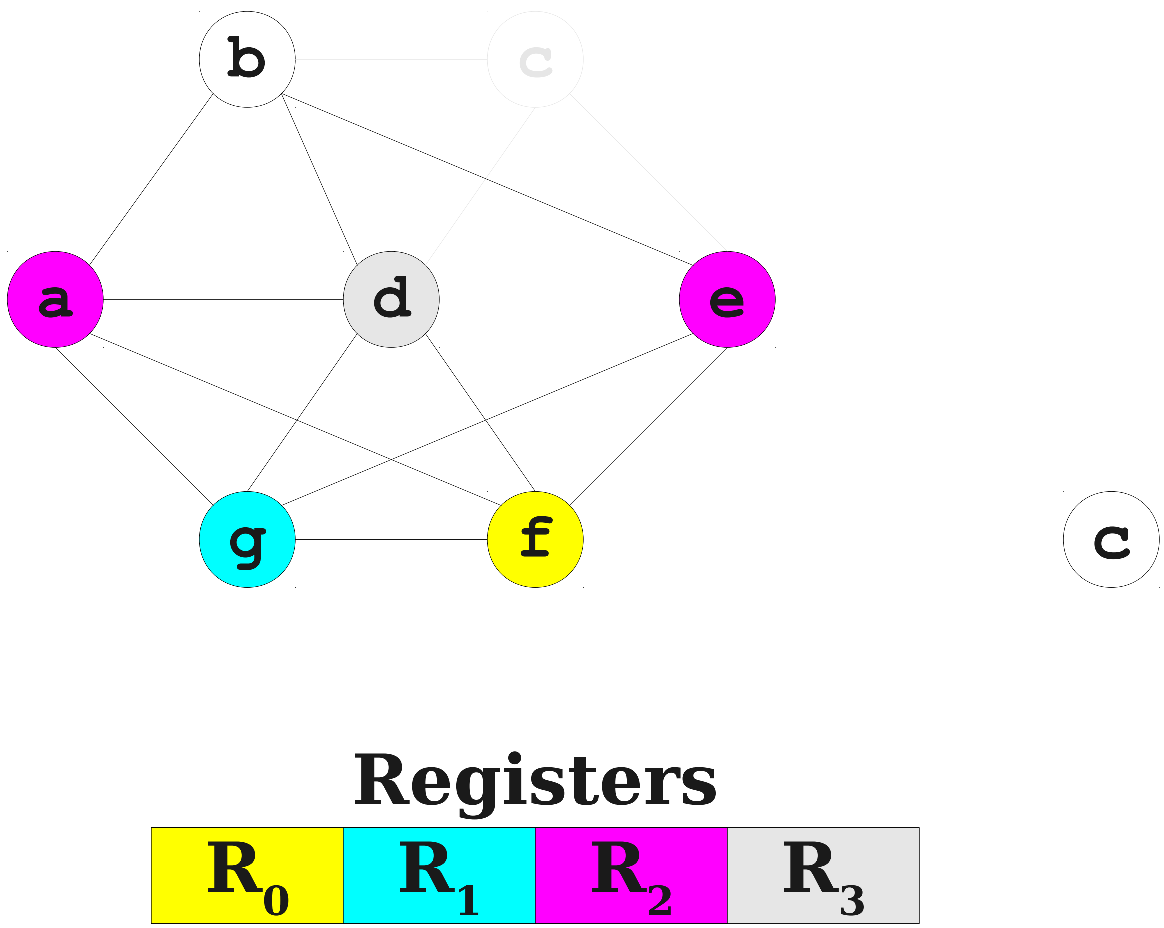

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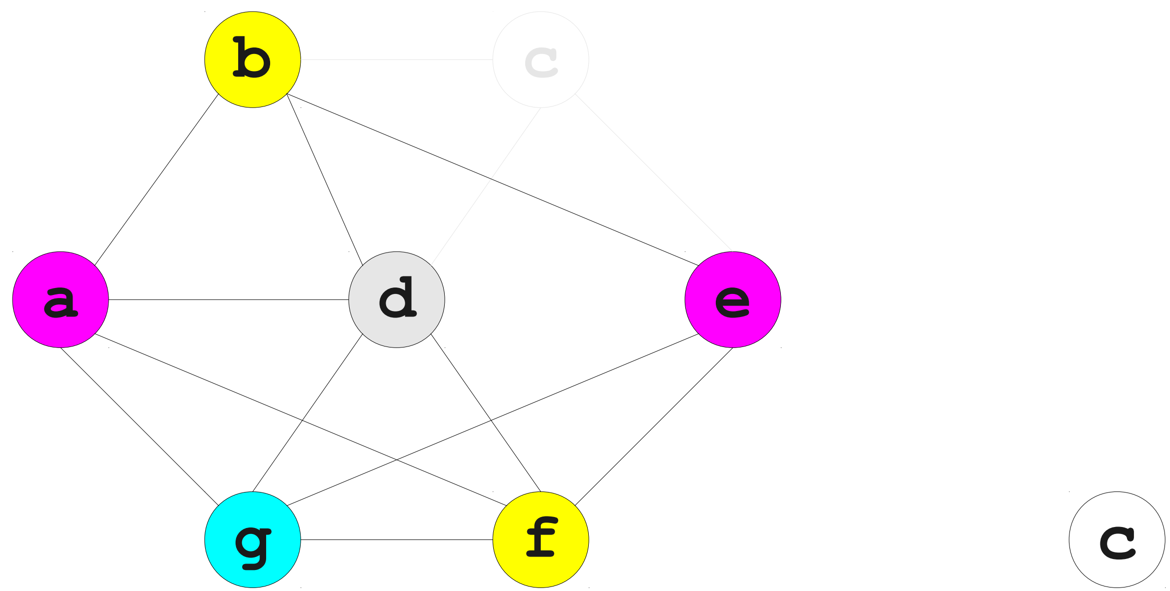
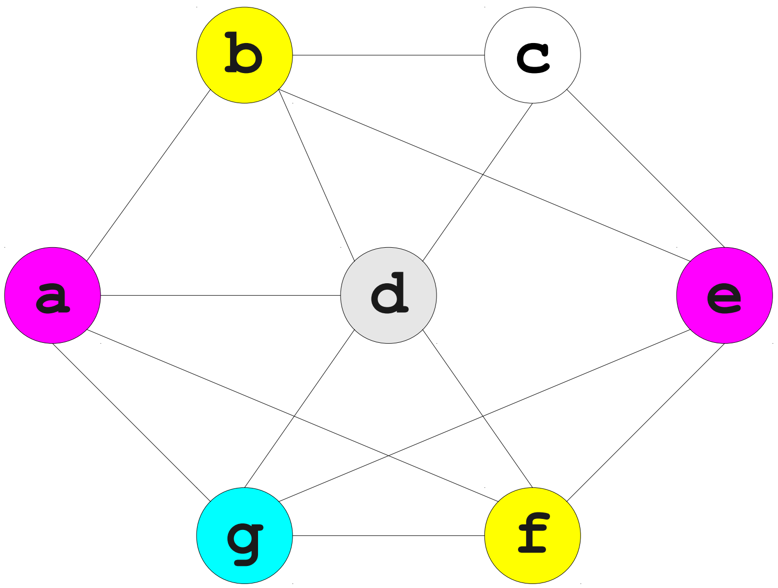


Registers

R_0	R_1	R_2	R_3

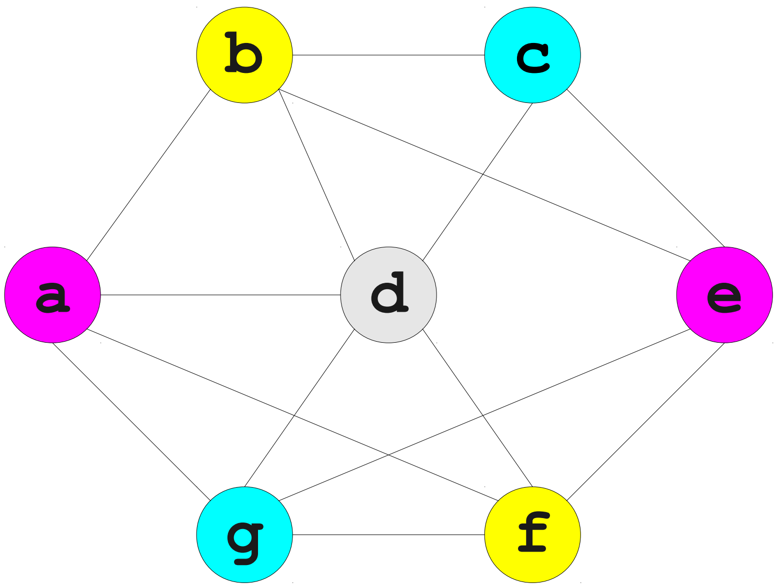
Chaitin's Algorithm



Registers

R_0	R_1	R_2	R_3

Chaitin's Algorithm



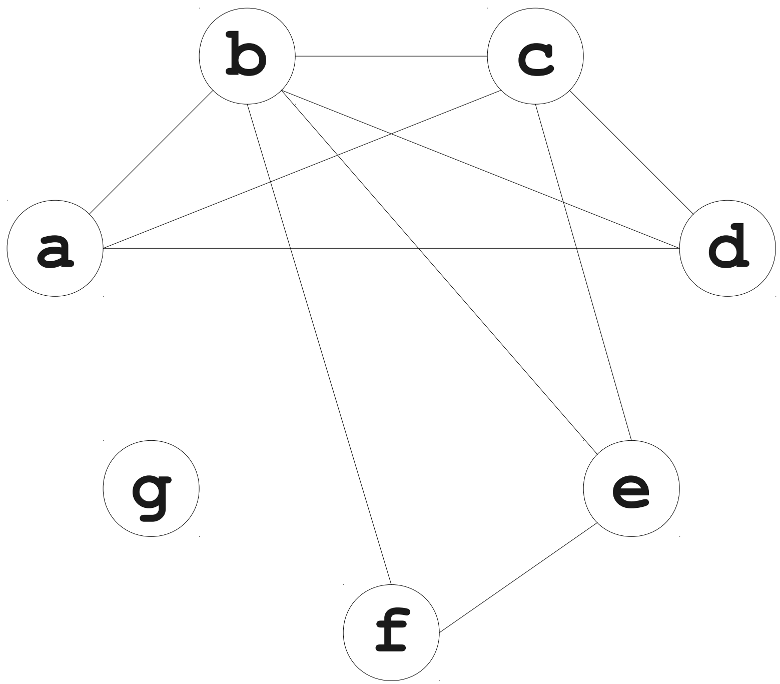
Registers

R_0	R_1	R_2	R_3

One Problem

- What if we can't find a node with fewer than *k* neighbors?
- Choose and remove an arbitrary node, marking it “troublesome.”
    - Use heuristics to choose which one.
- When adding node back in, it may be possible to find a valid color.
- Otherwise, we have to spill that node.

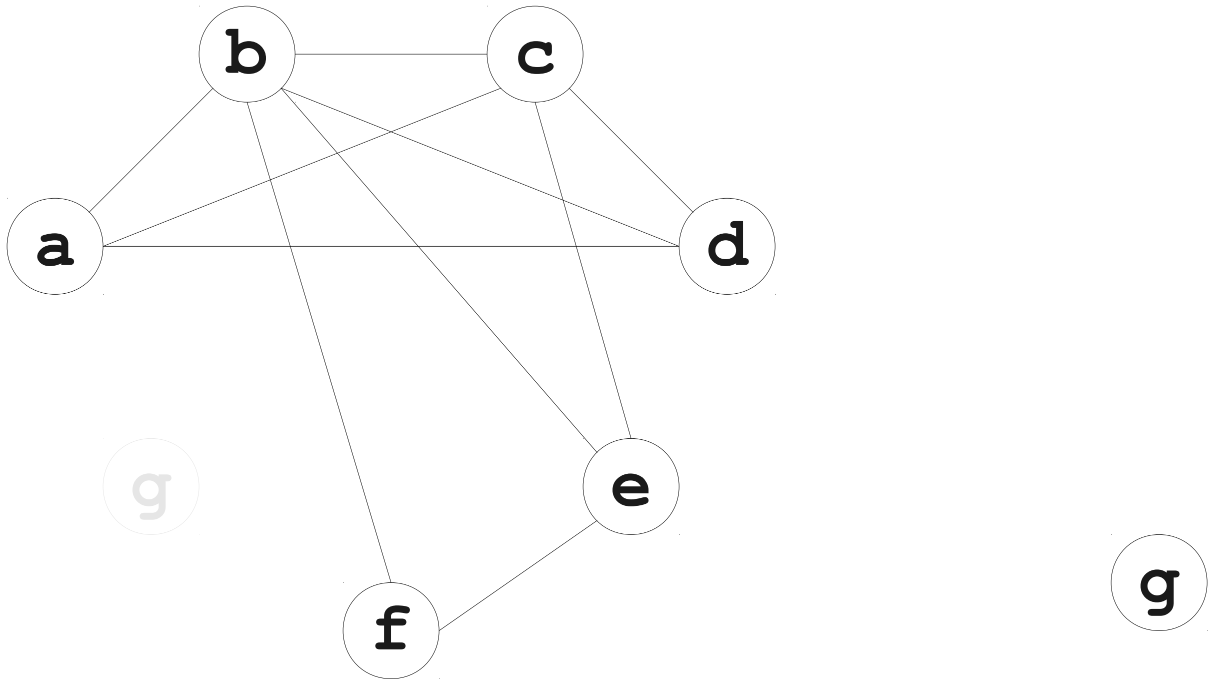
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

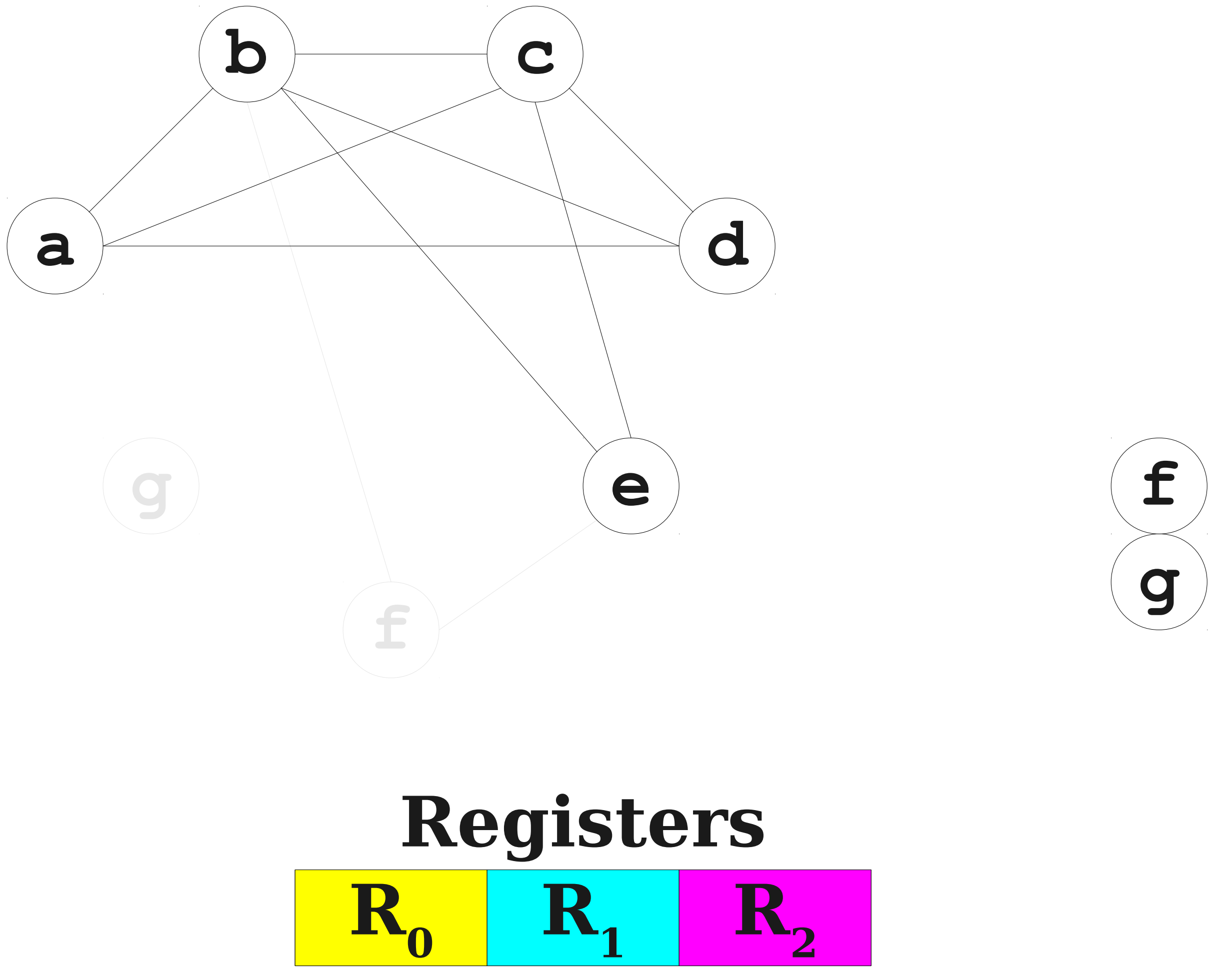
Chaitin's Algorithm Reloaded



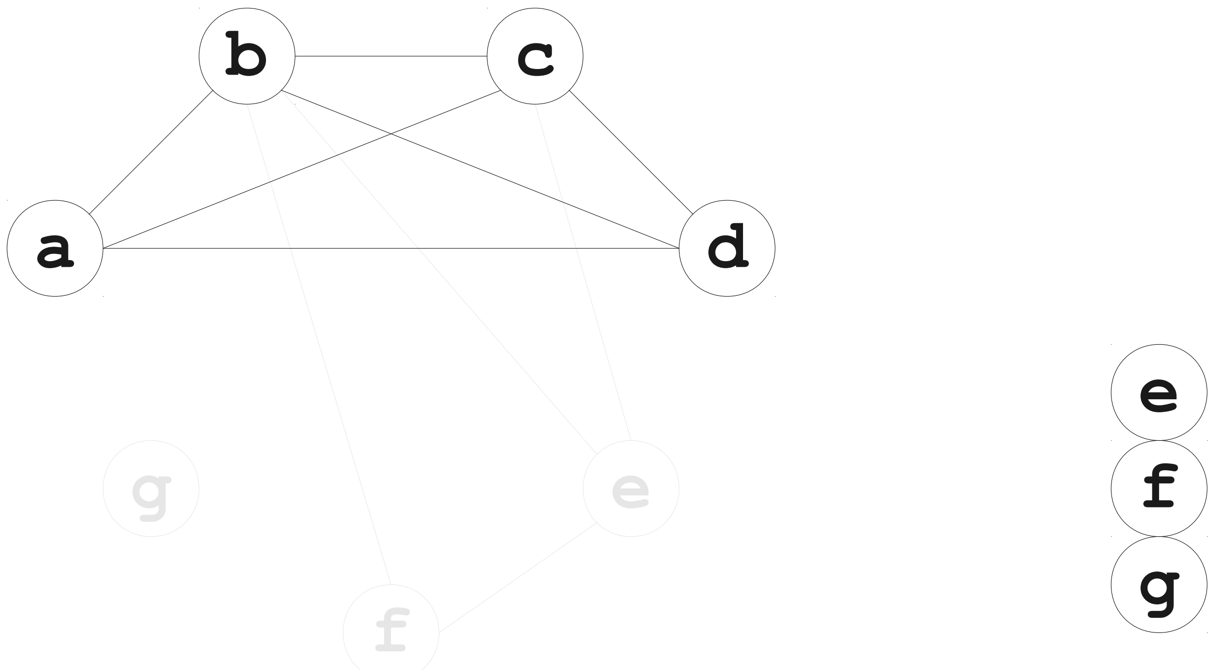
Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



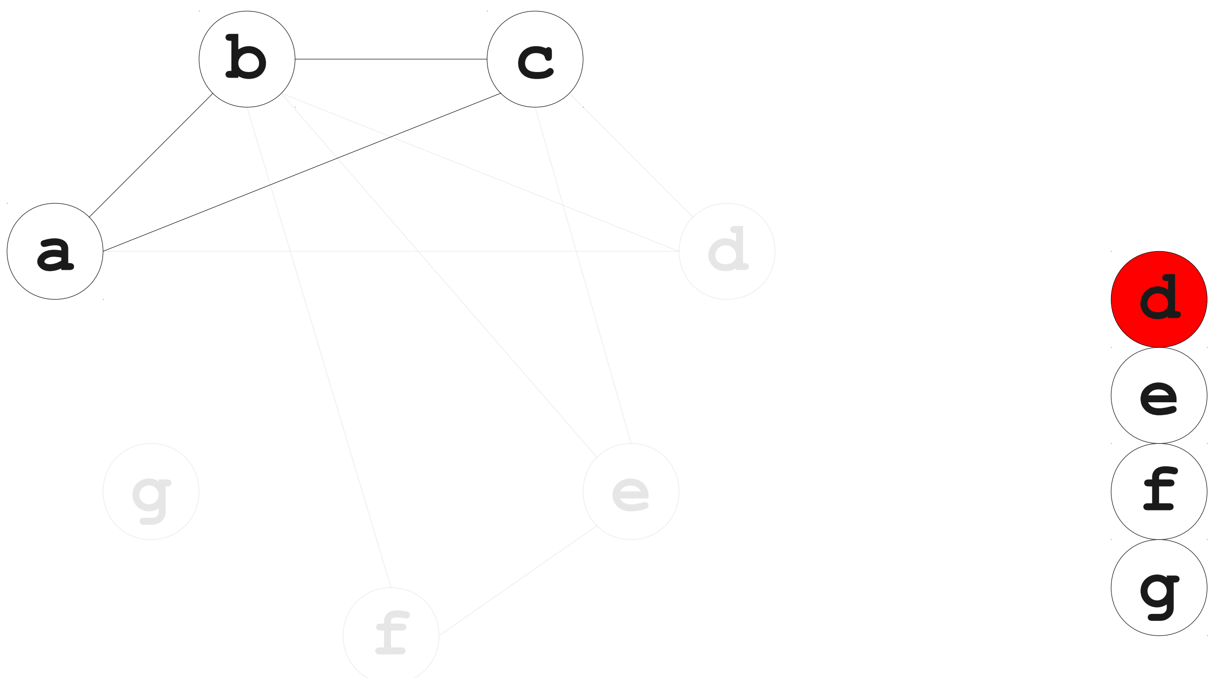
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



Registers

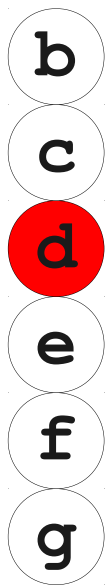
R_0	R_1	R_2

Chaitin's Algorithm Reloaded

Registers

R_0	R_1	R_2

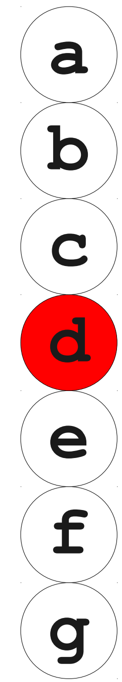
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded

Registers

R_0	R_1	R_2

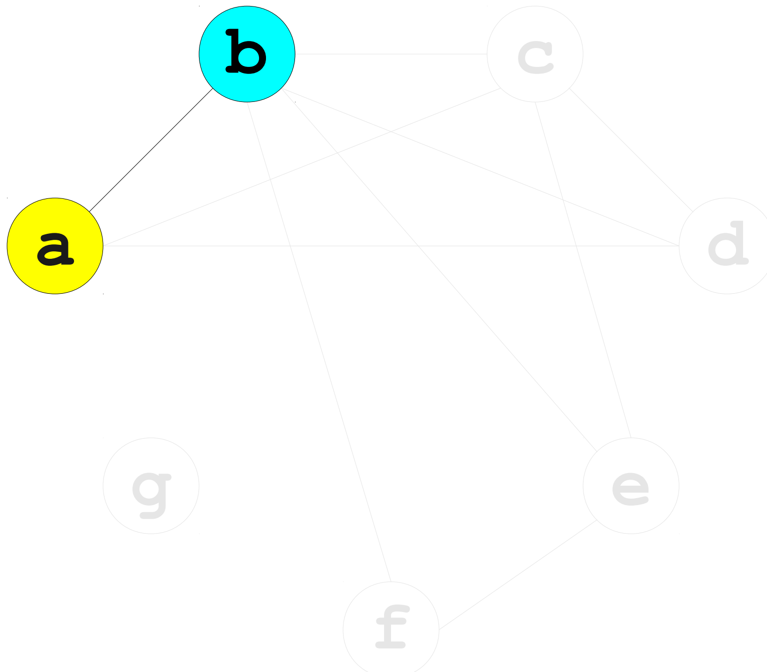
Chaitin's Algorithm Reloaded

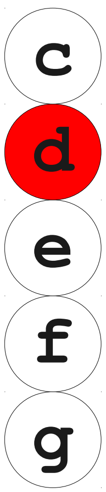
Chaitin's Algorithm Reloaded

Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded

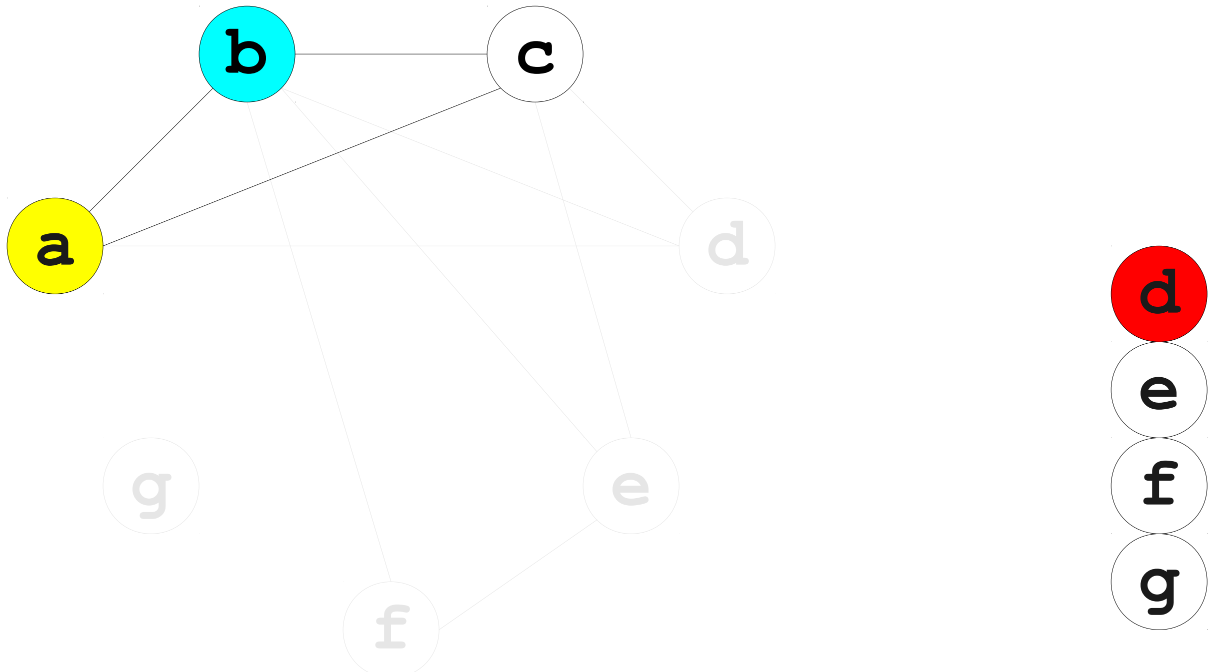




Registers

R_0	R_1	R_2

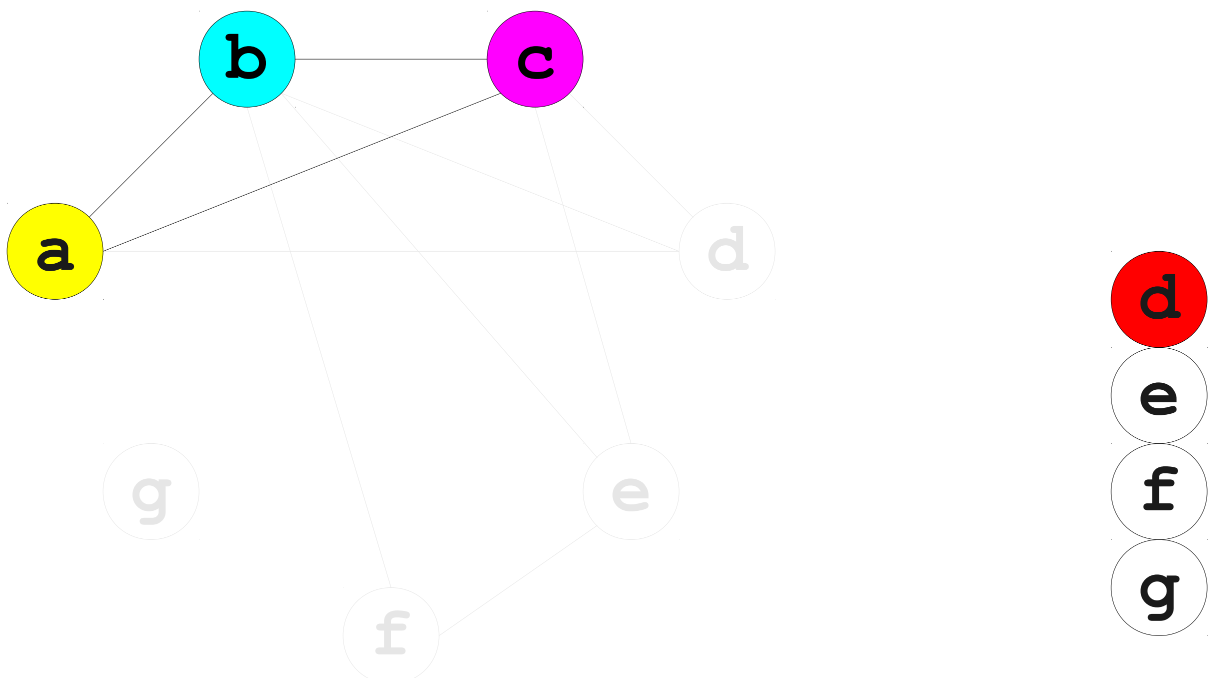
Chaitin's Algorithm Reloaded



Registers

R$_0$	R$_1$	R$_2$

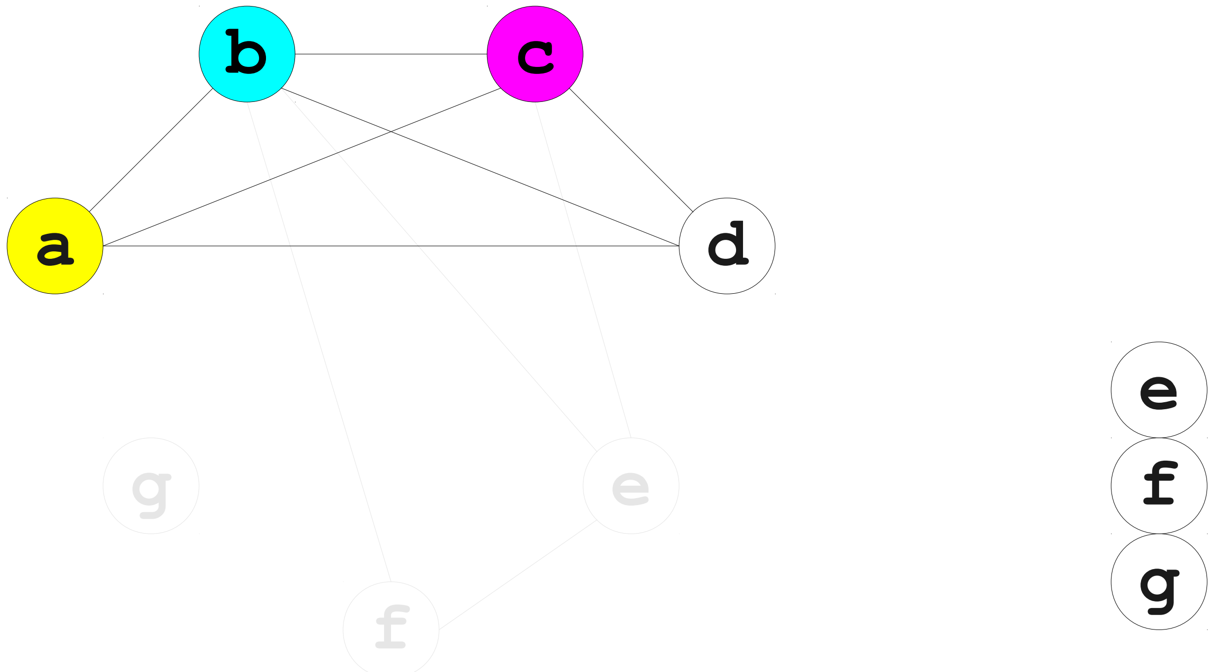
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

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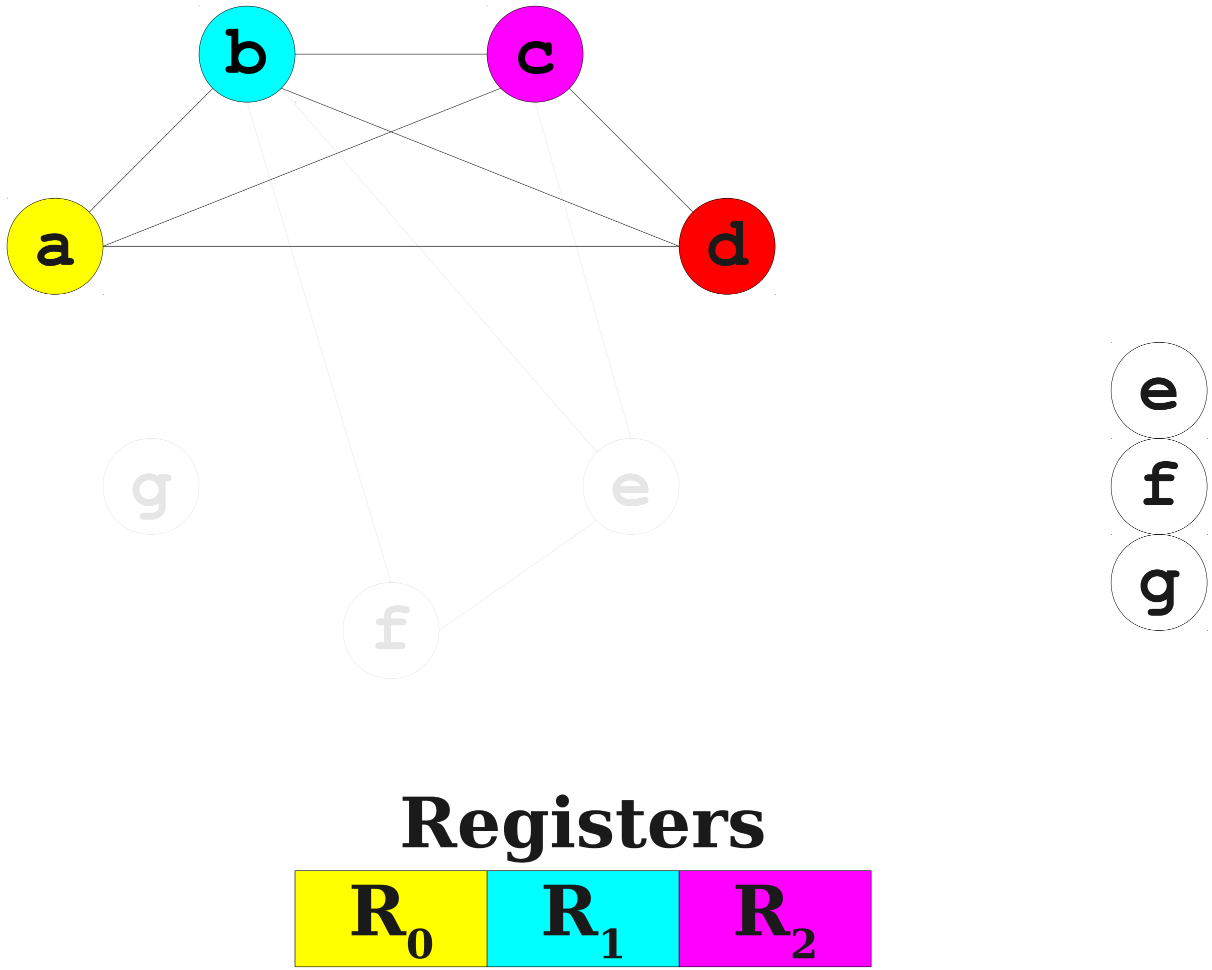

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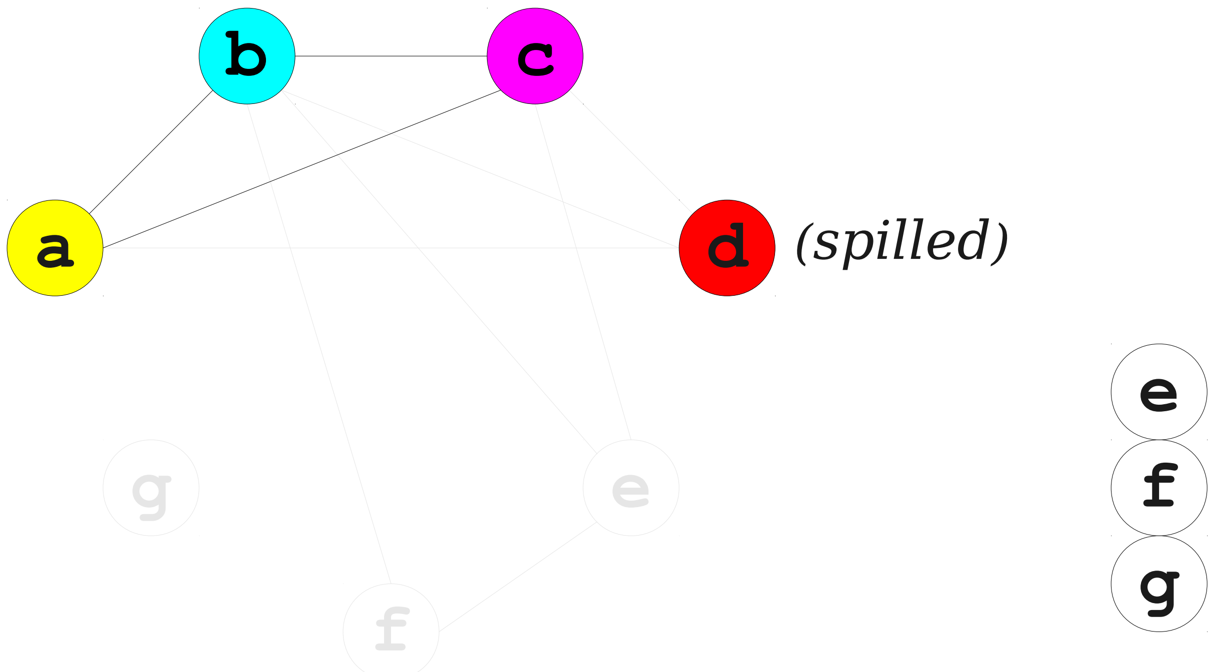
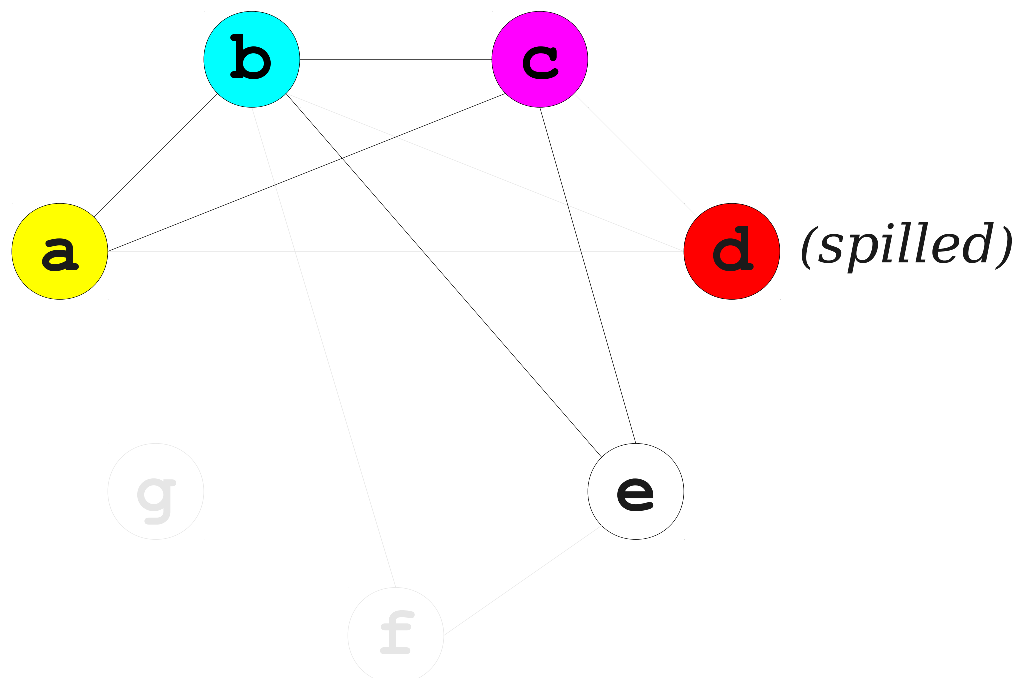


Registers

R_0	R_1	R_2

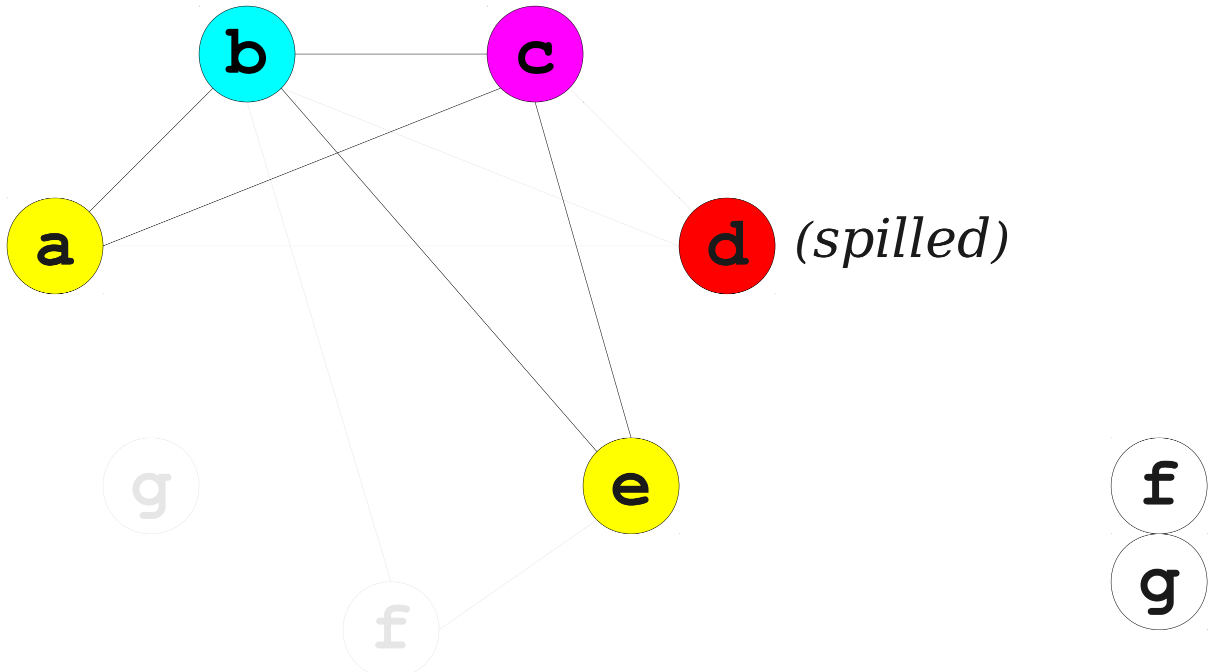
Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



Registers

R_0	R_1	R_2

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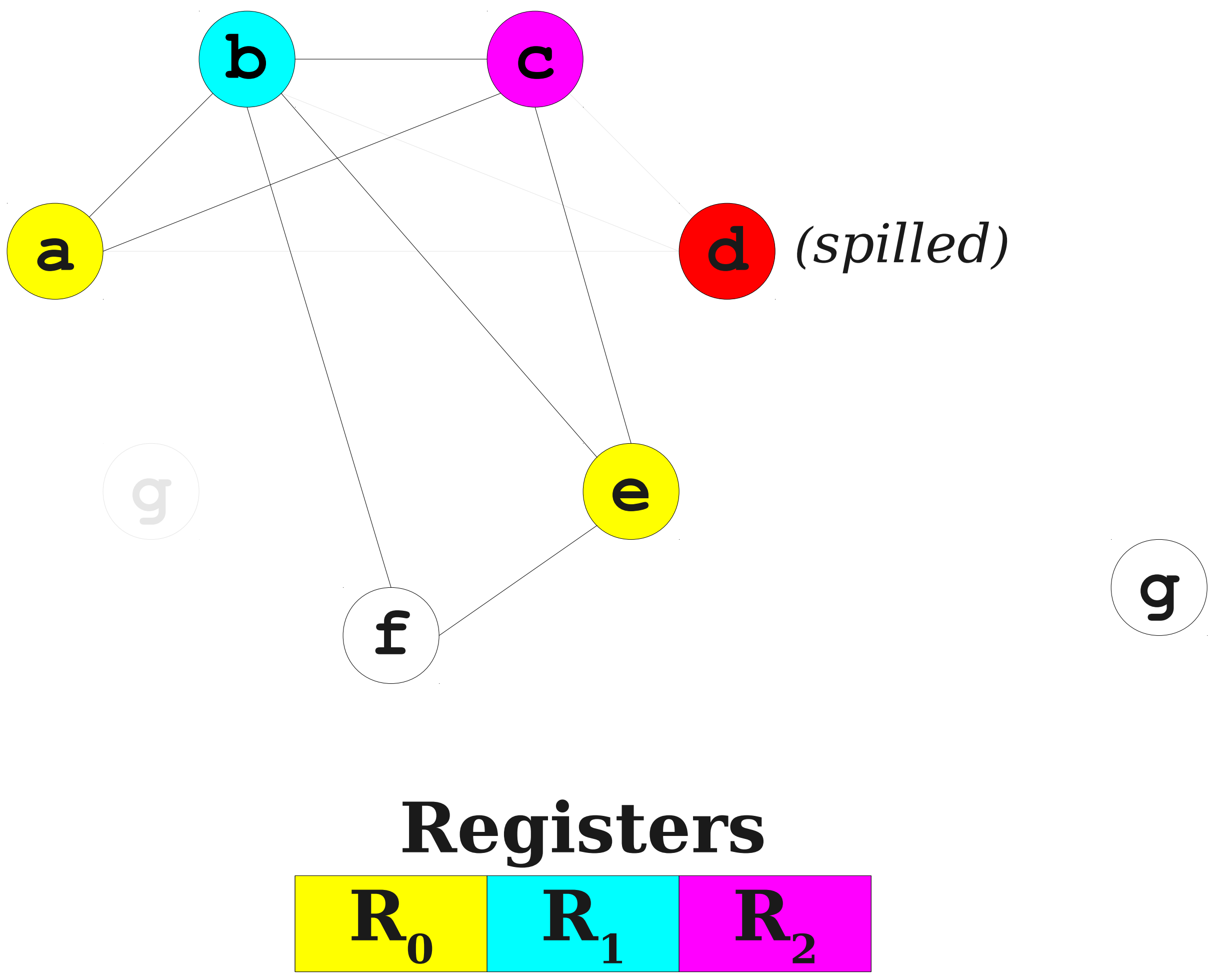

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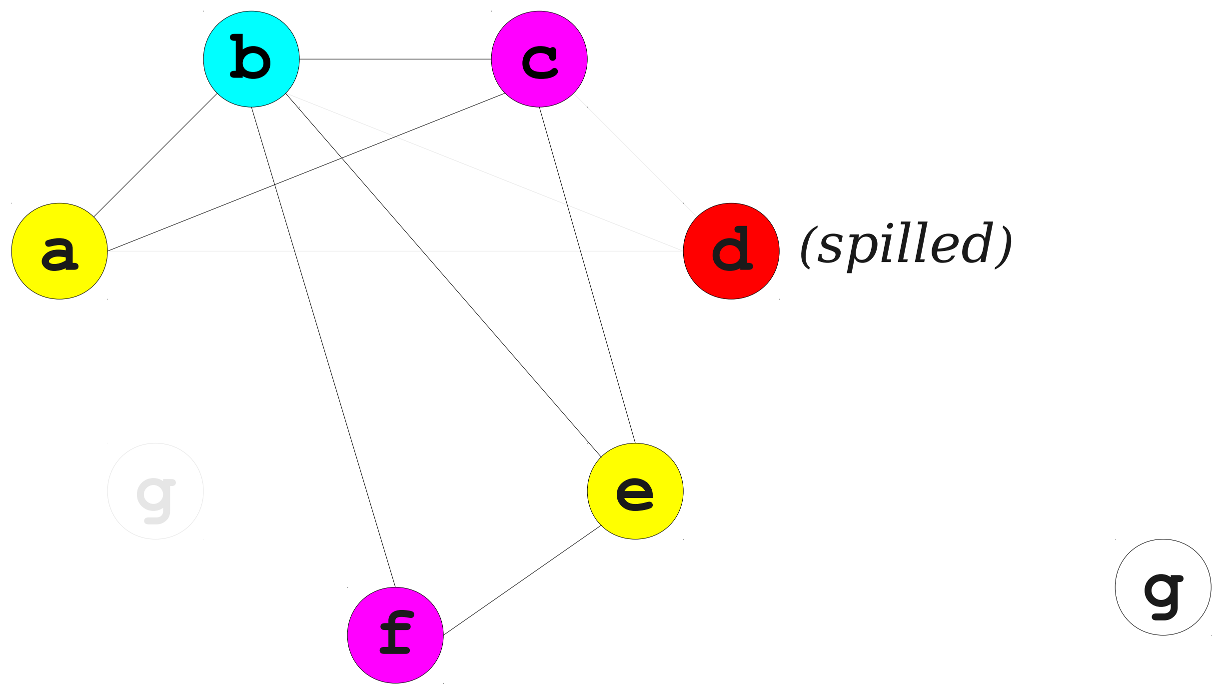
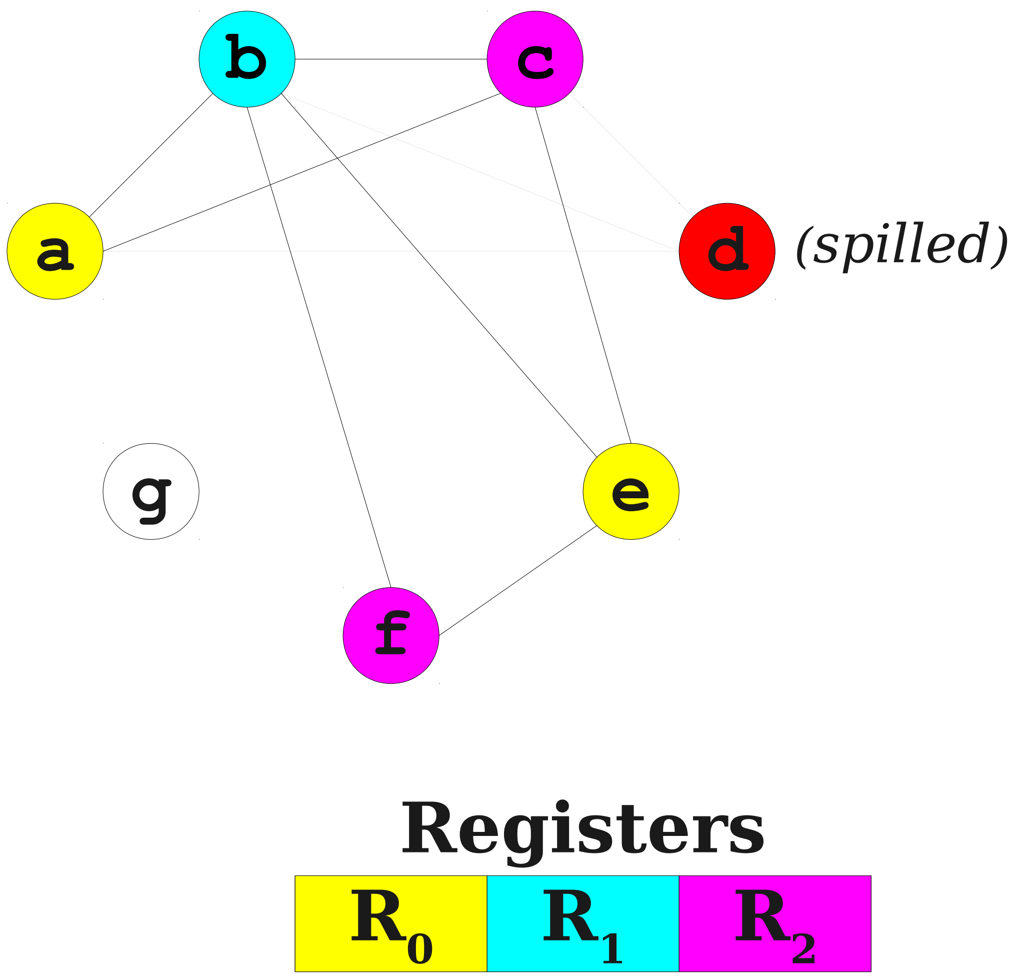


Registers

R_0	R_1	R_2

Chaitin's Algorithm Reloaded



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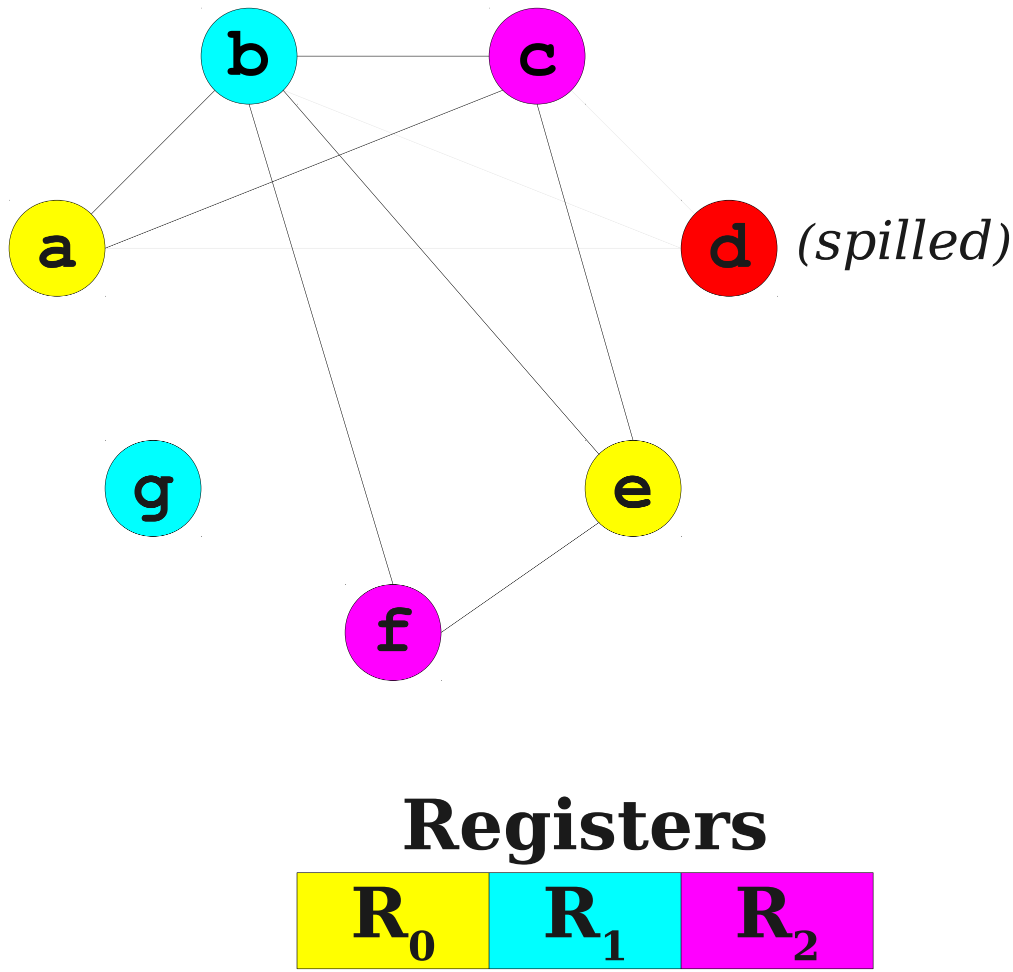
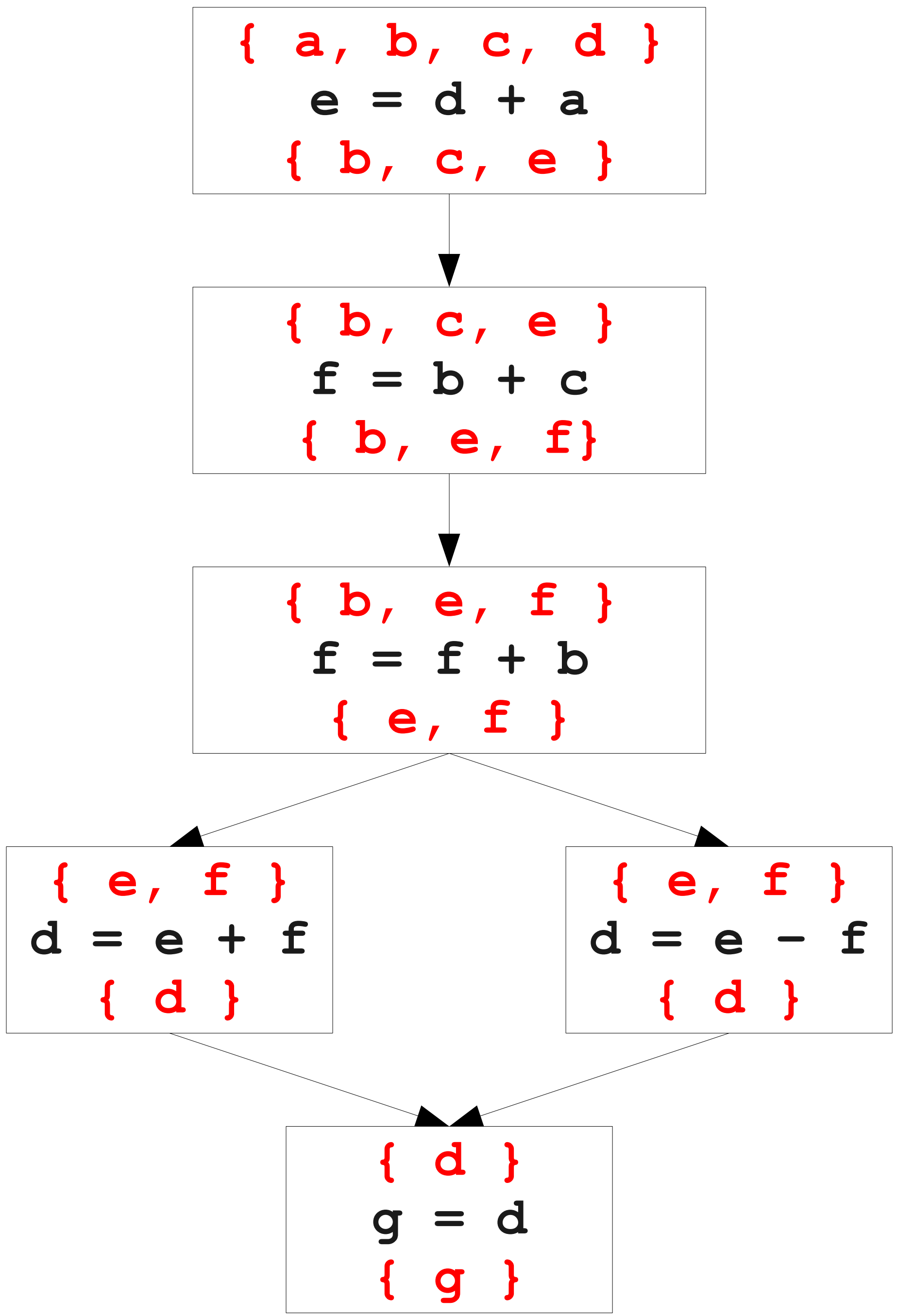
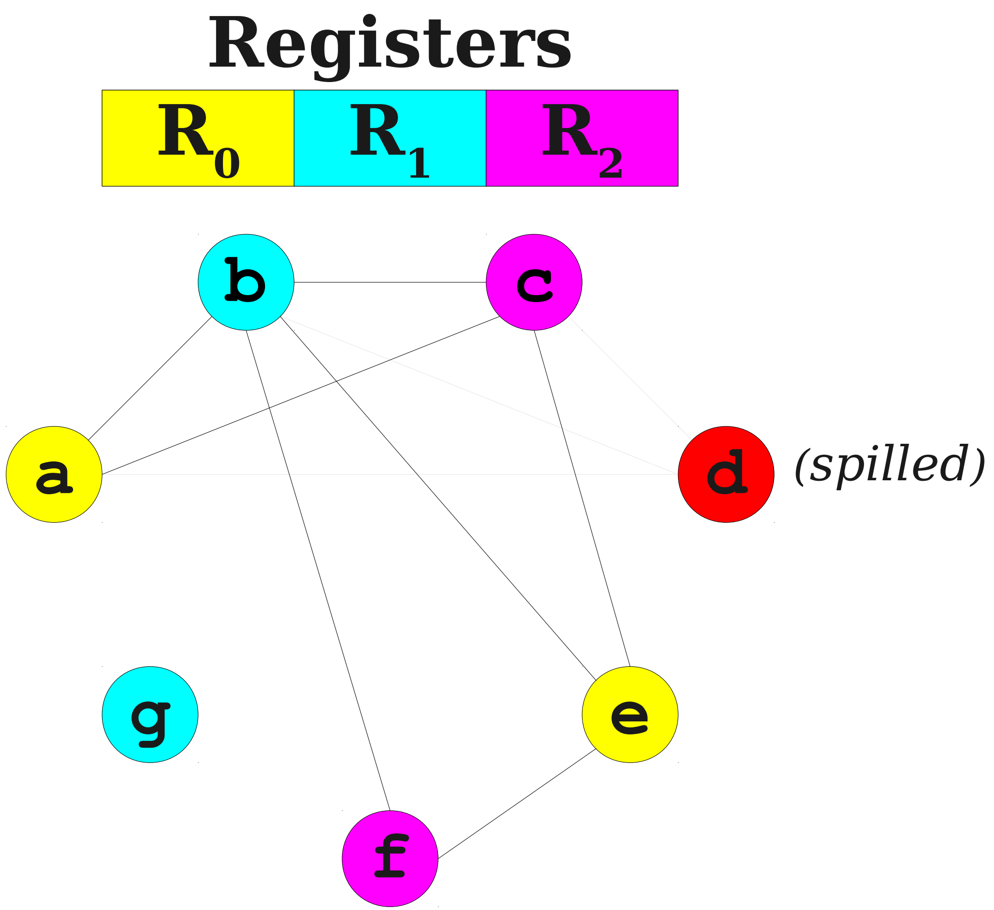
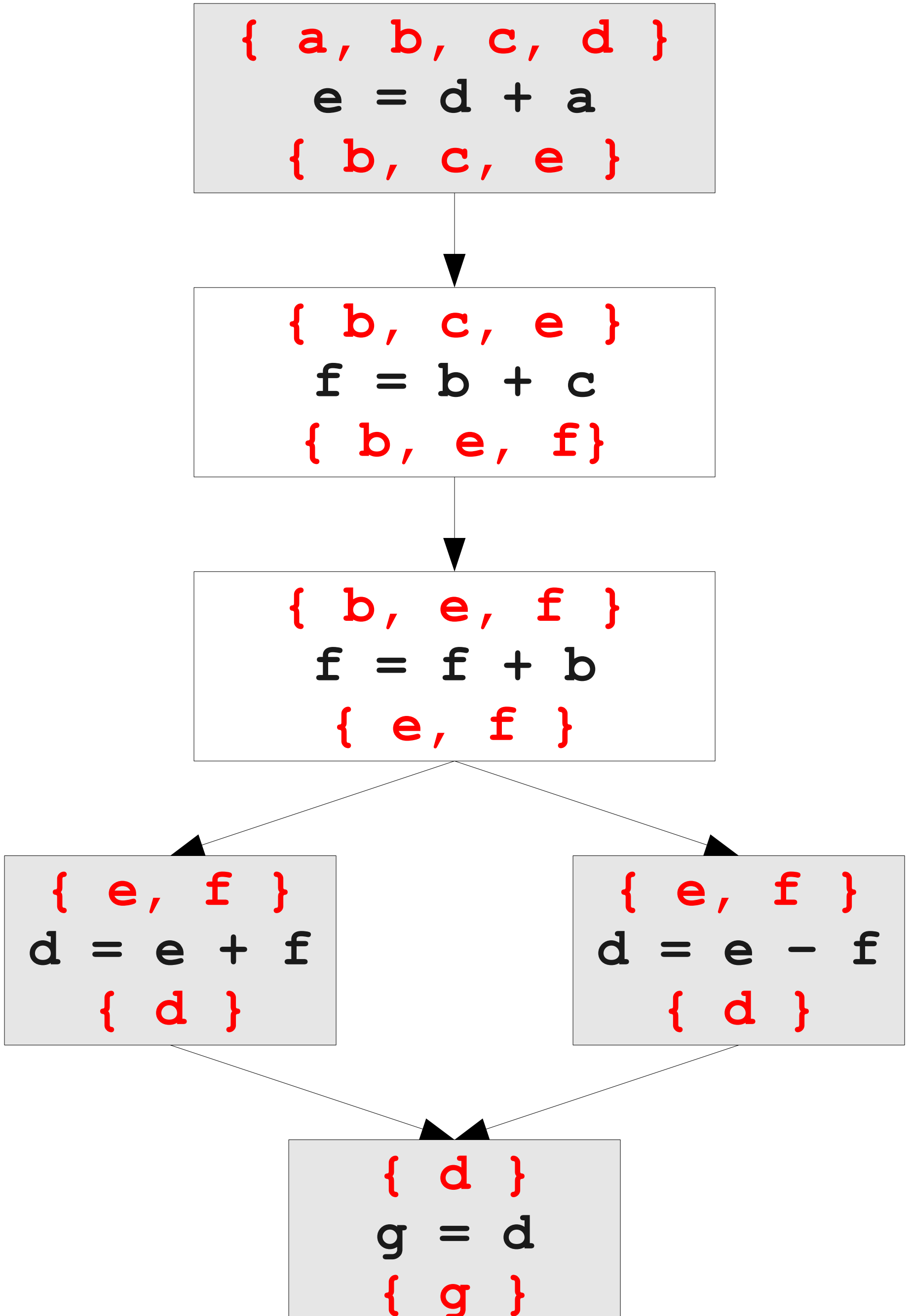


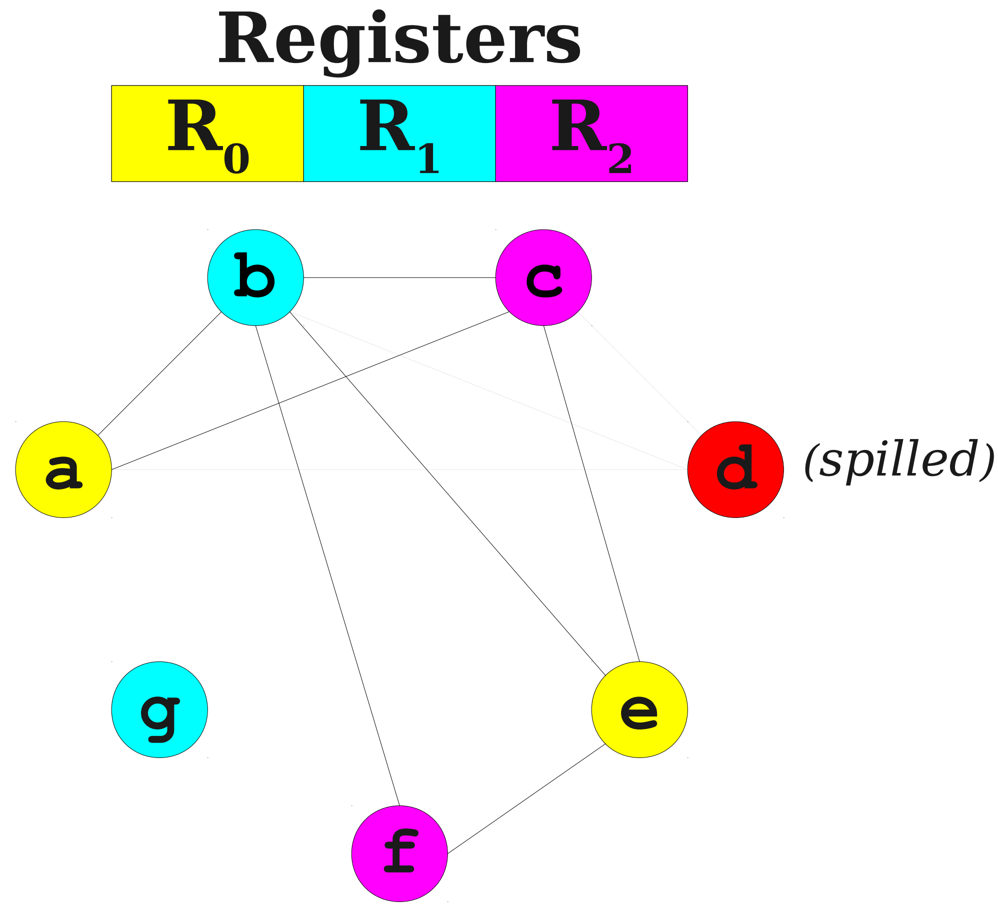
A Smarter Algorithm





A Smarter Algorithm





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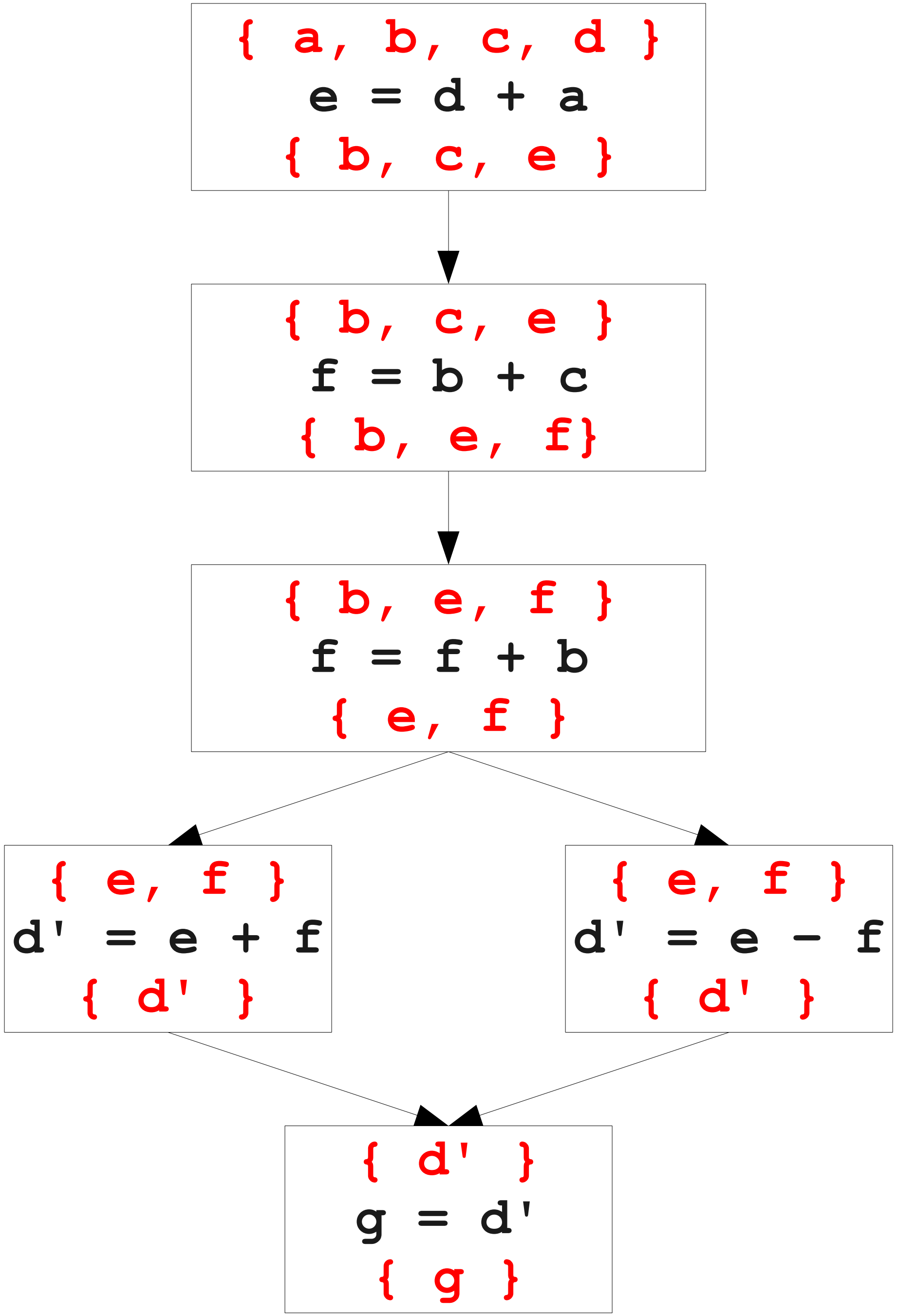
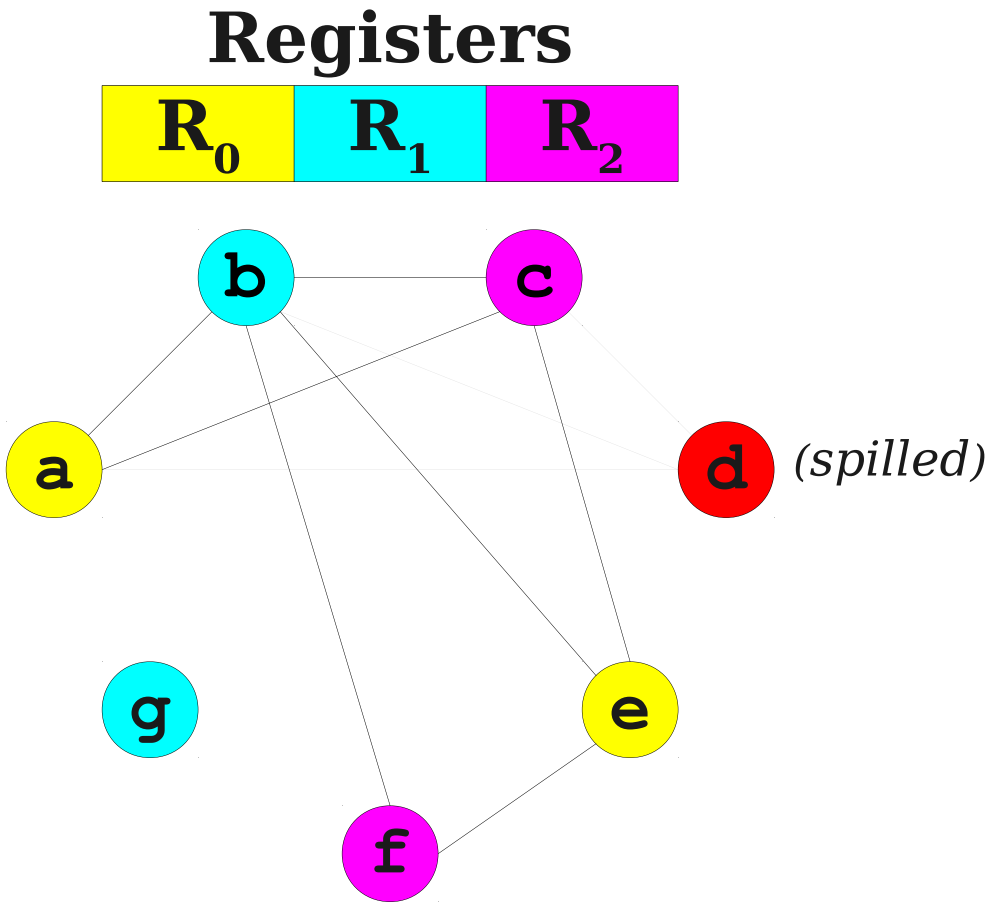
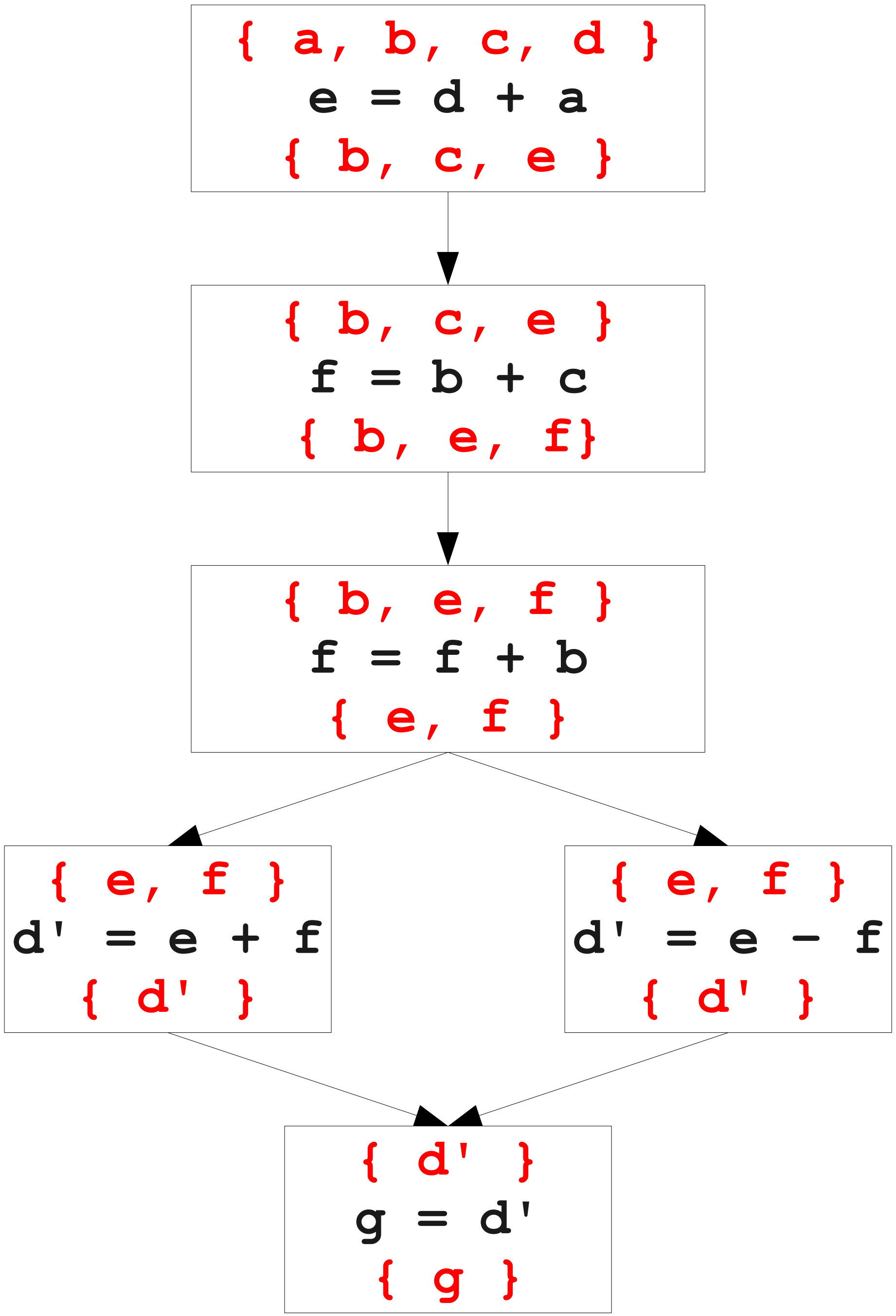
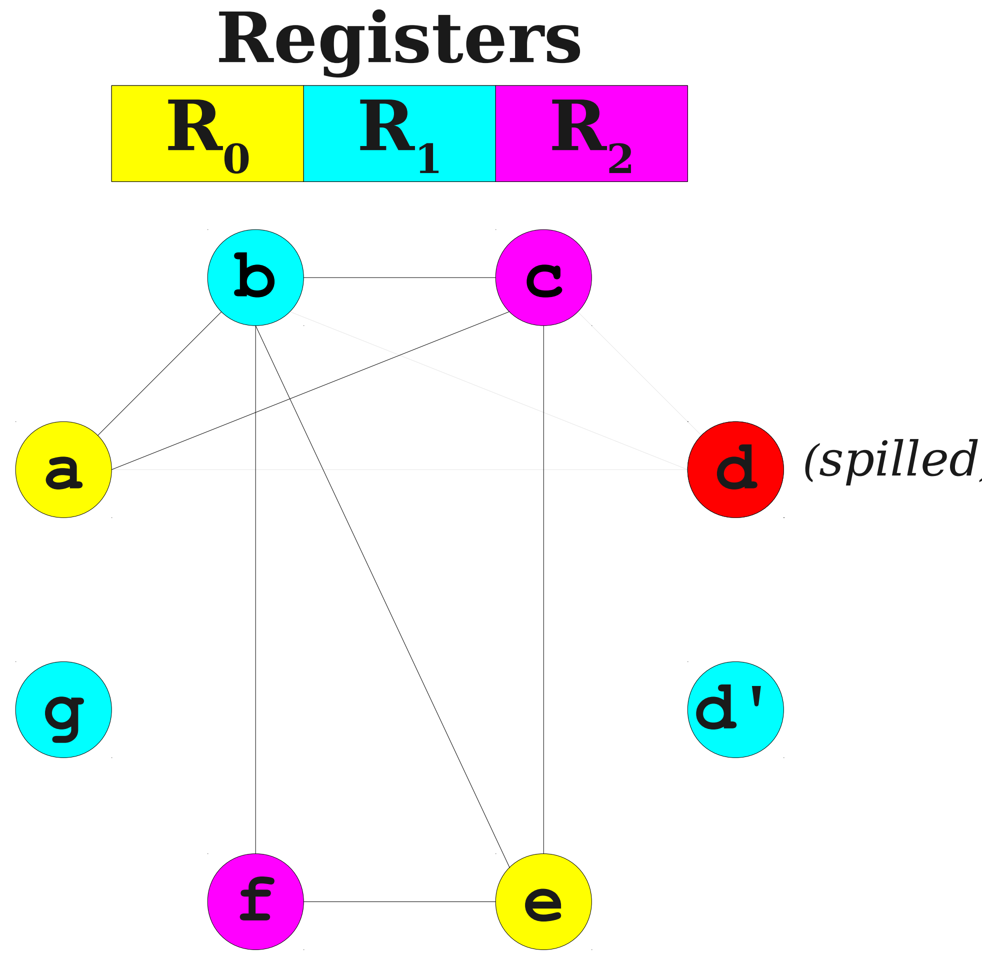




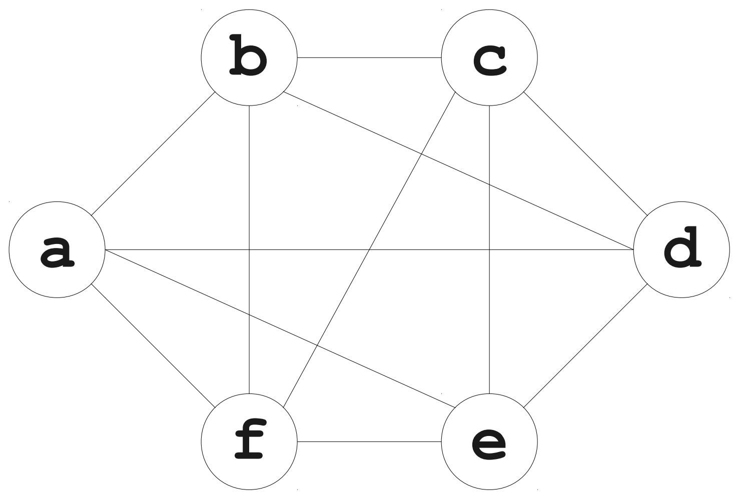
A Smarter Algorithm





Another Example

Another Example



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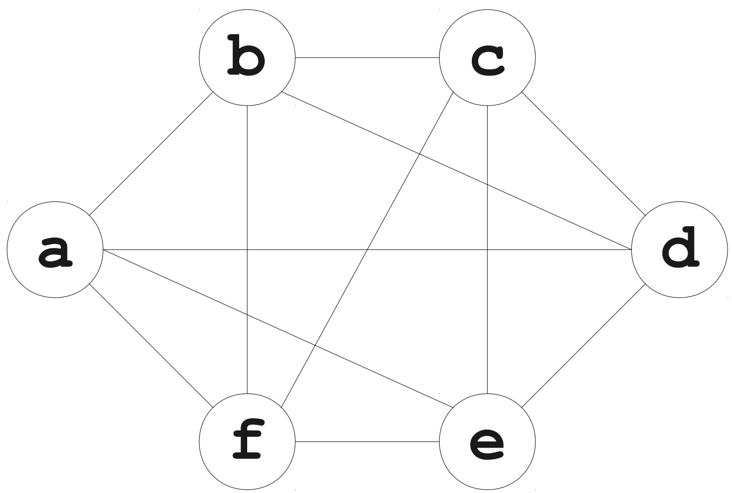
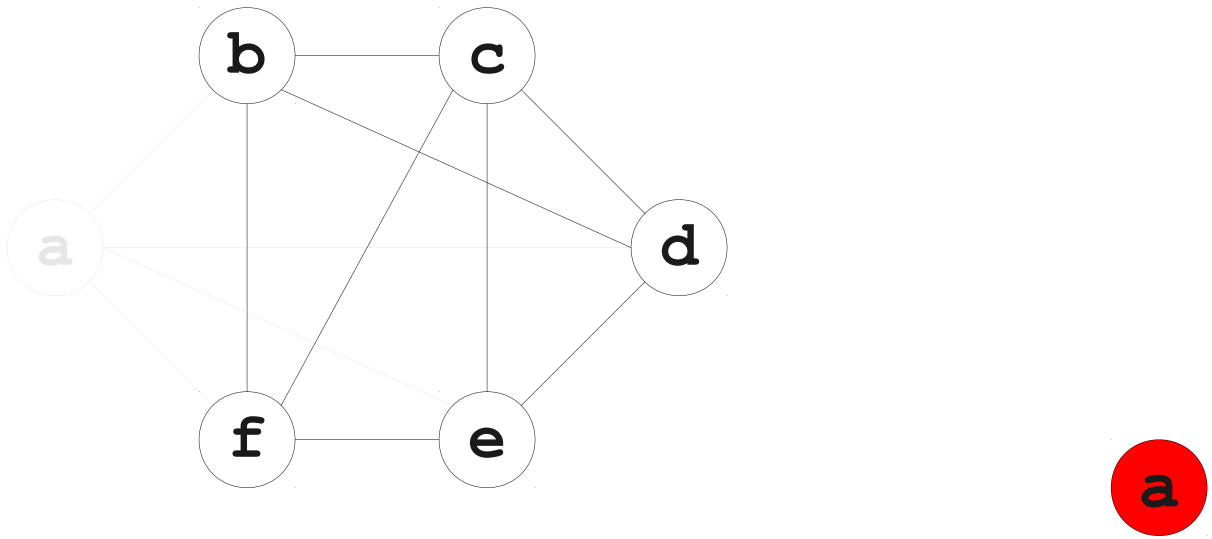


Registers

R_0	R_1	R_2

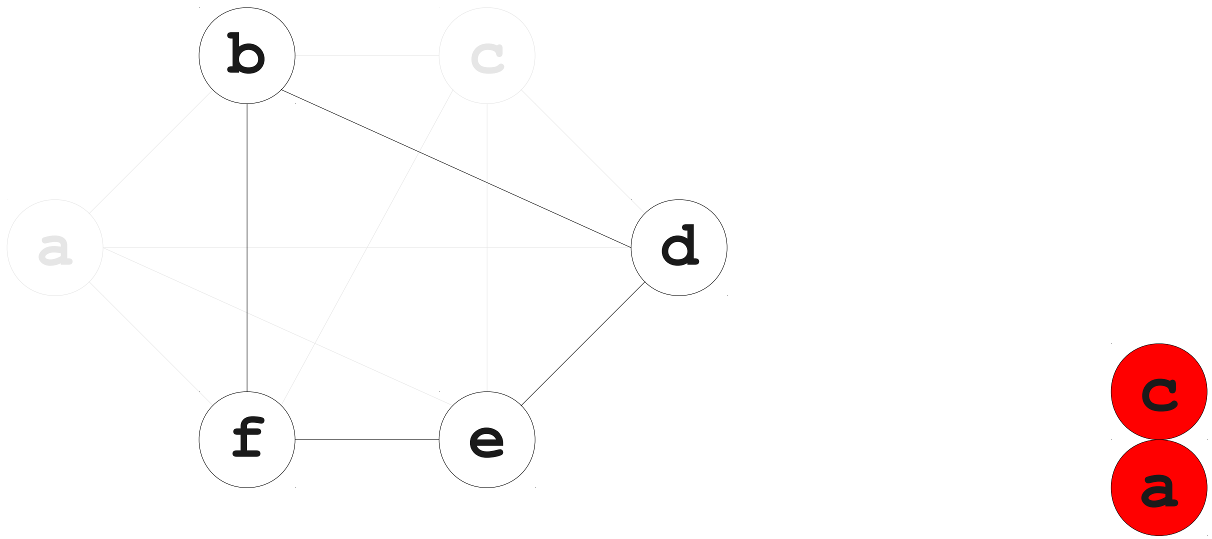
Another Example



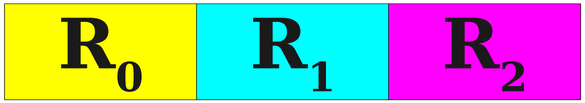
Registers

R_0	R_1	R_2

Another Example



Registers



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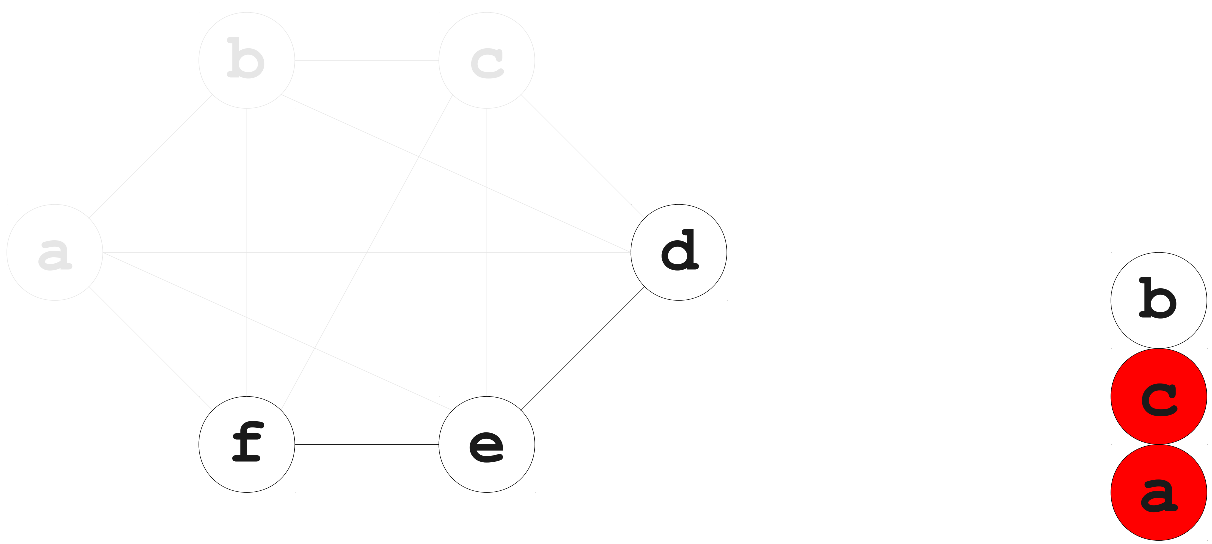

Registers

R_0	R_1	R_2

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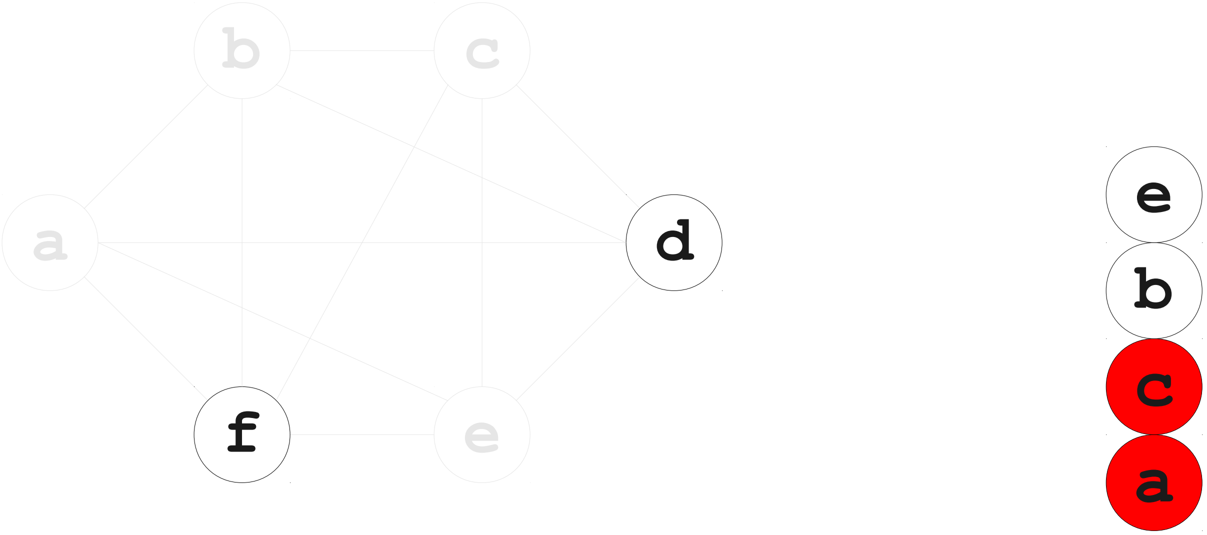
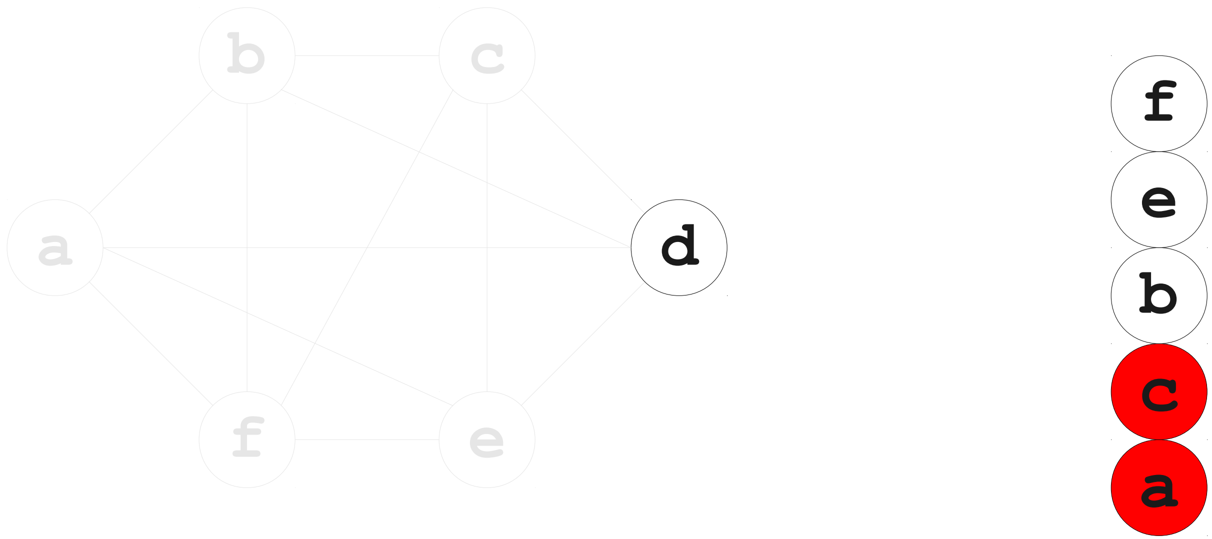


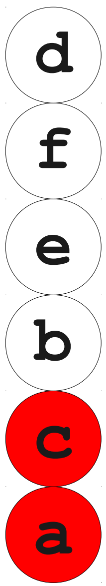
Registers

R_0	R_1	R_2

Another Example



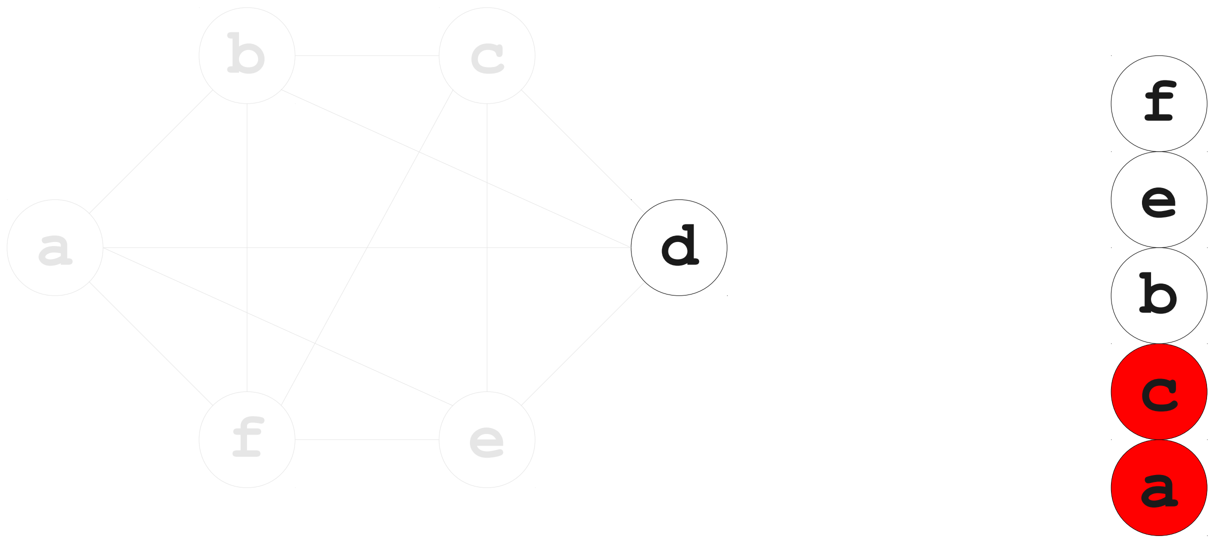
Another Example



Registers

R_0	R_1	R_2

Another Example



Registers

R_0	R_1	R_2

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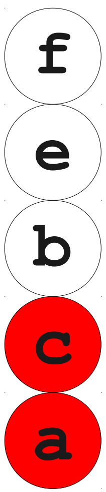

Registers

R_0	R_1	R_2

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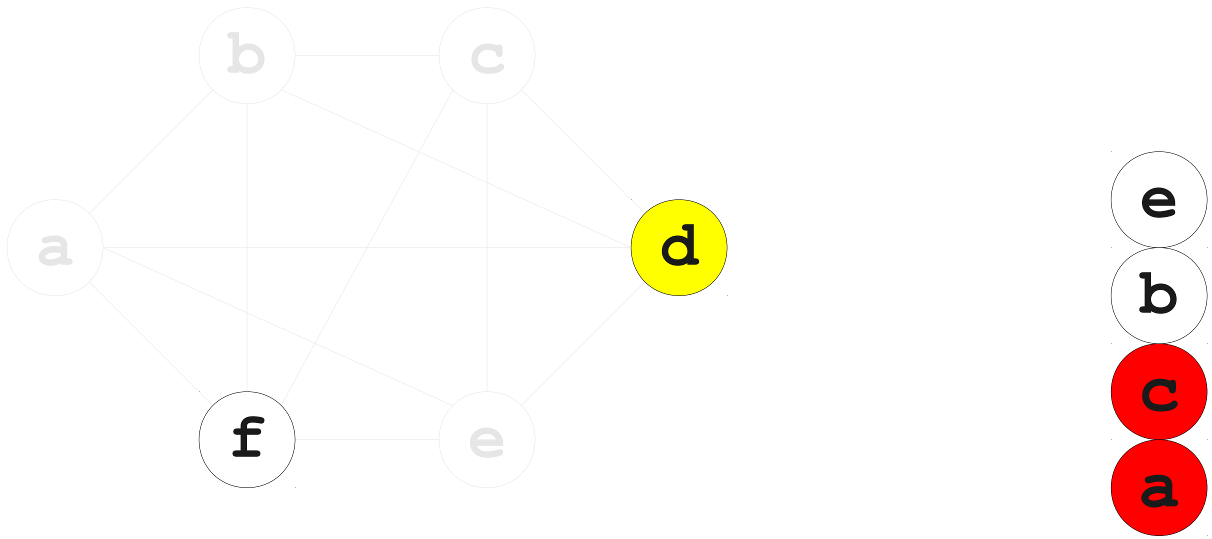

Registers

R_0	R_1	R_2

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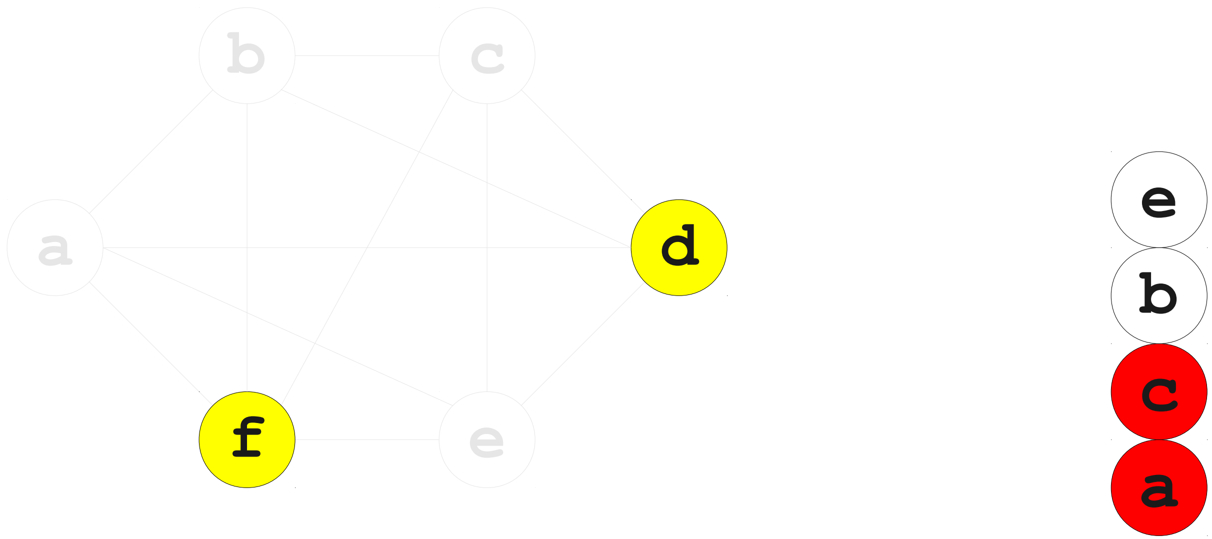
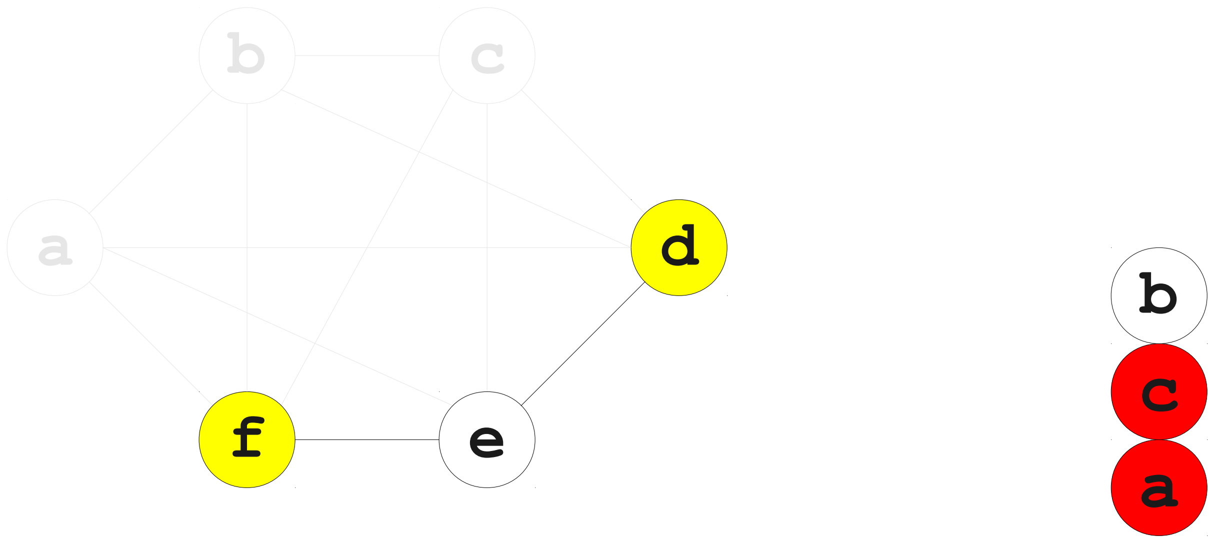


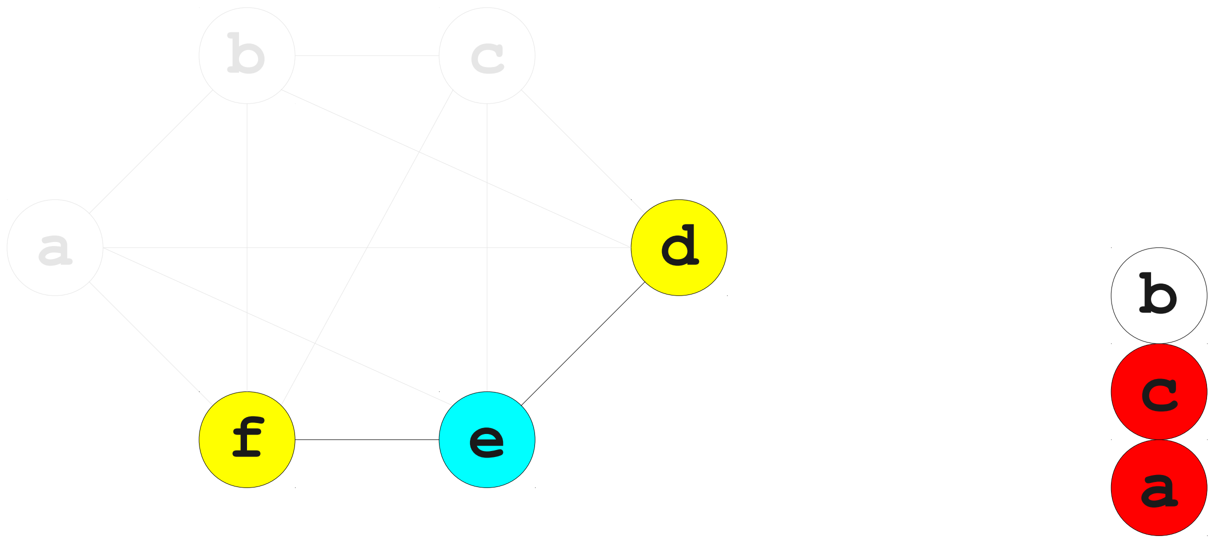
Registers

R_0	R_1	R_2

Another Example



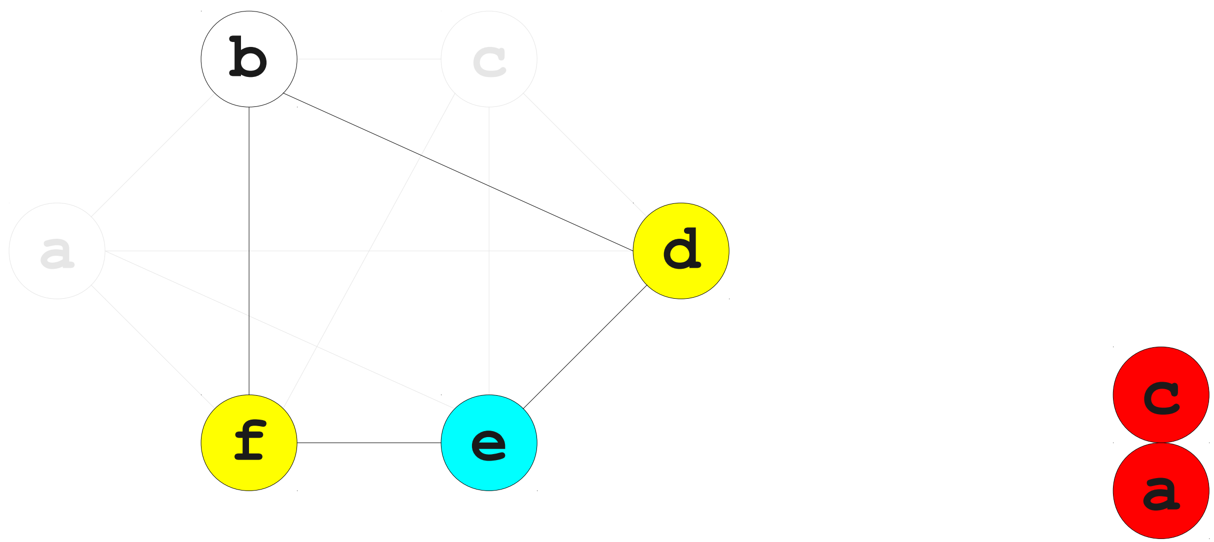
Another Example



Registers

R_0	R_1	R_2

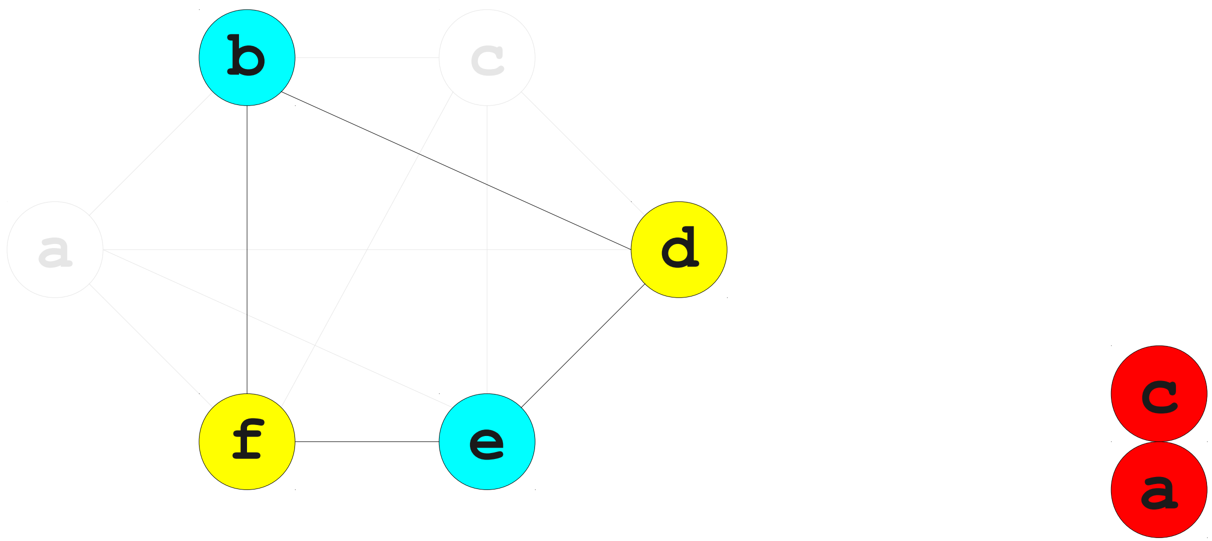
Another Example



Registers

R_0	R_1	R_2

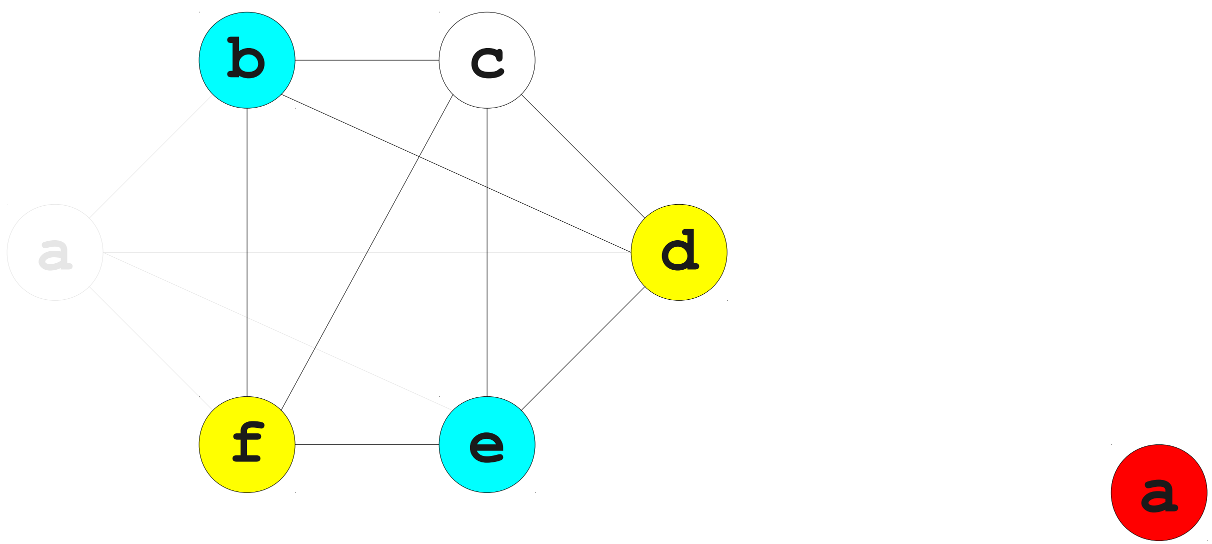
Another Example



Registers

R_0	R_1	R_2

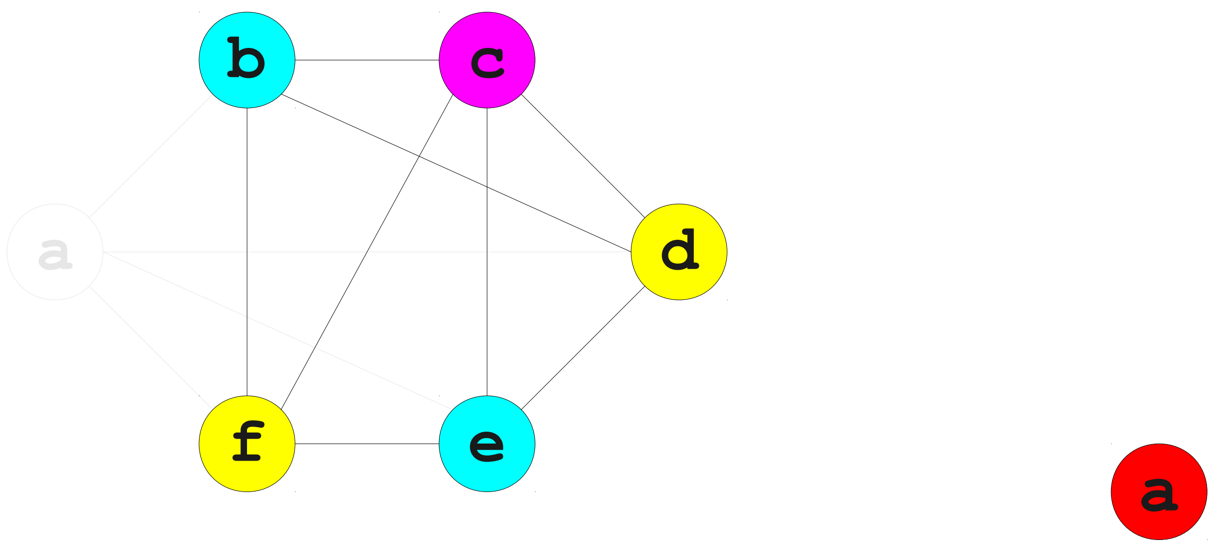
Another Example



Registers

R_0	R_1	R_2

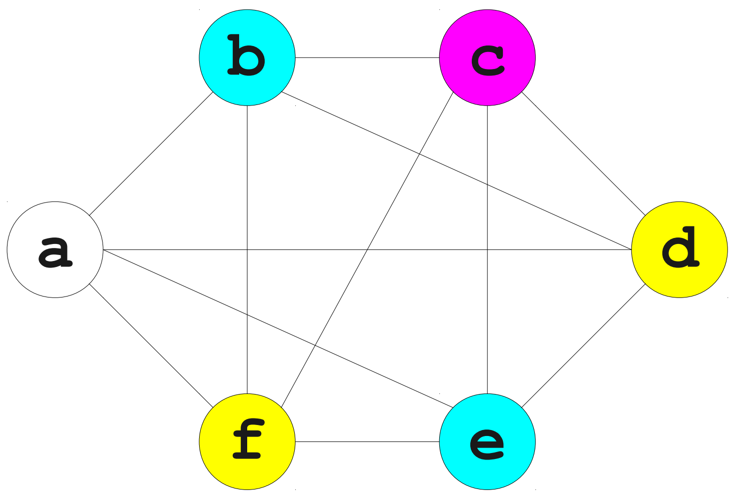
Another Example



Registers

R_0	R_1	R_2

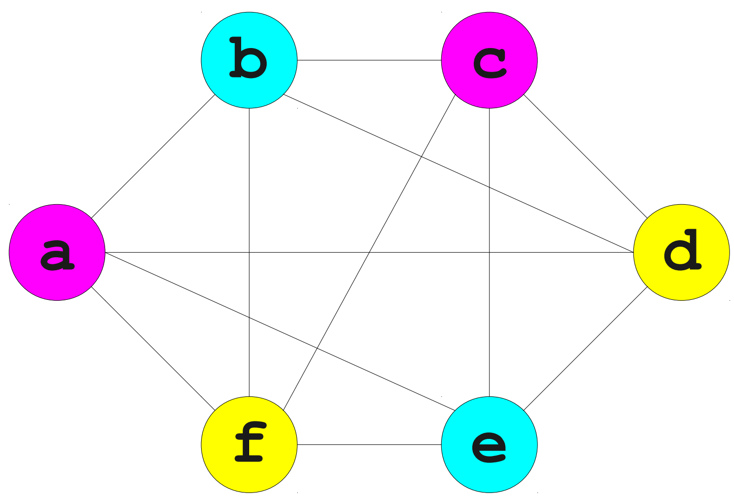
Another Example



Registers

R_0	R_1	R_2

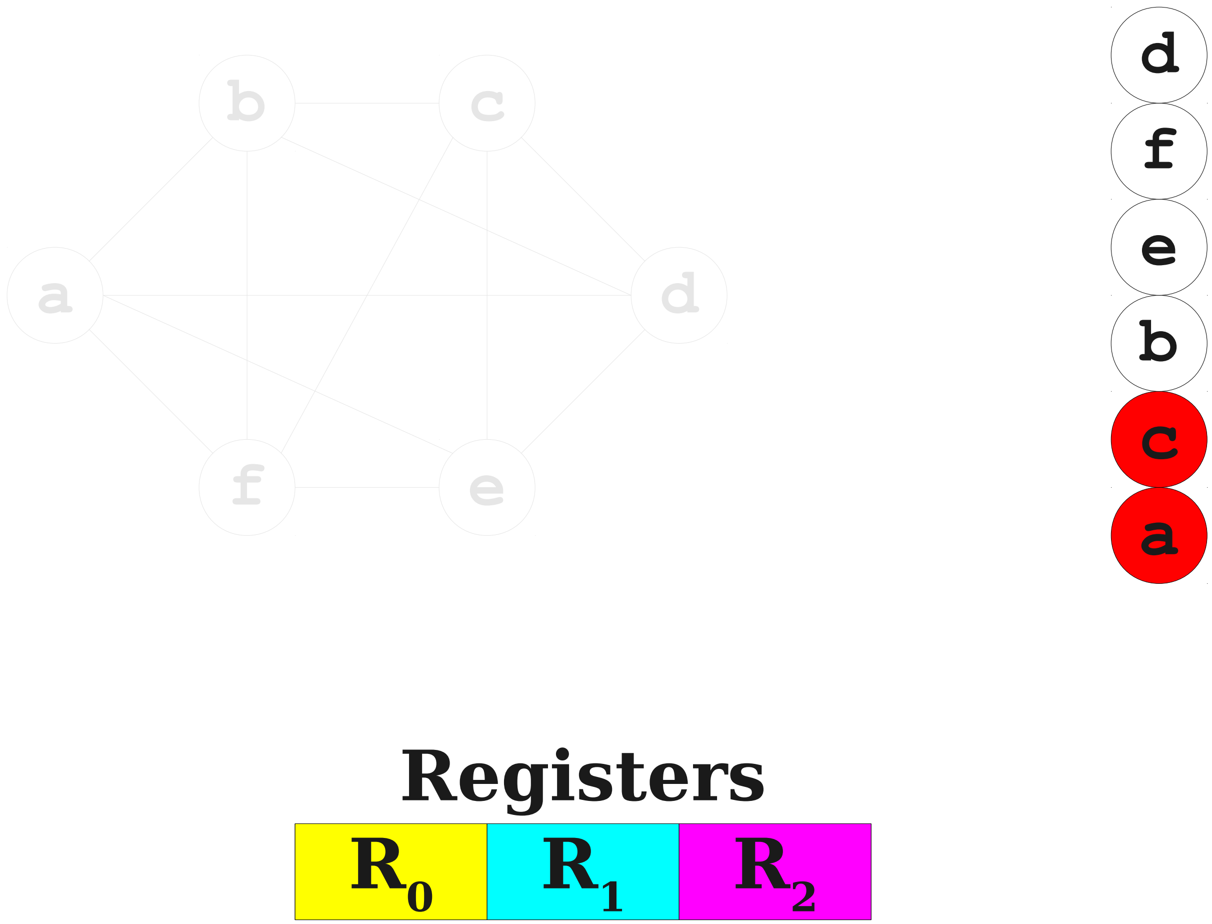
Another Example



Registers

R_0	R_1	R_2

Another Example



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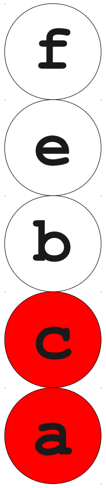
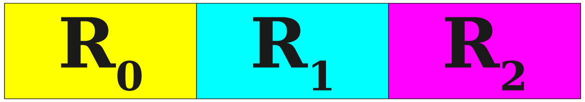
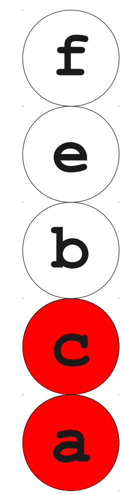


Registers



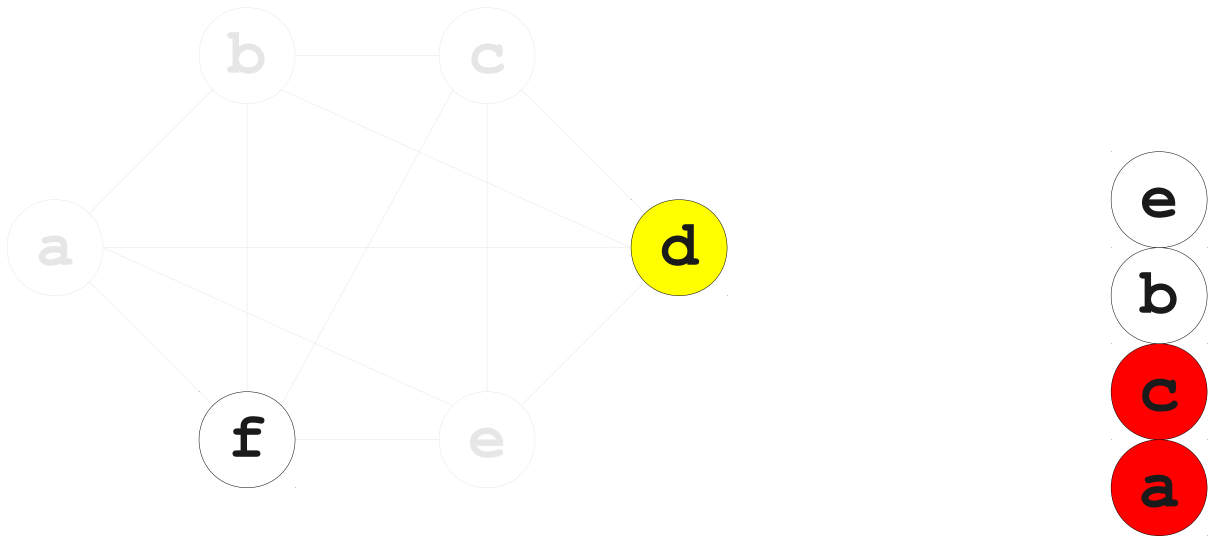
Another Example



Registers

R_0	R_1	R_2

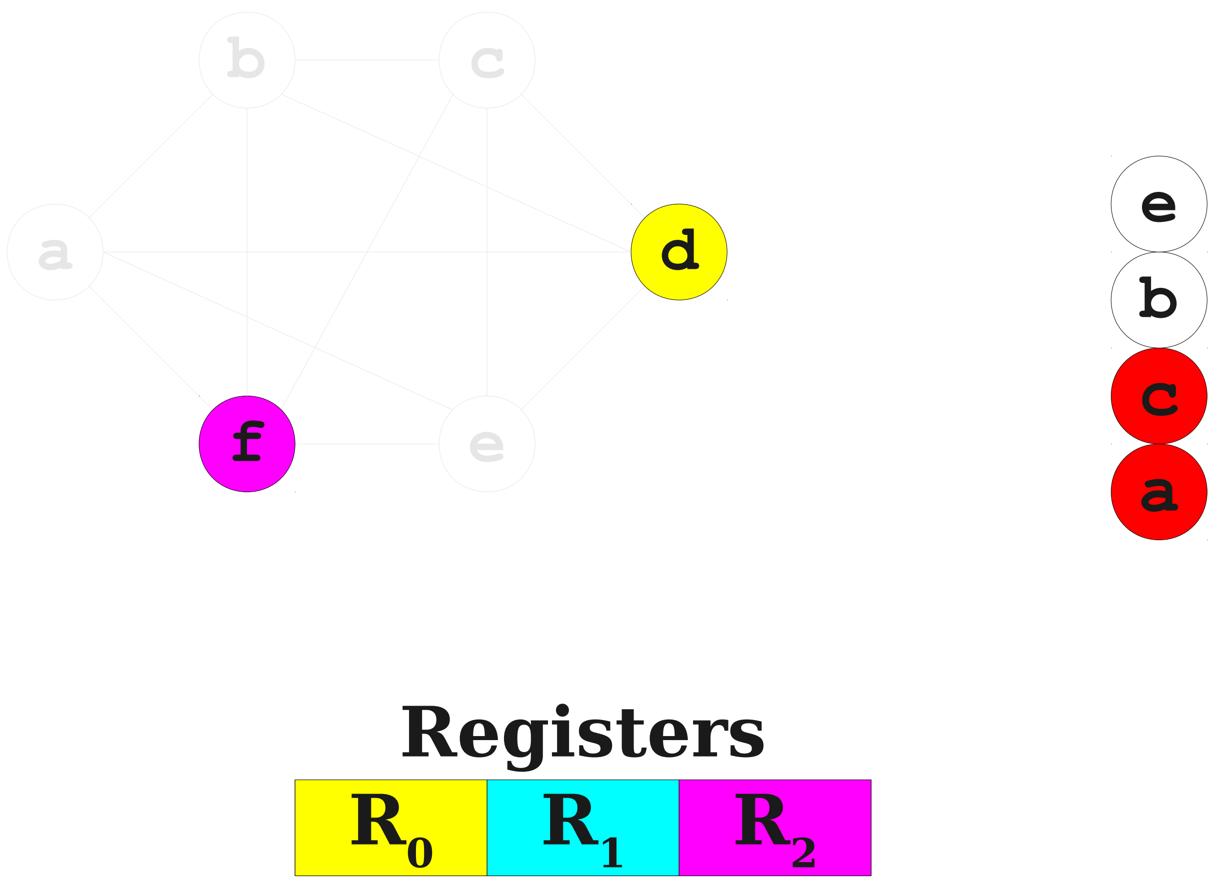
Another Example



Registers

R_0	R_1	R_2

Another Example



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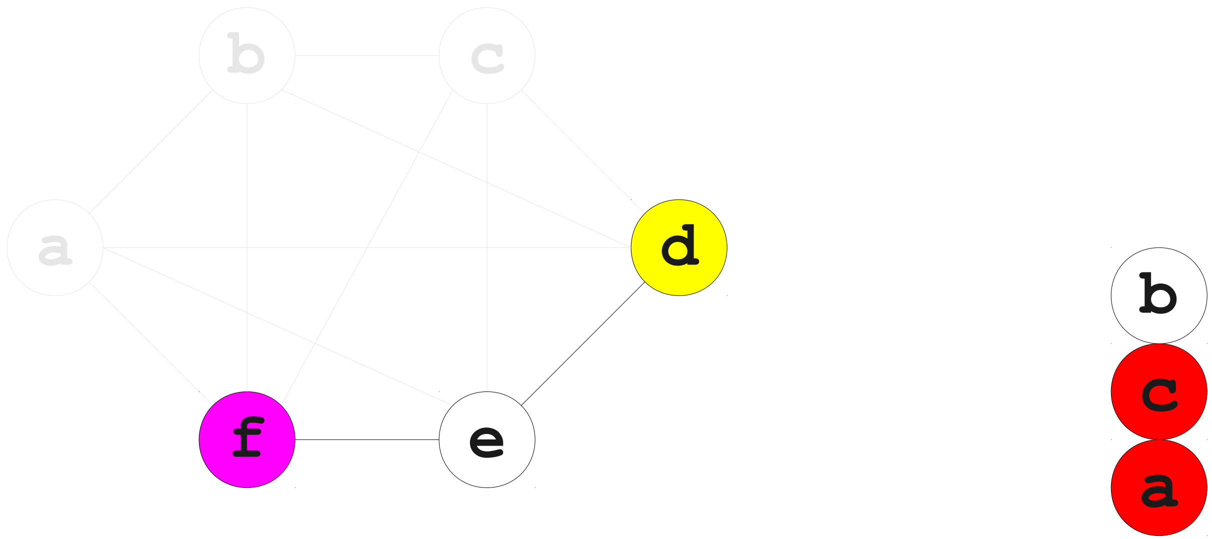
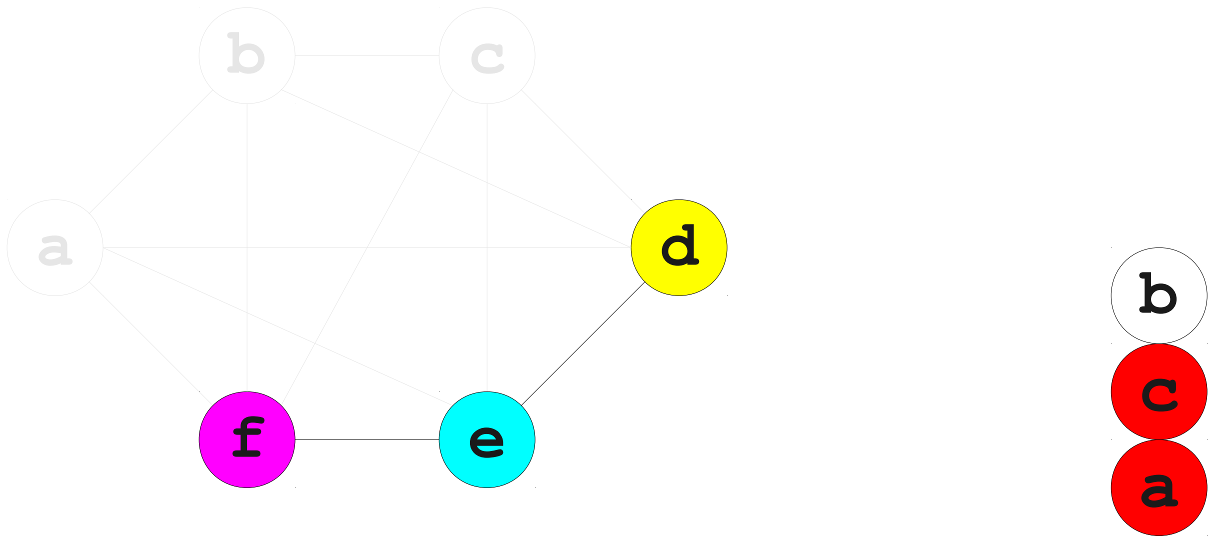


Registers

R_0	R_1	R_2

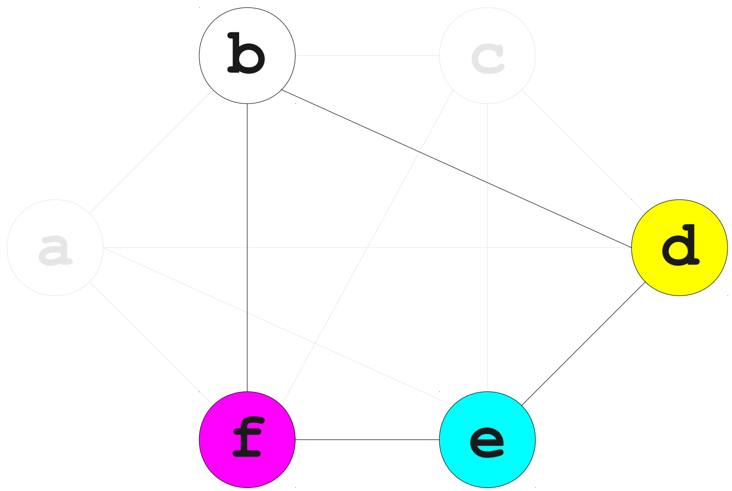
Another Example

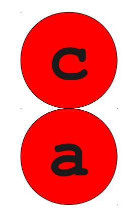


Registers

R_0	R_1	R_2

Another Example

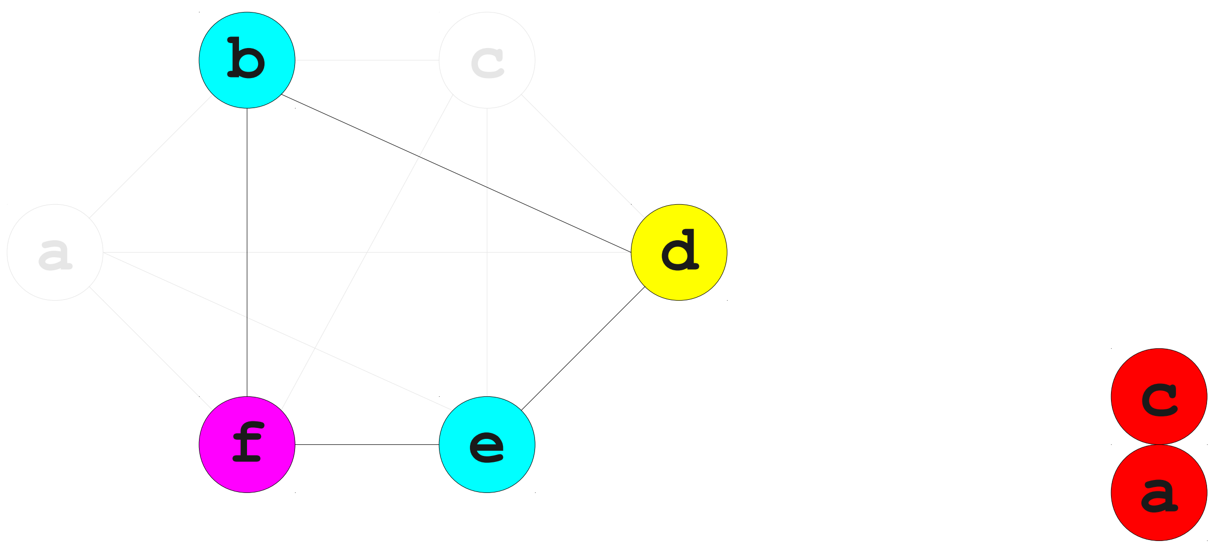




Registers

R_0	R_1	R_2

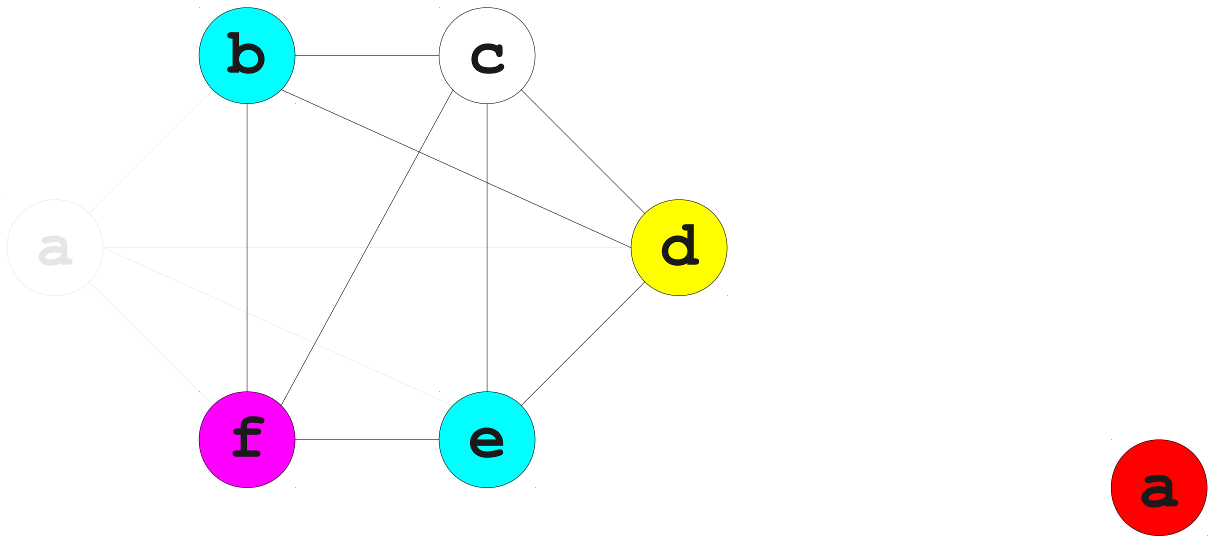
Another Example



Registers

R_0	R_1	R_2

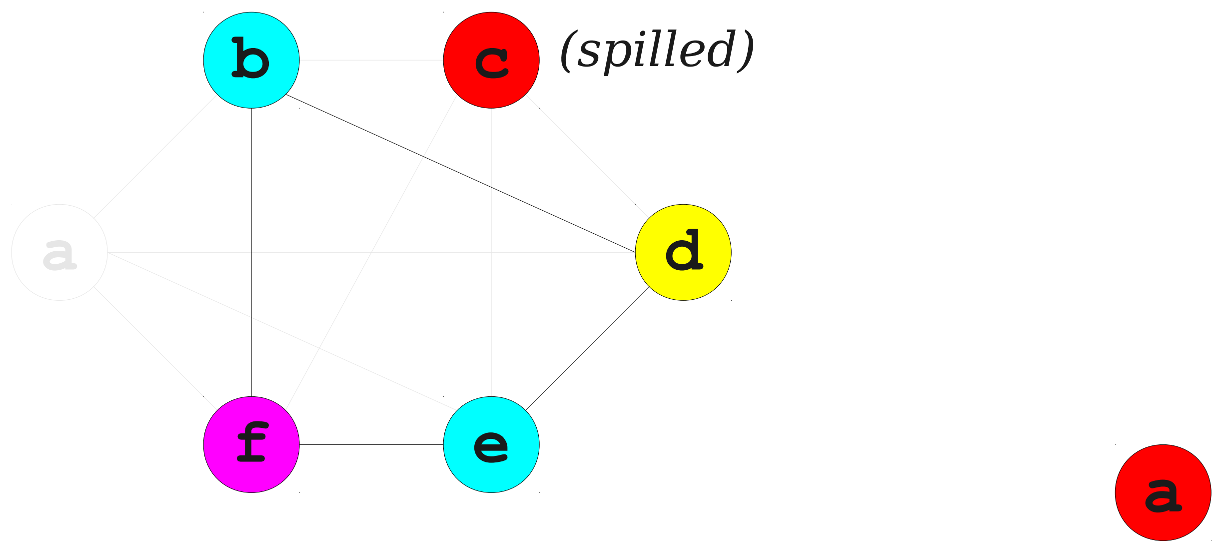
Another Example



Registers

R_0	R_1	R_2

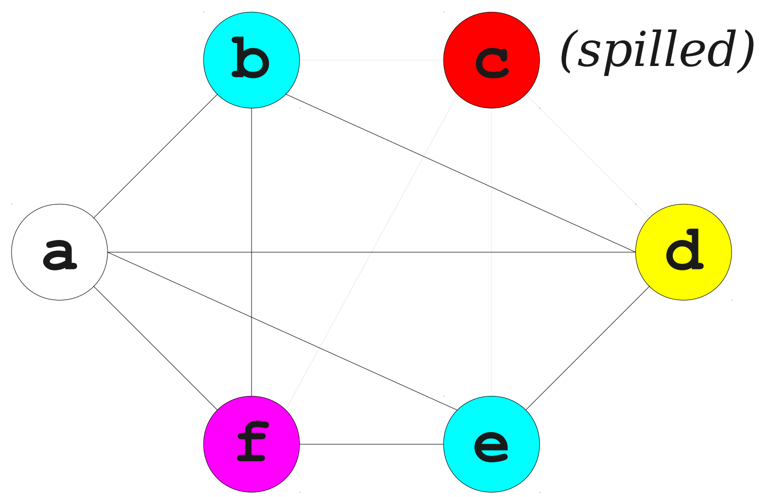
Another Example



Registers

R_0	R_1	R_2

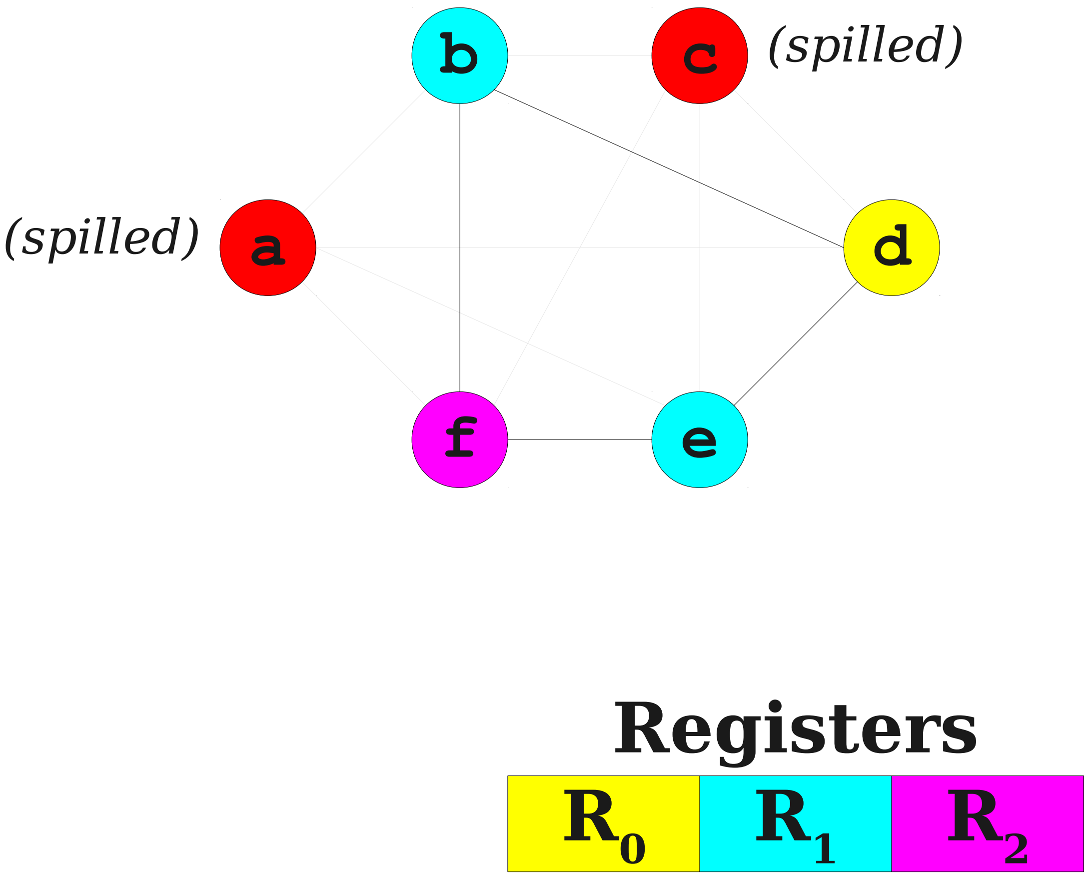
Another Example



Registers

R_0	R_1	R_2

Another Example



Chaitin's Algorithm

- Advantages:
  - For many control-flow graphs, finds an excellent assignment of variables to registers.
  - When distinguishing variables by use, produces a precise RIG.
  - Often used in production compilers like GCC.
- Disadvantages:
  - Core approach based on the NP-hard graph coloring problem.
  - Heuristic may produce pathologically worst-case assignments.

Correctness Proof Sketch

- No two variables live at some point are assigned the same register.
  - Forced by graph coloring.
- At any program point each variable is always in one location.
  - Automatic if we assign each variable one register.
  - Requires a few tricks if we separate by use case.

Improvements to the Algorithm

- Choose what to spill intelligently.
  - Use heuristics (least-commonly used, greatest improvement, etc.) to determine what to spill.
- Handle spilling intelligently.
  - When spilling a variable, recompute the RIG based on the spill and use a new coloring to find a register.

Summary of Register Allocation

- Critical step in all optimizing compilers.
- The **linear scan** algorithm uses **live intervals** to greedily assign variables to registers.
  - Often used in JIT compilers due to efficiency.
- **Chaitin's algorithm** uses the **register interference graph** (based on **live ranges**) and **graph coloring** to assign registers.
  - The basis for the technique used in GCC.