MA3K7 Week 6 Rubric

Rizwan Tariq (2107025)

1 Entry

KNOW

What I know about this problem is that we have a hat where we have 2024 papers in from numbers 1-2024 written on each paper so at least one number from 1-2024 is written on each of the papers and each paper had a unique number. So if i pick 2 numbers I then subtract the two numbers(where we subtract the smallest number from the bigger number to ensure we get a positive number) and then put the number on a piece of paper and put it in the hat. What we know is that we will after this will have 2 of the same number in the hat. And then we rinse and repeat until we are left with one paper.

WANT

This problem seems very interesting and at face value it seems like a probability based problem. Since we have lots of cases it would be absurd to try to brute force this due to the outstanding number of cases that are present. My first thought is to size down the problem to maybe multiples of 1024. A few questions pop up when I saw the problem:

- Does the order of the terms picked out matter?
- If so, is there multiple possible solutions for the final paper?
- And to add, would there be a certain pattern if there was multiple papers
- How long will it take to get to one paper?
- There is also a potential of multiple pairs that create the same subtraction result. So does this affect our end result?

INTRODUCE

In terms of how to speed up my attack phase, I am thinking of making a python code which would then imitate this game. HOWEVER, I would like to know if there is a trend if I tried this at multiple of 2024 and then compare it to 2024 and see if there is a theme with the end paper. However even though sizing down the problem would reduce the time for this game to finish, it would still take a long time to complete all the different multiples. Maybe I could do some of the potential multiples of 2024 and then create a table to compare the difference in the last piece of paper. The potential trends I can think of:

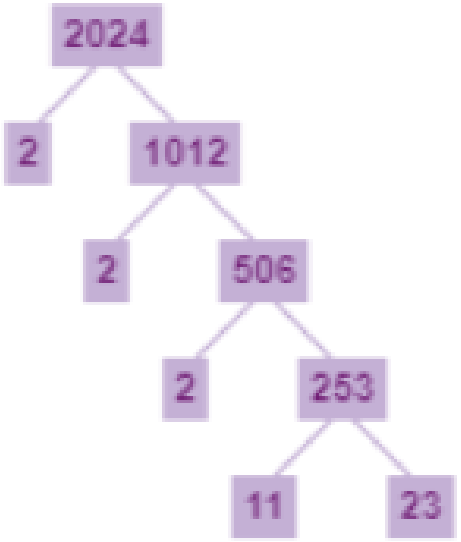
- All could be prime
- All could be a multiple of a certain number
- All could be the same number

and etc.

2 Attack

AHA!

My first plan of action is to start with laying out the multiples of 2024 and try to play this game with some multiples of 2024. But before this I may try some simple numbers like 6 to begin with.



CONJECTURING

I have a thought about one of the questions i asked myself in the Entry phase. the question was ' Does the order of the terms picked out matter?'.

JUSTIFYING

The problem's mathematical characteristic explains that, regardless of which two numbers are selected at each stage, the ultimate result—the value of the last piece of paper left in the hat—remains the same. This characteristic guarantees that, irrespective of the sequence in which the numbers are subtracted, the procedure produces the same result. This is due to the accumulation of differences.You deduct two numbers at each stage of the procedure, then add the difference back into the hat. The net change in the overall value of the numbers in the hat is represented by this difference.

Regardless of the sequence in which the numbers are selected, the total difference added to the hat at each stage will remain the same because the order of subtraction has no bearing on the outcome.

STUCK!

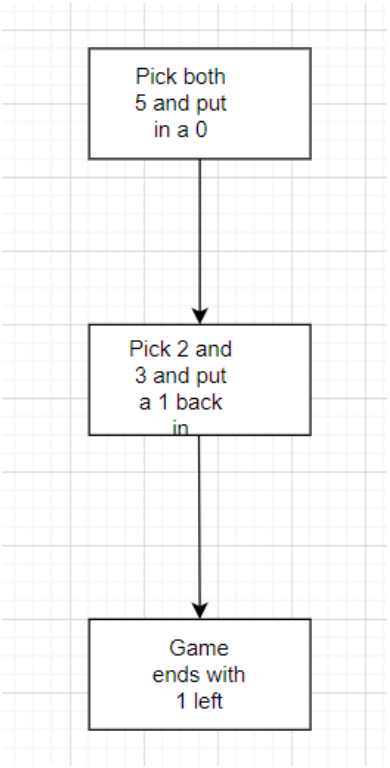
However, when i played the game for myself I was later proved wrong. Below I will illustrate how:

Suppose n =6 and we have numbers 1 to 6 placed in the hat.

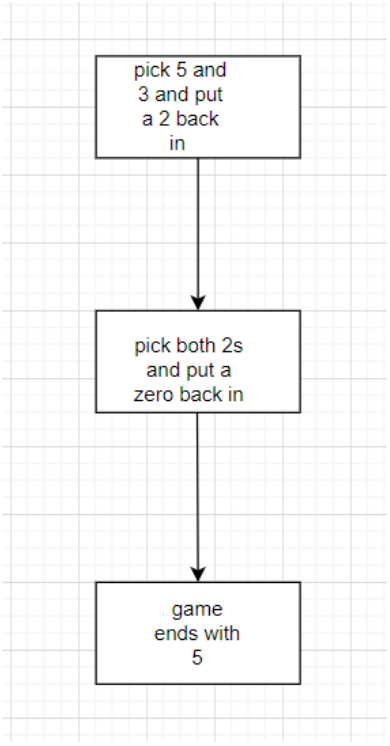
- Suppose I pick 6 and 1 first then the difference of 5 is placed in the hat and the other 2 are discarded.
- Then I pick 4 and 2 which leads a difference of 2 placed in the hat and the other 2 discarded
- We now have two 5s and a 2 and 3.

The game now has two possible paths for it to go.

Path 1:

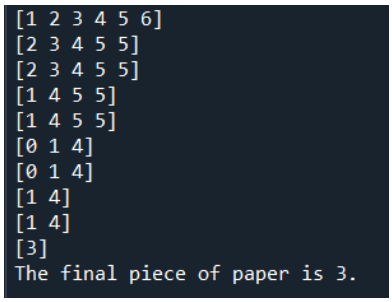


Path 2:



STUCK! As we can see we have 2 different results for the end paper which causes an issue.

I created a script with python to play this game for certain numbers. For example, even and odd numbers as above I got 2 different results which corresponded to odd numbers for n = 6. Running my code further I can also obtain the number 3 as the end number as shown below.



CONJECTURING The common trend is that all these numbers are odd numbers for this number 6. I initially thought that even numbers yield odd results but there are counter examples for that case. Which, with further thought, I created a conjecture which I will try to prove below:

Conjecture:

CONJECTURING **The final number will be odd if n = 1 or 2 mod 4 and the the number will be even if n = 0 or 3 mod 4**

We shall demonstrate that if n is the number of pieces of paper in the hat, then the JUSTIFYING number that results from n = 1 or 2 mod 4 will be odd, and if n = 0 or 3 mod 4 will be even.

JUSTIFYING First, we observe that the result is 0, which is even, if n = 1. The two possible final numbers, 1 and 2, are both odd, if n = 2. This validates the root case.

Let us now assume that for n, the assertion is true. We will demonstrate that for n+4, it is also true.

JUSTIFYING When there are n pieces of paper in the hat, let k be the last number. Consequently, k is odd by our induction hypothesis if n = 1 or 2 mod 4, as well as even if n = 0 or 3 mod 4. The numbers that can be drawn when we add four more pieces of paper to the hat are k, k+1, k+2, k+3, and k+4. The new final number is odd if k is odd and even if k is even if we draw k and k+1. The difference is even. The new final number is odd if k is odd and even if k is even if we draw k and k+2. The difference is even.

If we draw k and k+3, the new final number is even if k is even, and odd if k is odd. The difference is odd. The new final number is odd if k is odd and even if k is even if we JUSTIFYING draw k+1 and k+2. The difference is odd. The new final number is odd if k is odd and even if k is even if we draw k+2 and k+3. The difference is even. The new final number is odd if k is odd and even if k is even if we draw k+3 and k+4. The difference is odd.

Therefore, the new final number is odd if n = 1 or 2 mod 4 and even if n = 0 or 3 mod 4, which completes the induction step.

To bring this back to the game:

we know that 2024 is 0 mod 4 so by my above conjecture we know that the final JUSTIFYING paper must be even. Using my code i created 50 outputs of potential outputs of final papers. Below holds the first 20:

```
The final piece of paper is 150.
The final piece of paper is 188.
The final piece of paper is 12.
The final piece of paper is 344.
The final piece of paper is 462.
The final piece of paper is 172.
The final piece of paper is 34.
The final piece of paper is 664.
The final piece of paper is 576.
The final piece of paper is 14.
The final piece of paper is 666.
The final piece of paper is 2.
The final piece of paper is 66.
The final piece of paper is 368.
The final piece of paper is 22.
The final piece of paper is 122.
The final piece of paper is 214.
The final piece of paper is 64.
The final piece of paper is 180.
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This further proves the conjecture I stated.

3 Review

REFLECT This problem required a lot of thinking and made me brainstorm my ideas during the Review phase on how to correctly approach this problem.

CHECK I started out with trying some small numbers like n = 6 and seeing if there was any trend. I was first proven wrong ,that regardless of what you start off with you'll get the same number, so i had to work off the fact that my assumption was incorrect which led me to modify my approach.

EXTEND After this I tried to find different final numbers to find a trend which led me to make a conjecture and prove it with set the tone for the rest of this composition.

Overall, i felt like that this problem was the hardest one I've done and required a
REFLECT lot of critical thinking. the use of python I felt was also crucial as for higher numbers
EXTEND the game would've been impossible to get multiple final numbers and be able to make a coherent comparison.

Supplementary material

Below is attached my GitHub repository where my code resides:

`https://github.com/rizwan3254/PS-with-yth`