MA3K7 Week 8 Rubric

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1 Entry

KNOW

If we look and analyse this problem we can break it down. The problem states that we are on some infinite long line of integers and we posses a coin which is fair which also contains 1 on the one of the sides and the number 2 is on the other side. We progress on the line but he number that we obtain by flipping the coin. For example if we flip 1 we move one step across. The problem we are gonna analyse is the probability of landing on the number 25 in some point in the 24 flips we perform.

INTRODUCE

This problem seems quite interesting as I initially read the problem as the probability of landing on 25 in 24 flips which seemed pretty straight forward as you would just have to roll 1 all the time however landing on 25 in the midst of the 224 rolls seems like a more fun problem to tack. A few questions are raised when considering this:

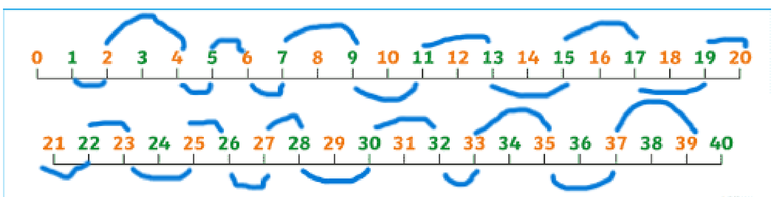
- What is the quickest way of getting 25?
- Do different flips/different patterns yield different results all the time?
- To follow from the above question, does there exists different patterns which yield the same amount of flips ?

WANT

My plan of attack for this problem is to try and play this game a few a times and see if what the results yields. to speed up the attack phase i will try to make a python code that would simulate this game and then see if there exists any trend that can be found. This seems like more like a purely combinatorics and probability based problems unlike some of the previous problems I have tackled on the previous assignments. I am also thinking of making some sort of bar chart which would project the most common number popping up as the result for when the number 25 occurs.

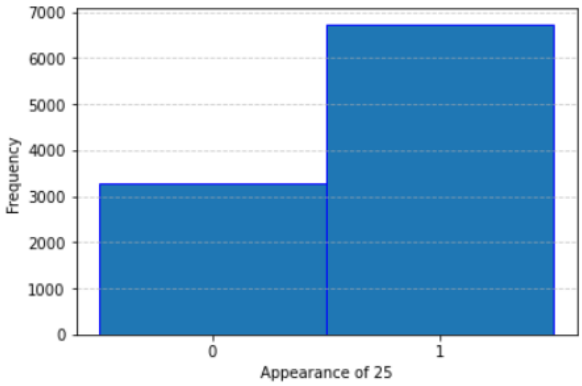
2 Attack

I'm gonna a start off with playing a simulation of this game with a piece of python code that i have created. I simply made the program to generate either 1 or 2 and see at what point I land on 25. In the first iteration I played I got:



AHA!

However, the number 25 is not always guaranteed. For example if I land on 24 and i role a 2 then I completed avoid 25. So I'm gonna do a numerical approach to see if i can get an approximate answer and then try a more analytical approach and compare the two answers.



Above is a histogram that shows the amount of times 25 appeared in the sequence of numbers. I used 10,000 in this case. I then divided that number by the total number of games which yielded a numerical estimate to the probability of 25 appearing in the sequence. In this above graph I had 100 games however when calculating the probability I used 100 games to yield a more close answer. The answer that I got was **0.665**

Lets try to do a more analytical approach to get a more accurate answer

Let p_n be the probability of landing on the number N in the sequence.

JUSTIFYING

We can consider the probability of trivial cases such as n = 3 as n = 2 and 1 are very trivial cases. So for n = 3, we can analyse the cases that will yield 3. The cases are:

- We get 1 twice.
- We get 2 once.

The only way i can miss 3 is if i get a 1 and then get a 2 straight after. The probability of that is $\frac{1}{4}$.

So the $p_3 = 1 - \frac{1}{4}$

But we can see that for n = 3, it relies on the probability of p_1 and p_2.

CONJECTURING

The 2 trivial cases $p_1 = 1$ and $p_2 = 1/2$ since 1 is always guaranteed since it started at 1 and we have a 50 percent chance of getting a 1 so we can land on 2. For n = 3,it can be rewritten as $p_3 = p_1 - (\frac{1}{2})p_2$. So we can **conjecture** a difference equation in the form:

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{2}x_{n-2}$$

We can solve the auxiliary equation $2\lambda^n - \lambda^{n-1} - \lambda^{n-2} = 0$

$2\lambda^2 - \lambda - 1 = 0$

$(\lambda - 1)(2\lambda + 1) = 0$

$\implies \lambda = 1$ or $\lambda = -\frac{1}{2}$

so...

$p_n = A + B(-\frac{1}{2})^n$

So now we need to solve for A and B. We can use our initial values of p_1 and p_2 and we then get: $1 = A - \frac{1}{2}B$

$\frac{1}{2} = A - \frac{1}{4}B$

Rearranging this equation we obtain:

$A = \frac{2}{3}$

$B = -\frac{2}{3}$

So...

$p_n = \frac{2}{3} - \frac{2}{3}(-\frac{1}{2})^n$

So we can use n = 25,

$p_{24} = \frac{2}{3} - \frac{2}{3}(-\frac{1}{2})^{25}$

JUSTIFYING

$= 0.66666664679$ which seems quite close to the numerical value we got before which was 0.665. This also yields the correct probability for n = 3 and so on.

3 Review

Although, this problem seemed to be a lot easier than I initially anticipated, a lot of thought was still required to progress through this problem.

EXTEND

I used python to play the game and used it to simulate the game that the problem discussed. i tried to do a numerical approach to this problem to see if i can get a solution for the problem by using a lot of iterations and seeing how many times the number 25 came up.

Check

I then used a analytical approach to make a conjecture about a difference equation which allowed me to solve it and get a solution to the problem. i used this solution to compare my solution to the analytical solution which was very close and had minimal error.

Reflect

Overall, this was quite fascinating and a joy to work on.

Supplementary material

Below is attached my GitHub repository where my code resides:
`https://github.com/rizwan3254/PS-with-yth`