MA3K7 Week 4 Rubric

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1 Entry

What i know about this problem is that it is a game based on a grid(which is almost similar to naughts and crosses) instead we need to make sure the resulting matrix is of the determinant 0.

WANT

So i am now thinking about the conditions that are required to make the matrix have a determinant of 0. the first thing that comes to my mind is to minimize the problem to shorter dimensions(for example a 2x2).

A couple of questions that come to my mind when thinking about this problem are:

- What is the probability of winning?
- What is the trend of the probability if i increase the dimension?
- How could I program the above?
- Is a strategy possible at such high dimensions?

To expand on the possible 'best' strategy, my next challenge would be to code this and try to create a graph to see if there is any trend present in this so called game.

INTRODUCE

Although, before all of this I'm interested into drawing out a grid and playing this little game to see if i can develop a strategy before even writing an code. However a though comes to my mind when i think about this approach.

KNOW

My 'strategy' may work for only smaller dimensions but for larger dimensions it may cause some problems. For example, it may be ineffective for larger dimensions due to the magnitude of the grid and may be easy to counter. My main point being that it could only work for smaller dimensions and could not be effective for higher dimension. I'll have to consider that when I attack the problem.

2 Attack

So lets start with my first plan of attack which is to reduce the problem to some base cases. In my case, I'm going to try to use 2x2 matrices(1x1 is a very trivial case).

2.1 2x2

If we look a 2x2 grid as a reference.

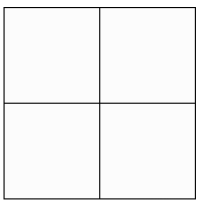

Figure 1: A normal 2x2 grid symbolising a 2x2 matrix

AHA!

The determinant for a 2x2 is fairly straight forward for entries a (1,1) ,b(1,2), c(2,1), d(2,2) would be ad - bc so for us to win we need to make sure that either a or c, a or b, d or c, d or b are 0 to ensure a determinant.

if we simulate a potential game, if I go fist and I play a 0 in (1,1) position then the other player will have no chance of winning the game at all as I have two possibles spaces to win the game. This is the same even if i pick any other position to start off with so we can conclude that i will always win unless...

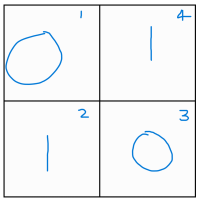

Figure 2: A possible loss with the numbers being the turn order

If the other player starts, then if he picks any square then i have only 1 option to have a potential of winning(which is the other square on the diagonal). However if we don't pick that square, then if the other player doesn't pick the square that on the diagonal then we win the game. This is one of the potential ways of winning the game assuming

JUSTIFYING

the other player chooses the wrong square. Below is the illustration of what i mean:

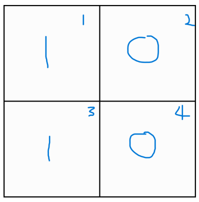

Figure 3: A possible win with the numbers being the turn order

STUCK

2.2 nxn

As we saw for the 2x2 we needed to get either a row or a column of all 0s to get a zero determinant. However, its not always that simple when it comes to nxn matrices as we have a counter example in the matrices of 3x3 below.

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

CONJECTURING

So as we can see we cant do an easy trial and error like we did before to get a winning matrix. So we need to develop a general strategy to try to 'force' a win from our side.

A possible strategy that comes to my mind that if the other player plays a move in the i,j position then we can play a 0 in the i,n-j position(basically a reflection in the y axis). If i go first or if the other player plays on the centre line then i play randomly on

JUSTIFYING

the centre line. If no centre line then i play randomly if i go first(for even nxn). This will guarantee a zero determinant as if you multiply it by the below vector:

$$\begin{bmatrix} 1 \\ -1 \\ 0 \\ \vdots \\ 0 \\ -1 \\ 1 \end{bmatrix} \tag{1}$$

This yields the zero vector which means that 0 must be an eigenvalue. Hence, zero determinant so i win. HOWEVER, this only works for $n \geq 4$ so we now have to develop a strategy for the case.

For the n = 3 case there is 3 cases which guarantees a win.

- A zero row
- A zero column
- zero determinants for all the 2x2 minors

If the other player plays first(eg 1,1) then let me take (2,2). This can be generalised due to permuatations.

$$\begin{pmatrix} 1 & \star & \star \\ \star & 0 & \star \\ \star & \star & \star \end{pmatrix}$$

Let take this for an example without lost of generality. After the other players second move, at least one of 2,3 and 3,2 remains empty. Assume 2,3 remains empty(WLOG) the I play there. After the other players next move, i win by playing at 2,1 if that position is empty. So assume instead that Player 1 has played there. Of the moves that the other player has played, two are at 1,1 and 2,1. Hence for row equal to one of 1 or 3, and for column equal to one of 2 or 3, the following are both true:

JUSTIFYING

- The 2x2 minor formed by rows 2 and either 1 or 3 and by columns 2 and 3 contains two 0s and two empty positions.
- Column 2 or 3 contains one zero and two empty positions.

So i must play at either (any of the above row, any of the above columns). To prevent a zero column, the other player must play in column i played in, upon which Player 0 completes the 2x2 minor for the win.

2.3 Probabilities

Now I am going to write some python code to create the total amount of possible results that can come from a nxn matrix for certain n to see how many winning possibility exist.

JUSTIFYING

Let try an easy example of 2x2 which we just analysed earlier.

I have printed out 2 probabilities with the top one of me starting and the bottom one of me going second. which answers the question on what the probabilities would be for if you started or if you didn't start.This yields an interesting result as it seems

```
0.6677700000000001
0.666955
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Figure 4: Probabilities of winning for 2x2

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0.71532
0.43031500000000006
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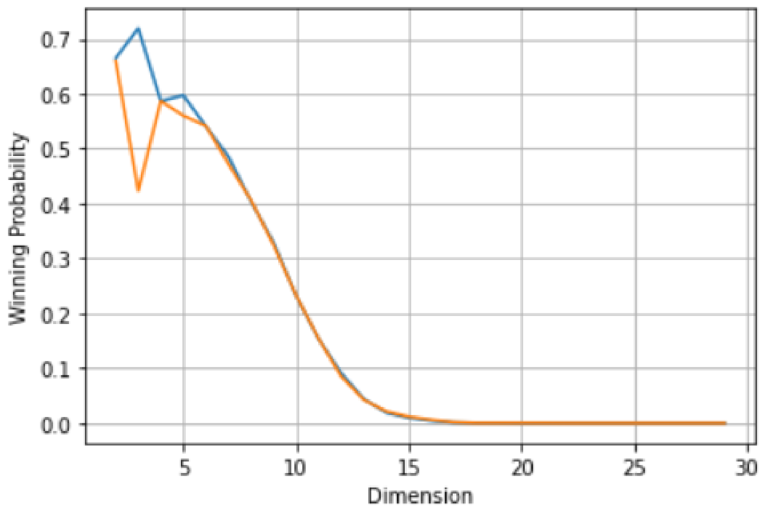
Figure 5: Probabilities of winning for 3x3

(with negligible difference) the probability seems to be same(with the probability of me starting a little bit higher) which seemed to be different to what i initially thought. But it seems that the probability of winning stays fairly constant regardless of who starts.

Conjecturing

i increased the dimension to 3 and saw that the probability of winning increased if i start but decreased if i didn't start which led to think if this trend continued. So i decided to code a graph to represent this.




```
Orange: If i start
Blue: If i dont start
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Justifying

As we can see by the trend(which i somewhat expected) that as the dimensions get bigger it gets a lot harder for me to win since it makes it a lot harder for the determinant to be 0 and the other player would have an easier time winning.

However, near the 3 to 5 range for the dimensions we get a higher probability for me winning when i start(we saw this earlier for 3x3 matrix when i outputted the probability above). But then this trend gets abolished as the dimensions get higher. However this makes for an interesting result as the curve then seems to converge to near 0 and flat out as the dimensions get a lot bigger.

3 Review

Reflect

During this problem required a lot of strategic thinking and out of the box thinking to think of potential strategies to come up with ways to win the game. This then led

to the thinking of the potential chances that a player may have due to the numerous
EXTEND possibilities possible when playing this game.

I then created strategies based on the dimensions of the matrices. I started small then slowly built up to higher dimensions to ensure that every possible dimension would
CHECK have a winning strategy if i played as the person who placed the 0s.

From this, this led me to use python to speed up the processes in finding the probabilities for dimensions higher than 2. this was because it took a lot of time deducing the 2x2 example and I realised that at higher dimensions lead to higher number of possibilities and combinations so i had to be smart in not wasting my time with a
EXTEND dimension higher than 2 and use Python to assist me in this endeavour.

From python I found out probabilities for a 2x2 and 3x3 matrix which led me to be curious about higher dimensions and what sort of numbers they would emit. So I then developed a graph which listed the dimensions and showed me the trend.this then led
REFLECT me to conclude in my search for how probability is affected by who starts the game.

Overall, this was a very interesting problem to work on and certainly was a little challenging but i thoroughly enjoyed myself while on this challenge.

Supplementary material

Below is attached my GitHub repository where my code resides:

`https://github.com/rizwan3254/PS-with-yth`