EM for Gaussian Mixtures

- Given: $\mathbf{X} = \{\mathbf{x}_1, \ldots \mathbf{x}_N\}$, a set of N observations
- Model: $p(\mathbf{X}|\ \boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\Sigma})$=$\prod_{i=1}^{N} \sum_{j=1}^{N} \pi_j \mathcal{N}(\mathbf{x}_i|\ \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$
- Parameters: 3 sets of parameters: $\boldsymbol{\mu}$,$\boldsymbol{\Sigma}$,$\boldsymbol{\pi}$
- Initialisation: $\boldsymbol{\mu}$,$\boldsymbol{\Sigma}$,$\boldsymbol{\pi}$: Estimate initial value of log-lh
- E-step (Expectation): $\gamma_j(\mathbf{x}_i) = \frac{\pi_j \mathcal{N}(\mathbf{x}_i|\boldsymbol{\mu}_j,\boldsymbol{\Sigma}_j)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(\mathbf{x}_i|\boldsymbol{\mu}_j,\boldsymbol{\Sigma}_j)}$: Eval responsibilities using current param values
- M-step (Maximisation): Re-estimate params using current $\gamma_j(\mathbf{x}_i)$'s. $N_j = \sum_{i=1}^{N} \gamma_j(\mathbf{x}_i)$ Estimation:
  - $\boldsymbol{\mu}_j^{new} = (1/N_j) \sum_{i=1}^{N} \gamma_j(\mathbf{x}_i).\mathbf{x}_i$
  - $\boldsymbol{\Sigma}_j^{new} = (1/N_j) \sum_{i=1}^{N} \gamma_j(\mathbf{x}_i).(\mathbf{x}_i - \boldsymbol{\mu}_j^{new})(\mathbf{x}_i - \boldsymbol{\mu}_j^{new})^T$
  - $\pi_j^{new} = N_j/N$
- Eval log-lh: $\log p(\mathbf{X}|\ \boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\Sigma})$=$\sum_{i=1}^{N} \log \sum_{j=1}^{K} \pi_j \mathcal{N}(\cdot)$; Check for convg of log-lh or params. Else, E-step

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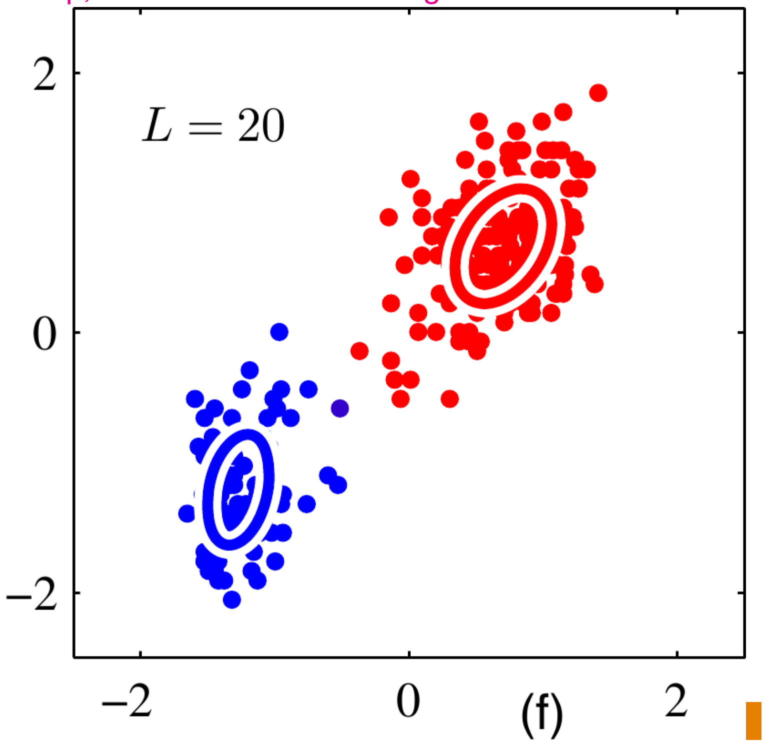
[C. M. Bishop, Pattern Recognition and Machine Learning]



(f)

- Initialisation: Use K-Means to initialise: $\boldsymbol{\mu}$ (sample means), $\boldsymbol{\Sigma}$ (sample Covs), $\boldsymbol{\pi}$ (rel. proportions)

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Stauffer-Grimson BG Subtraction

- The colour/grey value at a pixel is given by a distrn: modelled as a mixture of adaptive Gaussians.

$$p(\mathbf{x}) \triangleq \sum_{j=1}^{K} \pi_j \, \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$$ $\mathbf{x}$: $[r\ g\ b]^T$/grey value

https://upload.wikimedia.org/wikipedia/en/4/4b/Fifty-Gray-poster.jpg

  - K: 3-5, empirically
  - mixture: multiple entities can appear
  - adaptive: with lighting conditions
  - Given $\{\mathbf{x}^1, \ldots \mathbf{x}^t, \ldots \mathbf{x}^\tau\}$: history of colour/grey values at a pixel $t \in \{1, \tau\}$
- Heuristic at a pixel: to decide which Gaussians most likely to contribute to the background
- Pixels not matching BG Gaussians: foreground
- FG pixels grouped: 2-D ConnComp/K-Means