Kernels: Hard Nuts to Crack

http://www.kangarooblue.com/images/fragranceoils/coconut.jpg

- Kernels used in Classification/Regression
- [1964: Aizerman *et al.*], Soviet Control theorists
- [1992: Boser, Guyon, Vapnik]: Hard-M SVM

- Top-down Genesis: (from the SVM Theory):
  - Many linear models: recast into 'dual' form
  - Predictors (classification/regression): 'lin combo' of a kernel fn evaluated at (some) training pts
  - Of interest: models, non-lin feature space mapping, $\mathbf{x} \rightarrow \boldsymbol{\phi}(\mathbf{x})$. e.g., $\tilde{\mathbf{x}}_{3\times 1} = \boldsymbol{\phi}(\mathbf{x}_{2\times 1})$, $\boldsymbol{\phi}(\cdot)$: vector

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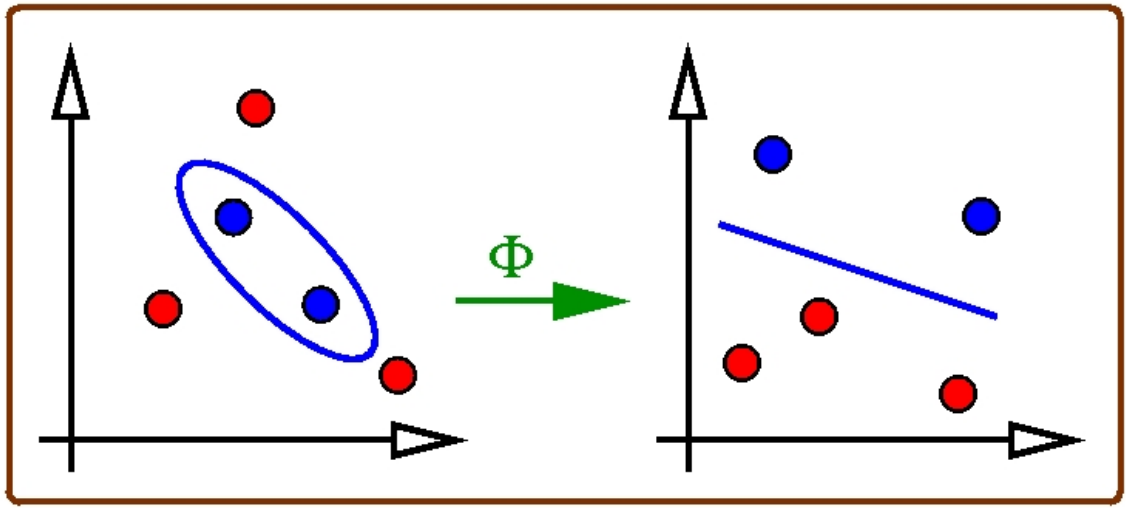
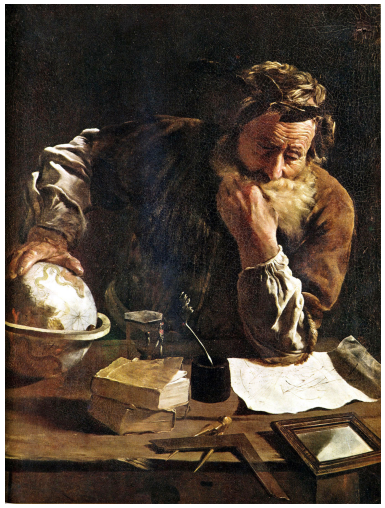


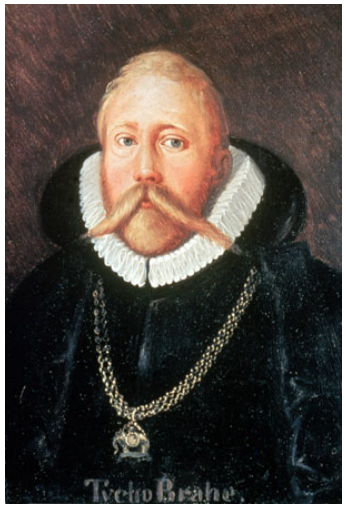
- Basic Idea: Use Φ to map the original patterns in $\mathscr{X}$ into a higher dimensional feature space $\mathscr{H}$
- Draw a separating hyperplane with max margin
- This corresponds to a non-linear decision boundary in the original pattern space
- Kernel trick: using $k(\cdot,\cdot)$, get separating hyperplane without explicitly using Φ to get $\mathscr{H}$

- $k(\mathbf{x}_i, \mathbf{x}_j) \stackrel{\triangle}{=} \boldsymbol{\phi}^T(\mathbf{x}_i)\boldsymbol{\phi}(\mathbf{x}_j) = \boldsymbol{\phi}^T(\mathbf{x}_j)\boldsymbol{\phi}(\mathbf{x}_i)$
- $\mathbf{x}_i$, $\mathbf{x}_j$: vecs in original space, $\boldsymbol{\phi}(\cdot)$: x'fm'd space
- $k(\mathbf{x}_i, \mathbf{x}_j)$: scalar, symmetric, 1st term: fn of $\mathbf{x}_i$, 2nd term: *same* fn of $\mathbf{x}_j$

Unfortunate Conclusions-1...



Archimedes
[287 BC-212 BC]



Tycho Brahe
[1546-1601]

Galileo Galilei
[1564-1642]

https://upload.wikimedia.org/wikipedia/commons/e/e7/Domenico-Fetti_Archimedes_1620.jpg

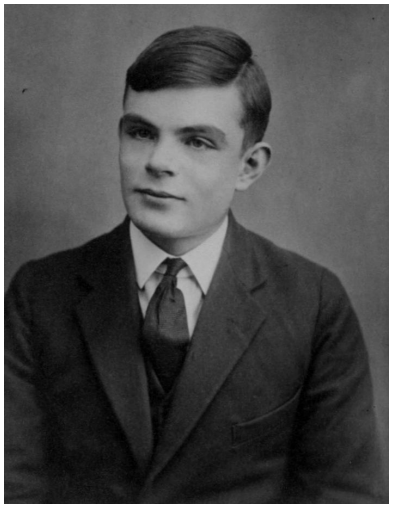
https://upload.wikimedia.org/wikipedia/commons/2/2b/Tycho_Brahe.JPG

https://upload.wikimedia.org/wikipedia/commons/d/d4/Justus_Sustermans_-_Portrait_of_Galileo_Galilei%2C_1636.jpg

Unfortunate Conclusions-2. . .

A. Lavoisier
[1743-1794]

E. Galois
[1811-1832]



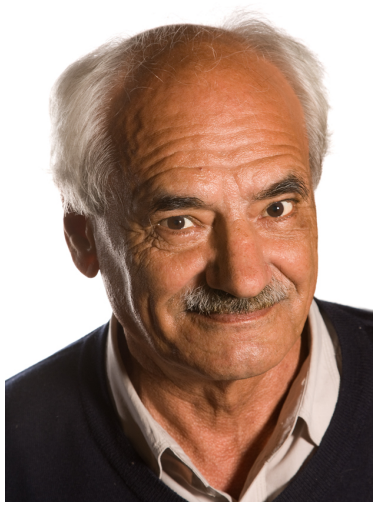
A. M. Turing
[1912-1954]

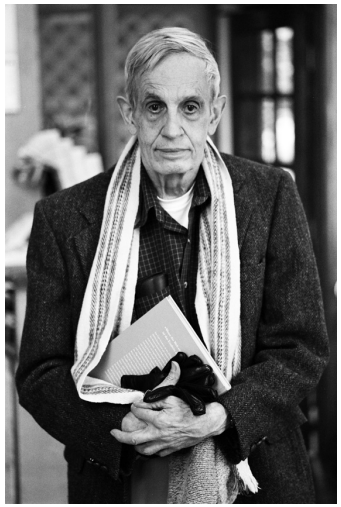
https://upload.wikimedia.org/wikipedia/commons/4/44/Lavoisier-statue.jpg

https://upload.wikimedia.org/wikipedia/commons/5/53/Evariste_galois.jpg

https://upload.wikimedia.org/wikipedia/commons/a/a1/Alan_Turing_Aged_16.jpg

Unfortunate Conclusions-3. . .





A. Y. Chervonenkis [1938-2014]

J. F. Nash [1928-2015]

http://www.clrc.rhul.ac.uk/people/photos/AClarge.JPG

https://upload.wikimedia.org/wikipedia/commons/a/a9/John_Forbes_Nash%2C_Jr._by_Peter_Badge.jpg