Name :- Rizwan Haider
Roll # 18i-0536
Section:- CS(E)

Course:- Probability & Statistics
Task:- Assignment #4
Date:-

Question #1

Many manfactures have quality control programs that include inspection of incoming materials for defects. Suppose a computer manufacturer recieves computer bou in lots of five. Two are selected from each lot for inspection. We can represent possible outcomes of the selection process by pairs. for example the pair (1,2) represents the selection of boards 1 and 2 for inspection.

(a) The possible outcomes are given below

$= \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(3,4),(3,5),(4,5)\}$

(b) Suppose that boards 1 and 2 are the only defective boards in a lot of five. Two boards are to be choosen at random. Define X to be the number of defective boards observed among those inspected. Find the probability distribution of X.

$X = 01 = \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$

$P(X=1) = \frac{6}{10} = 0.6$

$X = 0 = \{(3,4), (3,5), (4,5)\}$

$P(X=0) = \frac{3}{10} = 0.3$

$X = 2 = \{(1,2)\}$

$P(X=2) = \frac{1}{10} = 0.1$

c. Let $F(x)$ denote the cdf of X. first determine $F(1)$, and $F(2)$, obtain for other X.

$F(1) = P(X \leq 1) = P(X=0) + P(X=1)$

$= 0.3 + 0.6$

$= 0.9$

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.3 + 0.6 + 0.1$

$= 1$

$F(0) = P(X \leq 0) = 0.3$

Question #2

An automobile service facility specializing in engine tune-ups knows that 45% of all tune-up are done on four cylinder automobiles, 40% on six cylinder automobiles and 15% on eight cylinder automobiles. Let X = number of cylinders on the next car to be tuned.

a:- what is the pmf of X?

Probability of tuning four cylinder automobiles = 45%

$= \frac{45}{100} = 0.45$

Probability of tuning six cylinder automobiles = 40%

$= \frac{40}{100} = 0.40$

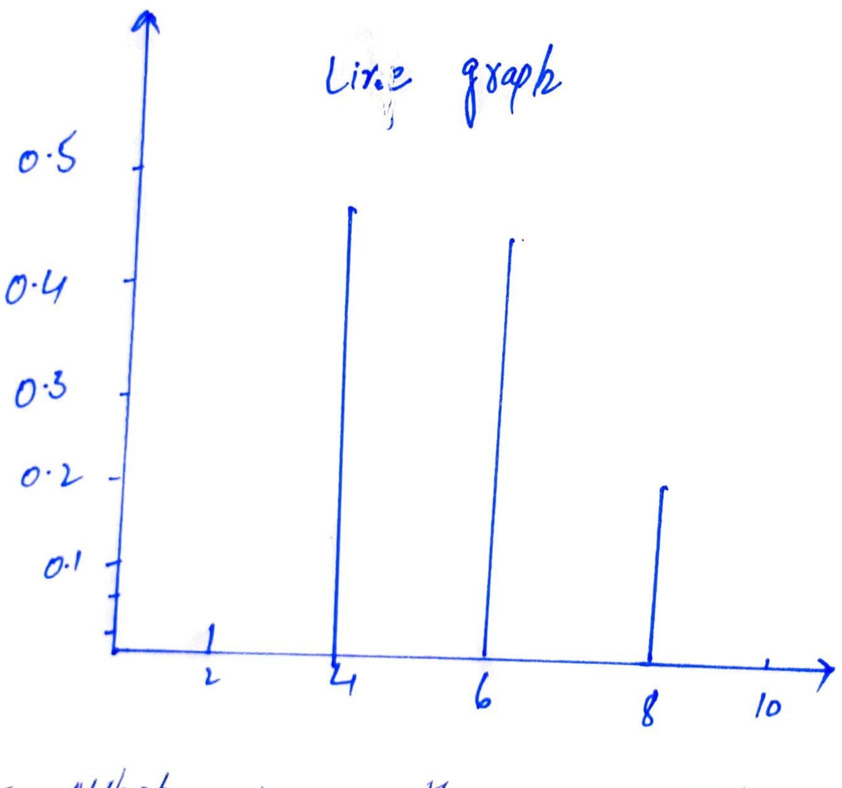
Probability of tuning eight cylinder automobiles = 15%

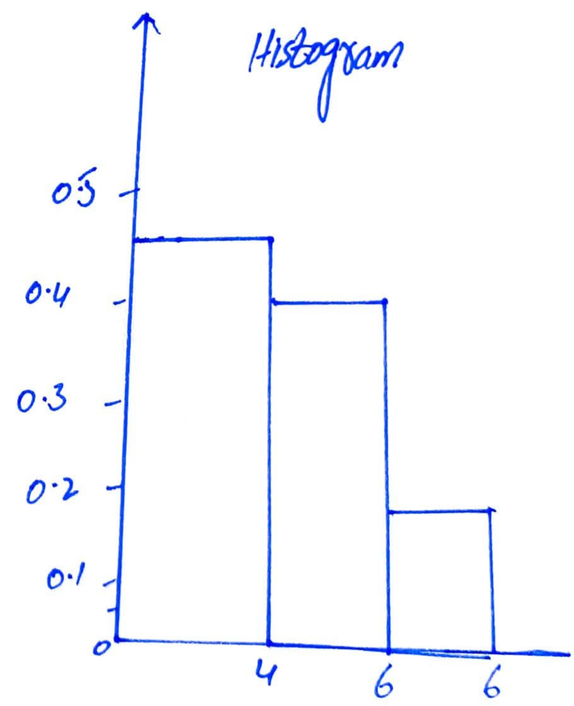
$= \frac{15}{100} = 0.15$

x	A	6	8
P(X)	0.45	0.40	0.15

Total = 1

b. Draw both a line graph and a probability histogram for the proof of part (a).





c. What is the probability that the next car tuned has at atleast six cylinder? More than six cylinder?

$P(\text{atleast six}) = P(X \geq 6) = 0.40 + 0.15 = 0.55$

$P(\text{more than six}) = P(X > 6)$

$= 0.15$

Question #3

An appliance deler sells three different models of upright freezers having 13.5, 15.9 and 19.1 cubic feet of storage space, respectively. Let of storage space purchased by the next customer to buy a freezer. Suppose that x has Pmf

x	13.5	15.9	19.1
$P(x)$	.2	.5	.3

a. Compute $E(X)$, $E(X2)$ and $V(X)$

$$E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

$$= (13.5 \times .2) + (15.9 \times .5) + (19.1 \times 0.3)$$

$$= 16.38$$

$$E(x^2) = \sum_{i=1}^{n} (x_i)^2 P(x_i)$$

$$= (13.5^2 \times .2) + (15.9^2 \times .5) + (19.1^2 \times 0.3)$$

$$= 272.3$$

$$V(x) = E(x^2) - E(x)^2$$

$$= 272.3 - (16.38)^2$$

$$= 3.99$$

b:- If the price of a freezer having capacity X cubi feet is $= 2x - 8.5$, what is the expected price paid by the next customer to buy a freezer?

$$E = (2X - 8.5)$$
$$= 2E(x) + (-8.5)$$
$$= 2(16.38) - 8.5$$
$$= 24.26$$

c:- What is the Variance of the Price = $22X - 8.5$ Paid by the next customer?

$$V(22X - 8.5)$$
$$= V(22X) + V(-8.5) + 2Cov(22X, -8.5)$$
$$= 22^2 V(X) + 0 + 0$$
$$= 22^2 \times 3.99$$
$$= 1931.16$$

d:- Suppose that although the rated capacity of a freezer is X, the actual capacity is $h(X) = X - 0.01X^2$ what is the expected actual capacity of the freezer purchased by the next customer?

$$h(X) = X - 0.01X^2$$
$$= E(X - 0.01x^2)$$
$$= E(x) - E(0.01x^2) = E(X) - 0.01E(x^2)$$
$$= 16.38 - 0.01(272.3)$$
$$= 13.657$$

Question #4

If x has the probability density function $f(x) = k$; $\{-4 \le x \le +4$ 0 elsewhere, find c such that $P(-c < x < +c) = 95\%$.

$$\int_{-4}^{4} k\,dx = 1$$

$$= (kx)\Big|_{-4}^{4} = 1$$

$$4x - (-4)k = 1$$
$$4k + 4k = 1$$

$$k = \frac{1}{8}$$

$$F(x) = \int_{-4}^{4} \frac{1}{8} d + \int_{-4}^{x} \frac{1}{8} d$$

$$= 0 + \left(\frac{1}{8}\right)\Big|_{-4}^{x}$$

$$= \frac{1}{8}x - \frac{1}{8}(-4)$$

$$= \frac{1}{8}x + \frac{4}{8} = \frac{1}{8}x + \frac{1}{4}$$

$$P(-c \le x \le c) = 0.95$$

$$\int_{-c}^{c} \frac{1}{8} dx) = 0.95$$

$$F(c) - F(-c) = 0.95$$

$$= c + \frac{4}{8} + -c + \frac{4}{8} = 0.95$$

$$\frac{c}{4} = 0.95$$

$$\boxed{c = 3.8}$$

Question # 5

A random variable X has the probability density function given by

$$f_x \begin{cases} \frac{1}{16}(3+x)^2, & -3 < x \leq -1 \\ \frac{1}{16}(6-2x^2), & -1 < x \leq 1 \\ \frac{1}{16}(3-x)^2 & 1 < x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Find the mean and the standard deviation of the random variable X.

Mean

$$E(x) = \int_{\infty}^{\infty} x f(x)\, dx$$

$$E(x) = \int_{-3}^{-1} x \frac{1}{16}(3+x)^2\, dx + \int_{-1}^{1} \frac{x\,1}{16}(6-2x^2)\, dx + \int_{1}^{3} x \frac{1}{16}(3-x)^2\, dx$$

$$= \frac{1}{16} \int_{-3}^{-1} x(9+6x+x^2)\, dx + \frac{1}{16}\int_{-1}^{1} x(6-2x^2)\, dx + \frac{1}{16}\int_{1}^{3} x(3-x)^2\, dx$$

$$= \frac{1}{16}\int_{-3}^{-1} 9x+6x^2+x^3)\, dx + \frac{1}{16}\int_{-1}^{1} (6x-2x^3)\, dx + \frac{1}{16}\int_{1}^{3} 9x-6x^2+x^3)\, dx$$

$$= \frac{1}{16}\left|\frac{9x}{2} + \frac{6x^2}{3} + \frac{x^4}{4}\right|_{-3}^{-1} + \frac{1}{16}\left|\frac{6x^2}{2} - \frac{2x^4}{4}\right|_{-1}^{1} + \frac{1}{16}\left|\frac{9x^2}{2} - \frac{6x^3}{3} + \frac{x^4}{4}\right|_{1}^{3}$$

$$= \frac{1}{16}(-4) + \frac{1}{16}\times 0 + \frac{1}{16}\times 4$$

$$= 0$$

$$\boxed{\text{Mean} = 0}$$

SD

$SD = \sqrt{var}$

$var = E(x^2) - [E(x)^2]$

$E(x^2) = \frac{1}{16}\left(\frac{9x^3}{3} + \frac{x^5}{5} + \frac{6x^4}{4}\right|_{-3}^{-1} + \frac{1}{16}\left|\frac{6x^3}{3} - \frac{2x^5}{5}\right|_{-1}^{1} + \frac{1}{16}\left|\frac{9x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right|_{1}^{3}$

$= \frac{1}{16}(6.4) + \frac{1}{16}(3.2) + \frac{1}{16}(6.4$

$= \frac{2}{5} + \frac{1}{5} + \frac{2}{5} = 1$

$var = E(x^2) - E(x)^2$

$= 1 - 0^2 = 1$

$\boxed{Var = 1}$

$SD = \sqrt{Var} = \sqrt{1} = 1$ $\boxed{SD = 1}$

Question #6

Suppose that in an automatic filling process of oil in cans, the content of cans the content of cans in gallons is $Y = 50 + X$ where x is a random variable withe density $f(x) = 1 - |x|$ when $|x| \le 1$ and 0 when $|x| > 1$ in a lot of 100 cans how many will contain 50 gallons or more? what is the probability of that a can will contain less than 49.5 gallons? less than 49 gallons? Also find $F(x)$.

CDF :-

$\cancel{F(x) = \int_{\infty}^{y}}$

$f(x) = 0 \quad |x| > 1$

$= x < -1, x > 1$

$f(x) = 1 - |x|$:

$|x| \leq 1 = x \geq -1, \; x \leq 1$

$-1 \leq x < 0$	$0 \leq x \leq 1$
$1-(-x)$	$= 1-(x)$
$= 1+x$	$= 1-x$

$$f(x) = \begin{cases} 0 & x < -1 \\ 1+x & 1 - \leq x < 0 \\ 1-x & 0 \leq x \leq 1 \\ 0 & x > \end{cases}$$

CDF $F(x) = \int_{\infty}^{y} f(y)\,dy$

a) if $x < -1$

$f(x) = 0$

$f(x) = \int_{\infty}^{x} 0\,dy = 0$

b) if $-1 < x < 0$:-

$f(x) = 1+x$

$F(x) = \int_{\infty}^{x} f(y)dy = \int_{\infty}^{-1} 0\,dy + \int_{-1}^{x} (1+y)\,dy$

$F(x) = \left(y + \frac{y^2}{2}\right)_{-1}^{x}$

$= x + \frac{x^2}{2} + \frac{1}{2}$

(c) if $0 < x < 1$

$f(x) = 1 - x$

$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{-1} 0\,dy + \int_{-1}^{0} (1+y)\,dy + \int_{0}^{x} (1-y)\,dy$

$F(x) = \left(y + \frac{y^2}{2}\right)\Big|_{-1}^{0} + \left(y - \frac{y^2}{2}\right)\Big|_{0}^{x}$

$= x - \frac{x^2}{2} + \frac{1}{2}$

d) if $x > 1$

$f(x) = 0$

$F(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{-1} 0\,dy + \int_{-1}^{0} (1+y)\,dy + \int_{0}^{1} (1-y)\,dy +$
$\int_{0}^{x} 0\,dy$

$F(x) = \left(y + \frac{y^2}{2}\right)\Big|_{-1}^{0} + \left(y - \frac{y^2}{2}\right)\Big|_{0}^{1}$

$F(x) = 1$

Now

$$F(x) = \begin{cases} 0, & x < -1 \\ x + \frac{x^2}{2} + \frac{1}{2}, & -1 \leq x < 0 \\ x - \frac{x^2}{2} + \frac{1}{2}, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$Y = 50 + X$

$P(Y \geq 50) = P(X + 50 \geq 50) = P(X \geq 0)$

$P(X \geq 0) = 1 - P(X < 0)$

$P(X < 0) = \int_{-\infty}^{0} f(x)\,dx = F(0) = 0 - \frac{0^2}{2} + \frac{1}{2} = \frac{1}{2}$

$\Rightarrow P(Y \geq 50) = P(X \geq 0) = 1 - P(X < 0) = \frac{1}{2}$

$\Rightarrow$ The probability of a can containing 50 gallons or more is 0.5, among 10 canss there will be $100 \times 0.5 = \boxed{50}$

$\boxed{\text{50 cans with 50 gallons or more}}$

$P(Y < 49.5) = P(X + 50 < 49.5) = P(X < -0.5) = P(X \leq -0.5)$

$P(Y < 49.5) = P(X \leq 0.5) = \int_{\infty}^{-0.5} f(x)\,dx = F(-0.5)$

$F(0.5) = -0.5 + -\frac{0.5^2}{2} + \frac{1}{2} = \boxed{0.125}$

$P(Y < 49) = P(X + 50 < 49) = P(X < -1) = P(X \leq -1)$

$P(X \leq -1) = \int_{-\infty}^{-1} f(x)\,dx = F(-1)$

$F(-1) = -1 + \frac{1^2}{2} + \frac{1}{2} = 0$

$F(-1) = 0$

$\boxed{P(Y < 49) = 0}$