1. Gathering the Network

```r
library("rtoot")
library("tidyr")
library("dplyr")
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library("igraph")
```

```
##
## Attaching package: 'igraph'

## The following objects are masked from 'package:dplyr':
##
##     as_data_frame, groups, union

## The following object is masked from 'package:tidyr':
##
##     crossing

## The following objects are masked from 'package:stats':
##
##     decompose, spectrum

## The following object is masked from 'package:base':
##
##     union
```

```r
library("poweRlaw")
```

```r
#toots <- get_timeline_hashtag(hashtag = "WSUCOMP7025", limit = 40L)
#save(toots, file = "toots.Rdata")
load("toots.Rdata")
```

```r
toots_table <- as.data.frame(toots)
```

```r
toots_table <- separate(toots_table, uri, into = paste0("uri_part", 1:10), sep = "/")
```

```r
filtered_table_mastodon.social <- toots_table %>% filter(uri_part3 == "mastodon.social")
count_mastodon.social <- nrow(filtered_table_mastodon.social)

filtered_table_mastodon.bot <- toots_table %>% filter(uri_part3 == "mastodon.bot")
count_mastodon.bot <- nrow(filtered_table_mastodon.bot)

filtered_table_mstdn.party <- toots_table %>% filter(uri_part3 == "mstdn.party")
count_mstdn.party <- nrow(filtered_table_mstdn.party)

filtered_table_universeodon.com <- toots_table %>% filter(uri_part3 == "universeodon.com")
count_universeodon.com <- nrow(filtered_table_universeodon.com)
```

```
filtered_table_convo.casa <- toots_table %>% filter(uri_part3 == "convo.casa")
count_convo.casa <- nrow(filtered_table_convo.casa)

filtered_table_scholar.social <- toots_table %>% filter(uri_part3 == "scholar.social")
count_scholar.social <- nrow(filtered_table_scholar.social)
```

```
server_data <- data.frame(
  Server = c("mastodon.social", "mastodon.bot", "mstdn.party", "universeodon.com",
             "convo.casa", "scholar.social"),
  Count = c(count_mastodon.social, count_mastodon.bot, count_mstdn.party,
            count_universeodon.com, count_convo.casa, count_scholar.social)
)
print(server_data)
```

```
##              Server Count
## 1  mastodon.social    20
## 2     mastodon.bot     1
## 3      mstdn.party     1
## 4 universeodon.com     1
## 5       convo.casa     1
## 6   scholar.social     1
```

From the table above we can observe that 20 toots belong to the mastodon.social server. The rest of the servers contain 1 toot in each among which the server mastodon.bot is a bot.

2. Following Graph

```
unique_usernames <- c()

for (i in 1:25) {
  username <- toots$account[[i]]$username
  if (!(username %in% unique_usernames)) {
    cat("Username at index", i, ":", username, "\n")
    unique_usernames <- c(unique_usernames, username)
  }
}
```

```
## Username at index 1 : harikrizhnavarma
## Username at index 2 : lapark
## Username at index 3 : trending
## Username at index 4 : BibekCh
## Username at index 5 : JessiaKyaw
## Username at index 6 : 22131755
## Username at index 7 : A_user
## Username at index 9 : HM69
## Username at index 10 : iamseemco
## Username at index 11 : bhubanpun
## Username at index 13 : 22105983
## Username at index 14 : riya_john
## Username at index 15 : chowdhuryshaheb66
## Username at index 16 : pakhi
## Username at index 17 : 17979535meow
## Username at index 18 : Ideree
## Username at index 19 : mgrishma17
## Username at index 20 : ayazkhanP
## Username at index 21 : 6sahani
## Username at index 22 : 22049113
## Username at index 23 : sushila
## Username at index 24 : nicole05
```

From the list above we can observe all the unique users/students who tooted with the hashtag #WSU-COMP7025. Among the users/students, trending (index 3) is a bot.

The following is done to find out who follows who:

```
toots_table$account[[1]]$username
```

```
## [1] "harikrizhnavarma"
```

```
harikrizhnavarma_follows <- get_account_following(toots_table$account[[1]]$id)
harikrizhnavarma_follows_df <- as.data.frame(harikrizhnavarma_follows)
harikrizhnavarma_follows_df$username
```

```
## [1] "ayazkhanP" "pakhi"
```

```
toots_table$account[[2]]$username
```

```
## [1] "lapark"
```

```
lapark_follows <- get_account_following(toots_table$account[[2]]$id)
lapark_follows_df <- as.data.frame(lapark_follows)
```

```
lapark_follows_df$username
```

```
## [1] "JessiaKyaw" "22131755"   "A_user"     "BibekCh"    "HM69"
## [6] "iamseemco"  "bhubanpun"  "22105983"
```

```
toots_table$account[[3]]$username
```

```
## [1] "trending"
```

```
trending_follows <- get_account_following(toots_table$account[[3]]$id)
trending_follows_df <- as.data.frame(trending_follows)
trending_follows_df
```

```
## data frame with 0 columns and 0 rows
```

User trending does not follow any of the accounts.

```
toots_table$account[[4]]$username
```

```
## [1] "BibekCh"
```

```
BibekCh_follows <- get_account_following(toots_table$account[[4]]$id)
BibekCh_follows_df <- as.data.frame(BibekCh_follows)
BibekCh_follows_df$username
```

```
## [1] "lapark"     "Deltanoob"  "DeltaNoob"  "nicole05"   "sushila"
## [6] "mgrishma17" "bhubanpun"
```

```
toots_table$account[[5]]$username
```

```
## [1] "JessiaKyaw"
```

```
JessiaKyaw_follows <- get_account_following(toots_table$account[[5]]$id)
JessiaKyaw_follows_df <- as.data.frame(JessiaKyaw_follows)
JessiaKyaw_follows_df$username
```

```
## [1] "Remant"    "mkst_2367" "lapark"    "riya_john" "iamseemco" "Ideree"
```

```
toots_table$account[[6]]$username
```

```
## [1] "22131755"
```

```
user22131755_follows <- get_account_following(toots_table$account[[6]]$id)
user22131755_follows_df <- as.data.frame(user22131755_follows)
user22131755_follows_df$username
```

```
## [1] "Ideree" "lapark"
```

```
toots_table$account[[7]]$username
```

```
## [1] "A_user"
```

```
A_user_follows <- get_account_following(toots_table$account[[7]]$id)
A_user_follows_df <- as.data.frame(A_user_follows)
A_user_follows_df$username
```

```
## [1] "Ideree" "lapark"
```

```
toots_table$account[[9]]$username
```

```
## [1] "HM69"
```

```r
HM69_follows <- get_account_following(toots_table$account[[9]]$id)
HM69_follows_df <- as.data.frame(HM69_follows)
HM69_follows_df$username
```

```
## [1] "lapark"
```

```r
toots_table$account[[10]]$username
```

```
## [1] "iamseemco"
```

```r
iamseemco_follows <- get_account_following(toots_table$account[[10]]$id)
iamseemco_follows_df <- as.data.frame(iamseemco_follows)
iamseemco_follows_df$username
```

```
## [1] "lapark"    "nicole05"  "riya_john" "Ideree"    "JessiaKyaw"
```

```r
toots_table$account[[11]]$username
```

```
## [1] "bhubanpun"
```

```r
bhubanpun_follows <- get_account_following(toots_table$account[[11]]$id)
bhubanpun_follows_df <- as.data.frame(bhubanpun_follows)
bhubanpun_follows_df$username
```

```
## [1] "lapark"    "BibekCh"   "Ideree"    "mgrishma17"
```

```r
toots_table$account[[13]]$username
```

```
## [1] "22105983"
```

```r
user22105983_follows <- get_account_following(toots_table$account[[13]]$id)
user22105983_follows_df <- as.data.frame(user22105983_follows)
user22105983_follows_df$username
```

```
## [1] "lapark"
```

```r
toots_table$account[[14]]$username
```

```
## [1] "riya_john"
```

```r
riya_john_follows <- get_account_following(toots_table$account[[14]]$id)
riya_john_follows_df <- as.data.frame(riya_john_follows)
riya_john_follows_df$username
```

```
## [1] "ayazkhanP"      "nicole05"        "harikrizhnavarma" "iamseemco"
## [5] "JessiaKyaw"     "pakhi"           "Ideree"
```

```r
toots_table$account[[15]]$username
```

```
## [1] "chowdhuryshaheb66"
```

```r
chowdhuryshaheb66_follows <- get_account_following(toots_table$account[[15]]$id)
chowdhuryshaheb66_follows_df <- as.data.frame(chowdhuryshaheb66_follows)
chowdhuryshaheb66_follows_df
```

```
## data frame with 0 columns and 0 rows
```

User chowdhuryshaheb66 does not follow any of the accounts.

```r
toots_table$account[[16]]$username
```

```
## [1] "pakhi"
```

```
pakhi_follows <- get_account_following(toots_table$account[[16]]$id)
pakhi_follows_df <- as.data.frame(pakhi_follows)
pakhi_follows_df$username
```

```
## [1] "harikrizhnavarma" "nicole05"         "ayazkhanP"        "riya_john"
```

```
toots_table$account[[17]]$username
```

```
## [1] "17979535meow"
```

```
user17979535meow_follows <- get_account_following(toots_table$account[[17]]$id)
user17979535meow_follows_df <- as.data.frame(user17979535meow_follows)
user17979535meow_follows_df
```

```
## data frame with 0 columns and 0 rows
```

User 17979535meow does not follow any of the accounts.

```
toots_table$account[[18]]$username
```

```
## [1] "Ideree"
```

```
Ideree_follows <- get_account_following(toots_table$account[[18]]$id)
Ideree_follows_df <- as.data.frame(Ideree_follows)
Ideree_follows_df$username
```

```
##  [1] "nicole05"          "sushila"           "22049113"
##  [4] "6sahani"           "ayazkhanP"         "mgrishma17"
##  [7] "17979535meow"      "pakhi"             "chowdhuryshaheb66"
## [10] "riya_john"         "22105983"          "bhubanpun"
## [13] "iamseemco"         "A_user"            "22131755"
## [16] "JessiaKyaw"        "BibekCh"
```

```
toots_table$account[[19]]$username
```

```
## [1] "mgrishma17"
```

```
mgrishma17_follows <- get_account_following(toots_table$account[[19]]$id)
mgrishma17_follows_df <- as.data.frame(mgrishma17_follows)
mgrishma17_follows_df$username
```

```
## [1] "Vtee"      "Ideree"    "nicole05"  "sushila"   "BibekCh"   "bhubanpun"
```

```
toots_table$account[[20]]$username
```

```
## [1] "ayazkhanP"
```

```
ayazkhanP_follows <- get_account_following(toots_table$account[[20]]$id)
ayazkhanP_follows_df <- as.data.frame(ayazkhanP_follows)
ayazkhanP_follows_df$username
```

```
## [1] "harikrizhnavarma" "pakhi"            "Ideree"
```

```
toots_table$account[[21]]$username
```

```
## [1] "6sahani"
```

```
user6sahani_follows <- get_account_following(toots_table$account[[21]]$id)
user6sahani_follows_df <- as.data.frame(user6sahani_follows)
user6sahani_follows_df$username
```

[1] "Ideree"

```
toots_table$account[[22]]$username
```

[1] "22049113"

```
user22049113_follows <- get_account_following(toots_table$account[[22]]$id)
user22049113_follows_df <- as.data.frame(user22049113_follows)
user22049113_follows_df
```

data frame with 0 columns and 0 rows

User 22049113 does not follow any of the accounts.

```
toots_table$account[[23]]$username
```

[1] "sushila"

```
sushila_follows <- get_account_following(toots_table$account[[23]]$id)
sushila_follows_df <- as.data.frame(sushila_follows)
sushila_follows_df$username
```

[1] "nicole05"   "BibekCh"   "mgrishma17" "Ideree"

```
toots_table$account[[24]]$username
```

[1] "nicole05"

```
nicole05_follows <- get_account_following(toots_table$account[[24]]$id)
nicole05_follows_df <- as.data.frame(nicole05_follows)
nicole05_follows_df$username
```

[1] "Ideree"   "pakhi"   "BibekCh"   "sushila"   "mgrishma17"
[6] "17979535meow" "iamseemco"

```
graph <- graph(edges=c(toots_table$account[[1]]$username,harikrizhnavarma_follows_df$username[[1]],
                       toots_table$account[[1]]$username,harikrizhnavarma_follows_df$username[[2]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[1]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[2]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[3]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[4]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[5]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[6]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[7]],
                       toots_table$account[[2]]$username,lapark_follows_df$username[[8]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[1]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[2]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[3]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[4]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[5]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[6]],
                       toots_table$account[[4]]$username,BibekCh_follows_df$username[[7]],
                       toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[1]],
                       toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[2]],
```

```
toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[3]],
toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[4]],
toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[5]],
toots_table$account[[5]]$username,JessiaKyaw_follows_df$username[[6]],
toots_table$account[[6]]$username,user22131755_follows_df$username[[1]],
toots_table$account[[6]]$username,user22131755_follows_df$username[[2]],
toots_table$account[[7]]$username,A_user_follows_df$username[[1]],
toots_table$account[[7]]$username,A_user_follows_df$username[[2]],
toots_table$account[[9]]$username,HM69_follows_df$username[[1]],
toots_table$account[[10]]$username,iamseemco_follows_df$username[[1]],
toots_table$account[[10]]$username,iamseemco_follows_df$username[[2]],
toots_table$account[[10]]$username,iamseemco_follows_df$username[[3]],
toots_table$account[[10]]$username,iamseemco_follows_df$username[[4]],
toots_table$account[[10]]$username,iamseemco_follows_df$username[[5]],
toots_table$account[[11]]$username,bhubanpun_follows_df$username[[1]],
toots_table$account[[11]]$username,bhubanpun_follows_df$username[[2]],
toots_table$account[[11]]$username,bhubanpun_follows_df$username[[3]],
toots_table$account[[11]]$username,bhubanpun_follows_df$username[[4]],
toots_table$account[[13]]$username,user22105983_follows_df$username[[1]],
toots_table$account[[14]]$username,riya_john_follows_df$username[[1]],
toots_table$account[[14]]$username,riya_john_follows_df$username[[2]],
toots_table$account[[14]]$username,riya_john_follows_df$username[[3]],
toots_table$account[[14]]$username,riya_john_follows_df$username[[4]],
toots_table$account[[16]]$username,pakhi_follows_df$username[[1]],
toots_table$account[[16]]$username,pakhi_follows_df$username[[2]],
toots_table$account[[16]]$username,pakhi_follows_df$username[[3]],
toots_table$account[[16]]$username,pakhi_follows_df$username[[4]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[1]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[2]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[3]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[4]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[5]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[6]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[7]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[8]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[9]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[10]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[11]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[12]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[13]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[14]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[15]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[16]],
toots_table$account[[18]]$username,Ideree_follows_df$username[[17]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[1]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[2]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[3]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[4]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[5]],
toots_table$account[[19]]$username,mgrishma17_follows_df$username[[6]],
toots_table$account[[20]]$username,ayazkhanP_follows_df$username[[1]],
toots_table$account[[20]]$username,ayazkhanP_follows_df$username[[2]],
toots_table$account[[20]]$username,ayazkhanP_follows_df$username[[3]],
```

```
                    toots_table$account[[21]]$username,user6sahani_follows_df$username[[1]],
                    toots_table$account[[23]]$username,sushila_follows_df$username[[1]],
                    toots_table$account[[23]]$username,sushila_follows_df$username[[2]],
                    toots_table$account[[23]]$username,sushila_follows_df$username[[3]],
                    toots_table$account[[23]]$username,sushila_follows_df$username[[4]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[1]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[2]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[3]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[4]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[5]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[6]],
                    toots_table$account[[24]]$username,nicole05_follows_df$username[[7]]),
           directed=TRUE)

plot(graph, layout = layout.kamada.kawai, vertex.size = 10, vertex.label.cex = 0.8,
     cex.lab = 0.8, edge.arrow.size = 0.3, main = "Student Network Graph")
```

Student Network Graph
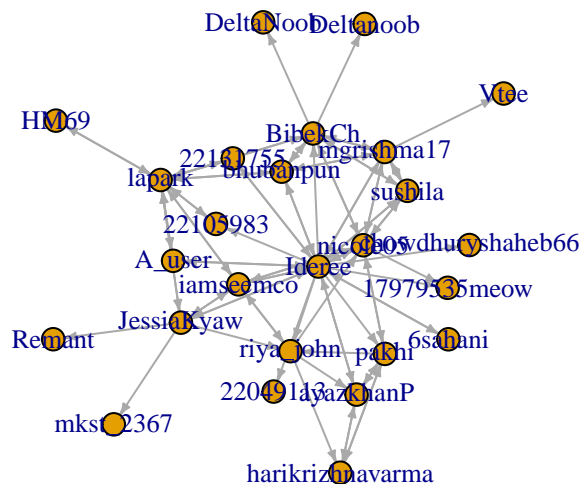

```
graph_components <- components(graph)
graph_components
```

```
## $membership
##  harikrizhnavarma          ayazkhanP              pakhi             lapark
##                 1                  1                  1                  1
##        JessiaKyaw           22131755             A_user            BibekCh
##                 1                  1                  1                  1
##              HM69          iamseemco          bhubanpun           22105983
```

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```
##                1                     1                  1                    1
##         Deltanoob             DeltaNoob           nicole05              sushila
##                1                     1                  1                    1
##        mgrishma17                Remant           mkst_2367            riya_john
##                1                     1                  1                    1
##            Ideree              22049113            6sahani          17979535meow
##                1                     1                  1                    1
## chowdhuryshaheb66                  Vtee
##                1                     1
##
## $csize
## [1] 26
##
## $no
## [1] 1
```

There is only 1 component in the graph with 26 nodes.

The graph has a central node representing the student "Ideree" with many outgoing and incoming connections. This indicates that this student has a key influence within the student network.

Several nodes are directly connected to the central node which represents a core group of friends who interact with each other frequently.

Some students including but not limited to "mkst_2367", "HM69", "Remant", "Vtee" have only one connection to the central or other nodes.

3. Graph Statistics

```
graph_diameter <- diameter(graph)
graph_diameter
```

```
## [1] 5
```

```
graph_density <- edge_density(graph)
graph_density
```
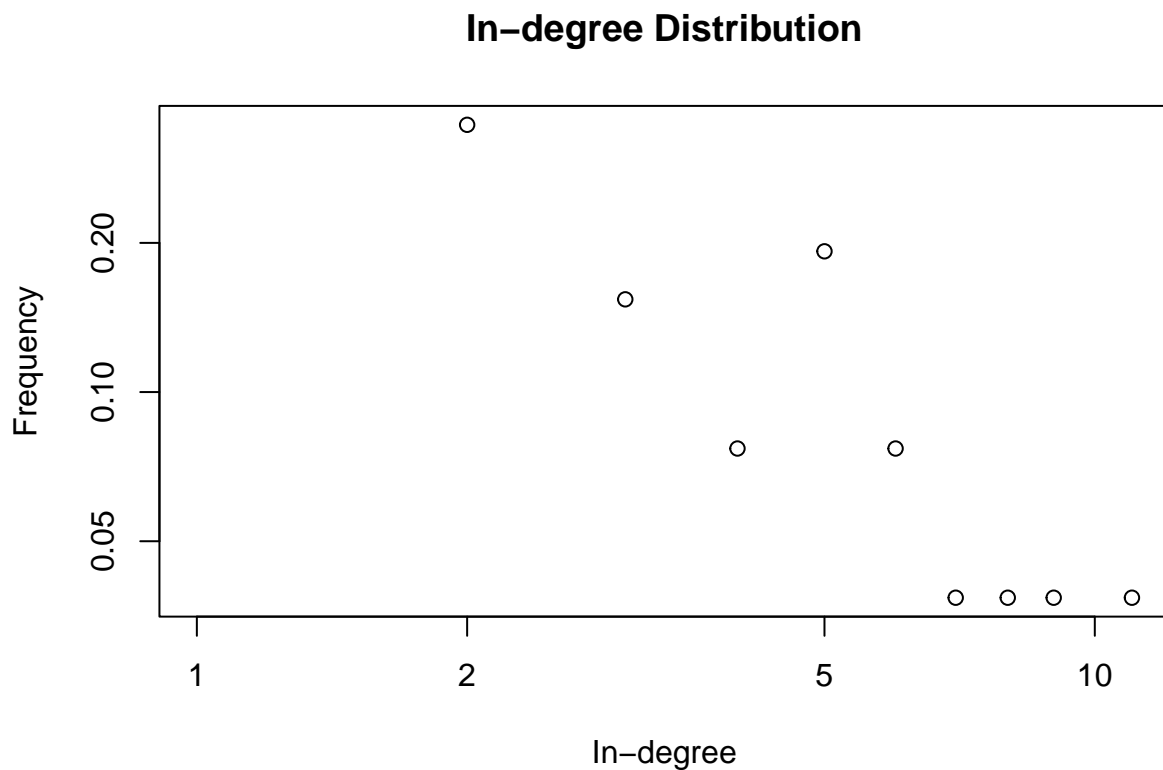
```
## [1] 0.1292308
```

The diameter of the graph is 5

The density of the graph is 0.1292308

```
in_degree_dist <- degree_distribution(graph, mode = "in")

plot(in_degree_dist, log = "xy", main = "In-degree Distribution", xlab = "In-degree",
     ylab = "Frequency")
```

```
## Warning in xy.coords(x, y, xlabel, ylabel, log): 2 y values <= 0 omitted from
## logarithmic plot
```



```
fit <- fit_power_law(degree(graph, mode = "in"))

power_law_coefficient <- fit$alpha
power_law_coefficient
```

```
## [1] 3.34032
```

Estimated Power Law coefficient (c): 3.3403197

The Power Law coefficient reveals that the student network has a few students with a lot of connections. The number of students with many friends drops off quickly. The students network is not dominated by a few highly connected students but rather has connections more evenly distributed among many students.

4. Graph Structure

```r
adj_matrix <- as_adjacency_matrix(graph, sparse = FALSE)

graph_adj <- graph_from_adjacency_matrix(adj_matrix, mode = "directed")

components <- components(graph_adj, mode = "strong")

gscc_index <- which.max(components$csize)
gscc_nodes <- which(components$membership == gscc_index)

in_nodes <- which(apply(distances(graph_adj, v = V(graph_adj),
                                  to = gscc_nodes, mode = "in"), 1, function(x)
                      any(x < Inf)) & !(V(graph_adj) %in% gscc_nodes))

out_nodes <- which(apply(distances(graph_adj, v = gscc_nodes,
                                   to = V(graph_adj), mode = "out"), 2, function(x)
                      any(x < Inf)) & !(V(graph_adj) %in% gscc_nodes))

tendrils_and_tubes_nodes <- setdiff(1:vcount(graph_adj), c(gscc_nodes, in_nodes, out_nodes))

V(graph_adj)$color <- "yellow"
V(graph_adj)$color[gscc_nodes] <- "red"
V(graph_adj)$color[in_nodes] <- "blue"
V(graph_adj)$color[out_nodes] <- "green"
V(graph_adj)$color[tendrils_and_tubes_nodes] <- "grey"

plot(graph_adj, layout = layout.kamada.kawai, vertex.size = 10, vertex.label.cex = 0.8,
     cex.lab = 0.8, vertex.color = V(graph_adj)$color, edge.arrow.size = 0.3,
     main = "Decomposition of the Student Network Graph",
     sub = "Red: GSCC, Blue: IN, Green: OUT, Grey: Tendrils and Tubes")
```
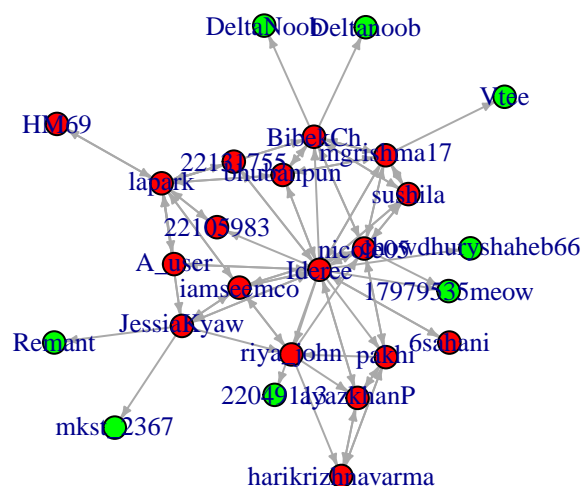
Decomposition of the Student Network Graph

Red: GSCC, Blue: IN, Green: OUT, Grey: Tendrils and Tubes

From the graph above we can observe that there are no IN's or Trendils and Tubes on the decomposed student network graph.

We can hence suggest that there is a core group of student (GSCC) those who share information and resources among each other leading to better academic performance. At the same time, there are students (OUT component) those who receive benefits from the GSCC but do not contribute back.

5. Account Popularity

```
page_rank_scores <- page_rank(graph, directed = TRUE, damping = 0.85)$vector

top_ten_accounts <- sort(page_rank_scores, decreasing = TRUE)[1:10]

in_deg <- degree(graph, mode = "in")

comparison_df <- data.frame(
  Account = names(page_rank_scores),
  PageRank = page_rank_scores,
  In_Degree = in_deg[names(page_rank_scores)]
)

top_ten_comparison <- comparison_df[comparison_df$Account %in% names(top_ten_accounts), ]
top_ten_comparison_sorted <- top_ten_comparison[order(-top_ten_comparison$PageRank), ]
print(top_ten_comparison_sorted[, -1])
```

```
##                     PageRank In_Degree
## Ideree            0.10655005        10
## lapark            0.10142472         8
## nicole05          0.06705909         7
## pakhi             0.06059762         4
## ayazkhanP         0.05778222         4
## BibekCh           0.05759432         6
## harikrizhnavarma  0.04829919         3
## iamseemco         0.04818828         5
## mgrishma17        0.04713420         5
## riya_john         0.04151589         4
```

High PageRank and High In-Degree:

lderee has the highest PageRank of 0.10655 and the highest in-degree of 10 followed by lapark (PageRank 0.1014247, in-degree 8). This indicates that lderee is not only highly connected but also important within the student network.

Moderate PageRank and In-Degree:

Several students exhibit moderate PageRank and in-degree, indicating notable influence and connectivity within the network. Notably, pakhi (PageRank 0.0605976, in-degree 4), nicole05 (PageRank 0.0670591, in-degree 7), BibekCh (PageRank 0.0575943, in-degree 6), and ayazkhanP (PageRank 0.0577822, in-degree 4) are key nodes with moderate incoming connections, reflecting their significance and steady influence within the network.

Lower PageRank and In-Degree:

Despite having fewer connections, some students exhibit a lower PageRank and in-degree, indicating their connections might be of high quality or importance. Notably, harikrizhnavarma (PageRank 0.0482992, in-degree 3), iamseemco (PageRank 0.0481883, in-degree 5), mgrishma17 (PageRank 0.0471342, in-degree 5) and riya_john (PageRank 0.0415159, in-degree 4) demonstrate this pattern, suggesting their significant yet specialized influence within the network.

6. Account Selection

Expected Payoff Calculation:

$$E_1 = 1 \cdot p_1 + 0.5 \cdot p_2 + 0.4 \cdot p_3$$
$$E_2 = 0.2 \cdot p_1 + 1 \cdot p_2 + 0.2 \cdot p_3$$
$$E_3 = 0.1 \cdot p_1 + 0.2 \cdot p_2 + 1 \cdot p_3$$

Equating the Expected Payoffs:

$$E_1 = E_2$$
$$1 \cdot p_1 + 0.5 \cdot p_2 + 0.4 \cdot p_3 = 0.2 \cdot p_1 + 1 \cdot p_2 + 0.2 \cdot p_3$$
$$4 \cdot p_1 - 2.5 \cdot p_2 + p_3 = 0 \quad \text{(Equation 1)}$$
$$E_1 = E_3$$
$$1 \cdot p_1 + 0.5 \cdot p_2 + 0.4 \cdot p_3 = 0.1 \cdot p_1 + 0.2 \cdot p_2 + 1 \cdot p_3$$
$$9 \cdot p_1 + 3 \cdot p_2 - 6 \cdot p_3 = 0 \quad \text{(Equation 2)}$$

Constraint equation:

$$p_1 + p_2 + p_3 = 1 \quad \text{(Equation 3)}$$

The system of equations can be represented in matrix form as:

$$\begin{pmatrix} 4 & -2.5 & 1 \\ 9 & 3 & -6 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Solving with R

```
m <- matrix(c(
  4, -2.5, 1,
  9, 3, -6,
  1, 1, 1
), nrow = 3, byrow = TRUE)

cons <- c(0, 0, 1)

solution <- solve(m, cons)

solution
```

[1] 0.1509434 0.4150943 0.4339623

Probability of choosing Parramatta: 0.1509434

Probability of choosing Kingswood: 0.4150943

Probability of choosing Campbelltown: 0.4339623

The higher probabilities of the Kingswood and Campbelltown (both above 40%) indicate a strategic focus on these campuses, perhaps due to their larger or more interconnected student populations, which can provide more comprehensive data for the study. The lower probability for Parramatta (approximately 15%) suggests it might be less central or have a smaller student population, reducing the risk of predictable and manipulable social networks.