1. Verify argument by Rules of inference:

P: lives in France.

q : speaks French.

r : drive a Dastun.

s : rides a bicycle

Formulation

$\sim p \to \sim q$

$\sim r$

$p \to s$

$q \vee r$

$\therefore s$

Validation.

Assertion	Reason.
1. $q \vee r$	Premise
2. $\sim r$	premise.
3. q	Disjunction syllogism
4. $\sim p \to \sim q$	Premise.
5. $\sim(\sim p)$	Moden's Tollen's
6. $p \to s$	Moden's Ponen's
7. s	

Conclusion: Since given statement follows rules of inferences, so it is Valid statement.

2. Consider

$M(x)$: x is a men.

$F(x)$: x is a falliable.

$K(x)$: x is a king.

Formulation:

$\forall x\, [M(x) \rightarrow F(x)]$

$\forall x\, [K(x) \rightarrow M(x)]$

$\therefore \forall x\, [K(x) \rightarrow F(x)]$

Validation :-

Assertion	Reason
$\forall x\, [M(x) \rightarrow F(x)]$	Premise
$M(c) \rightarrow F(c)$	universal Installisation. (1)
$\forall x\, [K(x) \rightarrow M(x)]$	Premise
$K(c) \rightarrow M(c)$	universal Installisation. (2)
$K(c) \rightarrow F(c)$	from. (1) and (2). By Hypothetical syllogism.
$\forall x\, [K(x) \rightarrow F(x)]$	by universal generalisation.

Conclusion :

$\therefore$ The given statement obeys the rules of inferences. Hence these are Valid.

3.

$M(x)$: x is a mother

$F(x)$: x is male.

$P(x)$: x is politician.

Formulation :-

$\forall x \cdot [M(x) \rightarrow \sim F(x)]$

$\exists x [F(x) \wedge P(x)]$

$\exists x [P(x) \wedge \sim F(x)]$

Validation

~~Assertion~~	Reason.
$\exists x [F(x) \wedge P(x)]$	Premice,
$F(c) \wedge P(c)$	Existential Installation.
$F(c)$	simplification (1)
$P(c)$	simplification (2)
$\forall x [M(x) \rightarrow \sim F(x)]$	Premice.
$M(c) \rightarrow \sim F(c)$	Universal Instatlation (3)
$\sim M(c)$	from 1 and 3. Modens Tollens
$\exists x [P(x) \wedge \sim F(x)]$	Premice

Conclusion :- Given statement is invalid, since it doesn't obeys rules of inference.

4. Given,

$$\forall x\, [P(x) \rightarrow Q(x)]$$
$$\exists x\, P(x)$$

$$\therefore \exists x\, Q(x)$$

Validation

Assertion	Reason
$\forall x\, [P(x) \rightarrow Q(x)]$	Premise.
$P(c) \rightarrow Q(c)$	universal Installisation. (1)
$\exists x\, P(x)$	Premise
$P(c)$	Existential Installisation. (2)
$Q(c)$	From ① and ② Moden Ponens
$\exists x\, Q(x)$	Existential Generalisation.

Conclusion :- $\therefore \exists x\, Q(x)$ is a valid conclusion for given statements.

5. Let,

$I(x)$: x be a integer.

$R(x)$: x be irrational

$P(x)$: x be a powers of 2

Formulation,

$$\forall x\, [I(x) \rightarrow R(x)]$$
$$\exists x\, [I(x) \land P(x)]$$
$$\therefore \exists x\, [R(x) \land P(x)]$$

Validation

Assertion	Reason
$\exists x[I(x) \wedge P(x)]$	Premise.
$I(c) \wedge P(c)$	Existential Installisation,
$I(c)$	simplification (1)
$P(c)$	simplification (2)
$\forall x[I(x) \to R(x)]$	Premise.
$I(c) \to R(c)$	Universal Installisation. (3)
$R(c)$	(1) and (3) Moden Ponen's. (4).
$P(c) \wedge R(c)$	(2) and (4) Conjuction.
$\exists x[P(x) \wedge R(x)]$	Existential Generalisation.

Conclusion : Given statements are Valid and obeys Rules of inferences,

6. Let :-

$p(x)$: x be a square,

$q(x)$: x be Equal sides.

$r(x)$: x be a Rombous

Formulation :-

$\forall x[p(x) \to q(x)]$

$r(x) \to q(x)$

$r(x) \to p(x)$

Validation

Assertion	Reason
$\forall x\,[p(x) \to q(x)]$	Premise. (1)
$p(c) \to q(c)$	universal instantiation.
$r(c) \to p(c)$	Premise. (2)
$r(c) \to q(c)$	(1) and (2) Hypothetical syllogism

Conclusion : Given statements follows Rules of inference Hence, they are valid.

7. Consider.

$p(x)$: x is mortal.

$q(x)$: x is a man.

Formulation :-

$$\frac{q(c)}{\therefore \forall x\,[q(x) \to}$$

p : Mortal

x : Man.

$$\frac{\exists x,\ p(x)}{\therefore \forall x\,[p(x) \vee \sim p(x)]}$$

By Addition rule.

8. p : advertisement is successful.

q : sales of product will go up

r : production will stopped.

$p \to q$.

$p \vee r$

$\sim q$

$\therefore r$

Validation :

Assertion	Reason
$p \to q$	premise
$\sim q$	premise
$\sim p$	By moden Tollen's
$p \vee r$	Premise
$\therefore r$	By Disjunction syllogism.

Conclusion:- It is a valid statement, since it follows rules of inference.

9. no. of 1 digits — $= {}^5P_1 = 5$

no. of 2 digits — — $= {}^5P_2 = 5 \times 4 = 20$

no. of 3 digits — — — $= {}^5P_3 = 5 \times 4 \times 3 = 60$

no. of 4 digits — — — — $= {}^5P_4 = 5 \times 4 \times 3 \times 2 = 120$

no. of 5 digits — — — — — $= {}^5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Total no. of ways $= 120 + 120 + 60 + 20 + 5 = 325$

10. no. of ways distributed n objects to r persons is

${}^{n+r-1}C_{r-1}$ here 2-boys, 10-rings

${}^{10+2-1}C_{2-1} = {}^{11}C_1 = 11$

no. of ways is 11.

11. no. of 1 letters $= 26 \times 10^4$

no. of 2 letters $= 26^2 \times 10^4$

no. of 3 letters ---- $= 26^3 \times 10^4$.

$\therefore$ Total no. of ways $= (26 + 26^2 + 26^3)\, 10^4$

$= 26 \times 10^4 (1 + 26 + 26^2)$

Total $= 18278 \times 10^4$

12. you can take pair as 1 set $\therefore$ 9 seats

therefore 9!.

However remaining cannot sit next to the other special person, so only 8 ways for him.

$\therefore$ Total no. of ways is $8 \times 9!$.

13. The 3 combination of a, b, c, d with infinite $\{\infty.a, \infty.b, \infty.c, \infty.d\}$.

There are 20 such 3-combinations,

aaa, aab, aac, aad,
bbb, bba, bbc, bbd,
ccc, cca, ccb, ccd,
ddd, dda, ddb, ddc,
abc, abd, acd, bcd.

14. a _ _ _ _ _ b (or) b _ _ _ _ _ a

$^{24}P_5 + {}^{24}P_5$ ways

$\therefore 2 \times {}^{24}P_5$

no. of ways $= 2 \times \frac{24!}{(19)!} = 2 \times 24 \times 23 \times 22 \times 21 \times 20$

15. Total players – 10

(a) include both strong and weak player

so, total – 8 players

required – 3 players since 2 confirmed,

$\Rightarrow$ no. of ways is $^8C_3 = 56$ ways.

(b) include strong, Exclude weak players

so, total – 8 players.

required – 4

no. of ways is $^8C_4 = 70$ ways.

(c) Exclude strong and weak players

so, total – 8 players

required – 5.

no. of ways is $^8C_5 = {^8C_3} = 56$ ways.

16..

Coefficient of x^4 in $(1+x+x^2+x^3)(1+x+x^2+x^3+x^4)(1+x+x^2+ \dots x^{12})$.

$$= \left(\frac{x^4-1}{x-1}\right)\left(\frac{x^5-1}{x-1}\right)\left(\frac{x^{15}-1}{x-1}\right)$$

$$= \frac{(x^4-1)(x^5-1)(x^{12}-1)}{(x-1)^3}$$

$$= (x^4-1)(x^5-1)(x^{12}-1)\left[{^{3+r-1}C_2}\, x^r\right)$$

$$= -3 \times {^{3+4-1}C_2} = -3 \times {^6C_2} = -3 \times \frac{6!}{4!\,2!} = -45.$$

17. R.R using generating function.

$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2 \longrightarrow (1)$

w.k.T $A(x) = \sum_{n=0}^{\infty} a_n x^n$

By multiply every term of (1) with x^n and do Summation from $n=2$ to ∞.

$$\sum_{n=2}^{\infty} a_n x^n - 7\sum_{n=2}^{\infty} a_{n-1} x^n + 10\sum_{n=2}^{\infty} a_{n-2} x^n = 0$$

$$A(x) - a_0 - a_1 x - 7x[A(x) - a_0] + 10x^2 A(x) = 0$$

$$A(x)[1 - 7x + 10x^2] - a_0 - a_1 x + 7a_0 x = 0$$

$$A(x) = \frac{a_0 + 7a_0 x + a_1 x}{1 - 7x + 10x^2}$$

$$A(x) = \frac{a_0(1+7x) + a_1 x}{1 - 7x + 10x^2}$$

$$= \frac{a_0(1+7x) + a_1 x}{(5x-1)(2x-1)} = \frac{a_0(1+7x) + a_1 x}{(1-5x)(1-2x)}$$

$$\therefore A(x) = \frac{c_1}{1-5x} + \frac{c_2}{1-2x}$$

$$\sum_{n=0}^{\infty} a_r x^r = c_1 \sum_{r=5}^{\infty} 5^r x^r + c_2 \sum_{r=2}^{\infty} 2^r x^r$$

$$\boxed{\therefore a_r = c_1 5^r + c_2 2^r}$$

18.

$a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0. \quad n \geq 3 \longrightarrow (1)$

multiply Every term of (1) with x^n and summation from $n=3$ to ∞

$$\sum_{n=3}^{\infty} a_n x^n - 9\sum_{n=3}^{\infty} a_{n-1}x^n + 26\sum_{n=3}^{\infty} a_{n-2}x^n - 24\sum_{n=3}^{\infty} a_{n-3} = 0.$$

$$A(x) - a_0 - a_1x - a_2x^2 - 9x[A(x) - a_0 - a_1x] + 26x^2[A(x) - a_0] - 24x^3 A(x) = 0$$

$$A(x)[1 - 9x + 26x^2 - 24x^3] - a_0 - a_1x - a_2x^2 + 9a_0x + 9a_1x^2 - 26a_0x^2 = 0$$

$$A(x) = \frac{a_0 + a_1x + a_2x^2 - 9a_0x - 9a_1x^2 + 26a_0x^2}{[1 - 9x + 26x^2 - 24x^3]}$$

$$\therefore A(x) = \frac{a_0(1 - 9x + 26x^2) + a_1(x + 9x^2) + a_2x^2}{(1 - 9x + 26x^2 - 24x^3)}$$

19.

$\cancel{a_n - 4a_{n-1} + 4a_{n-2} = 2^n.}$

$$A(x) = \frac{c_1}{1-2x} + \frac{c_2}{1-3x} + \frac{c_3}{1-4x}.$$

$$\sum_{n=r}^{\infty} a^r x^r = c_1\sum_{r=2}^{\infty} 2^r x^r + c_2\sum_{r=3}^{\infty} 3^r x^r + c_3\sum_{r=4}^{\infty} 4^r x^r$$

$$\boxed{\therefore a^r = c_1 2^r + c_2 3^r + c_3 4^r}$$

19. $a_n - 4a_{n-1} + 4a_{n-2} = 2^n \longrightarrow$ (1)

$c(t) = t^2 - 4t + 4$

$\therefore$ characteristics roots are 2, 2

$\therefore a_n^h = c_1 2^n + c_1 n 2^n$

$a_n^p = A \cdot n^2 2^n \quad [a_n^p = A n^m a^n]$

$a_n^p = A n^2 2^n$

a_n satisfies (1)

$An^2 2^n - A(n-1)^2 2^{n-1} + 4A(n-2)^2 2^{n-2} = 2^n$

if $\underline{n=2}$

$A(4)(4) - A(1)^2 2^1 + 0 = 4$

$16A - 2A = 4.$

$14A = 4$

$\boxed{A = 2/7}$

$a_n^p = \frac{2}{7} n^2 2^n$

$\boxed{a_n^p = \frac{n^2}{7} 2^{n+1}}$

20. $a_n - 7a_{n-1} + 10a_{n-2} = 7\cdot 3^n + 4^n$

$$A(x) = \frac{a_0(1-7x)+a_1x}{1-7x+12x^2} = \frac{P(x)}{Q(x)}$$

Ch. polynomial $= t^2 \cdot Q\left(\frac{1}{t}\right)$.

$$= t^2\left(1-\frac{7}{t}+\frac{10}{t^2}\right)$$

$$= t^2 - 7t + 10$$

$\therefore$ 5, 2 are characteristic roots.

Hence $\boxed{a_n^h = c_1 5^n + c_2 2^n} \rightarrow (*)$

Case 1

$a_n - 7a_{n-1} + 10a_{n-2} = 7\cdot 3^n \rightarrow (1)$

is of form. $D\cdot a^n$..

$$a_n^p = Aa^n$$

$$a_n^p = A\, 3^n \rightarrow (2)$$

sub (2) in (1).

$$A3^n - 7A3^{n-1} + 10\cdot 3^{n-2}A = 7\cdot 3^n$$

$$\cancel{3^{n-2}}\cdot A\left[3^2 - 7\cdot 3 + 10\right] = 7\cdot \cancel{3^n}\, 3^2$$

$$A\left[9-21+10\right] = 63.$$

$$-2A = 63$$

$$\boxed{A = \frac{-63}{2}}$$

$$\boxed{a_n^p = -\frac{63\cdot 3^n}{2}} \rightarrow (A)$$

$a_n - 7a_{n-1} + 10a_{n-2} = 4^n \rightarrow$ (3)

is of form a^n

$a_n^p = Aa^n = A4^n \rightarrow$ (4).

Sub (4) in (3)

$A4^n - 7A4^{n-1} + 10A4^{n-2} = 4^n$

$A\cancel{4^{n-2}}[4^2 - 7.4 + 10] = \cancel{4^{n-2}}.16$

$A[16-28+10] = 16.$

$-2A = 16$

$\boxed{A = -8}$

$\boxed{a_n^p = -8\,4^n} \rightarrow$ (B)

$\boxed{a_n^p = -\frac{63}{2}.3^n - 8.4^n}$ From (A) and (B)

$a_n = a_n^p + a_n^h$

$a_n = C_1 5^n + C_2 2^n - \frac{63}{2}.3^n - 8.4^n$

$\boxed{a_n = C_1 5^n + C_2 2^n - 32.5.3^n - 4^{n+2}}$