Definitions:

Group, Subgroup, Cyclic group, Abelian group, Homomorphism, Monomorphsim, Isomorphism, Epimorphism, Automorphism, Kernel, Index, Permutation group (S_n), Left coset and right coset of H in G, Index of H in G,

Theorems:

1. Prove that (Z_n, $+_n$) is a cyclic group.

2. Let G be a group and a$\in$ G. Then O(a) is the order of the cyclic group generated by a.

3. Let (G, *) be a group and S$\subseteq$ G. Then (S, *) is a subgroup of (G,*) if and only if $a*b^{-1} \in S$ for all a,b in S.

4. Let f: G $\longrightarrow$ G' be a group homomorphism from (G,*) to (G', o). Let e and e' be the identity elements of G and G' then (i) f(a) = e' (ii) $f(a^{-1}) = (f(a))^{-1}$ for all a in G. (iii) $f(a*b^{-1}) = f(a) \text{ o } (f(b))^{-1}$ for all a, b in G . (iv) f(H) is a subgroup of G whenever H is a subgroup of G.

5. A group homomorphism f is a monomorphism if and only if kerf = {e}.

6. (i) Any infinite cyclic group is isomorphic to (Z , +). (ii) Any cyclic group of order n is isomorphic to (Z_n ,$+_n$).

7. **Cayley's Theorem**:- A finite group (G,*) of order n is isomorphic to a group of permutations of G.

8. Let H be a subgroup of G. Then (i) a $\in$ H if and only if aH = H. (ii) aH = bH iff $a^{-1} *b \in H$ (iii) a $\in$ bH iff aH = bH.

9. **Lagrange's Theorem of finite groups:-**
   Statement:- Let G be a finite group and H be any subgroup of G. Then the order of H divides the order of G.

10. Every group of prime order is cyclic.

11. Let H be a subgroup of G. Then the following statements are equivalent.
    (i)$aH = Ha\ \forall\ a \in G$; (ii) $a^{-1}Ha = H\ \forall\ a \in G$; (iii) $a^{-1}Ha \subseteq H\ \forall\ a \in G$.