Assignment-II

1) $f(x) = x^2$

is not one-to-one because $f(-5)$ and $f(5)$ have the same image i.e 25 which contradicts the defination one-one.

Here $f(x)$ is not one-one.

2) Equivalance realation :- If a relation satisfies

(i) reflexive

(ii) ~~Reflex~~ Symmetry

(iii) Transitive

$aRb \rightarrow$ iff $l(a) = l(b)$

$\rightarrow$ It is reflexive since $l(a) = l(a)$

$\rightarrow$ let $a, b \in R$

if $aRb \Rightarrow l(a) = l(b)$

$\Rightarrow l(b) = l(a)$

$\Rightarrow bRa$

$\therefore$ It is symmetry

$\rightarrow$ let $a, b, c \in R$

let aRb & bRc

$\Rightarrow l(a) = l(b)$ and $l(b) = l(c)$

$\Rightarrow l(a) = l(c)$

$\Rightarrow aRc$

$\therefore$ It is transitive

3) $R = \{(a, b) / a \equiv b \pmod m\}$

$a \equiv b \pmod m$ means $(a-b)$ is divisible by m.

$\rightarrow$ since $a - a = 0$ is divible by any number (say m)

$\Rightarrow R$ is reflexive

→ let $a \equiv b \pmod m$

$\cancel{\frac{a-b}{m}}$ $a-b$ is divisibt by m

$-(a-b)$ is also divisible by m

$b-a$ " "

$\Rightarrow$ bRa

$\therefore$ It is symmetry

→ let $a,b,c \in R$

let aRb & bRc

$\frac{a-b}{m} = k$ $\qquad \frac{b-c}{m} = l$ (say l, k are constants)

$a-b = mk$ —① $\qquad b-c = ml$ —②

① + ② ⇒

$a-b+b-c = mk+ml$

$a-c = m(k+l)$ say x be another constant

$a-c = mx$

$\frac{a-c}{m} = x$

$\Rightarrow$ aRc

$\therefore$ It is transitive.

4) (i) Partial order set:- A set satisfies

1) Reflexive
2) Anti-Symmetry
3) Transitive.

ex:- $(Z^+, \leq)$

(ii) Total ordered set:-

If every pair of elements in a given poset are comparable wrt the relation is said to be totally ordered set.

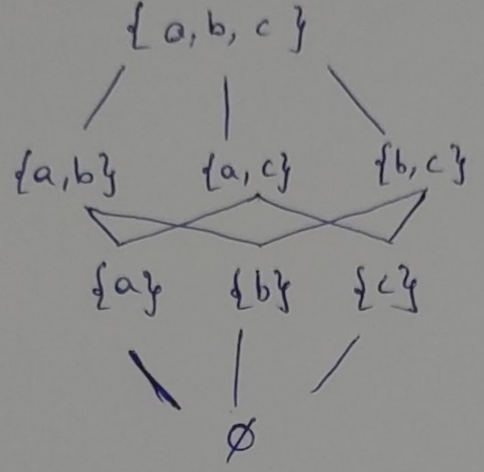
ex:- $(Z^+, \leq)$

(iii) Well ordered set:- A poset is said to be well ordered set if every subset of S must contain a minimal element.

ex:- $(Z^+, \leq)$

5) let $s = \{a, b, c\}$

$P(A) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\} \}$



If least element exists it should be unique

In this case least element is $\emptyset$ and greatest is $\{a,b,c\}$.

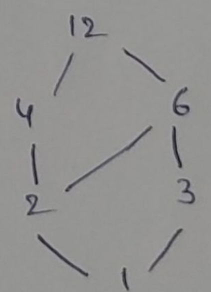
$\therefore$ least and Greatest elements does exist in $P(s)$ for any set s.

6)

<u>Hasse diagram</u>:- Graphical representation of relation of elements of poset.

$(D_{12}, /)$

$D_{12} = \{1, 2, 3, 4, 6, 12\}$



maximal, greatest element = 12

minimal, least element = 1

Problem: Prove that $(Z_n, +_n)$ is a cyclic group.

Solution: we know that $Z_n = \{0,1,2,\ldots..n-1\}$ for 1, 2 in Z_n, we have $1 +_n 2 \in Z_n$. Thus Z_n is closed under $+_n$ and also $a +_n b = b +_n a$ for all a, b in Z_n, Thus $+_n$ is commutative.

To prove that $+_n$ is associative: By division algorithm consider the following

$a + b = q1n + r1$ for $0 \le r1 \le n$ -----(1) ;

$b + c = q2\,n + r2$ for $0 \le r2 < n$ ------(2) and

$r1 + c = q3\,n + r3$ for $0 \le r3 < n$ -----(3).

So, $a +_n b = r1$ and $b +_n c = r2$; and $r1 +_n c = r3$.

Now $(a+b)+c = q1n + r1 + c$ [form (1)] $= q1n + q3n + r3$ [from (3)] ------(4)

So $(a +_n b) +_n c = r1 +_n c = r3$ [from (3)] also $a +_n (b +_n c) = a +_n r2$ -----(5)

Now $a + r2 = a + b + c - q2n$ [from 2] $= q1n + q3n + r3 - q2n = (q1+q3-q2)n + r3$ which implies $a +_n (b +_n c) = a +_n r2 = r3$ -------(6),

Thus from (5) & (6) we have $(a +_n b) +_n c = a +_n (b +_n c) = r3$., which proves that $+_n$ is associative. 0 is the identity element of Z_n .

The inverse of i for $1 < i < n$ is n-i.

Also any r in Z_n can be written as $1 +_n 1 +_n 1 +_n$ ---- --$+_n$ 1, 1 being repeated for r times ,

So $(Z_n, +_n)$ is a cyclic group.

Definition:- order of an element:- Let (G,*) be a group and a be an element of G, then the order of the element a is the smallest positive integer n for which $a^n = e$, if such an integer exists and is denoted by O(a).

Q) order of elements of $(Z_8, +_8)$

$Z_8 = \{0, 1, ---7\}$

* $0 +_8 0 = 0$ $\rightarrow$ order of 0 i.e $O(0) = 0$

* $1 +_8 1 = 2$

$1 +_8 1 +_8 1 = 3$

$1 +_8 1 +_8 1 +_8 1 = 4$

$1 +_8 1 +_8 1 +_8 1 +_8 1 = 5$

$= 6$

$= 7$

$1 +_8 1 +_8 1 +_8 1 +_8 1 +_8 1 +_8 1 +_8 1 = 8$

1 is raised 8 times to get identity

$O(1) = 8$

* $2 +_8 2 = 4$

$2 +_8 2 +_8 2 = 6$ $O(2) = 4$

$2 +_8 2 +_8 2 +_8 2 = 0$

* $3 +_8 3 = 6$

$3 +_8 3 +_8 3 = 1$

$3 +_8 3 +_8 3 +_8 3 +_8 3 +_8 3 +_8 3 +_8 3 = 0$ $O(3) = 8$

* $O(4) = 2$
* $O(5) = 8$
* $O(6) = 4$
* $O(7) = 8$