

UNIT-IV

VELAGAPUDI RAMAKRISHNA

SIDDHARTHA ENGINEERING COLLEGE

(AUTONOMOUS)

II/IV B.Tech. DEGREE EXAMINATION, NOVEMBER, 2017

Third Semester

14MA1301 COMPLEX ANALYSIS AND NUMERICAL METHODS

Time: 3 hours

Max. Marks: 70

Part-A is compulsory

Answer One Question from each Unit of Part - B

Answer to any single question or its part shall be written at one place only

PART-A

10 x 1 = 10M

8. a. Evaluate $\int_0^6 \frac{1}{1+x} dx$, using Simpson's 1/3 and 3/8 rules. 8M
- b. Compute $y(0.1)$ and $y(0.2)$ given that $\frac{dy}{dx} = x + y$ with $y(0) = 1$, by using Runge-Kutta fourth order method. 7M

(or)

9. a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ from the following table 8M

| | | | | | | | |
|---|------|------|------|------|------|------|------|
| x | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 | 2.2 |
| y | 2.72 | 3.32 | 4.06 | 4.96 | 6.05 | 7.39 | 9.02 |

- b. Compute $y(2)$ given that $\frac{dy}{dx} = 3x^2 + 1$ with $y(1) = 2$, using Euler's method by taking the step size $h = 0.5$. 7M

* * *

1. a. Define harmonic function.
- b. Write real and imaginary parts of $f(z) = z^3$.
- c. Show that $f(z) = \bar{z}$ is nowhere analytic.
- d. Find all poles of $f(z) = \frac{1}{\sin z}$.
- e. Evaluate $\oint \frac{1}{z-3} dz$, over the circle $|z| = \frac{3}{2}$.
- f. Write Newton-Raphson formula to find the cube root of N.
- g. Write the formula of Trapezoidal rule.
- h. Prove that symbolic relation between E and μ .
- i. Define boundary value problem.
- j. Write Newton's forward difference interpolation formula.

UNIT-I

2. a. Show that $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ is continuous and

satisfies C-R equations at origin but not analytic at origin. 8M

- b. If $f(z)$ is an analytic function of z , then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2. \quad 7M$$

(or)

3. a. Evaluate $\oint_C \frac{z+4}{z^2+2z+5} dz$ where $C: |z+1-i|=2$ using Cauchy's integral formula. 8M

- b. Evaluate $\oint_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+1)(z+2)} dz$, where C is a circle $|z|=3$. 7M

UNIT-II

4. a. Find the Taylor's series expansion of $f(z) = \frac{1}{z^2+z-6}$ about $z = -1$. 7M

- b. Show that $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{2(a+b)}, (a > 0, b > 0)$. 8M

(or)

5. a. Find the Laurent's series expansion of $f(z) = \frac{7z^2+9z-18}{z^3-9z}$ about $z = 3$. 8M

- b. Discuss about the transformation $f(z) = \frac{1}{z}$. 7M

UNIT-III

6. a. Using Newton-Raphson method, find the real root of $x \tan x + 1 = 0$. 8M

- b. Applying Gauss forward interpolation formula, obtain $f(x)$ at $x = 3.5$ from the following table: 7M

| | | | | |
|----|-------|-------|-------|-------|
| x: | 2 | 3 | 4 | 5 |
| y: | 2.626 | 3.454 | 4.784 | 6.986 |

(or)

7. a. Solve the following system of equations by using Gauss elimination method 7M

$$2x_1 + x_2 + 2x_3 + x_4 = 6 \quad ; \quad 6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$

$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1 \quad ; \quad 2x_1 + 2x_2 - x_3 + x_4 = 10$$

- b. Using Lagrange's formula, find $f(x)$ from the following data: 8M

| | | | | | | |
|------|---|----|----|---|---|----|
| x | 0 | 1 | 2 | 4 | 5 | 6 |
| f(x) | 1 | 14 | 15 | 5 | 6 | 19 |