Reliability Design

Design a system using devices.
Each device is having some cost and Reliability.

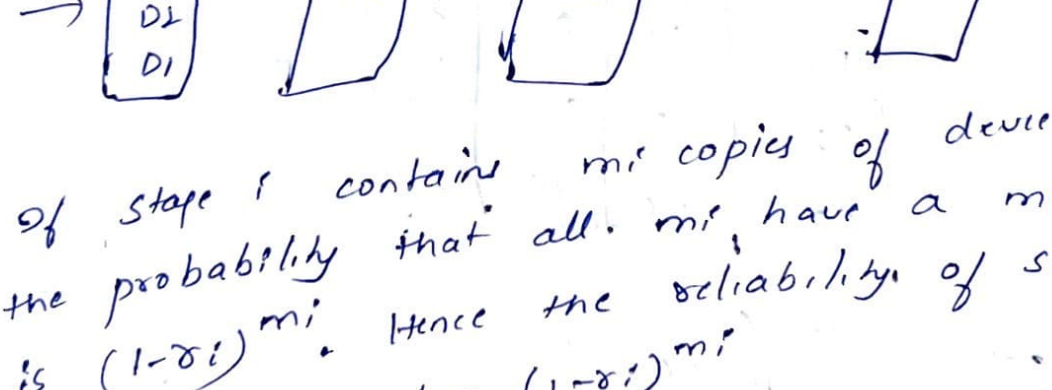
D1	D2	D3	D4
c_1	c_2	c_3	c_4
r_1	r_2	r_2	r_4
0.9	0.9	0.9	0.9

$\prod r_i = 0.9^4 = 0.6561$

The Reliability of devices may vary.
Set up a System with more reliability.

$\rightarrow$ [D1] $\rightarrow$ [D2] $\rightarrow$ [D3] - - - $\rightarrow$ [Dn] $\rightarrow$

Multiple copies of same device are connected in parallel.



If stage i contains m_i copies of device D_i, then the probability that all m_i have a malfunction is $(1-r_i)^{m_i}$. Hence the reliability of stage i becomes $1-(1-r_i)^{m_i}$

Reliability of stage i is given by $\phi_i(m_i)$ $1 \le n$

Reliability of system of stage is $\prod_{1 \le i \le n} \phi_i(m_i)$

maximize $\prod_{1 \le i \le n} \phi_i m_i$

subject to $\sum_{1 \le i \le n} c_i m_i \le C$

$m_i \ge 1$ and integer $1 \le i \le n$.

Ex

$r_1 = 0.9$

$1 - r_1 = 1 - 0.9 = 0.1$ (probability of not working)

$(1-r_1)^3 = (0.1)^3$ (3 copies)

$= 0.001$

$1 - (1-r_1)^3 = 0.999$ (probability that 3 copies are working properly)

(Reliability increased)

C is the cost (total amount).

Example

D_i	c_i	r_i	U_i (upper bound)
D1	30	0.9	2
D2	15	0.8	3
D3	20	0.5	3

$\sum c_i = c_1 + c_2 + c_3$

$30 + 15 + 20 = 65$

$C - \sum c_i = 105 - 65 = 40$ (Remaining amount)

$\frac{40}{30} = 1$

1st device $u_i = \left\lfloor \frac{C - \Sigma c_i}{c_i} \right\rfloor + 1 = \frac{40}{30} + 1 = 1 + 1 = 2$

2nd device $u_i = \frac{40}{15} = 2 + 1 = 3$ copies of device 2

3rd device $\frac{40}{20} = 2 + 1 = 3$

ordered pair (R, C)

$S^0 = \{(1, 0)\}$ Initially no device, no system so no cost

Consider D_1

(device 1 and 1 copy) $S_1^1 = \{(0.9, 30)\}$

$1 - (1 - r_1)^2 = 1 - (1 - 0.9)^2$

$= 1 - (0.1)^2$

$= 1 - 0.01$

$= 0.99$

(2 copies device 1) $S_2^1 = \{(0.99, 60)\}$

Combine these two to form

$S^1 = \{(0.9, 30), (0.99, 60)\}$

Consider D_2

$S_1^2 = \{(0.72, 45), (0.792, 75)\}$

$S_2^2 = \{(0.864, 60), (0.9504, 90)\}$

$1 - (1 - r_2)^2 = 1 - (1 - 0.8)^2$

$= 1 - (0.2)^2$

$= 1 - 0.04$

$= 0.96$

30 cost (2 copies)

$$1-(1-r_2)^3$$

$S_3^2 = (0.8928, 75), (\quad , 105)$ $\quad 1-(1-0.8)^3$

not feasible $\quad = 1-(0.2)^3$

$= 1-0.008$

$= 0.992, \quad 45$

$S^2 = \{(0.72, 45)\ \cancel{(0.792, 75)}\ (0.864, 60)\ (0.8928, 75)\}$ (3 copies)

(dominance Rule)

$S^2 = \{(0.72, 45)\ (0.864, 60)\ (0.8928, 75)\}$

Device D3

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$

$S_2^3 = \{(0.54, 85), (0.648, 100)\ (— , 115)\}$ (not feasible)

$r_3 = 0.5$

$1-(1-r_3)^2$

$1-(1-0.5)^2$

$1-(0.5)^2$

$1-0.25$

$0.75 \quad 40$ (2 copies)

$S_3^3 = \{(0.63, 105), (— , 120), (— , 135)\}$

$r_3 = 0.5$

$1-(1-0.5)^3$

$S_3^3 = \{(0.63, 105)\}$

$1-(0.5)^3$

$1-0.125$

$0.875 \quad 60$ (3 copies)

$S^3 = \{(0.36, 65)\ (0.432, 80)\ \cancel{(0.4464, 95)}$
$(0.54, 85)\ \cancel{(0.63, 105)}\ (0.648, 100)\}$

$S^3 = \{(0.36, 65)\ (0.432, 80)\ (0.54, 85)\ (0.648, 100)\}$

↳ Maximum Reliability is 0.648 and cost is 100.

D1	D2	D3
1	2	2

$\prod r_i = 0.648$

$\sum c_i = 100$