$$w_{ji} = w_{ji} + \Delta w_{ji}$$

where $\Delta w_{ji} = -n \frac{\partial E_d}{\partial w_{ji}}$ — ①

Also $E_d(\bar{w}) = \frac{1}{2} \sum_{k \in output} (t_k - o_k)^2$

$$\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} \times \frac{\partial net_j}{\partial w_{ji}}$$

$$net_j = \sum x_{ji} w_{ji}$$

$$\frac{\partial net_j}{\partial w_{ji}} = x_{ji}$$

$$\frac{\partial E_d}{\partial w_{ji}} = \frac{\partial E_d}{\partial net_j} \times x_{ji}$$

sub in ①

$$\boxed{\Delta w_{ji} = -n \frac{\partial E_d}{\partial net_j} x_{ji}}$$ — ②

$\frac{\partial E_d}{\partial net_j}$ expression depends on unit j — ① if j is output unit

② if j is internal unit.

(1) if j is output unit,

$$\frac{\partial E_d}{\partial net_j} = \frac{\partial E_d}{\partial o_j} \times \frac{\partial o_j}{\partial net_j}$$

$$= \frac{\partial E_d}{\partial o_j} \times o_j(1-o_j)$$

$$\left(\frac{\partial o_j}{\partial net_j} = \frac{\partial \sigma(net_j)}{\partial net_j} = \sigma(net_j)(1-\sigma(net_j)) = o_j(1-o_j) \right)$$

$$= -(t_j - o_j) \times o_j(1-o_j)$$

$$\left[\frac{\partial E_d}{\partial o_j} = \frac{\partial}{\partial o_j}\left(\frac{1}{2}(t_j - o_j)^2\right) = \frac{1}{2} \times 2(t_j - o_j)(-1) = -(t_j - o_j) \right]$$

Sub in (2)

$$\Delta w_{ij} = -n(-(t_j - o_j) \times o_j(1-o_j) \times x_{ji}$$

$$= n(t_j - o_j) \times o_j(1-o_j) \times x_{ji}$$

$$\boxed{\Delta w_{ij} = n\, \delta_j\, x_{ji}}$$

(2) if j is an internal node,

$$\frac{\partial E_d}{\partial net_j} = \sum_k \frac{\partial E_d}{\partial net_k} \times \frac{\partial net_k}{\partial net_j}$$

$$= \sum_k -\delta_k \times \frac{\partial net_k}{\partial net_j}$$

$$\frac{\partial net_k}{\partial net_j} = \frac{\partial net_k}{\partial o_j} \times \frac{\partial o_j}{\partial net_j}$$

$$= \frac{\partial net_k}{\partial o_j} \times o_j(1-o_j)$$

(from previou

$$= \frac{\partial \sum x_{jk} w_{jk}}{\partial o_j} \times o_j(1-o_j)$$

$$= \frac{\partial \sum o_j w_{jk}}{\partial o_j} \times o_j(1-o_j)$$

$$= w_{jk} \times o_j(1-o_j)$$

$$\therefore \frac{\partial E_d}{\partial net_j} = \sum_k -\delta_k \times w_{jk} \times o_j(1-o_j)$$

sub in 2

$$\Delta w_{ji} = \eta \sum_{k} \underbrace{\delta_k \, w_{jk} \; o_j (1 - o_j)}_{\delta} \, x_{ji}$$

$$\boxed{\Delta w_{ji} = \eta \, \delta \, x_{ji}}$$