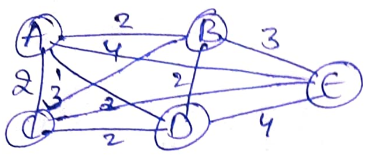
20IT4303-Advanced Data Structures-Home Assignment-3

① Travelling Salesperson problem to find optimal route using dynamic programming with 'A'.



A:-

	A	B	C	D	E
A	0	2	2	1	4
B	2	0	3	2	3
C	2	3	0	2	3
D	1	2	2	0	4
E	4	3	2	4	0

A is the starting point.

Iteration-1

Let the set of cities to be visited is zero

ie $|S|=0$

$P(B,\phi)=C_{BA}=2$

$P(C,\phi)=C_{CA}=2$

$P(D,\phi)=C_{DA}=1$

$P(E,\phi)=C_{EA}=4$

Iteration-2

$P(B,\{C\})=C_{BC}+P(C,\phi)=3+2=5$

$P(B,\{D\})=C_{BD}+P(D,\phi)=2+1=3$

$P(B,\{E\})=C_{BE}+P(E,\phi)=3+4=7$

$P(C,\{B\})=C_{CB}+P(B,\phi)=3+2=5$

$P(C,\{D\})=C_{CD}+P(D,\phi)=2+1=3$

$P(C,\{E\})=C_{CE}+P(E,\phi)=2+4=6$

$P(D,\{B\})=C_{DB}+P(B,\phi)=2+2=4$

$P(D,\{C\})=C_{DC}+P(C,\phi)=4$

$P(D,\{E\})=C_{DE}+P(E,\phi)=8$

$P(G,\{B\})=C_{EB}+P(B,\phi)=5$

$P(G,\{C\})=C_{EC}+P(C,\phi)=4$

$P(G,\{D\})=C_{ED}+P(D,\phi)=5$

Iteration-3

$|S|=2$

$$P(B,\{C,D\}) = \min \begin{array}{l} j=C \\ j=D \end{array} \begin{pmatrix} C_{B,C}+P(C,\{D\}) \\ C_{BD}+P(D,\{C\}) \end{pmatrix} = \min\begin{pmatrix} 3+3 \\ 2+4 \end{pmatrix} = 6$$

$$P(B,\{C,E\}) = \min \begin{array}{l} j=C \\ j=E \end{array} \begin{pmatrix} C_{BC}+P(C,\{E\}) \\ C_{BE}+P(E,\{C\}) \end{pmatrix} = \min\begin{pmatrix} 3+6 \\ 3+4 \end{pmatrix} = 7$$

$$P(B,\{D,E\}) = \min \begin{array}{l} j=D \\ j=E \end{array} \begin{pmatrix} C_{BD}+P(D,\{E\}) \\ C_{BE}+P(E,\{D\}) \end{pmatrix} = \min\begin{pmatrix} 2+8 \\ 3+5 \end{pmatrix} = 8$$

$$P(C,\{B,D\}) = \min \begin{array}{l} j=B \\ j=D \end{array} \begin{pmatrix} C_{CB}+P(B,\{D\}) \\ C_{CD}+P(D,\{B\}) \end{pmatrix} = \min\begin{pmatrix} 3+3 \\ 2+4 \end{pmatrix} = 6$$

$$P(C,\{B,E\}) = \min \begin{array}{l} j=B \\ j=E \end{array} \begin{pmatrix} C_{CB}+P(B,\{E\}) \\ C_{CE}+P(E,\{B\}) \end{pmatrix} = \min\begin{pmatrix} 3+7 \\ 2+5 \end{pmatrix} = 7$$

$$P(C,\{D,E\}) = \min \begin{array}{l} j=D \\ j=E \end{array} \begin{pmatrix} C_{CD}+P(D,\{E\}) \\ C_{CE}+P(E,\{D\}) \end{pmatrix} = \min\begin{pmatrix} 10 \\ 7 \end{pmatrix} = 7$$

$$P(D,\{B,C\}) = \min \begin{array}{l} j=B \\ j=C \end{array} \begin{pmatrix} C_{DB}+P(B,\{C\}) \\ C_{DC}+P(C,\{B\}) \end{pmatrix} = \min\begin{pmatrix} 2+5 \\ 2+5 \end{pmatrix} = 7$$

$$P(D,\{B,E\}) = \min \begin{array}{l} j=B \\ j=E \end{array} \begin{pmatrix} C_{DB}+P(B,\{E\}) \\ C_{DE}+P(E,\{B\}) \end{pmatrix} = 9$$

$$P(D,\{C,E\}) = \min = 8$$

$$P(E,\{B,C\}) = \min \begin{array}{l} j=B \\ j=C \end{array} \begin{pmatrix} C_{EB}+P(B,\{C\}) \\ C_{EC}+P(C,\{B\}) \end{pmatrix} = 7$$

$$P(E,\{B,D\}) = \min \begin{array}{l} j=B \\ j=D \end{array} \begin{pmatrix} C_{EB}+P(B,\{D\}) \\ C_{ED}+P(D,\{B\}) \end{pmatrix} = 6$$

$$P(E,\{C,D\}) = \min \begin{array}{l} j=C \\ j=D \end{array} \begin{pmatrix} C_{EC}+P(C,\{D\}) \\ C_{ED}+P(D,\{C\}) \end{pmatrix} = 6$$

Iteration-4

$|S|=3$

$$P(B,\{C,D,E\}) = \min \begin{array}{l} j=C \\ j=D \\ j=E \end{array} \begin{pmatrix} C_{BC}+P(C,\{D,E\}) \\ C_{BD}+P(D,\{C,E\}) \\ C_{BE}+P(E,\{C,D\}) \end{pmatrix} = \begin{pmatrix} 3+7 \\ 2+8 \\ 3+6 \end{pmatrix} = 9$$

$$P(C,\{B,D,E\}) = \min \begin{array}{l} j=B \\ j=D \\ j=E \end{array} \begin{pmatrix} C_{CB} + P(B,\{D,E\}) \\ C_{CD} + P(D,\{B,E\}) \\ C_{CE} + P(E,\{B,D\}) \end{pmatrix} = \min \begin{pmatrix} 3+8 \\ 2+9 \\ 2+6 \end{pmatrix} = 8$$

$$P(D,\{B,C,E\}) = \min \begin{array}{l} j=B \\ j=C \\ j=E \end{array} \begin{pmatrix} C_{DB} + P(B,\{C,E\}) \\ C_{DC} + P(C,\{B,E\}) \\ C_{DE} + P(E,\{B,C\}) \end{pmatrix} = \min \begin{pmatrix} 2+7 \\ 2+7 \\ 4+7 \end{pmatrix} = 9$$

$$P(E,\{B,C,D\}) = \min \begin{array}{l} j=B \\ j=C \\ j=D \end{array} \begin{pmatrix} C_{EB} + P(B,\{C,D\}) \\ C_{EC} + P(C,\{B,D\}) \\ C_{ED} + P(D,\{B,C\}) \end{pmatrix} = 8$$

<u>Iterations</u>

$|S| = 4$

$$P(A,\{B,C,D,E\}) = \min \begin{array}{l} j=B \\ j=C \\ j=D \\ j=E \end{array} \begin{pmatrix} C_{AB} + P(B,\{C,D,E\}) \\ C_{AC} + P(C,\{B,D,E\}) \\ C_{AD} + P(D,\{B,C,E\}) \\ C_{AE} + P(E,\{B,C,D\}) \end{pmatrix} = 10$$

The minimum cost to cover all cities is 10.

<u>Path</u>:

A-C-E-B-D-A

$2+2+3+2+1 = 10$ ✓

A-D-B-E-C-A = 10 ✓

A-D-C-E-B-A = 10 ✓

(10) Let us consider the capacity of the knapsack $w=60$ and the list of provides items are shown:

Item	A	B	C	D
Profit	280	100	120	120
Weight	40	10	20	24

Solve the problem as 0/1 knapsack problem, find optimal solution

A: Given $m=60$

$S^i = S^{i-1} \cup S_i^{i-1}$

$S^0 = (0,0)$

$S_1^0 = (280,40)$

$S^1 = S^0 \cup S_1^0$

$\boxed{S^1 = (0,0),(280,40)}$

$S^2 = S^1 \cup S_1^1$

$S_1^1 = (100,10),(380,50)$

$\boxed{S^2 = (0,0),(100,10),(180,40),(380,50)}$

$S^3 = S^2 \cup S_1^2$

$S_1^2 = (120,20),(220,30),(400,60),(500,70)$

$S^3 = (0,0),(100,10),(120,20),(220,30),(280,40),(380,50),(400,60),\cancel{(500,70)}$

$S^4 = S^3 \cup S_1^3$

$S_1^3 = (120,24),(220,34),(240,44),(340,54),(400,64),(500,74),(520,84)$

$S^4 = (0,0),(100,10),(120,20),(220,30),(280,40),(380,50),(400,60)$

$(400,60) \in S^3 \Rightarrow x_4 = 0$

$(400,60) \notin S^2 \Rightarrow x_3 = 1$

$(-120,20)$

$\Rightarrow (280,40) \in S^1 \Rightarrow x_2 = 0$

$(280,40) \notin S^0 \Rightarrow x_1 = 1$

$(280,40)$

$(0,0)$

$x = (1,0,1,0)$

$P_i = 280+0+120+0$
$= 400$
$W_i = 40+0+20+0$
$= 60$
$(P_i, W_i) = (400, 60)$

(15) Design a 3 stage system with device types D_1, D_2, D_3. The costs are Rs 10, Rs 15, Rs 10 respectively. The cost of the system is to be no more than Rs 95. The reliability of each device type is 0.5, 0.3 & 0.5 respectively.

A: Given $C = 95$

	D_1	D_2	D_3
r_i	0.5	0.5	0.5
c_i	10	15	10

$u_i = (C + c_i - \sum_{i=1}^{n} c_i) / c_i$

$u_1 = (95+10-35)/10 = 70/10 = 7$

$u_2 = (95+15-35)/15 = 5$

$u_3 = (95+10-35)/10 = 7.$

$S^0 = (1, 0)$

$S^1 = S_1^1 \cup S_2^1 \cup S_3^1 \cup S_4^1 \cup S_5^1 \cup S_6^1 \cup S_7^1$

$S_1^1 \rightarrow Sr_i = (1-(1-r_i)^m) = 1-(1-0.5)^1 = 1-0.5 = 0.5$

$Sc_1^1 = 10 \Rightarrow (0.5, 10)$

$\boxed{S_1^1 = (0.5, 10)}$

$S_2^1 \rightarrow Sr_i = (1-(1-r_i)^m) = 1-(1-0.5)^2 = 1-0.25$
$= 0.75$

$Sc_2^1 = 10+10 = 20$

$\boxed{S_2^1 = (0.75, 20)}$

$S_3^1 \rightarrow Sr = 1-(1-0.5)^3 = 1-0.125$
$= 0.875$

$Sc = 10+10+10 = 30$

$\boxed{S_3^1 = (0.875, 30)}$

$S_4^1 = (1-(1-0.5)^4) = 1-0.0625 = 0.9375$

$\boxed{S_4^1 = (0.9375, 40)}$

$\boxed{S_5^1 = (0.96875, 50)}$

$\boxed{S_6^1 = (0.98437, 60)}$

$\boxed{S_7^1 = (0.99218, 70)}$

$S^1 = (0.5, 10), (0.75, 20), (0.875, 30), (0.9375, 40), (0.96875, 50), (0.98437, 60),$
$(0.99218, 70)$

$S^2 = S_1^2 \cup S_2^2 \cup S_3^2 \cup S_4^2 \cup S_5^2$

$S_1^2 = S_1 = 1-(1-0.5) = 0.5$
$x = 15$

$S_1^2 = (0.25, 25), (0.375, 35), (0.4375, 45), (0.4687, 55), (0.4843, 65), (0.4921, 75),$
$(0.4960, 85)$

$S_2^2 (0.75, 30)$

$S_2^2 = (0.375, 40), (0.562, 50), (0.6562, 60), (0.7031, 70), (0.7265, 80), (0.7382, 90),$
$(0.7441, 100)^{x}$

$S_3^2 = (0.875, 45)$

$S_3^2 = (0.4375, 55), (0.656, 65), (0.7656, 75), (0.820, 85), (0.847, 95), (0.861, 105)^{x}$
$(0.868, 115)^{x}$

$S_4^2 = (0.975, 60)$

$S_4^2 = (0.468, 70), (0.702, 80), (0.819, 90), (0.877, 100)^{x}, (0.907, 110)^{x}, (0.922, 120)^{x},$
$(0.929, 130)^{x}$

$S_5^2 = (0.968, 75)$

$S_5^2 = (0.484, 85), (0.726, 95)$

$S^2 = (0.25, 25), (0.375, 35), (0.437, 45), (0.562, 50), (0.656, 60), (0.703, 70),$
$(0.765, 75), (0.820, 85), (0.8475, 95)$

$S_1^3 = (0.5, 10)$

$S_1^3 = (0.125, 35), (0.187, 45), (0.218, 55), (0.281, 60), (0.328, 70), (0.351, 80),$
$(0.382, 85), (0.41, 95), (0.423, 105)$
$S_2^3 = (0.75, 20)$
$S_2^3 = (0.187, 45), (0.281, 55), (0.327, 65), (0.421, 70), (0.492, 80), (0.527, 90),$
$(0.573, 95), (0.615, 105), (0.635, 115)$
$S_3^3 = (0.875, 30)$
$S_3^3 = (0.218, 55), (0.328, 65), (0.382, 75), (0.491, 80), (0.574, 90)$
$S_4^3 = (0.937, 40)$
$S_4^3 = (0.234, 65), (0.351, 75), (0.409, 85), (0.526, 90)$
$S_5^3 = (0.968, 50)$
$S_5^3 = (0.242, 75), (0.363, 85), (0.423, 95)$
$S_6^3 = (0.984, 60)$
$S_6^3 = (0.246, 85), (0.369, 95)$
$S_7^3 = 1 - (1 - 0.5)^7 = 0.992$
$x = 70$
$S_7^3 = (0.248, 95)$

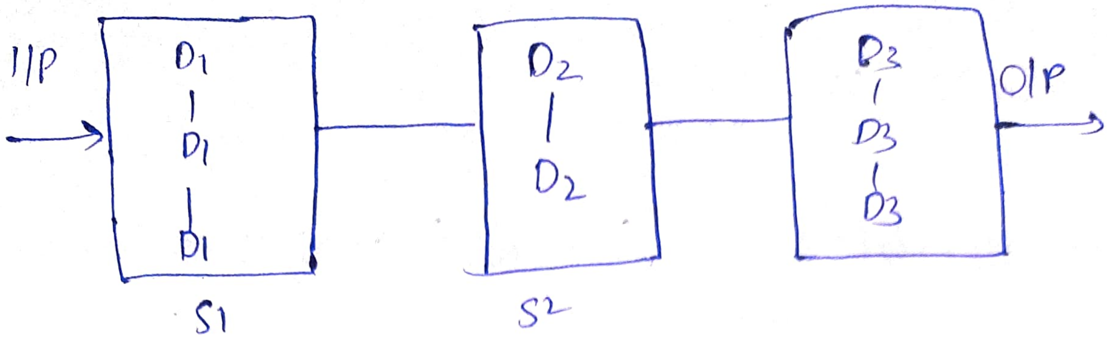
$S^3 = (0.125, 35), (0.187, 45), (0.281, 55), (0.328, 65), (0.421, 70)$
$(0.492, 80), (0.574, 90)$

$(0.574, 90) \in S^3$
$-30 \quad S_3^3 \quad x_3 = 3 \qquad (3 \times 10 = 30)$

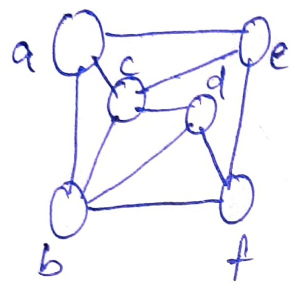
$(, 60) \in S^2$
$-30 \quad S_2^2 \qquad x_2 = 2 \qquad (2 \times 15 = 30)$

$(, 30) \in S^1$
$-30 \quad S_3^1 \qquad x_1 = 3 \qquad (3 \times 10 = 30)$

$(, 0)$
$x_i = (3, 2, 3)$
Total system can be designed with Rs 90 and with maximum reliability 0.574



(20) Identify any three ways of coloring the graph with Red, Green, Yellow & Blue Colors.



A:- Given Colors: R, G, Y, B

x_a =

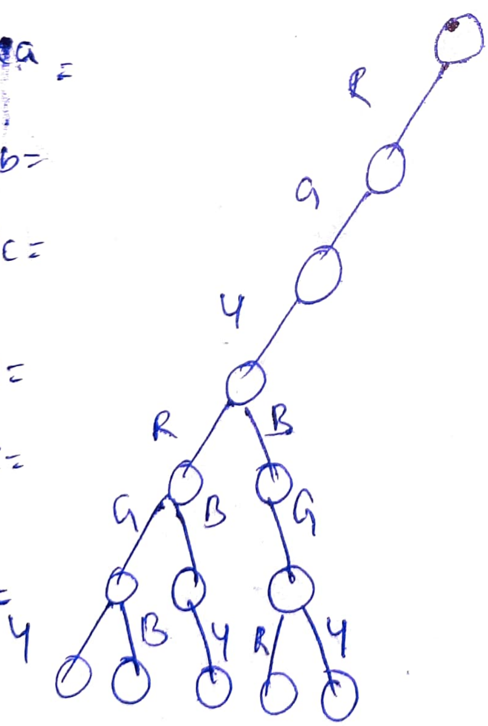
x_b =

x_c =

x_d =

x_e =

x_f =



a b c d e f

① R - G - Y - R - G - Y

② R - G - Y - R - G - B

③ R - G - Y - B - G - R

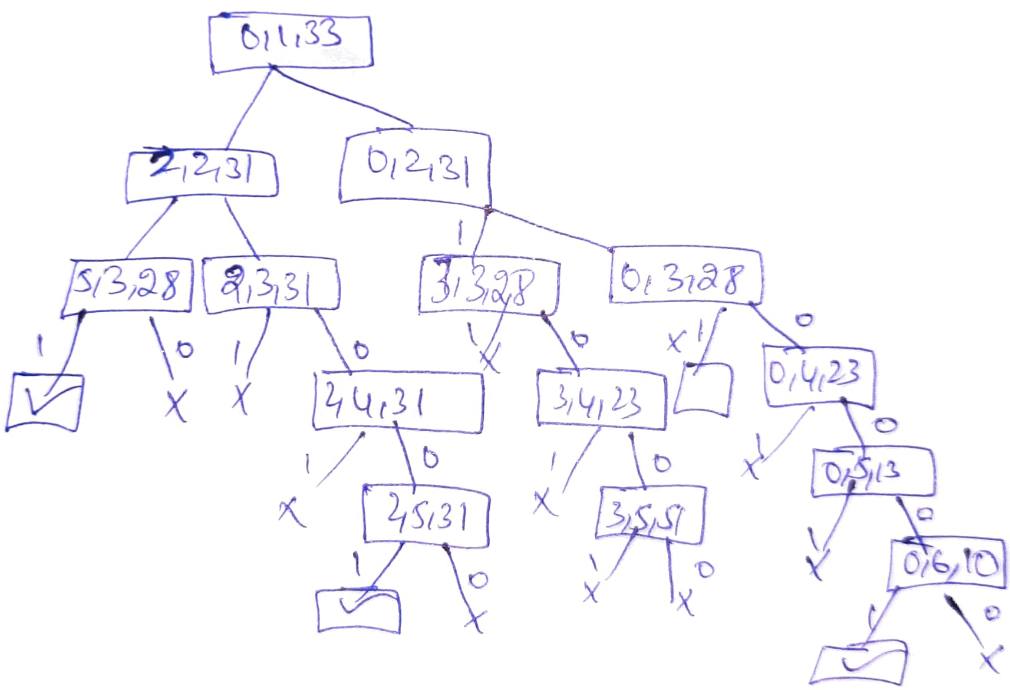
④ R - G - Y - B - G - Y

⑤ R - G - Y - R - B - Y

(25) Solve the following sums of subsets problem using backtracking N=6

$w[\,] = [2,3,5,6,8,10]$, $M=10$

A:-



⇒ [1,1,1,0,0,0]

⇒ [1,0,0,0,1,0]

[0,0,0,0,0,1]

} Solutions