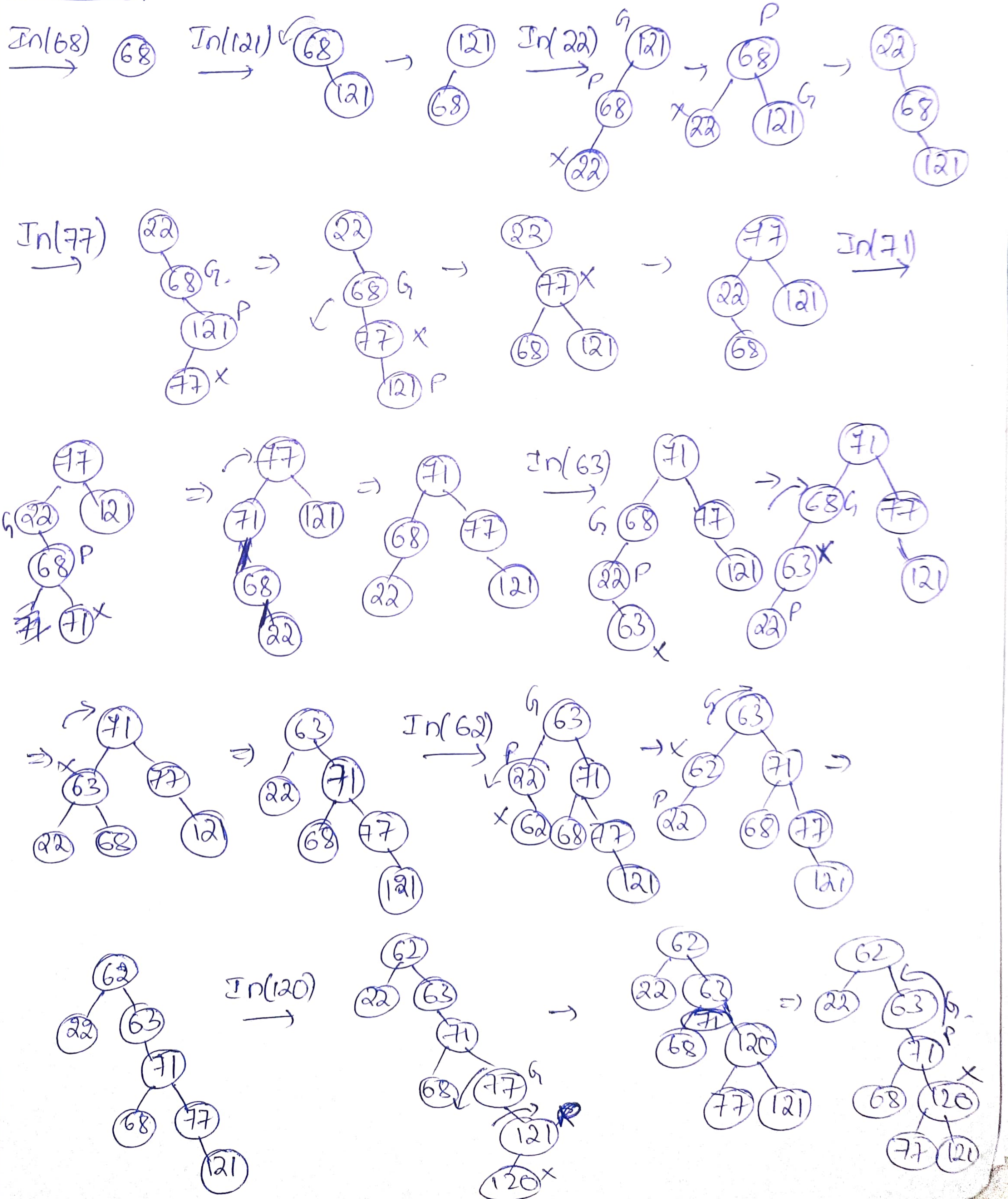
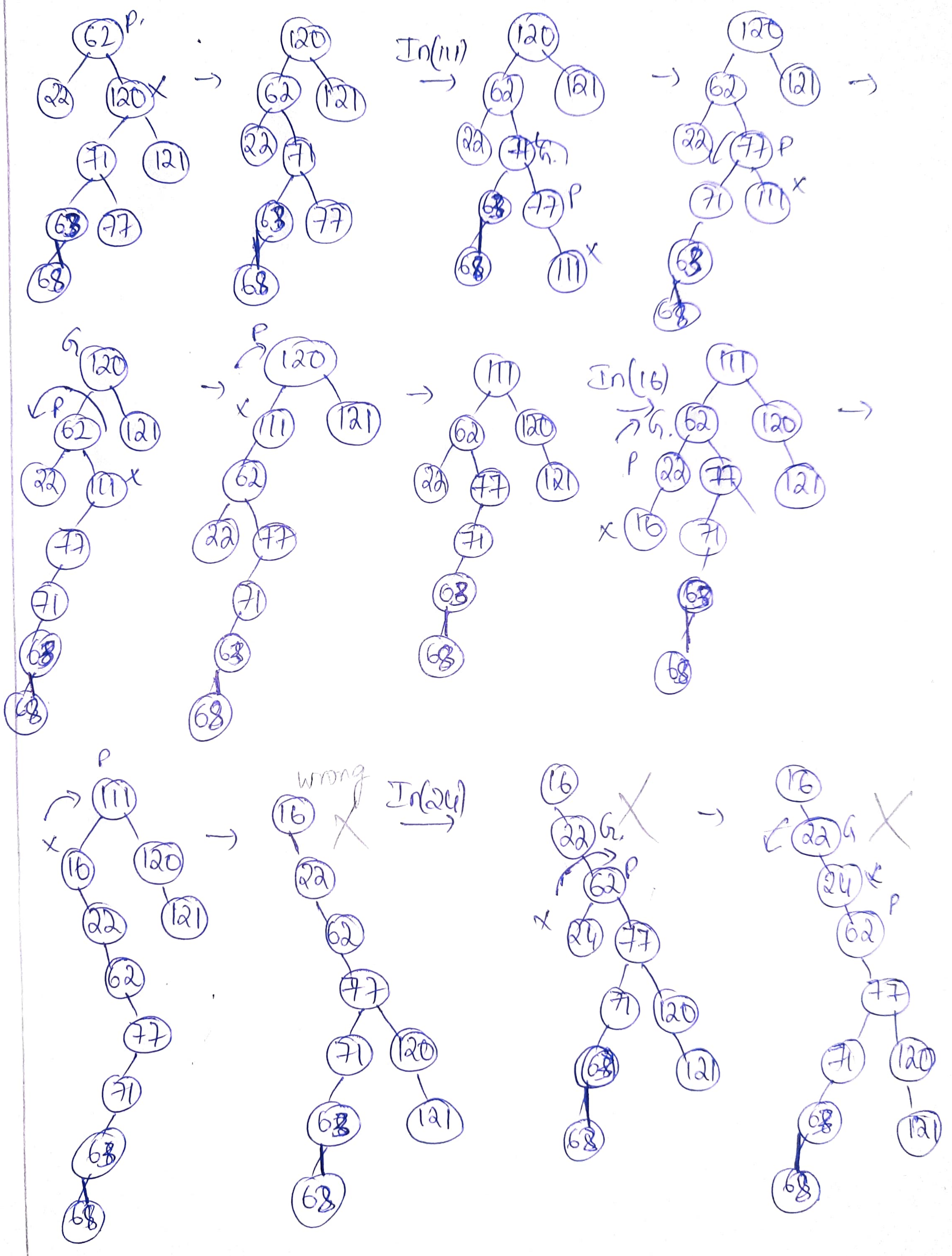
Advanced Data Structures Home Assignment-1

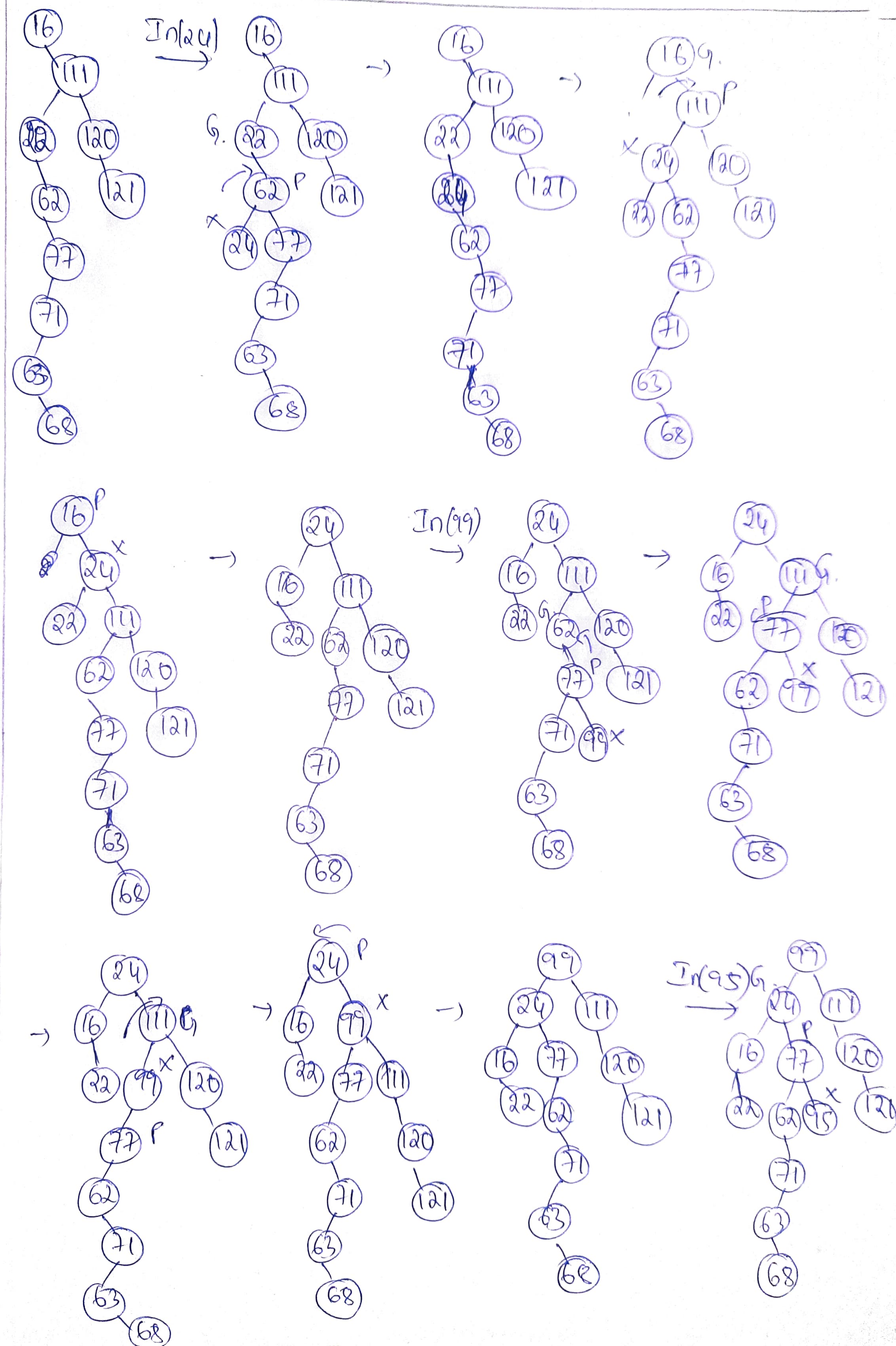
(1) Create a Splay Tree. After that delete 77,120,24

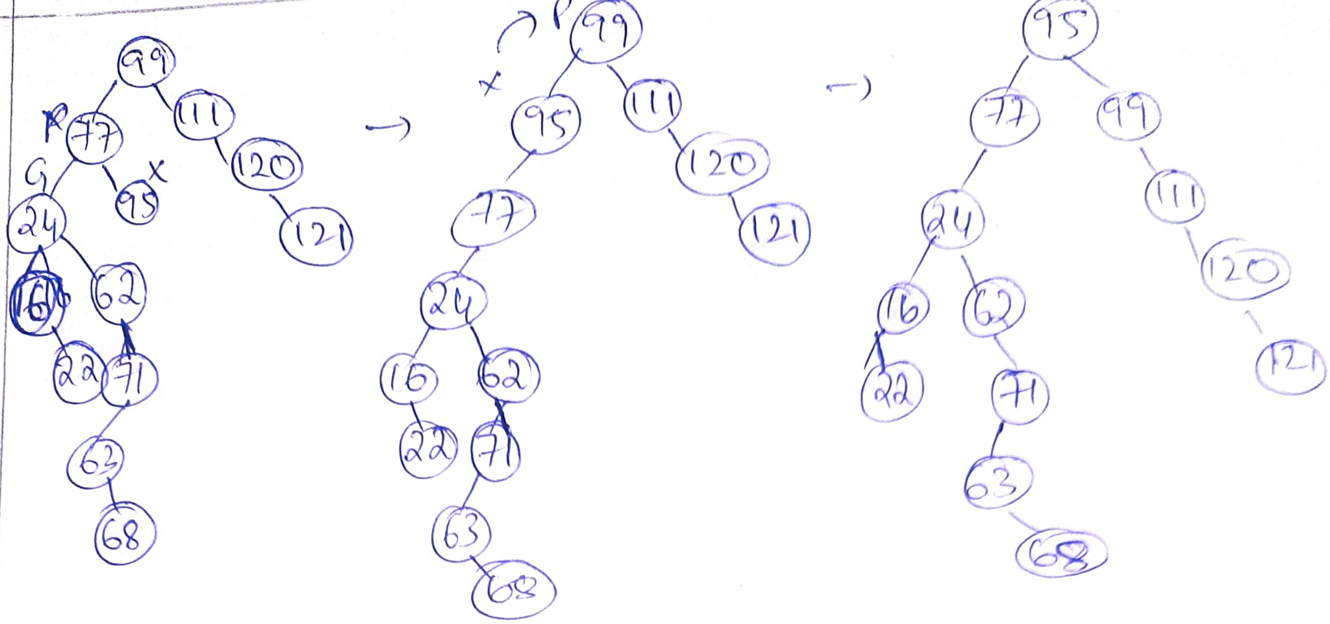
[68, 121, 22, 77, 71, 63, 62, 120, 111, 16, 24, 99, 95, 95]

Bottom Up:

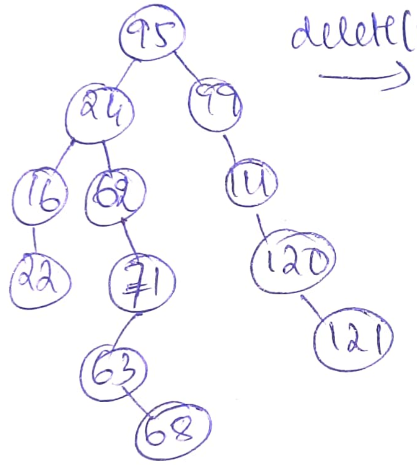



In(111)
In(16)
wrong
In(24)

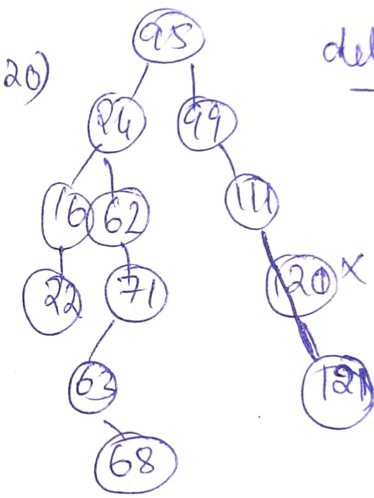

In(24)
In(99)
In(95)



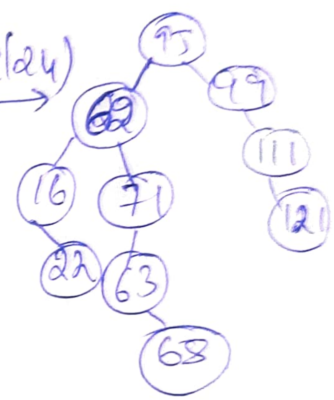
delete(77)



delete(120)

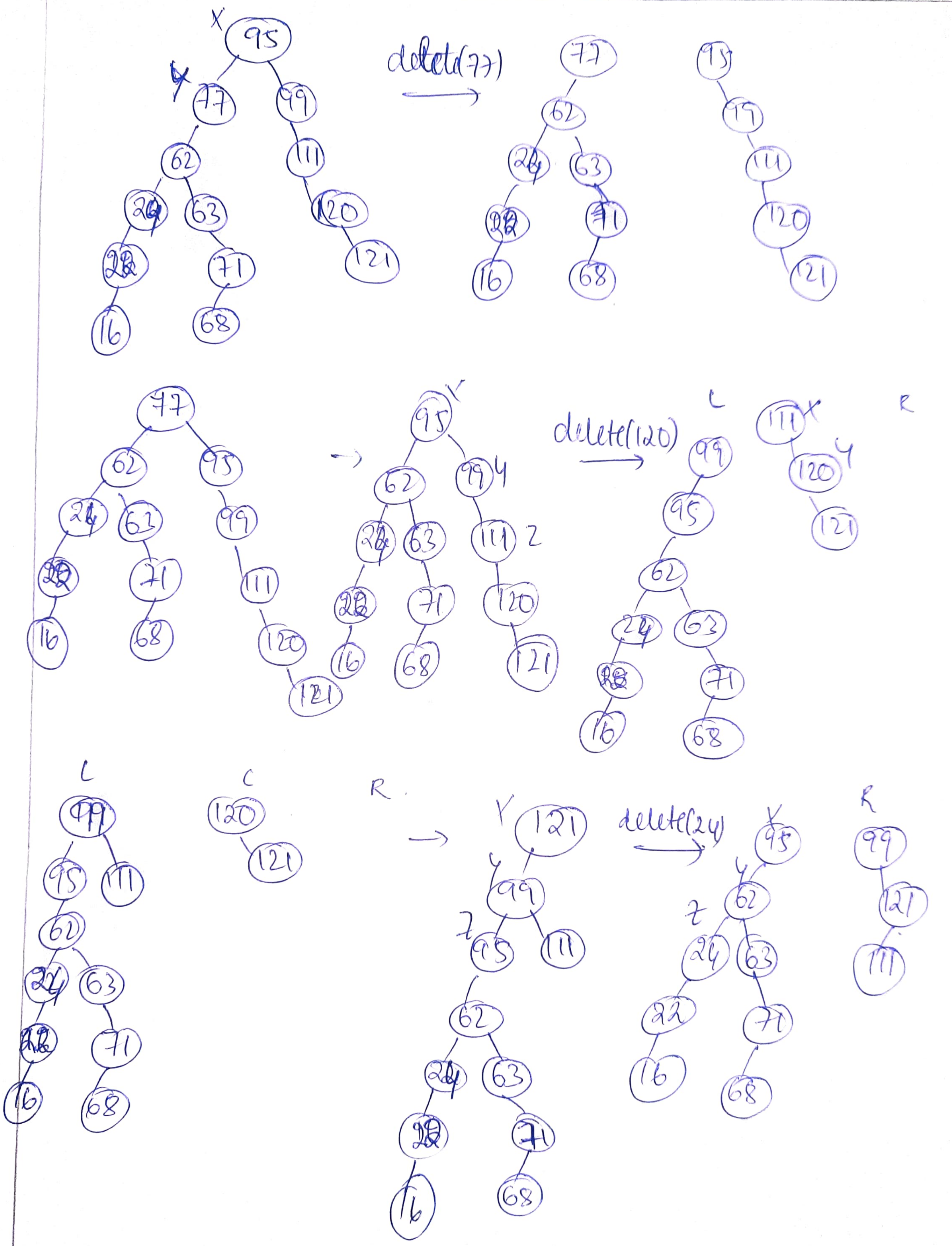


delete(24)



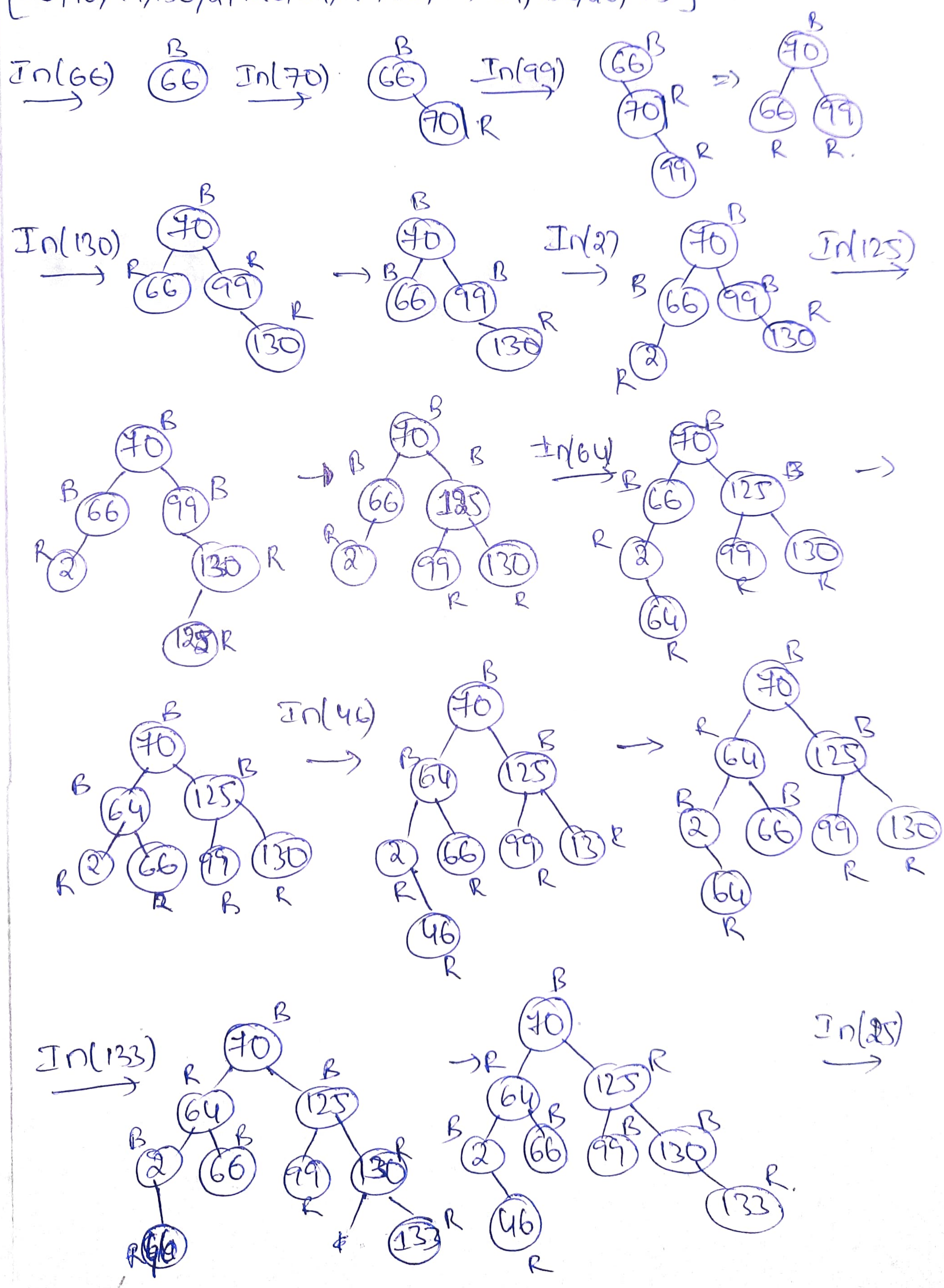
Top down

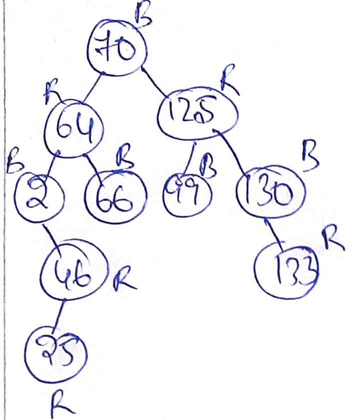

In(24)
In(99)
In(95)

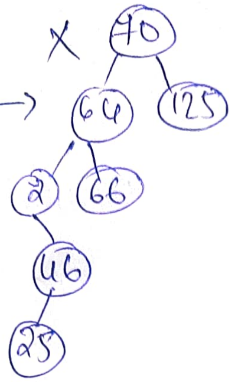

delete(77)
delete(120)
delete(24)

(8) Consider the following data and create a red black tree after that delete the elements 70,133,2

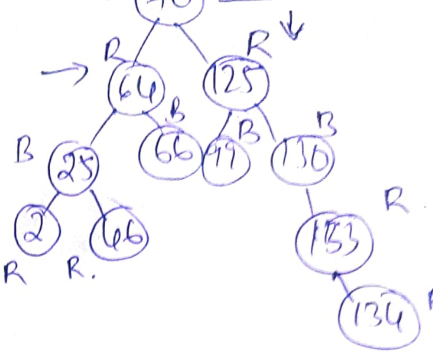
[66,70,99,130,2,125,64,46,133,25,134,136,40,95]

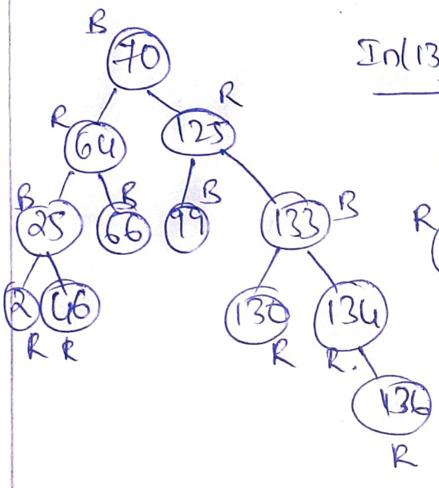




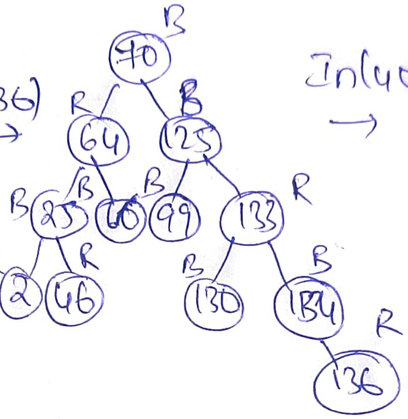


In(134)

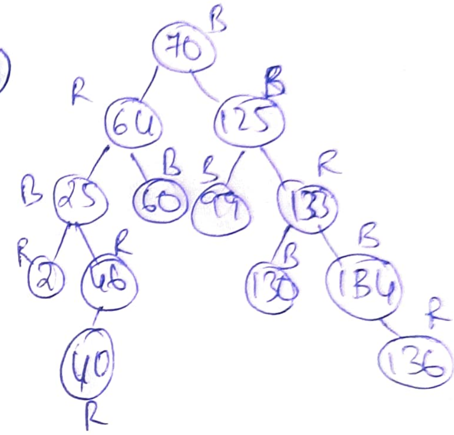




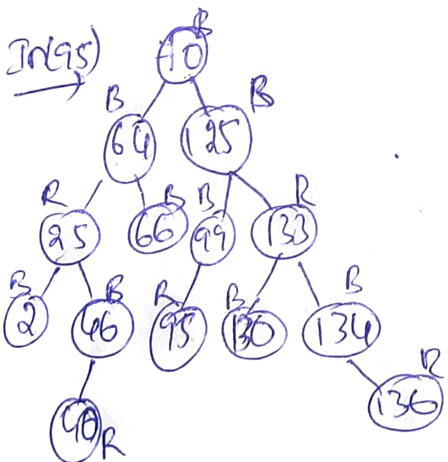
In(136)



In(40)

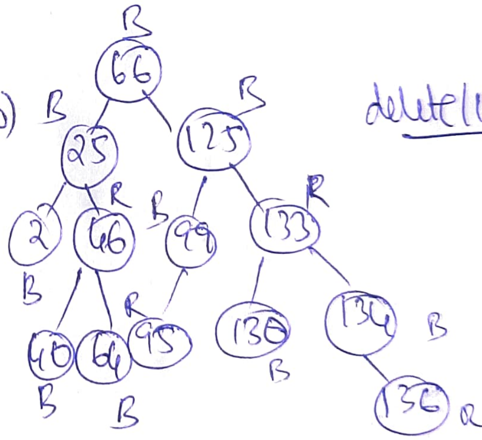


In(95)

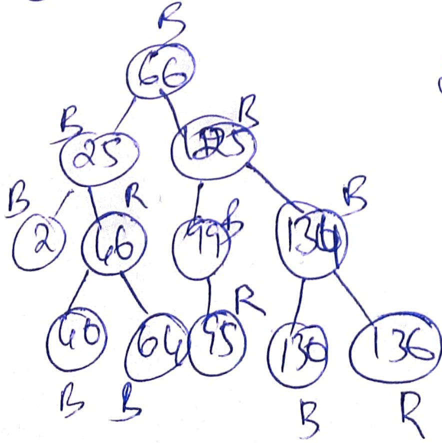


delete(70)

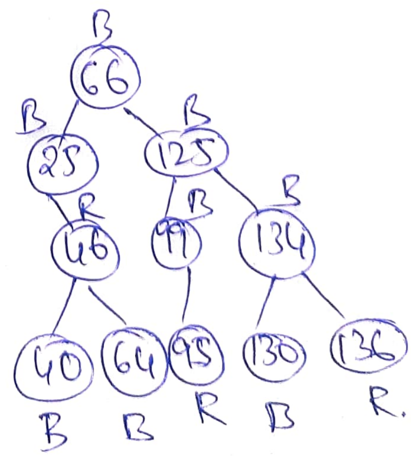
~~In(95)~~



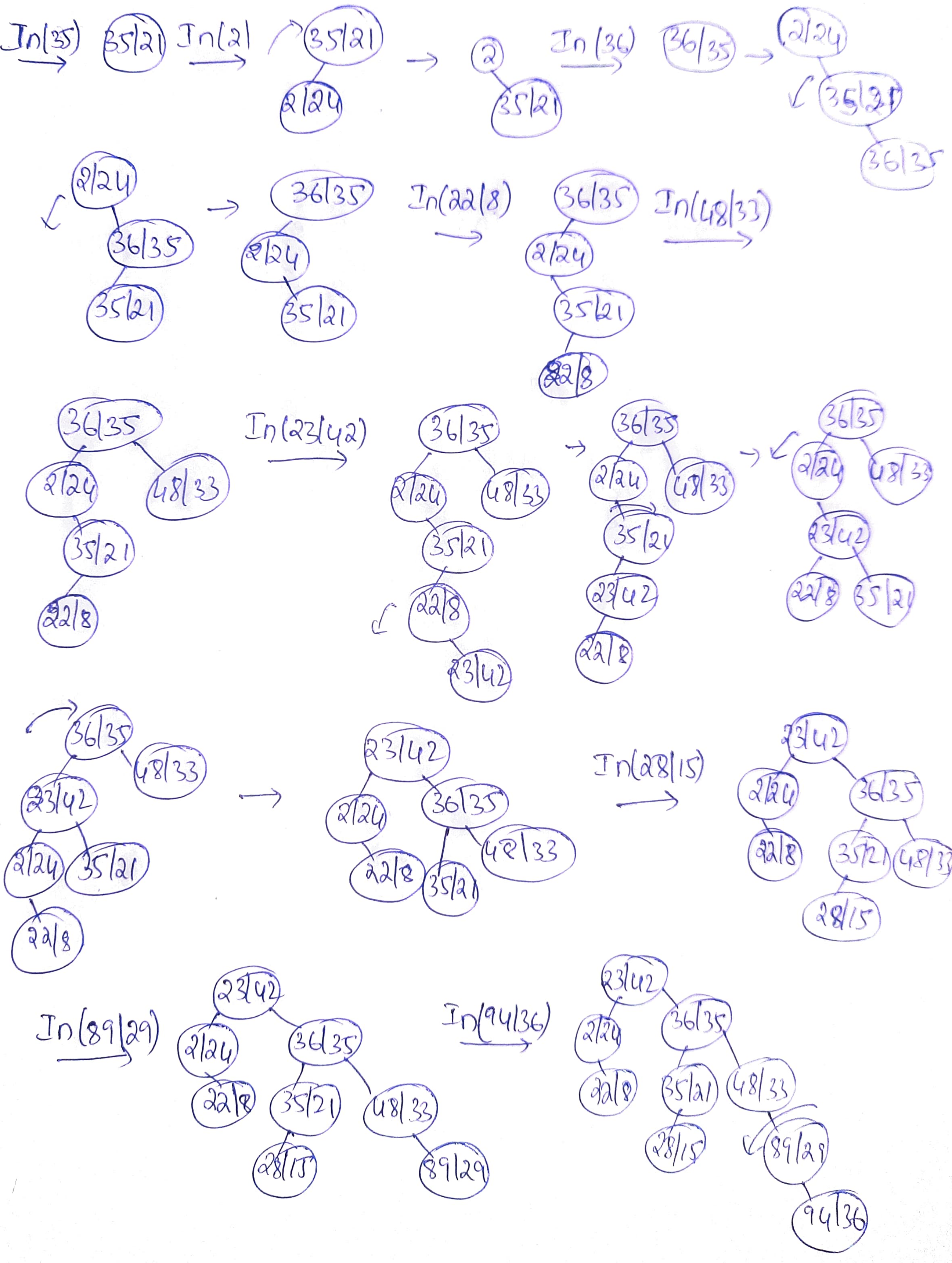
delete(133)

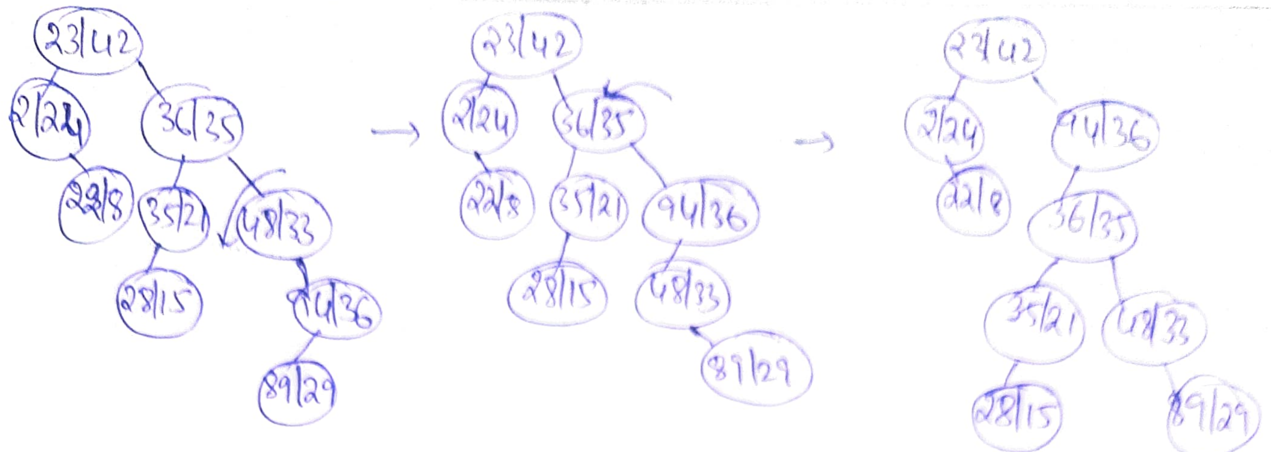


delete(2)

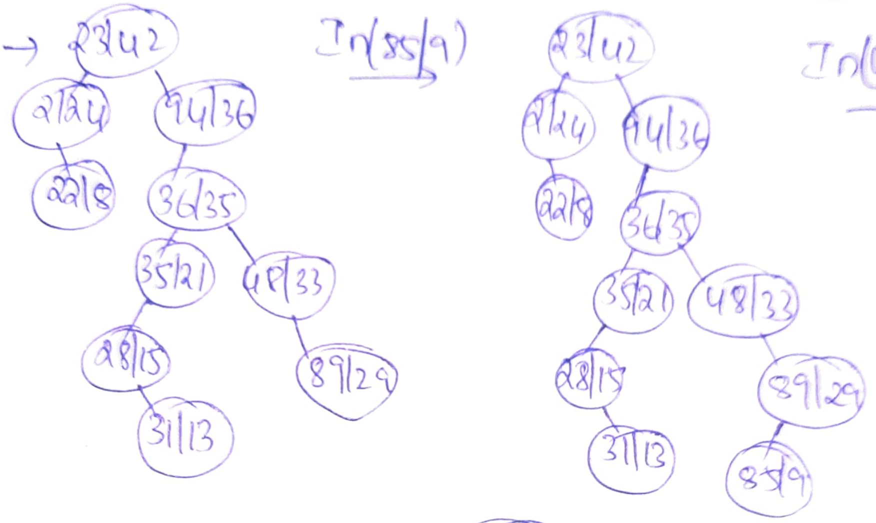


Max treap ~~35~~ 35,2,36,22,48,23,28,89,94,31,85,47,80,13,34 and delete 22,31,80

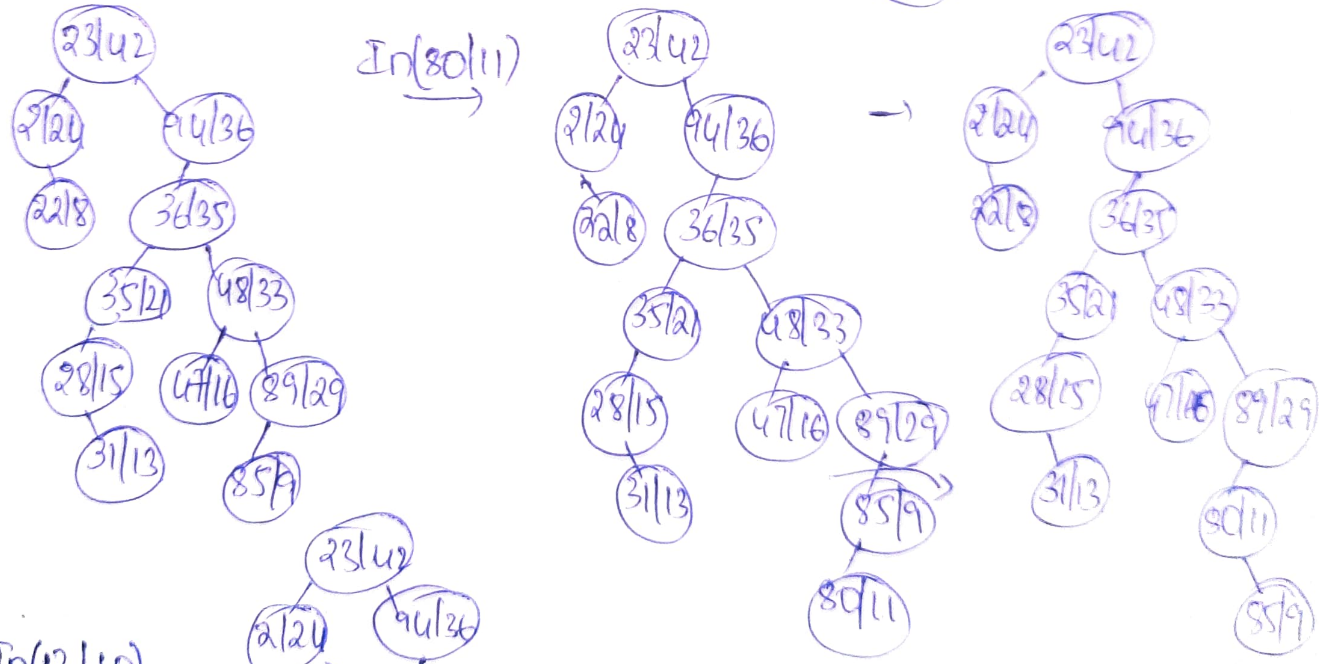




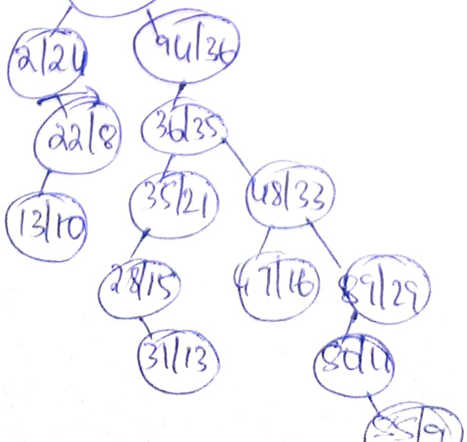
In(31/13) →

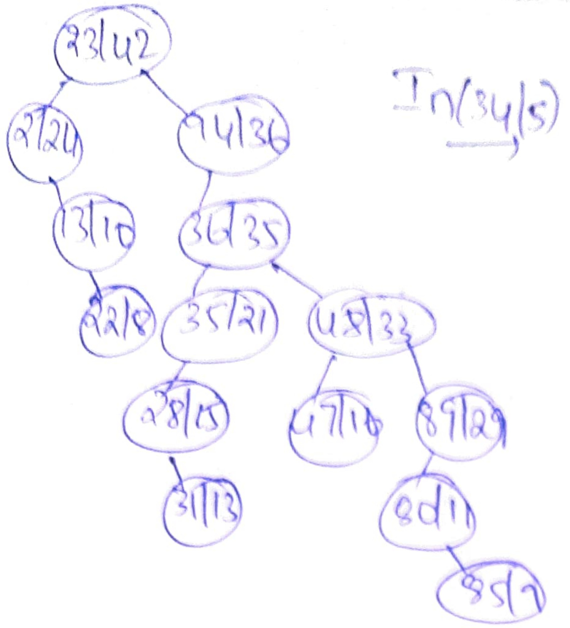


In(47/16)

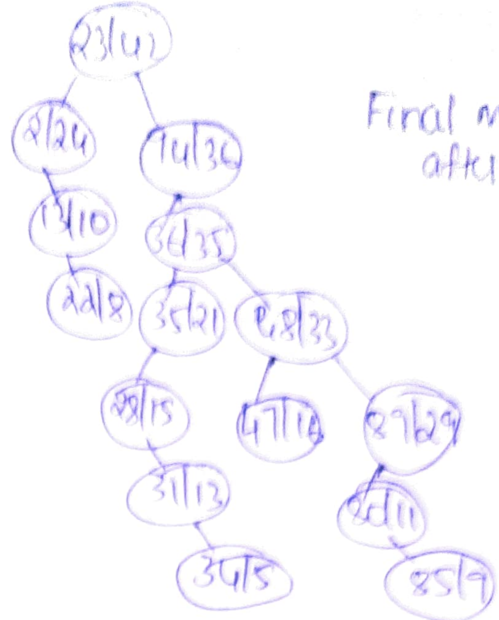


In(13/10)



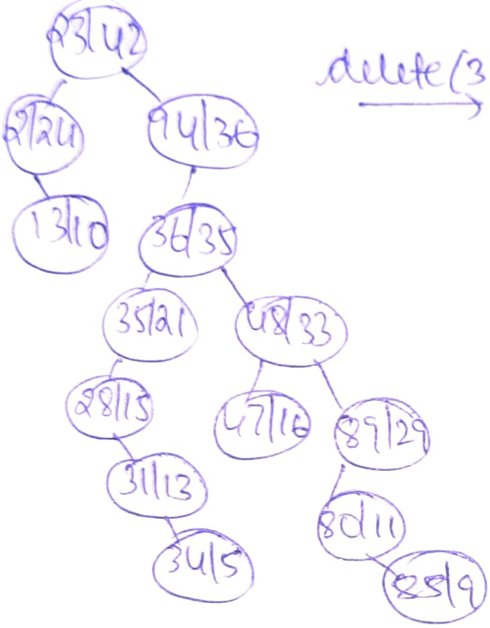


In(34|5)

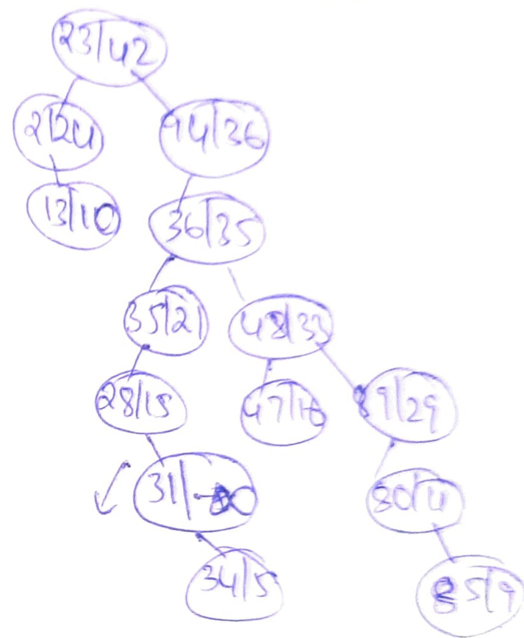


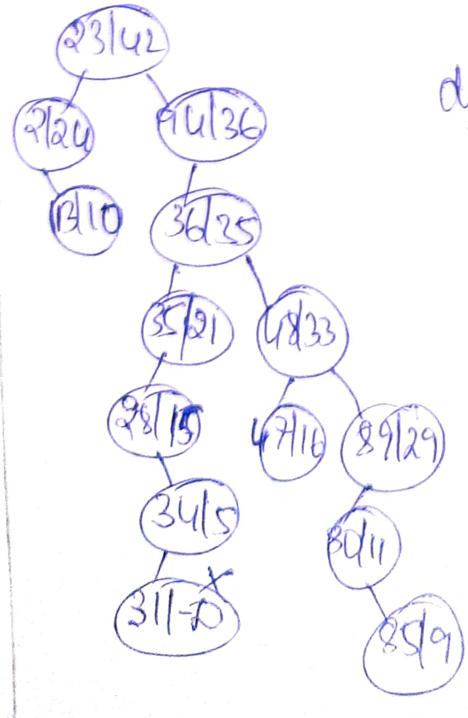
Final max treap after insertions

delete(22)

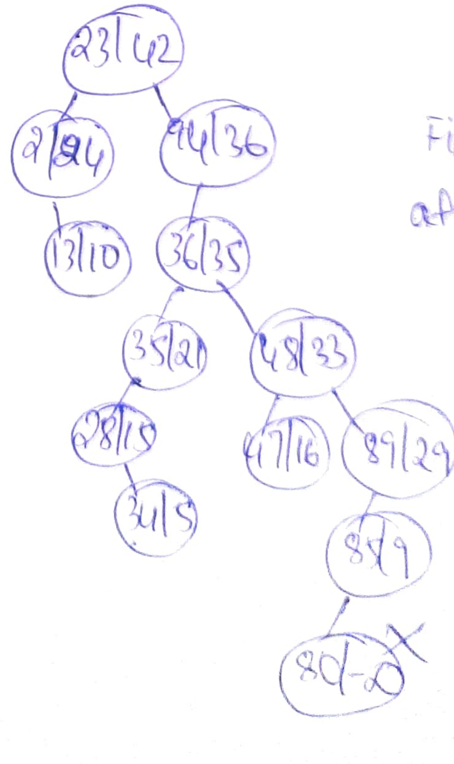


delete(31)





delete(80)



Final max treap after deletions

Find upper bound of running time of a cubic function

$f(n) = 2n^3+4n+5$

Upper bound

$f(n) \le c * g(n)$ for all $n \geq n_0$

$f(n) = 2n^3+4n+5$ given

$g(n) = 3n^3$

$\Rightarrow 2n^3+4n+5 \le c * g(n)$

$\Rightarrow 2n^3+4n+5 \le 2n^3+4n^3+5n^3$

$\Rightarrow 2n^3+4n+5 \le 11n^2$

$C = 11$

$g(n) = n^3$

$n_0 = 1$

n^2 satisfies

$2n^3+4n+5 \le 3n^3$

for $n \geq 3$ $f(n) \le c * g(n)$ holds true

So, $C=3$, $g(n) = n^3$ and $n_0 = 3$

$\therefore$ The upper bound is $2n^3+4n+5 = O(n^3)$

Find lower bound of running time of a linear function

$f(n) = 6n+3$.

Ω:- $f(n) \geq c * g(n)$ for all $n, n \geq n_0$

$6n+3 \geq c * g(n)$

$g(n) = n^2$

$6n+3 \geq c * n^2$

let $n=1$, $n_0 = 1$

$9 \geq c * 1$ for all $n \geq 1$

$c = 1$

$9 \geq 1$

not satisfies for n^2

$c = 4$ $n = 2$.

$6(2)+3 \geq 4*(2)^2$

$15 \geq 4*4$

$15 \geq 16$

false

$6n+3 \geq c*n$ for all $n \geq n_0$

$n=2, c=2, n_0 = 1$

$6(2)+3 \geq 4$ for all $2 \geq 1$

$15 \geq 4$ for all $2 \geq 1$

satisfies for n.

when. $c=4, n=2$

$15 \geq 4*2$

$15 \geq 8$ ✓

~~$\Omega(n)$:~~

$f(n) = \Omega(g(n)) = \Omega(n)$

21) Write an algorithm for matrix multiplication and calculate its time complexity

A:- Algorithm

Algorithm Matrix Multiplication

{

for i:=1 to n # n+1

for j:=1 to n # n(n+1)

for k:=1 to n # $n^2(n+1)$

c[i,j] = A[i,k] * B[k,j]

}

The General technique of finding time complexity for matrix multiplication it is $O(n^3)$

By divide & Conquer

$\frac{n}{2} * \frac{n}{2}$

$C_{11} = (A_{11} * B_{11}) + (A_{12} * B_{11})$

$C_{12} = (A_{11} * B_{12}) + (A_{12} * B_{22})$

$C_{21} = (A_{21} * B_{11}) + (A_{22} * B_{21})$

$C_{22} = (A_{21} * B_{12}) + (A_{22} * B_{22})$

The recurrence relation is

$T(n) = 2T(n/2) + Cn^2$

$$T(n) = 8T(n/2) + cn^2$$

$$= 8^2 T(n/4) + 2\frac{cn^2}{2^2} + cn^2$$

$$= 8^3 T(n/8) + 8^2 \frac{cn^2}{4^2} + 8\frac{cn^2}{2^2} + cn^2$$

$$= 8^4 T(n/16) + 2^3 cn^2 + 2^2 cn^2 + 2cn^2 + 8cn^2$$

$$= 8^i T(n/2^i) + cn^2 \left[2^{i-1} + 2^{i-2} + \cdots + 1\right]$$

Let $n = 2^i$

$i = \log_2 n$

$$= 8^{\log_2 n} T(1) + cn^2 \left[\frac{2^{i-1+1} - 1}{2-1}\right]$$

$$= 8^{\log_2 n} + cn^2 (2^i - 1)$$

$$= n^{\log_2 8} + cn^2(n-1) \quad [n = 2^i]$$

$$= n^3 + cn^2(n-1)$$

∴ Time complexity of matrix multiplication is $O(n^3)$