Assignment II Questions:: 2021-22

DMS (IT) VR 20

1. Is $f(x) = x^2$ is one to one from set of integers to set of integers?
2. Define an Equivalence relation. Suppose that R is the relation on the set of strings of English letters such that a R b if and only if l(a) = l(b), where l(x) is the length of the string x. Is R an equivalence relation
3. Show that the relation R = { (a,b) / a ≡ b(mod m) } is an equivalence relation.
4. Define (i) partial ordered set (ii) Total ordered set (iii) well ordered set with examples.
5. Let S be a set. Determine whether there is a greatest element and a least element in the poset $(P(S), \subseteq)$.
6. Define a Hasse diagram and draw Hasse diagram of $\{D_{12}, /\}$ then determine minimal, maximal, least and greatest elements in diagram.
7. Prove that $(Z_n, +_n)$ is a cyclic group.
8. What is an order of an element.Find the order of the elements of (Z_8 , $+_8$).
9. Define a group and prove that every cyclic group is abelian.
10. Let (G, *) be a group and $S \subseteq G$. Then (S, *) is a subgroup of (G,*) if and only if $a*b^{-1} \in S$ for all a,b in S.

Sessional II Question Bank

1. Prove that $(Z_n, +_n)$ is a cyclic group.

   Let G be a group and $a \subseteq G$. Then O(a) is the order of the cyclic group generated by a.

2. Let (G, *) be a group and $S \subseteq G$. Then (S, *) is a subgroup of (G,*) if and only if $a*b^{-1} \subseteq S$ for all a,b in S.

3. Let f: $G \rightarrow G'$ be a group homomorphism from (G,*) to (G', o). Let e and e' be the identity elements of G and G' then (i) f(a) = e' (ii) $f(a^{-1}) = (f(a))^{-1}$ for all a in G. (iii) $f(a*b^{-1}) = f(a) \circ (f(b))^{-1}$ for all a, b in G . (iv) f(H) is a subgroup of G whenever H is a subgroup of G.

4. A group homomorphism f is a monomorphism if and only if kerf = {e}.

5. (i) Any infinite cyclic group is isomorphic to (Z , +). (ii) Any cyclic group of order n is isomorphic to $(Z_n, +_n)$.

6. Cayley's Theorem:- A finite group (G,*) of order n is isomorphic to a group of permutations of G.

7. Let H be a subgroup of G. Then (i) $a \in H$ if and only if aH = H. (ii) aH = bH iff $a^{-1} * b \in H$ (iii) $a \subseteq bH$ iff aH = bH.

8. Lagrange's Theorem of finite groups:-
   Statement:- Let G be a finite group and H be any subgroup of G. Then the order of H divides the order of G.

9. Result: Let G= (V,E) be a graph then the number of vertices of odd degree is even.

10. Problem: Is there a simple graph with degree sequence (1,1,3,3,3,4,6,7)?

11. State and Prove Euler's Formula.

12 . In a connected plane graph G with $|E| > 1$, then we have a (i) $|E| \leq 3|V| - 6$ (ii) there is a vertex v in G such that $\deg(v) \leq 5$.

13. A complete graph K_n is planar iff $n \leq 4$.

14. Grinberg's Theorem.

15. Definitions of all types of graphs with examples. And definitions in Group theory.