

Assignment 3

Question 1:

Create a 3×3 matrix of integers from 1-9, then calculate its transpose, determinant, and trace.

Output

Transpose:

```
[[1 4 7]
 [2 5 8]
 [3 6 9]]
```

Determinant: 0.0

Trace: 15

Question 2:

Given two matrices, perform addition, subtraction, matrix multiplication, and element-wise multiplication.

A = [3, 1]
 [2, 4]

B = [2, 5]
 [1, 3]

Output:

A + B =
[[5 6]
 [3 7]]

A - B =
[[1 -4]
 [1 1]]

A × B (matrix product) =
[[7 18]
 [8 22]]

A * B (element-wise) =

[[6 5]
[2 12]]

Question 3: Solving Linear Equations

Solve the system of linear equations using NumPy:

$$\begin{aligned}2x + y + z &= 10 \\3x + 2y + 3z &= 18 \\x + 4y + 9z &= 16\end{aligned}$$

Solution:

$$\begin{aligned}x &= 4.0 \\y &= 2.0 \\z &= 0.0\end{aligned}$$

Verification ($A \times \text{solution}$):

$$[10. \ 18. \ 16.]$$

Question 4:

Given a dataset matrix, calculate the mean and standard deviation of each column, normalise the data, and find the correlation between columns.

$$\begin{aligned}&[5.1, 3.5, 1.4] \\&[4.9, 3.0, 1.4] \\A = &[4.7, 3.2, 1.3] \\&[4.6, 3.1, 1.5] \\&[5.0, 3.6, 1.4]\end{aligned}$$

Output:

Column means: [4.86 3.28 1.4]

Column standard deviations: [0.19595918 0.24899799 0.07071068]

Normalized data:

$$\begin{aligned}&[[1.22474487 \ 0.88354899 \ 0. \quad] \\&[0.20412414 \ -1.12451549 \ 0. \quad] \\&[-0.8164966 \ -0.32128999 \ -1.41421356] \\&[-1.32679698 \ -0.72290748 \ 1.41421356]\end{aligned}$$

[0.71443449 1.28516397 0.]]

Correlation matrix:

[[1. 0.65089018 -0.10566282]
[0.65089018 1. -0.00920389]
[-0.10566282 -0.00920389 1.]]

Question 5:

Calculate eigenvalues and eigenvectors of a given matrix, then reconstruct the original matrix.

[4, 2, 2]
A= [2, 5, 1]
[2, 1, 6]

Eigenvalues:

[2.34563298 4. 8.65436702]

Eigenvectors (as columns):

[[-0.5697282 0.81741256 0.08519846]
[-0.43976496 -0.28616193 0.85065781]
[-0.69597618 -0.5 -0.5185347]]

Matrix reconstruction:

[[4. 2. 2.]
[2. 5. 1.]
[2. 1. 6.]]

Verification of $Av = \lambda v$ for eigenvector 3

Av : [0.73731518 7.36364422 -4.48732634]

λv : [0.73731518 7.36364422 -4.48732634]

Question 6:

Calculate the covariance matrix for the following set of 2D data points:

(1,2), (2,3), (3,5), (4,7), (5,8).

output:

Means: $x = 3.0$, $y = 5.0$

Covariance matrix (manual calculation): $\begin{bmatrix} 2.5 & 3.5 \\ 3.5 & 6.5 \end{bmatrix}$

Question 7:

Calculate the correlation matrix for the data in Question 6 and interpret the results.

Interpretation

$\text{corr_xy} > 0.8$:

The correlation coefficient indicating a strong positive linear relationship.

$\text{corr_xy} > 0.5$:

"The correlation coefficient indicates a moderately positive linear relationship.

$\text{corr_xy} > 0$:

The correlation coefficient indicates a weak positive linear relationship.

$\text{corr_xy} == 0$:

The correlation coefficient indicates no linear relationship.

Else:

The correlation coefficient indicates a negative linear relationship.