HW#5 Decimation Using FIR Filters

A typical sampling rate for underwater acoustic signal processing systems is $f_s = 1$ kHz. Assume that a special purpose hardware FFT is available where NFFT = 256. Using the FFT hardware, it is desired to resolve two sinusoids which are located at $f_1 = 37.5$ Hz and $f_2 = 40.5$ Hz.

I. Preliminary Calculations

A. What is the FFT bin width in terms of analog frequency?

Recall $f_{\text{digital}} = f_{\text{analog}}(T) = f_{\text{analog}}\left(\frac{1}{f_s}\right)$ where T is the sampling interval.

B. In which bin (i.e. frequency index k = {0,.... N-1} does f_1 reside? In which bin does f_2 reside?

C. Will the two sinusoids be resolvable given that a rectangular window is used with the FFT? Speculate on whether or not any other type of window would be beneficial.

II. Decimation

Decimation of the time series by a factor of 8 is proposed in order to achieve higher resolution. A low pass filter must be designed and implemented. The output sequence will be decimated to achieve an effective sampling rate of $f_s' = \frac{f_s}{8} = 125\ Hz$. Repeat Part I A - C.

III. FIR Low Pass Filter Critical Frequencies

A 64-coefficient, linear phase, FIR low pass filter is to be designed for the decimator. An analog cutoff frequency $f_c = 50$ Hz is desired and defines the ideal passband edge. The filter stopband is defined by $\frac{f_s'}{2} = 62.5$ Hz. Compare the transition width specified and the bin width of a 64-point FFT. Speculate on how good this filter is likely to be.

IV. Equiripple Filter Design

A. Using an equiripple filter design algorithm, design the required decimation filter. Consider three cases: (1) passband / stopband weighting ratio = 10 (i.e. minimizing errors in the passband is 10 times more important than minimizing errors in the stopband), (2) passband / stopband weighting ratio = 1, and (3) passband / stopband weighting ratio = 1/10.

B. Plot the impulse response h(n) and frequency response |H(f)| (both linear magnitude and dB) of all three low pass filters. Indicate on the frequency response plots the locations of f_c, $\frac{f_s'}{2}$, f_1, and f_2 in terms of their digital frequency equivalents. Comment on the size of the passband and stopband ripples.

V. Filter Implementation

Implement the FIR filter with passband / stopband weighting ratio = 1 and use it to filter a time series obtained from sampling the sum of two equal amplitude sinusoids at frequencies $f_2 = 40.5$ Hz and $f_3 = 100$ Hz. Using a good window function, compute the NFFT = 256–point FFT of both the input and output time series. Plot these two spectra (dB) and comment on the sinusoid levels.