Appendix 1. Hand Calculation

I. Data Set

$f_1 := 160\ Hz$ $f_2 := 237\ Hz$ $f_3 := 240\ Hz$ Analog frequency components of the signal

$A_1 := 100$ $A_2 := 10$ $A_3 := 1$ Linear amplitude for each analog frequency component

$f_s := 1000\ Hz$ Sampling frequency

$NFFT := 256$

$BinWidth := \frac{f_s}{NFFT} = 3.906\ Hz$ Analog frequency bin width of the FFT

$Bin\_f_1 := \mathrm{round}\left(\frac{f_1}{BinWidth}\right) = 41$ Frequency bin index of analog frequency component f_1

$Bin\_f_2 := \mathrm{round}\left(\frac{f_2}{BinWidth}\right) = 61$ Frequency bin index of analog frequency component f_2

$Bin\_f_3 := \mathrm{round}\left(\frac{f_3}{BinWidth}\right) = 61$ Frequency bin index of analog frequency component f_3

Given the sampling frequency of 1000 Hz and NFFT = 256, the frequency bin indices for f_2 and f_3 are always 61, which is not affected by the window applied to the signal. Therefore, rectangular windowed FFT will not indicate distinct spectral peaks for f_2 and f_3.

$20 \cdot \log\left(A_1\right) = 40$ Magnitude of the peak corresponding to +/-f_1

$20 \cdot \log\left(A_2\right) + 20 \cdot \log\left(A_3\right) = 20$ Magnitude of the peak corresponding to +/-f_2 and +/-f_3 (same frequency bin)

Therefore, after the normalization, the magnitude of the peak corresponding to +/-f_1 is 0 dB and the magnitude of the peak corresponding to +/-f_2 and +/-f_3 (same frequency bin) is -20 dB.

III. Complex Basebanding and Desampling

1. Complex basebanding:

$f_0 := 250\ Hz$ Center frequency

Frequency components after complex basebanding:

$f_1' := f_1 - f_0 = -90\ Hz$ $f_2' := f_2 - f_0 = -13\ Hz$ $f_3' := f_3 - f_0 = -10\ Hz$

$f_1'' := -f_1 - f_0 = -410\ Hz$ $f_2'' := -f_2 - f_0 = -487\ Hz$ $f_3'' := -f_3 - f_0 = -490\ Hz$

Complex basebanding does not change the magnitude of the signal. Therefore, the magnitude of peaks corresponding to each frequency component remain the same as previous.

2. Low-pass filtering:
Low-pass filtering does not shift the frequency of the signal. Therefore, frequency values of the signal remain the same as previous.

$20 \cdot \log\left(A_1\right) - 40 = 0$ Magnitude of the peak corresponding to f_1

$20 \cdot \log\left(A_2\right) + 20 \cdot \log\left(A_3\right) = 20$ Magnitude of the peak corresponding to f_2 and f_3 (same frequency bin)

$20 \cdot \log\left(A_1\right) - 40 = 0$ Magnitude of the peak corresponding to $-f_1$

$20 \cdot \log\left(A_2\right) + 20 \cdot \log\left(A_3\right) - 40 = -20$ Magnitude of the peak corresponding to $-f_2$ and $-f_3$ (same frequency bin)

Therefore, after the normalization, the magnitude of the peak corresponding to f_1 is -20 dB; the magnitude of the peak corresponding to f_2 and f_3 (same frequency bin) is 0 dB; the magnitude of the peak corresponding to $-f_1$ is -20 dB; the magnitude of the peak corresponding to $-f_2$ and $-f_3$ (same frequency bin) is -40 dB;

IV. High Resolution Spectral Analysis

$f_s' := \dfrac{f_s}{8} = 125\ \mathbf{Hz}$ Sampling frequency after decimation

$BinWidth := \dfrac{f_s'}{NFFT} = 0.488\ \mathbf{Hz}$ Analog frequency bin width of the FFT after desampling

Frequency components after desampling:

$f_1' := f_1 - f_0 + f_s' = 35\ \mathbf{Hz}$ $f_2' := f_2 - f_0 = -13\ \mathbf{Hz}$ $f_3' := f_3 - f_0 = -10\ \mathbf{Hz}$

$f_1'' := -f_1 - f_0 + 3\ f_s' = -35\ \mathbf{Hz}$ $f_2'' := -f_2 - f_0 + 4\ f_s' = 13\ \mathbf{Hz}$ $f_3'' := -f_3 - f_0 + 4\ f_s' = 10\ \mathbf{Hz}$

$20 \cdot \log\left(A_1\right) - 40 = 0$ Magnitude of the peak corresponding to f_1 (-20 dB after normalization)

$20 \cdot \log\left(A_2\right) = 20$ Magnitude of the peak corresponding to f_2 (0 dB after normalization)

$20 \cdot \log\left(A_3\right) = 0$ Magnitude of the peak corresponding to f_3 (-20 dB after normalization)

$20 \cdot \log\left(A_1\right) - 40 = 0$ Magnitude of the peak corresponding to $-f_1$ (-20 dB after normalization)

$20 \cdot \log\left(A_2\right) - 40 = -20$ Magnitude of the peak corresponding to $-f_2$ (-40 dB after normalization)

$20 \cdot \log\left(A_3\right) - 40 = -40$ Magnitude of the peak corresponding to $-f_3$ (-60 dB after normalization)